Point-Group Theory Tables

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Preface

In his seminal book *Infrared and Raman Spectra of Polyatomic Molecules*, published in 1945, Herzberg asserted that 'The regular icosahedron and the regular pentagon dodecahedron belong to the point group $\mathbf{I}_h$. It is not likely that molecules of such a symmetry will ever be found.' Not yet half a century later we now know better: in the last few years very many molecules with novel symmetries have been found going well beyond the forty point groups or so that are usually tabulated. Not only do we now need more point groups but we also need far more detail about the groups that we use. In the 1950s, for instance, there was enormous confusion about the construction of eight hybrids of $\mathbf{D}_{2d}$ symmetry, some people claiming that f orbitals were necessary, a point that had to be elucidated by none less than Giulio Racah. This is a problem that can be sorted out in five minutes, given a table of the irreducible representations of the group where the spherical harmonic bases are properly identified, but it is not always easy, even now, to find tables that provide quick access to, say, f orbital bases or correct symmetrized expansions of spherical harmonics for a large number of groups. People also begin to require good sets of matrix representations, rather than mere characters, and these are not easily available. Neither are the Clebsch–Gordan coefficients, except for the crystallographic point groups. As regards the double groups, the situation is even more unsatisfactory, since the available tables are often incomplete and not always entirely reliable. If one is dealing with a group such as the double group of $\mathbf{D}_6$, one often needs to be sure that the subgroup corresponding to the double group of $\mathbf{D}_3$ is also properly treated, but this is not always the case since on subduction along the double groups it is possible that the characters cease to be constant over each class. And the question of a consistent definition of the multiplication rules in double groups is often a sore point.

Besides all these simple and basic problems very substantial difficulties remain to the user of existing tables. It is not always easy to identify uniquely the symmetry operations used in them, and the very many conventions that one requires in order to obtain consistent results have often to be guessed by working backwards from the results of the tables. As an example: the Jones faithful representation, obtained by acting on x, y, and z with each of the symmetry operations of a group, is some times given in order to allegedly identify uniquely the group operations and their multiplication rules. In this case, however, we must know whether the operations are active or passive and what the meaning of x, y, and z is. They can be the independent variables or they can be the functions x, y, and z (or, what is the same from the point of view of the transformation rules, body-fixed unit vectors along the three orthogonal directions of that name). We have already four possibilities and even then some further conventions might have to be added, such as correct phases. Yet, many of the published representations do not provide any indication at all of the conventions used.

Methods have become available in the last few years that provide consistent tables, not just for isolated groups but also for whole group chains and we have used these methods to treat 75 point groups, up to and including rotation axes of order ten. We have made sure that the symmetry operations are uniquely identified, that the multiplication rules of the groups are clear and correct (which is not trivial for the double groups), and that the matrix representations and Clebsch–Gordan coefficients provided are fully and explicitly defined (except that for the icosahedral groups limitations of size require the matrix representations to be given only by generators). One unusual feature of the tables is that full multiplication tables are given for all the point groups treated: not only will they be useful in teaching and illustrating group theory but, after all, a group is entirely defined by its multiplication table and it is only when this is available that no possible ambiguity can remain about the group definition and that consistent results in all applications required can be guaranteed. Tables of symmetrized harmonics or spin harmonics are given for all the point groups treated for all values of j, except for the cubic and icosahedral groups where we go up to $j = 18$. Also, and most importantly, correct subduction is achieved over as many group chains as possible, thus guaranteeing that phase factors are properly maintained.

We have ensured that absolutely all the conventions and definitions required in order to use the tables are given in Part 1 clearly and completely (in a dictionary style, in order to facilitate rapid use). The group-theoretical definitions given are often not the most general mathematical definitions available, but they have been chosen so as to be reasonably self-contained for a reader without specialized knowledge of the subject. Clear pictures are provided that permit the identification of the symmetry operations in each group. Likewise, pictures of molecular examples are provided for each group. Chapters **2**, **11**, and **12** of Part 1 contain a large collection of group-theoretical and matrix formulae, and Chapter **2** in particular will prove invaluable to the user of the tables to understand precisely the way in which the various tabulated items should to be used. Part 1 also contains complete information on the structure and properties of the point groups, including their generation. Chapter **17** contains worked-out examples that will ensure that the reader can see in practical cases how the tables are used. No proofs are given in Part 1 but they can normally be found in the references given at the end of each chapter; when this is not so concise proofs are provided.

A major problem when compiling and printing tables is that of avoiding errors and misprints. We have tried very hard to overcome this by obtaining the tables by computer, often in more than one way. The computer output has been directly transferred into print by the use of TEX, which has thus provided the final camera-ready copy. More details about the construction of the tables and comparisons with the literature may be found in Chapter **1**.

The completion of this book would have been impossible without the opportunity for one of us (S. L. A.) to spend the first half of 1992 in Vienna. He wishes to express his gratitude to The Royal Society for a grant for this purpose and to Professor A. Neckel for his kind and generous hospitality at the Institute of Physical Chemistry of the University of Vienna. Most of all, he is deeply indebted to Professor Peter Weinberger for finding the funds that made this visit possible, as well as for looking after all the practical details which made his stay in Vienna as enjoyable as it was useful. We should like to acknowledge gratefully the support of the Austrian Ministry of Sciences under Project No. GZ 49.731/2-24/91. We are also most indebted to Dr Peter Marksteiner for a critical reading of this work and to Florian Herzig for help in preparing the index and checking some of the tables.

It would not be right to finish these acknowledgements without expressing also our warmest thanks to our wives, Bocha and Ulli, for the gracefulness with which they accepted their roles of computer widows during the long years when this book was being prepared.

Oxford and Vienna S. L. A. and P. H.
October 1993

Note added in the Second Edition. Ten errata that have been found over the years in the previous printing have been corrected in this edition. We are grateful to Dr Nikolaos P. Konstantinidis for pointing out to us some errors in the tables of the icosahedral group $\mathbf{I}_h$. The digital version of this book was made possible thanks to the Phaidra Project of the University of Vienna. The digitalization process was conducted as part of the scheme of "E-Books on Demand". We thank the University Library of Vienna, in particular Dr Susanne Blumesberger, for their generous help. We are also grateful to Dr Peter Marksteiner of the Vienna University Computer Center for his kind help and advice.

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Contents

Part 2. The tables

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How to use this book. Notation

This book is divided in two parts. Part 1 is an introduction to the tables in 17 chapters. Part 2 contains the main body of the tables.

Chapters **1** to **15** of Part 1 contain concise definitions of all the properties tabulated in the tables plus some useful definitions and formulae related to point groups.

Chapter **16** of Part 1, **How to use the tables**, contains a statement of the notation used in each table, an explanation of the disposition of each table, and examples of its use. (Further examples of the use of the tables may be found in the Problems in Chapter **17**.) Each table contains a reference to the appropriate section of Chapter **16**.

The page number given at the heading of the tables for each point group (box at the top of the page) is a reference to the key for: (i) reading that heading; (ii) reading the sub-sections 1 to 5 (or 1 to 6) that follow that heading; (iii) using the footer at the bottom of each page of the tables.

All sections and page numbers in the headings of each sub-table for a point group refer to the place in Chapter **16** where full instructions for the use of that sub-table are given.

1 Table numbering and general cross-referencing

Table $\mathbf{m}.j$	A table in Chapter **m**, of Part 1. The digit j runs serially through the chapter. The chapter number is dropped in cross-references within the same chapter.
Fig. $\mathbf{m}.j$	A figure in Chapter **m**, of Part 1. The digit j runs serially through the chapter. The chapter number is dropped in cross-references within the same chapter.
T $\mathbf{n}.i$	A table in Part 2. The first number, **n**, in bold, individualizes the point group and runs from **1** to **75** in a specific order used in this book (see the Contents). The second number refers to the particular table (characters, bases, etc.).
F **n**	A figure in Part 2. The number **n**, in bold, individualizes the point group and runs from **1** to **75** in the specific order used in this book.
'Equations'	Displayed formulae, definitions, enunciations of theorems, comments, etc., are most often numbered on the right-hand side of the material in question. For brevity, all such material, when used in a cross-reference, is called an 'equation' here, and sometimes also in the body of the book.
($\mathbf{m}.i$)	In cross-references outside Chapter **m**, equation i of Chapter **m**. The number i runs serially through the chapter and the chapter number is dropped within a chapter.
(L $\mathbf{m}.i$), (R $\mathbf{m}.i$)	Left and right-hand sides, respectively of equation ($\mathbf{m}.i$).
§ $\mathbf{m}$–i	Section i of Chapter **m**. The chapter number is dropped within a chapter.

Cross-references on left margins of displayed lines

Numbering of equations	It is serial throughout each chapter. In the examples below all references are within the same chapter. Appropriate changes are otherwise introduced.
3	Equation (3) is used to derive the equation on the right.
$3'$	Equation (3), in a changed notation, is used to derive the equation on the right.
, 3	Equation (3) is used, but not immediately, to derive the equation on the right.

2, 3	Equations (2) and (3) are applied in that order to obtain the equation on the right.
2|3	Equation (2) applied on equation (3) gives the equation on the right.
F, T, P	On any of the above, indicate a Figure from Part 2, a Table from Part 2, or a Problem, respectively.

Literature references

Ono (1945)	Identifies a paper or book under that name in the alphabetic list of references at the end of this book.

2 Symbols used

$\forall$	For all.
$C(g_i)$	Class of the element g_i.
$\lvert C(G)\rvert$, $\lvert C\rvert$	Number of classes of a group G. The name of the group is often left implicit, as in the second symbol.
$\lvert C(\widetilde{G})\rvert$, $\lvert\widetilde{C}\rvert$	Number of classes of a double group $\widetilde{G}$. The name of the double group is often left implicit, as in the second symbol.
$\chi(g \mid u)$	Character of operation g in the irreducible basis $\langle u\rvert$.
$\chi(\mathring{g} \mid \hat{G})$, $\chi(\mathring{g} \mid \check{G})$	Character of the operation $\mathring{g}$ (written as g when unambiguous) in the representations $\hat{G}$ or $\check{G}$ of the group G.
δ_{ij}	Kronecker's delta.
e, E	Identity element of a group.
$\exists$	There exists.
$\in$	Belongs to.
G	Group of operations g.
$\lvert G\rvert$, $\lvert\mathbf{D}_{3h}\rvert$	Order of groups G, $\mathbf{D}_{3h}$, respectively.
$\widetilde{G}$	Double group of point group G.
$\hat{G}(\mathring{g})$, $\check{G}(\mathring{g})$	Matrix representative of the operator $\mathring{g}$ (written as g when unambiguous) in the representations $\hat{G}$ and $\check{G}$ respectively.
${}^{i}\hat{G}$	i-th irreducible (ordinary or *vector*) representation of G.
${}^{i}\check{G}$	i-th irreducible *projective* representation of G. Because vector representations are a particular case of projective ones this symbol often **denotes either vector or projective (unitary) representations.**
$\lvert{}^{i}\hat{G}\rvert$, $\lvert{}^{i}\check{G}\rvert$	Dimension of the above representations.
g	Configuration-space operator.
$\mathring{g}$	Function-space operator, written as g when unambiguous.
g_i^{-1}, $\bar{g}_i$	Inverse of element g_i.
$H \subset G$	H is a subgroup of G.
$H \triangleleft G$	H is an invariant subgroup of G.
$\lvert\mathsf{i}\rvert$, $\lvert\mathsf{i}(G)\rvert$	Number of irregular classes in the group. (Name of group in brackets if necessary.)
$\lvert I\rvert$, $\lvert I(G)\rvert$	Number of irreducible representations in the group. (Name of group in brackets if necessary.)
$\lvert\widetilde{I}\rvert$, $\lvert\widetilde{I}(G)\rvert$	Number of spinor irreducible representations in the group. (Name of group in brackets if necessary.)

i	Imaginary unit.
i	Inversion at the origin of coordinates. Also marker for improper operations.
Irreducible representations notation	**See Chapter 14.**
K	Conjugator operator.
$L \cap M$	Intersection.
$L \circledwedge M$	Semidirect product ($L \lhd (L \circledwedge M)$).
$L \otimes M$	Direct product.
$L \overline{\otimes} M$	Symmetrized direct product.
$L \underline{\otimes} M$	Antisymmetrized direct product.
Point-group notation	**See Chapter 5.**
Symmetry operations notation	**See Chapter 4.**
T	Time reversal operator.
$\mathbf{1}$	A unit matrix of appropriate dimension.
*	(Superscript.) Always a complex conjugate.
$^\mathsf{T}$	(Superscript on matrix.) Transpose.
†	(Superscript on matrix.) Adjoint: $A^\dagger = (A^*)^\mathsf{T}$.
$\mathbf{\vert} r \mathbf{\vert}$	Number of regular classes in the group. (Name of group in brackets if necessary.)
$\mathbf{\vert} n \mathbf{\vert}$	Upper limit of a running index n, not to be confused with an absolute value. **Notice the use of bold vertical bars to denote specific integers.**
$\{g_i\}$	Set of all elements $g_i, i = 1, 2, \ldots, n$.
$\vert\varphi(\mathbf{r})\rangle$	Ket.
$\langle\varphi(\mathbf{r})\vert$	Bra.
$\langle\varphi(\mathbf{r}) \mid \psi(\mathbf{r})\rangle$	Bra-ket or bracket.
$\boldsymbol{\langle}\varphi_1, \varphi_2, \ldots, \varphi_n\boldsymbol{\vert}$	Row vector of components $\varphi_1, \varphi_2, \ldots, \varphi_n$; basis of a representation.
$\boldsymbol{\langle}\varphi\boldsymbol{\vert}$	Abbreviated form of the above symbol, **not to be confused with a bra.**
$\boldsymbol{\vert}x, y, z\boldsymbol{\rangle}$	Column vector of components x, y, z, **not to be confused with a ket.**
$\oplus$, $\textcircled{$\Sigma$}$	Direct sums.
$\rightarrow$	Mapping: the set on the left of this symbol maps into the set on the right.
$\mapsto$	Mapping: the element on the left of this symbol maps to the element on the right.
$\Rightarrow$	*If then:* the statement on the left of this symbol implies the statement on the right.
$=_{\text{def}}$	The corresponding equality entails a definition.
$\twoheadrightarrow$	A table continues.

Part 1

Introduction to the Tables

1

Introduction

We shall first review briefly the literature and we shall then discuss the construction of the present tables.

1 Comparison with other tables

The best known tables for the point groups are probably those of Koster *et al.* (1963) which treat only the thirty-two crystallographic point groups. They have the merit that double groups and Clebsch–Gordan coefficients are provided. On the other hand, matrix representations are not explicitly given and the individual identification of the symmetry operations is not transparent. Since the Clebsch–Gordan coefficients depend on the matrix representations chosen, the use of these coefficients is not as easy as it would be desirable. Multiplication tables for the groups are not given, so that their definition remains a little loose, specially for the double groups. At the other end of the scale from the point of view of convenience of use, are the tables of Atkins *et al.* (1982). These authors deal with forty-seven point groups but they treat the double-group representations for only a few of these. Only character tables are given and bases are provided up to and including l equals 2. Subduction tables are also included. These tables are very convenient to use and almost free of error, the only mistake appearing in the table for $\mathbf{D}_6^*$ where the labels $E_{3/2}$ and $E_{5/2}$ should be interchanged.

The tables of the crystallographic point groups included in Bradley and Cracknell (1972) offer several advantages. The symmetry operations can be easily identified and they contain full matrix representations and symmetrized bases, although Clebsch–Gordan coefficients are not given. A drawback of the tables is that the matrix representative of an operation is not always directly related to it by the corresponding rotation operator. In other words, the matrices do not have a direct geometrical meaning in every case. Moreover, the double-group representations do not always subduce correctly to the corresponding subgroups. (Examples of these problems can be seen in Altmann 1986, Chapter 15.)

Harris and Bertolucci (1978) contains a large collection of tables of point groups both crystallographic and non-crystallographic. Only characters are given and no Clebsch–Gordan coefficients are provided. The tables of Pyykkö and Toivonen (1983) contain full matrix representations for the spinor (double group) representations of thirty-eight point groups and they are extremely accurate except that in Tables A3.10, A3.18, and A3.19 the surd ($\sqrt{}$) is missing in the characters for some of the operations. The symmetry operations are well identified and their matrices have the correct geometrical meaning. No Clebsch–Gordan coefficients are given, however. Perhaps the most comprehensive work on the point groups is that of Butler (1981), which contains extensive tables of Clebsch–Gordan coefficients. Whereas these depend on the bases chosen for the representations (or, what is largely the same, on the matrix representations themselves) Butler has provided Clebsch–Gordan coefficients with very well defined phase factors but which do not require explicit tabulation of the matrix representations. This is of course a very major advance but it makes the tables very difficult to use. The excellent book of Piepho and Schatz (1983), however, provides a very good introduction to Butler's method.

The tables of Thomas and Wood (1980) should also be mentioned, although they are not directly addressed to the point groups. They provide, however, full tables of group-theoretical properties (including multiplication tables) for all groups up to and including order 32 and although prepared from the point of view of the pure mathematician they provide quickly a variety of useful results.

2 Construction of the present tables

All the tables described above were constructed, whenever the symmetry operations are explicitly given, by using a parametrization of them based on the parametrization of rotations by Euler angles. For the finite point groups this parametrization is as cumbersome as it is defective, since the Euler parameters are not uniquely defined whenever the second Euler angle equals 0 or π, which is the case for all rotations in all dihedral groups and in all crystallographic point groups. Altmann (1986, Chapter 15; 1989*b*), shows examples of the difficulties thereby generated. We have used for this reason the quaternion parametrization of the symmetry operations as introduced by Altmann (1986). This parametrization permits very simply the construction of the multiplication tables of the groups and double groups and they are given here thus guaranteeing the precise definition of the groups and their representations.

Another advantage of the quaternion parametrization is that, with an adequate set of conventions (Altmann 1986) it guarantees that subduction of the matrix representations from a group to a subgroup can always be done correctly. This is important because, when spinor (or double-group representations) are subduced, the property that the character is constant along a class can break down, thus spoiling the subduction. Examples of this situation in the Euler parametrization can be seen in Altmann (1986, Chapter 15), and in Altmann (1989*b*). It should be understood, however, that subduction cannot always be guaranteed for all the subgroups of the same name of a given group. Thus the group $\mathbf{O}_h$ has four $\mathbf{C}_{3v}$ subgroups and subduction can only be ensured for two of them at the same time. (This is not an artifact of the method used: it is a mathematical necessity.) We have paid a great deal of attention to this question of subduction not just from one group to its subgroups but for whole group chains. First, we have very carefully developed a notation that, although in complete agreement with the standard notation for symmetry operations, is so chosen that the same operation does not change name along one group chain. Secondly, on using the quaternion parametrization coupled with a set of simple and well defined conventions, we have ensured that subduction is correct in all cases when this is at all possible.

For all groups treated we have provided complete sets of matrix representations. This means that a choice of bases has to be made. It was customary in the past to use for this purpose spherical harmonics in real form, as was done by Altmann and Bradley (1963*a,b*) and Bradley and Cracknell (1972). We have moved away from this approach for two reasons. First, with modern computing facilities there is no trouble whatever in dealing with complex functions. On the contrary, computer time is often thereby saved. Secondly, we decided to use bases which are as directly related as possible to the canonical bases of the full improper rotation group O(3). These are the harmonics and spin harmonics in the Condon and Shortley convention. The bases themselves have been chosen so that they, and therefore the representation matrices, change as little as possible along a group chain, thus making subduction as simple as it can be achieved. Also, we have ensured in this way that whenever a double-group representation contains representatives that should coincide with Pauli matrices this is actually the case.

The definition of the bases of the representations has also been simplified in the following way. Koster *et al.* (1963) and all their successors, have used bases of O(3) which are incomplete in the sense that only one spinor basis for $j = 1/2$ exists which is gerade. An ungerade basis for this value of j must then be obtained by vector coupling the spherical harmonic for $l = 1$ (ungerade) with the gerade spinor for $j = 1/2$. Altmann (1986, 1987) was able to construct directly an ungerade spinor for $j = 1/2$ which greatly rationalizes the presentation of the bases of O(3) and thus of the point groups.

In order to construct the tables with precise phase factors, we have used the method of the projective representations (Brown 1968, 1970; Altmann 1979; Altmann and Palacio 1979; Altmann and Herzig 1982; Altmann and Dirl 1984; Altmann 1986), which also, most importantly, permits printing of the tables, which would otherwise had been prohibitively bulky, in a compact form. Although it is perfectly possible to dispense entirely with the use of the double groups and work only with the spinor representations as given by the projective-representation method, the present tables have been displayed in such a way that no knowledge whatever of projective representations is required and that full details of the double groups are most easily obtained from them. Nevertheless, those who wish to use the spinor representations from the point of view of projective representations will have no trouble at all in doing so.

We present in these tables seventy-five groups and their corresponding double groups which cover all the cyclic, dihedral and related groups up to and including proper rotation axes of order 10, plus all the cubic and icosahedral groups. For each of these groups the stereographic projection is provided plus a three-dimensional depiction. All the group operations can be clearly identified from these figures and compared if desired with the full parameter tables given. In the three-dimensional figures, moreover, a molecular structure of the correct symmetry is shown. Molecular examples for each group, whenever possible, are also given. (Blanks have been left in the corresponding lines which users of the tables might wish to fill in with further examples when available.) For each group and double group the following tables are also given: multiplication tables, factor tables, character tables, tables of cartesian tensors and s, p, d, and f functions, symmetrized bases, matrix representations, direct products of representations, subduction tables, including subduction from O(3), and Clebsch–Gordan coefficients, except that the latter are not provided for the icosahedral groups since they are prohibitively bulky in that case and not very practical to use.

2

Basic group theory: definitions and formulae

1 Basic group definitions

Group properties (postulates)

Given	Set $\{g_i\}, i = 1, 2, \ldots, n$; binary operation $g_i g_j$.	
Associativity	$(g_i g_j)\, g_k = g_i\, (g_j g_k)$.	**(1)**
Closure	$g_i \in G, g_j \in G \Rightarrow g_i g_j \in G$.	**(2)**
Identity	$\exists E \in G\colon \quad g_i E = E g_i = g_i, \quad \forall g_i \in G$.	**(3)**
Inverse	$\forall g_i \in G \Rightarrow \exists g_i^{-1} \in G\colon \quad g_i^{-1} g_i = g_i g_i^{-1} = E$.	**(4)**

Group presentations

As set	$G = \{g_i\}, i = 1, 2, \ldots, n$.	**(5)**
As direct sum	$G = g_1 \oplus g_2 \oplus \cdots \oplus g_n = \sum_{i=1}^{n} g_i = \sum g_i$.	**(6)**

Group definitions

Order	**5** $\quad n =_{\text{def}}$ *order of* $G = \lvert G \rvert$.	**(7)**
Intersection	$G \cap H =_{\text{def}} \{k_i\} \quad \forall k_i \in G, k_i \in H$.	**(8)**
Conjugate of g_i by g	$g g_i g^{-1}$, $g_i \in G$, $g \in G$. **Notice that the inverse is always on the right in this book.**	**(9)**
Class of g_i	$C(g_i) = \sum_{\forall g \in G} g g_i g^{-1}$, *(no repetition)*. *No repetition* means that only one copy of each element is kept in the result of the summation.	**(10)**
	Notice that if $g_j \in C(g_i)$, then $C(g_j) \equiv C(g_i)$.	**(11)**
Classes of G	G is a sum of disjoint classes. Their number is $\lvert C(G) \rvert$, abbreviated as $\lvert C \rvert$ when the group in question can be identified from the context.	**(12)**
Subgroup H of G	If $\forall h \in H \Rightarrow h \in G$ and H a group, then $H \subset G$.	**(13)**
Proper subgroup	Given G, H is a proper subgroup of G if $H \subset G$ and $H \neq E$, $H \neq G$. **Do not confuse the word 'proper' as used here with the same word as used for proper point groups.**	**(14)**
Index of $H \subset G$	$\lvert G \rvert / \lvert H \rvert$. It is always an integer. (Lagrange's Theorem.)	**(15)**
Invariant subgroup	$H \subset G$ *and* $\forall g \in G, \forall h \in H \Rightarrow g h g^{-1} \in H$.	**(16)**
	Notation for invariant: $H \triangleleft G$.	**(17)**
Simple group	G is simple if it does not have a proper invariant subgroup. **Not to be confused with simple-reducible groups.** (See **100**.)	**(18)**
Cosets, $H \subset G$	*Left coset of H by $g \in G$*: $\quad gH = \sum_{\forall h \in H} gh$.	**(19)**
Property of cosets	$gH = H, \forall g \in H$.	**(20)**
Right cosets	Hg are similarly defined.	
Cosets, $H \triangleleft G$	**16** $\quad gH = Hg, \forall g \in G$. It follows that if $\lvert G \rvert / \lvert H \rvert$ is 2 (index 2), $H \triangleleft G$.	**(21)**

Coset expansion of G by $H \subset G$ — $G = \sum_i s_i H$, $i = 1, 2, \ldots, |G|/|H|$, $s_i \in G$, $s_i \notin H$ (except $s_i = E$). **(22)**

Coset representatives — The s_i in (**22**). Their choice is not unique. Convention in this book: one of the s_i is always taken to be E. **Notice:** $\{s_i\}$ **does not necessarily close and thus it is not necessarily a group.** **(23)**

Group products

GG — **6** $\quad GG =_{\text{def}} \underbrace{\sum_{i=1}^{n} g_i \sum_{j=1}^{n} g_j}_{\textit{no repetition}} = \sum_{k=1}^{n} g_k = G.$ **(24)**

Semidirect product — If $G = \sum_j s_j H$, $H \lhd G$, $\{s_j\} = S \subset G$, $S \cap H = E$,
then $G = \{h_i s_j\}_{\forall i,j} =_{\text{def}} H \circledwedge S.$ **(25)**
Notice: (i) The invariant is always first in the product symbol.
(ii) However, as in the above, G is always given in this book in *left* cosets of the invariant.

Direct product — First definition from (**25**):
If $G = \sum_j s_j H$, $H \lhd G$, $\{s_j\} = S \lhd G$, $S \cap H = E$,
then $G = \{h_i s_j\}_{\forall i,j} =_{\text{def}} H \otimes S.$ **(26)**
Second definition:
If $H \cap S = E$, $h_i s_j = s_j h_i$, $\forall i, j$,
then $\{h_i s_j\}_{\forall i,j} = G =_{\text{def}} H \otimes S.$ **(27)**

2 Operators

Configuration-space operators

x, **y**, **z** — Laboratory (space fixed) axes. **They are never transformed.** **(28)**

i, **j**, **k** — Body or configuration-space axes, fixed in the system studied. **(29)**

r — Position vector (tail at origin) of components x, y, z in the laboratory axes, fixed with respect to **i**, **j**, **k**. Their components in **i**, **j**, **k** *never change.* **(30)**

x, y, z — Independent variables (components of **r** in the laboratory axes). **(31)**

x, y, z — Functions such as $x(r, \theta, \varphi)$, etc. of any independent variables chosen. **(32)**

Operator g — Operations such as rotations, reflections, etc., transform *all* **r** into **r**′, with new components x', y', z' in the laboratory axes:
$g\mathbf{r} =_{\text{def}} \mathbf{r}'.$ **(33)**
g can also be defined as operating on **i**, **j**, **k**.

Active picture — The above definition of operators in the configuration space is called active. There is an alternative picture, the passive convention. **All operators used in this book are active. All symmetry elements, such as rotation axes and planes, are fixed in x, y, z and never transformed.** **(34)**

Warnings — (i) When g operates on **i**, **j**, **k**, notice that the transformation properties of **i**, **j**, **k** are not the same as the transformation properties of x, y, z implicit in (**33**). **(35)**
(ii) g cannot operate on the functions *x*, *y*, *z*. Operators on functions are denoted $\mathring{g}$ in this book (see **37** below). However, no such distinction is traditionally made when the conventional symbols for symmetry operations, C (for rotations), σ (for reflections), etc., are used. Therefore when such operators act on *x*, *y*, *z* they must be understood as function-space operators and their transformation rules are different from those of the configuration-space operators. **(36)**

(iii) **Results obtained in the passive picture are not directly compatible with the tables in this book.**

Function-space operators

$f'(\mathbf{r})$	The function $f(\mathbf{r})$ after the configuration space has been transformed by g.	
Function-space operator $\mathring{g}$	$\mathring{g}f(\mathbf{r}) =_{\text{def}} f'(\mathbf{r})$.	(**37**)
Defining relation for $\mathring{g}f$	$\mathring{g}f(\mathbf{r}) = f(g^{-1}\mathbf{r})$.	(**38**)
Warning	**Results obtained from the literature on using so-called alternative definitions of (38) are not necessarily compatible with the tables in this book.**	
Isomorphism of G *and* $\mathring{G}$	$\mathring{G} = \{\mathring{g}\}$. $\quad g_i g_j = g_k \quad \Rightarrow \quad \mathring{g}_i\mathring{g}_j = \mathring{g}_k$.	(**39**)
	Notice that because of this isomorphism it is usual in point groups to employ the same notation for g and $\mathring{g}$, as long as it is possible to recognize from the context which operator is meant. Accordingly, G and $\mathring{G}$, are usually treated as if they were one and the same group, rather than as two distinct realizations of the same abstract group. **This does not mean that the transformations effected by the operators g and $\mathring{g}$ are the same: they may entail different matrices. Be warned.**	
Conjugator operator K	Given complex numbers ω and u and the complex function $f(u)$,	
	$\mathrm{K}\omega f(u) =_{\text{def}} \omega^* f^*(u^*)$.	(**40**)
	K commutes with all geometrical symmetry operations.	
Time reversal operator T	It leaves invariant position vectors $\mathbf{r}$ and reverses the sign of the momentum $\mathbf{p}$ and spin $\mathbf{s}$:	
	$\mathrm{T}\,\mathbf{r}\,\mathrm{T}^{-1} = \mathbf{r}$, $\mathrm{T}\,\mathbf{p}\,\mathrm{T}^{-1} = -\mathbf{p}$, $\mathrm{T}\,\mathbf{s}\,\mathrm{T}^{-1} = -\mathbf{s}$.	(**41**)
	For any scalar α and the Pauli matrix $\boldsymbol{\sigma}_y$ (see **11**.18), it is given as	
	$\mathrm{T} = \alpha\,\boldsymbol{\sigma}_y\mathrm{K} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\mathrm{K}.$	(**42**)
	The matrix displayed here has been chosen for $\alpha = -\mathrm{i}$ and it is a binary rotation around the y axis in SU(2) (see **11**.16).	
	This operator is a symmetry operation for systems in free space and in the presence of electric fields but **not in the presence of a magnetic field**.	

3 Vector (ordinary) representations

Definition and properties

Definition	Given a group G of elements g, map a matrix $\hat{G}(g)$ to each g so as to conserve the multiplication rules of G:	
	$\hat{G}(g) \mapsto g$: $\quad \hat{G}(g_i)\,\hat{G}(g_j) = \hat{G}(g_i g_j)$.	(**43**)
	The set $\{\hat{G}(g)\} =_{\text{def}} \hat{G}$ is a (vector) representation of G.	
	Whenever the notation $\check{G}$ appears in this chapter the formulae given are valid either for vector representations $\hat{G}$ or for unitary projective representations with standardized and normalized factor systems. The reader who wishes to use the latter may refer to Chapter 10. The reader who does not wish to do so may read all inverted hats in this chapter as ordinary hats.	(44)
Alternative realization	$\hat{G}(\mathring{g}) \mapsto \mathring{g}$: $\quad \hat{G}(\mathring{g}_i)\,\hat{G}(\mathring{g}_j) = \hat{G}(\mathring{g}_i\mathring{g}_j)$.	(**45**)

Warning	**Even when the matrices $\hat{G}(\mathring{g}_i)$ are identical with the matrices $\hat{G}(g_i)$, they do not necessarily operate in the same way. All matrix representations in this book are given in the sense of (45) and even when this is not explicit and unless statements to the contrary, the operators to which they refer are function and not configuration-space operators.**	(**46**)
Unitary property	$\check{G}(g_i)^\dagger\,\check{G}(g_i) = \check{G}(g_i)\,\check{G}(g_i)^\dagger = \mathbf{1}$.	(**47**)
	All representations in this book (whether vector or projective) are unitary.	
Trivial representation	$\hat{G}(g) = 1, \quad \forall g.$	(**48**)
Faithful representation	All $\hat{G}(g)$ are distinct.	(**49**)
Regular representation	The matrix $\hat{G}(g)$ is the permutation matrix obtained by acting with g on $g_1, g_2, \ldots, g_{\lvert G\rvert}$.	(**50**)

Bases of the representations; representations

Invariant space under all $\mathring{g}$ of G	A set of functions $\varphi_1, \varphi_2, \ldots, \varphi_N$ such that the transform of any function of the set under any operation $\mathring{g}$ of G is a linear combination of the functions of the set:	
	$\mathring{g}\varphi_n = \sum_{m=1}^{N} \varphi_m\,\check{G}(\mathring{g})_{mn}$.	(**51**)
Row-vector notation	$\langle\varphi_1, \varphi_2, \ldots, \varphi_N\rvert$ = row vector of components $\varphi_1, \varphi_2, \ldots, \varphi_N$.	(**52**)
	Abbreviated as $\langle\varphi\rvert$.	
	Not to be confused with a bra (which is given in light brackets).	
Matrix notation	**52|51** $\quad \mathring{g}\langle\varphi_1, \varphi_2, \ldots, \varphi_N\rvert = \langle\varphi_1, \varphi_2, \ldots, \varphi_N\rvert\,\check{G}(\mathring{g})$.	(**53**)
	53 $\quad \mathring{g}\langle\varphi\rvert = \langle\varphi\rvert\,\check{G}(\mathring{g})$.	(**54**)
	46|54 $\quad g\langle\varphi\rvert = \langle\varphi\rvert\,\check{G}(g)$.	(**55**)
Basis	The functions $\langle\varphi_1, \varphi_2, \ldots, \varphi_N\rvert$.	(**56**)
Representation	The set $\check{G} =_{\text{def}} \{\check{G}(\mathring{g})\}$ (also written $\{\check{G}(g)\}, \quad \forall g \in G$).	(**57**)
	For vector representations these matrices satisfy (**45**).	
Dimension of the representation, $\lvert\check{G}\rvert$	It is the dimension of the matrices $\check{G}(g)$.	(**58**)
Function belonging to the i-th column of the representation	Any function that transforms by the same coefficients $\check{G}(\mathring{g})_{mi}$ as φ_i in (**51**).	
	Also said to to be the i-th partner of the basis.	
	It is the i-th component (that is, it is in the i-th column) of the row-vector basis.	
	Care must be exercised in comparing with other statements to this effect in the literature.	(**59**)
Independent variables basis x, y, z	Components of position vector **r** (see **31**).	
	They transform under g, not $\mathring{g}$. The 3 by 3 matrix $\hat{G}(g)$ forms a representation in the sense of (**43**) **but the transformation rule of the basis is not that given in (53):**	
	Basis (column vector of components x, y, z): $\lvert x, y, z\rangle$.	(**60**)
	Not to be confused with a ket (always given in light brackets).	
	Tranformation rule: $g\,\lvert x, y, z\rangle = \hat{G}(g)\,\lvert x, y, z\rangle$.	(**61**)
	Tensor bases formed from x, y, z lead to representations like (61) of dimension higher than 3, which must not be confused with (53).	
Basis x, y, z	See (**32**). Being functions, they must be written as $\langle x, y, z\rvert$ and transform under $\mathring{g}$ as in (**53**), **not as (61)**.	(**62**)

Direct sum of bases — Given bases $\langle\varphi^i| =_{\text{def}} \langle\varphi_1^i, \varphi_2^i, \ldots, \varphi_{N_i}^i|$, $i = 1, 2, \ldots$ (compare with **52**) the row vector of components φ_j^i, $\forall i, j$ is the direct sum of the bases, written as follows:

$\langle\Phi| = \bigoplus_i \langle\varphi^i|$. **(63)**

Direct sum of representations — If ${}^i\check{G}$ is the representation on the basis $\langle\varphi^i|$, the representation on the basis (**63**) is given by block-diagonal matrices with the matrices ${}^i\check{G}$ along the diagonal and it is called the direct sum of the representations:

$\check{G}(g) = \bigoplus_i {}^i\check{G}(g)$. **(64)**

Similarity and unitary transformation of representations

Dependence of the representation on the basis — $\check{G}(g)$ in (**55**) depends on the basis. Write it therefore as $\check{G}_{<\varphi|}(g)$:

$g\langle\varphi| = \langle\varphi|\,\check{G}_{<\varphi|}(g)$. **(65)**

Similarity transformation — Consider a second row basis $\langle\Phi| = \langle\varphi|\,C$, for some suitable matrix C:

$g\langle\varphi|\,C = \langle\varphi|\,C\,\check{G}_{<\varphi|C}(g) \qquad \Rightarrow \qquad g\langle\varphi| = \langle\varphi|\,C\,\check{G}_{<\varphi|C}(g)\,C^{-1}$. **(66)**

66,**65** $\qquad \check{G}_{<\varphi|C}(g) = C^{-1}\,\check{G}_{<\varphi|}(g)\,C$. **(67)**

Unitary transformation — **47**|**67** $\qquad \check{G}_{<\varphi|C}(g) = C^{\dagger}\,\check{G}_{<\varphi|}(g)\,C$, $\quad$ (C unitary). **(68)**

Warning — **Similarity and unitary transformations are often used in the literature with the inverse or adjoint on the right. For the work of this book they must be used as defined here.**

Characters

Definition, $\chi(g \mid \check{G})$ — $\chi(g \mid \check{G}) =_{\text{def}} \sum_{m=1}^{|\check{G}|} \check{G}(g)_{mm}$. **(69)**

As class functions — $g' \in C(g) \qquad \Rightarrow \qquad \chi(g' \mid \hat{G}) = \chi(g \mid \hat{G}), \quad \forall \hat{G}$. **(70)**

Invariance — $\chi(g \mid \hat{G}) = \chi(g \mid C^{-1}\,\hat{G}\,C) = \chi(g \mid C^{\dagger}\,\hat{G}\,C), \quad \forall g, \hat{G}$, a matrix C. **(71)**

Irreducible representations and their properties

Irreducible representation — One which cannot be taken into the form (**64**) by any similarity transformation. The corresponding basis is an irreducible basis. **(72)**

Number, $|I(G)|$ — $|I(G)|$ = number of irreducible representations of G, abbreviated $|I|$. **(73)**

$|I(G)| = |C(G)|$. **(74)**

Dimension relation — $\sum_{i=1}^{|I(G)|} |{}^i\check{G}|^2 = |G|$. **(75)**

Orthogonality relation for the representations — $\sum_g {}^i\check{G}(g)_{mn}^{*}\,{}^j\check{G}(g)_{pq} = |G|\,|{}^i\check{G}|^{-1}\,\delta_{ij}\,\delta_{mp}\,\delta_{nq}$. **(76)**

Orthogonality relations for the characters — $\sum_g \chi(g \mid {}^i\check{G})^{*}\,\chi(g \mid {}^j\check{G}) = |G|\,\delta_{ij}$. **(77)**

$\sum_{i=1}^{|I(G)|} \chi(g_m \mid {}^i\check{G})^{*}\,\chi(g_n \mid {}^i\check{G}) = |G|\,|C(g_m)|^{-1}\,\delta_{mn}$. **(78)**

Irreducibility condition — The representation $\check{G}$ is irreducible if and only if

$\sum_g \chi(g \mid \check{G})^{*}\,\chi(g \mid \check{G}) = |G|$. **(79)**

Schur's lemma — Given, for some matrix C,

${}^i\check{G}(g)\,C = C\,{}^j\check{G}(g), \quad \forall g \in G$. **(80)**

(i) $\quad |{}^i\check{G}| \neq |{}^j\check{G}| \qquad \Rightarrow \qquad C = 0$. **(81)**

(ii) $\quad |{}^i\check{G}| = |{}^j\check{G}| \qquad \Rightarrow \qquad$ (*either* ${}^j\check{G} = C^{-1}\,{}^i\check{G}(g)\,C$ *or* $C = 0$). **(82)**

(iii) $\quad i = j \qquad \Rightarrow \qquad C = c\,\mathbf{1}, \quad$ *c constant*, $\quad |\mathbf{1}| = |{}^i\check{G}|$. **(83)**

Corollary of Schur's lemma — Given $|{}^i\check{G}| = |{}^j\check{G}|$,

${}^j\hat{G} = C^{-1}\,{}^i\hat{G}(g)\,C \Rightarrow C$ *unique except for phase factor* $\omega\mathbf{1}$, *with* $|\omega| = 1$. **(84)**

4 Projection operators

Objective To generate, by acting on arbitrary functions ϕ (called the generators of the expansions), functions that belong to the n-th component of a basis of the i-th irreducible representation of a group, written φ^i_n, which therefore transform as follows:

51,**46** $g\varphi^i_n = \sum_m \varphi^i_m \, {}^i\check{G}(g)_{mn}$. (**85**)

Notice here that precisely the same transformation is valid if the basis is multiplied throughout by an arbitrary constant phase factor ω, ($|\omega| = 1$).

Definition $W^i_{np} = |{}^i\check{G}| \, |G|^{-1} \sum_g {}^i\check{G}(g)^*_{np} \, g$. (**86**)

$W^i_{np}\,\phi = \varphi^i_n \qquad \Rightarrow \qquad g_r\,(W^i_{np}\,\phi) = \sum_m (W^i_{mp}\,\phi)\, {}^i\check{G}(g_r)_{mn}$. (**87**)

Warnings (i) The first subscript in the projection operator determines the column to which the projected function belongs and **the second subscript has to be kept constant throughout the basis**. (**88**)

(ii) Notice also that if different functions of the basis for different n are generated from the first equation on (L **87**) their phase factors may differ, as follows from comparison of (**85**) and the second equation on (R **87**). (**89**)

Properties of the projection operators

Product $W^i_{np}\, W^j_{qr} = W^i_{nr}\,\delta_{ij}\,\delta_{pq}$. (**90**)

Transfer operator $W^i_{np}\,\varphi^j_q = \delta_{ij}\,\delta_{pq}\,\varphi^i_n$. (**91**)

Notice that the *transfer operator* W^i_{np} applied on the function φ^i_p of the basis transforms it into its partner φ^i_n of the same basis. This is the way in which, by allowing n to range over the whole dimension of the representation, all the functions of the basis are obtained with the correct phases: see the second warning above.

Adjoint $(W^i_{np})^\dagger = W^i_{pn}$. (**92**)

Projection operator over a representation

Definition $W^i = |{}^i\check{G}| \, |G|^{-1} \sum_g \chi(g \mid {}^i\check{G})^* \, g$. (**93**)

Property $W^i\,\phi =$ linear combination of the functions φ^i_n. (**94**)

5 Representation reduction

Objective Given the basis $\langle\varphi|$ in (**65**) to find a matrix C such that $\langle\varphi|\,C$ is reduced. (**95**)

Multiplicity or frequency **63** $\langle\varphi|\,C = \sum\!\!\!\!\bigoplus_{iu} \langle\Phi^{iu}|$, (**96**)

$i = 1, 2, \ldots, |I|; \quad u = 1, 2, \ldots, |i|; \quad |i| = 0, 1, 2, \ldots$ (**97**)

The notation in (**63**) has been expanded to recognize that the same irreducible basis i may appear in a number $|i|$ of copies which is called the *multiplicity* (or *frequency*) of the representation. The copies that thus appear may be either identical, or linearly independent, or related by a similarity transformation. The index u is called the *multiplicity index*. (**98**)

Double index iu Notice that the double index iu may be regarded as a single index which runs over all the bases that appear in the reduced representation. (**99**)

Simple-reducible groups The multiplicity is always unity for these groups. SO(3), O(3), and all point groups except cubic and icosahedral are simple reducible. **Do not confuse them with simple groups.** (See **18**.) (**100**)

Form of the representation	**67**, **64** **97**	$\check{G}_{\langle\varphi\mid C}(g) = C^{-1}\,\check{G}_{\langle\varphi\mid}(g)\,C = \textstyle\sum\!\!\!\!\bigcirc_{iu}\, {}^{iu}\check{G}(g).$ (**101**) $i = 1, 2, \ldots, \lvert I\rvert;\quad u = 1, 2, \ldots, \lvert i\rvert;\quad \lvert i\rvert = 0, 1, 2, \ldots$ (**102**)
Labelling of the matrix		The columns of the matrix C that effects the reduction in (**95**) must be labelled as those of the basis $\langle \Phi^{iu}\rvert$ in blocks of the form ium, where i and u run as in (**102**) and m runs from 1 to $\lvert {}^{i}\check{G}\rvert$. (**103**)
To bring C into unitary form		All the columns must be made orthogonal and each column normalized. **It is essential that this be done.** (**104**)
Orthogonalization of columns		In order to orthogonalize all the columns of all the blocks of the form ium, for a fixed i and u and m ranging as stated above, it is sufficient to orthogonalize one set of $\lvert i\rvert$ columns for all u and one fixed value of m. The same transformation that effects this orthogonalization will be valid for all other values of m. (**105**)
Normalization of columns		Once all the columns of C are orthogonalized, all the columns of a block ium for fixed iu and m ranging are normalized by obtaining the single normalization factor corresponding to any value of m. (See **167** below.) (**106**)
Uniqueness of C		From (**84**) C is unique **except for a phase factor**. (**107**)
Multiplicity calculation		Given (**97**) and (**98**), $\lvert i\rvert = \lvert G\rvert^{-1} \sum_g \chi(g \mid {}^{i}\check{G})^*\, \chi(g \mid \check{G}) = \lvert G\rvert^{-1} \sum_g \chi(g \mid {}^{i}\check{G})\, \chi(g \mid \check{G})^*.$ (**108**)

Notation used in this book for the indices

Irreducible representations	i, j, k, l.	(**109**)
Reducible representations	No index or α, β, γ.	(**110**)
Multiplicity indices	u, v, w.	(**111**)
Columns of bases, rows and columns of representations	m, n, p, q.	(**112**)

Representation reduction by projection operators

Objective	To form C as in (**95**) to (**102**).	
Step 1	Use (**108**) to determine the multiplicities. Use below only irreducible representations for which $\lvert i\rvert$ is different from zero.	(**113**)
Step 2	From (**87**), for $\varphi \in \langle\varphi\rvert$, form $W^{i}_{1p}\varphi = \varphi^{i}_{1}$, $\quad i = 1, \quad$ some p.	(**114**)
Step 3	From (**91**), form $W^{i}_{21}\varphi^{i}_{1} = \varphi^{i}_{2}$ and then apply W^{i}_{32} on the result and so on until the last partner of the basis (corresponding to the dimension of the representation) has been obtained.	(**115**)
Step 4	If $\lvert i\rvert$ is larger than unity, replace φ in Step 2 by one of its partners, until the resulting function is linearly independent from φ^{i}_{1} and then repeat Step 3 until a new basis is formed.	(**116**)
Step 5	If the multiplicity is two or larger, Step 4 must be repeated until the total number of bases required by $\lvert i\rvert$ is formed.	(**117**)
Step 6	Repeat Steps 2 to 5 until all irreducible representations are treated.	(**118**)

Orthonormalization The coefficients that appear in the symmetrized combinations give C, but the matrix must be made unitary by using (**105**) and (**106**).
For a given irreducible representation take one column, say the m-th one, in each of the copies ($|i|$ in number) of this representation and orthogonalize them (if necessary by Schmidt's procedure): the transformation thus obtained will be valid for all columns of the same representation. (See **105**.) Likewise, the normalization factor required for the columns is the same for all the columns of all the copies of the same irreducible representation. (See **106**.) (**119**)

Representation reduction by the internal method

Method Given a representation $\hat{G}(g)$, form the matrix M which is the sum (*not* the direct sum) of the matrices of one class of G. Find the matrix U which diagonalizes M. The matrix U reduces $\hat{G}(g)$. This method does not give any precise form of the irreducible matrices arising, whereas the projection operator method gives bases adapted to specific forms of the irreducible representations. On the other hand, it can be used when irreducible representation matrices are not available. (**120**)

6 Direct products

Representations of direct-product groups

Direct product, bases Given $G = H \otimes S = \{hs\}$, (**121**)
${}^i\check{H}(h)$, basis $\langle \varphi^i_m |$, $\quad |{}^i\check{H}| =_{\text{def}} |m|$, (**122**)
${}^j\check{S}(s)$, basis $\langle \psi^j_n |$, $\quad |{}^j\check{S}| =_{\text{def}} |n|$, (**123**)

Dictionary order The irreducible bases of G, in *dictionary order*, are of the form
$\langle \varphi^i_m | \otimes \langle \psi^j_n | = \langle \varphi^i_1\, \psi^j_1, \ldots, \varphi^i_{|m|}\, \psi^j_1, \ldots, \varphi^i_{|m|}\, \psi^j_{|n|} | =_{\text{def}} \langle \varphi^i_m\, \psi^j_n |.$ (**124**)

Double subscript The columns of the direct-product basis in (**124**) are labelled by a double subscript:
$\langle \varphi^i_m\, \psi^j_n | =_{\text{def}} \langle \xi^k_{mn} |.$ (**125**)

Labelling of matrix Correspondingly, the rows and columns of the irreducible representation of G spanned by (**125**) are labelled by double subscripts:
${}^k\check{G}(hs) = {}^i\check{H}(h) \otimes {}^j\check{S}(s) \qquad \Rightarrow \qquad {}^k\check{G}(hs)_{mn,pq} = {}^i\check{H}(h)_{mp}\, {}^j\check{S}(s)_{nq}.$ (**126**)

Characters $\chi(hs \mid {}^i\check{H} \otimes {}^j\check{S}) = \chi(h \mid {}^i\check{H})\, \chi(s \mid {}^j\check{S}).$ (**127**)

Number of irreducibles $|I(H \otimes S)| = |I(H)|\, |I(S)|.$ (**128**)

Direct product of two representations of the same group

Representations to multiply $g\, \langle \varphi^i_m | = \langle \varphi^i_m |\, {}^i\check{G}(g), \quad m = 1, 2, \ldots, |{}^i\check{G}| =_{\text{def}} |m|,$ (**129**)
$g\, \langle \varphi^j_n | = \langle \varphi^j_n |\, {}^j\check{G}(g), \quad n = 1, 2, \ldots, |{}^j\check{G}| =_{\text{def}} |n|,$ (**130**)

Direct-product basis **124** $\quad \langle \varphi^i_m | \otimes \langle \psi^j_n | = \langle \varphi^i_1 \psi^j_1, \ldots, \varphi^i_{|m|} \psi^j_1, \ldots, \varphi^i_{|m|} \psi^j_{|n|} |$ (**131**)
$=_{\text{def}} \langle \varphi^i_m\, \psi^j_n |.$ (**132**)

Direct-product representation (reducible) $\check{G}(g) = {}^i\check{G}(g) \otimes {}^j\check{G}(g), \qquad$ *basis* $\langle \varphi_{mn} | =_{\text{def}} \langle \varphi^i_m\, \psi^j_n |.$ (**133**)

Its matrix elements $\check{G}(g)_{mn,pq} = {}^i\check{G}(g)_{mp}\, {}^j\check{G}(g)_{nq},$ (**134**)
where the rows and columns of the direct-product matrix are labelled by the double subscripts used in the basis $\langle \varphi_{mn} |$ in (**133**).

Characters $\chi(g \mid {}^i\check{G} \otimes {}^j\check{G}) = \chi(g \mid {}^i\check{G})\, \chi(g \mid {}^j\check{G}).$ (**135**)

Symmetrized and antisymmetrized products of the same representation

Bases to multiply $\langle \varphi^i_m| =_{\text{def}} \langle \varphi_m|; \quad \langle \psi^i_n| =_{\text{def}} \langle \psi_n|.$ (**136**)

Direct-product basis **132** $\langle \varphi_m|\otimes\langle \psi_n| = \langle \varphi_m\, \psi_n| = \tfrac{1}{2}\langle \varphi_m\, \psi_n + \varphi_n\, \psi_m|\oplus\tfrac{1}{2}\langle \varphi_m\, \psi_n - \varphi_n\, \psi_m|$ (**137**)

$=_{\text{def}} \langle \varphi_m|\, \overline{\otimes}\, \langle \psi_n| \oplus \langle \varphi_m|\, \underline{\otimes}\, \langle \psi_n|.$ (**138**)

Symmetrized direct product $\langle \varphi_m|\, \overline{\otimes}\, \langle \psi_n| =_{\text{def}} \tfrac{1}{2}\langle \varphi_m\, \psi_n + \varphi_n\, \psi_m|.$ (**139**)

Antisymmetrized direct product $\langle \varphi_m|\, \underline{\otimes}\, \langle \psi_n| =_{\text{def}} \tfrac{1}{2}\langle \varphi_m\, \psi_n - \varphi_n\, \psi_m|, \quad (m \neq n).$ (**140**)

Characters $\chi(g \mid {}^i\check{G}\, \overline{\otimes}\, {}^i\check{G}) = \tfrac{1}{2}\left[\{\chi(g \mid {}^i\check{G})\}^2 + \chi(g^2 \mid {}^i\check{G})\right].$ (**141**)

$\chi(g \mid {}^i\check{G}\, \underline{\otimes}\, {}^i\check{G}) = \tfrac{1}{2}\left[\{\chi(g \mid {}^i\check{G})\}^2 - \chi(g^2 \mid {}^i\check{G})\right].$ (**142**)

Warnings The symbols ${}^i\check{G}\,\overline{\otimes}\,{}^i\check{G}$ and ${}^i\check{G}\,\underline{\otimes}\,{}^i\check{G}$ are given in the literature as $\left[{}^i\check{G}^2\right]$ and $\left\{{}^i\check{G}^2\right\}$ respectively, although this use is not always consistent. If the bases are **identical** the antisymmetrized direct product vanishes and **the only meaningful direct product is the symmetrized one.**

7 Clebsch–Gordan coefficients

Objective To reduce the direct product (**131**) by means of the transformation (**96**).

Notation

Representations Are given in the summation rather than the matrix notation.

Elements of the product bases. Kets Elements of a basis will be denoted by a *ket* holding the indices of the function:

129 $\varphi^i_m \equiv |im\rangle, \quad \psi^j_m \equiv |jm\rangle, \quad \varphi^i_m\, \psi^j_n \equiv |im\rangle\, |jn\rangle.$ (**143**)

Basis **129** $\langle \varphi^i_m| =_{\text{def}} \big\langle |im\rangle \big|, \qquad m = 1, 2, \ldots, |{}^i\check{G}| =_{\text{def}} |m|.$ (**144**)

The irreducible bases The symbols and indices of the irreducible bases which appear in the reduction of the direct product as well as those of the corresponding irreducible representations will be given in **capitals**. (**145**)

Thus the reduced basis $\langle \Phi^{iu}|$ which appears in (**96**) will be written with components

$\Phi^{IU}_P \equiv |IUP\rangle,$ (**146**)

where I: representation; U: multiplicity index; P: column index. (**147**)

Definition of the Clebsch–Gordan coefficients

The representations multiplied (or coupled) In the new notation,

129 $g\, |im\rangle = \sum_p |ip\rangle\, {}^i\check{G}(g)_{pm}, \qquad p, m = 1, 2, \ldots, |{}^i\check{G}| =_{\text{def}} |m|,$ (**148**)

130 $g\, |jn\rangle = \sum_q |jq\rangle\, {}^j\check{G}(g)_{qn}, \qquad q, n = 1, 2, \ldots, |{}^j\check{G}| =_{\text{def}} |n|.$ (**149**)

The direct-product basis (dictionary order) **131** $\big\langle |im\rangle \big| \otimes \big\langle |jn\rangle \big| =$

$\big\langle |i1\rangle\, |j1\rangle, \ldots, |i1\rangle\, |j|n|\rangle, \ldots, |im\rangle\, |jn\rangle, \ldots, |i|m|\rangle\, |j1\rangle, \ldots, |i|m|\rangle\, |j|n|\rangle \big|.$ (**150**)

The reduction matrix C (the Clebsch–Gordan matrix) **96′** $\sum_{mn} |im\rangle\, |jn\rangle\, C_{mn,P} = \Phi^{IU}_P \equiv |IUP\rangle.$ (**151**)

Notation for matrix elements of C **151** $C_{mn,P} =_{\text{def}} {}^{ij}\langle mn \mid IUP\rangle.$ (**152**)

Notation for the Clebsch–Gordan matrix

Left superscript ij It indicates the two irreducible representations the direct product of which is reduced by the Clebsch–Gordan matrix. (**153**)

Indices on bra The two indices mn used as a single subscript in dictionary order indicate the row of the Clebsch–Gordan matrix. (**154**)

Indices on ket — The first two are redundant from the point of view of the matrix-element labels: they indicate respectively the irreducible representation and multiplicity index for the reduced basis. These indices are introduced here for symmetry of the equations. The last index, P, denotes the element (column) of the reduced basis. (**155**)

The Clebsch–Gordan matrix

Full form of the reduced basis — **152|151**

$$|IUP\rangle = \sum_{mn} |im\rangle\, |jn\rangle\, {}^{ij}\langle mn \mid IUP\rangle. \qquad (\mathbf{156})$$

$$=_{\text{def}} \sum_{mn} |ijmn\rangle\, {}^{ij}\langle mn \mid IUP\rangle. \qquad (\mathbf{157})$$

Formula for the Clebsch–Gordan (unitary) matrix

$$ {}^{ij}\langle mn \mid IUP\rangle = |{}^{I}\check{G}|^{1/2}\, |G|^{-1/2} \left\{ \sum_g {}^{i}\check{G}(g)_{rr}\, {}^{j}\check{G}(g)_{ss}\, {}^{I}\check{G}(g)^{*}_{QQ} \right\}^{-1/2} \times \sum_g {}^{i}\check{G}(g)_{mr}\, {}^{j}\check{G}(g)_{ns}\, {}^{I}\check{G}(g)^{*}_{PQ}. \qquad (\mathbf{158})$$

Warning — It follows from (**84**) that the whole of the matrix of Clebsch–Gordan coefficients can be multiplied by an arbitrary phase factor exp(iω) without any observable change. Other phase factors, much more significant, arise through the fact that the irreducible representations (**148**) and (**149**) which are coupled are only defined within a similarity, but these factors can be eliminated if stated bases are used in every case, as is the case in the tables. Erratic phase factors may also appear in spinor representations owing to arbitrariness in the multiplication rules of the double group. These uncertain factors have been eliminated from our tables because of the accurate definition of the projective factors (or what is the same of the double-group multiplication rules). A number of schemes are given in the literature to fix the overall phase factor of the Clebsch–Gordan matrix and the user of the present tables can easily make his or her own choice. (**159**)

8 Matrix elements and selection rules

Definition — Given two functions ψ^i and ψ^j, which belong to the representations ${}^{i}\check{G}(g)$ and ${}^{j}\check{G}(g)$, respectively (this means that they are linear combinations of the functions of the corresponding bases) and some operator U^k, which belongs to the representation ${}^{k}\check{G}(g)$, write the *matrix element*

$$I_{ij} = \int (\psi^i)^*\, \mathrm{U}^k\, \psi^j\, \mathrm{d}\tau =_{\text{def}} \langle \psi^i \mid \mathrm{U}^k \mid \psi^j\rangle. \qquad (\mathbf{160})$$

This matrix element provides the transition probability between the two states ψ^i and ψ^j induced by a perturbation with the operator U^k. The vanishing of this element gives a *selection rule*.

Selection rule

$$I_{ij} \neq 0 \qquad \Rightarrow \qquad {}^{i}\check{G}(g) \otimes {}^{j}\check{G}(g)^{*} \text{ contains } {}^{k}\check{G}(g). \qquad (\mathbf{161})$$

Warning — If the bases ψ^i and ψ^j are **identical**, then the symmetrized direct product (see **139**) must be taken in (**161**). (**162**)

Matrix elements of the Hamiltonian — When U^k is the Hamiltonian H, the representation ${}^{k}\check{G}(g)$ is the totally symmetric (trivial) representation. The rule (**161**) becomes:

$$I_{ij} \neq 0 \qquad \Rightarrow \qquad {}^{i}\check{G}(g) = {}^{j}\check{G}(g), \text{ (within a similarity)}. \qquad (\mathbf{163})$$

Matrix elements of the Hamiltonian for fully symmetrized functions — Given two functions ψ^i_m and ψ^j_n, which belong to the m-th column of the representation ${}^{i}\check{G}(g)$ and to the n-th column of the representation ${}^{j}\check{G}(g)$, respectively, the *matrix element* of the Hamiltonian has the following orthogonality property:

$$H^{ij}_{mn} = \int (\psi^i_m)^*\, \mathrm{H}\, \psi^j_n\, \mathrm{d}\tau =_{\text{def}} \langle \psi^i_m \mid \mathrm{H} \mid \psi^j_n\rangle \qquad (\mathbf{164})$$

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$$= \langle W^i_{mp}\, \phi \mid W^j_{np}\, \phi\rangle = \langle \phi \mid \mathrm{H} \mid (W^i_{mp})^{\dagger}\, W^j_{np}\, \phi\rangle \qquad (\mathbf{165})$$

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$$= \langle \phi \mid \mathrm{H} \mid W^i_{pm}\, W^j_{np}\, \phi\rangle = \langle \phi \mid \mathrm{H} \mid W^i_{pp}\, \phi\rangle\, \delta_{ij}\, \delta_{mn} \qquad (\mathbf{166})$$

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$$=_{\text{def}} H^i\, \delta_{ij}\, \delta_{mn}, \quad (H^i \text{ independent of } m). \qquad (\mathbf{167})$$

Orthogonality of basis functions	**167**	$J^{ij}_{mn} = \int (\psi^i_m)^* \, \psi^j_n \, \mathrm{d}\tau =_{\mathrm{def}} \langle \psi^i_m \mid \psi^j_n \rangle$ $= J^i \, \delta_{ij} \, \delta_{mn}, \quad (J^i$ independent of $m)$.	(**168**)

9 The Wigner–Eckart theorem

Objective		To obtain the form of the matrix elements when one of the functions belongs to a direct-product basis $\lvert ijmn\rangle = \lvert im\rangle \, \lvert jn\rangle$. (See **157**.)	
The theorem	**157**	$\big\langle \lvert ijmn\rangle \big\rvert = \big\langle \lvert IUP\rangle \big\rvert \; {}^{ij}\langle mn \mid IUP\rangle^\dagger$	(**169**)
	169	$\langle I'P' \mid ijmn\rangle = \sum_{IUP} \langle I'P' \mid IUP\rangle \; {}^{ij}\langle mn \mid IUP\rangle^\dagger$	(**170**)
	168|**170**	$= \sum_U \langle I'P' \mid IUP\rangle \; {}^{ij}\langle mn \mid IUP\rangle^\dagger \, \delta_{I'I} \, \delta_{P'P}$	(**171**)
	168|**171**	$\langle IP \mid ijmn\rangle =_{\mathrm{def}} \sum_U \langle I \parallel IU\rangle \; {}^{ij}\langle mn \mid IUP\rangle^\dagger$.	(**172**)
Reduced matrix element		It is the term $\langle I \parallel IU\rangle$, which is independent of P.	(**173**)
Importance		In simple-reducible groups (see **100**) the summation over U disappears, in which case the ratio of two matrix elements belonging to the same irreducible representation I is provided by that of the Clebsch–Gordan coefficients. For a more specialized form of the theorem see Condon and Odabaşı (1980) or Messiah (1961).	

10 Subduced and induced representations

Subduced representations (descent of symmetry)

Definition — Given G, $H \subset G$, $\check{G} = \{\check{G}(g)\}$, $\forall g \in G$, the subduced representation or restriction of G down to H is the set $\{\check{G}(g)\}$, $\forall g \in H$. (**174**)

Warnings — If $\check{G}$ is irreducible it does not follow that the subduced representation is irreducible.
If the subduced representation is irreducible it does not follow that it is always identical with one of the irreducible representations tabulated for H. (A similarity might be entailed.)
For spinor representations (half-integral angular momenta) subduction might fail in the sense that the subduced representation does not satisfy the conservation of the characters as class functions.

Induced representations

Definition — Given $H \subset G$, and a representation $\check{H}(h)$ it is possible under certain conditions to construct a representation $\check{G}(g)$ starting from the matrices $\check{H}(h)$. This is called an induced representation. (**175**)

Bibliographical note

Most of the results quoted in this chapter may be found in all books on group theory, in particular Tinkham (1964) and Jansen and Boon (1967). A careful discussion of function-space operators is given in Wigner (1959). The distinction about the transformation properties of the independent variables x, y, z, the dependent variables x, y, z, and the unit vectors **i**, **j**, **k** can be studied in Altmann (1986). A complete discussion of projection operators and Clebsch–Gordan coefficients may be found in Altmann (1989*b*). A proof of the internal reduction method is given in Altmann (1962). The formula quoted for the calculation of the Clebsch–Gordan coefficients is due to Dirl (1979). A full discussion of induced representations is given in Altmann (1977). A careful discussion of phase factors for the Clebsch–Gordan matrix is given by König and Kremer (1973) and Butler (1981).

3

Parametrization of symmetry operations

1 Axes and general definitions

x, **y**, **z**	Axes fixed in space.	(1)
i, **j**, **k**	Axes fixed in the system. They coincide with **x**, **y**, **z** for the identity operation as performed on the system.	(2)
r	Position vector fixed in the system. (Its components in **i**, **j**, **k** do not change.)	(3)
Operation, g	Transforms **i**, **j**, **k** into **i**′, **j**′, **k**′ or **r** into **r**′.	(4)
Inverse, g^{-1}	Transforms **i**′, **j**′, **k**′ as defined for g into **i**, **j**, **k.**	(5)
Active or passive	All symmetry operations are treated as **active** in these tables, that is they refer to transformations with respect to the fixed axes **x**, **y**, **z**. (See **2**.34.)	(6)

2 Parametrization of proper rotations

Euler angles

$R(\alpha\beta\gamma)$	A rotation of **i**, **j**, **k** by γ around **z**, followed by a rotation of the transformed **i**, **j**, **k** by β about **y**, followed by a rotation of the transformed **i**, **j**, **k** by α about **z**. These combined operations take the original **i**, **j**, **k** into **i**′, **j**′, **k**′.	(7)
	Notice: γ first angle, β second angle, α third angle.	
Ranges	$-\pi < \gamma \le \pi, \qquad 0 \le \beta \le \pi, \qquad -\pi < \alpha \le \pi.$	(8)
Inverse $R(\alpha\beta\gamma)^{-1}$	$R(\alpha\beta\gamma)^{-1} = R(-\gamma \pm \pi, \beta, -\alpha \pm \pi).$	(9)
	Choose the $\pm$ signs so that the corresponding angles are in range.	
Ambiguities	$R(\alpha 0 \gamma) = R(\alpha + \gamma, 0, 0) = R(0, 0, \alpha + \gamma).$	(10)
	$R(\alpha\pi\gamma) = R(\alpha + \omega, \pi, \gamma + \omega)$, for an arbitrary ω, subject to range.	(11)
	The two cases above affect all Euler angles for all operations in all cyclic and dihedral groups. See also (67).	

Angle and axis of rotation

Axis	Place the object at the centre of a unit sphere. (Sphere of unit radius.) Assume that any rotation rotates the sphere solidly with the object, leaving fixed the centre of the sphere. The rotation axis is the diameter of the sphere which is left invariant under the rotation.	(12)
Positive and negative rotations	Except for the identity and binary rotations (rotations by π), all the proper rotations appear in pairs, one member of the pair being positive (counter-clockwise when looking from outside the sphere) and the other negative (clockwise when looking from outside the sphere). Notice that this distinction is not precise and that the conventions that follow tighten it up to avoid positive and negative rotations been mixed up.	(13)

Poles $\mathbf{n}$ — Each rotation of a group is mapped onto a unique position vector $\mathbf{n}$ on the unit sphere as follows. Given the axis common to the two rotations by $\pm\phi$, the end of this axis, called *the position vector* $\mathbf{n}$ *of each pole*, is assigned to each of these two rotations so that when viewing the head of this vector from outside the sphere the rotation is seen as counter-clockwise. (14)

Antipoles — Given the pole $\mathbf{n}$ for the rotation around an axis by $+\phi$, its antipole $-\mathbf{n}$ is the pole of the rotation by $-\phi$. **Notice, however, that the distinction between $+\phi$ and $-\phi$ is made only in the name of the operation. Both rotations are positive rotations by ϕ, the one labelled $+\phi$ around the pole $\mathbf{n}$ and the one labelled $-\phi$ around the pole $-\mathbf{n}$.** (15)

Binary rotations — The rotation by $-\pi$ is identical with the rotation by π and only one end of the rotation axis must be chosen conventionally as the pole, its other end or antipole being discarded. (16)

Identity — No pole is assigned to the identity. (But see 24 and 26.) (17)

Conjugate poles — If $\mathbf{n}_{g_i}$ is the pole of g_i, the point $g\mathbf{n}_{g_i}$, where g is a rotation of the group, is the pole of the conjugate operation of g_i by g, $\mathbf{n}_{gg_ig^{-1}}$. Two poles so related are called conjugate. (18)

Rules for choosing a set of poles as used in the tables

Rule 1. Conjugate poles — All poles of operations of a class must be transformed one into another by the group operations. **This is not trivial: if g and g' are in the same class, operations of the group might either leave $\mathbf{n}_g$ invariant or transform it into the antipole of g'.** (19)

Rule 2. Subgroups — If the group treated contains a subgroup and it is required that the representations of the group subduce properly to those of the subgroup, then the choice of poles must be made in such a way that Rule 1 is still valid when the group operations used in order to transform the poles are only operations of the subgroup. (20)

Warning — The choice of a set of poles for a group is conventional but not arbitrary. The rules given are necessary and sufficient to ensure that the characters are constant over a class of the group and remain so for the subgroup, a property that, although always true for ordinary group representations is not otherwise satisfied for spinor representations (half-integral angular momentum). (21)

The parameters ϕ, $\mathbf{n}$, and $\phi\mathbf{n}$

Rotation angle ϕ — As follows from (15), it is always positive (counter-clockwise) in the range $0 \le \phi \le \pi$. (22)

Rotation pole $\mathbf{n}$ *(Rotation axis)* — (i) For all proper rotations except the identity it is the vector $\mathbf{n}$ chosen in accordance with (14), (15), (16), and (19) to (21). (23)

(ii) For the identity it is the null vector. (24)

$R(\phi\mathbf{n})$ — A rotation by the angle ϕ about the pole $\mathbf{n}$. (25)

Vector parameter; the identity — ϕ and $\mathbf{n}$ are not, considered as a pair of parameters, good parameters for the rotation group, because any element of the infinite set $\{0, \mathbf{n}\}$, for all $\mathbf{n}$, maps the identity, thus breaking the required one-to-one property of the mapping. The single vector $\phi\mathbf{n}$, of modulus $|\phi\mathbf{n}| = \phi$ is instead a good parameter, since in this system the identity is mapped by the null vector only. This is the reason for the choice given above for the vector $\mathbf{n}$ for the identity. (26)

$R(\phi\mathbf{n})$	The symbol $R(\phi\mathbf{n})$ will when necessary be used in the present sense, $\phi\mathbf{n}$ indicating the vector parameter.	(27)

Quaternion (Euler–Rodrigues) parameters λ, $\mathbf{\Lambda}$

Requirements	A set of poles (see 13 to 17 and 21) and values of ϕ and $\mathbf{n}$ (see 22 to 24) must first be determined.	
λ	$\cos\frac{\phi}{2}$.	(28)
$\mathbf{\Lambda}$	$\sin\frac{\phi}{2}\,\mathbf{n}$.	(29)
$[\![\lambda, \mathbf{\Lambda}]\!]$	The quaternion $[\![\cos\frac{\phi}{2}, \sin\frac{\phi}{2}\,\mathbf{n}]\!]$. This quaternion corresponds always to a unique proper rotation g_i of the group but, when single rather than double groups are considered, then the operation g_i is parametrized by $\pm[\![\cos\frac{\phi}{2}, \sin\frac{\phi}{2}\,\mathbf{n}]\!]$. The parameters listed in the tables are always given for the positive sign in this expression.	(30)
Multiplication rule	If $g_i g_j = g_k$ and $g_i \mapsto [\![\lambda_i, \mathbf{\Lambda}_i]\!]$, $g_j \mapsto [\![\lambda_j, \mathbf{\Lambda}_j]\!]$, $g_k \mapsto [\![\lambda_k, \mathbf{\Lambda}_k]\!]$,	(31)
	then, $[\![\lambda_i, \mathbf{\Lambda}_i]\!][\![\lambda_j, \mathbf{\Lambda}_j]\!] = [\![\lambda_i\lambda_j - \mathbf{\Lambda}_i\cdot\mathbf{\Lambda}_j, \lambda_i\mathbf{\Lambda}_j + \lambda_j\mathbf{\Lambda}_i + \mathbf{\Lambda}_i\times\mathbf{\Lambda}_j]\!] = [\![\lambda_k, \mathbf{\Lambda}_k]\!]$.	(32)
Warning	In single groups, the quaternion on the right-hand side of the above multiplication rule may be multiplied by ± 1 without any change.	(33)
Note	**The only property of quaternions that the reader needs to use is that it is an object defined in terms of a (real) scalar and a vector with the multiplication rule (32).**	

Cayley–Klein parameters

Requirements	A set of poles (see 13 to 17 and 21) and values of ϕ and $\mathbf{n}$ (see 22 to 24) must first be determined.	
a	$\cos\frac{\phi}{2} - \mathrm{i}\, n_z \sin\frac{\phi}{2}$.	(34)
b	$-(n_y + \mathrm{i}\, n_x)\sin\frac{\phi}{2}$.	(35)

3 Parametrization of improper operations

Improper rotations	Written always as ig where g is a proper rotation. $ig = gi, \quad \forall g$.	(36)
Inversion	The inversion operator i is kept as a marker in the parameters of all improper rotations. In order to satisfy the commutation property above, the quaternion parameter for the inversion is $i\,[\![1, \mathbf{0}]\!]$. For the inversion operator $i^2 = E$, but for the marker i in the parameters the rule is $i^2 = 1$.	(37)
	Notice that no notational distinction is made between the operator i and the marker i. **When acting with the symmetry operations on polar vectors, i changes the signs of their components.**	
Warning	**For simplicity, the marker i is omitted from the tables but it is implicitly applied to all the parameters of all improper rotations.**	(38)
Reflections σ (mirrors)	$\sigma = i\,C_2$, $C_2 \perp \sigma$ or $\sigma = i\,R(\pi, \mathbf{n})$, $\mathbf{n} \perp \sigma$.	(39)
	Notice that in the tables the parameter for σ is listed as the parameter for C_2, the marker i being kept implicit.	
Rotoreflections S_m	$S_m = R(\frac{2\pi}{m}\mathbf{n})\sigma, \quad \sigma \perp \mathbf{n}$.	(40)
	$= R(\frac{2\pi}{m}\mathbf{n})\, i\, R(\pi\mathbf{n}) = i\, R(\frac{2\pi}{m} + \pi, \mathbf{n}) = i\, R\left(-(\pi - \frac{2\pi}{m}), \mathbf{n}\right)$.	(41)
	Notice that in the tables the parameter for S_m is given as the parameter for $R\left(\pi - \frac{2\pi}{m}, -\mathbf{n}\right)$, the marker i being kept implicit.	(42)

Multiplication rule It is the same as for proper rotations, except that the markers, which can be commuted and grouped as required, must be either kept, or multiplied on using the rule $i^2 = 1$. Example:
If $g_i g_j = g_k$ and $g_i \mapsto [\![\lambda_i, \mathbf{\Lambda}_i]\!]$, $g_j \mapsto i\,[\![\lambda_j, \mathbf{\Lambda}_j]\!]$, $g_k \mapsto i\,[\![\lambda_k, \mathbf{\Lambda}_k]\!]$,
then,

$$[\![\lambda_i, \mathbf{\Lambda}_i]\!]\, i\,[\![\lambda_j, \mathbf{\Lambda}_j]\!] = i\,[\![\lambda_i\lambda_j - \mathbf{\Lambda}_i \cdot \mathbf{\Lambda}_j, \lambda_i\mathbf{\Lambda}_j + \lambda_j\mathbf{\Lambda}_i + \mathbf{\Lambda}_i \times \mathbf{\Lambda}_j]\!] \quad (43)$$

$$= i\,[\![\lambda_k, \mathbf{\Lambda}_k]\!]. \quad (44)$$

Warning In single groups, the quaternion on the right-hand side of the above multiplication rule may be multiplied by ± 1 without any change.

4 Parametrization of double-group operations

Requirements Group $G = \{g_i\}$, set of quaternion parameters $[\![\lambda_i, \mathbf{\Lambda}_i]\!]$, for all $g_i \in G$. (See 28 to 30.)

Possible parametrizations Only the quaternion or Cayley-Klein parametrizations.

$\widetilde{E}$ $R(2\pi\mathbf{n})$, any $\mathbf{n}$. Take $R(2\pi\mathbf{0})$, quaternion parameter $[\![-1, \mathbf{0}]\!]$. (45)

$\widetilde{E}g_i = g_i\widetilde{E}$, $\forall g_i$. (46)

$\widetilde{g}_i$ $\widetilde{E}g_i$. (47)

$\widetilde{G}$ $\{g_i\} \oplus \{\widetilde{g}_i\}$. (48)

Parameter for g_i $[\![\lambda_i, \mathbf{\Lambda}_i]\!]$. (49)

Parameter for $\widetilde{g}_i$ Given the parameter for g_i as $+[\![\lambda_i, \mathbf{\Lambda}_i]\!]$, it is
$[\![-1, \mathbf{0}]\!][\![\lambda_i, \mathbf{\Lambda}_i]\!] = [\![-\lambda_i, -\mathbf{\Lambda}_i]\!]$. (50)
(See the multiplication rule 32 for the quaternion parameters.)
Notice that these parameters are not listed in the tables. They can most simply be obtained by changing the sign of the single-group parameters.

Multiplication rules The same as for proper rotations, where the tildes can be applied to any of the factors or may appear in the result through the quaternion of a $\widetilde{g}$ operation. The quaternions for the operations (with or without tildes) corresponding to the subscripts i and j, respectively, are multiplied in the normal manner:

$$[\![\lambda_i, \mathbf{\Lambda}_i]\!][\![\lambda_j, \mathbf{\Lambda}_j]\!] = [\![\lambda_i\lambda_j - \mathbf{\Lambda}_i \cdot \mathbf{\Lambda}_j, \lambda_i\mathbf{\Lambda}_j + \lambda_j\mathbf{\Lambda}_i + \mathbf{\Lambda}_i \times \mathbf{\Lambda}_j]\!] = [\![\lambda_k, \mathbf{\Lambda}_k]\!]. \quad (51)$$

The quaternion on the right-hand side gives the result of the product, the sign of the quaternion components revealing whether it is an ordinary or a tilde operation. Notice that the multiplication rules (37) for the inversion and its marker still obtain. (52)

Warning **See § 10–1 for the notational distinctions necessary for the multiplication rules of groups and their double groups.**

5 Calculation of the Euler angles

Objective To calculate the Euler angles from the angle and axis of rotation.

β $\cos\beta = 1 - 2(n_x^2 + n_y^2)\sin^2\frac{\phi}{2}, \qquad \sin\beta = +(1 - \cos^2\beta)^{1/2}$. (53)

α $\tan\alpha = (-n_x\sin\phi + 2n_yn_z\sin^2\frac{\phi}{2})\,(n_y\sin\phi + 2n_zn_x\sin^2\frac{\phi}{2})^{-1}$, (54)

$\sin\alpha = (-n_x\sin\phi + 2n_yn_z\sin^2\frac{\phi}{2})\,(\sin\beta)^{-1}$, (55)

$\cos\alpha = (n_y\sin\phi + 2n_zn_x\sin^2\frac{\phi}{2})\,(\sin\beta)^{-1}$. (56)

γ $\tan\gamma = (n_x\sin\phi + 2n_yn_z\sin^2\frac{\phi}{2})\,(n_y\sin\phi - 2n_zn_x\sin^2\frac{\phi}{2})^{-1}$, (57)

$\sin\gamma = (n_x\sin\phi + 2n_yn_z\sin^2\frac{\phi}{2})\,(\sin\beta)^{-1}$, (58)

$\cos\gamma = (n_y\sin\phi - 2n_zn_x\sin^2\frac{\phi}{2})\,(\sin\beta)^{-1}$. (59)

Special cases	The following special cases are important.	
$\beta = 0$, $n_z = +1$	$\alpha = 0$, $\gamma = \phi$.	(60)
$\beta = 0$, $n_z = -1$	$\alpha = 0$, $\gamma = -\phi$.	(61)
$\beta = \pi$, $\phi = \pi$	$\gamma = 2\tan^{-1}(n_x/n_y)$.	(62)
Note	The Euler angles are undetermined in the cases shown above. The results (60) to (62) are obtained on using (10) and (11) sensibly and are the formulae used in constructing the tables.	

6 Calculation of the angle and axis of rotation from the Euler angles

Comment	The following expressions will rarely be needed.	
ϕ	$\cos\frac{\phi}{2} = \cos\frac{\beta}{2}\cos\frac{1}{2}(\alpha+\gamma), \qquad \sin\frac{\phi}{2} = \pm(1-\cos^2\frac{\phi}{2})^{1/2}$.	(63)
n_z	$n_z = \ \ (\sin\frac{\phi}{2})^{-1}\cos\frac{\beta}{2}\sin\frac{1}{2}(\alpha+\gamma)$.	(64)
n_x	$n_x = -(\sin\frac{\phi}{2})^{-1}\sin\frac{\beta}{2}\sin\frac{1}{2}(\alpha-\gamma)$.	(65)
n_y	$n_y = \ \ (\sin\frac{\phi}{2})^{-1}\sin\frac{\beta}{2}\cos\frac{1}{2}(\alpha-\gamma)$.	(66)
Note	Notice that, given the Euler angles, the sign of the vector $\mathbf{n}$ remains undetermined, because of the $\pm$ sign in $\sin\frac{\phi}{2}$.	(67)

Bibliographical note

The definition of the Euler angles used here agrees exactly with the notation of Rose (1957), Brink and Satchler (1968), Biedenharn and Louck (1981), Butler (1981), and, with minor changes of notation, Fano and Racah (1959) and Messiah (1961). Further details of the definitions and conventions used in this chapter may be found in Altmann (1986). Poles are defined in that book as points on the unit sphere rather than as position vectors of it, but this does not entail any basic difference.

4

Symmetry operations: notation and properties

The principles which have guided the choice of the notation used in the tables and described below are as follows:

(i) To ensure agreement in notation, for the groups already found in published tables, with the notations most commonly used in the literature.

(ii) To emphasize whenever possible the importance of the binary rotations about the axes **x** and **y**, since, for the representations for $j = \frac{1}{2}$ they are, except for a numerical factor, the Pauli matrices.

(iii) To minimize the number of changes in notation required when going from a group to its subgroups. When different operations in the same class have to be distinguished by means of a numerical subscript, the actual choice of this subscript (which may be read from the stereographic projections) has been done with this purpose in mind.

1 Key to the symbols for symmetry operations

Basic notation

E	Identity.	(1)
i	Inversion.	(2)
C_2	Binary rotation, always understood as a rotation by π about the conventionally defined pole. (See **3**.16, **3**.19, **3**.21 for the rules used for this conventional choice.)	(3)
C_n^+, C_n^-	Rotations by $2\pi/n$ also called proper rotations. Given an axis of rotation (diameter in a unit sphere) one end of it is chosen conventionally as the pole and the other end is the antipole. (See **3**.14, **3**.19, **3**.21 for the rules used for this conventional choice.) Positive rotations are seen as counter-clockwise when looking at the sphere from outside the pole. Negative rotations are seen as counter-clockwise when looking at the sphere from outside the antipole.	(4)
S_n^+, S_n^-	Rotoreflections by $2\pi/n$. They are always given as a rotation C_n^+, C_n^- followed or preceded with a reflection on a plane perpendicular to the axis of rotation.	(5)
σ	Reflection plane, always treated in this book as the product iC_2, where the binary rotation is normal to the reflection plane and it is given a conventional pole. (See **3**.39, **3**.19, **3**.21 for the rules used for this conventional choice.) Also called a *mirror*.	(6)

Embellishments, subscripts, and superscripts

C_2 , C_n^+, C_n^- , S_n^+, S_n^-	Whenever the subscript is single and no embellishments further than those shown here are used, the rotation or rotoreflection is about the **z** axis, the poles for C_n^+ and C_n^- being along $+$**z** and $-$**z**, respectively.	(7)
$C_n^{m\pm}$, $S_n^{m\pm}$	Rotation and rotoreflection, respectively, by $\pm 2\pi m/n$.	(8)
C_{2x}, C_{2y}, C_{2z}	Binary axes along the **x**, **y**, and **z** axes respectively, always right-handed.	(9)

C'_{2p}	One class of binary axes, designated by different alphanumerical values of p. If p is a numerical index they are all perpendicular to the principal axis (21), that is they lie on the $\mathbf{x}, \mathbf{y}$ plane. C'_{21} is always chosen along the $\mathbf{x}$ axis.	(10)
C''_{2p}	Another class of m binary axes designated by different numerical values of p and perpendicular to the principal axis (21). If the binary rotation about the $\mathbf{y}$ axis belongs to this class it is always labelled C''_{21}.	(11)
σ_x, σ_y, σ_z	Reflection planes perpendicular to the axes $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$, respectively.	(12)
σ_h	Reflection plane perpendicular to the $\mathbf{z}$ axis (alternatively labelled σ_z if σ_x, σ_y belong to the group).	(13)
σ_{vp}	One class of m reflection planes that contain the $\mathbf{z}$ axis, designated by different numerical values of p.	(14)
	If the principal axis (21) is of order n, these planes are set as follows:	
	$\sigma_{vp} \perp C'_{2p}$, for $n = 4\nu$, ν integral.	(15)
	$\sigma_{vp} \perp C''_{2p}$, for $n = 4\nu + r$, ν integral, $r = 1, 2, 3$.	(16)
σ_{dp}	Either another class of m reflection planes that contain the $\mathbf{z}$ axis, designated by different numerical values of p, or the only class of such planes if they also intersect the angle between two binary axes perpendicular to $\mathbf{z}$.	(17)
	If the principal axis (21) is of order n, these planes are set as follows:	
	$\sigma_{dp} \perp C'_{2p}$, for $n = 4\nu + r$, ν integral, $r = 1, 2, 3$.	(18)
	$\sigma_{dp} \perp C''_{2p}$, for $n = 4\nu$, ν integral.	(19)
Subscripts h and v	The principal axis is always imagined to stand vertically along the $\mathbf{z}$ axis. With respect to this orientation h and v always stand for horizontal and vertical respectively.	(20)

2 Special rotations and rotoreflections

Principal axis	It is the axis of rotation or rotoreflection about the $\mathbf{z}$ axis. Except in the tetrahedral and icosahedral groups, it is always the axis of highest order in the group.	(21)
	Note that the use of this expression in the literature is not always identical to the one given here.	
Binary axis	An axis of rotation by π. It is always its own inverse, except when double-group notation is used.	(22)
Bilateral axis	An axis of rotation with a binary axis perpendicular to it or a mirror that contains it. The positive and negative rotations about a bilateral axis always belong in the same class.	(23)
Bilateral-binary	A binary axis that is bilateral.	(24)
Binary rotoreflection	It is identical with the inversion.	(25)
	The word rotoreflection in this book is always used excluding the case of binary rotoreflections.	

3 Commutation of symmetry operations

Two rotations	They commute if and only if they are either coaxial or bilateral-binary.	(26)
Two symmetry planes	They commute if and only if they are coincident or perpendicular.	(27)
A rotation and a symmetry plane	They commute if and only if either they are perpendicular or the rotation axis is binary and it lies in the symmetry plane.	(28)

Inversion	Commutes with all symmetry operations.	(29)
Two rotoreflections	They commute if and only if they are coaxial.	(30)
A rotoreflection and a symmetry plane	They commute if and only if they are perpendicular. S_2 is excluded from this rule. (See 33 below.)	(31)

4 Special relations for symmetry operations

$i\,C_2 = C_2\,i$	It equals σ_h. (See 6, 7, and 13.)	(32)
S_2	It equals i. (S_2 is never used in this book.)	(33)
$C_n^{m\pm}\,i = i\,C_n^{m\pm}$	It equals $\sigma_h\,C_{2n}^{(n-2m)\mp} = S_{2n}^{(n-2m)\mp}$, subject to division by a common factor of the upper and lower indices.	(34)
$S_n^{m\pm}$	It equals $i\,C_{2n}^{(n-2m)\mp} = C_{2n}^{(n-2m)\mp}\,i$, subject to division by a common factor of the upper and lower indices.	(35)

5

Notation for point groups, single and double

1 Cyclic, dihedral, and related groups

Group	Characteristic symmetry elements	
$\mathbf{C}_n$	n-fold axis C_n (see **4**.4) only. **Cyclic group.**	(1)
$\mathbf{S}_n$	n-fold alternating axis S_n (see **4**.5) only. **Cyclic group.**	(2)
$\mathbf{C}_{nh}$	n-fold axis C_n plus a reflection plane σ_h (see **4**.13) perpendicular to it.	(3)
$\mathbf{C}_{nv}$	n-fold axis C_n plus n reflection planes σ_v (see **4**.14 to **4**.16) through it.	(4)
$\mathbf{C}_{\infty v}$	Continuous rotation axis plus a continuous infinite number of reflection planes σ_v through it.	(5)
$\mathbf{D}_n$	n-fold axis C_n plus n binary axes C'_{2j}, C''_{2j} (see **4**.10, **4**.11) perpendicular to it. **Dihedral group.**	(6)
$\mathbf{D}_{nd}$	As for $\mathbf{D}_n$, plus reflection planes σ_d (see **4**.17 to **4**.19) bisecting the angles between the binary axes.	(7)
$\mathbf{D}_{nh}$	As for $\mathbf{D}_n$, plus a reflection plane σ_h (see **4**.13) perpendicular to C_n.	(8)
$\mathbf{D}_{\infty h}$	Continuous rotation axis plus a continuous infinite number of binary axes perpendicular to it, plus a continuous infinite number of reflection planes σ_v through it. The reflection plane σ_h appears as iC_2, where C_2 belongs to the continuous rotation axis.	(9)
$\mathbf{C}_i$	This is the notation used in the tables for $\mathbf{S}_2$ (identity plus i).	(10)
$\mathbf{C}_s$	This is the notation used in the tables for $\mathbf{C}_{1h}$ (identity plus σ_h).	(11)
$\mathbf{C}_{ni}$	n-fold axis C_n, plus the inversion i.	(12)
	Not used in the tables. For n odd, it is given as $\mathbf{S}_n$. For n even, it is given as $\mathbf{C}_{nh}$. Rule: first priority is given to the cyclic group notation. Second priority is given to σ_h.	(13)

2 Cubic groups

Group	Characteristic symmetry elements	
$\mathbf{O}$	Three C_4 (mutually perpendicular), four C_3 which permute the poles of the C_4 amongst themselves.	(14)
$\mathbf{T}$	Three C_2 (mutually perpendicular), four C_3 which permute the poles of the C_2 amongst themselves.	(15)
$\mathbf{O}_h$	Like $\mathbf{O}$, with the inversion. The reflection plane σ_h appears as iC_2, where C_2 belongs to the principal axis $\mathbf{z}$.	(16)
$\mathbf{T}_h$	Like $\mathbf{T}$, with the inversion. The reflection plane σ_h appears as iC_2, where C_2 belongs to the principal axis $\mathbf{z}$.	(17)
$\mathbf{T}_d$	Like $\mathbf{T}$, but the three C_2 are each in the subgroup of an S_4 axis.	(18)

3 Icosahedral groups

Group	Characteristic symmetry elements	
I	Six C_5, ten C_3, fifteen C_2.	(19)
$\mathbf{I}_h$	Like **I**, with the inversion.	(20)

4 Double groups

$\widetilde{\mathbf{G}}$	If **G** is any of the groups above, $\widetilde{\mathbf{G}}$ is its double group.	(21)

5 The Hermann–Mauguin or international notation

n	The principal axis (**z** axis) is a rotation axis of order n.	(22)
$\bar{n}$	The principal axis (**z** axis) is a rotoinversion axis of order n. (Rotation of order n followed or preceded by the inversion.)	(23)
$n2$ *or* $\bar{n}2$	A binary axis perpendicular to the principal axis.	(24)
nm *or* $\bar{n}m$	A mirror (reflection plane) parallel to the principal axis.	(25)
$\frac{n}{m}$ *or* $\frac{\bar{n}}{m}$	A mirror perpendicular to the principal axis.	(26)
Further entries	They refer to secondary axes.	

Bibliographical note

The notation used above is the standard Schönflies notation. For complete details of the Hermann–Mauguin international notation consult the *International tables for crystallography* (1989) where a description of the short form of this notation may also be found.

6

Derivation of the proper and improper point groups

1 Definitions for proper point groups

Proper point groups	Contain the identity and proper rotations C_n (rotations by $\frac{2\pi}{n}$) only.	(1)
$G = \{g\}$	Proper point group of order $\|G\| = N$.	(2)
Pole $\mathbf{n}_g$	The position vector of the unit sphere left invariant by g and such that g is seen as counter-clockwise when looking at $\mathbf{n}_g$ from outside the sphere.	(3)
Antipole of $\mathbf{n}_g$	The pole of g^{-1} (but see 6 below).	(4)
Pole of identity	None.	(5)
Pole of binary g	In this case g^{-1} is identical with g so that the antipole does not correspond to a different operation (but see 13 below).	(6)
Cyclic group associated with g	If $g = C_n$, all the operations of $\mathbf{C}_n$ (cyclic group of order n) leave $\mathbf{n}_g$ and $\mathbf{n}_{g^{-1}}$ invariant.	(7)
Conjugate poles	The vector $g\mathbf{n}_{g_i}$, $g_i \in G$, is $\mathbf{n}_{gg_ig^{-1}}$, $\forall g \in G$. $\mathbf{n}_{g_i}$ and $\mathbf{n}_{gg_ig^{-1}}$ are called conjugate poles. This property is transitive.	(8)
Set of conjugate poles	Given $\mathbf{n}_{g_i}$, the set $\{g\mathbf{n}_{g_i}\}$, $\forall g \in G$, is such that any two poles of the set are conjugate amongst themselves. This is a set of conjugate poles. All the rotations of the set belong to the class $C(g_i)$ and are of order n_i.	(9)
ν_i	The set $\{g\mathbf{n}_{g_i}\}$, $\forall g \in G$ contains repetitions. ν_i is the number of *distinct* poles in the set. (But see 15 below.)	(10)
Disjointness	Two sets of conjugate poles are either identical or disjoint.	(11)
Partition of the poles of G; P	The set of all poles of G separates out into P disjoint sets.	(12)
Double counting of poles	Each rotation g, binaries included, will henceforth be assigned **two** poles, namely $\mathbf{n}_g$ and its antipole. This means that the poles at the ends of all axes will always be counted in the set $\{\mathbf{n}_g\}$, $\forall g \in G$, even for g binary. (When g is not binary, the antipole of $\mathbf{n}_g$ is the pole of g^{-1} and it always belongs to the set.) The identity has no poles.	(13)
Total number of poles, ν	This is the number of poles in the set $\{\mathbf{n}_g\}$ in (13). Because each rotation in G, except the identity, has now two poles, $\nu = 2(N-1)$. Not all of these are distinct because two rotations about the same axis share the same poles.	(14)
New ν_i	**The number ν_i will henceforth be used for the number of distinct poles in a conjugate set, with the antipoles of binary rotations counting as poles in that set.**	(15)
Obtention of ν_i	Call $\mathbf{C}_i$ the cyclic group of order n_i that contains g_i. Form the coset expansion $G = \sum_{r=1}^{p} \sigma_r \mathbf{C}_i$. The coset representatives σ_r all generate *distinct* poles $\sigma_r \mathbf{n}_{g_i}$, whence $p = \nu_i$. From the coset expansion, $\nu_i\|\mathbf{C}_i\| = \|G\| = N \quad \Rightarrow \quad \nu_i n_i = N \quad \Rightarrow \quad \nu_i = N/n_i$.	(16)

Number of rotations that leave invariant the set conjugate to $\mathbf{n}_{g_i}$

$\mathbf{n}_{g_i}$ is the pole of $\mathbf{C}_i$ of order n_i. Subtracting the identity, the number of rotations that leave this pole invariant is $n_i - 1$. There are ν_i **distinct** poles in the set, whence the number sought is
$\nu_i\,(n_i - 1) = N\,(1 - \frac{1}{n_i})$. (17)

Condition to determine P and the n_i

The total number ν of poles given in (14) is the number obtained in equation (17), added up over all the P systems of conjugated poles:
$2(N-1) = \sum_{i=1}^{P} N\,(1 - \frac{1}{n_i}) \quad \Rightarrow \quad \sum_{i=1}^{P} \frac{1}{n_i} = P - 2 + \frac{2}{N}$. (18)

Auxiliary condition

$N \geq 2$, $n_i \geq 2$ (otherwise G, or $\mathbf{C}_i \subset G$, contain only E, which gives no poles). Therefore $N \geq n_i \geq 2$. (19)

Determination of P

For $n_i \geq 2$ equation (18) gives $P \leq 4 - \frac{4}{N}$. For $N \geq n_i$ it gives $P \geq 2$. (20)

Values of P

From (20), $P = 2$ or $P = 3$. ($P = 0$ for $N = 1$ is also obviously possible.) (21)

2 Derivation of the proper point groups

Objective

To obtain from the conditions (18) to (21) all the possible proper point groups, determining for each of them the number of systems of conjugate poles and the order and number of poles in each system.

Description of Table **6**.1

The column headed P gives conditions (21) and the summation in the second column is (18). For $P = 0$, the entry in the column headed N follows. For $P = 2$, (18) is written in the second column and leads to the solutions for n_1 and n_2 listed. The entry in the column N follows from (19). For $P = 3$, see the note at the foot of the table, which leads to the conditions for n_1, n_2, and n_3 displayed as a subheading in the table. Given n_1 and n_2 from these conditions, (18) is written in the second column and, from this relation, N is derived.

The table

Table **6**.1

P	$\sum_{i=1}^{P}\frac{1}{n_i} = P - 2 + \frac{2}{N}$	n_1	n_2	n_3	N	$\nu_i = N/n_i$: ν_1	ν_2	ν_3	*Group*
0					1				$\mathbf{C}_1$
2	$\frac{1}{n_1} + \frac{1}{n_2} = \frac{2}{N}$	N	N		≥ 2	1	1		$\mathbf{C}_N$
		2	2 or 3	$\geq n_2$					
3	$\frac{1}{n_3} = \frac{2}{N}$	2	2	$\frac{N}{2}$	even	$\frac{N}{2}$	$\frac{N}{2}$	2	$\mathbf{D}_{N/2}$
3	$\frac{1}{n_3} = \frac{1}{6} + \frac{2}{N}$	2	3	3	12	6	4	4	$\mathbf{T}$
3	$\frac{1}{n_3} = \frac{1}{6} + \frac{2}{N}$	2	3	4	24	12	8	6	$\mathbf{O}$
3	$\frac{1}{n_3} = \frac{1}{6} + \frac{2}{N}$	2	3	5	60	30	20	12	$\mathbf{I}$

n_i: order of a rotation, angle $\frac{2\pi}{n_i}$. (See 16.)
ν_i: number of poles in a system of conjugate poles of order n_i. (See 15.) If the axis is bilateral (see **4**.23) half of these poles will be antipoles. For binary rotations the set may consist entirely of poles or of antipoles but for bilateral-binaries (see **4**.24) both types will appear in equal numbers.
N: group order.
P: total number of systems of conjugated poles in the group. $P = 2$ or $P = 3$. (See 21.)
Auxiliary condition:
$N \geq n_i \geq 2$. (See 19.)
The case $P = 3$ *(see* 18*):*
$\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} = 1 + \frac{2}{N} \Rightarrow n_1 = n_2 = n_3 = 3$ *(impossible)* $\Rightarrow$
choose $n_1 = 2$, $n_2 \geq n_1$, $n_3 \geq n_2$ *(conventionally)* $\Rightarrow$
$\frac{1}{n_2} + \frac{1}{n_3} = \frac{1}{2} + \frac{2}{N} \quad \Rightarrow \quad n_2 = n_3 = 4$ *(impossible)* $\Rightarrow \quad n_2 = 2$ *or* 3.

3 Description of the proper point groups

Cyclic groups $\mathbf{C}_n$ (order $n \geq 1$)

Number of systems of conjugate poles	Two (except for $n = 1$, when this number is zero).	(22)
First system	One pole of order n, corresponding to the rotation C_n^+.	(23)
Second system	One pole of order n, corresponding to the rotation C_n^-. These two systems are not conjugate because the only rotation that will take a pole into its antipole is a binary rotation perpendicular to the axis, which does not belong to the group.	(24)
Invariant subgroups	For n even, $\mathbf{C}_{n/2}$. (Rule used: all the poles of an invariant subgroup must be interchanged by the operations of the group. The operations of $\mathbf{C}_n$ leave identically invariant the poles of $\mathbf{C}_{n/2}$.)	(25)

Dihedral groups $\mathbf{D}_n$ (order $2n$, $n \geq 2$)

Number of systems of conjugate poles	Three.	(26)
First system	n poles of order two (binary) perpendicular to C_n.	(27)
Second system	n poles of order two (binary) perpendicular to C_n.	(28)
Third system	Two poles of order n. They correspond to the rotations C_n^+ and C_n^-, in the same system because they are conjugated by any of the binaries.	(29)
n odd	The first and second sets correspond to poles and antipoles, respectively. They are not conjugate because C_2 does not belong to the subgroup $\mathbf{C}_n$.	(30)
n even	The first and second sets correspond to binaries C_2' and C_2'', respectively, separated by π/n. (These two sets are not conjugate because the rotation by π/n does not belong to the subgroup $\mathbf{C}_n$.) Each set contains $n/2$ poles and $n/2$ antipoles.	(31)
Invariant subgroups	$\mathbf{C}_n$ and, for n even, $\mathbf{D}_{n/2}$. If $n/2$ is even the process can be continued.	(32)

Tetrahedral group $\mathbf{T}$ (order 12)

Number of systems of conjugate poles	Three.	(33)
First system	Six binary poles (at the centres of the six tetrahedron edges) corresponding to three orthogonal axes.	(34)
Second system	Four poles of rotations C_3^+. Because they must interchange the binaries (which otherwise would not all belong to one conjugate set) they must be at the four vertices of the tetrahedron.	(35)
Third system	Four poles of rotations C_3^-, antipoles of the above, at the centres of the four tetrahedral faces. They are not in the same set as the C_3^+, because there are no binary axes perpendicular to the three-fold axes.	(36)
Invariant subgroups	$\mathbf{D}_2$. (Reason: the six binary poles, which correspond to this subgroup, are interchanged by all the other operations of the group.)	(37)

Octahedral group $\mathbf{O}$ (order 24)

Number of systems of conjugate poles	Three.	(38)
First system	Six four-fold poles corresponding to three orthogonal axes. (Centres of the six cube faces).	(39)

Second system	Eight poles of rotations C_3^+ and C_3^- at the eight vertices of the cube. The corresponding rotations interchange the above poles.	(40)
Third system	Twelve poles of six binary rotations. These poles are at the centres of the twelve cubic edges. The corresponding rotations interchange C_3^+ and C_3^-, thus justifying their belonging to the same system.	(41)
Invariant subgroups	$\mathbf{T}$ (because it is of index two) and $\mathbf{D}_2$ (for the same reason as given for $\mathbf{T}$).	(42)

Icosahedral group $\mathbf{I}$ (order 60)

Number of systems of conjugate poles	Three.	(43)
First system	Twelve five-fold poles at the twelve vertices of the icosahedron (centres of the twelve faces of the dodecahedron).	(44)
Second system	Twenty three-fold poles at the centres of the twenty triangular faces of the icosahedron (twenty vertices of the dodecahedron).	(45)
Third system	Thirty binary poles at the mid-points of the thirty edges of the icosahedron (thirty edges of the dodecahedron).	(46)
Invariant subgroups	None, the icosahedral group is the only simple proper point group. (See **2**.18.)	(47)

4 Improper groups: general structure

Definitions	Improper group G of proper operations $\{h\}$ and improper operations $\{u\}$. The product of two improper operations is always proper.	(48)
$\{h\} = H \subset G$	Because $h_m\, h_n = h_p$.	(49)
$u = s\,h$, $\forall u$, *one* $s \in \{u\}$, *some* h	$s \in \{u\} \;\Rightarrow\; s^{-1} \in \{u\} \;\Rightarrow\; s^{-1}\,u \in H \;\Rightarrow\; u = s\,h, \forall u \;\Rightarrow\; \{u\} = s\,H.$	(50)
Halving subgroup	The set of proper operations of G is a group of order $\lvert H\rvert = \lvert G\rvert/2$ (index 2) and therefore invariant. (See **2**.21.)	(51)
General structure	$G = H \oplus sH$, for some improper operation $s \in G$. $H \triangleleft G$, $\lvert H\rvert = \lvert G\rvert/2$.	(52)
	$G = H \oplus ih'H$, for some proper operation h' not necessarily in H.	(53)
Classification of improper groups	(i) $h' \in H \quad\Rightarrow\quad G = H \oplus iH \quad\Rightarrow\quad i \in G.$	(54)
	(ii) $h' \notin H \quad\Rightarrow\quad G = H \oplus ih'H \quad\Rightarrow\quad i \notin G.$	(55)
Improper groups with inversion	54 $\quad G = H \oplus iH = H \otimes \mathbf{C}_i$, $\mathbf{C}_i = E \oplus i$.	(56)
Improper groups without inversion	55 $\quad G = H \oplus ih'H$, $h' \notin G$.	(57)
Generation of improper groups G' from a proper group G	(i) Groups with inversion: form $G' = G \otimes \mathbf{C}_i$, $\forall G$.	(58)
	(ii) Groups without inversion: find all halving subgroups H of G. Write $G = H \oplus h'H$, $h' \notin H$, h' proper, $h' \in G$. $H \triangleleft G$.	(59)
	Write $G' = H \oplus ih'H$. $H \triangleleft G$.	(60)
Possible semidirect product form of improper groups without inversion	If, in (60), $E \oplus ih' = S$ (a group), then $G' = H \circledwedge S$. $H \triangleleft G$.	(61)

5 Improper groups with inversion

Generated from cyclic groups $\mathbf{C}_n$

General form	$G' = \mathbf{C}_n \otimes \mathbf{C}_i$.	(62)

n even, $\sigma_h \in G'$	n even $\Rightarrow$ $C_2 \in \mathbf{C}_n$ $\Rightarrow$ $i\,C_2 = \sigma_h \in G'$ $\Rightarrow$ $\mathbf{C}_n \otimes \mathbf{C}_i = \mathbf{C}_{nh}$.	(63)
	List: $\mathbf{C}_{2h}$, $\mathbf{C}_{4h}$, $\mathbf{C}_{6h}$, $\mathbf{C}_{8h}$, $\mathbf{C}_{10h}$.	(64)
n odd	$C_n^+\,i = C_n^+\,C_2\,\sigma_h = C^+_{\frac{2\pi}{n}+\pi}\,\sigma_h = C^-_{\pi-\frac{2\pi}{n}}\,\sigma_h = C^-_{\frac{\pi}{n}(n-2)}\sigma_h$ $= (C_{2n}^-)^{n-2}\,\sigma_h = (S_{2n}^-)^{n-2}$.	(65)
	$G' = \mathbf{C}_n \otimes \mathbf{C}_i = \mathbf{S}_{2n}$.	(66)
	List: $\mathbf{S}_2 = \mathbf{C}_i$, $\mathbf{S}_6$, $\mathbf{S}_{10}$, $\mathbf{S}_{14}$, $\mathbf{S}_{18}$.	(67)

Generated from dihedral groups $\mathbf{D}_n$

General form	$G' = \mathbf{D}_n \otimes \mathbf{C}_i$.	(68)
n even, $\sigma_h \in G'$	n even $\Rightarrow$ $C_2 \in \mathbf{C}_n$ $\Rightarrow$ $i\,C_2 = \sigma_h \in G'$ $\Rightarrow$ $\mathbf{D}_n \otimes \mathbf{C}_i = \mathbf{D}_{nh}$.	(69)
	List: $\mathbf{D}_{2h}$, $\mathbf{D}_{4h}$, $\mathbf{D}_{6h}$, $\mathbf{D}_{8h}$, $\mathbf{D}_{10h}$.	(70)
n odd	$iC_2' = \sigma_d \in \mathbf{D}_n$ $\Rightarrow$ $\mathbf{D}_n \otimes \mathbf{C}_i = \mathbf{D}_{nd}$.	(71)
	List: $\mathbf{D}_{3d}$, $\mathbf{D}_{5d}$, $\mathbf{D}_{7d}$, $\mathbf{D}_{9d}$.	(72)

Generated from the cubic groups $\mathbf{O}$, $\mathbf{T}$

General form	$C_2 \in \mathbf{O}, \mathbf{T}$ $\Rightarrow$ $i\,C_2 = \sigma_h \in G'$.	(73)
From $\mathbf{O}$	$\mathbf{O} \otimes \mathbf{C}_i = \mathbf{O}_h$.	(74)
From $\mathbf{T}$	$\mathbf{T} \otimes \mathbf{C}_i = \mathbf{T}_h$.	(75)

Generated from the icosahedral group $\mathbf{I}$

General form	$C_2 \in \mathbf{I}$ $\Rightarrow$ $i\,C_2 = \sigma_h \in G'$.	(76)
From $\mathbf{I}$	$\mathbf{I} \otimes \mathbf{C}_i = \mathbf{I}_h$.	(77)

6 Improper groups without inversion

Generated from cyclic groups $\mathbf{C}_n$

Halving subgroups	n even: $\mathbf{C}_{n/2}$. n odd: no halving subgroup exists.	(78)
n even, n/2 odd	$C_2 \notin \mathbf{C}_{n/2}$ $\Rightarrow$ $\mathbf{C}_n = \mathbf{C}_{n/2} \oplus C_2\,\mathbf{C}_{n/2}$ $\Rightarrow$ $\mathbf{C}_n = \mathbf{C}_{n/2} \circledwedge \mathbf{C}_2$.	(79)
	59, 60|79 $G' = \mathbf{C}_{n/2} \circledwedge \mathbf{C}_s$, $\mathbf{C}_s = E \oplus \sigma_h$ $\Rightarrow$ $G' = \mathbf{C}_{n/2,h}$.	(80)
	List: $\mathbf{C}_{1h} = \mathbf{C}_s$, $\mathbf{C}_{3h}$, $\mathbf{C}_{5h}$, $\mathbf{C}_{7h}$, $\mathbf{C}_{9h}$.	(81)
n even, n/2 even	$C_n^+ \notin \mathbf{C}_{n/2}$ $\Rightarrow$ $\mathbf{C}_n = \mathbf{C}_{n/2} \oplus C_n^+\,\mathbf{C}_{n/2}$.	(82)
	60|82; **4**.34 $G' = \mathbf{C}_{n/2} \oplus i\,C_n^+\,\mathbf{C}_{n/2} = \mathbf{C}_{n/2} \oplus S_n^{(n/2-1)-}\,\mathbf{C}_{n/2} = \mathbf{S}_n$.	(83)
	List: $\mathbf{S}_4$, $\mathbf{S}_8$, $\mathbf{S}_{12}$, $\mathbf{S}_{16}$, $\mathbf{S}_{20}$.	(84)

Generated from dihedral groups $\mathbf{D}_n$

Halving subgroups	$\forall n$: $\mathbf{C}_n$. n even: $\mathbf{D}_{n/2}$.	(85)
$\forall n$	$C_2' \in \mathbf{D}_n$, $C_2' \perp C_n$ $\Rightarrow$ $\mathbf{D}_n = \mathbf{C}_n \oplus C_2'\,\mathbf{C}_n$.	(86)
	$i\,C_2' = \sigma_v$ $\Rightarrow$ $G' = \mathbf{C}_n \oplus i\,C_2'\,\mathbf{C}_n = \mathbf{C}_n \oplus \sigma_v\,\mathbf{C}_n$ $\Rightarrow$	(87)
	$G' = \mathbf{C}_n \circledwedge \mathbf{C}_s = \mathbf{C}_{nv}$; $\mathbf{C}_s = E \oplus \sigma_v$.	(88)
	List: $\mathbf{C}_{2v}$, $\mathbf{C}_{3v}$, $\mathbf{C}_{4v}$, $\mathbf{C}_{5v}$, $\mathbf{C}_{6v}$, $\mathbf{C}_{7v}$, $\mathbf{C}_{8v}$, $\mathbf{C}_{9v}$, $\mathbf{C}_{10v}$.	(89)
n even, n/2 odd	$C_2 \in \mathbf{D}_n$, $C_2 \notin \mathbf{D}_{n/2}$ $\Rightarrow$ $\mathbf{D}_n = \mathbf{D}_{n/2} \oplus C_2\,\mathbf{D}_{n/2}$.	(90)
	$i\,C_2 = \sigma_h$ $\Rightarrow$ $G' = \mathbf{D}_{n/2} \oplus i\,C_2\,\mathbf{D}_{n/2} = \mathbf{D}_{n/2} \oplus \sigma_h\,\mathbf{D}_{n/2}$ $\Rightarrow$	(91)
	$G' = \mathbf{D}_{n/2} \circledwedge \mathbf{C}_s = \mathbf{D}_{n/2,h}$; $\mathbf{C}_s = E \oplus \sigma_h$.	(92)
	List: $\mathbf{D}_{3h}$, $\mathbf{D}_{5h}$, $\mathbf{D}_{7h}$, $\mathbf{D}_{9h}$.	(93)
n even, n/2 even	$C_2' \in \mathbf{D}_n$, $C_2' \perp C_n$ $\Rightarrow$ $\mathbf{D}_n = \mathbf{D}_{n/2} \oplus C_2'\,\mathbf{D}_{n/2}$.	(94)
	$i\,C_2' = \sigma_d$ $\Rightarrow$ $G' = \mathbf{D}_{n/2} \oplus i\,C_2'\,\mathbf{D}_{n/2} = \mathbf{D}_{n/2} \oplus \sigma_d\,\mathbf{D}_{n/2}$ $\Rightarrow$	(95)
	$G' = \mathbf{D}_{n/2} \circledwedge \mathbf{C}_s = \mathbf{D}_{n/2,d}$; $\mathbf{C}_s = E \oplus \sigma_d$.	(96)
	List: $\mathbf{D}_{2d}$, $\mathbf{D}_{4d}$, $\mathbf{D}_{6d}$, $\mathbf{D}_{8d}$, $\mathbf{D}_{10d}$.	(97)

Generated from the cubic groups **O**, **T**

Halving subgroups	For **O**: **T**. For **T**: none.	(98)
From **O**	$C_2' \in \mathbf{O},\ C_2' \perp C_{4z} \quad \Rightarrow \quad \mathbf{O} = \mathbf{T} \oplus C_2'\,\mathbf{T}.$	(99)
	$i\,C_2' = \sigma_d \Rightarrow \mathbf{O} = \mathbf{T} \oplus i\,C_2'\,\mathbf{T} = \mathbf{T} \oplus \sigma_d\,\mathbf{T} = \mathbf{T} \owedge \mathbf{C}_s = \mathbf{T}_d,\ \mathbf{C}_s = E \oplus \sigma_d.$	(100)
From **T**	None.	(101)

Generated from the icosahedral group **I**

Halving subgroups	None.	(102)
From **I**	None.	(103)

7 Summary. The point-group structure

Proper point groups	Improper point groups			
	With inversion		Without inversion	
Cyclic groups				
$\mathbf{C}_1$		$\mathbf{C}_1 \otimes \mathbf{C}_i = \mathbf{C}_i$		
$\mathbf{C}_2$	$\mathbf{C}_2 \otimes \mathbf{C}_i = \mathbf{C}_{2h}$		$\mathbf{C}_{1h} = \mathbf{C}_s$	
$\mathbf{C}_3$		$\mathbf{C}_3 \otimes \mathbf{C}_i = \mathbf{S}_6$		
$\mathbf{C}_4$	$\mathbf{C}_4 \otimes \mathbf{C}_i = \mathbf{C}_{4h}$			$\mathbf{S}_4$
$\mathbf{C}_5$		$\mathbf{C}_5 \otimes \mathbf{C}_i = \mathbf{S}_{10}$		
$\mathbf{C}_6$	$\mathbf{C}_6 \otimes \mathbf{C}_i = \mathbf{C}_{6h}$		$\mathbf{C}_3 \owedge \mathbf{C}_s = \mathbf{C}_{3h}$	
$\mathbf{C}_7$		$\mathbf{C}_7 \otimes \mathbf{C}_i = \mathbf{S}_{14}$		
$\mathbf{C}_8$	$\mathbf{C}_8 \otimes \mathbf{C}_i = \mathbf{C}_{8h}$			$\mathbf{S}_8$
$\mathbf{C}_9$		$\mathbf{C}_9 \otimes \mathbf{C}_i = \mathbf{S}_{18}$		
$\mathbf{C}_{10}$	$\mathbf{C}_{10} \otimes \mathbf{C}_i = \mathbf{C}_{10h}$		$\mathbf{C}_5 \owedge \mathbf{C}_s = \mathbf{C}_{5h}$	
$(\mathbf{C}_{12})$				$\mathbf{S}_{12}$
Dihedral groups				
$\mathbf{D}_2$	$\mathbf{D}_2 \otimes \mathbf{C}_i = \mathbf{D}_{2h}$		$\mathbf{C}_2 \owedge \mathbf{C}_s = \mathbf{C}_{2v}$	
$\mathbf{D}_3$		$\mathbf{D}_3 \otimes \mathbf{C}_i = \mathbf{D}_{3d}$	$\mathbf{C}_3 \owedge \mathbf{C}_s = \mathbf{C}_{3v}$	
$\mathbf{D}_4$	$\mathbf{D}_4 \otimes \mathbf{C}_i = \mathbf{D}_{4h}$		$\mathbf{C}_4 \owedge \mathbf{C}_s = \mathbf{C}_{4v}$	
$\mathbf{D}_4$				$\mathbf{D}_2 \owedge \mathbf{C}_s = \mathbf{D}_{2d}$
$\mathbf{D}_5$		$\mathbf{D}_5 \otimes \mathbf{C}_i = \mathbf{D}_{5d}$	$\mathbf{C}_5 \owedge \mathbf{C}_s = \mathbf{C}_{5v}$	
$\mathbf{D}_6$	$\mathbf{D}_6 \otimes \mathbf{C}_i = \mathbf{D}_{6h}$		$\mathbf{C}_6 \owedge \mathbf{C}_s = \mathbf{C}_{6v}$	
$\mathbf{D}_6$				$\mathbf{D}_3 \otimes \mathbf{C}_s = \mathbf{D}_{3h}$
$\mathbf{D}_7$		$\mathbf{D}_7 \otimes \mathbf{C}_i = \mathbf{D}_{7d}$	$\mathbf{C}_7 \owedge \mathbf{C}_s = \mathbf{C}_{7v}$	
$\mathbf{D}_8$	$\mathbf{D}_8 \otimes \mathbf{C}_i = \mathbf{D}_{8h}$		$\mathbf{C}_8 \owedge \mathbf{C}_s = \mathbf{C}_{8v}$	
$\mathbf{D}_8$				$\mathbf{D}_4 \owedge \mathbf{C}_s = \mathbf{D}_{4d}$
$\mathbf{D}_9$		$\mathbf{D}_9 \otimes \mathbf{C}_i = \mathbf{D}_{9d}$	$\mathbf{C}_9 \owedge \mathbf{C}_s = \mathbf{C}_{9v}$	
$\mathbf{D}_{10}$	$\mathbf{D}_{10} \otimes \mathbf{C}_i = \mathbf{D}_{10h}$		$\mathbf{C}_{10} \owedge \mathbf{C}_s = \mathbf{C}_{10v}$	
$\mathbf{D}_{10}$				$\mathbf{D}_5 \otimes \mathbf{C}_s = \mathbf{D}_{5h}$
$(\mathbf{D}_{12})$				$\mathbf{D}_6 \owedge \mathbf{C}_s = \mathbf{D}_{6d}$
$(\mathbf{D}_{16})$				$\mathbf{D}_8 \owedge \mathbf{C}_s = \mathbf{D}_{8d}$
Cubic groups				
$\mathbf{O}$	$\mathbf{O} \otimes \mathbf{C}_i = \mathbf{O}_h$		$\mathbf{T} \owedge \mathbf{C}_s = \mathbf{T}_d$	
$\mathbf{T}$	$\mathbf{T} \otimes \mathbf{C}_i = \mathbf{T}_h$			
Icosahedral group				
$\mathbf{I}$	$\mathbf{I} \otimes \mathbf{C}_i = \mathbf{I}_h$			

Notes The group $\mathbf{C}_s$ which appears in the third column, headed 'Without inversion', is given by $\mathbf{C}_s = E \oplus \sigma_r$, $r = h, v$, or d. This label must coincide with that of the group which appears as the result of the product stated.

Groups listed in brackets are not treated in the tables. On the other hand, for simplicity, the groups $\mathbf{S}_{16}$, $\mathbf{S}_{20}$, $\mathbf{D}_{7h}$, $\mathbf{D}_{9h}$, $\mathbf{D}_{10d}$, $\mathbf{C}_{7h}$, $\mathbf{C}_{9h}$, which are given in the tables, are not listed in the above scheme.

Bibliographical note

The theory used in § 4 is due to Zassenhaus (1949). See also Burckhardt (1947) and Coxeter and Moser (1984). For § 4 consult Weyl (1952) and Altmann (1977). This reference also covers much of the latter parts of this chapter. For a discussion of poles consult Altmann (1986).

7

Direct product, semidirect product, and coset expansion forms of the point groups

Group	Direct product form	Semidirect product form	Coset expansion
Proper cyclic groups			
$\mathbf{C}_1$			
$\mathbf{C}_2$			
$\mathbf{C}_3$			
$\mathbf{C}_4$			$\mathbf{C}_2 \oplus C_4\, \mathbf{C}_2$
$\mathbf{C}_5$			
$\mathbf{C}_6$	$\mathbf{C}_3 \otimes \mathbf{C}_2$		
$\mathbf{C}_7$			
$\mathbf{C}_8$			$\mathbf{C}_4 \oplus C_8\, \mathbf{C}_4$
$\mathbf{C}_9$			
$\mathbf{C}_{10}$	$\mathbf{C}_5 \otimes \mathbf{C}_2$		
Improper cyclic groups $\mathbf{C}_i$			
$\mathbf{C}_i$			
$\mathbf{C}_s$			
Improper cyclic groups $\mathbf{S}_n$			
$\mathbf{S}_4$			
$\mathbf{S}_6$	$\mathbf{C}_3 \otimes \mathbf{C}_i$		
$\mathbf{S}_8$			
$\mathbf{S}_{10}$	$\mathbf{C}_5 \otimes \mathbf{C}_i$		
$\mathbf{S}_{12}$			
$\mathbf{S}_{14}$	$\mathbf{C}_7 \otimes \mathbf{C}_i$		
$\mathbf{S}_{16}$			
$\mathbf{S}_{18}$	$\mathbf{C}_9 \otimes \mathbf{C}_i$		
$\mathbf{S}_{20}$			
Dihedral groups $\mathbf{D}_n$			
$\mathbf{D}_2$	$\mathbf{C}_2 \otimes \mathbf{C}_2'$		
$\mathbf{D}_3$		$\mathbf{C}_3 \owedge \mathbf{C}_2'$	
$\mathbf{D}_4$		$\mathbf{C}_4 \owedge \mathbf{C}_2'$, $\mathbf{D}_2 \owedge \mathbf{C}_2''$	
$\mathbf{D}_5$		$\mathbf{C}_5 \owedge \mathbf{C}_2'$	
$\mathbf{D}_6$		$\mathbf{C}_6 \owedge \mathbf{C}_2'$, $\mathbf{D}_3 \owedge \mathbf{C}_2''$	
$\mathbf{D}_7$		$\mathbf{C}_7 \owedge \mathbf{C}_2'$	
$\mathbf{D}_8$		$\mathbf{C}_8 \owedge \mathbf{C}_2'$, $\mathbf{D}_4 \owedge \mathbf{C}_2''$	

Group	Direct product form	Semidirect product form	Coset expansion
Dihedral groups $\mathbf{D}_n$ *(cont.)*			
$\mathbf{D}_9$		$\mathbf{C}_9 \circledwedge \mathbf{C}'_2$	
$\mathbf{D}_{10}$		$\mathbf{C}_{10} \circledwedge \mathbf{C}'_2$, $\mathbf{D}_5 \circledwedge \mathbf{C}''_2$	
The groups $\mathbf{D}_{nh}$			
$\mathbf{D}_{2h}$	$\mathbf{D}_2 \otimes \mathbf{C}_{i,s}$, $\mathbf{C}_{2v} \otimes \mathbf{C}_s$		
$\mathbf{D}_{3h}$	$\mathbf{D}_3 \otimes \mathbf{C}_s$, $\mathbf{C}_{3v} \otimes \mathbf{C}_s$		
$\mathbf{D}_{4h}$	$\mathbf{D}_4 \otimes \mathbf{C}_{i,s}$, $\mathbf{C}_{4v} \otimes \mathbf{C}_s$		
$\mathbf{D}_{5h}$	$\mathbf{D}_5 \otimes \mathbf{C}_s$, $\mathbf{C}_{5v} \otimes \mathbf{C}_s$		
$\mathbf{D}_{6h}$	$\mathbf{D}_6 \otimes \mathbf{C}_{i,s}$, $\mathbf{C}_{6v} \otimes \mathbf{C}_s$		
$\mathbf{D}_{7h}$	$\mathbf{D}_7 \otimes \mathbf{C}_s$, $\mathbf{C}_{7v} \otimes \mathbf{C}_s$		
$\mathbf{D}_{8h}$	$\mathbf{D}_8 \otimes \mathbf{C}_{i,s}$, $\mathbf{C}_{8v} \otimes \mathbf{C}_s$		
$\mathbf{D}_{9h}$	$\mathbf{D}_9 \otimes \mathbf{C}_s$, $\mathbf{C}_{9v} \otimes \mathbf{C}_s$		
$\mathbf{D}_{10h}$	$\mathbf{D}_{10} \otimes \mathbf{C}_{i,s}$, $\mathbf{C}_{10v} \otimes \mathbf{C}_s$		
$\mathbf{D}_{\infty h}$	$\mathbf{C}_{\infty v} \otimes \mathbf{C}_{i,s}$		
The groups $\mathbf{D}_{nd}$			
$\mathbf{D}_{2d}$		$\mathbf{D}_2 \circledwedge \mathbf{C}_s$	
$\mathbf{D}_{3d}$	$\mathbf{D}_3 \otimes \mathbf{C}_i$		
$\mathbf{D}_{4d}$		$\mathbf{D}_4 \circledwedge \mathbf{C}_s$	
$\mathbf{D}_{5d}$	$\mathbf{D}_5 \otimes \mathbf{C}_i$		
$\mathbf{D}_{6d}$		$\mathbf{D}_6 \circledwedge \mathbf{C}_s$	
$\mathbf{D}_{7d}$	$\mathbf{D}_7 \otimes \mathbf{C}_i$		
$\mathbf{D}_{8d}$		$\mathbf{D}_8 \circledwedge \mathbf{C}_s$	
$\mathbf{D}_{9d}$	$\mathbf{D}_9 \otimes \mathbf{C}_i$		
$\mathbf{D}_{10d}$		$\mathbf{D}_{10} \circledwedge \mathbf{C}_s$	
The groups $\mathbf{C}_{nv}$			
$\mathbf{C}_{2v}$	$\mathbf{C}_2 \otimes \mathbf{C}_s$		
$\mathbf{C}_{3v}$		$\mathbf{C}_3 \circledwedge \mathbf{C}_s$	
$\mathbf{C}_{4v}$		$\mathbf{C}_4 \circledwedge \mathbf{C}_s$	
$\mathbf{C}_{5v}$		$\mathbf{C}_5 \circledwedge \mathbf{C}_s$	
$\mathbf{C}_{6v}$		$\mathbf{C}_6 \circledwedge \mathbf{C}_s$	
$\mathbf{C}_{7v}$		$\mathbf{C}_7 \circledwedge \mathbf{C}_s$	
$\mathbf{C}_{8v}$		$\mathbf{C}_8 \circledwedge \mathbf{C}_s$	
$\mathbf{C}_{9v}$		$\mathbf{C}_9 \circledwedge \mathbf{C}_s$	
$\mathbf{C}_{10v}$		$\mathbf{C}_{10} \circledwedge \mathbf{C}_s$	
$\mathbf{C}_{\infty v}$		$\mathbf{C}_{\infty} \circledwedge \mathbf{C}_s$	
The groups $\mathbf{C}_{nh}$			
$\mathbf{C}_{2h}$	$\mathbf{C}_2 \otimes \mathbf{C}_{i,s}$		
$\mathbf{C}_{3h}$	$\mathbf{C}_3 \otimes \mathbf{C}_s$		
$\mathbf{C}_{4h}$	$\mathbf{C}_4 \otimes \mathbf{C}_{i,s}$		
$\mathbf{C}_{5h}$	$\mathbf{C}_5 \otimes \mathbf{C}_s$		
$\mathbf{C}_{6h}$	$\mathbf{C}_6 \otimes \mathbf{C}_{i,s}$		
$\mathbf{C}_{7h}$	$\mathbf{C}_7 \otimes \mathbf{C}_s$		
$\mathbf{C}_{8h}$	$\mathbf{C}_8 \otimes \mathbf{C}_{i,s}$		
$\mathbf{C}_{9h}$	$\mathbf{C}_9 \otimes \mathbf{C}_s$		
$\mathbf{C}_{10h}$	$\mathbf{C}_{10} \otimes \mathbf{C}_{i,s}$		

Group	Direct product form	Semidirect product form	Coset expansion
The octahedral and tetrahedral groups			
$\mathbf{O}$		$\mathbf{T} \circledwedge \mathbf{C}_2'$	
$\mathbf{T}$		$\mathbf{D}_2 \circledwedge \mathbf{C}_3'$	
$\mathbf{O}_h$	$\mathbf{O} \otimes \mathbf{C}_i$		
$\mathbf{T}_h$	$\mathbf{T} \otimes \mathbf{C}_i$		
$\mathbf{T}_d$		$\mathbf{T} \circledwedge \mathbf{C}_s'$	
The icosahedral groups			
$\mathbf{I}$			
$\mathbf{I}_h$	$\mathbf{I} \otimes \mathbf{C}_i$		

Notes

The symbol $\mathbf{C}_{i,s}$ means that either $\mathbf{C}_i$ or $\mathbf{C}_s$ may be used. The complete definition of the group $\mathbf{C}_s$ that appears in the products for $\mathbf{D}_{2h}$ and $\mathbf{C}_{2v}$ must be obtained from Subsection (1) of their respective tables. For all other products the group $\mathbf{C}_s$ is given by $\mathbf{C}_s = E \oplus \sigma_r$, $r = h, v$, or d. This label must coincide with that of the group which is the result of the product stated. The group $\mathbf{C}_2$ is $E \oplus C_2$, with C_2 along $\mathbf{z}$. When this group is listed as $\mathbf{C}_2'$ or $\mathbf{C}_2''$, the binary axis is perpendicular to $\mathbf{z}$, the prime and double prime indicating different orientations as defined in **4**.10 and **4**.11. C_3' in the group $\mathbf{C}_3'$ is an axis along the diagonal of the three binary rotations in $\mathbf{D}_2$.

Other product forms are listed in the tables for the individual groups.

8

The crystallographic point groups

Schönflies	International	
	Full	Short
Proper cyclic groups		
$\mathbf{C}_1$	1	1
$\mathbf{C}_2$	2	2
$\mathbf{C}_3$	3	3
$\mathbf{C}_4$	4	4
$\mathbf{C}_6$	6	6
Improper cyclic groups $\mathbf{C}_i$, $\mathbf{C}_s$		
$\mathbf{C}_i$ $(\mathbf{S}_2)$	$\bar{1}$	$\bar{1}$
$\mathbf{C}_s$ $(\mathbf{C}_{1h})$	m	m
Improper cyclic groups $\mathbf{S}_n$		
$\mathbf{S}_4$	$\bar{4}$	$\bar{4}$
$\mathbf{S}_6$ $(\mathbf{C}_{3i})$	$\bar{3}$	$\bar{3}$
Dihedral groups $\mathbf{D}_n$		
$\mathbf{D}_2$ $(\mathbf{V})$	222	222
$\mathbf{D}_3$	32	32
$\mathbf{D}_4$	422	422
$\mathbf{D}_6$	622	622

Schönflies	International	
	Full	Short
The groups $\mathbf{D}_{nh}$		
$\mathbf{D}_{2h}$ $(\mathbf{V}_h)$	$\frac{2}{m}\frac{2}{m}\frac{2}{m}$	mmm
$\mathbf{D}_{3h}$	$\bar{6}m2$	$\bar{6}m2$
$\mathbf{D}_{4h}$	$\frac{4}{m}\frac{2}{m}\frac{2}{m}$	$4/mmm$
$\mathbf{D}_{6h}$	$\frac{6}{m}\frac{2}{m}\frac{2}{m}$	$6/mmm$
The groups $\mathbf{D}_{nd}$		
$\mathbf{D}_{2d}$ $(\mathbf{V}_d)$	$\bar{4}2m$	$\bar{4}2m$
$\mathbf{D}_{3d}$	$\bar{3}\frac{2}{m}$	$\bar{3}m$
The groups $\mathbf{C}_{nv}$		
$\mathbf{C}_{2v}$	$mm2$	$mm2$
$\mathbf{C}_{3v}$	$3m$	$3m$
$\mathbf{C}_{4v}$	$4mm$	$4mm$
$\mathbf{C}_{6v}$	$6mm$	$6mm$
The groups $\mathbf{C}_{nh}$		
$\mathbf{C}_{2h}$	$\frac{2}{m}$	$2/m$
$\mathbf{C}_{3h}$	$\bar{6}$	$\bar{6}$
$\mathbf{C}_{4h}$	$\frac{4}{m}$	$4/m$
$\mathbf{C}_{6h}$	$\frac{6}{m}$	$6/m$
The octahedral and tetrahedral groups		
$\mathbf{O}$	432	432
$\mathbf{T}$	23	23
$\mathbf{O}_h$	$\frac{4}{m}\bar{3}\frac{2}{m}$	$m3m$
$\mathbf{T}_h$	$\frac{2}{m}\bar{3}$	$m3$
$\mathbf{T}_d$	$\bar{4}3m$	$\bar{4}3m$

9

Group chains

1 Definitions and structure of the tables

Group chains — A set of groups such as $A \supset B \supset C \supset D$. (1)

Invariance — In any of these relations a subgroup may or may not be an invariant subgroup of its supergroup. (2)

Possible difficulties in group chains, for $G \supset H$

Change of notation — An operation of G may change name when regarded as an operation of H. (3)

Change of setting — The subgroup H might not appear in the standard setting used in the tables. (4)

Subduction — Given G, $H \subset G$, $\check{G} = \{\check{G}(g)\}$, $\forall g \in G$, the subduced representation or restriction of G down to H is the set $\{\check{G}(g)\}$, $\forall g \in H$. (5)

Difficulties in subduction — (i) If $\check{G}$ is irreducible it does not follow that the subduced representation is irreducible. (This difficulty is an inherent one and it is not possible to plan the tables in order to avoid it.) If the subduced representation from $\check{G}$ is reducible the tables have been constructed so that whenever possible this representation is already reduced. (It is not always possible to achieve this because of the competing claims of two or more representations of H.) When the representation is not already reduced a **change of bases** arises. (6)

(ii) If the subduced representation is irreducible it does not follow that it is always identical with one of the irreducible representations tabulated for H. (A similarity might be entailed). In this case it means that in going from G to H a **change of bases** appears for some of the representations. The tables have been constructed so as to avoid this situation whenever possible. (This cannot always be achieved because of the competing claims of two or more representations of G.) (7)

(iii) For spinor representations (half-integral angular momenta) subduction might fail in the sense that the subduced representation does not satisfy the conservation of the characters as class functions. In this case there is a **subduction failure**. This can only happen in the cubic and icosahedral groups where several isomorphic subgroups (groups of the same name) appear in different settings. (See Herzig 1984.) (8)

Construction of the tables

Objective — To avoid as far as possible the difficulties listed above.

2 Description of the group-chain graphs

Description	Each of the 75 groups listed in the tables belongs to one or more group chains. These are displayed in twelve graphs. The groups are displayed in the graphs in columns that follow the conventional order used in the tables, except that $\mathbf{C}_i$ ($\mathbf{S}_2$) is listed under $\mathbf{S}_n$ and $\mathbf{C}_s$ ($\mathbf{C}_{1h}$) under $\mathbf{C}_{nh}$. The vertical scale (which uses a logarithmic scale base 2) gives the order of the groups. (9)
Note: $\mathbf{C}_{3v}$	**The group $\mathbf{C}_{3v}$ is given in the A setting in all graphs** (see F **51**A). The changes required for the B setting (see F **51**B) are as follows.

Graphs 1, 4.	$\mathbf{D}_{3h} \supset \mathbf{C}_{3v}$:	change of setting.	(10)
Graphs 1, 9, 11, 12.	$\mathbf{D}_{3d} \supset \mathbf{C}_{3v}$:	no change of setting, change of notation, $(\sigma_{di} \to \sigma_{vi},\ i = 1, 2, 3)$, no change of bases.	(11)
Graphs 1, 6.	$\mathbf{C}_{6v} \supset \mathbf{C}_{3v}$:	change of notation, $(\sigma_{di} \to \sigma_{vi},\ i = 1, 2, 3)$, change of basis.	(12)
Graphs 4, 9.	$\mathbf{C}_{9v} \supset \mathbf{C}_{3v}$:	change of setting.	(13)

Subduction	Possible changes of bases (see 6, 7) or failures of subduction (8) are coded in the graphs. When the change of bases is so coded, it normally happens to only one or two of the representations. For simplicity, when there is a change of setting, changes of notation or bases are not recorded. (14)
The graphs	They are listed below. The first line gives the group that is the head of all the group chains in the graph and the second gives the graph number. (15)

Group-chain head	$\mathbf{D}_{6h}$	$\mathbf{D}_{7h}$	$\mathbf{D}_{8h}$	$\mathbf{D}_{9h}$	$\mathbf{D}_{10h}$	$\mathbf{D}_{6d}$	$\mathbf{D}_{7d}$	$\mathbf{D}_{8d}$	$\mathbf{D}_{9d}$	$\mathbf{D}_{10d}$	$\mathbf{O}_h$	$\mathbf{I}_h$
Graph number	1	2	3	4	5	6	7	8	9	10	11	12

3 An index of the groups in the graphs

Group	Part 2 table	Graphs where the group appears
Proper cyclic groups $\mathbf{C}_n$		
$\mathbf{C}_1$	T **1**	1–12
$\mathbf{C}_2$	T **2**	1–12
$\mathbf{C}_3$	T **3**	1, 4, 6, 9, 11, 12
$\mathbf{C}_4$	T **4**	3, 8, 11
$\mathbf{C}_5$	T **5**	5, 10, 12
$\mathbf{C}_6$	T **6**	1, 6
$\mathbf{C}_7$	T **7**	2, 7
$\mathbf{C}_8$	T **8**	3, 8
$\mathbf{C}_9$	T **9**	4, 9
$\mathbf{C}_{10}$	T **10**	5, 10
Improper cyclic groups $\mathbf{C}_i$, $\mathbf{C}_s$		
$\mathbf{C}_i$	T **11**	1, 3, 5, 7, 9, 11, 12
$\mathbf{C}_s$	T **12**	1–12

Group	Part 2 table	Graphs where the group appears
Improper cyclic groups $\mathbf{S}_n$		
$\mathbf{S}_4$	T **13**	3, 6, 10, 11
$\mathbf{S}_6$	T **14**	1, 9, 11, 12
$\mathbf{S}_8$	T **15**	3
$\mathbf{S}_{10}$	T **16**	5, 12
$\mathbf{S}_{12}$	T **17**	6
$\mathbf{S}_{14}$	T **18**	7
$\mathbf{S}_{16}$	T **19**	8
$\mathbf{S}_{18}$	T **20**	9
$\mathbf{S}_{20}$	T **21**	10
Dihedral groups $\mathbf{D}_n$		
$\mathbf{D}_2$	T **22**	1, 3, 5, 6, 8, 10, 11, 12
$\mathbf{D}_3$	T **23**	1, 4, 6, 9, 11, 12
$\mathbf{D}_4$	T **24**	3, 8, 11
$\mathbf{D}_5$	T **25**	5, 10, 12
$\mathbf{D}_6$	T **26**	1, 6
$\mathbf{D}_7$	T **27**	2, 7
$\mathbf{D}_8$	T **28**	3, 8
$\mathbf{D}_9$	T **29**	4, 9
$\mathbf{D}_{10}$	T **30**	5, 10
The groups $\mathbf{D}_{nh}$		
$\mathbf{D}_{2h}$	T **31**	1, 3, 5, 11, 12
$\mathbf{D}_{3h}$	T **32**	1, 4
$\mathbf{D}_{4h}$	T **33**	3, 11
$\mathbf{D}_{5h}$	T **34**	5
$\mathbf{D}_{6h}$	T **35**	1
$\mathbf{D}_{7h}$	T **36**	2
$\mathbf{D}_{8h}$	T **37**	3
$\mathbf{D}_{9h}$	T **38**	4
$\mathbf{D}_{10h}$	T **39**	5
$\mathbf{D}_{\infty h}$	T **40**	
The groups $\mathbf{D}_{nd}$		
$\mathbf{D}_{2d}$	T **41**	3, 6, 10, 11
$\mathbf{D}_{3d}$	T **42**	1, 9, 11, 12
$\mathbf{D}_{4d}$	T **43**	3
$\mathbf{D}_{5d}$	T **44**	5, 12
$\mathbf{D}_{6d}$	T **45**	6
$\mathbf{D}_{7d}$	T **46**	7
$\mathbf{D}_{8d}$	T **47**	8
$\mathbf{D}_{9d}$	T **48**	9
$\mathbf{D}_{10d}$	T **49**	10

Group	Part 2 table	Graphs where the group appears
The groups $\mathbf{C}_{nv}$		
$\mathbf{C}_{2v}$	T **50**	1–6, 8, 10, 11, 12
$\mathbf{C}_{3v}$	T **51**	1, 4, 6, 9, 11, 12
$\mathbf{C}_{4v}$	T **52**	3, 8, 11
$\mathbf{C}_{5v}$	T **53**	5, 10, 12
$\mathbf{C}_{6v}$	T **54**	1, 6
$\mathbf{C}_{7v}$	T **55**	2, 7
$\mathbf{C}_{8v}$	T **56**	3, 8
$\mathbf{C}_{9v}$	T **57**	4, 9
$\mathbf{C}_{10v}$	T **58**	5, 10
$\mathbf{C}_{\infty v}$	T **59**	
The groups $\mathbf{C}_{nh}$		
$\mathbf{C}_{2h}$	T **60**	1, 3, 5, 7, 9, 11, 12
$\mathbf{C}_{3h}$	T **61**	1, 4
$\mathbf{C}_{4h}$	T **62**	3, 11
$\mathbf{C}_{5h}$	T **63**	5
$\mathbf{C}_{6h}$	T **64**	1
$\mathbf{C}_{7h}$	T **65**	2
$\mathbf{C}_{8h}$	T **66**	3
$\mathbf{C}_{9h}$	T **67**	4
$\mathbf{C}_{10h}$	T **68**	5
The octahedral and tetrahedral groups		
$\mathbf{O}$	T **69**	11
$\mathbf{T}$	T **70**	11, 12
$\mathbf{O}_h$	T **71**	11
$\mathbf{T}_h$	T **72**	11, 12
$\mathbf{T}_d$	T **73**	11
The icosahedral groups		
$\mathbf{I}$	T **74**	12
$\mathbf{I}_h$	T **75**	12

4 Examples

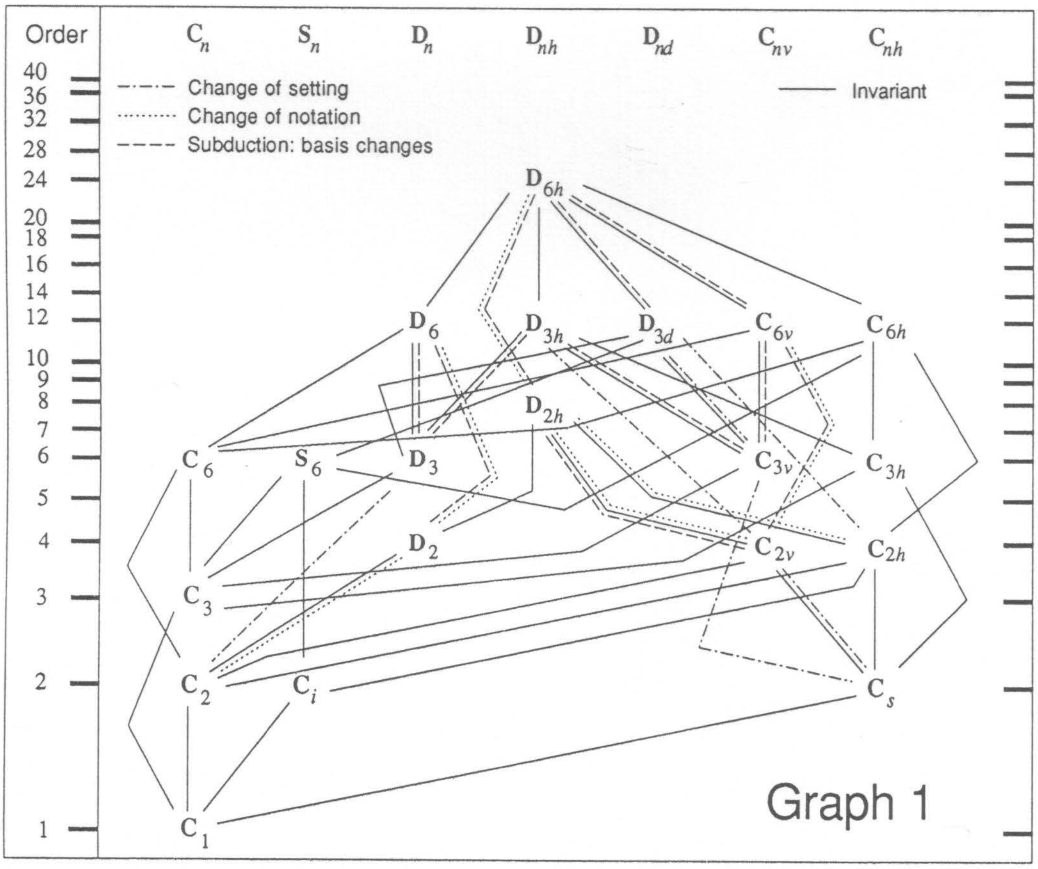
Graph 3

$\mathbf{D}_{8h} \supset \mathbf{C}_{8v} \supset \mathbf{C}_{4v} \supset \mathbf{C}_{2v} \supset \mathbf{C}_{2} \supset \mathbf{C}_{1}$.

$\mathbf{D}_{8h} \supset \mathbf{C}_{8h} \supset \mathbf{S}_{8} \supset \mathbf{C}_{4} \supset \mathbf{C}_{2} \supset \mathbf{C}_{1}$.

5 The graphs

They are shown below, on pages 45 to 50.


Order
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Cn
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Dn
Dnh
Dnd
Cnv
Cnh
Change of setting
Change of notation
Subduction: basis changes
Invariant
D6h
D6
D3h
D3d
C6v
C6h
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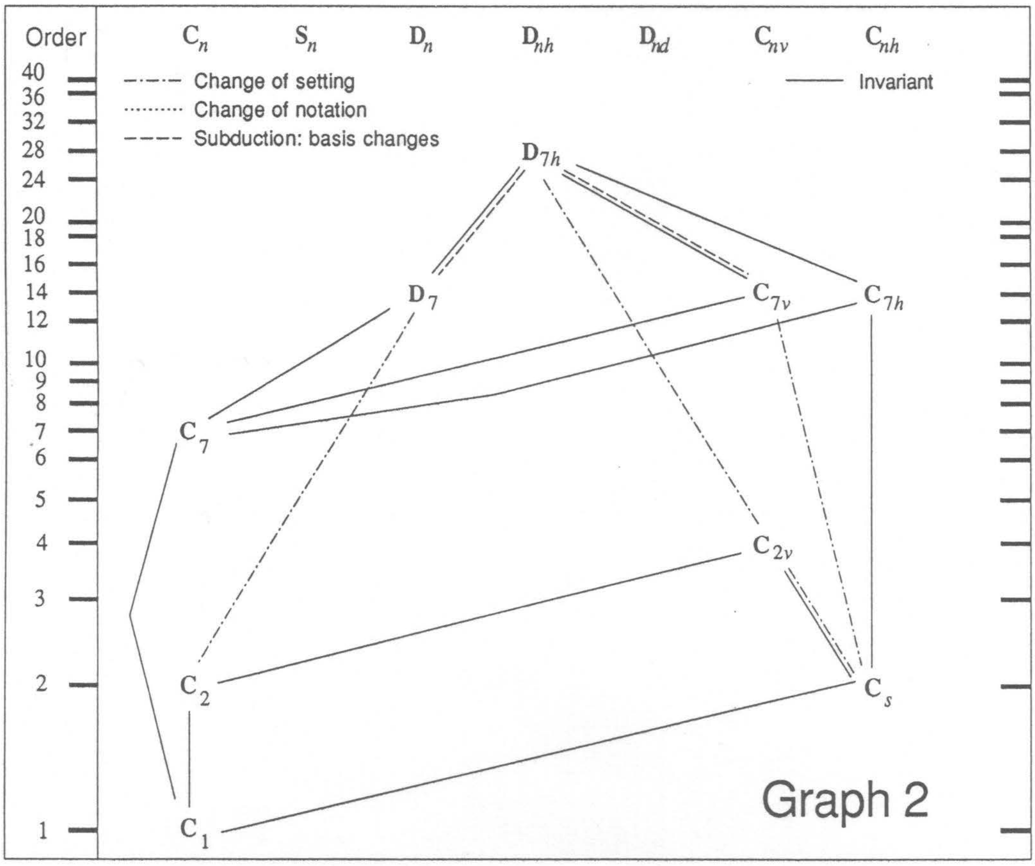
Cs
C1
Graph 1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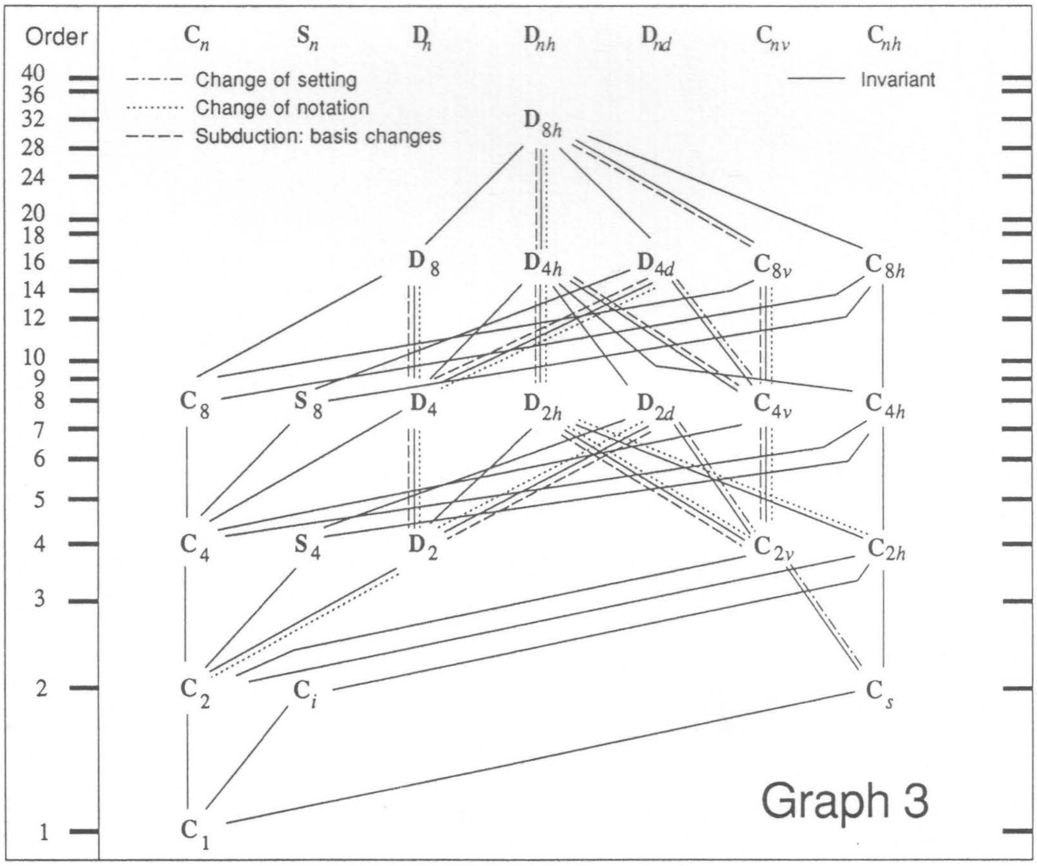
Cnv
Cnh
Change of setting
Change of notation
Subduction: basis changes
Invariant
D7h
D7
C7v
C7h
C7
C2v
C2
Cs
C1
Graph 2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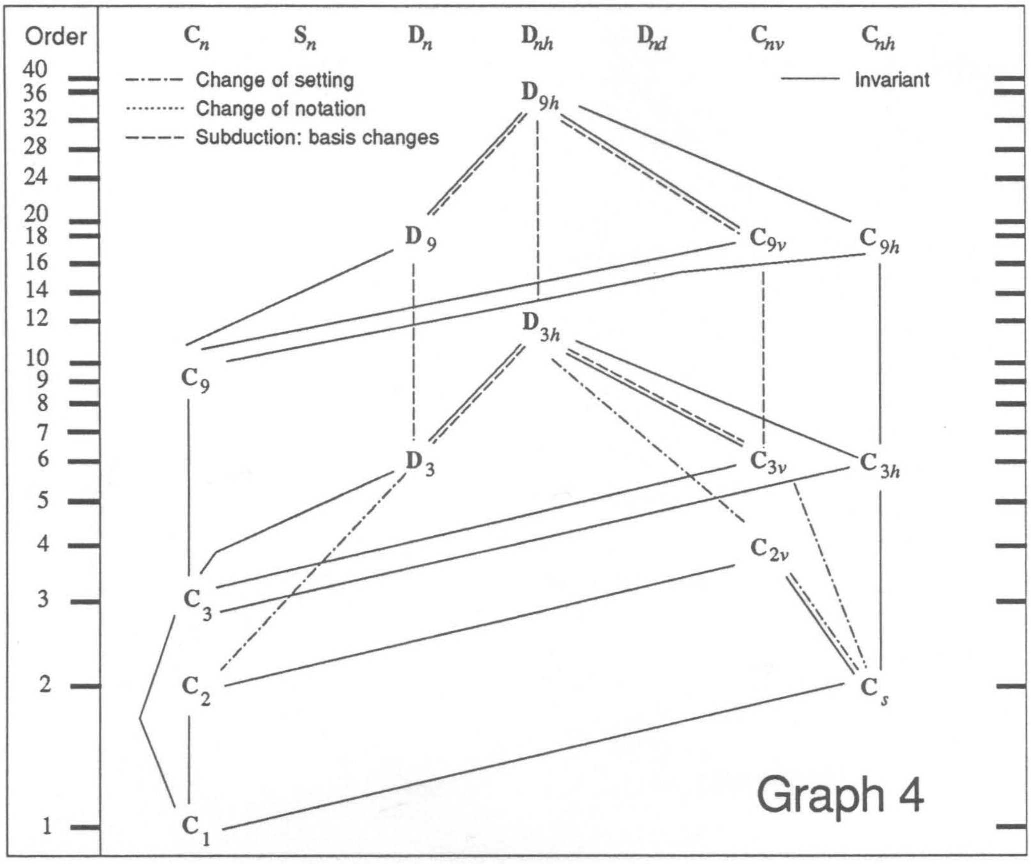
Change of setting
Change of notation
Subduction: basis changes
Invariant
D_{8h}
D_8 D_{4h} D_{4d} C_{8v} C_{8h}
C_8 S_8 D_4 D_{2h} D_{2d} C_{4v} C_{4h}
C_4 S_4 D_2 C_{2v} C_{2h}
C_2 C_i C_s
C_1
Graph 3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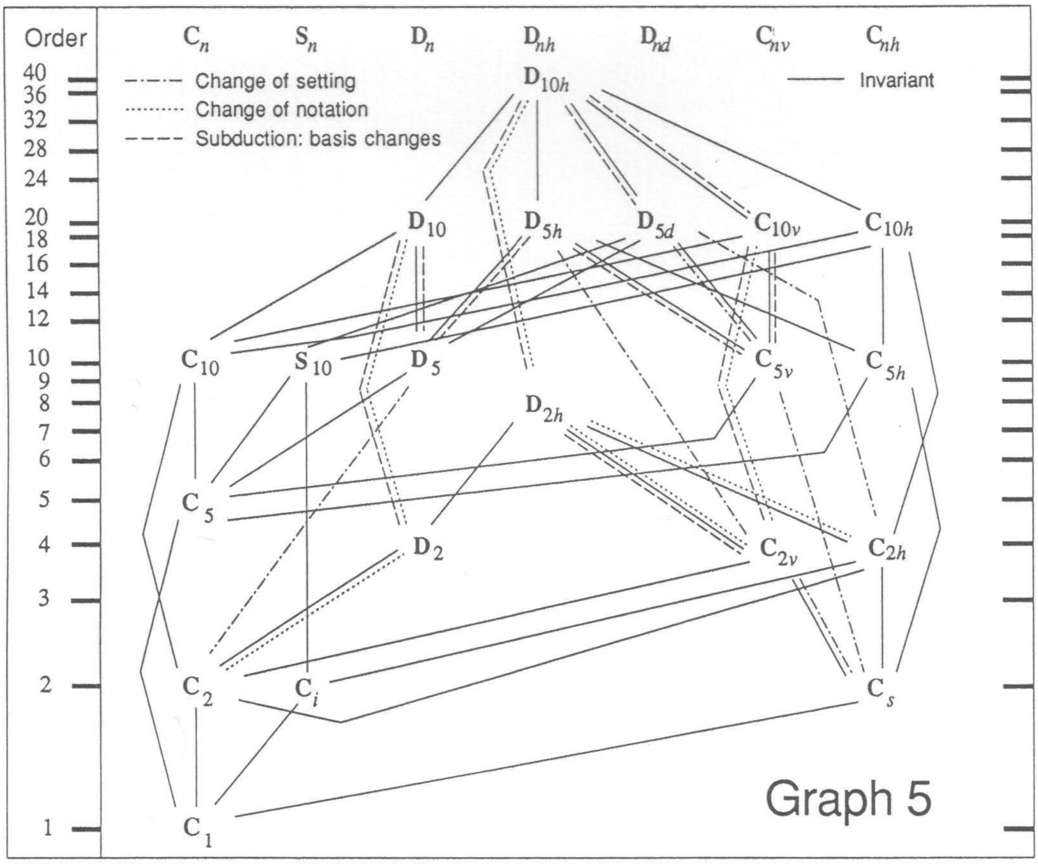
C_n S_n D_n D_{nh} D_{nd} C_{nv} C_{nh}
Change of setting
Change of notation
Subduction: basis changes
Invariant
D_{9h}
D_9 C_{9v} C_{9h}
D_{3h}
C_9
D_3 C_{3v} C_{3h}
C_{2v}
C_3
C_2 C_s
C_1
Graph 4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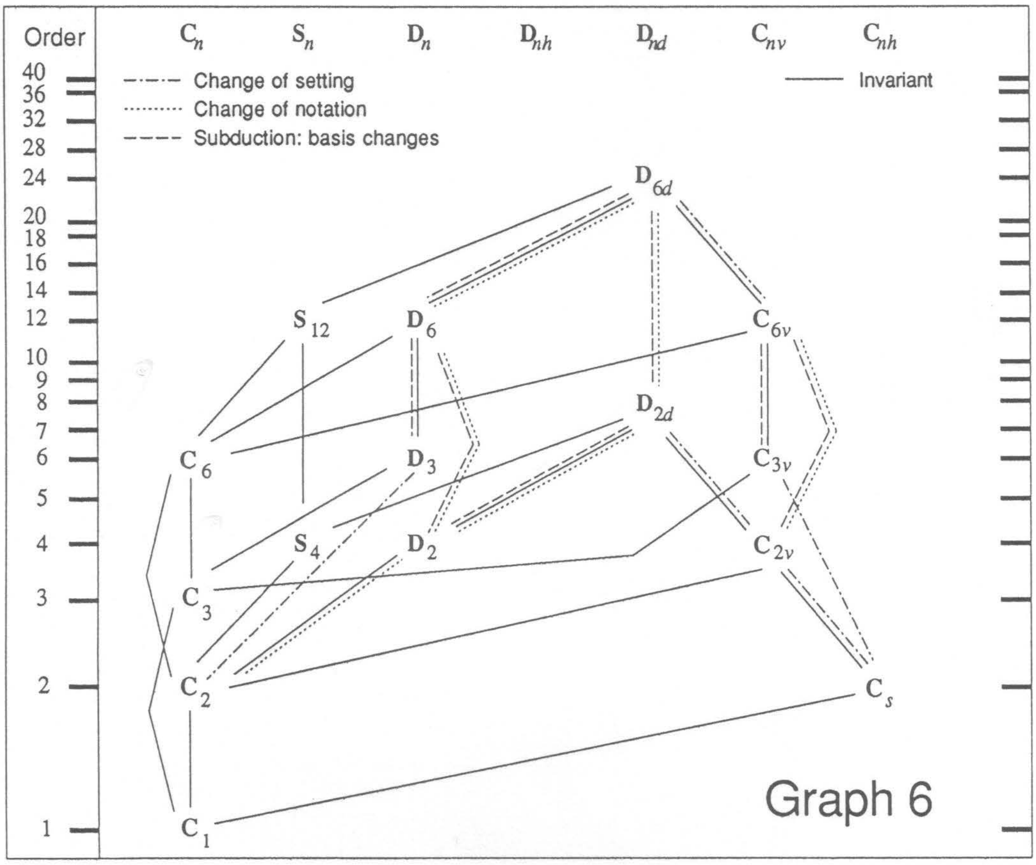
Cn Sn Dn Dnh Dnd Cnv Cnh
Change of setting
Change of notation
Subduction: basis changes
Invariant
D10h
D10 D5h D5d C10v C10h
C10 S10 D5 C5v C5h
D2h
C5
D2 C2v C2h
C2 Ci Cs
C1
Graph 5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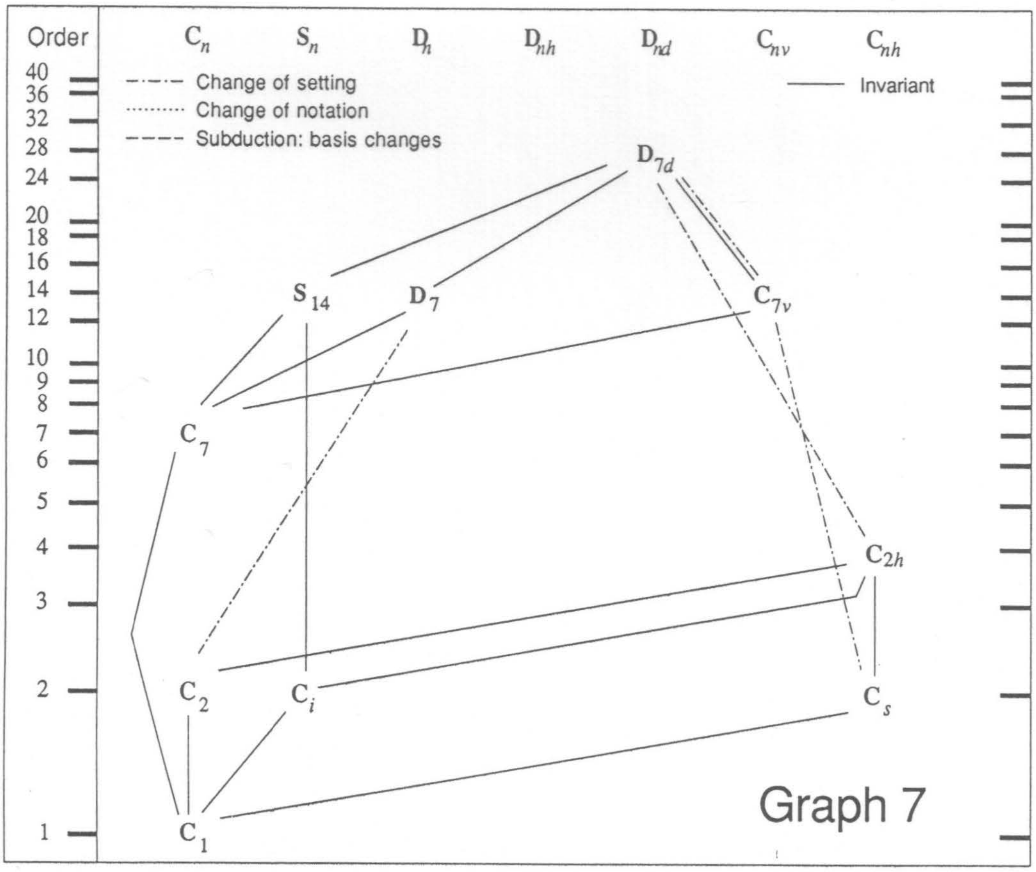
Cn Sn Dn Dnh Dnd Cnv Cnh
Change of setting
Change of notation
Subduction: basis changes
Invariant
D6d
S12 D6 C6v
D2d
C6 D3 C3v
S4 D2 C2v
C3
C2 Cs
C1
Graph 6



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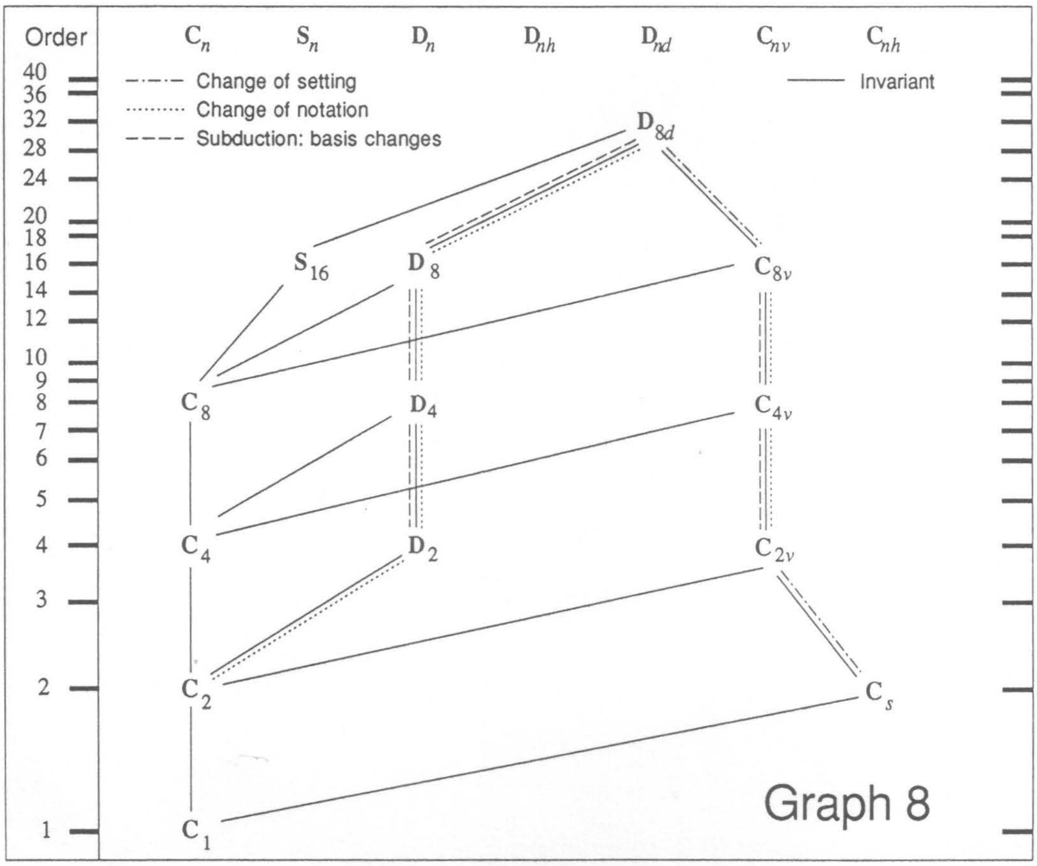
C_n
S_n
D_n
D_{nh}
D_{nd}
C_{nv}
C_{nh}
Change of setting
Change of notation
Subduction: basis changes
Invariant
D_{7d}
S_{14}
D_7
C_{7v}
C_7
C_{2h}
C_2
C_i
C_s
C_1
Graph 7



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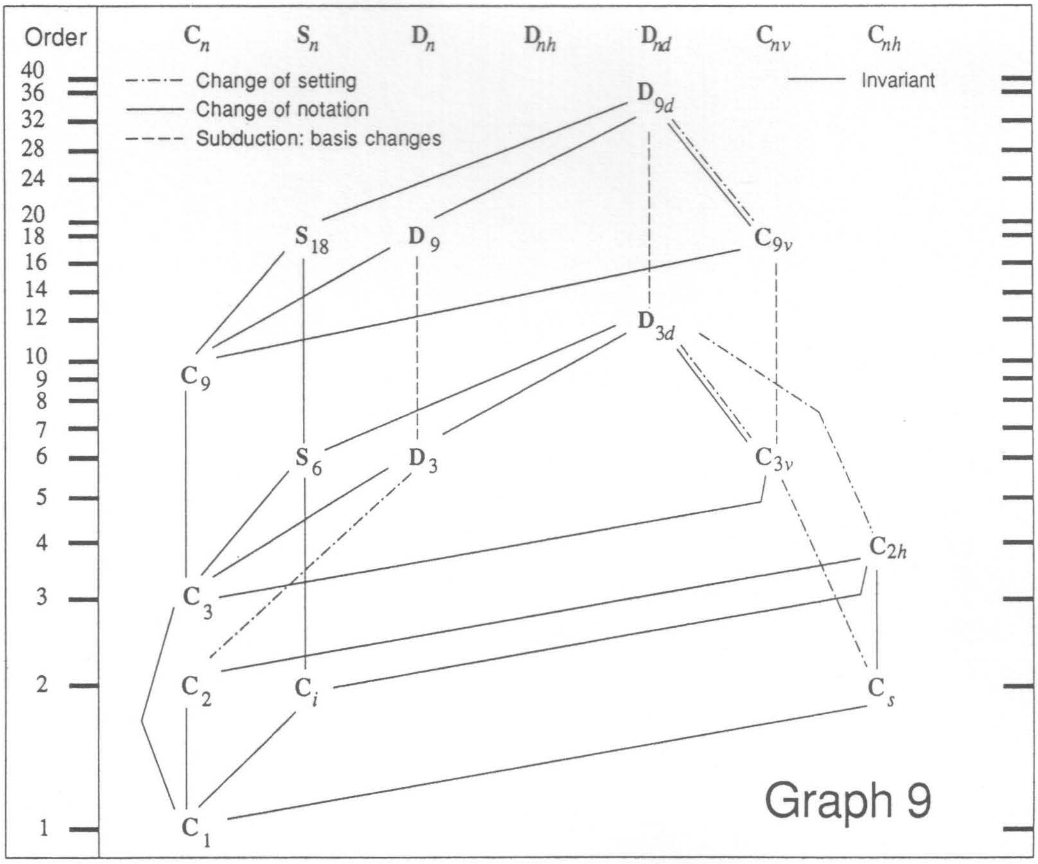
C_n
S_n
D_n
D_{nh}
D_{nd}
C_{nv}
C_{nh}
Change of setting
Change of notation
Subduction: basis changes
Invariant
D_{8d}
S_{16}
D_8
C_{8v}
C_8
D_4
C_{4v}
C_4
D_2
C_{2v}
C_2
C_s
C_1
Graph 8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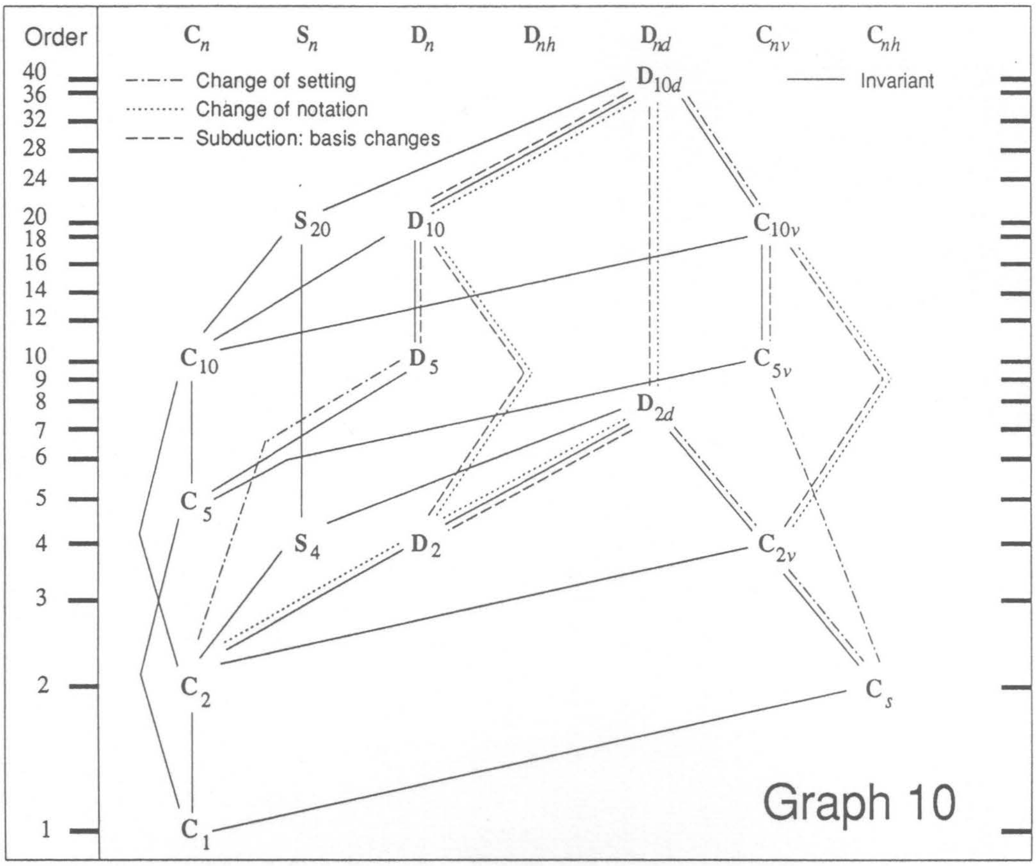
Cn Sn Dn Dnh Dnd Cnv Cnh
Change of setting
Change of notation
Subduction: basis changes
Invariant
D9d
S18
D9
C9v
D3d
C9
S6
D3
C3v
C2h
C3
C2
Ci
Cs
C1
Graph 9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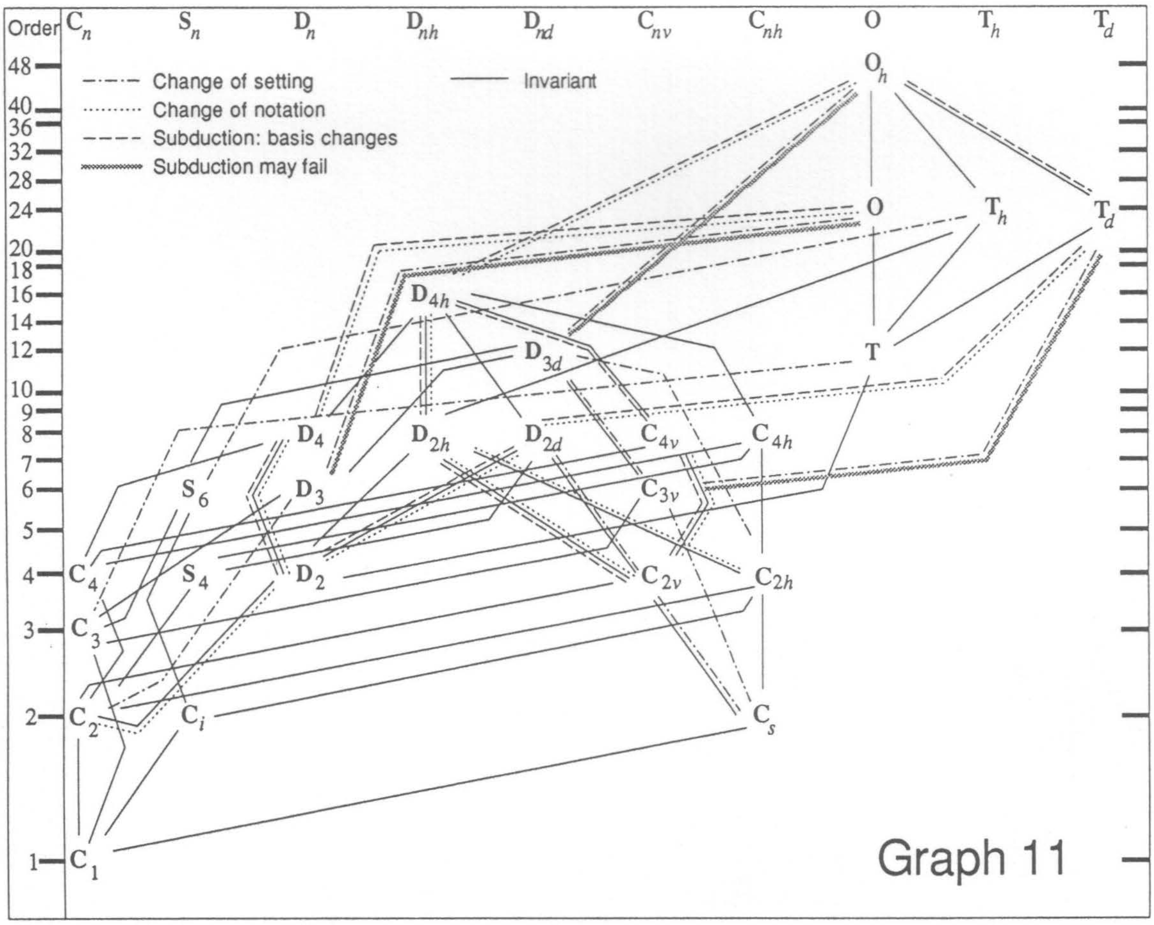
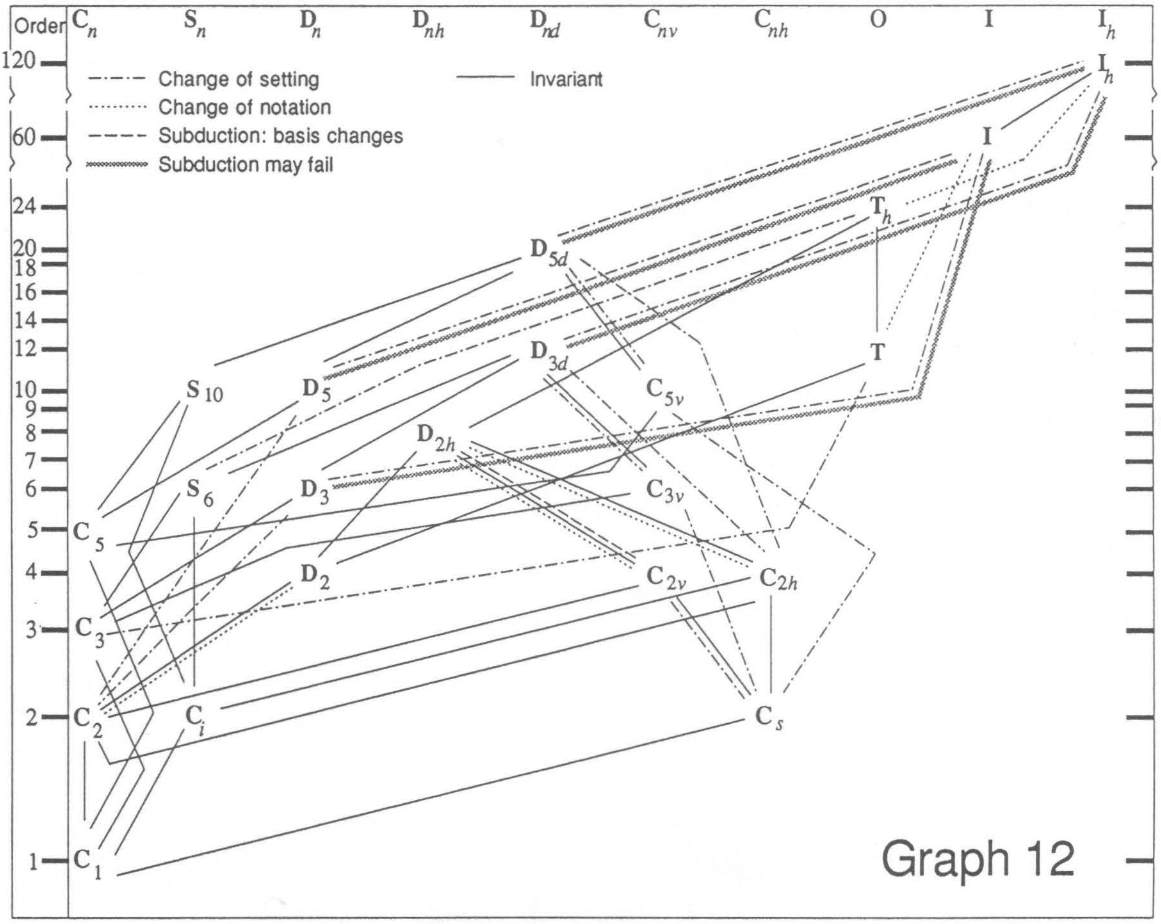
Cn Sn Dn Dnh Dnd Cnv Cnh
Change of setting
Change of notation
Subduction: basis changes
Invariant
D10d
S20
D10
C10v
C10
D5
C5v
D2d
C5
S4
D2
C2v
C2
Cs
C1
Graph 10


Order
C_n S_n D_n D_{nh} D_{nd} C_{nv} C_{nh} O T_h T_d
48 40 36 32 28 24 20 18 16 14 12 10 9 8 7 6 5 4 3 2 1
Change of setting
Change of notation
Subduction: basis changes
Subduction may fail
Invariant
O_h O T_h T_d T
D_{4h} D_{3d} D_4 D_{2h} D_{2d} C_{4v} C_{4h}
S_6 D_3 C_{3v}
C_4 S_4 D_2 C_{2v} C_{2h}
C_3 C_2 C_i C_s C_1
Graph 11


Order
C_n S_n D_n D_{nh} D_{nd} C_{nv} C_{nh} O I I_h
120 60 24 20 18 16 14 12 10 9 8 7 6 5 4 3 2 1
Change of setting
Change of notation
Subduction: basis changes
Subduction may fail
Invariant
I_h I T_h D_{5d} D_{3d} T
S_{10} D_5 C_{5v} D_{2h}
S_6 D_3 C_{3v}
C_5 D_2 C_{2v} C_{2h}
C_3 C_2 C_i C_s C_1
Graph 12

10

Double groups. Spinor and projective representations

1 The double group

Definitions

G	Group of operations $g_1 = E, g_2, \ldots, g_N$.	(1)
$\widetilde{E}$	An operation (which may be regarded as a rotation by 2π) which commutes with all $g \in G$ and such that $\widetilde{E}\,\widetilde{E} = E$.	(2)
$\widetilde{g}$	$\widetilde{g} =_{\text{def}} \widetilde{E}\,g = g\widetilde{E}, \forall g \in G$.	(3)
$\widetilde{G}$	The group of order $2N$ of operations $g, \widetilde{g}, \forall g \in G$.	(4)
Warning	G is not a subgroup of $\widetilde{G}$. The multiplication rules in G are **not preserved** in $\widetilde{G}$: $g_i\,g_j = g_k$ in G does not entail $g_i\,g_j = g_k$ in $\widetilde{G}$.	(5)
$=$	Equality in G, as in $g_i\,g_j = g_k$.	(6)
$\simeq$	Equality in $\widetilde{G}$, as in $g_i\,g_j \simeq g_k$ or $g_i\,g_j \simeq \widetilde{g}_k$. (Both results are possible.)	(7)
g^{-1}	Inverse in G.	(8)
$g^{\sim 1}$	Inverse in $\widetilde{G}$.	(9)
$g_i\ \mathbf{c}\ g_j$	Conjugation in G.	(10)
$g_i\ \widetilde{\mathbf{c}}\ g_j$	Conjugation in $\widetilde{G}$.	(11)
$C(g_i)$	Class of g_i in G.	(12)
$\widetilde{C}(g_i)$	Class of g_i in $\widetilde{G}$.	(13)
Irregular operations	Bilateral-binary rotations (see **4**.24) and one of a pair of orthogonal mirrors.	(14)
Regular operations	All operations of G which are not irregular.	(15)
Parametrization. The inversion	**See** § **3**–4. Notice that $i^2 = E$.	(16)

Class structure (Opechowski's theorem)

All operations	$g_i\ \mathbf{c}\ g_j \quad\Rightarrow\quad g_i\ \widetilde{\mathbf{c}}\ g_j; \qquad \widetilde{g}_i\ \mathbf{c}\ \widetilde{g}_j \quad\Rightarrow\quad \widetilde{g}_i\ \widetilde{\mathbf{c}}\ \widetilde{g}_j.$	(17)
Irregular operations ***only***	$g_i\ \mathbf{c}\ g_j \quad\Rightarrow\quad g_i\ \widetilde{\mathbf{c}}\ g_j$ **and** $g_i\ \widetilde{\mathbf{c}}\ \widetilde{g}_j$. $g_i\ \mathbf{c}\ \widetilde{g}_j \quad\Rightarrow\quad g_i\ \widetilde{\mathbf{c}}\ \widetilde{g}_j$ **and** $g_i\ \widetilde{\mathbf{c}}\ g_j$.	(18)
Regular operations	For each class $C(g_i)$ in G there are **two** classes $\widetilde{C}(g_i)$ and $\widetilde{C}(\widetilde{g}_i)$ in $\widetilde{G}$. (See 17.)	(19)
Irregular operations ***only***	For each class $C(g_i)$ in G there is only **one** class $\widetilde{C}(g_i) \equiv \widetilde{C}(\widetilde{g}_i)$. (See 18.)	(20)

Irreducible representations

Vector representations — They are the irreducible representations of a single group G. (21)

Spinor representations — Given a group G, its double group $\widetilde{G}$ contains all the vector representations of G plus a number of additional irreducible representations which are called spinor representations. They are also called *double-group representations*. They correspond to half-integral angular momenta. (22)

Number of irreducible spinor representations — It is equal to the number of regular classes of G. (23)

Dimensions — See (40) below.

Example — Consider the group $\mathbf{D}_2$, to be abbreviated as G. Although all the ordinary (vector) representations of this group are one-dimensional there is one two-dimensional spinor representation $E_{1/2}$, the matrices of which will be denoted $\hat{G}(g)$, $\forall g \in \widetilde{G}$.

T **22**.2 $\quad C_{2z}\, C_{2z} = \widetilde{E}.$ (24)

T **22**.7 $\quad \hat{G}(C_{2z})\,\hat{G}(C_{2z}) = \begin{bmatrix} -\mathrm{i} & 0 \\ 0 & \mathrm{i} \end{bmatrix} \begin{bmatrix} -\mathrm{i} & 0 \\ 0 & \mathrm{i} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \hat{G}(\widetilde{E}).$ (25)

2 Projective representations

Warning — **The user of these tables need not be concerned with projective representations at all and may skip this section.**

Motivation

The double-group approach — It is a property of the spinor representation of $\mathbf{D}_2$ (order 4) illustrated in (25) that the set of four matrices for the four operations of $\mathbf{D}_2$ **does not close**, since the matrix on (R 25) does not belong to the set given in T **22**.7. In the double-group method the group is doubled to order 8, the matrix on (R 25) is assigned to the artificial operation $\widetilde{E}$ and the new set of eight matrices closes. *Advantage*: no further theory is necessary, the spinor representations being merely vector representations of the double group. *Disadvantage*: the group is doubled, which increases the work. It is also inconvenient that the multiplication rules of the single group and its class structure are not preserved in the double group. More importantly, the multiplication rules of the double group are not uniquely defined. (26)

The projective representation approach — The group is not altered in any way at all, but a new type of representation (projective representation) is defined the matrices of which **do not close**. (R 24) will now be E. The matrix on (R 25), which does not belong to the set of four matrices of $\mathbf{D}_2$, will be written as the matrix of E multiplied by a numerical factor, which is called a projective factor. *Advantages*: (i) The group, its multiplication rules, and its class structure are not altered at all. (ii) The mathematical theory of projective representations is very precise and powerful, thus easily providing results which are obscure within the double-group framework. (iii) The work is quicker because there are fewer elements to use in the group. *Disadvantages*: one has to learn projective representation theory. However: **if the results of projective representation theory are accepted, the user of spinor representations can work with them exactly as if they were vector representations. It is enough to know that when two matrices are multiplied the matrix that appears is multiplied by a numerical factor.** (27)

Definitions

Projective representation — Given a group G of elements g, ($|G|$ in number), it is a set of $|G|$ matrices $\check{G}(g)$ that satisfy the relations
$\check{G}(g_i)\,\check{G}(g_j) = [g_i, g_j]\,\check{G}(g_i\,g_j),\ \forall g_i, g_j \in G.$ (28)

Projective factors — The complex numbers $[g_i, g_j]$, $|G|^2$ in number, are called projective factors. If they are all equal to unity, the above equation defines a *vector* (ordinary) representation $\hat{G}(g)$. With the conventions used in this book **all projective factors must be square roots of unity.** (29)

Factor system — The set of all projective factors for G, $|G|^2$ in number. (30)

Associativity conditions — $[g_i, g_j]\,[g_i\,g_j, g_k] = [g_i, g_j\,g_k]\,[g_j, g_k].$ (31)

Standardization condition — $[E, E] = [E, g_i] = [g_i, E] = 1,\ \forall g_i \in G.$ (32)

Normalization condition — $[g_i, g_j]\,[g_i, g_j]^* = 1,\ \forall g_i, g_j \in G.$ (33)

Inverse condition — $[g_i, \bar{g}_i] = [\bar{g}_i, g_i], \quad \bar{g}_i =_{\text{def}} g_i^{-1},\ \forall g_i \in G.$ (34)

Unitary condition — $\check{G}(g_i)^\dagger\,\check{G}(g_i) = \check{G}(g_i)\,\check{G}(g_i)^\dagger = \mathbf{1}.$ (35)

Notes — The associativity and inverse conditions are valid for all factor systems. **All the systems used in this book have been chosen standardized and normalized.** (36)

Properties

Characters — They are not necessarily class functions but the conventions given in this book for choosing poles of operations have been so designed that for all groups treated the characters are class functions and stay as class functions even when subduction is used, except in a few unavoidable cases where warnings to this effect are given. (Subduction in double groups is often extremely chancy because the same difficulty underlies the work but it is then not easy to control.) (37)

Orthogonality relations and others — All the relations given for vector representations are valid for the unitary projective representations with normalized and standardized factor systems and the pole conventions used in this book, except that the number of irreducible projectives is not equal to the number of classes in the group. (See next item.) Thus $|I(G)|$, which must be understood as the number of irreducible projective representations in G for the given factor system, is not $|C(G)|$ as in (**2**.74). With this proviso the condition for the dimensions (**2**.75) can still be used. (See 40 below.) (38)

Number of spinor representations — This is the number that we need of the irreducible projective representations in a group and it is equal to the number of regular classes in the group. (39)

Dimensions — The sum of the squares of the dimensions of all the spinor representations equals the order $|G|$ of the single group G. This rule is valid in this form also for a double group $\widetilde{G}$. (40)

Projection operators — The expressions given in (**2**.86), (**2**.87), and (**2**.90) to (**2**.92), are all valid with the conditions stated here. (41)

Bibliographical note

More details about double-group theory as presented here may be found in Altmann (1986). All the required properties of projective representations are given in Altmann (1977, 1986).

11

The matrices of SU(2) and SU′(2)

1 Definitions

Special matrices	Also called *unimodular*. Their determinant is +1.	(1)
SU(2)	The group of all 2 × 2 special unitary matrices.	(2)
SU′(2)	The group of all 2 × 2 unitary matrices with determinant ±1.	(3)

2 Form of the matrices

$A \in \mathrm{SU}(2)$	$A = \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}$, for a, b complex numbers and $a\,a^* + b\,b^* = 1$.	(4)
$A' \in \mathrm{SU}'(2)$	Either $A' = A$ *as defined above*, or $A' = \begin{bmatrix} a & b \\ b^* & -a^* \end{bmatrix}$, for a, b complex and $a\,a^* + b\,b^* = 1$.	(5)

3 Relation between SU(2) and SU′(2) to the rotation group

Definitions

SO(3)	The continuous group of all proper rotations of a sphere with a fixed centre.	(6)
O(3)	The continuous group of all proper and improper rotations of a sphere with a fixed centre. $\mathrm{O}(3) = \mathrm{SO}(3) \otimes \mathbf{C}_i$.	(7)

Relation between SO(3) and SU(2)

$g = R(\phi\,\mathbf{n}) \in \mathrm{SO}(3)$; *Cayley–Klein parameters* a, b	$\pm A \mapsto g$; $\quad a = \cos\frac{\phi}{2} - \mathrm{i}\,n_z \sin\frac{\phi}{2}$, $b = -(n_y + \mathrm{i}\,n_x)\sin\frac{\phi}{2}$.	(8)
Representation of the double group of SO(3)	SU(2) forms a (vector) representation of the double group of SO(3), with the conventions used in this book, as follows: $+A \mapsto g$, $-A \mapsto \widetilde{g}$.	(9)
Projective representation of SO(3)	The mapping $+A \mapsto g$ forms a projective representation of SO(3), with the conventions used in this book.	(10)

Relation between O(3), SU(2), and SU′(2)

$i \in \mathrm{O}(3)$	The conventional (*Pauli gauge*) SU(2) matrix that maps onto i is the unit matrix: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mapsto i$.	(11)
$g' = i\,g \in \mathrm{O}(3)$	With the above convention if $+A \mapsto g$, then $+A \mapsto g'$.	(12)

Representation of the double group of O(3) — The mapping $+A \mapsto g$, $+A \mapsto g'$, $-A \mapsto \widetilde{g}$, $-A \mapsto \widetilde{g}'$ forms a (vector) representation of the double group of SO(3), with the conventions used in this book. (13)

Projective representation of O(3) — The mapping $+A \mapsto g$, $+A \mapsto g'$ forms a projective representation of O(3), with the conventions used in this book. (14)

Note — In the scheme here described, which agrees with the one universally used in the literature, the matrices of SU′(2) are merely those of SU(2) used twice, to represent both the proper and the improper operations. (15)

The bilateral-binary rotation matrices. (See **4**.24)

$\check{R}(\pi\mathbf{x})$, $\check{R}(\pi\mathbf{y})$, $\check{R}(\pi\mathbf{z})$

$$\check{R}(\pi\mathbf{x}) = \begin{bmatrix} 0 & -\mathrm{i} \\ -\mathrm{i} & 0 \end{bmatrix}, \quad \check{R}(\pi\mathbf{y}) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \check{R}(\pi\mathbf{z}) = \begin{bmatrix} -\mathrm{i} & 0 \\ 0 & \mathrm{i} \end{bmatrix}. \quad (16)$$

In SO(3) both the matrices given above and their negatives represent the operations stated. The signs chosen for the matrices listed above are such that, with the conventions used in this book, they form a representation of the double group of $\mathbf{D}_2$ when the negatives of the the matrices shown are used for the tilde operations. (They also form, on their own, a projective representation of $\mathbf{D}_2$.) Notice that $\mathrm{i}\,\check{R}(\pi\,\mathbf{n})$ is the Pauli matrix $\boldsymbol{\sigma}_n$ (see 18 below). (17)

The Pauli matrices

$\boldsymbol{\sigma}_x$, $\boldsymbol{\sigma}_y$, $\boldsymbol{\sigma}_z$ — They are the following SU′(2) matrices, in the Condon and Shortley convention used in this book:

$$\boldsymbol{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \boldsymbol{\sigma}_y = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}, \quad \boldsymbol{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \quad (18)$$

Bibliographical note

Most of the results in this chapter may be obtained from Altmann (1986).

12

The continuous groups. Rotations, their matrices, and the irreducible representations of O(3)

1 The continuous groups

$\mathbf{C}_{\infty v}$	The group of the rectangular cone. A continuous axis of rotation at the cone axis and an infinite number of symmetry planes through this axis.	(1)
$\mathbf{D}_{\infty h} = \mathbf{C}_{\infty v} \otimes \mathbf{C}_i$	The group of the rectangular cylinder. A continuous axis of rotation at the cylinder axis, an infinite number of symmetry planes through this axis, and an infinite number of binary axes perpendicular to this axis, lying on a symmetry plane σ_h normal to the axis.	(2)
SO(3)	The group of proper rotations of the sphere with fixed centre.	(3)
O(3)= SO(3) $\otimes$ $\mathbf{C}_i$	The group of proper and improper rotations of the sphere with fixed centre.	(4)

2 Action of a rotation on a vector

$R(\phi\mathbf{n})$ $\qquad$ $R(\phi\mathbf{n})\,\mathbf{r} = \cos\phi\,\mathbf{r} + \sin\phi\,(\mathbf{n}\times\mathbf{r}) + (1-\cos\phi)\,(\mathbf{n}\cdot\mathbf{r})\,\mathbf{n}.$ (5)

3 Rotation matrices

Notation

$\langle a, b, \dots, z\vert$	A row basis of elements $a, b, \dots, z$ to be transformed.	(6)
$\vert a, b, \dots, z\rangle$	A column basis of elements $a, b, \dots, z$ to be transformed.	(7)
$\langle \bar{a}, \bar{b}, \dots, \bar{z}\vert$	A row basis of elements $\bar{a}, \bar{b}, \dots, \bar{z}$ obtained from $a, b, \dots, z$ after an active transformation.	(8)
$\vert \bar{a}, \bar{b}, \dots, \bar{z}\rangle$	A column basis of elements $\bar{a}, \bar{b}, \dots, \bar{z}$, obtained from $a, b, \dots, z$ after an active transformation.	(9)
Transformation for matrix A	$\langle \bar{a}, \bar{b}, \dots, \bar{z}\vert = \langle a, b, \dots, z\vert\, A,$	(10)
	$\vert \bar{a}, \bar{b}, \dots, \bar{z}\rangle = A\,\vert a, b, \dots, z\rangle.$	(11)

Notes For each matrix and for each basis stated the correct one of the two rules (10) or (11) must be used, depending on the nature of the basis used. (See eqns 12 to 18 below.)
See § **3**–2 for the notation for rotations and axes.

The matrices

$R(\phi\mathbf{z})$ $\qquad$ § **2**–2 $\qquad$ $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}.$ $\qquad$ Bases: $\vert x, y, z\rangle$, $\langle x, y, z\vert$, $\langle \mathbf{i}, \mathbf{j}, \mathbf{k}\vert$. (12)

$R(\phi\mathbf{n})$

$$\begin{bmatrix} 1-2(n_y^2+n_z^2)\sin^2\frac{\phi}{2} & -n_z\sin\phi+2n_xn_y\sin^2\frac{\phi}{2} & n_y\sin\phi+2n_zn_x\sin^2\frac{\phi}{2} \\ n_z\sin\phi+2n_xn_y\sin^2\frac{\phi}{2} & 1-2(n_z^2+n_x^2)\sin^2\frac{\phi}{2} & -n_x\sin\phi+2n_yn_z\sin^2\frac{\phi}{2} \\ -n_y\sin\phi+2n_zn_x\sin^2\frac{\phi}{2} & n_x\sin\phi+2n_yn_z\sin^2\frac{\phi}{2} & 1-2(n_x^2+n_y^2)\sin^2\frac{\phi}{2} \end{bmatrix}. \quad (13)$$

Bases: $|x,y,z\rangle$, $\langle x,y,z|$, $\langle \mathbf{i},\mathbf{j},\mathbf{k}|$. (14)

$R(\phi\mathbf{n})$

$$\begin{bmatrix} a^2 & 2^{1/2}ab & b^2 \\ -2^{1/2}ab^* & aa^*-bb^* & 2^{1/2}a^*b \\ b^{*2} & -2^{1/2}a^*b^* & a^{*2} \end{bmatrix}, \quad a=\cos\tfrac{\phi}{2}-\mathrm{i}n_z\sin\tfrac{\phi}{2},\ b=-(n_y+\mathrm{i}n_x)\sin\tfrac{\phi}{2}. \quad (15)$$

Basis: $\langle Y_1^1, Y_1^0, Y_1^{-1}|$. (See **13**.1 for the definition of the spherical harmonics.) (16)

$R(\alpha\beta\chi)$

$$\begin{bmatrix} \cos\alpha\cos\beta\cos\gamma-\sin\alpha\sin\gamma & -\cos\alpha\cos\beta\sin\gamma-\sin\alpha\cos\gamma & \cos\alpha\sin\beta \\ \sin\alpha\cos\beta\cos\gamma+\cos\alpha\sin\gamma & -\sin\alpha\cos\beta\sin\gamma+\cos\alpha\cos\gamma & \sin\alpha\sin\beta \\ -\sin\beta\cos\gamma & \sin\beta\sin\gamma & \cos\beta \end{bmatrix}. \quad (17)$$

Bases: $|x,y,z\rangle$, $\langle x,y,z|$, $\langle \mathbf{i},\mathbf{j},\mathbf{k}|$. (18)

4 The irreducible representations of O(3)

Basis and form of the representation

u_m^j — Eigenfunctions of the z component of the angular momentum operator in the Condon and Shortley phase convention. $j \geq m \geq -j$. Also written as $|jm\rangle$.
The detailed form of these functions is given in **13**.6, **13**.26, and **13**.28. (19)

$\langle u_j^j, \ldots, u_{-j}^j|$ — Row vector of the $2j+1$ functions $u_j^j, \ldots, u_{-j}^j$ also written in abbreviated form as $\langle u^j|$ or $\langle |jm\rangle|$. (20)

$R(\phi\mathbf{n})$, R — Active rotation by ϕ about axis $\mathbf{n}$, abbreviated R. (21)

Form of the representation — $R\,\langle u_j^j, \ldots, u_{-j}^j| = \langle u_j^j, \ldots, u_{-j}^j|\,\check{R} \quad \Rightarrow \quad R\,u_m^j = \sum_{m'=j}^{-j} u_{m'}^j\,\check{R}_{m'm}^j.$ (22)
When j is integral the inverted hat on the R can be read as a straight hat, denoting a vector (ordinary) representation. When j is half-integral the inverted hat denotes a spinor or double-group representation. (23)

Dimension of the representation — $2j+1$. (24)

Matrix element (parameters $\phi\mathbf{n}$) —

$$\check{R}_{m'm}^j = \{(j+m')!\,(j-m')!\,(j+m)!\,(j-m)!\}^{1/2} \times \sum_k \frac{a^{j+m-k}(a^*)^{j-m'-k}b^{m'-m+k}(-b^*)^k}{(j-m'-k)!(j+m-k)!(m'-m+k)!k!}. \quad (25)$$

$a = \cos\frac{\phi}{2} - \mathrm{i}\,n_z\,\sin\frac{\phi}{2}$, $b = -(n_y + \mathrm{i}\,n_x)\sin\frac{\phi}{2}$. (26)

Alternative form of the matrix element —

$$\check{R}_{m'm}^j = \sum_k \left\{\binom{j-m'}{k}\binom{j+m'}{m'-m+k}\binom{j-m}{m'-m+k}\binom{j+m}{k}\right\}^{1/2} \times a^{j+m-k}(a^*)^{j+m'-k}b^{m'-m+k}(-b^*)^k. \quad (27)$$

Choice of k — $k \geq 0$. In both cases the summation over k must be extended over all values of k for which, with the values of m' and m chosen, the arguments of the factorials do not become negative. (28)

Note — The second form of the matrix element is preferable, from the computational point of view, because the binomial coefficients are much smaller than the factorials. (29)

Matrix element (Euler parameters) —

$$\check{R}_{m'm}^j = \exp\{-\mathrm{i}(m'\alpha+m\gamma)\}\,\{(j+m')!\,(j-m')!\,(j+m)!\,(j-m)!\}^{1/2} \times \sum_k (-1)^{k+m'-m}\frac{\cos^{2j-m'+m-2k}(\frac{\beta}{2})\,\sin^{2k+m'-m}(\frac{\beta}{2})}{(j-m'-k)!\,(j+m-k)!\,(m'-m+k)!\,k!}. \quad (30)$$

Improper rotations

$g' = i\,R(\phi\mathbf{n})$	All improper rotations must be written in this form and $\check{R}^j_{m'm}$ obtained from the above expressions.	(31)
j integral	Multiply the matrix elements by $(-1)^j$.	(32)
j half-integral	Multiply the matrix elements by $+1$.	(33)

Special cases

$R(\phi\mathbf{z})$ ($b = 0$)	25	$\check{R}^j_{m'm} = a^{j+m}\,(a^*)^{j-m}\,\delta_{m'm}$.	(34)
$R(\phi\mathbf{m})$, $\mathbf{m}\perp\mathbf{z}$ ($a = 0$)	25	$\check{R}^j_{m'm} = b^{j-m}\,(-b^*)^{j+m}\,\delta_{m',-m}$.	(35)
Note		The above two cases cover all operations in all point groups except cubic and icosahedral.	(36)

The characters

$\chi^j(\phi)$	Character of $R(\phi\mathbf{n})$, which is independent of $\mathbf{n}$.	(37)
Formula	$\chi^j(\phi) = \sin(j+\frac{1}{2})\phi\,(\sin\frac{\phi}{2})^{-1}$, $\quad \phi \neq 0$ or 2π.	(38)
	$= 2j+1$, $\quad \phi = 0$.	(39)
	$= -(2j+1)$, $\quad \phi = 2\pi$.	(40)
	(The value $\phi = 2\pi$ is only used in the double-group method.)	(41)
Improper operation g'	Write $g' = i\,R(\phi\mathbf{n})$. Then: $\chi^j(g') = (-1)^j\,\chi^j(\phi)$.	(42)
Double group operation $\widetilde{g}$	Use the angle ϕ for the corresponding operation g, plus 2π.	(43)
The Clebsch–Gordan series	$\langle u^j\vert \otimes \langle u^{j'}\vert = \langle u^{j+j'}\vert \oplus \langle u^{j+j'-1}\vert \oplus \cdots \oplus \langle u^{j-j'}\vert, \qquad j \geq j'$.	(44)

Bibliographical note

Most of the results of this chapter may be obtained from Altmann (1986).

13

Bases: spherical harmonics, spinors, cartesian tensors, and the functions *s, p, d, f*

1 Integral angular momentum: the spherical harmonics

Spherical harmonics (normalized)
In the Condon and Shortley phase convention, always used in this book, they are given by the following expressions,

$$Y_l^m(\theta,\varphi) = \mathrm{i}^{m+|m|}\, P_l^{|m|}(\cos\theta)\, \exp(\mathrm{i}m\varphi), \tag{1}$$

$$P_l^{|m|}(\cos\theta) = \left\{\frac{(2l+1)\,(l-|m|)!}{4\pi\,(l+|m|)!}\right\}^{1/2} \frac{1}{2^l\, l!}\, \sin^{|m|}\theta\, \frac{\mathrm{d}^{l+|m|}(\cos^2\theta - 1)^l}{\mathrm{d}(\cos\theta)^{l+|m|}}. \tag{2}$$

l integral
$$l = 0, 1, 2, \ldots; \quad l \geq m \geq -l. \tag{3}$$

Effect of inversion
$$i\, Y_l^m(\theta,\varphi) = (-1)^l\, Y_l^m(\theta,\varphi). \tag{4}$$

Conjugation
$$Y_l^m(\theta,\varphi)^* = (-1)^m\, Y_l^{-m}(\theta,\varphi). \tag{5}$$

Notation
$$Y_l^m =_{\mathrm{def}} u_m^l =_{\mathrm{def}} |lm\rangle, \quad l = 0, 1, 2, \ldots; \quad l \geq m \geq -l. \tag{6}$$

Basis of irreducible representation of SO(3)
$\langle u_l^l, \ldots, u_{-l}^l|$, also written as $\big\langle |ll\rangle, \ldots, |l,-l\rangle \big|$ or in abbreviated notation as $\big\langle |lm\rangle \big|$ or $\big\langle |lm_l\rangle \big|$. (7)

Normalization
The basis (7) is normalized to $2l+1$. (8)

2 Half-integral angular momentum: spinors

Notation
$$u_{m_s}^s =_{\mathrm{def}} |sm_s\rangle, \quad s = \tfrac{1}{2}, \quad m_s = \pm\tfrac{1}{2}. \qquad \text{Bases: } \big\langle |sm_s\rangle \big|. \tag{9}$$

Spinors $j = 1/2$
Components of basis: $|\frac{1}{2}\frac{1}{2}\rangle$, $|\frac{1}{2}\frac{\bar{1}}{2}\rangle$. Basis: $\big\langle |\frac{1}{2}\frac{1}{2}\rangle |\frac{1}{2}\frac{\bar{1}}{2}\rangle \big|$. (10)

*Transformation under $R(\phi\mathbf{n})$ (see **11**.8)*
$$R(\phi\mathbf{n})\, \big\langle |\tfrac{1}{2}\tfrac{1}{2}\rangle |\tfrac{1}{2}\tfrac{\bar{1}}{2}\rangle \big| = \big\langle |\tfrac{1}{2}\tfrac{1}{2}\rangle |\tfrac{1}{2}\tfrac{\bar{1}}{2}\rangle \big| \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}, \tag{11}$$

$$a = \cos\tfrac{\phi}{2} - \mathrm{i}n_z \sin\tfrac{\phi}{2}, \quad b = -(n_y + \mathrm{i}n_x)\, \sin\tfrac{\phi}{2}. \tag{12}$$

Transformation under inversion
$$i\, \big\langle |\tfrac{1}{2}\tfrac{1}{2}\rangle |\tfrac{1}{2}\tfrac{\bar{1}}{2}\rangle \big| = \big\langle |\tfrac{1}{2}\tfrac{1}{2}\rangle |\tfrac{1}{2}\tfrac{\bar{1}}{2}\rangle \big| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{13}$$

Complex conjugate spinor
$$\big\langle |\tfrac{1}{2}\tfrac{\bar{1}}{2}\rangle^*, -|\tfrac{1}{2}\tfrac{1}{2}\rangle^* \big|. \tag{14}$$

Transformation under $R(\phi\mathbf{n})$ (see 11)
$$R(\phi\mathbf{n})\, \big\langle |\tfrac{1}{2}\tfrac{\bar{1}}{2}\rangle^*, -|\tfrac{1}{2}\tfrac{1}{2}\rangle^* \big| = \big\langle |\tfrac{1}{2}\tfrac{\bar{1}}{2}\rangle^*, -|\tfrac{1}{2}\tfrac{1}{2}\rangle^* \big| \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}, \tag{15}$$

$$a = \cos\tfrac{\phi}{2} - \mathrm{i}n_z \sin\tfrac{\phi}{2}, \quad b = -(n_y + \mathrm{i}n_x)\, \sin\tfrac{\phi}{2}. \tag{16}$$

Transformation under inversion
$$i\, \big\langle |\tfrac{1}{2}\tfrac{\bar{1}}{2}\rangle^*, -|\tfrac{1}{2}\tfrac{1}{2}\rangle^* \big| = \big\langle |\tfrac{1}{2}\tfrac{\bar{1}}{2}\rangle^*, -|\tfrac{1}{2}\tfrac{1}{2}\rangle^* \big| \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{17}$$

Notation
$$\big\langle |\tfrac{1}{2}\tfrac{\bar{1}}{2}\rangle^*, -|\tfrac{1}{2}\tfrac{1}{2}\rangle^* \big| =_{\mathrm{def}} \big\langle |\tfrac{1}{2}\tfrac{1}{2}\rangle |\tfrac{1}{2}\tfrac{\bar{1}}{2}\rangle \big|^{\bullet} =_{\mathrm{def}} \big\langle |\tfrac{1}{2}\tfrac{1}{2}\rangle^{\bullet} |\tfrac{1}{2}\tfrac{\bar{1}}{2}\rangle^{\bullet} \big|. \tag{18}$$

Remark
Notice that the complex conjugate spinor just defined behaves exactly like the ordinary spinor under rotations but is ungerade, whereas the ordinary spinor is gerade. (19)

Higher order spinors: spin harmonics

Notation $\big\langle |jm_j\rangle \big|$, j half-integral, $j \geq m_j \geq -j$. Always **gerade**. (20)
$\big\langle |jm_j\rangle \big|^{\bullet}$, j half-integral, $j \geq m_j \geq -j$. Always **ungerade**.
(See 18 and 19.) (21)

Normalization The bases (20) and (21) are normalized to $2j+1$. (22)

Note The bases $\big\langle |jm_j\rangle \big|$ and $\big\langle |jm_j\rangle \big|^{\bullet}$ transform identically under rotations, both being bases of the same irreducible representation of SO(3). (23)

Derivation By ls coupling: $\big\langle |lm_l\rangle \big| \otimes \big\langle |sm_s\rangle \big| = \big\langle |jm_j\rangle \big|$, $s = 1/2$, $m_s = \pm 1/2$.
In summation notation, with coefficients (Clebsch–Gordan coefficients, see § **2**–7) acting on the reducible basis on the left:

$$|jm_j\rangle = \sum_{m_l\, m_s} |lm_l\rangle |sm_s\rangle \langle m_l\, m_s \mid jm_j\rangle. \quad (24)$$

Alternative notation

$$|jm_j\rangle = \sum_{m_l\, m_s} |lm_l\rangle |sm_s\rangle \langle ls\, m_l\, m_s \mid jm_j\rangle. \quad (25)$$

Required couplings; bases functions

l **even:** $$|jm_j\rangle = \sum_{m_l\, m_s} Y_l^{m_l}\, |\tfrac{1}{2}m_s\rangle \langle l\tfrac{1}{2}\, m_l\, m_s \mid jm_j\rangle\, \delta_{m_j, m_l+m_s}. \quad (26)$$

$|jm_j\rangle$ **gerade,** $|l-s| \leq j \leq l+s$. (27)

l **odd:** $$|jm_j\rangle^{\bullet} = \sum_{m_l\, m_s} Y_l^{m_l}\, |\tfrac{1}{2}m_s\rangle \langle l\tfrac{1}{2}\, m_l\, m_s \mid jm_j\rangle\, \delta_{m_j, m_l+m_s}. \quad (28)$$

$|jm_j\rangle^{\bullet}$ **ungerade,** $|l-s| \leq j \leq l+s$. (29)

Clebsch–Gordan coefficients: properties (i) They are real. (30)
(ii) Phase-factor convention: $\langle l\tfrac{1}{2} l m_s \mid jj\rangle > 0$. (31)

The coefficients

Table **13**.1 The l, s coupling for $s = 1/2$. Clebsch–Gordan coefficients

j	m_s	$\langle l\tfrac{1}{2}\, m_l\, m_s \mid jm_j\rangle$
$l-\tfrac{1}{2}$	$\tfrac{1}{2}$	$-\big\{(l-m_j+\tfrac{1}{2})/(2l+1)\big\}^{1/2}$
$l-\tfrac{1}{2}$	$-\tfrac{1}{2}$	$\big\{(l+m_j+\tfrac{1}{2})/(2l+1)\big\}^{1/2}$
$l+\tfrac{1}{2}$	$\tfrac{1}{2}$	$\big\{(l+m_j+\tfrac{1}{2})/(2l+1)\big\}^{1/2}$
$l+\tfrac{1}{2}$	$-\tfrac{1}{2}$	$\big\{(l-m_j+\tfrac{1}{2})/(2l+1)\big\}^{1/2}$

$m_j = m_l + m_s$

3 Relation between the bases of SO(3) and those of O(3)

SO(3): $\big\langle |jm_j\rangle \big|$, $\big\langle |jm_j\rangle \big|^{\bullet}$ *(embellished bases)*

For the same half-integral j, bases $\big\langle |jm_j\rangle \big|$ and $\big\langle |jm_j\rangle \big|^{\bullet}$ may be formed from (26) and (28) which are g and u, respectively, but span the same irreducible representation $\check{R}^j$ of SO(3). (32)

SO(3): $\big\langle |lm_l\rangle \big|$, $\big\langle |lm_l\rangle \big|^{\blacksquare}$ *(embellished bases)*

From (4), on the other hand, it appears that for integral j, (l), the parity of the bases is fixed for each l. This is not so. By coupling of the form $\big\langle |\tfrac{1}{2}\tfrac{1}{2}\rangle |\tfrac{1}{2}\overline{\tfrac{1}{2}}\rangle \big| \otimes \big\langle |\tfrac{1}{2}\tfrac{1}{2}\rangle |\tfrac{1}{2}\overline{\tfrac{1}{2}}\rangle \big|$ and $\big\langle |\tfrac{1}{2}\tfrac{1}{2}\rangle |\tfrac{1}{2}\overline{\tfrac{1}{2}}\rangle \big| \otimes \big\langle |\tfrac{1}{2}\tfrac{1}{2}\rangle |\tfrac{1}{2}\overline{\tfrac{1}{2}}\rangle \big|^{\bullet}$ two bases are obtained for $l = 1$ which are g and u, respectively, the second of which is $\big\langle |11\rangle, |10\rangle, |1\bar{1}\rangle \big|$. In the same manner we can proceed for all l. We shall define bases
$\big\langle |lm_l\rangle \big|^{\blacksquare}$: g, $\forall l$ odd; u, $\forall l$ even, (33)
and such that they span identically the same representation as $\big\langle |lm_l\rangle \big|$ in SO(3).
The basis $\big\langle |11\rangle, |10\rangle, |1\bar{1}\rangle \big|^{\blacksquare}$ gives the functions S_x, S_y, S_z of Koster *et. al.* (1963). The functions R_x, R_y, R_z defined in §**16**–5 are another example of bases derived from $\big\langle |11\rangle, |10\rangle, |1\bar{1}\rangle \big|^{\blacksquare}$.

O(3)

For j half-integral, the bases $\langle|jm_j\rangle|$ and $\langle|jm_j\rangle|^\bullet$ belong respectively to gerade, $\breve{R}^j_g$, and ungerade, $\breve{R}^j_u$, irreducible representations of O(3). For integral j, (l), and l even, $\langle|lm_l\rangle|$ and $\langle|lm_l\rangle|^\bullet$ belong respectively to g and u irreducible representations of O(3). For l odd, $\langle|lm_l\rangle|$ and $\langle|lm_l\rangle|^\bullet$ belong respectively to u and g irreducible representations of O(3). (34)

Subduction to point groups

When subducing from O(3) down to a point group G, if G is a proper group then, whenever a basis such as $\langle|jm_j\rangle|$ and $\langle|lm_l\rangle|$ are listed in the table of the symmetrized bases of G, the embellished bases $\langle|jm_j\rangle|^\bullet$ and $\langle|lm_l\rangle|^\bullet$ also span identically the same representations but **are not listed in the tables. Notice also that in improper groups the bases $\langle|jm_j\rangle|^\bullet$ are always listed whenever they appear but, in order to simplify the tables, the bases $\langle|lm_l\rangle|^\bullet$ are not so listed. It should not be assumed that the symmetry assignment of $\langle|lm_l\rangle|^\bullet$ can be derived in a simple manner from that of $\langle|lm_l\rangle|$.** (35)

4 Cartesian tensors

Rank 0

The scalar number 1. (36)

Rank 1

The column vector $|x, y, z\rangle$ with x, y, z being the components of a unit position vector. They are **independent variables**, not functions. (37)

Higher rank

Obtained by forming **symmetrized** tensor products $|x, y, z\rangle \,\overline{\otimes}\, |x, y, z\rangle$ in which the symmetric component $xy + yx$ is written as xy, and the same for the others. Such products are repeated for the higher ranks. (38)

Surface condition

The condition

$$r^2 = x^2 + y^2 + z^2 = 1, \qquad (39)$$

which constrains the cartesian tensors to the surface of the unit sphere, is used in some particular cases. (See **16**.1.)

Harmonicity

Cartesian tensors that give the spherical harmonics in cartesian form must satisfy Laplace's equation. This reduces the number of independent cartesian tensors as follows:
Number of independent cartesian harmonic tensors of rank $k = 2k + 1$. (40)

Rank 2

x^2, y^2, z^2, xy, yz, zx. (Only five independent harmonics.) (41)

Rank 3

x^3, y^3, z^3, x^2y, xyz, zx^2, xy^2, y^2z, yz^2, z^2x. (Only seven independent harmonics.) (42)

Warning

The cartesian tensor bases transform under (**12**.11) as column and not as row bases.

5 The *s*, *p*, *d*, and *f* functions

Function type	Conventional subscript of the function	Expression in terms of the spherical harmonics in the Condon–Shortley convention	Full cartesian form ($r^2 = 1$)
s		Y_0^0	
p	x	$-\left(\frac{8\pi}{3}\right)^{1/2}\frac{1}{2}(Y_1^1 - Y_1^{-1})$	x
p	y	$\left(\frac{8\pi}{3}\right)^{1/2}\frac{1}{2}(Y_1^1 + Y_1^{-1})\,\mathrm{i}$	y
p	z	$\left(\frac{4\pi}{3}\right)^{1/2} Y_1^0$	z
d	zx	$-\left(\frac{8\pi}{15}\right)^{1/2}\frac{1}{2}(Y_2^1 - Y_2^{-1})$	zx
d	yz	$\left(\frac{8\pi}{15}\right)^{1/2}\frac{1}{2}(Y_2^1 + Y_2^{-1})\,\mathrm{i}$	yz
d	$x^2 - y^2$	$\left(\frac{32\pi}{15}\right)^{1/2}\frac{1}{2}(Y_2^2 + Y_2^{-2})$	$x^2 - y^2$
d	xy	$-\left(\frac{32\pi}{15}\right)^{1/2}\frac{1}{4}(Y_2^2 - Y_2^{-2})\,\mathrm{i}$	xy
d	z^2	$\left(\frac{16\pi}{5}\right)^{1/2} Y_2^0$	$3z^2 - 1$
f	xz^2	$-\left(\frac{64\pi}{21}\right)^{1/2}\frac{1}{2}(Y_3^1 - Y_3^{-1})$	$x(5z^2 - 1)$
f	yz^2	$\left(\frac{64\pi}{21}\right)^{1/2}\frac{1}{2}(Y_3^1 + Y_3^{-1})\,\mathrm{i}$	$y(5z^2 - 1)$
f	$z(x^2 - y^2)$	$\left(\frac{32\pi}{105}\right)^{1/2}\frac{1}{2}(Y_3^2 + Y_3^{-2})$	$z(x^2 - y^2)$
f	xyz	$-\left(\frac{32\pi}{105}\right)^{1/2}\frac{1}{4}(Y_3^2 - Y_3^{-2})\,\mathrm{i}$	xyz
f	$x(x^2 - y^2)$	$-\left(\frac{64\pi}{35}\right)^{1/2}\frac{1}{2}(Y_3^3 - Y_3^{-3})$	$x(x^2 - 3y^2)$
f	$y(x^2 - y^2)$	$\left(\frac{64\pi}{35}\right)^{1/2}\frac{1}{2}(Y_3^3 + Y_3^{-3})\,\mathrm{i}$	$y(3x^2 - y^2)$
f	z^3	$\left(\frac{16\pi}{7}\right)^{1/2} Y_3^0$	$5z^3 - 3z$

Bibliographical note

The ungerade spinor for $j = 1/2$ was introduced by Altmann (1986, 1987). There are innumerable notations for the Clebsch–Gordan coefficients in the full rotation group; the one used agrees with Condon and Shortley (1957) and Brink and Satchler (1968). The phase-factor condition given for the Clebsch–Gordan coefficients is standard (see Messiah 1961). The Clebsch–Gordan coefficients tabulated in Table 1 are given by Condon and Odabaşı (1980, p. 149). The coupling scheme used, although given in a more compact notation, agrees exactly with that adopted by Pyykkö and Toivonen (1983). A proof of eqn (40) may be obtained from Altmann (1986, p. 94). Further discussion of cartesian tensors and their symmetries may be found in Bhagavantam (1966) and Zheludev (1990).

14

Notation for the irreducible representations

1 The basic symbols

A	Singly-degenerate representation, symmetrical with respect to rotation about the principal axis. (See **4**.21.)	(1)
B	Singly-degenerate representation, antisymmetrical with respect to rotation about the principal axis. (See **4**.21.)	(2)
E	**Either:** one of a pair of singly-degenerate *conjugate* representations. (These are representations in which the representative of an operation in one representation is the complex conjugate of the representative of the same operation in the other representation.)	(3)
	Or: doubly-degenerate representation.	(4)
T	Triply-degenerate representation.	(5)
F	**Either:** one of a pair of doubly-degenerate *conjugate* representations. (See 3.)	(6)
	Or: four-fold degenerate representation.	(7)
H	Five-fold degenerate representation.	(8)
I	Six-fold degenerate representation.	(9)

2 Embellishments

Left superscript 1, 2 *on* E *or* F	Distinguishes between two conjugate representations.	(10)
	Whenever the distinction is possible, the left superscript 1 denotes the representation spanned by a spherical harmonic with positive m or by a combination of spherical harmonics with positive sign.	(11)
Subscripts 1, 2 *on* A *or* B	The subscript 1 indicates the most symmetrical of the A or B representations with respect to binary axes perpendicular to **z**.	(12)
	The A_1 representation is the trivial (totally symmetrical representation) in all groups.	(13)
Subscripts 1, 2, 3, *etc.* *on* E, T, F *or* H	Indicate representations spanned, whenever the distinction is possible, by spherical harmonics in ascending order of m.	(14)
Half-integral subscripts on A, E, T, F *or* I	They indicate double-group or spinor representations.	(15)
	The subscript $n/2$ indicates that the corresponding representation is spanned by spinor bases for $j = n/2$.	(16)
	For proper groups, the $E_{1/2}$ representation is a faithful (see **2**.49) spinor representation of the group.	(17)
Subscripts g *and* u *on any symbol*	They mean symmetrical and antisymmetrical representations, respectively, with respect to the inversion.	(18)
Primes and double primes on any symbol	They mean symmetrical and antisymmetrical representations, respectively, with respect to σ_h (symmetry plane perpendicular to **z**).	(19)

These prime and double-prime embellishments are not used if the inversion i belongs to the group, in which case the g and u subscripts take priority and are the only ones used. (20)

3 Lower-case symbols

a_{1g}, t_{2u} Symbols in lower case denote functions that belong to the basis of the corresponding irreducible representation. (21)

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Stereographic projections and three-dimensional drawings of point groups

1 Key to the symbols for the stereographic projections

In the stereographic projection the object is assumed placed at the centre of a unit sphere where the right-handed unit axes $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ meet. The axes and planes of symmetry are extended to cut the sphere, the axes at points (poles and antipoles) and the planes at great circles. The points and great circles of the sphere are projected onto the $\mathbf{x}, \mathbf{y}$ plane shown on the left (projection circle). The projection of a point is the intersection with the projection circle of the line which joins $-\mathbf{z}$ to the point. The poles project as points. The great circles corresponding to planes perpendicular to the $\mathbf{x}, \mathbf{y}$ plane project as straight lines and the great circles corresponding to other planes project as arcs, except that if the symmetry plane is on the $\mathbf{x}, \mathbf{y}$ plane then its projection is the projection circle itself. (1)

Alternative settings When alternative settings are used for the same group two stereographic projections, labelled A and B respectively, are shown. These labels are carried over in the corresponding tables. (2)

A full polygon of n sides denotes a rotation axis C_n. A full circle denotes an infinitely continuous rotation axis. (3)
This symbol entails all the rotation axes in the group of C_n, which are not explicitly labelled in the figures unless necessary for precisely positioning the poles. Thus, the symbol on the left corresponding to C_6 entails C_3 and C_2, not explicitly labelled. (4)
The principal axis is always along the $\mathbf{z}$ axis and it is not usually labelled in the stereographic projection. The poles of C_n^+ and C_n^- for the principal axis are along $+\mathbf{z}$ and $-\mathbf{z}$ respectively. (5)

A full digon denotes a rotation axis C_2 (binary axis). A large digon as shown here is used for binary axes along the $\mathbf{z}$ axis. (6)

A C_2 axis on the $\mathbf{x}, \mathbf{y}$ plane. The full digon is the pole (that is, the point used in the tables in order to determine uniquely the vector $\mathbf{n}$ that gives the axis of rotation) and the hatched digon is the antipole. (7)

An open n-sided polygon (or an open digon) denotes a rotoreflection axis S_n. Although this figure should always be a regular polygon, a stellated polygon is used when n is larger than ten in order to help readability. Notice that in this case some of the symmetry elements of the point group do not appear in this figure, although, of course, they are fully identified in the stereographic projection. Large open digons are used for S_2 axes along $\mathbf{z}$. (8)

Centre of inversion i. (9)

Reflection plane perpendicular to the $\mathbf{x}, \mathbf{y}$ plane. Its pole is normal to the line shown to denote the plane and it is such that the set 'pole, σ, $\mathbf{z}$' is always right-handed. (10)

A reflection plane coinciding with the $\mathbf{x}, \mathbf{y}$ plane. Except in the icosahedral group $\mathbf{I}_h$, it is always labelled σ_h unless two other planes exist, perpendicular to $\mathbf{x}$ (σ_x) and to $\mathbf{y}$ (σ_y), in which case it is called σ_z. In $\mathbf{I}_h$ this plane is labelled σ_c. (11)

A reflection plane not perpendicular to the $\mathbf{x}, \mathbf{y}$ plane. Such planes appear only in the cubic and icosahedral groups. **When grey, such planes are not symmetry planes.** (12)

2 Key to the symbols for the three-dimensional drawings

General All the above symbols are used, except that planes are represented as such, in full lines. (13)

These circles represent atoms or totally symmetric groups of atoms of different kinds. Not to be confused with the centre of inversion, a small circle always at the centre of coordinates. (14)

A short segment connected to a circle is used in order to facilitate the reading of the displacement from an edge of the particle indicated by the circle. (15)

Bibliographical note

The stereographic projection is discussed in all books on crystallography. See, for example, Kelly and Groves (1973). Notice that the drawings of stereographic projections provided in this book differ from those given in the *International tables for crystallography* (1989), where rotoinversion axes, rather than rotoreflection axes, are used.

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How to use the tables

General instructions

Use of this chapter	This chapter has been made as self-contained as possible and it should allow most people in most cases to use the tables without detailed study of the previous sections in Part 1 of this book.
Do you have to know projective representations?	**No.** Most readers will be unfamiliar with projective representations and will prefer to use double-group techniques. They can do so without any knowledge whatever of projective representations as long as they use the tables that involve the latter purely as auxiliary constructions (for which simple instructions are given in this chapter) in order to obtain the double-group tables that they require, which would otherwise have been too bulky to print.

Description of the tables

Headers	The page number on them gives a reference to the key for the header notation.
Subsections labelled (1), (2), ... *after the header*	The page number on the header gives a reference to the key for the notation used in these subsections.
Subsection (2). *Group chains. Subduction*	The group chains listed are given for one supergroup and one subgroup of the given group, both of the nearest order to that of the given group. Larger group chains can be constructed either by referring to the tables of the supergroup and subgroup or to the graphs in § **9**–5. Possible problems arising on subduction along the group chains are described in (**9**.6) to (**9**.8) and can be analyzed from the chains constructed by either of the methods here mentioned. See also Subsection (6) in the cubic and icosahedral groups.
The body of the tables	The point-group tables are labelled in the form T **n**.i where **n** is the number of the point group in the conventional order used in this book and the numeral i identifies a table providing some specific information, such as parameters, multiplication rules, etc. for each group. When there are two settings for the group, the labels A and B defined in (**15**.2) are used in order to distinguish the two different versions required for T **n**.1, T **n**.6, T **n**.7, and T **n**.11.
Instructions to use the tables	**They are given in the notes below, which are numbered with the same digit i as used in the tables**. They provide a complete key for each table T **n**.i and, where necessary for greater clarity, examples of the use of the table are given. The heading of each table T **n**.i contains a reference to § **16**–i which is section i of the present chapter, as well as to the page on which that section starts.
Footers	The page number on the header gives a reference to the use of the footers.

0 Subgroup elements

Purpose of the table This table appears only for point groups **n** that are heads of group chains so that their tables of parameters (T **n**.1), multiplication table (T **n**.2), factor system (T **n**.3), and matrix representations (T **n**.7) are used as master tables for all the subgroups within the chain. This table gives the correlation between the symbols of the operations of the master group and those of the subgroups. (Although the nomenclature has been chosen so as to minimize the number of changes of the symbol of the same operation in different groups, it is not possible to avoid all such changes.)

Instructions Read the operation h in the column corresponding to a subgroup H (group number **m**); the operation g of the master group in the extreme left of the same row is the one that must be used in order to generate the entry for h in T **m**.1, T **m**.2, T **m**.3 and T **m**.7.

References The subgroup tables T **m**.i ($i = 1, 2, 3, 7$) contain references to the master tables T **n**.i as well as to the subgroup elements table T **n**.0.

Absence of this table (i) It is not present in groups which are not heads of group chains.
(ii) It is not present in groups which are heads of group chains if there are no changes in nomenclature of the symmetry operations all through the group chain.

1 Parameters

Notation for the headers of T n.1

'Use T **m**.1*'* When this entry appears the parameters for the operations of the group **m** must be read for the operations of the same name in T **m**.1.

'Use T **m**.1 $\diamond$*'* When this entry appears:
(i) Identify the operations of the group **n** (listed at the top of T **n**, in subsections 3 and 4) in T **m**.0.
(ii) With this identification, obtain the parameters for **n** from T **m**.1.
See § 0.

Instructions

Operations g Given an operation g listed on the first column of the table, the entries under the columns labelled α, β, γ are the Euler parameters (see **3**.7). The columns ϕ and **n** give the angle and pole of rotation (see **3**.22, **3**.23). The columns λ and $\mathbf{\Lambda}$ give the quaternion parameters $\lambda = \cos\frac{\phi}{2}$, $\mathbf{\Lambda} = \sin\frac{\phi}{2}\,\mathbf{n}$ (see **3**.28 to **3**.30).

Improper operations All improper operations are given as ig for g proper, and their parameters are listed for g. The symbol i can be considered as a marker (see **3**.37), which **is not given in the tables**, such that $i^2 = 1$. When multiplying two operations of the group the appearance of the marker on the result indicates that the latter is an improper operation.

Operations $\widetilde{g}$ In order to obtain the parameters of the operation $\widetilde{g}$ proceed as follows, from the entries corresponding to g. The Euler angles should not be used since they cannot be defined uniquely for these operations. Add 2π to the angle ϕ and leave **n** unchanged. Change the signs of λ and $\mathbf{\Lambda}$.

2 Multiplication table

Notation for the headers of T **n**.2

'Use T **m**.2*'* — When this entry appears the products for the operations of the group **m** in T **m**.2 must be read for the operations of the same name in the group **n**.

'Use T **m**.2 ◇*'* — When this entry appears:
(i) Identify the operations of the group **n** (listed at the top of T **n**, in subsections 3 and 4) in T **m**.0.
(ii) With this identification, obtain the products for the group **n** from T **m**.2. See § 0.

'Use T **m**.2 ▪*'* — When this entry appears the group number **n** is of the form $L = H \otimes \mathbf{C}_i$, where H is the group number **m**. All operations of L are given by $l = h$ and $l = h\,i$, $\forall h \in H$. The necessary forms of the products of the operations of L are given in the following table:

	h'	$h'\,i$
h	$h\,h'$	$h\,h'\,i$
$h\,i$	$h\,h'\,i$	$h\,h'$

The names of the operations of the form $h\,i$ are obtained from the second column of the parameter table for L, T **n**.1. The products $h\,h'$ are obtained from the multiplication table for H, T **m**.2.

Instructions

Multiplication rules under the group G — The result of the product $g_i\,g_j$ appears in the intersection of the row g_i with the column g_j.

Multiplication rules under the group $\widetilde{G}$ — (i) Product $g_i\,g_j$. Read the entry, say g_k, in the intersection of the row g_i with the column g_j and read from Table **n**.3 (see § 3 below) the factor in the intersection of the row g_i with the column g_j. If this factor is 1 the desired product is g_k. If this factor is -1 the desired product is $\widetilde{g}_k$.
(ii) Product $\widetilde{g}_i\,g_j$ or $g_i\,\widetilde{g}_j$. Obtain first the product $g_i\,g_j$ as explained in (i) and add a tilde to the result. (Adding a tilde to $\widetilde{g}_k$ gives g_k.)
(iii) Product $\widetilde{g}_i\,\widetilde{g}_j$. Obtain first the product $g_i\,g_j$ as explained in (i) and take it without change.

Example. Obtention of the multiplication table for $\widetilde{\mathbf{D}}_2$

Necessary data

T **22**.2 Multiplication table

$\mathbf{D}_2$	E	C_{2z}	C_{2x}	C_{2y}
E	E	C_{2z}	C_{2x}	C_{2y}
C_{2z}	C_{2z}	E	C_{2y}	C_{2x}
C_{2x}	C_{2x}	C_{2y}	E	C_{2z}
C_{2y}	C_{2y}	C_{2x}	C_{2z}	E

T **22**.3 Factor table

$\mathbf{D}_2$	E	C_{2z}	C_{2x}	C_{2y}
E	1	1	1	1
C_{2z}	1	−1	1	−1
C_{2x}	1	−1	−1	1
C_{2y}	1	1	−1	−1

The result

Table **16**.1 Multiplication table for $\widetilde{\mathbf{D}}_2$

$\widetilde{\mathbf{D}}_2$	E	C_{2z}	C_{2x}	C_{2y}	$\widetilde{E}$	$\widetilde{C}_{2z}$	$\widetilde{C}_{2x}$	$\widetilde{C}_{2y}$
E	E	C_{2z}	C_{2x}	C_{2y}	$\widetilde{E}$	$\widetilde{C}_{2z}$	$\widetilde{C}_{2x}$	$\widetilde{C}_{2y}$
C_{2z}	C_{2z}	$\widetilde{E}$	C_{2y}	$\widetilde{C}_{2x}$	$\widetilde{C}_{2z}$	E	$\widetilde{C}_{2y}$	C_{2x}
C_{2x}	C_{2x}	$\widetilde{C}_{2y}$	$\widetilde{E}$	C_{2z}	$\widetilde{C}_{2x}$	C_{2y}	E	$\widetilde{C}_{2z}$
C_{2y}	C_{2y}	C_{2x}	$\widetilde{C}_{2z}$	$\widetilde{E}$	$\widetilde{C}_{2y}$	$\widetilde{C}_{2x}$	C_{2z}	E
E	$\widetilde{E}$	$\widetilde{C}_{2z}$	$\widetilde{C}_{2x}$	$\widetilde{C}_{2y}$	E	C_{2z}	C_{2x}	C_{2y}
C_{2z}	$\widetilde{C}_{2z}$	E	$\widetilde{C}_{2y}$	C_{2x}	C_{2z}	$\widetilde{E}$	C_{2y}	$\widetilde{C}_{2x}$
C_{2x}	$\widetilde{C}_{2x}$	C_{2y}	E	$\widetilde{C}_{2z}$	C_{2x}	$\widetilde{C}_{2y}$	$\widetilde{E}$	C_{2z}
C_{2y}	$\widetilde{C}_{2y}$	$\widetilde{C}_{2x}$	C_{2z}	E	C_{2y}	C_{2x}	$\widetilde{C}_{2z}$	$\widetilde{E}$

Construction of the table

Block in the box

The products $g_i\, g_j$ in the block in the box are obtained from the multiplication rule (i) above. For example: from T **22**.2, $C_{2z}\, C_{2y} = C_{2x}$. From T **22**.3 the factor corresponding to the product $C_{2z}\, C_{2y}$ is -1. Therefore, $C_{2z}\, C_{2y} = \widetilde{C}_{2x}$, as shown in the table.

Blocks on the right of and below the box

The products are obtained from the multiplication rule (ii) above, as a result of which both blocks are equal and they are the 'complement' of the basic block (boxed), in the sense that any operation g in the basic block is replaced by $\widetilde{g}$ and any operation $\widetilde{g}$ is replaced by g.

Block on the bottom right-hand corner

It is obtained from rule (iii) above, which makes it identical to the basic block of the table.

3 Factor table

Notation for the headers of T **n**.3

'Use T **m**.3*'*

When this entry appears the factors for the operations of the group **m** in T **m**.3 must be read for the operations of the same name in the group **n**.

'Use T **m**.3 ◇*'*

When this entry appears:
(i) Identify the operations of the group **n** (listed at the top of T **n**, in subsections 3 and 4) in T **m**.0.
(ii) With this identification, obtain the factors for the group **n** from T **m**.1.
See § 0.

'Use T **m**.3 ▪*'*

When this entry appears the group number **n** is of the form $L = H \otimes \mathbf{C}_i$, where H is the group number **m**. All operations of L are given by $l = h$ and $l = h\, i$, $\forall h \in H$. The necessary forms of the factors of the operations of L are given in the following table:

	h'	$h'\, i$
h	$[h, h']$	$-[h, h']$
$h\, i$	$-[h, h']$	$[h, h']$

The names of the operations of the form $h\, i$ are obtained from the second column of the parameter table for L, T **n**.1. The factors $[h, h']$ are obtained from the factor table for H, T **m**.3.

Instructions

Factor for the product $g_i\, g_j$

It appears in the intersection of the row g_i with the column g_j.

Use of the factor	This factor is used to obtain multiplication rules in $\widetilde{G}$ (see § 2 above) but it is also the projective factor $[g_i, g_j]$ which appears in the projective representations when the matrices corresponding to g_i and g_j are multiplied (see **10**.29).

4 Character table

First column	It lists all the ordinary (vector) representations as well as the spinor (or double-group) representations which appear in the group. The spinor representations are identified by half-integral subscripts. See §§ **14**–1 and **14**–2 for the notation.

Obtention of the character table for the double group

To form the head row of the table	This is given by the list of classes of $\widetilde{G}$ shown in Subsection 4 at the top of each point-group table T **n**.
To form the body of the table	You require the character table T **n**.4.
Vector representations	For classes that contain an operation g copy the character of the operation g in that class given in T **n**.4. For classes that contain **only** operations $\widetilde{g}$, copy the character given in T **n**.4 for the corresponding operation g.
Spinor representations	For classes that contain an operation g copy the character of the operation g in that class given in T **n**.4. For classes that contain **only** operations $\widetilde{g}$, copy the negative of the character given in T **n**.4 for the corresponding operation g.

Example. Obtention of the character table for $\widetilde{\mathbf{D}}_2$

Classes

T **22**, *Subsection* 4

E, $(C_{2z}, \widetilde{C}_{2z})$, $(C_{2x}, \widetilde{C}_{2x})$, $(C_{2y}, \widetilde{C}_{2y})$, $\widetilde{E}$.

Data and results

T **22**.4 Character table

$\mathbf{D}_2$	E	C_{2z}	C_{2x}	C_{2y}	τ
A	1	1	1	1	a
B_1	1	1	−1	−1	a
B_2	1	−1	−1	1	a
B_3	1	−1	1	−1	a
$E_{1/2}$	2	0	0	0	c

Table **16**.2 Character table for $\widetilde{\mathbf{D}}_2$

$\widetilde{\mathbf{D}}_2$	E	C_{2z} $\widetilde{C}_{2z}$	C_{2x} $\widetilde{C}_{2x}$	C_{2y} $\widetilde{C}_{2y}$	$\widetilde{E}$	τ
A	1	1	1	1	1	a
B_1	1	1	−1	−1	1	a
B_2	1	−1	−1	1	1	a
B_3	1	−1	1	−1	1	a
$E_{1/2}$	2	0	0	0	−2	c

Time reversal: column headed 'τ' in the tables

Notation	$\check{G}$: the representation listed in column 1 of the tables, vector (integral angular momentum, even number of electrons) or spinor representation (half-integral angular momentum, odd number of electrons). $\check{G}^*$: its complex conjugate.

Notation for 'τ'

Table **16**.3 Time-reversal classification

	$\check{G}$, $\check{G}^*$	Vector representation	Spinor representation
a	Real and equal	No extra degeneracy	Doubled degeneracy
b	Complex and inequivalent	Doubled degeneracy	Doubled degeneracy
c	Complex and equivalent	Doubled degeneracy	No extra degeneracy

5 Cartesian tensors. The *s*, *p*, *d*, and *f* functions

The cartesian tensors (up to and including rank 3)

Head row — Identifies the rank of the tensor.

First column — Identifies the irreducible representation. For the purpose of this table the conjugate one-dimensional representations ^1E and ^2E are joined together in a two-dimensional (reducible) representation $^1E \oplus {}^2E$. (This is necessary in order to construct the required real bases.)

Brackets — Cartesian tensors that span double or triple irreducible representations are joined within brackets either curved or curly. The type of bracket used has no notational significance.

Rank 0 — There is only one, the number 1, which always belongs to the totally symmetrical representation.

Rank 1 *(vectors)* — Under this heading the *axial* vectors that correspond to the irreducible representations are also given. The axial vector parallel to the **x** axis is listed as R_x and similarly for the others. These axial vectors have the same symmetry properties as rotation operations about the corresponding axes and are often called *rotation vectors*.

Rank 2 — On making, in this case, use of the surface condition (**13**.39)

$$x^2 + y^2 + z^2 = 1, \tag{1}$$

the combination shown on (L1), which strictly speaking is a tensor of rank 0, always belongs to the totally symmetric representation of the group and it is **not listed** on the tables unless it is the only expression involving tensors of rank 2 that belongs to that representation.

Rank 3 — Ten tensors of rank 3 are always listed in the tables.

Cartesian harmonics — The number of independent cartesian harmonics (see **13**.40) is: rank zero: 1; rank one: 3; rank two: 5; rank three: 7.

Warnings — (i) The 1-, 2-, or 3-dimensional bases formed by the cartesian tensors listed can be understood and used in two different senses. The elements of the bases can be taken to be the independent variables, in which case the bases must be written as column vectors and are pre-multiplied by the representative matrices. Or the bases can be taken to be functions, in which case x, y, and z should be understood as the functions x, y, and z (see **2**.32) and the bases must be written as row vectors, which are post-multiplied by the representative matrices.
(ii) The cartesian functions listed in the tables T **n**.5 span representations which are not necessarily identical with those listed in the representation tables T **n**.7, the latter always being constructed on bases derived from spherical harmonics.

The *s*, *p*, *d*, and *f* functions

Their identification — The cartesian tensors that allow the identification of the s, p, d, and f orbitals are recognized in T **n**.5 by a left superscript, such as in the symbol $^{\square}x$. Such entries lead to the correct subscript of the orbital function symbol, which must be read from Table 4 below. The full form of the orbital in terms of the spherical harmonics may be obtained from the table in § **13**–5.

Subscripts of the s, p, d, f functions

Table **16**.4 Correspondence between the cartesian tensors listed in the tables and the s, p, d, and f functions

Cartesian tensor listed in the tables with a left superscript □	Function	Subscript of the function
1	s	s
x	p	x
y	p	y
z	p	z
zx	d	zx
yz	d	yz
xy	d	xy
$2z^2 - x^2 - y^2$ or z^2	d	z^2
$x^2 - y^2$ or x^2, y^2	d	$x^2 - y^2$
xz^2 or $x(4z^2 - x^2 - y^2)$	f	xz^2
yz^2 or $y(4z^2 - x^2 - y^2)$	f	yz^2
$z(x^2 - y^2)$ or x^2z, y^2z	f	$z(x^2 - y^2)$
xyz	f	xyz
$x(x^2 - 3y^2)$ or x^3, xy^2	f	$x(x^2 - y^2)$
$y(3x^2 - y^2)$ or x^2y, y^3	f	$y(x^2 - y^2)$
z^3 or $z(2z^2 - 3x^2 - 3y^2)$	f	z^3

Warning

The functions obtained from this table span representations not necessarily identical with those listed in the representation tables T **n**.7, always constructed on bases derived from spherical harmonics.

Example. Cartesian tensors and *s*, *p*, *d*, and *f* functions for $\mathbf{D}_8$

T **28**.5 Cartesian tensors and s, p, d, and f functions

$\mathbf{D}_8$	0	1	2	3
A_1	□1		x^2+y^2, □z^2	
A_2		□z, R_z		$(x^2+y^2)z$, □z^3
B_1				
B_2				
E_1		□$(x,y), (R_x, R_y)$	□(zx, yz)	$\{x(x^2+y^2), y(x^2+y^2)\}$, □$(xz^2, yz^2)$
E_2			□(xy, x^2-y^2)	□$\{xyz, z(x^2-y^2)\}$
E_3				□$\{x(x^2-3y^2), y(3x^2-y^2)\}$

Cartesian tensors. Rank 2

For A_1: either x^2+y^2 or z^2, or a linear combination of these two bases.

Cartesian tensors. Rank 3

For E_1: either $\{x(x^2+y^2), y(x^2+y^2)\}$ or (xz^2, yz^2) or a linear combination of these two bases.

d functions

For A_1: from Table 4, d_{z^2}.
For E_1: from Table 4, (d_{zx}, d_{yz}).

f functions

For E_1: from Table 4, (f_{xz^2}, f_{yz^2}).
For E_2: from Table 4, $(f_{xyz}, f_{z(x^2-y^2)})$.
For E_3: from Table 4, $(f_{x(x^2-y^2)}, f_{y(x^2-y^2)})$.

6 Symmetrized bases

General notes

Ket symbols used

$\|jm\rangle$	j, m integral, normalized spherical harmonics, eqn (**13**.1).
$\|jm\rangle$	j, m half-integral, normalized gerade spin harmonics, eqn (**13**.26).
$\|jm\rangle^{\bullet}$	j, m half-integral, normalized ungerade spin harmonics, eqn (**13**.27).
$\|jm\rangle_{+}$	$=_{\text{def}}\ 2^{-1/2}\,(\|jm\rangle + \|j\,\overline{m}\rangle)\,, \quad m > 0.$
$\|j0\rangle_{+}$	$=_{\text{def}}\ \|j0\rangle.$
$\|jm\rangle_{-}$	$=_{\text{def}}\ 2^{-1/2}\,(\|jm\rangle - \|j\,\overline{m}\rangle)\,, \quad m > 0.$
$\langle \|j_1m_1\rangle, \ldots, \|j_nm_n\rangle \|$	Row vector of n components (columns) $\|j_1m_1\rangle, \|j_2m_2\rangle, \ldots, \|j_nm_n\rangle$. (See **2**.59.) Always gerade. (See **13**.20).
$\langle \|j_1m_1\rangle, \ldots, \|j_nm_n\rangle \|^{\bullet}$	All the components of this basis are ungerade spin harmonics. (See **13**.21).

Use of the bases

Function space	The only operators that can be applied on these bases are function space operators. (See **2**.38). The bases must always be post-multiplied by the representative matrices. (See **2**.55).

Proper groups

$\langle \|jm\rangle \|$	For proper groups, whenever this basis is listed, the bases $\langle \|jm\rangle \|^{\blacksquare}$ and $\langle \|jm\rangle \|^{\bullet}$ can be taken for j integral and half-integral, respectively. (See **13**.35.)

Improper groups with inversion

$\langle \|jm\rangle \|$	Whenever this basis is listed, for j integral, it is g and u for j even and odd, respectively. The bases $\langle \|jm\rangle \|^{\blacksquare}$ (see **13**.35) can then be taken with opposite parity. For j half-integral the bases $\langle \|jm\rangle \|$ are always g and the bases $\langle \|jm\rangle \|^{\bullet}$ (which are always listed) are always u.

Improper groups without inversion

$\langle \|jm\rangle \|$	The classification g and u is no longer valid. The symmetry assignment of the bases $\langle \|jm\rangle \|^{\bullet}$ is always given. The symmetry assignment of the bases $\langle \|jm\rangle \|^{\blacksquare}$ cannot in this case be obtained from the tables, since it is not readily derived from the symmetry of $\langle \|jm\rangle \|^{\bullet}$. These bases, however, are not of much practical interest, except for $l = 1$, for which identifications are provided in T **n**.5. (See **13**.33,**13**.35.)

The cyclic, dihedral, and related groups

Columns labelled ι and μ

Description	These are numbers which can repeatedly be added to or subtracted from the values of j and m, respectively, in the kets on their left, in accordance to the following rules.
No sign in μ	When no sign is given, ι and μ can only be added to (but not subtracted from) the values of j and $\|m\|$ in the kets on their left.
$\pm\mu$	When $\pm\mu$ appears, μ can be added to or subtracted from the values of m in the kets on its left. If there is more that one ket in the same basis it is permitted to use μ for one partner and $-\mu$ for another.
$\|m\| \leq j$	This condition must be satisfied in every case.

Example. Some symmetrized bases for $\mathbf{D}_{3h}$ *from* T **32**.6

Extract from T **32**.6

Table **16**.5 Symmetrized bases for some representations of $\mathbf{D}_{3h}$

$\mathbf{D}_{3h}$	$\big\langle \lvert j\,m\rangle \big\rvert$		ι	μ
A_2'	$\lvert 3\,3\rangle_+$	$\lvert 6\,6\rangle_-$	2	6
E'	$\big\langle \lvert 1\,1\rangle, \lvert 1\,\bar{1}\rangle \big\rvert$	$\big\langle \lvert 2\,\bar{2}\rangle, -\lvert 2\,2\rangle \big\rvert$	2	± 6
$E_{1/2}$	$\big\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\frac{\bar{1}}{2}\rangle \big\rvert$	$\big\langle \lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\frac{\bar{1}}{2}\rangle \big\rvert$	2	± 6
	$\big\langle \lvert \frac{5}{2}\,\frac{\bar{5}}{2}\rangle, \lvert \frac{5}{2}\,\frac{5}{2}\rangle \big\rvert^{\bullet}$	$\big\langle \lvert \frac{7}{2}\,\frac{\bar{5}}{2}\rangle, -\lvert \frac{7}{2}\,\frac{5}{2}\rangle \big\rvert^{\bullet}$	2	± 6

Bases for A_2' *(First 18, in increasing order of* j*)*

$2^{-1/2}\,(\lvert 3\,3\rangle + \lvert 3\,\bar{3}\rangle)$	$2^{-1/2}\,(\lvert 5\,3\rangle + \lvert 5\,\bar{3}\rangle)$	$2^{-1/2}\,(\lvert 6\,6\rangle - \lvert 6\,\bar{6}\rangle)$
$2^{-1/2}\,(\lvert 7\,3\rangle + \lvert 7\,\bar{3}\rangle)$	$2^{-1/2}\,(\lvert 8\,6\rangle - \lvert 8\,\bar{6}\rangle)$	$2^{-1/2}\,(\lvert 9\,3\rangle + \lvert 9\,\bar{3}\rangle)$
$2^{-1/2}\,(\lvert 9\,9\rangle + \lvert 9\,\bar{9}\rangle)$	$2^{-1/2}\,(\lvert 10\,6\rangle - \lvert 10\,\bar{6}\rangle)$	$2^{-1/2}\,(\lvert 11\,3\rangle + \lvert 11\,\bar{3}\rangle)$
$2^{-1/2}\,(\lvert 11\,9\rangle + \lvert 11\,\bar{9}\rangle)$	$2^{-1/2}\,(\lvert 12\,6\rangle - \lvert 12\,\bar{6}\rangle)$	$2^{-1/2}\,(\lvert 12\,12\rangle - \lvert 12\,\overline{12}\rangle)$
$2^{-1/2}\,(\lvert 13\,3\rangle + \lvert 13\,\bar{3}\rangle)$	$2^{-1/2}\,(\lvert 13\,9\rangle + \lvert 13\,\bar{9}\rangle)$	$2^{-1/2}\,(\lvert 14\,6\rangle - \lvert 14\,\bar{6}\rangle)$
$2^{-1/2}\,(\lvert 14\,12\rangle - \lvert 14\,\overline{12}\rangle)$	$2^{-1/2}\,(\lvert 15\,3\rangle + \lvert 15\,\bar{3}\rangle)$	$2^{-1/2}\,(\lvert 15\,9\rangle + \lvert 15\,\bar{9}\rangle)$

Bases for E' *(First 12, in increasing order of* j*)*

$\big\langle \lvert 1\,1\rangle, \lvert 1\,\bar{1}\rangle \big\rvert$	$\big\langle \lvert 2\,\bar{2}\rangle, -\lvert 2\,2\rangle \big\rvert$	$\big\langle \lvert 3\,1\rangle, \lvert 3\,\bar{1}\rangle \big\rvert$
$\big\langle \lvert 4\,\bar{2}\rangle, -\lvert 4\,2\rangle \big\rvert$	$\big\langle \lvert 5\,1\rangle, \lvert 5\,\bar{1}\rangle \big\rvert$	$\big\langle \lvert 5\,\bar{5}\rangle, \lvert 5\,5\rangle \big\rvert$
$\big\langle \lvert 6\,\bar{2}\rangle, -\lvert 6\,2\rangle \big\rvert$	$\big\langle \lvert 6\,4\rangle, -\lvert 6\,\bar{4}\rangle \big\rvert$	$\big\langle \lvert 7\,1\rangle, \lvert 7\,\bar{1}\rangle \big\rvert$
$\big\langle \lvert 7\,\bar{5}\rangle, \lvert 7\,5\rangle \big\rvert$	$\big\langle \lvert 7\,7\rangle, \lvert 7\,\bar{7}\rangle \big\rvert$	$\big\langle \lvert 8\,\bar{2}\rangle, -\lvert 8\,2\rangle \big\rvert$

Bases for $E_{1/2}$ *(First 18, in increasing order of* j*)*

$\big\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\frac{\bar{1}}{2}\rangle \big\rvert$	$\big\langle \lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\frac{\bar{1}}{2}\rangle \big\rvert$	$\big\langle \lvert \frac{5}{2}\,\frac{1}{2}\rangle, \lvert \frac{5}{2}\,\frac{\bar{1}}{2}\rangle \big\rvert$
$\big\langle \lvert \frac{5}{2}\,\frac{\bar{5}}{2}\rangle, \lvert \frac{5}{2}\,\frac{5}{2}\rangle \big\rvert^{\bullet}$	$\big\langle \lvert \frac{7}{2}\,\frac{1}{2}\rangle, -\lvert \frac{7}{2}\,\frac{\bar{1}}{2}\rangle \big\rvert$	$\big\langle \lvert \frac{7}{2}\,\frac{\bar{5}}{2}\rangle, -\lvert \frac{7}{2}\,\frac{5}{2}\rangle \big\rvert^{\bullet}$
$\big\langle \lvert \frac{7}{2}\,\frac{7}{2}\rangle, -\lvert \frac{7}{2}\,\frac{\bar{7}}{2}\rangle \big\rvert^{\bullet}$	$\big\langle \lvert \frac{9}{2}\,\frac{1}{2}\rangle, \lvert \frac{9}{2}\,\frac{\bar{1}}{2}\rangle \big\rvert$	$\big\langle \lvert \frac{9}{2}\,\frac{\bar{5}}{2}\rangle, \lvert \frac{9}{2}\,\frac{5}{2}\rangle \big\rvert^{\bullet}$
$\big\langle \lvert \frac{9}{2}\,\frac{7}{2}\rangle, \lvert \frac{9}{2}\,\frac{\bar{7}}{2}\rangle \big\rvert^{\bullet}$	$\big\langle \lvert \frac{11}{2}\,\frac{1}{2}\rangle, -\lvert \frac{11}{2}\,\frac{\bar{1}}{2}\rangle \big\rvert$	$\big\langle \lvert \frac{11}{2}\,\frac{\bar{5}}{2}\rangle, -\lvert \frac{11}{2}\,\frac{5}{2}\rangle \big\rvert^{\bullet}$
$\big\langle \lvert \frac{11}{2}\,\frac{\overline{11}}{2}\rangle, -\lvert \frac{11}{2}\,\frac{11}{2}\rangle \big\rvert$	$\big\langle \lvert \frac{13}{2}\,\frac{1}{2}\rangle, \lvert \frac{13}{2}\,\frac{\bar{1}}{2}\rangle \big\rvert$	$\big\langle \lvert \frac{13}{2}\,\frac{\bar{5}}{2}\rangle, \lvert \frac{13}{2}\,\frac{5}{2}\rangle \big\rvert^{\bullet}$
$\big\langle \lvert \frac{13}{2}\,\frac{\overline{11}}{2}\rangle, \lvert \frac{13}{2}\,\frac{11}{2}\rangle \big\rvert$	$\big\langle \lvert \frac{13}{2}\,\frac{13}{2}\rangle, \lvert \frac{13}{2}\,\frac{\overline{13}}{2}\rangle \big\rvert$	$\big\langle \lvert \frac{15}{2}\,\frac{1}{2}\rangle, -\lvert \frac{15}{2}\,\frac{\bar{1}}{2}\rangle \big\rvert$

Note $\big\langle \lvert \frac{5}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\frac{\bar{1}}{2}\rangle \big\rvert$

In the above bases, any first partner can be joined with any second partner as in the example shown on the left. Although such mixed bases span the correct representations, they are not useful in producing expansions in ascending order in $\lvert jm\rangle$ and the rules given are best used by ensuring that **the** j**'s and** m**'s in each basis keep their absolute values**, as we have done in the examples given.

The cubic and icosahedral groups

General description of the tables

Tables a, b, c

In the cubic and icosahedral groups T **n**.6 splits into three tables. T **n**.6*a* gives the bases used in order to span the irreducible representations listed in the tables. They are given in terms of the harmonics or spin harmonics of the lowest order belonging to the given irreducible representation. They satisfy the general conditions and conventions given in this section. This table is self-explanatory and requires no further description or instructions.

T **n**.6*b* gives the symmetrized harmonics (j integral) for all $j \le 18$ (cubic groups) or $j \le 15$ (icosahedral groups) for all (vector) representations. This table appears in full only for the master groups $\mathbf{O}_h$ (T **71**.6*b*) and $\mathbf{I}_h$ (T **75**.6*b*).

T **n**.6*c* gives the symmetrized spin harmonics (j half-integral) for all the spinor representations, in terms of the harmonics listed in T **n**.6*b*.

The bases	A basis of dimension n is a row vector of n columns.
Normalization	Each column of the basis (symmetrized harmonic or spin harmonic) is normalized to unity. Each basis is normalized to $2n+1$.
Orthogonality	When the multiplicity is larger than unity (two or more symmetrized bases for the same j, integral or half-integral) the successive bases are orthogonal.
Arbitrary phase factor	Each symmetrized basis of dimension n may be multiplied by an arbitrary phase factor.
Convention used	The phase factors have been chosen so that, for the first column of the successive bases belonging to the same j (multiplicity unity or larger): (i) The coefficient of the first column is positive in all cases. (ii) The coefficient of the first column of the first of the bases just mentioned is also real. (iii) For all three-dimensional bases of the cubic groups the coefficient of the second column is always ± 1 or 0.

Instructions for the use of T **n**.6*b*

Identification of representation	The symbol for the representation is given in the column headed by the group name.
The kets $\lvert jm\rangle_+$, $\lvert jm\rangle_-$	They are identified by the columns headed 'j' and 'm' and by the sign listed in a column headed '±' following one of the columns headed 'Coefficient'. These kets are defined at the beginning of this section.
The coefficients	They have been calculated to no less than fifteen significant figures and rounded off to twelve. Coefficients listed as 1 or 0 are correct to all orders.
Identification of a column of the basis	A pair of consecutive columns headed 'Coefficient', '±', respectively, identifies one column of the basis. In n-dimensional representations these pairs are labelled $1, 2, \ldots, n$ and identify the columns of the basis in that order.
Reading one column of the basis	(i) Identify the group and representation desired. Call X the symbol of the representation chosen. (ii) Number the rows of the table as follows. Row 1: the row that contains X. Row 2: the next row if and only if the representation symbol is blank. Row 3: the next row if and only if the representation symbol is blank. And so on. (iii) Each column of the basis is given by the coefficient in row 1 multiplied by $\lvert jm\rangle_\pm$, plus the coefficient in row 2 multiplied by $\lvert jm\rangle_\pm$, and so on.
${}^{1,2}E$	For the one-dimensional representations where this left superscript appears, proceed exactly as above but assign the 'first column' to ${}^1E$ and the 'second column' to ${}^2E$.
$E^\triangle$	For the representation where this superscript appears proceed in the normal way, but changing the sign of the second column.

Examples of the use of T **n**.6*b*

$\mathbf{O}_h$: A_{2g} *(pp.* 607, 608*)*	First basis: $0.829156197589\,\lvert 6\,2\rangle_+ - 0.559016994375\,\lvert 6\,6\rangle_+$. Second basis: $0.802015689788\,\lvert 10\,2\rangle_+ + 0.157288217401\,\lvert 10\,6\rangle_+ - 0.576221528581\,\lvert 10\,10\rangle_+$.
$\mathbf{T}$: ${}^2E$ *(pp.* 609, 610*)*	First basis (obtained from column 2): $0.707106781187\,\lvert 2\,0\rangle_+ + 0.707106781187\,\mathrm{i}\,\lvert 2\,2\rangle_+$. Sixth basis (obtained from column 2): $0.492125492126\,\lvert 8\,0\rangle_+ - 0.460101671793\,\mathrm{i}\,\lvert 8\,2\rangle_+ - 0.278605397905\,\lvert 8\,4\rangle_+ - 0.536941758120\,\mathrm{i}\,\lvert 8\,6\rangle_+ - 0.424489731629\,\lvert 8\,8\rangle_+$.

$\mathbf{T}_d$: T_1 (*p.* 613)

Second basis:
Column 1: $0.935414346693\,|4\,1\rangle_- - 0.353553390593\,|4\,3\rangle_-$.
Column 2: $-\,|4\,4\rangle_-$.
Column 3: $0.935414346693\,|4\,1\rangle_+ + 0.353553390593\,|4\,3\rangle_+$.

$\mathbf{I}_h$: F_g (*p.* 673)

First basis:
Column 1: $0.763762615826\,|4\,0\rangle_+ + 0.645497224368\,|4\,4\rangle_+$.
Column 2: $0.333776501991\,|4\,1\rangle_- - 0.942652240606\,|4\,3\rangle_-$.
Column 3: $-0.763762615826\,|4\,2\rangle_- - 0.645497224368\,|4\,4\rangle_-$.
Column 4: $0.873838226858\,|4\,1\rangle_+ - 0.486216776018\,|4\,3\rangle_+$.

Instructions for the use of T $\mathbf{n}$.6c

Symbols a_1, b_2, etc. They indicate functions belonging to the A_1, B_2, etc., one-dimensional representations of the group $\mathbf{n}$. They must be read from T $\mathbf{n}$.6b.

Symbols $t_1^{(3)}$, $t_2^{(3)}$, $h_g^{(4)}$, etc. They indicate functions corresponding to a given column of the multi-dimensional representations T_1, T_2, H_g, etc. The column is identified by the superscript. These functions must be read from T $\mathbf{n}$.6b.

Examples of the use of T $\mathbf{n}$.6c

$\mathbf{O}$: $E_{5/2}$ (*p.* 581)

Basis $\langle a_2\alpha, a_2\beta|$.
First basis: $\big\langle |3\,2\rangle_-|\tfrac12\,\tfrac12\rangle, |3\,2\rangle_-|\tfrac12\,\overline{\tfrac12}\rangle\big|$.
Second basis:

$$\big\langle(0.829156197589\,|6\,2\rangle_+ - 0.559016994375\,|6\,6\rangle_+)\,|\tfrac12\,\tfrac12\rangle,\\ (0.829156197589\,|6\,2\rangle_+ - 0.559016994375\,|6\,6\rangle_+)\,|\tfrac12\,\overline{\tfrac12}\rangle\big|.$$

Basis $\big\langle \frac{1}{\sqrt3}\,(t_2^{(1)}\beta - t_2^{(2)}\alpha + t_2^{(3)}\beta), \frac{1}{\sqrt3}\,(-t_2^{(1)}\alpha + t_2^{(2)}\beta + t_2^{(3)}\alpha)\big|$
First basis:

$$\big\langle \tfrac{1}{\sqrt3}\,(|2\,1\rangle_-|\tfrac12\,\overline{\tfrac12}\rangle + |2\,2\rangle_-|\tfrac12\,\tfrac12\rangle - |2\,1\rangle_+|\tfrac12\,\overline{\tfrac12}\rangle),\\ \tfrac{1}{\sqrt3}\,(-|2\,1\rangle_-|\tfrac12\,\tfrac12\rangle - |2\,2\rangle_-|\tfrac12\,\overline{\tfrac12}\rangle - |2\,1\rangle_+|\tfrac12\,\tfrac12\rangle)\big|.$$

$\mathbf{I}$: $E_{7/2}$ (*p.* 653)

Basis $\big\langle \frac12\,(f^{(1)}\alpha - f^{(2)}\beta + f^{(3)}\alpha - f^{(4)}\beta), \frac12\,(f^{(1)}\beta + f^{(2)}\alpha - f^{(3)}\beta - f^{(4)}\alpha)\big|$
First basis:

$$\big\langle \tfrac12\,\{|3\,2\rangle_-|\tfrac12\,\tfrac12\rangle - (0.925614793411\,|3\,1\rangle_+ + 0.378466979034\,|3\,3\rangle_+)\,|\tfrac12\,\overline{\tfrac12}\rangle\\ + (-0.866025403784\,|3\,0\rangle_+ + 0.5\,|3\,2\rangle_+)\,|\tfrac12\,\tfrac12\rangle\\ - (-0.135045378369\,|3\,1\rangle_- - 0.990839414729\,|3\,3\rangle_-)\,|\tfrac12\,\overline{\tfrac12}\rangle\},\\ \tfrac12\,\{|3\,2\rangle_-|\tfrac12\,\overline{\tfrac12}\rangle + (0.925614793411\,|3\,1\rangle_+ + 0.378466979034\,|3\,3\rangle_+)\,|\tfrac12\,\tfrac12\rangle\\ - (-0.866025403784\,|3\,0\rangle_+ + 0.5\,|3\,2\rangle_+)\,|\tfrac12\,\overline{\tfrac12}\rangle\\ - (-0.135045378369\,|3\,1\rangle_- - 0.990839414729\,|3\,3\rangle_-)\,|\tfrac12\,\tfrac12\rangle\}\big|.$$

7 Matrix representations

Notation for the headers of T $\mathbf{n}$.7, and for its first row

'Representation' **Please note that, for brevity, unless statements to the contrary, the word *'representation'* is always used in this book to denote an irreducible unitary representation.**

'Use T **m**.7 •'	When this entry appears, the matrices for the operations of the group **m** in T **m**.7 must be read for the operations of the group **n** as listed in T **m**.7. This entry appears in two cases. One is for the groups $\mathbf{C}_{nv}$, $(n = 8, 10)$, for which the tables are identical (except for the notation of the operations) to those of their isomorphs $\mathbf{D}_n$ and they are given with those of this latter group. (For groups of lower order the table is given explicitly for convenience.) The second case is for $\mathbf{T}_d$, for which the table is identical (except for the notation of the operations) to that for **O**, with which it is given.
'Use T **m**.7 ▪'	When this entry appears, the group number **n** is of the form $L = H \otimes \mathbf{C}_i$, where H is the group number **m**. All operations of L are given by $l = h$ and $l = h\,i$, $\forall h \in H$. Each representation $\check{H}$ of H splits into two representations $\check{H}_g$ and $\check{H}_u$ of L, given by the following rules:

Representation of L	$l = h \in H$	$l = h\,i,\ h \in H$
$\check{H}_g$	$\check{H}_g(l) = \check{H}(h)$	$\check{H}_g(l) = \check{H}(h)$
$\check{H}_u$	$\check{H}_u(l) = \check{H}(h)$	$\check{H}_u(l) = -\check{H}(h)$

In every case the matrix $\check{H}(h)$ is read directly from T **m**.7, the symbols $\check{H}$ being identically those in the first column of that table.
This entry appears only for direct products involving cubic or icosahedral groups or, in other cases, when the axis of highest symmetry of one of the factors is of order larger than 6. (For groups of lower order the table is given explicitly for convenience.)

'Use T **n**.4 ♠'	When this entry appears, the representations are all one-dimensional and therefore identical to their characters, which may be read from T **n**.4.
First row	It lists all the ordinary (vector) representations as well as the spinor (or double-group) representations which appear in the group. The spinor representations are identified by half-integral subscripts. See Chapter **14** for the notation.

Vector representations

One-dimensional	The matrix of the operation g for the group number **n** is the character listed in T **n**.4 for the class that contains g.
Multi-dimensional	For multi-dimensional representations the matrix representative is listed for each g in T **n**.7.
As required for the double group	In the double group, the matrices thus obtained for g are valid without change for $\widetilde{g}$.

Double-group representations

Notation	You must recognize in the tables the vector representations (symbols in the first row with no subscripts or integral subscripts) and the spinor representations (symbols in the first row with half-integral subscripts).
Operations	The operations g and $\widetilde{g}$ for $\widetilde{G}$ must be obtained from subsection 4 of T **n**, where the number **n** corresponds to the group G.
One-dimensional vector representations	For one-dimensional vector representations of the group number **n**, the matrix representative of the operations g and $\widetilde{g}$ are both equal to the character listed in T **n**.4 for the class that contains g.
Multi-dimensional vector representations	For multi-dimensional vector representations of the group number **n**, the matrix representative of the operations g and $\widetilde{g}$ are both equal to the matrix listed in T **n**.7 for the operation g.

One-dimensional spinor representations

For one-dimensional vector representations of the group number **n**, the matrix representative of the operations g and $\widetilde{g}$ are, respectively, the character listed in T **n**.4 for the class that contains g and the negative of this character.

Multi-dimensional vector representations

For multi-dimensional vector representations of the group number **n**, the matrix representative of the operations g and $\widetilde{g}$ are, respectively, the matrix listed in T **n**.7 for the operation g and the negative of this matrix.

Multiplication rules

The matrices obtained multiply by the multiplication rules obtained as in § 2 above.

Projective representations (full table, including vector representations)

One-dimensional representations

For one-dimensional representations of the group number **n**, the matrix representative of the operation g is equal to the character listed in T **n**.4 for the class that contains g.

Multi-dimensional representations

For multi-dimensional representations of the group number **n**, the matrix representative of the operation g is the matrix listed in T **n**.7 for this operation.

Multiplication rules

The matrices obtained multiply by the multiplication rules obtained from the multiplication table T **n**.2, without any change for the vector representations but, for the spinor representations, the multiplication of the matrices corresponding to g_i and g_j requires the insertion of the factor corresponding to the product $g_i\, g_j$, as read from T **n**.3.

Examples. Representations of $\mathbf{D}_3$

Necessary data

T **23**.4 Character table

$\mathbf{D}_3$	E	$2C_3$	$3C'_2$	τ
A_1	1	1	1	a
A_2	1	1	-1	a
E	2	-1	0	a
$E_{1/2}$	2	1	0	c
${}^1E_{3/2}$	1	-1	i	b
${}^2E_{3/2}$	1	-1	$-$i	b

T **23**.7 Matrix representations

$\mathbf{D}_3$	E	$E_{1/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_3^+	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$
C_3^-	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\mathrm{i}} \\ \overline{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon}^* \\ \mathrm{i}\overline{\epsilon} & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon} \\ \mathrm{i}\overline{\epsilon}^* & 0 \end{bmatrix}$

$\epsilon = \exp(2\pi\mathrm{i}/3)$

The double-group representations

Table **16**.6 The double-group representations for $\widetilde{\mathbf{D}}_3$

$\widetilde{\mathbf{D}}_3$	A_1	A_2	E	$E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$
E	1	1	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	1	1
C_3^+	1	1	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	-1	-1
C_3^-	1	1	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	-1	-1
C'_{21}	1	-1	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	i	$-$i
C'_{22}	1	-1	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	i	$-$i
C'_{23}	1	-1	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	i	$-$i
$\widetilde{E}$	1	1	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	-1	-1
$\widetilde{C}_3^+$	1	1	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	1	1
$\widetilde{C}_3^-$	1	1	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	1	1
$\widetilde{C}'_{21}$	1	-1	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$-$i	i
$\widetilde{C}'_{22}$	1	-1	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$-$i	i
$\widetilde{C}'_{23}$	1	-1	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$-$i	i

$\epsilon = \exp(2\pi\mathrm{i}/3)$

The projective representations

Table **16**.7 The vector and projective representations for $\widetilde{\mathbf{D}}_3$

$\mathbf{D}_3$	A_1	A_2	E	$E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$
E	1	1	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	1	1
C_3^+	1	1	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	-1	-1
C_3^-	1	1	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	-1	-1
C'_{21}	1	-1	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	i	$-$i
C'_{22}	1	-1	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	i	$-$i
C'_{23}	1	-1	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	i	$-$i

$\epsilon = \exp(2\pi\mathrm{i}/3)$

Icosahedral group $\mathbf{I}$

Description of the tables

T **74**.7 splits into two tables. T **74**.7*a* gives all group elements in terms of generators and T **74**.7*b* gives the matrices for the generators in all representations.

T **74**.7*a*. *Generators* — First column: lists the operations.
Second column: expression of the operations by the generators.

T **74**.7*b*. *Matrices for the generators* — This table uses the same conventions and notations described in the general part of this section.

8 Direct product of representations

Notation for the headers of T **n**.8, and for its first column

'Use T **m**.8 •' — When this entry appears, the products for the representations of the group **m** in T **m**.8 must be read for those of the group **n**.
This entry appears in two cases. One is for the groups $\mathbf{C}_{nv}$ ($n = 8, 10$), for which the tables are identical to those of their isomorphs $\mathbf{D}_n$ and they are given with those of this latter group. (For groups of lower order the table is given explicitly for convenience.) The second case is for $\mathbf{T}_d$ for which the table is identical to that for **O**, with which it is given.

First column and head row — They list all the ordinary (vector) representations as well as the spinor (or double-group) representations which appear in the group. The spinor representations are identified by half-integral subscripts. See Chapter **14** for the notation.

Use of the table

$A \otimes B$ — Given two distinct representations A and B, their direct product $A \otimes B$ appears in the intersection of the A row of the table with the B column.

$A \otimes A$ — Is given by all the representations listed in the intersection of the A row with the A column. Any brackets present (but not their contents) must be ignored.

$A \,\overline{\otimes}\, A$ — This is the *symmetrized product* of the representation A with itself. (See **2**.139.) It is given by all the representations listed in the intersection of the A row with the A column, *discarding those shown in curly brackets.*

$A \,\underline{\otimes}\, A$ — This is the *antisymmetrized product* of the representation A with itself. (See **2**.140.) It is given by all the representations listed *in curly brackets* in the intersection of the A row with the A column.

Note — For convenience of printing this table has often to be divided in blocks. In order to find a product such as $A \otimes B$ look for the block that contains B in its head row. **If $A \otimes B$ is listed $B \otimes A$ is equal to it but it is not listed.**

Example. Direct products for representations of $\mathbf{D}_{8h}$

Necessary data (Last four rows of the second block of T **37**.8*)*

Table **16**.8 Direct products for some representations of $\mathbf{D}_{8h}$

$\mathbf{D}_{8h}$	B_{2u}	E_{1u}	E_{2u}	E_{3u}
B_{2u}	A_{1g}	E_{3g}	E_{2g}	E_{1g}
E_{1u}		$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$	$E_{1g} \oplus E_{3g}$	$B_{1g} \oplus B_{2g} \oplus E_{2g}$
E_{2u}			$A_{1g} \oplus \{A_{2g}\} \oplus B_{1g} \oplus B_{2g}$	$E_{1g} \oplus E_{3g}$
E_{3u}				$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$

Results

$B_{2u} \otimes B_{2u} = A_{1g}$
$B_{2u} \otimes E_{1u} = E_{3g}$
$B_{2u} \otimes E_{2u} = E_{2g}$
$B_{2u} \otimes E_{3u} = E_{1g}$
$E_{1u} \otimes E_{1u} = A_{1g} \oplus A_{2g} \oplus E_{2g}$
$E_{1u} \,\overline{\otimes}\, E_{1u} = A_{1g} \oplus E_{2g}$
$E_{1u} \,\underline{\otimes}\, E_{1u} = A_{2g}$
$E_{1u} \otimes E_{2u} = E_{1g} \oplus E_{3g}$
$E_{1u} \otimes E_{3u} = B_{1g} \oplus B_{2g} \oplus E_{2g}$
$E_{2u} \otimes E_{2u} = A_{1g} \oplus A_{2g} \oplus B_{1g} \oplus B_{2g}$
$E_{2u} \,\overline{\otimes}\, E_{2u} = A_{1g} \oplus B_{1g} \oplus B_{2g}$
$E_{2u} \,\underline{\otimes}\, E_{2u} = A_{2g}$
$E_{2u} \otimes E_{3u} = E_{1g} \oplus E_{3g}$
$E_{3u} \otimes E_{3u} = A_{1g} \oplus A_{2g} \oplus E_{2g}$
$E_{3u} \,\overline{\otimes}\, E_{3u} = A_{1g} \oplus E_{2g}$
$E_{3u} \,\underline{\otimes}\, E_{3u} = A_{2g}$

9 Subduction (descent of symmetry)

Purpose of the table Given a representation of the group G, ${}^i\breve{G}$, it gives the representations of $H \subset G$ which appear in the reduction of ${}^i\breve{G}$.

Contents of the table The subgroups given in the table are only those which appear in the standard setting used in the tables.

Alternative entries For some subgroups H two isomorphic realizations are listed for different choices of the operations of G, which are given below the headings of H.

Other subgroups In most cases, all the subgroups $H \subset G$ which can be obtained from the graphs in § **9**–5 are treated in T **n**.9 (where **n** is the serial number of G). When this is not so the subgroups not so treated (H_i, say) are listed at the bottom of T **n**.9 with a reference to a supergroup of the H_i which does appear in the table. This permits the completion of a chain headed by G and which contains the desired subgroup H_i.

Subduction difficulty If, in going from G to $H \subset G$, there is a change of bases on reduction (see **9**.7) then the group H is listed in brackets in the heading, as (H). Please notice that in a few instances there is no change of bases but a change of notation (see **9**.3) arises. This has no important consequences and therefore it is not indicated in the table. That such a change of notation exists or not must in the first instance be ascertained from the graphs in § **9**–5. (See also the table of subgroup elements, labelled T **n**.0, that contains G.)

Example. Subgroups $\mathbf{D}_2$ of $\mathbf{O}$

Necessary data (from T **69**.9*)*

Table **16**.9 Subduction from $\mathbf{O}$ to $\mathbf{D}_2$

$\mathbf{O}$	$\mathbf{D}_2$	$(\mathbf{D}_2)$
	C_{2z}, C_{2x}, C_{2y}	C_{2z}, C'_{2a}, C'_{2b}
A_1	A	A
A_2	A	B_1
E	$2A$	$A \oplus B_1$
T_1	$B_1 \oplus B_2 \oplus B_3$	$B_1 \oplus B_2 \oplus B_3$
T_2	$B_1 \oplus B_2 \oplus B_3$	$A \oplus B_2 \oplus B_3$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{5/2}$	$E_{1/2}$	$E_{1/2}$
$F_{3/2}$	$2E_{1/2}$	$2E_{1/2}$

Results and comments In the first setting, T_2 reduces into $B_1 \oplus B_2 \oplus B_3$ of $\mathbf{D}_2$. The matrix representation T_2 may be obtained from T **69**.7, where it can be seen that the four matrices E, C_{2x}, C_{2y}, C_{2z} are already reduced in $\mathbf{D}_2$, the first, second, and third columns giving respectively the representations B_2, B_1, B_3 in T **22**.4. Notice that when, in the second setting, the matrices of E, C_{2z}, C'_{2a}, and C'_{2b} are extracted from T **69**.7 their characters equal correctly the characters of $A \oplus B_2 \oplus B_3$ in T **22**.4 but that a similarity is required to reduce this representation, as indicated by the bracket in the heading.

10 Subduction from O(3)

Purpose of the table Given representations of O(3), $\hat{R}^j$ (ordinary or vector representation) or $\check{R}^j$ (spinor representation, half-integral j) of dimension $2j+1$ (see **12**.22 and **12**.23), it gives the representations of a point group G which appear in the reduction of $\hat{R}^j$ or $\check{R}^j$ for each j.

Subduction details — They depend in general on the basis of O(3) chosen. For a given j this basis has the form $\langle|jm\rangle|$ but for the representations $\hat{R}^j$ and $\check{R}^j$ the embellished bases $\langle|jm\rangle|^{\blacksquare}$ and $\langle|jm\rangle|^{\bullet}$, respectively, may also be chosen. (See **13**.32 and **13**.33.)

Description of the table — The first column gives the value of j and the representations of G appear in the second column.

Subduction to proper groups — In this case, the basis $\langle|jm\rangle|$ of O(3) may be taken unembellished or embellished, no further changes being required in the representations listed in the table on reduction. (If corresponding bases are chosen, however, the appropriate embellishments must be inserted.)

Subduction to groups with i: table headings marked ♣ — When this symbol appears, the subduction from O(3) can be done in either of two ways:
(i) When the basis of O(3) chosen is unembellished, the subduction is exactly as given in the table.
(ii) When the basis of O(3) chosen is embellished, the line corresponding to the given j (or l) must be used with the subscripts g and u of the representations interchanged. (If corresponding bases are chosen, however, the appropriate embellishments must be inserted.)

Subduction to improper groups without the inversion — Notice that the rule above is not valid in this case because the g, u classification does not exist. The bases $\langle|jm\rangle|^{\bullet}$, in any case, have been explicitly identified in T **n**.6. As regards the bases $\langle|jm\rangle|^{\blacksquare}$ see **13**.33, **13**.35, and §6.

Example. Subduction from O(3) to $\mathbf{C}_{2h}$

Necessary data

T **60**.10 ♣ Subduction from O(3)

j	$\mathbf{C}_{2h}$
$2n$	$(2n+1)\,A_g \oplus 2n\,B_g$
$2n+1$	$(2n+1)\,A_u \oplus (2n+2)\,B_u$
$n+\frac{1}{2}$	$(n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g})$

$n = 0, 1, 2, \ldots$

Some results

$\langle|jm\rangle|$: $\hat{R}^0 = A_g$, $\hat{R}^1 = A_u \oplus 2B_u$, $\hat{R}^2 = 3A_g \oplus 2B_g$, $\hat{R}^3 = 3A_u \oplus 4B_u$, $\check{R}^{1/2} = {}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$, $\check{R}^{3/2} = 2\,{}^1E_{1/2,g} \oplus 2\,{}^2E_{1/2,g}$,

$\langle|jm\rangle|^{\blacksquare}$: $\hat{R}^0 = A_u$, $\hat{R}^1 = A_g \oplus 2B_g$, $\hat{R}^2 = 3A_u \oplus 2B_u$, $\hat{R}^3 = 3A_g \oplus 4B_g$,

$\langle|jm\rangle|^{\bullet}$: $\check{R}^{1/2} = {}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$, $\check{R}^{3/2} = 2\,{}^1E_{1/2,u} \oplus 2\,{}^2E_{1/2,u}$.

11 Clebsch–Gordan coefficients

Notation for the headers of T **n**.11

'Use T **m**.11 •' — When this entry appears, the Clebsch–Gordan coefficients for the group **m** in T **m**.11 must be read for those of the group **n**.
This entry appears in two cases. One is for the groups $\mathbf{C}_{nv}$ ($n = 4, 6, 8, 10$), for which the tables are identical to those of their isomorphs $\mathbf{D}_n$ and they are given with those of this latter group. The second case is for $\mathbf{T}_d$ for which the table is identical to that for $\mathbf{O}$, with which it is given.

'Use T **m**.11 ▪' — When this entry appears in the header of T **n**.11:
(i) Look up in T **n**.8 the direct product for which you want the Clebsch–Gordan coefficients, *disregarding the g, u subscripts in* T **n**.8.
(ii) Use T **m**.11.
This procedure is used for groups which are direct products.

♠ When this mark appears, the representations are all one-dimensional and therefore all the Clebsch–Gordan matrices are equal to the number 1.

Notation required to use the tables

Reference The following is a summary of § **2**.7.

Irreducible representations multiplied Labelled i, j. The direct product of their bases is formed, the representation i being the first factor.

$|im\rangle$, $|jn\rangle$ Function of the m column of the i basis and function of the n column of the j basis, respectively.

$|IUP\rangle$ Function which appears in the reduced product of the bases corresponding to i and j. I is the label of the irreducible representation, U is the multiplicity index, and P labels the column of that representation. U appears in the cubic and icosahedral groups only. (IU) works as a *single index*, as if the representation I were a new representation in each repetition. (Which would in fact be the case for the functions of the corresponding bases which, although belonging to the same representation and column, will in general be new functions for each value of U.)

${}^{ij}\langle mn \mid IUP\rangle$ Element of the matrix of the Clebsch–Gordan coefficients (Clebsch–Gordan matrix) corresponding to the row mn and the column $(IU)P$. (Notice that rows and columns are labelled by *double* subscripts in dictionary order.) The left superscript ij identifies the representations which are being multiplied.

Use of the coefficients $|IUP\rangle = \sum_{mn} |im\rangle\, |jn\rangle\, {}^{ij}\langle mn \mid IUP\rangle.$ (2)

Lower and upper case convention Notice that the irreducible representations which are the factors in the direct product are labelled in lower case and that the irreducible representations that appear in the reduction of the direct product are labelled in upper case.

Note The user of the tables may multiply the whole of the Clebsch–Gordan matrix ${}^{ij}\langle mn \mid IUP\rangle$ by any desired phase factor, constant for all its matrix elements.

Description of the tables

Their structure Each table is divided in four fields which shall be numbered 1 to 4 starting from top left and reading from left to right. Field 4 always forms a square matrix.

i and j They are given by the two representations in field 1.

m and n They are given in dictionary order of the numerical indices 1, 2, etc., in field 3. **They must be identified** by the correct labelling of the columns of the bases to which $|im\rangle$ and $|jn\rangle$ belong.

(IU) It is given in the first row of field 2. Notice that U does not appear explicitly. Its existence is recognized by the repetition of I.

P In field 2, the digits below each entry I give P. This digit labels the successive columns of the basis corresponding to I, **which must be identified** from the columns of that representation.

${}^{ij}\langle mn \mid IUP\rangle$ The Clebsch–Gordan matrix is given in field 4. The left superscript (obtained from field 1) merely identifies the whole matrix. The matrix element displayed on the left here appears in the intersection of the mn row in field 3 with the $(IU)P$ column from field 2.

Example. Coupling of the representations $E_{1/2}$ and $E_{5/2}$ of $\mathbf{D}_6$

Objective To form the direct product of the bases $e_{1/2} \otimes e_{5/2}$ and to reduce it.

Information needed

T **26**.6 $e_{1/2} = \big\langle |\tfrac{1}{2}\,\tfrac{1}{2}\rangle, |\tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle \big| = \big\langle |i\,1\rangle, |i\,2\rangle \big|$,

T **26**.6 $e_{5/2} = \big\langle |\tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle, |\tfrac{5}{2}\,\tfrac{5}{2}\rangle \big| = \big\langle |j\,1\rangle, |j\,2\rangle \big|$,

$$e_{1/2} \otimes e_{5/2} = \big\langle |\tfrac{1}{2}\,\tfrac{1}{2}\rangle\,|\tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle, |\tfrac{1}{2}\,\tfrac{1}{2}\rangle\,|\tfrac{5}{2}\,\tfrac{5}{2}\rangle, |\tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle\,|\tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle, |\tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle\,|\tfrac{5}{2}\,\tfrac{5}{2}\rangle \big|. \tag{3}$$

Identification of mn

3 11: $|\tfrac{1}{2}\,\tfrac{1}{2}\rangle\,|\tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle$, 12: $|\tfrac{1}{2}\,\tfrac{1}{2}\rangle\,|\tfrac{5}{2}\,\tfrac{5}{2}\rangle$, 21: $|\tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle\,|\tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle$, 22: $|\tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle\,|\tfrac{5}{2}\,\tfrac{5}{2}\rangle$.

Identification of $|(IU)P\rangle$

T **26**.8 $e_{1/2} \otimes e_{5/2} = B_1 \oplus B_2 \oplus E_2$. (Notice: no multiplicity.)
Because B_1 and B_2 are one-dimensional and E_2 two-dimensional these symbols are: $|B_1, 1\rangle$, $|B_2, 1\rangle$, $|E_2, 1\rangle$, $|E_2, 2\rangle$.

T **26**.6 $|B_1, 1\rangle = |3\,3\rangle_-$, $|B_2, 1\rangle = |3\,3\rangle_+$,
$|E_2, 1\rangle = |2\,\overline{2}\rangle$, $|E_2, 2\rangle = -|2\,2\rangle$. (4)

The table

The relevant table from T **26**.11 is transcribed below. In this transcription the components of the relevant bases are explicitly identified from the above and they are added to the table in boxes.

Table **16**.10 Clebsch–Gordan coefficients $\mathrm{u} = 2^{-1/2}$

$e_{1/2}$		$e_{5/2}$		B_1	B_2	E_2	
				1	1	1	2
$\big\langle \lvert\tfrac{1}{2}\,\tfrac{1}{2}\rangle, \lvert\tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle \big\rvert$		$\big\langle \lvert\tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle, \lvert\tfrac{5}{2}\,\tfrac{5}{2}\rangle \big\rvert$		$\lvert 3\,3\rangle_-$	$\lvert 3\,3\rangle_+$	$\lvert 2\,\overline{2}\rangle$	$-\lvert 2\,2\rangle$
1	$\lvert\tfrac{1}{2}\,\tfrac{1}{2}\rangle$	1	$\lvert\tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle$	0	0	1	0
1	$\lvert\tfrac{1}{2}\,\tfrac{1}{2}\rangle$	2	$\lvert\tfrac{5}{2}\,\tfrac{5}{2}\rangle$	u	u	0	0
2	$\lvert\tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle$	1	$\lvert\tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle$	$\overline{\mathrm{u}}$	u	0	0
2	$\lvert\tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle$	2	$\lvert\tfrac{5}{2}\,\tfrac{5}{2}\rangle$	0	0	0	1

Calculation of $|(IU)P\rangle$

From (2) multiply the basis (3) by the first column of the matrix:

$|B_1, 1\rangle$

$$|B_1, 1\rangle = |\tfrac{1}{2}\,\tfrac{1}{2}\rangle\,|\tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle\,0 + |\tfrac{1}{2}\,\tfrac{1}{2}\rangle\,|\tfrac{5}{2}\,\tfrac{5}{2}\rangle\,\mathrm{u} - |\tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle\,|\tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle\,\mathrm{u} + |\tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle\,|\tfrac{5}{2}\,\tfrac{5}{2}\rangle\,0$$
$$= 2^{-1/2}\Big(|\tfrac{1}{2}\,\tfrac{1}{2}\rangle\,|\tfrac{5}{2}\,\tfrac{5}{2}\rangle - |\tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle\,|\tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle\Big),$$

4 The above expression transforms like $|3\,3\rangle_-$.

$|B_2, 1\rangle$

$|B_2, 1\rangle = 2^{-1/2}\Big(|\tfrac{1}{2}\,\tfrac{1}{2}\rangle\,|\tfrac{5}{2}\,\tfrac{5}{2}\rangle + |\tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle\,|\tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle\Big)$, transforms like $|3\,3\rangle_+$.

$|E_2, 1\rangle$

$|E_2, 1\rangle = |\tfrac{1}{2}\,\tfrac{1}{2}\rangle\,|\tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle$, transforms like $|2\,\overline{2}\rangle$.

$|E_2, 2\rangle$

$|E_2, 2\rangle = |\tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle\,|\tfrac{5}{2}\,\tfrac{5}{2}\rangle$, transforms like $-|2\,2\rangle$.

Bibliographical note

A more detailed discussion of the three cases of time-reversal degeneracies given sketchily in the text may be found in Heine (1960).

17

Problems

Cross-references

All cross-references to material in Chapter **2** are given in bold without the chapter number.
All cross-references to material in the present chapter are given in light face without the chapter number.
All cross-references to material from other chapters are preceded by the chapter number in bold.

1 Multiplication rules

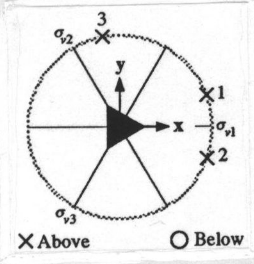

Fig. **17**.1

From T **51**.2 ($\mathbf{C}_{3v}$ group, *A* setting):

$$C_3^+\,\sigma_{v1} = \sigma_{v3}. \tag{1}$$

(C_3^+ is the counter-clockwise rotation seen from above Fig. 1.) Verify this result: (i) geometrically; (ii) from the quaternion multiplication rule. Interpret this result in the double group.

Part (i)

Construct Fig. 1 from F **51***A*. From Fig. 1:

$$\sigma_{v1}\,\mathbf{r}_1 = \mathbf{r}_2, \qquad C_3^+\,\mathbf{r}_2 = C_3^+\,\sigma_{v1}\,\mathbf{r}_1 = \mathbf{r}_3 = \sigma_{v3}\,\mathbf{r}_1 \qquad \Rightarrow \qquad C_3^+\,\sigma_{v1} = \sigma_{v3}. \tag{2}$$

Note. The above work should properly be done for three linearly independent vectors such as $\mathbf{r}_1$, $\mathbf{r}_1'$, $\mathbf{r}_1''$. In most cases, specially for the dihedral and related groups, the results are nevertheless fairly clear when operating on a single position vector, as done here.

Part (ii)

T **51**.1*A* $\qquad C_3^+ \mapsto \llbracket \tfrac{1}{2}, (00\tfrac{\sqrt{3}}{2})\rrbracket, \qquad \sigma_{v1} \mapsto i\,\llbracket 0, (010)\rrbracket, \qquad \sigma_{v3} \mapsto i\,\llbracket 0, (\tfrac{\sqrt{3}}{2}, -\tfrac{1}{2}, 0)\rrbracket. \qquad (3)$

$$\llbracket \tfrac{1}{2}, (00\tfrac{\sqrt{3}}{2})\rrbracket\; i\,\llbracket 0, (010)\rrbracket = i\,\llbracket 0, (0\tfrac{1}{2}0) + (00\tfrac{\sqrt{3}}{2}) \times (010)\rrbracket \tag{4}$$

$$= i\,\llbracket 0, (0\tfrac{1}{2}0) + (-\tfrac{\sqrt{3}}{2}00)\rrbracket \tag{5}$$

$$= i\,\llbracket 0, (-\tfrac{\sqrt{3}}{2}\tfrac{1}{2}0)\rrbracket \tag{6}$$

$$= i\,(-1)\,\llbracket 0, (\tfrac{\sqrt{3}}{2}, -\tfrac{1}{2}, 0)\rrbracket \tag{7}$$

In the single-group work the (-1) factor is disregarded. In the double group, the parameter for $\widetilde{\sigma}_{v3}$ is the negative of the parameter for σ_{v3}, whence (6) must be read as $C_3^+\,\sigma_{v1} = \widetilde{\sigma}_{v3}$. Notice that the factor (-1) in (7) is the factor that corresponds to the product $C_3^+\,\sigma_{v1}$ in T **51**.3.

2 The regular representation

Obtain the regular representation (see **50**) for C_{3v} (A setting). (i) Verify the multiplication rule (1). (ii) Determine the number of times each irreducible representation of $\mathbf{C}_{3v}$ appears in the regular representation.

Part (i)

T **51**.2

$$C_3^+ \langle E\, C_3^+\, C_3^-\, \sigma_{v1}\, \sigma_{v2}\, \sigma_{v3}| = \langle C_3^+\, C_3^-\, E\, \sigma_{v3}\, \sigma_{v1}\, \sigma_{v2}| = \langle E\, C_3^+\, C_3^-\, \sigma_{v1}\, \sigma_{v2}\, \sigma_{v3}| \begin{bmatrix} & & 1 & & & \\ 1 & & & & & \\ & 1 & & & & \\ & & & & 1 & \\ & & & & & 1 \\ & & & 1 & & \end{bmatrix}. \tag{8}$$

T **51**.2

$$\sigma_{v1} \langle E\, C_3^+\, C_3^-\, \sigma_{v1}\, \sigma_{v2}\, \sigma_{v3}| = \langle \sigma_{v1}\, \sigma_{v2}\, \sigma_{v3}\, E\, C_3^+\, C_3^-\, | = \langle E\, C_3^+\, C_3^-\, \sigma_{v1}\, \sigma_{v2}\, \sigma_{v3}| \begin{bmatrix} & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \\ 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \end{bmatrix}. \tag{9}$$

T **51**.2

$$\sigma_{v3} \langle E\, C_3^+\, C_3^-\, \sigma_{v1}\, \sigma_{v2}\, \sigma_{v3}| = \langle \sigma_{v3}\, \sigma_{v1}\, \sigma_{v2}\, C_3^+\, C_3^-\, E| = \langle E\, C_3^+\, C_3^-\, \sigma_{v1}\, \sigma_{v2}\, \sigma_{v3}| \begin{bmatrix} & & & & & 1 \\ & & & 1 & & \\ & & & & 1 & \\ & 1 & & & & \\ & & 1 & & & \\ 1 & & & & & \end{bmatrix}. \tag{10}$$

Multiply the matrices in (8) and (9) and you will obtain the matrix in (10).

Part (ii)

8, 9, 10

$$\chi(g \mid \hat{G}) = |G|,\ g = E; \qquad \chi(g \mid \hat{G}) = 0,\ g \neq E. \tag{11}$$

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$$|i| = |G|^{-1} \sum_g \chi(g \mid {}^i\hat{G})^*\, \chi(g \mid \hat{G}) = |G|^{-1} \chi(E \mid {}^i\hat{G})^*\, |G| = |{}^i\hat{G}|, \quad \forall i. \tag{12}$$

Notice that this result is valid for any group: it is a fundamental property of the regular representation.

3 Transformation of the components of a vector

Transform the components x, y, z of a position vector under the operations C_3^+ and σ_{v1} of $\mathbf{C}_{3v}$. Obtain the corresponding matrix representatives and check that they multiply correctly by eqn (1).

T **51**.1A

$$C_3^+ : \qquad \phi = \tfrac{2\pi}{3}, \quad \mathbf{n} = (001).$$

12.5

$$C_3^+\, \mathbf{r} = -\tfrac{1}{2}\, \mathbf{r} + \tfrac{\sqrt{3}}{2}\, (\mathbf{n} \times \mathbf{r}) + \tfrac{3}{2} (\mathbf{n} \cdot \mathbf{r})\, \mathbf{n} = -\tfrac{1}{2} |x, y, z\rangle + \tfrac{\sqrt{3}}{2} | -y, x, 0\rangle + \tfrac{3}{2} |0, 0, z\rangle = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} |x, y, z\rangle. \tag{13}$$

Proceed in the same way for σ_{v1} and σ_{v3} but **remember to include the inversion** i, which changes the signs of x, y, z.

4 A rotation acting on the function space

The cartesian components of a unit vector $\mathbf{r}$ are:

$$\mathsf{x} = \sin\theta\,\cos\varphi,\;\; \mathsf{y} = \sin\theta\,\sin\varphi,\;\; \mathsf{z} = \cos\theta. \tag{14}$$

(They are written in sans serif because they are not the independent variables but rather functions of θ, φ.) Find the matrix representative of a rotation $R(\alpha\mathbf{z})$ and compare it with the matrix for C_3^+ in (13).

38 $$R(\alpha\mathbf{z})\,\mathsf{x}(\theta\varphi) = \mathsf{x}(R(-\alpha\mathbf{z})\,\theta, R(-\alpha\mathbf{z})\,\varphi) = \mathsf{x}(\theta, \varphi-\alpha) = \sin\theta\,\cos(\varphi-\alpha) = \mathsf{x}\,\cos\alpha + \mathsf{y}\,\sin\alpha. \tag{15}$$

You will get in this way the matrix transformation:

$$R(\alpha\mathbf{z})\,\langle \mathsf{x}, \mathsf{y}, \mathsf{z}| = \langle \mathsf{x}, \mathsf{y}, \mathsf{z}| \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{16}$$

which will lead you to (13). Notice that it is essential to make the column-row distinction of the bases in (13) and (16) in order to get the correct agreement.

5 The faithful (Jones) representation

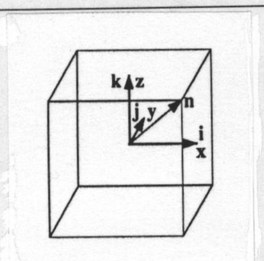


Fig. **17**.2

Consider a cube with right-handed space-fixed axes $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ at its centre and parallel to its edges. The cube-fixed axes $\mathbf{i}, \mathbf{j}, \mathbf{k}$ coincide with $\mathbf{x}, \mathbf{y}, \mathbf{z}$, respectively, for the identity E. Find the transform of $\mathbf{r} = |x, y, z\rangle$ under the rotation R by $-2\pi/3$ about the rotation axis of components $(1\bar{1}1)$ with respect to $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$. Compare your results with the tables of Onodera and Okazaki (1966).

It is easier to transform $\langle \mathbf{i}, \mathbf{j}, \mathbf{k}|$ than $|x, y, z\rangle$ (see **12**.14). From Fig. 2:

$$\langle \mathbf{i}, \mathbf{j}, \mathbf{k}| \to \langle -\mathbf{j}, -\mathbf{k}, \mathbf{i}| = \langle \mathbf{i}, \mathbf{j}, \mathbf{k}| \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}. \tag{17}$$

17 $$R\,|x, y, z\rangle = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} |x, y, z\rangle = |z, -x, -y\rangle. \tag{18}$$

This agrees with Onodera and Okazaki's transform as listed in their Table II for the *inverse* operation, that is the rotation by $+2\pi/3$ (See the heading of their Table II.) Notice the essential change from row to column vectors in dealing with the different bases in this problem.

6 Hybrids: general form

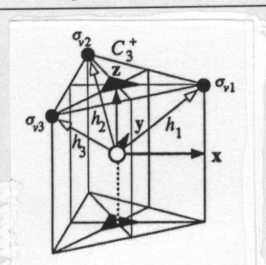


Fig. **17**.3

Obtain the type, in terms of orbitals, of the three hybrids that link the N atom to each of the three H atoms in ammonia.

The hybrids h_1, h_2, h_3 are displayed in Fig. 3 (constructed from F **51**A). Their symmetry group is $\mathbf{C}_{3v}$. From this figure their transformation properties are obtained in Table 1, where for simplicity the hybrids are denoted by their subscripts.

Table **17**.1

$\mathbf{C}_{3v}$	E	C_3^+	C_3^-	σ_{v1}	σ_{v2}	σ_{v3}
gh_1	1	2	3	1	3	2
gh_2	2	3	1	3	2	1
gh_3	3	1	2	2	1	3
$g\langle 123\vert$	$\langle 123\vert$	$\langle 231\vert$	$\langle 312\vert$	$\langle 132\vert$	$\langle 321\vert$	$\langle 213\vert$
$\hat{G}(g)$	$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	$\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$	$\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$	$\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$	$\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$	$\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$
χ	3	0	0	1	1	1

The reduction of the representation, from (**108**) is carried out in Table 2, on using T **51**.4.

Table **17**.2

$\mathbf{C}_{3v}$	E	$2C_3$	$3\sigma_v$	$\vert i\vert$
A_1	1	1	1	1
A_2	1	1	-1	0
E	2	-1	0	1
χ	3	0	1	$A_1 \oplus E$

T **51**.5*A* $\qquad s, p_z \in A_1,\ p_x, p_y \in E \quad \Rightarrow \quad$ hybrids are $p_x\, p_y\, p_z = p^3$.

(If the orbital s were chosen the hybrids would be planar.)

Note. Table 2 can be constructed at once on using the following rule. The character of the representation spanned by σ-type hybrids (hybrids which are symmetrical with respect to their own plane) equals the number of hybrids left invariant by the symmetry operation in question. **When using (108) remember to add up over all the group elements not merely over the characters.**

7 Reduction of a representation by the internal method

Find the matrix C that reduces the representation in Table 1.

From (**120**), add up the matrices of the $2C_3$ class. You will get the matrix M shown below. C is the matrix of its (normalized) eigenvectors.

$$M = \begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}, \qquad C = \begin{bmatrix}\frac{\sqrt{3}}{3} & -\frac{\sqrt{6}}{3} & 0\\ \frac{\sqrt{3}}{3} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{2}}{2}\\ \frac{\sqrt{3}}{3} & \frac{\sqrt{6}}{6} & \frac{\sqrt{2}}{2}\end{bmatrix}. \tag{19}$$

8 Cubic hybrids

Show that eight equivalent cubic hybrids of σ type have the form sp^3d^3f.

The symmetry group is $\mathbf{O}_h$. Construct Table 3 from T **71**.4. Use a drawing and the rule in the note to Table 2. The column headed '$|i|$' follows from (**108**).

Table **17**.3

$\mathbf{O}_h$	E	$3C_2$	$8C_3$	$6C_4$	$6C_2'$	i	3σ	$8S_6$	$6S_4$	$6\sigma_d$	$\lvert i\rvert$
A_{1g}	1	1	1	1	1	1	1	1	1	1	1
A_{2g}	1	1	1	−1	−1	1	1	1	−1	−1	0
E_g	2	2	−1	0	0	2	2	−1	0	0	0
T_{1g}	3	−1	0	1	−1	3	−1	0	1	−1	0
T_{2g}	3	−1	0	−1	1	3	−1	0	−1	1	1
A_{1u}	1	1	1	1	1	−1	−1	−1	−1	−1	0
A_{2u}	1	1	1	−1	−1	−1	−1	−1	1	1	1
E_u	2	2	−1	0	0	−2	−2	1	0	0	0
T_{1u}	3	−1	0	1	−1	−3	1	0	−1	1	1
T_{2u}	3	−1	0	−1	1	−3	1	0	1	−1	0
χ	8	0	2	0	0	0	0	0	0	4	

The result is $\chi = A_{1g} \oplus T_{2g} \oplus A_{2u} \oplus T_{1u}$.

From T **71**.5 the hybrids are $s\, d_{xy}\, d_{yz}\, d_{zx}\, f_{xyz}\, p_x\, p_y\, p_z$.

Note. It is because of the requirement for f orbitals that coordination number 8 is realized in the cubic structure only when the central atom in the bonding is a heavy atom, like U. One example is the complex $[(C_2H_5)_4N]_4[U(NCS)_8]$, (Countryman and McDonald 1971). See also Problem 9.

9 Eight equivalent hybrids not requiring *f* orbitals

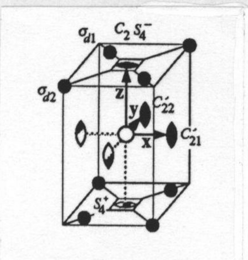


Fig. **17**.4

Show that the eight hybrids of the ion $[Mo(CN)_8]^{4-}$ have the form sp^3d^4. The structure of this molecule is illustrated in Fig. 4, constructed from F **41**. Notice that four of the bonds are slightly shorter than the other four. Its symmetry group is $\mathbf{D}_{2d}$.

Construct Table 4 from T **41**.4.

Table **17**.4

$\mathbf{D}_{2d}$	E	C_2	$2C_2'$	$2S_4$	$2\sigma_d$	$\lvert i\rvert$
A_1	1	1	1	1	1	2
A_2	1	1	−1	1	−1	0
B_1	1	1	1	−1	−1	0
B_2	1	1	−1	−1	1	2
E	2	−2	0	0	0	2
χ	8	0	0	0	4	

$$\chi = 2A_1 \oplus 2B_2 \oplus 2E.$$

T **41**.5 $$s, d_{z^2} \in A_1;\ p_z, d_{xy} \in B_2;\ (p_x, p_y), (d_{xz}, d_{yz}) \in E.$$

The characters are obtained from the rule at the end of § 6. The column headed '$|i|$' follows from (**108**).

Note. The molecule is very slightly distorted from the cubic structure (see Hoard and Nordsieck 1939) in order to avoid the use of f orbitals.

10 Hybrids: their full expression

Obtain an expression for the p^3 hybrids for ammonia in terms of the spherical harmonics for $l = 1$.

Form Table 5 in the same way as Table 1. Since you know that the hybrids will involve $A_1 \oplus E$, enter in the last two columns of the table their full representations from T **51**.7*A*.

Table **17**.5 $\qquad \epsilon = \exp(2\pi\mathrm{i}/3)$

$\mathbf{C}_{3v}$	E	C_3^+	C_3^-	σ_{v1}	σ_{v2}	σ_{v3}
gh_1	1	2	3	1	3	2
gh_2	2	3	1	3	2	1
gh_3	3	1	2	2	1	3
A_1	1	1	1	1	1	1
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$

86 $$A_1\colon\ W_{11}^{A_1} h_1 = \tfrac{1}{6}\, 2\,(h_1 + h_2 + h_3) =_{\text{def}} \phi_0 \quad\Rightarrow\quad \phi_0 = \tfrac{1}{3}\,(h_1 + h_2 + h_3). \tag{20}$$

86 $$E\colon\ W_{11}^{E} h_1 = 2\,\tfrac{1}{6}\,(h_1 + \epsilon h_2 + \epsilon^* h_3) =_{\text{def}} \phi_1 \quad\Rightarrow\quad \phi_1 = \tfrac{1}{3}\,(h_1 + \epsilon h_2 + \epsilon^* h_3). \tag{21}$$

91 $$E\colon\ W_{21}^{E} \phi_1 = \tfrac{1}{9}\left\{-(h_1 + \epsilon h_3 + \epsilon^* h_2) - \epsilon\,(h_3 + \epsilon h_2 + \epsilon^* h_1) - \epsilon^*\,(h_2 + \epsilon h_1 + \epsilon^* h_3)\right\} =_{\text{def}} \phi_2$$

$$\Rightarrow\quad \phi_2 = -\tfrac{1}{3}(h_1 + \epsilon^* h_2 + \epsilon h_3). \tag{22}$$

Eliminate h_1, h_2, h_3 from (20) to (22), remembering that $\epsilon + \epsilon^* = -1$:

$$h_1 = \phi_0 + \phi_1 - \phi_2. \tag{23}$$

$$h_2 = \phi_0 + \epsilon^* \phi_1 - \epsilon \phi_2. \tag{24}$$

$$h_3 = \phi_0 + \epsilon \phi_1 - \epsilon^* \phi_2. \tag{25}$$

From T **51**.6*A*, the basis function ϕ_0 can be chosen as Y_1^0 and the bases ϕ_1, ϕ_2 can be chosen as Y_1^1 and Y_1^{-1}, respectively. The spherical harmonics, if desired can be written in terms of the p_z, p_x, p_y functions from the expressions in § **13**–5.

11 Symmetrized molecular orbitals

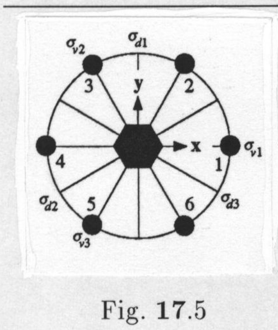


Fig. **17**.5

Obtain the symmetrized molecular orbitals for the π-electron system of benzene, C_6H_6.

Form Fig. 5 from F **54**. The numbers in the figure are the subscripts of the π orbitals ϕ_i $(i = 1, 2, \ldots, 6)$. All the positive rotations are counter-clockwise from above the figure.

The symmetry group

This is a point which requires attention. It is $\mathbf{D}_{6h}$ which must be written as

$$\mathbf{D}_{6h} = \mathbf{C}_{6v} \otimes \mathbf{C}_s, \quad \mathbf{C}_s = E \oplus \sigma_h. \tag{26}$$

When you operate on the π orbitals with σ_h you leave them invariant except for a change of sign. Thus, if you symmetrize with respect to $\mathbf{C}_{6v}$ (see Problem 14) you know that the symmetrized functions must belong to the irreducible representations of $\mathbf{D}_{6h}$ that are antisymmetrical with respect to σ_h and that subduce to the required representations of $\mathbf{C}_{6v}$.

Note. A very common approach here is to write $\mathbf{D}_{6h} = \mathbf{D}_6 \otimes \mathbf{C}_i$ and to symmetrize only with respect to $\mathbf{D}_6$. This is not good because the inversion changes not only the sign of the π orbitals *but also their labelling*. (It transforms a π orbital into the negative of a *different* π orbital.)

How to find the irreducible representations that appear in the molecular orbitals

You must form a basis $\langle \phi_1, \phi_2, \ldots, \phi_6|$. The characters of the representation spanned by this basis equals the number of orbitals left invariant by the corresponding operation. (This rule gives, in fact, the number of $+1$ along the diagonal. **Please note: this rule has to be changed if there are symmetry planes normal to the π-electron system in the symmetry group used.**) Form Table 6 from T **54**.4.

Table **17**.6

$\mathbf{C}_{6v}$	E	$2C_6$	$2C_3$	C_2	$3\sigma_d$	$3\sigma_v$	$\lvert i \rvert$
A_1	1	1	1	1	1	1	1
A_2	1	1	1	1	-1	-1	0
B_1	1	-1	1	-1	-1	1	1
B_2	1	-1	1	-1	1	-1	0
E_1	2	1	-1	-2	0	0	1
E_2	2	-1	-1	2	0	0	1
χ	6	0	0	0	0	2	

$$\chi = A_1 \oplus B_1 \oplus E_1 \oplus E_2. \tag{27}$$

Use of the projection operator

If the only object of this work is to factorize the secular determinant, phases of the bases are unimportant so that work is saved by not using the transfer operator. Also: although in principle the generators on which the projection operator is applied must be the six functions ϕ_i $(i = 1, 2, \ldots, 6)$, it will be found that ϕ_1 suffices and for brevity only this will be listed below. Remember that in order to apply the projection operator all operations of the group are necessary, not only those listed in the character table. The transformed functions $g\phi_i$ are obtained from Fig. 5. For simplicity, only the subscripts of the functions ϕ_i are entered in the table. The irreducible representations listed in (27) are obtained from T **54**.4 and T **54**.7.

Table **17**.7 $\epsilon = \exp(2\pi\mathrm{i}/3)$

$\mathbf{C}_{6v}$	E	C_6^+	C_6^-	C_3^+	C_3^-	C_2	σ_{d1}	σ_{d2}	σ_{d3}	σ_{v1}	σ_{v2}	σ_{v3}
$g\phi_1$	1	2	6	3	5	4	4	2	6	1	5	3
A_1	1	1	1	1	1	1	1	1	1	1	1	1
B_1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	1
E_1	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon} & 0\\0 & \bar{\epsilon}^*\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon}^* & 0\\0 & \bar{\epsilon}\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\bar{1} & 0\\0 & \bar{1}\end{bmatrix}$	$\begin{bmatrix}0 & \bar{1}\\\bar{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}\\\bar{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}^*\\\bar{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon\\\epsilon^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon^*\\\epsilon & 0\end{bmatrix}$
E_2	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}0 & \bar{1}\\\bar{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}\\\bar{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}^*\\\bar{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{1}\\\bar{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}\\\bar{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}^*\\\bar{\epsilon} & 0\end{bmatrix}$

The symmetrized functions (bases)

Call the bases as follows:

$$\psi_1 \in A_1; \qquad \psi_2 \in B_1; \qquad \psi_3, \psi_4 \in E_1; \qquad \psi_5, \psi_6 \in E_2. \tag{28}$$

86 A_1: $W_{11}^{A_1}\,\phi_1 = \frac{1}{12}\,2\,(\phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6).$ (29)

86 B_1: $W_{11}^{B_1}\,\phi_1 = \frac{1}{12}\,(\phi_1 - \phi_2 - \phi_6 + \phi_3 + \phi_5 - \phi_4 - \phi_4 - \phi_2 - \phi_6 + \phi_1 + \phi_5 + \phi_3).$ (30)

86 E_1: $W_{11}^{E_1}\,\phi_1 = 2\,\frac{1}{12}\,(\phi_1 - \epsilon^*\phi_2 - \epsilon\phi_6 + \epsilon\phi_3 + \epsilon^*\phi_5 - \phi_4).$ (31)

86 E_1: $W_{21}^{E_1}\,\phi_1 = 2\,\frac{1}{12}\,(-\phi_4 - \epsilon\phi_2 - \epsilon^*\phi_6 + \phi_1 + \epsilon\phi_5 + \epsilon^*\phi_3).$ (32)

86 E_2: $W_{11}^{E_2}\,\phi_1 = 2\,\frac{1}{12}\,(\phi_1 + \epsilon^*\phi_2 + \epsilon\phi_6 + \epsilon\phi_3 + \epsilon^*\phi_5 + \phi_4).$ (33)

86 E_2: $W_{21}^{E_2}\,\phi_1 = 2\,\frac{1}{12}\,(-\phi_4 - \epsilon\phi_2 - \epsilon^*\phi_6 - \phi_1 - \epsilon\phi_5 - \epsilon^*\phi_3).$ (34)

29 $\psi_1 = \frac{1}{6}\,(\phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6).$ (35)

30 $\psi_2 = \frac{1}{6}\,(\phi_1 - \phi_2 + \phi_3 - \phi_4 + \phi_5 - \phi_6).$ (36)

31 $\psi_3 = \frac{1}{6}\,(\phi_1 - \epsilon^*\phi_2 + \epsilon\phi_3 - \phi_4 + \epsilon^*\phi_5 - \epsilon\phi_6).$ (37)

32 $\psi_4 = \frac{1}{6}\,(\phi_1 - \epsilon\phi_2 + \epsilon^*\phi_3 - \phi_4 + \epsilon\phi_5 - \epsilon^*\phi_6).$ (38)

33 $\psi_5 = \frac{1}{6}\,(\phi_1 + \epsilon^*\phi_2 + \epsilon\phi_3 + \phi_4 + \epsilon^*\phi_5 + \epsilon\phi_6).$ (39)

34 $\psi_6 = -\frac{1}{6}\,(\phi_1 + \epsilon\phi_2 + \epsilon^*\phi_3 + \phi_4 + \epsilon\phi_5 + \epsilon^*\phi_6).$ (40)

Note. The functions ψ_1 to ψ_6 are all precisely orthogonal because they belong to different columns of different irreducible representations (see eqn **168**). Thus the secular determinant is fully diagonal. These functions must be normalized.

The full symmetry of the molecular orbitals in $\mathbf{D}_{6h}$

You must first find, from T **35**.4, the irreducible representations of $\mathbf{D}_{6h}$ which are antisymmetrical with respect to σ_h: B_{1g}, B_{2g}, E_{1g}, A_{1u}, A_{2u}, E_{2u}. You must then use the subduction table for $\mathbf{D}_{6h}$ the wrong way round, that is going from $\mathbf{C}_{6v}$ to $\mathbf{D}_{6h}$ in order to find the representations of $\mathbf{D}_{6h}$ in the above list that subduce to those in (27).

T **35**.9 $\psi_1 \in A_{2u}$; $\psi_2 \in B_{2g}$; $\psi_3, \psi_4 \in E_{1g}$; $\psi_5, \psi_6 \in E_{2u}$. (41)

12 Symmetrized molecular orbitals: projecting over the representations

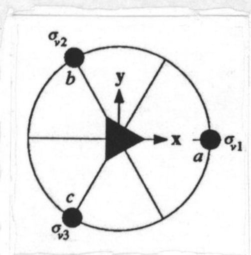


Fig. **17**.6

Obtain symmetrized molecular orbitals for the triangular molecule shown in the figure, where a, b, and c are normalized atomic functions symmetrical with respect to the respective symmetry planes shown. Form the diagonal matrix elements of the Hamiltonian for a degenerate representation. Why are they equal?

Write the symmetry group as $\mathbf{D}_{3h} = \mathbf{C}_{3v} \otimes \mathbf{C}_s$ and symmetrize with respect to $\mathbf{C}_{3v}$. (See Problems 11 and 14.) Obtain the characters of $\mathbf{C}_{3v}$ from T **51**.4. (Warning: in order to use **93** it is not sufficient to list one operation of each class: all are needed.) List, as it is done in rows 1 to 3 of Table 8, the transforms of the orbitals a, b, c. Obtain in row 4 the characters of the representation spanned by the basis $\langle a, b, c|$ and find the irreducible representations that will appear on symmetrization.

Table **17**.8

$\mathbf{C}_{3v}$	E	C_3^+	C_3^-	σ_{v1}	σ_{v2}	σ_{v3}		Row number
ga	a	b	c	a	c	b		1
gb	b	c	a	c	b	a		2
gc	c	a	b	b	a	c		3
χ	3	0	0	1	1	1	$A_1 \oplus E$	4
A_1	1	1	1	1	1	1		5
A_2	1	1	1	−1	−1	−1		6
E	2	−1	−1	0	0	0		7

From (**93**), on normalizing the molecular orbitals for A_1, you get:

$$\psi^{A_1} = \frac{1}{\sqrt{3}}\,(a + b + c). \tag{42}$$

For E, disregarding for the time being normalization and constant factors, you get:

$$\psi_1 = 2a - b - c, \qquad \psi_2 = 2b - c - a, \qquad \psi_3 = 2c - a - b, \tag{43}$$

You know that you must get only two independent functions belonging to this representation. You could therefore retain ψ_1 and ψ_2 and discard ψ_3, but you can do better than this since you would like to get two orthogonal functions. In order to do this, it is sufficient to construct out of the three functions in (43) two that are symmetrical and antisymmetrical respectively to one symmetry plane, say σ_{v1}. It is easy to guess that for this purpose it is enough to take ψ_1 (symmetrical), and $\psi_2 - \psi_3 = 3(b - c)$ (antisymmetrical). On normalizing,

$$\psi_1^E = \frac{1}{\sqrt{6}}\,(2a - b - c), \qquad \psi_2^E = \frac{1}{\sqrt{2}}\,(b - c). \tag{44}$$

Matrix elements:

$$H_{11}^E = \tfrac{1}{6}\,\langle 2a - b - c \mid H \mid 2a - b - c\rangle \tag{45}$$

$$= \tfrac{1}{6}\,(4H_{aa} - 2H_{ab} - 2H_{ac} - 2H_{ba} + H_{bb} + H_{bc} - 2H_{ca} + H_{cb} + H_{cc}) \tag{46}$$

$$= H_{aa} - H_{ab}. \qquad (H_{aa} = H_{bb},\ \text{etc.,} \quad H_{ab} = H_{bc},\ \text{etc.}) \tag{47}$$

$$H_{22}^E = H_{aa} - H_{ab}. \tag{48}$$

They are equal on account of (**167**).

13 A transition-metal complex

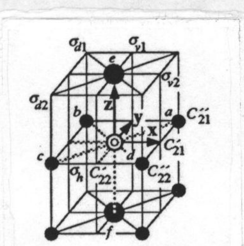


Fig. **17**.7

A transition-metal ion Me is surrounded by six s-type ligands, four of them, a, b, c, d, at the corners of a square of which Me is the centre, and two, e, f, at right angles to the square, above and below Me, respectively. The bond lengths Me–e and Me–f are equal but different from the other four. Form six symmetry-adapted molecular orbitals for the ligands and determine the orbitals of Me which can combine with them.

Figure 7, compared with F **33**, shows that the symmetry group is $\mathbf{D}_{4h}$. The work can easily be done by using the projection operator over the representations, eqn (**93**). Constant factors in it will be disregarded since in any case the functions will have to be normalized. Notice that when this approach is taken more than one generator must be used for the projection operator and that it is prudent to transform for this purpose all the six orbitals given. This transformation is done from Fig. 7. The characters come from T **33**.4.

Table **17**.9

$\mathbf{D}_{4h}$	E	C_4^+	C_4^-	C_2	C'_{21}	C'_{22}	C''_{21}	C''_{22}	i	S_4^-	S_4^+	σ_h	σ_{v1}	σ_{v2}	σ_{d1}	σ_{d2}	$\lvert i\rvert$
ga	a	b	d	c	d	b	a	c	c	d	b	a	b	d	c	a	
gb	b	c	a	d	c	a	d	b	d	a	c	b	a	c	b	d	
gc	c	d	b	a	b	d	c	a	a	b	d	c	d	b	a	c	
gd	d	a	c	b	a	c	b	d	b	c	a	d	c	a	d	b	
ge	e	e	e	e	f	f	f	f	f	f	f	f	e	e	e	e	
gf	f	f	f	f	e	e	e	e	e	e	e	e	f	f	f	f	
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2
A_{2g}	1	1	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	−1	0
B_{1g}	1	−1	−1	1	1	1	−1	−1	1	−1	−1	1	1	1	−1	−1	0
B_{2g}	1	−1	−1	1	−1	−1	1	1	1	−1	−1	1	−1	−1	1	1	1
E_g	2	0	0	−2	0	0	0	0	2	0	0	−2	0	0	0	0	0
A_{1u}	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	0
A_{2u}	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1
B_{1u}	1	−1	−1	1	1	1	−1	−1	−1	1	1	−1	−1	−1	1	1	0
B_{2u}	1	−1	−1	1	−1	−1	1	1	−1	1	1	−1	1	1	−1	−1	0
E_u	2	0	0	−2	0	0	0	0	−2	0	0	2	0	0	0	0	1
χ	6	2	2	2	0	0	2	2	0	0	0	4	2	2	4	4	

93, T **33**.5 A_{1g}: $\phi_1 = a + b + c + d,\quad \phi_2 = e + f.\quad s;\ d_{z^2}.$ (49)

93, T **33**.5 B_{2g}: $\phi_3 = a - b + c - d.\quad d_{xy}.$ (50)

93, T **33**.5 A_{2u}: $\phi_4 = e - f.\quad p_z;\ f_{z^3}.$ (51)

93, T **33**.5 E_u: $\phi_5 = a - c,\quad \phi_6 = b - d.\quad (p_x, p_y);\ (f_{xz^2}, f_{yz^2})$
or $(f_{x(x^2-y^2)}, f_{y(x^2-y^2)})$. (52)

14 Use of the projection operator on a direct product

Given $G = H \otimes S$, prove that in order to symmetrize with respect to G it is possible to symmetrize first with respect to H and then to symmetrize with respect to S the function thus obtained. Discuss the implication of their result in relation to the derivation of the symmetrized molecular orbitals in benzene (Problem 11).

86 $$W^i_{np} = |{}^i\check{G}|\,|G|^{-1} \sum\nolimits_g {}^i\check{G}(g)^*_{np}\, g \tag{53}$$

126 $$W^k_{mn,pq} = |{}^k\check{G}|\,|G|^{-1} \sum\nolimits_{hs} {}^k\check{G}(hs)^*_{mn,pq}\, hs \tag{54}$$

126 $$W^k_{mn,pq} = |{}^i\check{H}|\,|{}^j\check{S}|\,|H|^{-1}\,|S|^{-1} \sum\nolimits_{hs} {}^i\check{G}(hs)^*_{mn,pq}\, hs \tag{55}$$

126 $$= |{}^i\check{H}|\,|{}^j\check{S}|\,|H|^{-1}\,|S|^{-1} \sum\nolimits_{hs} {}^i\check{H}(h)^*_{mp}\, {}^j\check{S}(s)^*_{nq}\, hs \tag{56}$$

$$= |{}^j\check{S}|\,|S|^{-1} \sum\nolimits_s {}^j\check{S}(s)^*_{nq} \left\{ |{}^i\check{H}|\,|H|^{-1} \sum\nolimits_h {}^i\check{H}(h)^*_{mp}\, h \right\} s. \tag{57}$$

The curly bracket here is the projection operator over H. For $\mathbf{D}_{6h} = \mathbf{C}_{6v} \otimes \mathbf{C}_s$, the projection over $\mathbf{C}_s$ transforms each generator, in principle, into a symmetrical and an antisymmetrical function with respect to σ_h. In the case of benzene, the generators arising from the projection over $\mathbf{C}_{6v}$ are already antisymmetrical (π orbitals) and require no further symmetrization.

15 Selection rules

Consider the matrix element

160 $$I_{ij} = \int (\psi^i)^*\, \mathrm{U}^k\, \psi^j\, \mathrm{d}\tau, \qquad \mathrm{U}^k = x, y, \text{ or } z, \tag{58}$$

where ψ^i and ψ^j belong to irreducible representations of $\mathbf{O}$. Find all the permitted transitions.

T **69**.5

$$x, y, z \in T_1. \tag{59}$$

Use rule (**161**), but remember that when $i = j$ the symmetrized direct product must be formed, that is, representations in curly brackets in T **69**.8 must be disregarded. The result is:

$$A_1 \to T_1, \quad A_2 \to T_2, \quad E \to T_1, \quad E \to T_2, \quad T_1 \to T_2. \tag{60}$$

All the reverse transitions of those listed above are also permitted.

16 The form of the secular determinant

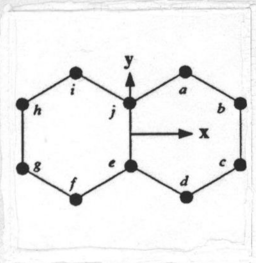


Fig. **17**.8

Find the structure of the secular determinant for the π-electron system of naphthalene, $C_{10}H_{10}$, in the molecular orbital approximation.

From Fig. 8, the symmetry group is $\mathbf{D}_{2h} = \mathbf{C}_{2v} \otimes \mathbf{C}_s$ and from F **31**, σ_x and σ_y are perpendicular to **x** and **y**, respectively. From Problem 14, symmetrize with respect to $\mathbf{C}_{2v}$, on using T **50**.4.

Table **17**.10

$\mathbf{C}_{2v}$	E	C_2	σ_x	σ_y	$\lvert i \rvert$
A_1	1	1	1	1	3
A_2	1	1	−1	−1	2
B_1	1	−1	−1	1	3
B_2	1	−1	1	−1	2
χ	10	0	0	2	

Since the representations that appear here are all one-dimensional, it follows from (**167**) that the determinant will be block-diagonal with two 3×3 and two 2×2 blocks along the diagonal.

17 Normal coordinates

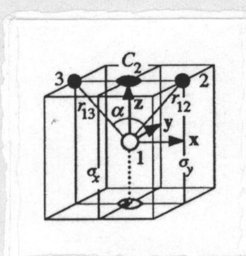


Fig. **17**.9

Construct normal coordinates for the water molecule: (i) in terms of internal coordinates, (ii) in cartesian coordinates.

The symmetry of the water molecule is $\mathbf{C}_{2v}$, as displayed in Fig. 9 (compare with F **50**).

Before you construct the coordinates you must determine the irreducible representations to which they belong. Address for this purpose the representation spanned by $x_1, y_1, z_1, x_2, y_2, z_2, x_3, y_3, z_3$. Rule for its character: Consider only such particles as are left invariant by the symmetry operation in question. Call n the number of their coordinates that are left invariant and m the number of those coordinates that change sign. The character is $n - m$. (See also Table 13 below.) We use T **50**.4 for the characters, and T **50**.5 to find the irreducible representations to which the three translations (along x, y, z) and the three rotations (R_x, R_y, R_z) of the molecule belong. In Table 11, $\lvert n \rvert$ is the number of normal coordinates in each representation and it is obtained by subtracting from the column labelled $\lvert i \rvert$ the two columns that follow $\lvert i \rvert$.

Table **17**.11

$\mathbf{C}_{2v}$	E	C_2	σ_x	σ_y	$\lvert i\rvert$	x, y, z	R_x, R_y, R_z	$\lvert n\rvert$
A_1	1	1	1	1	3	1		2
A_2	1	1	−1	−1	1		1	0
B_1	1	−1	−1	1	3	1	1	1
B_2	1	−1	1	−1	2	1	1	0
χ	9	−1	1	3				

Part (i)

Use the internal coordinates r_{12}, r_{13}, α in Fig. 9 as generators in (**93**), disregarding constant factors.

Table **17**.12

$\mathbf{C}_{2v}$	E	C_2	σ_x	σ_y
gr_{12}	r_{12}	r_{13}	r_{13}	r_{12}
gr_{13}	r_{13}	r_{12}	r_{12}	r_{13}
$g\alpha$	α	α	α	α
A_1	1	1	1	1
B_1	1	−1	−1	1

93 $$A_1\colon\ r_{12} + r_{13},\ \alpha. \tag{61}$$

93 $$B_1\colon\ r_{12} - r_{13}. \tag{62}$$

Notice that the number of normal coordinates in (61) and (62) agrees with $\lvert n\rvert$ in Table 11.

Part (ii)

Table **17**.13

$\mathbf{C}_{2v}$	E	C_2	σ_x	σ_y	$\lvert i\rvert$
x_1	x_1	$-x_1$	$-x_1$	x_1	
y_1	y_1	$-y_1$	y_1	$-y_1$	
z_1	z_1	z_1	z_1	z_1	
x_2	x_2	$-x_3$	$-x_3$	x_2	
y_2	y_2	$-y_3$	y_3	$-y_2$	
z_2	z_2	z_3	z_3	z_2	
x_3	x_3	$-x_2$	$-x_2$	x_3	
y_3	y_3	$-y_2$	y_2	$-y_3$	
z_3	z_3	z_2	z_2	z_3	
A_1	1	1	1	1	3
A_2	1	1	−1	−1	1
B_1	1	−1	−1	1	3
B_2	1	−1	1	−1	2
χ	9	−1	1	3	

93 $$A_1\colon\ z_1,\ x_2 - x_3,\ z_2 + z_3. \tag{63}$$

93 $$A_2\colon\ y_2 - y_3. \tag{64}$$

93 $$B_1\colon\ x_1,\ x_2 + x_3,\ z_2 - z_3. \tag{65}$$

93 $$B_2\colon\ y_1,\ y_2 + y_3. \tag{66}$$

As before, we have to remove from this coordinates three translations and three rotations. This can be done by using the Eckart conditions. See for instance Lomont (1959), p. 121.

18 Infrared and Raman activity of normal vibrations

Prove that all the normal vibrations of water, which belong to the irreducible representations $2A_1 \oplus B_1$ of $\mathbf{C}_{2v}$ (see Problem 17) are active both in infrared and in Raman transitions.

We require transition probability integrals

160 $$I_{ij} = \int (\psi^i)^* \, \mathrm{U}^k \, \psi^j \, \mathrm{d}\tau, \qquad (67)$$

with U^k equal to the dipole operator x, y, z for the infrared and to the polarizability tensor of components x^2, y^2, z^2, xy, yz, zx (or linear combinations thereof), for the Raman transitions. We form in Table 14 the terms required by the selection rule (**161**), noticing that all the representations are real.

Table **17**.14

${}^i\check{G}(g) \otimes {}^j\check{G}(g)$ (T **50**.8)	${}^k\check{G}(g)$ (T **50**.5)	
	Infrared	Raman
$A_1 \otimes A_1 = A_1$	$z \in A_1$	$x^2, y^2, z^2 \in A_1$
$A_1 \otimes B_1 = B_1$	$x \in B_1$	$xz \in B_1$
$B_1 \otimes B_1 = A_1$	$z \in A_1$	$x^2, y^2, z^2 \in A_1$

It can be seen at once from Table 14 that the infrared and Raman transitions are allowed under the selection rule (**161**).

19 Overtones and combination frequencies

The normal vibrations of ammonia (NH_3, symmetry $\mathbf{C}_{3v}$) are $2A_1 \oplus E$. Prove that all the normal vibrations of ammonia are active both in the infrared and in the Raman spectra and that the same property is valid also for all the overtones of A_1. Prove that this property is also valid for any combination frequency $A_1 \to E$.

Table **17**.15

${}^i\check{G}(g) \otimes {}^j\check{G}(g)$ (T **51**.8)	${}^k\check{G}(g)$ (T **51**.5A)	
	Infrared	Raman
$A_1 \otimes A_1 = A_1$	$z \in A_1$	$z^2 \in A_1$
$A_1 \otimes E = E$	$(x, y) \in E$	$(xy, x^2 - y^2), (zx, yz) \in E$
$E \,\overline{\otimes}\, E = A_1 \oplus E$	$z \in A_1$ $(x, y) \in E$	$x^2 \in A_1$ $(xy, x^2 - y^2), (zx, yz) \in E$

The results for the normal vibrations are established from the rows corresponding to the products $A_1 \otimes A_1$ and $E \,\overline{\otimes}\, E$. (See **162**.) Because $A_1 \otimes A_1 = A_1$, all the overtones $(A_1)^n$ are active in both cases, like A_1 is. The activity of the combination frequency $A_1 \to E$ follows from the row for $A_1 \otimes E$.

20 Normal vibrations in methane

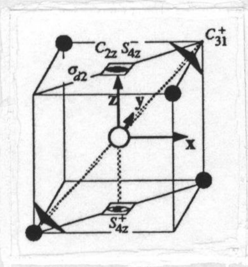


Fig. **17**.10

Methane (CH_4) is of symmetry $\mathbf{T}_d$. Find its normal vibrations and their infrared and Raman activity. Find the overtones and combination frequencies active in the infrared.

Draw, from F **73**, Fig. 10, where it is sufficient to identify an operation of each class only. Consider the basis spanned by the fifteen cartesian coordinates of the five particles and reduce it in Table 16, obtained from T **73**.4 and T **73**.5. $|n|$ is the number of normal coordinates in each representation.

Table **17**.16

$\mathbf{T}_d$	E	$3C_2$	$8C_3$	$6S_4$	$6\sigma_d$	$\|i\|$	x, y, z	R_x, R_y, R_z	$\|n\|$
A_1	1	1	1	1	1	1			1
A_2	1	1	1	−1	−1	0			0
E	2	2	−1	0	0	1			1
T_1	3	−1	0	1	−1	1		1	0
T_2	3	−1	0	−1	1	3	1		2
χ	15	−1	0	−1	3				

Notes. (i) In order to get χ it is sufficient to consider a single operation in each class. (ii) C_2 leaves only the central atom invariant: it leaves one coordinate invariant and changes the sign of the other two. Although S_4 leaves the central atom invariant it changes the sign of the coordinate along the S_4 axis, whereas the other two coordinates are not left invariant (subject to a possible change of sign) and thus do not contribute to the character. (iii) σ_d leaves invariant the z components of the central atom and of two H atoms.

The normal coordinates will be called $\phi_1 \in A_1$, $\phi_2 \in E$, $\phi_3 \in T_2$, $\phi_4 \in T_2$. ϕ_2 is doubly degenerate and ϕ_3 and ϕ_4 are both triply degenerate. See Problem 19 for the construction of Table 17.

Table **17**.17

${}^i\breve{G}(g) \otimes {}^j\breve{G}(g)$ (T **73**.8)	${}^k\breve{G}(g)$ (T **73**.5) Infrared	Raman	Comments
$A_1 \otimes A_1 = A_1$		$x^2, y^2, z^2 \in A_1$	Raman active
$A_1 \otimes E = E$			Comb. not active
$A_1 \otimes T_2 = T_2$	$(x, y, z) \in T_2$		Comb. active
$E \,\overline{\otimes}\, E = A_1 \oplus E$		$x^2, y^2, z^2 \in A_1$ $(x^2 - y^2, 2z^2 - x^2 - y^2) \in E$	Raman active
$E \otimes T_2 = T_1 \oplus T_2$	$(x, y, z) \in T_2$		Comb. active
$T_2 \,\overline{\otimes}\, T_2 = A_1 \oplus E \oplus T_2$	$(x, y, z) \in T_2$	$(xy, yz, zx) \in T_2$	Raman, infrared, act.
$T_2 \otimes T_2 = A_1 \oplus E \oplus T_1 \oplus T_2$	$(x, y, z) \in T_2$		$\phi_3 \rightarrow \phi_4$ active#

Notice that for this combination the ordinary direct product is required: the symmetrized direct product is used not just when the bases have the same symmetry but only when they are identical. (See **162**.)

If we require the matrix element of ϕ_3^2 with itself (overtone), we must form the direct product $(A_1 \oplus E \oplus T_2) \,\overline{\otimes}\, (A_1 \oplus E \oplus T_2)$, the term $T_2 \,\overline{\otimes}\, T_2$ of which will contain T_2. Since $x, y, z \in T_2$, this overtone will be active, and the same for ϕ_3^4.

21 Jahn–Teller effect

The methane molecule (CH_4) has $\mathbf{T}_d$ symmetry. Show that a molecular state for which the electronic wave function ψ^i is degenerate (that is, it belongs to the representations E, T_1 or T_2 of $\mathbf{T}_d$) is unstable.

From Problem 20, the normal vibrations φ^k are of symmetry A_1, E, or T_2. We have to deal with the coupling of the electronic state ψ^i with a normal vibration φ^k. This coupling will exist, and thus break the molecular symmetry, if the matrix element (67), for $i = j$, does not vanish for a vibration which is not totally symmetrical (that is of symmetry E, or T_2). It can be shown that the operator U^k in this integral has the same symmetry as φ^k. In order to use (**161**) we form the symmetrized direct products for the three possible degenerate electronic wave functions:

T **73**.8
$$E \mathbin{\overline{\otimes}} E = A_1 \oplus E. \tag{68}$$

T **73**.8
$$T_1 \mathbin{\overline{\otimes}} T_1 = T_2 \mathbin{\overline{\otimes}} T_2 = A_1 \oplus E \oplus T_2. \tag{69}$$

In all three cases the right-hand side is such that the matrix element will not vanish when U^k belongs to either E or T_2. Thus one of these two non-totally symmetrical vibrations is locked in, reducing the molecular symmetry: this is the Jahn–Teller effect.

22 Electronic states in an octahedral complex

Consider an octahedral complex (symmetry $\mathbf{O}_h$) in which we have two orbitals t_{2g} and e_g. A molecular electronic state in which there is one electron in the first and another electron in the second orbital is represented with the symbol $t_{2g}\, e_g$. Neglecting exclusions arising from the presence of identical electrons find the possible electronic states arising in the configurations t_{2g}^2, $t_{2g}\, e_g$, e_g^2.

Because we have assumed that the two states in t_{2g}^2 and e_g^2 are distinct, we do not require symmetrized direct products. (See **162**.)

T **71**.8
$$T_{2g} \otimes T_{2g} = A_{1g} \oplus E_g \oplus T_{1g} \oplus T_{2g}. \tag{70}$$
$$T_{2g} \otimes E_g = T_{1g} \oplus T_{2g}. \tag{71}$$
$$E_g \otimes E_g = A_{1g} \oplus A_{2g} \oplus E_g. \tag{72}$$

23 Splitting of a doublet in a magnetic field

Prove for an octahedral field that the transition $e_g \to t_{2g}$ is forbidden and that the doublets e_g and e_u do not split when placed in a magnetic field.

The first result follows from (71) and T **71**.5. For the second, consider the integral (67) with U^k equal to x, y, or z. From T **71**.5, these variables belong to T_{1u}. From T **71**.8, $E_g \mathbin{\overline{\otimes}} E_g = E_u \mathbin{\overline{\otimes}} E_u = A_{1g} \oplus E_g$. Because the result does not contain T_{1u} the interaction fails.

24 Subduction (descent of symmetry)

The normal vibrations of methane, (CH_4, symmetry $\mathbf{T}_d$), are of symmetry A_1, E, or T_2 (twice). Show that in CH_3D (symmetry $\mathbf{C}_{3v}$) the T_2 normal vibrations split into two vibrations, one singly degenerate and the other doubly degenerate.

From T **73**.9, T_2 of $\mathbf{T}_d$ goes into $A_1 \oplus E$ in $\mathbf{C}_{3v}$.

25 Double group: term splitting

Show that if a metal atom Me is at the centre of a complex with octahedral symmetry $\mathbf{O}$ its energy level for $j = 5/2$, which is six-fold degenerate in the free atom, splits into two levels of degeneracy 2 and 4, respectively.

This result is immediate from T **69**.10 which shows that $j = 5/2$ splits into $E_{5/2} \oplus F_{3/2}$. We shall derive this result below from first principles, however, for two reasons. First, it will provide an example of the construction of a character table for a double group in a case when there are irregular operations. Secondly, it will provide an example, for the more adventurous reader, of how the work with the double groups can be by-passed by using projective representations with substantial savings in labour and a minimal use of theory.

Double-group method

All the work required in this method is given in Table 18, the first half of which (until the first rule in the body of the table) consists of creating the character table of the double group.

Table **17**.18

$\widetilde{\mathbf{O}}$	E	$\widetilde{E}$	$3C_2, 3\widetilde{C}_2$	$8C_3$	$8\widetilde{C}_3$	$6C_4$	$6\widetilde{C}_4$	$6C_2', 6\widetilde{C}_2'$	$\lvert i \rvert$
A_1	1	1	1	1	1	1	1	1	0
A_2	1	1	1	1	1	-1	-1	-1	0
E	2	2	2	-1	-1	0	0	0	0
T_1	3	3	-1	0	0	1	1	-1	0
T_2	3	3	-1	0	0	-1	-1	1	0
$E_{1/2}$	2	-2	0	1	-1	$\sqrt{2}$	$-\sqrt{2}$	0	0
$E_{5/2}$	2	-2	0	1	-1	$-\sqrt{2}$	$\sqrt{2}$	0	1
$F_{3/2}$	4	-4	0	-1	1	0	0	0	1
ϕ	0	2π	π	$2\pi/3$	$8\pi/3$	$\pi/2$	$5\pi/2$	π	
3ϕ	0	6π	3π	2π	8π	$3\pi/2$	$3\pi/2$	3π	
$\phi/2$	0	π	$\pi/2$	$\pi/3$	$4\pi/3$	$\pi/4$	$5\pi/4$	$\pi/2$	
$\sin 3\phi$	0	0	0	0	0	-1	-1	0	
$\sin(\phi/2)$	0	0	1	$\sqrt{3}/2$	$-\sqrt{3}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	1	
$\chi^{5/2}$	6	-6	0	0	0	$-\sqrt{2}$	$\sqrt{2}$	0	

Notes about the construction of the table

Head row — The classes are obtained from T **69**, subsection 4.

Characters of vector representations — (Representations without fractional indices.) The character of a class that contains g or $\widetilde{g}$ is the same as the character of the class that contains g in T **69**.4.

Characters of spinor representations — The character of a class that contains g is the same as the character of the class that contains g in T **69**.4.
The character of a class that contains $\widetilde{g}$ is the negative of the character of the class that contains g in T **69**.4.
The character of a class that contains both g and $\widetilde{g}$ is always zero, so that the two rules above are not incompatible.

Values of ϕ for $\widetilde{g}$ — The rule is given in (**12**.43).

$\chi^{5/2}$ — Use (**12**.38) to (**12**.40). From the column $\lvert i \rvert$, it equals $E_{5/2} \oplus F_{3/2}$.

Projective-representation method

Table **17**.19

O	E	$3C_2$	$8C_3$	$6C_4$	$6C'_2$	$\lvert i \rvert$
A_1	1	1	1	1	1	0
A_2	1	1	1	-1	-1	0
E	2	2	-1	0	0	0
T_1	3	-1	0	1	-1	0
T_2	3	-1	0	-1	1	0
$E_{1/2}$	2	0	1	$\sqrt{2}$	0	0
$E_{5/2}$	2	0	1	$-\sqrt{2}$	0	1
$F_{3/2}$	4	0	-1	0	0	1
ϕ	0	π	$2\pi/3$	$\pi/2$	π	
3ϕ	0	3π	2π	$3\pi/2$	3π	
$\phi/2$	0	$\pi/2$	$\pi/3$	$\pi/4$	$\pi/2$	
$\sin 3\phi$	0	0	0	-1	0	
$\sin(\phi/2)$	0	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1	
$\chi^{5/2}$	6	0	0	$-\sqrt{2}$	0	

Notes about the construction of the table

Character table From T **69**.4.

$\chi^{5/2}$ Use (**12**.38) and (**12**.39). It equals $E_{5/2} \oplus F_{3/2}$.

The result is of course the same as in the double-group method, but with half the work. Notice that T **69**.4 can be used without any particular reference to the fact that the representations are projective rather than vector.

26 A crystal field

Cerium ethylsulphate, $Ce(C_2H_5SO_4)_3 \cdot 9H_2O$, has the following structure. The Ce^{3+} ion is at the centre of a hexagonal prism with three $(C_2H_5SO_4)^-$ ions at three alternate vertices of the central hexagon, the other three containing three H_2O. The remaining six H_2O are on the basal planes of the prism, three above and three below the $(C_2H_5SO_4)^-$ ions. All the reflection symmetry of the hexagonal prism, except that of the central plane, is broken by the ligands. The symmetry group is $\mathbf{C}_{3h}$. The lowest configuration of the Ce^{3+} ion is given by a $4f$ electron in a ^2F term ($l = 3$). This term splits first by LS coupling and secondly by the effect of the crystal field. Find the form of the crystal field V (in terms of spherical harmonics) required in order to calculate that interaction.

$$\mathrm{V} = \sum\nolimits_{lm} A_l^m(r)\, Y_l^m. \tag{73}$$

The matrix elements of the potential which will appear in the perturbation calculation are of the form

$$I = \int (\psi^{l=3})^* \, \mathrm{V} \, \psi^{l=3} \, \mathrm{d}\tau. \tag{74}$$

Because of the orthogonality of the spherical harmonics, terms with $l > 6$ will not contribute to I. Also, $(\psi^{l=3})^* \psi^{l=3}$ is gerade, so that V must also be gerade, whence it must contain even harmonics only. Therefore:

$$l = \text{even}, \quad l \leq 6. \tag{75}$$

We also know that V must have the symmetry of the totally symmetrical representation of $\mathbf{C}_{3h}$. The spherical harmonics belonging to this representation that satisfy (75) are:

T **61**.6 $$Y_0^0,\ Y_2^0,\ Y_4^0,\ Y_6^0,\ Y_6^6,\ Y_6^{-6}. \tag{76}$$

On taking the term for Y_0^0 as a reference state for the energy, the potential function be written as

76|73 $$\mathrm{V} = A_2^0\,Y_2^0 + A_4^0\,Y_4^0 + A_6^0\,Y_6^0 + A_6^6\,Y_6^6 + A_6^{-6}\,Y_6^{-6}. \tag{77}$$

27 Time reversal

Consider an electron in a level with $j = 3/2$ in a field of $\mathbf{D}_3$ symmetry and in the absence of magnetic fields. Prove that it splits into two doublets.

T **23**.10 $$j = 3/2 \to E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}. \tag{78}$$

T **23**.4, Table **16**.3 $$E_{1/2}\ \text{doublet, type } c, \text{ no extra degeneracy.} \tag{79}$$

T **23**.4, Table **16**.3 $${}^1E_{3/2},\ {}^2E_{3/2}\ \text{singlets, type } b, \text{ become a degenerate doublet.} \tag{80}$$

28 Vector coupling

Consider the coupling of one electron in an e state with another electron in a t_2 state in an octahedral field $\mathbf{O}$.

T **69**.8 $$e \otimes t_2 = T_1 \oplus T_2. \tag{81}$$

T **69**.11 $$T_{11} = \tfrac{1}{\sqrt{2}}\,\{e_1 - \exp(-2\pi\mathrm{i}/3)\,e_2\}\,t_{21}. \tag{82}$$

T **69**.11 $$T_{12} = \tfrac{1}{\sqrt{2}}\,\exp(2\pi\mathrm{i}/3)(e_1 - e_2)\,t_{22}. \tag{83}$$

T **69**.11 $$T_{13} = \tfrac{1}{\sqrt{2}}\,\{\exp(-2\pi\mathrm{i}/3)\,e_1 - e_2\}\,t_{23}. \tag{84}$$

T **69**.11 $$T_{21} = \tfrac{1}{\sqrt{2}}\,\{e_1 + \exp(-2\pi\mathrm{i}/3)\,e_2\}\,t_{21}. \tag{85}$$

T **69**.11 $$T_{22} = \tfrac{1}{\sqrt{2}}\,\exp(2\pi\mathrm{i}/3)(e_1 + e_2)\,t_{22}. \tag{86}$$

T **69**.11 $$T_{23} = \tfrac{1}{\sqrt{2}}\,\{\exp(-2\pi\mathrm{i}/3)\,e_1 + e_2\}\,t_{23}. \tag{87}$$

T **69**.6*a* $$e_1 = \tfrac{1}{\sqrt{2}}\,(|2\,0\rangle - \mathrm{i}\,|2\,2\rangle_+)\,, \quad e_2 = \tfrac{1}{\sqrt{2}}\,(|2\,0\rangle + \mathrm{i}\,|2\,2\rangle_+)\,. \tag{88}$$

T **69**.6*a* $$t_{21} = |2\,1\rangle_-, \quad t_{22} = -|2\,2\rangle_-, \quad t_{23} = -|2\,1\rangle_+. \tag{89}$$

All these functions can now be written in terms of the spherical harmonics by successive application of the expressions in § **16**–6 and § **13**–1.

Part 2

The Tables

The proper cyclic groups $\mathbf{C}_n$

Notation for headers

Items in header read from left to right

1		Hermann–Mauguin symbol for the point group.
2		$\lvert G\rvert$ order of the group.
3		$\lvert C\rvert$ number of classes in the group.
4		$\lvert\widetilde{C}\rvert$ number of classes in the double group.
5		Number of the table.
6		Page reference for the notation of the header, of the first five subsections below it, and of the footers.
7	□	This symbol indicates a crystallographic point group.
8		Schönflies notation for the point group.

Notation for the first five subsections below the header

(1) Product forms	Direct and semidirect product forms. $\otimes$ Direct product. ⓢ Semidirect product.
(2) Group chains *(See pp. 41, 67)*	Groups underlined: invariant. Groups in brackets: when subducing one or more of the representations in the tables to these subgroups in the setting used for them in the tables a change of bases (similarity transformation) is required.
(3) Operations of G	Lists all the operations of G, enclosing in brackets all the operations of the same class.
(4) Operations of $\widetilde{G}$	Lists all the operations of $\widetilde{G}$, enclosing in brackets all the operations of the same class.
(5) Classes and representations	$\lvert r\rvert$ number of regular classes in G (p. 51). $\lvert \mathrm{i}\rvert$ number of irregular classes in G (p. 51). $\lvert I\rvert$ number of irreducible representations in G. $\lvert\widetilde{I}\rvert$ number of spinor representations, also called the number of double-group representations.

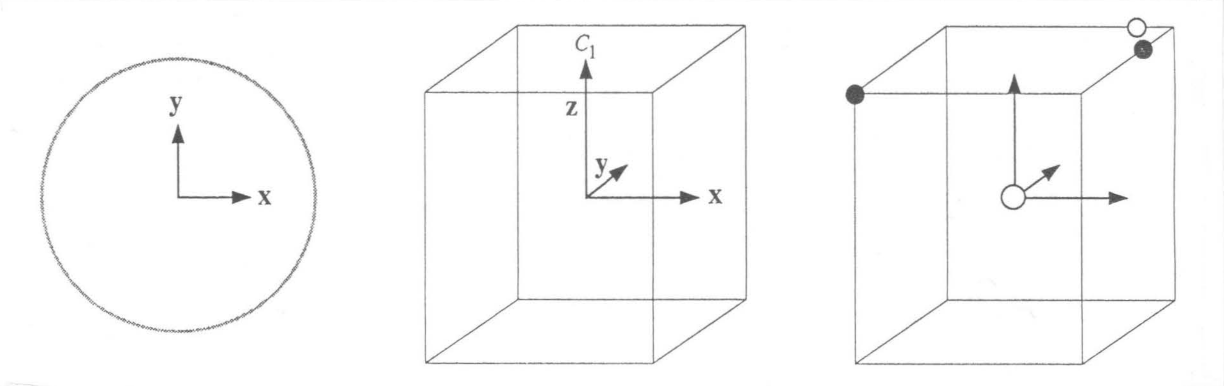
Use of the footers

Finding your way about the tables

Each page of the tables contains a footer giving the ordering of the group types listed in the tables. The current group type is recognized because it carries no page number under its name. On opening the tables at any page you should then see at a glance whether to move backwards or forwards to find the group type desired. More precisely, the page number for the group type desired gives the page, like the present one, which carries the contents for the group type wanted.

(1) Product forms: none.
(2) Group chains: $\mathbf{C}_2 \supset \underline{\mathbf{C}_1}$.
(3) Operations of G: E.
(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$.
(5) Classes and representations: $\lvert r\rvert = 1$, $\lvert i\rvert = 0$, $\lvert I\rvert = 1$, $\lvert\widetilde{I}\rvert = 1$.

F **1** See Chapter **15**, p. 65



Examples: CHFClBr, N_2H_4.

T **1**.1 Parameters § **16**–1, p. 68

$\mathbf{C}_1$	α	β	γ	ϕ	$\mathbf{n}$	λ	$\mathbf{\Lambda}$
E	0	0	0	0	(0 0 0)	⟦1,	(0 0 0)⟧

T **1**.2 Multiplication table
§ **16**–2, p. 69

$\mathbf{C}_1$	E
E	E

T **1**.3 Factor table
§ **16**–3, p. 70

$\mathbf{C}_1$	E
E	1

T **1**.4 Character table
§ **16**–4, p. 71

$\mathbf{C}_1$	E	τ
A	1	a
$A_{1/2}$	1	a

T **1**.5 Cartesian tensors and *s, p, d,* and *f* functions
§ **16**–5, p. 72
All functions $f(x, y, z)$ span representation A.

T **1**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_1$	$\vert j\,m\rangle$	ι	μ
A	$\vert 0\,0\rangle$	1	± 1
$A_{1/2}$	$\vert \frac{1}{2}\,\frac{1}{2}\rangle$	1	± 1

T **1**.7 Matrix representations

Use T **1**.4 ♠. § **16**–7, p. 77

T **1**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_1$	A	$A_{1/2}$
A	A	$A_{1/2}$
$A_{1/2}$		A

T **1**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

No subgroups.

T **1**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_1$
n	$(2n+1)\,A$
$n+\frac{1}{2}$	$(2n+2)\,A_{1/2}$

$n = 0, 1, 2, \ldots$

T **1**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

2 $|G| = 2$ $|C| = 2$ $|\widetilde{C}| = 4$ T **2** p. 107 $\square$ $\mathbf{C}_2$

(1) Product forms: none.

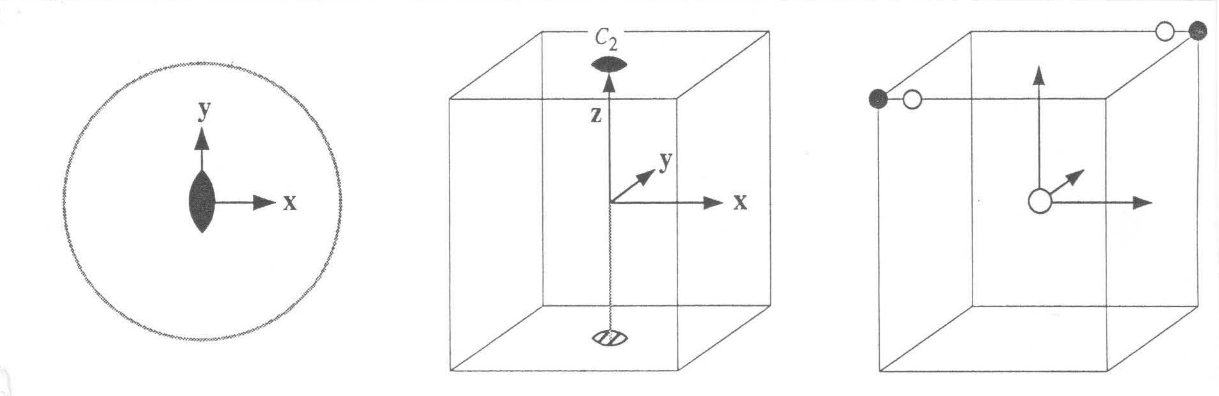
(2) Group chains: $\mathbf{C}_{2h} \supset \underline{\mathbf{C}_2} \supset \underline{\mathbf{C}_1}$, $\mathbf{C}_{2v} \supset \underline{\mathbf{C}_2} \supset \underline{\mathbf{C}_1}$, $\mathbf{D}_7 \supset (\mathbf{C}_2) \supset \underline{\mathbf{C}_1}$, $\mathbf{D}_5 \supset (\mathbf{C}_2) \supset \underline{\mathbf{C}_1}$,
$\mathbf{D}_3 \supset (\mathbf{C}_2) \supset \underline{\mathbf{C}_1}$, $\mathbf{D}_2 \supset \underline{\mathbf{C}_2} \supset \underline{\mathbf{C}_1}$, $\mathbf{S}_4 \supset \underline{\mathbf{C}_2} \supset \underline{\mathbf{C}_1}$, $\mathbf{C}_{10} \supset \underline{\mathbf{C}_2} \supset \underline{\mathbf{C}_1}$,
$\mathbf{C}_6 \supset \underline{\mathbf{C}_2} \supset \underline{\mathbf{C}_1}$, $\mathbf{C}_4 \supset \underline{\mathbf{C}_2} \supset \underline{\mathbf{C}_1}$.

(3) Operations of G: E, C_2.

(4) Operations of $\widetilde{G}$: E, C_2,
$\widetilde{E}$, $\widetilde{C}_2$.

(5) Classes and representations: $|r| = 2$, $|\text{i}| = 0$, $|I| = 2$, $|\widetilde{I}| = 2$.

F **2** See Chapter **15**, p. 65



Examples: Non planar H_2O_2, HClC=C=CHCl.

T 2.1 Parameters
Use T **31**.1 ◇. § **16**–1, p. 68

T 2.2 Multiplication table
Use T **31**.2 ◇. § **16**–2, p. 69

T 2.3 Factor table
Use T **31**.3 ◇. § **16**–3, p. 70

T 2.4 Character table
§ **16**–4, p. 71

$\mathbf{C}_2$	E	C_2	τ
A	1	1	a
B	1	−1	a
${}^1E_{1/2}$	1	i	b
${}^2E_{1/2}$	1	−i	b

T 2.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions
§ **16**–5, p. 72

$\mathbf{C}_2$	0	1	2	3
A	$^\square 1$	$^\square z, R_z$	$^\square x^2, y^2, {}^\square z^2, {}^\square xy$	$^\square x^2z, y^2z, {}^\square z^3, {}^\square xyz$
B		$^\square x, {}^\square y, R_x, R_y$	$^\square zx, {}^\square yz$	$^\square x^3, xy^2, {}^\square xz^2, {}^\square x^2y, y^3, {}^\square yz^2$

T **2**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_2$	$\lvert j\,m\rangle$	ι	μ
A	$\lvert 0\,0\rangle$	1	± 2
B	$\lvert 1\,1\rangle$	1	± 2
${}^1E_{1/2}$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	1	± 2
${}^2E_{1/2}$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle$	1	± 2

T **2**.7 Matrix representations

Use T **2**.4 ♠. § **16**–7, p. 77

T **2**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_2$	A	B	${}^1E_{1/2}$	${}^2E_{1/2}$
A	A	B	${}^1E_{1/2}$	${}^2E_{1/2}$
B		A	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^1E_{1/2}$			B	A
${}^2E_{1/2}$				B

T **2**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

No proper subgroups.

T **2**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_2$
$2n$	$(2n+1)\,A \oplus 2n\,B$
$2n+1$	$(2n+1)\,A \oplus (2n+2)\,B$
$n+\frac{1}{2}$	$(n+1)({}^1E_{1/2} \oplus {}^2E_{1/2})$

$n = 0, 1, 2, \ldots$

T **2**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: none.

(2) Group chains: $\mathbf{T} \supset (\mathbf{C}_3) \supset \underline{\mathbf{C}_1}$, $\mathbf{C}_{3h} \supset \underline{\mathbf{C}_3} \supset \underline{\mathbf{C}_1}$, $\mathbf{C}_{3v} \supset \underline{\mathbf{C}_3} \supset \underline{\mathbf{C}_1}$, $\mathbf{D}_3 \supset \underline{\mathbf{C}_3} \supset \underline{\mathbf{C}_1}$,
$\mathbf{S}_6 \supset \underline{\mathbf{C}_3} \supset \underline{\mathbf{C}_1}$, $\mathbf{C}_9 \supset \underline{\mathbf{C}_3} \supset \underline{\mathbf{C}_1}$, $\mathbf{C}_6 \supset \underline{\mathbf{C}_3} \supset \underline{\mathbf{C}_1}$.

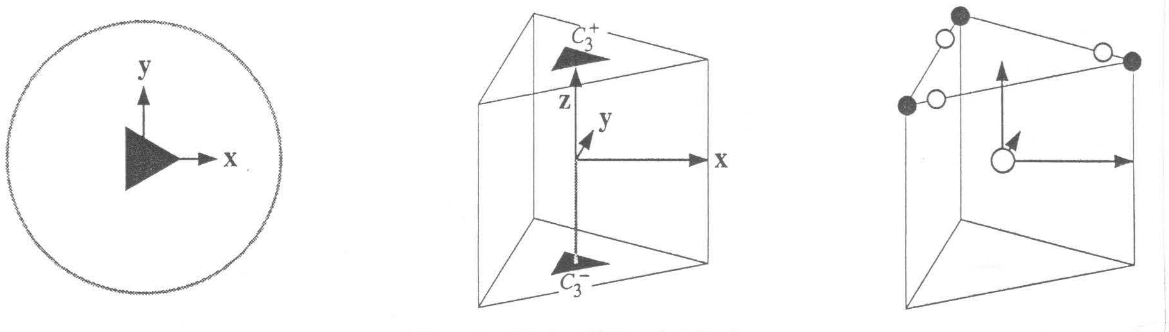
(3) Operations of G: E, C_3^+, C_3^-.

(4) Operations of $\widetilde{G}$: E, C_3^+, C_3^-,
$\widetilde{E}$, $\widetilde{C}_3^+$, $\widetilde{C}_3^-$.

(5) Classes and representations: $|r| = 3$, $|\text{i}| = 0$, $|I| = 3$, $|\widetilde{I}| = 3$.

F **3**

See Chapter **15**, p. 65



Examples: $H_3C{-}CCl_3$, partly rotated (not the ground state of this molecule).

T **3**.1 Parameters
Use T **35**.1. § **16**–1, p. 68

T **3**.2 Multiplication table
Use T **35**.2. § **16**–2, p. 69

T **3**.3 Factor table
Use T **35**.3. § **16**–3, p. 70

T **3**.4 Character table
§ **16**–4, p. 71

$\mathbf{C}_3$	E	C_3^+	C_3^-	τ
A	1	1	1	a
${}^1E$	1	ϵ^*	ϵ	b
${}^2E$	1	ϵ	ϵ^*	b
${}^1E_{1/2}$	1	$-\epsilon^*$	$-\epsilon$	b
${}^2E_{1/2}$	1	$-\epsilon$	$-\epsilon^*$	b
$A_{3/2}$	1	-1	-1	a

$\epsilon = \exp(2\pi\text{i}/3)$

T **3**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_3$	0	1	2	3
A	$\square 1$	$\square z, R_z$	$x^2+y^2, \square z^2$	$\square x(x^2-3y^2), \square y(3x^2-y^2), (x^2+y^2)z, \square z^3$
${}^1E \oplus {}^2E$		$\square(x,y),(R_x,R_y)$	$\square(xy, x^2-y^2)$, $\square(zx,yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, \square(xz^2, yz^2)$, $\square\{xyz, z(x^2-y^2)\}$

T **3**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_3$	$\lvert j\,m\rangle$	ι	μ
A	$\lvert 0\,0\rangle$	1	±3
${}^1E$	$\lvert 1\,1\rangle$	1	±3
${}^2E$	$\lvert 1\,\overline{1}\rangle$	1	±3
${}^1E_{1/2}$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	1	±3
${}^2E_{1/2}$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle$	1	±3
$A_{3/2}$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle$	1	±3

T **3**.7 Matrix representations

Use T **3**.4 ♠. § **16**–7, p. 77

T **3**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_3$	A	${}^1E$	${}^2E$	${}^1E_{1/2}$	${}^2E_{1/2}$	$A_{3/2}$
A	A	${}^1E$	${}^2E$	${}^1E_{1/2}$	${}^2E_{1/2}$	$A_{3/2}$
${}^1E$		${}^2E$	A	${}^2E_{1/2}$	$A_{3/2}$	${}^1E_{1/2}$
${}^2E$			${}^1E$	$A_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_{1/2}$				${}^2E$	A	${}^1E$
${}^2E_{1/2}$					${}^1E$	${}^2E$
$A_{3/2}$						A

T **3**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

No proper subgroups.

T **3**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_3$
$3n$	$(2n+1)\,A \oplus 2n\,({}^1E \oplus {}^2E)$
$3n+1$	$(2n+1)(A \oplus {}^1E \oplus {}^2E)$
$3n+2$	$(2n+1)\,A \oplus (2n+2)({}^1E \oplus {}^2E)$
$3n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus 2n\,A_{3/2}$
$3n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus (2n+2)\,A_{3/2}$
$3n+\frac{5}{2}$	$(2n+2)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus A_{3/2})$

$n = 0, 1, 2, \ldots$

T **3**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: none.

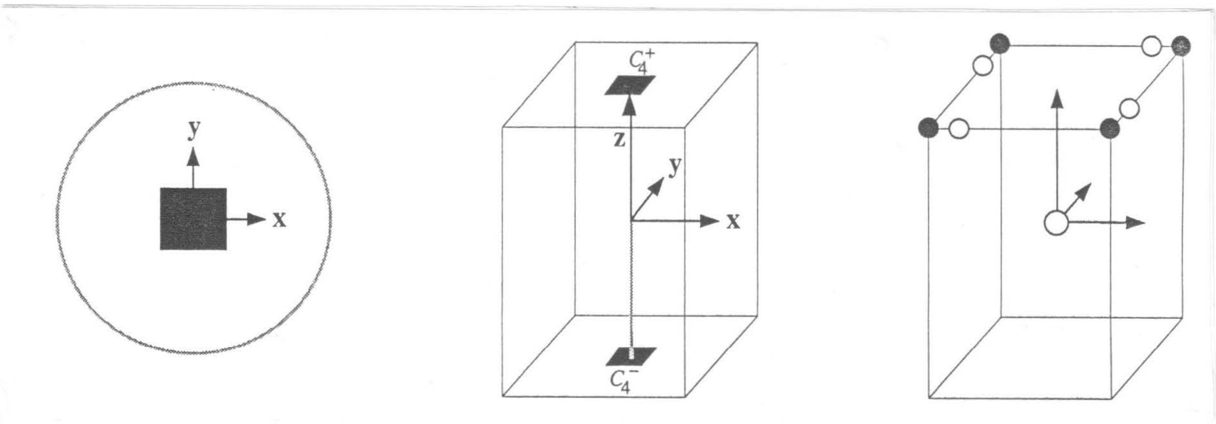
(2) Group chains: $\mathbf{C}_{4h} \supset \underline{\mathbf{C}_4} \supset \underline{\mathbf{C}_2}$, $\mathbf{C}_{4v} \supset \underline{\mathbf{C}_4} \supset \underline{\mathbf{C}_2}$, $\mathbf{D}_4 \supset \underline{\mathbf{C}_4} \supset \underline{\mathbf{C}_2}$, $\mathbf{S}_8 \supset \underline{\mathbf{C}_4} \supset \underline{\mathbf{C}_2}$, $\mathbf{C}_8 \supset \underline{\mathbf{C}_4} \supset \underline{\mathbf{C}_2}$.

(3) Operations of G: E, C_4^+, C_2, C_4^-.

(4) Operations of $\widetilde{G}$: E, C_4^+, C_2, C_4^-, $\widetilde{E}$, $\widetilde{C}_4^+$, $\widetilde{C}_2$, $\widetilde{C}_4^-$.

(5) Classes and representations: $|r| = 4$, $|\mathrm{i}| = 0$, $|I| = 4$, $|\widetilde{I}| = 4$.

F **4** See Chapter **15**, p. 65



Examples:

T **4**.1 Parameters
Use T **33**.1. § **16**–1, p. 68

T **4**.2 Multiplication table
Use T **33**.2. § **16**–2, p. 69

T **4**.3 Factor table
Use T **33**.3. § **16**–3, p. 70

T **4**.4 Character table
§ **16**–4, p. 71

$\mathbf{C}_4$	E	C_4^+	C_2	C_4^-	τ
A	1	1	1	1	a
B	1	-1	1	-1	a
${}^1E$	1	$-\mathrm{i}$	-1	i	b
${}^2E$	1	i	-1	$-\mathrm{i}$	b
${}^1E_{1/2}$	1	ϵ^*	$-\mathrm{i}$	ϵ	b
${}^2E_{1/2}$	1	ϵ	i	ϵ^*	b
${}^1E_{3/2}$	1	$-\epsilon^*$	$-\mathrm{i}$	$-\epsilon$	b
${}^2E_{3/2}$	1	$-\epsilon$	i	$-\epsilon^*$	b

$\epsilon = \exp(2\pi\mathrm{i}/8)$

T 4.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_4$	0	1	2	3
A	$\square 1$	$\square z, R_z$	$x^2+y^2, \square z^2$	$(x^2+y^2)z, \square z^3$
B			$\square x^2-y^2, \square xy$	$\square z(x^2-y^2), \square xyz$
${}^1E \oplus {}^2E$		$\square(x,y), (R_x,R_y)$	$\square(zx,yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, \square(xz^2,yz^2),$ $\square\{x(x^2-3y^2), y(3x^2-y^2)\}$

T 4.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_4$	$\|j\,m\rangle$	ι	μ	$\mathbf{C}_4$	$\|j\,m\rangle$	ι	μ
A	$\|0\,0\rangle$	1	± 4	${}^1E_{1/2}$	$\|\frac{1}{2}\,\frac{1}{2}\rangle$	1	± 4
B	$\|2\,2\rangle$	1	± 4	${}^2E_{1/2}$	$\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	1	± 4
${}^1E$	$\|1\,1\rangle$	1	± 4	${}^1E_{3/2}$	$\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	1	± 4
${}^2E$	$\|1\,\overline{1}\rangle$	1	± 4	${}^2E_{3/2}$	$\|\frac{3}{2}\,\frac{3}{2}\rangle$	1	± 4

T 4.7 Matrix representations

Use T **4**.4 ♠. § **16**–7, p. 77

T 4.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_4$	A	B	${}^1E$	${}^2E$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$
A	A	B	${}^1E$	${}^2E$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$
B		A	${}^2E$	${}^1E$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E$			B	A	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$
${}^2E$				B	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^1E_{1/2}$					${}^1E$	A	${}^2E$	B
${}^2E_{1/2}$						${}^2E$	B	${}^1E$
${}^1E_{3/2}$							${}^1E$	A
${}^2E_{3/2}$								${}^2E$

T 4.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_4$	$\mathbf{C}_2$
A	A
B	A
${}^1E$	B
${}^2E$	B
${}^1E_{1/2}$	${}^2E_{1/2}$
${}^2E_{1/2}$	${}^1E_{1/2}$
${}^1E_{3/2}$	${}^2E_{1/2}$
${}^2E_{3/2}$	${}^1E_{1/2}$

T 4.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_4$
$4n$	$(2n+1)\,A \oplus 2n\,(B \oplus {}^1E \oplus {}^2E)$
$4n+1$	$(2n+1)(A \oplus {}^1E \oplus {}^2E) \oplus 2n\,B$
$4n+2$	$(2n+1)(A \oplus {}^1E \oplus {}^2E) \oplus (2n+2)\,B$
$4n+3$	$(2n+1)\,A \oplus (2n+2)(B \oplus {}^1E \oplus {}^2E)$
$4n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus 2n\,({}^1E_{3/2} \oplus {}^2E_{3/2})$
$4n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2})$
$4n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus (2n+2)({}^1E_{3/2} \oplus {}^2E_{3/2})$
$4n+\frac{7}{2}$	$(2n+2)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2})$

$n = 0, 1, 2, \ldots$

T 4.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: none.

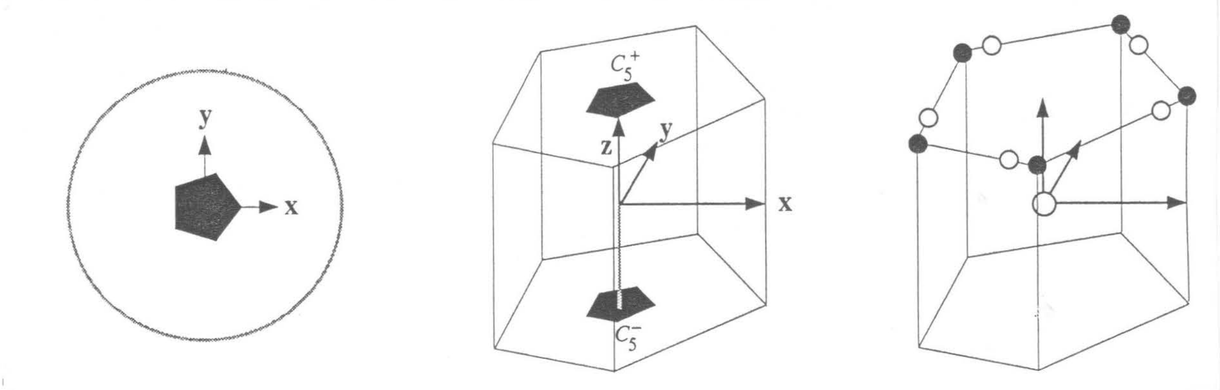
(2) Group chains: $\mathbf{C}_{5h} \supset \underline{\mathbf{C}_5} \supset \underline{\mathbf{C}_1}$, $\mathbf{C}_{5v} \supset \underline{\mathbf{C}_5} \supset \underline{\mathbf{C}_1}$, $\mathbf{D}_5 \supset \underline{\mathbf{C}_5} \supset \underline{\mathbf{C}_1}$, $\mathbf{S}_{10} \supset \underline{\mathbf{C}_5} \supset \underline{\mathbf{C}_1}$,
$\mathbf{C}_{10} \supset \underline{\mathbf{C}_5} \supset \underline{\mathbf{C}_1}$.

(3) Operations of G: E, C_5^+, C_5^{2+}, C_5^{2-}, C_5^-.

(4) Operations of $\widetilde{G}$: E, C_5^+, C_5^{2+}, C_5^{2-}, C_5^-,
$\widetilde{E}$, $\widetilde{C}_5^+$, $\widetilde{C}_5^{2+}$, $\widetilde{C}_5^{2-}$, $\widetilde{C}_5^-$.

(5) Classes and representations: $|r| = 5$, $|i| = 0$, $|I| = 5$, $|\widetilde{I}| = 5$.

F **5** See Chapter **15**, p. 65



Examples:

T 5.1 Parameters
Use T **39**.1. § **16**–1, p. 68

T 5.2 Multiplication table
Use T **39**.2. § **16**–2, p. 69

T 5.3 Factor table
Use T **39**.3. § **16**–3, p. 70

T 5.4 Character table
§ **16**–4, p. 71

$\mathbf{C}_5$	E	C_5^+	C_5^{2+}	C_5^{2-}	C_5^-	τ
A	1	1	1	1	1	a
${}^1E_1$	1	δ^*	ϵ^*	ϵ	δ	b
${}^2E_1$	1	δ	ϵ	ϵ^*	δ^*	b
${}^1E_2$	1	ϵ^*	δ	δ^*	ϵ	b
${}^2E_2$	1	ϵ	δ^*	δ	ϵ^*	b
${}^1E_{1/2}$	1	$-\epsilon^*$	δ	δ^*	$-\epsilon$	b
${}^2E_{1/2}$	1	$-\epsilon$	δ^*	δ	$-\epsilon^*$	b
${}^1E_{3/2}$	1	$-\delta$	ϵ	ϵ^*	$-\delta^*$	b
${}^2E_{3/2}$	1	$-\delta^*$	ϵ^*	ϵ	$-\delta$	b
$A_{5/2}$	1	-1	1	1	-1	a

$\delta = \exp(2\pi i/5)$, $\epsilon = \exp(4\pi i/5)$

T **5**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_5$	0	1	2	3
A	$^{\square}1$	$^{\square}z, R_z$	$x^2+y^2, {}^{\square}z^2$	$(x^2+y^2)z, {}^{\square}z^3$
${}^1E_1 \oplus {}^2E_1$		$^{\square}(x,y), (R_x, R_y)$	$^{\square}(zx, yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, {}^{\square}(xz^2, yz^2)$
${}^1E_2 \oplus {}^2E_2$			$^{\square}(xy, x^2-y^2)$	$^{\square}\{x(x^2-3y^2), y(3x^2-y^2)\},$ $^{\square}\{xyz, z(x^2-y^2)\}$

T **5**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_5$	$\vert j\, m\rangle$	ι	μ
A	$\vert 0\, 0\rangle$	1	± 5
${}^1E_1$	$\vert 1\, 1\rangle$	1	± 5
${}^2E_1$	$\vert 1\, \overline{1}\rangle$	1	± 5
${}^1E_2$	$\vert 2\, 2\rangle$	1	± 5
${}^2E_2$	$\vert 2\, \overline{2}\rangle$	1	± 5
${}^1E_{1/2}$	$\vert \frac{1}{2}\, \overline{\frac{1}{2}}\rangle$	1	± 5
${}^2E_{1/2}$	$\vert \frac{1}{2}\, \frac{1}{2}\rangle$	1	± 5
${}^1E_{3/2}$	$\vert \frac{3}{2}\, \frac{3}{2}\rangle$	1	± 5
${}^2E_{3/2}$	$\vert \frac{3}{2}\, \overline{\frac{3}{2}}\rangle$	1	± 5
$A_{5/2}$	$\vert \frac{5}{2}\, \frac{5}{2}\rangle$	1	± 5

T **5**.7 Matrix representations

Use T **5**.4 ♠. § **16**–7, p. 77

T **5**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_5$	A	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	$A_{5/2}$
A	A	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	$A_{5/2}$
${}^1E_1$		${}^1E_2$	A	${}^2E_2$	${}^2E_1$	${}^2E_{1/2}$	${}^1E_{3/2}$	$A_{5/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$
${}^2E_1$			${}^2E_2$	${}^1E_1$	${}^1E_2$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	$A_{5/2}$	${}^1E_{3/2}$
${}^1E_2$				${}^2E_1$	A	${}^1E_{3/2}$	$A_{5/2}$	${}^2E_{3/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_2$					${}^1E_1$	$A_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_{1/2}$						${}^2E_1$	A	${}^1E_1$	${}^2E_2$	${}^1E_2$
${}^2E_{1/2}$							${}^1E_1$	${}^1E_2$	${}^2E_1$	${}^2E_2$
${}^1E_{3/2}$								${}^2E_2$	A	${}^2E_1$
${}^2E_{3/2}$									${}^1E_2$	${}^1E_1$
$A_{5/2}$										A

T **5**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

No proper subgroups.

T **5**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_5$
$5n$	$(2n+1)\,A \oplus 2n\,({}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2)$
$5n+1$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1) \oplus 2n\,({}^1E_2 \oplus {}^2E_2)$
$5n+2$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2)$
$5n+3$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1) \oplus (2n+2)({}^1E_2 \oplus {}^2E_2)$
$5n+4$	$(2n+1)\,A \oplus (2n+2)({}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2)$
$5n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus 2n\,({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus A_{5/2})$
$5n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus 2n\,A_{5/2}$
$5n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus (2n+2)\,A_{5/2}$
$5n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus (2n+2)({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus A_{5/2})$
$5n+\frac{9}{2}$	$(2n+2)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus A_{5/2})$

$n = 0, 1, 2, \ldots$

T **5**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: $\mathbf{C}_3 \otimes \mathbf{C}_2$.

(2) Group chains: $\mathbf{C}_{6h} \supset \underline{\mathbf{C}_6} \supset \underline{\mathbf{C}_3}$, $\mathbf{C}_{6h} \supset \underline{\mathbf{C}_6} \supset \underline{\mathbf{C}_2}$, $\mathbf{C}_{6v} \supset \underline{\mathbf{C}_6} \supset \underline{\mathbf{C}_3}$, $\mathbf{C}_{6v} \supset \underline{\mathbf{C}_6} \supset \underline{\mathbf{C}_2}$,
$\mathbf{D}_6 \supset \underline{\mathbf{C}_6} \supset \underline{\mathbf{C}_3}$, $\mathbf{D}_6 \supset \underline{\mathbf{C}_6} \supset \underline{\mathbf{C}_2}$, $\mathbf{S}_{12} \supset \underline{\mathbf{C}_6} \supset \underline{\mathbf{C}_3}$, $\mathbf{S}_{12} \supset \underline{\mathbf{C}_6} \supset \underline{\mathbf{C}_2}$.

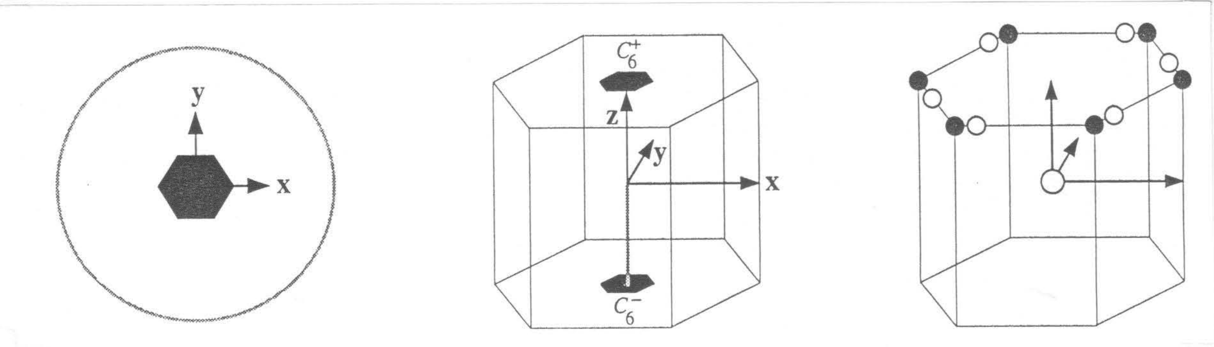
(3) Operations of G: E, C_6^+, C_3^+, C_2, C_3^-, C_6^-.

(4) Operations of $\widetilde{G}$: E, C_6^+, C_3^+, C_2, C_3^-, C_6^-,
$\widetilde{E}$, $\widetilde{C}_6^+$, $\widetilde{C}_3^+$, $\widetilde{C}_2$, $\widetilde{C}_3^-$, $\widetilde{C}_6^-$.

(5) Classes and representations: $|r| = 6$, $|\mathrm{i}| = 0$, $|I| = 6$, $|\widetilde{I}| = 6$.

F **6**

See Chapter **15**, p. 65



Examples:

T 6.1 Parameters
Use T **35**.1. § **16**–1, p. 68

T 6.2 Multiplication table
Use T **35**.2. § **16**–2, p. 69

T 6.3 Factor table
Use T **35**.3. § **16**–3, p. 70

T 6.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_6$	E	C_6^+	C_3^+	C_2	C_3^-	C_6^-	τ
A	1	1	1	1	1	1	a
B	1	−1	1	−1	1	−1	a
${}^1E_1$	1	$-\epsilon$	ϵ^*	−1	ϵ	$-\epsilon^*$	b
${}^2E_1$	1	$-\epsilon^*$	ϵ	−1	ϵ^*	$-\epsilon$	b
${}^1E_2$	1	ϵ	ϵ^*	1	ϵ	ϵ^*	b
${}^2E_2$	1	ϵ^*	ϵ	1	ϵ^*	ϵ	b
${}^1E_{1/2}$	1	$-\mathrm{i}\epsilon$	$-\epsilon^*$	i	$-\epsilon$	$\mathrm{i}\epsilon^*$	b
${}^2E_{1/2}$	1	$\mathrm{i}\epsilon^*$	$-\epsilon$	−i	$-\epsilon^*$	$-\mathrm{i}\epsilon$	b
${}^1E_{3/2}$	1	−i	−1	i	−1	i	b
${}^2E_{3/2}$	1	i	−1	−i	−1	−i	b
${}^1E_{5/2}$	1	$-\mathrm{i}\epsilon^*$	$-\epsilon$	i	$-\epsilon^*$	$\mathrm{i}\epsilon$	b
${}^2E_{5/2}$	1	$\mathrm{i}\epsilon$	$-\epsilon^*$	−i	$-\epsilon$	$-\mathrm{i}\epsilon^*$	b

$\epsilon = \exp(2\pi\mathrm{i}/3)$

T **6**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_6$	0	1	2	3
A	$^\square 1$	$^\square z, R_z$	$x^2+y^2, {}^\square z^2$	$(x^2+y^2)z, {}^\square z^3$
B				$^\square x(x^2-3y^2), {}^\square y(3x^2-y^2)$
$^1E_1 \oplus {}^2E_1$		$^\square(x,y), (R_x,R_y)$	$^\square(zx,yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2,yz^2)$
$^1E_2 \oplus {}^2E_2$			$^\square(xy, x^2-y^2)$	$^\square\{xyz, z(x^2-y^2)\}$

T **6**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_6$	$\|j\,m\rangle$	ι	μ
A	$\|0\,0\rangle$	1	±6
B	$\|3\,3\rangle$	1	±6
$^1E_1$	$\|1\,1\rangle$	1	±6
$^2E_1$	$\|1\,\overline{1}\rangle$	1	±6
$^1E_2$	$\|2\,\overline{2}\rangle$	1	±6
$^2E_2$	$\|2\,2\rangle$	1	±6
$^1E_{1/2}$	$\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	1	±6
$^2E_{1/2}$	$\|\frac{1}{2}\,\frac{1}{2}\rangle$	1	±6
$^1E_{3/2}$	$\|\frac{3}{2}\,\frac{3}{2}\rangle$	1	±6
$^2E_{3/2}$	$\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	1	±6
$^1E_{5/2}$	$\|\frac{5}{2}\,\overline{\frac{5}{2}}\rangle$	1	±6
$^2E_{5/2}$	$\|\frac{5}{2}\,\frac{5}{2}\rangle$	1	±6

T **6**.7 Matrix representations

Use T **6**.4 ♠. § **16**–7, p. 77

T **6**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_6$	A	B	$^1E_1$	$^2E_1$	$^1E_2$	$^2E_2$	$^1E_{1/2}$	$^2E_{1/2}$	$^1E_{3/2}$	$^2E_{3/2}$	$^1E_{5/2}$	$^2E_{5/2}$
A	A	B	$^1E_1$	$^2E_1$	$^1E_2$	$^2E_2$	$^1E_{1/2}$	$^2E_{1/2}$	$^1E_{3/2}$	$^2E_{3/2}$	$^1E_{5/2}$	$^2E_{5/2}$
B		A	$^1E_2$	$^2E_2$	$^1E_1$	$^2E_1$	$^2E_{5/2}$	$^1E_{5/2}$	$^2E_{3/2}$	$^1E_{3/2}$	$^2E_{1/2}$	$^1E_{1/2}$
$^1E_1$			$^2E_2$	A	$^2E_1$	B	$^2E_{1/2}$	$^1E_{3/2}$	$^2E_{5/2}$	$^1E_{1/2}$	$^2E_{3/2}$	$^1E_{5/2}$
$^2E_1$				$^1E_2$	B	$^1E_1$	$^2E_{3/2}$	$^1E_{1/2}$	$^2E_{1/2}$	$^1E_{5/2}$	$^2E_{5/2}$	$^1E_{3/2}$
$^1E_2$					$^2E_2$	A	$^1E_{5/2}$	$^2E_{3/2}$	$^1E_{1/2}$	$^2E_{5/2}$	$^1E_{3/2}$	$^2E_{1/2}$
$^2E_2$						$^1E_2$	$^1E_{3/2}$	$^2E_{5/2}$	$^1E_{5/2}$	$^2E_{1/2}$	$^1E_{1/2}$	$^2E_{3/2}$
$^1E_{1/2}$							$^2E_1$	A	$^1E_1$	$^1E_2$	B	$^2E_2$
$^2E_{1/2}$								$^1E_1$	$^2E_2$	$^2E_1$	$^1E_2$	B
$^1E_{3/2}$									B	A	$^2E_1$	$^1E_2$
$^2E_{3/2}$										B	$^2E_2$	$^1E_1$
$^1E_{5/2}$											$^1E_1$	A
$^2E_{5/2}$												$^2E_1$

T **6**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_6$	$\mathbf{C}_3$	$\mathbf{C}_2$
A	A	A
B	A	B
${}^1E_1$	${}^1E$	B
${}^2E_1$	${}^2E$	B
${}^1E_2$	${}^1E$	A
${}^2E_2$	${}^2E$	A
${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$
${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$
${}^1E_{3/2}$	$A_{3/2}$	${}^1E_{1/2}$
${}^2E_{3/2}$	$A_{3/2}$	${}^2E_{1/2}$
${}^1E_{5/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$

T **6**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_6$
$6n$	$(2n+1)\,A \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2)$
$6n+1$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1) \oplus 2n\,(B \oplus {}^1E_2 \oplus {}^2E_2)$
$6n+2$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2) \oplus 2n\,B$
$6n+3$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2) \oplus (2n+2)\,B$
$6n+4$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1) \oplus (2n+2)(B \oplus {}^1E_2 \oplus {}^2E_2)$
$6n+5$	$(2n+1)\,A \oplus (2n+2)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2)$
$6n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus 2n\,({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2})$
$6n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus 2n\,({}^1E_{5/2} \oplus {}^2E_{5/2})$
$6n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2})$
$6n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus (2n+2)({}^1E_{5/2} \oplus {}^2E_{5/2})$
$6n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus (2n+2)({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2})$
$6n+\frac{11}{2}$	$(2n+2)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2})$

$n = 0, 1, 2, \ldots$

T **6**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: none.

(2) Group chains: $\mathbf{C}_{7h} \supset \underline{\mathbf{C}_7} \supset \underline{\mathbf{C}_1}$, $\mathbf{C}_{7v} \supset \underline{\mathbf{C}_7} \supset \underline{\mathbf{C}_1}$, $\mathbf{D}_7 \supset \underline{\mathbf{C}_7} \supset \underline{\mathbf{C}_1}$, $\mathbf{S}_{14} \supset \underline{\mathbf{C}_7} \supset \underline{\mathbf{C}_1}$.

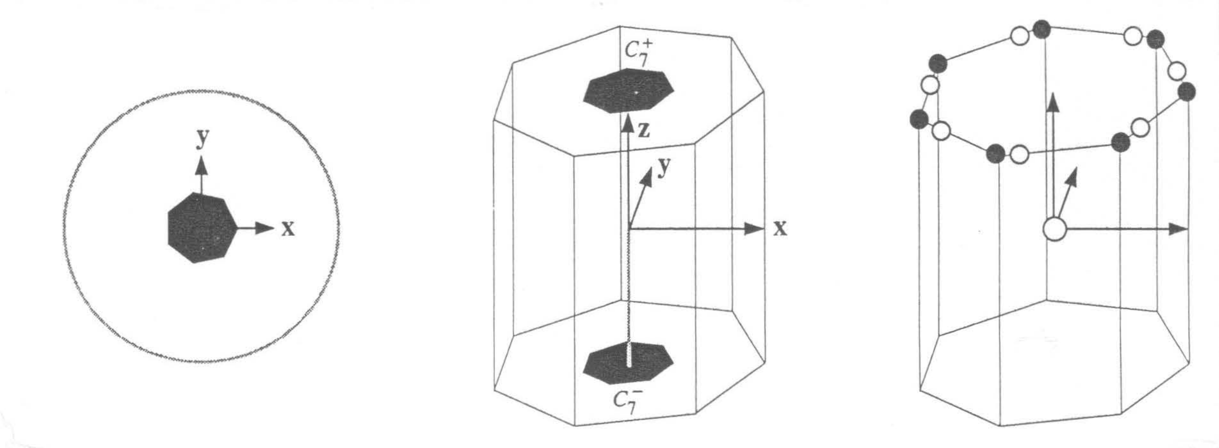
(3) Operations of G: E, C_7^+, C_7^{2+}, C_7^{3+}, C_7^{3-}, C_7^{2-}, C_7^-.

(4) Operations of $\widetilde{G}$: E, C_7^+, C_7^{2+}, C_7^{3+}, C_7^{3-}, C_7^{2-}, C_7^-,
$\widetilde{E}$, $\widetilde{C}_7^+$, $\widetilde{C}_7^{2+}$, $\widetilde{C}_7^{3+}$, $\widetilde{C}_7^{3-}$, $\widetilde{C}_7^{2-}$, $\widetilde{C}_7^-$.

(5) Classes and representations: $|r| = 7$, $|i| = 0$, $|I| = 7$, $|\widetilde{I}| = 7$.

F **7**

See Chapter **15**, p. 65



Examples:

T 7.1 Parameters
Use T **36**.1. § **16**–1, p. 68

T 7.2 Multiplication table
Use T **36**.2. § **16**–2, p. 69

T 7.3 Factor table
Use T **36**.3. § **16**–3, p. 70

T 7.4 Character table
§ **16**–4, p. 71

$\mathbf{C}_7$	E	C_7^+	C_7^{2+}	C_7^{3+}	C_7^{3-}	C_7^{2-}	C_7^-	τ
A	1	1	1	1	1	1	1	a
${}^1E_1$	1	δ^*	ϵ^*	η^*	η	ϵ	δ	b
${}^2E_1$	1	δ	ϵ	η	η^*	ϵ^*	δ^*	b
${}^1E_2$	1	ϵ^*	η	δ	δ^*	η^*	ϵ	b
${}^2E_2$	1	ϵ	η^*	δ^*	δ	η	ϵ^*	b
${}^1E_3$	1	η^*	δ	ϵ^*	ϵ	δ^*	η	b
${}^2E_3$	1	η	δ^*	ϵ	ϵ^*	δ	η^*	b
${}^1E_{1/2}$	1	$-\eta^*$	δ	$-\epsilon^*$	$-\epsilon$	δ^*	$-\eta$	b
${}^2E_{1/2}$	1	$-\eta$	δ^*	$-\epsilon$	$-\epsilon^*$	δ	$-\eta^*$	b
${}^1E_{3/2}$	1	$-\epsilon$	η^*	$-\delta^*$	$-\delta$	η	$-\epsilon^*$	b
${}^2E_{3/2}$	1	$-\epsilon^*$	η	$-\delta$	$-\delta^*$	η^*	$-\epsilon$	b
${}^1E_{5/2}$	1	$-\delta^*$	ϵ^*	$-\eta^*$	$-\eta$	ϵ	$-\delta$	b
${}^2E_{5/2}$	1	$-\delta$	ϵ	$-\eta$	$-\eta^*$	ϵ^*	$-\delta^*$	b
$A_{7/2}$	1	-1	1	-1	-1	1	-1	a

$\delta = \exp(2\pi i/7)$, $\epsilon = \exp(4\pi i/7)$, $\eta = \exp(6\pi i/7)$

T **7**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_7$	0	1	2	3
A	$^\square 1$	$^\square z, R_z$	$x^2+y^2, {}^\square z^2$	$(x^2+y^2)z, {}^\square z^3$
${}^1E_1 \oplus {}^2E_1$		$^\square(x,y), (R_x,R_y)$	$^\square(zx,yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2,yz^2)$
${}^1E_2 \oplus {}^2E_2$			$^\square(xy, x^2-y^2)$	$^\square\{xyz, z(x^2-y^2)\}$
${}^1E_3 \oplus {}^2E_3$				$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$

T **7**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_7$	$\lvert j\,m\rangle$	ι	μ
A	$\lvert 0\,0\rangle$	1	±7
${}^1E_1$	$\lvert 1\,1\rangle$	1	±7
${}^2E_1$	$\lvert 1\,\overline{1}\rangle$	1	±7
${}^1E_2$	$\lvert 2\,2\rangle$	1	±7
${}^2E_2$	$\lvert 2\,\overline{2}\rangle$	1	±7
${}^1E_3$	$\lvert 3\,3\rangle$	1	±7
${}^2E_3$	$\lvert 3\,\overline{3}\rangle$	1	±7
${}^1E_{1/2}$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	1	±7
${}^2E_{1/2}$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle$	1	±7
${}^1E_{3/2}$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle$	1	±7
${}^2E_{3/2}$	$\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	1	±7
${}^1E_{5/2}$	$\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle$	1	±7
${}^2E_{5/2}$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle$	1	±7
$A_{7/2}$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle$	1	±7

T **7**.7 Matrix representations

Use T **7**.4 ♠. § **16**–7, p. 77

T **7**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_7$	A	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	$A_{7/2}$
A	A	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	$A_{7/2}$
${}^1E_1$		${}^1E_2$	A	${}^1E_3$	${}^2E_1$	${}^2E_3$	${}^2E_2$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	$A_{7/2}$	${}^1E_{5/2}$
${}^2E_1$			${}^2E_2$	${}^1E_1$	${}^2E_3$	${}^1E_2$	${}^1E_3$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	$A_{7/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$
${}^1E_2$				${}^2E_3$	A	${}^2E_2$	${}^2E_1$	${}^1E_{3/2}$	${}^2E_{5/2}$	$A_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$
${}^2E_2$					${}^1E_3$	${}^1E_1$	${}^1E_2$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	$A_{7/2}$	${}^2E_{5/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$
${}^1E_3$						${}^2E_1$	A	${}^2E_{5/2}$	$A_{7/2}$	${}^1E_{5/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_3$							${}^1E_1$	$A_{7/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_{1/2}$								${}^2E_1$	A	${}^1E_1$	${}^2E_2$	${}^2E_3$	${}^1E_2$	${}^1E_3$
${}^2E_{1/2}$									${}^1E_1$	${}^1E_2$	${}^2E_1$	${}^2E_2$	${}^1E_3$	${}^2E_3$
${}^1E_{3/2}$										${}^1E_3$	A	${}^2E_1$	${}^2E_3$	${}^2E_2$
${}^2E_{3/2}$											${}^2E_3$	${}^1E_3$	${}^1E_1$	${}^1E_2$
${}^1E_{5/2}$												${}^1E_2$	A	${}^1E_1$
${}^2E_{5/2}$													${}^2E_2$	${}^2E_1$
$A_{7/2}$														A

T 7.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

No proper subgroups.

T 7.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_7$
$7n$	$(2n+1)\,A \oplus 2n\,({}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3)$
$7n+1$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1) \oplus 2n\,({}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3)$
$7n+2$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2) \oplus 2n\,({}^1E_3 \oplus {}^2E_3)$
$7n+3$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3)$
$7n+4$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2) \oplus (2n+2)({}^1E_3 \oplus {}^2E_3)$
$7n+5$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1) \oplus (2n+2)({}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3)$
$7n+6$	$(2n+1)\,A \oplus (2n+2)({}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3)$
$7n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus 2n\,({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus A_{7/2})$
$7n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus 2n\,({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus A_{7/2})$
$7n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus 2n\,A_{7/2}$
$7n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus (2n+2)\,A_{7/2}$
$7n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus (2n+2)({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus A_{7/2})$
$7n+\frac{11}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus (2n+2)({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus A_{7/2})$
$7n+\frac{13}{2}$	$(2n+2)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus A_{7/2})$

$n = 0, 1, 2, \ldots$

T 7.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: none.

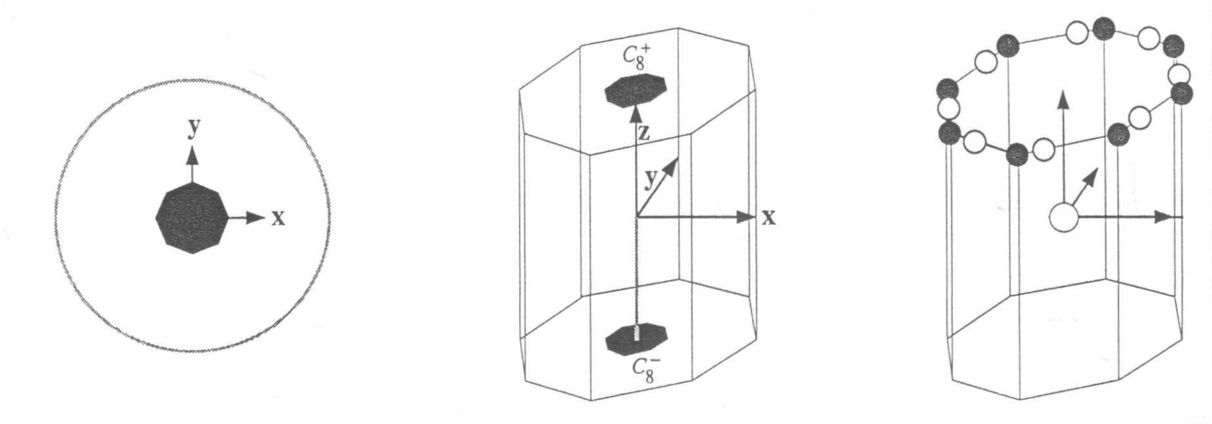
(2) Group chains: $\mathbf{C}_{8h} \supset \underline{\mathbf{C}_8} \supset \underline{\mathbf{C}_4}$, $\mathbf{C}_{8v} \supset \underline{\mathbf{C}_8} \supset \underline{\mathbf{C}_4}$, $\mathbf{D}_8 \supset \underline{\mathbf{C}_8} \supset \underline{\mathbf{C}_4}$, $\mathbf{S}_{16} \supset \underline{\mathbf{C}_8} \supset \underline{\mathbf{C}_4}$.

(3) Operations of G: E, C_8^+, C_4^+, C_8^{3+}, C_2, C_8^{3-}, C_4^-, C_8^-.

(4) Operations of $\widetilde{G}$: E, C_8^+, C_4^+, C_8^{3+}, C_2, C_8^{3-}, C_4^-, C_8^-,
$\widetilde{E}$, $\widetilde{C}_8^+$, $\widetilde{C}_4^+$, $\widetilde{C}_8^{3+}$, $\widetilde{C}_2$, $\widetilde{C}_8^{3-}$, $\widetilde{C}_4^-$, $\widetilde{C}_8^-$.

(5) Classes and representations: $|r| = 8$, $|i| = 0$, $|I| = 8$, $|\widetilde{I}| = 8$.

F **8** See Chapter **15**, p. 65



Examples:

T **8**.1 Parameters
Use T **37**.1. § **16**–1, p. 68

T **8**.2 Multiplication table
Use T **37**.2. § **16**–2, p. 69

T **8**.3 Factor table
Use T **37**.3. § **16**–3, p. 70

T **8**.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_8$	E	C_8^+	C_4^+	C_8^{3+}	C_2	C_8^{3-}	C_4^-	C_8^-	τ
A	1	1	1	1	1	1	1	1	a
B	1	-1	1	-1	1	-1	1	-1	a
${}^1E_1$	1	ϵ^*	$-\mathrm{i}$	$-\epsilon$	-1	$-\epsilon^*$	i	ϵ	b
${}^2E_1$	1	ϵ	i	$-\epsilon^*$	-1	$-\epsilon$	$-\mathrm{i}$	ϵ^*	b
${}^1E_2$	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	-1	i	b
${}^2E_2$	1	i	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	b
${}^1E_3$	1	$-\epsilon^*$	$-\mathrm{i}$	ϵ	-1	ϵ^*	i	$-\epsilon$	b
${}^2E_3$	1	$-\epsilon$	i	ϵ^*	-1	ϵ	$-\mathrm{i}$	$-\epsilon^*$	b
${}^1E_{1/2}$	1	δ	ϵ	$\mathrm{i}\delta^*$	i	$-\mathrm{i}\delta$	ϵ^*	δ^*	b
${}^2E_{1/2}$	1	δ^*	ϵ^*	$-\mathrm{i}\delta$	$-\mathrm{i}$	$\mathrm{i}\delta^*$	ϵ	δ	b
${}^1E_{3/2}$	1	$-\mathrm{i}\delta$	$-\epsilon$	$-\delta^*$	i	$-\delta$	$-\epsilon^*$	$\mathrm{i}\delta^*$	b
${}^2E_{3/2}$	1	$\mathrm{i}\delta^*$	$-\epsilon^*$	$-\delta$	$-\mathrm{i}$	$-\delta^*$	$-\epsilon$	$-\mathrm{i}\delta$	b
${}^1E_{5/2}$	1	$\mathrm{i}\delta$	$-\epsilon$	δ^*	i	δ	$-\epsilon^*$	$-\mathrm{i}\delta^*$	b
${}^2E_{5/2}$	1	$-\mathrm{i}\delta^*$	$-\epsilon^*$	δ	$-\mathrm{i}$	δ^*	$-\epsilon$	$\mathrm{i}\delta$	b
${}^1E_{7/2}$	1	$-\delta$	ϵ	$-\mathrm{i}\delta^*$	i	$\mathrm{i}\delta$	ϵ^*	$-\delta^*$	b
${}^2E_{7/2}$	1	$-\delta^*$	ϵ^*	$\mathrm{i}\delta$	$-\mathrm{i}$	$-\mathrm{i}\delta^*$	ϵ	$-\delta$	b

$\delta = \exp(2\pi\mathrm{i}/16)$, $\epsilon = \exp(2\pi\mathrm{i}/8)$

T **8**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_8$	0	1	2	3
A	$^\square 1$	$^\square z, R_z$	$x^2+y^2, {}^\square z^2$	$(x^2+y^2)z, {}^\square z^3$
B				
${}^1E_1 \oplus {}^2E_1$		$^\square(x,y), (R_x, R_y)$	$^\square(zx, yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
${}^1E_2 \oplus {}^2E_2$			$^\square(xy, x^2-y^2)$	$^\square\{xyz, z(x^2-y^2)\}$
${}^1E_3 \oplus {}^2E_3$				$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$

T **8**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_8$	$\|j\,m\rangle$	ι	μ
A	$\|0\,0\rangle$	1	± 8
B	$\|4\,4\rangle$	1	± 8
${}^1E_1$	$\|1\,1\rangle$	1	± 8
${}^2E_1$	$\|1\,\overline{1}\rangle$	1	± 8
${}^1E_2$	$\|2\,2\rangle$	1	± 8
${}^2E_2$	$\|2\,\overline{2}\rangle$	1	± 8
${}^1E_3$	$\|3\,\overline{3}\rangle$	1	± 8
${}^2E_3$	$\|3\,3\rangle$	1	± 8
${}^1E_{1/2}$	$\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	1	± 8
${}^2E_{1/2}$	$\|\frac{1}{2}\,\frac{1}{2}\rangle$	1	± 8
${}^1E_{3/2}$	$\|\frac{3}{2}\,\frac{3}{2}\rangle$	1	± 8
${}^2E_{3/2}$	$\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	1	± 8
${}^1E_{5/2}$	$\|\frac{5}{2}\,\overline{\frac{5}{2}}\rangle$	1	± 8
${}^2E_{5/2}$	$\|\frac{5}{2}\,\frac{5}{2}\rangle$	1	± 8
${}^1E_{7/2}$	$\|\frac{7}{2}\,\frac{7}{2}\rangle$	1	± 8
${}^2E_{7/2}$	$\|\frac{7}{2}\,\overline{\frac{7}{2}}\rangle$	1	± 8

T **8**.7 Matrix representations

Use T **8**.4 ♠. § **16**–7, p. 77

T **8**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_8$	A	B	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$
A	A	B	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$
B		A	${}^1E_3$	${}^2E_3$	${}^2E_2$	${}^1E_2$	${}^1E_1$	${}^2E_1$
${}^1E_1$			${}^1E_2$	A	${}^2E_3$	${}^2E_1$	${}^2E_2$	B
${}^2E_1$				${}^2E_2$	${}^1E_1$	${}^1E_3$	B	${}^1E_2$
${}^1E_2$					B	A	${}^2E_1$	${}^1E_3$
${}^2E_2$						B	${}^2E_3$	${}^1E_1$
${}^1E_3$							${}^1E_2$	A
${}^2E_3$								${}^2E_2$

↠

T 8.8 Direct products of representations *(cont.)*

$\mathbf{C}_8$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$
A	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$
B	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_1$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$
${}^2E_1$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$
${}^1E_2$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$
${}^2E_2$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{7/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$
${}^1E_3$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$
${}^2E_3$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^1E_{1/2}$	${}^2E_1$	A	${}^1E_1$	${}^2E_2$	${}^1E_3$	${}^1E_2$	${}^2E_3$	B
${}^2E_{1/2}$		${}^1E_1$	${}^1E_2$	${}^2E_1$	${}^2E_2$	${}^2E_3$	B	${}^1E_3$
${}^1E_{3/2}$			${}^2E_3$	A	${}^2E_1$	B	${}^1E_3$	${}^2E_2$
${}^2E_{3/2}$				${}^1E_3$	B	${}^1E_1$	${}^1E_2$	${}^2E_3$
${}^1E_{5/2}$					${}^2E_3$	A	${}^1E_1$	${}^1E_2$
${}^2E_{5/2}$						${}^1E_3$	${}^2E_2$	${}^2E_1$
${}^1E_{7/2}$							${}^2E_1$	A
${}^2E_{7/2}$								${}^1E_1$

T 8.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_8$	$\mathbf{C}_4$	$\mathbf{C}_2$
A	A	A
B	A	A
${}^1E_1$	${}^1E$	B
${}^2E_1$	${}^2E$	B
${}^1E_2$	B	A
${}^2E_2$	B	A
${}^1E_3$	${}^1E$	B
${}^2E_3$	${}^2E$	B
${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_{3/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_{7/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$

T 8.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_8$
$8n$	$(2n+1)\,A \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3)$
$8n+1$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1) \oplus 2n\,(B \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3)$
$8n+2$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2) \oplus 2n\,(B \oplus {}^1E_3 \oplus {}^2E_3)$
$8n+3$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3) \oplus 2n\,B$
$8n+4$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3) \oplus (2n+2)\,B$
$8n+5$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2) \oplus (2n+2)(B \oplus {}^1E_3 \oplus {}^2E_3)$
$8n+6$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1) \oplus (2n+2)(B \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3)$
$8n+7$	$(2n+1)\,A \oplus (2n+2)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3)$
$8n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus 2n\,({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$8n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus 2n\,({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$8n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus 2n\,({}^1E_{7/2} \oplus {}^2E_{7/2})$
$8n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$8n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus (2n+2)({}^1E_{7/2} \oplus {}^2E_{7/2})$
$8n+\frac{11}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus (2n+2)({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$8n+\frac{13}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus (2n+2)({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$8n+\frac{15}{2}$	$(2n+2)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$

$n = 0, 1, 2, \ldots$

T 8.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: none.

(2) Group chains: $\mathbf{C}_{9h} \supset \underline{\mathbf{C}_9} \supset \underline{\mathbf{C}_3}$, $\mathbf{C}_{9v} \supset \underline{\mathbf{C}_9} \supset \underline{\mathbf{C}_3}$, $\mathbf{D}_9 \supset \underline{\mathbf{C}_9} \supset \underline{\mathbf{C}_3}$, $\mathbf{S}_{18} \supset \underline{\mathbf{C}_9} \supset \underline{\mathbf{C}_3}$.

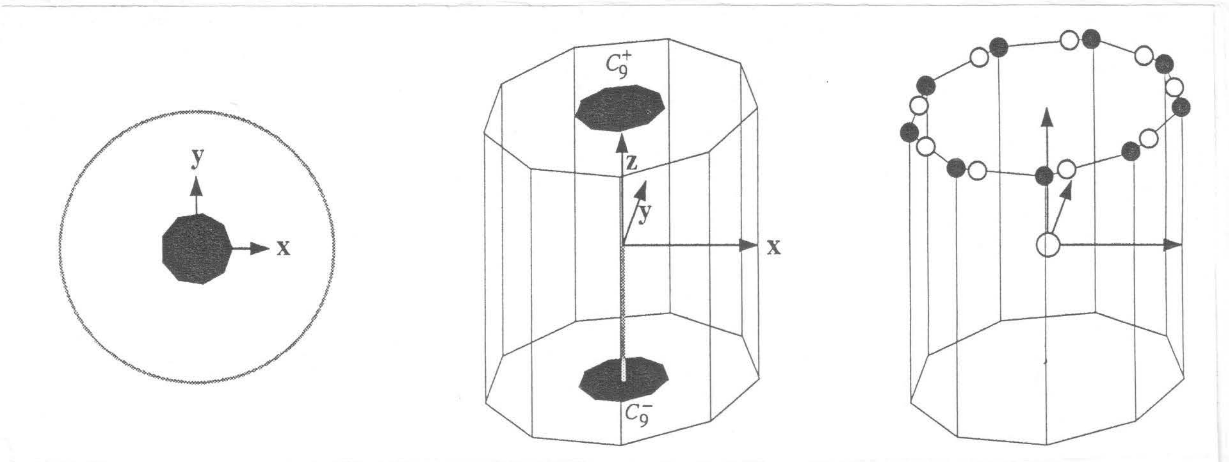
(3) Operations of G: E, C_9^+, C_9^{2+}, C_3^+, C_9^{4+}, C_9^{4-}, C_3^-, C_9^{2-}, C_9^-.

(4) Operations of $\widetilde{G}$: E, C_9^+, C_9^{2+}, C_3^+, C_9^{4+}, C_9^{4-}, C_3^-, C_9^{2-}, C_9^-,
$\widetilde{E}$, $\widetilde{C}_9^+$, $\widetilde{C}_9^{2+}$, $\widetilde{C}_3^+$, $\widetilde{C}_9^{4+}$, $\widetilde{C}_9^{4-}$, $\widetilde{C}_3^-$, $\widetilde{C}_9^{2-}$, $\widetilde{C}_9^-$.

(5) Classes and representations: $|r| = 9$, $|i| = 0$, $|I| = 9$, $|\widetilde{I}| = 9$.

F **9**

See Chapter **15**, p. 65



Examples:

T **9**.1 Parameters
Use T **38**.1. § **16**–1, p. 68

T **9**.2 Multiplication table
Use T **38**.2. § **16**–2, p. 69

T **9**.3 Factor table
Use T **38**.3. § **16**–3, p. 70

T **9**.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_9$	E	C_9^+	C_9^{2+}	C_3^+	C_9^{4+}	C_9^{4-}	C_3^-	C_9^{2-}	C_9^-	τ
A	1	1	1	1	1	1	1	1	1	a
$^1E_1$	1	δ^*	ϵ^*	η^*	θ^*	θ	η	ϵ	δ	b
$^2E_1$	1	δ	ϵ	η	θ	θ^*	η^*	ϵ^*	δ^*	b
$^1E_2$	1	ϵ^*	θ^*	η	δ	δ^*	η^*	θ	ϵ	b
$^2E_2$	1	ϵ	θ	η^*	δ^*	δ	η	θ^*	ϵ^*	b
$^1E_3$	1	η^*	η	1	η^*	η	1	η^*	η	b
$^2E_3$	1	η	η^*	1	η	η^*	1	η	η^*	b
$^1E_4$	1	θ^*	δ	η^*	ϵ	ϵ^*	η	δ^*	θ	b
$^2E_4$	1	θ	δ^*	η	ϵ^*	ϵ	η^*	δ	θ^*	b
$^1E_{1/2}$	1	$-\theta^*$	δ	$-\eta^*$	ϵ	ϵ^*	$-\eta$	δ^*	$-\theta$	b
$^2E_{1/2}$	1	$-\theta$	δ^*	$-\eta$	ϵ^*	ϵ	$-\eta^*$	δ	$-\theta^*$	b
$^1E_{3/2}$	1	$-\eta$	η^*	-1	η	η^*	-1	η	$-\eta^*$	b
$^2E_{3/2}$	1	$-\eta^*$	η	-1	η^*	η	-1	η^*	$-\eta$	b
$^1E_{5/2}$	1	$-\epsilon^*$	θ^*	$-\eta$	δ	δ^*	$-\eta^*$	θ	$-\epsilon$	b
$^2E_{5/2}$	1	$-\epsilon$	θ	$-\eta^*$	δ^*	δ	$-\eta$	θ^*	$-\epsilon^*$	b
$^1E_{7/2}$	1	$-\delta$	ϵ	$-\eta$	θ	θ^*	$-\eta^*$	ϵ^*	$-\delta^*$	b
$^2E_{7/2}$	1	$-\delta^*$	ϵ^*	$-\eta^*$	θ^*	θ	$-\eta$	ϵ	$-\delta$	b
$A_{9/2}$	1	-1	1	-1	1	1	-1	1	-1	a

$\delta = \exp(2\pi i/9)$, $\epsilon = \exp(4\pi i/9)$, $\eta = \exp(6\pi i/9)$, $\theta = \exp(8\pi i/9)$

T **9**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_9$	0	1	2	3
A	$^\square 1$	$^\square z, R_z$	$x^2+y^2, {}^\square z^2$	$(x^2+y^2)z, {}^\square z^3$
$^1E_1 \oplus {}^2E_1$		$^\square(x,y), (R_x, R_y)$	$^\square(zx, yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
$^1E_2 \oplus {}^2E_2$			$^\square(xy, x^2-y^2)$	$^\square\{xyz, z(x^2-y^2)\}$
$^1E_3 \oplus {}^2E_3$				$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$
$^1E_4 \oplus {}^2E_4$				

T **9**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_9$	$\lvert j\, m\rangle$	ι	μ
A	$\lvert 0\,0\rangle$	1	± 9
${}^1E_1$	$\lvert 1\,1\rangle$	1	± 9
${}^2E_1$	$\lvert 1\,\overline{1}\rangle$	1	± 9
${}^1E_2$	$\lvert 2\,2\rangle$	1	± 9
${}^2E_2$	$\lvert 2\,\overline{2}\rangle$	1	± 9
${}^1E_3$	$\lvert 3\,3\rangle$	1	± 9
${}^2E_3$	$\lvert 3\,\overline{3}\rangle$	1	± 9
${}^1E_4$	$\lvert 4\,4\rangle$	1	± 9
${}^2E_4$	$\lvert 4\,\overline{4}\rangle$	1	± 9
${}^1E_{1/2}$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	1	± 9
${}^2E_{1/2}$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle$	1	± 9
${}^1E_{3/2}$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle$	1	± 9
${}^2E_{3/2}$	$\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	1	± 9
${}^1E_{5/2}$	$\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle$	1	± 9
${}^2E_{5/2}$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle$	1	± 9
${}^1E_{7/2}$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle$	1	± 9
${}^2E_{7/2}$	$\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle$	1	± 9
$A_{9/2}$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle$	1	± 9

T **9**.7 Matrix representations

Use T **9**.4 ♠. § **16**–7, p. 77

T **9**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_9$	A	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$	${}^1E_4$	${}^2E_4$
A	A	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$	${}^1E_4$	${}^2E_4$
${}^1E_1$		${}^1E_2$	A	${}^1E_3$	${}^2E_1$	${}^1E_4$	${}^2E_2$	${}^2E_4$	${}^2E_3$
${}^2E_1$			${}^2E_2$	${}^1E_1$	${}^2E_3$	${}^1E_2$	${}^2E_4$	${}^1E_3$	${}^1E_4$
${}^1E_2$				${}^1E_4$	A	${}^2E_4$	${}^2E_1$	${}^2E_3$	${}^2E_2$
${}^2E_2$					${}^2E_4$	${}^1E_1$	${}^1E_4$	${}^1E_2$	${}^1E_3$
${}^1E_3$						${}^2E_3$	A	${}^2E_2$	${}^2E_1$
${}^2E_3$							${}^1E_3$	${}^1E_1$	${}^1E_2$
${}^1E_4$								${}^2E_1$	A
${}^2E_4$									${}^1E_1$

⇛

T **9**.8 Direct products of representations *(cont.)*

$\mathbf{C}_9$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	$A_{9/2}$
A	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	$A_{9/2}$
${}^1E_1$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{7/2}$	$A_{9/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$
${}^2E_1$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	$A_{9/2}$	${}^1E_{7/2}$
${}^1E_2$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	$A_{9/2}$	${}^2E_{7/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$
${}^2E_2$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{7/2}$	$A_{9/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$
${}^1E_3$	${}^2E_{5/2}$	${}^1E_{7/2}$	$A_{9/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$
${}^2E_3$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	$A_{9/2}$	${}^1E_{7/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$
${}^1E_4$	${}^1E_{7/2}$	$A_{9/2}$	${}^2E_{7/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_4$	$A_{9/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_{1/2}$	${}^2E_1$	A	${}^1E_1$	${}^2E_2$	${}^2E_3$	${}^1E_2$	${}^1E_3$	${}^2E_4$	${}^1E_4$
${}^2E_{1/2}$		${}^1E_1$	${}^1E_2$	${}^2E_1$	${}^2E_2$	${}^1E_3$	${}^1E_4$	${}^2E_3$	${}^2E_4$
${}^1E_{3/2}$			${}^1E_3$	A	${}^2E_1$	${}^1E_4$	${}^2E_4$	${}^2E_2$	${}^2E_3$
${}^2E_{3/2}$				${}^2E_3$	${}^2E_4$	${}^1E_1$	${}^1E_2$	${}^1E_4$	${}^1E_3$
${}^1E_{5/2}$					${}^1E_4$	A	${}^1E_1$	${}^1E_3$	${}^1E_2$
${}^2E_{5/2}$						${}^2E_4$	${}^2E_3$	${}^2E_1$	${}^2E_2$
${}^1E_{7/2}$							${}^2E_2$	A	${}^2E_1$
${}^2E_{7/2}$								${}^1E_2$	${}^1E_1$
$A_{9/2}$									A

T 9.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_9$	$\mathbf{C}_3$
A	A
${}^1E_1$	${}^1E$
${}^2E_1$	${}^2E$
${}^1E_2$	${}^2E$
${}^2E_2$	${}^1E$
${}^1E_3$	A
${}^2E_3$	A
${}^1E_4$	${}^1E$
${}^2E_4$	${}^2E$
${}^1E_{1/2}$	${}^1E_{1/2}$
${}^2E_{1/2}$	${}^2E_{1/2}$
${}^1E_{3/2}$	$A_{3/2}$
${}^2E_{3/2}$	$A_{3/2}$
${}^1E_{5/2}$	${}^2E_{1/2}$
${}^2E_{5/2}$	${}^1E_{1/2}$
${}^1E_{7/2}$	${}^2E_{1/2}$
${}^2E_{7/2}$	${}^1E_{1/2}$
$A_{9/2}$	$A_{3/2}$

T 9.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_9$
$9n$	$(2n+1)\,A \oplus 2n\,({}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$9n+1$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1) \oplus 2n\,({}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$9n+2$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2) \oplus 2n\,({}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$9n+3$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3) \oplus 2n\,({}^1E_4 \oplus {}^2E_4)$
$9n+4$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$9n+5$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3) \oplus (2n+2)({}^1E_4 \oplus {}^2E_4)$
$9n+6$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2) \oplus (2n+2)({}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$9n+7$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1) \oplus (2n+2)({}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$9n+8$	$(2n+1)\,A \oplus (2n+2)({}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$9n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus 2n\,({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus A_{9/2})$
$9n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus 2n\,({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus A_{9/2})$
$9n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus 2n\,({}^1E_{7/2} \oplus {}^2E_{7/2} \oplus A_{9/2})$
$9n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}) \oplus 2n\,A_{9/2}$
$9n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}) \oplus (2n+2)\,A_{9/2}$
$9n+\frac{11}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus (2n+2)({}^1E_{7/2} \oplus {}^2E_{7/2} \oplus A_{9/2})$
$9n+\frac{13}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus (2n+2)({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus A_{9/2})$
$9n+\frac{15}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus (2n+2)({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus A_{9/2})$
$9n+\frac{17}{2}$	$(2n+2)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus A_{9/2})$

$n = 0, 1, 2, \ldots$

T 9.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

10	$|G| = 10$	$|C| = 10$	$|\widetilde{C}| = 20$	T **10**	p. 107	$\mathbf{C}_{10}$

(1) Product forms: $\mathbf{C}_5 \otimes \mathbf{C}_2$.

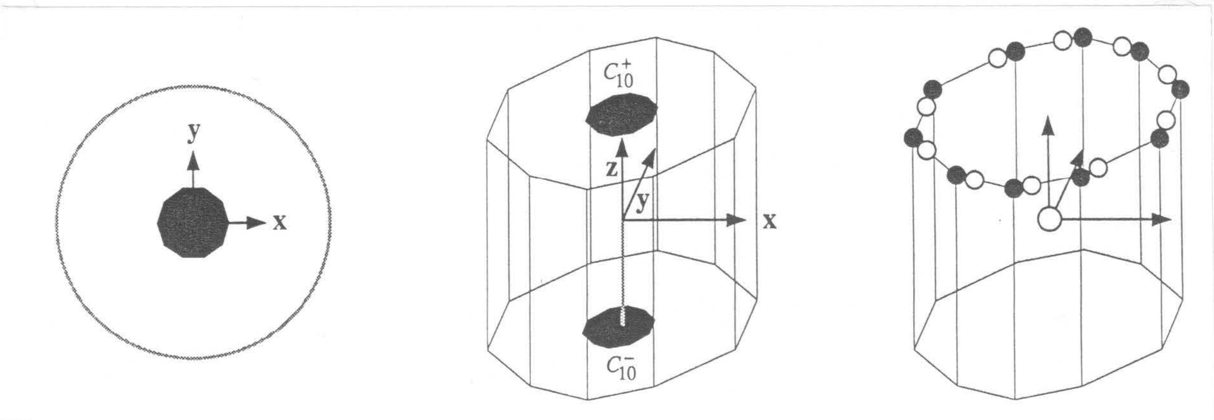
(2) Group chains: $\mathbf{C}_{10h} \supset \underline{\mathbf{C}_{10}} \supset \underline{\mathbf{C}_5}$, $\mathbf{C}_{10h} \supset \underline{\mathbf{C}_{10}} \supset \underline{\mathbf{C}_2}$, $\mathbf{C}_{10v} \supset \underline{\mathbf{C}_{10}} \supset \underline{\mathbf{C}_5}$, $\mathbf{C}_{10v} \supset \underline{\mathbf{C}_{10}} \supset \underline{\mathbf{C}_2}$, $\mathbf{D}_{10} \supset \underline{\mathbf{C}_{10}} \supset \underline{\mathbf{C}_5}$, $\mathbf{D}_{10} \supset \underline{\mathbf{C}_{10}} \supset \underline{\mathbf{C}_2}$, $\mathbf{S}_{20} \supset \underline{\mathbf{C}_{10}} \supset \underline{\mathbf{C}_5}$, $\mathbf{S}_{20} \supset \underline{\mathbf{C}_{10}} \supset \underline{\mathbf{C}_2}$.

(3) Operations of G: E, C_{10}^{+}, C_5^{+}, C_{10}^{3+}, C_5^{2+}, C_2, C_5^{2-}, C_{10}^{3-}, C_5^{-}, C_{10}^{-}.

(4) Operations of $\widetilde{G}$: E, C_{10}^{+}, C_5^{+}, C_{10}^{3+}, C_5^{2+}, C_2, C_5^{2-}, C_{10}^{3-}, C_5^{-}, C_{10}^{-},
$\widetilde{E}$, $\widetilde{C}_{10}^{+}$, $\widetilde{C}_5^{+}$, $\widetilde{C}_{10}^{3+}$, $\widetilde{C}_5^{2+}$, $\widetilde{C}_2$, $\widetilde{C}_5^{2-}$, $\widetilde{C}_{10}^{3-}$, $\widetilde{C}_5^{-}$, $\widetilde{C}_{10}^{-}$.

(5) Classes and representations: $|r| = 10$, $|\text{i}| = 0$, $|I| = 10$, $|\widetilde{I}| = 10$.

F **10** See Chapter **15**, p. 65



Examples:

T **10**.1 Parameters
Use T **39**.1. § **16**–1, p. 68

T **10**.2 Multiplication table
Use T **39**.2. § **16**–2, p. 69

T **10**.3 Factor table
Use T **39**.3. § **16**–3, p. 70

T **10**.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_{10}$	E	C_{10}^{+}	C_5^{+}	C_{10}^{3+}	C_5^{2+}	C_2	C_5^{2-}	C_{10}^{3-}	C_5^{-}	C_{10}^{-}	τ
A	1	1	1	1	1	1	1	1	1	1	a
B	1	-1	1	-1	1	-1	1	-1	1	-1	a
${}^1E_1$	1	$-\epsilon$	δ^*	$-\delta$	ϵ^*	-1	ϵ	$-\delta^*$	δ	$-\epsilon^*$	b
${}^2E_1$	1	$-\epsilon^*$	δ	$-\delta^*$	ϵ	-1	ϵ^*	$-\delta$	δ^*	$-\epsilon$	b
${}^1E_2$	1	δ^*	ϵ^*	ϵ	δ	1	δ^*	ϵ^*	ϵ	δ	b
${}^2E_2$	1	δ	ϵ	ϵ^*	δ^*	1	δ	ϵ	ϵ^*	δ^*	b
${}^1E_3$	1	$-\delta^*$	ϵ^*	$-\epsilon$	δ	-1	δ^*	$-\epsilon^*$	ϵ	$-\delta$	b
${}^2E_3$	1	$-\delta$	ϵ	$-\epsilon^*$	δ^*	-1	δ	$-\epsilon$	ϵ^*	$-\delta^*$	b
${}^1E_4$	1	ϵ	δ^*	δ	ϵ^*	1	ϵ	δ^*	δ	ϵ^*	b
${}^2E_4$	1	ϵ^*	δ	δ^*	ϵ	1	ϵ^*	δ	δ^*	ϵ	b
${}^1E_{1/2}$	1	$\mathrm{i}\delta^*$	$-\epsilon^*$	$-\mathrm{i}\epsilon$	δ	i	δ^*	$\mathrm{i}\epsilon^*$	$-\epsilon$	$-\mathrm{i}\delta$	b
${}^2E_{1/2}$	1	$-\mathrm{i}\delta$	$-\epsilon$	$\mathrm{i}\epsilon^*$	δ^*	$-\mathrm{i}$	δ	$-\mathrm{i}\epsilon$	$-\epsilon^*$	$\mathrm{i}\delta^*$	b
${}^1E_{3/2}$	1	$\mathrm{i}\epsilon^*$	$-\delta$	$-\mathrm{i}\delta^*$	ϵ	i	ϵ^*	$\mathrm{i}\delta$	$-\delta^*$	$-\mathrm{i}\epsilon$	b
${}^2E_{3/2}$	1	$-\mathrm{i}\epsilon$	$-\delta^*$	$\mathrm{i}\delta$	ϵ^*	$-\mathrm{i}$	ϵ	$-\mathrm{i}\delta^*$	$-\delta$	$\mathrm{i}\epsilon^*$	b
${}^1E_{5/2}$	1	i	-1	$-\mathrm{i}$	1	i	1	i	-1	$-\mathrm{i}$	b
${}^2E_{5/2}$	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	1	$-\mathrm{i}$	-1	i	b
${}^1E_{7/2}$	1	$\mathrm{i}\epsilon$	$-\delta^*$	$-\mathrm{i}\delta$	ϵ^*	i	ϵ	$\mathrm{i}\delta^*$	$-\delta$	$-\mathrm{i}\epsilon^*$	b
${}^2E_{7/2}$	1	$-\mathrm{i}\epsilon^*$	$-\delta$	$\mathrm{i}\delta^*$	ϵ	$-\mathrm{i}$	ϵ^*	$-\mathrm{i}\delta$	$-\delta^*$	$\mathrm{i}\epsilon$	b
${}^1E_{9/2}$	1	$\mathrm{i}\delta$	$-\epsilon$	$-\mathrm{i}\epsilon^*$	δ^*	i	δ	$\mathrm{i}\epsilon$	$-\epsilon^*$	$-\mathrm{i}\delta^*$	b
${}^2E_{9/2}$	1	$-\mathrm{i}\delta^*$	$-\epsilon^*$	$\mathrm{i}\epsilon$	δ	$-\mathrm{i}$	δ^*	$-\mathrm{i}\epsilon^*$	$-\epsilon$	$\mathrm{i}\delta$	b

$\delta = \exp(2\pi\mathrm{i}/5)$, $\epsilon = \exp(4\pi\mathrm{i}/5)$

T **10**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{10}$	0	1	2	3
A	$^\square 1$	$^\square z, R_z$	$x^2+y^2, {}^\square z^2$	$(x^2+y^2)z, {}^\square z^3$
B				
${}^1E_1 \oplus {}^2E_1$		$^\square(x,y), (R_x, R_y)$	$^\square(zx, yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
${}^1E_2 \oplus {}^2E_2$			$^\square(xy, x^2-y^2)$	$^\square\{xyz, z(x^2-y^2)\}$
${}^1E_3 \oplus {}^2E_3$				$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$
${}^1E_4 \oplus {}^2E_4$				

T **10**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{10}$	$\lvert j\,m\rangle$	ι	μ	$\mathbf{C}_{10}$	$\lvert j\,m\rangle$	ι	μ
A	$\lvert 0\,0\rangle$	1	± 10	${}^1E_{1/2}$	$\lvert \tfrac12\,\overline{\tfrac12}\rangle$	1	± 10
B	$\lvert 5\,5\rangle$	1	± 10	${}^2E_{1/2}$	$\lvert \tfrac12\,\tfrac12\rangle$	1	± 10
${}^1E_1$	$\lvert 1\,1\rangle$	1	± 10	${}^1E_{3/2}$	$\lvert \tfrac32\,\tfrac32\rangle$	1	± 10
${}^2E_1$	$\lvert 1\,\overline{1}\rangle$	1	± 10	${}^2E_{3/2}$	$\lvert \tfrac32\,\overline{\tfrac32}\rangle$	1	± 10
${}^1E_2$	$\lvert 2\,2\rangle$	1	± 10	${}^1E_{5/2}$	$\lvert \tfrac52\,\overline{\tfrac52}\rangle$	1	± 10
${}^2E_2$	$\lvert 2\,\overline{2}\rangle$	1	± 10	${}^2E_{5/2}$	$\lvert \tfrac52\,\tfrac52\rangle$	1	± 10
${}^1E_3$	$\lvert 3\,\overline{3}\rangle$	1	± 10	${}^1E_{7/2}$	$\lvert \tfrac72\,\tfrac72\rangle$	1	± 10
${}^2E_3$	$\lvert 3\,3\rangle$	1	± 10	${}^2E_{7/2}$	$\lvert \tfrac72\,\overline{\tfrac72}\rangle$	1	± 10
${}^1E_4$	$\lvert 4\,\overline{4}\rangle$	1	± 10	${}^1E_{9/2}$	$\lvert \tfrac92\,\overline{\tfrac92}\rangle$	1	± 10
${}^2E_4$	$\lvert 4\,4\rangle$	1	± 10	${}^2E_{9/2}$	$\lvert \tfrac92\,\tfrac92\rangle$	1	± 10

T **10**.8 Direct products of representations
§ **16**–8, p. 81

$\mathbf{C}_{10}$	A	B	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$	${}^1E_4$	${}^2E_4$
A	A	B	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$	${}^1E_4$	${}^2E_4$
B		A	${}^1E_4$	${}^2E_4$	${}^1E_3$	${}^2E_3$	${}^1E_2$	${}^2E_2$	${}^1E_1$	${}^2E_1$
${}^1E_1$			${}^1E_2$	A	${}^2E_3$	${}^2E_1$	${}^2E_2$	${}^2E_4$	${}^1E_3$	B
${}^2E_1$				${}^2E_2$	${}^1E_1$	${}^1E_3$	${}^1E_4$	${}^1E_2$	B	${}^2E_3$
${}^1E_2$					${}^2E_4$	A	${}^2E_1$	B	${}^2E_2$	${}^1E_4$
${}^2E_2$						${}^1E_4$	B	${}^1E_1$	${}^2E_4$	${}^1E_2$
${}^1E_3$							${}^2E_4$	A	${}^2E_3$	${}^1E_1$
${}^2E_3$								${}^1E_4$	${}^2E_1$	${}^1E_3$
${}^1E_4$									${}^1E_2$	A
${}^2E_4$										${}^2E_2$

⇛

T **10**.7 Matrix representations
Use T **10**.4 ♠. § **16**–7, p. 77

T **10**.8 Direct products of representations *(cont.)*

$\mathbf{C}_{10}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$
A	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$
B	${}^2E_{9/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^1E_1$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$
${}^2E_1$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$
${}^1E_2$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{9/2}$	${}^1E_{9/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$
${}^2E_2$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$
${}^1E_3$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$
${}^2E_3$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$
${}^1E_4$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^2E_4$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{9/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$
${}^1E_{1/2}$	${}^2E_1$	A	${}^1E_1$	${}^2E_2$	${}^1E_3$	${}^1E_2$	${}^2E_3$	${}^1E_4$	B	${}^2E_4$
${}^2E_{1/2}$		${}^1E_1$	${}^1E_2$	${}^2E_1$	${}^2E_2$	${}^2E_3$	${}^2E_4$	${}^1E_3$	${}^1E_4$	B
${}^1E_{3/2}$			${}^2E_3$	A	${}^2E_1$	${}^2E_4$	B	${}^2E_2$	${}^1E_3$	${}^1E_4$
${}^2E_{3/2}$				${}^1E_3$	${}^1E_4$	${}^1E_1$	${}^1E_2$	B	${}^2E_4$	${}^2E_3$
${}^1E_{5/2}$					B	A	${}^1E_1$	${}^2E_4$	${}^2E_3$	${}^1E_2$
${}^2E_{5/2}$						B	${}^1E_4$	${}^2E_1$	${}^2E_2$	${}^1E_3$
${}^1E_{7/2}$							${}^1E_3$	A	${}^2E_1$	${}^2E_2$
${}^2E_{7/2}$								${}^2E_3$	${}^1E_2$	${}^1E_1$
${}^1E_{9/2}$									${}^1E_1$	A
${}^2E_{9/2}$										${}^2E_1$

T **10**.9 Subduction (descent of symmetry)
§ **16**–9, p. 82

$\mathbf{C}_{10}$	$\mathbf{C}_5$	$\mathbf{C}_2$	$\mathbf{C}_{10}$	$\mathbf{C}_5$	$\mathbf{C}_2$
A	A	A	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$
B	A	B	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$
${}^1E_1$	${}^1E_1$	B	${}^1E_{3/2}$	${}^1E_{3/2}$	${}^1E_{1/2}$
${}^2E_1$	${}^2E_1$	B	${}^2E_{3/2}$	${}^2E_{3/2}$	${}^2E_{1/2}$
${}^1E_2$	${}^1E_2$	A	${}^1E_{5/2}$	$A_{5/2}$	${}^1E_{1/2}$
${}^2E_2$	${}^2E_2$	A	${}^2E_{5/2}$	$A_{5/2}$	${}^2E_{1/2}$
${}^1E_3$	${}^1E_2$	B	${}^1E_{7/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_3$	${}^2E_2$	B	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_4$	${}^1E_1$	A	${}^1E_{9/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_4$	${}^2E_1$	A	${}^2E_{9/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$

T **10**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{10}$
$10n$	$(2n+1)\,A \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$10n+1$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1) \oplus 2n\,(B \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$10n+2$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2) \oplus 2n\,(B \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$10n+3$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3) \oplus 2n\,(B \oplus {}^1E_4 \oplus {}^2E_4)$
$10n+4$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4) \oplus 2n\,B$
$10n+5$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4) \oplus (2n+2)\,B$
$10n+6$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3) \oplus (2n+2)(B \oplus {}^1E_4 \oplus {}^2E_4)$
$10n+7$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2) \oplus (2n+2)(B \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$10n+8$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1) \oplus (2n+2)(B \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$10n+9$	$(2n+1)\,A \oplus (2n+2)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$10n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus$ $2n\,({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus$ $2n\,({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus$ $2n\,({}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}) \oplus$ $2n\,({}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{11}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}) \oplus$ $(2n+2)({}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{13}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus$ $(2n+2)({}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{15}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus$ $(2n+2)({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{17}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus$ $(2n+2)({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{19}{2}$	$(2n+2)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$

$n = 0, 1, 2, \ldots$

T **10**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

The improper cyclic groups $\mathbf{C}_i$ and $\mathbf{C}_s$

Notation for headers

Items in header read from left to right

1		Hermann–Mauguin symbol for the point group.
2		$\|G\|$ order of the group.
3		$\|C\|$ number of classes in the group.
4		$\|\widetilde{C}\|$ number of classes in the double group.
5		Number of the table.
6		Page reference for the notation of the header, of the first five subsections below it, and of the footers.
7	□	This symbol indicates a crystallographic point group.
8		Schönflies notation for the point group.

Notation for the first five subsections below the header

(1) Product forms	Direct and semidirect product forms. ⊗ Direct product. ⓢ Semidirect product.
(2) Group chains *(See pp. 41, 67)*	Groups underlined: invariant. Groups in brackets: when subducing one or more of the representations in the tables to these subgroups in the setting used for them in the tables a change of bases (similarity transformation) is required.
(3) Operations of G	Lists all the operations of G, enclosing in brackets all the operations of the same class.
(4) Operations of $\widetilde{G}$	Lists all the operations of $\widetilde{G}$, enclosing in brackets all the operations of the same class.
(5) Classes and representations	$\|r\|$ number of regular classes in G (p. 51). $\|\mathrm{i}\|$ number of irregular classes in G (p. 51). $\|I\|$ number of irreducible representations in G. $\|\widetilde{I}\|$ number of spinor representations, also called the number of double-group representations.

Use of the footers

Finding your way about the tables

Each page of the tables contains a footer giving the ordering of the group types listed in the tables. The current group type is recognized because it carries no page number under its name. On opening the tables at any page you should then see at a glance whether to move backwards or forwards to find the group type desired. More precisely, the page number for the group type desired gives the page, like the present one, which carries the contents for the group type wanted.

(1) Product forms: none.

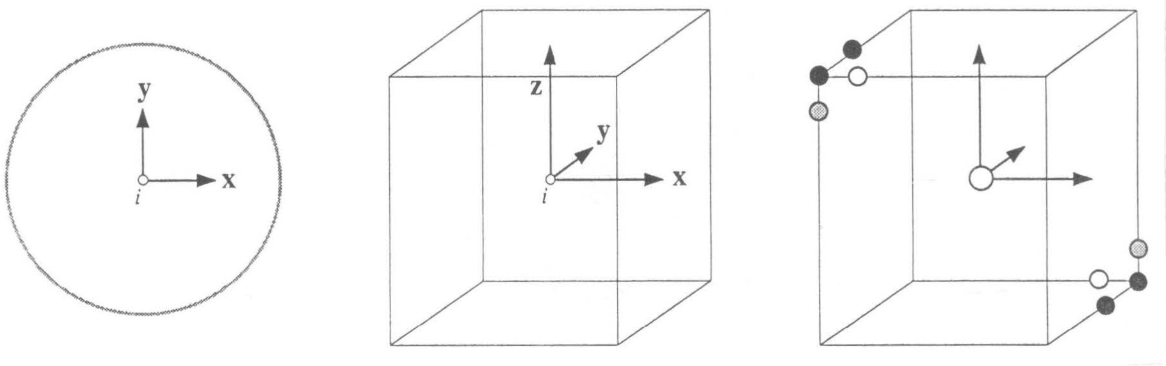
(2) Group chains: $\mathbf{C}_{2h} \supset \underline{\mathbf{C}_i} \supset \underline{\mathbf{C}_1}$, $\mathbf{S}_{14} \supset \underline{\mathbf{C}_i} \supset \underline{\mathbf{C}_1}$, $\mathbf{S}_{10} \supset \underline{\mathbf{C}_i} \supset \underline{\mathbf{C}_1}$, $\mathbf{S}_6 \supset \underline{\mathbf{C}_i} \supset \underline{\mathbf{C}_1}$.

(3) Operations of G: E, i.

(4) Operations of $\widetilde{G}$: E, i,
$\widetilde{E}$, $\widetilde{\imath}$.

(5) Classes and representations: $|r| = 2$, $|\mathrm{i}| = 0$, $|I| = 2$, $|\widetilde{I}| = 2$.

F **11** See Chapter **15**, p. 65



Examples: Staggered ClBrHC−CHBrCl.

T 11.1 Parameters

Use T **31**.1. § **16**–1, p. 68

T 11.2 Multiplication table

Use T **31**.2. § **16**–2, p. 69

T 11.3 Factor table

Use T **31**.3. § **16**–3, p. 70

T 11.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_i$	E	i	τ
A_g	1	1	a
A_u	1	−1	a
$A_{1/2,g}$	1	1	a
$A_{1/2,u}$	1	−1	a

T 11.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_i$	0	1	2	3
A_g	$\square 1$	R_x, R_y, R_z	$\square x^2, y^2, \square z^2,$ $\square zx, \square yz, \square xy$	
A_u		$\square x, \square y, \square z$		$\square x^3, xy^2, \square xz^2, \square x^2y, y^3,$ $\square yz^2, \square x^2z, y^2z, \square z^3, \square xyz$

T **11**.6 Symmetrized bases
§ **16**–6, p. 74

$\mathbf{C}_i$	$\vert j\,m\rangle$	ι	μ
A_g	$\vert 0\,0\rangle$	2	±1
A_u	$\vert 1\,0\rangle$	2	±1
$A_{1/2,g}$	$\vert \frac{1}{2}\,\frac{1}{2}\rangle$	1	±1
$A_{1/2,u}$	$\vert \frac{1}{2}\,\frac{1}{2}\rangle^\bullet$	1	±1

T **11**.7 Matrix representations
Use T **11**.4 ♠. § **16**–7, p. 77

T **11**.8 Direct products of representations
§ **16**–8, p. 81

$\mathbf{C}_i$	A_g	A_u	$A_{1/2,g}$	$A_{1/2,u}$
A_g	A_g	A_u	$A_{1/2,g}$	$A_{1/2,u}$
A_u		A_g	$A_{1/2,u}$	$A_{1/2,g}$
$A_{1/2,g}$			A_g	A_u
$A_{1/2,u}$				A_g

T **11**.9 Subduction (descent of symmetry)
§ **16**–9, p. 82

No proper subgroups.

T **11**.10 ♣ Subduction from O(3)
§ **16**–10, p. 82

j	$\mathbf{C}_i$
$2n$	$(4n+1)\,A_g$
$2n+1$	$(4n+3)\,A_u$
$n+\frac{1}{2}$	$(2n+2)\,A_{1/2,g}$

$n = 0, 1, 2, \ldots$

T **11**.11 Clebsch–Gordan coefficients
§ **16**–11 ♠, p. 83

(1) Product forms: none.

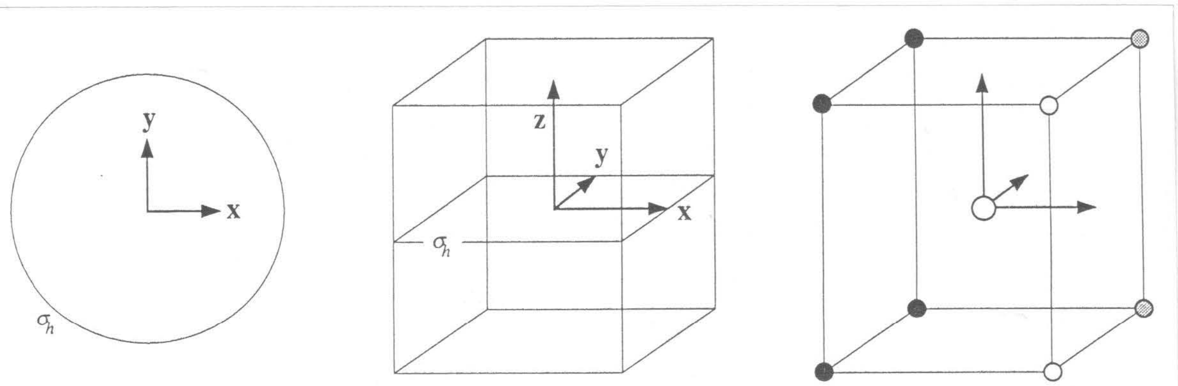
(2) Group chains: $\mathbf{C}_{7h} \supset \underline{\mathbf{C}_s} \supset \underline{\mathbf{C}_1}$, $\mathbf{C}_{5h} \supset \underline{\mathbf{C}_s} \supset \underline{\mathbf{C}_1}$, $\mathbf{C}_{3h} \supset \underline{\mathbf{C}_s} \supset \underline{\mathbf{C}_1}$, $\mathbf{C}_{2h} \supset \underline{\mathbf{C}_s} \supset \underline{\mathbf{C}_1}$,
$\mathbf{C}_{7v} \supset (\mathbf{C}_s) \supset \underline{\mathbf{C}_1}$, $\mathbf{C}_{5v} \supset (\mathbf{C}_s) \supset \underline{\mathbf{C}_1}$, $\mathbf{C}_{3v} \supset (\mathbf{C}_s) \supset \underline{\mathbf{C}_1}$, $\mathbf{C}_{2v} \supset (\underline{\mathbf{C}_s}) \supset \underline{\mathbf{C}_1}$.

(3) Operations of G: E, σ_h.

(4) Operations of $\widetilde{G}$: E, σ_h,
$\widetilde{E}$, $\widetilde{\sigma}_h$.

(5) Classes and representations: $|r| = 2$, $|\mathrm{i}| = 0$, $|I| = 2$, $|\widetilde{I}| = 2$.

F 12 See Chapter **15**, p. 65



Examples: Non-linear NOCl, planar N_3H, planar BFClBr, $C_2H_5NO_2$.

T **12**.1 Parameters
Use T **31**.1 ◇. § **16**–1, p. 68

T **12**.2 Multiplication table
Use T **31**.2 ◇. § **16**–2, p. 69

T **12**.3 Factor table
Use T **31**.3 ◇. § **16**–3, p. 70

T **12**.4 Character table
§ **16**–4, p. 71

$\mathbf{C}_s$	E	σ_h	τ
A'	1	1	a
A''	1	−1	a
${}^1E_{1/2}$	1	i	b
${}^2E_{1/2}$	1	−i	b

T **12**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions
§ **16**–5, p. 72

$\mathbf{C}_s$	0	1	2	3
A'	□1	□x, □y, R_z	□x^2, y^2, □z^2, □xy	□x^3, xy^2, □xz^2, □x^2y, y^3, □yz^2
A''		□z, R_x, R_y	□zx, □yz	□x^2z, y^2z, □z^3, □xyz

T **12**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_s$	$\|j\,m\rangle$		ι	μ
A'	$\|0\,0\rangle$	$\|1\,1\rangle$	2	± 2
A''	$\|1\,0\rangle$	$\|2\,1\rangle$	2	± 2
${}^1E_{1/2}$	$\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	$\|\frac{1}{2}\,\frac{1}{2}\rangle^\bullet$	1	± 2
${}^2E_{1/2}$	$\|\frac{1}{2}\,\frac{1}{2}\rangle$	$\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle^\bullet$	1	± 2

T **12**.7 Matrix representations

Use T **12**.4 ♠. § **16**–7, p. 77

T **12**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_s$	A'	A''	${}^1E_{1/2}$	${}^2E_{1/2}$
A'	A'	A''	${}^1E_{1/2}$	${}^2E_{1/2}$
A''		A'	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^1E_{1/2}$			A''	A'
${}^2E_{1/2}$				A''

T **12**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

No proper subgroups.

T **12**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_s$
n	$(n+1)\,A' \oplus n\,A''$
$n+\frac{1}{2}$	$(n+1)({}^1E_{1/2} \oplus {}^2E_{1/2})$

$n = 0, 1, 2, \ldots$

T **12**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

The improper cyclic groups $\mathbf{S}_n$

Notation for headers

Items in header read from left to right

1		Hermann–Mauguin symbol for the point group.
2		$\|G\|$ order of the group.
3		$\|C\|$ number of classes in the group.
4		$\|\widetilde{C}\|$ number of classes in the double group.
5		Number of the table.
6		Page reference for the notation of the header, of the first five subsections below it, and of the footers.
7	☐	This symbol indicates a crystallographic point group.
8		Schönflies notation for the point group.

Notation for the first five subsections below the header

(1) Product forms — Direct and semidirect product forms. $\otimes$ Direct product. ⓢ Semidirect product.

(2) Group chains *(See pp. 41, 67)* — Groups underlined: invariant. Groups in brackets: when subducing one or more of the representations in the tables to these subgroups in the setting used for them in the tables a change of bases (similarity transformation) is required.

(3) Operations of G — Lists all the operations of G, enclosing in brackets all the operations of the same class.

(4) Operations of $\widetilde{G}$ — Lists all the operations of $\widetilde{G}$, enclosing in brackets all the operations of the same class.

(5) Classes and representations — $|r|$ number of regular classes in G (p. 51).
$|\mathrm{i}|$ number of irregular classes in G (p. 51).
$|I|$ number of irreducible representations in G.
$|\widetilde{I}|$ number of spinor representations, also called the number of double-group representations.

Use of the footers

Finding your way about the tables — Each page of the tables contains a footer giving the ordering of the group types listed in the tables. The current group type is recognized because it carries no page number under its name. On opening the tables at any page you should then see at a glance whether to move backwards or forwards to find the group type desired. More precisely, the page number for the group type desired gives the page, like the present one, which carries the contents for the group type wanted.

(1) Product forms: none.

(2) Group chains: $\mathbf{C}_{4h} \supset \underline{\mathbf{S}_4} \supset \underline{\mathbf{C}_2}$, $\mathbf{D}_{2d} \supset \underline{\mathbf{S}_4} \supset \underline{\mathbf{C}_2}$, $\mathbf{S}_{20} \supset \underline{\mathbf{S}_4} \supset \underline{\mathbf{C}_2}$, $\mathbf{S}_{12} \supset \underline{\mathbf{S}_4} \supset \underline{\mathbf{C}_2}$.

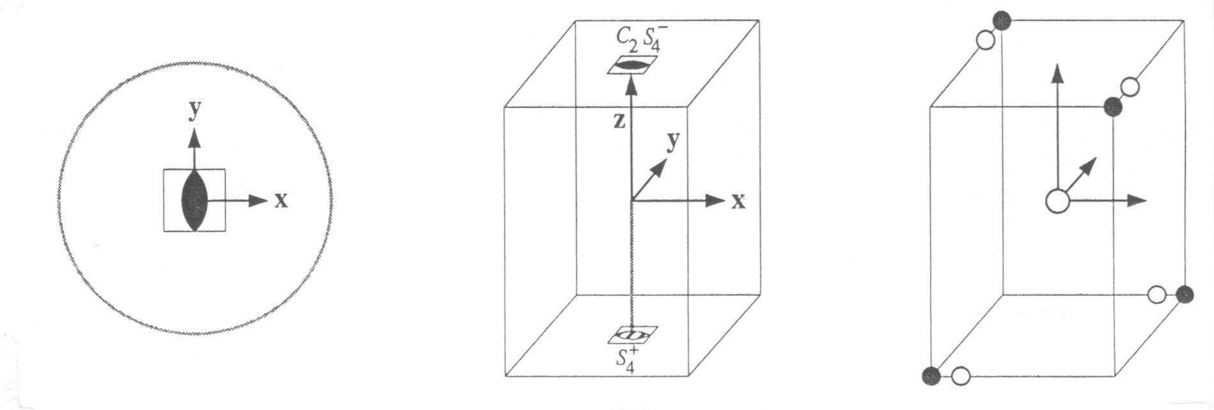
(3) Operations of G: E, S_4^-, C_2, S_4^+.

(4) Operations of $\widetilde{G}$: E, S_4^-, C_2, S_4^+
$\widetilde{E}$, $\widetilde{S}_4^-$, $\widetilde{C}_2$, $\widetilde{S}_4^+$.

(5) Classes and representations: $|r| = 4$, $|i| = 0$, $|I| = 4$, $|\widetilde{I}| = 4$.

F **13**

See Chapter **15**, p. 65



Examples: Tetraphenylmethane $C(C_6H_5)_4$.

T **13**.1 Parameters
Use T **33**.1. § **16**–1, p. 68

T **13**.2 Multiplication table
Use T **33**.2. § **16**–2, p. 69

T **13**.3 Factor table
Use T **33**.3. § **16**–3, p. 70

T **13**.4 Character table
§ **16**–4, p. 71

$\mathbf{S}_4$	E	S_4^-	C_2	S_4^+	τ
A	1	1	1	1	a
B	1	−1	1	−1	a
${}^1E$	1	−i	−1	i	b
${}^2E$	1	i	−1	−i	b
${}^1E_{1/2}$	1	ϵ^*	−i	ϵ	b
${}^2E_{1/2}$	1	ϵ	i	ϵ^*	b
${}^1E_{3/2}$	1	$-\epsilon^*$	−i	$-\epsilon$	b
${}^2E_{3/2}$	1	$-\epsilon$	i	$-\epsilon^*$	b

$\epsilon = \exp(2\pi i/8)$

T **13**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{S}_4$	0	1	2	3
A	$^\square 1$	R_z	$x^2+y^2, {}^\square z^2$	$^\square z(x^2-y^2), {}^\square xyz$
B		$^\square z$	$^\square x^2-y^2, {}^\square xy$	$(x^2+y^2)z, {}^\square z^3$
$^1E \oplus {}^2E$		$^\square(x,y), (R_x,R_y)$	$^\square(zx,yz)$	$^\square(xz^2,yz^2), \{x(x^2+y^2), y(x^2+y^2)\},$ $^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$

T **13**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{S}_4$	$\lvert j\,m\rangle$		ι	μ	$\mathbf{S}_4$	$\lvert j\,m\rangle$		ι	μ
A	$\lvert 0\,0\rangle$	$\lvert 3\,2\rangle$	2	± 4	$^1E_{1/2}$	$\lvert\frac{1}{2}\,\frac{1}{2}\rangle$	$\lvert\frac{3}{2}\,\overline{\frac{3}{2}}\rangle^\bullet$	1	± 4
B	$\lvert 1\,0\rangle$	$\lvert 2\,2\rangle$	2	± 4	$^2E_{1/2}$	$\lvert\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	$\lvert\frac{3}{2}\,\frac{3}{2}\rangle^\bullet$	1	± 4
^1E	$\lvert 1\,\overline{1}\rangle$	$\lvert 2\,1\rangle$	2	± 4	$^1E_{3/2}$	$\lvert\frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	$\lvert\frac{1}{2}\,\frac{1}{2}\rangle^\bullet$	1	± 4
^2E	$\lvert 1\,1\rangle$	$\lvert 2\,\overline{1}\rangle$	2	± 4	$^2E_{3/2}$	$\lvert\frac{3}{2}\,\frac{3}{2}\rangle$	$\lvert\frac{1}{2}\,\overline{\frac{1}{2}}\rangle^\bullet$	1	± 4

T **13**.7 Matrix representations

Use T **13**.4 ♠. § **16**–7, p. 77

T **13**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{S}_4$	A	B	^1E	^2E	$^1E_{1/2}$	$^2E_{1/2}$	$^1E_{3/2}$	$^2E_{3/2}$
A	A	B	^1E	^2E	$^1E_{1/2}$	$^2E_{1/2}$	$^1E_{3/2}$	$^2E_{3/2}$
B		A	^2E	^1E	$^1E_{3/2}$	$^2E_{3/2}$	$^1E_{1/2}$	$^2E_{1/2}$
^1E			B	A	$^2E_{3/2}$	$^1E_{1/2}$	$^2E_{1/2}$	$^1E_{3/2}$
^2E				B	$^2E_{1/2}$	$^1E_{3/2}$	$^2E_{3/2}$	$^1E_{1/2}$
$^1E_{1/2}$					^1E	A	^2E	B
$^2E_{1/2}$						^2E	B	^1E
$^1E_{3/2}$							^1E	A
$^2E_{3/2}$								^2E

T **13**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{S}_4$	$\mathbf{C}_2$
A	A
B	A
^1E	B
^2E	B
$^1E_{1/2}$	$^2E_{1/2}$
$^2E_{1/2}$	$^1E_{1/2}$
$^1E_{3/2}$	$^2E_{1/2}$
$^2E_{3/2}$	$^1E_{1/2}$

T **13**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{S}_4$
$4n$	$(2n+1)\,A \oplus 2n\,(B \oplus {}^1E \oplus {}^2E)$
$4n+1$	$2n\,A \oplus (2n+1)(B \oplus {}^1E \oplus {}^2E)$
$4n+2$	$(2n+1)(A \oplus {}^1E \oplus {}^2E) \oplus (2n+2)\,B$
$4n+3$	$(2n+2)(A \oplus {}^1E \oplus {}^2E) \oplus (2n+1)\,B$
$4n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus 2n\,({}^1E_{3/2} \oplus {}^2E_{3/2})$
$4n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2})$
$4n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus (2n+2)({}^1E_{3/2} \oplus {}^2E_{3/2})$
$4n+\frac{7}{2}$	$(2n+2)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2})$

$n = 0, 1, 2, \ldots$

T **13**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: $\mathbf{C}_3 \otimes \mathbf{C}_i$.

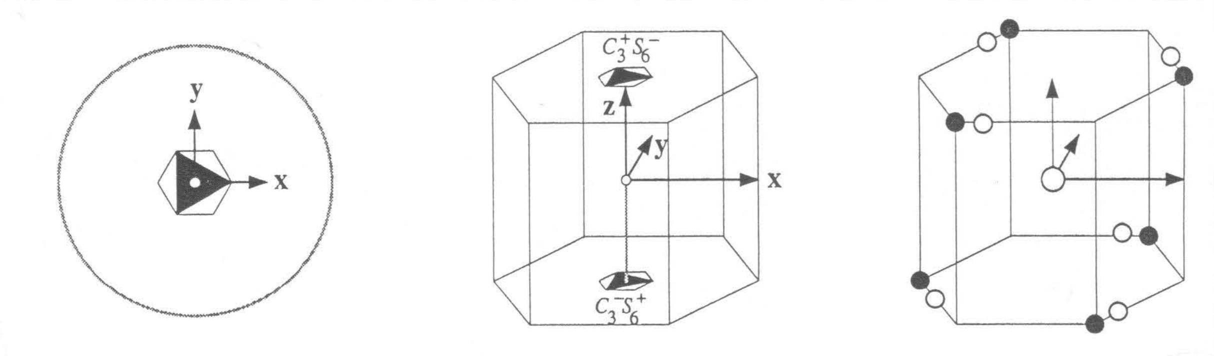
(2) Group chains: $\mathbf{T}_h \supset (\mathbf{S}_6) \supset \underline{\mathbf{C}_i}$, $\mathbf{T}_h \supset (\mathbf{S}_6) \supset \underline{\mathbf{C}_3}$, $\mathbf{C}_{6h} \supset \underline{\mathbf{S}_6} \supset \underline{\mathbf{C}_i}$, $\mathbf{C}_{6h} \supset \underline{\mathbf{S}_6} \supset \underline{\mathbf{C}_3}$,
$\mathbf{D}_{3d} \supset \underline{\mathbf{S}_6} \supset \underline{\mathbf{C}_i}$, $\mathbf{D}_{3d} \supset \underline{\mathbf{S}_6} \supset \underline{\mathbf{C}_3}$, $\mathbf{S}_{18} \supset \underline{\mathbf{S}_6} \supset \underline{\mathbf{C}_i}$, $\mathbf{S}_{18} \supset \underline{\mathbf{S}_6} \supset \underline{\mathbf{C}_3}$.

(3) Operations of G: E, C_3^+, C_3^-, i, S_6^-, S_6^+.

(4) Operations of $\widetilde{G}$: E, C_3^+, C_3^-, i, S_6^-, S_6^+,
$\widetilde{E}$, $\widetilde{C}_3^+$, $\widetilde{C}_3^-$, $\widetilde{\imath}$, $\widetilde{S}_6^-$, $\widetilde{S}_6^+$.

(5) Classes and representations: $|r| = 6$, $|\mathrm{i}| = 0$, $|I| = 6$, $|\widetilde{I}| = 6$.

F **14** See Chapter **15**, p. 65



Examples: Puckered C_6H_6 with the six H partly rotated (not the ground state of this molecule).

T **14**.1 Parameters

Use T **35**.1. § **16**–1, p. 68

T **14**.2 Multiplication table

Use T **35**.2. § **16**–2, p. 69

T **14**.3 Factor table

Use T **35**.3. § **16**–3, p. 70

T **14**.4 Character table

§ **16**–4, p. 71

$\mathbf{S}_6$	E	C_3^+	C_3^-	i	S_6^-	S_6^+	τ
A_g	1	1	1	1	1	1	a
${}^1E_g$	1	ϵ^*	ϵ	1	ϵ^*	ϵ	b
${}^2E_g$	1	ϵ	ϵ^*	1	ϵ	ϵ^*	b
A_u	1	1	1	-1	-1	-1	a
${}^1E_u$	1	ϵ^*	ϵ	-1	$-\epsilon^*$	$-\epsilon$	b
${}^2E_u$	1	ϵ	ϵ^*	-1	$-\epsilon$	$-\epsilon^*$	b
${}^1E_{1/2,g}$	1	$-\epsilon^*$	$-\epsilon$	1	$-\epsilon^*$	$-\epsilon$	b
${}^2E_{1/2,g}$	1	$-\epsilon$	$-\epsilon^*$	1	$-\epsilon$	$-\epsilon^*$	b
$A_{3/2,g}$	1	-1	-1	1	-1	-1	a
${}^1E_{1/2,u}$	1	$-\epsilon^*$	$-\epsilon$	-1	ϵ^*	ϵ	b
${}^2E_{1/2,u}$	1	$-\epsilon$	$-\epsilon^*$	-1	ϵ	ϵ^*	b
$A_{3/2,u}$	1	-1	-1	-1	1	1	a

$\epsilon = \exp(2\pi\mathrm{i}/3)$

T **14**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{S}_6$	0	1	2	3
A_g	$^{\square}1$	R_z	$x^2+y^2, {}^{\square}z^2$	
$^1E_g \oplus {}^2E_g$		(R_x, R_y)	$^{\square}(xy, x^2-y^2), {}^{\square}(zx, yz)$	
A_u		$^{\square}z$		$^{\square}x(x^2-3y^2), {}^{\square}y(3x^2-y^2), (x^2+y^2)z, {}^{\square}z^3$
$^1E_u \oplus {}^2E_u$		$^{\square}(x,y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^{\square}(xz^2, yz^2),$ $^{\square}\{xyz, z(x^2-y^2)\}$

T **14**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{S}_6$	$\lvert j\, m\rangle$	ι	μ
A_g	$\lvert 0\, 0\rangle$	2	± 3
^1E_g	$\lvert 2\, \overline{2}\rangle$	2	± 3
^2E_g	$\lvert 2\, 2\rangle$	2	± 3
A_u	$\lvert 1\, 0\rangle$	2	± 3
^1E_u	$\lvert 1\, 1\rangle$	2	± 3
^2E_u	$\lvert 1\, \overline{1}\rangle$	2	± 3
$^1E_{1/2,g}$	$\lvert \frac{1}{2}\, \overline{\frac{1}{2}}\rangle$	1	± 3
$^2E_{1/2,g}$	$\lvert \frac{1}{2}\, \frac{1}{2}\rangle$	1	± 3
$A_{3/2,g}$	$\lvert \frac{3}{2}\, \frac{3}{2}\rangle$	1	± 3
$^1E_{1/2,u}$	$\lvert \frac{1}{2}\, \overline{\frac{1}{2}}\rangle^{\bullet}$	1	± 3
$^2E_{1/2,u}$	$\lvert \frac{1}{2}\, \frac{1}{2}\rangle^{\bullet}$	1	± 3
$A_{3/2,u}$	$\lvert \frac{3}{2}\, \frac{3}{2}\rangle^{\bullet}$	1	± 3

T **14**.7 Matrix representations

Use T **14**.4 ♠. § **16**–7, p. 77

T **14**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{S}_6$	A_g	^1E_g	^2E_g	A_u	^1E_u	^2E_u	$^1E_{1/2,g}$	$^2E_{1/2,g}$	$A_{3/2,g}$	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$A_{3/2,u}$
A_g	A_g	^1E_g	^2E_g	A_u	^1E_u	^2E_u	$^1E_{1/2,g}$	$^2E_{1/2,g}$	$A_{3/2,g}$	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$A_{3/2,u}$
^1E_g		^2E_g	A_g	^1E_u	^2E_u	A_u	$^2E_{1/2,g}$	$A_{3/2,g}$	$^1E_{1/2,g}$	$^2E_{1/2,u}$	$A_{3/2,u}$	$^1E_{1/2,u}$
^2E_g			^1E_g	^2E_u	A_u	^1E_u	$A_{3/2,g}$	$^1E_{1/2,g}$	$^2E_{1/2,g}$	$A_{3/2,u}$	$^1E_{1/2,u}$	$^2E_{1/2,u}$
A_u				A_g	^1E_g	^2E_g	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$A_{3/2,u}$	$^1E_{1/2,g}$	$^2E_{1/2,g}$	$A_{3/2,g}$
^1E_u					^2E_g	A_g	$^2E_{1/2,u}$	$A_{3/2,u}$	$^1E_{1/2,u}$	$^2E_{1/2,g}$	$A_{3/2,g}$	$^1E_{1/2,g}$
^2E_u						^1E_g	$A_{3/2,u}$	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$A_{3/2,g}$	$^1E_{1/2,g}$	$^2E_{1/2,g}$
$^1E_{1/2,g}$							^2E_g	A_g	^1E_g	^2E_u	A_u	^1E_u
$^2E_{1/2,g}$								^1E_g	^2E_g	A_u	^1E_u	^2E_u
$A_{3/2,g}$									A_g	^1E_u	^2E_u	A_u
$^1E_{1/2,u}$										^2E_g	A_g	^1E_g
$^2E_{1/2,u}$											^1E_g	^2E_g
$A_{3/2,u}$												A_g

T **14**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{S}_6$	$\mathbf{C}_i$	$\mathbf{C}_3$
A_g	A_g	A
${}^1E_g$	A_g	${}^1E$
${}^2E_g$	A_g	${}^2E$
A_u	A_u	A
${}^1E_u$	A_u	${}^1E$
${}^2E_u$	A_u	${}^2E$
${}^1E_{1/2,g}$	$A_{1/2,g}$	${}^1E_{1/2}$
${}^2E_{1/2,g}$	$A_{1/2,g}$	${}^2E_{1/2}$
$A_{3/2,g}$	$A_{1/2,g}$	$A_{3/2}$
${}^1E_{1/2,u}$	$A_{1/2,u}$	${}^1E_{1/2}$
${}^2E_{1/2,u}$	$A_{1/2,u}$	${}^2E_{1/2}$
$A_{3/2,u}$	$A_{1/2,u}$	$A_{3/2}$

T **14**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{S}_6$
$6n$	$(4n+1)\,A_g \oplus 4n\,({}^1E_g \oplus {}^2E_g)$
$6n+1$	$(4n+1)(A_u \oplus {}^1E_u \oplus {}^2E_u)$
$6n+2$	$(4n+1)\,A_g \oplus (4n+2)({}^1E_g \oplus {}^2E_g)$
$6n+3$	$(4n+3)\,A_u \oplus (4n+2)({}^1E_u \oplus {}^2E_u)$
$6n+4$	$(4n+3)(A_g \oplus {}^1E_g \oplus {}^2E_g)$
$6n+5$	$(4n+3)\,A_u \oplus (4n+4)({}^1E_u \oplus {}^2E_u)$
$3n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus 2n\,A_{3/2,g}$
$3n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus (2n+2)\,A_{3/2,g}$
$3n+\frac{5}{2}$	$(2n+2)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus A_{3/2,g})$

$n = 0, 1, 2, \ldots$

T **14**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: none.

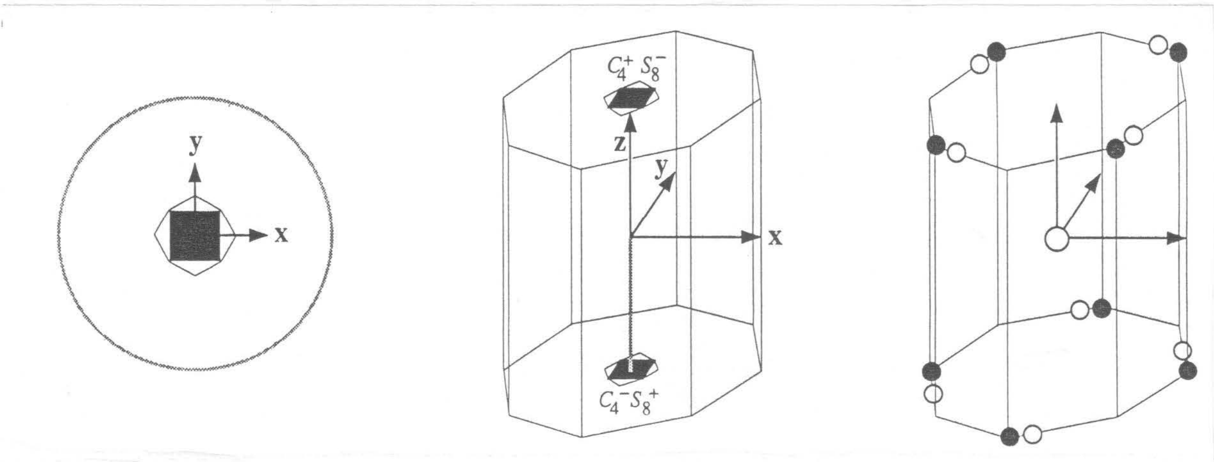
(2) Group chains: $\mathbf{C}_{8h} \supset \underline{\mathbf{S}_8} \supset \underline{\mathbf{C}_4}$, $\mathbf{D}_{4d} \supset \underline{\mathbf{S}_8} \supset \underline{\mathbf{C}_4}$.

(3) Operations of G: E, S_8^{3-}, C_4^+, S_8^-, C_2, S_8^+, C_4^-, S_8^{3+}.

(4) Operations of $\widetilde{G}$: E, S_8^{3-}, C_4^+, S_8^-, C_2, S_8^+, C_4^-, S_8^{3+},
$\widetilde{E}$, $\widetilde{S}_8^{3-}$, $\widetilde{C}_4^+$, $\widetilde{S}_8^-$, $\widetilde{C}_2$, $\widetilde{S}_8^+$, $\widetilde{C}_4^-$, $\widetilde{S}_8^{3+}$.

(5) Classes and representations: $|r| = 8$, $|\mathrm{i}| = 0$, $|I| = 8$, $|\widetilde{I}| = 8$.

F **15** See Chapter **15**, p. 65



Examples:

T **15**.1 Parameters
Use T **37**.1. § **16**–1, p. 68

T **15**.2 Multiplication table
Use T **37**.2. § **16**–2, p. 69

T **15**.3 Factor table
Use T **37**.3. § **16**–3, p. 70

T **15**.4 Character table

§ **16**–4, p. 71

$\mathbf{S}_8$	E	S_8^{3-}	C_4^+	S_8^-	C_2	S_8^+	C_4^-	S_8^{3+}	τ
A	1	1	1	1	1	1	1	1	a
B	1	-1	1	-1	1	-1	1	-1	a
${}^1E_1$	1	$-\epsilon^*$	$-\mathrm{i}$	ϵ	-1	ϵ^*	i	$-\epsilon$	b
${}^2E_1$	1	$-\epsilon$	i	ϵ^*	-1	ϵ	$-\mathrm{i}$	$-\epsilon^*$	b
${}^1E_2$	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	-1	i	b
${}^2E_2$	1	i	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	b
${}^1E_3$	1	ϵ^*	$-\mathrm{i}$	$-\epsilon$	-1	$-\epsilon^*$	i	ϵ	b
${}^2E_3$	1	ϵ	i	$-\epsilon^*$	-1	$-\epsilon$	$-\mathrm{i}$	ϵ^*	b
${}^1E_{1/2}$	1	δ	ϵ	$\mathrm{i}\delta^*$	i	$-\mathrm{i}\delta$	ϵ^*	δ^*	b
${}^2E_{1/2}$	1	δ^*	ϵ^*	$-\mathrm{i}\delta$	$-\mathrm{i}$	$\mathrm{i}\delta^*$	ϵ	δ	b
${}^1E_{3/2}$	1	$-\mathrm{i}\delta$	$-\epsilon$	$-\delta^*$	i	$-\delta$	$-\epsilon^*$	$\mathrm{i}\delta^*$	b
${}^2E_{3/2}$	1	$\mathrm{i}\delta^*$	$-\epsilon^*$	$-\delta$	$-\mathrm{i}$	$-\delta^*$	$-\epsilon$	$-\mathrm{i}\delta$	b
${}^1E_{5/2}$	1	$\mathrm{i}\delta$	$-\epsilon$	δ^*	i	δ	$-\epsilon^*$	$-\mathrm{i}\delta^*$	b
${}^2E_{5/2}$	1	$-\mathrm{i}\delta^*$	$-\epsilon^*$	δ	$-\mathrm{i}$	δ^*	$-\epsilon$	$\mathrm{i}\delta$	b
${}^1E_{7/2}$	1	$-\delta$	ϵ	$-\mathrm{i}\delta^*$	i	$\mathrm{i}\delta$	ϵ^*	$-\delta^*$	b
${}^2E_{7/2}$	1	$-\delta^*$	ϵ^*	$\mathrm{i}\delta$	$-\mathrm{i}$	$-\mathrm{i}\delta^*$	ϵ	$-\delta$	b

$\delta = \exp(2\pi\mathrm{i}/16)$, $\epsilon = \exp(2\pi\mathrm{i}/8)$

T **15**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{S}_8$	0	1	2	3
A	$^\square 1$	R_z	$x^2+y^2, {}^\square z^2$	
B		$^\square z$		$(x^2+y^2)z, {}^\square z^3$
${}^1E_1 \oplus {}^2E_1$		$^\square(x,y), (R_x,R_y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
${}^1E_2 \oplus {}^2E_2$			$^\square(xy, x^2-y^2)$	$^\square\{xyz, z(x^2-y^2)\}$
${}^1E_3 \oplus {}^2E_3$			$^\square(zx, yz)$	$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$

T **15**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{S}_8$	$\vert j\,m\rangle$		ι	μ
A	$\vert 0\,0\rangle$	$\vert 5\,4\rangle$	2	± 8
B	$\vert 1\,0\rangle$	$\vert 4\,4\rangle$	2	± 8
${}^1E_1$	$\vert 1\,1\rangle$	$\vert 4\,\overline{3}\rangle$	2	± 8
${}^2E_1$	$\vert 1\,\overline{1}\rangle$	$\vert 4\,3\rangle$	2	± 8
${}^1E_2$	$\vert 2\,2\rangle$	$\vert 3\,\overline{2}\rangle$	2	± 8
${}^2E_2$	$\vert 2\,\overline{2}\rangle$	$\vert 3\,2\rangle$	2	± 8
${}^1E_3$	$\vert 2\,1\rangle$	$\vert 3\,\overline{3}\rangle$	2	± 8
${}^2E_3$	$\vert 2\,\overline{1}\rangle$	$\vert 3\,3\rangle$	2	± 8
${}^1E_{1/2}$	$\vert \frac12\,\overline{\frac12}\rangle$	$\vert \frac72\,\frac72\rangle^\bullet$	1	± 8
${}^2E_{1/2}$	$\vert \frac12\,\frac12\rangle$	$\vert \frac72\,\overline{\frac72}\rangle^\bullet$	1	± 8
${}^1E_{3/2}$	$\vert \frac32\,\frac32\rangle$	$\vert \frac52\,\overline{\frac52}\rangle^\bullet$	1	± 8
${}^2E_{3/2}$	$\vert \frac32\,\overline{\frac32}\rangle$	$\vert \frac52\,\frac52\rangle^\bullet$	1	± 8
${}^1E_{5/2}$	$\vert \frac52\,\overline{\frac52}\rangle$	$\vert \frac32\,\frac32\rangle^\bullet$	1	± 8
${}^2E_{5/2}$	$\vert \frac52\,\frac52\rangle$	$\vert \frac32\,\overline{\frac32}\rangle^\bullet$	1	± 8
${}^1E_{7/2}$	$\vert \frac72\,\frac72\rangle$	$\vert \frac12\,\overline{\frac12}\rangle^\bullet$	1	± 8
${}^2E_{7/2}$	$\vert \frac72\,\overline{\frac72}\rangle$	$\vert \frac12\,\frac12\rangle^\bullet$	1	± 8

T **15**.7 Matrix representations

Use T **15**.4 ♠. § **16**–7, p. 77

T **15**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{S}_8$	A	B	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$
A	A	B	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$
B		A	${}^1E_3$	${}^2E_3$	${}^2E_2$	${}^1E_2$	${}^1E_1$	${}^2E_1$
${}^1E_1$			${}^1E_2$	A	${}^2E_3$	${}^2E_1$	${}^2E_2$	B
${}^2E_1$				${}^2E_2$	${}^1E_1$	${}^1E_3$	B	${}^1E_2$
${}^1E_2$					B	A	${}^2E_1$	${}^1E_3$
${}^2E_2$						B	${}^2E_3$	${}^1E_1$
${}^1E_3$							${}^1E_2$	A
${}^2E_3$								${}^2E_2$

$\twoheadrightarrow$

T **15**.8 Direct products of representations *(cont.)*

$\mathbf{S}_8$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$
A	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$
B	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_1$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$
${}^2E_1$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^1E_2$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$
${}^2E_2$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{7/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$
${}^1E_3$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$
${}^2E_3$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$
${}^1E_{1/2}$	${}^2E_3$	A	${}^1E_3$	${}^2E_2$	${}^1E_1$	${}^1E_2$	${}^2E_1$	B
${}^2E_{1/2}$		${}^1E_3$	${}^1E_2$	${}^2E_3$	${}^2E_2$	${}^2E_1$	B	${}^1E_1$
${}^1E_{3/2}$			${}^2E_1$	A	${}^2E_3$	B	${}^1E_1$	${}^2E_2$
${}^2E_{3/2}$				${}^1E_1$	B	${}^1E_3$	${}^1E_2$	${}^2E_1$
${}^1E_{5/2}$					${}^2E_1$	A	${}^1E_3$	${}^1E_2$
${}^2E_{5/2}$						${}^1E_1$	${}^2E_2$	${}^2E_3$
${}^1E_{7/2}$							${}^2E_3$	A
${}^2E_{7/2}$								${}^1E_3$

T **15**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{S}_8$	$\mathbf{C}_4$	$\mathbf{C}_2$
A	A	A
B	A	A
${}^1E_1$	${}^1E$	B
${}^2E_1$	${}^2E$	B
${}^1E_2$	B	A
${}^2E_2$	B	A
${}^1E_3$	${}^1E$	B
${}^2E_3$	${}^2E$	B
${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_{3/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_{7/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$

T **15**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{S}_8$
$8n$	$(2n+1)\,A \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3)$
$8n+1$	$2n\,(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1)$
$8n+2$	$(2n+1)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3) \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1)$
$8n+3$	$2n\,A \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3)$
$8n+4$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3) \oplus (2n+2)\,B$
$8n+5$	$(2n+2)(A \oplus {}^1E_3 \oplus {}^2E_3) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2)$
$8n+6$	$(2n+1)(A \oplus {}^1E_3 \oplus {}^2E_3) \oplus (2n+2)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2)$
$8n+7$	$(2n+2)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3) \oplus (2n+1)\,B$
$8n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus 2n\,({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$8n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus 2n\,({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$8n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus 2n\,({}^1E_{7/2} \oplus {}^2E_{7/2})$
$8n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$8n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus (2n+2)({}^1E_{7/2} \oplus {}^2E_{7/2})$
$8n+\frac{11}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus (2n+2)({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$8n+\frac{13}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus (2n+2)({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$8n+\frac{15}{2}$	$(2n+2)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$

$n = 0, 1, 2, \ldots$

T **15**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: $\mathbf{C}_5 \otimes \mathbf{C}_i$.

(2) Group chains: $\mathbf{C}_{10h} \supset \underline{\mathbf{S}_{10}} \supset \underline{\mathbf{C}_i}$, $\mathbf{C}_{10h} \supset \underline{\mathbf{S}_{10}} \supset \underline{\mathbf{C}_5}$, $\mathbf{D}_{5d} \supset \underline{\mathbf{S}_{10}} \supset \underline{\mathbf{C}_i}$, $\mathbf{D}_{5d} \supset \underline{\mathbf{S}_{10}} \supset \underline{\mathbf{C}_5}$.

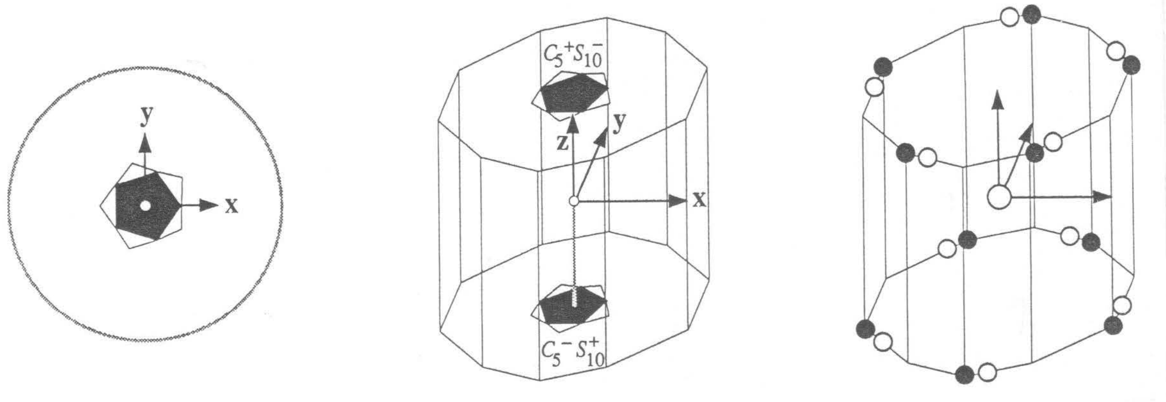
(3) Operations of G: E, C_5^+, C_5^{2+}, C_5^{2-}, C_5^-, i, S_{10}^{3-}, S_{10}^-, S_{10}^+, S_{10}^{3+}.

(4) Operations of $\widetilde{G}$: E, C_5^+, C_5^{2+}, C_5^{2-}, C_5^-, i, S_{10}^{3-}, S_{10}^-, S_{10}^+, S_{10}^{3+},
$\widetilde{E}$, $\widetilde{C}_5^+$, $\widetilde{C}_5^{2+}$, $\widetilde{C}_5^{2-}$, $\widetilde{C}_5^-$, $\widetilde{i}$, $\widetilde{S}_{10}^{3-}$, $\widetilde{S}_{10}^-$, $\widetilde{S}_{10}^+$, $\widetilde{S}_{10}^{3+}$.

(5) Classes and representations: $|r| = 10$, $|i| = 0$, $|I| = 10$, $|\widetilde{I}| = 10$.

F **16**

See Chapter **15**, p. 65



Examples:

T **16**.1 Parameters
Use T **39**.1. § **16**–1, p. 68

T **16**.2 Multiplication table
Use T **39**.2. § **16**–2, p. 69

T **16**.3 Factor table
Use T **39**.3. § **16**–3, p. 70

T **16**.4 Character table

§ **16**–4, p. 71

$\mathbf{S}_{10}$	E	C_5^+	C_5^{2+}	C_5^{2-}	C_5^-	i	S_{10}^{3-}	S_{10}^-	S_{10}^+	S_{10}^{3+}	τ
A_g	1	1	1	1	1	1	1	1	1	1	a
${}^1E_{1g}$	1	δ^*	ϵ^*	ϵ	δ	1	δ^*	ϵ^*	ϵ	δ	b
${}^2E_{1g}$	1	δ	ϵ	ϵ^*	δ^*	1	δ	ϵ	ϵ^*	δ^*	b
${}^1E_{2g}$	1	ϵ^*	δ	δ^*	ϵ	1	ϵ^*	δ	δ^*	ϵ	b
${}^2E_{2g}$	1	ϵ	δ^*	δ	ϵ^*	1	ϵ	δ^*	δ	ϵ^*	b
A_u	1	1	1	1	1	-1	-1	-1	-1	-1	a
${}^1E_{1u}$	1	δ^*	ϵ^*	ϵ	δ	-1	$-\delta^*$	$-\epsilon^*$	$-\epsilon$	$-\delta$	b
${}^2E_{1u}$	1	δ	ϵ	ϵ^*	δ^*	-1	$-\delta$	$-\epsilon$	$-\epsilon^*$	$-\delta^*$	b
${}^1E_{2u}$	1	ϵ^*	δ	δ^*	ϵ	-1	$-\epsilon^*$	$-\delta$	$-\delta^*$	$-\epsilon$	b
${}^2E_{2u}$	1	ϵ	δ^*	δ	ϵ^*	-1	$-\epsilon$	$-\delta^*$	$-\delta$	$-\epsilon^*$	b
${}^1E_{1/2,g}$	1	$-\epsilon^*$	δ	δ^*	$-\epsilon$	1	$-\epsilon^*$	δ	δ^*	$-\epsilon$	b
${}^2E_{1/2,g}$	1	$-\epsilon$	δ^*	δ	$-\epsilon^*$	1	$-\epsilon$	δ^*	δ	$-\epsilon^*$	b
${}^1E_{3/2,g}$	1	$-\delta$	ϵ	ϵ^*	$-\delta^*$	1	$-\delta$	ϵ	ϵ^*	$-\delta^*$	b
${}^2E_{3/2,g}$	1	$-\delta^*$	ϵ^*	ϵ	$-\delta$	1	$-\delta^*$	ϵ^*	ϵ	$-\delta$	b
$A_{5/2,g}$	1	-1	1	1	-1	1	-1	1	1	-1	a
${}^1E_{1/2,u}$	1	$-\epsilon^*$	δ	δ^*	$-\epsilon$	-1	ϵ^*	$-\delta$	$-\delta^*$	ϵ	b
${}^2E_{1/2,u}$	1	$-\epsilon$	δ^*	δ	$-\epsilon^*$	-1	ϵ	$-\delta^*$	$-\delta$	ϵ^*	b
${}^1E_{3/2,u}$	1	$-\delta$	ϵ	ϵ^*	$-\delta^*$	-1	δ	$-\epsilon$	$-\epsilon^*$	δ^*	b
${}^2E_{3/2,u}$	1	$-\delta^*$	ϵ^*	ϵ	$-\delta$	-1	δ^*	$-\epsilon^*$	$-\epsilon$	δ	b
$A_{5/2,u}$	1	-1	1	1	-1	-1	1	-1	-1	1	a

$\delta = \exp(2\pi i/5)$, $\epsilon = \exp(4\pi i/5)$

T **16**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{S}_{10}$	0	1	2	3
A_g	$^\square 1$	R_z	$x^2+y^2, {}^\square z^2$	
${}^1E_{1g} \oplus {}^2E_{1g}$		(R_x, R_y)	$^\square(zx, yz)$	
${}^1E_{2g} \oplus {}^2E_{2g}$			$^\square(xy, x^2-y^2)$	
A_u		$^\square z$		$(x^2+y^2)z, {}^\square z^3$
${}^1E_{1u} \oplus {}^2E_{1u}$		$^\square(x, y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
${}^1E_{2u} \oplus {}^2E_{2u}$				$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}, {}^\square\{xyz, z(x^2-y^2)\}$

T **16**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{S}_{10}$	$\|j\,m\rangle$	ι	μ	$\mathbf{S}_{10}$	$\|j\,m\rangle$	ι	μ
A_g	$\|0\,0\rangle$	2	±5	${}^1E_{1/2,g}$	$\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	1	±5
${}^1E_{1g}$	$\|2\,1\rangle$	2	±5	${}^2E_{1/2,g}$	$\|\frac{1}{2}\,\frac{1}{2}\rangle$	1	±5
${}^2E_{1g}$	$\|2\,\overline{1}\rangle$	2	±5	${}^1E_{3/2,g}$	$\|\frac{3}{2}\,\frac{3}{2}\rangle$	1	±5
${}^1E_{2g}$	$\|2\,2\rangle$	2	±5	${}^2E_{3/2,g}$	$\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	1	±5
${}^2E_{2g}$	$\|2\,\overline{2}\rangle$	2	±5	$A_{5/2,g}$	$\|\frac{5}{2}\,\frac{5}{2}\rangle$	1	±5
A_u	$\|1\,0\rangle$	2	±5	${}^1E_{1/2,u}$	$\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle^\bullet$	1	±5
${}^1E_{1u}$	$\|1\,1\rangle$	2	±5	${}^2E_{1/2,u}$	$\|\frac{1}{2}\,\frac{1}{2}\rangle^\bullet$	1	±5
${}^2E_{1u}$	$\|1\,\overline{1}\rangle$	2	±5	${}^1E_{3/2,u}$	$\|\frac{3}{2}\,\frac{3}{2}\rangle^\bullet$	1	±5
${}^1E_{2u}$	$\|3\,2\rangle$	2	±5	${}^2E_{3/2,u}$	$\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle^\bullet$	1	±5
${}^2E_{2u}$	$\|3\,\overline{2}\rangle$	2	±5	$A_{5/2,u}$	$\|\frac{5}{2}\,\frac{5}{2}\rangle^\bullet$	1	±5

T **16**.7 Matrix representations

Use T **16**.4 ♠. § **16**–7, p. 77

T **16**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{S}_{10}$	A_g	${}^1E_{1g}$	${}^2E_{1g}$	${}^1E_{2g}$	${}^2E_{2g}$	A_u	${}^1E_{1u}$	${}^2E_{1u}$	${}^1E_{2u}$	${}^2E_{2u}$
A_g	A_g	${}^1E_{1g}$	${}^2E_{1g}$	${}^1E_{2g}$	${}^2E_{2g}$	A_u	${}^1E_{1u}$	${}^2E_{1u}$	${}^1E_{2u}$	${}^2E_{2u}$
${}^1E_{1g}$		${}^1E_{2g}$	A_g	${}^2E_{2g}$	${}^2E_{1g}$	${}^1E_{1u}$	${}^1E_{2u}$	A_u	${}^2E_{2u}$	${}^2E_{1u}$
${}^2E_{1g}$			${}^2E_{2g}$	${}^1E_{1g}$	${}^1E_{2g}$	${}^2E_{1u}$	A_u	${}^2E_{2u}$	${}^1E_{1u}$	${}^1E_{2u}$
${}^1E_{2g}$				${}^2E_{1g}$	A_g	${}^1E_{2u}$	${}^2E_{2u}$	${}^1E_{1u}$	${}^2E_{1u}$	A_u
${}^2E_{2g}$					${}^1E_{1g}$	${}^2E_{2u}$	${}^2E_{1u}$	${}^1E_{2u}$	A_u	${}^1E_{1u}$
A_u						A_g	${}^1E_{1g}$	${}^2E_{1g}$	${}^1E_{2g}$	${}^2E_{2g}$
${}^1E_{1u}$							${}^1E_{2g}$	A_g	${}^2E_{2g}$	${}^2E_{1g}$
${}^2E_{1u}$								${}^2E_{2g}$	${}^1E_{1g}$	${}^1E_{2g}$
${}^1E_{2u}$									${}^2E_{1g}$	A_g
${}^2E_{2u}$										${}^1E_{1g}$

⇛

T **16**.8 Direct products of representations *(cont.)*

$\mathbf{S}_{10}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	$A_{5/2,g}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	$A_{5/2,u}$
A_g	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	$A_{5/2,g}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	$A_{5/2,u}$
${}^1E_{1g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	$A_{5/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	$A_{5/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$
${}^2E_{1g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	$A_{5/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	$A_{5/2,u}$	${}^1E_{3/2,u}$
${}^1E_{2g}$	${}^1E_{3/2,g}$	$A_{5/2,g}$	${}^2E_{3/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^1E_{3/2,u}$	$A_{5/2,u}$	${}^2E_{3/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$
${}^2E_{2g}$	$A_{5/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$	$A_{5/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$
A_u	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	$A_{5/2,u}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	$A_{5/2,g}$
${}^1E_{1u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	$A_{5/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	$A_{5/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$
${}^2E_{1u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	$A_{5/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	$A_{5/2,g}$	${}^1E_{3/2,g}$
${}^1E_{2u}$	${}^1E_{3/2,u}$	$A_{5/2,u}$	${}^2E_{3/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^1E_{3/2,g}$	$A_{5/2,g}$	${}^2E_{3/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$
${}^2E_{2u}$	$A_{5/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$	$A_{5/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$
${}^1E_{1/2,g}$	${}^2E_{1g}$	A_g	${}^1E_{1g}$	${}^2E_{2g}$	${}^1E_{2g}$	${}^2E_{1u}$	A_u	${}^1E_{1u}$	${}^2E_{2u}$	${}^1E_{2u}$
${}^2E_{1/2,g}$		${}^1E_{1g}$	${}^1E_{2g}$	${}^2E_{1g}$	${}^2E_{2g}$	A_u	${}^1E_{1u}$	${}^1E_{2u}$	${}^2E_{1u}$	${}^2E_{2u}$
${}^1E_{3/2,g}$			${}^2E_{2g}$	A_g	${}^2E_{1g}$	${}^1E_{1u}$	${}^1E_{2u}$	${}^2E_{2u}$	A_u	${}^2E_{1u}$
${}^2E_{3/2,g}$				${}^1E_{2g}$	${}^1E_{1g}$	${}^2E_{2u}$	${}^2E_{1u}$	A_u	${}^1E_{2u}$	${}^1E_{1u}$
$A_{5/2,g}$					A_g	${}^1E_{2u}$	${}^2E_{2u}$	${}^2E_{1u}$	${}^1E_{1u}$	A_u
${}^1E_{1/2,u}$						${}^2E_{1g}$	A_g	${}^1E_{1g}$	${}^2E_{2g}$	${}^1E_{2g}$
${}^2E_{1/2,u}$							${}^1E_{1g}$	${}^1E_{2g}$	${}^2E_{1g}$	${}^2E_{2g}$
${}^1E_{3/2,u}$								${}^2E_{2g}$	A_g	${}^2E_{1g}$
${}^2E_{3/2,u}$									${}^1E_{2g}$	${}^1E_{1g}$
$A_{5/2,u}$										A_g

T **16**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{S}_{10}$	$\mathbf{C}_i$	$\mathbf{C}_5$
A_g	A_g	A
${}^1E_{1g}$	A_g	${}^1E_1$
${}^2E_{1g}$	A_g	${}^2E_1$
${}^1E_{2g}$	A_g	${}^1E_2$
${}^2E_{2g}$	A_g	${}^2E_2$
A_u	A_u	A
${}^1E_{1u}$	A_u	${}^1E_1$
${}^2E_{1u}$	A_u	${}^2E_1$
${}^1E_{2u}$	A_u	${}^1E_2$
${}^2E_{2u}$	A_u	${}^2E_2$
${}^1E_{1/2,g}$	$A_{1/2,g}$	${}^1E_{1/2}$
${}^2E_{1/2,g}$	$A_{1/2,g}$	${}^2E_{1/2}$
${}^1E_{3/2,g}$	$A_{1/2,g}$	${}^1E_{3/2}$
${}^2E_{3/2,g}$	$A_{1/2,g}$	${}^2E_{3/2}$
$A_{5/2,g}$	$A_{1/2,g}$	$A_{5/2}$
${}^1E_{1/2,u}$	$A_{1/2,u}$	${}^1E_{1/2}$
${}^2E_{1/2,u}$	$A_{1/2,u}$	${}^2E_{1/2}$
${}^1E_{3/2,u}$	$A_{1/2,u}$	${}^1E_{3/2}$
${}^2E_{3/2,u}$	$A_{1/2,u}$	${}^2E_{3/2}$
$A_{5/2,u}$	$A_{1/2,u}$	$A_{5/2}$

T **16**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{S}_{10}$
$10n$	$(4n+1)\,A_g \oplus 4n\,({}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g})$
$10n+1$	$(4n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u}) \oplus 4n\,({}^1E_{2u} \oplus {}^2E_{2u})$
$10n+2$	$(4n+1)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g})$
$10n+3$	$(4n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u}) \oplus (4n+2)({}^1E_{2u} \oplus {}^2E_{2u})$
$10n+4$	$(4n+1)\,A_g \oplus (4n+2)({}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g})$
$10n+5$	$(4n+3)\,A_u \oplus (4n+2)({}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u})$
$10n+6$	$(4n+3)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g}) \oplus (4n+2)({}^1E_{2g} \oplus {}^2E_{2g})$
$10n+7$	$(4n+3)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u})$
$10n+8$	$(4n+3)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g}) \oplus (4n+4)({}^1E_{2g} \oplus {}^2E_{2g})$
$10n+9$	$(4n+3)\,A_u \oplus (4n+4)({}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u})$
$5n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus 2n\,({}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus A_{5/2,g})$
$5n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}) \oplus 2n\,A_{5/2,g}$
$5n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}) \oplus (2n+2)\,A_{5/2,g}$
$5n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus (2n+2)({}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus A_{5/2,g})$
$5n+\frac{9}{2}$	$(2n+2)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus A_{5/2,g})$

$n = 0, 1, 2, \ldots$

T **16**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: none.

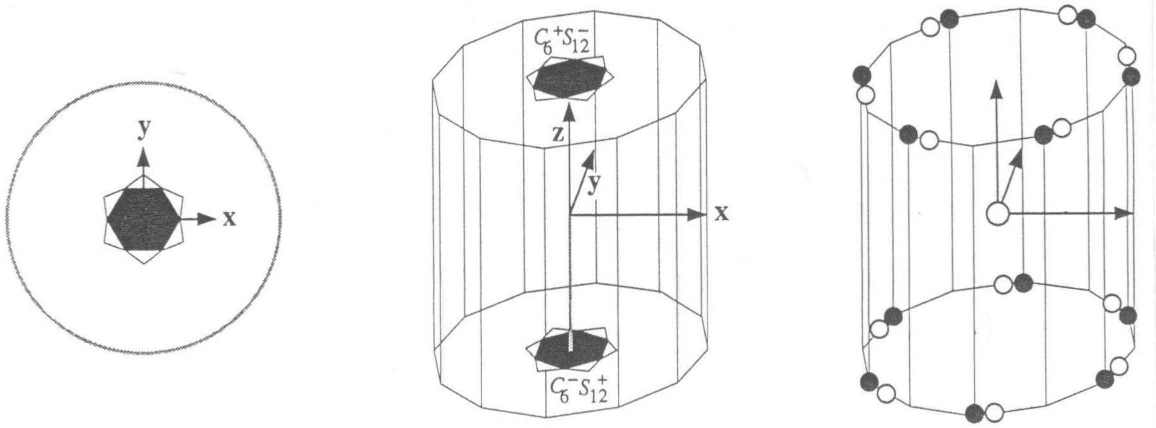
(2) Group chains: $\mathbf{D}_{6d} \supset \mathbf{S}_{12} \supset \mathbf{S}_4$, $\mathbf{D}_{6d} \supset \mathbf{S}_{12} \supset \mathbf{C}_6$.

(3) Operations of G: E, S_{12}^{5-}, C_6^+, S_4^-, C_3^+, S_{12}^-, C_2, S_{12}^+, C_3^-, S_4^+, C_6^-, S_{12}^{5+}.

(4) Operations of $\widetilde{G}$: E, S_{12}^{5-}, C_6^+, S_4^-, C_3^+, S_{12}^-, C_2, S_{12}^+, C_3^-, S_4^+, C_6^-, S_{12}^{5+},
$\widetilde{E}$, $\widetilde{S}_{12}^{5-}$, $\widetilde{C}_6^+$, $\widetilde{S}_4^-$, $\widetilde{C}_3^+$, $\widetilde{S}_{12}^-$, $\widetilde{C}_2$, $\widetilde{S}_{12}^+$, $\widetilde{C}_3^-$, $\widetilde{S}_4^+$, $\widetilde{C}_6^-$, $\widetilde{S}_{12}^{5+}$.

(5) Classes and representations: $|r| = 12$, $|\mathrm{i}| = 0$, $|I| = 12$, $|\widetilde{I}| = 12$.

F **17** See Chapter **15**, p. 65



Examples:

T **17**.1 Parameters
Use T **45**.1. § **16**–1, p. 68

T **17**.2 Multiplication table
Use T **45**.2. § **16**–2, p. 69

T **17**.3 Factor table
Use T **45**.3. § **16**–3, p. 70

T **17**.4 Character table

§ **16**–4, p. 71

$\mathbf{S}_{12}$	E	S_{12}^{5-}	C_6^+	S_4^-	C_3^+	S_{12}^-	C_2	S_{12}^+	C_3^-	S_4^+	C_6^-	S_{12}^{5+}	τ
A	1	1	1	1	1	1	1	1	1	1	1	1	a
B	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	a
${}^1E_1$	1	$-\mathrm{i}\eta^*$	$-\eta$	i	η^*	$-\mathrm{i}\eta$	-1	$\mathrm{i}\eta^*$	η	$-\mathrm{i}$	$-\eta^*$	$\mathrm{i}\eta$	b
${}^2E_1$	1	$\mathrm{i}\eta$	$-\eta^*$	$-\mathrm{i}$	η	$\mathrm{i}\eta^*$	-1	$-\mathrm{i}\eta$	η^*	i	$-\eta$	$-\mathrm{i}\eta^*$	b
${}^1E_2$	1	$-\eta^*$	η	-1	η^*	$-\eta$	1	$-\eta^*$	η	-1	η^*	$-\eta$	b
${}^2E_2$	1	$-\eta$	η^*	-1	η	$-\eta^*$	1	$-\eta$	η^*	-1	η	$-\eta^*$	b
${}^1E_3$	1	i	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	b
${}^2E_3$	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	-1	i	b
${}^1E_4$	1	η^*	η	1	η^*	η	1	η^*	η	1	η^*	η	b
${}^2E_4$	1	η	η^*	1	η	η^*	1	η	η^*	1	η	η^*	b
${}^1E_5$	1	$\mathrm{i}\eta^*$	$-\eta$	$-\mathrm{i}$	η^*	$\mathrm{i}\eta$	-1	$-\mathrm{i}\eta^*$	η	i	$-\eta^*$	$-\mathrm{i}\eta$	b
${}^2E_5$	1	$-\mathrm{i}\eta$	$-\eta^*$	i	η	$-\mathrm{i}\eta^*$	-1	$\mathrm{i}\eta$	η^*	$-\mathrm{i}$	$-\eta$	$\mathrm{i}\eta^*$	b
${}^1E_{1/2}$	1	δ	$-\mathrm{i}\eta$	ϵ	$-\eta^*$	$\mathrm{i}\delta^*$	i	$-\mathrm{i}\delta$	$-\eta$	ϵ^*	$\mathrm{i}\eta^*$	δ^*	b
${}^2E_{1/2}$	1	δ^*	$\mathrm{i}\eta^*$	ϵ^*	$-\eta$	$-\mathrm{i}\delta$	$-\mathrm{i}$	$\mathrm{i}\delta^*$	$-\eta^*$	ϵ	$-\mathrm{i}\eta$	δ	b
${}^1E_{3/2}$	1	ϵ^*	$-\mathrm{i}$	$-\epsilon$	-1	$-\epsilon^*$	i	$-\epsilon$	-1	$-\epsilon^*$	i	ϵ	b
${}^2E_{3/2}$	1	ϵ	i	$-\epsilon^*$	-1	$-\epsilon$	$-\mathrm{i}$	$-\epsilon^*$	-1	$-\epsilon$	$-\mathrm{i}$	ϵ^*	b
${}^1E_{5/2}$	1	$\mathrm{i}\delta^*$	$-\mathrm{i}\eta^*$	$-\epsilon$	$-\eta$	δ	i	δ^*	$-\eta^*$	$-\epsilon^*$	$\mathrm{i}\eta$	$-\mathrm{i}\delta$	b
${}^2E_{5/2}$	1	$-\mathrm{i}\delta$	$\mathrm{i}\eta$	$-\epsilon^*$	$-\eta^*$	δ^*	$-\mathrm{i}$	δ	$-\eta$	$-\epsilon$	$-\mathrm{i}\eta^*$	$\mathrm{i}\delta^*$	b
${}^1E_{7/2}$	1	$-\mathrm{i}\delta^*$	$-\mathrm{i}\eta^*$	ϵ	$-\eta$	$-\delta$	i	$-\delta^*$	$-\eta^*$	ϵ^*	$\mathrm{i}\eta$	$\mathrm{i}\delta$	b
${}^2E_{7/2}$	1	$\mathrm{i}\delta$	$\mathrm{i}\eta$	ϵ^*	$-\eta^*$	$-\delta^*$	$-\mathrm{i}$	$-\delta$	$-\eta$	ϵ	$-\mathrm{i}\eta^*$	$-\mathrm{i}\delta^*$	b
${}^1E_{9/2}$	1	$-\epsilon^*$	$-\mathrm{i}$	ϵ	-1	ϵ^*	i	ϵ	-1	ϵ^*	i	$-\epsilon$	b
${}^2E_{9/2}$	1	$-\epsilon$	i	ϵ^*	-1	ϵ	$-\mathrm{i}$	ϵ^*	-1	ϵ	$-\mathrm{i}$	$-\epsilon^*$	b
${}^1E_{11/2}$	1	$-\delta$	$-\mathrm{i}\eta$	$-\epsilon$	$-\eta^*$	$-\mathrm{i}\delta^*$	i	$\mathrm{i}\delta$	$-\eta$	$-\epsilon^*$	$\mathrm{i}\eta^*$	$-\delta^*$	b
${}^2E_{11/2}$	1	$-\delta^*$	$\mathrm{i}\eta^*$	$-\epsilon^*$	$-\eta$	$\mathrm{i}\delta$	$-\mathrm{i}$	$-\mathrm{i}\delta^*$	$-\eta^*$	$-\epsilon$	$-\mathrm{i}\eta$	$-\delta$	b

$\delta = \exp(2\pi\mathrm{i}/24)$, $\epsilon = \exp(2\pi\mathrm{i}/8)$, $\eta = \exp(2\pi\mathrm{i}/3)$

T **17**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{S}_{12}$	0	1	2	3
A	□1	R_z	x^2+y^2, □z^2	
B		□z		$(x^2+y^2)z$, □z^3
${}^1E_1 \oplus {}^2E_1$		□(x,y), (R_x,R_y)		$\{x(x^2+y^2), y(x^2+y^2)\}$, □$(xz^2, yz^2)$
${}^1E_2 \oplus {}^2E_2$			□(xy, x^2-y^2)	
${}^1E_3 \oplus {}^2E_3$				□$\{x(x^2-3y^2), y(3x^2-y^2)\}$
${}^1E_4 \oplus {}^2E_4$				□$\{xyz, z(x^2-y^2)\}$
${}^1E_5 \oplus {}^2E_5$			□(zx, yz)	

T **17**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{S}_{12}$	$\lvert j\,m\rangle$		ι	μ	$\mathbf{S}_{12}$	$\lvert j\,m\rangle$		ι	μ
A	$\lvert 0\,0\rangle$	$\lvert 7\,6\rangle$	2	± 12	${}^1E_{1/2}$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	$\lvert \frac{11}{2}\,\frac{11}{2}\rangle^\bullet$	1	± 12
B	$\lvert 1\,0\rangle$	$\lvert 6\,6\rangle$	2	± 12	${}^2E_{1/2}$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle$	$\lvert \frac{11}{2}\,\overline{\frac{11}{2}}\rangle^\bullet$	1	± 12
${}^1E_1$	$\lvert 1\,1\rangle$	$\lvert 6\,\bar{5}\rangle$	2	± 12	${}^1E_{3/2}$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle$	$\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle^\bullet$	1	± 12
${}^2E_1$	$\lvert 1\,\bar{1}\rangle$	$\lvert 6\,5\rangle$	2	± 12	${}^2E_{3/2}$	$\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle^\bullet$	1	± 12
${}^1E_2$	$\lvert 2\,\bar{2}\rangle$	$\lvert 5\,4\rangle$	2	± 12	${}^1E_{5/2}$	$\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle^\bullet$	1	± 12
${}^2E_2$	$\lvert 2\,2\rangle$	$\lvert 5\,\bar{4}\rangle$	2	± 12	${}^2E_{5/2}$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle$	$\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle^\bullet$	1	± 12
${}^1E_3$	$\lvert 3\,3\rangle$	$\lvert 4\,\bar{3}\rangle$	2	± 12	${}^1E_{7/2}$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle$	$\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle^\bullet$	1	± 12
${}^2E_3$	$\lvert 3\,\bar{3}\rangle$	$\lvert 4\,3\rangle$	2	± 12	${}^2E_{7/2}$	$\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle^\bullet$	1	± 12
${}^1E_4$	$\lvert 3\,\bar{2}\rangle$	$\lvert 4\,4\rangle$	2	± 12	${}^1E_{9/2}$	$\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle^\bullet$	1	± 12
${}^2E_4$	$\lvert 3\,2\rangle$	$\lvert 4\,\bar{4}\rangle$	2	± 12	${}^2E_{9/2}$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle$	$\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle^\bullet$	1	± 12
${}^1E_5$	$\lvert 2\,1\rangle$	$\lvert 5\,\bar{5}\rangle$	2	± 12	${}^1E_{11/2}$	$\lvert \frac{11}{2}\,\frac{11}{2}\rangle$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle^\bullet$	1	± 12
${}^2E_5$	$\lvert 2\,\bar{1}\rangle$	$\lvert 5\,5\rangle$	2	± 12	${}^2E_{11/2}$	$\lvert \frac{11}{2}\,\overline{\frac{11}{2}}\rangle$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle^\bullet$	1	± 12

T **17**.7 Matrix representations

Use T **17**.4 ♠. § **16**–7, p. 77

T **17**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{S}_{12}$	A	B	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$	${}^1E_4$	${}^2E_4$	${}^1E_5$	${}^2E_5$	${}^1E_{1/2}$	${}^2E_{1/2}$
A	A	B	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$	${}^1E_4$	${}^2E_4$	${}^1E_5$	${}^2E_5$	${}^1E_{1/2}$	${}^2E_{1/2}$
B		A	${}^1E_5$	${}^2E_5$	${}^1E_4$	${}^2E_4$	${}^2E_3$	${}^1E_3$	${}^1E_2$	${}^2E_2$	${}^1E_1$	${}^2E_1$	${}^1E_{11/2}$	${}^2E_{11/2}$
${}^1E_1$			${}^2E_2$	A	${}^2E_1$	${}^1E_3$	${}^1E_4$	${}^1E_2$	${}^2E_5$	${}^2E_3$	${}^2E_4$	B	${}^2E_{11/2}$	${}^1E_{9/2}$
${}^2E_1$				${}^1E_2$	${}^2E_3$	${}^1E_1$	${}^2E_2$	${}^2E_4$	${}^1E_3$	${}^1E_5$	B	${}^1E_4$	${}^2E_{9/2}$	${}^1E_{11/2}$
${}^1E_2$					${}^2E_4$	A	${}^1E_1$	${}^1E_5$	${}^2E_2$	B	${}^2E_5$	${}^1E_3$	${}^1E_{5/2}$	${}^2E_{3/2}$
${}^2E_2$						${}^1E_4$	${}^2E_5$	${}^2E_1$	B	${}^1E_2$	${}^2E_3$	${}^1E_5$	${}^1E_{3/2}$	${}^2E_{5/2}$
${}^1E_3$							B	A	${}^1E_5$	${}^2E_1$	${}^1E_2$	${}^2E_4$	${}^2E_{7/2}$	${}^1E_{5/2}$
${}^2E_3$								B	${}^1E_1$	${}^2E_5$	${}^1E_4$	${}^2E_2$	${}^2E_{5/2}$	${}^1E_{7/2}$
${}^1E_4$									${}^2E_4$	A	${}^2E_1$	${}^2E_3$	${}^1E_{7/2}$	${}^2E_{9/2}$
${}^2E_4$										${}^1E_4$	${}^1E_3$	${}^1E_1$	${}^1E_{9/2}$	${}^2E_{7/2}$
${}^1E_5$											${}^2E_2$	A	${}^2E_{1/2}$	${}^1E_{3/2}$
${}^2E_5$												${}^1E_2$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^1E_{1/2}$													${}^2E_5$	A
${}^2E_{1/2}$														${}^1E_5$

⇛

T **17**.8 Direct products of representations *(cont.)*

$\mathbf{S}_{12}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{11/2}$
A	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{11/2}$
B	${}^1E_{9/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_1$	${}^2E_{7/2}$	${}^1E_{11/2}$	${}^2E_{9/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$
${}^2E_1$	${}^2E_{11/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^1E_2$	${}^1E_{1/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{11/2}$	${}^1E_{11/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$
${}^2E_2$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$
${}^1E_3$	${}^2E_{3/2}$	${}^1E_{9/2}$	${}^2E_{11/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{11/2}$	${}^2E_{9/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$
${}^2E_3$	${}^2E_{9/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{11/2}$	${}^2E_{11/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$
${}^1E_4$	${}^1E_{11/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$
${}^2E_4$	${}^1E_{5/2}$	${}^2E_{11/2}$	${}^1E_{11/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$
${}^1E_5$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{11/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$
${}^2E_5$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{9/2}$	${}^2E_{11/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$
${}^1E_{1/2}$	${}^1E_5$	${}^1E_2$	${}^1E_3$	${}^2E_2$	${}^2E_3$	${}^2E_4$	${}^1E_1$	${}^1E_4$	${}^2E_1$	B
${}^2E_{1/2}$	${}^2E_2$	${}^2E_5$	${}^1E_2$	${}^2E_3$	${}^1E_4$	${}^1E_3$	${}^2E_4$	${}^2E_1$	B	${}^1E_1$
${}^1E_{3/2}$	${}^2E_3$	A	${}^2E_5$	${}^1E_4$	${}^2E_1$	${}^1E_2$	${}^1E_3$	B	${}^1E_1$	${}^2E_4$
${}^2E_{3/2}$		${}^1E_3$	${}^2E_4$	${}^1E_5$	${}^2E_2$	${}^1E_1$	B	${}^2E_3$	${}^1E_4$	${}^2E_1$
${}^1E_{5/2}$			${}^1E_1$	A	${}^1E_5$	B	${}^2E_1$	${}^2E_2$	${}^2E_3$	${}^1E_4$
${}^2E_{5/2}$				${}^2E_1$	B	${}^2E_5$	${}^1E_2$	${}^1E_1$	${}^2E_4$	${}^1E_3$
${}^1E_{7/2}$					${}^1E_1$	A	${}^2E_5$	${}^2E_4$	${}^1E_3$	${}^1E_2$
${}^2E_{7/2}$						${}^2E_1$	${}^1E_4$	${}^1E_5$	${}^2E_2$	${}^2E_3$
${}^1E_{9/2}$							${}^2E_3$	A	${}^1E_5$	${}^2E_2$
${}^2E_{9/2}$								${}^1E_3$	${}^1E_2$	${}^2E_5$
${}^1E_{11/2}$									${}^2E_5$	A
${}^2E_{11/2}$										${}^1E_5$

T **17**.9 Subduction (descent of symmetry) § **16**–9, p. 82

$\mathbf{S}_{12}$	$\mathbf{S}_4$	$\mathbf{C}_6$	$\mathbf{C}_3$	$\mathbf{C}_2$	$\mathbf{S}_{12}$	$\mathbf{S}_4$	$\mathbf{C}_6$	$\mathbf{C}_3$	$\mathbf{C}_2$
A	A	A	A	A	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$
B	B	A	A	A	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$
${}^1E_1$	${}^2E$	${}^1E_1$	${}^1E$	B	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{3/2}$	$A_{3/2}$	${}^1E_{1/2}$
${}^2E_1$	${}^1E$	${}^2E_1$	${}^2E$	B	${}^2E_{3/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	$A_{3/2}$	${}^2E_{1/2}$
${}^1E_2$	B	${}^1E_2$	${}^1E$	A	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_2$	B	${}^2E_2$	${}^2E$	A	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_3$	${}^1E$	B	A	B	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_3$	${}^2E$	B	A	B	${}^2E_{7/2}$	${}^1E_{1/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_4$	A	${}^1E_2$	${}^1E$	A	${}^1E_{9/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	$A_{3/2}$	${}^1E_{1/2}$
${}^2E_4$	A	${}^2E_2$	${}^2E$	A	${}^2E_{9/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	$A_{3/2}$	${}^2E_{1/2}$
${}^1E_5$	${}^1E$	${}^1E_1$	${}^1E$	B	${}^1E_{11/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$
${}^2E_5$	${}^2E$	${}^2E_1$	${}^2E$	B	${}^2E_{11/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$

T 17.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{S}_{12}$
$12n$	$(2n+1)\,A \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5)$
$12n+1$	$2n\,(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1)$
$12n+2$	$(2n+1)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_5 \oplus {}^2E_5) \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$12n+3$	$2n\,(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_5 \oplus {}^2E_5) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$12n+4$	$(2n+1)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5) \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1)$
$12n+5$	$2n\,A \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5)$
$12n+6$	$(2n+1)\,(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5) \oplus (2n+2)\,B$
$12n+7$	$(2n+2)\,(A \oplus {}^1E_5 \oplus {}^2E_5) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$12n+8$	$(2n+1)\,(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_5 \oplus {}^2E_5) \oplus (2n+2)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_4 \oplus {}^2E_4)$
$12n+9$	$(2n+2)\,(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_5 \oplus {}^2E_5) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_4 \oplus {}^2E_4)$
$12n+10$	$(2n+1)\,(A \oplus {}^1E_5 \oplus {}^2E_5) \oplus (2n+2)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4)$
$12n+11$	$(2n+2)\,(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5) \oplus (2n+1)\,B$
$12n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus$ $2n\,({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2})$
$12n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus$ $2n\,({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2})$
$12n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus$ $2n\,({}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2})$
$12n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}) \oplus$ $2n\,({}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2})$
$12n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2}) \oplus$ $2n\,({}^1E_{11/2} \oplus {}^2E_{11/2})$
$12n+\frac{11}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus$ ${}^1E_{11/2} \oplus {}^2E_{11/2})$
$12n+\frac{13}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2}) \oplus$ $(2n+2)({}^1E_{11/2} \oplus {}^2E_{11/2})$
$12n+\frac{15}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}) \oplus$ $(2n+2)({}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2})$
$12n+\frac{17}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus$ $(2n+2)({}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2})$
$12n+\frac{19}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus$ $(2n+2)({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2})$
$12n+\frac{21}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus (2n+2)({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus$ ${}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2})$
$12n+\frac{23}{2}$	$(2n+2)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus$ ${}^1E_{11/2} \oplus {}^2E_{11/2})$

$n = 0, 1, 2, \ldots$

T 17.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: $\mathbf{C}_7 \otimes \mathbf{C}_i$.

(2) Group chains: $\mathbf{D}_{7d} \supset \underline{\mathbf{S}_{14}} \supset \underline{\mathbf{C}_i}$, $\quad \mathbf{D}_{7d} \supset \underline{\mathbf{S}_{14}} \supset \underline{\mathbf{C}_7}$.

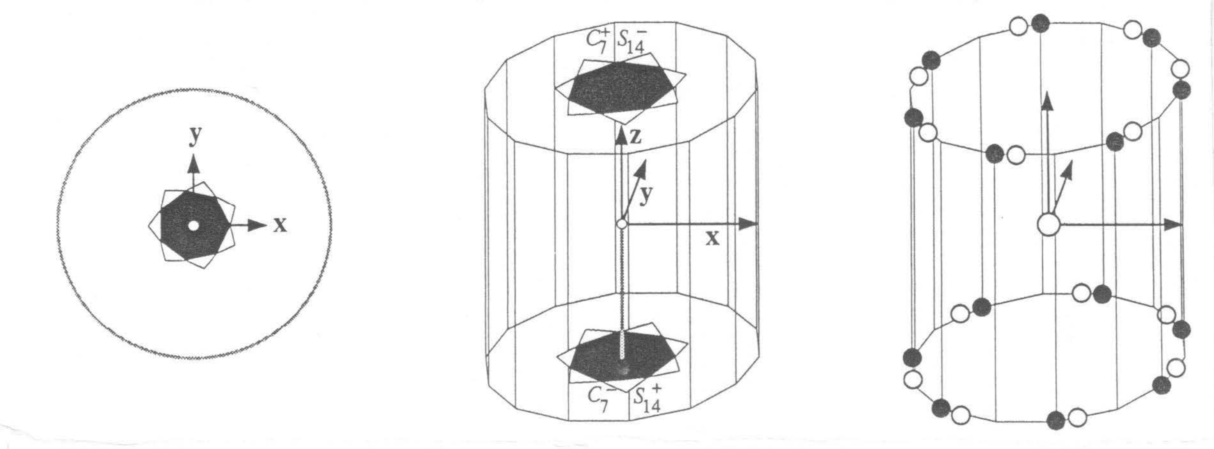
(3) Operations of G: E, C_7^+, C_7^{2+}, C_7^{3+}, C_7^{3-}, C_7^{2-}, C_7^-, i, S_{14}^{5-}, S_{14}^{3-}, S_{14}^-, S_{14}^+, S_{14}^{3+}, S_{14}^{5+}.

(4) Operations of $\widetilde{G}$: E, C_7^+, C_7^{2+}, C_7^{3+}, C_7^{3-}, C_7^{2-}, C_7^-, i, S_{14}^{5-}, S_{14}^{3-}, S_{14}^-, S_{14}^+, S_{14}^{3+}, S_{14}^{5+},
$\widetilde{E}$, $\widetilde{C}_7^+$, $\widetilde{C}_7^{2+}$, $\widetilde{C}_7^{3+}$, $\widetilde{C}_7^{3-}$, $\widetilde{C}_7^{2-}$, $\widetilde{C}_7^-$, $\widetilde{\imath}$, $\widetilde{S}_{14}^{5-}$, $\widetilde{S}_{14}^{3-}$, $\widetilde{S}_{14}^-$, $\widetilde{S}_{14}^+$, $\widetilde{S}_{14}^{3+}$, $\widetilde{S}_{14}^{5+}$.

(5) Classes and representations: $|r| = 14$, $\quad |i| = 0$, $\quad |I| = 14$, $\quad |\widetilde{I}| = 14$.

F **18**

See Chapter **15**, p. 65



Examples:

T **18**.1 Parameters
Use T **46**.1. § **16**–1, p. 68

T **18**.2 Multiplication table
Use T **46**.2. § **16**–2, p. 69

T **18**.3 Factor table
Use T **46**.3. § **16**–3, p. 70

T **18**.4 Character table

§ **16**–4, p. 71

$\mathbf{S}_{14}$	E	C_7^+	C_7^{2+}	C_7^{3+}	C_7^{3-}	C_7^{2-}	C_7^-	i	S_{14}^{5-}	S_{14}^{3-}	S_{14}^-	S_{14}^+	S_{14}^{3+}	S_{14}^{5+}	τ
A_g	1	1	1	1	1	1	1	1	1	1	1	1	1	1	a
${}^1E_{1g}$	1	δ^*	ϵ^*	η^*	η	ϵ	δ	1	δ^*	ϵ^*	η^*	η	ϵ	δ	b
${}^2E_{1g}$	1	δ	ϵ	η	η^*	ϵ^*	δ^*	1	δ	ϵ	η	η^*	ϵ^*	δ^*	b
${}^1E_{2g}$	1	ϵ^*	η	δ	δ^*	η^*	ϵ	1	ϵ^*	η	δ	δ^*	η^*	ϵ	b
${}^2E_{2g}$	1	ϵ	η^*	δ^*	δ	η	ϵ^*	1	ϵ	η^*	δ^*	δ	η	ϵ^*	b
${}^1E_{3g}$	1	η^*	δ	ϵ^*	ϵ	δ^*	η	1	η^*	δ	ϵ^*	ϵ	δ^*	η	b
${}^2E_{3g}$	1	η	δ^*	ϵ	ϵ^*	δ	η^*	1	η	δ^*	ϵ	ϵ^*	δ	η^*	b
A_u	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	a
${}^1E_{1u}$	1	δ^*	ϵ^*	η^*	η	ϵ	δ	−1	$-\delta^*$	$-\epsilon^*$	$-\eta^*$	$-\eta$	$-\epsilon$	$-\delta$	b
${}^2E_{1u}$	1	δ	ϵ	η	η^*	ϵ^*	δ^*	−1	$-\delta$	$-\epsilon$	$-\eta$	$-\eta^*$	$-\epsilon^*$	$-\delta^*$	b
${}^1E_{2u}$	1	ϵ^*	η	δ	δ^*	η^*	ϵ	−1	$-\epsilon^*$	$-\eta$	$-\delta$	$-\delta^*$	$-\eta^*$	$-\epsilon$	b
${}^2E_{2u}$	1	ϵ	η^*	δ^*	δ	η	ϵ^*	−1	$-\epsilon$	$-\eta^*$	$-\delta^*$	$-\delta$	$-\eta$	$-\epsilon^*$	b
${}^1E_{3u}$	1	η^*	δ	ϵ^*	ϵ	δ^*	η	−1	$-\eta^*$	$-\delta$	$-\epsilon^*$	$-\epsilon$	$-\delta^*$	$-\eta$	b
${}^2E_{3u}$	1	η	δ^*	ϵ	ϵ^*	δ	η^*	−1	$-\eta$	$-\delta^*$	$-\epsilon$	$-\epsilon^*$	$-\delta$	$-\eta^*$	b
${}^1E_{1/2,g}$	1	$-\eta^*$	δ	$-\epsilon^*$	$-\epsilon$	δ^*	$-\eta$	1	$-\eta^*$	δ	$-\epsilon^*$	$-\epsilon$	δ^*	$-\eta$	b
${}^2E_{1/2,g}$	1	$-\eta$	δ^*	$-\epsilon$	$-\epsilon^*$	δ	$-\eta^*$	1	$-\eta$	δ^*	$-\epsilon$	$-\epsilon^*$	δ	$-\eta^*$	b
${}^1E_{3/2,g}$	1	$-\epsilon$	η^*	$-\delta^*$	$-\delta$	η	$-\epsilon^*$	1	$-\epsilon$	η^*	$-\delta^*$	$-\delta$	η	$-\epsilon^*$	b
${}^2E_{3/2,g}$	1	$-\epsilon^*$	η	$-\delta$	$-\delta^*$	η^*	$-\epsilon$	1	$-\epsilon^*$	η	$-\delta$	$-\delta^*$	η^*	$-\epsilon$	b
${}^1E_{5/2,g}$	1	$-\delta^*$	ϵ^*	$-\eta^*$	$-\eta$	ϵ	$-\delta$	1	$-\delta^*$	ϵ^*	$-\eta^*$	$-\eta$	ϵ	$-\delta$	b
${}^2E_{5/2,g}$	1	$-\delta$	ϵ	$-\eta$	$-\eta^*$	ϵ^*	$-\delta^*$	1	$-\delta$	ϵ	$-\eta$	$-\eta^*$	ϵ^*	$-\delta^*$	b
$A_{7/2,g}$	1	−1	1	−1	−1	1	−1	1	−1	1	−1	−1	1	−1	a
${}^1E_{1/2,u}$	1	$-\eta^*$	δ	$-\epsilon^*$	$-\epsilon$	δ^*	$-\eta$	−1	η^*	$-\delta$	ϵ^*	ϵ	$-\delta^*$	η	b
${}^2E_{1/2,u}$	1	$-\eta$	δ^*	$-\epsilon$	$-\epsilon^*$	δ	$-\eta^*$	−1	η	$-\delta^*$	ϵ	ϵ^*	$-\delta$	η^*	b
${}^1E_{3/2,u}$	1	$-\epsilon$	η^*	$-\delta^*$	$-\delta$	η	$-\epsilon^*$	−1	ϵ	$-\eta^*$	δ^*	δ	$-\eta$	ϵ^*	b
${}^2E_{3/2,u}$	1	$-\epsilon^*$	η	$-\delta$	$-\delta^*$	η^*	$-\epsilon$	−1	ϵ^*	$-\eta$	δ	δ^*	$-\eta^*$	ϵ	b
${}^1E_{5/2,u}$	1	$-\delta^*$	ϵ^*	$-\eta^*$	$-\eta$	ϵ	$-\delta$	−1	δ^*	$-\epsilon^*$	η^*	η	$-\epsilon$	δ	b
${}^2E_{5/2,u}$	1	$-\delta$	ϵ	$-\eta$	$-\eta^*$	ϵ^*	$-\delta^*$	−1	δ	$-\epsilon$	η	η^*	$-\epsilon^*$	δ^*	b
$A_{7/2,u}$	1	−1	1	−1	−1	1	−1	−1	1	−1	1	1	−1	1	a

$\delta = \exp(2\pi i/7)$, $\epsilon = \exp(4\pi i/7)$, $\eta = \exp(6\pi i/7)$

T **18**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{S}_{14}$	0	1	2	3
A_g	$^\square 1$	R_z	$x^2+y^2, {}^\square z^2$	
${}^1E_{1g} \oplus {}^2E_{1g}$		(R_x, R_y)	$^\square(zx, yz)$	
${}^1E_{2g} \oplus {}^2E_{2g}$			$^\square(xy, x^2-y^2)$	
${}^1E_{3g} \oplus {}^2E_{3g}$				
A_u		$^\square z$		$(x^2+y^2)z, {}^\square z^3$
${}^1E_{1u} \oplus {}^2E_{1u}$		$^\square(x, y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
${}^1E_{2u} \oplus {}^2E_{2u}$				$^\square\{xyz, z(x^2-y^2)\}$
${}^1E_{3u} \oplus {}^2E_{3u}$				$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$

T 18.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{S}_{14}$	$\|j\,m\rangle$	ι	μ	$\mathbf{S}_{14}$	$\|j\,m\rangle$	ι	μ
A_g	$\|0\,0\rangle$	2	± 7	$^1E_{1/2,g}$	$\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	1	± 7
$^1E_{1g}$	$\|2\,1\rangle$	2	± 7	$^2E_{1/2,g}$	$\|\frac{1}{2}\,\frac{1}{2}\rangle$	1	± 7
$^2E_{1g}$	$\|2\,\overline{1}\rangle$	2	± 7	$^1E_{3/2,g}$	$\|\frac{3}{2}\,\frac{3}{2}\rangle$	1	± 7
$^1E_{2g}$	$\|2\,2\rangle$	2	± 7	$^2E_{3/2,g}$	$\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	1	± 7
$^2E_{2g}$	$\|2\,\overline{2}\rangle$	2	± 7	$^1E_{5/2,g}$	$\|\frac{5}{2}\,\overline{\frac{5}{2}}\rangle$	1	± 7
$^1E_{3g}$	$\|4\,3\rangle$	2	± 7	$^2E_{5/2,g}$	$\|\frac{5}{2}\,\frac{5}{2}\rangle$	1	± 7
$^2E_{3g}$	$\|4\,\overline{3}\rangle$	2	± 7	$A_{7/2,g}$	$\|\frac{7}{2}\,\frac{7}{2}\rangle$	1	± 7
A_u	$\|1\,0\rangle$	2	± 7	$^1E_{1/2,u}$	$\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle^\bullet$	1	± 7
$^1E_{1u}$	$\|1\,1\rangle$	2	± 7	$^2E_{1/2,u}$	$\|\frac{1}{2}\,\frac{1}{2}\rangle^\bullet$	1	± 7
$^2E_{1u}$	$\|1\,\overline{1}\rangle$	2	± 7	$^1E_{3/2,u}$	$\|\frac{3}{2}\,\frac{3}{2}\rangle^\bullet$	1	± 7
$^1E_{2u}$	$\|3\,2\rangle$	2	± 7	$^2E_{3/2,u}$	$\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle^\bullet$	1	± 7
$^2E_{2u}$	$\|3\,\overline{2}\rangle$	2	± 7	$^1E_{5/2,u}$	$\|\frac{5}{2}\,\overline{\frac{5}{2}}\rangle^\bullet$	1	± 7
$^1E_{3u}$	$\|3\,3\rangle$	2	± 7	$^2E_{5/2,u}$	$\|\frac{5}{2}\,\frac{5}{2}\rangle^\bullet$	1	± 7
$^2E_{3u}$	$\|3\,\overline{3}\rangle$	2	± 7	$A_{7/2,u}$	$\|\frac{7}{2}\,\frac{7}{2}\rangle^\bullet$	1	± 7

T 18.7 Matrix representations

Use T **18**.4 ♠. § **16**–7, p. 77

T 18.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{S}_{14}$	A_g	$^1E_{1g}$	$^2E_{1g}$	$^1E_{2g}$	$^2E_{2g}$	$^1E_{3g}$	$^2E_{3g}$	A_u	$^1E_{1u}$	$^2E_{1u}$	$^1E_{2u}$	$^2E_{2u}$	$^1E_{3u}$	$^2E_{3u}$
A_g	A_g	$^1E_{1g}$	$^2E_{1g}$	$^1E_{2g}$	$^2E_{2g}$	$^1E_{3g}$	$^2E_{3g}$	A_u	$^1E_{1u}$	$^2E_{1u}$	$^1E_{2u}$	$^2E_{2u}$	$^1E_{3u}$	$^2E_{3u}$
$^1E_{1g}$		$^1E_{2g}$	A_g	$^1E_{3g}$	$^2E_{1g}$	$^2E_{3g}$	$^2E_{2g}$	$^1E_{1u}$	$^1E_{2u}$	A_u	$^1E_{3u}$	$^2E_{1u}$	$^2E_{3u}$	$^2E_{2u}$
$^2E_{1g}$			$^2E_{2g}$	$^1E_{1g}$	$^2E_{3g}$	$^1E_{2g}$	$^1E_{3g}$	$^2E_{1u}$	A_u	$^2E_{2u}$	$^1E_{1u}$	$^2E_{3u}$	$^1E_{2u}$	$^1E_{3u}$
$^1E_{2g}$				$^2E_{3g}$	A_g	$^2E_{2g}$	$^2E_{1g}$	$^1E_{2u}$	$^1E_{3u}$	$^1E_{1u}$	$^2E_{3u}$	A_u	$^2E_{2u}$	$^2E_{1u}$
$^2E_{2g}$					$^1E_{3g}$	$^1E_{1g}$	$^1E_{2g}$	$^2E_{2u}$	$^2E_{1u}$	$^2E_{3u}$	A_u	$^1E_{3u}$	$^1E_{1u}$	$^1E_{2u}$
$^1E_{3g}$						$^2E_{1g}$	A_g	$^1E_{3u}$	$^2E_{3u}$	$^1E_{2u}$	$^2E_{2u}$	$^1E_{1u}$	$^2E_{1u}$	A_u
$^2E_{3g}$							$^1E_{1g}$	$^2E_{3u}$	$^2E_{2u}$	$^1E_{3u}$	$^2E_{1u}$	$^1E_{2u}$	A_u	$^1E_{1u}$
A_u								A_g	$^1E_{1g}$	$^2E_{1g}$	$^1E_{2g}$	$^2E_{2g}$	$^1E_{3g}$	$^2E_{3g}$
$^1E_{1u}$									$^1E_{2g}$	A_g	$^1E_{3g}$	$^2E_{1g}$	$^2E_{3g}$	$^2E_{2g}$
$^2E_{1u}$										$^2E_{2g}$	$^1E_{1g}$	$^2E_{3g}$	$^1E_{2g}$	$^1E_{3g}$
$^1E_{2u}$											$^2E_{3g}$	A_g	$^2E_{2g}$	$^2E_{1g}$
$^2E_{2u}$												$^1E_{3g}$	$^1E_{1g}$	$^1E_{2g}$
$^1E_{3u}$													$^2E_{1g}$	A_g
$^2E_{3u}$														$^1E_{1g}$

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T **18**.8 Direct products of representations *(cont.)*

$\mathbf{S}_{14}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$	$A_{7/2,g}$
A_g	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$	$A_{7/2,g}$
${}^1E_{1g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$	$A_{7/2,g}$	${}^1E_{5/2,g}$
${}^2E_{1g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{5/2,g}$	$A_{7/2,g}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$
${}^1E_{2g}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$	$A_{7/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$
${}^2E_{2g}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	$A_{7/2,g}$	${}^2E_{5/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$
${}^1E_{3g}$	${}^2E_{5/2,g}$	$A_{7/2,g}$	${}^1E_{5/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$
${}^2E_{3g}$	$A_{7/2,g}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$
A_u	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{5/2,u}$	$A_{7/2,u}$
${}^1E_{1u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$	$A_{7/2,u}$	${}^1E_{5/2,u}$
${}^2E_{1u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{5/2,u}$	$A_{7/2,u}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$
${}^1E_{2u}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$	$A_{7/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$
${}^2E_{2u}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	$A_{7/2,u}$	${}^2E_{5/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$
${}^1E_{3u}$	${}^2E_{5/2,u}$	$A_{7/2,u}$	${}^1E_{5/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$
${}^2E_{3u}$	$A_{7/2,u}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$
${}^1E_{1/2,g}$	${}^2E_{1g}$	A_g	${}^1E_{1g}$	${}^2E_{2g}$	${}^2E_{3g}$	${}^1E_{2g}$	${}^1E_{3g}$
${}^2E_{1/2,g}$		${}^1E_{1g}$	${}^1E_{2g}$	${}^2E_{1g}$	${}^2E_{2g}$	${}^1E_{3g}$	${}^2E_{3g}$
${}^1E_{3/2,g}$			${}^1E_{3g}$	A_g	${}^2E_{1g}$	${}^2E_{3g}$	${}^2E_{2g}$
${}^2E_{3/2,g}$				${}^2E_{3g}$	${}^1E_{3g}$	${}^1E_{1g}$	${}^1E_{2g}$
${}^1E_{5/2,g}$					${}^1E_{2g}$	A_g	${}^1E_{1g}$
${}^2E_{5/2,g}$						${}^2E_{2g}$	${}^2E_{1g}$
$A_{7/2,g}$							A_g

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T **18**.8 Direct products of representations *(cont.)*

$\mathbf{S}_{14}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{5/2,u}$	$A_{7/2,u}$
A_g	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{5/2,u}$	$A_{7/2,u}$
${}^1E_{1g}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$	$A_{7/2,u}$	${}^1E_{5/2,u}$
${}^2E_{1g}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{5/2,u}$	$A_{7/2,u}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$
${}^1E_{2g}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$	$A_{7/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$
${}^2E_{2g}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	$A_{7/2,u}$	${}^2E_{5/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$
${}^1E_{3g}$	${}^2E_{5/2,u}$	$A_{7/2,u}$	${}^1E_{5/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$
${}^2E_{3g}$	$A_{7/2,u}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$
A_u	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$	$A_{7/2,g}$
${}^1E_{1u}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$	$A_{7/2,g}$	${}^1E_{5/2,g}$
${}^2E_{1u}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{5/2,g}$	$A_{7/2,g}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$
${}^1E_{2u}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$	$A_{7/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$
${}^2E_{2u}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	$A_{7/2,g}$	${}^2E_{5/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$
${}^1E_{3u}$	${}^2E_{5/2,g}$	$A_{7/2,g}$	${}^1E_{5/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$
${}^2E_{3u}$	$A_{7/2,g}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$
${}^1E_{1/2,g}$	${}^2E_{1u}$	A_u	${}^1E_{1u}$	${}^2E_{2u}$	${}^2E_{3u}$	${}^1E_{2u}$	${}^1E_{3u}$
${}^2E_{1/2,g}$	A_u	${}^1E_{1u}$	${}^1E_{2u}$	${}^2E_{1u}$	${}^2E_{2u}$	${}^1E_{3u}$	${}^2E_{3u}$
${}^1E_{3/2,g}$	${}^1E_{1u}$	${}^1E_{2u}$	${}^1E_{3u}$	A_u	${}^2E_{1u}$	${}^2E_{3u}$	${}^2E_{2u}$
${}^2E_{3/2,g}$	${}^2E_{2u}$	${}^2E_{1u}$	A_u	${}^2E_{3u}$	${}^1E_{3u}$	${}^1E_{1u}$	${}^1E_{2u}$
${}^1E_{5/2,g}$	${}^2E_{3u}$	${}^2E_{2u}$	${}^2E_{1u}$	${}^1E_{3u}$	${}^1E_{2u}$	A_u	${}^1E_{1u}$
${}^2E_{5/2,g}$	${}^1E_{2u}$	${}^1E_{3u}$	${}^2E_{3u}$	${}^1E_{1u}$	A_u	${}^2E_{2u}$	${}^2E_{1u}$
$A_{7/2,g}$	${}^1E_{3u}$	${}^2E_{3u}$	${}^2E_{2u}$	${}^1E_{2u}$	${}^1E_{1u}$	${}^2E_{1u}$	A_u
${}^1E_{1/2,u}$	${}^2E_{1g}$	A_g	${}^1E_{1g}$	${}^2E_{2g}$	${}^2E_{3g}$	${}^1E_{2g}$	${}^1E_{3g}$
${}^2E_{1/2,u}$		${}^1E_{1g}$	${}^1E_{2g}$	${}^2E_{1g}$	${}^2E_{2g}$	${}^1E_{3g}$	${}^2E_{3g}$
${}^1E_{3/2,u}$			${}^1E_{3g}$	A_g	${}^2E_{1g}$	${}^2E_{3g}$	${}^2E_{2g}$
${}^2E_{3/2,u}$				${}^2E_{3g}$	${}^1E_{3g}$	${}^1E_{1g}$	${}^1E_{2g}$
${}^1E_{5/2,u}$					${}^1E_{2g}$	A_g	${}^1E_{1g}$
${}^2E_{5/2,u}$						${}^2E_{2g}$	${}^2E_{1g}$
$A_{7/2,u}$							A_g

T **18**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{S}_{14}$	$\mathbf{C}_i$	$\mathbf{C}_7$	$\mathbf{S}_{14}$	$\mathbf{C}_i$	$\mathbf{C}_7$
A_g	A_g	A	${}^1E_{1/2,g}$	$A_{1/2,g}$	${}^1E_{1/2}$
${}^1E_{1g}$	A_g	${}^1E_1$	${}^2E_{1/2,g}$	$A_{1/2,g}$	${}^2E_{1/2}$
${}^2E_{1g}$	A_g	${}^2E_1$	${}^1E_{3/2,g}$	$A_{1/2,g}$	${}^1E_{3/2}$
${}^1E_{2g}$	A_g	${}^1E_2$	${}^2E_{3/2,g}$	$A_{1/2,g}$	${}^2E_{3/2}$
${}^2E_{2g}$	A_g	${}^2E_2$	${}^1E_{5/2,g}$	$A_{1/2,g}$	${}^1E_{5/2}$
${}^1E_{3g}$	A_g	${}^1E_3$	${}^2E_{5/2,g}$	$A_{1/2,g}$	${}^2E_{5/2}$
${}^2E_{3g}$	A_g	${}^2E_3$	$A_{7/2,g}$	$A_{1/2,g}$	$A_{7/2}$
A_u	A_u	A	${}^1E_{1/2,u}$	$A_{1/2,u}$	${}^1E_{1/2}$
${}^1E_{1u}$	A_u	${}^1E_1$	${}^2E_{1/2,u}$	$A_{1/2,u}$	${}^2E_{1/2}$
${}^2E_{1u}$	A_u	${}^2E_1$	${}^1E_{3/2,u}$	$A_{1/2,u}$	${}^1E_{3/2}$
${}^1E_{2u}$	A_u	${}^1E_2$	${}^2E_{3/2,u}$	$A_{1/2,u}$	${}^2E_{3/2}$
${}^2E_{2u}$	A_u	${}^2E_2$	${}^1E_{5/2,u}$	$A_{1/2,u}$	${}^1E_{5/2}$
${}^1E_{3u}$	A_u	${}^1E_3$	${}^2E_{5/2,u}$	$A_{1/2,u}$	${}^2E_{5/2}$
${}^2E_{3u}$	A_u	${}^2E_3$	$A_{7/2,u}$	$A_{1/2,u}$	$A_{7/2}$

T **18**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{S}_{14}$
$14n$	$(4n+1)\,A_g \oplus 4n\,({}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g})$
$14n+1$	$(4n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u}) \oplus 4n\,({}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u})$
$14n+2$	$(4n+1)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g}) \oplus 4n\,({}^1E_{3g} \oplus {}^2E_{3g})$
$14n+3$	$(4n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u})$
$14n+4$	$(4n+1)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g}) \oplus (4n+2)({}^1E_{3g} \oplus {}^2E_{3g})$
$14n+5$	$(4n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u}) \oplus (4n+2)({}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u})$
$14n+6$	$(4n+1)\,A_g \oplus (4n+2)({}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g})$
$14n+7$	$(4n+3)\,A_u \oplus (4n+2)({}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u})$
$14n+8$	$(4n+3)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g}) \oplus (4n+2)({}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g})$
$14n+9$	$(4n+3)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u}) \oplus (4n+2)({}^1E_{3u} \oplus {}^2E_{3u})$
$14n+10$	$(4n+3)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g})$
$14n+11$	$(4n+3)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u}) \oplus (4n+4)({}^1E_{3u} \oplus {}^2E_{3u})$
$14n+12$	$(4n+3)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g}) \oplus (4n+4)({}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g})$
$14n+13$	$(4n+3)\,A_u \oplus (4n+4)({}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u})$
$7n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus 2n\,({}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus A_{7/2,g})$
$7n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}) \oplus 2n\,({}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus A_{7/2,g})$
$7n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g}) \oplus 2n\,A_{7/2,g}$
$7n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g}) \oplus (2n+2)\,A_{7/2,g}$
$7n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}) \oplus (2n+2)({}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus A_{7/2,g})$
$7n+\frac{11}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus (2n+2)({}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus A_{7/2,g})$
$7n+\frac{13}{2}$	$(2n+2)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus A_{7/2,g})$

$n = 0, 1, 2, \ldots$

T **18**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: none.

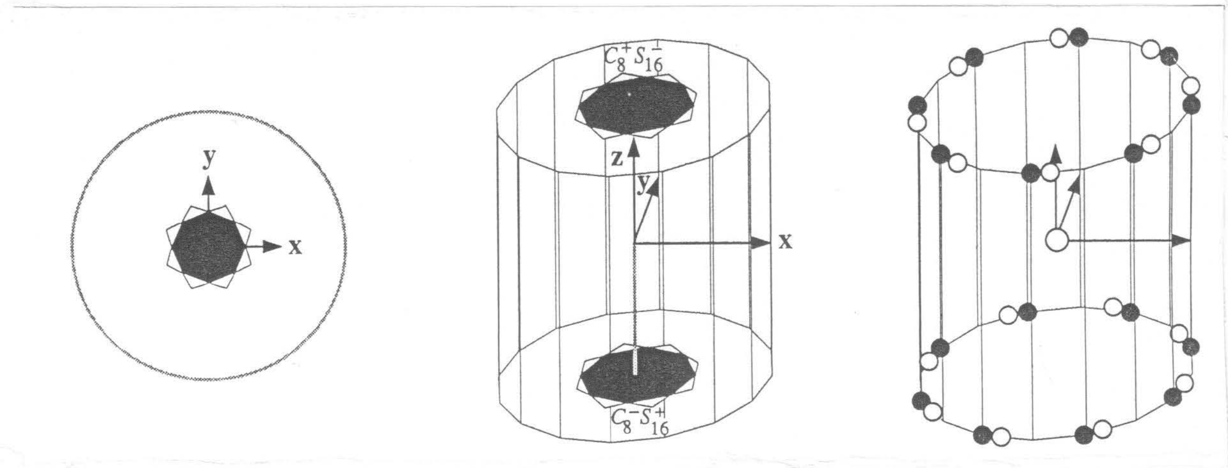
(2) Group chains: $\mathbf{D}_{8d} \supset \underline{\mathbf{S}_{16}} \supset \underline{\mathbf{C}_8}$.

(3) Operations of G: E, S_{16}^{7-}, C_8^{+}, S_{16}^{5-}, C_4^{+}, S_{16}^{3-}, C_8^{3+}, S_{16}^{-}, C_2, S_{16}^{+}, C_8^{3-}, S_{16}^{3+}, C_4^{-}, S_{16}^{5+}, C_8^{-}, S_{16}^{7+}.

(4) Operations of $\widetilde{G}$: E, S_{16}^{7-}, C_8^{+}, S_{16}^{5-}, C_4^{+}, S_{16}^{3-}, C_8^{3+}, S_{16}^{-}, C_2, S_{16}^{+}, C_8^{3-}, S_{16}^{3+}, C_4^{-}, S_{16}^{5+}, C_8^{-}, S_{16}^{7+},
$\widetilde{E}$, $\widetilde{S}_{16}^{7-}$, $\widetilde{C}_8^{+}$, $\widetilde{S}_{16}^{5-}$, $\widetilde{C}_4^{+}$, $\widetilde{S}_{16}^{3-}$, $\widetilde{C}_8^{3+}$, $\widetilde{S}_{16}^{-}$, $\widetilde{C}_2$, $\widetilde{S}_{16}^{+}$, $\widetilde{C}_8^{3-}$, $\widetilde{S}_{16}^{3+}$, $\widetilde{C}_4^{-}$, $\widetilde{S}_{16}^{5+}$, $\widetilde{C}_8^{-}$, $\widetilde{S}_{16}^{7+}$.

(5) Classes and representations: $|r| = 16$, $|i| = 0$, $|I| = 16$, $|\widetilde{I}| = 16$.

F **19** See Chapter **15**, p. 65



Examples:

T **19**.1 Parameters

Use T **47**.1. § **16**–1, p. 68

T **19**.2 Multiplication table

Use T **47**.2. § **16**–2, p. 69

T **19**.3 Factor table

Use T **47**.3. § **16**–3, p. 70

T **19**.4 Character table

§ **16**–4, p. 71

$\mathbf{S}_{16}$	E	S_{16}^{7-}	C_8^+	S_{16}^{5-}	C_4^+	S_{16}^{3-}	C_8^{3+}	S_{16}^-	C_2	S_{16}^+	C_8^{3-}	S_{16}^{3+}	C_4^-	S_{16}^{5+}	C_8^-	S_{16}^{7+}	τ
A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	a
B	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	a
${}^1E_1$	1	$-\epsilon^*$	θ^*	$\mathrm{i}\epsilon$	$-\mathrm{i}$	$\mathrm{i}\epsilon^*$	$-\theta$	ϵ	-1	ϵ^*	$-\theta^*$	$-\mathrm{i}\epsilon$	i	$-\mathrm{i}\epsilon^*$	θ	$-\epsilon$	b
${}^2E_1$	1	$-\epsilon$	θ	$-\mathrm{i}\epsilon^*$	i	$-\mathrm{i}\epsilon$	$-\theta^*$	ϵ^*	-1	ϵ	$-\theta$	$\mathrm{i}\epsilon^*$	$-\mathrm{i}$	$\mathrm{i}\epsilon$	θ^*	$-\epsilon^*$	b
${}^1E_2$	1	θ^*	$-\mathrm{i}$	$-\theta$	-1	$-\theta^*$	i	θ	1	θ^*	$-\mathrm{i}$	$-\theta$	-1	$-\theta^*$	i	θ	b
${}^2E_2$	1	θ	i	$-\theta^*$	-1	$-\theta$	$-\mathrm{i}$	θ^*	1	θ	i	$-\theta^*$	-1	$-\theta$	$-\mathrm{i}$	θ^*	b
${}^1E_3$	1	$-\mathrm{i}\epsilon^*$	$-\theta^*$	ϵ	$-\mathrm{i}$	$-\epsilon^*$	θ	$-\mathrm{i}\epsilon$	-1	$\mathrm{i}\epsilon^*$	θ^*	$-\epsilon$	i	ϵ^*	$-\theta$	$\mathrm{i}\epsilon$	b
${}^2E_3$	1	$\mathrm{i}\epsilon$	$-\theta$	ϵ^*	i	$-\epsilon$	θ^*	$\mathrm{i}\epsilon^*$	-1	$-\mathrm{i}\epsilon$	θ	$-\epsilon^*$	$-\mathrm{i}$	ϵ	$-\theta^*$	$-\mathrm{i}\epsilon^*$	b
${}^1E_4$	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	-1	i	b
${}^2E_4$	1	i	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	b
${}^1E_5$	1	$\mathrm{i}\epsilon^*$	$-\theta^*$	$-\epsilon$	$-\mathrm{i}$	ϵ^*	θ	$\mathrm{i}\epsilon$	-1	$-\mathrm{i}\epsilon^*$	θ^*	ϵ	i	$-\epsilon^*$	$-\theta$	$-\mathrm{i}\epsilon$	b
${}^2E_5$	1	$-\mathrm{i}\epsilon$	$-\theta$	$-\epsilon^*$	i	ϵ	θ^*	$-\mathrm{i}\epsilon^*$	-1	$\mathrm{i}\epsilon$	θ	ϵ^*	$-\mathrm{i}$	$-\epsilon$	$-\theta^*$	$\mathrm{i}\epsilon^*$	b
${}^1E_6$	1	$-\theta^*$	$-\mathrm{i}$	θ	-1	θ^*	i	$-\theta$	1	$-\theta^*$	$-\mathrm{i}$	θ	-1	θ^*	i	$-\theta$	b
${}^2E_6$	1	$-\theta$	i	θ^*	-1	θ	$-\mathrm{i}$	$-\theta^*$	1	$-\theta$	i	θ^*	-1	θ	$-\mathrm{i}$	$-\theta^*$	b
${}^1E_7$	1	ϵ^*	θ^*	$-\mathrm{i}\epsilon$	$-\mathrm{i}$	$-\mathrm{i}\epsilon^*$	$-\theta$	$-\epsilon$	-1	$-\epsilon^*$	$-\theta^*$	$\mathrm{i}\epsilon$	i	$\mathrm{i}\epsilon^*$	θ	ϵ	b
${}^2E_7$	1	ϵ	θ	$\mathrm{i}\epsilon^*$	i	$\mathrm{i}\epsilon$	$-\theta^*$	$-\epsilon^*$	-1	$-\epsilon$	$-\theta$	$-\mathrm{i}\epsilon^*$	$-\mathrm{i}$	$-\mathrm{i}\epsilon$	θ^*	ϵ^*	b
${}^1E_{1/2}$	1	δ	ϵ	η	θ	$\mathrm{i}\eta^*$	$\mathrm{i}\epsilon^*$	$\mathrm{i}\delta^*$	i	$-\mathrm{i}\delta$	$-\mathrm{i}\epsilon$	$-\mathrm{i}\eta$	θ^*	η^*	ϵ^*	δ^*	b
${}^2E_{1/2}$	1	δ^*	ϵ^*	η^*	θ^*	$-\mathrm{i}\eta$	$-\mathrm{i}\epsilon$	$-\mathrm{i}\delta$	$-\mathrm{i}$	$\mathrm{i}\delta^*$	$\mathrm{i}\epsilon^*$	$\mathrm{i}\eta^*$	θ	η	ϵ	δ	b
${}^1E_{3/2}$	1	η^*	$-\mathrm{i}\epsilon$	$-\mathrm{i}\delta^*$	$-\theta$	$-\delta$	$-\epsilon^*$	$\mathrm{i}\eta$	i	$-\mathrm{i}\eta^*$	$-\epsilon$	$-\delta^*$	$-\theta^*$	$\mathrm{i}\delta$	$\mathrm{i}\epsilon^*$	η	b
${}^2E_{3/2}$	1	η	$\mathrm{i}\epsilon^*$	$\mathrm{i}\delta$	$-\theta^*$	$-\delta^*$	$-\epsilon$	$-\mathrm{i}\eta^*$	$-\mathrm{i}$	$\mathrm{i}\eta$	$-\epsilon^*$	$-\delta$	$-\theta$	$-\mathrm{i}\delta^*$	$-\mathrm{i}\epsilon$	η^*	b
${}^1E_{5/2}$	1	$\mathrm{i}\eta^*$	$\mathrm{i}\epsilon$	$-\delta^*$	$-\theta$	$-\mathrm{i}\delta$	ϵ^*	η	i	η^*	ϵ	$\mathrm{i}\delta^*$	$-\theta^*$	$-\delta$	$-\mathrm{i}\epsilon^*$	$-\mathrm{i}\eta$	b
${}^2E_{5/2}$	1	$-\mathrm{i}\eta$	$-\mathrm{i}\epsilon^*$	$-\delta$	$-\theta^*$	$\mathrm{i}\delta^*$	ϵ	η^*	$-\mathrm{i}$	η	ϵ^*	$-\mathrm{i}\delta$	$-\theta$	$-\delta^*$	$\mathrm{i}\epsilon$	$\mathrm{i}\eta^*$	b
${}^1E_{7/2}$	1	$-\mathrm{i}\delta$	$-\epsilon$	$\mathrm{i}\eta$	θ	η^*	$-\mathrm{i}\epsilon^*$	$-\delta^*$	i	$-\delta$	$\mathrm{i}\epsilon$	η	θ^*	$-\mathrm{i}\eta^*$	$-\epsilon^*$	$\mathrm{i}\delta^*$	b
${}^2E_{7/2}$	1	$\mathrm{i}\delta^*$	$-\epsilon^*$	$-\mathrm{i}\eta^*$	θ^*	η	$\mathrm{i}\epsilon$	$-\delta$	$-\mathrm{i}$	$-\delta^*$	$-\mathrm{i}\epsilon^*$	η^*	θ	$\mathrm{i}\eta$	$-\epsilon$	$-\mathrm{i}\delta$	b
${}^1E_{9/2}$	1	$\mathrm{i}\delta$	$-\epsilon$	$-\mathrm{i}\eta$	θ	$-\eta^*$	$-\mathrm{i}\epsilon^*$	δ^*	i	δ	$\mathrm{i}\epsilon$	$-\eta$	θ^*	$\mathrm{i}\eta^*$	$-\epsilon^*$	$-\mathrm{i}\delta^*$	b
${}^2E_{9/2}$	1	$-\mathrm{i}\delta^*$	$-\epsilon^*$	$\mathrm{i}\eta^*$	θ^*	$-\eta$	$\mathrm{i}\epsilon$	δ	$-\mathrm{i}$	δ^*	$-\mathrm{i}\epsilon^*$	$-\eta^*$	θ	$-\mathrm{i}\eta$	$-\epsilon$	$\mathrm{i}\delta$	b
${}^1E_{11/2}$	1	$-\mathrm{i}\eta^*$	$\mathrm{i}\epsilon$	δ^*	$-\theta$	$\mathrm{i}\delta$	ϵ^*	$-\eta$	i	$-\eta^*$	ϵ	$-\mathrm{i}\delta^*$	$-\theta^*$	δ	$-\mathrm{i}\epsilon^*$	$\mathrm{i}\eta$	b
${}^2E_{11/2}$	1	$\mathrm{i}\eta$	$-\mathrm{i}\epsilon^*$	δ	$-\theta^*$	$-\mathrm{i}\delta^*$	ϵ	$-\eta^*$	$-\mathrm{i}$	$-\eta$	ϵ^*	$\mathrm{i}\delta$	$-\theta$	δ^*	$\mathrm{i}\epsilon$	$-\mathrm{i}\eta^*$	b
${}^1E_{13/2}$	1	$-\eta^*$	$-\mathrm{i}\epsilon$	$\mathrm{i}\delta^*$	$-\theta$	δ	$-\epsilon^*$	$-\mathrm{i}\eta$	i	$\mathrm{i}\eta^*$	$-\epsilon$	δ^*	$-\theta^*$	$-\mathrm{i}\delta$	$\mathrm{i}\epsilon^*$	$-\eta$	b
${}^2E_{13/2}$	1	$-\eta$	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\delta$	$-\theta^*$	δ^*	$-\epsilon$	$\mathrm{i}\eta^*$	$-\mathrm{i}$	$-\mathrm{i}\eta$	$-\epsilon^*$	δ	$-\theta$	$\mathrm{i}\delta^*$	$-\mathrm{i}\epsilon$	$-\eta^*$	b
${}^1E_{15/2}$	1	$-\delta$	ϵ	$-\eta$	θ	$-\mathrm{i}\eta^*$	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\delta^*$	i	$\mathrm{i}\delta$	$-\mathrm{i}\epsilon$	$\mathrm{i}\eta$	θ^*	$-\eta^*$	ϵ^*	$-\delta^*$	b
${}^2E_{15/2}$	1	$-\delta^*$	ϵ^*	$-\eta^*$	θ^*	$\mathrm{i}\eta$	$-\mathrm{i}\epsilon$	$\mathrm{i}\delta$	$-\mathrm{i}$	$-\mathrm{i}\delta^*$	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\eta^*$	θ	$-\eta$	ϵ	$-\delta$	b

$\delta = \exp(2\pi\mathrm{i}/32)$, $\epsilon = \exp(4\pi\mathrm{i}/32)$, $\eta = \exp(6\pi\mathrm{i}/32)$, $\theta = \exp(8\pi\mathrm{i}/32)$

T **19**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{S}_{16}$	0	1	2	3
A	$^\square 1$	R_z	$x^2+y^2, {}^\square z^2$	
B		$^\square z$		$(x^2+y^2)z, {}^\square z^3$
${}^1E_1 \oplus {}^2E_1$		$^\square(x,y), (R_x,R_y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
${}^1E_2 \oplus {}^2E_2$			$^\square(xy, x^2-y^2)$	
${}^1E_3 \oplus {}^2E_3$				$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$
${}^1E_4 \oplus {}^2E_4$				
${}^1E_5 \oplus {}^2E_5$				
${}^1E_6 \oplus {}^2E_6$				$^\square\{xyz, z(x^2-y^2)\}$
${}^1E_7 \oplus {}^2E_7$			$^\square(zx, yz)$	

T **19**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{S}_{16}$	$\lvert j\,m\rangle$		ι	μ
A	$\lvert 0\,0\rangle$	$\lvert 9\,8\rangle$	2	± 16
B	$\lvert 1\,0\rangle$	$\lvert 8\,8\rangle$	2	± 16
${}^1E_1$	$\lvert 1\,1\rangle$	$\lvert 8\,\overline{7}\rangle$	2	± 16
${}^2E_1$	$\lvert 1\,\overline{1}\rangle$	$\lvert 8\,7\rangle$	2	± 16
${}^1E_2$	$\lvert 2\,2\rangle$	$\lvert 7\,\overline{6}\rangle$	2	± 16
${}^2E_2$	$\lvert 2\,\overline{2}\rangle$	$\lvert 7\,6\rangle$	2	± 16
${}^1E_3$	$\lvert 3\,\overline{3}\rangle$	$\lvert 6\,5\rangle$	2	± 16
${}^2E_3$	$\lvert 3\,3\rangle$	$\lvert 6\,\overline{5}\rangle$	2	± 16
${}^1E_4$	$\lvert 4\,4\rangle$	$\lvert 5\,\overline{4}\rangle$	2	± 16
${}^2E_4$	$\lvert 4\,\overline{4}\rangle$	$\lvert 5\,4\rangle$	2	± 16
${}^1E_5$	$\lvert 4\,\overline{3}\rangle$	$\lvert 5\,5\rangle$	2	± 16
${}^2E_5$	$\lvert 4\,3\rangle$	$\lvert 5\,\overline{5}\rangle$	2	± 16
${}^1E_6$	$\lvert 3\,2\rangle$	$\lvert 6\,\overline{6}\rangle$	2	± 16
${}^2E_6$	$\lvert 3\,\overline{2}\rangle$	$\lvert 6\,6\rangle$	2	± 16
${}^1E_7$	$\lvert 2\,1\rangle$	$\lvert 7\,\overline{7}\rangle$	2	± 16
${}^2E_7$	$\lvert 2\,\overline{1}\rangle$	$\lvert 7\,7\rangle$	2	± 16
${}^1E_{1/2}$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	$\lvert \frac{15}{2}\,\frac{15}{2}\rangle^\bullet$	1	± 16
${}^2E_{1/2}$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle$	$\lvert \frac{15}{2}\,\overline{\frac{15}{2}}\rangle^\bullet$	1	± 16
${}^1E_{3/2}$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle$	$\lvert \frac{13}{2}\,\overline{\frac{13}{2}}\rangle^\bullet$	1	± 16
${}^2E_{3/2}$	$\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	$\lvert \frac{13}{2}\,\frac{13}{2}\rangle^\bullet$	1	± 16
${}^1E_{5/2}$	$\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle$	$\lvert \frac{11}{2}\,\frac{11}{2}\rangle^\bullet$	1	± 16
${}^2E_{5/2}$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle$	$\lvert \frac{11}{2}\,\overline{\frac{11}{2}}\rangle^\bullet$	1	± 16
${}^1E_{7/2}$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle$	$\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle^\bullet$	1	± 16
${}^2E_{7/2}$	$\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle^\bullet$	1	± 16
${}^1E_{9/2}$	$\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle^\bullet$	1	± 16
${}^2E_{9/2}$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle$	$\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle^\bullet$	1	± 16
${}^1E_{11/2}$	$\lvert \frac{11}{2}\,\frac{11}{2}\rangle$	$\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle^\bullet$	1	± 16
${}^2E_{11/2}$	$\lvert \frac{11}{2}\,\overline{\frac{11}{2}}\rangle$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle^\bullet$	1	± 16
${}^1E_{13/2}$	$\lvert \frac{13}{2}\,\overline{\frac{13}{2}}\rangle$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle^\bullet$	1	± 16
${}^2E_{13/2}$	$\lvert \frac{13}{2}\,\frac{13}{2}\rangle$	$\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle^\bullet$	1	± 16
${}^1E_{15/2}$	$\lvert \frac{15}{2}\,\frac{15}{2}\rangle$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle^\bullet$	1	± 16
${}^2E_{15/2}$	$\lvert \frac{15}{2}\,\overline{\frac{15}{2}}\rangle$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle^\bullet$	1	± 16

T **19**.7 Matrix representations

Use T **19**.4 ♠. § **16**–7, p. 77

T **19**.8 Direct products of representations § **16**–8, p. 81

$\mathbf{S}_{16}$	A	B	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$	${}^1E_4$	${}^2E_4$	${}^1E_5$	${}^2E_5$	${}^1E_6$	${}^2E_6$	${}^1E_7$	${}^2E_7$
A	A	B	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$	${}^1E_4$	${}^2E_4$	${}^1E_5$	${}^2E_5$	${}^1E_6$	${}^2E_6$	${}^1E_7$	${}^2E_7$
B		A	${}^1E_7$	${}^2E_7$	${}^1E_6$	${}^2E_6$	${}^1E_5$	${}^2E_5$	${}^2E_4$	${}^1E_4$	${}^1E_3$	${}^2E_3$	${}^1E_2$	${}^2E_2$	${}^1E_1$	${}^2E_1$
${}^1E_1$			${}^1E_2$	A	${}^2E_3$	${}^2E_1$	${}^2E_2$	${}^1E_4$	${}^1E_5$	${}^1E_3$	${}^2E_6$	${}^2E_4$	${}^2E_5$	${}^2E_7$	${}^1E_6$	B
${}^2E_1$				${}^2E_2$	${}^1E_1$	${}^1E_3$	${}^2E_4$	${}^1E_2$	${}^2E_3$	${}^2E_5$	${}^1E_4$	${}^1E_6$	${}^1E_7$	${}^1E_5$	B	${}^2E_6$
${}^1E_2$					${}^1E_4$	A	${}^2E_1$	${}^1E_5$	${}^2E_6$	${}^2E_2$	${}^2E_7$	${}^1E_3$	${}^2E_4$	B	${}^2E_5$	${}^1E_7$
${}^2E_2$						${}^2E_4$	${}^2E_5$	${}^1E_1$	${}^1E_2$	${}^1E_6$	${}^2E_3$	${}^1E_7$	B	${}^1E_4$	${}^2E_7$	${}^1E_5$
${}^1E_3$							${}^1E_6$	A	${}^1E_1$	${}^1E_7$	${}^1E_2$	B	${}^2E_7$	${}^2E_3$	${}^2E_6$	${}^1E_4$
${}^2E_3$								${}^2E_6$	${}^2E_7$	${}^2E_1$	B	${}^2E_2$	${}^1E_3$	${}^1E_7$	${}^2E_4$	${}^1E_6$
${}^1E_4$									B	A	${}^1E_7$	${}^2E_1$	${}^2E_2$	${}^1E_6$	${}^1E_3$	${}^2E_5$
${}^2E_4$										B	${}^1E_1$	${}^2E_7$	${}^2E_6$	${}^1E_2$	${}^1E_5$	${}^2E_3$
${}^1E_5$											${}^1E_6$	A	${}^2E_1$	${}^2E_5$	${}^2E_2$	${}^2E_4$
${}^2E_5$												${}^2E_6$	${}^1E_5$	${}^1E_1$	${}^1E_4$	${}^1E_2$
${}^1E_6$													${}^1E_4$	A	${}^2E_3$	${}^1E_1$
${}^2E_6$														${}^2E_4$	${}^2E_1$	${}^1E_3$
${}^1E_7$															${}^1E_2$	A
${}^2E_7$																${}^2E_2$

$\twoheadrightarrow$

T **19**.8 Direct products of representations *(cont.)*

$\mathbf{S}_{16}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$
A	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$
B	${}^1E_{15/2}$	${}^2E_{15/2}$	${}^1E_{13/2}$	${}^2E_{13/2}$	${}^1E_{11/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$
${}^1E_1$	${}^2E_{15/2}$	${}^1E_{13/2}$	${}^2E_{11/2}$	${}^1E_{15/2}$	${}^2E_{13/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{11/2}$
${}^2E_1$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{15/2}$	${}^1E_{11/2}$	${}^2E_{9/2}$	${}^1E_{13/2}$	${}^2E_{11/2}$	${}^1E_{7/2}$
${}^1E_2$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{3/2}$
${}^2E_2$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{1/2}$	${}^2E_{11/2}$	${}^1E_{13/2}$
${}^1E_3$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{13/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{15/2}$	${}^2E_{15/2}$	${}^1E_{3/2}$
${}^2E_3$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{13/2}$	${}^2E_{15/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{15/2}$
${}^1E_4$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{1/2}$
${}^2E_4$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{11/2}$	${}^1E_{13/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{15/2}$
${}^1E_5$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{9/2}$	${}^2E_{11/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{13/2}$
${}^2E_5$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{11/2}$	${}^2E_{13/2}$	${}^1E_{1/2}$
${}^1E_6$	${}^1E_{13/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{15/2}$	${}^1E_{15/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{13/2}$
${}^2E_6$	${}^1E_{11/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$	${}^2E_{15/2}$	${}^1E_{13/2}$	${}^2E_{5/2}$
${}^1E_7$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{5/2}$
${}^2E_7$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{9/2}$
${}^1E_{1/2}$	${}^2E_7$	A	${}^1E_7$	${}^2E_2$	${}^1E_5$	${}^1E_2$	${}^2E_5$	${}^2E_4$
${}^2E_{1/2}$		${}^1E_7$	${}^1E_2$	${}^2E_7$	${}^2E_2$	${}^2E_5$	${}^1E_4$	${}^1E_5$
${}^1E_{3/2}$			${}^2E_5$	A	${}^2E_7$	${}^1E_4$	${}^1E_3$	${}^2E_2$
${}^2E_{3/2}$				${}^1E_5$	${}^2E_4$	${}^1E_7$	${}^1E_2$	${}^2E_3$
${}^1E_{5/2}$					${}^2E_3$	A	${}^1E_7$	${}^1E_6$
${}^2E_{5/2}$						${}^1E_3$	${}^2E_6$	${}^2E_7$
${}^1E_{7/2}$							${}^2E_1$	A
${}^2E_{7/2}$								${}^1E_1$

$\twoheadrightarrow$

T 19.8 Direct products of representations *(cont.)*

$\mathbf{S}_{16}$	${}^1E_{9/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{11/2}$	${}^1E_{13/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{15/2}$
A	${}^1E_{9/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{11/2}$	${}^1E_{13/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{15/2}$
B	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_1$	${}^2E_{9/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$
${}^2E_1$	${}^2E_{5/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^1E_2$	${}^1E_{5/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{15/2}$	${}^1E_{13/2}$	${}^2E_{11/2}$
${}^2E_2$	${}^1E_{13/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{15/2}$	${}^1E_{15/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{13/2}$
${}^1E_3$	${}^2E_{1/2}$	${}^1E_{13/2}$	${}^2E_{11/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$
${}^2E_3$	${}^2E_{13/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{11/2}$	${}^2E_{9/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$
${}^1E_4$	${}^1E_{1/2}$	${}^2E_{15/2}$	${}^1E_{13/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$
${}^2E_4$	${}^1E_{15/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{13/2}$	${}^1E_{11/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$
${}^1E_5$	${}^2E_{15/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{15/2}$	${}^2E_{13/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$
${}^2E_5$	${}^2E_{3/2}$	${}^1E_{15/2}$	${}^2E_{15/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{13/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$
${}^1E_6$	${}^1E_{11/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$
${}^2E_6$	${}^1E_{3/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$
${}^1E_7$	${}^2E_{7/2}$	${}^1E_{11/2}$	${}^2E_{13/2}$	${}^1E_{9/2}$	${}^2E_{11/2}$	${}^1E_{15/2}$	${}^2E_{15/2}$	${}^1E_{13/2}$
${}^2E_7$	${}^2E_{11/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{13/2}$	${}^2E_{15/2}$	${}^1E_{11/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$
${}^1E_{1/2}$	${}^2E_3$	${}^1E_4$	${}^1E_3$	${}^1E_6$	${}^1E_1$	${}^2E_6$	${}^2E_1$	B
${}^2E_{1/2}$	${}^2E_4$	${}^1E_3$	${}^2E_6$	${}^2E_3$	${}^1E_6$	${}^2E_1$	B	${}^1E_1$
${}^1E_{3/2}$	${}^1E_5$	${}^2E_6$	${}^2E_1$	${}^2E_4$	${}^2E_3$	B	${}^1E_1$	${}^1E_6$
${}^2E_{3/2}$	${}^1E_6$	${}^2E_5$	${}^1E_4$	${}^1E_1$	B	${}^1E_3$	${}^2E_6$	${}^2E_1$
${}^1E_{5/2}$	${}^1E_1$	${}^1E_2$	${}^2E_5$	B	${}^2E_1$	${}^1E_4$	${}^1E_3$	${}^2E_6$
${}^2E_{5/2}$	${}^2E_2$	${}^2E_1$	B	${}^1E_5$	${}^2E_4$	${}^1E_1$	${}^1E_6$	${}^2E_3$
${}^1E_{7/2}$	${}^1E_7$	B	${}^1E_1$	${}^2E_2$	${}^1E_5$	${}^1E_6$	${}^2E_3$	${}^2E_4$
${}^2E_{7/2}$	B	${}^1E_7$	${}^1E_2$	${}^2E_1$	${}^2E_6$	${}^2E_5$	${}^1E_4$	${}^1E_3$
${}^1E_{9/2}$	${}^2E_1$	A	${}^1E_7$	${}^2E_6$	${}^1E_3$	${}^1E_2$	${}^2E_5$	${}^1E_4$
${}^2E_{9/2}$		${}^1E_1$	${}^1E_6$	${}^2E_7$	${}^2E_2$	${}^2E_3$	${}^2E_4$	${}^1E_5$
${}^1E_{11/2}$			${}^2E_3$	A	${}^2E_7$	${}^2E_4$	${}^1E_5$	${}^2E_2$
${}^2E_{11/2}$				${}^1E_3$	${}^1E_4$	${}^1E_7$	${}^1E_2$	${}^2E_5$
${}^1E_{13/2}$					${}^2E_5$	A	${}^1E_7$	${}^1E_2$
${}^2E_{13/2}$						${}^1E_5$	${}^2E_2$	${}^2E_7$
${}^1E_{15/2}$							${}^2E_7$	A
${}^2E_{15/2}$								${}^1E_7$

T 19.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{S}_{16}$	$\mathbf{C}_8$	$\mathbf{C}_4$	$\mathbf{C}_2$	$\mathbf{S}_{16}$	$\mathbf{C}_8$	$\mathbf{C}_4$	$\mathbf{C}_2$
A	A	A	A	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
B	A	A	A	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_1$	${}^1E_1$	${}^1E$	B	${}^1E_{3/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_1$	${}^2E_1$	${}^2E$	B	${}^2E_{3/2}$	${}^2E_{3/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_2$	${}^1E_2$	B	A	${}^1E_{5/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_2$	${}^2E_2$	B	A	${}^2E_{5/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_3$	${}^1E_3$	${}^1E$	B	${}^1E_{7/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_3$	${}^2E_3$	${}^2E$	B	${}^2E_{7/2}$	${}^2E_{7/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_4$	B	A	A	${}^1E_{9/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_4$	B	A	A	${}^2E_{9/2}$	${}^2E_{7/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_5$	${}^1E_3$	${}^1E$	B	${}^1E_{11/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_5$	${}^2E_3$	${}^2E$	B	${}^2E_{11/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_6$	${}^1E_2$	B	A	${}^1E_{13/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_6$	${}^2E_2$	B	A	${}^2E_{13/2}$	${}^2E_{3/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_7$	${}^1E_1$	${}^1E$	B	${}^1E_{15/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_7$	${}^2E_1$	${}^2E$	B	${}^2E_{15/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$

T **19**.10 Subduction from O(3) § **16**–10, p. 82

j	$\mathbf{S}_{16}$
$16n$	$(2n+1)\,A \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7)$
$16n+1$	$2n\,(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1)$
$16n+2$	$(2n+1)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_7 \oplus {}^2E_7) \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6)$
$16n+3$	$2n\,(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_7 \oplus {}^2E_7) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_6 \oplus {}^2E_6)$
$16n+4$	$(2n+1)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_7 \oplus {}^2E_7) \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_6 \oplus {}^2E_6)$
$16n+5$	$2n\,(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_7 \oplus {}^2E_7) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6)$
$16n+6$	$(2n+1)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7) \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1)$
$16n+7$	$2n\,A \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7)$
$16n+8$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7) \oplus (2n+2)\,B$
$16n+9$	$(2n+2)(A \oplus {}^1E_7 \oplus {}^2E_7) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6)$
$16n+10$	$(2n+1)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_7 \oplus {}^2E_7) \oplus (2n+2)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_6 \oplus {}^2E_6)$
$16n+11$	$(2n+2)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_7 \oplus {}^2E_7) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_6 \oplus {}^2E_6)$
$16n+12$	$(2n+1)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_7 \oplus {}^2E_7) \oplus (2n+2)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_6 \oplus {}^2E_6)$
$16n+13$	$(2n+2)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_7 \oplus {}^2E_7) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_6 \oplus {}^2E_6)$
$16n+14$	$(2n+1)(A \oplus {}^1E_7 \oplus {}^2E_7) \oplus (2n+2)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6)$
$16n+15$	$(2n+2)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7) \oplus (2n+1)\,B$

$n = 0, 1, 2, \dots$ ↠

T 19.10 Subduction from O(3) *(cont.)*

j	$\mathbf{S}_{16}$
$16n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus 2n\,({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2})$
$16n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus 2n\,({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2})$
$16n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus 2n\,({}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2})$
$16n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}) \oplus 2n\,({}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2})$
$16n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2}) \oplus 2n\,({}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2})$
$16n+\frac{11}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2}) \oplus 2n\,({}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2})$
$16n+\frac{13}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2}) \oplus 2n\,({}^1E_{15/2} \oplus {}^2E_{15/2})$
$16n+\frac{15}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2})$
$16n+\frac{17}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2}) \oplus (2n+2)({}^1E_{15/2} \oplus {}^2E_{15/2})$
$16n+\frac{19}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2}) \oplus (2n+2)({}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2})$
$16n+\frac{21}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2}) \oplus (2n+2)({}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2})$
$16n+\frac{23}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}) \oplus (2n+2)({}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2})$
$16n+\frac{25}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus (2n+2)({}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2})$
$16n+\frac{27}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus (2n+2)({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2})$
$16n+\frac{29}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus (2n+2)({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2})$
$16n+\frac{31}{2}$	$(2n+2)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2})$

$n = 0, 1, 2, \ldots$

T 19.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: $\mathbf{C}_9 \otimes \mathbf{C}_i$.

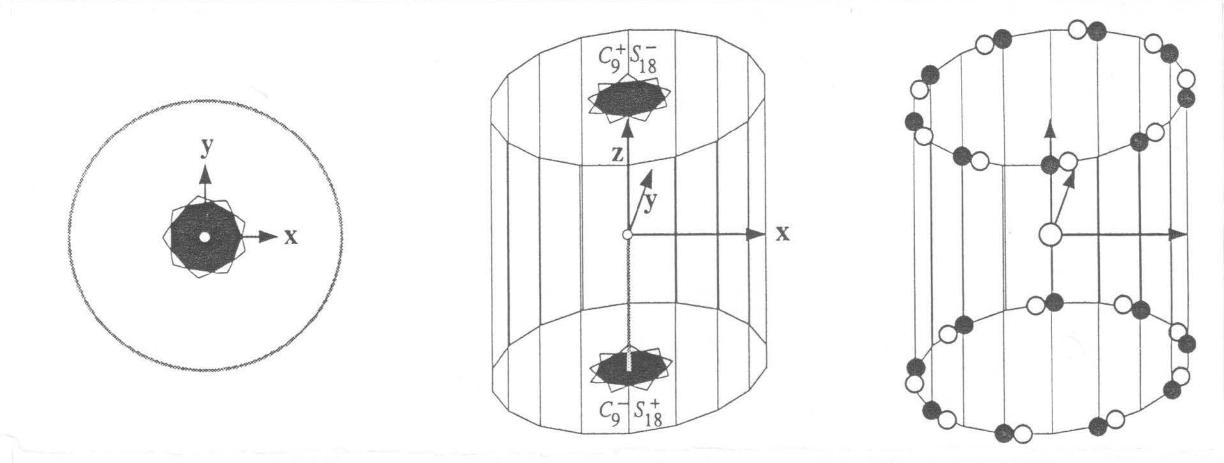
(2) Group chains: $\mathbf{D}_{9d} \supset \underline{\mathbf{S}_{18}} \supset \underline{\mathbf{S}_6}$, $\mathbf{D}_{9d} \supset \underline{\mathbf{S}_{18}} \supset \underline{\mathbf{C}_9}$.

(3) Operations of G: E, C_9^+, C_9^{2+}, C_3^+, C_9^{4+}, C_9^{4-}, C_3^-, C_9^{2-}, C_9^-, i, S_{18}^{7-}, S_{18}^{5-}, S_6^-, S_{18}^-, S_{18}^+, S_6^+, S_{18}^{5+}, S_{18}^{7+}.

(4) Operations of $\widetilde{G}$: E, C_9^+, C_9^{2+}, C_3^+, C_9^{4+}, C_9^{4-}, C_3^-, C_9^{2-}, C_9^-, i, S_{18}^{7-}, S_{18}^{5-}, S_6^-, S_{18}^-, S_{18}^+, S_6^+, S_{18}^{5+}, S_{18}^{7+},
$\widetilde{E}$, $\widetilde{C}_9^+$, $\widetilde{C}_9^{2+}$, $\widetilde{C}_3^+$, $\widetilde{C}_9^{4+}$, $\widetilde{C}_9^{4-}$, $\widetilde{C}_3^-$, $\widetilde{C}_9^{2-}$, $\widetilde{C}_9^-$, $\widetilde{i}$, $\widetilde{S}_{18}^{7-}$, $\widetilde{S}_{18}^{5-}$, $\widetilde{S}_6^-$, $\widetilde{S}_{18}^-$, $\widetilde{S}_{18}^+$, $\widetilde{S}_6^+$, $\widetilde{S}_{18}^{5+}$, $\widetilde{S}_{18}^{7+}$.

(5) Classes and representations: $|r| = 18$, $|i| = 0$, $|I| = 18$, $|\widetilde{I}| = 18$.

F **20** See Chapter **15**, p. 65



Examples:

T **20**.1 Parameters
Use T **48**.1. § **16**–1, p. 68

T **20**.2 Multiplication table
Use T **48**.2. § **16**–2, p. 69

T **20**.3 Factor table
Use T **48**.3. § **16**–3, p. 70

T **20**.4 Character table

§ **16**–4, p. 71

$\mathbf{S}_{18}$	E	C_9^+	C_9^{2+}	C_3^+	C_9^{4+}	C_9^{4-}	C_3^-	C_9^{2-}	C_9^-	τ
A_g	1	1	1	1	1	1	1	1	1	a
${}^1E_{1g}$	1	δ^*	ϵ^*	η^*	θ^*	θ	η	ϵ	δ	b
${}^2E_{1g}$	1	δ	ϵ	η	θ	θ^*	η^*	ϵ^*	δ^*	b
${}^1E_{2g}$	1	ϵ^*	θ^*	η	δ	δ^*	η^*	θ	ϵ	b
${}^2E_{2g}$	1	ϵ	θ	η^*	δ^*	δ	η	θ^*	ϵ^*	b
${}^1E_{3g}$	1	η^*	η	1	η^*	η	1	η^*	η	b
${}^2E_{3g}$	1	η	η^*	1	η	η^*	1	η	η^*	b
${}^1E_{4g}$	1	θ^*	δ	η^*	ϵ	ϵ^*	η	δ^*	θ	b
${}^2E_{4g}$	1	θ	δ^*	η	ϵ^*	ϵ	η^*	δ	θ^*	b
A_u	1	1	1	1	1	1	1	1	1	a
${}^1E_{1u}$	1	δ^*	ϵ^*	η^*	θ^*	θ	η	ϵ	δ	b
${}^2E_{1u}$	1	δ	ϵ	η	θ	θ^*	η^*	ϵ^*	δ^*	b
${}^1E_{2u}$	1	ϵ^*	θ^*	η	δ	δ^*	η^*	θ	ϵ	b
${}^2E_{2u}$	1	ϵ	θ	η^*	δ^*	δ	η	θ^*	ϵ^*	b
${}^1E_{3u}$	1	η^*	η	1	η^*	η	1	η^*	η	b
${}^2E_{3u}$	1	η	η^*	1	η	η^*	1	η	η^*	b
${}^1E_{4u}$	1	θ^*	δ	η^*	ϵ	ϵ^*	η	δ^*	θ	b
${}^2E_{4u}$	1	θ	δ^*	η	ϵ^*	ϵ	η^*	δ	θ^*	b
${}^1E_{1/2,g}$	1	$-\theta^*$	δ	$-\eta^*$	ϵ	ϵ^*	$-\eta$	δ^*	$-\theta$	b
${}^2E_{1/2,g}$	1	$-\theta$	δ^*	$-\eta$	ϵ^*	ϵ	$-\eta^*$	δ	$-\theta^*$	b
${}^1E_{3/2,g}$	1	$-\eta$	η^*	-1	η	η^*	-1	η	$-\eta^*$	b
${}^2E_{3/2,g}$	1	$-\eta^*$	η	-1	η^*	η	-1	η^*	$-\eta$	b
${}^1E_{5/2,g}$	1	$-\epsilon^*$	θ^*	$-\eta$	δ	δ^*	$-\eta^*$	θ	$-\epsilon$	b
${}^2E_{5/2,g}$	1	$-\epsilon$	θ	$-\eta^*$	δ^*	δ	$-\eta$	θ^*	$-\epsilon^*$	b
${}^1E_{7/2,g}$	1	$-\delta$	ϵ	$-\eta$	θ	θ^*	$-\eta^*$	ϵ^*	$-\delta^*$	b
${}^2E_{7/2,g}$	1	$-\delta^*$	ϵ^*	$-\eta^*$	θ^*	θ	$-\eta$	ϵ	$-\delta$	b
$A_{9/2,g}$	1	-1	1	-1	1	1	-1	1	-1	a
${}^1E_{1/2,u}$	1	$-\theta^*$	δ	$-\eta^*$	ϵ	ϵ^*	$-\eta$	δ^*	$-\theta$	b
${}^2E_{1/2,u}$	1	$-\theta$	δ^*	$-\eta$	ϵ^*	ϵ	$-\eta^*$	δ	$-\theta^*$	b
${}^1E_{3/2,u}$	1	$-\eta$	η^*	-1	η	η^*	-1	η	$-\eta^*$	b
${}^2E_{3/2,u}$	1	$-\eta^*$	η	-1	η^*	η	-1	η^*	$-\eta$	b
${}^1E_{5/2,u}$	1	$-\epsilon^*$	θ^*	$-\eta$	δ	δ^*	$-\eta^*$	θ	$-\epsilon$	b
${}^2E_{5/2,u}$	1	$-\epsilon$	θ	$-\eta^*$	δ^*	δ	$-\eta$	θ^*	$-\epsilon^*$	b
${}^1E_{7/2,u}$	1	$-\delta$	ϵ	$-\eta$	θ	θ^*	$-\eta^*$	ϵ^*	$-\delta^*$	b
${}^2E_{7/2,u}$	1	$-\delta^*$	ϵ^*	$-\eta^*$	θ^*	θ	$-\eta$	ϵ	$-\delta$	b
$A_{9/2,u}$	1	-1	1	-1	1	1	-1	1	-1	a

$\delta = \exp(2\pi i/9)$, $\epsilon = \exp(4\pi i/9)$, $\eta = \exp(6\pi i/9)$, $\theta = \exp(8\pi i/9)$ $\twoheadrightarrow$

T **20**.4 Character table *(cont.)*

$\mathbf{S}_{18}$	i	S_{18}^{7-}	S_{18}^{5-}	S_6^-	S_{18}^-	S_{18}^+	S_6^+	S_{18}^{5+}	S_{18}^{7+}	τ
A_g	1	1	1	1	1	1	1	1	1	a
${}^1E_{1g}$	1	δ^*	ϵ^*	η^*	θ^*	θ	η	ϵ	δ	b
${}^2E_{1g}$	1	δ	ϵ	η	θ	θ^*	η^*	ϵ^*	δ^*	b
${}^1E_{2g}$	1	ϵ^*	θ^*	η	δ	δ^*	η^*	θ	ϵ	b
${}^2E_{2g}$	1	ϵ	θ	η^*	δ^*	δ	η	θ^*	ϵ^*	b
${}^1E_{3g}$	1	η^*	η	1	η^*	η	1	η^*	η	b
${}^2E_{3g}$	1	η	η^*	1	η	η^*	1	η	η^*	b
${}^1E_{4g}$	1	θ^*	δ	η^*	ϵ	ϵ^*	η	δ^*	θ	b
${}^2E_{4g}$	1	θ	δ^*	η	ϵ^*	ϵ	η^*	δ	θ^*	b
A_u	-1	-1	-1	-1	-1	-1	-1	-1	-1	a
${}^1E_{1u}$	-1	$-\delta^*$	$-\epsilon^*$	$-\eta^*$	$-\theta^*$	$-\theta$	$-\eta$	$-\epsilon$	$-\delta$	b
${}^2E_{1u}$	-1	$-\delta$	$-\epsilon$	$-\eta$	$-\theta$	$-\theta^*$	$-\eta^*$	$-\epsilon^*$	$-\delta^*$	b
${}^1E_{2u}$	-1	$-\epsilon^*$	$-\theta^*$	$-\eta$	$-\delta$	$-\delta^*$	$-\eta^*$	$-\theta$	$-\epsilon$	b
${}^2E_{2u}$	-1	$-\epsilon$	$-\theta$	$-\eta^*$	$-\delta^*$	$-\delta$	$-\eta$	$-\theta^*$	$-\epsilon^*$	b
${}^1E_{3u}$	-1	$-\eta^*$	$-\eta$	-1	$-\eta^*$	$-\eta$	-1	$-\eta^*$	$-\eta$	b
${}^2E_{3u}$	-1	$-\eta$	$-\eta^*$	-1	$-\eta$	$-\eta^*$	-1	$-\eta$	$-\eta^*$	b
${}^1E_{4u}$	-1	$-\theta^*$	$-\delta$	$-\eta^*$	$-\epsilon$	$-\epsilon^*$	$-\eta$	$-\delta^*$	$-\theta$	b
${}^2E_{4u}$	-1	$-\theta$	$-\delta^*$	$-\eta$	$-\epsilon^*$	$-\epsilon$	$-\eta^*$	$-\delta$	$-\theta^*$	b
${}^1E_{1/2,g}$	1	$-\theta^*$	δ	$-\eta^*$	ϵ	ϵ^*	$-\eta$	δ^*	$-\theta$	b
${}^2E_{1/2,g}$	1	$-\theta$	δ^*	$-\eta$	ϵ^*	ϵ	$-\eta^*$	δ	$-\theta^*$	b
${}^1E_{3/2,g}$	1	$-\eta$	η^*	-1	η	η^*	-1	η	$-\eta^*$	b
${}^2E_{3/2,g}$	1	$-\eta^*$	η	-1	η^*	η	-1	η^*	$-\eta$	b
${}^1E_{5/2,g}$	1	$-\epsilon^*$	θ^*	$-\eta$	δ	δ^*	$-\eta^*$	θ	$-\epsilon$	b
${}^2E_{5/2,g}$	1	$-\epsilon$	θ	$-\eta^*$	δ^*	δ	$-\eta$	θ^*	$-\epsilon^*$	b
${}^1E_{7/2,g}$	1	$-\delta$	ϵ	$-\eta$	θ	θ^*	$-\eta^*$	ϵ^*	$-\delta^*$	b
${}^2E_{7/2,g}$	1	$-\delta^*$	ϵ^*	$-\eta^*$	θ^*	θ	$-\eta$	ϵ	$-\delta$	b
$A_{9/2,g}$	1	-1	1	-1	1	1	-1	1	-1	a
${}^1E_{1/2,u}$	-1	θ^*	$-\delta$	η^*	$-\epsilon$	$-\epsilon^*$	η	$-\delta^*$	θ	b
${}^2E_{1/2,u}$	-1	θ	$-\delta^*$	η	$-\epsilon^*$	$-\epsilon$	η^*	$-\delta$	θ^*	b
${}^1E_{3/2,u}$	-1	η	$-\eta^*$	1	$-\eta$	$-\eta^*$	1	$-\eta$	η^*	b
${}^2E_{3/2,u}$	-1	η^*	$-\eta$	1	$-\eta^*$	$-\eta$	1	$-\eta^*$	η	b
${}^1E_{5/2,u}$	-1	ϵ^*	$-\theta^*$	η	$-\delta$	$-\delta^*$	η^*	$-\theta$	ϵ	b
${}^2E_{5/2,u}$	-1	ϵ	$-\theta$	η^*	$-\delta^*$	$-\delta$	η	$-\theta^*$	ϵ^*	b
${}^1E_{7/2,u}$	-1	δ	$-\epsilon$	η	$-\theta$	$-\theta^*$	η^*	$-\epsilon^*$	δ^*	b
${}^2E_{7/2,u}$	-1	δ^*	$-\epsilon^*$	η^*	$-\theta^*$	$-\theta$	η	$-\epsilon$	δ	b
$A_{9/2,u}$	-1	1	-1	1	-1	-1	1	-1	1	a

$\delta = \exp(2\pi i/9)$, $\epsilon = \exp(4\pi i/9)$, $\eta = \exp(6\pi i/9)$, $\theta = \exp(8\pi i/9)$

T **20**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions § **16**–5, p. 72

$\mathbf{S}_{18}$	0	1	2	3
A_g	$^{\square}1$	R_z	$x^2+y^2, {}^{\square}z^2$	
${}^1E_{1g} \oplus {}^2E_{1g}$		(R_x, R_y)	${}^{\square}(zx, yz)$	
${}^1E_{2g} \oplus {}^2E_{2g}$			${}^{\square}(xy, x^2-y^2)$	
${}^1E_{3g} \oplus {}^2E_{3g}$				
${}^1E_{4g} \oplus {}^2E_{4g}$				
A_u		${}^{\square}z$		$(x^2+y^2)z, {}^{\square}z^3$
${}^1E_{1u} \oplus {}^2E_{1u}$		${}^{\square}(x, y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^{\square}(xz^2, yz^2)$
${}^1E_{2u} \oplus {}^2E_{2u}$				${}^{\square}\{xyz, z(x^2-y^2)\}$
${}^1E_{3u} \oplus {}^2E_{3u}$				${}^{\square}\{x(x^2-3y^2), y(3x^2-y^2)\}$
${}^1E_{4u} \oplus {}^2E_{4u}$				

T **20**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{S}_{18}$	$\lvert j\, m\rangle$	ι	μ	$\mathbf{S}_{18}$	$\lvert j\, m\rangle$	ι	μ
A_g	$\lvert 0\, 0\rangle$	2	± 9	${}^1E_{1/2,g}$	$\lvert \tfrac12\, \overline{\tfrac12}\rangle$	1	± 9
${}^1E_{1g}$	$\lvert 2\, 1\rangle$	2	± 9	${}^2E_{1/2,g}$	$\lvert \tfrac12\, \tfrac12\rangle$	1	± 9
${}^2E_{1g}$	$\lvert 2\, \bar{1}\rangle$	2	± 9	${}^1E_{3/2,g}$	$\lvert \tfrac32\, \tfrac32\rangle$	1	± 9
${}^1E_{2g}$	$\lvert 2\, 2\rangle$	2	± 9	${}^2E_{3/2,g}$	$\lvert \tfrac32\, \overline{\tfrac32}\rangle$	1	± 9
${}^2E_{2g}$	$\lvert 2\, \bar{2}\rangle$	2	± 9	${}^1E_{5/2,g}$	$\lvert \tfrac52\, \overline{\tfrac52}\rangle$	1	± 9
${}^1E_{3g}$	$\lvert 4\, 3\rangle$	2	± 9	${}^2E_{5/2,g}$	$\lvert \tfrac52\, \tfrac52\rangle$	1	± 9
${}^2E_{3g}$	$\lvert 4\, \bar{3}\rangle$	2	± 9	${}^1E_{7/2,g}$	$\lvert \tfrac72\, \tfrac72\rangle$	1	± 9
${}^1E_{4g}$	$\lvert 4\, 4\rangle$	2	± 9	${}^2E_{7/2,g}$	$\lvert \tfrac72\, \overline{\tfrac72}\rangle$	1	± 9
${}^2E_{4g}$	$\lvert 4\, \bar{4}\rangle$	2	± 9	$A_{9/2,g}$	$\lvert \tfrac92\, \tfrac92\rangle$	1	± 9
A_u	$\lvert 1\, 0\rangle$	2	± 9	${}^1E_{1/2,u}$	$\lvert \tfrac12\, \overline{\tfrac12}\rangle^\bullet$	1	± 9
${}^1E_{1u}$	$\lvert 1\, 1\rangle$	2	± 9	${}^2E_{1/2,u}$	$\lvert \tfrac12\, \tfrac12\rangle^\bullet$	1	± 9
${}^2E_{1u}$	$\lvert 1\, \bar{1}\rangle$	2	± 9	${}^1E_{3/2,u}$	$\lvert \tfrac32\, \tfrac32\rangle^\bullet$	1	± 9
${}^1E_{2u}$	$\lvert 3\, 2\rangle$	2	± 9	${}^2E_{3/2,u}$	$\lvert \tfrac32\, \overline{\tfrac32}\rangle^\bullet$	1	± 9
${}^2E_{2u}$	$\lvert 3\, \bar{2}\rangle$	2	± 9	${}^1E_{5/2,u}$	$\lvert \tfrac52\, \overline{\tfrac52}\rangle^\bullet$	1	± 9
${}^1E_{3u}$	$\lvert 3\, 3\rangle$	2	± 9	${}^2E_{5/2,u}$	$\lvert \tfrac52\, \tfrac52\rangle^\bullet$	1	± 9
${}^2E_{3u}$	$\lvert 3\, \bar{3}\rangle$	2	± 9	${}^1E_{7/2,u}$	$\lvert \tfrac72\, \tfrac72\rangle^\bullet$	1	± 9
${}^1E_{4u}$	$\lvert 5\, 4\rangle$	2	± 9	${}^2E_{7/2,u}$	$\lvert \tfrac72\, \overline{\tfrac72}\rangle^\bullet$	1	± 9
${}^2E_{4u}$	$\lvert 5\, \bar{4}\rangle$	2	± 9	$A_{9/2,u}$	$\lvert \tfrac92\, \tfrac92\rangle^\bullet$	1	± 9

T **20**.7 Matrix representations

Use T **20**.4 ♠. § **16**–7, p. 77

T **20**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{S}_{18}$	A_g	${}^1E_{1g}$	${}^2E_{1g}$	${}^1E_{2g}$	${}^2E_{2g}$	${}^1E_{3g}$	${}^2E_{3g}$	${}^1E_{4g}$	${}^2E_{4g}$
A_g	A_g	${}^1E_{1g}$	${}^2E_{1g}$	${}^1E_{2g}$	${}^2E_{2g}$	${}^1E_{3g}$	${}^2E_{3g}$	${}^1E_{4g}$	${}^2E_{4g}$
${}^1E_{1g}$		${}^1E_{2g}$	A_g	${}^1E_{3g}$	${}^2E_{1g}$	${}^1E_{4g}$	${}^2E_{2g}$	${}^2E_{4g}$	${}^2E_{3g}$
${}^2E_{1g}$			${}^2E_{2g}$	${}^1E_{1g}$	${}^2E_{3g}$	${}^1E_{2g}$	${}^2E_{4g}$	${}^1E_{3g}$	${}^1E_{4g}$
${}^1E_{2g}$				${}^1E_{4g}$	A_g	${}^2E_{4g}$	${}^2E_{1g}$	${}^2E_{3g}$	${}^2E_{2g}$
${}^2E_{2g}$					${}^2E_{4g}$	${}^1E_{1g}$	${}^1E_{4g}$	${}^1E_{2g}$	${}^1E_{3g}$
${}^1E_{3g}$						${}^2E_{3g}$	A_g	${}^2E_{2g}$	${}^2E_{1g}$
${}^2E_{3g}$							${}^1E_{3g}$	${}^1E_{1g}$	${}^1E_{2g}$
${}^1E_{4g}$								${}^2E_{1g}$	A_g
${}^2E_{4g}$									${}^1E_{1g}$

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T **20**.8 Direct products of representations *(cont.)*

$\mathbf{S}_{18}$	A_u	${}^1E_{1u}$	${}^2E_{1u}$	${}^1E_{2u}$	${}^2E_{2u}$	${}^1E_{3u}$	${}^2E_{3u}$	${}^1E_{4u}$	${}^2E_{4u}$
A_g	A_u	${}^1E_{1u}$	${}^2E_{1u}$	${}^1E_{2u}$	${}^2E_{2u}$	${}^1E_{3u}$	${}^2E_{3u}$	${}^1E_{4u}$	${}^2E_{4u}$
${}^1E_{1g}$	${}^1E_{1u}$	${}^1E_{2u}$	A_u	${}^1E_{3u}$	${}^2E_{1u}$	${}^1E_{4u}$	${}^2E_{2u}$	${}^2E_{4u}$	${}^2E_{3u}$
${}^2E_{1g}$	${}^2E_{1u}$	A_u	${}^2E_{2u}$	${}^1E_{1u}$	${}^2E_{3u}$	${}^1E_{2u}$	${}^2E_{4u}$	${}^1E_{3u}$	${}^1E_{4u}$
${}^1E_{2g}$	${}^1E_{2u}$	${}^1E_{3u}$	${}^1E_{1u}$	${}^1E_{4u}$	A_u	${}^2E_{4u}$	${}^2E_{1u}$	${}^2E_{3u}$	${}^2E_{2u}$
${}^2E_{2g}$	${}^2E_{2u}$	${}^2E_{1u}$	${}^2E_{3u}$	A_u	${}^2E_{4u}$	${}^1E_{1u}$	${}^1E_{4u}$	${}^1E_{2u}$	${}^1E_{3u}$
${}^1E_{3g}$	${}^1E_{3u}$	${}^1E_{4u}$	${}^1E_{2u}$	${}^2E_{4u}$	${}^1E_{1u}$	${}^2E_{3u}$	A_u	${}^2E_{2u}$	${}^2E_{1u}$
${}^2E_{3g}$	${}^2E_{3u}$	${}^2E_{2u}$	${}^2E_{4u}$	${}^2E_{1u}$	${}^1E_{4u}$	A_u	${}^1E_{3u}$	${}^1E_{1u}$	${}^1E_{2u}$
${}^1E_{4g}$	${}^1E_{4u}$	${}^2E_{4u}$	${}^1E_{3u}$	${}^2E_{3u}$	${}^1E_{2u}$	${}^2E_{2u}$	${}^1E_{1u}$	${}^2E_{1u}$	A_u
${}^2E_{4g}$	${}^2E_{4u}$	${}^2E_{3u}$	${}^1E_{4u}$	${}^2E_{2u}$	${}^1E_{3u}$	${}^2E_{1u}$	${}^1E_{2u}$	A_u	${}^1E_{1u}$
A_u	A_g	${}^1E_{1g}$	${}^2E_{1g}$	${}^1E_{2g}$	${}^2E_{2g}$	${}^1E_{3g}$	${}^2E_{3g}$	${}^1E_{4g}$	${}^2E_{4g}$
${}^1E_{1u}$		${}^1E_{2g}$	A_g	${}^1E_{3g}$	${}^2E_{1g}$	${}^1E_{4g}$	${}^2E_{2g}$	${}^2E_{4g}$	${}^2E_{3g}$
${}^2E_{1u}$			${}^2E_{2g}$	${}^1E_{1g}$	${}^2E_{3g}$	${}^1E_{2g}$	${}^2E_{4g}$	${}^1E_{3g}$	${}^1E_{4g}$
${}^1E_{2u}$				${}^1E_{4g}$	A_g	${}^2E_{4g}$	${}^2E_{1g}$	${}^2E_{3g}$	${}^2E_{2g}$
${}^2E_{2u}$					${}^2E_{4g}$	${}^1E_{1g}$	${}^1E_{4g}$	${}^1E_{2g}$	${}^1E_{3g}$
${}^1E_{3u}$						${}^2E_{3g}$	A_g	${}^2E_{2g}$	${}^2E_{1g}$
${}^2E_{3u}$							${}^1E_{3g}$	${}^1E_{1g}$	${}^1E_{2g}$
${}^1E_{4u}$								${}^2E_{1g}$	A_g
${}^2E_{4u}$									${}^1E_{1g}$

$\twoheadrightarrow$

T **20**.8 Direct products of representations *(cont.)*

$\mathbf{S}_{18}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$	${}^1E_{7/2,g}$	${}^2E_{7/2,g}$	$A_{9/2,g}$
A_g	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$	${}^1E_{7/2,g}$	${}^2E_{7/2,g}$	$A_{9/2,g}$
${}^1E_{1g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$	${}^1E_{7/2,g}$	$A_{9/2,g}$	${}^1E_{5/2,g}$	${}^2E_{7/2,g}$
${}^2E_{1g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{5/2,g}$	${}^2E_{7/2,g}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$	$A_{9/2,g}$	${}^1E_{7/2,g}$
${}^1E_{2g}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{7/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	$A_{9/2,g}$	${}^2E_{7/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$
${}^2E_{2g}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{7/2,g}$	$A_{9/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^1E_{7/2,g}$	${}^2E_{5/2,g}$
${}^1E_{3g}$	${}^2E_{5/2,g}$	${}^1E_{7/2,g}$	$A_{9/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$	${}^2E_{7/2,g}$	${}^1E_{5/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$
${}^2E_{3g}$	${}^2E_{7/2,g}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	$A_{9/2,g}$	${}^1E_{7/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^2E_{5/2,g}$	${}^1E_{3/2,g}$
${}^1E_{4g}$	${}^1E_{7/2,g}$	$A_{9/2,g}$	${}^2E_{7/2,g}$	${}^2E_{5/2,g}$	${}^1E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$
${}^2E_{4g}$	$A_{9/2,g}$	${}^2E_{7/2,g}$	${}^1E_{5/2,g}$	${}^1E_{7/2,g}$	${}^2E_{5/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$
A_u	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{5/2,u}$	${}^1E_{7/2,u}$	${}^2E_{7/2,u}$	$A_{9/2,u}$
${}^1E_{1u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$	${}^1E_{7/2,u}$	$A_{9/2,u}$	${}^1E_{5/2,u}$	${}^2E_{7/2,u}$
${}^2E_{1u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{5/2,u}$	${}^2E_{7/2,u}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$	$A_{9/2,u}$	${}^1E_{7/2,u}$
${}^1E_{2u}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{7/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	$A_{9/2,u}$	${}^2E_{7/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$
${}^2E_{2u}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{7/2,u}$	$A_{9/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^1E_{7/2,u}$	${}^2E_{5/2,u}$
${}^1E_{3u}$	${}^2E_{5/2,u}$	${}^1E_{7/2,u}$	$A_{9/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$	${}^2E_{7/2,u}$	${}^1E_{5/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$
${}^2E_{3u}$	${}^2E_{7/2,u}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	$A_{9/2,u}$	${}^1E_{7/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^2E_{5/2,u}$	${}^1E_{3/2,u}$
${}^1E_{4u}$	${}^1E_{7/2,u}$	$A_{9/2,u}$	${}^2E_{7/2,u}$	${}^2E_{5/2,u}$	${}^1E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$
${}^2E_{4u}$	$A_{9/2,u}$	${}^2E_{7/2,u}$	${}^1E_{5/2,u}$	${}^1E_{7/2,u}$	${}^2E_{5/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$
${}^1E_{1/2,g}$	${}^2E_{1g}$	A_g	${}^1E_{1g}$	${}^2E_{2g}$	${}^2E_{3g}$	${}^1E_{2g}$	${}^1E_{3g}$	${}^2E_{4g}$	${}^1E_{4g}$
${}^2E_{1/2,g}$		${}^1E_{1g}$	${}^1E_{2g}$	${}^2E_{1g}$	${}^2E_{2g}$	${}^1E_{3g}$	${}^1E_{4g}$	${}^2E_{3g}$	${}^2E_{4g}$
${}^1E_{3/2,g}$			${}^1E_{3g}$	A_g	${}^2E_{1g}$	${}^1E_{4g}$	${}^2E_{4g}$	${}^2E_{2g}$	${}^2E_{3g}$
${}^2E_{3/2,g}$				${}^2E_{3g}$	${}^2E_{4g}$	${}^1E_{1g}$	${}^1E_{2g}$	${}^1E_{4g}$	${}^1E_{3g}$
${}^1E_{5/2,g}$					${}^1E_{4g}$	A_g	${}^1E_{1g}$	${}^1E_{3g}$	${}^1E_{2g}$
${}^2E_{5/2,g}$						${}^2E_{4g}$	${}^2E_{3g}$	${}^2E_{1g}$	${}^2E_{2g}$
${}^1E_{7/2,g}$							${}^2E_{2g}$	A_g	${}^2E_{1g}$
${}^2E_{7/2,g}$								${}^1E_{2g}$	${}^1E_{1g}$
$A_{9/2,g}$									A_g

$\twoheadrightarrow$

T **20**.8 Direct products of representations *(cont.)*

$\mathbf{S}_{18}$	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$^1E_{3/2,u}$	$^2E_{3/2,u}$	$^1E_{5/2,u}$	$^2E_{5/2,u}$	$^1E_{7/2,u}$	$^2E_{7/2,u}$	$A_{9/2,u}$
A_g	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$^1E_{3/2,u}$	$^2E_{3/2,u}$	$^1E_{5/2,u}$	$^2E_{5/2,u}$	$^1E_{7/2,u}$	$^2E_{7/2,u}$	$A_{9/2,u}$
$^1E_{1g}$	$^2E_{1/2,u}$	$^1E_{3/2,u}$	$^2E_{5/2,u}$	$^1E_{1/2,u}$	$^2E_{3/2,u}$	$^1E_{7/2,u}$	$A_{9/2,u}$	$^1E_{5/2,u}$	$^2E_{7/2,u}$
$^2E_{1g}$	$^2E_{3/2,u}$	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$^1E_{5/2,u}$	$^2E_{7/2,u}$	$^1E_{3/2,u}$	$^2E_{5/2,u}$	$A_{9/2,u}$	$^1E_{7/2,u}$
$^1E_{2g}$	$^1E_{3/2,u}$	$^2E_{5/2,u}$	$^1E_{7/2,u}$	$^2E_{1/2,u}$	$^1E_{1/2,u}$	$A_{9/2,u}$	$^2E_{7/2,u}$	$^2E_{3/2,u}$	$^1E_{5/2,u}$
$^2E_{2g}$	$^1E_{5/2,u}$	$^2E_{3/2,u}$	$^1E_{1/2,u}$	$^2E_{7/2,u}$	$A_{9/2,u}$	$^2E_{1/2,u}$	$^1E_{3/2,u}$	$^1E_{7/2,u}$	$^2E_{5/2,u}$
$^1E_{3g}$	$^2E_{5/2,u}$	$^1E_{7/2,u}$	$A_{9/2,u}$	$^1E_{3/2,u}$	$^2E_{1/2,u}$	$^2E_{7/2,u}$	$^1E_{5/2,u}$	$^1E_{1/2,u}$	$^2E_{3/2,u}$
$^2E_{3g}$	$^2E_{7/2,u}$	$^1E_{5/2,u}$	$^2E_{3/2,u}$	$A_{9/2,u}$	$^1E_{7/2,u}$	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$^2E_{5/2,u}$	$^1E_{3/2,u}$
$^1E_{4g}$	$^1E_{7/2,u}$	$A_{9/2,u}$	$^2E_{7/2,u}$	$^2E_{5/2,u}$	$^1E_{3/2,u}$	$^1E_{5/2,u}$	$^2E_{3/2,u}$	$^2E_{1/2,u}$	$^1E_{1/2,u}$
$^2E_{4g}$	$A_{9/2,u}$	$^2E_{7/2,u}$	$^1E_{5/2,u}$	$^1E_{7/2,u}$	$^2E_{5/2,u}$	$^2E_{3/2,u}$	$^1E_{1/2,u}$	$^1E_{3/2,u}$	$^2E_{1/2,u}$
A_u	$^1E_{1/2,g}$	$^2E_{1/2,g}$	$^1E_{3/2,g}$	$^2E_{3/2,g}$	$^1E_{5/2,g}$	$^2E_{5/2,g}$	$^1E_{7/2,g}$	$^2E_{7/2,g}$	$A_{9/2,g}$
$^1E_{1u}$	$^2E_{1/2,g}$	$^1E_{3/2,g}$	$^2E_{5/2,g}$	$^1E_{1/2,g}$	$^2E_{3/2,g}$	$^1E_{7/2,g}$	$A_{9/2,g}$	$^1E_{5/2,g}$	$^2E_{7/2,g}$
$^2E_{1u}$	$^2E_{3/2,g}$	$^1E_{1/2,g}$	$^2E_{1/2,g}$	$^1E_{5/2,g}$	$^2E_{7/2,g}$	$^1E_{3/2,g}$	$^2E_{5/2,g}$	$A_{9/2,g}$	$^1E_{7/2,g}$
$^1E_{2u}$	$^1E_{3/2,g}$	$^2E_{5/2,g}$	$^1E_{7/2,g}$	$^2E_{1/2,g}$	$^1E_{1/2,g}$	$A_{9/2,g}$	$^2E_{7/2,g}$	$^2E_{3/2,g}$	$^1E_{5/2,g}$
$^2E_{2u}$	$^1E_{5/2,g}$	$^2E_{3/2,g}$	$^1E_{1/2,g}$	$^2E_{7/2,g}$	$A_{9/2,g}$	$^2E_{1/2,g}$	$^1E_{3/2,g}$	$^1E_{7/2,g}$	$^2E_{5/2,g}$
$^1E_{3u}$	$^2E_{5/2,g}$	$^1E_{7/2,g}$	$A_{9/2,g}$	$^1E_{3/2,g}$	$^2E_{1/2,g}$	$^2E_{7/2,g}$	$^1E_{5/2,g}$	$^1E_{1/2,g}$	$^2E_{3/2,g}$
$^2E_{3u}$	$^2E_{7/2,g}$	$^1E_{5/2,g}$	$^2E_{3/2,g}$	$A_{9/2,g}$	$^1E_{7/2,g}$	$^1E_{1/2,g}$	$^2E_{1/2,g}$	$^2E_{5/2,g}$	$^1E_{3/2,g}$
$^1E_{4u}$	$^1E_{7/2,g}$	$A_{9/2,g}$	$^2E_{7/2,g}$	$^2E_{5/2,g}$	$^1E_{3/2,g}$	$^1E_{5/2,g}$	$^2E_{3/2,g}$	$^2E_{1/2,g}$	$^1E_{1/2,g}$
$^2E_{4u}$	$A_{9/2,g}$	$^2E_{7/2,g}$	$^1E_{5/2,g}$	$^1E_{7/2,g}$	$^2E_{5/2,g}$	$^2E_{3/2,g}$	$^1E_{1/2,g}$	$^1E_{3/2,g}$	$^2E_{1/2,g}$
$^1E_{1/2,g}$	$^2E_{1u}$	A_u	$^1E_{1u}$	$^2E_{2u}$	$^2E_{3u}$	$^1E_{2u}$	$^1E_{3u}$	$^2E_{4u}$	$^1E_{4u}$
$^2E_{1/2,g}$	A_u	$^1E_{1u}$	$^1E_{2u}$	$^2E_{1u}$	$^2E_{2u}$	$^1E_{3u}$	$^1E_{4u}$	$^2E_{3u}$	$^2E_{4u}$
$^1E_{3/2,g}$	$^1E_{1u}$	$^1E_{2u}$	$^1E_{3u}$	A_u	$^2E_{1u}$	$^1E_{4u}$	$^2E_{4u}$	$^2E_{2u}$	$^2E_{3u}$
$^2E_{3/2,g}$	$^2E_{2u}$	$^2E_{1u}$	A_u	$^2E_{3u}$	$^2E_{4u}$	$^1E_{1u}$	$^1E_{2u}$	$^1E_{4u}$	$^1E_{3u}$
$^1E_{5/2,g}$	$^2E_{3u}$	$^2E_{2u}$	$^2E_{1u}$	$^2E_{4u}$	$^1E_{4u}$	A_u	$^1E_{1u}$	$^1E_{3u}$	$^1E_{2u}$
$^2E_{5/2,g}$	$^1E_{2u}$	$^1E_{3u}$	$^1E_{4u}$	$^1E_{1u}$	A_u	$^2E_{4u}$	$^2E_{3u}$	$^2E_{1u}$	$^2E_{2u}$
$^1E_{7/2,g}$	$^1E_{3u}$	$^1E_{4u}$	$^2E_{4u}$	$^1E_{2u}$	$^1E_{1u}$	$^2E_{3u}$	$^2E_{2u}$	A_u	$^2E_{1u}$
$^2E_{7/2,g}$	$^2E_{4u}$	$^2E_{3u}$	$^2E_{2u}$	$^1E_{4u}$	$^1E_{3u}$	$^2E_{1u}$	A_u	$^1E_{2u}$	$^1E_{1u}$
$A_{9/2,g}$	$^1E_{4u}$	$^2E_{4u}$	$^2E_{3u}$	$^1E_{3u}$	$^1E_{2u}$	$^2E_{2u}$	$^2E_{1u}$	$^1E_{1u}$	A_u
$^1E_{1/2,u}$	$^2E_{1g}$	A_g	$^1E_{1g}$	$^2E_{2g}$	$^2E_{3g}$	$^1E_{2g}$	$^1E_{3g}$	$^2E_{4g}$	$^1E_{4g}$
$^2E_{1/2,u}$		$^1E_{1g}$	$^1E_{2g}$	$^2E_{1g}$	$^2E_{2g}$	$^1E_{3g}$	$^1E_{4g}$	$^2E_{3g}$	$^2E_{4g}$
$^1E_{3/2,u}$			$^1E_{3g}$	A_g	$^2E_{1g}$	$^1E_{4g}$	$^2E_{4g}$	$^2E_{2g}$	$^2E_{3g}$
$^2E_{3/2,u}$				$^2E_{3g}$	$^2E_{4g}$	$^1E_{1g}$	$^1E_{2g}$	$^1E_{4g}$	$^1E_{3g}$
$^1E_{5/2,u}$					$^1E_{4g}$	A_g	$^1E_{1g}$	$^1E_{3g}$	$^1E_{2g}$
$^2E_{5/2,u}$						$^2E_{4g}$	$^2E_{3g}$	$^2E_{1g}$	$^2E_{2g}$
$^1E_{7/2,u}$							$^2E_{2g}$	A_g	$^2E_{1g}$
$^2E_{7/2,u}$								$^1E_{2g}$	$^1E_{1g}$
$A_{9/2,u}$									A_g

T **20**.9 Subduction
(descent of symmetry)
§ **16**–9, p. 82

$\mathbf{S}_{18}$	$\mathbf{S}_6$	$\mathbf{C}_i$	$\mathbf{C}_9$	$\mathbf{C}_3$
A_g	A_g	A_g	A	A
$^1E_{1g}$	^1E_g	A_g	$^1E_1$	^1E
$^2E_{1g}$	^2E_g	A_g	$^2E_1$	^2E
$^1E_{2g}$	^2E_g	A_g	$^1E_2$	^2E
$^2E_{2g}$	^1E_g	A_g	$^2E_2$	^1E
$^1E_{3g}$	A_g	A_g	$^1E_3$	A
$^2E_{3g}$	A_g	A_g	$^2E_3$	A
$^1E_{4g}$	^1E_g	A_g	$^1E_4$	^1E
$^2E_{4g}$	^2E_g	A_g	$^2E_4$	^2E
A_u	A_u	A_u	A	A
$^1E_{1u}$	^1E_u	A_u	$^1E_1$	^1E
$^2E_{1u}$	^2E_u	A_u	$^2E_1$	^2E
$^1E_{2u}$	^2E_u	A_u	$^1E_2$	^2E
$^2E_{2u}$	^1E_u	A_u	$^2E_2$	^1E
$^1E_{3u}$	A_u	A_u	$^1E_3$	A
$^2E_{3u}$	A_u	A_u	$^2E_3$	A
$^1E_{4u}$	^1E_u	A_u	$^1E_4$	^1E
$^2E_{4u}$	^2E_u	A_u	$^2E_4$	^2E
$^1E_{1/2,g}$	$^1E_{1/2,g}$	$A_{1/2,g}$	$^1E_{1/2}$	$^1E_{1/2}$
$^2E_{1/2,g}$	$^2E_{1/2,g}$	$A_{1/2,g}$	$^2E_{1/2}$	$^2E_{1/2}$
$^1E_{3/2,g}$	$A_{3/2,g}$	$A_{1/2,g}$	$^1E_{3/2}$	$A_{3/2}$
$^2E_{3/2,g}$	$A_{3/2,g}$	$A_{1/2,g}$	$^2E_{3/2}$	$A_{3/2}$
$^1E_{5/2,g}$	$^2E_{1/2,g}$	$A_{1/2,g}$	$^1E_{5/2}$	$^2E_{1/2}$
$^2E_{5/2,g}$	$^1E_{1/2,g}$	$A_{1/2,g}$	$^2E_{5/2}$	$^1E_{1/2}$
$^1E_{7/2,g}$	$^2E_{1/2,g}$	$A_{1/2,g}$	$^1E_{7/2}$	$^2E_{1/2}$
$^2E_{7/2,g}$	$^1E_{1/2,g}$	$A_{1/2,g}$	$^2E_{7/2}$	$^1E_{1/2}$
$A_{9/2,g}$	$A_{3/2,g}$	$A_{1/2,g}$	$A_{9/2}$	$A_{3/2}$
$^1E_{1/2,u}$	$^1E_{1/2,u}$	$A_{1/2,u}$	$^1E_{1/2}$	$^1E_{1/2}$
$^2E_{1/2,u}$	$^2E_{1/2,u}$	$A_{1/2,u}$	$^2E_{1/2}$	$^2E_{1/2}$
$^1E_{3/2,u}$	$A_{3/2,u}$	$A_{1/2,u}$	$^1E_{3/2}$	$A_{3/2}$
$^2E_{3/2,u}$	$A_{3/2,u}$	$A_{1/2,u}$	$^2E_{3/2}$	$A_{3/2}$
$^1E_{5/2,u}$	$^2E_{1/2,u}$	$A_{1/2,u}$	$^1E_{5/2}$	$^2E_{1/2}$
$^2E_{5/2,u}$	$^1E_{1/2,u}$	$A_{1/2,u}$	$^2E_{5/2}$	$^1E_{1/2}$
$^1E_{7/2,u}$	$^2E_{1/2,u}$	$A_{1/2,u}$	$^1E_{7/2}$	$^2E_{1/2}$
$^2E_{7/2,u}$	$^1E_{1/2,u}$	$A_{1/2,u}$	$^2E_{7/2}$	$^1E_{1/2}$
$A_{9/2,u}$	$A_{3/2,u}$	$A_{1/2,u}$	$A_{9/2}$	$A_{3/2}$

T **20**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{S}_{18}$
$18n$	$(4n+1)\,A_g \oplus 4n\,({}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g} \oplus {}^1E_{4g} \oplus {}^2E_{4g})$
$18n+1$	$(4n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u}) \oplus 4n\,({}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u} \oplus {}^1E_{4u} \oplus {}^2E_{4u})$
$18n+2$	$(4n+1)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g}) \oplus 4n\,({}^1E_{3g} \oplus {}^2E_{3g} \oplus {}^1E_{4g} \oplus {}^2E_{4g})$
$18n+3$	$(4n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u}) \oplus 4n\,({}^1E_{4u} \oplus {}^2E_{4u})$
$18n+4$	$(4n+1)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g} \oplus {}^1E_{4g} \oplus {}^2E_{4g})$
$18n+5$	$(4n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u}) \oplus (4n+2)({}^1E_{4u} \oplus {}^2E_{4u})$
$18n+6$	$(4n+1)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g}) \oplus (4n+2)({}^1E_{3g} \oplus {}^2E_{3g} \oplus {}^1E_{4g} \oplus {}^2E_{4g})$
$18n+7$	$(4n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u}) \oplus (4n+2)({}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u} \oplus {}^1E_{4u} \oplus {}^2E_{4u})$
$18n+8$	$(4n+1)\,A_g \oplus (4n+2)({}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g} \oplus {}^1E_{4g} \oplus {}^2E_{4g})$
$18n+9$	$(4n+3)\,A_u \oplus (4n+2)({}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u} \oplus {}^1E_{4u} \oplus {}^2E_{4u})$
$18n+10$	$(4n+3)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g}) \oplus (4n+2)({}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g} \oplus {}^1E_{4g} \oplus {}^2E_{4g})$
$18n+11$	$(4n+3)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u}) \oplus (4n+2)({}^1E_{3u} \oplus {}^2E_{3u} \oplus {}^1E_{4u} \oplus {}^2E_{4u})$
$18n+12$	$(4n+3)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g}) \oplus (4n+2)({}^1E_{4g} \oplus {}^2E_{4g})$
$18n+13$	$(4n+3)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u} \oplus {}^1E_{4u} \oplus {}^2E_{4u})$
$18n+14$	$(4n+3)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g}) \oplus (4n+4)({}^1E_{4g} \oplus {}^2E_{4g})$
$18n+15$	$(4n+3)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u}) \oplus (4n+4)({}^1E_{3u} \oplus {}^2E_{3u} \oplus {}^1E_{4u} \oplus {}^2E_{4u})$
$18n+16$	$(4n+3)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g}) \oplus (4n+4)({}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g} \oplus {}^1E_{4g} \oplus {}^2E_{4g})$
$18n+17$	$(4n+3)\,A_u \oplus (4n+4)({}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u} \oplus {}^1E_{4u} \oplus {}^2E_{4u})$
$9n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus 2n\,({}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g} \oplus A_{9/2,g})$
$9n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}) \oplus 2n\,({}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g} \oplus A_{9/2,g})$
$9n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g}) \oplus 2n\,({}^1E_{7/2,g} \oplus {}^2E_{7/2,g} \oplus A_{9/2,g})$
$9n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g}) \oplus 2n\,A_{9/2,g}$
$9n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g}) \oplus (2n+2)\,A_{9/2,g}$
$9n+\frac{11}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g}) \oplus (2n+2)({}^1E_{7/2,g} \oplus {}^2E_{7/2,g} \oplus A_{9/2,g})$
$9n+\frac{13}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}) \oplus (2n+2)({}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g} \oplus A_{9/2,g})$
$9n+\frac{15}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus (2n+2)({}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g} \oplus A_{9/2,g})$
$9n+\frac{17}{2}$	$(2n+2)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g} \oplus A_{9/2,g})$

$n = 0, 1, 2, \ldots$

T **20**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: none.

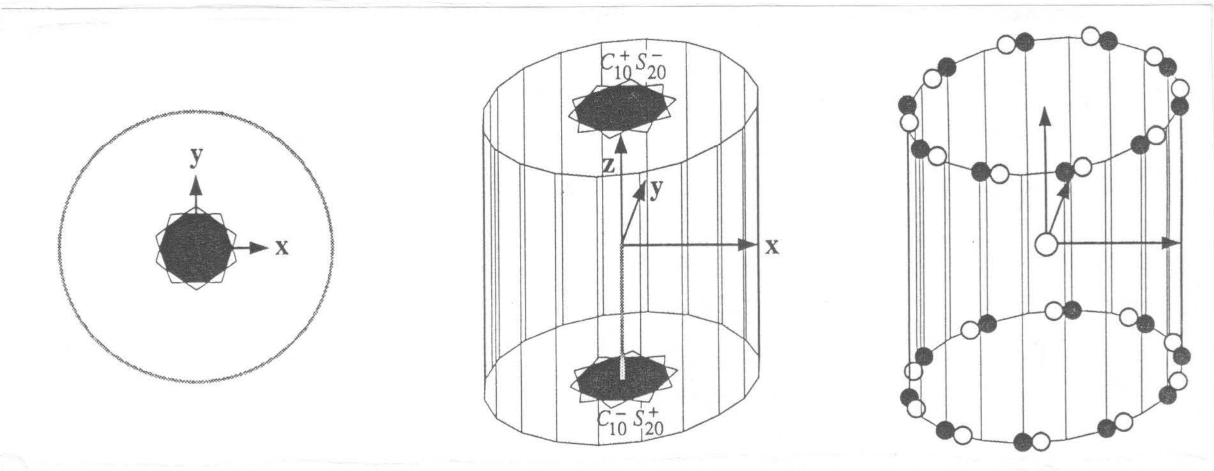
(2) Group chains: $\mathbf{D}_{10d} \supset \underline{\mathbf{S}_{20}} \supset \underline{\mathbf{S}_4}$, $\mathbf{D}_{10d} \supset \underline{\mathbf{S}_{20}} \supset \underline{\mathbf{C}_{10}}$.

(3) Operations of G: $E,\ S_{20}^{9-},\ C_{10}^{+},\ S_{20}^{7-},\ C_5^{+},\ S_4^{-},\ C_{10}^{3+},\ S_{20}^{3-},\ C_5^{2+},\ S_{20}^{-},$
$C_2,\ S_{20}^{+},\ C_5^{2-},\ S_{20}^{3+},\ C_{10}^{3-},\ S_4^{+},\ C_5^{-},\ S_{20}^{7+},\ C_{10}^{-},\ S_{20}^{9+}.$

(4) Operations of $\widetilde{G}$: $E,\ S_{20}^{9-},\ C_{10}^{+},\ S_{20}^{7-},\ C_5^{+},\ S_4^{-},\ C_{10}^{3+},\ S_{20}^{3-},\ C_5^{2+},\ S_{20}^{-},$
$C_2,\ S_{20}^{+},\ C_5^{2-},\ S_{20}^{3+},\ C_{10}^{3-},\ S_4^{+},\ C_5^{-},\ S_{20}^{7+},\ C_{10}^{-},\ S_{20}^{9+},$
$\widetilde{E},\ \widetilde{S}_{20}^{9-},\ \widetilde{C}_{10}^{+},\ \widetilde{S}_{20}^{7-},\ \widetilde{C}_5^{+},\ \widetilde{S}_4^{-},\ \widetilde{C}_{10}^{3+},\ \widetilde{S}_{20}^{3-},\ \widetilde{C}_5^{2+},\ \widetilde{S}_{20}^{-},$
$\widetilde{C}_2,\ \widetilde{S}_{20}^{+},\ \widetilde{C}_5^{2-},\ \widetilde{S}_{20}^{3+},\ \widetilde{C}_{10}^{3-},\ \widetilde{S}_4^{+},\ \widetilde{C}_5^{-},\ \widetilde{S}_{20}^{7+},\ \widetilde{C}_{10}^{-},\ \widetilde{S}_{20}^{9+}.$

(5) Classes and representations: $|r| = 20$, $|i| = 0$, $|I| = 20$, $|\widetilde{I}| = 20$.

F **21** See Chapter **15**, p. 65



Examples:

T **21**.1 Parameters
Use T **49**.1. § **16**–1, p. 68

T **21**.2 Multiplication table
Use T **49**.2. § **16**–2, p. 69

T **21**.3 Factor table
Use T **49**.3. § **16**–3, p. 70

T **21**.4 Character table

§ **16**–4, p. 71

$\mathbf{S}_{20}$	E	S_{20}^{9-}	C_{10}^{+}	S_{20}^{7-}	C_{5}^{+}	S_{4}^{-}	C_{10}^{3+}	S_{20}^{3-}	C_{5}^{2+}	S_{20}^{-}	τ
A	1	1	1	1	1	1	1	1	1	1	a
B	1	-1	1	-1	1	-1	1	-1	1	-1	a
${}^{1}E_{1}$	1	$\mathrm{i}\eta$	$-\theta$	$-\mathrm{i}\theta^*$	η^*	i	$-\eta$	$-\mathrm{i}\theta$	θ^*	$\mathrm{i}\eta^*$	b
${}^{2}E_{1}$	1	$-\mathrm{i}\eta^*$	$-\theta^*$	$\mathrm{i}\theta$	η	$-\mathrm{i}$	$-\eta^*$	$\mathrm{i}\theta^*$	θ	$-\mathrm{i}\eta$	b
${}^{1}E_{2}$	1	$-\theta$	η^*	$-\eta$	θ^*	-1	θ	$-\eta^*$	η	$-\theta^*$	b
${}^{2}E_{2}$	1	$-\theta^*$	η	$-\eta^*$	θ	-1	θ^*	$-\eta$	η^*	$-\theta$	b
${}^{1}E_{3}$	1	$\mathrm{i}\theta$	$-\eta^*$	$-\mathrm{i}\eta$	θ^*	i	$-\theta$	$-\mathrm{i}\eta^*$	η	$\mathrm{i}\theta^*$	b
${}^{2}E_{3}$	1	$-\mathrm{i}\theta^*$	$-\eta$	$\mathrm{i}\eta^*$	θ	$-\mathrm{i}$	$-\theta^*$	$\mathrm{i}\eta$	η^*	$-\mathrm{i}\theta$	b
${}^{1}E_{4}$	1	η	θ	θ^*	η^*	1	η	θ	θ^*	η^*	b
${}^{2}E_{4}$	1	η^*	θ^*	θ	η	1	η^*	θ^*	θ	η	b
${}^{1}E_{5}$	1	i	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	1	i	b
${}^{2}E_{5}$	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	b
${}^{1}E_{6}$	1	$-\eta$	θ	$-\theta^*$	η^*	-1	η	$-\theta$	θ^*	$-\eta^*$	b
${}^{2}E_{6}$	1	$-\eta^*$	θ^*	$-\theta$	η	-1	η^*	$-\theta^*$	θ	$-\eta$	b
${}^{1}E_{7}$	1	$-\mathrm{i}\theta$	$-\eta^*$	$\mathrm{i}\eta$	θ^*	$-\mathrm{i}$	$-\theta$	$\mathrm{i}\eta^*$	η	$-\mathrm{i}\theta^*$	b
${}^{2}E_{7}$	1	$\mathrm{i}\theta^*$	$-\eta$	$-\mathrm{i}\eta^*$	θ	i	$-\theta^*$	$-\mathrm{i}\eta$	η^*	$\mathrm{i}\theta$	b
${}^{1}E_{8}$	1	θ	η^*	η	θ^*	1	θ	η^*	η	θ^*	b
${}^{2}E_{8}$	1	θ^*	η	η^*	θ	1	θ^*	η	η^*	θ	b
${}^{1}E_{9}$	1	$-\mathrm{i}\eta$	$-\theta$	$\mathrm{i}\theta^*$	η^*	$-\mathrm{i}$	$-\eta$	$\mathrm{i}\theta$	θ^*	$-\mathrm{i}\eta^*$	b
${}^{2}E_{9}$	1	$\mathrm{i}\eta^*$	$-\theta^*$	$-\mathrm{i}\theta$	η	i	$-\eta^*$	$-\mathrm{i}\theta^*$	θ	$\mathrm{i}\eta$	b
${}^{1}E_{1/2}$	1	δ	$\mathrm{i}\eta^*$	ϵ	$-\theta^*$	ζ	$-\mathrm{i}\theta$	$\mathrm{i}\epsilon^*$	η	$\mathrm{i}\delta^*$	b
${}^{2}E_{1/2}$	1	δ^*	$-\mathrm{i}\eta$	ϵ^*	$-\theta$	ζ^*	$\mathrm{i}\theta^*$	$-\mathrm{i}\epsilon$	η^*	$-\mathrm{i}\delta$	b
${}^{1}E_{3/2}$	1	ϵ^*	$\mathrm{i}\theta^*$	$-\mathrm{i}\delta$	$-\eta$	$-\zeta$	$-\mathrm{i}\eta^*$	$-\delta^*$	θ	$\mathrm{i}\epsilon$	b
${}^{2}E_{3/2}$	1	ϵ	$-\mathrm{i}\theta$	$\mathrm{i}\delta^*$	$-\eta^*$	$-\zeta^*$	$\mathrm{i}\eta$	$-\delta$	θ^*	$-\mathrm{i}\epsilon^*$	b
${}^{1}E_{5/2}$	1	ζ	i	$-\zeta^*$	-1	$-\zeta$	$-\mathrm{i}$	ζ^*	1	ζ	b
${}^{2}E_{5/2}$	1	ζ^*	$-\mathrm{i}$	$-\zeta$	-1	$-\zeta^*$	i	ζ	1	ζ^*	b
${}^{1}E_{7/2}$	1	$-\mathrm{i}\epsilon$	$\mathrm{i}\theta$	$-\delta^*$	$-\eta^*$	ζ	$-\mathrm{i}\eta$	$-\mathrm{i}\delta$	θ^*	$-\epsilon^*$	b
${}^{2}E_{7/2}$	1	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\theta^*$	$-\delta$	$-\eta$	ζ^*	$\mathrm{i}\eta^*$	$\mathrm{i}\delta^*$	θ	$-\epsilon$	b
${}^{1}E_{9/2}$	1	$\mathrm{i}\delta^*$	$\mathrm{i}\eta$	$-\mathrm{i}\epsilon^*$	$-\theta$	ζ	$-\mathrm{i}\theta^*$	$-\epsilon$	η^*	δ	b
${}^{2}E_{9/2}$	1	$-\mathrm{i}\delta$	$-\mathrm{i}\eta^*$	$\mathrm{i}\epsilon$	$-\theta^*$	ζ^*	$\mathrm{i}\theta$	$-\epsilon^*$	η	δ^*	b
${}^{1}E_{11/2}$	1	$-\mathrm{i}\delta^*$	$\mathrm{i}\eta$	$\mathrm{i}\epsilon^*$	$-\theta$	$-\zeta$	$-\mathrm{i}\theta^*$	ϵ	η^*	$-\delta$	b
${}^{2}E_{11/2}$	1	$\mathrm{i}\delta$	$-\mathrm{i}\eta^*$	$-\mathrm{i}\epsilon$	$-\theta^*$	$-\zeta^*$	$\mathrm{i}\theta$	ϵ^*	η	$-\delta^*$	b
${}^{1}E_{13/2}$	1	$\mathrm{i}\epsilon$	$\mathrm{i}\theta$	δ^*	$-\eta^*$	$-\zeta$	$-\mathrm{i}\eta$	$\mathrm{i}\delta$	θ^*	ϵ^*	b
${}^{2}E_{13/2}$	1	$-\mathrm{i}\epsilon^*$	$-\mathrm{i}\theta^*$	δ	$-\eta$	$-\zeta^*$	$\mathrm{i}\eta^*$	$-\mathrm{i}\delta^*$	θ	ϵ	b
${}^{1}E_{15/2}$	1	$-\zeta$	i	ζ^*	-1	ζ	$-\mathrm{i}$	$-\zeta^*$	1	$-\zeta$	b
${}^{2}E_{15/2}$	1	$-\zeta^*$	$-\mathrm{i}$	ζ	-1	ζ^*	i	$-\zeta$	1	$-\zeta^*$	b
${}^{1}E_{17/2}$	1	$-\epsilon^*$	$\mathrm{i}\theta^*$	$\mathrm{i}\delta$	$-\eta$	ζ	$-\mathrm{i}\eta^*$	δ^*	θ	$-\mathrm{i}\epsilon$	b
${}^{2}E_{17/2}$	1	$-\epsilon$	$-\mathrm{i}\theta$	$-\mathrm{i}\delta^*$	$-\eta^*$	ζ^*	$\mathrm{i}\eta$	δ	θ^*	$\mathrm{i}\epsilon^*$	b
${}^{1}E_{19/2}$	1	$-\delta$	$\mathrm{i}\eta^*$	$-\epsilon$	$-\theta^*$	$-\zeta$	$-\mathrm{i}\theta$	$-\mathrm{i}\epsilon^*$	η	$-\mathrm{i}\delta^*$	b
${}^{2}E_{19/2}$	1	$-\delta^*$	$-\mathrm{i}\eta$	$-\epsilon^*$	$-\theta$	$-\zeta^*$	$\mathrm{i}\theta^*$	$\mathrm{i}\epsilon$	η^*	$\mathrm{i}\delta$	b

$\delta = \exp(2\pi\mathrm{i}/40)$, $\epsilon = \exp(6\pi\mathrm{i}/40)$, $\zeta = \exp(2\pi\mathrm{i}/8)$, $\eta = \exp(2\pi\mathrm{i}/5)$, $\theta = \exp(4\pi\mathrm{i}/5)$

$\twoheadrightarrow$

T **21**.4 Character table *(cont.)*

$\mathbf{S}_{20}$	C_2	S_{20}^{+}	C_5^{2-}	S_{20}^{3+}	C_{10}^{3-}	S_4^{+}	C_5^{-}	S_{20}^{7+}	C_{10}^{-}	S_{20}^{9+}	τ
A	1	1	1	1	1	1	1	1	1	1	a
B	1	-1	1	-1	1	-1	1	-1	1	-1	a
${}^1E_1$	-1	$-\mathrm{i}\eta$	θ	$\mathrm{i}\theta^*$	$-\eta^*$	$-\mathrm{i}$	η	$\mathrm{i}\theta$	$-\theta^*$	$-\mathrm{i}\eta^*$	b
${}^2E_1$	-1	$\mathrm{i}\eta^*$	θ^*	$-\mathrm{i}\theta$	$-\eta$	i	η^*	$-\mathrm{i}\theta^*$	$-\theta$	$\mathrm{i}\eta$	b
${}^1E_2$	1	$-\theta$	η^*	$-\eta$	θ^*	-1	θ	$-\eta^*$	η	$-\theta^*$	b
${}^2E_2$	1	$-\theta^*$	η	$-\eta^*$	θ	-1	θ^*	$-\eta$	η^*	$-\theta$	b
${}^1E_3$	-1	$-\mathrm{i}\theta$	η^*	$\mathrm{i}\eta$	$-\theta^*$	$-\mathrm{i}$	θ	$\mathrm{i}\eta^*$	$-\eta$	$-\mathrm{i}\theta^*$	b
${}^2E_3$	-1	$\mathrm{i}\theta^*$	η	$-\mathrm{i}\eta^*$	$-\theta$	i	θ^*	$-\mathrm{i}\eta$	$-\eta^*$	$\mathrm{i}\theta$	b
${}^1E_4$	1	η	θ	θ^*	η^*	1	η	θ	θ^*	η^*	b
${}^2E_4$	1	η^*	θ^*	θ	η	1	η^*	θ^*	θ	η	b
${}^1E_5$	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	b
${}^2E_5$	-1	i	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	-1	i	b
${}^1E_6$	1	$-\eta$	θ	$-\theta^*$	η^*	-1	η	$-\theta$	θ^*	$-\eta^*$	b
${}^2E_6$	1	$-\eta^*$	θ^*	$-\theta$	η	-1	η^*	$-\theta^*$	θ	$-\eta$	b
${}^1E_7$	-1	$\mathrm{i}\theta$	η^*	$-\mathrm{i}\eta$	$-\theta^*$	i	θ	$-\mathrm{i}\eta^*$	$-\eta$	$\mathrm{i}\theta^*$	b
${}^2E_7$	-1	$-\mathrm{i}\theta^*$	η	$\mathrm{i}\eta^*$	$-\theta$	$-\mathrm{i}$	θ^*	$\mathrm{i}\eta$	$-\eta^*$	$-\mathrm{i}\theta$	b
${}^1E_8$	1	θ	η^*	η	θ^*	1	θ	η^*	η	θ^*	b
${}^2E_8$	1	θ^*	η	η^*	θ	1	θ^*	η	η^*	θ	b
${}^1E_9$	-1	$\mathrm{i}\eta$	θ	$-\mathrm{i}\theta^*$	$-\eta^*$	i	η	$-\mathrm{i}\theta$	$-\theta^*$	$\mathrm{i}\eta^*$	b
${}^2E_9$	-1	$-\mathrm{i}\eta^*$	θ^*	$\mathrm{i}\theta$	$-\eta$	$-\mathrm{i}$	η^*	$\mathrm{i}\theta^*$	$-\theta$	$-\mathrm{i}\eta$	b
${}^1E_{1/2}$	i	$-\mathrm{i}\delta$	η^*	$-\mathrm{i}\epsilon$	$\mathrm{i}\theta^*$	ζ^*	$-\theta$	ϵ^*	$-\mathrm{i}\eta$	δ^*	b
${}^2E_{1/2}$	$-\mathrm{i}$	$\mathrm{i}\delta^*$	η	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\theta$	ζ	$-\theta^*$	ϵ	$\mathrm{i}\eta^*$	δ	b
${}^1E_{3/2}$	i	$-\mathrm{i}\epsilon^*$	θ^*	$-\delta$	$\mathrm{i}\eta$	$-\zeta^*$	$-\eta^*$	$\mathrm{i}\delta^*$	$-\mathrm{i}\theta$	ϵ	b
${}^2E_{3/2}$	$-\mathrm{i}$	$\mathrm{i}\epsilon$	θ	$-\delta^*$	$-\mathrm{i}\eta^*$	$-\zeta$	$-\eta$	$-\mathrm{i}\delta$	$\mathrm{i}\theta^*$	ϵ^*	b
${}^1E_{5/2}$	i	ζ^*	1	ζ	i	$-\zeta^*$	-1	$-\zeta$	$-\mathrm{i}$	ζ^*	b
${}^2E_{5/2}$	$-\mathrm{i}$	ζ	1	ζ^*	$-\mathrm{i}$	$-\zeta$	-1	$-\zeta^*$	i	ζ	b
${}^1E_{7/2}$	i	$-\epsilon$	θ	$\mathrm{i}\delta^*$	$\mathrm{i}\eta^*$	ζ^*	$-\eta$	$-\delta$	$-\mathrm{i}\theta^*$	$\mathrm{i}\epsilon^*$	b
${}^2E_{7/2}$	$-\mathrm{i}$	$-\epsilon^*$	θ^*	$-\mathrm{i}\delta$	$-\mathrm{i}\eta$	ζ	$-\eta^*$	$-\delta^*$	$\mathrm{i}\theta$	$-\mathrm{i}\epsilon$	b
${}^1E_{9/2}$	i	δ^*	η	$-\epsilon^*$	$\mathrm{i}\theta$	ζ^*	$-\theta^*$	$\mathrm{i}\epsilon$	$-\mathrm{i}\eta^*$	$-\mathrm{i}\delta$	b
${}^2E_{9/2}$	$-\mathrm{i}$	δ	η^*	$-\epsilon$	$-\mathrm{i}\theta^*$	ζ	$-\theta$	$-\mathrm{i}\epsilon^*$	$\mathrm{i}\eta$	$\mathrm{i}\delta^*$	b
${}^1E_{11/2}$	i	$-\delta^*$	η	ϵ^*	$\mathrm{i}\theta$	$-\zeta^*$	$-\theta^*$	$-\mathrm{i}\epsilon$	$-\mathrm{i}\eta^*$	$\mathrm{i}\delta$	b
${}^2E_{11/2}$	$-\mathrm{i}$	$-\delta$	η^*	ϵ	$-\mathrm{i}\theta^*$	$-\zeta$	$-\theta$	$\mathrm{i}\epsilon^*$	$\mathrm{i}\eta$	$-\mathrm{i}\delta^*$	b
${}^1E_{13/2}$	i	ϵ	θ	$-\mathrm{i}\delta^*$	$\mathrm{i}\eta^*$	$-\zeta^*$	$-\eta$	δ	$-\mathrm{i}\theta^*$	$-\mathrm{i}\epsilon^*$	b
${}^2E_{13/2}$	$-\mathrm{i}$	ϵ^*	θ^*	$\mathrm{i}\delta$	$-\mathrm{i}\eta$	$-\zeta$	$-\eta^*$	δ^*	$\mathrm{i}\theta$	$\mathrm{i}\epsilon$	b
${}^1E_{15/2}$	i	$-\zeta^*$	1	$-\zeta$	i	ζ^*	-1	ζ	$-\mathrm{i}$	$-\zeta^*$	b
${}^2E_{15/2}$	$-\mathrm{i}$	$-\zeta$	1	$-\zeta^*$	$-\mathrm{i}$	ζ	-1	ζ^*	i	$-\zeta$	b
${}^1E_{17/2}$	i	$\mathrm{i}\epsilon^*$	θ^*	δ	$\mathrm{i}\eta$	ζ^*	$-\eta^*$	$-\mathrm{i}\delta^*$	$-\mathrm{i}\theta$	$-\epsilon$	b
${}^2E_{17/2}$	$-\mathrm{i}$	$-\mathrm{i}\epsilon$	θ	δ^*	$-\mathrm{i}\eta^*$	ζ	$-\eta$	$\mathrm{i}\delta$	$\mathrm{i}\theta^*$	$-\epsilon^*$	b
${}^1E_{19/2}$	i	$\mathrm{i}\delta$	η^*	$\mathrm{i}\epsilon$	$\mathrm{i}\theta^*$	$-\zeta^*$	$-\theta$	$-\epsilon^*$	$-\mathrm{i}\eta$	$-\delta^*$	b
${}^2E_{19/2}$	$-\mathrm{i}$	$-\mathrm{i}\delta^*$	η	$-\mathrm{i}\epsilon^*$	$-\mathrm{i}\theta$	$-\zeta$	$-\theta^*$	$-\epsilon$	$\mathrm{i}\eta^*$	$-\delta$	b

$\delta = \exp(2\pi\mathrm{i}/40)$, $\epsilon = \exp(6\pi\mathrm{i}/40)$, $\zeta = \exp(2\pi\mathrm{i}/8)$, $\eta = \exp(2\pi\mathrm{i}/5)$, $\theta = \exp(4\pi\mathrm{i}/5)$

T **21**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{S}_{20}$	0	1	2	3
A	$^{\square}1$	R_z	$x^2+y^2, {}^{\square}z^2$	
B		$^{\square}z$		$(x^2+y^2)z, {}^{\square}z^3$
$^1E_1 \oplus {}^2E_1$		$^{\square}(x,y),(R_x,R_y)$		$\{x(x^2+y^2),y(x^2+y^2)\}, {}^{\square}(xz^2,yz^2)$
$^1E_2 \oplus {}^2E_2$			$^{\square}(xy,x^2-y^2)$	
$^1E_3 \oplus {}^2E_3$				$^{\square}\{x(x^2-3y^2),y(3x^2-y^2)\}$
$^1E_4 \oplus {}^2E_4$				
$^1E_5 \oplus {}^2E_5$				
$^1E_6 \oplus {}^2E_6$				
$^1E_7 \oplus {}^2E_7$				
$^1E_8 \oplus {}^2E_8$				$^{\square}\{xyz,(x^2-y^2)z\}$
$^1E_9 \oplus {}^2E_9$			$^{\square}(zx,yz)$	

T **21**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{S}_{20}$	$\|j\,m\rangle$		ι	μ	$\mathbf{S}_{20}$	$\|j\,m\rangle$		ι	μ
A	$\|0\,0\rangle$	$\|11\,10\rangle$	2	±20	$^1E_{1/2}$	$\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	$\|\frac{19}{2}\,\frac{19}{2}\rangle^\bullet$	1	±20
B	$\|1\,0\rangle$	$\|10\,10\rangle$	2	±20	$^2E_{1/2}$	$\|\frac{1}{2}\,\frac{1}{2}\rangle$	$\|\frac{19}{2}\,\overline{\frac{19}{2}}\rangle^\bullet$	1	±20
$^1E_1$	$\|1\,1\rangle$	$\|10\,\bar{9}\rangle$	2	±20	$^1E_{3/2}$	$\|\frac{3}{2}\,\frac{3}{2}\rangle$	$\|\frac{17}{2}\,\overline{\frac{17}{2}}\rangle^\bullet$	1	±20
$^2E_1$	$\|1\,\bar{1}\rangle$	$\|10\,9\rangle$	2	±20	$^2E_{3/2}$	$\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	$\|\frac{17}{2}\,\frac{17}{2}\rangle^\bullet$	1	±20
$^1E_2$	$\|2\,2\rangle$	$\|9\,\bar{8}\rangle$	2	±20	$^1E_{5/2}$	$\|\frac{5}{2}\,\overline{\frac{5}{2}}\rangle$	$\|\frac{15}{2}\,\frac{15}{2}\rangle^\bullet$	1	±20
$^2E_2$	$\|2\,\bar{2}\rangle$	$\|9\,8\rangle$	2	±20	$^2E_{5/2}$	$\|\frac{5}{2}\,\frac{5}{2}\rangle$	$\|\frac{15}{2}\,\overline{\frac{15}{2}}\rangle^\bullet$	1	±20
$^1E_3$	$\|3\,\bar{3}\rangle$	$\|8\,7\rangle$	2	±20	$^1E_{7/2}$	$\|\frac{7}{2}\,\frac{7}{2}\rangle$	$\|\frac{13}{2}\,\overline{\frac{13}{2}}\rangle^\bullet$	1	±20
$^2E_3$	$\|3\,3\rangle$	$\|8\,\bar{7}\rangle$	2	±20	$^2E_{7/2}$	$\|\frac{7}{2}\,\overline{\frac{7}{2}}\rangle$	$\|\frac{13}{2}\,\frac{13}{2}\rangle^\bullet$	1	±20
$^1E_4$	$\|4\,\bar{4}\rangle$	$\|7\,6\rangle$	2	±20	$^1E_{9/2}$	$\|\frac{9}{2}\,\overline{\frac{9}{2}}\rangle$	$\|\frac{11}{2}\,\frac{11}{2}\rangle^\bullet$	1	±20
$^2E_4$	$\|4\,4\rangle$	$\|7\,\bar{6}\rangle$	2	±20	$^2E_{9/2}$	$\|\frac{9}{2}\,\frac{9}{2}\rangle$	$\|\frac{11}{2}\,\overline{\frac{11}{2}}\rangle^\bullet$	1	±20
$^1E_5$	$\|5\,5\rangle$	$\|6\,\bar{5}\rangle$	2	±20	$^1E_{11/2}$	$\|\frac{11}{2}\,\frac{11}{2}\rangle$	$\|\frac{9}{2}\,\overline{\frac{9}{2}}\rangle^\bullet$	1	±20
$^2E_5$	$\|5\,\bar{5}\rangle$	$\|6\,5\rangle$	2	±20	$^2E_{11/2}$	$\|\frac{11}{2}\,\overline{\frac{11}{2}}\rangle$	$\|\frac{9}{2}\,\frac{9}{2}\rangle^\bullet$	1	±20
$^1E_6$	$\|5\,\bar{4}\rangle$	$\|6\,6\rangle$	2	±20	$^1E_{13/2}$	$\|\frac{13}{2}\,\overline{\frac{13}{2}}\rangle$	$\|\frac{7}{2}\,\frac{7}{2}\rangle^\bullet$	1	±20
$^2E_6$	$\|5\,4\rangle$	$\|6\,\bar{6}\rangle$	2	±20	$^2E_{13/2}$	$\|\frac{13}{2}\,\frac{13}{2}\rangle$	$\|\frac{7}{2}\,\overline{\frac{7}{2}}\rangle^\bullet$	1	±20
$^1E_7$	$\|4\,\bar{3}\rangle$	$\|7\,7\rangle$	2	±20	$^1E_{15/2}$	$\|\frac{15}{2}\,\frac{15}{2}\rangle$	$\|\frac{5}{2}\,\overline{\frac{5}{2}}\rangle^\bullet$	1	±20
$^2E_7$	$\|4\,3\rangle$	$\|7\,\bar{7}\rangle$	2	±20	$^2E_{15/2}$	$\|\frac{15}{2}\,\overline{\frac{15}{2}}\rangle$	$\|\frac{5}{2}\,\frac{5}{2}\rangle^\bullet$	1	±20
$^1E_8$	$\|3\,2\rangle$	$\|8\,\bar{8}\rangle$	2	±20	$^1E_{17/2}$	$\|\frac{17}{2}\,\overline{\frac{17}{2}}\rangle$	$\|\frac{3}{2}\,\frac{3}{2}\rangle^\bullet$	1	±20
$^2E_8$	$\|3\,\bar{2}\rangle$	$\|8\,8\rangle$	2	±20	$^2E_{17/2}$	$\|\frac{17}{2}\,\frac{17}{2}\rangle$	$\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle^\bullet$	1	±20
$^1E_9$	$\|2\,1\rangle$	$\|9\,\bar{9}\rangle$	2	±20	$^1E_{19/2}$	$\|\frac{19}{2}\,\frac{19}{2}\rangle$	$\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle^\bullet$	1	±20
$^2E_9$	$\|2\,\bar{1}\rangle$	$\|9\,9\rangle$	2	±20	$^2E_{19/2}$	$\|\frac{19}{2}\,\overline{\frac{19}{2}}\rangle$	$\|\frac{1}{2}\,\frac{1}{2}\rangle^\bullet$	1	±20

T **21**.7 Matrix representations

Use T **21**.4 ♠. § **16**–7, p. 77

T **21**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{S}_{20}$	A	B	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$	${}^1E_4$	${}^2E_4$
A	A	B	${}^1E_1$	${}^2E_1$	${}^1E_2$	${}^2E_2$	${}^1E_3$	${}^2E_3$	${}^1E_4$	${}^2E_4$
B		A	${}^1E_9$	${}^2E_9$	${}^1E_8$	${}^2E_8$	${}^1E_7$	${}^2E_7$	${}^1E_6$	${}^2E_6$
${}^1E_1$			${}^1E_2$	A	${}^2E_3$	${}^2E_1$	${}^2E_2$	${}^2E_4$	${}^1E_3$	${}^1E_5$
${}^2E_1$				${}^2E_2$	${}^1E_1$	${}^1E_3$	${}^1E_4$	${}^1E_2$	${}^2E_5$	${}^2E_3$
${}^1E_2$					${}^2E_4$	A	${}^2E_1$	${}^1E_5$	${}^2E_2$	${}^1E_6$
${}^2E_2$						${}^1E_4$	${}^2E_5$	${}^1E_1$	${}^2E_6$	${}^1E_2$
${}^1E_3$							${}^2E_6$	A	${}^2E_7$	${}^1E_1$
${}^2E_3$								${}^1E_6$	${}^2E_1$	${}^1E_7$
${}^1E_4$									${}^1E_8$	A
${}^2E_4$										${}^2E_8$

⇉

T **21**.8 Direct products of representations *(cont.)*

$\mathbf{S}_{20}$	${}^1E_5$	${}^2E_5$	${}^1E_6$	${}^2E_6$	${}^1E_7$	${}^2E_7$	${}^1E_8$	${}^2E_8$	${}^1E_9$	${}^2E_9$
A	${}^1E_5$	${}^2E_5$	${}^1E_6$	${}^2E_6$	${}^1E_7$	${}^2E_7$	${}^1E_8$	${}^2E_8$	${}^1E_9$	${}^2E_9$
B	${}^2E_5$	${}^1E_5$	${}^1E_4$	${}^2E_4$	${}^1E_3$	${}^2E_3$	${}^1E_2$	${}^2E_2$	${}^1E_1$	${}^2E_1$
${}^1E_1$	${}^1E_6$	${}^1E_4$	${}^1E_7$	${}^2E_5$	${}^2E_8$	${}^2E_6$	${}^2E_7$	${}^2E_9$	${}^1E_8$	B
${}^2E_1$	${}^2E_4$	${}^2E_6$	${}^1E_5$	${}^2E_7$	${}^1E_6$	${}^1E_8$	${}^1E_9$	${}^1E_7$	B	${}^2E_8$
${}^1E_2$	${}^1E_7$	${}^1E_3$	${}^2E_8$	${}^1E_4$	${}^2E_9$	${}^2E_5$	${}^2E_6$	B	${}^2E_7$	${}^1E_9$
${}^2E_2$	${}^2E_3$	${}^2E_7$	${}^2E_4$	${}^1E_8$	${}^1E_5$	${}^1E_9$	B	${}^1E_6$	${}^2E_9$	${}^1E_7$
${}^1E_3$	${}^1E_2$	${}^1E_8$	${}^2E_3$	${}^1E_9$	${}^2E_4$	B	${}^2E_9$	${}^1E_5$	${}^2E_8$	${}^1E_6$
${}^2E_3$	${}^2E_8$	${}^2E_2$	${}^2E_9$	${}^1E_3$	B	${}^1E_4$	${}^2E_5$	${}^1E_9$	${}^2E_6$	${}^1E_8$
${}^1E_4$	${}^1E_1$	${}^1E_9$	${}^1E_2$	B	${}^2E_3$	${}^2E_9$	${}^2E_8$	${}^2E_4$	${}^1E_7$	${}^1E_5$
${}^2E_4$	${}^2E_9$	${}^2E_1$	B	${}^2E_2$	${}^1E_9$	${}^1E_3$	${}^1E_4$	${}^1E_8$	${}^2E_5$	${}^2E_7$
${}^1E_5$	B	A	${}^1E_9$	${}^2E_1$	${}^1E_8$	${}^2E_2$	${}^1E_3$	${}^2E_7$	${}^1E_4$	${}^2E_6$
${}^2E_5$		B	${}^1E_1$	${}^2E_9$	${}^1E_2$	${}^2E_8$	${}^1E_7$	${}^2E_3$	${}^1E_6$	${}^2E_4$
${}^1E_6$			${}^1E_8$	A	${}^2E_7$	${}^2E_1$	${}^2E_2$	${}^2E_6$	${}^1E_3$	${}^2E_5$
${}^2E_6$				${}^2E_8$	${}^1E_1$	${}^1E_7$	${}^1E_6$	${}^1E_2$	${}^1E_5$	${}^2E_3$
${}^1E_7$					${}^2E_6$	A	${}^2E_1$	${}^2E_5$	${}^2E_2$	${}^1E_4$
${}^2E_7$						${}^1E_6$	${}^1E_5$	${}^1E_1$	${}^2E_4$	${}^1E_2$
${}^1E_8$							${}^2E_4$	A	${}^2E_3$	${}^1E_1$
${}^2E_8$								${}^1E_4$	${}^2E_1$	${}^1E_3$
${}^1E_9$									${}^1E_2$	A
${}^2E_9$										${}^2E_2$

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T **21**.8 Direct products of representations *(cont.)*

$\mathbf{S}_{20}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$
A	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$
B	${}^1E_{19/2}$	${}^2E_{19/2}$	${}^1E_{17/2}$	${}^2E_{17/2}$	${}^1E_{15/2}$	${}^2E_{15/2}$	${}^1E_{13/2}$	${}^2E_{13/2}$	${}^1E_{11/2}$	${}^2E_{11/2}$
${}^1E_1$	${}^2E_{19/2}$	${}^1E_{17/2}$	${}^2E_{15/2}$	${}^1E_{19/2}$	${}^2E_{17/2}$	${}^1E_{13/2}$	${}^2E_{11/2}$	${}^1E_{15/2}$	${}^2E_{13/2}$	${}^1E_{9/2}$
${}^2E_1$	${}^2E_{17/2}$	${}^1E_{19/2}$	${}^2E_{19/2}$	${}^1E_{15/2}$	${}^2E_{13/2}$	${}^1E_{17/2}$	${}^2E_{15/2}$	${}^1E_{11/2}$	${}^2E_{9/2}$	${}^1E_{13/2}$
${}^1E_2$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{13/2}$
${}^2E_2$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{11/2}$	${}^1E_{13/2}$	${}^2E_{5/2}$
${}^1E_3$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{17/2}$	${}^1E_{11/2}$	${}^2E_{9/2}$	${}^1E_{19/2}$	${}^2E_{19/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{17/2}$
${}^2E_3$	${}^2E_{15/2}$	${}^1E_{13/2}$	${}^2E_{11/2}$	${}^1E_{17/2}$	${}^2E_{19/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{19/2}$	${}^2E_{17/2}$	${}^1E_{5/2}$
${}^1E_4$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{11/2}$	${}^1E_{13/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{15/2}$	${}^1E_{17/2}$	${}^2E_{1/2}$
${}^2E_4$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{17/2}$
${}^1E_5$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{13/2}$	${}^2E_{15/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{17/2}$	${}^2E_{19/2}$	${}^1E_{1/2}$
${}^2E_5$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{13/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{15/2}$	${}^2E_{17/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{19/2}$
${}^1E_6$	${}^1E_{11/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$	${}^2E_{17/2}$	${}^1E_{19/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{19/2}$
${}^2E_6$	${}^1E_{13/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{15/2}$	${}^1E_{17/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{19/2}$	${}^1E_{19/2}$	${}^2E_{3/2}$
${}^1E_7$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{9/2}$	${}^2E_{11/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{13/2}$	${}^2E_{15/2}$	${}^1E_{3/2}$
${}^2E_7$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{11/2}$	${}^2E_{13/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{15/2}$
${}^1E_8$	${}^1E_{17/2}$	${}^2E_{15/2}$	${}^1E_{13/2}$	${}^2E_{19/2}$	${}^1E_{19/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{17/2}$	${}^1E_{15/2}$	${}^2E_{7/2}$
${}^2E_8$	${}^1E_{15/2}$	${}^2E_{17/2}$	${}^1E_{19/2}$	${}^2E_{13/2}$	${}^1E_{11/2}$	${}^2E_{19/2}$	${}^1E_{17/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$	${}^2E_{15/2}$
${}^1E_9$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{11/2}$
${}^2E_9$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{9/2}$	${}^2E_{11/2}$	${}^1E_{7/2}$
${}^1E_{1/2}$	${}^2E_9$	A	${}^1E_9$	${}^2E_2$	${}^1E_7$	${}^1E_2$	${}^2E_7$	${}^1E_4$	${}^1E_5$	${}^2E_4$
${}^2E_{1/2}$		${}^1E_9$	${}^1E_2$	${}^2E_9$	${}^2E_2$	${}^2E_7$	${}^2E_4$	${}^1E_7$	${}^1E_4$	${}^2E_5$
${}^1E_{3/2}$			${}^2E_7$	A	${}^2E_9$	${}^2E_4$	${}^2E_5$	${}^2E_2$	${}^1E_7$	${}^1E_6$
${}^2E_{3/2}$				${}^1E_7$	${}^1E_4$	${}^1E_9$	${}^1E_2$	${}^1E_5$	${}^2E_6$	${}^2E_7$
${}^1E_{5/2}$					${}^1E_5$	A	${}^1E_9$	${}^2E_6$	${}^2E_3$	${}^1E_2$
${}^2E_{5/2}$						${}^2E_5$	${}^1E_6$	${}^2E_9$	${}^2E_2$	${}^1E_3$
${}^1E_{7/2}$							${}^1E_3$	A	${}^2E_9$	${}^2E_8$
${}^2E_{7/2}$								${}^2E_3$	${}^1E_8$	${}^1E_9$
${}^1E_{9/2}$									${}^1E_1$	A
${}^2E_{9/2}$										${}^2E_1$

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T **21**.8 Direct products of representations *(cont.)*

$\mathbf{S}_{20}$	${}^1E_{11/2}$	${}^2E_{11/2}$	${}^1E_{13/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{15/2}$	${}^1E_{17/2}$	${}^2E_{17/2}$	${}^1E_{19/2}$	${}^2E_{19/2}$
A	${}^1E_{11/2}$	${}^2E_{11/2}$	${}^1E_{13/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{15/2}$	${}^1E_{17/2}$	${}^2E_{17/2}$	${}^1E_{19/2}$	${}^2E_{19/2}$
B	${}^1E_{9/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_1$	${}^2E_{7/2}$	${}^1E_{11/2}$	${}^2E_{9/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$
${}^2E_1$	${}^2E_{11/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^1E_2$	${}^1E_{15/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{17/2}$	${}^1E_{19/2}$	${}^2E_{11/2}$	${}^1E_{13/2}$	${}^2E_{19/2}$	${}^1E_{17/2}$	${}^2E_{15/2}$
${}^2E_2$	${}^1E_{7/2}$	${}^2E_{15/2}$	${}^1E_{17/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{19/2}$	${}^1E_{19/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{17/2}$
${}^1E_3$	${}^2E_{15/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{13/2}$	${}^2E_{11/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$
${}^2E_3$	${}^2E_{3/2}$	${}^1E_{15/2}$	${}^2E_{13/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{11/2}$	${}^2E_{9/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$
${}^1E_4$	${}^1E_{3/2}$	${}^2E_{19/2}$	${}^1E_{19/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{17/2}$	${}^1E_{15/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{13/2}$
${}^2E_4$	${}^1E_{19/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{19/2}$	${}^1E_{17/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{15/2}$	${}^1E_{13/2}$	${}^2E_{11/2}$
${}^1E_5$	${}^2E_{1/2}$	${}^1E_{19/2}$	${}^2E_{17/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{15/2}$	${}^2E_{13/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$
${}^2E_5$	${}^2E_{19/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{17/2}$	${}^2E_{15/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{13/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$
${}^1E_6$	${}^1E_{17/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{15/2}$	${}^1E_{13/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$
${}^2E_6$	${}^1E_{1/2}$	${}^2E_{17/2}$	${}^1E_{15/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{13/2}$	${}^1E_{11/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$
${}^1E_7$	${}^2E_{5/2}$	${}^1E_{17/2}$	${}^2E_{19/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{19/2}$	${}^2E_{17/2}$	${}^1E_{11/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$
${}^2E_7$	${}^2E_{17/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{19/2}$	${}^2E_{19/2}$	${}^1E_{9/2}$	${}^2E_{11/2}$	${}^1E_{17/2}$	${}^2E_{15/2}$	${}^1E_{13/2}$
${}^1E_8$	${}^1E_{5/2}$	${}^2E_{13/2}$	${}^1E_{11/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$
${}^2E_8$	${}^1E_{13/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$
${}^1E_9$	${}^2E_{13/2}$	${}^1E_{9/2}$	${}^2E_{11/2}$	${}^1E_{15/2}$	${}^2E_{17/2}$	${}^1E_{13/2}$	${}^2E_{15/2}$	${}^1E_{19/2}$	${}^2E_{19/2}$	${}^1E_{17/2}$
${}^2E_9$	${}^2E_{9/2}$	${}^1E_{13/2}$	${}^2E_{15/2}$	${}^1E_{11/2}$	${}^2E_{13/2}$	${}^1E_{17/2}$	${}^2E_{19/2}$	${}^1E_{15/2}$	${}^2E_{17/2}$	${}^1E_{19/2}$
${}^1E_{1/2}$	${}^2E_5$	${}^2E_6$	${}^2E_3$	${}^1E_6$	${}^1E_3$	${}^1E_8$	${}^1E_1$	${}^2E_8$	${}^2E_1$	B
${}^2E_{1/2}$	${}^1E_6$	${}^1E_5$	${}^2E_6$	${}^1E_3$	${}^2E_8$	${}^2E_3$	${}^1E_8$	${}^2E_1$	B	${}^1E_1$
${}^1E_{3/2}$	${}^1E_3$	${}^1E_4$	${}^1E_5$	${}^2E_8$	${}^2E_1$	${}^2E_6$	${}^2E_3$	B	${}^1E_1$	${}^1E_8$
${}^2E_{3/2}$	${}^2E_4$	${}^2E_3$	${}^1E_8$	${}^2E_5$	${}^1E_6$	${}^1E_1$	B	${}^1E_3$	${}^2E_8$	${}^2E_1$
${}^1E_{5/2}$	${}^2E_7$	${}^1E_8$	${}^1E_1$	${}^2E_4$	${}^2E_5$	B	${}^2E_1$	${}^1E_6$	${}^1E_3$	${}^2E_8$
${}^2E_{5/2}$	${}^2E_8$	${}^1E_7$	${}^1E_4$	${}^2E_1$	B	${}^1E_5$	${}^2E_6$	${}^1E_1$	${}^1E_8$	${}^2E_3$
${}^1E_{7/2}$	${}^2E_1$	${}^2E_2$	${}^1E_7$	B	${}^1E_1$	${}^1E_4$	${}^1E_5$	${}^1E_8$	${}^2E_3$	${}^2E_6$
${}^2E_{7/2}$	${}^1E_2$	${}^1E_1$	B	${}^2E_7$	${}^2E_4$	${}^2E_1$	${}^2E_8$	${}^2E_5$	${}^1E_6$	${}^1E_3$
${}^1E_{9/2}$	${}^1E_9$	B	${}^2E_1$	${}^1E_2$	${}^2E_7$	${}^2E_8$	${}^1E_3$	${}^2E_4$	${}^2E_5$	${}^1E_6$
${}^2E_{9/2}$	B	${}^2E_9$	${}^2E_2$	${}^1E_1$	${}^1E_8$	${}^1E_7$	${}^1E_4$	${}^2E_3$	${}^2E_6$	${}^1E_5$
${}^1E_{11/2}$	${}^1E_1$	A	${}^2E_9$	${}^1E_8$	${}^2E_3$	${}^2E_2$	${}^1E_7$	${}^2E_6$	${}^1E_5$	${}^1E_4$
${}^2E_{11/2}$		${}^2E_1$	${}^2E_8$	${}^1E_9$	${}^1E_2$	${}^1E_3$	${}^1E_6$	${}^2E_7$	${}^2E_4$	${}^2E_5$
${}^1E_{13/2}$			${}^1E_3$	A	${}^1E_9$	${}^1E_6$	${}^2E_5$	${}^1E_2$	${}^2E_7$	${}^2E_4$
${}^2E_{13/2}$				${}^2E_3$	${}^2E_6$	${}^2E_9$	${}^2E_2$	${}^1E_5$	${}^1E_4$	${}^1E_7$
${}^1E_{15/2}$					${}^1E_5$	A	${}^2E_9$	${}^1E_4$	${}^1E_7$	${}^2E_2$
${}^2E_{15/2}$						${}^2E_5$	${}^2E_4$	${}^1E_9$	${}^1E_2$	${}^2E_7$
${}^1E_{17/2}$							${}^2E_7$	A	${}^1E_9$	${}^1E_2$
${}^2E_{17/2}$								${}^1E_7$	${}^2E_2$	${}^2E_9$
${}^1E_{19/2}$									${}^2E_9$	A
${}^2E_{19/2}$										${}^1E_9$

T **21**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{S}_{20}$	$\mathbf{S}_4$	$\mathbf{C}_{10}$	$\mathbf{C}_5$	$\mathbf{C}_2$
A	A	A	A	A
B	B	A	A	A
${}^1E_1$	${}^2E$	${}^1E_1$	${}^1E_1$	B
${}^2E_1$	${}^1E$	${}^2E_1$	${}^2E_1$	B
${}^1E_2$	B	${}^1E_2$	${}^1E_2$	A
${}^2E_2$	B	${}^2E_2$	${}^2E_2$	A
${}^1E_3$	${}^2E$	${}^1E_3$	${}^1E_2$	B
${}^2E_3$	${}^1E$	${}^2E_3$	${}^2E_2$	B
${}^1E_4$	A	${}^1E_4$	${}^1E_1$	A
${}^2E_4$	A	${}^2E_4$	${}^2E_1$	A
${}^1E_5$	${}^2E$	B	A	B
${}^2E_5$	${}^1E$	B	A	B
${}^1E_6$	B	${}^1E_4$	${}^1E_1$	A
${}^2E_6$	B	${}^2E_4$	${}^2E_1$	A
${}^1E_7$	${}^2E$	${}^1E_3$	${}^1E_2$	B
${}^2E_7$	${}^1E$	${}^2E_3$	${}^2E_2$	B
${}^1E_8$	A	${}^1E_2$	${}^1E_2$	A
${}^2E_8$	A	${}^2E_2$	${}^2E_2$	A
${}^1E_9$	${}^1E$	${}^1E_1$	${}^1E_1$	B
${}^2E_9$	${}^2E$	${}^2E_1$	${}^2E_1$	B
${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$
${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$
${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{3/2}$	${}^1E_{3/2}$	${}^1E_{1/2}$
${}^2E_{3/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^2E_{3/2}$	${}^2E_{1/2}$
${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	$A_{5/2}$	${}^1E_{1/2}$
${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	$A_{5/2}$	${}^2E_{1/2}$
${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{7/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_{7/2}$	${}^1E_{1/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_{9/2}$	${}^2E_{1/2}$	${}^1E_{9/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_{9/2}$	${}^1E_{1/2}$	${}^2E_{9/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_{11/2}$	${}^2E_{3/2}$	${}^1E_{9/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_{11/2}$	${}^1E_{3/2}$	${}^2E_{9/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_{13/2}$	${}^2E_{3/2}$	${}^1E_{7/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_{13/2}$	${}^1E_{3/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_{15/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	$A_{5/2}$	${}^1E_{1/2}$
${}^2E_{15/2}$	${}^1E_{1/2}$	${}^2E_{5/2}$	$A_{5/2}$	${}^2E_{1/2}$
${}^1E_{17/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^1E_{3/2}$	${}^1E_{1/2}$
${}^2E_{17/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^2E_{3/2}$	${}^2E_{1/2}$
${}^1E_{19/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$
${}^2E_{19/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$

T **21**.10 Subduction from O(3) § **16**–10, p. 82

j	$\mathbf{S}_{20}$
$20n$	$(2n+1)\,A \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_8 \oplus {}^2E_8 \oplus {}^1E_9 \oplus {}^2E_9)$
$20n+1$	$2n\,(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_8 \oplus {}^2E_8 \oplus {}^1E_9 \oplus {}^2E_9) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1)$
$20n+2$	$(2n+1)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_9 \oplus {}^2E_9) \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_8 \oplus {}^2E_8)$
$20n+3$	$2n\,(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_9 \oplus {}^2E_9) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_8 \oplus {}^2E_8)$
$20n+4$	$(2n+1)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_9 \oplus {}^2E_9) \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_8 \oplus {}^2E_8)$
$20n+5$	$2n\,(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_9 \oplus {}^2E_9) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_8 \oplus {}^2E_8)$
$20n+6$	$(2n+1)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_9 \oplus {}^2E_9) \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_8 \oplus {}^2E_8)$
$20n+7$	$2n\,(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_9 \oplus {}^2E_9) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_8 \oplus {}^2E_8)$
$20n+8$	$(2n+1)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_8 \oplus {}^2E_8 \oplus {}^1E_9 \oplus {}^2E_9) \oplus 2n\,(B \oplus {}^1E_1 \oplus {}^2E_1)$
$20n+9$	$2n\,A \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_8 \oplus {}^2E_8 \oplus {}^1E_9 \oplus {}^2E_9)$
$20n+10$	$(2n+1)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_8 \oplus {}^2E_8 \oplus {}^1E_9 \oplus {}^2E_9) \oplus (2n+2)\,B$
$20n+11$	$(2n+2)(A \oplus {}^1E_9 \oplus {}^2E_9) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_8 \oplus {}^2E_8)$
$20n+12$	$(2n+1)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_9 \oplus {}^2E_9) \oplus (2n+2)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_8 \oplus {}^2E_8)$
$20n+13$	$(2n+2)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_9 \oplus {}^2E_9) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_8 \oplus {}^2E_8)$
$20n+14$	$(2n+1)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_9 \oplus {}^2E_9) \oplus (2n+2)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_8 \oplus {}^2E_8)$
$20n+15$	$(2n+2)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_9 \oplus {}^2E_9) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_8 \oplus {}^2E_8)$
$20n+16$	$(2n+1)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_9 \oplus {}^2E_9) \oplus (2n+2)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_8 \oplus {}^2E_8)$
$20n+17$	$(2n+2)(A \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_9 \oplus {}^2E_9) \oplus (2n+1)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_8 \oplus {}^2E_8)$
$20n+18$	$(2n+1)(A \oplus {}^1E_9 \oplus {}^2E_9) \oplus (2n+2)(B \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_8 \oplus {}^2E_8)$
$20n+19$	$(2n+2)(A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2 \oplus {}^1E_3 \oplus {}^2E_3 \oplus {}^1E_4 \oplus {}^2E_4 \oplus {}^1E_5 \oplus {}^2E_5 \oplus {}^1E_6 \oplus {}^2E_6 \oplus {}^1E_7 \oplus {}^2E_7 \oplus {}^1E_8 \oplus {}^2E_8 \oplus {}^1E_9 \oplus {}^2E_9) \oplus (2n+1)\,B$

$n = 0, 1, 2, \ldots$ $\twoheadrightarrow$

T **21**.10 Subduction from O(3) *(cont.)*

j	$\mathbf{S}_{20}$
$20n+\frac{1}{2}$	$(2n+1)(^1E_{1/2} \oplus {}^2E_{1/2}) \oplus 2n\,(^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2} \oplus {}^1E_{17/2} \oplus {}^2E_{17/2} \oplus {}^1E_{19/2} \oplus {}^2E_{19/2})$
$20n+\frac{3}{2}$	$(2n+1)(^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus 2n\,(^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2} \oplus {}^1E_{17/2} \oplus {}^2E_{17/2} \oplus {}^1E_{19/2} \oplus {}^2E_{19/2})$
$20n+\frac{5}{2}$	$(2n+1)(^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus 2n\,(^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2} \oplus {}^1E_{17/2} \oplus {}^2E_{17/2} \oplus {}^1E_{19/2} \oplus {}^2E_{19/2})$
$20n+\frac{7}{2}$	$(2n+1)(^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}) \oplus 2n\,(^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2} \oplus {}^1E_{17/2} \oplus {}^2E_{17/2} \oplus {}^1E_{19/2} \oplus {}^2E_{19/2})$
$20n+\frac{9}{2}$	$(2n+1)(^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2}) \oplus 2n\,(^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2} \oplus {}^1E_{17/2} \oplus {}^2E_{17/2} \oplus {}^1E_{19/2} \oplus {}^2E_{19/2})$
$20n+\frac{11}{2}$	$(2n+1)(^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2}) \oplus 2n\,(^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2} \oplus {}^1E_{17/2} \oplus {}^2E_{17/2} \oplus {}^1E_{19/2} \oplus {}^2E_{19/2})$
$20n+\frac{13}{2}$	$(2n+1)(^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2}) \oplus 2n\,(^1E_{15/2} \oplus {}^2E_{15/2} \oplus {}^1E_{17/2} \oplus {}^2E_{17/2} \oplus {}^1E_{19/2} \oplus {}^2E_{19/2})$
$20n+\frac{15}{2}$	$(2n+1)(^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2}) \oplus 2n\,(^1E_{17/2} \oplus {}^2E_{17/2} \oplus {}^1E_{19/2} \oplus {}^2E_{19/2})$
$20n+\frac{17}{2}$	$(2n+1)(^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2} \oplus {}^1E_{17/2} \oplus {}^2E_{17/2}) \oplus 2n\,(^1E_{19/2} \oplus {}^2E_{19/2})$
$20n+\frac{19}{2}$	$(2n+1)(^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2} \oplus {}^1E_{17/2} \oplus {}^2E_{17/2} \oplus {}^1E_{19/2} \oplus {}^2E_{19/2})$
$20n+\frac{21}{2}$	$(2n+1)(^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2} \oplus {}^1E_{17/2} \oplus {}^2E_{17/2}) \oplus (2n+2)(^1E_{19/2} \oplus {}^2E_{19/2})$
$20n+\frac{23}{2}$	$(2n+1)(^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2} \oplus {}^1E_{15/2} \oplus {}^2E_{15/2}) \oplus (2n+2)(^1E_{17/2} \oplus {}^2E_{17/2} \oplus {}^1E_{19/2} \oplus {}^2E_{19/2})$
$20n+\frac{25}{2}$	$(2n+1)(^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2} \oplus {}^1E_{11/2} \oplus {}^2E_{11/2} \oplus {}^1E_{13/2} \oplus {}^2E_{13/2}) \oplus (2n+2)(^1E_{15/2} \oplus {}^2E_{15/2} \oplus {}^1E_{17/2} \oplus {}^2E_{17/2} \oplus {}^1E_{19/2} \oplus {}^2E_{19/2})$

$n = 0, 1, 2, \ldots$ $\twoheadrightarrow$

T **21**.10 Subduction from O(3) *(cont.)*

j	$\mathbf{S}_{20}$
$20n+\frac{27}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2})\oplus(2n+2)({}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2}\oplus{}^1E_{19/2}\oplus{}^2E_{19/2})$
$20n+\frac{29}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2})\oplus(2n+2)({}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2}\oplus{}^1E_{19/2}\oplus{}^2E_{19/2})$
$20n+\frac{31}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2})\oplus(2n+2)({}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2}\oplus{}^1E_{19/2}\oplus{}^2E_{19/2})$
$20n+\frac{33}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2})\oplus(2n+2)({}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2}\oplus{}^1E_{19/2}\oplus{}^2E_{19/2})$
$20n+\frac{35}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2})\oplus(2n+2)({}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2}\oplus{}^1E_{19/2}\oplus{}^2E_{19/2})$
$20n+\frac{37}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2})\oplus(2n+2)({}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2}\oplus{}^1E_{19/2}\oplus{}^2E_{19/2})$
$20n+\frac{39}{2}$	$(2n+2)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2}\oplus{}^1E_{19/2}\oplus{}^2E_{19/2})$

$n=0,1,2,\ldots$

T **21**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

The dihedral groups $\mathbf{D}_n$

Notation for headers

Items in header read from left to right

1	Hermann–Mauguin symbol for the point group.
2	$\|G\|$ order of the group.
3	$\|C\|$ number of classes in the group.
4	$\|\widetilde{C}\|$ number of classes in the double group.
5	Number of the table.
6	Page reference for the notation of the header, of the first five subsections below it, and of the footers.
7 ☐	This symbol indicates a crystallographic point group.
8	Schönflies notation for the point group.

Notation for the first five subsections below the header

(1) Product forms	Direct and semidirect product forms (p. 37, note on p. 39).
(2) Group chains *(See pp.* 41, 67*)*	Groups underlined: invariant. Groups in brackets: when subducing one or more of the representations in the tables to these subgroups in the setting used for them in the tables a change of bases (similarity transformation) is required.
(3) Operations of G	Lists all the operations of G, enclosing in brackets all the operations of the same class.
(4) Operations of $\widetilde{G}$	Lists all the operations of $\widetilde{G}$, enclosing in brackets all the operations of the same class.
(5) Classes and representations	$\|r\|$ number of regular classes in G (p. 51). $\|\mathrm{i}\|$ number of irregular classes in G (p. 51). $\|I\|$ number of irreducible representations in G. $\|\widetilde{I}\|$ number of spinor representations, also called the number of double-group representations.

Use of the footers

Finding your way about the tables

Each page of the tables contains a footer giving the ordering of the group types listed in the tables. The current group type is recognized because it carries no page number under its name. On opening the tables at any page you should then see at a glance whether to move backwards or forwards to find the group type desired. More precisely, the page number for the group type desired gives the page, like the present one, which carries the contents for the group type wanted.

222	$|G| = 4$	$|C| = 4$	$|\widetilde{C}| = 5$	T **22**	p. 193	□	$\mathbf{D}_2$

(1) Product forms: $\mathbf{C}_2 \otimes \mathbf{C}_2'$.

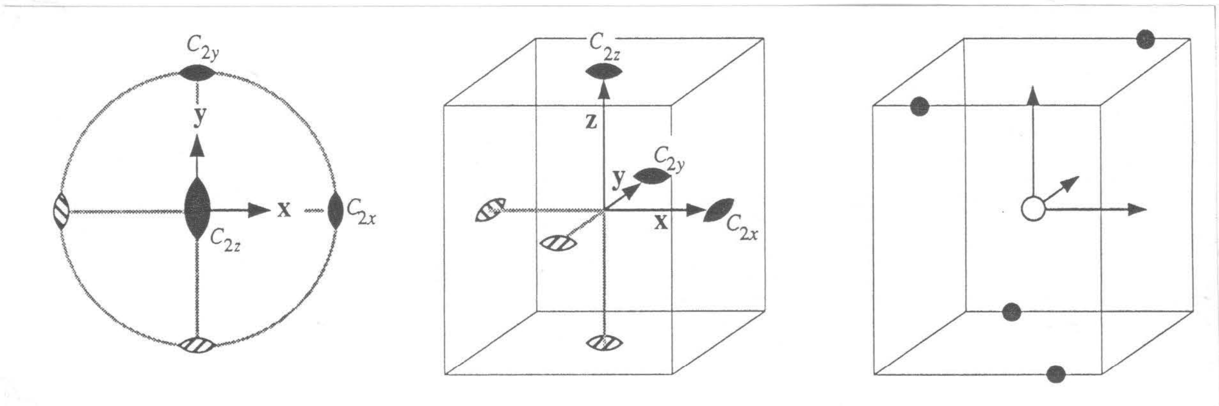
(2) Group chains: $\mathbf{T} \supset \underline{\mathbf{D}_2} \supset \underline{\mathbf{C}_2}$, $\mathbf{D}_{2d} \supset (\underline{\mathbf{D}_2}) \supset \underline{\mathbf{C}_2}$, $\mathbf{D}_{2h} \supset \underline{\mathbf{D}_2} \supset \underline{\mathbf{C}_2}$, $\mathbf{D}_{10} \supset (\mathbf{D}_2) \supset \underline{\mathbf{C}_2}$, $\mathbf{D}_6 \supset (\mathbf{D}_2) \supset \underline{\mathbf{C}_2}$, $\mathbf{D}_4 \supset (\underline{\mathbf{D}_2}) \supset \underline{\mathbf{C}_2}$.

(3) Operations of G: E, C_{2z}, C_{2x}, C_{2y}.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, $(C_{2z}, \widetilde{C}_{2z})$, $(C_{2x}, \widetilde{C}_{2x})$, $(C_{2y}, \widetilde{C}_{2y})$.

(5) Classes and representations: $|r| = 1$, $|i| = 3$, $|I| = 4$, $|\widetilde{I}| = 1$.

F **22** See Chapter **15**, p. 65



Examples: Possible excited state of C_2H_4 partly rotated.

T **22**.1 Parameters
Use T **31**.1. § **16**–1, p. 68

T **22**.2 Multiplication table
Use T **31**.2. § **16**–2, p. 69

T **22**.3 Factor table
Use T **31**.3. § **16**–3, p. 70

T **22**.4 Character table
§ **16**–4, p. 71

$\mathbf{D}_2$	E	C_{2z}	C_{2x}	C_{2y}	τ
A	1	1	1	1	a
B_1	1	1	−1	−1	a
B_2	1	−1	−1	1	a
B_3	1	−1	1	−1	a
$E_{1/2}$	2	0	0	0	c

T **22**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions
§ **16**–5, p. 72

$\mathbf{D}_2$	0	1	2	3
A	$^\square 1$		$^\square x^2, y^2, {}^\square z^2$	$^\square xyz$
B_1		$^\square z, R_z$	$^\square xy$	$^\square x^2z, y^2z, {}^\square z^3$
B_2		$^\square y, R_y$	$^\square zx$	$^\square x^2y, y^3, {}^\square yz^2$
B_3		$^\square x, R_x$	$^\square yz$	$^\square x^3, xy^2, {}^\square xz^2$

T **22**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_2$	$E_{1/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_{2z}	$\begin{bmatrix} \bar{\text{i}} & 0 \\ 0 & \text{i} \end{bmatrix}$
C_{2x}	$\begin{bmatrix} 0 & \bar{\text{i}} \\ \bar{\text{i}} & 0 \end{bmatrix}$
C_{2y}	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$

T **22**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_2$	$\langle \vert j\, m\rangle \vert$		ι	μ
A	$\vert 0\,0\rangle_+$	$\vert 3\,2\rangle_-$	2	2
B_1	$\vert 1\,0\rangle_+$	$\vert 2\,2\rangle_-$	2	2
B_2	$\vert 1\,1\rangle_+$	$\vert 2\,1\rangle_-$	2	2
B_3	$\vert 1\,1\rangle_-$	$\vert 2\,1\rangle_+$	2	2
$E_{1/2}$	$\langle \vert\frac{1}{2}\,\frac{1}{2}\rangle, \vert\frac{1}{2}\,\overline{\frac{1}{2}}\rangle \vert$	$\langle \vert\frac{3}{2}\,\overline{\frac{3}{2}}\rangle, -\vert\frac{3}{2}\,\frac{3}{2}\rangle \vert$	2	± 2

T **22**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{D}_2$	A	B_1	B_2	B_3	$E_{1/2}$
A	A	B_1	B_2	B_3	$E_{1/2}$
B_1		A	B_3	B_2	$E_{1/2}$
B_2			A	B_1	$E_{1/2}$
B_3				A	$E_{1/2}$
$E_{1/2}$					$\{A\} \oplus B_1 \oplus B_2 \oplus B_3$

T **22**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_2$	$\mathbf{C}_2$	$(\mathbf{C}_2)$	$(\mathbf{C}_2)$
	C_{2z}	C_{2x}	C_{2y}
A	A	A	A
B_1	A	B	B
B_2	B	B	A
B_3	B	A	B
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **22**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_2$
$2n$	$(n+1)\,A \oplus n\,(B_1 \oplus B_2 \oplus B_3)$
$2n+1$	$n\,A \oplus (n+1)(B_1 \oplus B_2 \oplus B_3)$
$n+\frac{1}{2}$	$(n+1)\,E_{1/2}$

$n = 0, 1, 2, \ldots$

T **22**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{D}_2$

b_1	$e_{1/2}$	$E_{1/2}$ 1	$E_{1/2}$ 2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{1/2}$	$E_{1/2}$ 1	$E_{1/2}$ 2
1	1	0	$\bar{1}$
1	2	1	0

b_3	$e_{1/2}$	$E_{1/2}$ 1	$E_{1/2}$ 2
1	1	0	1
1	2	1	0

$e_{1/2}$	$e_{1/2}$	A 1	B_1 1	B_2 1	B_3 1
1	1	0	0	u	u
1	2	u	u	0	0
2	1	$\bar{\text{u}}$	u	0	0
2	2	0	0	u	$\bar{\text{u}}$

$\text{u} = 2^{-1/2}$

(1) Product forms: $\mathbf{C}_3 \otimes \mathbf{C}'_2$.

(2) Group chains: $\mathbf{I} \supset (\mathbf{D}_3) \supset \underline{\mathbf{C}_3}$, $\mathbf{I} \supset (\mathbf{D}_3) \supset (\mathbf{C}_2)$, $\mathbf{O} \supset (\mathbf{D}_3) \supset \underline{\mathbf{C}_3}$, $\mathbf{O} \supset (\mathbf{D}_3) \supset (\mathbf{C}_2)$,
$\mathbf{D}_{3d} \supset \underline{\mathbf{D}_3} \supset \underline{\mathbf{C}_3}$, $\mathbf{D}_{3d} \supset \underline{\mathbf{D}_3} \supset (\mathbf{C}_2)$, $\mathbf{D}_{3h} \supset (\underline{\mathbf{D}_3}) \supset \underline{\mathbf{C}_3}$, $\mathbf{D}_{3h} \supset (\underline{\mathbf{D}_3}) \supset (\mathbf{C}_2)$,
$\mathbf{D}_9 \supset (\mathbf{D}_3) \supset \underline{\mathbf{C}_3}$, $\mathbf{D}_9 \supset (\mathbf{D}_3) \supset (\mathbf{C}_2)$, $\mathbf{D}_6 \supset (\underline{\mathbf{D}_3}) \supset \underline{\mathbf{C}_3}$, $\mathbf{D}_6 \supset (\underline{\mathbf{D}_3}) \supset (\mathbf{C}_2)$.

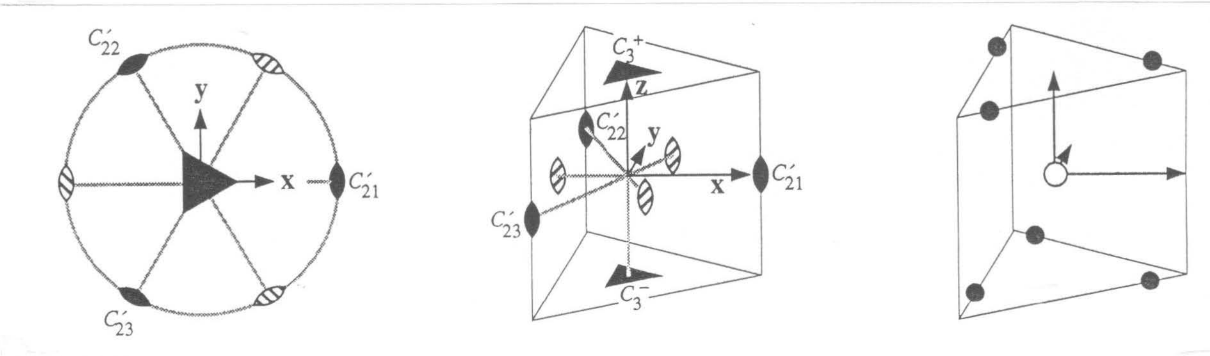
(3) Operations of G: E, (C_3^+, C_3^-), $(C'_{21}, C'_{22}, C'_{23})$.

(4) Operations of $\widetilde{G}$: E, (C_3^+, C_3^-), $(C'_{21}, C'_{22}, C'_{23})$,
$\widetilde{E}$, $(\widetilde{C}_3^+, \widetilde{C}_3^-)$, $(\widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23})$.

(5) Classes and representations: $|r| = 3$, $|i| = 0$, $|I| = 3$, $|\widetilde{I}| = 3$.

F 23

See Chapter **15**, p. 65



Examples: Possible excited state of C_2H_6 partly rotated.

T 23.1 Parameters
Use T **35**.1. § **16**–1, p. 68

T 23.2 Multiplication table
Use T **35**.2. § **16**–2, p. 69

T 23.3 Factor table
Use T **35**.3. § **16**–3, p. 70

T 23.4 Character table
§ **16**–4, p. 71

$\mathbf{D}_3$	E	$2C_3$	$3C'_2$	τ
A_1	1	1	1	a
A_2	1	1	−1	a
E	2	−1	0	a
$E_{1/2}$	2	1	0	c
${}^1E_{3/2}$	1	−1	i	b
${}^2E_{3/2}$	1	−1	−i	b

T **23**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_3$	0	1	2	3
A_1	$\square 1$		$x^2+y^2, \square z^2$	$\square x(x^2-3y^2)$
A_2		$\square z, R_z$		$\square y(3x^2-y^2), (x^2+y^2)z, \square z^3$
E		$\square(x,y), (R_x,R_y)$	$\square(xy, x^2-y^2), \square(zx, yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, \square(xz^2, yz^2),$ $\square\{xyz, z(x^2-y^2)\}$

T **23**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_3$	E	$E_{1/2}$
E	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$
C_3^+	$\begin{bmatrix}\epsilon^* & 0\\ 0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon} & 0\\ 0 & \bar{\epsilon}^*\end{bmatrix}$
C_3^-	$\begin{bmatrix}\epsilon & 0\\ 0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon}^* & 0\\ 0 & \bar{\epsilon}\end{bmatrix}$
C'_{21}	$\begin{bmatrix}0 & \bar{1}\\ \bar{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\mathrm{i}}\\ \bar{\mathrm{i}} & 0\end{bmatrix}$
C'_{22}	$\begin{bmatrix}0 & \bar{\epsilon}\\ \bar{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \mathrm{i}\bar{\epsilon}^*\\ \mathrm{i}\bar{\epsilon} & 0\end{bmatrix}$
C'_{23}	$\begin{bmatrix}0 & \bar{\epsilon}^*\\ \bar{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \mathrm{i}\bar{\epsilon}\\ \mathrm{i}\bar{\epsilon}^* & 0\end{bmatrix}$

$\epsilon = \exp(2\pi\mathrm{i}/3)$

T **23**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_3$	$\langle\lvert j\,m\rangle\rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$	$\lvert 3\,3\rangle_-$	2	3
A_2	$\lvert 1\,0\rangle_+$	$\lvert 4\,3\rangle_-$	2	3
E	$\langle\lvert 1\,1\rangle, \lvert 1\,\bar{1}\rangle\rvert$	$\langle\lvert 2\,\bar{2}\rangle, -\lvert 2\,2\rangle\rvert$	2	± 3
$E_{1/2}$	$\langle\lvert \tfrac12\,\tfrac12\rangle, \lvert \tfrac12\,\overline{\tfrac12}\rangle\rvert$	$\langle\lvert \tfrac32\,\tfrac12\rangle, -\lvert \tfrac32\,\overline{\tfrac12}\rangle\rvert$	2	± 3
${}^1E_{3/2}$	$\lvert \tfrac32\,\tfrac32\rangle_+$	$\lvert \tfrac52\,\tfrac32\rangle_-$	2	3
${}^2E_{3/2}$	$\lvert \tfrac32\,\tfrac32\rangle_-$	$\lvert \tfrac52\,\tfrac32\rangle_+$	2	3

T **23**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{D}_3$	A_1	A_2	E	$E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$
A_1	A_1	A_2	E	$E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$
A_2		A_1	E	$E_{1/2}$	${}^2E_{3/2}$	${}^1E_{3/2}$
E			$A_1 \oplus \{A_2\} \oplus E$	$E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{1/2}$	$E_{1/2}$
$E_{1/2}$				$\{A_1\} \oplus A_2 \oplus E$	E	E
${}^1E_{3/2}$					A_2	A_1
${}^2E_{3/2}$						A_2

T **23**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_3$
$6n$	$(2n+1)\,A_1 \oplus 2n\,(A_2 \oplus 2E)$
$6n+1$	$2n\,(A_1 \oplus E) \oplus (2n+1)(A_2 \oplus E)$
$6n+2$	$(2n+1)(A_1 \oplus 2E) \oplus 2n\,A_2$
$6n+3$	$(2n+1)(A_1 \oplus 2E) \oplus (2n+2)\,A_2$
$6n+4$	$(2n+2)(A_1 \oplus E) \oplus (2n+1)(A_2 \oplus E)$
$6n+5$	$(2n+1)\,A_1 \oplus (2n+2)(A_2 \oplus 2E)$
$3n+\frac12$	$(2n+1)\,E_{1/2} \oplus n\,({}^1E_{3/2} \oplus {}^2E_{3/2})$
$3n+\frac32$	$(2n+1)\,E_{1/2} \oplus (n+1)({}^1E_{3/2} \oplus {}^2E_{3/2})$
$3n+\frac52$	$(n+1)(2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2})$

$n = 0, 1, 2, \ldots$

T **23**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_3$	$\mathbf{C}_3$	$(\mathbf{C}_2)$
A_1	A	A
A_2	A	B
E	${}^1E \oplus {}^2E$	$A \oplus B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1E_{3/2}$	$A_{3/2}$	${}^1E_{1/2}$
${}^2E_{3/2}$	$A_{3/2}$	${}^2E_{1/2}$

T **23**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83

a_2	e	E 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

e	e	A_1 1	A_2 1	E 1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{\mathrm{u}}$	0	0
2	2	0	0	1	0

e	$e_{1/2}$	$E_{1/2}$ 1	2	${}^1E_{3/2}$ 1	${}^2E_{3/2}$ 1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	u	$\bar{\mathrm{u}}$

e	${}^1e_{3/2}$	$E_{1/2}$ 1	2
1	1	0	1
2	1	1	0

e	${}^2e_{3/2}$	$E_{1/2}$ 1	2
1	1	0	$\bar{1}$
2	1	1	0

$e_{1/2}$	$e_{1/2}$	A_1 1	A_2 1	E 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\mathrm{u}}$	u	0	0
2	2	0	0	0	$\bar{1}$

$e_{1/2}$	${}^1e_{3/2}$	E 1	2
1	1	0	$\bar{1}$
2	1	1	0

$e_{1/2}$	${}^2e_{3/2}$	E 1	2
1	1	0	1
2	1	1	0

$\mathrm{u} = 2^{-1/2}$

(1) Product forms: $\mathbf{C}_4 \otimes \mathbf{C}_2'$, $\mathbf{D}_2 \otimes \mathbf{C}_2''$.

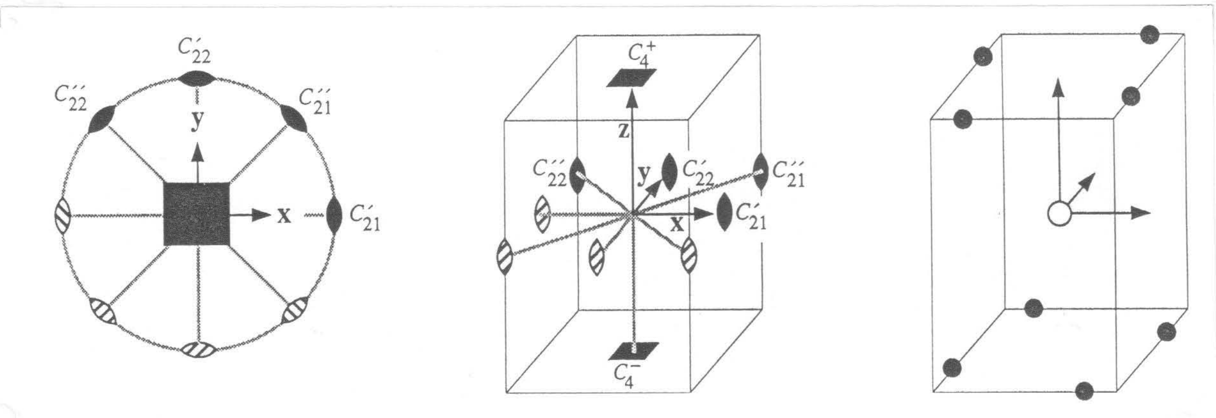
(2) Group chains: $\mathbf{O} \supset (\mathbf{D}_4) \supset (\underline{\mathbf{D}_2})$, $\mathbf{O} \supset (\mathbf{D}_4) \supset \underline{\mathbf{C}_4}$, $\mathbf{D}_{4d} \supset (\underline{\mathbf{D}_4}) \supset (\underline{\mathbf{D}_2})$, $\mathbf{D}_{4d} \supset (\underline{\mathbf{D}_4}) \supset \underline{\mathbf{C}_4}$, $\mathbf{D}_{4h} \supset \underline{\mathbf{D}_4} \supset (\underline{\mathbf{D}_2})$, $\mathbf{D}_{4h} \supset \underline{\mathbf{D}_4} \supset \underline{\mathbf{C}_4}$.

(3) Operations of G: E, (C_4^+, C_4^-), C_2, (C_{21}', C_{22}'), (C_{21}'', C_{22}'').

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_4^+, C_4^-), $(\widetilde{C}_4^+, \widetilde{C}_4^-)$, $(C_2, \widetilde{C}_2)$, $(C_{21}', C_{22}', \widetilde{C}_{21}', \widetilde{C}_{22}')$, $(C_{21}'', C_{22}'', \widetilde{C}_{21}'', \widetilde{C}_{22}'')$.

(5) Classes and representations: $|r| = 2$, $|\text{i}| = 3$, $|I| = 5$, $|\widetilde{I}| = 2$.

F **24** See Chapter **15**, p. 65



Examples:

T 24.1 Parameters
Use T **33**.1. § **16**–1, p. 68

T 24.2 Multiplication table
Use T **33**.2. § **16**–2, p. 69

T 24.3 Factor table
Use T **33**.3. § **16**–3, p. 70

T 24.4 Character table
§ **16**–4, p. 71

$\mathbf{D}_4$	E	$2C_4$	C_2	$2C_2'$	$2C_2''$	τ
A_1	1	1	1	1	1	a
A_2	1	1	1	−1	−1	a
B_1	1	−1	1	1	−1	a
B_2	1	−1	1	−1	1	a
E	2	0	−2	0	0	a
$E_{1/2}$	2	$\sqrt{2}$	0	0	0	c
$E_{3/2}$	2	$-\sqrt{2}$	0	0	0	c

T **24**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_4$	0	1	2	3
A_1	$^\square 1$		$x^2+y^2, {}^\square z^2$	
A_2		$^\square z, R_z$		$(x^2+y^2)z, {}^\square z^3$
B_1			$^\square x^2-y^2$	$^\square xyz$
B_2			$^\square xy$	$^\square z(x^2-y^2)$
E		$^\square(x,y),(R_x,R_y)$	$^\square(zx,yz)$	$\{x(x^2+y^2),y(x^2+y^2)\}, {}^\square(xz^2,yz^2)$, $^\square\{x(x^2-3y^2),y(3x^2-y^2)\}$

T **24**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_4$	$\langle \lvert j\,m\rangle \rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$	$\lvert 5\,4\rangle_-$	2	4
A_2	$\lvert 1\,0\rangle_+$	$\lvert 4\,4\rangle_-$	2	4
B_1	$\lvert 2\,2\rangle_+$	$\lvert 3\,2\rangle_-$	2	4
B_2	$\lvert 2\,2\rangle_-$	$\lvert 3\,2\rangle_+$	2	4
E	$\langle \lvert 1\,1\rangle, \lvert 1\,\bar{1}\rangle \rvert$	$\langle \lvert 2\,1\rangle, -\lvert 2\,\bar{1}\rangle \rvert$	2	± 4
$E_{1/2}$	$\langle \lvert \tfrac12\,\tfrac12\rangle, \lvert \tfrac12\,\tfrac{\bar1}{2}\rangle \rvert$	$\langle \lvert \tfrac32\,\tfrac12\rangle, -\lvert \tfrac32\,\tfrac{\bar1}{2}\rangle \rvert$	2	± 4
$E_{3/2}$	$\langle \lvert \tfrac32\,\tfrac{\bar3}{2}\rangle, -\lvert \tfrac32\,\tfrac32\rangle \rvert$	$\langle \lvert \tfrac52\,\tfrac{\bar3}{2}\rangle, \lvert \tfrac52\,\tfrac32\rangle \rvert$	2	± 4

T **24**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_4$	E	$E_{1/2}$	$E_{3/2}$
E	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$
C_4^+	$\begin{bmatrix}\bar{\mathrm{i}}&0\\0&\mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\bar\epsilon^*&0\\0&\bar\epsilon\end{bmatrix}$
C_4^-	$\begin{bmatrix}\mathrm{i}&0\\0&\bar{\mathrm{i}}\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\bar\epsilon&0\\0&\bar\epsilon^*\end{bmatrix}$
C_2	$\begin{bmatrix}\bar1&0\\0&\bar1\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}}&0\\0&\mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}}&0\\0&\mathrm{i}\end{bmatrix}$
C'_{21}	$\begin{bmatrix}0&\bar1\\\bar1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\mathrm{i}}\\\bar{\mathrm{i}}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\mathrm{i}}\\\bar{\mathrm{i}}&0\end{bmatrix}$
C'_{22}	$\begin{bmatrix}0&1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\1&0\end{bmatrix}$
C''_{21}	$\begin{bmatrix}0&\mathrm{i}\\\bar{\mathrm{i}}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\epsilon\\\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon\\\bar\epsilon^*&0\end{bmatrix}$
C''_{22}	$\begin{bmatrix}0&\bar{\mathrm{i}}\\\mathrm{i}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\epsilon^*\\\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon^*\\\bar\epsilon&0\end{bmatrix}$

$\epsilon = \exp(2\pi\mathrm{i}/8)$

T 24.8 Direct products of representations

§ 16–8, p. 81

$\mathbf{D}_4$	A_1	A_2	B_1	B_2	E	$E_{1/2}$	$E_{3/2}$
A_1	A_1	A_2	B_1	B_2	E	$E_{1/2}$	$E_{3/2}$
A_2		A_1	B_2	B_1	E	$E_{1/2}$	$E_{3/2}$
B_1			A_1	A_2	E	$E_{3/2}$	$E_{1/2}$
B_2				A_1	E	$E_{3/2}$	$E_{1/2}$
E					$A_1 \oplus \{A_2\} \oplus B_1 \oplus B_2$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{3/2}$
$E_{1/2}$						$\{A_1\} \oplus A_2 \oplus E$	$B_1 \oplus B_2 \oplus E$
$E_{3/2}$							$\{A_1\} \oplus A_2 \oplus E$

T 24.9 Subduction (descent of symmetry)

§ 16–9, p. 82

$\mathbf{D}_4$	$(\mathbf{D}_2)$ C_2'	$(\mathbf{D}_2)$ C_2''	$\mathbf{C}_4$	$\mathbf{C}_2$ C_2	$(\mathbf{C}_2)$ C_2'	$(\mathbf{C}_2)$ C_2''
A_1	A	A	A	A	A	A
A_2	B_1	B_1	A	A	B	B
B_1	A	B_1	B	A	A	B
B_2	B_1	A	B	A	B	A
E	$B_2 \oplus B_3$	$B_2 \oplus B_3$	${}^1E \oplus {}^2E$	$2B$	$A \oplus B$	$A \oplus B$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T 24.10 Subduction from O(3)

§ 16–10, p. 82

j	$\mathbf{D}_4$
$4n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E)$
$4n+1$	$n\,(A_1 \oplus B_1 \oplus B_2 \oplus E) \oplus (n+1)(A_2 \oplus E)$
$4n+2$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E) \oplus n\,(A_2 \oplus E)$
$4n+3$	$n\,A_1 \oplus (n+1)(A_2 \oplus B_1 \oplus B_2 \oplus 2E)$
$4n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,E_{3/2}$
$4n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2})$
$4n+\frac{5}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)\,E_{3/2}$
$4n+\frac{7}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2})$

$n = 0, 1, 2, \ldots$

T 24.11 Clebsch–Gordan coefficients

§ 16–11, p. 83 $\mathbf{D}_4$, $\mathbf{C}_{4v}$

a_2	e	E 1	E 2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$ 1	$E_{1/2}$ 2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{3/2}$	$E_{3/2}$ 1	$E_{3/2}$ 2
1	1	1	0
1	2	0	$\bar{1}$

$\twoheadrightarrow$

T **24**.11 Clebsch–Gordan coefficients *(cont.)* $\mathbf{D}_4,\ \mathbf{C}_{4v}$

b_1	e	E 1	2
1	1	0	1
1	2	1	0

b_1	$e_{1/2}$	$E_{3/2}$ 1	2
1	1	1	0
1	2	0	1

b_1	$e_{3/2}$	$E_{1/2}$ 1	2
1	1	1	0
1	2	0	1

b_2	e	E 1	2
1	1	0	$\bar{1}$
1	2	1	0

b_2	$e_{1/2}$	$E_{3/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{3/2}$	$E_{1/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

e	e	A_1 1	A_2 1	B_1 1	B_2 1
1	1	0	0	u	u
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	u	$\bar{u}$

e	$e_{1/2}$	$E_{1/2}$ 1	2	$E_{3/2}$ 1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e	$e_{3/2}$	$E_{1/2}$ 1	2	$E_{3/2}$ 1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{1/2}$	$e_{1/2}$	A_1 1	A_2 1	E 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{3/2}$	B_1 1	B_2 1	E 1	2
1	1	0	0	0	1
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{3/2}$	A_1 1	A_2 1	E 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

(1) Product forms: $\mathbf{C}_5 \otimes \mathbf{C}'_2$.

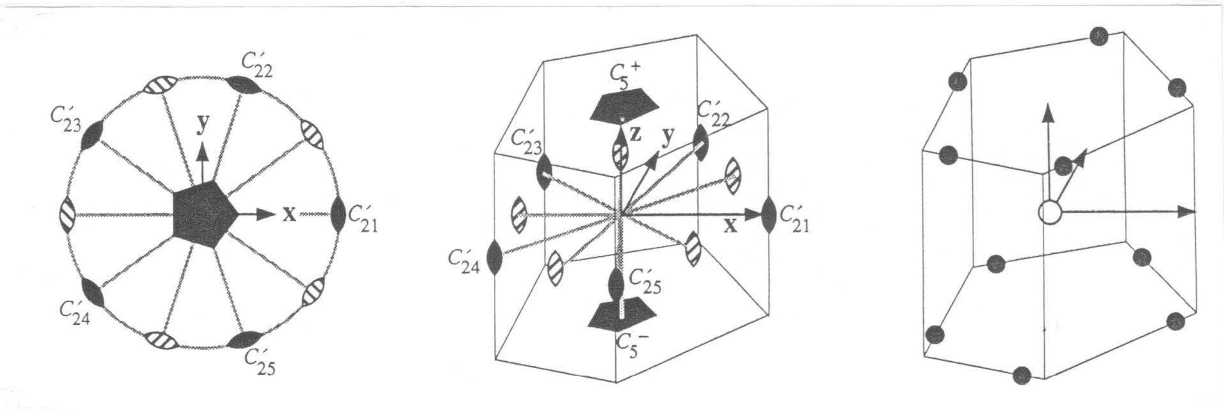
(2) Group chains: $\mathbf{I} \supset (\mathbf{D}_5) \supset \underline{\mathbf{C}_5}$, $\mathbf{I} \supset (\mathbf{D}_5) \supset (\mathbf{C}_2)$, $\mathbf{D}_{5d} \supset \underline{\mathbf{D}_5} \supset \underline{\mathbf{C}_5}$, $\mathbf{D}_{5d} \supset \underline{\mathbf{D}_5} \supset (\mathbf{C}_2)$,
$\mathbf{D}_{5h} \supset (\underline{\mathbf{D}_5}) \supset \underline{\mathbf{C}_5}$, $\mathbf{D}_{5h} \supset (\underline{\mathbf{D}_5}) \supset (\mathbf{C}_2)$, $\mathbf{D}_{10} \supset (\underline{\mathbf{D}_5}) \supset \underline{\mathbf{C}_5}$, $\mathbf{D}_{10} \supset (\underline{\mathbf{D}_5}) \supset (\mathbf{C}_2)$.

(3) Operations of G: E, (C_5^+, C_5^-), (C_5^{2+}, C_5^{2-}), $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25})$.

(4) Operations of $\widetilde{G}$: E, (C_5^+, C_5^-), (C_5^{2+}, C_5^{2-}), $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25})$,
$\widetilde{E}$, $(\widetilde{C}_5^+, \widetilde{C}_5^-)$, $(\widetilde{C}_5^{2+}, \widetilde{C}_5^{2-})$, $(\widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24}, \widetilde{C}'_{25})$.

(5) Classes and representations: $|r| = 4$, $|\mathrm{i}| = 0$, $|I| = 4$, $|\widetilde{I}| = 4$.

F **25** See Chapter **15**, p. 65



Examples:

T **25**.1 Parameters

Use T **39**.1. § **16**–1, p. 68

T **25**.2 Multiplication table

Use T **39**.2. § **16**–2, p. 69

T **25**.3 Factor table

Use T **39**.3. § **16**–3, p. 70

T **25**.4 Character table

§ **16**–4, p. 71

$\mathbf{D}_5$	E	$2C_5$	$2C_5^2$	$5C'_2$	τ
A_1	1	1	1	1	a
A_2	1	1	1	-1	a
E_1	2	$2c_5^2$	$2c_5^4$	0	a
E_2	2	$2c_5^4$	$2c_5^2$	0	a
$E_{1/2}$	2	$-2c_5^4$	$2c_5^2$	0	c
$E_{3/2}$	2	$-2c_5^2$	$2c_5^4$	0	c
${}^1E_{5/2}$	1	-1	1	i	b
${}^2E_{5/2}$	1	-1	1	$-$i	b

$c_n^m = \cos \frac{m}{n}\pi$

T **25**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_5$	0	1	2	3
A_1	$^\square 1$		$x^2+y^2, {}^\square z^2$	
A_2		$^\square z, R_z$		$(x^2+y^2)z, {}^\square z^3$
E_1		$^\square(x,y),(R_x,R_y)$	$^\square(zx,yz)$	$\{x(x^2+y^2),y(x^2+y^2)\}, {}^\square(xz^2,yz^2)$
E_2			$^\square(xy,x^2-y^2)$	$^\square\{x(x^2-3y^2),y(3x^2-y^2)\}, {}^\square\{xyz,z(x^2-y^2)\}$

T **25**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_5$	$\langle \lvert j\,m\rangle \rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$	$\lvert 5\,5\rangle_-$	2	5
A_2	$\lvert 1\,0\rangle_+$	$\lvert 6\,5\rangle_-$	2	5
E_1	$\langle \lvert 1\,1\rangle, \lvert 1\,\bar{1}\rangle \rvert$	$\langle \lvert 2\,1\rangle, -\lvert 2\,\bar{1}\rangle \rvert$	2	± 5
E_2	$\langle \lvert 2\,2\rangle, \lvert 2\,\bar{2}\rangle \rvert$	$\langle \lvert 3\,\bar{3}\rangle, -\lvert 3\,3\rangle \rvert$	2	± 5
$E_{1/2}$	$\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \rvert$	$\langle \lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle \rvert$	2	± 5
$E_{3/2}$	$\langle \lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle \rvert$	$\langle \lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle \rvert$	2	± 5
$^1E_{5/2}$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle_-$	$\lvert \frac{7}{2}\,\frac{5}{2}\rangle_+$	2	5
$^2E_{5/2}$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle_+$	$\lvert \frac{7}{2}\,\frac{5}{2}\rangle_-$	2	5

T **25**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_5$	E_1	E_2	$E_{1/2}$	$E_{3/2}$
E	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$
C_5^+	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon}&0\\0&\bar{\epsilon}^*\end{bmatrix}$	$\begin{bmatrix}\bar{\delta}&0\\0&\bar{\delta}^*\end{bmatrix}$
C_5^-	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon}^*&0\\0&\bar{\epsilon}\end{bmatrix}$	$\begin{bmatrix}\bar{\delta}^*&0\\0&\bar{\delta}\end{bmatrix}$
C_5^{2+}	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$
C_5^{2-}	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$
C'_{21}	$\begin{bmatrix}0&\bar{1}\\\bar{1}&0\end{bmatrix}$	$\begin{bmatrix}0&1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\mathrm{i}}\\\bar{\mathrm{i}}&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\\\mathrm{i}&0\end{bmatrix}$
C'_{22}	$\begin{bmatrix}0&\bar{\epsilon}^*\\\bar{\epsilon}&0\end{bmatrix}$	$\begin{bmatrix}0&\delta\\\delta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\bar{\delta}^*\\\mathrm{i}\bar{\delta}&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\epsilon\\\mathrm{i}\epsilon^*&0\end{bmatrix}$
C'_{23}	$\begin{bmatrix}0&\bar{\delta}\\\bar{\delta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon\\\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\bar{\epsilon}^*\\\mathrm{i}\bar{\epsilon}&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\delta^*\\\mathrm{i}\delta&0\end{bmatrix}$
C'_{24}	$\begin{bmatrix}0&\bar{\delta}^*\\\bar{\delta}&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon^*\\\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\bar{\epsilon}\\\mathrm{i}\bar{\epsilon}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\delta\\\mathrm{i}\delta^*&0\end{bmatrix}$
C'_{25}	$\begin{bmatrix}0&\bar{\epsilon}\\\bar{\epsilon}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\delta^*\\\delta&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\bar{\delta}\\\mathrm{i}\bar{\delta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\epsilon^*\\\mathrm{i}\epsilon&0\end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/5)$, $\epsilon = \exp(4\pi\mathrm{i}/5)$

T **25**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{D}_5$	A_1	A_2	E_1	E_2	$E_{1/2}$	$E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$
A_1	A_1	A_2	E_1	E_2	$E_{1/2}$	$E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$
A_2		A_1	E_1	E_2	$E_{1/2}$	$E_{3/2}$	${}^2E_{5/2}$	${}^1E_{5/2}$
E_1			$A_1 \oplus \{A_2\} \oplus E_2$	$E_1 \oplus E_2$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{3/2}$	$E_{3/2}$
E_2				$A_1 \oplus \{A_2\} \oplus E_1$	$E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2}$	$E_{1/2}$
$E_{1/2}$					$\{A_1\} \oplus A_2 \oplus E_1$	$E_1 \oplus E_2$	E_2	E_2
$E_{3/2}$						$\{A_1\} \oplus A_2 \oplus E_2$	E_1	E_1
${}^1E_{5/2}$							A_2	A_1
${}^2E_{5/2}$								A_2

T **25**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_5$	$\mathbf{C}_5$	$(\mathbf{C}_2)$
A_1	A	A
A_2	A	B
E_1	${}^1E_1 \oplus {}^2E_1$	$A \oplus B$
E_2	${}^1E_2 \oplus {}^2E_2$	$A \oplus B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1E_{5/2}$	$A_{5/2}$	${}^1E_{1/2}$
${}^2E_{5/2}$	$A_{5/2}$	${}^2E_{1/2}$

T **25**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_5$
$10n$	$(2n+1)\,A_1 \oplus 2n\,(A_2 \oplus 2E_1 \oplus 2E_2)$
$10n+1$	$2n\,(A_1 \oplus E_1 \oplus 2E_2) \oplus (2n+1)(A_2 \oplus E_1)$
$10n+2$	$(2n+1)(A_1 \oplus E_1 \oplus E_2) \oplus 2n\,(A_2 \oplus E_1 \oplus E_2)$
$10n+3$	$2n\,(A_1 \oplus E_1) \oplus (2n+1)(A_2 \oplus E_1 \oplus 2E_2)$
$10n+4$	$(2n+1)(A_1 \oplus 2E_1 \oplus 2E_2) \oplus 2n\,A_2$
$10n+5$	$(2n+1)(A_1 \oplus 2E_1 \oplus 2E_2) \oplus (2n+2)\,A_2$
$10n+6$	$(2n+2)(A_1 \oplus E_1) \oplus (2n+1)(A_2 \oplus E_1 \oplus 2E_2)$
$10n+7$	$(2n+1)(A_1 \oplus E_1 \oplus E_2) \oplus (2n+2)(A_2 \oplus E_1 \oplus E_2)$
$10n+8$	$(2n+2)(A_1 \oplus E_1 \oplus 2E_2) \oplus (2n+1)(A_2 \oplus E_1)$
$10n+9$	$(2n+1)\,A_1 \oplus (2n+2)(A_2 \oplus 2E_1 \oplus 2E_2)$
$5n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus n\,(2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2})$
$5n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus n\,({}^1E_{5/2} \oplus {}^2E_{5/2})$
$5n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (n+1)({}^1E_{5/2} \oplus {}^2E_{5/2})$
$5n+\frac{7}{2}$	$(2n+1)\,E_{1/2} \oplus (n+1)(2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2})$
$5n+\frac{9}{2}$	$(n+1)(2E_{1/2} \oplus 2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2})$

$n = 0, 1, 2, \ldots$

T **25**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{D}_5$

a_2	e_1	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_2	E_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e_1	e_1	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_1	e_2	E_1		E_2	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	$e_{1/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_1	$e_{3/2}$	$E_{1/2}$		${}^1E_{5/2}$	${}^2E_{5/2}$
		1	2	1	1
1	1	0	0	u	u
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	$\bar{u}$	u

e_1	${}^1e_{5/2}$	$E_{3/2}$	
		1	2
1	1	0	$\bar{1}$
2	1	1	0

e_1	${}^2e_{5/2}$	$E_{3/2}$	
		1	2
1	1	0	1
2	1	1	0

e_2	e_2	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_2	$e_{1/2}$	$E_{3/2}$		${}^1E_{5/2}$	${}^2E_{5/2}$
		1	2	1	1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	$\bar{u}$	u

e_2	$e_{3/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_2	${}^1e_{5/2}$	$E_{1/2}$	
		1	2
1	1	0	$\bar{1}$
2	1	1	0

e_2	${}^2e_{5/2}$	$E_{1/2}$	
		1	2
1	1	0	1
2	1	1	0

$e_{1/2}$	$e_{1/2}$	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{3/2}$	E_1		E_2	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{1/2}$	${}^1e_{5/2}$	E_2	
		1	2
1	1	0	1
2	1	1	0

$e_{1/2}$	${}^2e_{5/2}$	E_2	
		1	2
1	1	0	$\bar{1}$
2	1	1	0

$e_{3/2}$	$e_{3/2}$	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{3/2}$	${}^1e_{5/2}$	E_1	
		1	2
1	1	0	1
2	1	1	0

$e_{3/2}$	${}^2e_{5/2}$	E_1	
		1	2
1	1	0	$\bar{1}$
2	1	1	0

$u = 2^{-1/2}$

(1) Product forms: $\mathbf{C}_6 \otimes \mathbf{C}'_2$, $\mathbf{D}_3 \otimes \mathbf{C}''_2$.

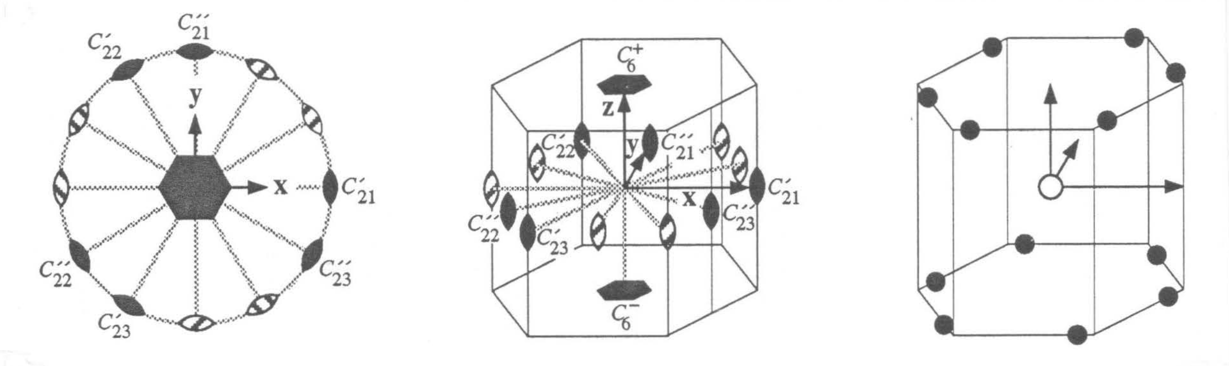
(2) Group chains: $\mathbf{D}_{6d} \supset (\underline{\mathbf{D}_6}) \supset (\underline{\mathbf{D}_3})$, $\mathbf{D}_{6d} \supset (\underline{\mathbf{D}_6}) \supset (\mathbf{D}_2)$, $\mathbf{D}_{6d} \supset (\underline{\mathbf{D}_6}) \supset \underline{\mathbf{C}_6}$,
$\mathbf{D}_{6h} \supset \underline{\mathbf{D}_6} \supset (\underline{\mathbf{D}_3})$, $\mathbf{D}_{6h} \supset \underline{\mathbf{D}_6} \supset (\mathbf{D}_2)$, $\mathbf{D}_{6h} \supset \underline{\mathbf{D}_6} \supset \underline{\mathbf{C}_6}$.

(3) Operations of G: E, (C_6^+, C_6^-), (C_3^+, C_3^-), C_2, $(C'_{21}, C'_{22}, C'_{23})$, $(C''_{21}, C''_{22}, C''_{23})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_6^+, C_6^-), $(\widetilde{C}_6^+, \widetilde{C}_6^-)$, (C_3^+, C_3^-), $(\widetilde{C}_3^+, \widetilde{C}_3^-)$, $(C_2, \widetilde{C}_2)$,
$(C'_{21}, C'_{22}, C'_{23}, \widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23})$, $(C''_{21}, C''_{22}, C''_{23}, \widetilde{C}''_{21}, \widetilde{C}''_{22}, \widetilde{C}''_{23})$.

(5) Classes and representations: $|r| = 3$, $|\text{i}| = 3$, $|I| = 6$, $|\widetilde{I}| = 3$.

F **26** See Chapter **15**, p. 65



Examples:

T **26**.1 Parameters
Use T **35**.1. § **16**–1, p. 68

T **26**.2 Multiplication table
Use T **35**.2. § **16**–2, p. 69

T **26**.3 Factor table
Use T **35**.3. § **16**–3, p. 70

T **26**.4 Character table
§ **16**–4, p. 71

$\mathbf{D}_6$	E	$2C_6$	$2C_3$	C_2	$3C'_2$	$3C''_2$	τ
A_1	1	1	1	1	1	1	a
A_2	1	1	1	1	−1	−1	a
B_1	1	−1	1	−1	1	−1	a
B_2	1	−1	1	−1	−1	1	a
E_1	2	1	−1	−2	0	0	a
E_2	2	−1	−1	2	0	0	a
$E_{1/2}$	2	$\sqrt{3}$	1	0	0	0	c
$E_{3/2}$	2	0	−2	0	0	0	c
$E_{5/2}$	2	$-\sqrt{3}$	1	0	0	0	c

T **26**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_6$	0	1	2	3
A_1	$^\square 1$		$x^2+y^2, {}^\square z^2$	
A_2		$^\square z, R_z$		$(x^2+y^2)z, {}^\square z^3$
B_1				$^\square x(x^2-3y^2)$
B_2				$^\square y(3x^2-y^2)$
E_1		$^\square(x,y),(R_x,R_y)$	$^\square(zx,yz)$	$\{x(x^2+y^2),y(x^2+y^2)\}, {}^\square(xz^2,yz^2)$
E_2			$^\square(xy,x^2-y^2)$	$^\square\{xyz,z(x^2-y^2)\}$

T **26**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_6$	$\langle\|j\,m\rangle\|$		ι	μ
A_1	$\|0\,0\rangle_+$	$\|7\,6\rangle_-$	2	6
A_2	$\|1\,0\rangle_+$	$\|6\,6\rangle_-$	2	6
B_1	$\|3\,3\rangle_-$	$\|4\,3\rangle_+$	2	6
B_2	$\|3\,3\rangle_+$	$\|4\,3\rangle_-$	2	6
E_1	$\langle\|1\,1\rangle,\|1\,\overline{1}\rangle\|$	$\langle\|2\,1\rangle,-\|2\,\overline{1}\rangle\|$	2	±6
E_2	$\langle\|2\,\overline{2}\rangle,-\|2\,2\rangle\|$	$\langle\|3\,\overline{2}\rangle,\|3\,2\rangle\|$	2	±6
$E_{1/2}$	$\langle\|\frac{1}{2}\,\frac{1}{2}\rangle,\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle\|$	$\langle\|\frac{3}{2}\,\frac{1}{2}\rangle,-\|\frac{3}{2}\,\overline{\frac{1}{2}}\rangle\|$	2	±6
$E_{3/2}$	$\langle\|\frac{3}{2}\,\frac{3}{2}\rangle,\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle\|$	$\langle\|\frac{5}{2}\,\frac{3}{2}\rangle,-\|\frac{5}{2}\,\overline{\frac{3}{2}}\rangle\|$	2	±6
$E_{5/2}$	$\langle\|\frac{5}{2}\,\overline{\frac{5}{2}}\rangle,\|\frac{5}{2}\,\frac{5}{2}\rangle\|$	$\langle\|\frac{7}{2}\,\overline{\frac{5}{2}}\rangle,-\|\frac{7}{2}\,\frac{5}{2}\rangle\|$	2	±6

T **26**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_6$	E_1	E_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_6^+	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$
C_6^-	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar{\epsilon}^* \end{bmatrix}$
C_3^+	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$
C_3^-	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$
C_2	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$
C''_{21}	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$
C''_{22}	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$
C''_{23}	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$

$\epsilon = \exp(2\pi\mathrm{i}/3)$

T **26**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{D}_6$	A_1	A_2	B_1	B_2	E_1	E_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
A_1	A_1	A_2	B_1	B_2	E_1	E_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
A_2		A_1	B_2	B_1	E_1	E_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
B_1			A_1	A_2	E_2	E_1	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
B_2				A_1	E_2	E_1	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
E_1					$A_1 \oplus \{A_2\} \oplus E_2$	$B_1 \oplus B_2 \oplus E_1$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus E_{5/2}$
E_2						$A_1 \oplus \{A_2\} \oplus E_2$	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$
$E_{1/2}$							$\{A_1\} \oplus A_2 \oplus E_1$	$E_1 \oplus E_2$	$B_1 \oplus B_2 \oplus E_2$
$E_{3/2}$								$\{A_1\} \oplus A_2 \oplus B_1 \oplus B_2$	$E_1 \oplus E_2$
$E_{5/2}$									$\{A_1\} \oplus A_2 \oplus E_1$

T **26**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_6$	$(\mathbf{D}_3)$ C_2'	$(\mathbf{D}_3)$ C_2''	$(\mathbf{D}_2)$	$\mathbf{C}_6$
A_1	A_1	A_1	A	A
A_2	A_2	A_2	B_1	A
B_1	A_1	A_2	B_3	B
B_2	A_2	A_1	B_2	B
E_1	E	E	$B_2 \oplus B_3$	${}^1E_1 \oplus {}^2E_1$
E_2	E	E	$A \oplus B_1$	${}^1E_2 \oplus {}^2E_2$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$
$E_{5/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$

$\twoheadrightarrow$

T **26**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_6$	$\mathbf{C}_3$	$\mathbf{C}_2$ C_2	$(\mathbf{C}_2)$ C_2'	$(\mathbf{C}_2)$ C_2''
A_1	A	A	A	A
A_2	A	A	B	B
B_1	A	B	A	B
B_2	A	B	B	A
E_1	${}^1E \oplus {}^2E$	$2B$	$A \oplus B$	$A \oplus B$
E_2	${}^1E \oplus {}^2E$	$2A$	$A \oplus B$	$A \oplus B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	$2A_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **26**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_6$
$6n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2)$
$6n+1$	$n\,(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus 2E_2) \oplus (n+1)(A_2 \oplus E_1)$
$6n+2$	$(n+1)(A_1 \oplus E_1 \oplus E_2) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2)$
$6n+3$	$n\,(A_1 \oplus E_1 \oplus E_2) \oplus (n+1)(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2)$
$6n+4$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus 2E_2) \oplus n\,(A_2 \oplus E_1)$
$6n+5$	$n\,A_1 \oplus (n+1)(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2)$
$6n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,(E_{3/2} \oplus E_{5/2})$
$6n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus 2n\,E_{5/2}$
$6n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2})$
$6n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (2n+2)\,E_{5/2}$
$6n+\frac{9}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)(E_{3/2} \oplus E_{5/2})$
$6n+\frac{11}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2})$

$n = 0, 1, 2, \ldots$

T **26**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{D}_6$, $\mathbf{C}_{6v}$

a_2	e_1	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_2	E_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{5/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_1	e_1	E_2	
		1	2
1	1	1	0
1	2	0	1

b_1	e_2	E_1	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{1/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	0	1
1	2	1	0

b_1	$e_{5/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	1

b_2	e_1	E_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_2	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{1/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	0	$\bar{1}$
1	2	1	0

b_2	$e_{5/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e_1	e_1	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_1	e_2	B_1	B_2	E_1	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_1	$e_{1/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_1	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	$e_{5/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_2	e_2	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_2	$e_{1/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_2	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_2	$e_{5/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

↠

T **26**.11 Clebsch–Gordan coefficients *(cont.)* $\mathbf{D}_6,\ \mathbf{C}_{6v}$

$e_{1/2}$	$e_{1/2}$	A_1 1	A_2 1	E_1 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{3/2}$	E_1 1	2	E_2 1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{1/2}$	$e_{5/2}$	B_1 1	B_2 1	E_2 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{3/2}$	A_1 1	A_2 1	B_1 1	B_2 1
1	1	0	0	u	u
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	$\bar{u}$	u

$e_{3/2}$	$e_{5/2}$	E_1 1	2	E_2 1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{5/2}$	$e_{5/2}$	A_1 1	A_2 1	E_1 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

(1) Product forms: $\mathbf{C}_7 \otimes \mathbf{C}'_2$.

(2) Group chains: $\mathbf{D}_{7d} \supset \underline{\mathbf{D}_7} \supset \underline{\mathbf{C}_7}$, $\mathbf{D}_{7d} \supset \underline{\mathbf{D}_7} \supset (\mathbf{C}_2)$, $\mathbf{D}_{7h} \supset (\underline{\mathbf{D}_7}) \supset \underline{\mathbf{C}_7}$, $\mathbf{D}_{7h} \supset (\underline{\mathbf{D}_7}) \supset (\mathbf{C}_2)$.

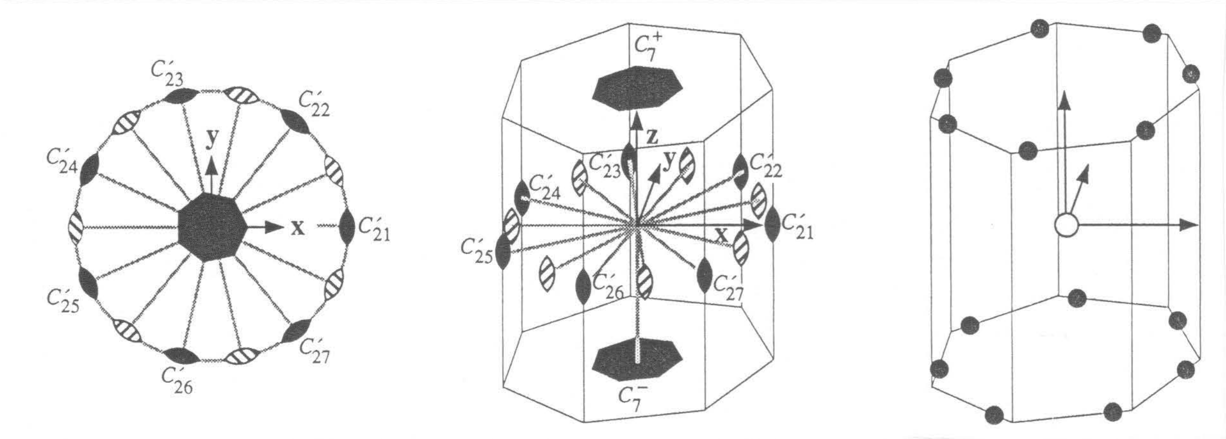
(3) Operations of G: E, (C_7^+, C_7^-), (C_7^{2+}, C_7^{2-}), (C_7^{3+}, C_7^{3-}), $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27})$.

(4) Operations of $\widetilde{G}$: E, (C_7^+, C_7^-), (C_7^{2+}, C_7^{2-}), (C_7^{3+}, C_7^{3-}), $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27})$,
$\widetilde{E}$, $(\widetilde{C}_7^+, \widetilde{C}_7^-)$, $(\widetilde{C}_7^{2+}, \widetilde{C}_7^{2-})$, $(\widetilde{C}_7^{3+}, \widetilde{C}_7^{3-})$, $(\widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24}, \widetilde{C}'_{24}, \widetilde{C}'_{26}, \widetilde{C}'_{27})$.

(5) Classes and representations: $|r| = 5$, $|\mathrm{i}| = 0$, $|I| = 5$, $|\widetilde{I}| = 5$.

F **27**

See Chapter **15**, p. 65



Examples:

T **27**.1 Parameters

Use T **36**.1. § **16**–1, p. 68

T **27**.2 Multiplication table

Use T **36**.2. § **16**–2, p. 69

T **27**.3 Factor table

Use T **36**.3. § **16**–3, p. 70

T **27**.4 Character table

§ **16**–4, p. 71

$\mathbf{D}_7$	E	$2C_7$	$2C_7^2$	$2C_7^3$	$7C'_2$	τ
A_1	1	1	1	1	1	a
A_2	1	1	1	1	-1	a
E_1	2	$2c_7^2$	$2c_7^4$	$2c_7^6$	0	a
E_2	2	$2c_7^4$	$2c_7^6$	$2c_7^2$	0	a
E_3	2	$2c_7^6$	$2c_7^2$	$2c_7^4$	0	a
$E_{1/2}$	2	$-2c_7^6$	$2c_7^2$	$-2c_7^4$	0	c
$E_{3/2}$	2	$-2c_7^4$	$2c_7^6$	$-2c_7^2$	0	c
$E_{5/2}$	2	$-2c_7^2$	$2c_7^4$	$-2c_7^6$	0	c
${}^1E_{7/2}$	1	-1	1	-1	i	b
${}^2E_{7/2}$	1	-1	1	-1	$-$i	b

$c_n^m = \cos \frac{m}{n}\pi$

T **27**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_7$	0	1	2	3
A_1	$^\square 1$		$x^2+y^2, {}^\square z^2$	
A_2		$^\square z, R_z$		$(x^2+y^2)z, {}^\square z^3$
E_1		$^\square(x,y),(R_x,R_y)$	$^\square(zx,yz)$	$\{x(x^2+y^2),y(x^2+y^2)\}, {}^\square(xz^2,yz^2)$
E_2			$^\square(xy,x^2-y^2)$	$^\square\{xyz,z(x^2-y^2)\}$
E_3				$^\square\{x(x^2-3y^2),y(3x^2-y^2)\}$

T **27**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_7$	$\langle \lvert j\,m\rangle \rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$	$\lvert 7\,7\rangle_-$	2	7
A_2	$\lvert 1\,0\rangle_+$	$\lvert 8\,7\rangle_-$	2	7
E_1	$\langle \lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle \rvert$	$\langle \lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle \rvert$	2	± 7
E_2	$\langle \lvert 2\,2\rangle, \lvert 2\,\overline{2}\rangle \rvert$	$\langle \lvert 3\,2\rangle, -\lvert 3\,\overline{2}\rangle \rvert$	2	± 7
E_3	$\langle \lvert 3\,3\rangle, \lvert 3\,\overline{3}\rangle \rvert$	$\langle \lvert 4\,\overline{4}\rangle, -\lvert 4\,4\rangle \rvert$	2	± 7
$E_{1/2}$	$\langle \lvert \tfrac12\,\tfrac12\rangle, \lvert \tfrac12\,\overline{\tfrac12}\rangle \rvert$	$\langle \lvert \tfrac32\,\tfrac12\rangle, -\lvert \tfrac32\,\overline{\tfrac12}\rangle \rvert$	2	± 7
$E_{3/2}$	$\langle \lvert \tfrac32\,\tfrac32\rangle, \lvert \tfrac32\,\overline{\tfrac32}\rangle \rvert$	$\langle \lvert \tfrac52\,\tfrac32\rangle, -\lvert \tfrac52\,\overline{\tfrac32}\rangle \rvert$	2	± 7
$E_{5/2}$	$\langle \lvert \tfrac52\,\tfrac52\rangle, \lvert \tfrac52\,\overline{\tfrac52}\rangle \rvert$	$\langle \lvert \tfrac72\,\tfrac52\rangle, -\lvert \tfrac72\,\overline{\tfrac52}\rangle \rvert$	2	± 7
${}^1E_{7/2}$	$\lvert \tfrac72\,\tfrac72\rangle_+$	$\lvert \tfrac92\,\tfrac72\rangle_-$	2	7
${}^2E_{7/2}$	$\lvert \tfrac72\,\tfrac72\rangle_-$	$\lvert \tfrac92\,\tfrac72\rangle_+$	2	7

T **27**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_7$	E_1	E_2	E_3	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_7^+	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\delta} & 0 \\ 0 & \overline{\delta}^* \end{bmatrix}$
C_7^-	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \overline{\delta}^* & 0 \\ 0 & \overline{\delta} \end{bmatrix}$
C_7^{2+}	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
C_7^{2-}	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
C_7^{3+}	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\delta}^* & 0 \\ 0 & \overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$
C_7^{3-}	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \overline{\delta} & 0 \\ 0 & \overline{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\imath} \\ \bar{\imath} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\imath} \\ \bar{\imath} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon} \\ \mathrm{i}\overline{\epsilon}^* & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon}^* \\ \mathrm{i}\overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & \overline{\delta} \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$
C'_{25}	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$
C'_{26}	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon} \\ \mathrm{i}\overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$
C'_{27}	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon}^* \\ \mathrm{i}\overline{\epsilon} & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/7)$, $\epsilon = \exp(4\pi\mathrm{i}/7)$, $\eta = \exp(6\pi\mathrm{i}/7)$

T **27**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{D}_7$	A_1	A_2	E_1	E_2	E_3
A_1	A_1	A_2	E_1	E_2	E_3
A_2		A_1	E_1	E_2	E_3
E_1			$A_1 \oplus \{A_2\} \oplus E_2$	$E_1 \oplus E_3$	$E_2 \oplus E_3$
E_2				$A_1 \oplus \{A_2\} \oplus E_3$	$E_1 \oplus E_2$
E_3					$A_1 \oplus \{A_2\} \oplus E_1$

$\twoheadrightarrow$

T **27**.8 Direct products of representations *(cont.)*

$\mathbf{D}_7$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$
A_1	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$
A_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	${}^2E_{7/2}$	${}^1E_{7/2}$
E_1	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}$	$E_{5/2}$	$E_{5/2}$
E_2	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2}$	$E_{3/2}$
E_3	$E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}$	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2}$	$E_{1/2}$
$E_{1/2}$	$\{A_1\} \oplus A_2 \oplus E_1$	$E_1 \oplus E_2$	$E_2 \oplus E_3$	E_3	E_3
$E_{3/2}$		$\{A_1\} \oplus A_2 \oplus E_3$	$E_1 \oplus E_3$	E_2	E_2
$E_{5/2}$			$\{A_1\} \oplus A_2 \oplus E_2$	E_1	E_1
${}^1E_{7/2}$				A_2	A_1
${}^2E_{7/2}$					A_2

T **27**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_7$	$\mathbf{C}_7$	$(\mathbf{C}_2)$
A_1	A	A
A_2	A	B
E_1	${}^1E_1 \oplus {}^2E_1$	$A \oplus B$
E_2	${}^1E_2 \oplus {}^2E_2$	$A \oplus B$
E_3	${}^1E_3 \oplus {}^2E_3$	$A \oplus B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1E_{7/2}$	$A_{7/2}$	${}^1E_{1/2}$
${}^2E_{7/2}$	$A_{7/2}$	${}^2E_{1/2}$

T **27**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_7$
$14n$	$(2n+1)\,A_1 \oplus 2n\,(A_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3)$
$14n+1$	$2n\,(A_1 \oplus E_1 \oplus 2E_2 \oplus 2E_3) \oplus (2n+1)(A_2 \oplus E_1)$
$14n+2$	$(2n+1)(A_1 \oplus E_1 \oplus E_2) \oplus 2n\,(A_2 \oplus E_1 \oplus E_2 \oplus 2E_3)$
$14n+3$	$2n\,(A_1 \oplus E_1 \oplus E_2 \oplus E_3) \oplus (2n+1)(A_2 \oplus E_1 \oplus E_2 \oplus E_3)$
$14n+4$	$(2n+1)(A_1 \oplus E_1 \oplus E_2 \oplus 2E_3) \oplus 2n\,(A_2 \oplus E_1 \oplus E_2)$
$14n+5$	$2n\,(A_1 \oplus E_1) \oplus (2n+1)(A_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3)$
$14n+6$	$(2n+1)(A_1 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3) \oplus 2n\,A_2$
$14n+7$	$(2n+1)(A_1 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3) \oplus (2n+2)\,A_2$
$14n+8$	$(2n+2)(A_1 \oplus E_1) \oplus (2n+1)(A_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3)$
$14n+9$	$(2n+1)(A_1 \oplus E_1 \oplus E_2 \oplus 2E_3) \oplus (2n+2)(A_2 \oplus E_1 \oplus E_2)$
$14n+10$	$(2n+2)(A_1 \oplus E_1 \oplus E_2 \oplus E_3) \oplus (2n+1)(A_2 \oplus E_1 \oplus E_2 \oplus E_3)$
$14n+11$	$(2n+1)(A_1 \oplus E_1 \oplus E_2) \oplus (2n+2)(A_2 \oplus E_1 \oplus E_2 \oplus 2E_3)$
$14n+12$	$(2n+2)(A_1 \oplus E_1 \oplus 2E_2 \oplus 2E_3) \oplus (2n+1)(A_2 \oplus E_1)$
$14n+13$	$(2n+1)\,A_1 \oplus (2n+2)(A_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3)$
$7n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus n\,(2E_{3/2} \oplus 2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$7n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus n\,(2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$7n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus n\,({}^1E_{7/2} \oplus {}^2E_{7/2})$
$7n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus (n+1)({}^1E_{7/2} \oplus {}^2E_{7/2})$
$7n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (n+1)(2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$7n+\frac{11}{2}$	$(2n+1)\,E_{1/2} \oplus (n+1)(2E_{3/2} \oplus 2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$7n+\frac{13}{2}$	$(n+1)(2E_{1/2} \oplus 2E_{3/2} \oplus 2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$

$n = 0, 1, 2, \ldots$

T **27**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{D}_7$

a_2	e_1	E_1 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_2	E_2 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_3	E_3 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{3/2}$	$E_{3/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{5/2}$	$E_{5/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

e_1	e_1	A_1 1	A_2 1	E_2 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	0	1

e_1	e_2	E_1 1	2	E_3 1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_1	e_3	E_2 1	2	E_3 1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	$e_{1/2}$	$E_{1/2}$ 1	2	$E_{3/2}$ 1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$\text{u} = 2^{-1/2}$

↠

T 27.11 Clebsch–Gordan coefficients *(cont.)*

e_1	$e_{3/2}$	$E_{1/2}$ 1	2	$E_{5/2}$ 1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_1	$e_{5/2}$	$E_{3/2}$ 1	2	${}^1E_{7/2}$ 1	${}^2E_{7/2}$ 1
1	1	0	0	u	u
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	u	$\overline{\text{u}}$

e_1	${}^1e_{7/2}$	$E_{5/2}$ 1	2
1	1	0	1
2	1	1	0

e_1	${}^2e_{7/2}$	$E_{5/2}$ 1	2
1	1	0	$\overline{1}$
2	1	1	0

e_2	e_2	A_1 1	A_2 1	E_3 1	2
1	1	0	0	0	$\overline{1}$
1	2	u	u	0	0
2	1	u	$\overline{\text{u}}$	0	0
2	2	0	0	1	0

e_2	e_3	E_1 1	2	E_2 1	2
1	1	0	0	0	$\overline{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_2	$e_{1/2}$	$E_{3/2}$ 1	2	$E_{5/2}$ 1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\overline{1}$	0	0
2	2	0	0	0	1

e_2	$e_{3/2}$	$E_{1/2}$ 1	2	${}^1E_{7/2}$ 1	${}^2E_{7/2}$ 1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	$\overline{1}$	0	0
2	2	0	0	u	$\overline{\text{u}}$

e_2	$e_{5/2}$	$E_{1/2}$ 1	2	$E_{5/2}$ 1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_2	${}^1e_{7/2}$	$E_{3/2}$ 1	2
1	1	0	1
2	1	1	0

e_2	${}^2e_{7/2}$	$E_{3/2}$ 1	2
1	1	0	$\overline{1}$
2	1	1	0

e_3	e_3	A_1 1	A_2 1	E_1 1	2
1	1	0	0	0	$\overline{1}$
1	2	u	u	0	0
2	1	u	$\overline{\text{u}}$	0	0
2	2	0	0	1	0

e_3	$e_{1/2}$	$E_{5/2}$ 1	2	${}^1E_{7/2}$ 1	${}^2E_{7/2}$ 1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	$\overline{1}$	0	0
2	2	0	0	u	$\overline{\text{u}}$

e_3	$e_{3/2}$	$E_{3/2}$ 1	2	$E_{5/2}$ 1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\overline{1}$	0	0
2	2	0	0	1	0

e_3	$e_{5/2}$	$E_{1/2}$ 1	2	$E_{3/2}$ 1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\overline{1}$	0	0
2	2	0	0	1	0

e_3	${}^1e_{7/2}$	$E_{1/2}$ 1	2
1	1	0	1
2	1	1	0

e_3	${}^2e_{7/2}$	$E_{1/2}$ 1	2
1	1	0	$\overline{1}$
2	1	1	0

$e_{1/2}$	$e_{1/2}$	A_1 1	A_2 1	E_1 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\overline{\text{u}}$	u	0	0
2	2	0	0	0	1

$\text{u} = 2^{-1/2}$

↠

T 27.11 Clebsch–Gordan coefficients *(cont.)*

$e_{1/2}$	$e_{3/2}$	E_1 1	2	E_2 1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{5/2}$	E_2 1	2	E_3 1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{1/2}$	${}^1e_{7/2}$	E_3 1	2
1	1	0	$\bar{1}$
2	1	1	0

$e_{1/2}$	${}^2e_{7/2}$	E_3 1	2
1	1	0	1
2	1	1	0

$e_{3/2}$	$e_{3/2}$	A_1 1	A_2 1	E_3 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\text{u}}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{5/2}$	E_1 1	2	E_3 1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{3/2}$	${}^1e_{7/2}$	E_2 1	2
1	1	0	$\bar{1}$
2	1	1	0

$e_{3/2}$	${}^2e_{7/2}$	E_2 1	2
1	1	0	1
2	1	1	0

$e_{5/2}$	$e_{5/2}$	A_1 1	A_2 1	E_2 1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	$\bar{\text{u}}$	u	0	0
2	2	0	0	1	0

$e_{5/2}$	${}^1e_{7/2}$	E_1 1	2
1	1	0	$\bar{1}$
2	1	1	0

$e_{5/2}$	${}^2e_{7/2}$	E_1 1	2
1	1	0	1
2	1	1	0

$\text{u} = 2^{-1/2}$

(1) Product forms: $\mathbf{C}_8 \otimes \mathbf{C}_2'$, $\mathbf{D}_4 \otimes \mathbf{C}_2''$.

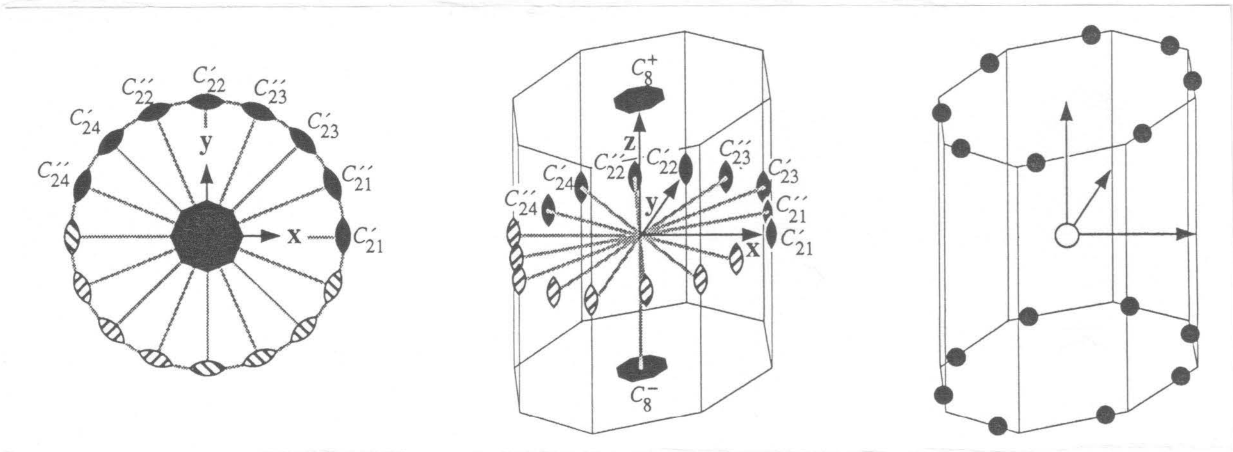
(2) Group chains: $\mathbf{D}_{8d} \supset (\underline{\mathbf{D}_8}) \supset (\underline{\mathbf{D}_4})$, $\mathbf{D}_{8d} \supset (\underline{\mathbf{D}_8}) \supset \underline{\mathbf{C}_8}$, $\mathbf{D}_{8h} \supset \underline{\mathbf{D}_8} \supset (\underline{\mathbf{D}_4})$, $\mathbf{D}_{8h} \supset \underline{\mathbf{D}_8} \supset \underline{\mathbf{C}_8}$.

(3) Operations of G: E, (C_8^+, C_8^-), (C_4^+, C_4^-), (C_8^{3+}, C_8^{3-}), C_2, $(C_{21}', C_{22}', C_{23}', C_{24}')$, $(C_{21}'', C_{22}'', C_{23}'', C_{24}'')$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_8^+, C_8^-), $(\widetilde{C}_8^+, \widetilde{C}_8^-)$, (C_4^+, C_4^-), $(\widetilde{C}_4^+, \widetilde{C}_4^-)$, (C_8^{3+}, C_8^{3-}), $(\widetilde{C}_8^{3+}, \widetilde{C}_8^{3-})$,
$(C_2, \widetilde{C}_2)$, $(C_{21}', C_{22}', C_{23}', C_{24}', \widetilde{C}_{21}', \widetilde{C}_{22}', \widetilde{C}_{23}', \widetilde{C}_{24}')$,
$(C_{21}'', C_{22}'', C_{23}'', C_{24}'', \widetilde{C}_{21}'', \widetilde{C}_{22}'', \widetilde{C}_{23}'', \widetilde{C}_{24}'')$.

(5) Classes and representations: $|r| = 4$, $|i| = 3$, $|I| = 7$, $|\widetilde{I}| = 4$.

F **28** See Chapter **15**, p. 65



Examples:

T **28**.1 Parameters
Use T **37**.1. § **16**–1, p. 68

T **28**.2 Multiplication table
Use T **37**.2. § **16**–2, p. 69

T **28**.3 Factor table
Use T **37**.3. § **16**–3, p. 70

T **28**.4 Character table
§ **16**–4, p. 71

$\mathbf{D}_8$	E	$2C_8$	$2C_4$	$2C_8^3$	C_2	$4C_2'$	$4C_2''$	τ
A_1	1	1	1	1	1	1	1	a
A_2	1	1	1	1	1	−1	−1	a
B_1	1	−1	1	−1	1	1	−1	a
B_2	1	−1	1	−1	1	−1	1	a
E_1	2	$\sqrt{2}$	0	$-\sqrt{2}$	−2	0	0	a
E_2	2	0	−2	0	2	0	0	a
E_3	2	$-\sqrt{2}$	0	$\sqrt{2}$	−2	0	0	a
$E_{1/2}$	2	$2c_8$	$\sqrt{2}$	$2c_8^3$	0	0	0	c
$E_{3/2}$	2	$2c_8^3$	$-\sqrt{2}$	$-2c_8$	0	0	0	c
$E_{5/2}$	2	$-2c_8^3$	$-\sqrt{2}$	$2c_8$	0	0	0	c
$E_{7/2}$	2	$-2c_8$	$\sqrt{2}$	$-2c_8^3$	0	0	0	c

$c_n^m = \cos \frac{m}{n}\pi$

T **28**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_8$	0	1	2	3
A_1	$^{\square}1$		$x^2+y^2, {}^{\square}z^2$	
A_2		$^{\square}z, R_z$		$(x^2+y^2)z, {}^{\square}z^3$
B_1				
B_2				
E_1		$^{\square}(x,y), (R_x,R_y)$	$^{\square}(zx,yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, {}^{\square}(xz^2,yz^2)$
E_2			$^{\square}(xy, x^2-y^2)$	$^{\square}\{xyz, z(x^2-y^2)\}$
E_3				$^{\square}\{x(x^2-3y^2), y(3x^2-y^2)\}$

T **28**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_8$	$\langle \lvert j\,m\rangle \rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$	$\lvert 9\,8\rangle_-$	2	8
A_2	$\lvert 1\,0\rangle_+$	$\lvert 8\,8\rangle_-$	2	8
B_1	$\lvert 4\,4\rangle_+$	$\lvert 5\,4\rangle_-$	2	8
B_2	$\lvert 4\,4\rangle_-$	$\lvert 5\,4\rangle_+$	2	8
E_1	$\langle \lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle \rvert$	$\langle \lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle \rvert$	2	± 8
E_2	$\langle \lvert 2\,2\rangle, \lvert 2\,\overline{2}\rangle \rvert$	$\langle \lvert 3\,2\rangle, -\lvert 3\,\overline{2}\rangle \rvert$	2	± 8
E_3	$\langle \lvert 3\,\overline{3}\rangle, \lvert 3\,3\rangle \rvert$	$\langle \lvert 4\,\overline{3}\rangle, -\lvert 4\,3\rangle \rvert$	2	± 8
$E_{1/2}$	$\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \rvert$	$\langle \lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle \rvert$	2	± 8
$E_{3/2}$	$\langle \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle, -\lvert \frac{3}{2}\,\frac{3}{2}\rangle \rvert$	$\langle \lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle, \lvert \frac{5}{2}\,\frac{3}{2}\rangle \rvert$	2	± 8
$E_{5/2}$	$\langle \lvert \frac{5}{2}\,\frac{5}{2}\rangle, \lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle \rvert$	$\langle \lvert \frac{7}{2}\,\frac{5}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle \rvert$	2	± 8
$E_{7/2}$	$\langle \lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle, -\lvert \frac{7}{2}\,\frac{7}{2}\rangle \rvert$	$\langle \lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle, \lvert \frac{9}{2}\,\frac{7}{2}\rangle \rvert$	2	± 8

T 28.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_8$	$\mathbf{C}_{8v}$	E_1	E_2	E_3	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$
E	E	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$
C_8^+	C_8^+	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}}&0\\0&\mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\bar\epsilon^*&0\\0&\bar\epsilon\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\mathrm{i}\delta^*&0\\0&\mathrm{i}\bar\delta\end{bmatrix}$	$\begin{bmatrix}\mathrm{i}\bar\delta^*&0\\0&\mathrm{i}\delta\end{bmatrix}$	$\begin{bmatrix}\bar\delta^*&0\\0&\bar\delta\end{bmatrix}$
C_8^-	C_8^-	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\mathrm{i}&0\\0&\bar{\mathrm{i}}\end{bmatrix}$	$\begin{bmatrix}\bar\epsilon&0\\0&\bar\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\mathrm{i}\bar\delta&0\\0&\mathrm{i}\delta^*\end{bmatrix}$	$\begin{bmatrix}\mathrm{i}\delta&0\\0&\mathrm{i}\bar\delta^*\end{bmatrix}$	$\begin{bmatrix}\bar\delta&0\\0&\bar\delta^*\end{bmatrix}$
C_4^+	C_4^+	$\begin{bmatrix}\bar{\mathrm{i}}&0\\0&\mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\bar1&0\\0&\bar1\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}}&0\\0&\mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\bar\epsilon^*&0\\0&\bar\epsilon\end{bmatrix}$	$\begin{bmatrix}\bar\epsilon^*&0\\0&\bar\epsilon\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$
C_4^-	C_4^-	$\begin{bmatrix}\mathrm{i}&0\\0&\bar{\mathrm{i}}\end{bmatrix}$	$\begin{bmatrix}\bar1&0\\0&\bar1\end{bmatrix}$	$\begin{bmatrix}\mathrm{i}&0\\0&\bar{\mathrm{i}}\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\bar\epsilon&0\\0&\bar\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\bar\epsilon&0\\0&\bar\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$
C_8^{3+}	C_8^{3+}	$\begin{bmatrix}\bar\epsilon&0\\0&\bar\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\mathrm{i}&0\\0&\bar{\mathrm{i}}\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\mathrm{i}\bar\delta&0\\0&\mathrm{i}\delta^*\end{bmatrix}$	$\begin{bmatrix}\bar\delta&0\\0&\bar\delta^*\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\mathrm{i}\delta&0\\0&\mathrm{i}\bar\delta^*\end{bmatrix}$
C_8^{3-}	C_8^{3-}	$\begin{bmatrix}\bar\epsilon^*&0\\0&\bar\epsilon\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}}&0\\0&\mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\mathrm{i}\delta^*&0\\0&\mathrm{i}\bar\delta\end{bmatrix}$	$\begin{bmatrix}\bar\delta^*&0\\0&\bar\delta\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\mathrm{i}\bar\delta^*&0\\0&\mathrm{i}\delta\end{bmatrix}$
C_2	C_2	$\begin{bmatrix}\bar1&0\\0&\bar1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}\bar1&0\\0&\bar1\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}}&0\\0&\mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}}&0\\0&\mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}}&0\\0&\mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}}&0\\0&\mathrm{i}\end{bmatrix}$
C'_{21}	σ_{v1}	$\begin{bmatrix}0&\bar1\\\bar1&0\end{bmatrix}$	$\begin{bmatrix}0&1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\\bar1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\mathrm{i}}\\\bar{\mathrm{i}}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\mathrm{i}}\\\bar{\mathrm{i}}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\mathrm{i}}\\\bar{\mathrm{i}}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\mathrm{i}}\\\bar{\mathrm{i}}&0\end{bmatrix}$
C'_{22}	σ_{v2}	$\begin{bmatrix}0&1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\1&0\end{bmatrix}$
C'_{23}	σ_{v3}	$\begin{bmatrix}0&\mathrm{i}\\\bar{\mathrm{i}}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\\bar1&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\\\bar{\mathrm{i}}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\epsilon\\\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon\\\bar\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon\\\bar\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\epsilon\\\epsilon^*&0\end{bmatrix}$
C'_{24}	σ_{v4}	$\begin{bmatrix}0&\bar{\mathrm{i}}\\\mathrm{i}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\\bar1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\mathrm{i}}\\\mathrm{i}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\epsilon^*\\\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon^*\\\bar\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon^*\\\bar\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\epsilon^*\\\epsilon&0\end{bmatrix}$
C''_{21}	σ_{d1}	$\begin{bmatrix}0&\bar\epsilon^*\\\bar\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\mathrm{i}}\\\mathrm{i}&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon^*\\\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\bar\delta^*\\\mathrm{i}\bar\delta&0\end{bmatrix}$	$\begin{bmatrix}0&\delta^*\\\bar\delta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\delta^*\\\delta&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\delta^*\\\mathrm{i}\delta&0\end{bmatrix}$
C''_{22}	σ_{d2}	$\begin{bmatrix}0&\epsilon^*\\\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\mathrm{i}}\\\mathrm{i}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\epsilon^*\\\bar\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\delta^*\\\delta&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\bar\delta^*\\\mathrm{i}\bar\delta&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\delta^*\\\mathrm{i}\delta&0\end{bmatrix}$	$\begin{bmatrix}0&\delta^*\\\bar\delta&0\end{bmatrix}$
C''_{23}	σ_{d3}	$\begin{bmatrix}0&\epsilon\\\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\\\bar{\mathrm{i}}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\epsilon\\\bar\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\delta\\\delta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\delta\\\mathrm{i}\delta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\bar\delta\\\mathrm{i}\bar\delta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\delta\\\bar\delta^*&0\end{bmatrix}$
C''_{24}	σ_{d4}	$\begin{bmatrix}0&\bar\epsilon\\\bar\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\\\bar{\mathrm{i}}&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon\\\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\delta\\\mathrm{i}\delta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\delta\\\bar\delta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\delta\\\delta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\mathrm{i}\bar\delta\\\mathrm{i}\bar\delta^*&0\end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/16)$, $\epsilon = \exp(2\pi\mathrm{i}/8)$

T 28.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{D}_8$, $\mathbf{C}_{8v}$	A_1	A_2	B_1	B_2	E_1	E_2	E_3
A_1	A_1	A_2	B_1	B_2	E_1	E_2	E_3
A_2		A_1	B_2	B_1	E_1	E_2	E_3
B_1			A_1	A_2	E_3	E_2	E_1
B_2				A_1	E_3	E_2	E_1
E_1					$A_1 \oplus \{A_2\} \oplus E_2$	$E_1 \oplus E_3$	$B_1 \oplus B_2 \oplus E_2$
E_2						$A_1 \oplus \{A_2\} \oplus B_1 \oplus B_2$	$E_1 \oplus E_3$
E_3							$A_1 \oplus \{A_2\} \oplus E_2$

⇛

T **28**.8 Direct products of representations *(cont.)*

$\mathbf{D}_8$, $\mathbf{C}_{8v}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$
A_1	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$
A_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$
B_1	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
B_2	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
E_1	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{5/2} \oplus E_{7/2}$
E_2	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{5/2}$
E_3	$E_{5/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$
$E_{1/2}$	$\{A_1\} \oplus A_2 \oplus E_1$	$E_1 \oplus E_2$	$E_2 \oplus E_3$	$B_1 \oplus B_2 \oplus E_3$
$E_{3/2}$		$\{A_1\} \oplus A_2 \oplus E_3$	$B_1 \oplus B_2 \oplus E_1$	$E_2 \oplus E_3$
$E_{5/2}$			$\{A_1\} \oplus A_2 \oplus E_3$	$E_1 \oplus E_2$
$E_{7/2}$				$\{A_1\} \oplus A_2 \oplus E_1$

T **28**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_8$	$(\mathbf{D}_4)$ C_2'	$(\mathbf{D}_4)$ C_2''	$(\mathbf{D}_2)$ C_2'	$(\mathbf{D}_2)$ C_2''	$\mathbf{C}_8$
A_1	A_1	A_1	A	A	A
A_2	A_2	A_2	B_1	B_1	A
B_1	A_1	A_2	A	B_1	B
B_2	A_2	A_1	B_1	A	B
E_1	E	E	$B_2 \oplus B_3$	$B_2 \oplus B_3$	${}^1E_1 \oplus {}^2E_1$
E_2	$B_1 \oplus B_2$	$B_1 \oplus B_2$	$A \oplus B_1$	$A \oplus B_1$	${}^1E_2 \oplus {}^2E_2$
E_3	E	E	$B_2 \oplus B_3$	$B_2 \oplus B_3$	${}^1E_3 \oplus {}^2E_3$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$
$E_{5/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$
$E_{7/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$

$\twoheadrightarrow$

T **28**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_8$	$\mathbf{C}_4$	$\mathbf{C}_2$ C_2	$(\mathbf{C}_2)$ C_2'	$(\mathbf{C}_2)$ C_2''
A_1	A	A	A	A
A_2	A	A	B	B
B_1	A	A	A	B
B_2	A	A	B	A
E_1	${}^1E \oplus {}^2E$	$2B$	$A \oplus B$	$A \oplus B$
E_2	$2B$	$2A$	$A \oplus B$	$A \oplus B$
E_3	${}^1E \oplus {}^2E$	$2B$	$A \oplus B$	$A \oplus B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **28**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_8$
$8n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3)$
$8n+1$	$n\,(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3) \oplus (n+1)(A_2 \oplus E_1)$
$8n+2$	$(n+1)(A_1 \oplus E_1 \oplus E_2) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus 2E_3)$
$8n+3$	$n\,(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3) \oplus (n+1)(A_2 \oplus E_1 \oplus E_2 \oplus E_3)$
$8n+4$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3) \oplus n\,(A_2 \oplus E_1 \oplus E_2 \oplus E_3)$
$8n+5$	$n\,(A_1 \oplus E_1 \oplus E_2) \oplus (n+1)\,(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus 2E_3)$
$8n+6$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3) \oplus n\,(A_2 \oplus E_1)$
$8n+7$	$n\,A_1 \oplus (n+1)(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3)$
$8n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,(E_{3/2} \oplus E_{5/2} \oplus E_{7/2})$
$8n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus 2n\,(E_{5/2} \oplus E_{7/2})$
$8n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus 2n\,E_{7/2}$
$8n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2})$
$8n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus (2n+2)\,E_{7/2}$
$8n+\frac{11}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (2n+2)(E_{5/2} \oplus E_{7/2})$
$8n+\frac{13}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)(E_{3/2} \oplus E_{5/2} \oplus E_{7/2})$
$8n+\frac{15}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2})$

$n = 0, 1, 2, \ldots$

T **28**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83

$\mathbf{D}_8$, $\mathbf{C}_{8v}$

a_2	e_1	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_2	E_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_3	E_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{5/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{7/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_1	e_1	E_3	
		1	2
1	1	1	0
1	2	0	1

b_1	e_2	E_2	
		1	2
1	1	0	1
1	2	1	0

b_1	e_3	E_1	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{1/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{3/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{5/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{7/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	1

b_2	e_1	E_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_2	E_2	
		1	2
1	1	0	$\bar{1}$
1	2	1	0

$\twoheadrightarrow$

T **28**.11 Clebsch–Gordan coefficients *(cont.)* $\mathbf{D}_8,\ \mathbf{C}_{8v}$

b_2	e_3	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{1/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{3/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{5/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{7/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e_1	e_1	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	0	1

e_1	e_2	E_1		E_3	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	e_3	B_1	B_2	E_2	
		1	1	1	2
1	1	0	0	0	1
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	1	0

e_1	$e_{1/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_1	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_1	$e_{5/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_1	$e_{7/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_2	e_2	A_1	A_2	B_1	B_2
		1	1	1	1
1	1	0	0	u	u
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	u	$\bar{\text{u}}$

e_2	e_3	E_1		E_3	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	1	0	0	0

e_2	$e_{1/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_2	$e_{3/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_2	$e_{5/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_2	$e_{7/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_3	e_3	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	0	1

$\text{u} = 2^{-1/2}$

↠

T **28**.11 Clebsch–Gordan coefficients *(cont.)* $\mathbf{D}_8$, $\mathbf{C}_{8v}$

e_3	$e_{1/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_3	$e_{3/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_3	$e_{5/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_3	$e_{7/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{1/2}$	$e_{1/2}$	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{3/2}$	E_1		E_2	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{1/2}$	$e_{5/2}$	E_2		E_3	
		1	2	1	2
1	1	0	0	0	1
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{1/2}$	$e_{7/2}$	B_1	B_2	E_3	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{3/2}$	A_1	A_2	E_3	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{5/2}$	B_1	B_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{7/2}$	E_2		E_3	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{5/2}$	$e_{5/2}$	A_1	A_2	E_3	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{5/2}$	$e_{7/2}$	E_1		E_2	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{7/2}$	$e_{7/2}$	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

(1) Product forms: $\mathbf{C}_9 \otimes \mathbf{C}'_2$.

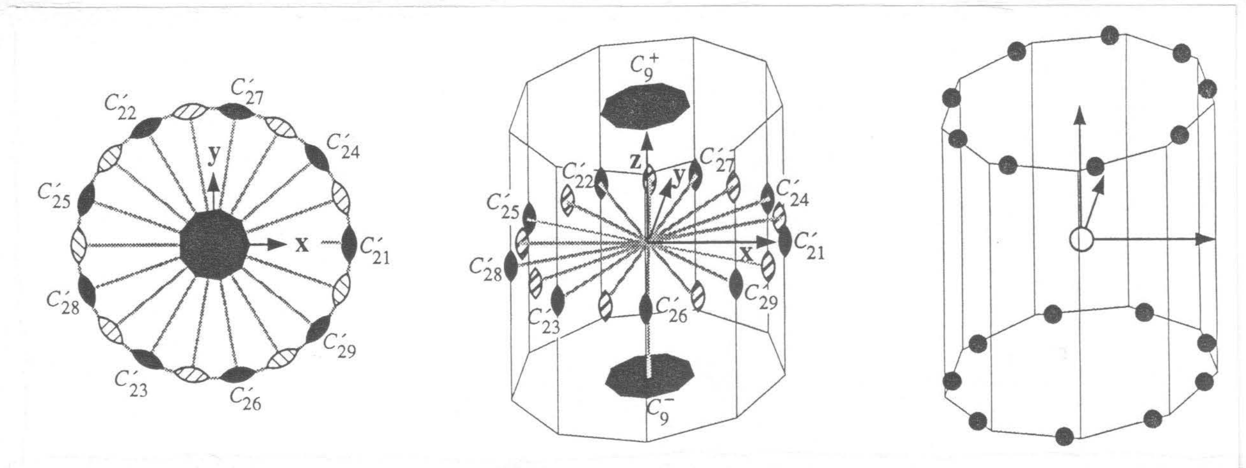
(2) Group chains: $\mathbf{D}_{9d} \supset \underline{\mathbf{D}_9} \supset (\mathbf{D}_3)$, $\mathbf{D}_{9d} \supset \underline{\mathbf{D}_9} \supset \underline{\mathbf{C}_9}$, $\mathbf{D}_{9h} \supset (\underline{\mathbf{D}_9}) \supset (\mathbf{D}_3)$, $\mathbf{D}_{9h} \supset (\underline{\mathbf{D}_9}) \supset \underline{\mathbf{C}_9}$.

(3) Operations of G: E, (C_9^+, C_9^-), (C_9^{2+}, C_9^{2-}), (C_3^+, C_3^-), (C_9^{4+}, C_9^{4-}),
$(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27}, C'_{28}, C'_{29})$.

(4) Operations of $\widetilde{G}$: E, (C_9^+, C_9^-), (C_9^{2+}, C_9^{2-}), (C_3^+, C_3^-), (C_9^{4+}, C_9^{4-}),
$(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27}, C'_{28}, C'_{29})$,
$\widetilde{E}$, $(\widetilde{C}_9^+, \widetilde{C}_9^-)$, $(\widetilde{C}_9^{2+}, \widetilde{C}_9^{2-})$, $(\widetilde{C}_3^+, \widetilde{C}_3^-)$, $(\widetilde{C}_9^{4+}, \widetilde{C}_9^{4-})$,
$(\widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24}, \widetilde{C}'_{25}, \widetilde{C}'_{26}, \widetilde{C}'_{27}, \widetilde{C}'_{28}, \widetilde{C}'_{29}$.

(5) Classes and representations: $|r| = 6$, $|\mathrm{i}| = 0$, $|I| = 6$, $|\widetilde{I}| = 6$.

F **29** See Chapter **15**, p. 65



Examples:

T 29.1 Parameters
Use T **38**.1. § **16**–1, p. 68

T 29.2 Multiplication table
Use T **38**.2. § **16**–2, p. 69

T 29.3 Factor table
Use T **38**.3. § **16**–3, p. 70

T **29**.4 Character table

§ **16**–4, p. 71

$\mathbf{D}_9$	E	$2C_9$	$2C_9^2$	$2C_3$	$2C_9^4$	$9C_2'$	τ
A_1	1	1	1	1	1	1	a
A_2	1	1	1	1	1	-1	a
E_1	2	$2c_9^2$	$2c_9^4$	-1	$2c_9^8$	0	a
E_2	2	$2c_9^4$	$2c_9^8$	-1	$2c_9^2$	0	a
E_3	2	-1	-1	2	-1	0	a
E_4	2	$2c_9^8$	$2c_9^2$	-1	$2c_9^4$	0	a
$E_{1/2}$	2	$-2c_9^8$	$2c_9^2$	1	$2c_9^4$	0	c
$E_{3/2}$	2	1	-1	-2	-1	0	c
$E_{5/2}$	2	$-2c_9^4$	$2c_9^8$	1	$2c_9^2$	0	c
$E_{7/2}$	2	$-2c_9^2$	$2c_9^4$	1	$2c_9^8$	0	c
${}^1E_{9/2}$	1	-1	1	-1	1	i	b
${}^2E_{9/2}$	1	-1	1	-1	1	$-$i	b

$c_n^m = \cos \frac{m}{n}\pi$

T **29**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_9$	0	1	2	3
A_1	$^\square 1$		$x^2+y^2, {}^\square z^2$	
A_2		$^\square z, R_z$		$(x^2+y^2)z, {}^\square z^3$
E_1		$^\square(x,y),(R_x,R_y)$	$^\square(zx,yz)$	$\{x(x^2+y^2),y(x^2+y^2)\}, {}^\square(xz^2,yz^2)$
E_2			$^\square(xy,x^2-y^2)$	$^\square\{xyz,z(x^2-y^2)\}$
E_3				$^\square\{x(x^2-3y^2),y(3x^2-y^2)\}$
E_4				

T **29**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_9$	$\langle\lvert j\,m\rangle\rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$	$\lvert 9\,9\rangle_-$	2	9
A_2	$\lvert 1\,0\rangle_+$	$\lvert 10\,9\rangle_-$	2	9
E_1	$\langle\lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle\rvert$	$\langle\lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle\rvert$	2	± 9
E_2	$\langle\lvert 2\,\overline{2}\rangle, -\lvert 2\,2\rangle\rvert$	$\langle\lvert 3\,\overline{2}\rangle, \lvert 3\,2\rangle\rvert$	2	± 9
E_3	$\langle\lvert 3\,3\rangle, \lvert 3\,\overline{3}\rangle\rvert$	$\langle\lvert 4\,3\rangle, -\lvert 4\,\overline{3}\rangle\rvert$	2	± 9
E_4	$\langle\lvert 4\,4\rangle, -\lvert 4\,\overline{4}\rangle\rvert$	$\langle\lvert 5\,\overline{5}\rangle, \lvert 5\,5\rangle\rvert$	2	± 9
$E_{1/2}$	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	2	± 9
$E_{3/2}$	$\langle\lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle\rvert$	$\langle\lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle\rvert$	2	± 9
$E_{5/2}$	$\langle\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle, \lvert \frac{5}{2}\,\frac{5}{2}\rangle\rvert$	$\langle\lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle, -\lvert \frac{7}{2}\,\frac{5}{2}\rangle\rvert$	2	± 9
$E_{7/2}$	$\langle\lvert \frac{7}{2}\,\frac{7}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle\rvert$	$\langle\lvert \frac{9}{2}\,\frac{7}{2}\rangle, \lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle\rvert$	2	± 9
${}^1E_{9/2}$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle_-$	$\lvert \frac{11}{2}\,\frac{9}{2}\rangle_+$	2	9
${}^2E_{9/2}$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle_+$	$\lvert \frac{11}{2}\,\frac{9}{2}\rangle_-$	2	9

T **29**.7 Matrix representations

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$\mathbf{D}_9$	E_1	E_2	E_3	E_4	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_9^+	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \overline{\delta} & 0 \\ 0 & \overline{\delta}^* \end{bmatrix}$
C_9^-	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\delta}^* & 0 \\ 0 & \overline{\delta} \end{bmatrix}$
C_9^{2+}	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
C_9^{2-}	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
C_3^+	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$
C_3^-	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$
C_9^{4+}	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$
C_9^{4-}	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta}^* \\ \mathrm{i}\overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon} \\ \mathrm{i}\overline{\epsilon}^* & 0 \end{bmatrix}$
C'_{25}	$\begin{bmatrix} 0 & \overline{\delta} \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta}^* \\ \mathrm{i}\overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon} \\ \mathrm{i}\overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$
C'_{26}	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon} \\ \mathrm{i}\overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta}^* \\ \mathrm{i}\overline{\theta} & 0 \end{bmatrix}$
C'_{27}	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon}^* \\ \mathrm{i}\overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta} \\ \mathrm{i}\overline{\theta}^* & 0 \end{bmatrix}$
C'_{28}	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta} \\ \mathrm{i}\overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon}^* \\ \mathrm{i}\overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$
C'_{29}	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta} \\ \mathrm{i}\overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon}^* \\ \mathrm{i}\overline{\epsilon} & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/9)$, $\epsilon = \exp(4\pi\mathrm{i}/9)$, $\eta = \exp(6\pi\mathrm{i}/9)$, $\theta = \exp(8\pi\mathrm{i}/9)$

T **29**.8 Direct products of representations § **16**–8, p. 81

$\mathbf{D}_9$	A_1	A_2	E_1	E_2	E_3	E_4
A_1	A_1	A_2	E_1	E_2	E_3	E_4
A_2		A_1	E_1	E_2	E_3	E_4
E_1			$A_1 \oplus \{A_2\} \oplus E_2$	$E_1 \oplus E_3$	$E_2 \oplus E_4$	$E_3 \oplus E_4$
E_2				$A_1 \oplus \{A_2\} \oplus E_4$	$E_1 \oplus E_4$	$E_2 \oplus E_3$
E_3					$A_1 \oplus \{A_2\} \oplus E_3$	$E_1 \oplus E_2$
E_4						$A_1 \oplus \{A_2\} \oplus E_1$

$\twoheadrightarrow$

T **29**.8 Direct products of representations *(cont.)*

$\mathbf{D}_9$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$
A_1	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$
A_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	${}^2E_{9/2}$	${}^1E_{9/2}$
E_1	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{5/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2}$	$E_{7/2}$	$E_{7/2}$
E_2	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{1/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{5/2}$	$E_{5/2}$
E_3	$E_{5/2} \oplus E_{7/2}$	$E_{3/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2}$	$E_{3/2}$
E_4	$E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2}$	$E_{5/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2}$	$E_{1/2}$
$E_{1/2}$	$\{A_1\} \oplus A_2 \oplus E_1$	$E_1 \oplus E_2$	$E_2 \oplus E_3$	$E_3 \oplus E_4$	E_4	E_4
$E_{3/2}$		$\{A_1\} \oplus A_2 \oplus E_3$	$E_1 \oplus E_4$	$E_2 \oplus E_4$	E_3	E_3
$E_{5/2}$			$\{A_1\} \oplus A_2 \oplus E_4$	$E_1 \oplus E_3$	E_2	E_2
$E_{7/2}$				$\{A_1\} \oplus A_2 \oplus E_2$	E_1	E_1
${}^1E_{9/2}$					A_2	A_1
${}^2E_{9/2}$						A_2

T **29**.9 Subduction (descent of symmetry) § **16**–9, p. 82

$\mathbf{D}_9$	$(\mathbf{D}_3)$	$\mathbf{C}_9$	$\mathbf{C}_3$	$(\mathbf{C}_2)$
A_1	A_1	A	A	A
A_2	A_2	A	A	B
E_1	E	${}^1E_1 \oplus {}^2E_1$	${}^1E \oplus {}^2E$	$A \oplus B$
E_2	E	${}^1E_2 \oplus {}^2E_2$	${}^1E \oplus {}^2E$	$A \oplus B$
E_3	$A_1 \oplus A_2$	${}^1E_3 \oplus {}^2E_3$	$2A$	$A \oplus B$
E_4	E	${}^1E_4 \oplus {}^2E_4$	${}^1E \oplus {}^2E$	$A \oplus B$
$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$2A_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	$E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	$E_{1/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1E_{9/2}$	${}^1E_{3/2}$	$A_{9/2}$	$A_{3/2}$	${}^1E_{1/2}$
${}^2E_{9/2}$	${}^2E_{3/2}$	$A_{9/2}$	$A_{3/2}$	${}^2E_{1/2}$

T **29**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_9$
$18n$	$(2n+1)\,A_1 \oplus 2n\,(A_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4)$
$18n+1$	$2n\,(A_1 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4) \oplus (2n+1)(A_2 \oplus E_1)$
$18n+2$	$(2n+1)(A_1 \oplus E_1 \oplus E_2) \oplus 2n\,(A_2 \oplus E_1 \oplus E_2 \oplus 2E_3 \oplus 2E_4)$
$18n+3$	$2n\,(A_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus 2E_4) \oplus (2n+1)(A_2 \oplus E_1 \oplus E_2 \oplus E_3)$
$18n+4$	$(2n+1)(A_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4) \oplus 2n\,(A_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4)$
$18n+5$	$2n\,(A_1 \oplus E_1 \oplus E_2 \oplus E_3) \oplus (2n+1)(A_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus 2E_4)$
$18n+6$	$(2n+1)(A_1 \oplus E_1 \oplus E_2 \oplus 2E_3 \oplus 2E_4) \oplus 2n\,(A_2 \oplus E_1 \oplus E_2)$
$18n+7$	$2n\,(A_1 \oplus E_1) \oplus (2n+1)(A_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4)$
$18n+8$	$(2n+1)(A_1 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4) \oplus 2n\,A_2$
$18n+9$	$(2n+1)(A_1 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4) \oplus (2n+2)\,A_2$
$18n+10$	$(2n+2)(A_1 \oplus E_1) \oplus (2n+1)(A_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4)$
$18n+11$	$(2n+1)(A_1 \oplus E_1 \oplus E_2 \oplus 2E_3 \oplus 2E_4) \oplus (2n+2)(A_2 \oplus E_1 \oplus E_2)$
$18n+12$	$(2n+2)(A_1 \oplus E_1 \oplus E_2 \oplus E_3) \oplus (2n+1)(A_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus 2E_4)$
$18n+13$	$(2n+1)(A_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4) \oplus (2n+2)(A_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4)$
$18n+14$	$(2n+2)(A_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus 2E_4) \oplus (2n+1)(A_2 \oplus E_1 \oplus E_2 \oplus E_3)$
$18n+15$	$(2n+1)(A_1 \oplus E_1 \oplus E_2) \oplus (2n+2)(A_2 \oplus E_1 \oplus E_2 \oplus 2E_3 \oplus 2E_4)$
$18n+16$	$(2n+2)(A_1 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4) \oplus (2n+1)(A_2 \oplus E_1)$
$18n+17$	$(2n+1)\,A_1 \oplus (2n+2)(A_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4)$
$9n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus n\,(2E_{3/2} \oplus 2E_{5/2} \oplus 2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus n\,(2E_{5/2} \oplus 2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus n\,(2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus n\,({}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus (n+1)({}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{11}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus (n+1)(2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{13}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (n+1)(2E_{5/2} \oplus 2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{15}{2}$	$(2n+1)\,E_{1/2} \oplus (n+1)(2E_{3/2} \oplus 2E_{5/2} \oplus 2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{17}{2}$	$(n+1)(2E_{1/2} \oplus 2E_{3/2} \oplus 2E_{5/2} \oplus 2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$

$n = 0, 1, 2, \ldots$

T **29**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83

$\mathbf{D}_9$

a_2	e_1	E_1 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_2	E_2 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_3	E_3 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_4	E_4 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{3/2}$	$E_{3/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{5/2}$	$E_{5/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{7/2}$	$E_{7/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

↠

T **29**.11 Clebsch–Gordan coefficients *(cont.)*

e_1	e_1	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_1	e_2	E_1		E_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_1	e_3	E_2		E_4	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	$\bar{1}$

e_1	e_4	E_3		E_4	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	$e_{1/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_1	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	$e_{5/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_1	$e_{7/2}$	$E_{5/2}$		${}^1E_{9/2}$	${}^2E_{9/2}$
		1	2	1	1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	u	$\bar{u}$

e_1	${}^1e_{9/2}$	$E_{7/2}$	
		1	2
1	1	0	1
2	1	1	0

e_1	${}^2e_{9/2}$	$E_{7/2}$	
		1	2
1	1	0	$\bar{1}$
2	1	1	0

e_2	e_2	A_1	A_2	E_4	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_2	e_3	E_1		E_4	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	$\bar{1}$	0	0

e_2	e_4	E_2		E_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_2	$e_{1/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_2	$e_{3/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_2	$e_{5/2}$	$E_{1/2}$		${}^1E_{9/2}$	${}^2E_{9/2}$
		1	2	1	1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	u	$\bar{u}$

e_2	$e_{7/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_2	${}^1e_{9/2}$	$E_{5/2}$	
		1	2
1	1	0	1
2	1	1	0

$u = 2^{-1/2}$

↠

T 29.11 Clebsch–Gordan coefficients *(cont.)*

e_2	${}^2e_{9/2}$	$E_{5/2}$ 1	2
1	1	0	$\bar{1}$
2	1	1	0

e_3	e_3	A_1 1	A_2 1	E_3 1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_3	e_4	E_1 1	2	E_2 1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e_3	$e_{1/2}$	$E_{5/2}$ 1	2	$E_{7/2}$ 1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e_3	$e_{3/2}$	$E_{3/2}$ 1	2	${}^1E_{9/2}$ 1	${}^2E_{9/2}$ 1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	$\bar{u}$	u

e_3	$e_{5/2}$	$E_{1/2}$ 1	2	$E_{7/2}$ 1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e_3	$e_{7/2}$	$E_{1/2}$ 1	2	$E_{5/2}$ 1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e_3	${}^1e_{9/2}$	$E_{3/2}$ 1	2
1	1	0	$\bar{1}$
2	1	1	0

e_3	${}^2e_{9/2}$	$E_{3/2}$ 1	2
1	1	0	1
2	1	1	0

e_4	e_4	A_1 1	A_2 1	E_1 1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_4	$e_{1/2}$	$E_{7/2}$ 1	2	${}^1E_{9/2}$ 1	${}^2E_{9/2}$ 1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	u	$\bar{u}$

e_4	$e_{3/2}$	$E_{5/2}$ 1	2	$E_{7/2}$ 1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_4	$e_{5/2}$	$E_{3/2}$ 1	2	$E_{5/2}$ 1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_4	$e_{7/2}$	$E_{1/2}$ 1	2	$E_{3/2}$ 1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_4	${}^1e_{9/2}$	$E_{1/2}$ 1	2
1	1	0	1
2	1	1	0

e_4	${}^2e_{9/2}$	$E_{1/2}$ 1	2
1	1	0	$\bar{1}$
2	1	1	0

$e_{1/2}$	$e_{1/2}$	A_1 1	A_2 1	E_1 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{3/2}$	E_1 1	2	E_2 1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$u = 2^{-1/2}$

↠

T **29**.11 Clebsch–Gordan coefficients *(cont.)*

$e_{1/2}$	$e_{5/2}$	E_2 1	E_2 2	E_3 1	E_3 2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	0	1	0	0

$e_{1/2}$	$e_{7/2}$	E_3 1	E_3 2	E_4 1	E_4 2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{1/2}$	${}^1e_{9/2}$	E_4 1	E_4 2
1	1	0	$\bar{1}$
2	1	1	0

$e_{1/2}$	${}^2e_{9/2}$	E_4 1	E_4 2
1	1	0	1
2	1	1	0

$e_{3/2}$	$e_{3/2}$	A_1 1	A_2 1	E_3 1	E_3 2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{5/2}$	E_1 1	E_1 2	E_4 1	E_4 2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{3/2}$	$e_{7/2}$	E_2 1	E_2 2	E_4 1	E_4 2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{3/2}$	${}^1e_{9/2}$	E_3 1	E_3 2
1	1	0	1
2	1	1	0

$e_{3/2}$	${}^2e_{9/2}$	E_3 1	E_3 2
1	1	0	$\bar{1}$
2	1	1	0

$e_{5/2}$	$e_{5/2}$	A_1 1	A_2 1	E_4 1	E_4 2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{5/2}$	$e_{7/2}$	E_1 1	E_1 2	E_3 1	E_3 2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	0	1	0	0

$e_{5/2}$	${}^1e_{9/2}$	E_2 1	E_2 2
1	1	0	$\bar{1}$
2	1	1	0

$e_{5/2}$	${}^2e_{9/2}$	E_2 1	E_2 2
1	1	0	1
2	1	1	0

$e_{7/2}$	$e_{7/2}$	A_1 1	A_2 1	E_2 1	E_2 2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{7/2}$	${}^1e_{9/2}$	E_1 1	E_1 2
1	1	0	$\bar{1}$
2	1	1	0

$e_{7/2}$	${}^2e_{9/2}$	E_1 1	E_1 2
1	1	0	1
2	1	1	0

$u = 2^{-1/2}$

(1) Product forms: $\mathbf{C}_{10} \otimes \mathbf{C}'_2$, $\mathbf{D}_5 \otimes \mathbf{C}''_2$.

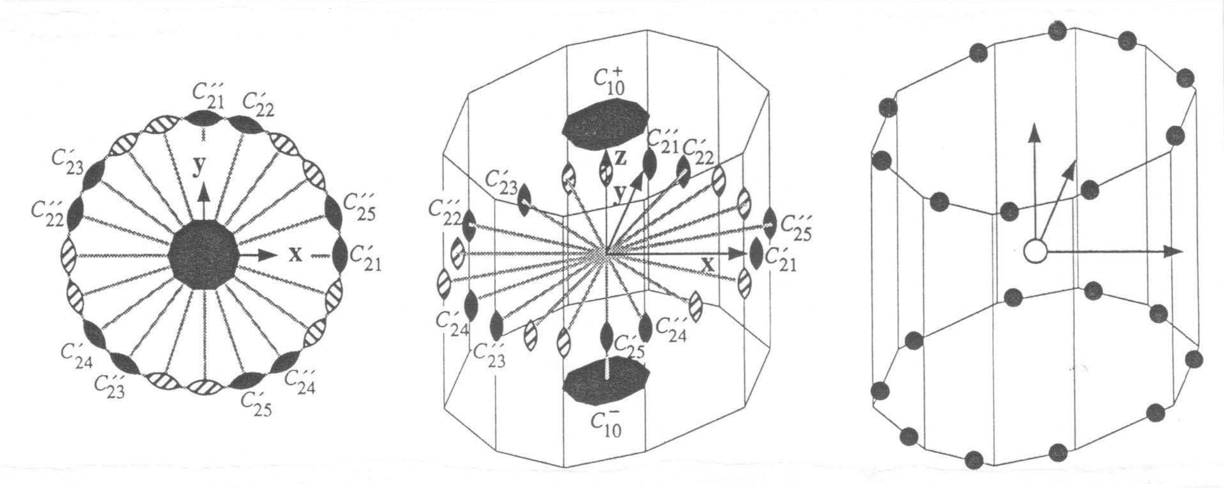
(2) Group chains: $\mathbf{D}_{10d} \supset (\underline{\mathbf{D}_{10}}) \supset (\underline{\mathbf{D}_5})$, $\mathbf{D}_{10d} \supset (\underline{\mathbf{D}_{10}}) \supset (\mathbf{D}_2)$, $\mathbf{D}_{10d} \supset (\underline{\mathbf{D}_{10}}) \supset \underline{\mathbf{C}_{10}}$,
$\mathbf{D}_{10h} \supset \underline{\mathbf{D}_{10}} \supset (\underline{\mathbf{D}_5})$, $\mathbf{D}_{10h} \supset \underline{\mathbf{D}_{10}} \supset (\mathbf{D}_2)$, $\mathbf{D}_{10h} \supset \underline{\mathbf{D}_{10}} \supset \underline{\mathbf{C}_{10}}$.

(3) Operations of G: E, (C_{10}^+, C_{10}^-), (C_5^+, C_5^-), $(C_{10}^{3+}, C_{10}^{3-})$, (C_5^{2+}, C_5^{2-}), C_2,
$(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25})$, $(C''_{21}, C''_{22}, C''_{23}, C''_{24}, C''_{25})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_{10}^+, C_{10}^-), $(\widetilde{C}_{10}^+, \widetilde{C}_{10}^-)$, (C_5^+, C_5^-), $(\widetilde{C}_5^+, \widetilde{C}_5^-)$, $(C_{10}^{3+}, C_{10}^{3-})$, $(\widetilde{C}_{10}^{3+}, \widetilde{C}_{10}^{3-})$,
(C_5^{2+}, C_5^{2-}), $(\widetilde{C}_5^{2+}, \widetilde{C}_5^{2-})$, $(C_2, \widetilde{C}_2)$, $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, \widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24}, \widetilde{C}'_{25})$,
$(C''_{21}, C''_{22}, C''_{23}, C''_{24}, C''_{25}, \widetilde{C}''_{21}, \widetilde{C}''_{22}, \widetilde{C}''_{23}, \widetilde{C}''_{24}, \widetilde{C}''_{25})$.

(5) Classes and representations: $|r| = 5$, $|\mathrm{i}| = 5$, $|I| = 10$, $|\widetilde{I}| = 5$.

F **30** See Chapter **15**, p. 65



Examples:

T **30**.1 Parameters
Use T **39**.1. § **16**–1, p. 68

T **30**.2 Multiplication table
Use T **39**.2. § **16**–2, p. 69

T **30**.3 Factor table
Use T **39**.3. § **16**–3, p. 70

T **30**.4 Character table

§ **16**–4, p. 71

$\mathbf{D}_{10}$	E	$2C_{10}$	$2C_5$	$2C_{10}^3$	$2C_5^2$	C_2	$5C_2'$	$5C_2''$	τ
A_1	1	1	1	1	1	1	1	1	a
A_2	1	1	1	1	1	1	-1	-1	a
B_1	1	-1	1	-1	1	-1	1	-1	a
B_2	1	-1	1	-1	1	-1	-1	1	a
E_1	2	$2c_5$	$2c_5^2$	$-2c_5^2$	$-2c_5$	-2	0	0	a
E_2	2	$2c_5^2$	$-2c_5$	$-2c_5$	$2c_5^2$	2	0	0	a
E_3	2	$-2c_5^2$	$-2c_5$	$2c_5$	$2c_5^2$	-2	0	0	a
E_4	2	$-2c_5$	$2c_5^2$	$2c_5^2$	$-2c_5$	2	0	0	a
$E_{1/2}$	2	$2c_{10}$	$2c_5$	$2c_{10}^3$	$2c_5^2$	0	0	0	c
$E_{3/2}$	2	$2c_{10}^3$	$-2c_5^2$	$-2c_{10}$	$-2c_5$	0	0	0	c
$E_{5/2}$	2	0	-2	0	2	0	0	0	c
$E_{7/2}$	2	$-2c_{10}^3$	$-2c_5^2$	$2c_{10}$	$-2c_5$	0	0	0	c
$E_{9/2}$	2	$-2c_{10}$	$2c_5$	$-2c_{10}^3$	$2c_5^2$	0	0	0	c

$c_n^m = \cos \frac{m}{n}\pi$

T **30**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{10}$	0	1	2	3
A_1	$^\square 1$		$x^2+y^2, {}^\square z^2$	
A_2		$^\square z, R_z$		$(x^2+y^2)z, {}^\square z^3$
B_1				
B_2				
E_1		$^\square(x,y),(R_x,R_y)$	$^\square(zx,yz)$	$\{x(x^2+y^2),y(x^2+y^2)\}, {}^\square(xz^2,yz^2)$
E_2			$^\square(xy,x^2-y^2)$	$^\square\{xyz,z(x^2-y^2)\}$
E_3				$^\square\{x(x^2-3y^2),y(3x^2-y^2)\}$
E_4				

T **30**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{10}$	$\langle \lvert j\,m\rangle \rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$	$\lvert 11\,10\rangle_-$	2	10
A_2	$\lvert 1\,0\rangle_+$	$\lvert 10\,10\rangle_-$	2	10
B_1	$\lvert 5\,5\rangle_-$	$\lvert 6\,5\rangle_+$	2	10
B_2	$\lvert 5\,5\rangle_+$	$\lvert 6\,5\rangle_-$	2	10
E_1	$\langle \lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle \rvert$	$\langle \lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle \rvert$	2	± 10
E_2	$\langle \lvert 2\,2\rangle, \lvert 2\,\overline{2}\rangle \rvert$	$\langle \lvert 3\,2\rangle, -\lvert 3\,\overline{2}\rangle \rvert$	2	± 10
E_3	$\langle \lvert 3\,\overline{3}\rangle, -\lvert 3\,3\rangle \rvert$	$\langle \lvert 4\,\overline{3}\rangle, \lvert 4\,3\rangle \rvert$	2	± 10
E_4	$\langle \lvert 4\,\overline{4}\rangle, -\lvert 4\,4\rangle \rvert$	$\langle \lvert 5\,\overline{4}\rangle, \lvert 5\,4\rangle \rvert$	2	± 10
$E_{1/2}$	$\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \rvert$	$\langle \lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle \rvert$	2	± 10
$E_{3/2}$	$\langle \lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle \rvert$	$\langle \lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle \rvert$	2	± 10
$E_{5/2}$	$\langle \lvert \frac{5}{2}\,\frac{5}{2}\rangle, \lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle \rvert$	$\langle \lvert \frac{7}{2}\,\frac{5}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle \rvert$	2	± 10
$E_{7/2}$	$\langle \lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle, \lvert \frac{7}{2}\,\frac{7}{2}\rangle \rvert$	$\langle \lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle, -\lvert \frac{9}{2}\,\frac{7}{2}\rangle \rvert$	2	± 10
$E_{9/2}$	$\langle \lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle, \lvert \frac{9}{2}\,\frac{9}{2}\rangle \rvert$	$\langle \lvert \frac{11}{2}\,\overline{\frac{9}{2}}\rangle, -\lvert \frac{11}{2}\,\frac{9}{2}\rangle \rvert$	2	± 10

T **30**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_{10}$	$\mathbf{C}_{10v}$	E_1	E_2	E_3	E_4
E	E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_{10}^{+}	C_{10}^{+}	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
C_{10}^{-}	C_{10}^{-}	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
C_{5}^{+}	C_{5}^{+}	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$
C_{5}^{-}	C_{5}^{-}	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$
C_{10}^{3+}	C_{10}^{3+}	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$
C_{10}^{3-}	C_{10}^{3-}	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$
C_{5}^{2+}	C_{5}^{2+}	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
C_{5}^{2-}	C_{5}^{2-}	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
C_2	C_2	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C'_{21}	σ_{d1}	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$
C'_{22}	σ_{d2}	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$
C'_{23}	σ_{d3}	$\begin{bmatrix} 0 & \bar{\delta} \\ \bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta} \\ \bar{\delta}^* & 0 \end{bmatrix}$
C'_{24}	σ_{d4}	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \bar{\delta} & 0 \end{bmatrix}$
C'_{25}	σ_{d5}	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$
C''_{21}	σ_{v1}	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$
C''_{22}	σ_{v2}	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta} \\ \bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$
C''_{23}	σ_{v3}	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta} \\ \bar{\delta}^* & 0 \end{bmatrix}$
C''_{24}	σ_{v4}	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \bar{\delta} & 0 \end{bmatrix}$
C''_{25}	σ_{v5}	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$

$\delta = \exp(2\pi \mathrm{i}/5)$, $\epsilon = \exp(4\pi \mathrm{i}/5)$ $\twoheadrightarrow$

T **30**.7 Matrix representations *(cont.)*

$\mathbf{D}_{10}$	$\mathbf{C}_{10v}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$
E	E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_{10}^{+}	C_{10}^{+}	$\begin{bmatrix} \mathrm{i}\overline{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \overline{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\overline{\delta}^* \end{bmatrix}$
C_{10}^{-}	C_{10}^{-}	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \overline{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta}^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$
C_{5}^{+}	C_{5}^{+}	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\delta} & 0 \\ 0 & \overline{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\delta} & 0 \\ 0 & \overline{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$
C_{5}^{-}	C_{5}^{-}	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \overline{\delta}^* & 0 \\ 0 & \overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\delta}^* & 0 \\ 0 & \overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$
C_{10}^{3+}	C_{10}^{3+}	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta}^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \overline{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$
C_{10}^{3-}	C_{10}^{3-}	$\begin{bmatrix} \mathrm{i}\overline{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\overline{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\overline{\epsilon}^* \end{bmatrix}$
C_{5}^{2+}	C_{5}^{2+}	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$
C_{5}^{2-}	C_{5}^{2-}	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$
C_2	C_2	$\begin{bmatrix} \overline{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \overline{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \overline{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \overline{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \overline{\mathrm{i}} \end{bmatrix}$
C'_{21}	σ_{d1}	$\begin{bmatrix} 0 & \overline{\mathrm{i}} \\ \overline{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\mathrm{i}} \\ \overline{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\mathrm{i}} \\ \overline{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	σ_{d2}	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\mathrm{i}} \\ \overline{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$
C'_{23}	σ_{d3}	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon}^* \\ \mathrm{i}\overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\mathrm{i}} \\ \overline{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon}^* \\ \mathrm{i}\overline{\epsilon} & 0 \end{bmatrix}$
C'_{24}	σ_{d4}	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon} \\ \mathrm{i}\overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\mathrm{i}} \\ \overline{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon} \\ \mathrm{i}\overline{\epsilon}^* & 0 \end{bmatrix}$
C'_{25}	σ_{d5}	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\mathrm{i}} \\ \overline{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$
C''_{21}	σ_{v1}	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \overline{1} & 0 \end{bmatrix}$
C''_{22}	σ_{v2}	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \overline{\delta} & 0 \end{bmatrix}$
C''_{23}	σ_{v3}	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \overline{\epsilon} & 0 \end{bmatrix}$
C''_{24}	σ_{v4}	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \overline{\epsilon}^* & 0 \end{bmatrix}$
C''_{25}	σ_{v5}	$\begin{bmatrix} 0 & \overline{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \overline{\delta}^* & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/5)$, $\epsilon = \exp(4\pi\mathrm{i}/5)$

T **30**.8 Direct products of representations

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$\mathbf{D}_{10}$, $\mathbf{C}_{10v}$	A_1	A_2	B_1	B_2	E_1	E_2	E_3	E_4
A_1	A_1	A_2	B_1	B_2	E_1	E_2	E_3	E_4
A_2		A_1	B_2	B_1	E_1	E_2	E_3	E_4
B_1			A_1	A_2	E_4	E_3	E_2	E_1
B_2				A_1	E_4	E_3	E_2	E_1
E_1					$A_1 \oplus \{A_2\} \oplus E_2$	$E_1 \oplus E_3$	$E_2 \oplus E_4$	$B_1 \oplus B_2 \oplus E_3$
E_2						$A_1 \oplus \{A_2\} \oplus E_4$	$B_1 \oplus B_2 \oplus E_1$	$E_2 \oplus E_4$
E_3							$A_1 \oplus \{A_2\} \oplus E_4$	$E_1 \oplus E_3$
E_4								$A_1 \oplus \{A_2\} \oplus E_2$

$\twoheadrightarrow$

T **30**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{10}$, $\mathbf{C}_{10v}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$
A_1	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$
A_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$
B_1	$E_{9/2}$	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
B_2	$E_{9/2}$	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
E_1	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{5/2} \oplus E_{9/2}$	$E_{7/2} \oplus E_{9/2}$
E_2	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{9/2}$	$E_{3/2} \oplus E_{9/2}$	$E_{5/2} \oplus E_{7/2}$
E_3	$E_{5/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{9/2}$	$E_{1/2} \oplus E_{9/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{5/2}$
E_4	$E_{7/2} \oplus E_{9/2}$	$E_{5/2} \oplus E_{9/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$
$E_{1/2}$	$\{A_1\} \oplus A_2 \oplus E_1$	$E_1 \oplus E_2$	$E_2 \oplus E_3$	$E_3 \oplus E_4$	$B_1 \oplus B_2 \oplus E_4$
$E_{3/2}$		$\{A_1\} \oplus A_2 \oplus E_3$	$E_1 \oplus E_4$	$B_1 \oplus B_2 \oplus E_2$	$E_3 \oplus E_4$
$E_{5/2}$			$\{A_1\} \oplus A_2 \oplus B_1 \oplus B_2$	$E_1 \oplus E_4$	$E_2 \oplus E_3$
$E_{7/2}$				$\{A_1\} \oplus A_2 \oplus E_3$	$E_1 \oplus E_2$
$E_{9/2}$					$\{A_1\} \oplus A_2 \oplus E_1$

T **30**.9 Subduction (descent of symmetry)

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$\mathbf{D}_{10}$	$(\mathbf{D}_5)$ C_2'	$(\mathbf{D}_5)$ C_2''	$(\mathbf{D}_2)$	$\mathbf{C}_{10}$
A_1	A_1	A_1	A	A
A_2	A_2	A_2	B_1	A
B_1	A_1	A_2	B_3	B
B_2	A_2	A_1	B_2	B
E_1	E_1	E_1	$B_2 \oplus B_3$	${}^1E_1 \oplus {}^2E_1$
E_2	E_2	E_2	$A \oplus B_1$	${}^1E_2 \oplus {}^2E_2$
E_3	E_2	E_2	$B_2 \oplus B_3$	${}^1E_3 \oplus {}^2E_3$
E_4	E_1	E_1	$A \oplus B_1$	${}^1E_4 \oplus {}^2E_4$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$
$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$
$E_{7/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$
$E_{9/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{9/2} \oplus {}^2E_{9/2}$

$\twoheadrightarrow$

T **30**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{10}$	$\mathbf{C}_5$	$\mathbf{C}_2$	$(\mathbf{C}_2)$	$(\mathbf{C}_2)$
		C_2	C_2'	C_2''
A_1	A	A	A	A
A_2	A	A	B	B
B_1	A	B	A	B
B_2	A	B	B	A
E_1	${}^1E_1 \oplus {}^2E_1$	$2B$	$A \oplus B$	$A \oplus B$
E_2	${}^1E_2 \oplus {}^2E_2$	$2A$	$A \oplus B$	$A \oplus B$
E_3	${}^1E_2 \oplus {}^2E_2$	$2B$	$A \oplus B$	$A \oplus B$
E_4	${}^1E_1 \oplus {}^2E_1$	$2A$	$A \oplus B$	$A \oplus B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	$2A_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{9/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **30**.10 Subduction from O(3) § **16**–10, p. 82

j	$\mathbf{D}_{10}$
$10n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4)$
$10n+1$	$n\,(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4) \oplus (n+1)(A_2 \oplus E_1)$
$10n+2$	$(n+1)(A_1 \oplus E_1 \oplus E_2) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus 2E_3 \oplus 2E_4)$
$10n+3$	$n\,(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus 2E_4) \oplus (n+1)(A_2 \oplus E_1 \oplus E_2 \oplus E_3)$
$10n+4$	$(n+1)(A_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4)$
$10n+5$	$n\,(A_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4) \oplus (n+1)(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4)$
$10n+6$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus 2E_4) \oplus n\,(A_2 \oplus E_1 \oplus E_2 \oplus E_3)$
$10n+7$	$n\,(A_1 \oplus E_1 \oplus E_2) \oplus (n+1)(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus 2E_3 \oplus 2E_4)$
$10n+8$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4) \oplus n\,(A_2 \oplus E_1)$
$10n+9$	$n\,A_1 \oplus (n+1)(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4)$
$10n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$
$10n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus 2n\,(E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$
$10n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus 2n\,(E_{7/2} \oplus E_{9/2})$
$10n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus 2n\,E_{9/2}$
$10n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$
$10n+\frac{11}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus (2n+2)\,E_{9/2}$
$10n+\frac{13}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus (2n+2)(E_{7/2} \oplus E_{9/2})$
$10n+\frac{15}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (2n+2)(E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$
$10n+\frac{17}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$
$10n+\frac{19}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$

$n = 0, 1, 2, \ldots$

T **30**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{D}_{10}$, $\mathbf{C}_{10v}$

a_2	e_1	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_2	E_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_3	E_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_4	E_4	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{5/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{7/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{9/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_1	e_1	E_4	
		1	2
1	1	1	0
1	2	0	1

b_1	e_2	E_3	
		1	2
1	1	1	0
1	2	0	1

b_1	e_3	E_2	
		1	2
1	1	1	0
1	2	0	1

b_1	e_4	E_1	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{1/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{3/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{5/2}$	$E_{5/2}$	
		1	2
1	1	0	1
1	2	1	0

b_1	$e_{7/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{9/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	1

b_2	e_1	E_4	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_2	E_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_3	E_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_4	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{1/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{3/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{5/2}$	$E_{5/2}$	
		1	2
1	1	0	$\bar{1}$
1	2	1	0

b_2	$e_{7/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{9/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e_1	e_1	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{\mathrm{u}}$	0	0
2	2	0	0	0	1

e_1	e_2	E_1		E_3	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	e_3	E_2		E_4	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

$\mathrm{u} = 2^{-1/2}$ ↠

T **30**.11 Clebsch–Gordan coefficients *(cont.)* $\mathbf{D}_{10}$, $\mathbf{C}_{10v}$

e_1	e_4	B_1	B_2	E_3	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_1	$e_{1/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_1	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_1	$e_{5/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	$e_{7/2}$	$E_{5/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_1	$e_{9/2}$	$E_{7/2}$		$E_{9/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_2	e_2	A_1	A_2	E_4	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_2	e_3	B_1	B_2	E_1	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_2	e_4	E_2		E_4	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	1	0	0	0

e_2	$e_{1/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_2	$e_{3/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_2	$e_{5/2}$	$E_{1/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_2	$e_{7/2}$	$E_{3/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_2	$e_{9/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_3	e_3	A_1	A_2	E_4	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_3	e_4	E_1		E_3	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

e_3	$e_{1/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_3	$e_{3/2}$	$E_{3/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$u = 2^{-1/2}$

↠

T **30**.11 Clebsch–Gordan coefficients *(cont.)* $\mathbf{D}_{10},\ \mathbf{C}_{10v}$

e_3	$e_{5/2}$	$E_{1/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_3	$e_{7/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_3	$e_{9/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_4	e_4	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_4	$e_{1/2}$	$E_{7/2}$		$E_{9/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_4	$e_{3/2}$	$E_{5/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_4	$e_{5/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_4	$e_{7/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_4	$e_{9/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{1/2}$	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{3/2}$	E_1		E_2	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{5/2}$	E_2		E_3	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{1/2}$	$e_{7/2}$	E_3		E_4	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	1	0	0

$e_{1/2}$	$e_{9/2}$	B_1	B_2	E_4	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{3/2}$	A_1	A_2	E_3	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{5/2}$	E_1		E_4	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{7/2}$	B_1	B_2	E_2	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{9/2}$	E_3		E_4	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

$u = 2^{-1/2}$

⋙

T **30**.11 Clebsch–Gordan coefficients *(cont.)* $\mathbf{D}_{10}$, $\mathbf{C}_{10v}$

$e_{5/2}$	$e_{5/2}$	A_1 1	A_2 1	B_1 1	B_2 1
1	1	0	0	u	u
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	$\bar{u}$	u

$e_{5/2}$	$e_{7/2}$	E_1 1	2	E_4 1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{5/2}$	$e_{9/2}$	E_2 1	2	E_3 1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{7/2}$	$e_{7/2}$	A_1 1	A_2 1	E_3 1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{7/2}$	$e_{9/2}$	E_1 1	2	E_2 1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$e_{9/2}$	$e_{9/2}$	A_1 1	A_2 1	E_1 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

The groups $\mathbf{D}_{nh}$

Notation for headers

Items in header read from left to right

1	Hermann–Mauguin symbol for the point group.
2	$\lvert G\rvert$ order of the group.
3	$\lvert C\rvert$ number of classes in the group.
4	$\lvert \widetilde{C}\rvert$ number of classes in the double group.
5	Number of the table.
6	Page reference for the notation of the header, of the first five subsections below it, and of the footers.
7 ☐	This symbol indicates a crystallographic point group.
8	Schönflies notation for the point group.

Notation for the first five subsections below the header

(1) Product forms	Direct and semidirect product forms. $\otimes$ Direct product. Ⓢ Semidirect product.
(2) Group chains *(See pp.* 41, 67*)*	Groups underlined: invariant. Groups in brackets: when subducing one or more of the representations in the tables to these subgroups in the setting used for them in the tables a change of bases (similarity transformation) is required.
(3) Operations of G	Lists all the operations of G, enclosing in brackets all the operations of the same class.
(4) Operations of $\widetilde{G}$	Lists all the operations of $\widetilde{G}$, enclosing in brackets all the operations of the same class.
(5) Classes and representations	$\lvert r\rvert$ number of regular classes in G (p. 51). $\lvert \mathrm{i}\rvert$ number of irregular classes in G (p. 51). $\lvert I\rvert$ number of irreducible representations in G. $\lvert \widetilde{I}\rvert$ number of spinor representations, also called the number of double-group representations.

Use of the footers

Finding your way about the tables — Each page of the tables contains a footer giving the ordering of the group types listed in the tables. The current group type is recognized because it carries no page number under its name. On opening the tables at any page you should then see at a glance whether to move backwards or forwards to find the group type desired. More precisely, the page number for the group type desired gives the page, like the present one, which carries the contents for the group type wanted.

mmm	$\lvert G\rvert = 8$	$\lvert C\rvert = 8$	$\lvert\widetilde{C}\rvert = 10$	T **31**	p. 245	□	$\mathbf{D}_{2h}$

(1) Product forms: $\mathbf{D}_2 \otimes \mathbf{C}_i$, $\mathbf{D}_2 \otimes \mathbf{C}_s$, $\mathbf{C}_{2v} \otimes \mathbf{C}_s$.

(2) Group chains: $\mathbf{T}_h \supset \underline{\mathbf{D}_{2h}} \supset \underline{\mathbf{C}_{2h}}$, $\mathbf{T}_h \supset \underline{\mathbf{D}_{2h}} \supset (\underline{\mathbf{C}_{2v}})$, $\mathbf{T}_h \supset \underline{\mathbf{D}_{2h}} \supset \underline{\mathbf{D}_2}$,
$\mathbf{D}_{10h} \supset (\mathbf{D}_{2h}) \supset \underline{\mathbf{C}_{2h}}$, $\mathbf{D}_{10h} \supset (\mathbf{D}_{2h}) \supset (\underline{\mathbf{C}_{2v}})$, $\mathbf{D}_{10h} \supset (\mathbf{D}_{2h}) \supset (\underline{\mathbf{D}_2})$,
$\mathbf{D}_{6h} \supset (\mathbf{D}_{2h}) \supset \underline{\mathbf{C}_{2h}}$, $\mathbf{D}_{6h} \supset (\mathbf{D}_{2h}) \supset (\underline{\mathbf{C}_{2v}})$, $\mathbf{D}_{6h} \supset (\mathbf{D}_{2h}) \supset \underline{\mathbf{D}_2}$,
$\mathbf{D}_{4h} \supset (\underline{\mathbf{D}_{2h}}) \supset \underline{\mathbf{C}_{2h}}$, $\mathbf{D}_{4h} \supset (\underline{\mathbf{D}_{2h}}) \supset (\underline{\mathbf{C}_{2v}})$, $\mathbf{D}_{4h} \supset (\underline{\mathbf{D}_{2h}}) \supset \underline{\mathbf{D}_2}$.

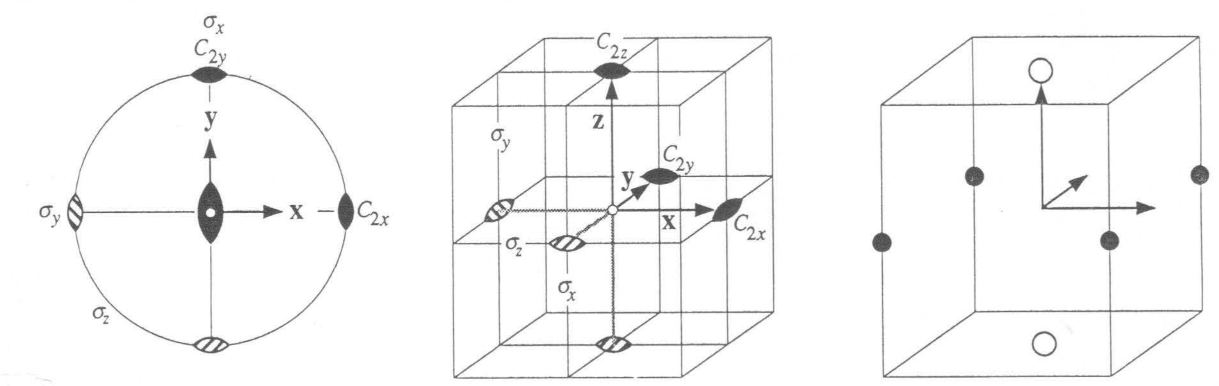
(3) Operations of G: E, C_{2z}, C_{2x}, C_{2y}, i, σ_z, σ_x, σ_y.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, $(C_{2z}, \widetilde{C}_{2z})$, $(C_{2x}, \widetilde{C}_{2x})$, $(C_{2y}, \widetilde{C}_{2y})$, i, $\widetilde{i}$, $(\sigma_z, \widetilde{\sigma}_z)$, $(\sigma_x, \widetilde{\sigma}_x)$, $(\sigma_y, \widetilde{\sigma}_y)$.

(5) Classes and representations: $\lvert r\rvert = 2$, $\lvert i\rvert = 6$, $\lvert I\rvert = 8$, $\lvert\widetilde{I}\rvert = 2$.

F **31**

See Chapter **15**, p. 65



Examples: Planar C_2H_4, planar N_2O_4.

T **31**.0 Subgroup elements

§ **16**–0, p. 68

$\mathbf{D}_{2h}$	$\mathbf{C}_{2h}$	$\mathbf{C}_{2v}$	$\mathbf{D}_2$	$\mathbf{C}_s$	$\mathbf{C}_i$	$\mathbf{C}_2$	$\mathbf{C}_1$
E	E	E	E	E	E	E	E
C_{2z}	C_2	C_2	C_{2z}			C_2	
C_{2x}			C_{2x}				
C_{2y}			C_{2y}				
i	i				i		
σ_z	σ_h			σ_h			
σ_x		σ_x					
σ_y		σ_y					

T **31**.1 Parameters

§ **16**–1, p. 68

$\mathbf{D}_{2h}$		α	β	γ	ϕ	$\mathbf{n}$	λ $\mathbf{\Lambda}$
E	i	0	0	0	0	(0 0 0)	⟦1, (0 0 0)⟧
C_{2z}	σ_z	0	0	π	π	(0 0 1)	⟦0, (0 0 1)⟧
C_{2x}	σ_x	0	π	π	π	(1 0 0)	⟦0, (1 0 0)⟧
C_{2y}	σ_y	0	π	0	π	(0 1 0)	⟦0, (0 1 0)⟧

T **31**.2 Multiplication table

§ **16**–2, p. 69

$\mathbf{D}_{2h}$	E	C_{2z}	C_{2x}	C_{2y}	i	σ_z	σ_x	σ_y
E	E	C_{2z}	C_{2x}	C_{2y}	i	σ_z	σ_x	σ_y
C_{2z}	C_{2z}	E	C_{2y}	C_{2x}	σ_z	i	σ_y	σ_x
C_{2x}	C_{2x}	C_{2y}	E	C_{2z}	σ_x	σ_y	i	σ_z
C_{2y}	C_{2y}	C_{2x}	C_{2z}	E	σ_y	σ_x	σ_z	i
i	i	σ_z	σ_x	σ_y	E	C_{2z}	C_{2x}	C_{2y}
σ_z	σ_z	i	σ_y	σ_x	C_{2z}	E	C_{2y}	C_{2x}
σ_x	σ_x	σ_y	i	σ_z	C_{2x}	C_{2y}	E	C_{2z}
σ_y	σ_y	σ_x	σ_z	i	C_{2y}	C_{2x}	C_{2z}	E

T **31**.3 Factor table

§ **16**–3, p. 70

$\mathbf{D}_{2h}$	E	C_{2z}	C_{2x}	C_{2y}	i	σ_z	σ_x	σ_y
E	1	1	1	1	1	1	1	1
C_{2z}	1	−1	1	−1	1	−1	1	−1
C_{2x}	1	−1	−1	1	1	−1	−1	1
C_{2y}	1	1	−1	−1	1	1	−1	−1
i	1	1	1	1	1	1	1	1
σ_z	1	−1	1	−1	1	−1	1	−1
σ_x	1	−1	−1	1	1	−1	−1	1
σ_y	1	1	−1	−1	1	1	−1	−1

T **31**.4 Character table

§ **16**–4, p. 71

$\mathbf{D}_{2h}$	E	C_{2z}	C_{2x}	C_{2y}	i	σ_z	σ_x	σ_y	τ
A_g	1	1	1	1	1	1	1	1	a
B_{1g}	1	1	−1	−1	1	1	−1	−1	a
B_{2g}	1	−1	−1	1	1	−1	−1	1	a
B_{3g}	1	−1	1	−1	1	−1	1	−1	a
A_u	1	1	1	1	−1	−1	−1	−1	a
B_{1u}	1	1	−1	−1	−1	−1	1	1	a
B_{2u}	1	−1	−1	1	−1	1	1	−1	a
B_{3u}	1	−1	1	−1	−1	1	−1	1	a
$E_{1/2,g}$	2	0	0	0	2	0	0	0	c
$E_{1/2,u}$	2	0	0	0	−2	0	0	0	c

T **31**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{2h}$	0	1	2	3
A_g	$\square 1$		$\square x^2, y^2, \square z^2$	
B_{1g}		R_z	$\square xy$	
B_{2g}		R_y	$\square zx$	
B_{3g}		R_x	$\square yz$	
A_u				$\square xyz$
B_{1u}		$\square z$		$\square x^2z, y^2z, \square z^3$
B_{2u}		$\square y$		$\square x^2y, y^3, \square yz^2$
B_{3u}		$\square x$		$\square x^3, xy^2, \square xz^2$

T **31**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_{2h}$	$E_{1/2,g}$	$E_{1/2,u}$
E	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$
C_{2z}	$\begin{bmatrix}\bar{\mathrm{i}} & 0\\ 0 & \mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}} & 0\\ 0 & \mathrm{i}\end{bmatrix}$
C_{2x}	$\begin{bmatrix}0 & \bar{\mathrm{i}}\\ \bar{\mathrm{i}} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\mathrm{i}}\\ \bar{\mathrm{i}} & 0\end{bmatrix}$
C_{2y}	$\begin{bmatrix}0 & \bar{1}\\ 1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{1}\\ 1 & 0\end{bmatrix}$
i	$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}\bar{1} & 0\\ 0 & \bar{1}\end{bmatrix}$
σ_z	$\begin{bmatrix}\bar{\mathrm{i}} & 0\\ 0 & \mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\mathrm{i} & 0\\ 0 & \bar{\mathrm{i}}\end{bmatrix}$
σ_x	$\begin{bmatrix}0 & \bar{\mathrm{i}}\\ \bar{\mathrm{i}} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \mathrm{i}\\ \mathrm{i} & 0\end{bmatrix}$
σ_y	$\begin{bmatrix}0 & \bar{1}\\ 1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 1\\ \bar{1} & 0\end{bmatrix}$

T **31**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{2h}$	$\langle \lvert j\,m\rangle \rvert$		ι	μ
A_g	$\lvert 0\,0\rangle_+$		2	2
B_{1g}	$\lvert 2\,2\rangle_-$		2	2
B_{2g}	$\lvert 2\,1\rangle_-$		2	2
B_{3g}	$\lvert 2\,1\rangle_+$		2	2
A_u	$\lvert 3\,2\rangle_-$		2	2
B_{1u}	$\lvert 1\,0\rangle_+$		2	2
B_{2u}	$\lvert 1\,1\rangle_+$		2	2
B_{3u}	$\lvert 1\,1\rangle_-$		2	2
$E_{1/2,g}$	$\langle \lvert \tfrac12\,\tfrac12\rangle, \lvert \tfrac12\,\overline{\tfrac12}\rangle \rvert$	$\langle \lvert \tfrac32\,\overline{\tfrac32}\rangle, -\lvert \tfrac32\,\tfrac32\rangle \rvert$	2	± 2
$E_{1/2,u}$	$\langle \lvert \tfrac12\,\tfrac12\rangle, \lvert \tfrac12\,\overline{\tfrac12}\rangle \rvert^\bullet$	$\langle \lvert \tfrac32\,\overline{\tfrac32}\rangle, -\lvert \tfrac32\,\tfrac32\rangle \rvert^\bullet$	2	± 2

T **31**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{D}_{2h}$	A_g	B_{1g}	B_{2g}	B_{3g}	A_u	B_{1u}	B_{2u}	B_{3u}	$E_{1/2,g}$	$E_{1/2,u}$
A_g	A_g	B_{1g}	B_{2g}	B_{3g}	A_u	B_{1u}	B_{2u}	B_{3u}	$E_{1/2,g}$	$E_{1/2,u}$
B_{1g}		A_g	B_{3g}	B_{2g}	B_{1u}	A_u	B_{3u}	B_{2u}	$E_{1/2,g}$	$E_{1/2,u}$
B_{2g}			A_g	B_{1g}	B_{2u}	B_{3u}	A_u	B_{1u}	$E_{1/2,g}$	$E_{1/2,u}$
B_{3g}				A_g	B_{3u}	B_{2u}	B_{1u}	A_u	$E_{1/2,g}$	$E_{1/2,u}$
A_u					A_g	B_{1g}	B_{2g}	B_{3g}	$E_{1/2,u}$	$E_{1/2,g}$
B_{1u}						A_g	B_{3g}	B_{2g}	$E_{1/2,u}$	$E_{1/2,g}$
B_{2u}							A_g	B_{1g}	$E_{1/2,u}$	$E_{1/2,g}$
B_{3u}								A_g	$E_{1/2,u}$	$E_{1/2,g}$
$E_{1/2,g}$									$\{A_g\} \oplus B_{1g} \oplus B_{2g} \oplus B_{3g}$	$A_u \oplus B_{1u} \oplus B_{2u} \oplus B_{3u}$
$E_{1/2,u}$										$\{A_g\} \oplus B_{1g} \oplus B_{2g} \oplus B_{3g}$

T **31**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_{2h}$	$\mathbf{C}_{2h}$	$(\mathbf{C}_{2h})$	$(\mathbf{C}_{2h})$	$(\mathbf{C}_{2v})$	$(\mathbf{C}_{2v})$	$(\mathbf{C}_{2v})$	$\mathbf{D}_2$
	C_{2z}	C_{2x}	C_{2y}	C_{2z}	C_{2x}	C_{2y}	
A_g	A_g	A_g	A_g	A_1	A_1	A_1	A
B_{1g}	A_g	B_g	B_g	A_2	B_2	B_2	B_1
B_{2g}	B_g	B_g	A_g	B_1	B_1	A_2	B_2
B_{3g}	B_g	A_g	B_g	B_2	A_2	B_1	B_3
A_u	A_u	A_u	A_u	A_2	A_2	A_2	A
B_{1u}	A_u	B_u	B_u	A_1	B_1	B_1	B_1
B_{2u}	B_u	B_u	A_u	B_2	B_2	A_1	B_2
B_{3u}	B_u	A_u	B_u	B_1	A_1	B_2	B_3
$E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$

$\twoheadrightarrow$

T **31**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{2h}$	$\mathbf{C}_s$	$(\mathbf{C}_s)$	$(\mathbf{C}_s)$	$\mathbf{C}_i$
	σ_z	σ_x	σ_y	
A_g	A'	A'	A'	A_g
B_{1g}	A'	A''	A''	A_g
B_{2g}	A''	A''	A'	A_g
B_{3g}	A''	A'	A''	A_g
A_u	A''	A''	A''	A_u
B_{1u}	A''	A'	A'	A_u
B_{2u}	A'	A'	A''	A_u
B_{3u}	A'	A''	A'	A_u
$E_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,g}$
$E_{1/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,u}$

$\twoheadrightarrow$

T **31**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{2h}$	$\mathbf{C}_2$	$(\mathbf{C}_2)$	$(\mathbf{C}_2)$
	C_{2z}	C_{2x}	C_{2y}
A_g	A	A	A
B_{1g}	A	B	B
B_{2g}	B	B	A
B_{3g}	B	A	B
A_u	A	A	A
B_{1u}	A	B	B
B_{2u}	B	B	A
B_{3u}	B	A	B
$E_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{1/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **31**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{2h}$
$2n$	$(n+1)\,A_g \oplus n\,(B_{1g} \oplus B_{2g} \oplus B_{3g})$
$2n+1$	$n\,A_u \oplus (n+1)(B_{1u} \oplus B_{2u} \oplus B_{3u})$
$n+\frac{1}{2}$	$(n+1)\,E_{1/2,g}$

$n = 0, 1, 2, \ldots$

T **31**.11 Clebsch–Gordan coefficients

Use T **22**.11 ▪. § **16**–11, p. 83

(1) Product forms: $\mathbf{D}_3 \otimes \mathbf{C}_s$, $\mathbf{C}_{3v} \otimes \mathbf{C}_s$.

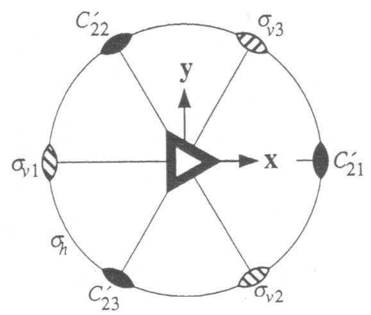
(2) Group chains: $\mathbf{D}_{9h} \supset (\mathbf{D}_{3h}) \supset \underline{\mathbf{C}_{3h}}$, $\mathbf{D}_{9h} \supset (\mathbf{D}_{3h}) \supset (\underline{\mathbf{C}_{3v}})$, $\mathbf{D}_{9h} \supset (\mathbf{D}_{3h}) \supset (\mathbf{C}_{2v})$, $\mathbf{D}_{9h} \supset (\mathbf{D}_{3h}) \supset (\underline{\mathbf{D}_3})$, $\mathbf{D}_{6h} \supset \underline{\mathbf{D}_{3h}} \supset \underline{\mathbf{C}_{3h}}$, $\mathbf{D}_{6h} \supset \underline{\mathbf{D}_{3h}} \supset (\underline{\mathbf{C}_{3v}})$, $\mathbf{D}_{6h} \supset \underline{\mathbf{D}_{3h}} \supset (\mathbf{C}_{2v})$, $\mathbf{D}_{6h} \supset \underline{\mathbf{D}_{3h}} \supset (\underline{\mathbf{D}_3})$.

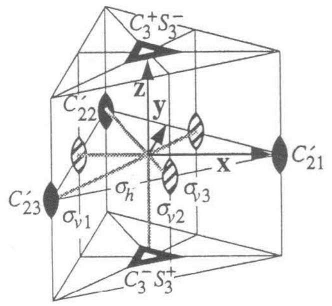
(3) Operations of G: E, (C_3^+, C_3^-), $(C'_{21}, C'_{22}, C'_{23})$, σ_h, (S_3^+, S_3^-), $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3})$.

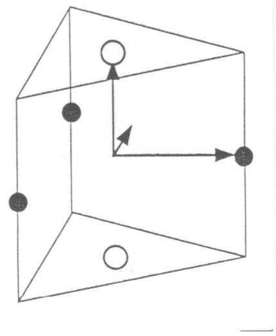
(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_3^+, C_3^-), $(\widetilde{C}_3^+, \widetilde{C}_3^-)$, $(C'_{21}, C'_{22}, C'_{23}, \widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23})$, $(\sigma_h, \widetilde{\sigma}_h)$, (S_3^+, S_3^-), $(\widetilde{S}_3^+, \widetilde{S}_3^-)$, $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2}, \widetilde{\sigma}_{v3})$.

(5) Classes and representations: $|r| = 3$, $|\mathrm{i}| = 3$, $|I| = 6$, $|\widetilde{I}| = 3$.

F **32** See Chapter **15**, p. 65







Examples: BCl_3, eclipsed C_2H_6, 1,3,5-trichlorobenzene $C_6H_3Cl_3$, $B_3N_3H_6$.

T **32**.1 Parameters
Use T **35**.1. § **16**–1, p. 68

T **32**.2 Multiplication table
Use T **35**.2. § **16**–2, p. 69

T **32**.3 Factor table
Use T **35**.3. § **16**–3, p. 70

T **32**.4 Character table
§ **16**–4, p. 71

$\mathbf{D}_{3h}$	E	$2C_3$	$3C'_2$	σ_h	$2S_3$	$3\sigma_v$	τ
A'_1	1	1	1	1	1	1	a
A'_2	1	1	−1	1	1	−1	a
E'	2	−1	0	2	−1	0	a
A''_1	1	1	1	−1	−1	−1	a
A''_2	1	1	−1	−1	−1	1	a
E''	2	−1	0	−2	1	0	a
$E_{1/2}$	2	1	0	0	$\sqrt{3}$	0	c
$E_{3/2}$	2	−2	0	0	0	0	c
$E_{5/2}$	2	1	0	0	$-\sqrt{3}$	0	c

T **32**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{3h}$	0	1	2	3
A_1'	$^{\square}1$		$x^2+y^2, {}^{\square}z^2$	$^{\square}x(x^2-3y^2)$
A_2'		R_z		$^{\square}y(3x^2-y^2)$
E'		$^{\square}(x,y)$	$^{\square}(xy, x^2-y^2)$	$\{x(x^2+y^2), y(x^2+y^2)\}, {}^{\square}(xz^2, yz^2)$
A_1''				
A_2''		$^{\square}z$		$(x^2+y^2)z, {}^{\square}z^3$
E''		(R_x, R_y)	$^{\square}(zx, yz)$	$^{\square}\{xyz, z(x^2-y^2)\}$

T **32**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{3h}$	$\langle\lvert j\,m\rangle\rvert$		ι	μ
A_1'	$\lvert 0\,0\rangle_+$	$\lvert 3\,3\rangle_-$	2	6
A_2'	$\lvert 3\,3\rangle_+$	$\lvert 6\,6\rangle_-$	2	6
E'	$\langle\lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle\rvert$	$\langle\lvert 2\,\overline{2}\rangle, -\lvert 2\,2\rangle\rvert$	2	± 6
A_1''	$\lvert 4\,3\rangle_+$	$\lvert 7\,6\rangle_-$	2	6
A_2''	$\lvert 1\,0\rangle_+$	$\lvert 4\,3\rangle_-$	2	6
E''	$\langle\lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle\rvert$	$\langle\lvert 3\,\overline{2}\rangle, \lvert 3\,2\rangle\rvert$	2	± 6
$E_{1/2}$	$\langle\lvert \tfrac12\,\tfrac12\rangle, \lvert \tfrac12\,\overline{\tfrac12}\rangle\rvert$	$\langle\lvert \tfrac32\,\tfrac12\rangle, -\lvert \tfrac32\,\overline{\tfrac12}\rangle\rvert$	2	± 6
	$\langle\lvert \tfrac52\,\overline{\tfrac52}\rangle, \lvert \tfrac52\,\tfrac52\rangle\rvert^{\bullet}$	$\langle\lvert \tfrac72\,\overline{\tfrac52}\rangle, -\lvert \tfrac72\,\tfrac52\rangle\rvert^{\bullet}$	2	± 6
$E_{3/2}$	$\langle\lvert \tfrac32\,\tfrac32\rangle, \lvert \tfrac32\,\overline{\tfrac32}\rangle\rvert$	$\langle\lvert \tfrac52\,\tfrac32\rangle, -\lvert \tfrac52\,\overline{\tfrac32}\rangle\rvert$	2	± 6
	$\langle\lvert \tfrac32\,\overline{\tfrac32}\rangle, \lvert \tfrac32\,\tfrac32\rangle\rvert^{\bullet}$	$\langle\lvert \tfrac52\,\overline{\tfrac32}\rangle, -\lvert \tfrac52\,\tfrac32\rangle\rvert^{\bullet}$	2	± 6
$E_{5/2}$	$\langle\lvert \tfrac52\,\overline{\tfrac52}\rangle, \lvert \tfrac52\,\tfrac52\rangle\rvert$	$\langle\lvert \tfrac72\,\overline{\tfrac52}\rangle, -\lvert \tfrac72\,\tfrac52\rangle\rvert$	2	± 6
	$\langle\lvert \tfrac12\,\tfrac12\rangle, \lvert \tfrac12\,\overline{\tfrac12}\rangle\rvert^{\bullet}$	$\langle\lvert \tfrac32\,\tfrac12\rangle, -\lvert \tfrac32\,\overline{\tfrac12}\rangle\rvert^{\bullet}$	2	± 6

T **32**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_{3h}$	E'	E''	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_3^+	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$
C_3^-	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$
σ_h	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$
S_3^+	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar{\epsilon}^* \end{bmatrix}$
S_3^-	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$
σ_{v1}	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$
σ_{v2}	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$
σ_{v3}	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$

$\epsilon = \exp(2\pi\mathrm{i}/3)$

T **32**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{D}_{3h}$	A'_1	A'_2	E'	A''_1	A''_2	E''	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
A'_1	A'_1	A'_2	E'	A''_1	A''_2	E''	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
A'_2		A'_1	E'	A''_2	A''_1	E''	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
E'			$A'_1 \oplus \{A'_2\} \oplus E'$	E''	E''	$A''_1 \oplus A''_2 \oplus E''$	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$
A''_1				A'_1	A'_2	E'	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
A''_2					A'_1	E'	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
E''						$A'_1 \oplus \{A'_2\} \oplus E'$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus E_{5/2}$
$E_{1/2}$							$\{A'_1\} \oplus A'_2 \oplus E''$	$E' \oplus E''$	$E' \oplus A''_1 \oplus A''_2$
$E_{3/2}$								$\{A'_1\} \oplus A'_2 \oplus A''_1 \oplus A''_2$	$E' \oplus E''$
$E_{5/2}$									$\{A'_1\} \oplus A'_2 \oplus E''$

T **32**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_{3h}$	$\mathbf{C}_{3h}$	$(\mathbf{C}_{3v})$	$(\mathbf{C}_{2v})$ C'_2, σ_v, σ_h	$(\mathbf{D}_3)$
A'_1	A'	A_1	A_1	A_1
A'_2	A'	A_2	B_1	A_2
E'	${}^1E' \oplus {}^2E'$	E	$A_1 \oplus B_1$	E
A''_1	A''	A_2	A_2	A_1
A''_2	A''	A_1	B_2	A_2
E''	${}^1E'' \oplus {}^2E''$	E	$A_2 \oplus B_2$	E
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$
$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$

$\twoheadrightarrow$

T **32**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{3h}$	$\mathbf{C}_s$ σ_h	$(\mathbf{C}_s)$ σ_v	$\mathbf{C}_3$	$(\mathbf{C}_2)$
A'_1	A'	A'	A	A
A'_2	A'	A''	A	B
E'	$2A'$	$A' \oplus A''$	${}^1E \oplus {}^2E$	$A \oplus B$
A''_1	A''	A''	A	A
A''_2	A''	A'	A	B
E''	$2A''$	$A' \oplus A''$	${}^1E \oplus {}^2E$	$A \oplus B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **32**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{3h}$
$6n$	$(n+1)\,A'_1 \oplus n\,(A'_2 \oplus 2E' \oplus A''_1 \oplus A''_2 \oplus 2E'')$
$6n+1$	$(n+1)(E' \oplus A''_2) \oplus n\,(A'_1 \oplus A'_2 \oplus E' \oplus A''_1 \oplus 2E'')$
$6n+2$	$(n+1)(A'_1 \oplus E' \oplus E'') \oplus n\,(A'_2 \oplus E' \oplus A''_1 \oplus A''_2 \oplus E'')$
$6n+3$	$(n+1)(A'_1 \oplus A'_2 \oplus E' \oplus A''_2 \oplus E'') \oplus n\,(E' \oplus A''_1 \oplus E'')$
$6n+4$	$(n+1)(A'_1 \oplus 2E' \oplus A''_1 \oplus A''_2 \oplus E'') \oplus n\,(A'_2 \oplus E'')$
$6n+5$	$(n+1)(A'_1 \oplus A'_2 \oplus 2E' \oplus A''_2 \oplus 2E'') \oplus n\,A''_1$
$6n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,(E_{3/2} \oplus E_{5/2})$
$6n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus 2n\,E_{5/2}$
$6n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2})$
$6n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (2n+2)\,E_{5/2}$
$6n+\frac{9}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)(E_{3/2} \oplus E_{5/2})$
$6n+\frac{11}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2})$

$n = 0, 1, 2, \ldots$

T **32**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{D}_{3h}$

a_2'	e'	E'	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2'	e''	E''	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2'	$e_{1/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2'	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2'	$e_{5/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e'	e'	A_1'	A_2'	E'	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e'	a_1''	E''	
		1	2
1	1	1	0
2	1	0	1

e'	a_2''	E''	
		1	2
1	1	1	0
2	1	0	$\bar{1}$

e'	e''	A_1''	A_2''	E''	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e'	$e_{1/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e'	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e'	$e_{5/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

a_1''	e''	E'	
		1	2
1	1	1	0
1	2	0	1

a_1''	$e_{1/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	1

a_1''	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	0	1
1	2	1	0

a_1''	$e_{5/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	1

a_2''	e''	E'	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2''	$e_{1/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2''	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	0	$\bar{1}$
1	2	1	0

a_2''	$e_{5/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e''	e''	A_1'	A_2'	E'	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e''	$e_{1/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e''	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e''	$e_{5/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$u = 2^{-1/2}$

$\twoheadrightarrow$

T **32**.11 Clebsch–Gordan coefficients *(cont.)*

$e_{1/2}$	$e_{1/2}$	A_1'	A_2'	E''	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{3/2}$	E'		E''	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{1/2}$	$e_{5/2}$	E'		A_1''	A_2''
		1	2	1	1
1	1	1	0	0	0
1	2	0	0	u	u
2	1	0	0	$\bar{u}$	u
2	2	0	1	0	0

$e_{3/2}$	$e_{3/2}$	A_1'	A_2'	A_1''	A_2''
		1	1	1	1
1	1	0	0	u	u
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	$\bar{u}$	u

$e_{3/2}$	$e_{5/2}$	E'		E''	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{5/2}$	$e_{5/2}$	A_1'	A_2'	E''	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

$4/mmm$	$\lvert G\rvert = 16$	$\lvert C\rvert = 10$	$\lvert\widetilde{C}\rvert = 14$	T **33**	p. 245	□	$\mathbf{D}_{4h}$

(1) Product forms: $\mathbf{D}_4 \otimes \mathbf{C}_i$, $\mathbf{D}_4 \otimes \mathbf{C}_s$, $\mathbf{C}_{4v} \otimes \mathbf{C}_s$.

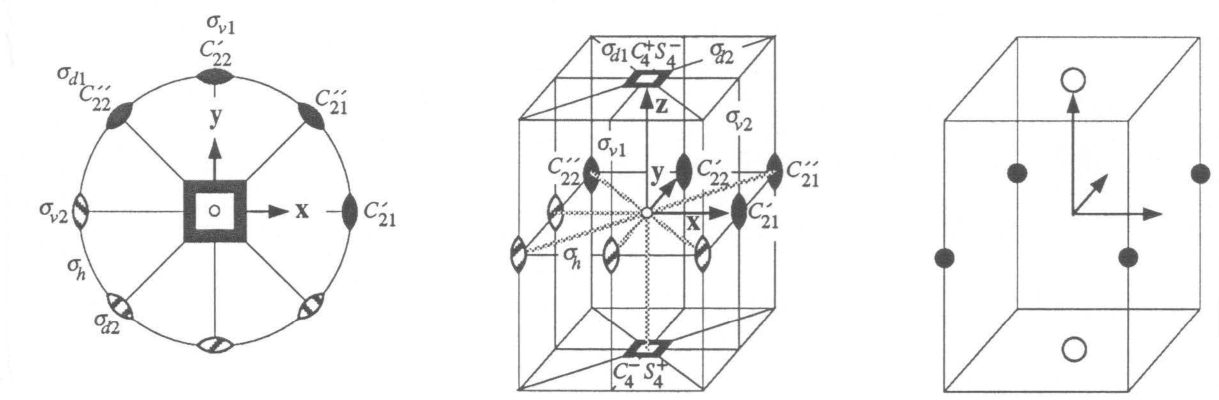
(2) Group chains: $\mathbf{O}_h \supset (\mathbf{D}_{4h}) \supset \underline{\mathbf{C}_{4h}}$, $\mathbf{O}_h \supset (\mathbf{D}_{4h}) \supset (\underline{\mathbf{C}_{4v}})$, $\mathbf{O}_h \supset (\mathbf{D}_{4h}) \supset \underline{\mathbf{D}_{2d}}$,
$\mathbf{O}_h \supset (\mathbf{D}_{4h}) \supset (\underline{\mathbf{D}_{2h}})$, $\mathbf{O}_h \supset (\mathbf{D}_{4h}) \supset \underline{\mathbf{D}_4}$,
$\mathbf{D}_{8h} \supset (\underline{\mathbf{D}_{4h}}) \supset \underline{\mathbf{C}_{4h}}$, $\mathbf{D}_{8h} \supset (\underline{\mathbf{D}_{4h}}) \supset (\underline{\mathbf{C}_{4v}})$, $\mathbf{D}_{8h} \supset (\underline{\mathbf{D}_{4h}}) \supset \underline{\mathbf{D}_{2d}}$,
$\mathbf{D}_{8h} \supset (\underline{\mathbf{D}_{4h}}) \supset (\underline{\mathbf{D}_{2h}})$, $\mathbf{D}_{8h} \supset (\underline{\mathbf{D}_{4h}}) \supset \underline{\mathbf{D}_4}$.

(3) Operations of G: E, (C_4^+, C_4^-), C_2, (C'_{21}, C'_{22}), (C''_{21}, C''_{22}), i, (S_4^-, S_4^+), σ_h, $(\sigma_{v1}, \sigma_{v2})$, $(\sigma_{d1}, \sigma_{d2})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_4^+, C_4^-), $(\widetilde{C}_4^+, \widetilde{C}_4^-)$, $(C_2, \widetilde{C}_2)$, $(C'_{21}, C'_{22}, \widetilde{C}'_{21}, \widetilde{C}'_{22})$, $(C''_{21}, C''_{22}, \widetilde{C}''_{21}, \widetilde{C}''_{22})$,
i, $\widetilde{\imath}$, (S_4^-, S_4^+), $(\widetilde{S}_4^-, \widetilde{S}_4^+)$, $(\sigma_h, \widetilde{\sigma}_h)$, $(\sigma_{v1}, \sigma_{v2}, \widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2})$, $(\sigma_{d1}, \sigma_{d2}, \widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2})$.

(5) Classes and representations: $\lvert r\rvert = 4$, $\lvert \mathrm{i}\rvert = 6$, $\lvert I\rvert = 10$, $\lvert\widetilde{I}\rvert = 4$.

F **33** See Chapter **15**, p. 65



Examples: Cyclobutane C_4H_8, square planar $AuCl_4$, $HW_2(CO)_{10}$.

T 33.0 Subgroup elements

§ **16**–0, p. 68

$\mathbf{D}_{4h}$	$\mathbf{C}_{4h}$	$\mathbf{C}_{2h}$	$\mathbf{C}_{4v}$	$\mathbf{C}_{2v}$	$\mathbf{D}_{2d}$	$\mathbf{D}_{2h}$	$\mathbf{D}_4$	$\mathbf{D}_2$	$\mathbf{S}_4$	$\mathbf{C}_s$	$\mathbf{C}_i$	$\mathbf{C}_4$	$\mathbf{C}_2$
E	E	E	E	E	E	E	E	E	E	E	E	E	E
C_4^+	C_4^+		C_4^+				C_4^+					C_4^+	
C_4^-	C_4^-		C_4^-				C_4^-					C_4^-	
C_2	C_2	C_2	C_2	C_2	C_2	C_{2z}	C_2	C_{2z}	C_2			C_2	C_2
C'_{21}					C'_{21}	C_{2x}	C'_{21}	C_{2x}					
C'_{22}					C'_{22}	C_{2y}	C'_{22}	C_{2y}					
C''_{21}							C''_{21}						
C''_{22}							C''_{22}						
i	i	i				i					i		
S_4^-	S_4^-				S_4^-				S_4^-				
S_4^+	S_4^+				S_4^+				S_4^+				
σ_h	σ_h	σ_h				σ_z				σ_h			
σ_{v1}			σ_{v1}	σ_x		σ_x							
σ_{v2}			σ_{v2}	σ_y		σ_y							
σ_{d1}			σ_{d1}		σ_{d1}								
σ_{d2}			σ_{d2}		σ_{d2}								

T **33**.1 Parameters

§ **16**–1, p. 68

$\mathbf{D}_{4h}$		α	β	γ	ϕ	$\mathbf{n}$	λ	$\boldsymbol{\Lambda}$
E	i	0	0	0	0	$(0\ \ 0\ \ 0)$	$[\![\ 1,$	$(0\ \ 0\ \ 0)]\!]$
C_4^+	S_4^-	0	0	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$(0\ \ 0\ \ 1)$	$[\![\frac{1}{\sqrt{2}},$	$(0\ \ 0\ \ \frac{1}{\sqrt{2}})]\!]$
C_4^-	S_4^+	0	0	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	$(0\ \ 0\ -1)$	$[\![\frac{1}{\sqrt{2}},$	$(0\ \ 0\ -\frac{1}{\sqrt{2}})]\!]$
C_2	σ_h	0	0	π	π	$(0\ \ 0\ \ 1)$	$[\![\ 0,$	$(0\ \ 0\ \ 1)]\!]$
C'_{21}	σ_{v1}	0	π	π	π	$(1\ \ 0\ \ 0)$	$[\![\ 0,$	$(1\ \ 0\ \ 0)]\!]$
C'_{22}	σ_{v2}	0	π	0	π	$(0\ \ 1\ \ 0)$	$[\![\ 0,$	$(0\ \ 1\ \ 0)]\!]$
C''_{21}	σ_{d1}	0	π	$\frac{\pi}{2}$	π	$(\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)$	$[\![\ 0,$	$(\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)]\!]$
C''_{22}	σ_{d2}	0	π	$-\frac{\pi}{2}$	π	$(-\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)$	$[\![\ 0,$	$(-\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)]\!]$

T **33**.2 Multiplication table

§ **16**–2, p. 69

$\mathbf{D}_{4h}$	E	C_4^+	C_4^-	C_2	C'_{21}	C'_{22}	C''_{21}	C''_{22}	i	S_4^-	S_4^+	σ_h	σ_{v1}	σ_{v2}	σ_{d1}	σ_{d2}
E	E	C_4^+	C_4^-	C_2	C'_{21}	C'_{22}	C''_{21}	C''_{22}	i	S_4^-	S_4^+	σ_h	σ_{v1}	σ_{v2}	σ_{d1}	σ_{d2}
C_4^+	C_4^+	C_2	E	C_4^-	C''_{21}	C''_{22}	C'_{22}	C'_{21}	S_4^-	σ_h	i	S_4^+	σ_{d1}	σ_{d2}	σ_{v2}	σ_{v1}
C_4^-	C_4^-	E	C_2	C_4^+	C''_{22}	C''_{21}	C'_{21}	C'_{22}	S_4^+	i	σ_h	S_4^-	σ_{d2}	σ_{d1}	σ_{v1}	σ_{v2}
C_2	C_2	C_4^-	C_4^+	E	C'_{22}	C'_{21}	C''_{22}	C''_{21}	σ_h	S_4^+	S_4^-	i	σ_{v2}	σ_{v1}	σ_{d2}	σ_{d1}
C'_{21}	C'_{21}	C''_{22}	C''_{21}	C'_{22}	E	C_2	C_4^-	C_4^+	σ_{v1}	σ_{d2}	σ_{d1}	σ_{v2}	i	σ_h	S_4^+	S_4^-
C'_{22}	C'_{22}	C''_{21}	C''_{22}	C'_{21}	C_2	E	C_4^+	C_4^-	σ_{v2}	σ_{d1}	σ_{d2}	σ_{v1}	σ_h	i	S_4^-	S_4^+
C''_{21}	C''_{21}	C'_{21}	C'_{22}	C''_{22}	C_4^+	C_4^-	E	C_2	σ_{d1}	σ_{v1}	σ_{v2}	σ_{d2}	S_4^-	S_4^+	i	σ_h
C''_{22}	C''_{22}	C'_{22}	C'_{21}	C''_{21}	C_4^-	C_4^+	C_2	E	σ_{d2}	σ_{v2}	σ_{v1}	σ_{d1}	S_4^+	S_4^-	σ_h	i
i	i	S_4^-	S_4^+	σ_h	σ_{v1}	σ_{v2}	σ_{d1}	σ_{d2}	E	C_4^+	C_4^-	C_2	C'_{21}	C'_{22}	C''_{21}	C''_{22}
S_4^-	S_4^-	σ_h	i	S_4^+	σ_{d1}	σ_{d2}	σ_{v2}	σ_{v1}	C_4^+	C_2	E	C_4^-	C''_{21}	C''_{22}	C'_{22}	C'_{21}
S_4^+	S_4^+	i	σ_h	S_4^-	σ_{d2}	σ_{d1}	σ_{v1}	σ_{v2}	C_4^-	E	C_2	C_4^+	C''_{22}	C''_{21}	C'_{21}	C'_{22}
σ_h	σ_h	S_4^+	S_4^-	i	σ_{v2}	σ_{v1}	σ_{d2}	σ_{d1}	C_2	C_4^-	C_4^+	E	C'_{22}	C'_{21}	C''_{22}	C''_{21}
σ_{v1}	σ_{v1}	σ_{d2}	σ_{d1}	σ_{v2}	i	σ_h	S_4^+	S_4^-	C'_{21}	C''_{22}	C''_{21}	C'_{22}	E	C_2	C_4^-	C_4^+
σ_{v2}	σ_{v2}	σ_{d1}	σ_{d2}	σ_{v1}	σ_h	i	S_4^-	S_4^+	C'_{22}	C''_{21}	C''_{22}	C'_{21}	C_2	E	C_4^+	C_4^-
σ_{d1}	σ_{d1}	σ_{v1}	σ_{v2}	σ_{d2}	S_4^-	S_4^+	i	σ_h	C''_{21}	C'_{21}	C'_{22}	C''_{22}	C_4^+	C_4^-	E	C_2
σ_{d2}	σ_{d2}	σ_{v2}	σ_{v1}	σ_{d1}	S_4^+	S_4^-	σ_h	i	C''_{22}	C'_{22}	C'_{21}	C''_{21}	C_4^-	C_4^+	C_2	E

T **33**.3 Factor table

§ **16**–3, p. 70

$\mathbf{D}_{4h}$	E	C_4^+	C_4^-	C_2	C'_{21}	C'_{22}	C''_{21}	C''_{22}	i	S_4^-	S_4^+	σ_h	σ_{v1}	σ_{v2}	σ_{d1}	σ_{d2}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_4^+	1	1	1	−1	1	1	1	−1	1	1	1	−1	1	1	1	−1
C_4^-	1	1	−1	1	−1	1	1	1	1	1	−1	1	−1	1	1	1
C_2	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1
C'_{21}	1	−1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	1
C'_{22}	1	1	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	−1
C''_{21}	1	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	−1	1
C''_{22}	1	1	−1	1	1	−1	−1	−1	1	1	−1	1	1	−1	−1	−1
i	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_4^-	1	1	1	−1	1	1	1	−1	1	1	1	−1	1	1	1	−1
S_4^+	1	1	−1	1	−1	1	1	1	1	1	−1	1	−1	1	1	1
σ_h	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1
σ_{v1}	1	−1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	1
σ_{v2}	1	1	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	−1
σ_{d1}	1	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	−1	1
σ_{d2}	1	1	−1	1	1	−1	−1	−1	1	1	−1	1	1	−1	−1	−1

T **33**.4 Character table

§ **16**–4, p. 71

$\mathbf{D}_{4h}$	E	$2C_4$	C_2	$2C_2'$	$2C_2''$	i	$2S_4$	σ_h	$2\sigma_v$	$2\sigma_d$	τ
A_{1g}	1	1	1	1	1	1	1	1	1	1	a
A_{2g}	1	1	1	−1	−1	1	1	1	−1	−1	a
B_{1g}	1	−1	1	1	−1	1	−1	1	1	−1	a
B_{2g}	1	−1	1	−1	1	1	−1	1	−1	1	a
E_g	2	0	−2	0	0	2	0	−2	0	0	a
A_{1u}	1	1	1	1	1	−1	−1	−1	−1	−1	a
A_{2u}	1	1	1	−1	−1	−1	−1	−1	1	1	a
B_{1u}	1	−1	1	1	−1	−1	1	−1	−1	1	a
B_{2u}	1	−1	1	−1	1	−1	1	−1	1	−1	a
E_u	2	0	−2	0	0	−2	0	2	0	0	a
$E_{1/2,g}$	2	$\sqrt{2}$	0	0	0	2	$\sqrt{2}$	0	0	0	c
$E_{3/2,g}$	2	$-\sqrt{2}$	0	0	0	2	$-\sqrt{2}$	0	0	0	c
$E_{1/2,u}$	2	$\sqrt{2}$	0	0	0	−2	$-\sqrt{2}$	0	0	0	c
$E_{3/2,u}$	2	$-\sqrt{2}$	0	0	0	−2	$\sqrt{2}$	0	0	0	c

T **33**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{4h}$	0	1	2	3
A_{1g}	$^{\square}1$		$x^2+y^2, {}^{\square}z^2$	
A_{2g}		R_z		
B_{1g}			$^{\square}x^2-y^2$	
B_{2g}			$^{\square}xy$	
E_g		(R_x, R_y)	$^{\square}(zx, yz)$	
A_{1u}				
A_{2u}		$^{\square}z$		$(x^2+y^2)z, {}^{\square}z^3$
B_{1u}				$^{\square}xyz$
B_{2u}				$^{\square}z(x^2-y^2)$
E_u		$^{\square}(x, y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^{\square}(xz^2, yz^2), {}^{\square}\{x(x^2-3y^2), y(3x^2-y^2)\}$

T **33**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{4h}$	$\langle\lvert j\,m\rangle\rvert$		ι	μ
A_{1g}	$\lvert 0\,0\rangle_+$		2	4
A_{2g}	$\lvert 4\,4\rangle_-$		2	4
B_{1g}	$\lvert 2\,2\rangle_+$		2	4
B_{2g}	$\lvert 2\,2\rangle_-$		2	4
E_g	$\langle\lvert 2\,1\rangle, -\lvert 2\,\bar{1}\rangle\rvert$		2	±4
A_{1u}	$\lvert 5\,4\rangle_-$		2	4
A_{2u}	$\lvert 1\,0\rangle_+$		2	4
B_{1u}	$\lvert 3\,2\rangle_-$		2	4
B_{2u}	$\lvert 3\,2\rangle_+$		2	4
E_u	$\langle\lvert 1\,1\rangle, \lvert 1\,\bar{1}\rangle\rvert$		2	±4
$E_{1/2,g}$	$\langle\lvert \tfrac{1}{2}\,\tfrac{1}{2}\rangle, \lvert \tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle\rvert$	$\langle\lvert \tfrac{3}{2}\,\tfrac{1}{2}\rangle, -\lvert \tfrac{3}{2}\,\overline{\tfrac{1}{2}}\rangle\rvert$	2	±4
$E_{3/2,g}$	$\langle\lvert \tfrac{3}{2}\,\overline{\tfrac{3}{2}}\rangle, -\lvert \tfrac{3}{2}\,\tfrac{3}{2}\rangle\rvert$	$\langle\lvert \tfrac{5}{2}\,\overline{\tfrac{3}{2}}\rangle, \lvert \tfrac{5}{2}\,\tfrac{3}{2}\rangle\rvert$	2	±4
$E_{1/2,u}$	$\langle\lvert \tfrac{1}{2}\,\tfrac{1}{2}\rangle, \lvert \tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle\rvert^{\bullet}$	$\langle\lvert \tfrac{3}{2}\,\tfrac{1}{2}\rangle, -\lvert \tfrac{3}{2}\,\overline{\tfrac{1}{2}}\rangle\rvert^{\bullet}$	2	±4
$E_{3/2,u}$	$\langle\lvert \tfrac{3}{2}\,\overline{\tfrac{3}{2}}\rangle, -\lvert \tfrac{3}{2}\,\tfrac{3}{2}\rangle\rvert^{\bullet}$	$\langle\lvert \tfrac{5}{2}\,\overline{\tfrac{3}{2}}\rangle, \lvert \tfrac{5}{2}\,\tfrac{3}{2}\rangle\rvert^{\bullet}$	2	±4

T **33**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_{4h}$	E_g	E_u	$E_{1/2,g}$	$E_{3/2,g}$	$E_{1/2,u}$	$E_{3/2,u}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_4^+	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$
C_4^-	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$
C_2	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$
C''_{21}	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$
C''_{22}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$
i	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$
S_4^-	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
S_4^+	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
σ_h	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$
σ_{v1}	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$
σ_{v2}	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$
σ_{d1}	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$
σ_{d2}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$

$\epsilon = \exp(2\pi\mathrm{i}/8)$

T **33**.8 Direct products of representations

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$\mathbf{D}_{4h}$	A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_g	A_{1u}	A_{2u}	B_{1u}	B_{2u}	E_u
A_{1g}	A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_g	A_{1u}	A_{2u}	B_{1u}	B_{2u}	E_u
A_{2g}		A_{1g}	B_{2g}	B_{1g}	E_g	A_{2u}	A_{1u}	B_{2u}	B_{1u}	E_u
B_{1g}			A_{1g}	A_{2g}	E_g	B_{1u}	B_{2u}	A_{1u}	A_{2u}	E_u
B_{2g}				A_{1g}	E_g	B_{2u}	B_{1u}	A_{2u}	A_{1u}	E_u
E_g					$A_{1g} \oplus \{A_{2g}\} \oplus B_{1g} \oplus B_{2g}$	E_u	E_u	E_u	E_u	$A_{1u} \oplus A_{2u} \oplus B_{1u} \oplus B_{2u}$
A_{1u}						A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_g
A_{2u}							A_{1g}	B_{2g}	B_{1g}	E_g
B_{1u}								A_{1g}	A_{2g}	E_g
B_{2u}									A_{1g}	E_g
E_u										$A_{1g} \oplus \{A_{2g}\} \oplus B_{1g} \oplus B_{2g}$

$\twoheadrightarrow$

T **33**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{4h}$	$E_{1/2,g}$	$E_{3/2,g}$	$E_{1/2,u}$	$E_{3/2,u}$
A_{1g}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{1/2,u}$	$E_{3/2,u}$
A_{2g}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{1/2,u}$	$E_{3/2,u}$
B_{1g}	$E_{3/2,g}$	$E_{1/2,g}$	$E_{3/2,u}$	$E_{1/2,u}$
B_{2g}	$E_{3/2,g}$	$E_{1/2,g}$	$E_{3/2,u}$	$E_{1/2,u}$
E_g	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$
A_{1u}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{1/2,g}$	$E_{3/2,g}$
A_{2u}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{1/2,g}$	$E_{3/2,g}$
B_{1u}	$E_{3/2,u}$	$E_{1/2,u}$	$E_{3/2,g}$	$E_{1/2,g}$
B_{2u}	$E_{3/2,u}$	$E_{1/2,u}$	$E_{3/2,g}$	$E_{1/2,g}$
E_u	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$
$E_{1/2,g}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_g$	$B_{1g} \oplus B_{2g} \oplus E_g$	$A_{1u} \oplus A_{2u} \oplus E_u$	$B_{1u} \oplus B_{2u} \oplus E_u$
$E_{3/2,g}$		$\{A_{1g}\} \oplus A_{2g} \oplus E_g$	$B_{1u} \oplus B_{2u} \oplus E_u$	$A_{1u} \oplus A_{2u} \oplus E_u$
$E_{1/2,u}$			$\{A_{1g}\} \oplus A_{2g} \oplus E_g$	$B_{1g} \oplus B_{2g} \oplus E_g$
$E_{3/2,u}$				$\{A_{1g}\} \oplus A_{2g} \oplus E_g$

T **33**.9 Subduction (descent of symmetry)

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$\mathbf{D}_{4h}$	$\mathbf{C}_{4h}$	$\mathbf{C}_{2h}$	$(\mathbf{C}_{2h})$	$(\mathbf{C}_{2h})$	$(\mathbf{C}_{4v})$
		C_2	C_2'	C_2''	
A_{1g}	A_g	A_g	A_g	A_g	A_1
A_{2g}	A_g	A_g	B_g	B_g	A_2
B_{1g}	B_g	A_g	A_g	B_g	B_1
B_{2g}	B_g	A_g	B_g	A_g	B_2
E_g	${}^1E_g \oplus {}^2E_g$	$2B_g$	$A_g \oplus B_g$	$A_g \oplus B_g$	E
A_{1u}	A_u	A_u	A_u	A_u	A_2
A_{2u}	A_u	A_u	B_u	B_u	A_1
B_{1u}	B_u	A_u	A_u	B_u	B_2
B_{2u}	B_u	A_u	B_u	A_u	B_1
E_u	${}^1E_u \oplus {}^2E_u$	$2B_u$	$A_u \oplus B_u$	$A_u \oplus B_u$	E
$E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{1/2}$
$E_{3/2,g}$	${}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{3/2}$
$E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{1/2}$
$E_{3/2,u}$	${}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{3/2}$

$\twoheadrightarrow$

T **33**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{4h}$	$(\mathbf{C}_{2v})$	$(\mathbf{C}_{2v})$	$(\mathbf{C}_{2v})$	$(\mathbf{C}_{2v})$	$\mathbf{D}_{2d}$	$(\mathbf{D}_{2d})$	$(\mathbf{D}_{2h})$	$(\mathbf{D}_{2h})$	$\mathbf{D}_4$
	C_2, σ_v	C_2, σ_d	$C'_{21}, \sigma_{v2}, \sigma_h$	$C''_{21}, \sigma_{d2}, \sigma_h$	C'_2	C''_2	C'_2	C''_2	
A_{1g}	A_1	A_1	A_1	A_1	A_1	A_1	A_g	A_g	A_1
A_{2g}	A_2	A_2	B_1	B_1	A_2	A_2	B_{1g}	B_{1g}	A_2
B_{1g}	A_1	A_2	A_1	B_1	B_1	B_2	A_g	B_{1g}	B_1
B_{2g}	A_2	A_1	B_1	A_1	B_2	B_1	B_{1g}	A_g	B_2
E_g	$B_1 \oplus B_2$	$B_1 \oplus B_2$	$A_2 \oplus B_2$	$A_2 \oplus B_2$	E	E	$B_{2g} \oplus B_{3g}$	$B_{2g} \oplus B_{3g}$	E
A_{1u}	A_2	A_2	A_2	A_2	B_1	B_1	A_u	A_u	A_1
A_{2u}	A_1	A_1	B_2	B_2	B_2	B_2	B_{1u}	B_{1u}	A_2
B_{1u}	A_2	A_1	A_2	B_2	A_1	A_2	A_u	B_{1u}	B_1
B_{2u}	A_1	A_2	B_2	A_2	A_2	A_1	B_{1u}	A_u	B_2
E_u	$B_1 \oplus B_2$	$B_1 \oplus B_2$	$A_1 \oplus B_1$	$A_1 \oplus B_1$	E	E	$B_{2u} \oplus B_{3u}$	$B_{2u} \oplus B_{3u}$	E
$E_{1/2,g}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2,g}$	$E_{1/2}$
$E_{3/2,g}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2,g}$	$E_{1/2,g}$	$E_{3/2}$
$E_{1/2,u}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2,u}$	$E_{1/2,u}$	$E_{1/2}$
$E_{3/2,u}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2,u}$	$E_{1/2,u}$	$E_{3/2}$

$\twoheadrightarrow$

T **33**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{4h}$	$(\mathbf{D}_2)$	$(\mathbf{D}_2)$	$\mathbf{S}_4$	$\mathbf{C}_s$	$(\mathbf{C}_s)$	$(\mathbf{C}_s)$
	C'_2	C''_2		σ_h	σ_v	σ_d
A_{1g}	A	A	A	A'	A'	A'
A_{2g}	B_1	B_1	A	A'	A''	A''
B_{1g}	A	B_1	B	A'	A'	A''
B_{2g}	B_1	A	B	A'	A''	A'
E_g	$B_2 \oplus B_3$	$B_2 \oplus B_3$	${}^1E \oplus {}^2E$	$2A''$	$A' \oplus A''$	$A' \oplus A''$
A_{1u}	A	A	B	A''	A''	A''
A_{2u}	B_1	B_1	B	A''	A'	A'
B_{1u}	A	B_1	A	A''	A''	A'
B_{2u}	B_1	A	A	A''	A'	A''
E_u	$B_2 \oplus B_3$	$B_2 \oplus B_3$	${}^1E \oplus {}^2E$	$2A'$	$A' \oplus A''$	$A' \oplus A''$
$E_{1/2,g}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2,g}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{1/2,u}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2,u}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

$\twoheadrightarrow$

T **33**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{4h}$	$\mathbf{C}_i$	$\mathbf{C}_4$	$\mathbf{C}_2$	$(\mathbf{C}_2)$	$(\mathbf{C}_2)$
			C_2	C_2'	C_2''
A_{1g}	A_g	A	A	A	A
A_{2g}	A_g	A	A	B	B
B_{1g}	A_g	B	A	A	B
B_{2g}	A_g	B	A	B	A
E_g	$2A_g$	${}^1E \oplus {}^2E$	$2B$	$A \oplus B$	$A \oplus B$
A_{1u}	A_u	A	A	A	A
A_{2u}	A_u	A	A	B	B
B_{1u}	A_u	B	A	A	B
B_{2u}	A_u	B	A	B	A
E_u	$2A_u$	${}^1E \oplus {}^2E$	$2B$	$A \oplus B$	$A \oplus B$
$E_{1/2,g}$	$2A_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2,g}$	$2A_{1/2,g}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{1/2,u}$	$2A_{1/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2,u}$	$2A_{1/2,u}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **33**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{4h}$
$4n$	$(n+1)\, A_{1g} \oplus n\,(A_{2g} \oplus B_{1g} \oplus B_{2g} \oplus 2E_g)$
$4n+1$	$n\,(A_{1u} \oplus B_{1u} \oplus B_{2u} \oplus E_u) \oplus (n+1)(A_{2u} \oplus E_u)$
$4n+2$	$(n+1)(A_{1g} \oplus B_{1g} \oplus B_{2g} \oplus E_g) \oplus n\,(A_{2g} \oplus E_g)$
$4n+3$	$n\, A_{1u} \oplus (n+1)(A_{2u} \oplus B_{1u} \oplus B_{2u} \oplus 2E_u)$
$4n+\frac{1}{2}$	$(2n+1)\, E_{1/2,g} \oplus 2n\, E_{3/2,g}$
$4n+\frac{3}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g})$
$4n+\frac{5}{2}$	$(2n+1)\, E_{1/2,g} \oplus (2n+2)\, E_{3/2,g}$
$4n+\frac{7}{2}$	$(2n+2)(E_{1/2,g} \oplus E_{3/2,g})$

$n = 0, 1, 2, \ldots$

T **33**.11 Clebsch–Gordan coefficients

Use T **24**.11 ▪. § **16**–11, p. 83

$\overline{10}\,m2$	$\vert G\vert = 20$	$\vert C\vert = 8$	$\vert \widetilde{C}\vert = 13$	T **34**	p. 245	$\mathbf{D}_{5h}$

(1) Product forms: $\mathbf{D}_5 \otimes \mathbf{C}_s$, $\mathbf{C}_{5v} \otimes \mathbf{C}_s$.

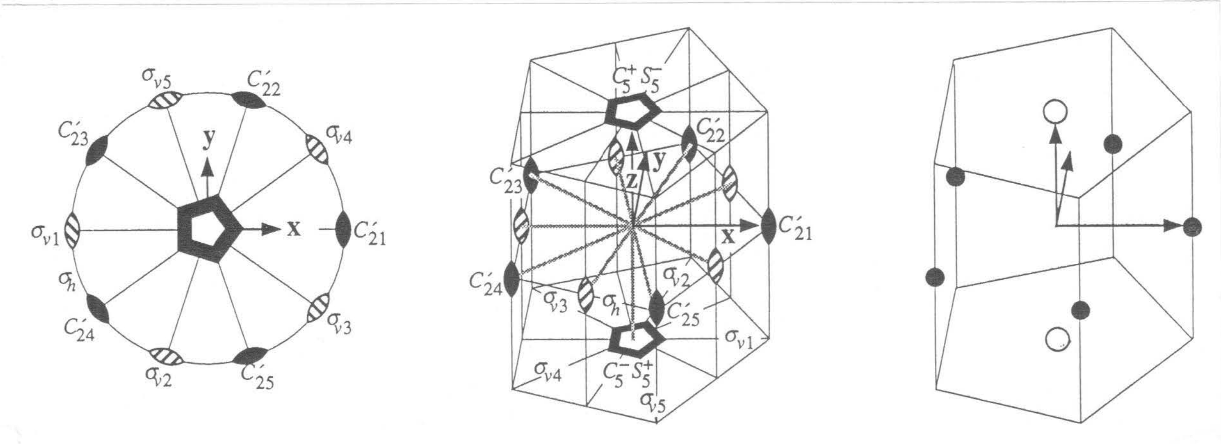
(2) Group chains: $\mathbf{D}_{10h} \supset \underline{\mathbf{D}_{5h}} \supset \underline{\mathbf{C}_{5h}}$, $\mathbf{D}_{10h} \supset \underline{\mathbf{D}_{5h}} \supset (\underline{\mathbf{C}_{5v}})$,
$\mathbf{D}_{10h} \supset \underline{\mathbf{D}_{5h}} \supset (\mathbf{C}_{2v})$, $\mathbf{D}_{10h} \supset \underline{\mathbf{D}_{5h}} \supset (\underline{\mathbf{D}_5})$.

(3) Operations of G: E, (C_5^+, C_5^-), (C_5^{2+}, C_5^{2-}), $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25})$,
σ_h, (S_5^+, S_5^-), (S_5^{2+}, S_5^{2-}), $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_5^+, C_5^-), $(\widetilde{C}_5^+, \widetilde{C}_5^-)$, (C_5^{2+}, C_5^{2-}), $(\widetilde{C}_5^{2+}, \widetilde{C}_5^{2-})$,
$(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, \widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24}, \widetilde{C}'_{25})$,
$(\sigma_h, \widetilde{\sigma}_h)$, (S_5^+, S_5^-), $(\widetilde{S}_5^+, \widetilde{S}_5^-)$, (S_5^{2+}, S_5^{2-}), $(\widetilde{S}_5^{2+}, \widetilde{S}_5^{2-})$,
$(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5}, \widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2}, \widetilde{\sigma}_{v3}, \widetilde{\sigma}_{v4}, \widetilde{\sigma}_{v5})$.

(5) Classes and representations: $|r| = 5$, $|\mathrm{i}| = 3$, $|I| = 8$, $|\widetilde{I}| = 5$.

F **34** See Chapter **15**, p. 65



Examples: $B_7H_7^{2-}$ (*closo*-borane), IF_7 (pentagonal bipyramid).

T **34**.1 Parameters
Use T **39**.1. § **16**–1, p. 68

T **34**.2 Multiplication table
Use T **39**.2. § **16**–2, p. 69

T **34**.3 Factor table
Use T **39**.3. § **16**–3, p. 70

T **34**.4 Character table

§ **16**–4, p. 71

$\mathbf{D}_{5h}$	E	$2C_5$	$2C_5^2$	$5C_2'$	σ_h	$2S_5$	$2S_5^2$	$5\sigma_v$	τ
A_1'	1	1	1	1	1	1	1	1	a
A_2'	1	1	1	-1	1	1	1	-1	a
E_1'	2	$2c_5^2$	$2c_5^4$	0	2	$2c_5^2$	$2c_5^4$	0	a
E_2'	2	$2c_5^4$	$2c_5^2$	0	2	$2c_5^4$	$2c_5^2$	0	a
A_1''	1	1	1	1	-1	-1	-1	-1	a
A_2''	1	1	1	-1	-1	-1	-1	1	a
E_1''	2	$2c_5^2$	$2c_5^4$	0	-2	$-2c_5^2$	$-2c_5^4$	0	a
E_2''	2	$2c_5^4$	$2c_5^2$	0	-2	$-2c_5^4$	$-2c_5^2$	0	a
$E_{1/2}$	2	$-2c_5^4$	$2c_5^2$	0	0	$2c_{10}$	$2c_{10}^3$	0	c
$E_{3/2}$	2	$-2c_5^2$	$2c_5^4$	0	0	$-2c_{10}^3$	$2c_{10}$	0	c
$E_{5/2}$	2	-2	2	0	0	0	0	0	c
$E_{7/2}$	2	$-2c_5^2$	$2c_5^4$	0	0	$2c_{10}^3$	$-2c_{10}$	0	c
$E_{9/2}$	2	$-2c_5^4$	$2c_5^2$	0	0	$-2c_{10}$	$-2c_{10}^3$	0	c

$c_n^m = \cos \frac{m}{n}\pi$

T **34**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{5h}$	0	1	2	3
A_1'	$^{\square}1$		$x^2+y^2, {}^{\square}z^2$	
A_2'		R_z		
E_1'		$^{\square}(x,y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^{\square}(xz^2, yz^2)$
E_2'			$^{\square}(xy, x^2-y^2)$	$^{\square}\{x(x^2-3y^2), y(3x^2-y^2)\}$
A_1''				
A_2''		$^{\square}z$		$(x^2+y^2)z, {}^{\square}z^3$
E_1''		(R_x, R_y)	$^{\square}(zx, yz)$	
E_2''				$^{\square}\{xyz, z(x^2-y^2)\}$

T **34**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{5h}$	$\langle\lvert j\,m\rangle\rvert$		ι	μ
A_1'	$\lvert 0\,0\rangle_+$	$\lvert 5\,5\rangle_-$	2	10
A_2'	$\lvert 5\,5\rangle_+$	$\lvert 10\,10\rangle_-$	2	10
E_1'	$\langle\lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle\rvert$	$\langle\lvert 4\,\overline{4}\rangle, -\lvert 4\,4\rangle\rvert$	2	± 10
E_2'	$\langle\lvert 2\,2\rangle, \lvert 2\,\overline{2}\rangle\rvert$	$\langle\lvert 3\,\overline{3}\rangle, -\lvert 3\,3\rangle\rvert$	2	± 10
A_1''	$\lvert 6\,5\rangle_+$	$\lvert 11\,10\rangle_-$	2	10
A_2''	$\lvert 1\,0\rangle_+$	$\lvert 6\,5\rangle_-$	2	10
E_1''	$\langle\lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle\rvert$	$\langle\lvert 5\,\overline{4}\rangle, \lvert 5\,4\rangle\rvert$	2	± 10
E_2''	$\langle\lvert 3\,2\rangle, -\lvert 3\,\overline{2}\rangle\rvert$	$\langle\lvert 4\,\overline{3}\rangle, \lvert 4\,3\rangle\rvert$	2	± 10
$E_{1/2}$	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	2	± 10
	$\langle\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle, \lvert \frac{9}{2}\,\frac{9}{2}\rangle\rvert^{\bullet}$	$\langle\lvert \frac{11}{2}\,\overline{\frac{9}{2}}\rangle, -\lvert \frac{11}{2}\,\frac{9}{2}\rangle\rvert^{\bullet}$	2	± 10
$E_{3/2}$	$\langle\lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle\rvert$	$\langle\lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle\rvert$	2	± 10
	$\langle\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle, \lvert \frac{7}{2}\,\frac{7}{2}\rangle\rvert^{\bullet}$	$\langle\lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle, -\lvert \frac{9}{2}\,\frac{7}{2}\rangle\rvert^{\bullet}$	2	± 10
$E_{5/2}$	$\langle\lvert \frac{5}{2}\,\frac{5}{2}\rangle, \lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle\rvert$	$\langle\lvert \frac{7}{2}\,\frac{5}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle\rvert$	2	± 10
	$\langle\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle, \lvert \frac{5}{2}\,\frac{5}{2}\rangle\rvert^{\bullet}$	$\langle\lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle, -\lvert \frac{7}{2}\,\frac{5}{2}\rangle\rvert^{\bullet}$	2	± 10
$E_{7/2}$	$\langle\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle, \lvert \frac{7}{2}\,\frac{7}{2}\rangle\rvert$	$\langle\lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle, -\lvert \frac{9}{2}\,\frac{7}{2}\rangle\rvert$	2	± 10
	$\langle\lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle\rvert^{\bullet}$	$\langle\lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle\rvert^{\bullet}$	2	± 10
$E_{9/2}$	$\langle\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle, \lvert \frac{9}{2}\,\frac{9}{2}\rangle\rvert$	$\langle\lvert \frac{11}{2}\,\overline{\frac{9}{2}}\rangle, -\lvert \frac{11}{2}\,\frac{9}{2}\rangle\rvert$	2	± 10
	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert^{\bullet}$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert^{\bullet}$	2	± 10

T **34**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_{5h}$	E'_1	E'_2	E''_1	E''_2
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_5^+	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
C_5^-	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
C_5^{2+}	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$
C_5^{2-}	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \overline{\delta} \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$
C'_{25}	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$
σ_h	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$
S_5^+	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \overline{\delta}^* & 0 \\ 0 & \overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$
S_5^-	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \overline{\delta} & 0 \\ 0 & \overline{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$
S_5^{2+}	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \overline{\delta} & 0 \\ 0 & \overline{\delta}^* \end{bmatrix}$
S_5^{2-}	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\delta}^* & 0 \\ 0 & \overline{\delta} \end{bmatrix}$
σ_{v1}	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$
σ_{v2}	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \overline{\delta}^* & 0 \end{bmatrix}$
σ_{v3}	$\begin{bmatrix} 0 & \overline{\delta} \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \overline{\epsilon}^* & 0 \end{bmatrix}$
σ_{v4}	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \overline{\epsilon} & 0 \end{bmatrix}$
σ_{v5}	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \overline{\delta} & 0 \end{bmatrix}$

$\delta = \exp(2\pi i/5)$, $\epsilon = \exp(4\pi i/5)$ $\twoheadrightarrow$

T **34**.7 Matrix representations *(cont.)*

$\mathbf{D}_{5h}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_5^+	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$
C_5^-	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$
C_5^{2+}	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$
C_5^{2-}	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta}^* \\ \mathrm{i}\bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta}^* \\ \mathrm{i}\bar{\delta} & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$
C'_{25}	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta} \\ \mathrm{i}\bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta} \\ \mathrm{i}\bar{\delta}^* & 0 \end{bmatrix}$
σ_h	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$
S_5^+	$\begin{bmatrix} \mathrm{i}\bar{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar{\epsilon}^* \end{bmatrix}$
S_5^-	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\delta}^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$
S_5^{2+}	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\delta}^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$
S_5^{2-}	$\begin{bmatrix} \mathrm{i}\bar{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\bar{\delta}^* \end{bmatrix}$
σ_{v1}	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$
σ_{v2}	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \bar{\delta} & 0 \end{bmatrix}$
σ_{v3}	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$
σ_{v4}	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$
σ_{v5}	$\begin{bmatrix} 0 & \bar{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \bar{\delta}^* & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/5)$, $\epsilon = \exp(4\pi\mathrm{i}/5)$

T **34**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{D}_{5h}$	A_1'	A_2'	E_1'	E_2'	A_1''	A_2''	E_1''	E_2''
A_1'	A_1'	A_2'	E_1'	E_2'	A_1''	A_2''	E_1''	E_2''
A_2'		A_1'	E_1'	E_2'	A_2''	A_1''	E_1''	E_2''
E_1'			$A_1' \oplus \{A_2'\} \oplus E_2'$	$E_1' \oplus E_2'$	E_1''	E_1''	$A_1'' \oplus A_2'' \oplus E_2''$	$E_1'' \oplus E_2''$
E_2'				$A_1' \oplus \{A_2'\} \oplus E_1'$	E_2''	E_2''	$E_1'' \oplus E_2''$	$A_1'' \oplus A_2'' \oplus E_1''$
A_1''					A_1'	A_2'	E_1'	E_2'
A_2''						A_1'	E_1'	E_2'
E_1''							$A_1' \oplus \{A_2'\} \oplus E_2'$	$E_1' \oplus E_2'$
E_2''								$A_1' \oplus \{A_2'\} \oplus E_1'$

$\twoheadrightarrow$

T **34**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{5h}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$
A_1'	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$
A_2'	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$
E_1'	$E_{7/2} \oplus E_{9/2}$	$E_{5/2} \oplus E_{9/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$
E_2'	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{9/2}$	$E_{3/2} \oplus E_{9/2}$	$E_{5/2} \oplus E_{7/2}$
A_1''	$E_{9/2}$	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
A_2''	$E_{9/2}$	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
E_1''	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{5/2} \oplus E_{9/2}$	$E_{7/2} \oplus E_{9/2}$
E_2''	$E_{5/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{9/2}$	$E_{1/2} \oplus E_{9/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{5/2}$
$E_{1/2}$	$\{A_1'\} \oplus A_2' \oplus E_1''$	$E_2' \oplus E_1''$	$E_2' \oplus E_2''$	$E_1' \oplus E_2''$	$E_1' \oplus A_1'' \oplus A_2''$
$E_{3/2}$		$\{A_1'\} \oplus A_2' \oplus E_2''$	$E_1' \oplus E_1''$	$E_2' \oplus A_1'' \oplus A_2''$	$E_1' \oplus E_2''$
$E_{5/2}$			$\{A_1'\} \oplus A_2' \oplus A_1'' \oplus A_2''$	$E_1' \oplus E_1''$	$E_2' \oplus E_2''$
$E_{7/2}$				$\{A_1'\} \oplus A_2' \oplus E_2''$	$E_2' \oplus E_1''$
$E_{9/2}$					$\{A_1'\} \oplus A_2' \oplus E_1''$

T **34**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_{5h}$	$\mathbf{C}_{5h}$	$(\mathbf{C}_{5v})$	$(\mathbf{C}_{2v})$	$(\mathbf{D}_5)$
			C_2', σ_v, σ_h	
A_1'	A'	A_1	A_1	A_1
A_2'	A'	A_2	B_1	A_2
E_1'	${}^1E_1' \oplus {}^2E_1'$	E_1	$A_1 \oplus B_1$	E_1
E_2'	${}^1E_2' \oplus {}^2E_2'$	E_2	$A_1 \oplus B_1$	E_2
A_1''	A''	A_2	A_2	A_1
A_2''	A''	A_1	B_2	A_2
E_1''	${}^1E_1'' \oplus {}^2E_1''$	E_1	$A_2 \oplus B_2$	E_1
E_2''	${}^1E_2'' \oplus {}^2E_2''$	E_2	$A_2 \oplus B_2$	E_2
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{3/2}$	$E_{1/2}$	$E_{3/2}$
$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$
$E_{7/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	$E_{3/2}$	$E_{1/2}$	$E_{3/2}$
$E_{9/2}$	${}^1E_{9/2} \oplus {}^2E_{9/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$

$\twoheadrightarrow$

T **34**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{5h}$	$\mathbf{C}_s$	$(\mathbf{C}_s)$	$\mathbf{C}_5$	$(\mathbf{C}_2)$
	σ_h	σ_v		
A'_1	A'	A'	A	A
A'_2	A'	A''	A	B
E'_1	$2A'$	$A' \oplus A''$	${}^1E_1 \oplus {}^2E_1$	$A \oplus B$
E'_2	$2A'$	$A' \oplus A''$	${}^1E_2 \oplus {}^2E_2$	$A \oplus B$
A''_1	A''	A''	A	A
A''_2	A''	A'	A	B
E''_1	$2A''$	$A' \oplus A''$	${}^1E_1 \oplus {}^2E_1$	$A \oplus B$
E''_2	$2A''$	$A' \oplus A''$	${}^1E_2 \oplus {}^2E_2$	$A \oplus B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{9/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **34**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{5h}$
$10n$	$(n+1)\,A'_1 \oplus n\,(A'_2 \oplus 2E'_1 \oplus 2E'_2 \oplus A''_1 \oplus A''_2 \oplus 2E''_1 \oplus 2E''_2)$
$10n+1$	$n\,(A'_1 \oplus A'_2 \oplus E'_1 \oplus 2E'_2 \oplus A''_1 \oplus 2E''_1 \oplus 2E''_2) \oplus (n+1)(E'_1 \oplus A''_2)$
$10n+2$	$(n+1)(A'_1 \oplus E'_2 \oplus E''_1) \oplus n\,(A'_2 \oplus 2E'_1 \oplus E'_2 \oplus A''_1 \oplus A''_2 \oplus E''_1 \oplus 2E''_2)$
$10n+3$	$n\,(A'_1 \oplus A'_2 \oplus E'_1 \oplus E'_2 \oplus A''_1 \oplus 2E''_1 \oplus E''_2) \oplus (n+1)(E'_1 \oplus E'_2 \oplus A''_2 \oplus E''_2)$
$10n+4$	$(n+1)(A'_1 \oplus E'_1 \oplus E'_2 \oplus E''_1 \oplus E''_2) \oplus n\,(A'_2 \oplus E'_1 \oplus E'_2 \oplus A''_1 \oplus A''_2 \oplus E''_1 \oplus E''_2)$
$10n+5$	$(n+1)(A'_1 \oplus A'_2 \oplus E'_1 \oplus E'_2 \oplus A''_2 \oplus E''_1 \oplus E''_2) \oplus n\,(E'_1 \oplus E'_2 \oplus A''_1 \oplus E''_1 \oplus E''_2)$
$10n+6$	$(n+1)(A'_1 \oplus 2E'_1 \oplus E'_2 \oplus A''_1 \oplus A''_2 \oplus E''_1 \oplus E''_2) \oplus n\,(A'_2 \oplus E'_2 \oplus E''_1 \oplus E''_2)$
$10n+7$	$(n+1)(A'_1 \oplus A'_2 \oplus E'_1 \oplus 2E'_2 \oplus A''_2 \oplus 2E''_1 \oplus E''_2) \oplus n\,(E'_1 \oplus A''_1 \oplus E''_2)$
$10n+8$	$(n+1)(A'_1 \oplus 2E'_1 \oplus 2E'_2 \oplus A''_1 \oplus A''_2 \oplus E''_1 \oplus 2E''_2) \oplus n\,(A'_2 \oplus E''_1)$
$10n+9$	$(n+1)(A'_1 \oplus A'_2 \oplus 2E'_1 \oplus 2E'_2 \oplus A''_2 \oplus 2E''_1 \oplus 2E''_2) \oplus n\,A''_1$
$10n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$
$10n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus 2n\,(E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$
$10n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus 2n\,(E_{7/2} \oplus E_{9/2})$
$10n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus 2n\,E_{9/2}$
$10n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$
$10n+\frac{11}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus (2n+2)\,E_{9/2}$
$10n+\frac{13}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus (2n+2)(E_{7/2} \oplus E_{9/2})$
$10n+\frac{15}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (2n+2)(E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$
$10n+\frac{17}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$
$10n+\frac{19}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$

$n = 0, 1, 2, \ldots$

T **34**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{D}_{5h}$

a'_2	e'_1	E'_1 1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	e'_2	E'_2 1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	e''_1	E''_1 1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	e''_2	E''_2 1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{1/2}$	$E_{1/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{3/2}$	$E_{3/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{5/2}$	$E_{5/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{7/2}$	$E_{7/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{9/2}$	$E_{9/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

e'_1	e'_1	A'_1 1	A'_2 1	E'_2 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e'_1	e'_2	E'_1 1	2	E'_2 1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e'_1	e''_1	A''_1 1	A''_2 1	E''_2 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e'_1	a''_1	E''_1 1	2
1	1	1	0
2	1	0	1

e'_1	a''_2	E''_1 1	2
1	1	1	0
2	1	0	$\bar{1}$

e'_1	e''_2	E''_1 1	2	E''_2 1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e'_1	$e_{1/2}$	$E_{7/2}$ 1	2	$E_{9/2}$ 1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e'_1	$e_{3/2}$	$E_{5/2}$ 1	2	$E_{9/2}$ 1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e'_1	$e_{5/2}$	$E_{3/2}$ 1	2	$E_{7/2}$ 1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e'_1	$e_{7/2}$	$E_{1/2}$ 1	2	$E_{5/2}$ 1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e'_1	$e_{9/2}$	$E_{1/2}$ 1	2	$E_{3/2}$ 1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e'_2	e'_2	A'_1 1	A'_2 1	E'_1 1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e'_2	a''_1	E''_2 1	2
1	1	1	0
2	1	0	1

e'_2	a''_2	E''_2 1	2
1	1	1	0
2	1	0	$\bar{1}$

$u = 2^{-1/2}$

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T **34**.11 Clebsch–Gordan coefficients *(cont.)*

e'_2	e''_1	E''_1		E''_2	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

e'_2	e''_2	A''_1	A''_2	E''_1	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e'_2	$e_{1/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e'_2	$e_{3/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e'_2	$e_{5/2}$	$E_{1/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e'_2	$e_{7/2}$	$E_{3/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e'_2	$e_{9/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

a''_1	e''_1	E'_1	
		1	2
1	1	1	0
1	2	0	1

a''_1	e''_2	E'_2	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{1/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{3/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{5/2}$	$E_{5/2}$	
		1	2
1	1	0	1
1	2	1	0

a''_1	$e_{7/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{9/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_2	e''_1	E'_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	e''_2	E'_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	$e_{1/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	$e_{3/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	$e_{5/2}$	$E_{5/2}$	
		1	2
1	1	0	$\bar{1}$
1	2	1	0

a''_2	$e_{7/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	$e_{9/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e''_1	e''_1	A'_1	A'_2	E'_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e''_1	e''_2	E'_1		E'_2	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e''_1	$e_{1/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e''_1	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

↠

T **34**.11 Clebsch–Gordan coefficients *(cont.)*

e''_1	$e_{5/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e''_1	$e_{7/2}$	$E_{5/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e''_1	$e_{9/2}$	$E_{7/2}$		$E_{9/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e''_2	e''_2	A'_1	A'_2	E'_1	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e''_2	$e_{1/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e''_2	$e_{3/2}$	$E_{3/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e''_2	$e_{5/2}$	$E_{1/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e''_2	$e_{7/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e''_2	$e_{9/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{1/2}$	A'_1	A'_2	E''_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{3/2}$	E'_2		E''_1	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	1	0	0

$e_{1/2}$	$e_{5/2}$	E'_2		E''_2	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{1/2}$	$e_{7/2}$	E'_1		E''_2	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{9/2}$	E'_1		A''_1	A''_2
		1	2	1	1
1	1	1	0	0	0
1	2	0	0	u	u
2	1	0	0	$\bar{u}$	u
2	2	0	1	0	0

$e_{3/2}$	$e_{3/2}$	A'_1	A'_2	E''_2	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{5/2}$	E'_1		E''_1	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{3/2}$	$e_{7/2}$	E'_2		A''_1	A''_2
		1	2	1	1
1	1	0	$\bar{1}$	0	0
1	2	0	0	u	u
2	1	0	0	$\bar{u}$	u
2	2	1	0	0	0

$e_{3/2}$	$e_{9/2}$	E'_1		E''_2	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

$\twoheadrightarrow$

T **34**.11 Clebsch–Gordan coefficients *(cont.)*

$e_{5/2}$	$e_{5/2}$	A_1'	A_2'	A_1''	A_2''
		1	1	1	1
1	1	0	0	u	u
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	$\bar{u}$	u

$e_{5/2}$	$e_{7/2}$	E_1'		E_1''	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{5/2}$	$e_{9/2}$	E_2'		E_2''	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{7/2}$	$e_{7/2}$	A_1'	A_2'	E_2''	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{7/2}$	$e_{9/2}$	E_2'		E_1''	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

$e_{9/2}$	$e_{9/2}$	A_1'	A_2'	E_1''	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

(1) Product forms: $\mathbf{D}_6 \otimes \mathbf{C}_i$, $\mathbf{D}_6 \otimes \mathbf{C}_s$, $\mathbf{C}_{6v} \otimes \mathbf{C}_s$.

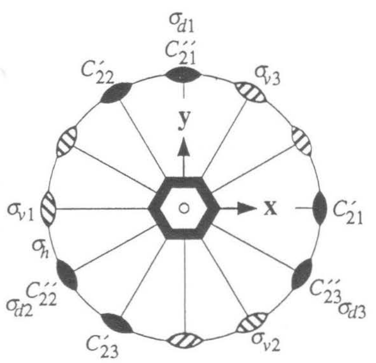
(2) Group chains: $\mathbf{D}_{6h} \supset \underline{\mathbf{C}_{6h}}$, $\mathbf{D}_{6h} \supset (\underline{\mathbf{C}_{6v}})$, $\mathbf{D}_{6h} \supset (\underline{\mathbf{D}_{3d}})$,
$\mathbf{D}_{6h} \supset \underline{\mathbf{D}_{3h}}$, $\mathbf{D}_{6h} \supset (\mathbf{D}_{2h})$, $\mathbf{D}_{6h} \supset \underline{\mathbf{D}_6}$.

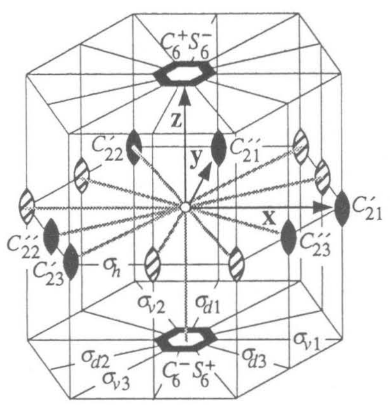
(3) Operations of G: E, (C_6^+, C_6^-), (C_3^+, C_3^-), C_2, $(C'_{21}, C'_{22}, C'_{23})$, $(C''_{21}, C''_{22}, C''_{23})$,
i, (S_3^-, S_3^+), (S_6^-, S_6^+), σ_h, $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3})$, $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3})$.

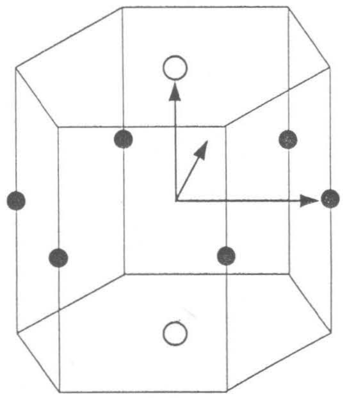
(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_6^+, C_6^-), $(\widetilde{C}_6^+, \widetilde{C}_6^-)$, (C_3^+, C_3^-), $(\widetilde{C}_3^+, \widetilde{C}_3^-)$, $(C_2, \widetilde{C}_2)$,
$(C'_{21}, C'_{22}, C'_{23}, \widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23})$, $(C''_{21}, C''_{22}, C''_{23}, \widetilde{C}''_{21}, \widetilde{C}''_{22}, \widetilde{C}''_{23})$,
i, $\widetilde{\imath}$, (S_3^-, S_3^+), $(\widetilde{S}_3^-, \widetilde{S}_3^+)$, (S_6^-, S_6^+), $(\widetilde{S}_6^-, \widetilde{S}_6^+)$, $(\sigma_h, \widetilde{\sigma}_h)$,
$(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3})$, $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2}, \widetilde{\sigma}_{v3})$.

(5) Classes and representations: $|r| = 6$, $|\mathrm{i}| = 6$, $|I| = 12$, $|\widetilde{I}| = 6$.

F **35** See Chapter **15**, p. 65







Examples: Benzene C_6H_6, C_6Cl_6, bis-benzene chromium $Cr(C_6H_6)_2$.

T **35**.0 Subgroup elements

§ **16**–0, p. 68

$\mathbf{D}_{6h}$	$\mathbf{C}_{6h}$	$\mathbf{C}_{3h}$	$\mathbf{C}_{2h}$	$\mathbf{C}_{6v}$	$\mathbf{C}_{3v}^{A}$	$\mathbf{C}_{3v}^{B}$	$\mathbf{C}_{2v}$	$\mathbf{D}_{3d}$	$\mathbf{D}_{3h}$	$\mathbf{D}_{2h}$	$\mathbf{D}_{6}$	$\mathbf{D}_{3}$	$\mathbf{D}_{2}$	$\mathbf{S}_{6}$	$\mathbf{C}_{s}$	$\mathbf{C}_{i}$	$\mathbf{C}_{6}$	$\mathbf{C}_{3}$	$\mathbf{C}_{2}$
E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
C_6^+	C_6^+			C_6^+							C_6^+						C_6^+		
C_6^-	C_6^-			C_6^-							C_6^-						C_6^-		
C_3^+	C_3^+	C_3^+		C_3^+	C_3^+	C_3^+		C_3^+	C_3^+		C_3^+	C_3^+		C_3^+			C_3^+	C_3^+	
C_3^-	C_3^-	C_3^-		C_3^-	C_3^-	C_3^-		C_3^-	C_3^-		C_3^-	C_3^-		C_3^-			C_3^-	C_3^-	
C_2	C_2		C_2	C_2			C_2			C_{2z}	C_2		C_{2z}				C_2		C_2
C'_{21}								C'_{21}	C'_{21}	C_{2x}	C'_{21}	C'_{21}	C_{2x}						
C'_{22}								C'_{22}	C'_{22}		C'_{22}	C'_{22}							
C'_{23}								C'_{23}	C'_{23}		C'_{23}	C'_{23}							
C''_{21}										C_{2y}	C''_{21}		C_{2y}						
C''_{22}											C''_{22}								
C''_{23}											C''_{23}								
i	i		i					i		i				i		i			
S_3^-	S_3^-	S_3^-							S_3^-										
S_3^+	S_3^+	S_3^+							S_3^+										
S_6^-	S_6^-							S_6^-						S_6^-					
S_6^+	S_6^+							S_6^+						S_6^+					
σ_h	σ_h	σ_h	σ_h						σ_h	σ_z					σ_h				
σ_{d1}				σ_{d1}		σ_{v1}	σ_x	σ_{d1}		σ_x									
σ_{d2}				σ_{d2}		σ_{v2}		σ_{d2}											
σ_{d3}				σ_{d3}		σ_{v3}		σ_{d3}											
σ_{v1}				σ_{v1}	σ_{v1}		σ_y		σ_{v1}	σ_y									
σ_{v2}				σ_{v2}	σ_{v2}				σ_{v2}										
σ_{v3}				σ_{v3}	σ_{v3}				σ_{v3}										

T **35**.1 Parameters

§ **16**–1, p. 68

$\mathbf{D}_{6h}$		α	β	γ	ϕ	$\mathbf{n}$	λ	$\mathbf{\Lambda}$
E	i	0	0	0	0	$(0\ \ 0\ \ 0)$	$[\![\ 1,$	$(0\ \ 0\ \ 0)]\!]$
C_6^+	S_3^-	0	0	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$(0\ \ 0\ \ 1)$	$[\![\frac{\sqrt{3}}{2},$	$(0\ \ 0\ \ \frac{1}{2})]\!]$
C_6^-	S_3^+	0	0	$-\frac{\pi}{3}$	$\frac{\pi}{3}$	$(0\ \ 0\ -1)$	$[\![\frac{\sqrt{3}}{2},$	$(0\ \ 0\ -\frac{1}{2})]\!]$
C_3^+	S_6^-	0	0	$\frac{2\pi}{3}$	$\frac{2\pi}{3}$	$(0\ \ 0\ \ 1)$	$[\![\ \frac{1}{2},$	$(0\ \ 0\ \ \frac{\sqrt{3}}{2})]\!]$
C_3^-	S_6^+	0	0	$-\frac{2\pi}{3}$	$\frac{2\pi}{3}$	$(0\ \ 0\ -1)$	$[\![\ \frac{1}{2},$	$(0\ \ 0\ -\frac{\sqrt{3}}{2})]\!]$
C_2	σ_h	0	0	π	π	$(0\ \ 0\ \ 1)$	$[\![\ 0,$	$(0\ \ 0\ \ 1)]\!]$
C'_{21}	σ_{d1}	0	π	π	π	$(1\ \ 0\ \ 0)$	$[\![\ 0,$	$(1\ \ 0\ \ 0)]\!]$
C'_{22}	σ_{d2}	0	π	$-\frac{\pi}{3}$	π	$(-\frac{1}{2}\ \ \frac{\sqrt{3}}{2}\ \ 0)$	$[\![\ 0,$	$(-\frac{1}{2}\ \ \frac{\sqrt{3}}{2}\ \ 0)]\!]$
C'_{23}	σ_{d3}	0	π	$\frac{\pi}{3}$	π	$(-\frac{1}{2}\ -\frac{\sqrt{3}}{2}\ \ 0)$	$[\![\ 0,$	$(-\frac{1}{2}\ -\frac{\sqrt{3}}{2}\ \ 0)]\!]$
C''_{21}	σ_{v1}	0	π	0	π	$(0\ \ 1\ \ 0)$	$[\![\ 0,$	$(0\ \ 1\ \ 0)]\!]$
C''_{22}	σ_{v2}	0	π	$\frac{2\pi}{3}$	π	$(-\frac{\sqrt{3}}{2}\ -\frac{1}{2}\ \ 0)$	$[\![\ 0,$	$(-\frac{\sqrt{3}}{2}\ -\frac{1}{2}\ \ 0)]\!]$
C''_{23}	σ_{v3}	0	π	$-\frac{2\pi}{3}$	π	$(\frac{\sqrt{3}}{2}\ -\frac{1}{2}\ \ 0)$	$[\![\ 0,$	$(\frac{\sqrt{3}}{2}\ -\frac{1}{2}\ \ 0)]\!]$

T **35**.2 Multiplication table

§ **16**–2, p. 69

$\mathbf{D}_{6h}$	E	C_6^+	C_6^-	C_3^+	C_3^-	C_2	C'_{21}	C'_{22}	C'_{23}	C''_{21}	C''_{22}	C''_{23}	i	S_3^-	S_3^+	S_6^-	S_6^+	σ_h	σ_{d1}	σ_{d2}	σ_{d3}	σ_{v1}	σ_{v2}	σ_{v3}
E	E	C_6^+	C_6^-	C_3^+	C_3^-	C_2	C'_{21}	C'_{22}	C'_{23}	C''_{21}	C''_{22}	C''_{23}	i	S_3^-	S_3^+	S_6^-	S_6^+	σ_h	σ_{d1}	σ_{d2}	σ_{d3}	σ_{v1}	σ_{v2}	σ_{v3}
C_6^+	C_6^+	C_3^+	E	C_2	C_6^-	C_3^-	C''_{22}	C''_{23}	C''_{21}	C'_{22}	C'_{23}	C'_{21}	S_3^-	S_6^-	i	σ_h	S_3^+	S_6^+	σ_{v2}	σ_{v3}	σ_{v1}	σ_{d2}	σ_{d3}	σ_{d1}
C_6^-	C_6^-	E	C_3^-	C_6^+	C_2	C_3^+	C''_{23}	C''_{21}	C''_{22}	C'_{23}	C'_{21}	C'_{22}	S_3^+	i	S_6^+	S_3^-	σ_h	S_6^-	σ_{v3}	σ_{v1}	σ_{v2}	σ_{d3}	σ_{d1}	σ_{d2}
C_3^+	C_3^+	C_2	C_6^+	C_3^-	E	C_6^-	C'_{23}	C'_{21}	C'_{22}	C''_{23}	C''_{21}	C''_{22}	S_6^-	σ_h	S_3^-	S_6^+	i	S_3^+	σ_{d3}	σ_{d1}	σ_{d2}	σ_{v3}	σ_{v1}	σ_{v2}
C_3^-	C_3^-	C_6^-	C_2	E	C_3^+	C_6^+	C'_{22}	C'_{23}	C'_{21}	C''_{22}	C''_{23}	C''_{21}	S_6^+	S_3^+	σ_h	i	S_6^-	S_3^-	σ_{d2}	σ_{d3}	σ_{d1}	σ_{v2}	σ_{v3}	σ_{v1}
C_2	C_2	C_3^-	C_3^+	C_6^-	C_6^+	E	C''_{21}	C''_{22}	C''_{23}	C'_{21}	C'_{22}	C'_{23}	σ_h	S_6^+	S_6^-	S_3^+	S_3^-	i	σ_{v1}	σ_{v2}	σ_{v3}	σ_{d1}	σ_{d2}	σ_{d3}
C'_{21}	C'_{21}	C''_{23}	C''_{22}	C'_{22}	C'_{23}	C''_{21}	E	C_3^+	C_3^-	C_2	C_6^-	C_6^+	σ_{d1}	σ_{v3}	σ_{v2}	σ_{d2}	σ_{d3}	σ_{v1}	i	S_6^-	S_6^+	σ_h	S_3^+	S_3^-
C'_{22}	C'_{22}	C''_{21}	C''_{23}	C'_{23}	C'_{21}	C''_{22}	C_3^-	E	C_3^+	C_6^+	C_2	C_6^-	σ_{d2}	σ_{v1}	σ_{v3}	σ_{d3}	σ_{d1}	σ_{v2}	S_6^+	i	S_6^-	S_3^-	σ_h	S_3^+
C'_{23}	C'_{23}	C''_{22}	C''_{21}	C'_{21}	C'_{22}	C''_{23}	C_3^+	C_3^-	E	C_6^-	C_6^+	C_2	σ_{d3}	σ_{v2}	σ_{v1}	σ_{d1}	σ_{d2}	σ_{v3}	S_6^-	S_6^+	i	S_3^+	S_3^-	σ_h
C''_{21}	C''_{21}	C'_{23}	C'_{22}	C''_{22}	C''_{23}	C'_{21}	C_2	C_6^-	C_6^+	E	C_3^+	C_3^-	σ_{v1}	σ_{d3}	σ_{d2}	σ_{v2}	σ_{v3}	σ_{d1}	σ_h	S_3^+	S_3^-	i	S_6^-	S_6^+
C''_{22}	C''_{22}	C'_{21}	C'_{23}	C''_{23}	C''_{21}	C'_{22}	C_6^+	C_2	C_6^-	C_3^-	E	C_3^+	σ_{v2}	σ_{d1}	σ_{d3}	σ_{v3}	σ_{v1}	σ_{d2}	S_3^-	σ_h	S_3^+	S_6^+	i	S_6^-
C''_{23}	C''_{23}	C'_{22}	C'_{21}	C''_{21}	C''_{22}	C'_{23}	C_6^-	C_6^+	C_2	C_3^+	C_3^-	E	σ_{v3}	σ_{d2}	σ_{d1}	σ_{v1}	σ_{v2}	σ_{d3}	S_3^+	S_3^-	σ_h	S_6^-	S_6^+	i
i	i	S_3^-	S_3^+	S_6^-	S_6^+	σ_h	σ_{d1}	σ_{d2}	σ_{d3}	σ_{v1}	σ_{v2}	σ_{v3}	E	C_6^+	C_6^-	C_3^+	C_3^-	C_2	C'_{21}	C'_{22}	C'_{23}	C''_{21}	C''_{22}	C''_{23}
S_3^-	S_3^-	S_6^-	i	σ_h	S_3^+	S_6^+	σ_{v2}	σ_{v3}	σ_{v1}	σ_{d2}	σ_{d3}	σ_{d1}	C_6^+	C_3^+	E	C_2	C_6^-	C_3^-	C''_{22}	C''_{23}	C''_{21}	C'_{22}	C'_{23}	C'_{21}
S_3^+	S_3^+	i	S_6^+	S_3^-	σ_h	S_6^-	σ_{v3}	σ_{v1}	σ_{v2}	σ_{d3}	σ_{d1}	σ_{d2}	C_6^-	E	C_3^-	C_6^+	C_2	C_3^+	C''_{23}	C''_{21}	C''_{22}	C'_{23}	C'_{21}	C'_{22}
S_6^-	S_6^-	σ_h	S_3^-	S_6^+	i	S_3^+	σ_{d3}	σ_{d1}	σ_{d2}	σ_{v3}	σ_{v1}	σ_{v2}	C_3^+	C_2	C_6^+	C_3^-	E	C_6^-	C'_{23}	C'_{21}	C'_{22}	C''_{23}	C''_{21}	C''_{22}
S_6^+	S_6^+	S_3^+	σ_h	i	S_6^-	S_3^-	σ_{d2}	σ_{d3}	σ_{d1}	σ_{v2}	σ_{v3}	σ_{v1}	C_3^-	C_6^-	C_2	E	C_3^+	C_6^+	C'_{22}	C'_{23}	C'_{21}	C''_{22}	C''_{23}	C''_{21}
σ_h	σ_h	S_6^+	S_6^-	S_3^+	S_3^-	i	σ_{v1}	σ_{v2}	σ_{v3}	σ_{d1}	σ_{d2}	σ_{d3}	C_2	C_3^-	C_3^+	C_6^-	C_6^+	E	C''_{21}	C''_{22}	C''_{23}	C'_{21}	C'_{22}	C'_{23}
σ_{d1}	σ_{d1}	σ_{v3}	σ_{v2}	σ_{d2}	σ_{d3}	σ_{v1}	i	S_6^-	S_6^+	σ_h	S_3^+	S_3^-	C'_{21}	C''_{23}	C''_{22}	C'_{22}	C'_{23}	C''_{21}	E	C_3^+	C_3^-	C_2	C_6^-	C_6^+
σ_{d2}	σ_{d2}	σ_{v1}	σ_{v3}	σ_{d3}	σ_{d1}	σ_{v2}	S_6^+	i	S_6^-	S_3^-	σ_h	S_3^+	C'_{22}	C''_{21}	C''_{23}	C'_{23}	C'_{21}	C''_{22}	C_3^-	E	C_3^+	C_6^+	C_2	C_6^-
σ_{d3}	σ_{d3}	σ_{v2}	σ_{v1}	σ_{d1}	σ_{d2}	σ_{v3}	S_6^-	S_6^+	i	S_3^+	S_3^-	σ_h	C'_{23}	C''_{22}	C''_{21}	C'_{21}	C'_{22}	C''_{23}	C_3^+	C_3^-	E	C_6^-	C_6^+	C_2
σ_{v1}	σ_{v1}	σ_{d3}	σ_{d2}	σ_{v2}	σ_{v3}	σ_{d1}	σ_h	S_3^+	S_3^-	i	S_6^-	S_6^+	C''_{21}	C'_{23}	C'_{22}	C''_{22}	C''_{23}	C'_{21}	C_2	C_6^-	C_6^+	E	C_3^+	C_3^-
σ_{v2}	σ_{v2}	σ_{d1}	σ_{d3}	σ_{v3}	σ_{v1}	σ_{d2}	S_3^-	σ_h	S_3^+	S_6^+	i	S_6^-	C''_{22}	C'_{21}	C'_{23}	C''_{23}	C''_{21}	C'_{22}	C_6^+	C_2	C_6^-	C_3^-	E	C_3^+
σ_{v3}	σ_{v3}	σ_{d2}	σ_{d1}	σ_{v1}	σ_{v2}	σ_{d3}	S_3^+	S_3^-	σ_h	S_6^-	S_6^+	i	C''_{23}	C'_{22}	C'_{21}	C''_{21}	C''_{22}	C'_{23}	C_6^-	C_6^+	C_2	C_3^+	C_3^-	E

T **35**.3 Factor table

§ **16**–3, p. 70

$\mathbf{D}_{6h}$	E	C_6^+	C_6^-	C_3^+	C_3^-	C_2	C'_{21}	C'_{22}	C'_{23}	C''_{21}	C''_{22}	C''_{23}	i	S_3^-	S_3^+	S_6^-	S_6^+	σ_h	σ_{d1}	σ_{d2}	σ_{d3}	σ_{v1}	σ_{v2}	σ_{v3}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_6^+	1	1	1	1	1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	1	1	1
C_6^-	1	1	1	1	−1	1	1	1	1	−1	−1	−1	1	1	1	1	−1	1	1	1	1	−1	−1	−1
C_3^+	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1
C_3^-	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1
C_2	1	−1	1	−1	1	−1	1	1	1	−1	−1	−1	1	−1	1	−1	1	−1	1	1	1	−1	−1	−1
C'_{21}	1	1	−1	−1	−1	−1	−1	1	1	1	1	−1	1	1	−1	−1	−1	−1	−1	1	1	1	1	−1
C'_{22}	1	1	−1	−1	−1	−1	1	−1	1	−1	1	1	1	1	−1	−1	−1	−1	1	−1	1	−1	1	1
C'_{23}	1	1	−1	−1	−1	−1	1	1	−1	1	−1	1	1	1	−1	−1	−1	−1	1	1	−1	1	−1	1
C''_{21}	1	−1	1	−1	−1	1	−1	−1	1	−1	1	1	1	−1	1	−1	−1	1	−1	−1	1	−1	1	1
C''_{22}	1	−1	1	−1	−1	1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	1	−1	−1	1	−1	1
C''_{23}	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1
i	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_3^-	1	1	1	1	1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	1	1	1
S_3^+	1	1	1	1	−1	1	1	1	1	−1	−1	−1	1	1	1	1	−1	1	1	1	1	−1	−1	−1
S_6^-	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1
S_6^+	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1
σ_h	1	−1	1	−1	1	−1	1	1	1	−1	−1	−1	1	−1	1	−1	1	−1	1	1	1	−1	−1	−1
σ_{d1}	1	1	−1	−1	−1	−1	−1	1	1	1	1	−1	1	1	−1	−1	−1	−1	−1	1	1	1	1	−1
σ_{d2}	1	1	−1	−1	−1	−1	1	−1	1	−1	1	1	1	1	−1	−1	−1	−1	1	−1	1	−1	1	1
σ_{d3}	1	1	−1	−1	−1	−1	1	1	−1	1	−1	1	1	1	−1	−1	−1	−1	1	1	−1	1	−1	1
σ_{v1}	1	−1	1	−1	−1	1	−1	−1	1	−1	1	1	1	−1	1	−1	−1	1	−1	−1	1	−1	1	1
σ_{v2}	1	−1	1	−1	−1	1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	1	−1	−1	1	−1	1
σ_{v3}	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1

T **35**.4 Character table

§ **16**–4, p. 71

$\mathbf{D}_{6h}$	E	$2C_6$	$2C_3$	C_2	$3C'_2$	$3C''_2$	i	$2S_3$	$2S_6$	σ_h	$3\sigma_d$	$3\sigma_v$	τ
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1	a
A_{2g}	1	1	1	1	−1	−1	1	1	1	1	−1	−1	a
B_{1g}	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	a
B_{2g}	1	−1	1	−1	−1	1	1	−1	1	−1	−1	1	a
E_{1g}	2	1	−1	−2	0	0	2	1	−1	−2	0	0	a
E_{2g}	2	−1	−1	2	0	0	2	−1	−1	2	0	0	a
A_{1u}	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	a
A_{2u}	1	1	1	1	−1	−1	−1	−1	−1	−1	1	1	a
B_{1u}	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	a
B_{2u}	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1	a
E_{1u}	2	1	−1	−2	0	0	−2	−1	1	2	0	0	a
E_{2u}	2	−1	−1	2	0	0	−2	1	1	−2	0	0	a
$E_{1/2,g}$	2	$\sqrt{3}$	1	0	0	0	2	$\sqrt{3}$	1	0	0	0	c
$E_{3/2,g}$	2	0	−2	0	0	0	2	0	−2	0	0	0	c
$E_{5/2,g}$	2	$-\sqrt{3}$	1	0	0	0	2	$-\sqrt{3}$	1	0	0	0	c
$E_{1/2,u}$	2	$\sqrt{3}$	1	0	0	0	−2	$-\sqrt{3}$	−1	0	0	0	c
$E_{3/2,u}$	2	0	−2	0	0	0	−2	0	2	0	0	0	c
$E_{5/2,u}$	2	$-\sqrt{3}$	1	0	0	0	−2	$\sqrt{3}$	−1	0	0	0	c

T **35**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{6h}$	0	1	2	3
A_{1g}	□1		x^2+y^2, □z^2	
A_{2g}		R_z		
B_{1g}				
B_{2g}				
E_{1g}		(R_x, R_y)	□(zx, yz)	
E_{2g}			□(xy, x^2-y^2)	
A_{1u}				
A_{2u}		□z		$(x^2+y^2)z$, □z^3
B_{1u}				□$x(x^2-3y^2)$
B_{2u}				□$y(3x^2-y^2)$
E_{1u}		□(x, y)		$\{x(x^2+y^2), y(x^2+y^2)\}$, □$(xz^2, yz^2)$
E_{2u}				□$\{xyz, z(x^2-y^2)\}$

T **35**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{6h}$	$\langle \lvert j\,m\rangle \rvert$		ι	μ
A_{1g}	$\lvert 0\,0\rangle_+$		2	6
A_{2g}	$\lvert 6\,6\rangle_-$		2	6
B_{1g}	$\lvert 4\,3\rangle_+$		2	6
B_{2g}	$\lvert 4\,3\rangle_-$		2	6
E_{1g}	$\langle \lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle \rvert$		2	± 6
E_{2g}	$\langle \lvert 2\,\overline{2}\rangle, -\lvert 2\,2\rangle \rvert$		2	± 6
A_{1u}	$\lvert 7\,6\rangle_-$		2	6
A_{2u}	$\lvert 1\,0\rangle_+$		2	6
B_{1u}	$\lvert 3\,3\rangle_-$		2	6
B_{2u}	$\lvert 3\,3\rangle_+$		2	6
E_{1u}	$\langle \lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle \rvert$		2	± 6
E_{2u}	$\langle \lvert 3\,\overline{2}\rangle, \lvert 3\,2\rangle \rvert$		2	± 6
$E_{1/2,g}$	$\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \rvert$	$\langle \lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle \rvert$	2	± 6
$E_{3/2,g}$	$\langle \lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle \rvert$	$\langle \lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle \rvert$	2	± 6
$E_{5/2,g}$	$\langle \lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle, \lvert \frac{5}{2}\,\frac{5}{2}\rangle \rvert$	$\langle \lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle, -\lvert \frac{7}{2}\,\frac{5}{2}\rangle \rvert$	2	± 6
$E_{1/2,u}$	$\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \rvert^\bullet$	$\langle \lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle \rvert^\bullet$	2	± 6
$E_{3/2,u}$	$\langle \lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle \rvert^\bullet$	$\langle \lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle \rvert^\bullet$	2	± 6
$E_{5/2,u}$	$\langle \lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle, \lvert \frac{5}{2}\,\frac{5}{2}\rangle \rvert^\bullet$	$\langle \lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle, -\lvert \frac{7}{2}\,\frac{5}{2}\rangle \rvert^\bullet$	2	± 6

T **35**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_{6h}$	E_{1g}	E_{2g}	E_{1u}	E_{2u}
E	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
C_6^+	$\begin{bmatrix}\bar{\epsilon} & 0\\0 & \bar{\epsilon}^*\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon} & 0\\0 & \bar{\epsilon}^*\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$
C_6^-	$\begin{bmatrix}\bar{\epsilon}^* & 0\\0 & \bar{\epsilon}\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon}^* & 0\\0 & \bar{\epsilon}\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$
C_3^+	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$
C_3^-	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$
C_2	$\begin{bmatrix}\bar{1} & 0\\0 & \bar{1}\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}\bar{1} & 0\\0 & \bar{1}\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
C'_{21}	$\begin{bmatrix}0 & \bar{1}\\\bar{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{1}\\\bar{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{1}\\\bar{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{1}\\\bar{1} & 0\end{bmatrix}$
C'_{22}	$\begin{bmatrix}0 & \bar{\epsilon}\\\bar{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}\\\bar{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}\\\bar{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}\\\bar{\epsilon}^* & 0\end{bmatrix}$
C'_{23}	$\begin{bmatrix}0 & \bar{\epsilon}^*\\\bar{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}^*\\\bar{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}^*\\\bar{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}^*\\\bar{\epsilon} & 0\end{bmatrix}$
C''_{21}	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{1}\\\bar{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{1}\\\bar{1} & 0\end{bmatrix}$
C''_{22}	$\begin{bmatrix}0 & \epsilon\\\epsilon^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}\\\bar{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon\\\epsilon^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}\\\bar{\epsilon}^* & 0\end{bmatrix}$
C''_{23}	$\begin{bmatrix}0 & \epsilon^*\\\epsilon & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}^*\\\bar{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon^*\\\epsilon & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}^*\\\bar{\epsilon} & 0\end{bmatrix}$
i	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}\bar{1} & 0\\0 & \bar{1}\end{bmatrix}$	$\begin{bmatrix}\bar{1} & 0\\0 & \bar{1}\end{bmatrix}$
S_3^-	$\begin{bmatrix}\bar{\epsilon} & 0\\0 & \bar{\epsilon}^*\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon} & 0\\0 & \bar{\epsilon}^*\end{bmatrix}$
S_3^+	$\begin{bmatrix}\bar{\epsilon}^* & 0\\0 & \bar{\epsilon}\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon}^* & 0\\0 & \bar{\epsilon}\end{bmatrix}$
S_6^-	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon}^* & 0\\0 & \bar{\epsilon}\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon}^* & 0\\0 & \bar{\epsilon}\end{bmatrix}$
S_6^+	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon} & 0\\0 & \bar{\epsilon}^*\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon} & 0\\0 & \bar{\epsilon}^*\end{bmatrix}$
σ_h	$\begin{bmatrix}\bar{1} & 0\\0 & \bar{1}\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}\bar{1} & 0\\0 & \bar{1}\end{bmatrix}$
σ_{d1}	$\begin{bmatrix}0 & \bar{1}\\\bar{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{1}\\\bar{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$
σ_{d2}	$\begin{bmatrix}0 & \bar{\epsilon}\\\bar{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}\\\bar{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon\\\epsilon^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon\\\epsilon^* & 0\end{bmatrix}$
σ_{d3}	$\begin{bmatrix}0 & \bar{\epsilon}^*\\\bar{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}^*\\\bar{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon^*\\\epsilon & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon^*\\\epsilon & 0\end{bmatrix}$
σ_{v1}	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{1}\\\bar{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{1}\\\bar{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$
σ_{v2}	$\begin{bmatrix}0 & \epsilon\\\epsilon^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}\\\bar{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}\\\bar{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon\\\epsilon^* & 0\end{bmatrix}$
σ_{v3}	$\begin{bmatrix}0 & \epsilon^*\\\epsilon & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}^*\\\bar{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}^*\\\bar{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon^*\\\epsilon & 0\end{bmatrix}$

$\epsilon = \exp(2\pi i/3)$ $\twoheadrightarrow$

T **35**.7 Matrix representations *(cont.)*

$\mathbf{D}_{6h}$	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_6^+	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$
C_6^-	$\begin{bmatrix} \mathrm{i}\bar{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar{\epsilon}^* \end{bmatrix}$
C_3^+	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$
C_3^-	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$
C_2	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$
C''_{21}	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$
C''_{22}	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$
C''_{23}	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$
i	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$
S_3^-	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar{\epsilon} \end{bmatrix}$
S_3^+	$\begin{bmatrix} \mathrm{i}\bar{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$
S_6^-	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
S_6^+	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
σ_h	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$
σ_{d1}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$
σ_{d2}	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$
σ_{d3}	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$
σ_{v1}	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$
σ_{v2}	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$
σ_{v3}	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$

$\epsilon = \exp(2\pi\mathrm{i}/3)$

T **35**.8 Direct products of representations § **16**–8, p. 81

$\mathbf{D}_{6h}$	A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_{1g}	E_{2g}
A_{1g}	A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_{1g}	E_{2g}
A_{2g}		A_{1g}	B_{2g}	B_{1g}	E_{1g}	E_{2g}
B_{1g}			A_{1g}	A_{2g}	E_{2g}	E_{1g}
B_{2g}				A_{1g}	E_{2g}	E_{1g}
E_{1g}					$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$	$B_{1g} \oplus B_{2g} \oplus E_{1g}$
E_{2g}						$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$

$\twoheadrightarrow$

T **35**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{6h}$	A_{1u}	A_{2u}	B_{1u}	B_{2u}	E_{1u}	E_{2u}
A_{1g}	A_{1u}	A_{2u}	B_{1u}	B_{2u}	E_{1u}	E_{2u}
A_{2g}	A_{2u}	A_{1u}	B_{2u}	B_{1u}	E_{1u}	E_{2u}
B_{1g}	B_{1u}	B_{2u}	A_{1u}	A_{2u}	E_{2u}	E_{1u}
B_{2g}	B_{2u}	B_{1u}	A_{2u}	A_{1u}	E_{2u}	E_{1u}
E_{1g}	E_{1u}	E_{1u}	E_{2u}	E_{2u}	$A_{1u} \oplus A_{2u} \oplus E_{2u}$	$B_{1u} \oplus B_{2u} \oplus E_{1u}$
E_{2g}	E_{2u}	E_{2u}	E_{1u}	E_{1u}	$B_{1u} \oplus B_{2u} \oplus E_{1u}$	$A_{1u} \oplus A_{2u} \oplus E_{2u}$
A_{1u}	A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_{1g}	E_{2g}
A_{2u}		A_{1g}	B_{2g}	B_{1g}	E_{1g}	E_{2g}
B_{1u}			A_{1g}	A_{2g}	E_{2g}	E_{1g}
B_{2u}				A_{1g}	E_{2g}	E_{1g}
E_{1u}					$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$	$B_{1g} \oplus B_{2g} \oplus E_{1g}$
E_{2u}						$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$

$\twoheadrightarrow$

T **35**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{6h}$	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$
A_{1g}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$
A_{2g}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$
B_{1g}	$E_{5/2,g}$	$E_{3/2,g}$	$E_{1/2,g}$
B_{2g}	$E_{5/2,g}$	$E_{3/2,g}$	$E_{1/2,g}$
E_{1g}	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{3/2,g} \oplus E_{5/2,g}$
E_{2g}	$E_{3/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$
A_{1u}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$
A_{2u}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$
B_{1u}	$E_{5/2,u}$	$E_{3/2,u}$	$E_{1/2,u}$
B_{2u}	$E_{5/2,u}$	$E_{3/2,u}$	$E_{1/2,u}$
E_{1u}	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{3/2,u} \oplus E_{5/2,u}$
E_{2u}	$E_{3/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$
$E_{1/2,g}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$	$E_{1g} \oplus E_{2g}$	$B_{1g} \oplus B_{2g} \oplus E_{2g}$
$E_{3/2,g}$		$\{A_{1g}\} \oplus A_{2g} \oplus B_{1g} \oplus B_{2g}$	$E_{1g} \oplus E_{2g}$
$E_{5/2,g}$			$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$

$\twoheadrightarrow$

T **35**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{6h}$	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$
A_{1g}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$
A_{2g}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$
B_{1g}	$E_{5/2,u}$	$E_{3/2,u}$	$E_{1/2,u}$
B_{2g}	$E_{5/2,u}$	$E_{3/2,u}$	$E_{1/2,u}$
E_{1g}	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{3/2,u} \oplus E_{5/2,u}$
E_{2g}	$E_{3/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$
A_{1u}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$
A_{2u}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$
B_{1u}	$E_{5/2,g}$	$E_{3/2,g}$	$E_{1/2,g}$
B_{2u}	$E_{5/2,g}$	$E_{3/2,g}$	$E_{1/2,g}$
E_{1u}	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{3/2,g} \oplus E_{5/2,g}$
E_{2u}	$E_{3/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$
$E_{1/2,g}$	$A_{1u} \oplus A_{2u} \oplus E_{1u}$	$E_{1u} \oplus E_{2u}$	$B_{1u} \oplus B_{2u} \oplus E_{2u}$
$E_{3/2,g}$	$E_{1u} \oplus E_{2u}$	$A_{1u} \oplus A_{2u} \oplus B_{1u} \oplus B_{2u}$	$E_{1u} \oplus E_{2u}$
$E_{5/2,g}$	$B_{1u} \oplus B_{2u} \oplus E_{2u}$	$E_{1u} \oplus E_{2u}$	$A_{1u} \oplus A_{2u} \oplus E_{1u}$
$E_{1/2,u}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$	$E_{1g} \oplus E_{2g}$	$B_{1g} \oplus B_{2g} \oplus E_{2g}$
$E_{3/2,u}$		$\{A_{1g}\} \oplus A_{2g} \oplus B_{1g} \oplus B_{2g}$	$E_{1g} \oplus E_{2g}$
$E_{5/2,u}$			$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$

T **35**.9 Subduction (descent of symmetry) § **16**–9, p. 82

$\mathbf{D}_{6h}$	$\mathbf{C}_{6h}$	$\mathbf{C}_{3h}$	$(\mathbf{C}_{6v})$	$(\mathbf{C}_{3v}^{A})$	$(\mathbf{C}_{3v}^{B})$	$(\mathbf{D}_{3d})$
				σ_v	σ_d	C_2', σ_d
A_{1g}	A_g	A'	A_1	A_1	A_1	A_{1g}
A_{2g}	A_g	A'	A_2	A_2	A_2	A_{2g}
B_{1g}	B_g	A''	B_2	A_2	A_1	A_{1g}
B_{2g}	B_g	A''	B_1	A_1	A_2	A_{2g}
E_{1g}	${}^1E_{1g} \oplus {}^2E_{1g}$	${}^1E'' \oplus {}^2E''$	E_1	E	E	E_g
E_{2g}	${}^1E_{2g} \oplus {}^2E_{2g}$	${}^1E' \oplus {}^2E'$	E_2	E	E	E_g
A_{1u}	A_u	A''	A_2	A_2	A_2	A_{1u}
A_{2u}	A_u	A''	A_1	A_1	A_1	A_{2u}
B_{1u}	B_u	A'	B_1	A_1	A_2	A_{1u}
B_{2u}	B_u	A'	B_2	A_2	A_1	A_{2u}
E_{1u}	${}^1E_{1u} \oplus {}^2E_{1u}$	${}^1E' \oplus {}^2E'$	E_1	E	E	E_u
E_{2u}	${}^1E_{2u} \oplus {}^2E_{2u}$	${}^1E'' \oplus {}^2E''$	E_2	E	E	E_u
$E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2,g}$
$E_{3/2,g}$	${}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$
$E_{5/2,g}$	${}^1E_{5/2,g} \oplus {}^2E_{5/2,g}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{5/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2,g}$
$E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2,u}$
$E_{3/2,u}$	${}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$
$E_{5/2,u}$	${}^1E_{5/2,u} \oplus {}^2E_{5/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$E_{5/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2,u}$

$\twoheadrightarrow$

T **35**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{6h}$	$(\mathbf{D}_{3d})$ C_2'', σ_v	$\mathbf{D}_{3h}$ C_2', σ_v	$(\mathbf{D}_{3h})$ C_2'', σ_d	$(\mathbf{D}_{2h})$ C_2, C_2', C_2''	$\mathbf{D}_6$	$\mathbf{S}_6$
A_{1g}	A_{1g}	A_1'	A_1'	A_g	A_1	A_g
A_{2g}	A_{2g}	A_2'	A_2'	B_{1g}	A_2	A_g
B_{1g}	A_{2g}	A_1''	A_2''	B_{3g}	B_1	A_g
B_{2g}	A_{1g}	A_2''	A_1''	B_{2g}	B_2	A_g
E_{1g}	E_g	E''	E''	$B_{2g} \oplus B_{3g}$	E_1	${}^1E_g \oplus {}^2E_g$
E_{2g}	E_g	E'	E'	$A_g \oplus B_{1g}$	E_2	${}^1E_g \oplus {}^2E_g$
A_{1u}	A_{1u}	A_1''	A_1''	A_u	A_1	A_u
A_{2u}	A_{2u}	A_2''	A_2''	B_{1u}	A_2	A_u
B_{1u}	A_{2u}	A_1'	A_2'	B_{3u}	B_1	A_u
B_{2u}	A_{1u}	A_2'	A_1'	B_{2u}	B_2	A_u
E_{1u}	E_u	E'	E'	$B_{2u} \oplus B_{3u}$	E_1	${}^1E_u \oplus {}^2E_u$
E_{2u}	E_u	E''	E''	$A_u \oplus B_{1u}$	E_2	${}^1E_u \oplus {}^2E_u$
$E_{1/2,g}$	$E_{1/2,g}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$
$E_{3/2,g}$	${}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2,g}$	$E_{3/2}$	$2A_{3/2,g}$
$E_{5/2,g}$	$E_{1/2,g}$	$E_{5/2}$	$E_{5/2}$	$E_{1/2,g}$	$E_{5/2}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$
$E_{1/2,u}$	$E_{1/2,u}$	$E_{5/2}$	$E_{5/2}$	$E_{1/2,u}$	$E_{1/2}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$
$E_{3/2,u}$	${}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2,u}$	$E_{3/2}$	$2A_{3/2,u}$
$E_{5/2,u}$	$E_{1/2,u}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2,u}$	$E_{5/2}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$

Other subgroups: $3\mathbf{C}_{2h}$, $3\mathbf{C}_{2v}$, $3\mathbf{C}_s$, $\mathbf{C}_i$ (see $\mathbf{D}_{2h}$); $2\mathbf{D}_3$, $\mathbf{D}_2$, $\mathbf{C}_6$, $\mathbf{C}_3$, $3\mathbf{C}_2$ (see $\mathbf{D}_6$).

T **35**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{6h}$
$6n$	$(n+1)\,A_{1g} \oplus n\,(A_{2g} \oplus B_{1g} \oplus B_{2g} \oplus 2E_{1g} \oplus 2E_{2g})$
$6n+1$	$n\,(A_{1u} \oplus B_{1u} \oplus B_{2u} \oplus E_{1u} \oplus 2E_{2u}) \oplus (n+1)(A_{2u} \oplus E_{1u})$
$6n+2$	$(n+1)(A_{1g} \oplus E_{1g} \oplus E_{2g}) \oplus n\,(A_{2g} \oplus B_{1g} \oplus B_{2g} \oplus E_{1g} \oplus E_{2g})$
$6n+3$	$n\,(A_{1u} \oplus E_{1u} \oplus E_{2u}) \oplus (n+1)(A_{2u} \oplus B_{1u} \oplus B_{2u} \oplus E_{1u} \oplus E_{2u})$
$6n+4$	$(n+1)(A_{1g} \oplus B_{1g} \oplus B_{2g} \oplus E_{1g} \oplus 2E_{2g}) \oplus n\,(A_{2g} \oplus E_{1g})$
$6n+5$	$n\,A_{1u} \oplus (n+1)(A_{2u} \oplus B_{1u} \oplus B_{2u} \oplus 2E_{1u} \oplus 2E_{2u})$
$6n+\frac{1}{2}$	$(2n+1)\,E_{1/2,g} \oplus 2n\,(E_{3/2,g} \oplus E_{5/2,g})$
$6n+\frac{3}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g}) \oplus 2n\,E_{5/2,g}$
$6n+\frac{5}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g})$
$6n+\frac{7}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g}) \oplus (2n+2)\,E_{5/2,g}$
$6n+\frac{9}{2}$	$(2n+1)\,E_{1/2,g} \oplus (2n+2)(E_{3/2,g} \oplus E_{5/2,g})$
$6n+\frac{11}{2}$	$(2n+2)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g})$

$n = 0, 1, 2, \ldots$

T **35**.11 Clebsch–Gordan coefficients

Use T **26**.11 ▪. § **16**–11, p. 83

(1) Product forms: $\mathbf{D}_7 \otimes \mathbf{C}_s$, $\mathbf{C}_{7v} \otimes \mathbf{C}_s$.

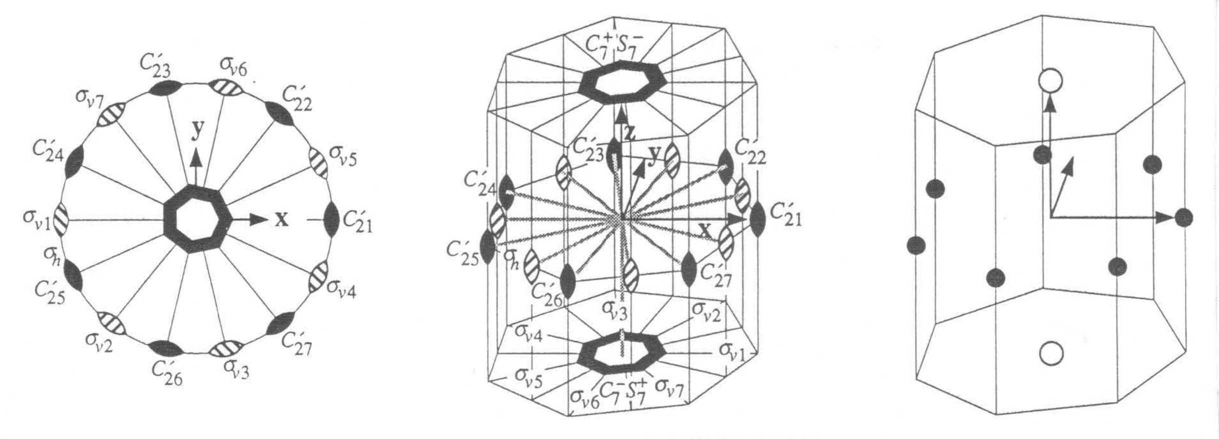
(2) Group chains: $\mathbf{D}_{7h} \supset \underline{\mathbf{C}_{7h}}$, $\mathbf{D}_{7h} \supset (\underline{\mathbf{C}_{7v}})$, $\mathbf{D}_{7h} \supset (\mathbf{C}_{2v})$, $\mathbf{D}_{7h} \supset (\underline{\mathbf{D}_7})$.

(3) Operations of G: E, (C_7^+, C_7^-), (C_7^{2+}, C_7^{2-}), (C_7^{3+}, C_7^{3-}), $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27})$,
σ_h, (S_7^+, S_7^-), (S_7^{2+}, S_7^{2-}), (S_7^{3+}, S_7^{3-}), $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5}, \sigma_{v6}, \sigma_{v7})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_7^+, C_7^-), $(\widetilde{C}_7^+, \widetilde{C}_7^-)$, (C_7^{2+}, C_7^{2-}), $(\widetilde{C}_7^{2+}, \widetilde{C}_7^{2-})$, (C_7^{3+}, C_7^{3-}), $(\widetilde{C}_7^{3+}, \widetilde{C}_7^{3-})$,
$(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27}, \widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24}, \widetilde{C}'_{25}, \widetilde{C}'_{26}, \widetilde{C}'_{27})$,
σ_h, $\widetilde{\sigma}_h$, (S_7^+, S_7^-), $(\widetilde{S}_7^+, \widetilde{S}_7^-)$, (S_7^{2+}, S_7^{2-}), $(\widetilde{S}_7^{2+}, \widetilde{S}_7^{2-})$, (S_7^{3+}, S_7^{3-}), $(\widetilde{S}_7^{3+}, \widetilde{S}_7^{3-})$,
$(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5}, \sigma_{v6}, \sigma_{v7}, \widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2}, \widetilde{\sigma}_{v3}, \widetilde{\sigma}_{v4}, \widetilde{\sigma}_{v5}, \widetilde{\sigma}_{v6}, \widetilde{\sigma}_{v7})$.

(5) Classes and representations: $|r| = 7$, $|\mathrm{i}| = 3$, $|I| = 10$, $|\widetilde{I}| = 7$.

F **36** See Chapter **15**, p. 65



Examples:

T **36**.1 Parameters

§ **16**–1, p. 68

$\mathbf{D}_{7h}$	α	β	γ	ϕ	$\mathbf{n}$	λ	$\mathbf{\Lambda}$
E	0	0	0	0	$(0\ \ 0\ \ 0)$	$\llbracket 1,$	$(0\ \ 0\ \ 0)\rrbracket$
C_7^+	0	0	$\frac{2\pi}{7}$	$\frac{2\pi}{7}$	$(0\ \ 0\ \ 1)$	$\llbracket c_7,$	$(0\ \ 0\ \ s_7)\rrbracket$
C_7^-	0	0	$-\frac{2\pi}{7}$	$\frac{2\pi}{7}$	$(0\ \ 0\ -1)$	$\llbracket c_7,$	$(0\ \ 0\ -s_7)\rrbracket$
C_7^{2+}	0	0	$\frac{4\pi}{7}$	$\frac{4\pi}{7}$	$(0\ \ 0\ \ 1)$	$\llbracket c_7^2,$	$(0\ \ 0\ \ s_7^2)\rrbracket$
C_7^{2-}	0	0	$-\frac{4\pi}{7}$	$\frac{4\pi}{7}$	$(0\ \ 0\ -1)$	$\llbracket c_7^2,$	$(0\ \ 0\ -s_7^2)\rrbracket$
C_7^{3+}	0	0	$\frac{6\pi}{7}$	$\frac{6\pi}{7}$	$(0\ \ 0\ \ 1)$	$\llbracket c_7^3,$	$(0\ \ 0\ \ s_7^3)\rrbracket$
C_7^{3-}	0	0	$-\frac{6\pi}{7}$	$\frac{6\pi}{7}$	$(0\ \ 0\ -1)$	$\llbracket c_7^3,$	$(0\ \ 0\ -s_7^3)\rrbracket$
C'_{21}	0	π	π	π	$(1\ \ 0\ \ 0)$	$\llbracket 0,$	$(1\ \ 0\ \ 0)\rrbracket$
C'_{22}	0	π	$\frac{3\pi}{7}$	π	$(c_7^2\ \ s_7^2\ \ 0)$	$\llbracket 0,$	$(c_7^2\ \ s_7^2\ \ 0)\rrbracket$
C'_{23}	0	π	$-\frac{\pi}{7}$	π	$(-c_7^3\ \ s_7^3\ \ 0)$	$\llbracket 0,$	$(-c_7^3\ \ s_7^3\ \ 0)\rrbracket$
C'_{24}	0	π	$-\frac{5\pi}{7}$	π	$(-c_7\ \ s_7\ \ 0)$	$\llbracket 0,$	$(-c_7\ \ s_7\ \ 0)\rrbracket$
C'_{25}	0	π	$\frac{5\pi}{7}$	π	$(-c_7\ -s_7\ \ 0)$	$\llbracket 0,$	$(-c_7\ -s_7\ \ 0)\rrbracket$
C'_{26}	0	π	$\frac{\pi}{7}$	π	$(-c_7^3\ -s_7^3\ \ 0)$	$\llbracket 0,$	$(-c_7^3\ -s_7^3\ \ 0)\rrbracket$
C'_{27}	0	π	$-\frac{3\pi}{7}$	π	$(c_7^2\ -s_7^2\ \ 0)$	$\llbracket 0,$	$(c_7^2\ -s_7^2\ \ 0)\rrbracket$
σ_h	0	0	π	π	$(0\ \ 0\ \ 1)$	$\llbracket 0,$	$(0\ \ 0\ \ 1)\rrbracket$
S_7^+	0	0	$-\frac{5\pi}{7}$	$\frac{5\pi}{7}$	$(0\ \ 0\ -1)$	$\llbracket s_7,$	$(0\ \ 0\ -c_7)\rrbracket$
S_7^-	0	0	$\frac{5\pi}{7}$	$\frac{5\pi}{7}$	$(0\ \ 0\ \ 1)$	$\llbracket s_7,$	$(0\ \ 0\ \ c_7)\rrbracket$
S_7^{2+}	0	0	$-\frac{3\pi}{7}$	$\frac{3\pi}{7}$	$(0\ \ 0\ -1)$	$\llbracket s_7^2,$	$(0\ \ 0\ -c_7^2)\rrbracket$
S_7^{2-}	0	0	$\frac{3\pi}{7}$	$\frac{3\pi}{7}$	$(0\ \ 0\ \ 1)$	$\llbracket s_7^2,$	$(0\ \ 0\ \ c_7^2)\rrbracket$
S_7^{3+}	0	0	$-\frac{\pi}{7}$	$\frac{\pi}{7}$	$(0\ \ 0\ -1)$	$\llbracket s_7^3,$	$(0\ \ 0\ -c_7^3)\rrbracket$
S_7^{3-}	0	0	$\frac{\pi}{7}$	$\frac{\pi}{7}$	$(0\ \ 0\ \ 1)$	$\llbracket s_7^3,$	$(0\ \ 0\ \ c_7^3)\rrbracket$
σ_{v1}	0	π	0	π	$(0\ \ 1\ \ 0)$	$\llbracket 0,$	$(0\ \ 1\ \ 0)\rrbracket$
σ_{v2}	0	π	$-\frac{4\pi}{7}$	π	$(-s_7^2\ \ c_7^2\ \ 0)$	$\llbracket 0,$	$(-s_7^2\ \ c_7^2\ \ 0)\rrbracket$
σ_{v3}	0	π	$\frac{6\pi}{7}$	π	$(-s_7^3\ -c_7^3\ \ 0)$	$\llbracket 0,$	$(-s_7^3\ -c_7^3\ \ 0)\rrbracket$
σ_{v4}	0	π	$\frac{2\pi}{7}$	π	$(-s_7\ -c_7\ \ 0)$	$\llbracket 0,$	$(-s_7\ -c_7\ \ 0)\rrbracket$
σ_{v5}	0	π	$-\frac{2\pi}{7}$	π	$(s_7\ -c_7\ \ 0)$	$\llbracket 0,$	$(s_7\ -c_7\ \ 0)\rrbracket$
σ_{v6}	0	π	$-\frac{6\pi}{7}$	π	$(s_7^3\ -c_7^3\ \ 0)$	$\llbracket 0,$	$(s_7^3\ -c_7^3\ \ 0)\rrbracket$
σ_{v7}	0	π	$\frac{4\pi}{7}$	π	$(s_7^2\ \ c_7^2\ \ 0)$	$\llbracket 0,$	$(s_7^2\ \ c_7^2\ \ 0)\rrbracket$

$c_n^m = \cos \frac{m}{n}\pi$, $s_n^m = \sin \frac{m}{n}\pi$

T **36**.2 Multiplication table

§ **16**–2, p. 69

$\mathbf{D}_{7h}$	E	C_7^+	C_7^-	C_7^{2+}	C_7^{2-}	C_7^{3+}	C_7^{3-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	σ_h	S_7^+	S_7^-	S_7^{2+}	S_7^{2-}	S_7^{3+}	S_7^{3-}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}
E	E	C_7^+	C_7^-	C_7^{2+}	C_7^{2-}	C_7^{3+}	C_7^{3-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	σ_h	S_7^+	S_7^-	S_7^{2+}	S_7^{2-}	S_7^{3+}	S_7^{3-}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}
C_7^+	C_7^+	C_7^{2+}	E	C_7^{3+}	C_7^-	C_7^{3-}	C_7^{2-}	C'_{25}	C'_{26}	C'_{27}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	S_7^+	S_7^{2+}	σ_h	S_7^{3+}	S_7^-	S_7^{3-}	S_7^{2-}	σ_{v5}	σ_{v6}	σ_{v7}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}
C_7^-	C_7^-	E	C_7^{2-}	C_7^+	C_7^{3-}	C_7^{2+}	C_7^{3+}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{21}	C'_{22}	C'_{23}	S_7^-	σ_h	S_7^{2-}	S_7^+	S_7^{3-}	S_7^{2+}	S_7^{3+}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}	σ_{v1}	σ_{v2}	σ_{v3}
C_7^{2+}	C_7^{2+}	C_7^{3+}	C_7^+	C_7^{3-}	E	C_7^{2-}	C_7^-	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{21}	S_7^{2+}	S_7^{3+}	S_7^+	S_7^{3-}	σ_h	S_7^{2-}	S_7^-	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}	σ_{v1}
C_7^{2-}	C_7^{2-}	C_7^-	C_7^{3-}	E	C_7^{3+}	C_7^+	C_7^{2+}	C'_{27}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	S_7^{2-}	S_7^-	S_7^{3-}	σ_h	S_7^{3+}	S_7^+	S_7^{2+}	σ_{v7}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}
C_7^{3+}	C_7^{3+}	C_7^{3-}	C_7^{2+}	C_7^{2-}	C_7^+	C_7^-	E	C'_{26}	C'_{27}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	S_7^{3+}	S_7^{3-}	S_7^{2+}	S_7^{2-}	S_7^+	S_7^-	σ_h	σ_{v6}	σ_{v7}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}
C_7^{3-}	C_7^{3-}	C_7^{2-}	C_7^{3+}	C_7^-	C_7^{2+}	E	C_7^+	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{21}	C'_{22}	S_7^{3-}	S_7^{2-}	S_7^{3+}	S_7^-	S_7^{2+}	σ_h	S_7^+	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}	σ_{v1}	σ_{v2}
C'_{21}	C'_{21}	C'_{24}	C'_{25}	C'_{27}	C'_{22}	C'_{23}	C'_{26}	E	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}	σ_{v1}	σ_{v4}	σ_{v5}	σ_{v7}	σ_{v2}	σ_{v3}	σ_{v6}	σ_h	S_7^{2-}	S_7^{3+}	S_7^+	S_7^-	S_7^{3-}	S_7^{2+}
C'_{22}	C'_{22}	C'_{25}	C'_{26}	C'_{21}	C'_{23}	C'_{24}	C'_{27}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}	σ_{v2}	σ_{v5}	σ_{v6}	σ_{v1}	σ_{v3}	σ_{v4}	σ_{v7}	S_7^{2+}	σ_h	S_7^{2-}	S_7^{3+}	S_7^+	S_7^-	S_7^{3-}
C'_{23}	C'_{23}	C'_{26}	C'_{27}	C'_{22}	C'_{24}	C'_{25}	C'_{21}	C_7^{3-}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-	σ_{v3}	σ_{v6}	σ_{v7}	σ_{v2}	σ_{v4}	σ_{v5}	σ_{v1}	S_7^{3-}	S_7^{2+}	σ_h	S_7^{2-}	S_7^{3+}	S_7^+	S_7^-
C'_{24}	C'_{24}	C'_{27}	C'_{21}	C'_{23}	C'_{25}	C'_{26}	C'_{22}	C_7^-	C_7^{3-}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}	C_7^+	σ_{v4}	σ_{v7}	σ_{v1}	σ_{v3}	σ_{v5}	σ_{v6}	σ_{v2}	S_7^-	S_7^{3-}	S_7^{2+}	σ_h	S_7^{2-}	S_7^{3+}	S_7^+
C'_{25}	C'_{25}	C'_{21}	C'_{22}	C'_{24}	C'_{26}	C'_{27}	C'_{23}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}	σ_{v5}	σ_{v1}	σ_{v2}	σ_{v4}	σ_{v6}	σ_{v7}	σ_{v3}	S_7^+	S_7^-	S_7^{3-}	S_7^{2+}	σ_h	S_7^{2-}	S_7^{3+}
C'_{26}	C'_{26}	C'_{22}	C'_{23}	C'_{25}	C'_{27}	C'_{21}	C'_{24}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}	E	C_7^{2-}	σ_{v6}	σ_{v2}	σ_{v3}	σ_{v5}	σ_{v7}	σ_{v1}	σ_{v4}	S_7^{3+}	S_7^+	S_7^-	S_7^{3-}	S_7^{2+}	σ_h	S_7^{2-}
C'_{27}	C'_{27}	C'_{23}	C'_{24}	C'_{26}	C'_{21}	C'_{22}	C'_{25}	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}	E	σ_{v7}	σ_{v3}	σ_{v4}	σ_{v6}	σ_{v1}	σ_{v2}	σ_{v5}	S_7^{2-}	S_7^{3+}	S_7^+	S_7^-	S_7^{3-}	S_7^{2+}	σ_h
σ_h	σ_h	S_7^+	S_7^-	S_7^{2+}	S_7^{2-}	S_7^{3+}	S_7^{3-}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}	E	C_7^+	C_7^-	C_7^{2+}	C_7^{2-}	C_7^{3+}	C_7^{3-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}
S_7^+	S_7^+	S_7^{2+}	σ_h	S_7^{3+}	S_7^-	S_7^{3-}	S_7^{2-}	σ_{v5}	σ_{v6}	σ_{v7}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	C_7^+	C_7^{2+}	E	C_7^{3+}	C_7^-	C_7^{3-}	C_7^{2-}	C'_{25}	C'_{26}	C'_{27}	C'_{21}	C'_{22}	C'_{23}	C'_{24}
S_7^-	S_7^-	σ_h	S_7^{2-}	S_7^+	S_7^{3-}	S_7^{2+}	S_7^{3+}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}	σ_{v1}	σ_{v2}	σ_{v3}	C_7^-	E	C_7^{2-}	C_7^+	C_7^{3-}	C_7^{2+}	C_7^{3+}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{21}	C'_{22}	C'_{23}
S_7^{2+}	S_7^{2+}	S_7^{3+}	S_7^+	S_7^{3-}	σ_h	S_7^{2-}	S_7^-	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}	σ_{v1}	C_7^{2+}	C_7^{3+}	C_7^+	C_7^{3-}	E	C_7^{2-}	C_7^-	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{21}
S_7^{2-}	S_7^{2-}	S_7^-	S_7^{3-}	σ_h	S_7^{3+}	S_7^+	S_7^{2+}	σ_{v7}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}	C_7^{2-}	C_7^-	C_7^{3-}	E	C_7^{3+}	C_7^+	C_7^{2+}	C'_{27}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}
S_7^{3+}	S_7^{3+}	S_7^{3-}	S_7^{2+}	S_7^{2-}	S_7^+	S_7^-	σ_h	σ_{v6}	σ_{v7}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	C_7^{3+}	C_7^{3-}	C_7^{2+}	C_7^{2-}	C_7^+	C_7^-	E	C'_{26}	C'_{27}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}
S_7^{3-}	S_7^{3-}	S_7^{2-}	S_7^{3+}	S_7^-	S_7^{2+}	σ_h	S_7^+	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}	σ_{v1}	σ_{v2}	C_7^{3-}	C_7^{2-}	C_7^{3+}	C_7^-	C_7^{2+}	E	C_7^+	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{21}	C'_{22}
σ_{v1}	σ_{v1}	σ_{v4}	σ_{v5}	σ_{v7}	σ_{v2}	σ_{v3}	σ_{v6}	σ_h	S_7^{2-}	S_7^{3+}	S_7^+	S_7^-	S_7^{3-}	S_7^{2+}	C'_{21}	C'_{24}	C'_{25}	C'_{27}	C'_{22}	C'_{23}	C'_{26}	E	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}
σ_{v2}	σ_{v2}	σ_{v5}	σ_{v6}	σ_{v1}	σ_{v3}	σ_{v4}	σ_{v7}	S_7^{2+}	σ_h	S_7^{2-}	S_7^{3+}	S_7^+	S_7^-	S_7^{3-}	C'_{22}	C'_{25}	C'_{26}	C'_{21}	C'_{23}	C'_{24}	C'_{27}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}
σ_{v3}	σ_{v3}	σ_{v6}	σ_{v7}	σ_{v2}	σ_{v4}	σ_{v5}	σ_{v1}	S_7^{3-}	S_7^{2+}	σ_h	S_7^{2-}	S_7^{3+}	S_7^+	S_7^-	C'_{23}	C'_{26}	C'_{27}	C'_{22}	C'_{24}	C'_{25}	C'_{21}	C_7^{3-}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-
σ_{v4}	σ_{v4}	σ_{v7}	σ_{v1}	σ_{v3}	σ_{v5}	σ_{v6}	σ_{v2}	S_7^-	S_7^{3-}	S_7^{2+}	σ_h	S_7^{2-}	S_7^{3+}	S_7^+	C'_{24}	C'_{27}	C'_{21}	C'_{23}	C'_{25}	C'_{26}	C'_{22}	C_7^-	C_7^{3-}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}	C_7^+
σ_{v5}	σ_{v5}	σ_{v1}	σ_{v2}	σ_{v4}	σ_{v6}	σ_{v7}	σ_{v3}	S_7^+	S_7^-	S_7^{3-}	S_7^{2+}	σ_h	S_7^{2-}	S_7^{3+}	C'_{25}	C'_{21}	C'_{22}	C'_{24}	C'_{26}	C'_{27}	C'_{23}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}
σ_{v6}	σ_{v6}	σ_{v2}	σ_{v3}	σ_{v5}	σ_{v7}	σ_{v1}	σ_{v4}	S_7^{3+}	S_7^+	S_7^-	S_7^{3-}	S_7^{2+}	σ_h	S_7^{2-}	C'_{26}	C'_{22}	C'_{23}	C'_{25}	C'_{27}	C'_{21}	C'_{24}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}	E	C_7^{2-}
σ_{v7}	σ_{v7}	σ_{v3}	σ_{v4}	σ_{v6}	σ_{v1}	σ_{v2}	σ_{v5}	S_7^{2-}	S_7^{3+}	S_7^+	S_7^-	S_7^{3-}	S_7^{2+}	σ_h	C'_{27}	C'_{23}	C'_{24}	C'_{26}	C'_{21}	C'_{22}	C'_{25}	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}	E

T **36**.3 Factor table

§ **16**–3, p. 70

$\mathbf{D}_{7h}$	E	C_7^+	C_7^-	C_7^{2+}	C_7^{2-}	C_7^{3+}	C_7^{3-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	σ_h	S_7^+	S_7^-	S_7^{2+}	S_7^{2-}	S_7^{3+}	S_7^{3-}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_7^+	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1
C_7^-	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	−1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1
C_7^{2+}	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1
C_7^{2-}	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
C_7^{3+}	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	−1	1	−1	1	1	−1	−1	−1	−1	−1	−1	−1
C_7^{3-}	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1
C'_{21}	1	−1	−1	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	1	1	−1	−1	1	1	1	1	1	−1	−1	−1
C'_{22}	1	−1	−1	1	1	−1	−1	−1	−1	−1	1	1	1	1	−1	−1	1	1	−1	−1	1	−1	1	1	1	1	−1	−1
C'_{23}	1	−1	−1	1	1	−1	−1	1	−1	−1	−1	1	1	1	−1	−1	1	1	−1	−1	1	−1	−1	1	1	1	1	−1
C'_{24}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	−1	−1	1	1	1	1
C'_{25}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	−1	1	−1	−1	1	1	−1	−1	1	1	−1	−1	−1	1	1	1
C'_{26}	1	−1	−1	1	1	−1	−1	1	1	1	1	−1	−1	−1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	−1	1	1
C'_{27}	1	−1	−1	1	1	−1	−1	−1	1	1	1	1	−1	−1	−1	−1	1	1	−1	−1	1	1	1	1	−1	−1	−1	1
σ_h	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1
S_7^+	1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	−1	−1	−1	−1	−1	−1	−1
S_7^-	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1
S_7^{2+}	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1	1
S_7^{2-}	1	1	1	1	1	−1	1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1
S_7^{3+}	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1
S_7^{3-}	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	1	1	1	1	1
σ_{v1}	1	−1	−1	1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	−1	−1	1	1	−1	−1	−1	1	1	1	1	−1
σ_{v2}	1	−1	−1	1	1	−1	−1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	1	1	−1	−1	−1	−1	1	1	1	1
σ_{v3}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	−1	−1	1	1	1	−1	−1	1	1	−1	1	−1	−1	−1	1	1	1
σ_{v4}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	−1	−1	1	1	−1	−1	1	1	−1	1	1	−1	−1	−1	1	1
σ_{v5}	1	−1	−1	1	1	−1	−1	−1	1	1	1	−1	−1	−1	1	1	−1	−1	1	1	−1	1	1	1	−1	−1	−1	1
σ_{v6}	1	−1	−1	1	1	−1	−1	−1	−1	1	1	1	−1	−1	1	1	−1	−1	1	1	−1	1	1	1	1	−1	−1	−1
σ_{v7}	1	−1	−1	1	1	−1	−1	−1	−1	−1	1	1	1	−1	1	1	−1	−1	1	1	−1	−1	1	1	1	1	−1	−1

T **36**.4 Character table

§ **16**–4, p. 71

$\mathbf{D}_{7h}$	E	$2C_7$	$2C_7^2$	$2C_7^3$	$7C_2'$	σ_h	$2S_7$	$2S_7^2$	$2S_7^3$	$7\sigma_v$	τ
A_1'	1	1	1	1	1	1	1	1	1	1	a
A_2'	1	1	1	1	-1	1	1	1	1	-1	a
E_1'	2	$2c_7^2$	$2c_7^4$	$2c_7^6$	0	2	$2c_7^2$	$2c_7^4$	$2c_7^6$	0	a
E_2'	2	$2c_7^4$	$2c_7^6$	$2c_7^2$	0	2	$2c_7^4$	$2c_7^6$	$2c_7^2$	0	a
E_3'	2	$2c_7^6$	$2c_7^2$	$2c_7^4$	0	2	$2c_7^6$	$2c_7^2$	$2c_7^4$	0	a
A_1''	1	1	1	1	1	-1	-1	-1	-1	-1	a
A_2''	1	1	1	1	-1	-1	-1	-1	-1	1	a
E_1''	2	$2c_7^2$	$2c_7^4$	$2c_7^6$	0	-2	$-2c_7^2$	$-2c_7^4$	$-2c_7^6$	0	a
E_2''	2	$2c_7^4$	$2c_7^6$	$2c_7^2$	0	-2	$-2c_7^4$	$-2c_7^6$	$-2c_7^2$	0	a
E_3''	2	$2c_7^6$	$2c_7^2$	$2c_7^4$	0	-2	$-2c_7^6$	$-2c_7^2$	$-2c_7^4$	0	a
$E_{1/2}$	2	$-2c_7^6$	$2c_7^2$	$-2c_7^4$	0	0	$2c_{14}^5$	$2c_{14}^3$	$2c_{14}$	0	c
$E_{3/2}$	2	$-2c_7^4$	$2c_7^6$	$-2c_7^2$	0	0	$-2c_{14}$	$-2c_{14}^5$	$2c_{14}^3$	0	c
$E_{5/2}$	2	$-2c_7^2$	$2c_7^4$	$-2c_7^6$	0	0	$2c_{14}^3$	$-2c_{14}$	$2c_{14}^5$	0	c
$E_{7/2}$	2	-2	2	-2	0	0	0	0	0	0	c
$E_{9/2}$	2	$-2c_7^2$	$2c_7^4$	$-2c_7^6$	0	0	$-2c_{14}^3$	$2c_{14}$	$-2c_{14}^5$	0	c
$E_{11/2}$	2	$-2c_7^4$	$2c_7^6$	$-2c_7^2$	0	0	$2c_{14}$	$2c_{14}^5$	$-2c_{14}^3$	0	c
$E_{13/2}$	2	$-2c_7^6$	$2c_7^2$	$-2c_7^4$	0	0	$-2c_{14}^5$	$-2c_{14}^3$	$-2c_{14}$	0	c

$c_n^m = \cos \frac{m}{n}\pi$

T **36**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{7h}$	0	1	2	3
A_1'	$\square 1$		$x^2+y^2, \square z^2$	
A_2'		R_z		
E_1'		$\square(x,y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, \square(xz^2, yz^2)$
E_2'			$\square(xy, x^2-y^2)$	
E_3'				$\square\{x(x^2-3y^2), y(3x^2-y^2)\}$
A_1''				
A_2''		$\square z$		$(x^2+y^2)z, \square z^3$
E_1''		(R_x, R_y)	$\square(zx, yz)$	
E_2''				$\square\{xyz, (x^2-y^2)z\}$
E_3''				

T **36**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{7h}$	$\langle \vert j\,m\rangle \vert$		ι	μ
A_1'	$\vert 0\,0\rangle_+$	$\vert 7\,7\rangle_-$	2	14
A_2'	$\vert 7\,7\rangle_+$	$\vert 14\,14\rangle_-$	2	14
E_1'	$\langle \vert 1\,1\rangle, \vert 1\,\overline{1}\rangle \vert$	$\langle \vert 6\,\overline{6}\rangle, -\vert 6\,6\rangle \vert$	2	± 14
E_2'	$\langle \vert 2\,2\rangle, \vert 2\,\overline{2}\rangle \vert$	$\langle \vert 5\,\overline{5}\rangle, -\vert 5\,5\rangle \vert$	2	± 14
E_3'	$\langle \vert 3\,3\rangle, \vert 3\,\overline{3}\rangle \vert$	$\langle \vert 4\,\overline{4}\rangle, -\vert 4\,4\rangle \vert$	2	± 14
A_1''	$\vert 8\,7\rangle_+$	$\vert 15\,14\rangle_-$	2	14
A_2''	$\vert 1\,0\rangle_+$	$\vert 8\,7\rangle_-$	2	14
E_1''	$\langle \vert 2\,1\rangle, -\vert 2\,\overline{1}\rangle \vert$	$\langle \vert 7\,\overline{6}\rangle, \vert 7\,6\rangle \vert$	2	± 14
E_2''	$\langle \vert 3\,2\rangle, -\vert 3\,\overline{2}\rangle \vert$	$\langle \vert 6\,\overline{5}\rangle, \vert 6\,5\rangle \vert$	2	± 14
E_3''	$\langle \vert 4\,3\rangle, -\vert 4\,\overline{3}\rangle \vert$	$\langle \vert 5\,\overline{4}\rangle, \vert 5\,4\rangle \vert$	2	± 14
$E_{1/2}$	$\langle \vert \frac{1}{2}\,\frac{1}{2}\rangle, \vert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \vert$	$\langle \vert \frac{3}{2}\,\frac{1}{2}\rangle, -\vert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle \vert$	2	± 14
	$\langle \vert \frac{13}{2}\,\overline{\frac{13}{2}}\rangle, \vert \frac{13}{2}\,\frac{13}{2}\rangle \vert^\bullet$	$\langle \vert \frac{15}{2}\,\overline{\frac{13}{2}}\rangle, -\vert \frac{15}{2}\,\frac{13}{2}\rangle \vert^\bullet$	2	± 14
$E_{3/2}$	$\langle \vert \frac{3}{2}\,\frac{3}{2}\rangle, \vert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle \vert$	$\langle \vert \frac{5}{2}\,\frac{3}{2}\rangle, -\vert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle \vert$	2	± 14
	$\langle \vert \frac{11}{2}\,\overline{\frac{11}{2}}\rangle, \vert \frac{11}{2}\,\frac{11}{2}\rangle \vert^\bullet$	$\langle \vert \frac{13}{2}\,\overline{\frac{11}{2}}\rangle, -\vert \frac{13}{2}\,\frac{11}{2}\rangle \vert^\bullet$	2	± 14
$E_{5/2}$	$\langle \vert \frac{5}{2}\,\frac{5}{2}\rangle, \vert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle \vert$	$\langle \vert \frac{7}{2}\,\frac{5}{2}\rangle, -\vert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle \vert$	2	± 14
	$\langle \vert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle, \vert \frac{9}{2}\,\frac{9}{2}\rangle \vert^\bullet$	$\langle \vert \frac{11}{2}\,\overline{\frac{9}{2}}\rangle, -\vert \frac{11}{2}\,\frac{9}{2}\rangle \vert^\bullet$	2	± 14
$E_{7/2}$	$\langle \vert \frac{7}{2}\,\frac{7}{2}\rangle, \vert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle \vert$	$\langle \vert \frac{9}{2}\,\frac{7}{2}\rangle, -\vert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle \vert$	2	± 14
	$\langle \vert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle, \vert \frac{7}{2}\,\frac{7}{2}\rangle \vert^\bullet$	$\langle \vert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle, -\vert \frac{9}{2}\,\frac{7}{2}\rangle \vert^\bullet$	2	± 14
$E_{9/2}$	$\langle \vert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle, \vert \frac{9}{2}\,\frac{9}{2}\rangle \vert$	$\langle \vert \frac{11}{2}\,\overline{\frac{9}{2}}\rangle, -\vert \frac{11}{2}\,\frac{9}{2}\rangle \vert$	2	± 14
	$\langle \vert \frac{5}{2}\,\frac{5}{2}\rangle, \vert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle \vert^\bullet$	$\langle \vert \frac{7}{2}\,\frac{5}{2}\rangle, -\vert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle \vert^\bullet$	2	± 14
$E_{11/2}$	$\langle \vert \frac{11}{2}\,\overline{\frac{11}{2}}\rangle, \vert \frac{11}{2}\,\frac{11}{2}\rangle \vert$	$\langle \vert \frac{13}{2}\,\overline{\frac{11}{2}}\rangle, -\vert \frac{13}{2}\,\frac{11}{2}\rangle \vert$	2	± 14
	$\langle \vert \frac{3}{2}\,\frac{3}{2}\rangle, \vert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle \vert^\bullet$	$\langle \vert \frac{5}{2}\,\frac{3}{2}\rangle, -\vert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle \vert^\bullet$	2	± 14
$E_{13/2}$	$\langle \vert \frac{13}{2}\,\overline{\frac{13}{2}}\rangle, \vert \frac{13}{2}\,\frac{13}{2}\rangle \vert$	$\langle \vert \frac{15}{2}\,\overline{\frac{13}{2}}\rangle, -\vert \frac{15}{2}\,\frac{13}{2}\rangle \vert$	2	± 14
	$\langle \vert \frac{1}{2}\,\frac{1}{2}\rangle, \vert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \vert^\bullet$	$\langle \vert \frac{3}{2}\,\frac{1}{2}\rangle, -\vert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle \vert^\bullet$	2	± 14

T **36**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_{7h}$	E'_1	E'_2	E'_3	E''_1	E''_2	E''_3
E	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$
C_7^+	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$
C_7^-	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$
C_7^{2+}	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$
C_7^{2-}	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$
C_7^{3+}	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$
C_7^{3-}	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$
C'_{21}	$\begin{bmatrix}0&\overline{1}\\\overline{1}&0\end{bmatrix}$	$\begin{bmatrix}0&1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{1}\\\overline{1}&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{1}\\\overline{1}&0\end{bmatrix}$	$\begin{bmatrix}0&1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{1}\\\overline{1}&0\end{bmatrix}$
C'_{22}	$\begin{bmatrix}0&\overline{\epsilon}^*\\\overline{\epsilon}&0\end{bmatrix}$	$\begin{bmatrix}0&\eta\\\eta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\delta}\\\overline{\delta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\epsilon}^*\\\overline{\epsilon}&0\end{bmatrix}$	$\begin{bmatrix}0&\eta\\\eta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\delta}\\\overline{\delta}^*&0\end{bmatrix}$
C'_{23}	$\begin{bmatrix}0&\overline{\eta}\\\overline{\eta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\delta^*\\\delta&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\epsilon}\\\overline{\epsilon}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\eta}\\\overline{\eta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\delta^*\\\delta&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\epsilon}\\\overline{\epsilon}^*&0\end{bmatrix}$
C'_{24}	$\begin{bmatrix}0&\overline{\delta}\\\overline{\delta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon\\\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\eta}\\\overline{\eta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\delta}\\\overline{\delta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon\\\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\eta}\\\overline{\eta}^*&0\end{bmatrix}$
C'_{25}	$\begin{bmatrix}0&\overline{\delta}^*\\\overline{\delta}&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon^*\\\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\eta}^*\\\overline{\eta}&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\delta}^*\\\overline{\delta}&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon^*\\\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\eta}^*\\\overline{\eta}&0\end{bmatrix}$
C'_{26}	$\begin{bmatrix}0&\overline{\eta}^*\\\overline{\eta}&0\end{bmatrix}$	$\begin{bmatrix}0&\delta\\\delta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\epsilon}^*\\\overline{\epsilon}&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\eta}^*\\\overline{\eta}&0\end{bmatrix}$	$\begin{bmatrix}0&\delta\\\delta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\epsilon}^*\\\overline{\epsilon}&0\end{bmatrix}$
C'_{27}	$\begin{bmatrix}0&\overline{\epsilon}\\\overline{\epsilon}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\eta^*\\\eta&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\delta}^*\\\overline{\delta}&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\epsilon}\\\overline{\epsilon}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\eta^*\\\eta&0\end{bmatrix}$	$\begin{bmatrix}0&\overline{\delta}^*\\\overline{\delta}&0\end{bmatrix}$

$\delta = \exp(2\pi i/7)$, $\epsilon = \exp(4\pi i/7)$, $\eta = \exp(6\pi i/7)$ ⟫

T **36**.7 Matrix representations *(cont.)*

$\mathbf{D}_{7h}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$	$E_{11/2}$	$E_{13/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_7^+	$\begin{bmatrix} \bar{\eta} & 0 \\ 0 & \bar{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\eta} & 0 \\ 0 & \bar{\eta}^* \end{bmatrix}$
C_7^-	$\begin{bmatrix} \bar{\eta}^* & 0 \\ 0 & \bar{\eta} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\eta}^* & 0 \\ 0 & \bar{\eta} \end{bmatrix}$
C_7^{2+}	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$
C_7^{2-}	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$
C_7^{3+}	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \bar{\eta} & 0 \\ 0 & \bar{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\eta} & 0 \\ 0 & \bar{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$
C_7^{3-}	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\eta}^* & 0 \\ 0 & \bar{\eta} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\eta}^* & 0 \\ 0 & \bar{\eta} \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta}^* \\ \mathrm{i}\bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta}^* \\ \mathrm{i}\bar{\delta} & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta}^* \\ \mathrm{i}\bar{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta}^* \\ \mathrm{i}\bar{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta}^* \\ \mathrm{i}\bar{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta}^* \\ \mathrm{i}\bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta}^* \\ \mathrm{i}\bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta}^* \\ \mathrm{i}\bar{\eta} & 0 \end{bmatrix}$
C'_{25}	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta} \\ \mathrm{i}\bar{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta} \\ \mathrm{i}\bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta} \\ \mathrm{i}\bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta} \\ \mathrm{i}\bar{\eta}^* & 0 \end{bmatrix}$
C'_{26}	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta} \\ \mathrm{i}\bar{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta} \\ \mathrm{i}\bar{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$
C'_{27}	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta} \\ \mathrm{i}\bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta} \\ \mathrm{i}\bar{\delta}^* & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/7)$, $\epsilon = \exp(4\pi\mathrm{i}/7)$, $\eta = \exp(6\pi\mathrm{i}/7)$ $\twoheadrightarrow$

T **36**.7 Matrix representations *(cont.)*

$\mathbf{D}_{7h}$	E'_1	E'_2	E'_3	E''_1	E''_2	E''_3
σ_h	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}\overline{1} & 0\\0 & \overline{1}\end{bmatrix}$	$\begin{bmatrix}\overline{1} & 0\\0 & \overline{1}\end{bmatrix}$	$\begin{bmatrix}\overline{1} & 0\\0 & \overline{1}\end{bmatrix}$
S_7^+	$\begin{bmatrix}\delta^* & 0\\0 & \delta\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\eta^* & 0\\0 & \eta\end{bmatrix}$	$\begin{bmatrix}\overline{\delta}^* & 0\\0 & \overline{\delta}\end{bmatrix}$	$\begin{bmatrix}\overline{\epsilon}^* & 0\\0 & \overline{\epsilon}\end{bmatrix}$	$\begin{bmatrix}\overline{\eta}^* & 0\\0 & \overline{\eta}\end{bmatrix}$
S_7^-	$\begin{bmatrix}\delta & 0\\0 & \delta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\eta & 0\\0 & \eta^*\end{bmatrix}$	$\begin{bmatrix}\overline{\delta} & 0\\0 & \overline{\delta}^*\end{bmatrix}$	$\begin{bmatrix}\overline{\epsilon} & 0\\0 & \overline{\epsilon}^*\end{bmatrix}$	$\begin{bmatrix}\overline{\eta} & 0\\0 & \overline{\eta}^*\end{bmatrix}$
S_7^{2+}	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\eta & 0\\0 & \eta^*\end{bmatrix}$	$\begin{bmatrix}\delta & 0\\0 & \delta^*\end{bmatrix}$	$\begin{bmatrix}\overline{\epsilon}^* & 0\\0 & \overline{\epsilon}\end{bmatrix}$	$\begin{bmatrix}\overline{\eta} & 0\\0 & \overline{\eta}^*\end{bmatrix}$	$\begin{bmatrix}\overline{\delta} & 0\\0 & \overline{\delta}^*\end{bmatrix}$
S_7^{2-}	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\eta^* & 0\\0 & \eta\end{bmatrix}$	$\begin{bmatrix}\delta^* & 0\\0 & \delta\end{bmatrix}$	$\begin{bmatrix}\overline{\epsilon} & 0\\0 & \overline{\epsilon}^*\end{bmatrix}$	$\begin{bmatrix}\overline{\eta}^* & 0\\0 & \overline{\eta}\end{bmatrix}$	$\begin{bmatrix}\overline{\delta}^* & 0\\0 & \overline{\delta}\end{bmatrix}$
S_7^{3+}	$\begin{bmatrix}\eta^* & 0\\0 & \eta\end{bmatrix}$	$\begin{bmatrix}\delta & 0\\0 & \delta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\overline{\eta}^* & 0\\0 & \overline{\eta}\end{bmatrix}$	$\begin{bmatrix}\overline{\delta} & 0\\0 & \overline{\delta}^*\end{bmatrix}$	$\begin{bmatrix}\overline{\epsilon}^* & 0\\0 & \overline{\epsilon}\end{bmatrix}$
S_7^{3-}	$\begin{bmatrix}\eta & 0\\0 & \eta^*\end{bmatrix}$	$\begin{bmatrix}\delta^* & 0\\0 & \delta\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\overline{\eta} & 0\\0 & \overline{\eta}^*\end{bmatrix}$	$\begin{bmatrix}\overline{\delta}^* & 0\\0 & \overline{\delta}\end{bmatrix}$	$\begin{bmatrix}\overline{\epsilon} & 0\\0 & \overline{\epsilon}^*\end{bmatrix}$
σ_{v1}	$\begin{bmatrix}0 & \overline{1}\\\overline{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{1}\\\overline{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{1}\\\overline{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$
σ_{v2}	$\begin{bmatrix}0 & \overline{\epsilon}^*\\\overline{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \eta\\\eta^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\delta}\\\overline{\delta}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon^*\\\epsilon & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\eta}\\\overline{\eta}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \delta\\\delta^* & 0\end{bmatrix}$
σ_{v3}	$\begin{bmatrix}0 & \overline{\eta}\\\overline{\eta}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \delta^*\\\delta & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\epsilon}\\\overline{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \eta\\\eta^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\delta}^*\\\overline{\delta} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon\\\epsilon^* & 0\end{bmatrix}$
σ_{v4}	$\begin{bmatrix}0 & \overline{\delta}\\\overline{\delta}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon\\\epsilon^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\eta}\\\overline{\eta}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \delta\\\delta^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\epsilon}\\\overline{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \eta\\\eta^* & 0\end{bmatrix}$
σ_{v5}	$\begin{bmatrix}0 & \overline{\delta}^*\\\overline{\delta} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon^*\\\epsilon & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\eta}^*\\\overline{\eta} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \delta^*\\\delta & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\epsilon}^*\\\overline{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \eta^*\\\eta & 0\end{bmatrix}$
σ_{v6}	$\begin{bmatrix}0 & \overline{\eta}^*\\\overline{\eta} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \delta\\\delta^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\epsilon}^*\\\overline{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \eta^*\\\eta & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\delta}\\\overline{\delta}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon^*\\\epsilon & 0\end{bmatrix}$
σ_{v7}	$\begin{bmatrix}0 & \overline{\epsilon}\\\overline{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \eta^*\\\eta & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\delta}^*\\\overline{\delta} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon\\\epsilon^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\eta}^*\\\overline{\eta} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \delta^*\\\delta & 0\end{bmatrix}$

$\delta = \exp(2\pi i/7)$, $\epsilon = \exp(4\pi i/7)$, $\eta = \exp(6\pi i/7)$ $\twoheadrightarrow$

T **36**.7 Matrix representations *(cont.)*

$\mathbf{D}_{7h}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$	$E_{11/2}$	$E_{13/2}$
σ_h	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$
S_7^+	$\begin{bmatrix} \mathrm{i}\overline{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\overline{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\overline{\eta}^* \end{bmatrix}$
S_7^-	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta}^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$
S_7^{2+}	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta}^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$
S_7^{2-}	$\begin{bmatrix} \mathrm{i}\overline{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\overline{\delta}^* \end{bmatrix}$
S_7^{3+}	$\begin{bmatrix} \mathrm{i}\overline{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta}^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\overline{\epsilon}^* \end{bmatrix}$
S_7^{3-}	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\overline{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$
σ_{v1}	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$
σ_{v2}	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \overline{\delta} & 0 \end{bmatrix}$
σ_{v3}	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \overline{\epsilon} & 0 \end{bmatrix}$
σ_{v4}	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \overline{\eta} & 0 \end{bmatrix}$
σ_{v5}	$\begin{bmatrix} 0 & \overline{\eta} \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \overline{\eta}^* & 0 \end{bmatrix}$
σ_{v6}	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \overline{\epsilon}^* & 0 \end{bmatrix}$
σ_{v7}	$\begin{bmatrix} 0 & \overline{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \overline{\delta}^* & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/7)$, $\epsilon = \exp(4\pi\mathrm{i}/7)$, $\eta = \exp(6\pi\mathrm{i}/7)$

T **36**.8 Direct products of representations § **16**–8, p. 81

$\mathbf{D}_{7h}$	A'_1	A'_2	E'_1	E'_2	E'_3
A'_1	A'_1	A'_2	E'_1	E'_2	E'_3
A'_2		A'_1	E'_1	E'_2	E'_3
E'_1			$A'_1 \oplus \{A'_2\} \oplus E'_2$	$E'_1 \oplus E'_3$	$E'_2 \oplus E'_3$
E'_2				$A'_1 \oplus \{A'_2\} \oplus E'_3$	$E'_1 \oplus E'_2$
E'_3					$A'_1 \oplus \{A'_2\} \oplus E'_1$

$\twoheadrightarrow$

T **36**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{7h}$	A_1''	A_2''	E_1''	E_2''	E_3''
A_1'	A_1''	A_2''	E_1''	E_2''	E_3''
A_2'	A_2''	A_1''	E_1''	E_2''	E_3''
E_1'	E_1''	E_1''	$A_1'' \oplus A_2'' \oplus E_2''$	$E_1'' \oplus E_3''$	$E_2'' \oplus E_3''$
E_2'	E_2''	E_2''	$E_1'' \oplus E_3''$	$A_1'' \oplus A_2'' \oplus E_3''$	$E_1'' \oplus E_2''$
E_3'	E_3''	E_3''	$E_2'' \oplus E_3''$	$E_1'' \oplus E_2''$	$A_1'' \oplus A_2'' \oplus E_1''$
A_1''	A_1'	A_2'	E_1'	E_2'	E_3'
A_2''		A_1'	E_1'	E_2'	E_3'
E_1''			$A_1' \oplus \{A_2'\} \oplus E_2'$	$E_1' \oplus E_3'$	$E_2' \oplus E_3'$
E_2''				$A_1' \oplus \{A_2'\} \oplus E_3'$	$E_1' \oplus E_2'$
E_3''					$A_1' \oplus \{A_2'\} \oplus E_1'$

$\twoheadrightarrow$

T **36**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{7h}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
A_1'	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
A_2'	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
E_1'	$E_{11/2} \oplus E_{13/2}$	$E_{9/2} \oplus E_{13/2}$	$E_{7/2} \oplus E_{11/2}$
E_2'	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{9/2}$
E_3'	$E_{7/2} \oplus E_{9/2}$	$E_{5/2} \oplus E_{11/2}$	$E_{3/2} \oplus E_{13/2}$
A_1''	$E_{13/2}$	$E_{11/2}$	$E_{9/2}$
A_2''	$E_{13/2}$	$E_{11/2}$	$E_{9/2}$
E_1''	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus E_{7/2}$
E_2''	$E_{9/2} \oplus E_{11/2}$	$E_{7/2} \oplus E_{13/2}$	$E_{5/2} \oplus E_{13/2}$
E_3''	$E_{5/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{9/2}$	$E_{1/2} \oplus E_{11/2}$
$E_{1/2}$	$\{A_1'\} \oplus A_2' \oplus E_1''$	$E_2' \oplus E_1''$	$E_2' \oplus E_3''$
$E_{3/2}$		$\{A_1'\} \oplus A_2' \oplus E_3''$	$E_3' \oplus E_1''$
$E_{5/2}$			$\{A_1'\} \oplus A_2' \oplus E_2''$

$\twoheadrightarrow$

T **36**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{7h}$	$E_{7/2}$	$E_{9/2}$	$E_{11/2}$	$E_{13/2}$
A_1'	$E_{1/2}$	$E_{9/2}$	$E_{11/2}$	$E_{13/2}$
A_2'	$E_{1/2}$	$E_{9/2}$	$E_{11/2}$	$E_{13/2}$
E_1'	$E_{5/2} \oplus E_{9/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$
E_2'	$E_{3/2} \oplus E_{11/2}$	$E_{5/2} \oplus E_{13/2}$	$E_{7/2} \oplus E_{13/2}$	$E_{9/2} \oplus E_{11/2}$
E_3'	$E_{1/2} \oplus E_{13/2}$	$E_{1/2} \oplus E_{11/2}$	$E_{3/2} \oplus E_{9/2}$	$E_{5/2} \oplus E_{7/2}$
A_1''	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
A_2''	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
E_1''	$E_{5/2} \oplus E_{9/2}$	$E_{7/2} \oplus E_{11/2}$	$E_{9/2} \oplus E_{13/2}$	$E_{11/2} \oplus E_{13/2}$
E_2''	$E_{3/2} \oplus E_{11/2}$	$E_{1/2} \oplus E_{9/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{5/2}$
E_3''	$E_{1/2} \oplus E_{13/2}$	$E_{3/2} \oplus E_{13/2}$	$E_{5/2} \oplus E_{11/2}$	$E_{7/2} \oplus E_{9/2}$
$E_{1/2}$	$E_3' \oplus E_3''$	$E_3' \oplus E_2''$	$E_1' \oplus E_2''$	$E_1' \oplus A_1'' \oplus A_2''$
$E_{3/2}$	$E_2' \oplus E_2''$	$E_1' \oplus E_3''$	$E_3' \oplus A_1'' \oplus A_2''$	$E_1' \oplus E_2''$
$E_{5/2}$	$E_1' \oplus E_1''$	$E_2' \oplus A_1'' \oplus A_2''$	$E_1' \oplus E_3''$	$E_3' \oplus E_2''$
$E_{7/2}$	$\{A_1'\} \oplus A_2' \oplus A_1'' \oplus A_2''$	$E_1' \oplus E_1''$	$E_2' \oplus E_2''$	$E_3' \oplus E_3''$
$E_{9/2}$		$\{A_1'\} \oplus A_2' \oplus E_2''$	$E_3' \oplus E_1''$	$E_2' \oplus E_3''$
$E_{11/2}$			$\{A_1'\} \oplus A_2' \oplus E_3''$	$E_2' \oplus E_1''$
$E_{13/2}$				$\{A_1'\} \oplus A_2' \oplus E_1''$

T **36**.9 Subduction (descent of symmetry) § **16**–9, p. 82

$\mathbf{D}_{7h}$	$\mathbf{C}_{7h}$	$(\mathbf{C}_{7v})$	$(\mathbf{C}_{2v})$ C_2', σ_v, σ_h	$(\mathbf{D}_7)$
A_1'	A'	A_1	A_1	A_1
A_2'	A'	A_2	B_1	A_2
E_1'	${}^1E_1' \oplus {}^2E_1'$	E_1	$A_1 \oplus B_1$	E_1
E_2'	${}^1E_2' \oplus {}^2E_2'$	E_2	$A_1 \oplus B_1$	E_2
E_3'	${}^1E_3' \oplus {}^2E_3'$	E_3	$A_1 \oplus B_1$	E_3
A_1''	A''	A_2	A_2	A_1
A_2''	A''	A_1	B_2	A_2
E_1''	${}^1E_1'' \oplus {}^2E_1''$	E_1	$A_2 \oplus B_2$	E_1
E_2''	${}^1E_2'' \oplus {}^2E_2''$	E_2	$A_2 \oplus B_2$	E_2
E_3''	${}^1E_3'' \oplus {}^2E_3''$	E_3	$A_2 \oplus B_2$	E_3
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{3/2}$	$E_{1/2}$	$E_{3/2}$
$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{5/2}$	$E_{1/2}$	$E_{5/2}$
$E_{7/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	$E_{1/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$
$E_{9/2}$	${}^1E_{9/2} \oplus {}^2E_{9/2}$	$E_{5/2}$	$E_{1/2}$	$E_{5/2}$
$E_{11/2}$	${}^1E_{11/2} \oplus {}^2E_{11/2}$	$E_{3/2}$	$E_{1/2}$	$E_{3/2}$
$E_{13/2}$	${}^1E_{13/2} \oplus {}^2E_{13/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$

$\twoheadrightarrow$

T **36**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{7h}$	$\mathbf{C}_s$ σ_h	$(\mathbf{C}_s)$ σ_v	$\mathbf{C}_7$	$(\mathbf{C}_2)$
A_1'	A'	A'	A	A
A_2'	A'	A''	A	B
E_1'	$2A'$	$A' \oplus A''$	${}^1E_1 \oplus {}^2E_1$	$A \oplus B$
E_2'	$2A'$	$A' \oplus A''$	${}^1E_2 \oplus {}^2E_2$	$A \oplus B$
E_3'	$2A'$	$A' \oplus A''$	${}^1E_3 \oplus {}^2E_3$	$A \oplus B$
A_1''	A''	A''	A	A
A_2''	A''	A'	A	B
E_1''	$2A''$	$A' \oplus A''$	${}^1E_1 \oplus {}^2E_1$	$A \oplus B$
E_2''	$2A''$	$A' \oplus A''$	${}^1E_2 \oplus {}^2E_2$	$A \oplus B$
E_3''	$2A''$	$A' \oplus A''$	${}^1E_3 \oplus {}^2E_3$	$A \oplus B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{9/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{11/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{13/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **36**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{7h}$
$14n$	$(n+1)\,A_1' \oplus n\,(A_2' \oplus 2E_1' \oplus 2E_2' \oplus 2E_3' \oplus A_1'' \oplus A_2'' \oplus 2E_1'' \oplus 2E_2'' \oplus 2E_3'')$
$14n+1$	$n\,(A_1' \oplus A_2' \oplus E_1' \oplus 2E_2' \oplus 2E_3' \oplus A_1'' \oplus 2E_1'' \oplus 2E_2'' \oplus 2E_3'') \oplus (n+1)(E_1' \oplus A_2'')$
$14n+2$	$(n+1)(A_1' \oplus E_2' \oplus E_1'') \oplus n\,(A_2' \oplus 2E_1' \oplus E_2' \oplus 2E_3' \oplus A_1'' \oplus A_2'' \oplus E_1'' \oplus 2E_2'' \oplus 2E_3'')$
$14n+3$	$n\,(A_1' \oplus A_2' \oplus E_1' \oplus 2E_2' \oplus E_3' \oplus A_1'' \oplus 2E_1'' \oplus E_2'' \oplus 2E_3'') \oplus (n+1)(E_1' \oplus E_3' \oplus A_2'' \oplus E_2'')$
$14n+4$	$(n+1)(A_1' \oplus E_2' \oplus E_3' \oplus E_1'' \oplus E_3'') \oplus$ $n\,(A_2' \oplus 2E_1' \oplus E_2' \oplus E_3' \oplus A_1'' \oplus A_2'' \oplus E_1'' \oplus 2E_2'' \oplus E_3'')$
$14n+5$	$n\,(A_1' \oplus A_2' \oplus E_1' \oplus E_2' \oplus E_3' \oplus A_1'' \oplus 2E_1'' \oplus E_2'' \oplus E_3'') \oplus$ $(n+1)(E_1' \oplus E_2' \oplus E_3' \oplus A_2'' \oplus E_2'' \oplus E_3'')$
$14n+6$	$(n+1)(A_1' \oplus E_1' \oplus E_2' \oplus E_3' \oplus E_1'' \oplus E_2'' \oplus E_3'') \oplus$ $n\,(A_2' \oplus E_1' \oplus E_2' \oplus E_3' \oplus A_1'' \oplus A_2'' \oplus E_1'' \oplus E_2'' \oplus E_3'')$
$14n+7$	$(n+1)(A_1' \oplus A_2' \oplus E_1' \oplus E_2' \oplus E_3' \oplus A_2'' \oplus E_1'' \oplus E_2'' \oplus E_3'') \oplus$ $n\,(E_1' \oplus E_2' \oplus E_3' \oplus A_1'' \oplus E_1'' \oplus E_2'' \oplus E_3'')$
$14n+8$	$(n+1)(A_1' \oplus 2E_1' \oplus E_2' \oplus E_3' \oplus A_1'' \oplus A_2'' \oplus E_1'' \oplus E_2'' \oplus E_3'') \oplus$ $n\,(A_2' \oplus E_2' \oplus E_3' \oplus E_1'' \oplus E_2'' \oplus E_3'')$
$14n+9$	$(n+1)(A_1' \oplus A_2' \oplus E_1' \oplus 2E_2' \oplus E_3' \oplus A_2'' \oplus 2E_1'' \oplus E_2'' \oplus E_3'') \oplus$ $n\,(E_1' \oplus E_3' \oplus A_1'' \oplus E_2'' \oplus E_3'')$
$14n+10$	$(n+1)(A_1' \oplus 2E_1' \oplus E_2' \oplus 2E_3' \oplus A_1'' \oplus A_2'' \oplus E_1'' \oplus 2E_2'' \oplus E_3'') \oplus n\,(A_2' \oplus E_2' \oplus E_1'' \oplus E_3'')$
$14n+11$	$(n+1)(A_1' \oplus A_2' \oplus E_1' \oplus 2E_2' \oplus 2E_3' \oplus A_2'' \oplus 2E_1'' \oplus E_2'' \oplus 2E_3'') \oplus n\,(E_1' \oplus A_1'' \oplus E_2'')$
$14n+12$	$(n+1)(A_1' \oplus 2E_1' \oplus 2E_2' \oplus 2E_3' \oplus A_1'' \oplus A_2'' \oplus E_1'' \oplus 2E_2'' \oplus 2E_3'') \oplus n\,(A_2' \oplus E_1'')$
$14n+13$	$(n+1)(A_1' \oplus A_2' \oplus 2E_1' \oplus 2E_2' \oplus 2E_3' \oplus A_2'' \oplus 2E_1'' \oplus 2E_2'' \oplus 2E_3'') \oplus n\,A_1''$
$14n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2})$
$14n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus 2n\,(E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2})$
$14n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus 2n\,(E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2})$
$14n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus 2n\,(E_{9/2} \oplus E_{11/2} \oplus E_{13/2})$
$14n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2}) \oplus 2n\,(E_{11/2} \oplus E_{13/2})$
$14n+\frac{11}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2}) \oplus 2n\,E_{13/2}$
$14n+\frac{13}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2})$
$14n+\frac{15}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2}) \oplus (2n+2)\,E_{13/2}$
$14n+\frac{17}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2}) \oplus (2n+2)(E_{11/2} \oplus E_{13/2})$
$14n+\frac{19}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus (2n+2)(E_{9/2} \oplus E_{11/2} \oplus E_{13/2})$
$14n+\frac{21}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus (2n+2)(E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2})$
$14n+\frac{23}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (2n+2)(E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2})$
$14n+\frac{25}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2})$
$14n+\frac{27}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2})$

$n = 0, 1, 2, \ldots$

T **36**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{D}_{7h}$

a'_2	e'_1	E'_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	e'_2	E'_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	e'_3	E'_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	e''_1	E''_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	e''_2	E''_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	e''_3	E''_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{1/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{5/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{7/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{9/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{11/2}$	$E_{11/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{13/2}$	$E_{13/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e'_1	e'_1	A'_1	A'_2	E'_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e'_1	e'_2	E'_1		E'_3	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e'_1	e'_3	E'_2		E'_3	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e'_1	a''_1	E''_1	
		1	2
1	1	1	0
2	1	0	1

e'_1	a''_2	E''_1	
		1	2
1	1	1	0
2	1	0	$\bar{1}$

e'_1	e''_1	A''_1	A''_2	E''_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e'_1	e''_2	E''_1		E''_3	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e'_1	e''_3	E''_2		E''_3	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e'_1	$e_{1/2}$	$E_{11/2}$		$E_{13/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e'_1	$e_{3/2}$	$E_{9/2}$		$E_{13/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e'_1	$e_{5/2}$	$E_{7/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

$\mathrm{u} = 2^{-1/2}$

↠

T **36**.11 Clebsch–Gordan coefficients *(cont.)*

e'_1	$e_{7/2}$	$E_{5/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e'_1	$e_{9/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e'_1	$e_{11/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e'_1	$e_{13/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e'_2	e'_2	A'_1	A'_2	E'_3	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e'_2	e'_3	E'_1		E'_2	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e'_2	a''_1	E''_2	
		1	2
1	1	1	0
2	1	0	1

e'_2	a''_2	E''_2	
		1	2
1	1	1	0
2	1	0	$\bar{1}$

e'_2	e''_1	E''_1		E''_3	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	0	1

e'_2	e''_2	A''_1	A''_2	E''_3	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e'_2	e''_3	E''_1		E''_2	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e'_2	$e_{1/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e'_2	$e_{3/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e'_2	$e_{5/2}$	$E_{1/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e'_2	$e_{7/2}$	$E_{3/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e'_2	$e_{9/2}$	$E_{5/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e'_2	$e_{11/2}$	$E_{7/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e'_2	$e_{13/2}$	$E_{9/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

$u = 2^{-1/2}$

↠

T **36**.11 Clebsch–Gordan coefficients *(cont.)*

e'_3	e'_3	A'_1	A'_2	E'_1	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e'_3	a''_1	E''_3	
		1	2
1	1	1	0
2	1	0	1

e'_3	a''_2	E''_3	
		1	2
1	1	1	0
2	1	0	$\bar{1}$

e'_3	e''_1	E''_2		E''_3	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

e'_3	e''_2	E''_1		E''_2	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

e'_3	e''_3	A''_1	A''_2	E''_1	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e'_3	$e_{1/2}$	$E_{7/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e'_3	$e_{3/2}$	$E_{5/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e'_3	$e_{5/2}$	$E_{3/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e'_3	$e_{7/2}$	$E_{1/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e'_3	$e_{9/2}$	$E_{1/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e'_3	$e_{11/2}$	$E_{3/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e'_3	$e_{13/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

a''_1	e''_1	E'_1	
		1	2
1	1	1	0
1	2	0	1

a''_1	e''_2	E'_2	
		1	2
1	1	1	0
1	2	0	1

a''_1	e''_3	E'_3	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{1/2}$	$E_{13/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{3/2}$	$E_{11/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{5/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{7/2}$	$E_{7/2}$	
		1	2
1	1	0	1
1	2	1	0

$u = 2^{-1/2}$

↠

T **36**.11 Clebsch–Gordan coefficients *(cont.)*

a''_1	$e_{9/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{11/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{13/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_2	e''_1	E'_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	e''_2	E'_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	e''_3	E'_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	$e_{1/2}$	$E_{13/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	$e_{3/2}$	$E_{11/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	$e_{5/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	$e_{7/2}$	$E_{7/2}$	
		1	2
1	1	0	$\bar{1}$
1	2	1	0

a''_2	$e_{9/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	$e_{11/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	$e_{13/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e''_1	e''_1	A'_1	A'_2	E'_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e''_1	e''_2	E'_1		E'_3	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e''_1	e''_3	E'_2		E'_3	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e''_1	$e_{1/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e''_1	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e''_1	$e_{5/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e''_1	$e_{7/2}$	$E_{5/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e''_1	$e_{9/2}$	$E_{7/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e''_1	$e_{11/2}$	$E_{9/2}$		$E_{13/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e''_1	$e_{13/2}$	$E_{11/2}$		$E_{13/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e''_2	e''_2	A'_1	A'_2	E'_3	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

$u = 2^{-1/2}$

↠

T **36**.11 Clebsch–Gordan coefficients *(cont.)*

e''_2	e''_3	E'_1		E'_2	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e''_2	$e_{1/2}$	$E_{9/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e''_2	$e_{3/2}$	$E_{7/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e''_2	$e_{5/2}$	$E_{5/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e''_2	$e_{7/2}$	$E_{3/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e''_2	$e_{9/2}$	$E_{1/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e''_2	$e_{11/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e''_2	$e_{13/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e''_3	e''_3	A'_1	A'_2	E'_1	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e''_3	$e_{1/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e''_3	$e_{3/2}$	$E_{3/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e''_3	$e_{5/2}$	$E_{1/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e''_3	$e_{7/2}$	$E_{1/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e''_3	$e_{9/2}$	$E_{3/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e''_3	$e_{11/2}$	$E_{5/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e''_3	$e_{13/2}$	$E_{7/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{1/2}$	$e_{1/2}$	A'_1	A'_2	E''_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{3/2}$	E'_2		E''_1	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	1	0	0

$u = 2^{-1/2}$

↠

T **36**.11 Clebsch–Gordan coefficients *(cont.)*

$e_{1/2}$	$e_{5/2}$	E'_2		E''_3	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{7/2}$	E'_3		E''_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{1/2}$	$e_{9/2}$	E'_3		E''_2	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	1	0	0

$e_{1/2}$	$e_{11/2}$	E'_1		E''_2	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{13/2}$	E'_1		A''_1	A''_2
		1	2	1	1
1	1	1	0	0	0
1	2	0	0	u	u
2	1	0	0	$\bar{u}$	u
2	2	0	1	0	0

$e_{3/2}$	$e_{3/2}$	A'_1	A'_2	E''_3	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{5/2}$	E'_3		E''_1	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{3/2}$	$e_{7/2}$	E'_2		E''_2	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{9/2}$	E'_1		E''_3	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{11/2}$	E'_3		A''_1	A''_2
		1	2	1	1
1	1	1	0	0	0
1	2	0	0	u	u
2	1	0	0	$\bar{u}$	u
2	2	0	1	0	0

$e_{3/2}$	$e_{13/2}$	E'_1		E''_2	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$e_{5/2}$	$e_{5/2}$	A'_1	A'_2	E''_2	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{5/2}$	$e_{7/2}$	E'_1		E''_1	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{5/2}$	$e_{9/2}$	E'_2		A''_1	A''_2
		1	2	1	1
1	1	0	$\bar{1}$	0	0
1	2	0	0	u	u
2	1	0	0	$\bar{u}$	u
2	2	1	0	0	0

$e_{5/2}$	$e_{11/2}$	E'_1		E''_3	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{5/2}$	$e_{13/2}$	E'_3		E''_2	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

$e_{7/2}$	$e_{7/2}$	A'_1	A'_2	A''_1	A''_2
		1	1	1	1
1	1	0	0	u	u
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	$\bar{u}$	u

$e_{7/2}$	$e_{9/2}$	E'_1		E''_1	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$u = 2^{-1/2}$

↠

T **36**.11 Clebsch–Gordan coefficients *(cont.)*

$e_{7/2}$	$e_{11/2}$	E'_2 1	2	E''_2 1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{7/2}$	$e_{13/2}$	E'_3 1	2	E''_3 1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{9/2}$	$e_{9/2}$	A'_1 1	A'_2 1	E''_2 1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{9/2}$	$e_{11/2}$	E'_3 1	2	E''_1 1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{9/2}$	$e_{13/2}$	E'_2 1	2	E''_3 1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$e_{11/2}$	$e_{11/2}$	A'_1 1	A'_2 1	E''_3 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{11/2}$	$e_{13/2}$	E'_2 1	2	E''_1 1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

$e_{13/2}$	$e_{13/2}$	A'_1 1	A'_2 1	E''_1 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

$8/mmm$	$\lvert G\rvert = 32$	$\lvert C\rvert = 14$	$\lvert\widetilde{C}\rvert = 22$	T **37**	p. 245	$\mathbf{D}_{8h}$

(1) Product forms: $\mathbf{D}_8 \otimes \mathbf{C}_i$, $\mathbf{D}_8 \otimes \mathbf{C}_s$, $\mathbf{C}_{8v} \otimes \mathbf{C}_s$.

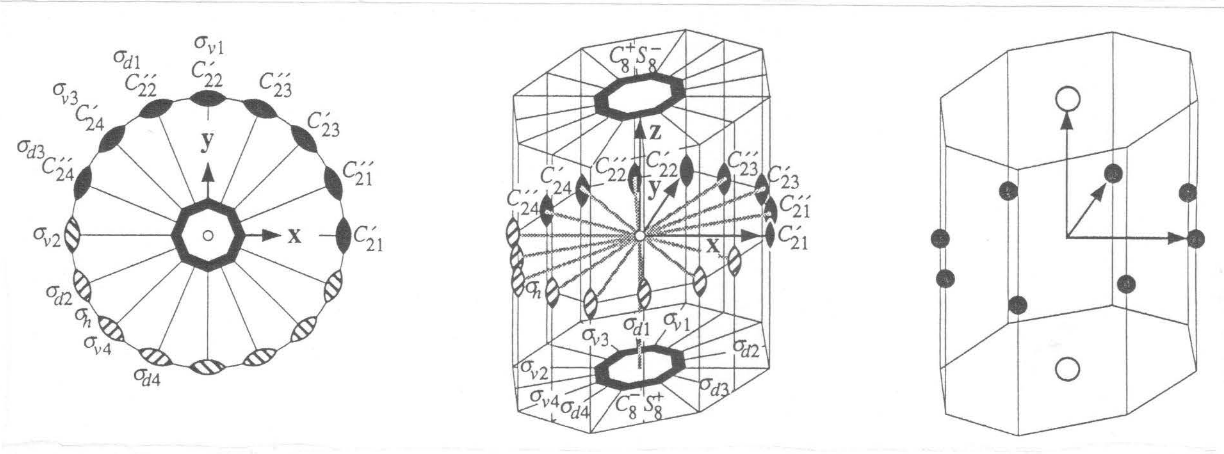
(2) Group chains: $\mathbf{D}_{8h} \supset \underline{\mathbf{C}_{8h}}$, $\mathbf{D}_{8h} \supset (\underline{\mathbf{C}_{8v}})$, $\mathbf{D}_{8h} \supset \underline{\mathbf{D}_{4d}}$, $\mathbf{D}_{8h} \supset (\underline{\mathbf{D}_{4h}})$, $\mathbf{D}_{8h} \supset \underline{\mathbf{D}_8}$.

(3) Operations of G: E, (C_8^+, C_8^-), (C_4^+, C_4^-), (C_8^{3+}, C_8^{3-}),
C_2, $(C'_{21}, C'_{22}, C'_{23}, C'_{24})$, $(C''_{21}, C''_{22}, C''_{23}, C''_{24})$,
i, (S_8^{3-}, S_8^{3+}), (S_4^-, S_4^+), (S_8^-, S_8^+), σ_h, $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4})$, $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_8^+, C_8^-), $(\widetilde{C}_8^+, \widetilde{C}_8^-)$, (C_4^+, C_4^-), $(\widetilde{C}_4^+, \widetilde{C}_4^-)$, (C_8^{3+}, C_8^{3-}), $(\widetilde{C}_8^{3+}, \widetilde{C}_8^{3-})$,
$(C_2, \widetilde{C}_2)$, $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, \widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24})$,
$(C''_{21}, C''_{22}, C''_{23}, C''_{24}, \widetilde{C}''_{21}, \widetilde{C}''_{22}, \widetilde{C}''_{23}, \widetilde{C}''_{24})$,
i, $\widetilde{\imath}$, (S_8^{3-}, S_8^{3+}), $(\widetilde{S}_8^{3-}, \widetilde{S}_8^{3+})$, (S_4^-, S_4^+), $(\widetilde{S}_4^-, \widetilde{S}_4^+)$, (S_8^-, S_8^+), $(\widetilde{S}_8^-, \widetilde{S}_8^+)$, $(\sigma_h, \widetilde{\sigma}_h)$,
$(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2}, \widetilde{\sigma}_{v3}, \widetilde{\sigma}_{v4})$, $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3}, \widetilde{\sigma}_{d4})$.

(5) Classes and representations: $\lvert r\rvert = 8$, $\lvert i\rvert = 6$, $\lvert I\rvert = 14$, $\lvert\widetilde{I}\rvert = 8$.

F **37** See Chapter **15**, p. 65



Examples: Uranocene $U(C_8H_8)_2$.

T **37**.0 Subgroup elements

§ **16**–0, p. 68

$\mathbf{D}_{8h}$	$\mathbf{C}_{8h}$	$\mathbf{C}_{4h}$	$\mathbf{C}_{2h}$	$\mathbf{C}_{8v}$	$\mathbf{C}_{4v}$	$\mathbf{C}_{2v}$	$\mathbf{D}_{4d}$	$\mathbf{D}_{2d}$	$\mathbf{D}_{4h}$	$\mathbf{D}_{2h}$	$\mathbf{D}_{8}$	$\mathbf{D}_{4}$	$\mathbf{D}_{2}$	$\mathbf{S}_{8}$	$\mathbf{S}_{4}$	$\mathbf{C}_{s}$	$\mathbf{C}_{i}$	$\mathbf{C}_{8}$	$\mathbf{C}_{4}$	$\mathbf{C}_{2}$
E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
C_8^+	C_8^+			C_8^+							C_8^+							C_8^+		
C_8^-	C_8^-			C_8^-							C_8^-							C_8^-		
C_4^+	C_4^+	C_4^+		C_4^+	C_4^+		C_4^+		C_4^+		C_4^+	C_4^+		C_4^+				C_4^+	C_4^+	
C_4^-	C_4^-	C_4^-		C_4^-	C_4^-		C_4^-		C_4^-		C_4^-	C_4^-		C_4^-				C_4^-	C_4^-	
C_8^{3+}	C_8^{3+}			C_8^{3+}							C_8^{3+}							C_8^{3+}		
C_8^{3-}	C_8^{3-}			C_8^{3-}							C_8^{3-}							C_8^{3-}		
C_2	C_2	C_2	C_2	C_2	C_2	C_2	C_2	C_2	C_2	C_{2z}	C_2	C_2	C_{2z}	C_2	C_2			C_2	C_2	C_2
C'_{21}							C'_{21}	C'_{21}	C'_{21}	C_{2x}	C'_{21}	C'_{21}	C_{2x}							
C'_{22}							C'_{22}	C'_{22}	C'_{22}	C_{2y}	C'_{22}	C'_{22}	C_{2y}							
C'_{23}							C'_{23}		C''_{21}		C'_{23}	C''_{21}								
C'_{24}							C'_{24}		C''_{22}		C'_{24}	C''_{22}								
C''_{21}											C''_{21}									
C''_{22}											C''_{22}									
C''_{23}											C''_{23}									
C''_{24}											C''_{24}									
i	i	i	i						i	i							i			
S_8^{3-}	S_8^{3-}						S_8^{3-}							S_8^{3-}						
S_8^{3+}	S_8^{3+}						S_8^{3+}							S_8^{3+}						
S_4^-	S_4^-	S_4^-						S_4^-	S_4^-						S_4^-					
S_4^+	S_4^+	S_4^+						S_4^+	S_4^+						S_4^+					
S_8^-	S_8^-						S_8^-							S_8^-						
S_8^+	S_8^+						S_8^+							S_8^+						
σ_h	σ_h	σ_h	σ_h						σ_h	σ_z						σ_h				
σ_{v1}				σ_{v1}	σ_{v1}	σ_x			σ_{v1}	σ_x										
σ_{v2}				σ_{v2}	σ_{v2}	σ_y			σ_{v2}	σ_y										
σ_{v3}				σ_{v3}	σ_{d1}			σ_{d1}	σ_{d1}											
σ_{v4}				σ_{v4}	σ_{d2}			σ_{d2}	σ_{d2}											
σ_{d1}				σ_{d1}			σ_{d1}													
σ_{d2}				σ_{d2}			σ_{d2}													
σ_{d3}				σ_{d3}			σ_{d3}													
σ_{d4}				σ_{d4}			σ_{d4}													

T **37**.1 Parameters

§ **16**–1, p. 68

$\mathbf{D}_{8h}$		α	β	γ	ϕ	$\mathbf{n}$	λ	$\mathbf{\Lambda}$
E	i	0	0	0	0	$(0\ 0\ 0)$	$[\![1,$	$(0\ 0\ 0)]\!]$
C_8^+	S_8^{3-}	0	0	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$(0\ 0\ 1)$	$[\![c_8,$	$(0\ 0\ s_8)]\!]$
C_8^-	S_8^{3+}	0	0	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$(0\ 0\ {-1})$	$[\![c_8,$	$(0\ 0\ {-s_8})]\!]$
C_4^+	S_4^-	0	0	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$(0\ 0\ 1)$	$[\![\frac{1}{\sqrt{2}},$	$(0\ 0\ \frac{1}{\sqrt{2}})]\!]$
C_4^-	S_4^+	0	0	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	$(0\ 0\ {-1})$	$[\![\frac{1}{\sqrt{2}},$	$(0\ 0\ {-\frac{1}{\sqrt{2}}})]\!]$
C_8^{3+}	S_8^-	0	0	$\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$(0\ 0\ 1)$	$[\![s_8,$	$(0\ 0\ c_8)]\!]$
C_8^{3-}	S_8^+	0	0	$-\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$(0\ 0\ {-1})$	$[\![s_8,$	$(0\ 0\ {-c_8})]\!]$
C_2	σ_h	0	0	π	π	$(0\ 0\ 1)$	$[\![0,$	$(0\ 0\ 1)]\!]$
C'_{21}	σ_{v1}	0	π	π	π	$(1\ 0\ 0)$	$[\![0,$	$(1\ 0\ 0)]\!]$
C'_{22}	σ_{v2}	0	π	0	π	$(0\ 1\ 0)$	$[\![0,$	$(0\ 1\ 0)]\!]$
C'_{23}	σ_{v3}	0	π	$\frac{\pi}{2}$	π	$(\frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}}\ 0)$	$[\![0,$	$(\frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}}\ 0)]\!]$
C'_{24}	σ_{v4}	0	π	$-\frac{\pi}{2}$	π	$(-\frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}}\ 0)$	$[\![0,$	$(-\frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}}\ 0)]\!]$
C''_{21}	σ_{d1}	0	π	$\frac{3\pi}{4}$	π	$(c_8\ s_8\ 0)$	$[\![0,$	$(c_8\ s_8\ 0)]\!]$
C''_{22}	σ_{d2}	0	π	$-\frac{\pi}{4}$	π	$(-s_8\ c_8\ 0)$	$[\![0,$	$(-s_8\ c_8\ 0)]\!]$
C''_{23}	σ_{d3}	0	π	$\frac{\pi}{4}$	π	$(s_8\ c_8\ 0)$	$[\![0,$	$(s_8\ c_8\ 0)]\!]$
C''_{24}	σ_{d4}	0	π	$-\frac{3\pi}{4}$	π	$(-c_8\ s_8\ 0)$	$[\![0,$	$(-c_8\ s_8\ 0)]\!]$

$c_n = \cos\frac{\pi}{n}$, $s_n = \sin\frac{\pi}{n}$

T **37**.2 Multiplication table

§ **16**–2, p. 69

$\mathbf{D}_{8h}$	E	C_8^+	C_8^-	C_4^+	C_4^-	C_8^{3+}	C_8^{3-}	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C''_{21}	C''_{22}	C''_{23}	C''_{24}	i	S_8^{3-}	S_8^{3+}	S_4^-	S_4^+	S_8^-	S_8^+	σ_h	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}
E	E	C_8^+	C_8^-	C_4^+	C_4^-	C_8^{3+}	C_8^{3-}	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C''_{21}	C''_{22}	C''_{23}	C''_{24}	i	S_8^{3-}	S_8^{3+}	S_4^-	S_4^+	S_8^-	S_8^+	σ_h	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}
C_8^+	C_8^+	C_4^+	E	C_8^{3+}	C_8^-	C_2	C_4^-	C_8^{3-}	C''_{21}	C''_{22}	C''_{23}	C''_{24}	C'_{23}	C'_{24}	C'_{22}	C'_{21}	S_8^{3-}	S_4^-	i	S_8^-	S_8^{3+}	σ_h	S_4^+	S_8^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{v3}	σ_{v4}	σ_{v2}	σ_{v1}
C_8^-	C_8^-	E	C_4^-	C_8^+	C_8^{3-}	C_4^+	C_2	C_8^{3+}	C''_{24}	C''_{23}	C''_{21}	C''_{22}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	S_8^{3+}	i	S_4^+	S_8^{3-}	S_8^+	S_4^-	σ_h	S_8^-	σ_{d4}	σ_{d3}	σ_{d1}	σ_{d2}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}
C_4^+	C_4^+	C_8^{3+}	C_8^+	C_2	E	C_8^{3-}	C_8^-	C_4^-	C'_{23}	C'_{24}	C'_{22}	C'_{21}	C''_{23}	C''_{24}	C''_{22}	C''_{21}	S_4^-	S_8^-	S_8^{3-}	σ_h	i	S_8^+	S_8^{3+}	S_4^+	σ_{v3}	σ_{v4}	σ_{v2}	σ_{v1}	σ_{d3}	σ_{d4}	σ_{d2}	σ_{d1}
C_4^-	C_4^-	C_8^-	C_8^{3-}	E	C_2	C_8^+	C_8^{3+}	C_4^+	C'_{24}	C'_{23}	C'_{21}	C'_{22}	C''_{24}	C''_{23}	C''_{21}	C''_{22}	S_4^+	S_8^{3+}	S_8^+	i	σ_h	S_8^{3-}	S_8^-	S_4^-	σ_{v4}	σ_{v3}	σ_{v1}	σ_{v2}	σ_{d4}	σ_{d3}	σ_{d1}	σ_{d2}
C_8^{3+}	C_8^{3+}	C_2	C_4^+	C_8^{3-}	C_8^+	C_4^-	E	C_8^-	C''_{23}	C''_{24}	C''_{22}	C''_{21}	C'_{22}	C'_{21}	C'_{24}	C'_{23}	S_8^-	σ_h	S_4^-	S_8^+	S_8^{3-}	S_4^+	i	S_8^{3+}	σ_{d3}	σ_{d4}	σ_{d2}	σ_{d1}	σ_{v2}	σ_{v1}	σ_{v4}	σ_{v3}
C_8^{3-}	C_8^{3-}	C_4^-	C_2	C_8^-	C_8^{3+}	E	C_4^+	C_8^+	C''_{22}	C''_{21}	C''_{24}	C''_{23}	C'_{24}	C'_{23}	C'_{21}	C'_{22}	S_8^+	S_4^+	σ_h	S_8^{3+}	S_8^-	i	S_4^-	S_8^{3-}	σ_{d2}	σ_{d1}	σ_{d4}	σ_{d3}	σ_{v4}	σ_{v3}	σ_{v1}	σ_{v2}
C_2	C_2	C_8^{3-}	C_8^{3+}	C_4^-	C_4^+	C_8^-	C_8^+	E	C'_{22}	C'_{21}	C'_{24}	C'_{23}	C''_{22}	C''_{21}	C''_{24}	C''_{23}	σ_h	S_8^+	S_8^-	S_4^+	S_4^-	S_8^{3+}	S_8^{3-}	i	σ_{v2}	σ_{v1}	σ_{v4}	σ_{v3}	σ_{d2}	σ_{d1}	σ_{d4}	σ_{d3}
C'_{21}	C'_{21}	C''_{24}	C''_{21}	C'_{24}	C'_{23}	C''_{22}	C''_{23}	C'_{22}	E	C_2	C_4^-	C_4^+	C_8^-	C_8^{3+}	C_8^{3-}	C_8^+	σ_{v1}	σ_{d4}	σ_{d1}	σ_{v4}	σ_{v3}	σ_{d2}	σ_{d3}	σ_{v2}	i	σ_h	S_4^+	S_4^-	S_8^{3+}	S_8^-	S_8^+	S_8^{3-}
C'_{22}	C'_{22}	C''_{23}	C''_{22}	C'_{23}	C'_{24}	C''_{21}	C''_{24}	C'_{21}	C_2	E	C_4^+	C_4^-	C_8^{3+}	C_8^-	C_8^+	C_8^{3-}	σ_{v2}	σ_{d3}	σ_{d2}	σ_{v3}	σ_{v4}	σ_{d1}	σ_{d4}	σ_{v1}	σ_h	i	S_4^-	S_4^+	S_8^-	S_8^{3+}	S_8^{3-}	S_8^+
C'_{23}	C'_{23}	C''_{21}	C''_{23}	C'_{21}	C'_{22}	C''_{24}	C''_{22}	C'_{24}	C_4^+	C_4^-	E	C_2	C_8^+	C_8^{3-}	C_8^-	C_8^{3+}	σ_{v3}	σ_{d1}	σ_{d3}	σ_{v1}	σ_{v2}	σ_{d4}	σ_{d2}	σ_{v4}	S_4^-	S_4^+	i	σ_h	S_8^{3-}	S_8^+	S_8^{3+}	S_8^-
C'_{24}	C'_{24}	C''_{22}	C''_{24}	C'_{22}	C'_{21}	C''_{23}	C''_{21}	C'_{23}	C_4^-	C_4^+	C_2	E	C_8^{3-}	C_8^+	C_8^{3+}	C_8^-	σ_{v4}	σ_{d2}	σ_{d4}	σ_{v2}	σ_{v1}	σ_{d3}	σ_{d1}	σ_{v3}	S_4^+	S_4^-	σ_h	i	S_8^+	S_8^{3-}	S_8^-	S_8^{3+}
C''_{21}	C''_{21}	C'_{21}	C'_{23}	C''_{24}	C''_{23}	C'_{24}	C'_{22}	C''_{22}	C_8^+	C_8^{3-}	C_8^-	C_8^{3+}	E	C_2	C_4^-	C_4^+	σ_{d1}	σ_{v1}	σ_{v3}	σ_{d4}	σ_{d3}	σ_{v4}	σ_{v2}	σ_{d2}	S_8^{3-}	S_8^+	S_8^{3+}	S_8^-	i	σ_h	S_4^+	S_4^-
C''_{22}	C''_{22}	C'_{22}	C'_{24}	C''_{23}	C''_{24}	C'_{23}	C'_{21}	C''_{21}	C_8^{3-}	C_8^+	C_8^{3+}	C_8^-	C_2	E	C_4^+	C_4^-	σ_{d2}	σ_{v2}	σ_{v4}	σ_{d3}	σ_{d4}	σ_{v3}	σ_{v1}	σ_{d1}	S_8^+	S_8^{3-}	S_8^-	S_8^{3+}	σ_h	i	S_4^-	S_4^+
C''_{23}	C''_{23}	C'_{23}	C'_{22}	C''_{21}	C''_{22}	C'_{21}	C'_{24}	C''_{24}	C_8^{3+}	C_8^-	C_8^+	C_8^{3-}	C_4^+	C_4^-	E	C_2	σ_{d3}	σ_{v3}	σ_{v2}	σ_{d1}	σ_{d2}	σ_{v1}	σ_{v4}	σ_{d4}	S_8^-	S_8^{3+}	S_8^{3-}	S_8^+	S_4^-	S_4^+	i	σ_h
C''_{24}	C''_{24}	C'_{24}	C'_{21}	C''_{22}	C''_{21}	C'_{22}	C'_{23}	C''_{23}	C_8^-	C_8^{3+}	C_8^{3-}	C_8^+	C_4^-	C_4^+	C_2	E	σ_{d4}	σ_{v4}	σ_{v1}	σ_{d2}	σ_{d1}	σ_{v2}	σ_{v3}	σ_{d3}	S_8^{3+}	S_8^-	S_8^+	S_8^{3-}	S_4^+	S_4^-	σ_h	i
i	i	S_8^{3-}	S_8^{3+}	S_4^-	S_4^+	S_8^-	S_8^+	σ_h	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	E	C_8^+	C_8^-	C_4^+	C_4^-	C_8^{3+}	C_8^{3-}	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C''_{21}	C''_{22}	C''_{23}	C''_{24}
S_8^{3-}	S_8^{3-}	S_4^-	i	S_8^-	S_8^{3+}	σ_h	S_4^+	S_8^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{v3}	σ_{v4}	σ_{v2}	σ_{v1}	C_8^+	C_4^+	E	C_8^{3+}	C_8^-	C_2	C_4^-	C_8^{3-}	C''_{21}	C''_{22}	C''_{23}	C''_{24}	C'_{23}	C'_{24}	C'_{22}	C'_{21}
S_8^{3+}	S_8^{3+}	i	S_4^+	S_8^{3-}	S_8^+	S_4^-	σ_h	S_8^-	σ_{d4}	σ_{d3}	σ_{d1}	σ_{d2}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	C_8^-	E	C_4^-	C_8^+	C_8^{3-}	C_4^+	C_2	C_8^{3+}	C''_{24}	C''_{23}	C''_{21}	C''_{22}	C'_{21}	C'_{22}	C'_{23}	C'_{24}
S_4^-	S_4^-	S_8^-	S_8^{3-}	σ_h	i	S_8^+	S_8^{3+}	S_4^+	σ_{v3}	σ_{v4}	σ_{v2}	σ_{v1}	σ_{d3}	σ_{d4}	σ_{d2}	σ_{d1}	C_4^+	C_8^{3+}	C_8^+	C_2	E	C_8^{3-}	C_8^-	C_4^-	C'_{23}	C'_{24}	C'_{22}	C'_{21}	C''_{23}	C''_{24}	C''_{22}	C''_{21}
S_4^+	S_4^+	S_8^{3+}	S_8^+	i	σ_h	S_8^{3-}	S_8^-	S_4^-	σ_{v4}	σ_{v3}	σ_{v1}	σ_{v2}	σ_{d4}	σ_{d3}	σ_{d1}	σ_{d2}	C_4^-	C_8^-	C_8^{3-}	E	C_2	C_8^+	C_8^{3+}	C_4^+	C'_{24}	C'_{23}	C'_{21}	C'_{22}	C''_{24}	C''_{23}	C''_{21}	C''_{22}
S_8^-	S_8^-	σ_h	S_4^-	S_8^+	S_8^{3-}	S_4^+	i	S_8^{3+}	σ_{d3}	σ_{d4}	σ_{d2}	σ_{d1}	σ_{v2}	σ_{v1}	σ_{v4}	σ_{v3}	C_8^{3+}	C_2	C_4^+	C_8^{3-}	C_8^+	C_4^-	E	C_8^-	C''_{23}	C''_{24}	C''_{22}	C''_{21}	C'_{22}	C'_{21}	C'_{24}	C'_{23}
S_8^+	S_8^+	S_4^+	σ_h	S_8^{3+}	S_8^-	i	S_4^-	S_8^{3-}	σ_{d2}	σ_{d1}	σ_{d4}	σ_{d3}	σ_{v4}	σ_{v3}	σ_{v1}	σ_{v2}	C_8^{3-}	C_4^-	C_2	C_8^-	C_8^{3+}	E	C_4^+	C_8^+	C''_{22}	C''_{21}	C''_{24}	C''_{23}	C'_{24}	C'_{23}	C'_{21}	C'_{22}
σ_h	σ_h	S_8^+	S_8^-	S_4^+	S_4^-	S_8^{3+}	S_8^{3-}	i	σ_{v2}	σ_{v1}	σ_{v4}	σ_{v3}	σ_{d2}	σ_{d1}	σ_{d4}	σ_{d3}	C_2	C_8^{3-}	C_8^{3+}	C_4^-	C_4^+	C_8^-	C_8^+	E	C'_{22}	C'_{21}	C'_{24}	C'_{23}	C''_{22}	C''_{21}	C''_{24}	C''_{23}
σ_{v1}	σ_{v1}	σ_{d4}	σ_{d1}	σ_{v4}	σ_{v3}	σ_{d2}	σ_{d3}	σ_{v2}	i	σ_h	S_4^+	S_4^-	S_8^{3+}	S_8^-	S_8^+	S_8^{3-}	C'_{21}	C''_{24}	C''_{21}	C'_{24}	C'_{23}	C''_{22}	C''_{23}	C'_{22}	E	C_2	C_4^-	C_4^+	C_8^-	C_8^{3+}	C_8^{3-}	C_8^+
σ_{v2}	σ_{v2}	σ_{d3}	σ_{d2}	σ_{v3}	σ_{v4}	σ_{d1}	σ_{d4}	σ_{v1}	σ_h	i	S_4^-	S_4^+	S_8^-	S_8^{3+}	S_8^{3-}	S_8^+	C'_{22}	C''_{23}	C''_{22}	C'_{23}	C'_{24}	C''_{21}	C''_{24}	C'_{21}	C_2	E	C_4^+	C_4^-	C_8^{3+}	C_8^-	C_8^+	C_8^{3-}
σ_{v3}	σ_{v3}	σ_{d1}	σ_{d3}	σ_{v1}	σ_{v2}	σ_{d4}	σ_{d2}	σ_{v4}	S_4^-	S_4^+	i	σ_h	S_8^{3-}	S_8^+	S_8^{3+}	S_8^-	C'_{23}	C''_{21}	C''_{23}	C'_{21}	C'_{22}	C''_{24}	C''_{22}	C'_{24}	C_4^+	C_4^-	E	C_2	C_8^+	C_8^{3-}	C_8^-	C_8^{3+}
σ_{v4}	σ_{v4}	σ_{d2}	σ_{d4}	σ_{v2}	σ_{v1}	σ_{d3}	σ_{d1}	σ_{v3}	S_4^+	S_4^-	σ_h	i	S_8^+	S_8^{3-}	S_8^-	S_8^{3+}	C'_{24}	C''_{22}	C''_{24}	C'_{22}	C'_{21}	C''_{23}	C''_{21}	C'_{23}	C_4^-	C_4^+	C_2	E	C_8^{3-}	C_8^+	C_8^{3+}	C_8^-
σ_{d1}	σ_{d1}	σ_{v1}	σ_{v3}	σ_{d4}	σ_{d3}	σ_{v4}	σ_{v2}	σ_{d2}	S_8^{3-}	S_8^+	S_8^{3+}	S_8^-	i	σ_h	S_4^+	S_4^-	C''_{21}	C'_{21}	C'_{23}	C''_{24}	C''_{23}	C'_{24}	C'_{22}	C''_{22}	C_8^+	C_8^{3-}	C_8^-	C_8^{3+}	E	C_2	C_4^-	C_4^+
σ_{d2}	σ_{d2}	σ_{v2}	σ_{v4}	σ_{d3}	σ_{d4}	σ_{v3}	σ_{v1}	σ_{d1}	S_8^+	S_8^{3-}	S_8^-	S_8^{3+}	σ_h	i	S_4^-	S_4^+	C''_{22}	C'_{22}	C'_{24}	C''_{23}	C''_{24}	C'_{23}	C'_{21}	C''_{21}	C_8^{3-}	C_8^+	C_8^{3+}	C_8^-	C_2	E	C_4^+	C_4^-
σ_{d3}	σ_{d3}	σ_{v3}	σ_{v2}	σ_{d1}	σ_{d2}	σ_{v1}	σ_{v4}	σ_{d4}	S_8^-	S_8^{3+}	S_8^{3-}	S_8^+	S_4^-	S_4^+	i	σ_h	C''_{23}	C'_{23}	C'_{22}	C''_{21}	C''_{22}	C'_{21}	C'_{24}	C''_{24}	C_8^{3+}	C_8^-	C_8^+	C_8^{3-}	C_4^+	C_4^-	E	C_2
σ_{d4}	σ_{d4}	σ_{v4}	σ_{v1}	σ_{d2}	σ_{d1}	σ_{v2}	σ_{v3}	σ_{d3}	S_8^{3+}	S_8^-	S_8^+	S_8^{3-}	S_4^+	S_4^-	σ_h	i	C''_{24}	C'_{24}	C'_{21}	C''_{22}	C''_{21}	C'_{22}	C'_{23}	C''_{23}	C_8^-	C_8^{3+}	C_8^{3-}	C_8^+	C_4^-	C_4^+	C_2	E

T **37**.3 Factor table

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$\mathbf{D}_{8h}$	E	C_8^+	C_8^-	C_4^+	C_4^-	C_8^{3+}	C_8^{3-}	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C''_{21}	C''_{22}	C''_{23}	C''_{24}	i	S_8^{3-}	S_8^{3+}	S_4^-	S_4^+	S_8^-	S_8^+	σ_h	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_8^+	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	−1
C_8^-	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1
C_4^+	1	1	1	1	1	−1	1	−1	1	1	1	−1	1	1	1	−1	1	1	1	1	1	−1	1	−1	1	1	1	−1	1	1	1	−1
C_4^-	1	1	1	1	−1	1	−1	1	−1	1	1	1	−1	1	1	1	1	1	1	1	−1	1	−1	1	−1	1	1	1	−1	1	1	1
C_8^{3+}	1	1	1	−1	1	−1	1	−1	1	1	1	−1	1	−1	1	−1	1	1	1	−1	1	−1	1	−1	1	1	1	−1	1	−1	1	−1
C_8^{3-}	1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	1	1
C_2	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1
C'_{21}	1	−1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	−1	1
C'_{22}	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1
C'_{23}	1	1	1	1	1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	1
C'_{24}	1	1	1	1	−1	1	−1	1	1	−1	−1	−1	1	−1	−1	−1	1	1	1	1	−1	1	−1	1	1	−1	−1	−1	1	−1	−1	−1
C''_{21}	1	1	1	−1	1	−1	1	−1	−1	−1	−1	1	−1	1	−1	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	1	−1	1	−1	1
C''_{22}	1	1	1	1	1	1	−1	1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	−1	1	1	−1	−1	−1	−1	−1	−1	−1
C''_{23}	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1
C''_{24}	1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	−1	−1	1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	−1	−1
i	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_8^{3-}	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	−1
S_8^{3+}	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1
S_4^-	1	1	1	1	1	−1	1	−1	1	1	1	−1	1	1	1	−1	1	1	1	1	1	−1	1	−1	1	1	1	−1	1	1	1	−1
S_4^+	1	1	1	1	−1	1	−1	1	−1	1	1	1	−1	1	1	1	1	1	1	1	−1	1	−1	1	−1	1	1	1	−1	1	1	1
S_8^-	1	1	1	−1	1	−1	1	−1	1	1	1	−1	1	−1	1	−1	1	1	1	−1	1	−1	1	−1	1	1	1	−1	1	−1	1	−1
S_8^+	1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	1	1
σ_h	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1
σ_{v1}	1	−1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	−1	1
σ_{v2}	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1
σ_{v3}	1	1	1	1	1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	1
σ_{v4}	1	1	1	1	−1	1	−1	1	1	−1	−1	−1	1	−1	−1	−1	1	1	1	1	−1	1	−1	1	1	−1	−1	−1	1	−1	−1	−1
σ_{d1}	1	1	1	−1	1	−1	1	−1	−1	−1	−1	1	−1	1	−1	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	1	−1	1	−1	1
σ_{d2}	1	1	1	1	1	1	−1	1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	−1	1	1	−1	−1	−1	−1	−1	−1	−1
σ_{d3}	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1
σ_{d4}	1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	−1	−1	1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	−1	−1

T **37**.4 Character table

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$\mathbf{D}_{8h}$	E	$2C_8$	$2C_4$	$2C_8^3$	C_2	$4C_2'$	$4C_2''$	i	$2S_8^3$	$2S_4$	$2S_8$	σ_h	$4\sigma_v$	$4\sigma_d$	τ
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	a
A_{2g}	1	1	1	1	1	−1	−1	1	1	1	1	1	−1	−1	a
B_{1g}	1	−1	1	−1	1	1	−1	1	−1	1	−1	1	1	−1	a
B_{2g}	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	1	a
E_{1g}	2	$\sqrt{2}$	0	$-\sqrt{2}$	−2	0	0	2	$\sqrt{2}$	0	$-\sqrt{2}$	−2	0	0	a
E_{2g}	2	0	−2	0	2	0	0	2	0	−2	0	2	0	0	a
E_{3g}	2	$-\sqrt{2}$	0	$\sqrt{2}$	−2	0	0	2	$-\sqrt{2}$	0	$\sqrt{2}$	−2	0	0	a
A_{1u}	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	a
A_{2u}	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	1	1	a
B_{1u}	1	−1	1	−1	1	1	−1	−1	1	−1	1	−1	−1	1	a
B_{2u}	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	a
E_{1u}	2	$\sqrt{2}$	0	$-\sqrt{2}$	−2	0	0	−2	$-\sqrt{2}$	0	$\sqrt{2}$	2	0	0	a
E_{2u}	2	0	−2	0	2	0	0	−2	0	2	0	−2	0	0	a
E_{3u}	2	$-\sqrt{2}$	0	$\sqrt{2}$	−2	0	0	−2	$\sqrt{2}$	0	$-\sqrt{2}$	2	0	0	a
$E_{1/2,g}$	2	$2c_8$	$\sqrt{2}$	$2c_8^3$	0	0	0	2	$2c_8$	$\sqrt{2}$	$2c_8^3$	0	0	0	c
$E_{3/2,g}$	2	$2c_8^3$	$-\sqrt{2}$	$-2c_8$	0	0	0	2	$2c_8^3$	$-\sqrt{2}$	$-2c_8$	0	0	0	c
$E_{5/2,g}$	2	$-2c_8^3$	$-\sqrt{2}$	$2c_8$	0	0	0	2	$-2c_8^3$	$-\sqrt{2}$	$2c_8$	0	0	0	c
$E_{7/2,g}$	2	$-2c_8$	$\sqrt{2}$	$-2c_8^3$	0	0	0	2	$-2c_8$	$\sqrt{2}$	$-2c_8^3$	0	0	0	c
$E_{1/2,u}$	2	$2c_8$	$\sqrt{2}$	$2c_8^3$	0	0	0	−2	$-2c_8$	$-\sqrt{2}$	$-2c_8^3$	0	0	0	c
$E_{3/2,u}$	2	$2c_8^3$	$-\sqrt{2}$	$-2c_8$	0	0	0	−2	$-2c_8^3$	$\sqrt{2}$	$2c_8$	0	0	0	c
$E_{5/2,u}$	2	$-2c_8^3$	$-\sqrt{2}$	$2c_8$	0	0	0	−2	$2c_8^3$	$\sqrt{2}$	$-2c_8$	0	0	0	c
$E_{7/2,u}$	2	$-2c_8$	$\sqrt{2}$	$-2c_8^3$	0	0	0	−2	$2c_8$	$-\sqrt{2}$	$2c_8^3$	0	0	0	c

$c_n^m = \cos \frac{m}{n}\pi$

T **37**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{8h}$	0	1	2	3
A_{1g}	□1		$x^2+y^2, {}^{\square}z^2$	
A_{2g}		R_z		
B_{1g}				
B_{2g}				
E_{1g}		(R_x, R_y)	□(zx, yz)	
E_{2g}			□(xy, x^2-y^2)	
E_{3g}				
A_{1u}				
A_{2u}		□z		$(x^2+y^2)z, {}^{\square}z^3$
B_{1u}				
B_{2u}				
E_{1u}		□(x, y)		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^{\square}(xz^2, yz^2)$
E_{2u}				□$\{xyz, z(x^2-y^2)\}$
E_{3u}				□$\{x(x^2-3y^2), y(3x^2-y^2)\}$

T **37**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{8h}$	$\langle\vert j\, m\rangle\vert$		ι	μ
A_{1g}	$\vert 0\,0\rangle_+$		2	8
A_{2g}	$\vert 8\,8\rangle_-$		2	8
B_{1g}	$\vert 4\,4\rangle_+$		2	8
B_{2g}	$\vert 4\,4\rangle_-$		2	8
E_{1g}	$\langle\vert 2\,1\rangle, -\vert 2\,\overline{1}\rangle\vert$		2	± 8
E_{2g}	$\langle\vert 2\,2\rangle, \vert 2\,\overline{2}\rangle\vert$		2	± 8
E_{3g}	$\langle\vert 4\,\overline{3}\rangle, -\vert 4\,3\rangle\vert$		2	± 8
A_{1u}	$\vert 9\,8\rangle_-$		2	8
A_{2u}	$\vert 1\,0\rangle_+$		2	8
B_{1u}	$\vert 5\,4\rangle_-$		2	8
B_{2u}	$\vert 5\,4\rangle_+$		2	8
E_{1u}	$\langle\vert 1\,1\rangle, \vert 1\,\overline{1}\rangle\vert$		2	± 8
E_{2u}	$\langle\vert 3\,2\rangle, -\vert 3\,\overline{2}\rangle\vert$		2	± 8
E_{3u}	$\langle\vert 3\,\overline{3}\rangle, \vert 3\,3\rangle\vert$		2	± 8
$E_{1/2,g}$	$\langle\vert\frac{1}{2}\,\frac{1}{2}\rangle, \vert\frac{1}{2}\,\overline{\frac{1}{2}}\rangle\vert$	$\langle\vert\frac{3}{2}\,\frac{1}{2}\rangle, -\vert\frac{3}{2}\,\overline{\frac{1}{2}}\rangle\vert$	2	± 8
$E_{3/2,g}$	$\langle\vert\frac{3}{2}\,\overline{\frac{3}{2}}\rangle, -\vert\frac{3}{2}\,\frac{3}{2}\rangle\vert$	$\langle\vert\frac{5}{2}\,\overline{\frac{3}{2}}\rangle, \vert\frac{5}{2}\,\frac{3}{2}\rangle\vert$	2	± 8
$E_{5/2,g}$	$\langle\vert\frac{5}{2}\,\frac{5}{2}\rangle, \vert\frac{5}{2}\,\overline{\frac{5}{2}}\rangle\vert$	$\langle\vert\frac{7}{2}\,\frac{5}{2}\rangle, -\vert\frac{7}{2}\,\overline{\frac{5}{2}}\rangle\vert$	2	± 8
$E_{7/2,g}$	$\langle\vert\frac{7}{2}\,\overline{\frac{7}{2}}\rangle, -\vert\frac{7}{2}\,\frac{7}{2}\rangle\vert$	$\langle\vert\frac{9}{2}\,\overline{\frac{7}{2}}\rangle, \vert\frac{9}{2}\,\frac{7}{2}\rangle\vert$	2	± 8
$E_{1/2,u}$	$\langle\vert\frac{1}{2}\,\frac{1}{2}\rangle, \vert\frac{1}{2}\,\overline{\frac{1}{2}}\rangle\vert^\bullet$	$\langle\vert\frac{3}{2}\,\frac{1}{2}\rangle, -\vert\frac{3}{2}\,\overline{\frac{1}{2}}\rangle\vert^\bullet$	2	± 8
$E_{3/2,u}$	$\langle\vert\frac{3}{2}\,\overline{\frac{3}{2}}\rangle, -\vert\frac{3}{2}\,\frac{3}{2}\rangle\vert^\bullet$	$\langle\vert\frac{5}{2}\,\overline{\frac{3}{2}}\rangle, \vert\frac{5}{2}\,\frac{3}{2}\rangle\vert^\bullet$	2	± 8
$E_{5/2,u}$	$\langle\vert\frac{5}{2}\,\frac{5}{2}\rangle, \vert\frac{5}{2}\,\overline{\frac{5}{2}}\rangle\vert^\bullet$	$\langle\vert\frac{7}{2}\,\frac{5}{2}\rangle, -\vert\frac{7}{2}\,\overline{\frac{5}{2}}\rangle\vert^\bullet$	2	± 8
$E_{7/2,u}$	$\langle\vert\frac{7}{2}\,\overline{\frac{7}{2}}\rangle, -\vert\frac{7}{2}\,\frac{7}{2}\rangle\vert^\bullet$	$\langle\vert\frac{9}{2}\,\overline{\frac{7}{2}}\rangle, \vert\frac{9}{2}\,\frac{7}{2}\rangle\vert^\bullet$	2	± 8

T **37**.7 Matrix representations

Use T **28**.7 ▪. § **16**–7, p. 77

T **37**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{D}_{8h}$	A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_{1g}	E_{2g}	E_{3g}
A_{1g}	A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_{1g}	E_{2g}	E_{3g}
A_{2g}		A_{1g}	B_{2g}	B_{1g}	E_{1g}	E_{2g}	E_{3g}
B_{1g}			A_{1g}	A_{2g}	E_{3g}	E_{2g}	E_{1g}
B_{2g}				A_{1g}	E_{3g}	E_{2g}	E_{1g}
E_{1g}					$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$	$E_{1g} \oplus E_{3g}$	$B_{1g} \oplus B_{2g} \oplus E_{2g}$
E_{2g}						$A_{1g} \oplus \{A_{2g}\} \oplus B_{1g} \oplus B_{2g}$	$E_{1g} \oplus E_{3g}$
E_{3g}							$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$

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T **37**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{8h}$	A_{1u}	A_{2u}	B_{1u}	B_{2u}	E_{1u}	E_{2u}	E_{3u}
A_{1g}	A_{1u}	A_{2u}	B_{1u}	B_{2u}	E_{1u}	E_{2u}	E_{3u}
A_{2g}	A_{2u}	A_{1u}	B_{2u}	B_{1u}	E_{1u}	E_{2u}	E_{3u}
B_{1g}	B_{1u}	B_{2u}	A_{1u}	A_{2u}	E_{3u}	E_{2u}	E_{1u}
B_{2g}	B_{2u}	B_{1u}	A_{2u}	A_{1u}	E_{3u}	E_{2u}	E_{1u}
E_{1g}	E_{1u}	E_{1u}	E_{3u}	E_{3u}	$A_{1u} \oplus A_{2u} \oplus E_{2u}$	$E_{1u} \oplus E_{3u}$	$B_{1u} \oplus B_{2u} \oplus E_{2u}$
E_{2g}	E_{2u}	E_{2u}	E_{2u}	E_{2u}	$E_{1u} \oplus E_{3u}$	$A_{1u} \oplus A_{2u} \oplus B_{1u} \oplus B_{2u}$	$E_{1u} \oplus E_{3u}$
E_{3g}	E_{3u}	E_{3u}	E_{1u}	E_{1u}	$B_{1u} \oplus B_{2u} \oplus E_{2u}$	$E_{1u} \oplus E_{3u}$	$A_{1u} \oplus A_{2u} \oplus E_{2u}$
A_{1u}	A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_{1g}	E_{2g}	E_{3g}
A_{2u}		A_{1g}	B_{2g}	B_{1g}	E_{1g}	E_{2g}	E_{3g}
B_{1u}			A_{1g}	A_{2g}	E_{3g}	E_{2g}	E_{1g}
B_{2u}				A_{1g}	E_{3g}	E_{2g}	E_{1g}
E_{1u}					$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$	$E_{1g} \oplus E_{3g}$	$B_{1g} \oplus B_{2g} \oplus E_{2g}$
E_{2u}						$A_{1g} \oplus \{A_{2g}\} \oplus B_{1g} \oplus B_{2g}$	$E_{1g} \oplus E_{3g}$
E_{3u}							$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$

$\twoheadrightarrow$

T **37**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{8h}$	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{7/2,g}$
A_{1g}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{7/2,g}$
A_{2g}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{7/2,g}$
B_{1g}	$E_{7/2,g}$	$E_{5/2,g}$	$E_{3/2,g}$	$E_{1/2,g}$
B_{2g}	$E_{7/2,g}$	$E_{5/2,g}$	$E_{3/2,g}$	$E_{1/2,g}$
E_{1g}	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{3/2,g} \oplus E_{7/2,g}$	$E_{5/2,g} \oplus E_{7/2,g}$
E_{2g}	$E_{3/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{7/2,g}$	$E_{1/2,g} \oplus E_{7/2,g}$	$E_{3/2,g} \oplus E_{5/2,g}$
E_{3g}	$E_{5/2,g} \oplus E_{7/2,g}$	$E_{3/2,g} \oplus E_{7/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$
A_{1u}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	$E_{7/2,u}$
A_{2u}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	$E_{7/2,u}$
B_{1u}	$E_{7/2,u}$	$E_{5/2,u}$	$E_{3/2,u}$	$E_{1/2,u}$
B_{2u}	$E_{7/2,u}$	$E_{5/2,u}$	$E_{3/2,u}$	$E_{1/2,u}$
E_{1u}	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{3/2,u} \oplus E_{7/2,u}$	$E_{5/2,u} \oplus E_{7/2,u}$
E_{2u}	$E_{3/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{7/2,u}$	$E_{1/2,u} \oplus E_{7/2,u}$	$E_{3/2,u} \oplus E_{5/2,u}$
E_{3u}	$E_{5/2,u} \oplus E_{7/2,u}$	$E_{3/2,u} \oplus E_{7/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$
$E_{1/2,g}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$	$E_{1g} \oplus E_{2g}$	$E_{2g} \oplus E_{3g}$	$B_{1g} \oplus B_{2g} \oplus E_{3g}$
$E_{3/2,g}$		$\{A_{1g}\} \oplus A_{2g} \oplus E_{3g}$	$B_{1g} \oplus B_{2g} \oplus E_{1g}$	$E_{2g} \oplus E_{3g}$
$E_{5/2,g}$			$\{A_{1g}\} \oplus A_{2g} \oplus E_{3g}$	$E_{1g} \oplus E_{2g}$
$E_{7/2,g}$				$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$

$\twoheadrightarrow$

T **37**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{8h}$	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	$E_{7/2,u}$
A_{1g}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	$E_{7/2,u}$
A_{2g}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	$E_{7/2,u}$
B_{1g}	$E_{7/2,u}$	$E_{5/2,u}$	$E_{3/2,u}$	$E_{1/2,u}$
B_{2g}	$E_{7/2,u}$	$E_{5/2,u}$	$E_{3/2,u}$	$E_{1/2,u}$
E_{1g}	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{3/2,u} \oplus E_{7/2,u}$	$E_{5/2,u} \oplus E_{7/2,u}$
E_{2g}	$E_{3/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{7/2,u}$	$E_{1/2,u} \oplus E_{7/2,u}$	$E_{3/2,u} \oplus E_{5/2,u}$
E_{3g}	$E_{5/2,u} \oplus E_{7/2,u}$	$E_{3/2,u} \oplus E_{7/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$
A_{1u}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{7/2,g}$
A_{2u}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{7/2,g}$
B_{1u}	$E_{7/2,g}$	$E_{5/2,g}$	$E_{3/2,g}$	$E_{1/2,g}$
B_{2u}	$E_{7/2,g}$	$E_{5/2,g}$	$E_{3/2,g}$	$E_{1/2,g}$
E_{1u}	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{3/2,g} \oplus E_{7/2,g}$	$E_{5/2,g} \oplus E_{7/2,g}$
E_{2u}	$E_{3/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{7/2,g}$	$E_{1/2,g} \oplus E_{7/2,g}$	$E_{3/2,g} \oplus E_{5/2,g}$
E_{3u}	$E_{5/2,g} \oplus E_{7/2,g}$	$E_{3/2,g} \oplus E_{7/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$
$E_{1/2,g}$	$A_{1u} \oplus A_{2u} \oplus E_{1u}$	$E_{1u} \oplus E_{2u}$	$E_{2u} \oplus E_{3u}$	$B_{1u} \oplus B_{2u} \oplus E_{3u}$
$E_{3/2,g}$	$E_{1u} \oplus E_{2u}$	$A_{1u} \oplus A_{2u} \oplus E_{3u}$	$B_{1u} \oplus B_{2u} \oplus E_{1u}$	$E_{2u} \oplus E_{3u}$
$E_{5/2,g}$	$E_{2u} \oplus E_{3u}$	$B_{1u} \oplus B_{2u} \oplus E_{1u}$	$A_{1u} \oplus A_{2u} \oplus E_{3u}$	$E_{1u} \oplus E_{2u}$
$E_{7/2,g}$	$B_{1u} \oplus B_{2u} \oplus E_{3u}$	$E_{2u} \oplus E_{3u}$	$E_{1u} \oplus E_{2u}$	$A_{1u} \oplus A_{2u} \oplus E_{1u}$
$E_{1/2,u}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$	$E_{1g} \oplus E_{2g}$	$E_{2g} \oplus E_{3g}$	$B_{1g} \oplus B_{2g} \oplus E_{3g}$
$E_{3/2,u}$		$\{A_{1g}\} \oplus A_{2g} \oplus E_{3g}$	$B_{1g} \oplus B_{2g} \oplus E_{1g}$	$E_{2g} \oplus E_{3g}$
$E_{5/2,u}$			$\{A_{1g}\} \oplus A_{2g} \oplus E_{3g}$	$E_{1g} \oplus E_{2g}$
$E_{7/2,u}$				$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$

T **37**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_{8h}$	$\mathbf{C}_{8h}$	$(\mathbf{C}_{8v})$	$(\mathbf{C}_{4v})$	$(\mathbf{C}_{4v})$	$\mathbf{D}_{4d}$
			σ_v	σ_d	C_2', σ_d
A_{1g}	A_g	A_1	A_1	A_1	A_1
A_{2g}	A_g	A_2	A_2	A_2	A_2
B_{1g}	B_g	B_1	A_1	A_2	B_1
B_{2g}	B_g	B_2	A_2	A_1	B_2
E_{1g}	${}^1E_{1g} \oplus {}^2E_{1g}$	E_1	E	E	E_3
E_{2g}	${}^1E_{2g} \oplus {}^2E_{2g}$	E_2	$B_1 \oplus B_2$	$B_1 \oplus B_2$	E_2
E_{3g}	${}^1E_{3g} \oplus {}^2E_{3g}$	E_3	E	E	E_1
A_{1u}	A_u	A_2	A_2	A_2	B_1
A_{2u}	A_u	A_1	A_1	A_1	B_2
B_{1u}	B_u	B_2	A_2	A_1	A_1
B_{2u}	B_u	B_1	A_1	A_2	A_2
E_{1u}	${}^1E_{1u} \oplus {}^2E_{1u}$	E_1	E	E	E_1
E_{2u}	${}^1E_{2u} \oplus {}^2E_{2u}$	E_2	$B_1 \oplus B_2$	$B_1 \oplus B_2$	E_2
E_{3u}	${}^1E_{3u} \oplus {}^2E_{3u}$	E_3	E	E	E_3
$E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{3/2,g}$	${}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$	$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{3/2}$
$E_{5/2,g}$	${}^1E_{5/2,g} \oplus {}^2E_{5/2,g}$	$E_{5/2}$	$E_{3/2}$	$E_{3/2}$	$E_{5/2}$
$E_{7/2,g}$	${}^1E_{7/2,g} \oplus {}^2E_{7/2,g}$	$E_{7/2}$	$E_{1/2}$	$E_{1/2}$	$E_{7/2}$
$E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{7/2}$
$E_{3/2,u}$	${}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$	$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{5/2}$
$E_{5/2,u}$	${}^1E_{5/2,u} \oplus {}^2E_{5/2,u}$	$E_{5/2}$	$E_{3/2}$	$E_{3/2}$	$E_{3/2}$
$E_{7/2,u}$	${}^1E_{7/2,u} \oplus {}^2E_{7/2,u}$	$E_{7/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$

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T **37**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{8h}$	$(\mathbf{D}_{4d})$ C_2'', σ_v	$(\mathbf{D}_{4h})$ C_2', σ_v	$(\mathbf{D}_{4h})$ C_2'', σ_d	$\mathbf{D}_8$	$\mathbf{S}_8$
A_{1g}	A_1	A_{1g}	A_{1g}	A_1	A
A_{2g}	A_2	A_{2g}	A_{2g}	A_2	A
B_{1g}	B_2	A_{1g}	A_{2g}	B_1	B
B_{2g}	B_1	A_{2g}	A_{1g}	B_2	B
E_{1g}	E_3	E_g	E_g	E_1	${}^1E_3 \oplus {}^2E_3$
E_{2g}	E_2	$B_{1g} \oplus B_{2g}$	$B_{1g} \oplus B_{2g}$	E_2	${}^1E_2 \oplus {}^2E_2$
E_{3g}	E_1	E_g	E_g	E_3	${}^1E_1 \oplus {}^2E_1$
A_{1u}	B_1	A_{1u}	A_{1u}	A_1	B
A_{2u}	B_2	A_{2u}	A_{2u}	A_2	B
B_{1u}	A_2	A_{1u}	A_{2u}	B_1	A
B_{2u}	A_1	A_{2u}	A_{1u}	B_2	A
E_{1u}	E_1	E_u	E_u	E_1	${}^1E_1 \oplus {}^2E_1$
E_{2u}	E_2	$B_{1u} \oplus B_{2u}$	$B_{1u} \oplus B_{2u}$	E_2	${}^1E_2 \oplus {}^2E_2$
E_{3u}	E_3	E_u	E_u	E_3	${}^1E_3 \oplus {}^2E_3$
$E_{1/2,g}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2,g}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2,g}$	$E_{3/2}$	$E_{3/2,g}$	$E_{3/2,g}$	$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$
$E_{5/2,g}$	$E_{5/2}$	$E_{3/2,g}$	$E_{3/2,g}$	$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$
$E_{7/2,g}$	$E_{7/2}$	$E_{1/2,g}$	$E_{1/2,g}$	$E_{7/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$
$E_{1/2,u}$	$E_{7/2}$	$E_{1/2,u}$	$E_{1/2,u}$	$E_{1/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$
$E_{3/2,u}$	$E_{5/2}$	$E_{3/2,u}$	$E_{3/2,u}$	$E_{3/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$
$E_{5/2,u}$	$E_{3/2}$	$E_{3/2,u}$	$E_{3/2,u}$	$E_{5/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$
$E_{7/2,u}$	$E_{1/2}$	$E_{1/2,u}$	$E_{1/2,u}$	$E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

Other subgroups: $\mathbf{C}_{4h}$, $3\mathbf{C}_{2h}$, $4\mathbf{C}_{2v}$, $2\mathbf{D}_{2d}$, $2\mathbf{D}_{2h}$, $\mathbf{S}_4$, $3\mathbf{C}_s$, $\mathbf{C}_i$ (see $\mathbf{D}_{4h}$);
$2\mathbf{D}_4$, $2\mathbf{D}_2$, $\mathbf{C}_8$, $\mathbf{C}_4$, $3\mathbf{C}_2$ (see $\mathbf{D}_8$).

T **37**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{8h}$
$8n$	$(n+1)\,A_{1g} \oplus n\,(A_{2g} \oplus B_{1g} \oplus B_{2g} \oplus 2E_{1g} \oplus 2E_{2g} \oplus 2E_{3g})$
$8n+1$	$n\,(A_{1u} \oplus B_{1u} \oplus B_{2u} \oplus E_{1u} \oplus 2E_{2u} \oplus 2E_{3u}) \oplus (n+1)(A_{2u} \oplus E_{1u})$
$8n+2$	$(n+1)(A_{1g} \oplus E_{1g} \oplus E_{2g}) \oplus n\,(A_{2g} \oplus B_{1g} \oplus B_{2g} \oplus E_{1g} \oplus E_{2g} \oplus 2E_{3g})$
$8n+3$	$n\,(A_{1u} \oplus B_{1u} \oplus B_{2u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u}) \oplus (n+1)(A_{2u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u})$
$8n+4$	$(n+1)(A_{1g} \oplus B_{1g} \oplus B_{2g} \oplus E_{1g} \oplus E_{2g} \oplus E_{3g}) \oplus n\,(A_{2g} \oplus E_{1g} \oplus E_{2g} \oplus E_{3g})$
$8n+5$	$n\,(A_{1u} \oplus E_{1u} \oplus E_{2u}) \oplus (n+1)\,(A_{2u} \oplus B_{1u} \oplus B_{2u} \oplus E_{1u} \oplus E_{2u} \oplus 2E_{3u})$
$8n+6$	$(n+1)(A_{1g} \oplus B_{1g} \oplus B_{2g} \oplus E_{1g} \oplus 2E_{2g} \oplus 2E_{3g}) \oplus n\,(A_{2g} \oplus E_{1g})$
$8n+7$	$n\,A_{1u} \oplus (n+1)(A_{2u} \oplus B_{1u} \oplus B_{2u} \oplus 2E_{1u} \oplus 2E_{2u} \oplus 2E_{3u})$
$8n+\frac{1}{2}$	$(2n+1)\,E_{1/2,g} \oplus 2n\,(E_{3/2,g} \oplus E_{5/2,g} \oplus E_{7/2,g})$
$8n+\frac{3}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g}) \oplus 2n\,(E_{5/2,g} \oplus E_{7/2,g})$
$8n+\frac{5}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g}) \oplus 2n\,E_{7/2,g}$
$8n+\frac{7}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g} \oplus E_{7/2,g})$
$8n+\frac{9}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g}) \oplus (2n+2)\,E_{7/2,g}$
$8n+\frac{11}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g}) \oplus (2n+2)(E_{5/2,g} \oplus E_{7/2,g})$
$8n+\frac{13}{2}$	$(2n+1)\,E_{1/2,g} \oplus (2n+2)(E_{3/2,g} \oplus E_{5/2,g} \oplus E_{7/2,g})$
$8n+\frac{15}{2}$	$(2n+2)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g} \oplus E_{7/2,g})$

$n = 0, 1, 2, \ldots$

T **37**.11 Clebsch–Gordan coefficients

Use T **28**.11 ■. § **16**–11, p. 83

$\overline{18}\,m2$	$\lvert G\rvert = 36$	$\lvert C\rvert = 12$	$\lvert\widetilde{C}\rvert = 21$	T **38**	p. 245	$\mathbf{D}_{9h}$

(1) Product forms: $\mathbf{D}_9 \otimes \mathbf{C}_s$, $\mathbf{C}_{9v} \otimes \mathbf{C}_s$.

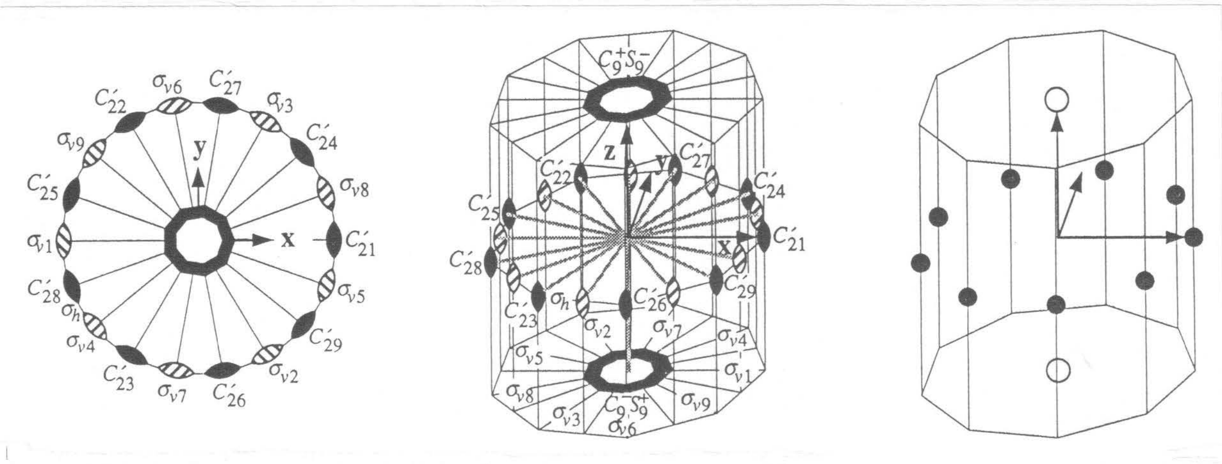
(2) Group chains: $\mathbf{D}_{9h} \supset \underline{\mathbf{C}_{9h}}$, $\mathbf{D}_{9h} \supset (\underline{\mathbf{C}_{9v}})$, $\mathbf{D}_{9h} \supset (\mathbf{D}_{3h})$, $\mathbf{D}_{9h} \supset (\underline{\mathbf{D}_9})$.

(3) Operations of G: E, (C_9^+, C_9^-), (C_9^{2+}, C_9^{2-}), (C_3^+, C_3^-), (C_9^{4+}, C_9^{4-}),
$(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27}, C'_{28}, C'_{29})$,
σ_h, (S_9^+, S_9^-), (S_9^{2+}, S_9^{2-}), (S_3^+, S_3^-), (S_9^{4+}, S_9^{4-}),
$(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5}, \sigma_{v6}, \sigma_{v7}, \sigma_{v8}, \sigma_{v9})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_9^+, C_9^-), $(\widetilde{C}_9^+, \widetilde{C}_9^-)$, (C_9^{2+}, C_9^{2-}), $(\widetilde{C}_9^{2+}, \widetilde{C}_9^{2-})$,
(C_3^+, C_3^-), $(\widetilde{C}_3^+, \widetilde{C}_3^-)$, (C_9^{4+}, C_9^{4-}), $(\widetilde{C}_9^{4+}, \widetilde{C}_9^{4-})$,
$(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27}, C'_{28}, C'_{29}, \widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24}, \widetilde{C}'_{25}, \widetilde{C}'_{26}, \widetilde{C}'_{27}, \widetilde{C}'_{28}, \widetilde{C}'_{29})$,
σ_h, $\widetilde{\sigma}_h$, (S_9^+, S_9^-), $(\widetilde{S}_9^+, \widetilde{S}_9^-)$, (S_9^{2+}, S_9^{2-}), $(\widetilde{S}_9^{2+}, \widetilde{S}_9^{2-})$,
(S_3^+, S_3^-), $(\widetilde{S}_3^+, \widetilde{S}_3^-)$, (S_9^{4+}, S_9^{4-}), $(\widetilde{S}_9^{4+}, \widetilde{S}_9^{4-})$,
$(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5}, \sigma_{v6}, \sigma_{v7}, \sigma_{v8}, \sigma_{v9}, \widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2}, \widetilde{\sigma}_{v3}, \widetilde{\sigma}_{v4}, \widetilde{\sigma}_{v5}, \widetilde{\sigma}_{v6}, \widetilde{\sigma}_{v7}, \widetilde{\sigma}_{v8}, \widetilde{\sigma}_{v9})$.

(5) Classes and representations: $\lvert r\rvert = 9$, $\lvert i\rvert = 3$, $\lvert I\rvert = 12$, $\lvert\widetilde{I}\rvert = 9$.

F **38** See Chapter **15**, p. 65



Examples:

T **38**.1 Parameters

§ **16**–1, p. 68

$\mathbf{D}_{9h}$	α	β	γ	ϕ	$\mathbf{n}$	λ	$\mathbf{\Lambda}$
E	0	0	0	0	$(0 \;\; 0 \;\; 0)$	$[\![\, 1,$	$(0 \;\; 0 \;\; 0)]\!]$
C_9^{+}	0	0	$\frac{2\pi}{9}$	$\frac{2\pi}{9}$	$(0 \;\; 0 \;\; 1)$	$[\![\, c_9,$	$(0 \;\; 0 \;\; s_9)]\!]$
C_9^{-}	0	0	$-\frac{2\pi}{9}$	$\frac{2\pi}{9}$	$(0 \;\; 0 \; -1)$	$[\![\, c_9,$	$(0 \;\; 0 \; -s_9)]\!]$
C_9^{2+}	0	0	$\frac{4\pi}{9}$	$\frac{4\pi}{9}$	$(0 \;\; 0 \;\; 1)$	$[\![\, c_9^2,$	$(0 \;\; 0 \;\; s_9^2)]\!]$
C_9^{2-}	0	0	$-\frac{4\pi}{9}$	$\frac{4\pi}{9}$	$(0 \;\; 0 \; -1)$	$[\![\, c_9^2,$	$(0 \;\; 0 \; -s_9^2)]\!]$
C_3^{+}	0	0	$\frac{2\pi}{3}$	$\frac{2\pi}{3}$	$(0 \;\; 0 \;\; 1)$	$[\![\, \frac{1}{2},$	$(0 \;\; 0 \;\; \frac{\sqrt{3}}{2})]\!]$
C_3^{-}	0	0	$-\frac{2\pi}{3}$	$\frac{2\pi}{3}$	$(0 \;\; 0 \; -1)$	$[\![\, \frac{1}{2},$	$(0 \;\; 0 \; -\frac{\sqrt{3}}{2})]\!]$
C_9^{4+}	0	0	$\frac{8\pi}{9}$	$\frac{8\pi}{9}$	$(0 \;\; 0 \;\; 1)$	$[\![\, c_9^4,$	$(0 \;\; 0 \;\; s_9^4)]\!]$
C_9^{4-}	0	0	$-\frac{8\pi}{9}$	$\frac{8\pi}{9}$	$(0 \;\; 0 \; -1)$	$[\![\, c_9^4,$	$(0 \;\; 0 \; -s_9^4)]\!]$
C'_{21}	0	π	π	π	$(1 \;\; 0 \;\; 0)$	$[\![\, 0,$	$(1 \;\; 0 \;\; 0)]\!]$
C'_{22}	0	π	$-\frac{\pi}{3}$	π	$(-\frac{1}{2} \;\; \frac{\sqrt{3}}{2} \;\; 0)$	$[\![\, 0,$	$(-\frac{1}{2} \;\; \frac{\sqrt{3}}{2} \;\; 0)]\!]$
C'_{23}	0	π	$\frac{\pi}{3}$	π	$(-\frac{1}{2} \; -\frac{\sqrt{3}}{2} \;\; 0)$	$[\![\, 0,$	$(-\frac{1}{2} \; -\frac{\sqrt{3}}{2} \;\; 0)]\!]$
C'_{24}	0	π	$\frac{5\pi}{9}$	π	$(c_9^2 \;\; s_9^2 \;\; 0)$	$[\![\, 0,$	$(c_9^2 \;\; s_9^2 \;\; 0)]\!]$
C'_{25}	0	π	$-\frac{7\pi}{9}$	π	$(-c_9 \;\; s_9 \;\; 0)$	$[\![\, 0,$	$(-c_9 \;\; s_9 \;\; 0)]\!]$
C'_{26}	0	π	$-\frac{\pi}{9}$	π	$(c_9^4 \; -s_9^4 \;\; 0)$	$[\![\, 0,$	$(c_9^4 \; -s_9^4 \;\; 0)]\!]$
C'_{27}	0	π	$\frac{\pi}{9}$	π	$(c_9^4 \;\; s_9^4 \;\; 0)$	$[\![\, 0,$	$(c_9^4 \;\; s_9^4 \;\; 0)]\!]$
C'_{28}	0	π	$\frac{7\pi}{9}$	π	$(-c_9 \; -s_9 \;\; 0)$	$[\![\, 0,$	$(-c_9 \; -s_9 \;\; 0)]\!]$
C'_{29}	0	π	$-\frac{5\pi}{9}$	π	$(c_9^2 \; -s_9^2 \;\; 0)$	$[\![\, 0,$	$(c_9^2 \; -s_9^2 \;\; 0)]\!]$
σ_h	0	0	π	π	$(0 \;\; 0 \;\; 1)$	$[\![\, 0,$	$(0 \;\; 0 \;\; 1)]\!]$
S_9^{+}	0	0	$-\frac{7\pi}{9}$	$\frac{7\pi}{9}$	$(0 \;\; 0 \; -1)$	$[\![\, s_9,$	$(0 \;\; 0 \; -c_9)]\!]$
S_9^{-}	0	0	$\frac{7\pi}{9}$	$\frac{7\pi}{9}$	$(0 \;\; 0 \;\; 1)$	$[\![\, s_9,$	$(0 \;\; 0 \;\; c_9)]\!]$
S_9^{2+}	0	0	$-\frac{5\pi}{9}$	$\frac{5\pi}{9}$	$(0 \;\; 0 \; -1)$	$[\![\, s_9^2,$	$(0 \;\; 0 \; -c_9^2)]\!]$
S_9^{2-}	0	0	$\frac{5\pi}{9}$	$\frac{5\pi}{9}$	$(0 \;\; 0 \;\; 1)$	$[\![\, s_9^2,$	$(0 \;\; 0 \;\; c_9^2)]\!]$
S_3^{+}	0	0	$-\frac{\pi}{3}$	$\frac{\pi}{3}$	$(0 \;\; 0 \; -1)$	$[\![\, \frac{\sqrt{3}}{2},$	$(0 \;\; 0 \; -\frac{1}{2})]\!]$
S_3^{-}	0	0	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$(0 \;\; 0 \;\; 1)$	$[\![\, \frac{\sqrt{3}}{2},$	$(0 \;\; 0 \;\; \frac{1}{2})]\!]$
S_9^{4+}	0	0	$-\frac{\pi}{9}$	$\frac{\pi}{9}$	$(0 \;\; 0 \; -1)$	$[\![\, s_9^4,$	$(0 \;\; 0 \; -c_9^4)]\!]$
S_9^{4-}	0	0	$\frac{\pi}{9}$	$\frac{\pi}{9}$	$(0 \;\; 0 \;\; 1)$	$[\![\, s_9^4,$	$(0 \;\; 0 \;\; c_9^4)]\!]$
σ_{v1}	0	π	0	π	$(0 \;\; 1 \;\; 0)$	$[\![\, 0,$	$(0 \;\; 1 \;\; 0)]\!]$
σ_{v2}	0	π	$\frac{2\pi}{3}$	π	$(-\frac{\sqrt{3}}{2} \; -\frac{1}{2} \;\; 0)$	$[\![\, 0,$	$(-\frac{\sqrt{3}}{2} \; -\frac{1}{2} \;\; 0)]\!]$
σ_{v3}	0	π	$-\frac{2\pi}{3}$	π	$(\frac{\sqrt{3}}{2} \; -\frac{1}{2} \;\; 0)$	$[\![\, 0,$	$(\frac{\sqrt{3}}{2} \; -\frac{1}{2} \;\; 0)]\!]$
σ_{v4}	0	π	$-\frac{4\pi}{9}$	π	$(-s_9^2 \;\; c_9^2 \;\; 0)$	$[\![\, 0,$	$(-s_9^2 \;\; c_9^2 \;\; 0)]\!]$
σ_{v5}	0	π	$\frac{2\pi}{9}$	π	$(-s_9 \; -c_9 \;\; 0)$	$[\![\, 0,$	$(-s_9 \; -c_9 \;\; 0)]\!]$
σ_{v6}	0	π	$\frac{8\pi}{9}$	π	$(s_9^4 \;\; c_9^4 \;\; 0)$	$[\![\, 0,$	$(s_9^4 \;\; c_9^4 \;\; 0)]\!]$
σ_{v7}	0	π	$-\frac{8\pi}{9}$	π	$(-s_9^4 \;\; c_9^4 \;\; 0)$	$[\![\, 0,$	$(-s_9^4 \;\; c_9^4 \;\; 0)]\!]$
σ_{v8}	0	π	$-\frac{2\pi}{9}$	π	$(s_9 \; -c_9 \;\; 0)$	$[\![\, 0,$	$(s_9 \; -c_9 \;\; 0)]\!]$
σ_{v9}	0	π	$\frac{4\pi}{9}$	π	$(s_9^2 \;\; c_9^2 \;\; 0)$	$[\![\, 0,$	$(s_9^2 \;\; c_9^2 \;\; 0)]\!]$

$c_n^m = \cos \frac{m}{n}\pi$, $s_n^m = \sin \frac{m}{n}\pi$

T **38**.2 Multiplication table

§ **16**–2, p. 69

$\mathbf{D}_{9h}$	E	C_9^{+}	C_9^{-}	C_9^{2+}	C_9^{2-}	C_3^{+}	C_3^{-}	C_9^{4+}	C_9^{4-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}
E	E	C_9^{+}	C_9^{-}	C_9^{2+}	C_9^{2-}	C_3^{+}	C_3^{-}	C_9^{4+}	C_9^{4-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}
C_9^{+}	C_9^{+}	C_9^{2+}	E	C_3^{+}	C_9^{-}	C_9^{4+}	C_9^{2-}	C_9^{4-}	C_3^{-}	C'_{28}	C'_{29}	C'_{27}	C'_{23}	C'_{21}	C'_{22}	C'_{26}	C'_{24}	C'_{25}
C_9^{-}	C_9^{-}	E	C_9^{2-}	C_9^{+}	C_3^{-}	C_9^{2+}	C_9^{4-}	C_3^{+}	C_9^{4+}	C'_{25}	C'_{26}	C'_{24}	C'_{28}	C'_{29}	C'_{27}	C'_{23}	C'_{21}	C'_{22}
C_9^{2+}	C_9^{2+}	C_3^{+}	C_9^{+}	C_9^{4+}	E	C_9^{4-}	C_9^{-}	C_3^{-}	C_9^{2-}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}	C'_{22}	C'_{23}	C'_{21}
C_9^{2-}	C_9^{2-}	C_9^{-}	C_3^{-}	E	C_9^{4-}	C_9^{+}	C_9^{4+}	C_9^{2+}	C_3^{+}	C'_{29}	C'_{27}	C'_{28}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}
C_3^{+}	C_3^{+}	C_9^{4+}	C_9^{2+}	C_9^{4-}	C_9^{+}	C_3^{-}	E	C_9^{2-}	C_9^{-}	C'_{23}	C'_{21}	C'_{22}	C'_{26}	C'_{24}	C'_{25}	C'_{29}	C'_{27}	C'_{28}
C_3^{-}	C_3^{-}	C_9^{2-}	C_9^{4-}	C_9^{-}	C_9^{4+}	E	C_3^{+}	C_9^{+}	C_9^{2+}	C'_{22}	C'_{23}	C'_{21}	C'_{25}	C'_{26}	C'_{24}	C'_{28}	C'_{29}	C'_{27}
C_9^{4+}	C_9^{4+}	C_9^{4-}	C_3^{+}	C_3^{-}	C_9^{2+}	C_9^{2-}	C_9^{+}	C_9^{-}	E	C'_{27}	C'_{28}	C'_{29}	C'_{22}	C'_{23}	C'_{21}	C'_{25}	C'_{26}	C'_{24}
C_9^{4-}	C_9^{4-}	C_3^{-}	C_9^{4+}	C_9^{2-}	C_3^{+}	C_9^{-}	C_9^{2+}	E	C_9^{+}	C'_{26}	C'_{24}	C'_{25}	C'_{29}	C'_{27}	C'_{28}	C'_{21}	C'_{22}	C'_{23}
C'_{21}	C'_{21}	C'_{25}	C'_{28}	C'_{29}	C'_{24}	C'_{22}	C'_{23}	C'_{26}	C'_{27}	E	C_3^{+}	C_3^{-}	C_9^{2-}	C_9^{+}	C_9^{4+}	C_9^{4-}	C_9^{-}	C_9^{2+}
C'_{22}	C'_{22}	C'_{26}	C'_{29}	C'_{27}	C'_{25}	C'_{23}	C'_{21}	C'_{24}	C'_{28}	C_3^{-}	E	C_3^{+}	C_9^{4+}	C_9^{2-}	C_9^{+}	C_9^{2+}	C_9^{4-}	C_9^{-}
C'_{23}	C'_{23}	C'_{24}	C'_{27}	C'_{28}	C'_{26}	C'_{21}	C'_{22}	C'_{25}	C'_{29}	C_3^{+}	C_3^{-}	E	C_9^{+}	C_9^{4+}	C_9^{2-}	C_9^{-}	C_9^{2+}	C_9^{4-}
C'_{24}	C'_{24}	C'_{28}	C'_{23}	C'_{21}	C'_{27}	C'_{25}	C'_{26}	C'_{29}	C'_{22}	C_9^{2+}	C_9^{4-}	C_9^{-}	E	C_3^{+}	C_3^{-}	C_9^{2-}	C_9^{+}	C_9^{4+}
C'_{25}	C'_{25}	C'_{29}	C'_{21}	C'_{22}	C'_{28}	C'_{26}	C'_{24}	C'_{27}	C'_{23}	C_9^{-}	C_9^{2+}	C_9^{4-}	C_3^{-}	E	C_3^{+}	C_9^{4+}	C_9^{2-}	C_9^{+}
C'_{26}	C'_{26}	C'_{27}	C'_{22}	C'_{23}	C'_{29}	C'_{24}	C'_{25}	C'_{28}	C'_{21}	C_9^{4-}	C_9^{-}	C_9^{2+}	C_3^{+}	C_3^{-}	E	C_9^{+}	C_9^{4+}	C_9^{2-}
C'_{27}	C'_{27}	C'_{23}	C'_{26}	C'_{24}	C'_{22}	C'_{28}	C'_{29}	C'_{21}	C'_{25}	C_9^{4+}	C_9^{2-}	C_9^{+}	C_9^{2+}	C_9^{4-}	C_9^{-}	E	C_3^{+}	C_3^{-}
C'_{28}	C'_{28}	C'_{21}	C'_{24}	C'_{25}	C'_{23}	C'_{29}	C'_{27}	C'_{22}	C'_{26}	C_9^{+}	C_9^{4+}	C_9^{2-}	C_9^{-}	C_9^{2+}	C_9^{4-}	C_3^{-}	E	C_3^{+}
C'_{29}	C'_{29}	C'_{22}	C'_{25}	C'_{26}	C'_{21}	C'_{27}	C'_{28}	C'_{23}	C'_{24}	C_9^{2-}	C_9^{+}	C_9^{4+}	C_9^{4-}	C_9^{-}	C_9^{2+}	C_3^{+}	C_3^{-}	E
σ_h	σ_h	S_9^{+}	S_9^{-}	S_9^{2+}	S_9^{2-}	S_3^{+}	S_3^{-}	S_9^{4+}	S_9^{4-}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}	σ_{v8}	σ_{v9}
S_9^{+}	S_9^{+}	S_9^{2+}	σ_h	S_3^{+}	S_9^{-}	S_9^{4+}	S_9^{2-}	S_9^{4-}	S_3^{-}	σ_{v8}	σ_{v9}	σ_{v7}	σ_{v3}	σ_{v1}	σ_{v2}	σ_{v6}	σ_{v4}	σ_{v5}
S_9^{-}	S_9^{-}	σ_h	S_9^{2-}	S_9^{+}	S_3^{-}	S_9^{2+}	S_9^{4-}	S_3^{+}	S_9^{4+}	σ_{v5}	σ_{v6}	σ_{v4}	σ_{v8}	σ_{v9}	σ_{v7}	σ_{v3}	σ_{v1}	σ_{v2}
S_9^{2+}	S_9^{2+}	S_3^{+}	S_9^{+}	S_9^{4+}	σ_h	S_9^{4-}	S_9^{-}	S_3^{-}	S_9^{2-}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}	σ_{v8}	σ_{v9}	σ_{v2}	σ_{v3}	σ_{v1}
S_9^{2-}	S_9^{2-}	S_9^{-}	S_3^{-}	σ_h	S_9^{4-}	S_9^{+}	S_9^{4+}	S_9^{2+}	S_3^{+}	σ_{v9}	σ_{v7}	σ_{v8}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}
S_3^{+}	S_3^{+}	S_9^{4+}	S_9^{2+}	S_9^{4-}	S_9^{+}	S_3^{-}	σ_h	S_9^{2-}	S_9^{-}	σ_{v3}	σ_{v1}	σ_{v2}	σ_{v6}	σ_{v4}	σ_{v5}	σ_{v9}	σ_{v7}	σ_{v8}
S_3^{-}	S_3^{-}	S_9^{2-}	S_9^{4-}	S_9^{-}	S_9^{4+}	σ_h	S_3^{+}	S_9^{+}	S_9^{2+}	σ_{v2}	σ_{v3}	σ_{v1}	σ_{v5}	σ_{v6}	σ_{v4}	σ_{v8}	σ_{v9}	σ_{v7}
S_9^{4+}	S_9^{4+}	S_9^{4-}	S_3^{+}	S_3^{-}	S_9^{2+}	S_9^{2-}	S_9^{+}	S_9^{-}	σ_h	σ_{v7}	σ_{v8}	σ_{v9}	σ_{v2}	σ_{v3}	σ_{v1}	σ_{v5}	σ_{v6}	σ_{v4}
S_9^{4-}	S_9^{4-}	S_3^{-}	S_9^{4+}	S_9^{2-}	S_3^{+}	S_9^{-}	S_9^{2+}	σ_h	S_9^{+}	σ_{v6}	σ_{v4}	σ_{v5}	σ_{v9}	σ_{v7}	σ_{v8}	σ_{v1}	σ_{v2}	σ_{v3}
σ_{v1}	σ_{v1}	σ_{v5}	σ_{v8}	σ_{v9}	σ_{v4}	σ_{v2}	σ_{v3}	σ_{v6}	σ_{v7}	σ_h	S_3^{+}	S_3^{-}	S_9^{2-}	S_9^{+}	S_9^{4+}	S_9^{4-}	S_9^{-}	S_9^{2+}
σ_{v2}	σ_{v2}	σ_{v6}	σ_{v9}	σ_{v7}	σ_{v5}	σ_{v3}	σ_{v1}	σ_{v4}	σ_{v8}	S_3^{-}	σ_h	S_3^{+}	S_9^{4+}	S_9^{2-}	S_9^{+}	S_9^{2+}	S_9^{4-}	S_9^{-}
σ_{v3}	σ_{v3}	σ_{v4}	σ_{v7}	σ_{v8}	σ_{v6}	σ_{v1}	σ_{v2}	σ_{v5}	σ_{v9}	S_3^{+}	S_3^{-}	σ_h	S_9^{+}	S_9^{4+}	S_9^{2-}	S_9^{-}	S_9^{2+}	S_9^{4-}
σ_{v4}	σ_{v4}	σ_{v8}	σ_{v3}	σ_{v1}	σ_{v7}	σ_{v5}	σ_{v6}	σ_{v9}	σ_{v2}	S_9^{2+}	S_9^{4-}	S_9^{-}	σ_h	S_3^{+}	S_3^{-}	S_9^{2-}	S_9^{+}	S_9^{4+}
σ_{v5}	σ_{v5}	σ_{v9}	σ_{v1}	σ_{v2}	σ_{v8}	σ_{v6}	σ_{v4}	σ_{v7}	σ_{v3}	S_9^{-}	S_9^{2+}	S_9^{4-}	S_3^{-}	σ_h	S_3^{+}	S_9^{4+}	S_9^{2-}	S_9^{+}
σ_{v6}	σ_{v6}	σ_{v7}	σ_{v2}	σ_{v3}	σ_{v9}	σ_{v4}	σ_{v5}	σ_{v8}	σ_{v1}	S_9^{4-}	S_9^{-}	S_9^{2+}	S_3^{+}	S_3^{-}	σ_h	S_9^{+}	S_9^{4+}	S_9^{2-}
σ_{v7}	σ_{v7}	σ_{v3}	σ_{v6}	σ_{v4}	σ_{v2}	σ_{v8}	σ_{v9}	σ_{v1}	σ_{v5}	S_9^{4+}	S_9^{2-}	S_9^{+}	S_9^{2+}	S_9^{4-}	S_9^{-}	σ_h	S_3^{+}	S_3^{-}
σ_{v8}	σ_{v8}	σ_{v1}	σ_{v4}	σ_{v5}	σ_{v3}	σ_{v9}	σ_{v7}	σ_{v2}	σ_{v6}	S_9^{+}	S_9^{4+}	S_9^{2-}	S_9^{-}	S_9^{2+}	S_9^{4-}	S_3^{-}	σ_h	S_3^{+}
σ_{v9}	σ_{v9}	σ_{v2}	σ_{v5}	σ_{v6}	σ_{v1}	σ_{v7}	σ_{v8}	σ_{v3}	σ_{v4}	S_9^{2-}	S_9^{+}	S_9^{4+}	S_9^{4-}	S_9^{-}	S_9^{2+}	S_3^{+}	S_3^{-}	σ_h

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T **38**.2 Multiplication table *(cont.)*

$\mathbf{D}_{9h}$	σ_h	S_9^{+}	S_9^{-}	S_9^{2+}	S_9^{2-}	S_3^{+}	S_3^{-}	S_9^{4+}	S_9^{4-}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}	σ_{v8}	σ_{v9}
E	σ_h	S_9^{+}	S_9^{-}	S_9^{2+}	S_9^{2-}	S_3^{+}	S_3^{-}	S_9^{4+}	S_9^{4-}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}	σ_{v8}	σ_{v9}
C_9^{+}	S_9^{+}	S_9^{2+}	σ_h	S_3^{+}	S_9^{-}	S_9^{4+}	S_9^{2-}	S_9^{4-}	S_3^{-}	σ_{v8}	σ_{v9}	σ_{v7}	σ_{v3}	σ_{v1}	σ_{v2}	σ_{v6}	σ_{v4}	σ_{v5}
C_9^{-}	S_9^{-}	σ_h	S_9^{2-}	S_9^{+}	S_3^{-}	S_9^{2+}	S_9^{4-}	S_3^{+}	S_9^{4+}	σ_{v5}	σ_{v6}	σ_{v4}	σ_{v8}	σ_{v9}	σ_{v7}	σ_{v3}	σ_{v1}	σ_{v2}
C_9^{2+}	S_9^{2+}	S_3^{+}	S_9^{+}	S_9^{4+}	σ_h	S_9^{4-}	S_9^{-}	S_3^{-}	S_9^{2-}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}	σ_{v8}	σ_{v9}	σ_{v2}	σ_{v3}	σ_{v1}
C_9^{2-}	S_9^{2-}	S_9^{-}	S_3^{-}	σ_h	S_9^{4-}	S_9^{+}	S_9^{4+}	S_9^{2+}	S_3^{+}	σ_{v9}	σ_{v7}	σ_{v8}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}
C_3^{+}	S_3^{+}	S_9^{4+}	S_9^{2+}	S_9^{4-}	S_9^{+}	S_3^{-}	σ_h	S_9^{2-}	S_9^{-}	σ_{v3}	σ_{v1}	σ_{v2}	σ_{v6}	σ_{v4}	σ_{v5}	σ_{v9}	σ_{v7}	σ_{v8}
C_3^{-}	S_3^{-}	S_9^{2-}	S_9^{4-}	S_9^{-}	S_9^{4+}	σ_h	S_3^{+}	S_9^{+}	S_9^{2+}	σ_{v2}	σ_{v3}	σ_{v1}	σ_{v5}	σ_{v6}	σ_{v4}	σ_{v8}	σ_{v9}	σ_{v7}
C_9^{4+}	S_9^{4+}	S_9^{4-}	S_3^{+}	S_3^{-}	S_9^{2+}	S_9^{2-}	S_9^{+}	S_9^{-}	σ_h	σ_{v7}	σ_{v8}	σ_{v9}	σ_{v2}	σ_{v3}	σ_{v1}	σ_{v5}	σ_{v6}	σ_{v4}
C_9^{4-}	S_9^{4-}	S_3^{-}	S_9^{4+}	S_9^{2-}	S_3^{+}	S_9^{-}	S_9^{2+}	σ_h	S_9^{+}	σ_{v6}	σ_{v4}	σ_{v5}	σ_{v9}	σ_{v7}	σ_{v8}	σ_{v1}	σ_{v2}	σ_{v3}
C'_{21}	σ_{v1}	σ_{v5}	σ_{v8}	σ_{v9}	σ_{v4}	σ_{v2}	σ_{v3}	σ_{v6}	σ_{v7}	σ_h	S_3^{+}	S_3^{-}	S_9^{2-}	S_9^{+}	S_9^{4+}	S_9^{4-}	S_9^{-}	S_9^{2+}
C'_{22}	σ_{v2}	σ_{v6}	σ_{v9}	σ_{v7}	σ_{v5}	σ_{v3}	σ_{v1}	σ_{v4}	σ_{v8}	S_3^{-}	σ_h	S_3^{+}	S_9^{4+}	S_9^{2-}	S_9^{+}	S_9^{2+}	S_9^{4-}	S_9^{-}
C'_{23}	σ_{v3}	σ_{v4}	σ_{v7}	σ_{v8}	σ_{v6}	σ_{v1}	σ_{v2}	σ_{v5}	σ_{v9}	S_3^{+}	S_3^{-}	σ_h	S_9^{+}	S_9^{4+}	S_9^{2-}	S_9^{-}	S_9^{2+}	S_9^{4-}
C'_{24}	σ_{v4}	σ_{v8}	σ_{v3}	σ_{v1}	σ_{v7}	σ_{v5}	σ_{v6}	σ_{v9}	σ_{v2}	S_9^{2+}	S_9^{4-}	S_9^{-}	σ_h	S_3^{+}	S_3^{-}	S_9^{2-}	S_9^{+}	S_9^{4+}
C'_{25}	σ_{v5}	σ_{v9}	σ_{v1}	σ_{v2}	σ_{v8}	σ_{v6}	σ_{v4}	σ_{v7}	σ_{v3}	S_9^{-}	S_9^{2+}	S_9^{4-}	S_3^{-}	σ_h	S_3^{+}	S_9^{4+}	S_9^{2-}	S_9^{+}
C'_{26}	σ_{v6}	σ_{v7}	σ_{v2}	σ_{v3}	σ_{v9}	σ_{v4}	σ_{v5}	σ_{v8}	σ_{v1}	S_9^{4-}	S_9^{-}	S_9^{2+}	S_3^{+}	S_3^{-}	σ_h	S_9^{+}	S_9^{4+}	S_9^{2-}
C'_{27}	σ_{v7}	σ_{v3}	σ_{v6}	σ_{v4}	σ_{v2}	σ_{v8}	σ_{v9}	σ_{v1}	σ_{v5}	S_9^{4+}	S_9^{2-}	S_9^{+}	S_9^{2+}	S_9^{4-}	S_9^{-}	σ_h	S_3^{+}	S_3^{-}
C'_{28}	σ_{v8}	σ_{v1}	σ_{v4}	σ_{v5}	σ_{v3}	σ_{v9}	σ_{v7}	σ_{v2}	σ_{v6}	S_9^{+}	S_9^{4+}	S_9^{2-}	S_9^{-}	S_9^{2+}	S_9^{4-}	S_3^{-}	σ_h	S_3^{+}
C'_{29}	σ_{v9}	σ_{v2}	σ_{v5}	σ_{v6}	σ_{v1}	σ_{v7}	σ_{v8}	σ_{v3}	σ_{v4}	S_9^{2-}	S_9^{+}	S_9^{4+}	S_9^{4-}	S_9^{-}	S_9^{2+}	S_3^{+}	S_3^{-}	σ_h
σ_h	E	C_9^{+}	C_9^{-}	C_9^{2+}	C_9^{2-}	C_3^{+}	C_3^{-}	C_9^{4+}	C_9^{4-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}
S_9^{+}	C_9^{+}	C_9^{2+}	E	C_3^{+}	C_9^{-}	C_9^{4+}	C_9^{2-}	C_9^{4-}	C_3^{-}	C'_{28}	C'_{29}	C'_{27}	C'_{23}	C'_{21}	C'_{22}	C'_{26}	C'_{24}	C'_{25}
S_9^{-}	C_9^{-}	E	C_9^{2-}	C_9^{+}	C_3^{-}	C_9^{2+}	C_9^{4-}	C_3^{+}	C_9^{4+}	C'_{25}	C'_{26}	C'_{24}	C'_{28}	C'_{29}	C'_{27}	C'_{23}	C'_{21}	C'_{22}
S_9^{2+}	C_9^{2+}	C_3^{+}	C_9^{+}	C_9^{4+}	E	C_9^{4-}	C_9^{-}	C_3^{-}	C_9^{2-}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}	C'_{22}	C'_{23}	C'_{21}
S_9^{2-}	C_9^{2-}	C_9^{-}	C_3^{-}	E	C_9^{4-}	C_9^{+}	C_9^{4+}	C_9^{2+}	C_3^{+}	C'_{29}	C'_{27}	C'_{28}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}
S_3^{+}	C_3^{+}	C_9^{4+}	C_9^{2+}	C_9^{4-}	C_9^{+}	C_3^{-}	E	C_9^{2-}	C_9^{-}	C'_{23}	C'_{21}	C'_{22}	C'_{26}	C'_{24}	C'_{25}	C'_{29}	C'_{27}	C'_{28}
S_3^{-}	C_3^{-}	C_9^{2-}	C_9^{4-}	C_9^{-}	C_9^{4+}	E	C_3^{+}	C_9^{+}	C_9^{2+}	C'_{22}	C'_{23}	C'_{21}	C'_{25}	C'_{26}	C'_{24}	C'_{28}	C'_{29}	C'_{27}
S_9^{4+}	C_9^{4+}	C_9^{4-}	C_3^{+}	C_3^{-}	C_9^{2+}	C_9^{2-}	C_9^{+}	C_9^{-}	E	C'_{27}	C'_{28}	C'_{29}	C'_{22}	C'_{23}	C'_{21}	C'_{25}	C'_{26}	C'_{24}
S_9^{4-}	C_9^{4-}	C_3^{-}	C_9^{4+}	C_9^{2-}	C_3^{+}	C_9^{-}	C_9^{2+}	E	C_9^{+}	C'_{26}	C'_{24}	C'_{25}	C'_{29}	C'_{27}	C'_{28}	C'_{21}	C'_{22}	C'_{23}
σ_{v1}	C'_{21}	C'_{25}	C'_{28}	C'_{29}	C'_{24}	C'_{22}	C'_{23}	C'_{26}	C'_{27}	E	C_3^{+}	C_3^{-}	C_9^{2-}	C_9^{+}	C_9^{4+}	C_9^{4-}	C_9^{-}	C_9^{2+}
σ_{v2}	C'_{22}	C'_{26}	C'_{29}	C'_{27}	C'_{25}	C'_{23}	C'_{21}	C'_{24}	C'_{28}	C_3^{-}	E	C_3^{+}	C_9^{4+}	C_9^{2-}	C_9^{+}	C_9^{2+}	C_9^{4-}	C_9^{-}
σ_{v3}	C'_{23}	C'_{24}	C'_{27}	C'_{28}	C'_{26}	C'_{21}	C'_{22}	C'_{25}	C'_{29}	C_3^{+}	C_3^{-}	E	C_9^{+}	C_9^{4+}	C_9^{2-}	C_9^{-}	C_9^{2+}	C_9^{4-}
σ_{v4}	C'_{24}	C'_{28}	C'_{23}	C'_{21}	C'_{27}	C'_{25}	C'_{26}	C'_{29}	C'_{22}	C_9^{2+}	C_9^{4-}	C_9^{-}	E	C_3^{+}	C_3^{-}	C_9^{2-}	C_9^{+}	C_9^{4+}
σ_{v5}	C'_{25}	C'_{29}	C'_{21}	C'_{22}	C'_{28}	C'_{26}	C'_{24}	C'_{27}	C'_{23}	C_9^{-}	C_9^{2+}	C_9^{4-}	C_3^{-}	E	C_3^{+}	C_9^{4+}	C_9^{2-}	C_9^{+}
σ_{v6}	C'_{26}	C'_{27}	C'_{22}	C'_{23}	C'_{29}	C'_{24}	C'_{25}	C'_{28}	C'_{21}	C_9^{4-}	C_9^{-}	C_9^{2+}	C_3^{+}	C_3^{-}	E	C_9^{+}	C_9^{4+}	C_9^{2-}
σ_{v7}	C'_{27}	C'_{23}	C'_{26}	C'_{24}	C'_{22}	C'_{28}	C'_{29}	C'_{21}	C'_{25}	C_9^{4+}	C_9^{2-}	C_9^{+}	C_9^{2+}	C_9^{4-}	C_9^{-}	E	C_3^{+}	C_3^{-}
σ_{v8}	C'_{28}	C'_{21}	C'_{24}	C'_{25}	C'_{23}	C'_{29}	C'_{27}	C'_{22}	C'_{26}	C_9^{+}	C_9^{4+}	C_9^{2-}	C_9^{-}	C_9^{2+}	C_9^{4-}	C_3^{-}	E	C_3^{+}
σ_{v9}	C'_{29}	C'_{22}	C'_{25}	C'_{26}	C'_{21}	C'_{27}	C'_{28}	C'_{23}	C'_{24}	C_9^{2-}	C_9^{+}	C_9^{4+}	C_9^{4-}	C_9^{-}	C_9^{2+}	C_3^{+}	C_3^{-}	E

T **38**.3 Factor table

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$\mathbf{D}_{9h}$	E	C_9^+	C_9^-	C_9^{2+}	C_9^{2-}	C_3^+	C_3^-	C_9^{4+}	C_9^{4-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_9^+	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_9^-	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_9^{2+}	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
C_9^{2-}	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1
C_3^+	1	1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_3^-	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_9^{4+}	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
C_9^{4-}	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1
C'_{21}	1	−1	−1	1	1	−1	−1	1	1	−1	1	1	−1	1	−1	−1	1	−1
C'_{22}	1	−1	−1	1	1	−1	−1	1	1	1	−1	1	−1	−1	1	−1	−1	1
C'_{23}	1	−1	−1	1	1	−1	−1	1	1	1	1	−1	1	−1	−1	1	−1	−1
C'_{24}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	1	1	−1	1	−1
C'_{25}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	1	−1	1	−1	−1	1
C'_{26}	1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	1	1	−1	1	−1	−1
C'_{27}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	−1	1	−1	1	1
C'_{28}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	1	−1	−1	1	−1	1
C'_{29}	1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	−1	1	−1	1	1	−1
σ_h	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
S_9^+	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1
S_9^-	1	1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_9^{2+}	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_9^{2-}	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
S_3^+	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1
S_3^-	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_9^{4+}	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_9^{4-}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
σ_{v1}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	−1	1	−1	1	1
σ_{v2}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	1	−1	−1	1	−1	1
σ_{v3}	1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	−1	1	−1	1	1	−1
σ_{v4}	1	−1	−1	1	1	−1	−1	1	1	1	−1	1	−1	−1	1	−1	−1	1
σ_{v5}	1	−1	−1	1	1	−1	−1	1	1	1	1	−1	1	−1	−1	1	−1	−1
σ_{v6}	1	−1	−1	1	1	−1	−1	1	1	−1	1	1	−1	1	−1	−1	1	−1
σ_{v7}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	1	−1	1	−1	−1	1
σ_{v8}	1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	1	1	−1	1	−1	−1
σ_{v9}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	1	1	−1	1	−1

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T **38**.3 Factor table *(cont.)*

$\mathbf{D}_{9h}$	σ_h	S_9^+	S_9^-	S_9^{2+}	S_9^{2-}	S_3^+	S_3^-	S_9^{4+}	S_9^{4-}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v6}	σ_{v7}	σ_{v8}	σ_{v9}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_9^+	−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_9^-	1	−1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_9^{2+}	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_9^{2-}	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_3^+	−1	1	−1	1	−1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_3^-	1	−1	1	−1	1	−1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_9^{4+}	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1
C_9^{4-}	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
C'_{21}	−1	−1	1	1	−1	−1	1	1	−1	1	1	−1	1	1	−1	1	−1	−1
C'_{22}	−1	−1	1	1	−1	−1	1	1	−1	−1	1	1	−1	1	1	−1	1	−1
C'_{23}	−1	−1	1	1	−1	−1	1	1	−1	1	−1	1	1	−1	1	−1	−1	1
C'_{24}	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	1	1	−1	1	1	−1
C'_{25}	−1	−1	1	1	−1	−1	1	1	−1	−1	−1	1	−1	1	1	−1	1	1
C'_{26}	−1	−1	1	1	−1	−1	1	1	−1	1	−1	−1	1	−1	1	1	−1	1
C'_{27}	−1	−1	1	1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	1	1	−1
C'_{28}	−1	−1	1	1	−1	−1	1	1	−1	1	−1	1	−1	−1	1	−1	1	1
C'_{29}	−1	−1	1	1	−1	−1	1	1	−1	1	1	−1	1	−1	−1	1	−1	1
σ_h	−1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_9^+	1	−1	1	−1	1	−1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_9^-	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1
S_9^{2+}	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_9^{2-}	−1	1	−1	1	−1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_3^+	1	−1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_3^-	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_9^{4+}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_9^{4-}	−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
σ_{v1}	1	1	−1	−1	1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	−1
σ_{v2}	1	1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	−1	1	−1	−1	1
σ_{v3}	1	1	−1	−1	1	1	−1	−1	1	1	1	−1	1	−1	−1	1	−1	−1
σ_{v4}	1	1	−1	−1	1	1	−1	−1	1	−1	−1	1	−1	1	1	−1	1	−1
σ_{v5}	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	1	−1	−1	1
σ_{v6}	1	1	−1	−1	1	1	−1	−1	1	−1	1	−1	1	1	−1	1	−1	−1
σ_{v7}	1	1	−1	−1	1	1	−1	−1	1	−1	−1	1	−1	−1	1	−1	1	1
σ_{v8}	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	−1	1	−1	1
σ_{v9}	1	1	−1	−1	1	1	−1	−1	1	−1	1	−1	−1	1	−1	1	1	−1

T **38**.4 Character table

§ **16**–4, p. 71

$\mathbf{D}_{9h}$	E	$2C_9$	$2C_9^2$	$2C_3$	$2C_9^4$	$9C_2'$	σ_h	$2S_9$	$2S_9^2$	$2S_3$	$2S_9^4$	$9\sigma_v$	τ
A_1'	1	1	1	1	1	1	1	1	1	1	1	1	a
A_2'	1	1	1	1	1	-1	1	1	1	1	1	-1	a
E_1'	2	$2c_9^2$	$2c_9^4$	-1	$2c_9^8$	0	2	$2c_9^2$	$2c_9^4$	-1	$2c_9^8$	0	a
E_2'	2	$2c_9^4$	$2c_9^8$	-1	$2c_9^2$	0	2	$2c_9^4$	$2c_9^8$	-1	$2c_9^2$	0	a
E_3'	2	-1	-1	2	-1	0	2	-1	-1	2	-1	0	a
E_4'	2	$2c_9^8$	$2c_9^2$	-1	$2c_9^4$	0	2	$2c_9^8$	$2c_9^2$	-1	$2c_9^4$	0	a
A_1''	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	a
A_2''	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	a
E_1''	2	$2c_9^2$	$2c_9^4$	-1	$2c_9^8$	0	-2	$-2c_9^2$	$-2c_9^4$	1	$-2c_9^8$	0	a
E_2''	2	$2c_9^4$	$2c_9^8$	-1	$2c_9^2$	0	-2	$-2c_9^4$	$-2c_9^8$	1	$-2c_9^2$	0	a
E_3''	2	-1	-1	2	-1	0	-2	1	1	-2	1	0	a
E_4''	2	$2c_9^8$	$2c_9^2$	-1	$2c_9^4$	0	-2	$-2c_9^8$	$-2c_9^2$	1	$-2c_9^4$	0	a
$E_{1/2}$	2	$-2c_9^8$	$2c_9^2$	1	$2c_9^4$	0	0	$2c_{18}^7$	$2c_{18}^5$	$\sqrt{3}$	$2c_{18}$	0	c
$E_{3/2}$	2	1	-1	-2	-1	0	0	$-\sqrt{3}$	$-\sqrt{3}$	0	$\sqrt{3}$	0	c
$E_{5/2}$	2	$-2c_9^4$	$2c_9^8$	1	$2c_9^2$	0	0	$2c_{18}$	$-2c_{18}^7$	$-\sqrt{3}$	$2c_{18}^5$	0	c
$E_{7/2}$	2	$-2c_9^2$	$2c_9^4$	1	$2c_9^8$	0	0	$-2c_{18}^5$	$2c_{18}$	$-\sqrt{3}$	$2c_{18}^7$	0	c
$E_{9/2}$	2	-2	2	-2	2	0	0	0	0	0	0	0	c
$E_{11/2}$	2	$-2c_9^2$	$2c_9^4$	1	$2c_9^8$	0	0	$2c_{18}^5$	$-2c_{18}$	$\sqrt{3}$	$-2c_{18}^7$	0	c
$E_{13/2}$	2	$-2c_9^4$	$2c_9^8$	1	$2c_9^2$	0	0	$-2c_{18}$	$2c_{18}^7$	$\sqrt{3}$	$-2c_{18}^5$	0	c
$E_{15/2}$	2	1	-1	-2	-1	0	0	$\sqrt{3}$	$\sqrt{3}$	0	$-\sqrt{3}$	0	c
$E_{17/2}$	2	$-2c_9^8$	$2c_9^2$	1	$2c_9^4$	0	0	$-2c_{18}^7$	$-2c_{18}^5$	$-\sqrt{3}$	$-2c_{18}$	0	c

$c_n^m = \cos \frac{m}{n}\pi$

T **38**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{9h}$	0	1	2	3
A_1'	$\square 1$		$x^2+y^2, \square z^2$	
A_2'		R_z		
E_1'		$\square(x,y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, \square(xz^2, yz^2)$
E_2'			$\square(xy, x^2-y^2)$	
E_3'				$\square\{x(x^2-3y^2), y(3x^2-y^2)\}$
E_4'				
A_1''				
A_2''		$\square z$		$(x^2+y^2)z, \square z^3$
E_1''		(R_x, R_y)	$\square(zx, yz)$	
E_2''				$\square\{xyz, z(x^2-y^2)\}$
E_3''				
E_4''				

T **38**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{9h}$	$\langle \vert j\, m\rangle \vert$		ι	μ
A_1'	$\vert 0\,0\rangle_+$	$\vert 9\,9\rangle_-$	2	18
A_2'	$\vert 9\,9\rangle_+$	$\vert 18\,18\rangle_-$	2	18
E_1'	$\langle \vert 1\,1\rangle, \vert 1\,\overline{1}\rangle \vert$	$\langle \vert 8\,\overline{8}\rangle, -\vert 8\,8\rangle \vert$	2	± 18
E_2'	$\langle \vert 2\,\overline{2}\rangle, -\vert 2\,2\rangle \vert$	$\langle \vert 7\,7\rangle, \vert 7\,\overline{7}\rangle \vert$	2	± 18
E_3'	$\langle \vert 3\,3\rangle, \vert 3\,\overline{3}\rangle \vert$	$\langle \vert 6\,\overline{6}\rangle, -\vert 6\,6\rangle \vert$	2	± 18
E_4'	$\langle \vert 4\,4\rangle, -\vert 4\,\overline{4}\rangle \vert$	$\langle \vert 5\,\overline{5}\rangle, \vert 5\,5\rangle \vert$	2	± 18
A_1''	$\vert 10\,9\rangle_+$	$\vert 19\,18\rangle_-$	2	18
A_2''	$\vert 1\,0\rangle_+$	$\vert 10\,9\rangle_-$	2	18
E_1''	$\langle \vert 2\,1\rangle, -\vert 2\,\overline{1}\rangle \vert$	$\langle \vert 9\,\overline{8}\rangle, \vert 9\,8\rangle \vert$	2	± 18
E_2''	$\langle \vert 3\,\overline{2}\rangle, \vert 3\,2\rangle \vert$	$\langle \vert 8\,7\rangle, -\vert 8\,\overline{7}\rangle \vert$	2	± 18
E_3''	$\langle \vert 4\,3\rangle, -\vert 4\,\overline{3}\rangle \vert$	$\langle \vert 7\,\overline{6}\rangle, \vert 7\,6\rangle \vert$	2	± 18
E_4''	$\langle \vert 5\,4\rangle, \vert 5\,\overline{4}\rangle \vert$	$\langle \vert 6\,\overline{5}\rangle, -\vert 6\,5\rangle \vert$	2	± 18
$E_{1/2}$	$\langle \vert \tfrac{1}{2}\,\tfrac{1}{2}\rangle, \vert \tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle \vert$	$\langle \vert \tfrac{3}{2}\,\tfrac{1}{2}\rangle, -\vert \tfrac{3}{2}\,\overline{\tfrac{1}{2}}\rangle \vert$	2	± 18
	$\langle \vert \tfrac{17}{2}\,\overline{\tfrac{17}{2}}\rangle, \vert \tfrac{17}{2}\,\tfrac{17}{2}\rangle \vert^\bullet$	$\langle \vert \tfrac{19}{2}\,\overline{\tfrac{17}{2}}\rangle, -\vert \tfrac{19}{2}\,\tfrac{17}{2}\rangle \vert^\bullet$	2	± 18
$E_{3/2}$	$\langle \vert \tfrac{3}{2}\,\tfrac{3}{2}\rangle, \vert \tfrac{3}{2}\,\overline{\tfrac{3}{2}}\rangle \vert$	$\langle \vert \tfrac{5}{2}\,\tfrac{3}{2}\rangle, -\vert \tfrac{5}{2}\,\overline{\tfrac{3}{2}}\rangle \vert$	2	± 18
	$\langle \vert \tfrac{15}{2}\,\overline{\tfrac{15}{2}}\rangle, \vert \tfrac{15}{2}\,\tfrac{15}{2}\rangle \vert^\bullet$	$\langle \vert \tfrac{17}{2}\,\overline{\tfrac{15}{2}}\rangle, -\vert \tfrac{17}{2}\,\tfrac{15}{2}\rangle \vert^\bullet$	2	± 18
$E_{5/2}$	$\langle \vert \tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle, \vert \tfrac{5}{2}\,\tfrac{5}{2}\rangle \vert$	$\langle \vert \tfrac{7}{2}\,\overline{\tfrac{5}{2}}\rangle, -\vert \tfrac{7}{2}\,\tfrac{5}{2}\rangle \vert$	2	± 18
	$\langle \vert \tfrac{13}{2}\,\tfrac{13}{2}\rangle, \vert \tfrac{13}{2}\,\overline{\tfrac{13}{2}}\rangle \vert^\bullet$	$\langle \vert \tfrac{15}{2}\,\tfrac{13}{2}\rangle, -\vert \tfrac{15}{2}\,\overline{\tfrac{13}{2}}\rangle \vert^\bullet$	2	± 18
$E_{7/2}$	$\langle \vert \tfrac{7}{2}\,\tfrac{7}{2}\rangle, -\vert \tfrac{7}{2}\,\overline{\tfrac{7}{2}}\rangle \vert$	$\langle \vert \tfrac{9}{2}\,\tfrac{7}{2}\rangle, \vert \tfrac{9}{2}\,\overline{\tfrac{7}{2}}\rangle \vert$	2	± 18
	$\langle \vert \tfrac{11}{2}\,\overline{\tfrac{11}{2}}\rangle, -\vert \tfrac{11}{2}\,\tfrac{11}{2}\rangle \vert^\bullet$	$\langle \vert \tfrac{13}{2}\,\overline{\tfrac{11}{2}}\rangle, \vert \tfrac{13}{2}\,\tfrac{11}{2}\rangle \vert^\bullet$	2	± 18
$E_{9/2}$	$\langle \vert \tfrac{9}{2}\,\tfrac{9}{2}\rangle, \vert \tfrac{9}{2}\,\overline{\tfrac{9}{2}}\rangle \vert$	$\langle \vert \tfrac{11}{2}\,\tfrac{9}{2}\rangle, -\vert \tfrac{11}{2}\,\overline{\tfrac{9}{2}}\rangle \vert$	2	± 18
	$\langle \vert \tfrac{9}{2}\,\overline{\tfrac{9}{2}}\rangle, \vert \tfrac{9}{2}\,\tfrac{9}{2}\rangle \vert^\bullet$	$\langle \vert \tfrac{11}{2}\,\overline{\tfrac{9}{2}}\rangle, -\vert \tfrac{11}{2}\,\tfrac{9}{2}\rangle \vert^\bullet$	2	± 18
$E_{11/2}$	$\langle \vert \tfrac{11}{2}\,\overline{\tfrac{11}{2}}\rangle, -\vert \tfrac{11}{2}\,\tfrac{11}{2}\rangle \vert$	$\langle \vert \tfrac{13}{2}\,\overline{\tfrac{11}{2}}\rangle, \vert \tfrac{13}{2}\,\tfrac{11}{2}\rangle \vert$	2	± 18
	$\langle \vert \tfrac{7}{2}\,\tfrac{7}{2}\rangle, -\vert \tfrac{7}{2}\,\overline{\tfrac{7}{2}}\rangle \vert^\bullet$	$\langle \vert \tfrac{9}{2}\,\tfrac{7}{2}\rangle, \vert \tfrac{9}{2}\,\overline{\tfrac{7}{2}}\rangle \vert^\bullet$	2	± 18
$E_{13/2}$	$\langle \vert \tfrac{13}{2}\,\tfrac{13}{2}\rangle, \vert \tfrac{13}{2}\,\overline{\tfrac{13}{2}}\rangle \vert$	$\langle \vert \tfrac{15}{2}\,\tfrac{13}{2}\rangle, -\vert \tfrac{15}{2}\,\overline{\tfrac{13}{2}}\rangle \vert$	2	± 18
	$\langle \vert \tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle, \vert \tfrac{5}{2}\,\tfrac{5}{2}\rangle \vert^\bullet$	$\langle \vert \tfrac{7}{2}\,\overline{\tfrac{5}{2}}\rangle, -\vert \tfrac{7}{2}\,\tfrac{5}{2}\rangle \vert^\bullet$	2	± 18
$E_{15/2}$	$\langle \vert \tfrac{15}{2}\,\overline{\tfrac{15}{2}}\rangle, \vert \tfrac{15}{2}\,\tfrac{15}{2}\rangle \vert$	$\langle \vert \tfrac{17}{2}\,\overline{\tfrac{15}{2}}\rangle, -\vert \tfrac{17}{2}\,\tfrac{15}{2}\rangle \vert$	2	± 18
	$\langle \vert \tfrac{3}{2}\,\tfrac{3}{2}\rangle, \vert \tfrac{3}{2}\,\overline{\tfrac{3}{2}}\rangle \vert^\bullet$	$\langle \vert \tfrac{5}{2}\,\tfrac{3}{2}\rangle, -\vert \tfrac{5}{2}\,\overline{\tfrac{3}{2}}\rangle \vert^\bullet$	2	± 18
$E_{17/2}$	$\langle \vert \tfrac{17}{2}\,\overline{\tfrac{17}{2}}\rangle, \vert \tfrac{17}{2}\,\tfrac{17}{2}\rangle \vert$	$\langle \vert \tfrac{19}{2}\,\overline{\tfrac{17}{2}}\rangle, -\vert \tfrac{19}{2}\,\tfrac{17}{2}\rangle \vert$	2	± 18
	$\langle \vert \tfrac{1}{2}\,\tfrac{1}{2}\rangle, \vert \tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle \vert^\bullet$	$\langle \vert \tfrac{3}{2}\,\tfrac{1}{2}\rangle, -\vert \tfrac{3}{2}\,\overline{\tfrac{1}{2}}\rangle \vert^\bullet$	2	± 18

T **38**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_{9h}$	E'_1	E'_2	E'_3	E'_4	E''_1	E''_2
E	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$
C_9^+	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\theta^*&0\\0&\theta\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$
C_9^-	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\theta&0\\0&\theta^*\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$
C_9^{2+}	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\theta&0\\0&\theta^*\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\theta&0\\0&\theta^*\end{bmatrix}$
C_9^{2-}	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\theta^*&0\\0&\theta\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\theta^*&0\\0&\theta\end{bmatrix}$
C_3^+	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$
C_3^-	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$
C_9^{4+}	$\begin{bmatrix}\theta^*&0\\0&\theta\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\theta^*&0\\0&\theta\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$
C_9^{4-}	$\begin{bmatrix}\theta&0\\0&\theta^*\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\theta&0\\0&\theta^*\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$
C'_{21}	$\begin{bmatrix}0&\bar{1}\\\bar{1}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{1}\\\bar{1}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{1}\\\bar{1}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{1}\\\bar{1}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{1}\\\bar{1}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{1}\\\bar{1}&0\end{bmatrix}$
C'_{22}	$\begin{bmatrix}0&\bar{\eta}\\\bar{\eta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\eta}\\\bar{\eta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{1}\\\bar{1}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\eta}\\\bar{\eta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\eta}\\\bar{\eta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\eta}\\\bar{\eta}^*&0\end{bmatrix}$
C'_{23}	$\begin{bmatrix}0&\bar{\eta}^*\\\bar{\eta}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\eta}^*\\\bar{\eta}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{1}\\\bar{1}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\eta}^*\\\bar{\eta}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\eta}^*\\\bar{\eta}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\eta}^*\\\bar{\eta}&0\end{bmatrix}$
C'_{24}	$\begin{bmatrix}0&\bar{\epsilon}^*\\\bar{\epsilon}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\theta}\\\bar{\theta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\eta}\\\bar{\eta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\delta}\\\bar{\delta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\epsilon}^*\\\bar{\epsilon}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\theta}\\\bar{\theta}^*&0\end{bmatrix}$
C'_{25}	$\begin{bmatrix}0&\bar{\delta}\\\bar{\delta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\epsilon}^*\\\bar{\epsilon}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\eta}\\\bar{\eta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\theta}\\\bar{\theta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\delta}\\\bar{\delta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\epsilon}^*\\\bar{\epsilon}&0\end{bmatrix}$
C'_{26}	$\begin{bmatrix}0&\bar{\theta}\\\bar{\theta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\delta}\\\bar{\delta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\eta}\\\bar{\eta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\epsilon}^*\\\bar{\epsilon}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\theta}\\\bar{\theta}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\delta}\\\bar{\delta}^*&0\end{bmatrix}$
C'_{27}	$\begin{bmatrix}0&\bar{\theta}^*\\\bar{\theta}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\delta}^*\\\bar{\delta}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\eta}^*\\\bar{\eta}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\epsilon}\\\bar{\epsilon}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\theta}^*\\\bar{\theta}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\delta}^*\\\bar{\delta}&0\end{bmatrix}$
C'_{28}	$\begin{bmatrix}0&\bar{\delta}^*\\\bar{\delta}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\epsilon}\\\bar{\epsilon}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\eta}^*\\\bar{\eta}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\theta}^*\\\bar{\theta}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\delta}^*\\\bar{\delta}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\epsilon}\\\bar{\epsilon}^*&0\end{bmatrix}$
C'_{29}	$\begin{bmatrix}0&\bar{\epsilon}\\\bar{\epsilon}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\theta}^*\\\bar{\theta}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\eta}^*\\\bar{\eta}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\delta}^*\\\bar{\delta}&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\epsilon}\\\bar{\epsilon}^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar{\theta}^*\\\bar{\theta}&0\end{bmatrix}$

$\delta = \exp(2\pi i/9)$, $\epsilon = \exp(4\pi i/9)$, $\eta = \exp(6\pi i/9)$, $\theta = \exp(8\pi i/9)$

↠

T **38**.7 Matrix representations *(cont.)*

$\mathbf{D}_{9h}$	E''_3	E''_4	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_9^+	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \overline{\delta} & 0 \\ 0 & \overline{\delta}^* \end{bmatrix}$
C_9^-	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\delta}^* & 0 \\ 0 & \overline{\delta} \end{bmatrix}$
C_9^{2+}	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
C_9^{2-}	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
C_3^+	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$
C_3^-	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$
C_9^{4+}	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$
C_9^{4-}	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta}^* \\ \mathrm{i}\overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon} \\ \mathrm{i}\overline{\epsilon}^* & 0 \end{bmatrix}$
C'_{25}	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta}^* \\ \mathrm{i}\overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon} \\ \mathrm{i}\overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$
C'_{26}	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon} \\ \mathrm{i}\overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta}^* \\ \mathrm{i}\overline{\theta} & 0 \end{bmatrix}$
C'_{27}	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon}^* \\ \mathrm{i}\overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta} \\ \mathrm{i}\overline{\theta}^* & 0 \end{bmatrix}$
C'_{28}	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta} \\ \mathrm{i}\overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon}^* \\ \mathrm{i}\overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$
C'_{29}	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta} \\ \mathrm{i}\overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon}^* \\ \mathrm{i}\overline{\epsilon} & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/9)$, $\epsilon = \exp(4\pi\mathrm{i}/9)$, $\eta = \exp(6\pi\mathrm{i}/9)$, $\theta = \exp(8\pi\mathrm{i}/9)$ $\ggg$

T **38**.7 Matrix representations *(cont.)*

$\mathbf{D}_{9h}$	$E_{9/2}$	$E_{11/2}$	$E_{13/2}$	$E_{15/2}$	$E_{17/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_9^+	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\delta} & 0 \\ 0 & \overline{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$
C_9^-	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\delta}^* & 0 \\ 0 & \overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$
C_9^{2+}	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$
C_9^{2-}	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$
C_3^+	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$
C_3^-	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$
C_9^{4+}	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
C_9^{4-}	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon} \\ \mathrm{i}\overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta}^* \\ \mathrm{i}\overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$
C'_{25}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon} \\ \mathrm{i}\overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta}^* \\ \mathrm{i}\overline{\theta} & 0 \end{bmatrix}$
C'_{26}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta}^* \\ \mathrm{i}\overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon} \\ \mathrm{i}\overline{\epsilon}^* & 0 \end{bmatrix}$
C'_{27}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta} \\ \mathrm{i}\overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon}^* \\ \mathrm{i}\overline{\epsilon} & 0 \end{bmatrix}$
C'_{28}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon}^* \\ \mathrm{i}\overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta} \\ \mathrm{i}\overline{\theta}^* & 0 \end{bmatrix}$
C'_{29}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon}^* \\ \mathrm{i}\overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta} \\ \mathrm{i}\overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/9)$, $\epsilon = \exp(4\pi\mathrm{i}/9)$, $\eta = \exp(6\pi\mathrm{i}/9)$, $\theta = \exp(8\pi\mathrm{i}/9)$ ⇛

T **38**.7 Matrix representations *(cont.)*

$\mathbf{D}_{9h}$	E_1'	E_2'	E_3'	E_4'	E_1''	E_2''
σ_h	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$
S_9^+	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \overline{\delta}^* & 0 \\ 0 & \overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$
S_9^-	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\delta} & 0 \\ 0 & \overline{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$
S_9^{2+}	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$
S_9^{2-}	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$
S_3^+	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$
S_3^-	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$
S_9^{4+}	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \overline{\delta}^* & 0 \\ 0 & \overline{\delta} \end{bmatrix}$
S_9^{4-}	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\delta} & 0 \\ 0 & \overline{\delta}^* \end{bmatrix}$
σ_{v1}	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
σ_{v2}	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$
σ_{v3}	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$
σ_{v4}	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$
σ_{v5}	$\begin{bmatrix} 0 & \overline{\delta} \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$
σ_{v6}	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$
σ_{v7}	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$
σ_{v8}	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$
σ_{v9}	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$

$\delta = \exp(2\pi i/9)$, $\epsilon = \exp(4\pi i/9)$, $\eta = \exp(6\pi i/9)$, $\theta = \exp(8\pi i/9)$ $\twoheadrightarrow$

T **38**.7 Matrix representations *(cont.)*

$\mathbf{D}_{9h}$	E_3''	E_4''	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$
σ_h	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$
S_9^+	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\theta} & 0 \\ 0 & \mathrm{i}\theta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\overline{\delta}^* \end{bmatrix}$
S_9^-	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta^* & 0 \\ 0 & \mathrm{i}\overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta}^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$
S_9^{2+}	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\delta} & 0 \\ 0 & \overline{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\theta}^* & 0 \\ 0 & \mathrm{i}\theta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$
S_9^{2-}	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{\delta}^* & 0 \\ 0 & \overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta & 0 \\ 0 & \mathrm{i}\overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\overline{\epsilon} \end{bmatrix}$
S_3^+	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\overline{\eta}^* \end{bmatrix}$
S_3^-	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$
S_9^{4+}	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\theta} & 0 \\ 0 & \mathrm{i}\theta^* \end{bmatrix}$
S_9^{4-}	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta^* & 0 \\ 0 & \mathrm{i}\overline{\theta} \end{bmatrix}$
σ_{v1}	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \overline{1} & 0 \end{bmatrix}$
σ_{v2}	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \overline{\eta} & 0 \end{bmatrix}$
σ_{v3}	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \overline{\eta}^* & 0 \end{bmatrix}$
σ_{v4}	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \overline{\epsilon}^* & 0 \end{bmatrix}$
σ_{v5}	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \overline{\delta} & 0 \end{bmatrix}$
σ_{v6}	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \overline{\theta} & 0 \end{bmatrix}$
σ_{v7}	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \overline{\theta}^* & 0 \end{bmatrix}$
σ_{v8}	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \overline{\delta}^* & 0 \end{bmatrix}$
σ_{v9}	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \overline{\epsilon} & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/9)$, $\epsilon = \exp(4\pi\mathrm{i}/9)$, $\eta = \exp(6\pi\mathrm{i}/9)$, $\theta = \exp(8\pi\mathrm{i}/9)$ $\twoheadrightarrow$

T **38**.7 Matrix representations *(cont.)*

$\mathbf{D}_{9h}$	$E_{9/2}$	$E_{11/2}$	$E_{13/2}$	$E_{15/2}$	$E_{17/2}$
σ_h	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$
S_9^+	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta & 0 \\ 0 & \mathrm{i}\overline{\theta}^* \end{bmatrix}$
S_9^-	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\theta}^* & 0 \\ 0 & \mathrm{i}\theta \end{bmatrix}$
S_9^{2+}	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta^* & 0 \\ 0 & \mathrm{i}\overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta}^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$
S_9^{2-}	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\theta} & 0 \\ 0 & \mathrm{i}\theta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\overline{\delta}^* \end{bmatrix}$
S_3^+	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\overline{\eta}^* \end{bmatrix}$
S_3^-	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$
S_9^{4+}	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta & 0 \\ 0 & \mathrm{i}\overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\overline{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$
S_9^{4-}	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\theta}^* & 0 \\ 0 & \mathrm{i}\theta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta}^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\overline{\epsilon}^* \end{bmatrix}$
σ_{v1}	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$
σ_{v2}	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \overline{\eta} & 0 \end{bmatrix}$
σ_{v3}	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \overline{\eta}^* & 0 \end{bmatrix}$
σ_{v4}	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \overline{\delta} & 0 \end{bmatrix}$
σ_{v5}	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \overline{\theta} & 0 \end{bmatrix}$
σ_{v6}	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \overline{\epsilon}^* & 0 \end{bmatrix}$
σ_{v7}	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \overline{\epsilon} & 0 \end{bmatrix}$
σ_{v8}	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \overline{\theta}^* & 0 \end{bmatrix}$
σ_{v9}	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \overline{\delta}^* & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/9)$, $\epsilon = \exp(4\pi\mathrm{i}/9)$, $\eta = \exp(6\pi\mathrm{i}/9)$, $\theta = \exp(8\pi\mathrm{i}/9)$

T **38**.8 Direct products of representations § **16**–8, p. 81

$\mathbf{D}_{9h}$	A'_1	A'_2	E'_1	E'_2	E'_3	E'_4
A'_1	A'_1	A'_2	E'_1	E'_2	E'_3	E'_4
A'_2		A'_1	E'_1	E'_2	E'_3	E'_4
E'_1			$A'_1 \oplus \{A'_2\} \oplus E'_2$	$E'_1 \oplus E'_3$	$E'_2 \oplus E'_4$	$E'_3 \oplus E'_4$
E'_2				$A'_1 \oplus \{A'_2\} \oplus E'_4$	$E'_1 \oplus E'_4$	$E'_2 \oplus E'_3$
E'_3					$A'_1 \oplus \{A'_2\} \oplus E'_3$	$E'_1 \oplus E'_2$
E'_4						$A'_1 \oplus \{A'_2\} \oplus E'_1$

⇾

T **38**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{9h}$	A''_1	A''_2	E''_1	E''_2	E''_3	E''_4
A'_1	A''_1	A''_2	E''_1	E''_2	E''_3	E''_4
A'_2	A''_2	A''_1	E''_1	E''_2	E''_3	E''_4
E'_1	E''_1	E''_1	$A''_1 \oplus A''_2 \oplus E''_2$	$E''_1 \oplus E''_3$	$E''_2 \oplus E''_4$	$E''_3 \oplus E''_4$
E'_2	E''_2	E''_2	$E''_1 \oplus E''_3$	$A''_1 \oplus A''_2 \oplus E''_4$	$E''_1 \oplus E''_4$	$E''_2 \oplus E''_3$
E'_3	E''_3	E''_3	$E''_2 \oplus E''_4$	$E''_1 \oplus E''_4$	$A''_1 \oplus A''_2 \oplus E''_3$	$E''_1 \oplus E''_2$
E'_4	E''_4	E''_4	$E''_3 \oplus E''_4$	$E''_2 \oplus E''_3$	$E''_1 \oplus E''_2$	$A''_1 \oplus A''_2 \oplus E''_1$
A''_1	A'_1	A'_2	E'_1	E'_2	E'_3	E'_4
A''_2		A'_1	E'_1	E'_2	E'_3	E'_4
E''_1			$A'_1 \oplus \{A'_2\} \oplus E'_2$	$E'_1 \oplus E'_3$	$E'_2 \oplus E'_4$	$E'_3 \oplus E'_4$
E''_2				$A'_1 \oplus \{A'_2\} \oplus E'_4$	$E'_1 \oplus E'_4$	$E'_2 \oplus E'_3$
E''_3					$A'_1 \oplus \{A'_2\} \oplus E'_3$	$E'_1 \oplus E'_2$
E''_4						$A'_1 \oplus \{A'_2\} \oplus E'_1$

⇾

T **38**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{9h}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$
A'_1	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$
A'_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$
E'_1	$E_{15/2} \oplus E_{17/2}$	$E_{13/2} \oplus E_{17/2}$	$E_{11/2} \oplus E_{15/2}$	$E_{9/2} \oplus E_{13/2}$	$E_{7/2} \oplus E_{11/2}$
E'_2	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{9/2}$	$E_{3/2} \oplus E_{11/2}$	$E_{5/2} \oplus E_{13/2}$
E'_3	$E_{11/2} \oplus E_{13/2}$	$E_{9/2} \oplus E_{15/2}$	$E_{7/2} \oplus E_{17/2}$	$E_{5/2} \oplus E_{17/2}$	$E_{3/2} \oplus E_{15/2}$
E'_4	$E_{7/2} \oplus E_{9/2}$	$E_{5/2} \oplus E_{11/2}$	$E_{3/2} \oplus E_{13/2}$	$E_{1/2} \oplus E_{15/2}$	$E_{1/2} \oplus E_{17/2}$
A''_1	$E_{17/2}$	$E_{15/2}$	$E_{13/2}$	$E_{11/2}$	$E_{9/2}$
A''_2	$E_{17/2}$	$E_{15/2}$	$E_{13/2}$	$E_{11/2}$	$E_{9/2}$
E''_1	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{5/2} \oplus E_{9/2}$	$E_{7/2} \oplus E_{11/2}$
E''_2	$E_{13/2} \oplus E_{15/2}$	$E_{11/2} \oplus E_{17/2}$	$E_{9/2} \oplus E_{17/2}$	$E_{7/2} \oplus E_{15/2}$	$E_{5/2} \oplus E_{13/2}$
E''_3	$E_{5/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{9/2}$	$E_{1/2} \oplus E_{11/2}$	$E_{1/2} \oplus E_{13/2}$	$E_{3/2} \oplus E_{15/2}$
E''_4	$E_{9/2} \oplus E_{11/2}$	$E_{7/2} \oplus E_{13/2}$	$E_{5/2} \oplus E_{15/2}$	$E_{3/2} \oplus E_{17/2}$	$E_{1/2} \oplus E_{17/2}$
$E_{1/2}$	$\{A'_1\} \oplus A'_2 \oplus E''_1$	$E'_2 \oplus E''_1$	$E'_2 \oplus E''_3$	$E'_4 \oplus E''_3$	$E'_4 \oplus E''_4$
$E_{3/2}$		$\{A'_1\} \oplus A'_2 \oplus E''_3$	$E'_4 \oplus E''_1$	$E'_2 \oplus E''_4$	$E'_3 \oplus E''_3$
$E_{5/2}$			$\{A'_1\} \oplus A'_2 \oplus E''_4$	$E'_3 \oplus E''_1$	$E'_2 \oplus E''_2$
$E_{7/2}$				$\{A'_1\} \oplus A'_2 \oplus E''_2$	$E'_1 \oplus E''_1$
$E_{9/2}$					$\{A'_1\} \oplus A'_2 \oplus A''_1 \oplus A''_2$

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T **38**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{9h}$	$E_{11/2}$	$E_{13/2}$	$E_{15/2}$	$E_{17/2}$
A'_1	$E_{11/2}$	$E_{13/2}$	$E_{15/2}$	$E_{17/2}$
A'_2	$E_{11/2}$	$E_{13/2}$	$E_{15/2}$	$E_{17/2}$
E'_1	$E_{5/2} \oplus E_{9/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$
E'_2	$E_{7/2} \oplus E_{15/2}$	$E_{9/2} \oplus E_{17/2}$	$E_{11/2} \oplus E_{17/2}$	$E_{13/2} \oplus E_{15/2}$
E'_3	$E_{1/2} \oplus E_{13/2}$	$E_{1/2} \oplus E_{11/2}$	$E_{3/2} \oplus E_{9/2}$	$E_{5/2} \oplus E_{7/2}$
E'_4	$E_{3/2} \oplus E_{17/2}$	$E_{5/2} \oplus E_{15/2}$	$E_{7/2} \oplus E_{13/2}$	$E_{9/2} \oplus E_{11/2}$
A''_1	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
A''_2	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
E''_1	$E_{9/2} \oplus E_{13/2}$	$E_{11/2} \oplus E_{15/2}$	$E_{13/2} \oplus E_{17/2}$	$E_{15/2} \oplus E_{17/2}$
E''_2	$E_{3/2} \oplus E_{11/2}$	$E_{1/2} \oplus E_{9/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{5/2}$
E''_3	$E_{5/2} \oplus E_{17/2}$	$E_{7/2} \oplus E_{17/2}$	$E_{9/2} \oplus E_{15/2}$	$E_{11/2} \oplus E_{13/2}$
E''_4	$E_{1/2} \oplus E_{15/2}$	$E_{3/2} \oplus E_{13/2}$	$E_{5/2} \oplus E_{11/2}$	$E_{7/2} \oplus E_{9/2}$
$E_{1/2}$	$E'_3 \oplus E''_4$	$E'_3 \oplus E''_2$	$E'_1 \oplus E''_2$	$E'_1 \oplus A''_1 \oplus A''_2$
$E_{3/2}$	$E'_4 \oplus E''_2$	$E'_1 \oplus E''_4$	$E'_3 \oplus A''_1 \oplus A''_2$	$E'_1 \oplus E''_2$
$E_{5/2}$	$E'_1 \oplus E''_3$	$E'_4 \oplus A''_1 \oplus A''_2$	$E'_1 \oplus E''_4$	$E'_3 \oplus E''_2$
$E_{7/2}$	$E'_2 \oplus A''_1 \oplus A''_2$	$E'_1 \oplus E''_3$	$E'_4 \oplus E''_2$	$E'_3 \oplus E''_4$
$E_{9/2}$	$E'_1 \oplus E''_1$	$E'_2 \oplus E''_2$	$E'_3 \oplus E''_3$	$E'_4 \oplus E''_4$
$E_{11/2}$	$\{A'_1\} \oplus A'_2 \oplus E''_2$	$E'_3 \oplus E''_1$	$E'_2 \oplus E''_4$	$E'_4 \oplus E''_3$
$E_{13/2}$		$\{A'_1\} \oplus A'_2 \oplus E''_4$	$E'_4 \oplus E''_1$	$E'_2 \oplus E''_3$
$E_{15/2}$			$\{A'_1\} \oplus A'_2 \oplus E''_3$	$E'_2 \oplus E''_1$
$E_{17/2}$				$\{A'_1\} \oplus A'_2 \oplus E''_1$

T **38**.9 Subduction (descent of symmetry) § **16**–9, p. 82

$\mathbf{D}_{9h}$	$\mathbf{C}_{9h}$	$\mathbf{C}_{3h}$	$(\mathbf{C}_{9v})$	$(\mathbf{C}_{3v})$	$(\mathbf{C}_{2v})$ C'_2, σ_v, σ_h	$(\mathbf{D}_{3h})$	$(\mathbf{D}_9)$
A'_1	A'	A'	A_1	A_1	A_1	A'_1	A_1
A'_2	A'	A'	A_2	A_2	B_1	A'_2	A_2
E'_1	${}^1E'_1 \oplus {}^2E'_1$	${}^1E' \oplus {}^2E'$	E_1	E	$A_1 \oplus B_1$	E'	E_1
E'_2	${}^1E'_2 \oplus {}^2E'_2$	${}^1E' \oplus {}^2E'$	E_2	E	$A_1 \oplus B_1$	E'	E_2
E'_3	${}^1E'_3 \oplus {}^2E'_3$	$2A'$	E_3	$A_1 \oplus A_2$	$A_1 \oplus B_1$	$A'_1 \oplus A'_2$	E_3
E'_4	${}^1E'_4 \oplus {}^2E'_4$	${}^1E' \oplus {}^2E'$	E_4	E	$A_1 \oplus B_1$	E'	E_4
A''_1	A''	A''	A_2	A_2	A_2	A''_1	A_1
A''_2	A''	A''	A_1	A_1	B_2	A''_2	A_2
E''_1	${}^1E''_1 \oplus {}^2E''_1$	${}^1E'' \oplus {}^2E''$	E_1	E	$A_2 \oplus B_2$	E''	E_1
E''_2	${}^1E''_2 \oplus {}^2E''_2$	${}^1E'' \oplus {}^2E''$	E_2	E	$A_2 \oplus B_2$	E''	E_2
E''_3	${}^1E''_3 \oplus {}^2E''_3$	$2A''$	E_3	$A_1 \oplus A_2$	$A_2 \oplus B_2$	$A''_1 \oplus A''_2$	E_3
E''_4	${}^1E''_4 \oplus {}^2E''_4$	${}^1E'' \oplus {}^2E''$	E_4	E	$A_2 \oplus B_2$	E''	E_4
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{1/2}$	$E_{3/2}$	$E_{3/2}$
$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{5/2}$	$E_{1/2}$	$E_{1/2}$	$E_{5/2}$	$E_{5/2}$
$E_{7/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{7/2}$	$E_{1/2}$	$E_{1/2}$	$E_{5/2}$	$E_{7/2}$
$E_{9/2}$	${}^1E_{9/2} \oplus {}^2E_{9/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{9/2} \oplus {}^2E_{9/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{1/2}$	$E_{3/2}$	${}^1E_{9/2} \oplus {}^2E_{9/2}$
$E_{11/2}$	${}^1E_{11/2} \oplus {}^2E_{11/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$E_{7/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{7/2}$
$E_{13/2}$	${}^1E_{13/2} \oplus {}^2E_{13/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$E_{5/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{5/2}$
$E_{15/2}$	${}^1E_{15/2} \oplus {}^2E_{15/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{1/2}$	$E_{3/2}$	$E_{3/2}$
$E_{17/2}$	${}^1E_{17/2} \oplus {}^2E_{17/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{5/2}$	$E_{1/2}$

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T **38**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{9h}$	$(\mathbf{D}_3)$	$\mathbf{C}_s$	$(\mathbf{C}_s)$	$\mathbf{C}_9$	$\mathbf{C}_3$	$(\mathbf{C}_2)$
		σ_h	σ_v			
A'_1	A_1	A'	A'	A	A	A
A'_2	A_2	A'	A''	A	A	B
E'_1	E	$2A'$	$A' \oplus A''$	${}^1E_1 \oplus {}^2E_1$	${}^1E \oplus {}^2E$	$A \oplus B$
E'_2	E	$2A'$	$A' \oplus A''$	${}^1E_2 \oplus {}^2E_2$	${}^1E \oplus {}^2E$	$A \oplus B$
E'_3	$A_1 \oplus A_2$	$2A'$	$A' \oplus A''$	${}^1E_3 \oplus {}^2E_3$	$2A$	$A \oplus B$
E'_4	E	$2A'$	$A' \oplus A''$	${}^1E_4 \oplus {}^2E_4$	${}^1E \oplus {}^2E$	$A \oplus B$
A''_1	A_1	A''	A''	A	A	A
A''_2	A_2	A''	A'	A	A	B
E''_1	E	$2A''$	$A' \oplus A''$	${}^1E_1 \oplus {}^2E_1$	${}^1E \oplus {}^2E$	$A \oplus B$
E''_2	E	$2A''$	$A' \oplus A''$	${}^1E_2 \oplus {}^2E_2$	${}^1E \oplus {}^2E$	$A \oplus B$
E''_3	$A_1 \oplus A_2$	$2A''$	$A' \oplus A''$	${}^1E_3 \oplus {}^2E_3$	$2A$	$A \oplus B$
E''_4	E	$2A''$	$A' \oplus A''$	${}^1E_4 \oplus {}^2E_4$	${}^1E \oplus {}^2E$	$A \oplus B$
$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$2A_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{9/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{9/2}$	$2A_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{11/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{13/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{15/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$2A_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{17/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **38**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{9h}$
$18n$	$(n+1)\,A'_1 \oplus n\,(A'_2 \oplus 2E'_1 \oplus 2E'_2 \oplus 2E'_3 \oplus 2E'_4 \oplus A''_1 \oplus A''_2 \oplus 2E''_1 \oplus 2E''_2 \oplus 2E''_3 \oplus 2E''_4)$
$18n+1$	$n\,(A'_1 \oplus A'_2 \oplus E'_1 \oplus 2E'_2 \oplus 2E'_3 \oplus 2E'_4 \oplus A''_1 \oplus 2E''_1 \oplus 2E''_2 \oplus 2E''_3 \oplus 2E''_4) \oplus$ $(n+1)(E'_1 \oplus A''_2)$
$18n+2$	$(n+1)(A'_1 \oplus E'_2 \oplus E''_1) \oplus$ $n\,(A'_2 \oplus 2E'_1 \oplus E'_2 \oplus 2E'_3 \oplus 2E'_4 \oplus A''_1 \oplus A''_2 \oplus E''_1 \oplus 2E''_2 \oplus 2E''_3 \oplus 2E''_4)$
$18n+3$	$n\,(A'_1 \oplus A'_2 \oplus E'_1 \oplus 2E'_2 \oplus E'_3 \oplus 2E'_4 \oplus A''_1 \oplus 2E''_1 \oplus E''_2 \oplus 2E''_3 \oplus 2E''_4) \oplus$ $(n+1)(E'_1 \oplus E'_3 \oplus A''_2 \oplus E''_2)$
$18n+4$	$(n+1)(A'_1 \oplus E'_2 \oplus E'_4 \oplus E''_1 \oplus E''_3) \oplus$ $n\,(A'_2 \oplus 2E'_1 \oplus E'_2 \oplus 2E'_3 \oplus E'_4 \oplus A''_1 \oplus A''_2 \oplus E''_1 \oplus 2E''_2 \oplus E''_3 \oplus 2E''_4)$
$18n+5$	$n\,(A'_1 \oplus A'_2 \oplus E'_1 \oplus 2E'_2 \oplus E'_3 \oplus E'_4 \oplus A''_1 \oplus 2E''_1 \oplus E''_2 \oplus 2E''_3 \oplus E''_4) \oplus$ $(n+1)(E'_1 \oplus E'_3 \oplus E'_4 \oplus A''_2 \oplus E''_2 \oplus E''_4)$
$18n+6$	$(n+1)(A'_1 \oplus E'_2 \oplus E'_3 \oplus E'_4 \oplus E''_1 \oplus E''_3 \oplus E''_4) \oplus$ $n\,(A'_2 \oplus 2E'_1 \oplus E'_2 \oplus E'_3 \oplus E'_4 \oplus A''_1 \oplus A''_2 \oplus E''_1 \oplus 2E''_2 \oplus E''_3 \oplus E''_4)$
$18n+7$	$n\,(A'_1 \oplus A'_2 \oplus E'_1 \oplus E'_2 \oplus E'_3 \oplus E'_4 \oplus A''_1 \oplus 2E''_1 \oplus E''_2 \oplus E''_3 \oplus E''_4) \oplus$ $(n+1)(E'_1 \oplus E'_2 \oplus E'_3 \oplus E'_4 \oplus A''_2 \oplus E''_2 \oplus E''_3 \oplus E''_4)$
$18n+8$	$(n+1)(A'_1 \oplus E'_1 \oplus E'_2 \oplus E'_3 \oplus E'_4 \oplus E''_1 \oplus E''_2 \oplus E''_3 \oplus E''_4) \oplus$ $n\,(A'_2 \oplus E'_1 \oplus E'_2 \oplus E'_3 \oplus E'_4 \oplus A''_1 \oplus A''_2 \oplus E''_1 \oplus E''_2 \oplus E''_3 \oplus E''_4)$

$n = 0, 1, 2, \ldots$ $\twoheadrightarrow$

T **38**.10 Subduction from O(3) *(cont.)*

j	$\mathbf{D}_{9h}$
$18n+9$	$(n+1)(A_1' \oplus A_2' \oplus E_1' \oplus E_2' \oplus E_3' \oplus E_4' \oplus A_2'' \oplus E_1'' \oplus E_2'' \oplus E_3'' \oplus E_4'') \oplus$ $n\,(E_1' \oplus E_2' \oplus E_3' \oplus E_4' \oplus A_1'' \oplus E_1'' \oplus E_2'' \oplus E_3'' \oplus E_4'')$
$18n+10$	$(n+1)(A_1' \oplus 2E_1' \oplus E_2' \oplus E_3' \oplus E_4' \oplus A_1'' \oplus A_2'' \oplus E_1'' \oplus E_2'' \oplus E_3'' \oplus E_4'') \oplus$ $n\,(A_2' \oplus E_2' \oplus E_3' \oplus E_4' \oplus E_1'' \oplus E_2'' \oplus E_3'' \oplus E_4'')$
$18n+11$	$(n+1)(A_1' \oplus A_2' \oplus E_1' \oplus 2E_2' \oplus E_3' \oplus E_4' \oplus A_2'' \oplus 2E_1'' \oplus E_2'' \oplus E_3'' \oplus E_4'') \oplus$ $n\,(E_1' \oplus E_3' \oplus E_4' \oplus A_1'' \oplus E_2'' \oplus E_3'' \oplus E_4'')$
$18n+12$	$(n+1)(A_1' \oplus 2E_1' \oplus E_2' \oplus 2E_3' \oplus E_4' \oplus A_1'' \oplus A_2'' \oplus E_1'' \oplus 2E_2'' \oplus E_3'' \oplus E_4'') \oplus$ $n\,(A_2' \oplus E_2' \oplus E_4' \oplus E_1'' \oplus E_3'' \oplus E_4'')$
$18n+13$	$(n+1)(A_1' \oplus A_2' \oplus E_1' \oplus 2E_2' \oplus E_3' \oplus 2E_4' \oplus A_2'' \oplus 2E_1'' \oplus E_2'' \oplus 2E_3'' \oplus E_4'') \oplus$ $n\,(E_1' \oplus E_3' \oplus A_1'' \oplus E_2'' \oplus E_4'')$
$18n+14$	$(n+1)(A_1' \oplus 2E_1' \oplus E_2' \oplus 2E_3' \oplus 2E_4' \oplus A_1'' \oplus A_2'' \oplus E_1'' \oplus 2E_2'' \oplus E_3'' \oplus 2E_4'') \oplus$ $n\,(A_2' \oplus E_2' \oplus E_1'' \oplus E_3'')$
$18n+15$	$(n+1)(A_1' \oplus A_2' \oplus E_1' \oplus 2E_2' \oplus 2E_3' \oplus 2E_4' \oplus A_2'' \oplus 2E_1'' \oplus E_2'' \oplus 2E_3'' \oplus 2E_4'') \oplus$ $n\,(E_1' \oplus A_1'' \oplus E_2'')$
$18n+16$	$(n+1)(A_1' \oplus 2E_1' \oplus 2E_2' \oplus 2E_3' \oplus 2E_4' \oplus A_1'' \oplus A_2'' \oplus E_1'' \oplus 2E_2'' \oplus 2E_3'' \oplus 2E_4'') \oplus$ $n\,(A_2' \oplus E_1'')$
$18n+17$	$(n+1)(A_1' \oplus A_2' \oplus 2E_1' \oplus 2E_2' \oplus 2E_3' \oplus 2E_4' \oplus A_2'' \oplus 2E_1'' \oplus 2E_2'' \oplus 2E_3'' \oplus 2E_4'') \oplus n\,A_1''$
$14n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2})$
$18n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus 2n\,(E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2})$
$18n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus 2n\,(E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2})$
$18n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus 2n\,(E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2})$
$18n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2}) \oplus 2n\,(E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2})$
$18n+\frac{11}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2}) \oplus 2n\,(E_{13/2} \oplus E_{15/2} \oplus E_{17/2})$
$18n+\frac{13}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2}) \oplus 2n\,(E_{15/2} \oplus E_{17/2})$
$18n+\frac{15}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2}) \oplus 2n\,E_{17/2}$
$18n+\frac{17}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2})$
$18n+\frac{19}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2}) \oplus (2n+2)\,E_{17/2}$
$18n+\frac{21}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2}) \oplus (2n+2)(E_{15/2} \oplus E_{17/2})$
$18n+\frac{23}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2}) \oplus (2n+2)(E_{13/2} \oplus E_{15/2} \oplus E_{17/2})$
$18n+\frac{25}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2}) \oplus (2n+2)(E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2})$
$18n+\frac{27}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus (2n+2)(E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2})$
$18n+\frac{29}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus (2n+2)(E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2})$
$18n+\frac{31}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (2n+2)(E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2})$
$18n+\frac{33}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2})$
$18n+\frac{35}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2})$

$n = 0, 1, 2, \ldots$

T **38**.11 Clebsch–Gordan coefficients § **16**–11, p. 83 $\mathbf{D}_{9h}$

a'_2	e'_1	E'_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	e'_2	E'_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	e'_3	E'_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	e'_4	E'_4	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	e''_1	E''_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	e''_2	E''_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	e''_3	E''_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	e''_4	E''_4	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{1/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{5/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{7/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{9/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{11/2}$	$E_{11/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{13/2}$	$E_{13/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{15/2}$	$E_{15/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a'_2	$e_{17/2}$	$E_{17/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e'_1	e'_1	A'_1	A'_2	E'_2	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e'_1	e'_2	E'_1		E'_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e'_1	e'_3	E'_2		E'_4	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	$\bar{1}$

e'_1	e'_4	E'_3		E'_4	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e'_1	a''_1	E''_1	
		1	2
1	1	1	0
2	1	0	1

e'_1	a''_2	E''_1	
		1	2
1	1	1	0
2	1	0	$\bar{1}$

e'_1	e''_1	A''_1	A''_2	E''_2	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e'_1	e''_2	E''_1		E''_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$u = 2^{-1/2}$ ↠

T **38**.11 Clebsch–Gordan coefficients *(cont.)*

e'_1	e''_3	E''_2		E''_4	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	$\bar{1}$

e'_1	e''_4	E''_3		E''_4	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e'_1	$e_{1/2}$	$E_{15/2}$		$E_{17/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e'_1	$e_{3/2}$	$E_{13/2}$		$E_{17/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e'_1	$e_{5/2}$	$E_{11/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e'_1	$e_{7/2}$	$E_{9/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e'_1	$e_{9/2}$	$E_{7/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

e'_1	$e_{11/2}$	$E_{5/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	$\bar{1}$

e'_1	$e_{13/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e'_1	$e_{15/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e'_1	$e_{17/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e'_2	e'_2	A'_1	A'_2	E'_4	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e'_2	e'_3	E'_1		E'_4	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	$\bar{1}$	0	0

e'_2	e'_4	E'_2		E'_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e'_2	a''_1	E''_2	
		1	2
1	1	1	0
2	1	0	1

e'_2	a''_2	E''_2	
		1	2
1	1	1	0
2	1	0	$\bar{1}$

e'_2	e''_1	E''_1		E''_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

e'_2	e''_2	A''_1	A''_2	E''_4	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

$u = 2^{-1/2}$

⇻

T **38**.11 Clebsch–Gordan coefficients *(cont.)*

e'_2	e''_3	E''_1		E''_4	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	$\bar{1}$	0	0

e'_2	e''_4	E''_2		E''_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e'_2	$e_{1/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e'_2	$e_{3/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e'_2	$e_{5/2}$	$E_{1/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e'_2	$e_{7/2}$	$E_{3/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e'_2	$e_{9/2}$	$E_{5/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

e'_2	$e_{11/2}$	$E_{7/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e'_2	$e_{13/2}$	$E_{9/2}$		$E_{17/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	$\bar{1}$	0	0

e'_2	$e_{15/2}$	$E_{11/2}$		$E_{17/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e'_2	$e_{17/2}$	$E_{13/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e'_3	e'_3	A'_1	A'_2	E'_3	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e'_3	e'_4	E'_1		E'_2	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e'_3	a''_1	E''_3	
		1	2
1	1	1	0
2	1	0	1

e'_3	a''_2	E''_3	
		1	2
1	1	1	0
2	1	0	$\bar{1}$

e'_3	e''_1	E''_2		E''_4	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e'_3	e''_2	E''_1		E''_4	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e'_3	e''_3	A''_1	A''_2	E''_3	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

$u = 2^{-1/2}$

↠

T **38**.11 Clebsch–Gordan coefficients *(cont.)*

e'_3	e''_4	E''_1		E''_2	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e'_3	$e_{1/2}$	$E_{11/2}$		$E_{13/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e'_3	$e_{3/2}$	$E_{9/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e'_3	$e_{5/2}$	$E_{7/2}$		$E_{17/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e'_3	$e_{7/2}$	$E_{5/2}$		$E_{17/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e'_3	$e_{9/2}$	$E_{3/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e'_3	$e_{11/2}$	$E_{1/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e'_3	$e_{13/2}$	$E_{1/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e'_3	$e_{15/2}$	$E_{3/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e'_3	$e_{17/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e'_4	e'_4	A'_1	A'_2	E'_1	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{\mathrm{u}}$	0	0
2	2	0	0	1	0

e'_4	a''_1	E''_4	
		1	2
1	1	1	0
2	1	0	1

e'_4	a''_2	E''_4	
		1	2
1	1	1	0
2	1	0	$\bar{1}$

e'_4	e''_1	E''_3		E''_4	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e'_4	e''_2	E''_2		E''_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

e'_4	e''_3	E''_1		E''_2	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	$\bar{1}$

e'_4	e''_4	A''_1	A''_2	E''_1	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{\mathrm{u}}$	0	0
2	2	0	0	1	0

e'_4	$e_{1/2}$	$E_{7/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	$\bar{1}$

$\mathrm{u} = 2^{-1/2}$

↠

T **38**.11 Clebsch–Gordan coefficients *(cont.)*

e'_4	$e_{3/2}$	$E_{5/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e'_4	$e_{5/2}$	$E_{3/2}$		$E_{13/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e'_4	$e_{7/2}$	$E_{1/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e'_4	$e_{9/2}$	$E_{1/2}$		$E_{17/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e'_4	$e_{11/2}$	$E_{3/2}$		$E_{17/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e'_4	$e_{13/2}$	$E_{5/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e'_4	$e_{15/2}$	$E_{7/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e'_4	$e_{17/2}$	$E_{9/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

a''_1	e''_1	E'_1	
		1	2
1	1	1	0
1	2	0	1

a''_1	e''_2	E'_2	
		1	2
1	1	1	0
1	2	0	1

a''_1	e''_3	E'_3	
		1	2
1	1	1	0
1	2	0	1

a''_1	e''_4	E'_4	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{1/2}$	$E_{17/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{3/2}$	$E_{15/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{5/2}$	$E_{13/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{7/2}$	$E_{11/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{9/2}$	$E_{9/2}$	
		1	2
1	1	0	1
1	2	1	0

a''_1	$e_{11/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{13/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{15/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_1	$e_{17/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	1

a''_2	e''_1	E'_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	e''_2	E'_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	e''_3	E'_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a''_2	e''_4	E'_4	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

↠

T **38**.11 Clebsch–Gordan coefficients *(cont.)*

a_2''	$e_{1/2}$	$E_{17/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2''	$e_{3/2}$	$E_{15/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2''	$e_{5/2}$	$E_{13/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2''	$e_{7/2}$	$E_{11/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2''	$e_{9/2}$	$E_{9/2}$	
		1	2
1	1	0	$\bar{1}$
1	2	1	0

a_2''	$e_{11/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2''	$e_{13/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2''	$e_{15/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2''	$e_{17/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e_1''	e_1''	A_1'	A_2'	E_2'	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{\mathrm{u}}$	0	0
2	2	0	0	1	0

e_1''	e_2''	E_1'		E_3'	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_1''	e_3''	E_2'		E_4'	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	$\bar{1}$

e_1''	e_4''	E_3'		E_4'	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1''	$e_{1/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_1''	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1''	$e_{5/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_1''	$e_{7/2}$	$E_{5/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	$\bar{1}$

e_1''	$e_{9/2}$	$E_{7/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1''	$e_{11/2}$	$E_{9/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_1''	$e_{13/2}$	$E_{11/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$\mathrm{u} = 2^{-1/2}$

$\twoheadrightarrow$

T **38**.11 Clebsch–Gordan coefficients *(cont.)*

e''_1	$e_{15/2}$	$E_{13/2}$		$E_{17/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e''_1	$e_{17/2}$	$E_{15/2}$		$E_{17/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e''_2	e''_2	A'_1	A'_2	E'_4	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	1	0

e''_2	e''_3	E'_1		E'_4	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	$\bar{1}$	0	0

e''_2	e''_4	E'_2		E'_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e''_2	$e_{1/2}$	$E_{13/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e''_2	$e_{3/2}$	$E_{11/2}$		$E_{17/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e''_2	$e_{5/2}$	$E_{9/2}$		$E_{17/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	$\bar{1}$	0	0

e''_2	$e_{7/2}$	$E_{7/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e''_2	$e_{9/2}$	$E_{5/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e''_2	$e_{11/2}$	$E_{3/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e''_2	$e_{13/2}$	$E_{1/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e''_2	$e_{15/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e''_2	$e_{17/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e''_3	e''_3	A'_1	A'_2	E'_3	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	1	0

e''_3	e''_4	E'_1		E'_2	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e''_3	$e_{1/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e''_3	$e_{3/2}$	$E_{3/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$\text{u} = 2^{-1/2}$

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T **38**.11 Clebsch–Gordan coefficients *(cont.)*

e''_3	$e_{5/2}$	$E_{1/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e''_3	$e_{7/2}$	$E_{1/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e''_3	$e_{9/2}$	$E_{3/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e''_3	$e_{11/2}$	$E_{5/2}$		$E_{17/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e''_3	$e_{13/2}$	$E_{7/2}$		$E_{17/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e''_3	$e_{15/2}$	$E_{9/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e''_3	$e_{17/2}$	$E_{11/2}$		$E_{13/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e''_4	e''_4	A'_1	A'_2	E'_1	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e''_4	$e_{1/2}$	$E_{9/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e''_4	$e_{3/2}$	$E_{7/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e''_4	$e_{5/2}$	$E_{5/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e''_4	$e_{7/2}$	$E_{3/2}$		$E_{17/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e''_4	$e_{9/2}$	$E_{1/2}$		$E_{17/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

e''_4	$e_{11/2}$	$E_{1/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e''_4	$e_{13/2}$	$E_{3/2}$		$E_{13/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e''_4	$e_{15/2}$	$E_{5/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e''_4	$e_{17/2}$	$E_{7/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	$\bar{1}$

$e_{1/2}$	$e_{1/2}$	A'_1	A'_2	E''_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

↠

T **38**.11 Clebsch–Gordan coefficients *(cont.)*

$e_{1/2}$	$e_{3/2}$	E'_2		E''_1	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{1/2}$	$e_{5/2}$	E'_2		E''_3	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	0	1	0	0

$e_{1/2}$	$e_{7/2}$	E'_4		E''_3	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

$e_{1/2}$	$e_{9/2}$	E'_4		E''_4	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{1/2}$	$e_{11/2}$	E'_3		E''_4	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{13/2}$	E'_3		E''_2	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{15/2}$	E'_1		E''_2	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{1/2}$	$e_{17/2}$	E'_1		A''_1	A''_2
		1	2	1	1
1	1	1	0	0	0
1	2	0	0	u	u
2	1	0	0	$\bar{u}$	u
2	2	0	1	0	0

$e_{3/2}$	$e_{3/2}$	A'_1	A'_2	E''_3	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{5/2}$	E'_4		E''_1	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{7/2}$	E'_2		E''_4	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{9/2}$	E'_3		E''_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{3/2}$	$e_{11/2}$	E'_4		E''_2	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{3/2}$	$e_{13/2}$	E'_1		E''_4	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{3/2}$	$e_{15/2}$	E'_3		A''_1	A''_2
		1	2	1	1
1	1	1	0	0	0
1	2	0	0	u	u
2	1	0	0	$\bar{u}$	u
2	2	0	1	0	0

$e_{3/2}$	$e_{17/2}$	E'_1		E''_2	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{5/2}$	$e_{5/2}$	A'_1	A'_2	E''_4	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{5/2}$	$e_{7/2}$	E'_3		E''_1	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$ ↠

T **38**.11 Clebsch–Gordan coefficients *(cont.)*

$e_{5/2}$	$e_{9/2}$	E'_2		E''_2	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

$e_{5/2}$	$e_{11/2}$	E'_1		E''_3	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	0	1	0	0

$e_{5/2}$	$e_{13/2}$	E'_4		A''_1	A''_2
		1	2	1	1
1	1	1	0	0	0
1	2	0	0	u	u
2	1	0	0	$\bar{u}$	u
2	2	0	1	0	0

$e_{5/2}$	$e_{15/2}$	E'_1		E''_4	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{5/2}$	$e_{17/2}$	E'_3		E''_2	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{7/2}$	$e_{7/2}$	A'_1	A'_2	E''_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{7/2}$	$e_{9/2}$	E'_1		E''_1	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

$e_{7/2}$	$e_{11/2}$	E'_2		A''_1	A''_2
		1	2	1	1
1	1	1	0	0	0
1	2	0	0	u	u
2	1	0	0	$\bar{u}$	u
2	2	0	1	0	0

$e_{7/2}$	$e_{13/2}$	E'_1		E''_3	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

$e_{7/2}$	$e_{15/2}$	E'_4		E''_2	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{7/2}$	$e_{17/2}$	E'_3		E''_4	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	0	1

$e_{9/2}$	$e_{9/2}$	A'_1	A'_2	A''_1	A''_2
		1	1	1	1
1	1	0	0	u	u
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	$\bar{u}$	u

$e_{9/2}$	$e_{11/2}$	E'_1		E''_1	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

$e_{9/2}$	$e_{13/2}$	E'_2		E''_2	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

$e_{9/2}$	$e_{15/2}$	E'_3		E''_3	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{9/2}$	$e_{17/2}$	E'_4		E''_4	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	1	0	0	0

$e_{11/2}$	$e_{11/2}$	A'_1	A'_2	E''_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{11/2}$	$e_{13/2}$	E'_3		E''_1	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

↠

T **38**.11 Clebsch–Gordan coefficients *(cont.)*

$e_{11/2}$	$e_{15/2}$	E'_2 1	2	E''_4 1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{11/2}$	$e_{17/2}$	E'_4 1	2	E''_3 1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	0	1	0	0

$e_{13/2}$	$e_{13/2}$	A'_1 1	A'_2 1	E''_4 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\text{u}}$	u	0	0
2	2	0	0	0	1

$e_{13/2}$	$e_{15/2}$	E'_4 1	2	E''_1 1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{13/2}$	$e_{17/2}$	E'_2 1	2	E''_3 1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

$e_{15/2}$	$e_{15/2}$	A'_1 1	A'_2 1	E''_3 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\text{u}}$	u	0	0
2	2	0	0	0	1

$e_{15/2}$	$e_{17/2}$	E'_2 1	2	E''_1 1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{17/2}$	$e_{17/2}$	A'_1 1	A'_2 1	E''_1 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\text{u}}$	u	0	0
2	2	0	0	0	1

$\text{u} = 2^{-1/2}$

(1) Product forms: $\mathbf{D}_{10} \otimes \mathbf{C}_i$, $\mathbf{D}_{10} \otimes \mathbf{C}_s$, $\mathbf{C}_{10v} \otimes \mathbf{C}_s$.

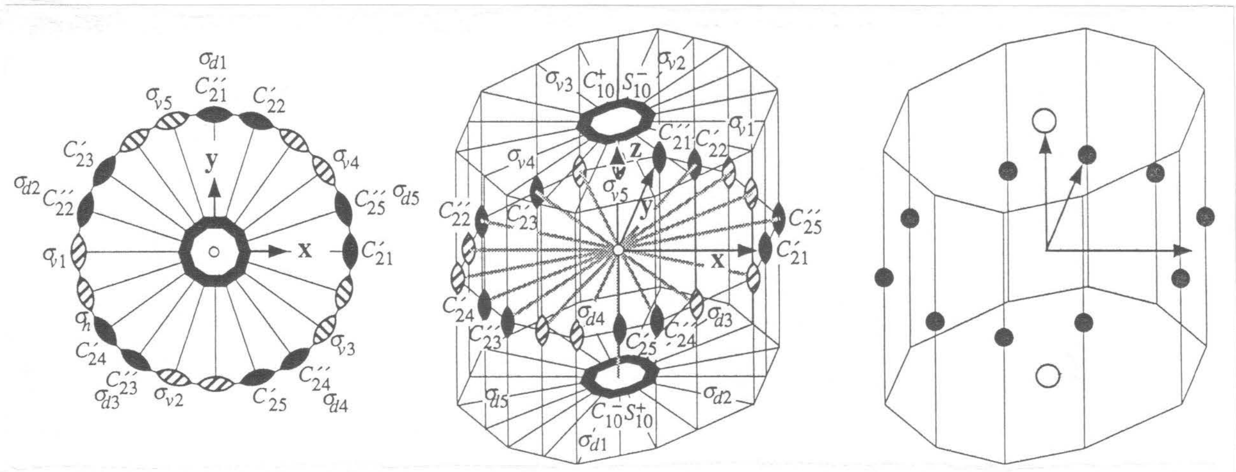
(2) Group chains: $\mathbf{D}_{10h} \supset \underline{\mathbf{C}_{10h}}$, $\mathbf{D}_{10h} \supset (\underline{\mathbf{C}_{10v}})$, $\mathbf{D}_{10h} \supset (\underline{\mathbf{D}_{5d}})$,
$\mathbf{D}_{10h} \supset \underline{\mathbf{D}_{5h}}$, $\mathbf{D}_{10h} \supset (\mathbf{D}_{2h})$, $\mathbf{D}_{10h} \supset \underline{\mathbf{D}_{10}}$.

(3) Operations of G: E, (C_{10}^+, C_{10}^-), (C_5^+, C_5^-), $(C_{10}^{3+}, C_{10}^{3-})$, (C_5^{2+}, C_5^{2-}),
C_2, $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25})$, $(C''_{21}, C''_{22}, C''_{23}, C''_{24}, C''_{25})$,
i, (S_5^{2-}, S_5^{2+}), $(S_{10}^{3-}, S_{10}^{3+})$, (S_5^-, S_5^+), (S_{10}^-, S_{10}^+),
σ_h, $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5})$, $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_{10}^+, C_{10}^-), $(\widetilde{C}_{10}^+, \widetilde{C}_{10}^-)$, (C_5^+, C_5^-), $(\widetilde{C}_5^+, \widetilde{C}_5^-)$,
$(C_{10}^{3+}, C_{10}^{3-})$, $(\widetilde{C}_{10}^{3+}, \widetilde{C}_{10}^{3-})$, (C_5^{2+}, C_5^{2-}), $(\widetilde{C}_5^{2+}, \widetilde{C}_5^{2-})$,
$(C_2, \widetilde{C}_2)$, $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, \widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24}, \widetilde{C}'_{25})$,
$(C''_{21}, C''_{22}, C''_{23}, C''_{24}, C''_{25}, \widetilde{C}''_{21}, \widetilde{C}''_{22}, \widetilde{C}''_{23}, \widetilde{C}''_{24}, \widetilde{C}''_{25})$,
i, $\widetilde{\imath}$, (S_5^{2-}, S_5^{2+}), $(\widetilde{S}_5^{2-}, \widetilde{S}_5^{2+})$, $(S_{10}^{3-}, S_{10}^{3+})$, $(\widetilde{S}_{10}^{3-}, \widetilde{S}_{10}^{3+})$,
(S_5^-, S_5^+), $(\widetilde{S}_5^-, \widetilde{S}_5^+)$, (S_{10}^-, S_{10}^+), $(\widetilde{S}_{10}^-, \widetilde{S}_{10}^+)$,
$(\sigma_h, \widetilde{\sigma}_h)$, $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5}, \widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2}, \widetilde{\sigma}_{v3}, \widetilde{\sigma}_{v4}, \widetilde{\sigma}_{v5})$,
$(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3}, \widetilde{\sigma}_{d4}, \widetilde{\sigma}_{d5})$.

(5) Classes and representations: $|r| = 10$, $|i| = 6$, $|I| = 16$, $|\widetilde{I}| = 10$.

F **39** See Chapter **15**, p. 65



Examples:

T **39**.0 Subgroup elements § **16**–0, p. 68

$\mathbf{D}_{10h}$	$\mathbf{C}_{10h}$	$\mathbf{C}_{5h}$	$\mathbf{C}_{2h}$	$\mathbf{C}_{10v}$	$\mathbf{C}_{5v}$	$\mathbf{C}_{2v}$	$\mathbf{D}_{5d}$	$\mathbf{D}_{5h}$	$\mathbf{D}_{2h}$	$\mathbf{D}_{10}$	$\mathbf{D}_{5}$	$\mathbf{D}_{2}$	$\mathbf{S}_{10}$	$\mathbf{C}_{s}$	$\mathbf{C}_{i}$	$\mathbf{C}_{10}$	$\mathbf{C}_{5}$	$\mathbf{C}_{2}$
E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
C_{10}^{+}	C_{10}^{+}			C_{10}^{+}						C_{10}^{+}						C_{10}^{+}		
C_{10}^{-}	C_{10}^{-}			C_{10}^{-}						C_{10}^{-}						C_{10}^{-}		
C_{5}^{+}	C_{5}^{+}	C_{5}^{+}		C_{5}^{+}	C_{5}^{+}		C_{5}^{+}	C_{5}^{+}		C_{5}^{+}	C_{5}^{+}		C_{5}^{+}			C_{5}^{+}	C_{5}^{+}	
C_{5}^{-}	C_{5}^{-}	C_{5}^{-}		C_{5}^{-}	C_{5}^{-}		C_{5}^{-}	C_{5}^{-}		C_{5}^{-}	C_{5}^{-}		C_{5}^{-}			C_{5}^{-}	C_{5}^{-}	
C_{10}^{3+}	C_{10}^{3+}			C_{10}^{3+}						C_{10}^{3+}						C_{10}^{3+}		
C_{10}^{3-}	C_{10}^{3-}			C_{10}^{3-}						C_{10}^{3-}						C_{10}^{3-}		
C_{5}^{2+}	C_{5}^{2+}	C_{5}^{2+}		C_{5}^{2+}	C_{5}^{2+}		C_{5}^{2+}	C_{5}^{2+}		C_{5}^{2+}	C_{5}^{2+}		C_{5}^{2+}			C_{5}^{2+}	C_{5}^{2+}	
C_{5}^{2-}	C_{5}^{2-}	C_{5}^{2-}		C_{5}^{2-}	C_{5}^{2-}		C_{5}^{2-}	C_{5}^{2-}		C_{5}^{2-}	C_{5}^{2-}		C_{5}^{2-}			C_{5}^{2-}	C_{5}^{2-}	
C_2	C_2		C_2	C_2		C_2			C_{2z}	C_2		C_{2z}				C_2		C_2
C'_{21}							C'_{21}	C'_{21}	C_{2x}	C'_{21}	C'_{21}	C_{2x}						
C'_{22}							C'_{22}	C'_{22}		C'_{22}	C'_{22}							
C'_{23}							C'_{23}	C'_{23}		C'_{23}	C'_{23}							
C'_{24}							C'_{24}	C'_{24}		C'_{24}	C'_{24}							
C'_{25}							C'_{25}	C'_{25}		C'_{25}	C'_{25}							
C''_{21}									C_{2y}	C''_{21}		C_{2y}						
C''_{22}										C''_{22}								
C''_{23}										C''_{23}								
C''_{24}										C''_{24}								
C''_{25}										C''_{25}								
i	i		i				i		i				i		i			
S_{5}^{2-}	S_{5}^{2-}	S_{5}^{2-}						S_{5}^{2-}										
S_{5}^{2+}	S_{5}^{2+}	S_{5}^{2+}						S_{5}^{2+}										
S_{10}^{3-}	S_{10}^{3-}						S_{10}^{3-}						S_{10}^{3-}					
S_{10}^{3+}	S_{10}^{3+}						S_{10}^{3+}						S_{10}^{3+}					
S_{5}^{-}	S_{5}^{-}	S_{5}^{-}						S_{5}^{-}										
S_{5}^{+}	S_{5}^{+}	S_{5}^{+}						S_{5}^{+}										
S_{10}^{-}	S_{10}^{-}						S_{10}^{-}						S_{10}^{-}					
S_{10}^{+}	S_{10}^{+}						S_{10}^{+}						S_{10}^{+}					
σ_h	σ_h	σ_h	σ_h					σ_h	σ_z					σ_h				
σ_{d1}				σ_{d1}		σ_x	σ_{d1}		σ_x									
σ_{d2}				σ_{d2}			σ_{d2}											
σ_{d3}				σ_{d3}			σ_{d3}											
σ_{d4}				σ_{d4}			σ_{d4}											
σ_{d5}				σ_{d5}			σ_{d5}											
σ_{v1}				σ_{v1}	σ_{v1}	σ_y		σ_{v1}	σ_y									
σ_{v2}				σ_{v2}	σ_{v2}			σ_{v2}										
σ_{v3}				σ_{v3}	σ_{v3}			σ_{v3}										
σ_{v4}				σ_{v4}	σ_{v4}			σ_{v4}										
σ_{v5}				σ_{v5}	σ_{v5}			σ_{v5}										

T **39**.1 Parameters

§ **16**–1, p. 68

$\mathbf{D}_{10h}$		α	β	γ	ϕ	$\mathbf{n}$	λ	$\mathbf{\Lambda}$
E	i	0	0	0	0	(0 0 0)	[[1,	(0 0 0)]]
C_{10}^{+}	S_5^{2-}	0	0	$\frac{\pi}{5}$	$\frac{\pi}{5}$	(0 0 1)	[[c_{10},	(0 0 s_{10})]]
C_{10}^{-}	S_5^{2+}	0	0	$-\frac{\pi}{5}$	$\frac{\pi}{5}$	(0 0 −1)	[[c_{10},	(0 0 $-s_{10}$)]]
C_5^{+}	S_{10}^{3-}	0	0	$\frac{2\pi}{5}$	$\frac{2\pi}{5}$	(0 0 1)	[[c_5,	(0 0 s_5)]]
C_5^{-}	S_{10}^{3+}	0	0	$-\frac{2\pi}{5}$	$\frac{2\pi}{5}$	(0 0 −1)	[[c_5,	(0 0 $-s_5$)]]
C_{10}^{3+}	S_5^{-}	0	0	$\frac{3\pi}{5}$	$\frac{3\pi}{5}$	(0 0 1)	[[s_5,	(0 0 c_5)]]
C_{10}^{3-}	S_5^{+}	0	0	$-\frac{3\pi}{5}$	$\frac{3\pi}{5}$	(0 0 −1)	[[s_5,	(0 0 $-c_5$)]]
C_5^{2+}	S_{10}^{-}	0	0	$\frac{4\pi}{5}$	$\frac{4\pi}{5}$	(0 0 1)	[[s_{10},	(0 0 c_{10})]]
C_5^{2-}	S_{10}^{+}	0	0	$-\frac{4\pi}{5}$	$\frac{4\pi}{5}$	(0 0 −1)	[[s_{10},	(0 0 $-c_{10}$)]]
C_2	σ_h	0	0	π	π	(0 0 1)	[[0,	(0 0 1)]]
C'_{21}	σ_{d1}	0	π	π	π	(1 0 0)	[[0,	(1 0 0)]]
C'_{22}	σ_{d2}	0	π	$\frac{\pi}{5}$	π	(s_{10} c_{10} 0)	[[0,	(s_{10} c_{10} 0)]]
C'_{23}	σ_{d3}	0	π	$-\frac{3\pi}{5}$	π	($-c_5$ s_5 0)	[[0,	($-c_5$ s_5 0)]]
C'_{24}	σ_{d4}	0	π	$\frac{3\pi}{5}$	π	($-c_5$ $-s_5$ 0)	[[0,	($-c_5$ $-s_5$ 0)]]
C'_{25}	σ_{d5}	0	π	$-\frac{\pi}{5}$	π	(s_{10} $-c_{10}$ 0)	[[0,	(s_{10} $-c_{10}$ 0)]]
C''_{21}	σ_{v1}	0	π	0	π	(0 1 0)	[[0,	(0 1 0)]]
C''_{22}	σ_{v2}	0	π	$-\frac{4\pi}{5}$	π	($-c_{10}$ s_{10} 0)	[[0,	($-c_{10}$ s_{10} 0)]]
C''_{23}	σ_{v3}	0	π	$\frac{2\pi}{5}$	π	($-s_5$ $-c_5$ 0)	[[0,	($-s_5$ $-c_5$ 0)]]
C''_{24}	σ_{v4}	0	π	$-\frac{2\pi}{5}$	π	(s_5 $-c_5$ 0)	[[0,	(s_5 $-c_5$ 0)]]
C''_{25}	σ_{v5}	0	π	$\frac{4\pi}{5}$	π	(c_{10} s_{10} 0)	[[0,	(c_{10} s_{10} 0)]]

$c_n = \cos\frac{\pi}{n}$, $s_n = \sin\frac{\pi}{n}$

T **39**.2 Multiplication table

§ **16**–2, p. 69

$\mathbf{D}_{10h}$	E	C_{10}^{+}	C_{10}^{-}	C_5^{+}	C_5^{-}	C_{10}^{3+}	C_{10}^{3-}	C_5^{2+}	C_5^{2-}	C_2	C_{21}'	C_{22}'	C_{23}'	C_{24}'	C_{25}'	C_{21}''	C_{22}''	C_{23}''	C_{24}''	C_{25}''
E	E	C_{10}^{+}	C_{10}^{-}	C_5^{+}	C_5^{-}	C_{10}^{3+}	C_{10}^{3-}	C_5^{2+}	C_5^{2-}	C_2	C_{21}'	C_{22}'	C_{23}'	C_{24}'	C_{25}'	C_{21}''	C_{22}''	C_{23}''	C_{24}''	C_{25}''
C_{10}^{+}	C_{10}^{+}	C_5^{+}	E	C_{10}^{3+}	C_{10}^{-}	C_5^{2+}	C_5^{-}	C_2	C_{10}^{3-}	C_5^{2-}	C_{25}''	C_{21}''	C_{22}''	C_{23}''	C_{24}''	C_{25}'	C_{21}'	C_{22}'	C_{23}'	C_{24}'
C_{10}^{-}	C_{10}^{-}	E	C_5^{-}	C_{10}^{+}	C_{10}^{3-}	C_5^{+}	C_5^{2-}	C_{10}^{3+}	C_2	C_5^{2+}	C_{22}''	C_{23}''	C_{24}''	C_{25}''	C_{21}''	C_{22}'	C_{23}'	C_{24}'	C_{25}'	C_{21}'
C_5^{+}	C_5^{+}	C_{10}^{3+}	C_{10}^{+}	C_5^{2+}	E	C_2	C_{10}^{-}	C_5^{2-}	C_5^{-}	C_{10}^{3-}	C_{24}'	C_{25}'	C_{21}'	C_{22}'	C_{23}'	C_{24}''	C_{25}''	C_{21}''	C_{22}''	C_{23}''
C_5^{-}	C_5^{-}	C_{10}^{-}	C_{10}^{3-}	E	C_5^{2-}	C_{10}^{+}	C_2	C_5^{+}	C_5^{2+}	C_{10}^{3+}	C_{23}'	C_{24}'	C_{25}'	C_{21}'	C_{22}'	C_{23}''	C_{24}''	C_{25}''	C_{21}''	C_{22}''
C_{10}^{3+}	C_{10}^{3+}	C_5^{2+}	C_5^{+}	C_2	C_{10}^{+}	C_5^{2-}	E	C_{10}^{3-}	C_{10}^{-}	C_5^{-}	C_{23}''	C_{24}''	C_{25}''	C_{21}''	C_{22}''	C_{23}'	C_{24}'	C_{25}'	C_{21}'	C_{22}'
C_{10}^{3-}	C_{10}^{3-}	C_5^{-}	C_5^{2-}	C_{10}^{-}	C_2	E	C_5^{2+}	C_{10}^{+}	C_{10}^{3+}	C_5^{+}	C_{24}''	C_{25}''	C_{21}''	C_{22}''	C_{23}''	C_{24}'	C_{25}'	C_{21}'	C_{22}'	C_{23}'
C_5^{2+}	C_5^{2+}	C_2	C_{10}^{3+}	C_5^{2-}	C_5^{+}	C_{10}^{3-}	C_{10}^{+}	C_5^{-}	E	C_{10}^{-}	C_{22}'	C_{23}'	C_{24}'	C_{25}'	C_{21}'	C_{22}''	C_{23}''	C_{24}''	C_{25}''	C_{21}''
C_5^{2-}	C_5^{2-}	C_{10}^{3-}	C_2	C_5^{-}	C_5^{2+}	C_{10}^{-}	C_{10}^{3+}	E	C_5^{+}	C_{10}^{+}	C_{25}'	C_{21}'	C_{22}'	C_{23}'	C_{24}'	C_{25}''	C_{21}''	C_{22}''	C_{23}''	C_{24}''
C_2	C_2	C_5^{2-}	C_5^{2+}	C_{10}^{3-}	C_{10}^{3+}	C_5^{-}	C_5^{+}	C_{10}^{-}	C_{10}^{+}	E	C_{21}''	C_{22}''	C_{23}''	C_{24}''	C_{25}''	C_{21}'	C_{22}'	C_{23}'	C_{24}'	C_{25}'
C_{21}'	C_{21}'	C_{22}''	C_{25}''	C_{23}'	C_{24}'	C_{24}''	C_{23}''	C_{25}'	C_{22}'	C_{21}''	E	C_5^{2-}	C_5^{+}	C_5^{-}	C_5^{2+}	C_2	C_{10}^{+}	C_{10}^{3-}	C_{10}^{3+}	C_{10}^{-}
C_{22}'	C_{22}'	C_{23}''	C_{21}''	C_{24}'	C_{25}'	C_{25}''	C_{24}''	C_{21}'	C_{23}'	C_{22}''	C_5^{2+}	E	C_5^{2-}	C_5^{+}	C_5^{-}	C_{10}^{-}	C_2	C_{10}^{+}	C_{10}^{3-}	C_{10}^{3+}
C_{23}'	C_{23}'	C_{24}''	C_{22}''	C_{25}'	C_{21}'	C_{21}''	C_{25}''	C_{22}'	C_{24}'	C_{23}''	C_5^{-}	C_5^{2+}	E	C_5^{2-}	C_5^{+}	C_{10}^{3+}	C_{10}^{-}	C_2	C_{10}^{+}	C_{10}^{3-}
C_{24}'	C_{24}'	C_{25}''	C_{23}''	C_{21}'	C_{22}'	C_{22}''	C_{21}''	C_{23}'	C_{25}'	C_{24}''	C_5^{+}	C_5^{-}	C_5^{2+}	E	C_5^{2-}	C_{10}^{3-}	C_{10}^{3+}	C_{10}^{-}	C_2	C_{10}^{+}
C_{25}'	C_{25}'	C_{21}''	C_{24}''	C_{22}'	C_{23}'	C_{23}''	C_{22}''	C_{24}'	C_{21}'	C_{25}''	C_5^{2-}	C_5^{+}	C_5^{-}	C_5^{2+}	E	C_{10}^{+}	C_{10}^{3-}	C_{10}^{3+}	C_{10}^{-}	C_2
C_{21}''	C_{21}''	C_{22}'	C_{25}'	C_{23}''	C_{24}''	C_{24}'	C_{23}'	C_{25}''	C_{22}''	C_{21}'	C_2	C_{10}^{+}	C_{10}^{3-}	C_{10}^{3+}	C_{10}^{-}	E	C_5^{2-}	C_5^{+}	C_5^{-}	C_5^{2+}
C_{22}''	C_{22}''	C_{23}'	C_{21}'	C_{24}''	C_{25}''	C_{25}'	C_{24}'	C_{21}''	C_{23}''	C_{22}'	C_{10}^{-}	C_2	C_{10}^{+}	C_{10}^{3-}	C_{10}^{3+}	C_5^{2+}	E	C_5^{2-}	C_5^{+}	C_5^{-}
C_{23}''	C_{23}''	C_{24}'	C_{22}'	C_{25}''	C_{21}''	C_{21}'	C_{25}'	C_{22}''	C_{24}''	C_{23}'	C_{10}^{3+}	C_{10}^{-}	C_2	C_{10}^{+}	C_{10}^{3-}	C_5^{-}	C_5^{2+}	E	C_5^{2-}	C_5^{+}
C_{24}''	C_{24}''	C_{25}'	C_{23}'	C_{21}''	C_{22}''	C_{22}'	C_{21}'	C_{23}''	C_{25}''	C_{24}'	C_{10}^{3-}	C_{10}^{3+}	C_{10}^{-}	C_2	C_{10}^{+}	C_5^{+}	C_5^{-}	C_5^{2+}	E	C_5^{2-}
C_{25}''	C_{25}''	C_{21}'	C_{24}'	C_{22}''	C_{23}''	C_{23}'	C_{22}'	C_{24}''	C_{21}''	C_{25}'	C_{10}^{+}	C_{10}^{3-}	C_{10}^{3+}	C_{10}^{-}	C_2	C_5^{2-}	C_5^{+}	C_5^{-}	C_5^{2+}	E
i	i	S_5^{2-}	S_5^{2+}	S_{10}^{3-}	S_{10}^{3+}	S_5^{-}	S_5^{+}	S_{10}^{-}	S_{10}^{+}	σ_h	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}
S_5^{2-}	S_5^{2-}	S_{10}^{3-}	i	S_5^{-}	S_5^{2+}	S_{10}^{-}	S_{10}^{3+}	σ_h	S_5^{+}	S_{10}^{+}	σ_{v5}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}
S_5^{2+}	S_5^{2+}	i	S_{10}^{3+}	S_5^{2-}	S_5^{+}	S_{10}^{3-}	S_{10}^{+}	S_5^{-}	σ_h	S_{10}^{-}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}
S_{10}^{3-}	S_{10}^{3-}	S_5^{-}	S_5^{2-}	S_{10}^{-}	i	σ_h	S_5^{2+}	S_{10}^{+}	S_{10}^{3+}	S_5^{+}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{v4}	σ_{v5}	σ_{v1}	σ_{v2}	σ_{v3}
S_{10}^{3+}	S_{10}^{3+}	S_5^{2+}	S_5^{+}	i	S_{10}^{+}	S_5^{2-}	σ_h	S_{10}^{3-}	S_{10}^{-}	S_5^{-}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v1}	σ_{v2}
S_5^{-}	S_5^{-}	S_{10}^{-}	S_{10}^{3-}	σ_h	S_5^{2-}	S_{10}^{+}	i	S_5^{+}	S_5^{2+}	S_{10}^{3+}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v1}	σ_{v2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}
S_5^{+}	S_5^{+}	S_{10}^{3+}	S_{10}^{+}	S_5^{2+}	σ_h	i	S_{10}^{-}	S_5^{2-}	S_5^{-}	S_{10}^{3-}	σ_{v4}	σ_{v5}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}
S_{10}^{-}	S_{10}^{-}	σ_h	S_5^{-}	S_{10}^{+}	S_{10}^{3-}	S_5^{+}	S_5^{2-}	S_{10}^{3+}	i	S_5^{2+}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v1}
S_{10}^{+}	S_{10}^{+}	S_5^{+}	σ_h	S_{10}^{3+}	S_{10}^{-}	S_5^{2+}	S_5^{-}	i	S_{10}^{3-}	S_5^{2-}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{v5}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}
σ_h	σ_h	S_{10}^{+}	S_{10}^{-}	S_5^{+}	S_5^{-}	S_{10}^{3+}	S_{10}^{3-}	S_5^{2+}	S_5^{2-}	i	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}
σ_{d1}	σ_{d1}	σ_{v2}	σ_{v5}	σ_{d3}	σ_{d4}	σ_{v4}	σ_{v3}	σ_{d5}	σ_{d2}	σ_{v1}	i	S_{10}^{+}	S_{10}^{3-}	S_{10}^{3+}	S_{10}^{-}	σ_h	S_5^{2-}	S_5^{+}	S_5^{-}	S_5^{2+}
σ_{d2}	σ_{d2}	σ_{v3}	σ_{v1}	σ_{d4}	σ_{d5}	σ_{v5}	σ_{v4}	σ_{d1}	σ_{d3}	σ_{v2}	S_{10}^{-}	i	S_{10}^{+}	S_{10}^{3-}	S_{10}^{3+}	S_5^{2+}	σ_h	S_5^{2-}	S_5^{+}	S_5^{-}
σ_{d3}	σ_{d3}	σ_{v4}	σ_{v2}	σ_{d5}	σ_{d1}	σ_{v1}	σ_{v5}	σ_{d2}	σ_{d4}	σ_{v3}	S_{10}^{3+}	S_{10}^{-}	i	S_{10}^{+}	S_{10}^{3-}	S_5^{-}	S_5^{2+}	σ_h	S_5^{2-}	S_5^{+}
σ_{d4}	σ_{d4}	σ_{v5}	σ_{v3}	σ_{d1}	σ_{d2}	σ_{v2}	σ_{v1}	σ_{d3}	σ_{d5}	σ_{v4}	S_{10}^{3-}	S_{10}^{3+}	S_{10}^{-}	i	S_{10}^{+}	S_5^{+}	S_5^{-}	S_5^{2+}	σ_h	S_5^{2-}
σ_{d5}	σ_{d5}	σ_{v1}	σ_{v4}	σ_{d2}	σ_{d3}	σ_{v3}	σ_{v2}	σ_{d4}	σ_{d1}	σ_{v5}	S_{10}^{+}	S_{10}^{3-}	S_{10}^{3+}	S_{10}^{-}	i	S_5^{2-}	S_5^{+}	S_5^{-}	S_5^{2+}	σ_h
σ_{v1}	σ_{v1}	σ_{d2}	σ_{d5}	σ_{v3}	σ_{v4}	σ_{d4}	σ_{d3}	σ_{v5}	σ_{v2}	σ_{d1}	σ_h	S_5^{2-}	S_5^{+}	S_5^{-}	S_5^{2+}	i	S_{10}^{+}	S_{10}^{3-}	S_{10}^{3+}	S_{10}^{-}
σ_{v2}	σ_{v2}	σ_{d3}	σ_{d1}	σ_{v4}	σ_{v5}	σ_{d5}	σ_{d4}	σ_{v1}	σ_{v3}	σ_{d2}	S_5^{2+}	σ_h	S_5^{2-}	S_5^{+}	S_5^{-}	S_{10}^{-}	i	S_{10}^{+}	S_{10}^{3-}	S_{10}^{3+}
σ_{v3}	σ_{v3}	σ_{d4}	σ_{d2}	σ_{v5}	σ_{v1}	σ_{d1}	σ_{d5}	σ_{v2}	σ_{v4}	σ_{d3}	S_5^{-}	S_5^{2+}	σ_h	S_5^{2-}	S_5^{+}	S_{10}^{3+}	S_{10}^{-}	i	S_{10}^{+}	S_{10}^{3-}
σ_{v4}	σ_{v4}	σ_{d5}	σ_{d3}	σ_{v1}	σ_{v2}	σ_{d2}	σ_{d1}	σ_{v3}	σ_{v5}	σ_{d4}	S_5^{+}	S_5^{-}	S_5^{2+}	σ_h	S_5^{2-}	S_{10}^{3-}	S_{10}^{3+}	S_{10}^{-}	i	S_{10}^{+}
σ_{v5}	σ_{v5}	σ_{d1}	σ_{d4}	σ_{v2}	σ_{v3}	σ_{d3}	σ_{d2}	σ_{v4}	σ_{v1}	σ_{d5}	S_5^{2-}	S_5^{+}	S_5^{-}	S_5^{2+}	σ_h	S_{10}^{+}	S_{10}^{3-}	S_{10}^{3+}	S_{10}^{-}	i

$\twoheadrightarrow$

T **39**.2 Multiplication table *(cont.)*

$\mathbf{D}_{10h}$	i	S_5^{2-}	S_5^{2+}	S_{10}^{3-}	S_{10}^{3+}	S_5^-	S_5^+	S_{10}^-	S_{10}^+	σ_h	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}
E	i	S_5^{2-}	S_5^{2+}	S_{10}^{3-}	S_{10}^{3+}	S_5^-	S_5^+	S_{10}^-	S_{10}^+	σ_h	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}
C_{10}^+	S_5^{2-}	S_{10}^{3-}	i	S_5^-	S_5^{2+}	S_{10}^-	S_{10}^{3+}	σ_h	S_5^+	S_{10}^+	σ_{v5}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}
C_{10}^-	S_5^{2+}	i	S_{10}^{3+}	S_5^{2-}	S_5^+	S_{10}^{3-}	S_{10}^+	S_5^-	σ_h	S_{10}^-	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}
C_5^+	S_{10}^{3-}	S_5^-	S_5^{2-}	S_{10}^-	i	σ_h	S_5^{2+}	S_{10}^+	S_{10}^{3+}	S_5^+	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{v4}	σ_{v5}	σ_{v1}	σ_{v2}	σ_{v3}
C_5^-	S_{10}^{3+}	S_5^{2+}	S_5^+	i	S_{10}^+	S_5^{2-}	σ_h	S_{10}^{3-}	S_{10}^-	S_5^-	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v1}	σ_{v2}
C_{10}^{3+}	S_5^-	S_{10}^-	S_{10}^{3-}	σ_h	S_5^{2-}	S_{10}^+	i	S_5^+	S_5^{2+}	S_{10}^{3+}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v1}	σ_{v2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}
C_{10}^{3-}	S_5^+	S_{10}^{3+}	S_{10}^+	S_5^{2+}	σ_h	i	S_{10}^-	S_5^{2-}	S_5^-	S_{10}^{3-}	σ_{v4}	σ_{v5}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}
C_5^{2+}	S_{10}^-	σ_h	S_5^-	S_{10}^+	S_{10}^{3-}	S_5^+	S_5^{2-}	S_{10}^{3+}	i	S_5^{2+}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{v1}
C_5^{2-}	S_{10}^+	S_5^+	σ_h	S_{10}^{3+}	S_{10}^-	S_5^{2+}	S_5^-	i	S_{10}^{3-}	S_5^{2-}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{v5}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}
C_2	σ_h	S_{10}^+	S_{10}^-	S_5^+	S_5^-	S_{10}^{3+}	S_{10}^{3-}	S_5^{2+}	S_5^{2-}	i	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}
C'_{21}	σ_{d1}	σ_{v2}	σ_{v5}	σ_{d3}	σ_{d4}	σ_{v4}	σ_{v3}	σ_{d5}	σ_{d2}	σ_{v1}	i	S_{10}^+	S_{10}^{3-}	S_{10}^{3+}	S_{10}^-	σ_h	S_5^{2-}	S_5^+	S_5^-	S_5^{2+}
C'_{22}	σ_{d2}	σ_{v3}	σ_{v1}	σ_{d4}	σ_{d5}	σ_{v5}	σ_{v4}	σ_{d1}	σ_{d3}	σ_{v2}	S_{10}^-	i	S_{10}^+	S_{10}^{3-}	S_{10}^{3+}	S_5^{2+}	σ_h	S_5^{2-}	S_5^+	S_5^-
C'_{23}	σ_{d3}	σ_{v4}	σ_{v2}	σ_{d5}	σ_{d1}	σ_{v1}	σ_{v5}	σ_{d2}	σ_{d4}	σ_{v3}	S_{10}^{3+}	S_{10}^-	i	S_{10}^+	S_{10}^{3-}	S_5^-	S_5^{2+}	σ_h	S_5^{2-}	S_5^+
C'_{24}	σ_{d4}	σ_{v5}	σ_{v3}	σ_{d1}	σ_{d2}	σ_{v2}	σ_{v1}	σ_{d3}	σ_{d5}	σ_{v4}	S_{10}^{3-}	S_{10}^{3+}	S_{10}^-	i	S_{10}^+	S_5^+	S_5^-	S_5^{2+}	σ_h	S_5^{2-}
C'_{25}	σ_{d5}	σ_{v1}	σ_{v4}	σ_{d2}	σ_{d3}	σ_{v3}	σ_{v2}	σ_{d4}	σ_{d1}	σ_{v5}	S_{10}^+	S_{10}^{3-}	S_{10}^{3+}	S_{10}^-	i	S_5^{2-}	S_5^+	S_5^-	S_5^{2+}	σ_h
C''_{21}	σ_{v1}	σ_{d2}	σ_{d5}	σ_{v3}	σ_{v4}	σ_{d4}	σ_{d3}	σ_{v5}	σ_{v2}	σ_{d1}	σ_h	S_5^{2-}	S_5^+	S_5^-	S_5^{2+}	i	S_{10}^+	S_{10}^{3-}	S_{10}^{3+}	S_{10}^-
C''_{22}	σ_{v2}	σ_{d3}	σ_{d1}	σ_{v4}	σ_{v5}	σ_{d5}	σ_{d4}	σ_{v1}	σ_{v3}	σ_{d2}	S_5^{2+}	σ_h	S_5^{2-}	S_5^+	S_5^-	S_{10}^-	i	S_{10}^+	S_{10}^{3-}	S_{10}^{3+}
C''_{23}	σ_{v3}	σ_{d4}	σ_{d2}	σ_{v5}	σ_{v1}	σ_{d1}	σ_{d5}	σ_{v2}	σ_{v4}	σ_{d3}	S_5^-	S_5^{2+}	σ_h	S_5^{2-}	S_5^+	S_{10}^{3+}	S_{10}^-	i	S_{10}^+	S_{10}^{3-}
C''_{24}	σ_{v4}	σ_{d5}	σ_{d3}	σ_{v1}	σ_{v2}	σ_{d2}	σ_{d1}	σ_{v3}	σ_{v5}	σ_{d4}	S_5^+	S_5^-	S_5^{2+}	σ_h	S_5^{2-}	S_{10}^{3-}	S_{10}^{3+}	S_{10}^-	i	S_{10}^+
C''_{25}	σ_{v5}	σ_{d1}	σ_{d4}	σ_{v2}	σ_{v3}	σ_{d3}	σ_{d2}	σ_{v4}	σ_{v1}	σ_{d5}	S_5^{2-}	S_5^+	S_5^-	S_5^{2+}	σ_h	S_{10}^+	S_{10}^{3-}	S_{10}^{3+}	S_{10}^-	i
i	E	C_{10}^+	C_{10}^-	C_5^+	C_5^-	C_{10}^{3+}	C_{10}^{3-}	C_5^{2+}	C_5^{2-}	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C''_{21}	C''_{22}	C''_{23}	C''_{24}	C''_{25}
S_5^{2-}	C_{10}^+	C_5^+	E	C_{10}^{3+}	C_{10}^-	C_5^{2+}	C_5^-	C_2	C_{10}^{3-}	C_5^{2-}	C''_{25}	C''_{21}	C''_{22}	C''_{23}	C''_{24}	C'_{25}	C'_{21}	C'_{22}	C'_{23}	C'_{24}
S_5^{2+}	C_{10}^-	E	C_5^-	C_{10}^+	C_{10}^{3-}	C_5^+	C_5^{2-}	C_{10}^{3+}	C_2	C_5^{2+}	C''_{22}	C''_{23}	C''_{24}	C''_{25}	C''_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{21}
S_{10}^{3-}	C_5^+	C_{10}^{3+}	C_{10}^+	C_5^{2+}	E	C_2	C_{10}^-	C_5^{2-}	C_5^-	C_{10}^{3-}	C'_{24}	C'_{25}	C'_{21}	C'_{22}	C'_{23}	C''_{24}	C''_{25}	C''_{21}	C''_{22}	C''_{23}
S_{10}^{3+}	C_5^-	C_{10}^-	C_{10}^{3-}	E	C_5^{2-}	C_{10}^+	C_2	C_5^+	C_5^{2+}	C_{10}^{3+}	C'_{23}	C'_{24}	C'_{25}	C'_{21}	C'_{22}	C''_{23}	C''_{24}	C''_{25}	C''_{21}	C''_{22}
S_5^-	C_{10}^{3+}	C_5^{2+}	C_5^+	C_2	C_{10}^+	C_5^{2-}	E	C_{10}^{3-}	C_{10}^-	C_5^-	C''_{23}	C''_{24}	C''_{25}	C''_{21}	C''_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{21}	C'_{22}
S_5^+	C_{10}^{3-}	C_5^-	C_5^{2-}	C_{10}^-	C_2	E	C_5^{2+}	C_{10}^+	C_{10}^{3+}	C_5^+	C''_{24}	C''_{25}	C''_{21}	C''_{22}	C''_{23}	C'_{24}	C'_{25}	C'_{21}	C'_{22}	C'_{23}
S_{10}^-	C_5^{2+}	C_2	C_{10}^{3+}	C_5^{2-}	C_5^+	C_{10}^{3-}	C_{10}^+	C_5^-	E	C_{10}^-	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{21}	C''_{22}	C''_{23}	C''_{24}	C''_{25}	C''_{21}
S_{10}^+	C_5^{2-}	C_{10}^{3-}	C_2	C_5^-	C_5^{2+}	C_{10}^-	C_{10}^{3+}	E	C_5^+	C_{10}^+	C'_{25}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C''_{25}	C''_{21}	C''_{22}	C''_{23}	C''_{24}
σ_h	C_2	C_5^{2-}	C_5^{2+}	C_{10}^{3-}	C_{10}^{3+}	C_5^-	C_5^+	C_{10}^-	C_{10}^+	E	C''_{21}	C''_{22}	C''_{23}	C''_{24}	C''_{25}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}
σ_{d1}	C'_{21}	C''_{22}	C''_{25}	C'_{23}	C'_{24}	C''_{24}	C''_{23}	C'_{25}	C'_{22}	C''_{21}	E	C_5^{2-}	C_5^+	C_5^-	C_5^{2+}	C_2	C_{10}^+	C_{10}^{3-}	C_{10}^{3+}	C_{10}^-
σ_{d2}	C'_{22}	C''_{23}	C''_{21}	C'_{24}	C'_{25}	C''_{25}	C''_{24}	C'_{21}	C'_{23}	C''_{22}	C_5^{2+}	E	C_5^{2-}	C_5^+	C_5^-	C_{10}^-	C_2	C_{10}^+	C_{10}^{3-}	C_{10}^{3+}
σ_{d3}	C'_{23}	C''_{24}	C''_{22}	C'_{25}	C'_{21}	C''_{21}	C''_{25}	C'_{22}	C'_{24}	C''_{23}	C_5^-	C_5^{2+}	E	C_5^{2-}	C_5^+	C_{10}^{3+}	C_{10}^-	C_2	C_{10}^+	C_{10}^{3-}
σ_{d4}	C'_{24}	C''_{25}	C''_{23}	C'_{21}	C'_{22}	C''_{22}	C''_{21}	C'_{23}	C'_{25}	C''_{24}	C_5^+	C_5^-	C_5^{2+}	E	C_5^{2-}	C_{10}^{3-}	C_{10}^{3+}	C_{10}^-	C_2	C_{10}^+
σ_{d5}	C'_{25}	C''_{21}	C''_{24}	C'_{22}	C'_{23}	C''_{23}	C''_{22}	C'_{24}	C'_{21}	C''_{25}	C_5^{2-}	C_5^+	C_5^-	C_5^{2+}	E	C_{10}^+	C_{10}^{3-}	C_{10}^{3+}	C_{10}^-	C_2
σ_{v1}	C''_{21}	C'_{22}	C'_{25}	C''_{23}	C''_{24}	C'_{24}	C'_{23}	C''_{25}	C''_{22}	C'_{21}	C_2	C_{10}^+	C_{10}^{3-}	C_{10}^{3+}	C_{10}^-	E	C_5^{2-}	C_5^+	C_5^-	C_5^{2+}
σ_{v2}	C''_{22}	C'_{23}	C'_{21}	C''_{24}	C''_{25}	C'_{25}	C'_{24}	C''_{21}	C''_{23}	C'_{22}	C_{10}^-	C_2	C_{10}^+	C_{10}^{3-}	C_{10}^{3+}	C_5^{2+}	E	C_5^{2-}	C_5^+	C_5^-
σ_{v3}	C''_{23}	C'_{24}	C'_{22}	C''_{25}	C''_{21}	C'_{21}	C'_{25}	C''_{22}	C''_{24}	C'_{23}	C_{10}^{3+}	C_{10}^-	C_2	C_{10}^+	C_{10}^{3-}	C_5^-	C_5^{2+}	E	C_5^{2-}	C_5^+
σ_{v4}	C''_{24}	C'_{25}	C'_{23}	C''_{21}	C''_{22}	C'_{22}	C'_{21}	C''_{23}	C''_{25}	C'_{24}	C_{10}^{3-}	C_{10}^{3+}	C_{10}^-	C_2	C_{10}^+	C_5^+	C_5^-	C_5^{2+}	E	C_5^{2-}
σ_{v5}	C''_{25}	C'_{21}	C'_{24}	C''_{22}	C''_{23}	C'_{23}	C'_{22}	C''_{24}	C''_{21}	C'_{25}	C_{10}^+	C_{10}^{3-}	C_{10}^{3+}	C_{10}^-	C_2	C_5^{2-}	C_5^+	C_5^-	C_5^{2+}	E

T **39**.3 Factor table

§ **16**–3, p. 70

$\mathbf{D}_{10h}$	E	C_{10}^{+}	C_{10}^{-}	C_5^{+}	C_5^{-}	C_{10}^{3+}	C_{10}^{3-}	C_5^{2+}	C_5^{2-}	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C''_{21}	C''_{22}	C''_{23}	C''_{24}	C''_{25}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_{10}^{+}	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	−1	−1	−1	−1	−1
C_{10}^{-}	1	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	1	1	1	1	1
C_5^{+}	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_5^{-}	1	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_{10}^{3+}	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	1
C_{10}^{3-}	1	1	1	1	−1	1	−1	1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1
C_5^{2+}	1	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
C_5^{2-}	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1
C_2	1	−1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	−1	−1	−1	−1	−1
C'_{21}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	−1	1	1	−1	1	1	1	−1	−1
C'_{22}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	−1	−1	1	1	−1	1	1	1	−1
C'_{23}	1	−1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	−1	1	−1	−1	1	1	1
C'_{24}	1	−1	1	−1	−1	1	−1	1	1	−1	1	1	−1	−1	−1	1	−1	−1	1	1
C'_{25}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1
C''_{21}	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	1	1	−1	−1	1	1	−1
C''_{22}	1	1	−1	−1	−1	−1	1	1	1	1	1	−1	−1	−1	1	−1	−1	−1	1	1
C''_{23}	1	1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	−1	1	−1	−1	−1	1
C''_{24}	1	1	−1	−1	−1	−1	1	1	1	1	−1	1	1	−1	−1	1	1	−1	−1	−1
C''_{25}	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1
i	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_5^{2-}	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	−1	−1	−1	−1	−1
S_5^{2+}	1	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	1	1	1	1	1
S_{10}^{3-}	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_{10}^{3+}	1	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_5^{-}	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	1
S_5^{+}	1	1	1	1	−1	1	−1	1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1
S_{10}^{-}	1	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
S_{10}^{+}	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1
σ_h	1	−1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	−1	−1	−1	−1	−1
σ_{d1}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	−1	1	1	−1	1	1	1	−1	−1
σ_{d2}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	−1	−1	1	1	−1	1	1	1	−1
σ_{d3}	1	−1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	−1	1	−1	−1	1	1	1
σ_{d4}	1	−1	1	−1	−1	1	−1	1	1	−1	1	1	−1	−1	−1	1	−1	−1	1	1
σ_{d5}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1
σ_{v1}	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	1	1	−1	−1	1	1	−1
σ_{v2}	1	1	−1	−1	−1	−1	1	1	1	1	1	−1	−1	−1	1	−1	−1	−1	1	1
σ_{v3}	1	1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	−1	1	−1	−1	−1	1
σ_{v4}	1	1	−1	−1	−1	−1	1	1	1	1	−1	1	1	−1	−1	1	1	−1	−1	−1
σ_{v5}	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1

$\twoheadrightarrow$

T **39**.3 Factor table *(cont.)*

$\mathbf{D}_{10h}$	i	S_5^{2-}	S_5^{2+}	S_{10}^{3-}	S_{10}^{3+}	S_5^-	S_5^+	S_{10}^-	S_{10}^+	σ_h	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{v1}	σ_{v2}	σ_{v3}	σ_{v4}	σ_{v5}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_{10}^+	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	−1	−1	−1	−1	−1
C_{10}^-	1	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	1	1	1	1	1
C_5^+	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_5^-	1	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_{10}^{3+}	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	1
C_{10}^{3-}	1	1	1	1	−1	1	−1	1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1
C_5^{2+}	1	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
C_5^{2-}	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1
C_2	1	−1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	−1	−1	−1	−1	−1
C'_{21}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	−1	1	1	−1	1	1	1	−1	−1
C'_{22}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	−1	−1	1	1	−1	1	1	1	−1
C'_{23}	1	−1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	−1	1	−1	−1	1	1	1
C'_{24}	1	−1	1	−1	−1	1	−1	1	1	−1	1	1	−1	−1	−1	1	−1	−1	1	1
C'_{25}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1
C''_{21}	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	1	1	−1	−1	1	1	−1
C''_{22}	1	1	−1	−1	−1	−1	1	1	1	1	1	−1	−1	−1	1	−1	−1	−1	1	1
C''_{23}	1	1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	−1	1	−1	−1	−1	1
C''_{24}	1	1	−1	−1	−1	−1	1	1	1	1	−1	1	1	−1	−1	1	1	−1	−1	−1
C''_{25}	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1
i	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_5^{2-}	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	−1	−1	−1	−1	−1
S_5^{2+}	1	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	1	1	1	1	1
S_{10}^{3-}	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_{10}^{3+}	1	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_5^-	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	1
S_5^+	1	1	1	1	−1	1	−1	1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1
S_{10}^-	1	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
S_{10}^+	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1
σ_h	1	−1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	−1	−1	−1	−1	−1
σ_{d1}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	−1	1	1	−1	1	1	1	−1	−1
σ_{d2}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	−1	−1	1	1	−1	1	1	1	−1
σ_{d3}	1	−1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	−1	1	−1	−1	1	1	1
σ_{d4}	1	−1	1	−1	−1	1	−1	1	1	−1	1	1	−1	−1	−1	1	−1	−1	1	1
σ_{d5}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1
σ_{v1}	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	1	1	−1	−1	1	1	−1
σ_{v2}	1	1	−1	−1	−1	−1	1	1	1	1	1	−1	−1	−1	1	−1	−1	−1	1	1
σ_{v3}	1	1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	−1	1	−1	−1	−1	1
σ_{v4}	1	1	−1	−1	−1	−1	1	1	1	1	−1	1	1	−1	−1	1	1	−1	−1	−1
σ_{v5}	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1

T **39**.4 Character table

§ **16**–4, p. 71

$\mathbf{D}_{10h}$	E	$2C_{10}$	$2C_5$	$2C_{10}^3$	$2C_5^2$	C_2	$5C_2'$	$5C_2''$	i	$2S_5^2$	$2S_{10}^3$	$2S_5$	$2S_{10}$	σ_h	$5\sigma_d$	$5\sigma_v$	τ
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	a
A_{2g}	1	1	1	1	1	1	−1	−1	1	1	1	1	1	1	−1	−1	a
B_{1g}	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	a
B_{2g}	1	−1	1	−1	1	−1	−1	1	1	−1	1	−1	1	−1	−1	1	a
E_{1g}	2	$2c_5$	$2c_5^2$	$-2c_5^2$	$-2c_5$	−2	0	0	2	$2c_5$	$2c_5^2$	$-2c_5^2$	$-2c_5$	−2	0	0	a
E_{2g}	2	$2c_5^2$	$-2c_5$	$-2c_5$	$2c_5^2$	2	0	0	2	$2c_5^2$	$-2c_5$	$-2c_5$	$2c_5^2$	2	0	0	a
E_{3g}	2	$-2c_5^2$	$-2c_5$	$2c_5$	$2c_5^2$	−2	0	0	2	$-2c_5^2$	$-2c_5$	$2c_5$	$2c_5^2$	−2	0	0	a
E_{4g}	2	$-2c_5$	$2c_5^2$	$2c_5^2$	$-2c_5$	2	0	0	2	$-2c_5$	$2c_5^2$	$2c_5^2$	$-2c_5$	2	0	0	a
A_{1u}	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	a
A_{2u}	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	a
B_{1u}	1	−1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	−1	1	a
B_{2u}	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	−1	1	1	−1	a
E_{1u}	2	$2c_5$	$2c_5^2$	$-2c_5^2$	$-2c_5$	−2	0	0	−2	$-2c_5$	$-2c_5^2$	$2c_5^2$	$2c_5$	2	0	0	a
E_{2u}	2	$2c_5^2$	$-2c_5$	$-2c_5$	$2c_5^2$	2	0	0	−2	$-2c_5^2$	$2c_5$	$2c_5$	$-2c_5^2$	−2	0	0	a
E_{3u}	2	$-2c_5^2$	$-2c_5$	$2c_5$	$2c_5^2$	−2	0	0	−2	$2c_5^2$	$2c_5$	$-2c_5$	$-2c_5^2$	2	0	0	a
E_{4u}	2	$-2c_5$	$2c_5^2$	$2c_5^2$	$-2c_5$	2	0	0	−2	$2c_5$	$-2c_5^2$	$-2c_5^2$	$2c_5$	−2	0	0	a
$E_{1/2,g}$	2	$2c_{10}$	$2c_5$	$2c_{10}^3$	$2c_5^2$	0	0	0	2	$2c_{10}$	$2c_5$	$2c_{10}^3$	$2c_5^2$	0	0	0	c
$E_{3/2,g}$	2	$2c_{10}^3$	$-2c_5^2$	$-2c_{10}$	$-2c_5$	0	0	0	2	$2c_{10}^3$	$-2c_5^2$	$-2c_{10}$	$-2c_5$	0	0	0	c
$E_{5/2,g}$	2	0	−2	0	2	0	0	0	2	0	−2	0	2	0	0	0	c
$E_{7/2,g}$	2	$-2c_{10}^3$	$-2c_5^2$	$2c_{10}$	$-2c_5$	0	0	0	2	$-2c_{10}^3$	$-2c_5^2$	$2c_{10}$	$-2c_5$	0	0	0	c
$E_{9/2,g}$	2	$-2c_{10}$	$2c_5$	$-2c_{10}^3$	$2c_5^2$	0	0	0	2	$-2c_{10}$	$2c_5$	$-2c_{10}^3$	$2c_5^2$	0	0	0	c
$E_{1/2,u}$	2	$2c_{10}$	$2c_5$	$2c_{10}^3$	$2c_5^2$	0	0	0	−2	$-2c_{10}$	$-2c_5$	$-2c_{10}^3$	$-2c_5^2$	0	0	0	c
$E_{3/2,u}$	2	$2c_{10}^3$	$-2c_5^2$	$-2c_{10}$	$-2c_5$	0	0	0	−2	$-2c_{10}^3$	$2c_5^2$	$2c_{10}$	$2c_5$	0	0	0	c
$E_{5/2,u}$	2	0	−2	0	2	0	0	0	−2	0	2	0	−2	0	0	0	c
$E_{7/2,u}$	2	$-2c_{10}^3$	$-2c_5^2$	$2c_{10}$	$-2c_5$	0	0	0	−2	$2c_{10}^3$	$2c_5^2$	$-2c_{10}$	$2c_5$	0	0	0	c
$E_{9/2,u}$	2	$-2c_{10}$	$2c_5$	$-2c_{10}^3$	$2c_5^2$	0	0	0	−2	$2c_{10}$	$-2c_5$	$2c_{10}^3$	$-2c_5^2$	0	0	0	c

$c_n^m = \cos \frac{m}{n}\pi$

T **39**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{10h}$	0	1	2	3
A_{1g}	$^\square 1$		$x^2+y^2, {}^\square z^2$	
A_{2g}		R_z		
B_{1g}				
B_{2g}				
E_{1g}		(R_x, R_y)	$^\square(zx, yz)$	
E_{2g}			$^\square(xy, x^2-y^2)$	
E_{3g}				
E_{4g}				
A_{1u}				
A_{2u}		$^\square z$		$(x^2+y^2)z, {}^\square z^3$
B_{1u}				
B_{2u}				
E_{1u}		$^\square(x, y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
E_{2u}				$^\square\{xyz, z(x^2-y^2)\}$
E_{3u}				$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$
E_{4u}				

T **39**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{10h}$	$\langle\lvert j\,m\rangle\rvert$		ι	μ
A_{1g}	$\lvert 0\,0\rangle_+$		2	10
A_{2g}	$\lvert 10\,10\rangle_-$		2	10
B_{1g}	$\lvert 6\,5\rangle_+$		2	10
B_{2g}	$\lvert 6\,5\rangle_-$		2	10
E_{1g}	$\langle\lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle\rvert$		2	± 10
E_{2g}	$\langle\lvert 2\,2\rangle, \lvert 2\,\overline{2}\rangle\rvert$		2	± 10
E_{3g}	$\langle\lvert 4\,\overline{3}\rangle, \lvert 4\,3\rangle\rvert$		2	± 10
E_{4g}	$\langle\lvert 4\,\overline{4}\rangle, -\lvert 4\,4\rangle\rvert$		2	± 10
A_{1u}	$\lvert 11\,10\rangle_-$		2	10
A_{2u}	$\lvert 1\,0\rangle_+$		2	10
B_{1u}	$\lvert 5\,5\rangle_-$		2	10
B_{2u}	$\lvert 5\,5\rangle_+$		2	10
E_{1u}	$\langle\lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle\rvert$		2	± 10
E_{2u}	$\langle\lvert 3\,2\rangle, -\lvert 3\,\overline{2}\rangle\rvert$		2	± 10
E_{3u}	$\langle\lvert 3\,\overline{3}\rangle, -\lvert 3\,3\rangle\rvert$		2	± 10
E_{4u}	$\langle\lvert 5\,\overline{4}\rangle, \lvert 5\,4\rangle\rvert$		2	± 10
$E_{1/2,g}$	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	2	± 10
$E_{3/2,g}$	$\langle\lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle\rvert$	$\langle\lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle\rvert$	2	± 10
$E_{5/2,g}$	$\langle\lvert \frac{5}{2}\,\frac{5}{2}\rangle, \lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle\rvert$	$\langle\lvert \frac{7}{2}\,\frac{5}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle\rvert$	2	± 10
$E_{7/2,g}$	$\langle\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle, \lvert \frac{7}{2}\,\frac{7}{2}\rangle\rvert$	$\langle\lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle, -\lvert \frac{9}{2}\,\frac{7}{2}\rangle\rvert$	2	± 10
$E_{9/2,g}$	$\langle\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle, \lvert \frac{9}{2}\,\frac{9}{2}\rangle\rvert$	$\langle\lvert \frac{11}{2}\,\overline{\frac{9}{2}}\rangle, -\lvert \frac{11}{2}\,\frac{9}{2}\rangle\rvert$	2	± 10
$E_{1/2,u}$	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert^{\bullet}$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert^{\bullet}$	2	± 10
$E_{3/2,u}$	$\langle\lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle\rvert^{\bullet}$	$\langle\lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle\rvert^{\bullet}$	2	± 10
$E_{5/2,u}$	$\langle\lvert \frac{5}{2}\,\frac{5}{2}\rangle, \lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle\rvert^{\bullet}$	$\langle\lvert \frac{7}{2}\,\frac{5}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle\rvert^{\bullet}$	2	± 10
$E_{7/2,u}$	$\langle\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle, \lvert \frac{7}{2}\,\frac{7}{2}\rangle\rvert^{\bullet}$	$\langle\lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle, -\lvert \frac{9}{2}\,\frac{7}{2}\rangle\rvert^{\bullet}$	2	± 10
$E_{9/2,u}$	$\langle\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle, \lvert \frac{9}{2}\,\frac{9}{2}\rangle\rvert^{\bullet}$	$\langle\lvert \frac{11}{2}\,\overline{\frac{9}{2}}\rangle, -\lvert \frac{11}{2}\,\frac{9}{2}\rangle\rvert^{\bullet}$	2	± 10

T **39**.7 Matrix representations

Use T **30**.7 ▪. § **16**–7, p. 77

T **39**.8 Direct products of representations § **16**–8, p. 81

$\mathbf{D}_{10h}$	A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_{1g}	E_{2g}	E_{3g}	E_{4g}
A_{1g}	A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_{1g}	E_{2g}	E_{3g}	E_{4g}
A_{2g}		A_{1g}	B_{2g}	B_{1g}	E_{1g}	E_{2g}	E_{3g}	E_{4g}
B_{1g}			A_{1g}	A_{2g}	E_{4g}	E_{3g}	E_{2g}	E_{1g}
B_{2g}				A_{1g}	E_{4g}	E_{3g}	E_{2g}	E_{1g}
E_{1g}					$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$	$E_{1g} \oplus E_{3g}$	$E_{2g} \oplus E_{4g}$	$B_{1g} \oplus B_{2g} \oplus E_{3g}$
E_{2g}						$A_{1g} \oplus \{A_{2g}\} \oplus E_{4g}$	$B_{1g} \oplus B_{2g} \oplus E_{1g}$	$E_{2g} \oplus E_{4g}$
E_{3g}							$A_{1g} \oplus \{A_{2g}\} \oplus E_{4g}$	$E_{1g} \oplus E_{3g}$
E_{4g}								$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$

$\twoheadrightarrow$

T **39**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{10h}$	A_{1u}	A_{2u}	B_{1u}	B_{2u}	E_{1u}	E_{2u}	E_{3u}	E_{4u}
A_{1g}	A_{1u}	A_{2u}	B_{1u}	B_{2u}	E_{1u}	E_{2u}	E_{3u}	E_{4u}
A_{2g}	A_{2u}	A_{1u}	B_{2u}	B_{1u}	E_{1u}	E_{2u}	E_{3u}	E_{4u}
B_{1g}	B_{1u}	B_{2u}	A_{1u}	A_{2u}	E_{4u}	E_{3u}	E_{2u}	E_{1u}
B_{2g}	B_{2u}	B_{1u}	A_{2u}	A_{1u}	E_{4u}	E_{3u}	E_{2u}	E_{1u}
E_{1g}	E_{1u}	E_{1u}	E_{4u}	E_{4u}	$A_{1u} \oplus A_{2u} \oplus E_{2u}$	$E_{1u} \oplus E_{3u}$	$E_{2u} \oplus E_{4u}$	$B_{1u} \oplus B_{2u} \oplus E_{3u}$
E_{2g}	E_{2u}	E_{2u}	E_{3u}	E_{3u}	$E_{1u} \oplus E_{3u}$	$A_{1u} \oplus A_{2u} \oplus E_{4u}$	$B_{1u} \oplus B_{2u} \oplus E_{1u}$	$E_{2u} \oplus E_{4u}$
E_{3g}	E_{3u}	E_{3u}	E_{2u}	E_{2u}	$E_{2u} \oplus E_{4u}$	$B_{1u} \oplus B_{2u} \oplus E_{1u}$	$A_{1u} \oplus A_{2u} \oplus E_{4u}$	$E_{1u} \oplus E_{3u}$
E_{4g}	E_{4u}	E_{4u}	E_{1u}	E_{1u}	$B_{1u} \oplus B_{2u} \oplus E_{3u}$	$E_{2u} \oplus E_{4u}$	$E_{1u} \oplus E_{3u}$	$A_{1u} \oplus A_{2u} \oplus E_{2u}$
A_{1u}	A_{1g}	A_{2g}	B_{1g}	B_{2g}	E_{1g}	E_{2g}	E_{3g}	E_{4g}
A_{2u}		A_{1g}	B_{2g}	B_{1g}	E_{1g}	E_{2g}	E_{3g}	E_{4g}
B_{1u}			A_{1g}	A_{2g}	E_{4g}	E_{3g}	E_{2g}	E_{1g}
B_{2u}				A_{1g}	E_{4g}	E_{3g}	E_{2g}	E_{1g}
E_{1u}					$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$	$E_{1g} \oplus E_{3g}$	$E_{2g} \oplus E_{4g}$	$B_{1g} \oplus B_{2g} \oplus E_{3g}$
E_{2u}						$A_{1g} \oplus \{A_{2g}\} \oplus E_{4g}$	$B_{1g} \oplus B_{2g} \oplus E_{1g}$	$E_{2g} \oplus E_{4g}$
E_{3u}							$A_{1g} \oplus \{A_{2g}\} \oplus E_{4g}$	$E_{1g} \oplus E_{3g}$
E_{4u}								$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$

$\twoheadrightarrow$

T **39**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{10h}$	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{7/2,g}$	$E_{9/2,g}$
A_{1g}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{7/2,g}$	$E_{9/2,g}$
A_{2g}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{7/2,g}$	$E_{9/2,g}$
B_{1g}	$E_{9/2,g}$	$E_{7/2,g}$	$E_{5/2,g}$	$E_{3/2,g}$	$E_{1/2,g}$
B_{2g}	$E_{9/2,g}$	$E_{7/2,g}$	$E_{5/2,g}$	$E_{3/2,g}$	$E_{1/2,g}$
E_{1g}	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{3/2,g} \oplus E_{7/2,g}$	$E_{5/2,g} \oplus E_{9/2,g}$	$E_{7/2,g} \oplus E_{9/2,g}$
E_{2g}	$E_{3/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{7/2,g}$	$E_{1/2,g} \oplus E_{9/2,g}$	$E_{3/2,g} \oplus E_{9/2,g}$	$E_{5/2,g} \oplus E_{7/2,g}$
E_{3g}	$E_{5/2,g} \oplus E_{7/2,g}$	$E_{3/2,g} \oplus E_{9/2,g}$	$E_{1/2,g} \oplus E_{9/2,g}$	$E_{1/2,g} \oplus E_{7/2,g}$	$E_{3/2,g} \oplus E_{5/2,g}$
E_{4g}	$E_{7/2,g} \oplus E_{9/2,g}$	$E_{5/2,g} \oplus E_{9/2,g}$	$E_{3/2,g} \oplus E_{7/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$
A_{1u}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	$E_{7/2,u}$	$E_{9/2,u}$
A_{2u}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	$E_{7/2,u}$	$E_{9/2,u}$
B_{1u}	$E_{9/2,u}$	$E_{7/2,u}$	$E_{5/2,u}$	$E_{3/2,u}$	$E_{1/2,u}$
B_{2u}	$E_{9/2,u}$	$E_{7/2,u}$	$E_{5/2,u}$	$E_{3/2,u}$	$E_{1/2,u}$
E_{1u}	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{3/2,u} \oplus E_{7/2,u}$	$E_{5/2,u} \oplus E_{9/2,u}$	$E_{7/2,u} \oplus E_{9/2,u}$
E_{2u}	$E_{3/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{7/2,u}$	$E_{1/2,u} \oplus E_{9/2,u}$	$E_{3/2,u} \oplus E_{9/2,u}$	$E_{5/2,u} \oplus E_{7/2,u}$
E_{3u}	$E_{5/2,u} \oplus E_{7/2,u}$	$E_{3/2,u} \oplus E_{9/2,u}$	$E_{1/2,u} \oplus E_{9/2,u}$	$E_{1/2,u} \oplus E_{7/2,u}$	$E_{3/2,u} \oplus E_{5/2,u}$
E_{4u}	$E_{7/2,u} \oplus E_{9/2,u}$	$E_{5/2,u} \oplus E_{9/2,u}$	$E_{3/2,u} \oplus E_{7/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$
$E_{1/2,g}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$	$E_{1g} \oplus E_{2g}$	$E_{2g} \oplus E_{3g}$	$E_{3g} \oplus E_{4g}$	$B_{1g} \oplus B_{2g} \oplus E_{4g}$
$E_{3/2,g}$		$\{A_{1g}\} \oplus A_{2g} \oplus E_{3g}$	$E_{1g} \oplus E_{4g}$	$B_{1g} \oplus B_{2g} \oplus E_{2g}$	$E_{3g} \oplus E_{4g}$
$E_{5/2,g}$			$\{A_{1g}\} \oplus A_{2g} \oplus B_{1g} \oplus B_{2g}$	$E_{1g} \oplus E_{4g}$	$E_{2g} \oplus E_{3g}$
$E_{7/2,g}$				$\{A_{1g}\} \oplus A_{2g} \oplus E_{3g}$	$E_{1g} \oplus E_{2g}$
$E_{9/2,g}$					$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$

$\twoheadrightarrow$

T **39**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{10h}$	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	$E_{7/2,u}$	$E_{9/2,u}$
A_{1g}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	$E_{7/2,u}$	$E_{9/2,u}$
A_{2g}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	$E_{7/2,u}$	$E_{9/2,u}$
B_{1g}	$E_{9/2,u}$	$E_{7/2,u}$	$E_{5/2,u}$	$E_{3/2,u}$	$E_{1/2,u}$
B_{2g}	$E_{9/2,u}$	$E_{7/2,u}$	$E_{5/2,u}$	$E_{3/2,u}$	$E_{1/2,u}$
E_{1g}	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{3/2,u} \oplus E_{7/2,u}$	$E_{5/2,u} \oplus E_{9/2,u}$	$E_{7/2,u} \oplus E_{9/2,u}$
E_{2g}	$E_{3/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{7/2,u}$	$E_{1/2,u} \oplus E_{9/2,u}$	$E_{3/2,u} \oplus E_{9/2,u}$	$E_{5/2,u} \oplus E_{7/2,u}$
E_{3g}	$E_{5/2,u} \oplus E_{7/2,u}$	$E_{3/2,u} \oplus E_{9/2,u}$	$E_{1/2,u} \oplus E_{9/2,u}$	$E_{1/2,u} \oplus E_{7/2,u}$	$E_{3/2,u} \oplus E_{5/2,u}$
E_{4g}	$E_{7/2,u} \oplus E_{9/2,u}$	$E_{5/2,u} \oplus E_{9/2,u}$	$E_{3/2,u} \oplus E_{7/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$
A_{1u}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{7/2,g}$	$E_{9/2,g}$
A_{2u}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{7/2,g}$	$E_{9/2,g}$
B_{1u}	$E_{9/2,g}$	$E_{7/2,g}$	$E_{5/2,g}$	$E_{3/2,g}$	$E_{1/2,g}$
B_{2u}	$E_{9/2,g}$	$E_{7/2,g}$	$E_{5/2,g}$	$E_{3/2,g}$	$E_{1/2,g}$
E_{1u}	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{3/2,g} \oplus E_{7/2,g}$	$E_{5/2,g} \oplus E_{9/2,g}$	$E_{7/2,g} \oplus E_{9/2,g}$
E_{2u}	$E_{3/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{7/2,g}$	$E_{1/2,g} \oplus E_{9/2,g}$	$E_{3/2,g} \oplus E_{9/2,g}$	$E_{5/2,g} \oplus E_{7/2,g}$
E_{3u}	$E_{5/2,g} \oplus E_{7/2,g}$	$E_{3/2,g} \oplus E_{9/2,g}$	$E_{1/2,g} \oplus E_{9/2,g}$	$E_{1/2,g} \oplus E_{7/2,g}$	$E_{3/2,g} \oplus E_{5/2,g}$
E_{4u}	$E_{7/2,g} \oplus E_{9/2,g}$	$E_{5/2,g} \oplus E_{9/2,g}$	$E_{3/2,g} \oplus E_{7/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$
$E_{1/2,g}$	$A_{1u} \oplus A_{2u} \oplus E_{1u}$	$E_{1u} \oplus E_{2u}$	$E_{2u} \oplus E_{3u}$	$E_{3u} \oplus E_{4u}$	$B_{1u} \oplus B_{2u} \oplus E_{4u}$
$E_{3/2,g}$	$E_{1u} \oplus E_{2u}$	$A_{1u} \oplus A_{2u} \oplus E_{3u}$	$E_{1u} \oplus E_{4u}$	$B_{1u} \oplus B_{2u} \oplus E_{2u}$	$E_{3u} \oplus E_{4u}$
$E_{5/2,g}$	$E_{2u} \oplus E_{3u}$	$E_{1u} \oplus E_{4u}$	$A_{1u} \oplus A_{2u} \oplus B_{1u} \oplus B_{2u}$	$E_{1u} \oplus E_{4u}$	$E_{2u} \oplus E_{3u}$
$E_{7/2,g}$	$E_{3u} \oplus E_{4u}$	$B_{1u} \oplus B_{2u} \oplus E_{2u}$	$E_{1u} \oplus E_{4u}$	$A_{1u} \oplus A_{2u} \oplus E_{3u}$	$E_{1u} \oplus E_{2u}$
$E_{9/2,g}$	$B_{1u} \oplus B_{2u} \oplus E_{4u}$	$E_{3u} \oplus E_{4u}$	$E_{2u} \oplus E_{3u}$	$E_{1u} \oplus E_{2u}$	$A_{1u} \oplus A_{2u} \oplus E_{1u}$
$E_{1/2,u}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$	$E_{1g} \oplus E_{2g}$	$E_{2g} \oplus E_{3g}$	$E_{3g} \oplus E_{4g}$	$B_{1g} \oplus B_{2g} \oplus E_{4g}$
$E_{3/2,u}$		$\{A_{1g}\} \oplus A_{2g} \oplus E_{3g}$	$E_{1g} \oplus E_{4g}$	$B_{1g} \oplus B_{2g} \oplus E_{2g}$	$E_{3g} \oplus E_{4g}$
$E_{5/2,u}$			$\{A_{1g}\} \oplus A_{2g} \oplus B_{1g} \oplus B_{2g}$	$E_{1g} \oplus E_{4g}$	$E_{2g} \oplus E_{3g}$
$E_{7/2,u}$				$\{A_{1g}\} \oplus A_{2g} \oplus E_{3g}$	$E_{1g} \oplus E_{2g}$
$E_{9/2,u}$					$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$

T **39**.9 Subduction (descent of symmetry) § **16**–9, p. 82

$\mathbf{D}_{10h}$	$\mathbf{C}_{10h}$	$\mathbf{C}_{5h}$	$(\mathbf{C}_{10v})$	$(\mathbf{C}_{5v})$	$(\mathbf{C}_{5v})$	$(\mathbf{D}_{5d})$	$(\mathbf{D}_{5d})$	$\mathbf{D}_{5h}$	$(\mathbf{D}_{5h})$	$(\mathbf{D}_{2h})$	$\mathbf{D}_{10}$	$\mathbf{S}_{10}$
				σ_v	σ_d	C_2', σ_d	C_2'', σ_v	C_2', σ_v	C_2'', σ_d	C_2, C_2', C_2''		
A_{1g}	A_g	A'	A_1	A_1	A_1	A_{1g}	A_{1g}	A_1'	A_1'	A_g	A_1	A_g
A_{2g}	A_g	A'	A_2	A_2	A_2	A_{2g}	A_{2g}	A_2'	A_2'	B_{1g}	A_2	A_g
B_{1g}	B_g	A''	B_2	A_2	A_1	A_{1g}	A_{2g}	A_1''	A_2''	B_{3g}	B_1	A_g
B_{2g}	B_g	A''	B_1	A_1	A_2	A_{2g}	A_{1g}	A_2''	A_1''	B_{2g}	B_2	A_g
E_{1g}	${}^1E_{1g} \oplus {}^2E_{1g}$	${}^1E_1'' \oplus {}^2E_1''$	E_1	E_1	E_1	E_{1g}	E_{1g}	E_1''	E_1''	$B_{2g} \oplus B_{3g}$	E_1	${}^1E_{1g} \oplus {}^2E_{1g}$
E_{2g}	${}^1E_{2g} \oplus {}^2E_{2g}$	${}^1E_2' \oplus {}^2E_2'$	E_2	E_2	E_2	E_{2g}	E_{2g}	E_2'	E_2'	$A_g \oplus B_{1g}$	E_2	${}^1E_{2g} \oplus {}^2E_{2g}$
E_{3g}	${}^1E_{3g} \oplus {}^2E_{3g}$	${}^1E_2'' \oplus {}^2E_2''$	E_3	E_2	E_2	E_{2g}	E_{2g}	E_2''	E_2''	$B_{2g} \oplus B_{3g}$	E_3	${}^1E_{2g} \oplus {}^2E_{2g}$
E_{4g}	${}^1E_{4g} \oplus {}^2E_{4g}$	${}^1E_1' \oplus {}^2E_1'$	E_4	E_1	E_1	E_{1g}	E_{1g}	E_1'	E_1'	$A_g \oplus B_{1g}$	E_4	${}^1E_{1g} \oplus {}^2E_{1g}$
A_{1u}	A_u	A''	A_2	A_2	A_2	A_{1u}	A_{1u}	A_1''	A_1''	A_u	A_1	A_u
A_{2u}	A_u	A''	A_1	A_1	A_1	A_{2u}	A_{2u}	A_2''	A_2''	B_{1u}	A_2	A_u
B_{1u}	B_u	A'	B_1	A_1	A_2	A_{1u}	A_{2u}	A_1'	A_2'	B_{3u}	B_1	A_u
B_{2u}	B_u	A'	B_2	A_2	A_1	A_{2u}	A_{1u}	A_2'	A_1'	B_{2u}	B_2	A_u
E_{1u}	${}^1E_{1u} \oplus {}^2E_{1u}$	${}^1E_1' \oplus {}^2E_1'$	E_1	E_1	E_1	E_{1u}	E_{1u}	E_1'	E_1'	$B_{2u} \oplus B_{3u}$	E_1	${}^1E_{1u} \oplus {}^2E_{1u}$
E_{2u}	${}^1E_{2u} \oplus {}^2E_{2u}$	${}^1E_2'' \oplus {}^2E_2''$	E_2	E_2	E_2	E_{2u}	E_{2u}	E_2''	E_2''	$A_u \oplus B_{1u}$	E_2	${}^1E_{2u} \oplus {}^2E_{2u}$
E_{3u}	${}^1E_{3u} \oplus {}^2E_{3u}$	${}^1E_2' \oplus {}^2E_2'$	E_3	E_2	E_2	E_{2u}	E_{2u}	E_2'	E_2'	$B_{2u} \oplus B_{3u}$	E_3	${}^1E_{2u} \oplus {}^2E_{2u}$
E_{4u}	${}^1E_{4u} \oplus {}^2E_{4u}$	${}^1E_1'' \oplus {}^2E_1''$	E_4	E_1	E_1	E_{1u}	E_{1u}	E_1''	E_1''	$A_u \oplus B_{1u}$	E_4	${}^1E_{1u} \oplus {}^2E_{1u}$
$E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2,g}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$
$E_{3/2,g}$	${}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{3/2,g}$	$E_{3/2,g}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2,g}$	$E_{3/2}$	${}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$
$E_{5/2,g}$	${}^1E_{5/2,g} \oplus {}^2E_{5/2,g}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{5/2,g} \oplus {}^2E_{5/2,g}$	${}^1E_{5/2,g} \oplus {}^2E_{5/2,g}$	$E_{5/2}$	$E_{5/2}$	$E_{1/2,g}$	$E_{5/2}$	$2A_{5/2,g}$
$E_{7/2,g}$	${}^1E_{7/2,g} \oplus {}^2E_{7/2,g}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	$E_{7/2}$	$E_{3/2}$	$E_{3/2}$	$E_{3/2,g}$	$E_{3/2,g}$	$E_{7/2}$	$E_{7/2}$	$E_{1/2,g}$	$E_{7/2}$	${}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$
$E_{9/2,g}$	${}^1E_{9/2,g} \oplus {}^2E_{9/2,g}$	${}^1E_{9/2} \oplus {}^2E_{9/2}$	$E_{9/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2,g}$	$E_{9/2}$	$E_{9/2}$	$E_{1/2,g}$	$E_{9/2}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$
$E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	${}^1E_{9/2} \oplus {}^2E_{9/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2,u}$	$E_{1/2,u}$	$E_{9/2}$	$E_{9/2}$	$E_{1/2,u}$	$E_{1/2}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$
$E_{3/2,u}$	${}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{3/2,u}$	$E_{3/2,u}$	$E_{7/2}$	$E_{7/2}$	$E_{1/2,u}$	$E_{3/2}$	${}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$
$E_{5/2,u}$	${}^1E_{5/2,u} \oplus {}^2E_{5/2,u}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{5/2,u} \oplus {}^2E_{5/2,u}$	${}^1E_{5/2,u} \oplus {}^2E_{5/2,u}$	$E_{5/2}$	$E_{5/2}$	$E_{1/2,u}$	$E_{5/2}$	$2A_{5/2,u}$
$E_{7/2,u}$	${}^1E_{7/2,u} \oplus {}^2E_{7/2,u}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{7/2}$	$E_{3/2}$	$E_{3/2}$	$E_{3/2,u}$	$E_{3/2,u}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2,u}$	$E_{7/2}$	${}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$
$E_{9/2,u}$	${}^1E_{9/2,u} \oplus {}^2E_{9/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$E_{9/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2,u}$	$E_{1/2,u}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2,u}$	$E_{9/2}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$

Other subgroups: $3\mathbf{C}_{2h}$, $3\mathbf{C}_{2v}$, $3\mathbf{C}_s$, $\mathbf{C}_i$ (see $\mathbf{D}_{2h}$); $2\mathbf{D}_5$, $\mathbf{D}_2$, $\mathbf{C}_{10}$, $\mathbf{C}_5$, $3\mathbf{C}_2$ (see $\mathbf{D}_{10}$).

T **39**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{10h}$
$10n$	$(n+1)\,A_{1g} \oplus n\,(A_{2g} \oplus B_{1g} \oplus B_{2g} \oplus 2E_{1g} \oplus 2E_{2g} \oplus 2E_{3g} \oplus 2E_{4g})$
$10n+1$	$n\,(A_{1u} \oplus B_{1u} \oplus B_{2u} \oplus E_{1u} \oplus 2E_{2u} \oplus 2E_{3u} \oplus 2E_{4u}) \oplus (n+1)(A_{2u} \oplus E_{1u})$
$10n+2$	$(n+1)(A_{1g} \oplus E_{1g} \oplus E_{2g}) \oplus n\,(A_{2g} \oplus B_{1g} \oplus B_{2g} \oplus E_{1g} \oplus E_{2g} \oplus 2E_{3g} \oplus 2E_{4g})$
$10n+3$	$n\,(A_{1u} \oplus B_{1u} \oplus B_{2u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u} \oplus 2E_{4u}) \oplus (n+1)(A_{2u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u})$
$10n+4$	$(n+1)(A_{1g} \oplus E_{1g} \oplus E_{2g} \oplus E_{3g} \oplus E_{4g}) \oplus n\,(A_{2g} \oplus B_{1g} \oplus B_{2g} \oplus E_{1g} \oplus E_{2g} \oplus E_{3g} \oplus E_{4g})$
$10n+5$	$n\,(A_{1u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u} \oplus E_{4u}) \oplus (n{+}1)(A_{2u} \oplus B_{1u} \oplus B_{2u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u} \oplus E_{4u})$
$10n+6$	$(n+1)(A_{1g} \oplus B_{1g} \oplus B_{2g} \oplus E_{1g} \oplus E_{2g} \oplus E_{3g} \oplus 2E_{4g}) \oplus n\,(A_{2g} \oplus E_{1g} \oplus E_{2g} \oplus E_{3g})$
$10n+7$	$n\,(A_{1u} \oplus E_{1u} \oplus E_{2u}) \oplus (n+1)(A_{2u} \oplus B_{1u} \oplus B_{2u} \oplus E_{1u} \oplus E_{2u} \oplus 2E_{3u} \oplus 2E_{4u})$
$10n+8$	$(n+1)(A_{1g} \oplus B_{1g} \oplus B_{2g} \oplus E_{1g} \oplus 2E_{2g} \oplus 2E_{3g} \oplus 2E_{4g}) \oplus n\,(A_{2g} \oplus E_{1g})$
$10n+9$	$n\,A_{1u} \oplus (n+1)(A_{2u} \oplus B_{1u} \oplus B_{2u} \oplus 2E_{1u} \oplus 2E_{2u} \oplus 2E_{3u} \oplus 2E_{4u})$
$10n+\frac{1}{2}$	$(2n+1)\,E_{1/2,g} \oplus 2n\,(E_{3/2,g} \oplus E_{5/2,g} \oplus E_{7/2,g} \oplus E_{9/2,g})$
$10n+\frac{3}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g}) \oplus 2n\,(E_{5/2,g} \oplus E_{7/2,g} \oplus E_{9/2,g})$
$10n+\frac{5}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g}) \oplus 2n\,(E_{7/2,g} \oplus E_{9/2,g})$
$10n+\frac{7}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g} \oplus E_{7/2,g}) \oplus 2n\,E_{9/2,g}$
$10n+\frac{9}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g} \oplus E_{7/2,g} \oplus E_{9/2,g})$
$10n+\frac{11}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g} \oplus E_{7/2,g}) \oplus (2n+2)\,E_{9/2,g}$
$10n+\frac{13}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g}) \oplus (2n+2)(E_{7/2,g} \oplus E_{9/2,g})$
$10n+\frac{15}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g}) \oplus (2n+2)(E_{5/2,g} \oplus E_{7/2,g} \oplus E_{9/2,g})$
$10n+\frac{17}{2}$	$(2n+1)\,E_{1/2,g} \oplus (2n+2)(E_{3/2,g} \oplus E_{5/2,g} \oplus E_{7/2,g} \oplus E_{9/2,g})$
$10n+\frac{19}{2}$	$(2n+2)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g} \oplus E_{7/2,g} \oplus E_{9/2,g})$

$n = 0, 1, 2, \ldots$

T **39**.11 Clebsch–Gordan coefficients

Use T **30**.11 ▪. § **16**–11, p. 83

(1) Product forms: $\mathbf{C}_{\infty v} \otimes \mathbf{C}_i$, $\mathbf{C}_{\infty v} \otimes \mathbf{C}_s$.

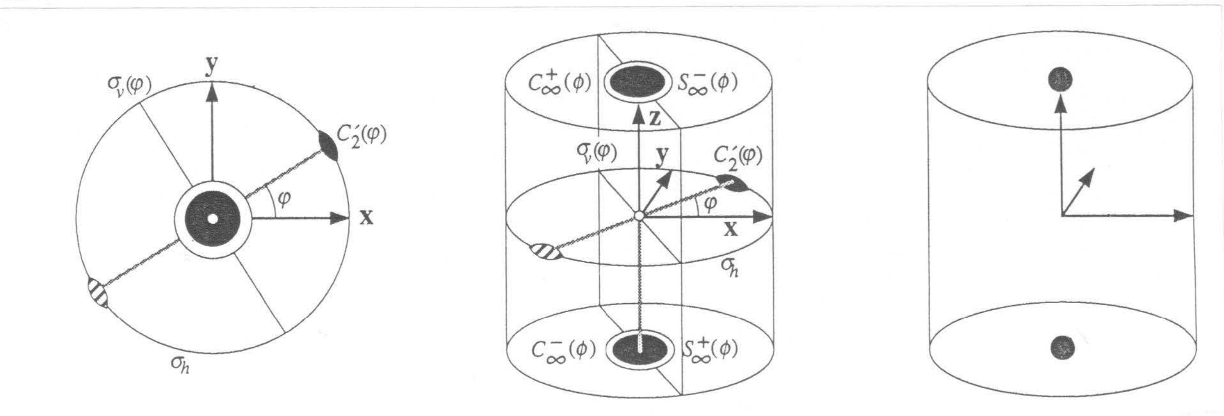
(2) Group chains: $\mathbf{D}_{\infty h} \supset \underline{\mathbf{C}_{nv}}$, $\mathbf{D}_{\infty h} \supset (\mathbf{D}_{nh})$; $(n = 2, 3, \ldots, 10)$.

(3) Operations of G: E, $(C_\infty^+(\phi), C_\infty^-(\phi))$, C_2, $(\sigma_v(\varphi))$, σ_h, $(S_\infty^+(\phi), S_\infty^-(\phi))$, i, $(C_2'(\varphi + \frac{\pi}{2}))$;
$0 < \phi < \pi$; $0 \le \varphi < \pi$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, $(C_\infty^+(\phi), C_\infty^-(\phi))$, $(\widetilde{C}_\infty^+(\phi), \widetilde{C}_\infty^-(\phi))$, $(C_2, \widetilde{C}_2)$, $(\sigma_v(\varphi), \widetilde{\sigma}_v(\varphi))$,
$(\sigma_h, \widetilde{\sigma}_h)$, $(S_\infty^+(\phi), S_\infty^-(\phi))$, $(\widetilde{S}_\infty^+(\phi), \widetilde{S}_\infty^-(\phi))$, i, $\widetilde{\imath}$, $(C_2'(\varphi + \frac{\pi}{2}), \widetilde{C}_2'(\varphi + \frac{\pi}{2}))$;
$0 < \phi < \pi$; $0 \le \varphi < \pi$.

(5) Classes and representations: $|r| = \infty$, $|i| = \infty$, $|I| = \infty$, $|\widetilde{I}| = \infty$.

F **40** See Chapter **15**, p. 65



Examples: H_2, O_2, CO_2, acetylene C_2H_2, $HgBr_2$.

T **40**.1 Parameters § **16**–1, p. 68

$\mathbf{D}_{\infty h}$	α	β	γ	ϕ	$\mathbf{n}$	λ	$\mathbf{\Lambda}$
E	0	0	0	0	$(0\ \ 0\ \ 0)$	$[\![1,$	$(0\ \ 0\ \ 0)]\!]$
$\sigma_h = E\,\sigma_h$	0	0	π	π	$(0\ \ 0\ \ 1)$	$[\![0,$	$(0\ \ 0\ \ 1)]\!]$
$C_\infty^+(\phi)$	0	0	ϕ	ϕ	$(0\ \ 0\ \ 1)$	$[\![c_2^\phi,$	$(0\ \ 0\ \ s_2^\phi)]\!]$
$S_\infty^+(\phi) = C_\infty^+(\phi)\,\sigma_h$	0	0	$\phi - \pi$	$\pi - \phi$	$(0\ \ 0\ \ -1)$	$[\![s_2^\phi,$	$(0\ \ 0\ \ -c_2^\phi)]\!]$
$C_\infty^-(\phi)$	0	0	$-\phi$	ϕ	$(0\ \ 0\ \ -1)$	$[\![c_2^\phi,$	$(0\ \ 0\ \ -s_2^\phi)]\!]$
$S_\infty^-(\phi) = C_\infty^-(\phi)\,\sigma_h$	0	0	$\pi - \phi$	$\pi - \phi$	$(0\ \ 0\ \ 1)$	$[\![s_2^\phi,$	$(0\ \ 0\ \ c_2^\phi)]\!]$
C_2	0	0	π	π	$(0\ \ 0\ \ 1)$	$[\![0,$	$(0\ \ 0\ \ 1)]\!]$
$i = C_2\,\sigma_h$	0	0	0	0	$(0\ \ 0\ \ 0)$	$[\![1,$	$(0\ \ 0\ \ 0)]\!]$
$\sigma_v(\varphi)$	0	π	$\pi - 2\varphi$	π	$(c^\varphi\ \ s^\varphi\ \ 0)$	$[\![0,$	$(c^\varphi\ \ s^\varphi\ \ 0)]\!]$
$C_2'(\varphi + \frac{\pi}{2}) = \sigma_v(\varphi)\,\sigma_h$	0	π	-2φ	π	$(-s^\varphi\ \ c^\varphi\ \ 0)$	$[\![0,$	$(-s^\varphi\ \ c^\varphi\ \ 0)]\!]$

$c_2^\phi = \cos\frac{\phi}{2}$, $s_2^\phi = \sin\frac{\phi}{2}$, $0 < \phi < \pi$; $c^\varphi = \cos\varphi$, $s^\varphi = \sin\varphi$, $0 \le \varphi < \pi$.

T 40.2 Multiplication table

All operations of $\mathbf{D}_{\infty h} = \mathbf{C}_{\infty v} \otimes \mathbf{C}_s$ are of the form h or $h\,\sigma_h$, $\forall h \in \mathbf{C}_{\infty v}$. Their products are given in the following table.

	h'	$h'\,\sigma_h$
h	$h\,h'$	$h\,h'\,\sigma_h$
$h\,\sigma_h$	$h\,h'\,\sigma_h$	$h\,h'$

The products $h\,h'$ (say h'') in the body of the table must be obtained from T **59**.2. The operations $h''\,\sigma_h$ are defined from the second column of T **40**.1.

T 40.3 Factor table

All operations of $\mathbf{D}_{\infty h} = \mathbf{C}_{\infty v} \otimes \mathbf{C}_s$ are of the form h or $h\,\sigma_h$, $\forall h \in \mathbf{C}_{\infty v}$. Their factors are given in the following table.

	h'	$h'\,\sigma_h$
h	$[h, h']$	$[h, h'\,C_2]$
$h\,\sigma_h$	$[h\,C_2, h']$	$[h\,C_2, h'\,C_2]$

The products $h\,C_2$, $h'\,C_2$ must be obtained from T **59**.2, and the resulting factors from T **59**.3.

T 40.4 Character table

§ **16**–4, p. 71

$\mathbf{D}_{\infty h}$	E	$2C_\infty(\phi)$	C_2	$\infty\sigma_v(\varphi)$	σ_h	$2S_\infty(\phi)$	i	$\infty C_2'(\varphi+\frac{\pi}{2})$	τ
A_{1g} (Σ_g^+)	1	1	1	1	1	1	1	1	a
A_{2g} (Σ_g^-)	1	1	1	-1	1	1	1	-1	a
E_{1g} (Π_g)	2	$2\cos\phi$	-2	0	-2	$-2\cos\phi$	2	0	a
E_{2g} (Δ_g)	2	$2\cos 2\phi$	2	0	2	$2\cos 2\phi$	2	0	a
E_{3g} (Φ_g)	2	$2\cos 3\phi$	-2	0	-2	$-2\cos 3\phi$	2	0	a
$E_{n,g}$	2	$2\cos n\phi$	$2(-1)^n$	0	$2(-1)^n$	$2(-1)^n\cos n\phi$	2	0	a
A_{1u} (Σ_u^+)	1	1	1	1	-1	-1	-1	-1	a
A_{2u} (Σ_u^-)	1	1	1	-1	-1	-1	-1	1	a
E_{1u} (Π_u)	2	$2\cos\phi$	-2	0	2	$2\cos\phi$	-2	0	a
E_{2u} (Δ_u)	2	$2\cos 2\phi$	2	0	-2	$-2\cos 2\phi$	-2	0	a
E_{3u} (Φ_u)	2	$2\cos 3\phi$	-2	0	2	$2\cos 3\phi$	-2	0	a
$E_{n,u}$	2	$2\cos n\phi$	$2(-1)^n$	0	$-2(-1)^n$	$-2(-1)^n\cos n\phi$	-2	0	a
$E_{1/2,g}$	2	$2\cos\frac{1}{2}\phi$	0	0	0	$2\sin\frac{1}{2}\phi$	2	0	c
$E_{3/2,g}$	2	$2\cos\frac{3}{2}\phi$	0	0	0	$2\sin\frac{3}{2}\phi$	2	0	c
$E_{5/2,g}$	2	$2\cos\frac{5}{2}\phi$	0	0	0	$2\sin\frac{5}{2}\phi$	2	0	c
$E_{7/2,g}$	2	$2\cos\frac{7}{2}\phi$	0	0	0	$2\sin\frac{7}{2}\phi$	2	0	c
$E_{n+1/2,g}$	2	$2\cos(n+\frac{1}{2})\phi$	0	0	0	$2\sin(n+\frac{1}{2})\phi$	2	0	c
$E_{1/2,u}$	2	$2\cos\frac{1}{2}\phi$	0	0	0	$-2\sin\frac{1}{2}\phi$	-2	0	c
$E_{3/2,u}$	2	$2\cos\frac{3}{2}\phi$	0	0	0	$-2\sin\frac{3}{2}\phi$	-2	0	c
$E_{5/2,u}$	2	$2\cos\frac{5}{2}\phi$	0	0	0	$-2\sin\frac{5}{2}\phi$	-2	0	c
$E_{7/2,u}$	2	$2\cos\frac{7}{2}\phi$	0	0	0	$-2\sin\frac{7}{2}\phi$	-2	0	c
$E_{n+1/2,u}$	2	$2\cos(n+\frac{1}{2})\phi$	0	0	0	$-2\sin(n+\frac{1}{2})\phi$	-2	0	c

$0 < \phi < \pi, \quad 0 \le \varphi < \pi, \quad n = 4, 5, 6, \dots$

T **40**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{\infty h}$	0	1	2	3
A_{1g} (Σ_g^+)	$^\square 1$		$x^2+y^2, {}^\square z^2$	
A_{2g} (Σ_g^-)		R_z		
E_{1g} (Π_g)		(R_x, R_y)	$^\square(zx, yz)$	
E_{2g} (Δ_g)			$^\square(xy, x^2-y^2)$	
E_{3g} (Φ_g)				
A_{1u} (Σ_u^+)		$^\square z$		$(x^2+y^2)z, {}^\square z^3$
A_{2u} (Σ_u^-)				
E_{1u} (Π_u)		$^\square(x, y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
E_{2u} (Δ_u)				$^\square\{xyz, z(x^2-y^2)\}$
E_{3u} (Φ_u)				$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$

T **40**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{\infty h}$	$\langle \|j\,m\rangle \|$		ι
A_{1g} (Σ_g^+)	$\|0\,0\rangle$		2
A_{2g} (Σ_g^-)			
E_{1g} (Π_g)	$\langle \|2\,1\rangle, -\|2\,\overline{1}\rangle \|$		2
E_{2g} (Δ_g)	$\langle \|2\,2\rangle, \|2\,\overline{2}\rangle \|$		2
E_{3g} (Φ_g)	$\langle \|4\,3\rangle, -\|4\,\overline{3}\rangle \|$		2
$E_{2n,g}$	$\langle \|2n, 2n\rangle, \|2n, -2n\rangle \|$		2
$E_{2n+1,g}$	$\langle \|2n+2, 2n+1\rangle, -\|2n+2, -2n-1\rangle \|$		2
A_{1u} (Σ_u^+)	$\|1\,0\rangle$		2
A_{2u} (Σ_u^-)			
E_{1u} (Π_u)	$\langle \|1\,1\rangle, \|1\,\overline{1}\rangle \|$		2
E_{2u} (Δ_u)	$\langle \|3\,2\rangle, -\|3\,\overline{2}\rangle \|$		2
E_{3u} (Φ_u)	$\langle \|3\,3\rangle, \|3\,\overline{3}\rangle \|$		2
$E_{2n,u}$	$\langle \|2n+1, 2n\rangle, -\|2n+1, -2n\rangle \|$		2
$E_{2n+1,u}$	$\langle \|2n+1, 2n+1\rangle, \|2n+1, -2n-1\rangle \|$		2
$E_{1/2,g}$	$\langle \|\frac{1}{2}\,\frac{1}{2}\rangle, \|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle \|$	$\langle \|\frac{3}{2}\,\frac{1}{2}\rangle, -\|\frac{3}{2}\,\overline{\frac{1}{2}}\rangle \|$	2
$E_{3/2,g}$	$\langle \|\frac{3}{2}\,\frac{3}{2}\rangle, \|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle \|$	$\langle \|\frac{5}{2}\,\frac{3}{2}\rangle, -\|\frac{5}{2}\,\overline{\frac{3}{2}}\rangle \|$	2
$E_{5/2,g}$	$\langle \|\frac{5}{2}\,\frac{5}{2}\rangle, \|\frac{5}{2}\,\overline{\frac{5}{2}}\rangle \|$	$\langle \|\frac{7}{2}\,\frac{5}{2}\rangle, -\|\frac{7}{2}\,\overline{\frac{5}{2}}\rangle \|$	2
$E_{7/2,g}$	$\langle \|\frac{7}{2}\,\frac{7}{2}\rangle, \|\frac{7}{2}\,\overline{\frac{7}{2}}\rangle \|$	$\langle \|\frac{9}{2}\,\frac{7}{2}\rangle, -\|\frac{9}{2}\,\overline{\frac{7}{2}}\rangle \|$	2
$E_{n+1/2,g}$	$\langle \|n+\frac{1}{2}, n+\frac{1}{2}\rangle, \|n+\frac{1}{2}, -n-\frac{1}{2}\rangle \|$	$\langle \|n+\frac{3}{2}, n+\frac{1}{2}\rangle, -\|n+\frac{3}{2}, -n-\frac{1}{2}\rangle \|$	2
$E_{1/2,u}$	$\langle \|\frac{1}{2}\,\frac{1}{2}\rangle, \|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle \|^\bullet$	$\langle \|\frac{3}{2}\,\frac{1}{2}\rangle, -\|\frac{3}{2}\,\overline{\frac{1}{2}}\rangle \|^\bullet$	2
$E_{3/2,u}$	$\langle \|\frac{3}{2}\,\frac{3}{2}\rangle, \|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle \|^\bullet$	$\langle \|\frac{5}{2}\,\frac{3}{2}\rangle, -\|\frac{5}{2}\,\overline{\frac{3}{2}}\rangle \|^\bullet$	2
$E_{5/2,u}$	$\langle \|\frac{5}{2}\,\frac{5}{2}\rangle, \|\frac{5}{2}\,\overline{\frac{5}{2}}\rangle \|^\bullet$	$\langle \|\frac{7}{2}\,\frac{5}{2}\rangle, -\|\frac{7}{2}\,\overline{\frac{5}{2}}\rangle \|^\bullet$	2
$E_{7/2,u}$	$\langle \|\frac{7}{2}\,\frac{7}{2}\rangle, \|\frac{7}{2}\,\overline{\frac{7}{2}}\rangle \|^\bullet$	$\langle \|\frac{9}{2}\,\frac{7}{2}\rangle, -\|\frac{9}{2}\,\overline{\frac{7}{2}}\rangle \|^\bullet$	2
$E_{n+1/2,u}$	$\langle \|n+\frac{1}{2}, n+\frac{1}{2}\rangle, \|n+\frac{1}{2}, -n-\frac{1}{2}\rangle \|^\bullet$	$\langle \|n+\frac{3}{2}, n+\frac{1}{2}\rangle, -\|n+\frac{3}{2}, -n-\frac{1}{2}\rangle \|^\bullet$	2

The μ column mentioned on p. 74 is not relevant here.
$n = 4, 5, 6, \ldots$

T **40**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_{\infty h}$	$E_{n,g}$ $(n = 1, 3, 5, \ldots)$	$E_{p,g}$ $(p = 2, 4, 6, \ldots)$	$E_{n,u}$ $(n = 1, 3, 5, \ldots)$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$C_\infty^+(\phi)$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}n\phi} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}n\phi} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}p\phi} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}p\phi} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}n\phi} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}n\phi} \end{bmatrix}$
$C_\infty^-(\phi)$	$\begin{bmatrix} \mathrm{e}^{\mathrm{i}n\phi} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}n\phi} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{\mathrm{i}p\phi} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}p\phi} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{\mathrm{i}n\phi} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}n\phi} \end{bmatrix}$
C_2	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$
$\sigma_v(\varphi)$	$\begin{bmatrix} 0 & -\mathrm{e}^{-2\mathrm{i}n\varphi} \\ -\mathrm{e}^{2\mathrm{i}n\varphi} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{e}^{-2\mathrm{i}p\varphi} \\ \mathrm{e}^{2\mathrm{i}p\varphi} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{e}^{-2\mathrm{i}n\varphi} \\ \mathrm{e}^{2\mathrm{i}n\varphi} & 0 \end{bmatrix}$
σ_h	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$S_\infty^+(\phi)$	$\begin{bmatrix} -\mathrm{e}^{-\mathrm{i}n\phi} & 0 \\ 0 & -\mathrm{e}^{\mathrm{i}n\phi} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}p\phi} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}p\phi} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}n\phi} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}n\phi} \end{bmatrix}$
$S_\infty^-(\phi)$	$\begin{bmatrix} -\mathrm{e}^{\mathrm{i}n\phi} & 0 \\ 0 & -\mathrm{e}^{-\mathrm{i}n\phi} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{\mathrm{i}p\phi} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}p\phi} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{\mathrm{i}n\phi} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}n\phi} \end{bmatrix}$
i	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$
$C_2'(\varphi + \frac{\pi}{2})$	$\begin{bmatrix} 0 & \mathrm{e}^{-2\mathrm{i}n\varphi} \\ \mathrm{e}^{2\mathrm{i}n\varphi} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{e}^{-2\mathrm{i}p\varphi} \\ \mathrm{e}^{2\mathrm{i}p\varphi} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{e}^{-2\mathrm{i}n\varphi} \\ \mathrm{e}^{2\mathrm{i}n\varphi} & 0 \end{bmatrix}$

⇒

T **40**.7 Matrix representations *(cont.)*

$\mathbf{D}_{\infty h}$	$E_{p,u}$ $(p = 2, 4, 6, \ldots)$	$E_{n/2,g}$ $(n = 1, 3, 5, \ldots)$	$E_{n/2,u}$ $(n = 1, 3, 5, \ldots)$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$C_\infty^+(\phi)$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}p\phi} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}p\phi} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}n\phi/2} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}n\phi/2} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}n\phi/2} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}n\phi/2} \end{bmatrix}$
$C_\infty^-(\phi)$	$\begin{bmatrix} \mathrm{e}^{\mathrm{i}p\phi} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}p\phi} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{\mathrm{i}n\phi/2} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}n\phi/2} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{\mathrm{i}n\phi/2} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}n\phi/2} \end{bmatrix}$
C_2	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}n\pi/2} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}n\pi/2} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}n\pi/2} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}n\pi/2} \end{bmatrix}$
$\sigma_v(\varphi)$	$\begin{bmatrix} 0 & -\mathrm{e}^{-2\mathrm{i}p\varphi} \\ -\mathrm{e}^{2\mathrm{i}p\varphi} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{e}^{-\mathrm{i}n(\frac{\pi}{2}+\varphi)} \\ \mathrm{e}^{-\mathrm{i}n(\frac{\pi}{2}-\varphi)} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -\mathrm{e}^{-\mathrm{i}n(\frac{\pi}{2}+\varphi)} \\ -\mathrm{e}^{-\mathrm{i}n(\frac{\pi}{2}-\varphi)} & 0 \end{bmatrix}$
σ_h	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}n\pi/2} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}n\pi/2} \end{bmatrix}$	$\begin{bmatrix} -\mathrm{e}^{-\mathrm{i}n\pi/2} & 0 \\ 0 & -\mathrm{e}^{\mathrm{i}n\pi/2} \end{bmatrix}$
$S_\infty^+(\phi)$	$\begin{bmatrix} -\mathrm{e}^{-\mathrm{i}p\phi} & 0 \\ 0 & -\mathrm{e}^{\mathrm{i}p\phi} \end{bmatrix}$	$\begin{bmatrix} -\mathrm{e}^{-\mathrm{i}n(\frac{\phi+\pi}{2})} & 0 \\ 0 & -\mathrm{e}^{\mathrm{i}n(\frac{\phi+\pi}{2})} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}n(\frac{\phi+\pi}{2})} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}n(\frac{\phi+\pi}{2})} \end{bmatrix}$
$S_\infty^-(\phi)$	$\begin{bmatrix} -\mathrm{e}^{\mathrm{i}p\phi} & 0 \\ 0 & -\mathrm{e}^{-\mathrm{i}p\phi} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{\mathrm{i}n(\frac{\phi-\pi}{2})} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}n(\frac{\phi-\pi}{2})} \end{bmatrix}$	$\begin{bmatrix} -\mathrm{e}^{\mathrm{i}n(\frac{\phi-\pi}{2})} & 0 \\ 0 & -\mathrm{e}^{-\mathrm{i}n(\frac{\phi-\pi}{2})} \end{bmatrix}$
i	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$
$C_2'(\varphi + \frac{\pi}{2})$	$\begin{bmatrix} 0 & \mathrm{e}^{-2\mathrm{i}p\varphi} \\ \mathrm{e}^{2\mathrm{i}p\varphi} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -\mathrm{e}^{-\mathrm{i}n\varphi} \\ \mathrm{e}^{\mathrm{i}n\varphi} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -\mathrm{e}^{-\mathrm{i}n\varphi} \\ \mathrm{e}^{\mathrm{i}n\varphi} & 0 \end{bmatrix}$

T **40**.8 Direct products of representations § **16**–8, p. 81

$\mathbf{D}_{\infty h}$	A_{1g}	A_{2g}	E_{1g}	$E_{n,g}$	$E_{p,g}$
A_{1g}	A_{1g}	A_{2g}	E_{1g}	$E_{n,g}$	$E_{p,g}$
A_{2g}		A_{1g}	E_{1g}	$E_{n,g}$	$E_{p,g}$
E_{1g}			$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$	$E_{n-1,g} \oplus E_{n+1,g}$	$E_{p-1,g} \oplus E_{p+1,g}$
$E_{n,g}$				$A_{1g} \oplus \{A_{2g}\} \oplus E_{2n,g}$	$E_{p-n,g} \oplus E_{p+n,g}$

$n = 2, 3, 4, \ldots,$ $p = 3, 4, 5, \ldots,$ $p > n.$ $\twoheadrightarrow$

T **40**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{\infty h}$	A_{1u}	A_{2u}	E_{1u}	$E_{n,u}$	$E_{p,u}$
A_{1g}	A_{1u}	A_{2u}	E_{1u}	$E_{n,u}$	$E_{p,u}$
A_{2g}	A_{2u}	A_{1u}	E_{1u}	$E_{n,u}$	$E_{p,u}$
E_{1g}	E_{1u}	E_{1u}	$A_{1u} \oplus A_{2u} \oplus E_{2u}$	$E_{n-1,u} \oplus E_{n+1,u}$	$E_{p-1,u} \oplus E_{p+1,u}$
$E_{n,g}$	$E_{n,u}$	$E_{n,u}$	$E_{n-1,u} \oplus E_{n+1,u}$	$A_{1u} \oplus A_{2u} \oplus E_{2n,u}$	$E_{p-n,u} \oplus E_{p+n,u}$
A_{1u}	A_{1g}	A_{2g}	E_{1g}	$E_{n,g}$	$E_{p,g}$
A_{2u}		A_{1g}	E_{1g}	$E_{n,g}$	$E_{p,g}$
E_{1u}			$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$	$E_{n-1,g} \oplus E_{n+1,g}$	$E_{p-1,g} \oplus E_{p+1,g}$
$E_{n,u}$				$A_{1g} \oplus \{A_{2g}\} \oplus E_{2n,g}$	$E_{p-n,g} \oplus E_{p+n,g}$

$n = 2, 3, 4, \ldots,$ $p = 3, 4, 5, \ldots,$ $p > n.$ $\twoheadrightarrow$

T **40**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{\infty h}$	$E_{1/2,g}$	$E_{3/2,g}$	$E_{n+1/2,g}$	$E_{p+1/2,g}$
A_{1g}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{n+1/2,g}$	$E_{p+1/2,g}$
A_{2g}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{n+1/2,g}$	$E_{p+1/2,g}$
E_{1g}	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{n-1/2,g} \oplus E_{n+3/2,g}$	$E_{p-1/2,g} \oplus E_{p+3/2,g}$
$E_{n,g}$	$E_{n-1/2,g} \oplus E_{n+1/2,g}$	$E_{n-3/2,g} \oplus E_{n+3/2,g}$	$E_{1/2,g} \oplus E_{2n+1/2,g}$	$E_{p-n+1/2,g} \oplus E_{p+n+1/2,g}$
A_{1u}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{n+1/2,u}$	$E_{p+1/2,u}$
A_{2u}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{n+1/2,u}$	$E_{p+1/2,u}$
E_{1u}	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{n-1/2,u} \oplus E_{n+3/2,u}$	$E_{p-1/2,u} \oplus E_{p+3/2,u}$
$E_{n,u}$	$E_{n-1/2,u} \oplus E_{n+1/2,u}$	$E_{n-3/2,u} \oplus E_{n+3/2,u}$	$E_{1/2,u} \oplus E_{2n+1/2,u}$	$E_{p-n+1/2,u} \oplus E_{p+n+1/2,u}$
$E_{1/2,g}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$	$E_{1g} \oplus E_{2g}$	$E_{n,g} \oplus E_{n+1,g}$	$E_{p,g} \oplus E_{p+1,g}$
$E_{3/2,g}$		$\{A_{1g}\} \oplus A_{2g} \oplus E_{3g}$	$E_{n-1,g} \oplus E_{n+2,g}$	$E_{p-1,g} \oplus E_{p+2,g}$
$E_{n+1/2,g}$			$\{A_{1g}\} \oplus A_{2g} \oplus E_{2n+1,g}$	$E_{p-n,g} \oplus E_{p+n+1,g}$

$n = 2, 3, 4, \ldots,$ $p = 3, 4, 5, \ldots,$ $p > n.$ $\twoheadrightarrow$

T **40**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{\infty h}$	$E_{1/2,u}$	$E_{3/2,u}$	$E_{n+1/2,u}$	$E_{p+1/2,u}$
A_{1g}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{n+1/2,u}$	$E_{p+1/2,u}$
A_{2g}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{n+1/2,u}$	$E_{p+1/2,u}$
E_{1g}	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{n-1/2,u} \oplus E_{n+3/2,u}$	$E_{p-1/2,u} \oplus E_{p+3/2,u}$
$E_{n,g}$	$E_{n-1/2,u} \oplus E_{n+1/2,u}$	$E_{n-3/2,u} \oplus E_{n+3/2,u}$	$E_{1/2,u} \oplus E_{2n+1/2,u}$	$E_{p-n+1/2,u} \oplus E_{p+n+1/2,u}$
A_{1u}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{n+1/2,g}$	$E_{p+1/2,g}$
A_{2u}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{n+1/2,g}$	$E_{p+1/2,g}$
E_{1u}	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{n-1/2,g} \oplus E_{n+3/2,g}$	$E_{p-1/2,g} \oplus E_{p+3/2,g}$
$E_{n,u}$	$E_{n-1/2,g} \oplus E_{n+1/2,g}$	$E_{n-3/2,g} \oplus E_{n+3/2,g}$	$E_{1/2,g} \oplus E_{2n+1/2,g}$	$E_{p-n+1/2,g} \oplus E_{p+n+1/2,g}$
$E_{1/2,g}$	$A_{1u} \oplus A_{2u} \oplus E_{1u}$	$E_{1u} \oplus E_{2u}$	$E_{n,u} \oplus E_{n+1,u}$	$E_{p,u} \oplus E_{p+1,u}$
$E_{3/2,g}$	$E_{1u} \oplus E_{2u}$	$A_{1u} \oplus A_{2u} \oplus E_{3u}$	$E_{n-1,u} \oplus E_{n+2,u}$	$E_{p-1,u} \oplus E_{p+2,u}$
$E_{n+1/2,g}$	$E_{n,u} \oplus E_{n+1,u}$	$E_{p,u} \oplus E_{p+1,u}$	$A_{1u} \oplus A_{2u} \oplus E_{2n+1,u}$	$E_{p-n,u} \oplus E_{p+n+1,u}$
$E_{1/2,u}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$	$E_{1g} \oplus E_{2g}$	$E_{n,g} \oplus E_{n+1,g}$	$E_{p,g} \oplus E_{p+1,g}$
$E_{3/2,u}$		$\{A_{1g}\} \oplus A_{2g} \oplus E_{3g}$	$E_{n-1,g} \oplus E_{n+2,g}$	$E_{p-1,g} \oplus E_{p+2,g}$
$E_{n+1/2,u}$			$\{A_{1g}\} \oplus A_{2g} \oplus E_{2n+1,g}$	$E_{p-n,g} \oplus E_{p+n+1,g}$

$n = 2, 3, 4, \ldots,\quad p = 3, 4, 5, \ldots,\quad p > n.$

T **40**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_{\infty h}$	$\mathbf{C}_{\infty v}$	$(\mathbf{D}_{10h})$	$(\mathbf{D}_{9h})$	$(\mathbf{D}_{8h})$	$(\mathbf{D}_{7h})$	$(\mathbf{D}_{6h})$	$(\mathbf{D}_{5h})$	$(\mathbf{D}_{4h})$	$(\mathbf{D}_{3h})$	$(\mathbf{D}_{2h})$
A_{1g}	A_1	A_{1g}	A'_1	A_{1g}	A'_1	A_{1g}	A'_1	A_{1g}	A'_1	A_g
A_{2g}	A_2	A_{2g}	A'_2	A_{2g}	A'_2	A_{2g}	A'_2	A_{2g}	A'_2	B_{1g}
E_{1g}	E_1	E_{1g}	E''_1	E_{1g}	E''_1	E_{1g}	E''_1	E_g	E''	$B_{2g} \oplus B_{3g}$
E_{2g}	E_2	E_{2g}	E'_2	E_{2g}	E'_2	E_{2g}	E'_2	$B_{1g} \oplus B_{2g}$	E'	$A_g \oplus B_{1g}$
E_{3g}	E_3	E_{3g}	E''_3	E_{3g}	E''_3	$B_{1g} \oplus B_{2g}$	E''_2	E_g	$A''_1 \oplus A''_2$	$B_{2g} \oplus B_{3g}$
E_{4g}	E_4	E_{4g}	E'_4	$B_{1g} \oplus B_{2g}$	E'_3	E_{2g}	E'_1	$A_{1g} \oplus A_{2g}$	E'	$A_g \oplus B_{1g}$
E_{5g}	E_5	$B_{1g} \oplus B_{2g}$	E''_4	E_{3g}	E''_2	E_{1g}	$A''_1 \oplus A''_2$	E_g	E''	$B_{2g} \oplus B_{3g}$
E_{6g}	E_6	E_{4g}	E'_3	E_{2g}	E'_1	$A_{1g} \oplus A_{2g}$	E'_1	$B_{1g} \oplus B_{2g}$	$A'_1 \oplus A'_2$	$A_g \oplus B_{1g}$
E_{7g}	E_7	E_{3g}	E''_2	E_{1g}	$A''_1 \oplus A''_2$	E_{1g}	E''_2	E_g	E''	$B_{2g} \oplus B_{3g}$
E_{8g}	E_8	E_{2g}	E'_1	$A_{1g} \oplus A_{2g}$	E'_1	E_{2g}	E'_2	$A_{1g} \oplus A_{2g}$	E'	$A_g \oplus B_{1g}$
E_{9g}	E_9	E_{1g}	$A''_1 \oplus A''_2$	E_{1g}	E''_2	$B_{1g} \oplus B_{2g}$	E''_1	E_g	$A''_1 \oplus A''_2$	$B_{2g} \oplus B_{3g}$
E_{10g}	E_{10}	$A_{1g} \oplus A_{2g}$	E'_1	E_{2g}	E'_3	E_{2g}	$A'_1 \oplus A'_2$	$B_{1g} \oplus B_{2g}$	E'	$A_g \oplus B_{1g}$
$\vdots$										
A_{1u}	A_1	A_{2u}	A''_2	A_{2u}	A''_2	A_{2u}	A''_2	A_{2u}	A''_2	B_{1u}
A_{2u}	A_2	A_{1u}	A''_1	A_{1u}	A''_1	A_{1u}	A''_1	A_{1u}	A''_1	A_u
E_{1u}	E_1	E_{1u}	E'_1	E_{1u}	E'_1	E_{1u}	E'_1	E_u	E'	$B_{2u} \oplus B_{3u}$
E_{2u}	E_2	E_{2u}	E''_2	E_{2u}	E''_2	E_{2u}	E''_2	$B_{1u} \oplus B_{2u}$	E''	$A_u \oplus B_{1u}$
E_{3u}	E_3	E_{3u}	E'_3	E_{3u}	E'_3	$B_{1u} \oplus B_{2u}$	E'_2	E_u	$A'_1 \oplus A'_2$	$B_{2u} \oplus B_{3u}$
E_{4u}	E_4	E_{4u}	E''_4	$B_{1u} \oplus B_{2u}$	E''_3	E_{2u}	E''_1	$A_{1u} \oplus A_{2u}$	E''	$A_u \oplus B_{1u}$
E_{5u}	E_5	$B_{1u} \oplus B_{2u}$	E'_4	E_{3u}	E'_2	E_{1u}	$A'_1 \oplus A'_2$	E_u	E'	$B_{2u} \oplus B_{3u}$
E_{6u}	E_6	E_{4u}	E''_3	E_{2u}	E''_1	$A_{1u} \oplus A_{2u}$	E''_1	$B_{1u} \oplus B_{2u}$	$A''_1 \oplus A''_2$	$A_u \oplus B_{1u}$
E_{7u}	E_7	E_{3u}	E'_2	E_{1u}	$A'_1 \oplus A'_2$	E_{1u}	E'_2	E_u	E'	$B_{2u} \oplus B_{3u}$
E_{8u}	E_8	E_{2u}	E''_1	$A_{1u} \oplus A_{2u}$	E''_1	E_{2u}	E''_2	$A_{1u} \oplus A_{2u}$	E''	$A_u \oplus B_{1u}$
E_{9u}	E_9	E_{1u}	$A'_1 \oplus A'_2$	E_{1u}	E'_2	$B_{1u} \oplus B_{2u}$	E''_1	E_u	$A'_1 \oplus A'_2$	$B_{2u} \oplus B_{3u}$
E_{10u}	E_{10}	$A_{1u} \oplus A_{2u}$	E''_1	E_{2u}	E''_3	E_{2u}	$A''_1 \oplus A''_2$	$B_{1u} \oplus B_{2u}$	E''	$A_u \oplus B_{1u}$
$\vdots$										

$\twoheadrightarrow$

T **40**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{\infty h}$	$\mathbf{C}_{\infty v}$	$(\mathbf{D}_{10h})$	$(\mathbf{D}_{9h})$	$(\mathbf{D}_{8h})$	$(\mathbf{D}_{7h})$	$(\mathbf{D}_{6h})$	$(\mathbf{D}_{5h})$	$(\mathbf{D}_{4h})$	$(\mathbf{D}_{3h})$	$(\mathbf{D}_{2h})$
$E_{1/2,g}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2}$	$E_{1/2,g}$
$E_{3/2,g}$	$E_{3/2}$	$E_{3/2,g}$	$E_{3/2}$	$E_{3/2,g}$	$E_{3/2}$	$E_{3/2,g}$	$E_{3/2}$	$E_{3/2,g}$	$E_{3/2}$	$E_{1/2,g}$
$E_{5/2,g}$	$E_{5/2}$	$E_{5/2,g}$	$E_{5/2}$	$E_{5/2,g}$	$E_{5/2}$	$E_{5/2,g}$	$E_{5/2}$	$E_{3/2,g}$	$E_{5/2}$	$E_{1/2,g}$
$E_{7/2,g}$	$E_{7/2}$	$E_{7/2,g}$	$E_{7/2}$	$E_{7/2,g}$	$E_{7/2}$	$E_{5/2,g}$	$E_{7/2}$	$E_{1/2,g}$	$E_{5/2}$	$E_{1/2,g}$
$E_{9/2,g}$	$E_{9/2}$	$E_{9/2,g}$	$E_{9/2}$	$E_{7/2,g}$	$E_{9/2}$	$E_{3/2,g}$	$E_{9/2}$	$E_{1/2,g}$	$E_{3/2}$	$E_{1/2,g}$
$E_{11/2,g}$	$E_{11/2}$	$E_{9/2,g}$	$E_{11/2}$	$E_{5/2,g}$	$E_{11/2}$	$E_{1/2,g}$	$E_{9/2}$	$E_{3/2,g}$	$E_{1/2}$	$E_{1/2,g}$
$E_{13/2,g}$	$E_{13/2}$	$E_{7/2,g}$	$E_{13/2}$	$E_{3/2,g}$	$E_{13/2}$	$E_{1/2,g}$	$E_{7/2}$	$E_{3/2,g}$	$E_{1/2}$	$E_{1/2,g}$
$E_{15/2,g}$	$E_{15/2}$	$E_{5/2,g}$	$E_{15/2}$	$E_{1/2,g}$	$E_{13/2}$	$E_{3/2,g}$	$E_{5/2}$	$E_{1/2,g}$	$E_{3/2}$	$E_{1/2,g}$
$E_{17/2,g}$	$E_{17/2}$	$E_{3/2,g}$	$E_{17/2}$	$E_{1/2,g}$	$E_{11/2}$	$E_{5/2,g}$	$E_{3/2}$	$E_{1/2,g}$	$E_{5/2}$	$E_{1/2,g}$
$E_{19/2,g}$	$E_{19/2}$	$E_{1/2,g}$	$E_{17/2}$	$E_{3/2,g}$	$E_{9/2}$	$E_{5/2,g}$	$E_{1/2}$	$E_{3/2,g}$	$E_{5/2}$	$E_{1/2,g}$
$\vdots$										
$E_{1/2,u}$	$E_{1/2}$	$E_{1/2,u}$	$E_{17/2}$	$E_{1/2,u}$	$E_{13/2}$	$E_{1/2,u}$	$E_{9/2}$	$E_{1/2,u}$	$E_{5/2}$	$E_{1/2,u}$
$E_{3/2,u}$	$E_{3/2}$	$E_{3/2,u}$	$E_{15/2}$	$E_{3/2,u}$	$E_{11/2}$	$E_{3/2,u}$	$E_{7/2}$	$E_{3/2,u}$	$E_{3/2}$	$E_{1/2,u}$
$E_{5/2,u}$	$E_{5/2}$	$E_{5/2,u}$	$E_{13/2}$	$E_{5/2,u}$	$E_{9/2}$	$E_{5/2,u}$	$E_{5/2}$	$E_{3/2,u}$	$E_{1/2}$	$E_{1/2,u}$
$E_{7/2,u}$	$E_{7/2}$	$E_{7/2,u}$	$E_{11/2}$	$E_{7/2,u}$	$E_{7/2}$	$E_{5/2,u}$	$E_{3/2}$	$E_{1/2,u}$	$E_{1/2}$	$E_{1/2,u}$
$E_{9/2,u}$	$E_{9/2}$	$E_{9/2,u}$	$E_{9/2}$	$E_{7/2,u}$	$E_{5/2}$	$E_{3/2,u}$	$E_{1/2}$	$E_{1/2,u}$	$E_{3/2}$	$E_{1/2,u}$
$E_{11/2,u}$	$E_{11/2}$	$E_{9/2,u}$	$E_{7/2}$	$E_{5/2,u}$	$E_{3/2}$	$E_{1/2,u}$	$E_{1/2}$	$E_{3/2,u}$	$E_{5/2}$	$E_{1/2,u}$
$E_{13/2,u}$	$E_{13/2}$	$E_{7/2,u}$	$E_{5/2}$	$E_{3/2,u}$	$E_{1/2}$	$E_{1/2,u}$	$E_{3/2}$	$E_{3/2,u}$	$E_{5/2}$	$E_{1/2,u}$
$E_{15/2,u}$	$E_{15/2}$	$E_{5/2,u}$	$E_{3/2}$	$E_{1/2,u}$	$E_{1/2}$	$E_{3/2,u}$	$E_{5/2}$	$E_{1/2,u}$	$E_{3/2}$	$E_{1/2,u}$
$E_{17/2,u}$	$E_{17/2}$	$E_{3/2,u}$	$E_{1/2}$	$E_{1/2,u}$	$E_{3/2}$	$E_{5/2,u}$	$E_{7/2}$	$E_{1/2,u}$	$E_{1/2}$	$E_{1/2,u}$
$E_{19/2,u}$	$E_{19/2}$	$E_{1/2,u}$	$E_{1/2}$	$E_{3/2,u}$	$E_{5/2}$	$E_{5/2,u}$	$E_{9/2}$	$E_{3/2,u}$	$E_{1/2}$	$E_{1/2,u}$
$\vdots$										

Other subgroups: see $\mathbf{D}_{nh}$; $n = 2, 3, \ldots, 10$.

T **40**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{\infty h}$
0	A_{1g}
1	$A_{1u} \oplus E_{1u}$
2	$A_{1g} \oplus E_{1g} \oplus E_{2g}$
3	$A_{1u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u}$
$2n$	$A_{1g} \oplus E_{1g} \oplus E_{2g} \oplus E_{3g} \oplus \cdots \oplus E_{2n,g}$
$2n+1$	$A_{1u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u} \oplus \cdots \oplus E_{2n+1,u}$
$\frac{1}{2}$	$E_{1/2,g}$
$\frac{3}{2}$	$E_{1/2,g} \oplus E_{3/2,g}$
$n+\frac{1}{2}$	$E_{1/2,g} \oplus E_{3/2,g} \oplus \cdots \oplus E_{n+1/2,g}$

$n = 2, 3, 4, \ldots$

T **40**.11 Clebsch–Gordan coefficients

Use T **59**.11 ▪. § **16**–11, p. 83

The groups $\mathbf{D}_{nd}$

Notation for headers

Items in header read from left to right

1		Hermann–Mauguin symbol for the point group.
2		$|G|$ order of the group.
3		$|C|$ number of classes in the group.
4		$|\widetilde{C}|$ number of classes in the double group.
5		Number of the table.
6		Page reference for the notation of the header, of the first five subsections below it, and of the footers.
7	☐	This symbol indicates a crystallographic point group.
8		Schönflies notation for the point group.

Notation for the first five subsections below the header

(1) Product forms	Direct and semidirect product forms. ⊗ Direct product. ⓢ Semidirect product.
(2) Group chains *(See pp. 41, 67)*	Groups underlined: invariant. Groups in brackets: when subducing one or more of the representations in the tables to these subgroups in the setting used for them in the tables a change of bases (similarity transformation) is required.
(3) Operations of G	Lists all the operations of G, enclosing in brackets all the operations of the same class.
(4) Operations of $\widetilde{G}$	Lists all the operations of $\widetilde{G}$, enclosing in brackets all the operations of the same class.
(5) Classes and representations	$|r|$ number of regular classes in G (p. 51). $|\mathsf{i}|$ number of irregular classes in G (p. 51). $|I|$ number of irreducible representations in G. $|\widetilde{I}|$ number of spinor representations, also called the number of double-group representations.

Use of the footers

Finding your way about the tables — Each page of the tables contains a footer giving the ordering of the group types listed in the tables. The current group type is recognized because it carries no page number under its name. On opening the tables at any page you should then see at a glance whether to move backwards or forwards to find the group type desired. More precisely, the page number for the group type desired gives the page, like the present one, which carries the contents for the group type wanted.

(1) Product forms: $\mathbf{D}_2 \otimes \mathbf{C}_s$.

(2) Group chains: $\mathbf{D}_{10d} \supset (\mathbf{D}_{2d}) \supset (\underline{\mathbf{C}_{2v}})$, $\mathbf{D}_{10d} \supset (\mathbf{D}_{2d}) \supset (\underline{\mathbf{D}_2})$, $\mathbf{D}_{10d} \supset (\mathbf{D}_{2d}) \supset \underline{\mathbf{S}_4}$,
$\mathbf{D}_{6d} \supset (\mathbf{D}_{2d}) \supset (\underline{\mathbf{C}_{2v}})$, $\mathbf{D}_{6d} \supset (\mathbf{D}_{2d}) \supset (\underline{\mathbf{D}_2})$, $\mathbf{D}_{6d} \supset (\mathbf{D}_{2d}) \supset \underline{\mathbf{S}_4}$,
$\mathbf{D}_{4h} \supset \underline{\mathbf{D}_{2d}} \supset (\underline{\mathbf{C}_{2v}})$, $\mathbf{D}_{4h} \supset \underline{\mathbf{D}_{2d}} \supset (\underline{\mathbf{D}_2})$, $\mathbf{D}_{4h} \supset \underline{\mathbf{D}_{2d}} \supset \underline{\mathbf{S}_4}$.

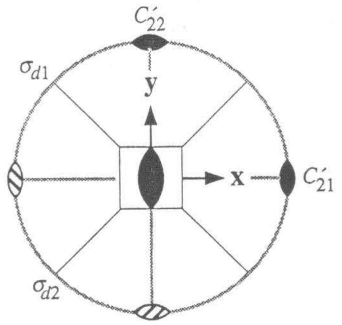
(3) Operations of G: E, C_2, (C'_{21}, C'_{22}), (S_4^-, S_4^+), $(\sigma_{d1}, \sigma_{d2})$.

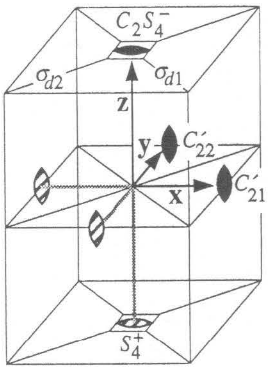
(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, $(C_2, \widetilde{C}_2)$, $(C'_{21}, C'_{22}, \widetilde{C}'_{21}, \widetilde{C}'_{22})$, (S_4^-, S_4^+), $(\widetilde{S}_4^-, \widetilde{S}_4^+)$, $(\sigma_{d1}, \sigma_{d2}, \widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2})$.

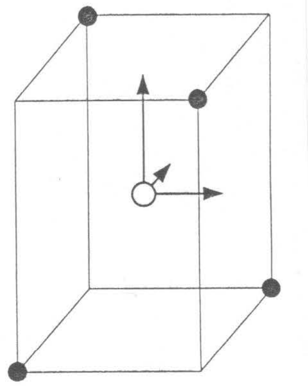
(5) Classes and representations: $|r| = 2$, $|i| = 3$, $|I| = 5$, $|\widetilde{I}| = 2$.

F **41**

See Chapter **15**, p. 65







Examples: Allene $H_2C{=}C{=}CH_2$.

T **41**.1 Parameters
Use T **33**.1. § **16**–1, p. 68

T **41**.2 Multiplication table
Use T **33**.2. § **16**–2, p. 69

T **41**.3 Factor table
Use T **33**.3. § **16**–3, p. 70

T **41**.4 Character table
§ **16**–4, p. 71

$\mathbf{D}_{2d}$	E	C_2	$2C'_2$	$2S_4$	$2\sigma_d$	τ
A_1	1	1	1	1	1	a
A_2	1	1	−1	1	−1	a
B_1	1	1	1	−1	−1	a
B_2	1	1	−1	−1	1	a
E	2	−2	0	0	0	a
$E_{1/2}$	2	0	0	$\sqrt{2}$	0	c
$E_{3/2}$	2	0	0	$-\sqrt{2}$	0	c

T **41**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{2d}$	0	1	2	3
A_1	$\square 1$		$x^2+y^2, \square z^2$	$\square xyz$
A_2		R_z		$\square z(x^2-y^2)$
B_1			$\square x^2-y^2$	
B_2		$\square z$	$\square xy$	$(x^2+y^2)z, \square z^3$
E		$\square(x,y),(R_x,R_y)$	$\square(zx,yz)$	$\{x(x^2+y^2),y(x^2+y^2)\}, \square(xz^2,yz^2),$ $\square\{x(x^2-3y^2),y(3x^2-y^2)\}$

T **41**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{2d}$	$\langle\lvert j\,m\rangle\rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$	$\lvert 3\,2\rangle_-$	2	4
A_2	$\lvert 3\,2\rangle_+$	$\lvert 4\,4\rangle_-$	2	4
B_1	$\lvert 2\,2\rangle_+$	$\lvert 5\,4\rangle_-$	2	4
B_2	$\lvert 1\,0\rangle_+$	$\lvert 2\,2\rangle_-$	2	4
E	$\langle\lvert 1\,\overline{1}\rangle, \lvert 1\,1\rangle\rvert$	$\langle\lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle\rvert$	2	± 4
$E_{1/2}$	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	2	± 4
	$\langle\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle, -\lvert \frac{3}{2}\,\frac{3}{2}\rangle\rvert^\bullet$	$\langle\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle, \lvert \frac{5}{2}\,\frac{3}{2}\rangle\rvert^\bullet$	2	± 4
$E_{3/2}$	$\langle\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle, -\lvert \frac{3}{2}\,\frac{3}{2}\rangle\rvert$	$\langle\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle, \lvert \frac{5}{2}\,\frac{3}{2}\rangle\rvert$	2	± 4
	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert^\bullet$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert^\bullet$	2	± 4

T **41**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_{2d}$	E	$E_{1/2}$	$E_{3/2}$
E	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
C_2	$\begin{bmatrix}\overline{1} & 0\\0 & \overline{1}\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}} & 0\\0 & \mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}} & 0\\0 & \mathrm{i}\end{bmatrix}$
C'_{21}	$\begin{bmatrix}0 & \overline{1}\\\overline{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\mathrm{i}}\\\bar{\mathrm{i}} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\mathrm{i}}\\\bar{\mathrm{i}} & 0\end{bmatrix}$
C'_{22}	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{1}\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{1}\\1 & 0\end{bmatrix}$
S_4^-	$\begin{bmatrix}\bar{\mathrm{i}} & 0\\0 & \mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\overline{\epsilon}^* & 0\\0 & \overline{\epsilon}\end{bmatrix}$
S_4^+	$\begin{bmatrix}\mathrm{i} & 0\\0 & \bar{\mathrm{i}}\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\overline{\epsilon} & 0\\0 & \overline{\epsilon}^*\end{bmatrix}$
σ_{d1}	$\begin{bmatrix}0 & \mathrm{i}\\\bar{\mathrm{i}} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\epsilon}\\\epsilon^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon\\\overline{\epsilon}^* & 0\end{bmatrix}$
σ_{d2}	$\begin{bmatrix}0 & \bar{\mathrm{i}}\\\mathrm{i} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\epsilon}^*\\\epsilon & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon^*\\\overline{\epsilon} & 0\end{bmatrix}$

$\epsilon = \exp(2\pi\mathrm{i}/8)$

T **41**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{D}_{2d}$	A_1	A_2	B_1	B_2	E	$E_{1/2}$	$E_{3/2}$
A_1	A_1	A_2	B_1	B_2	E	$E_{1/2}$	$E_{3/2}$
A_2		A_1	B_2	B_1	E	$E_{1/2}$	$E_{3/2}$
B_1			A_1	A_2	E	$E_{3/2}$	$E_{1/2}$
B_2				A_1	E	$E_{3/2}$	$E_{1/2}$
E					$A_1 \oplus \{A_2\} \oplus B_1 \oplus B_2$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{3/2}$
$E_{1/2}$						$\{A_1\} \oplus A_2 \oplus E$	$B_1 \oplus B_2 \oplus E$
$E_{3/2}$							$\{A_1\} \oplus A_2 \oplus E$

T **41**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_{2d}$	$(\mathbf{C}_{2v})$	$(\mathbf{D}_2)$	$\mathbf{S}_4$	$(\mathbf{C}_s)$	$\mathbf{C}_2$	$(\mathbf{C}_2)$
				σ_d	C_2	C_2'
A_1	A_1	A	A	A'	A	A
A_2	A_2	B_1	A	A''	A	B
B_1	A_2	A	B	A''	A	A
B_2	A_1	B_1	B	A'	A	B
E	$B_1 \oplus B_2$	$B_2 \oplus B_3$	${}^1E \oplus {}^2E$	$A' \oplus A''$	$2B$	$A \oplus B$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **41**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{2d}$
$4n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E)$
$4n+1$	$n\,(A_1 \oplus A_2 \oplus B_1 \oplus E) \oplus (n+1)(B_2 \oplus E)$
$4n+2$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E) \oplus n\,(A_2 \oplus E)$
$4n+3$	$(n+1)(A_1 \oplus A_2 \oplus B_2 \oplus 2E) \oplus n\,B_1$
$4n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,E_{3/2}$
$4n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2})$
$4n+\frac{5}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)\,E_{3/2}$
$4n+\frac{7}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2})$

$n = 0, 1, 2, \ldots$

T **41**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83

$\mathbf{D}_{2d}$

a_2	e	E 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{3/2}$	$E_{3/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

b_1	e	E 1	2
1	1	0	1
1	2	1	0

⇛

T **41**.11 Clebsch–Gordan coefficients *(cont.)*

b_1	$e_{1/2}$	$E_{3/2}$ 1	2
1	1	1	0
1	2	0	1

b_1	$e_{3/2}$	$E_{1/2}$ 1	2
1	1	1	0
1	2	0	1

b_2	e	E 1	2
1	1	0	$\bar{1}$
1	2	1	0

b_2	$e_{1/2}$	$E_{3/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{3/2}$	$E_{1/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

e	e	A_1 1	A_2 1	B_1 1	B_2 1
1	1	0	0	u	u
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	u	$\bar{\text{u}}$

e	$e_{1/2}$	$E_{1/2}$ 1	2	$E_{3/2}$ 1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e	$e_{3/2}$	$E_{1/2}$ 1	2	$E_{3/2}$ 1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{1/2}$	$e_{1/2}$	A_1 1	A_2 1	E 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\text{u}}$	u	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{3/2}$	B_1 1	B_2 1	E 1	2
1	1	0	0	0	1
1	2	u	u	0	0
2	1	$\bar{\text{u}}$	u	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{3/2}$	A_1 1	A_2 1	E 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\text{u}}$	u	0	0
2	2	0	0	0	1

$\text{u} = 2^{-1/2}$

(1) Product forms: $\mathbf{D}_3 \otimes \mathbf{C}_i$.

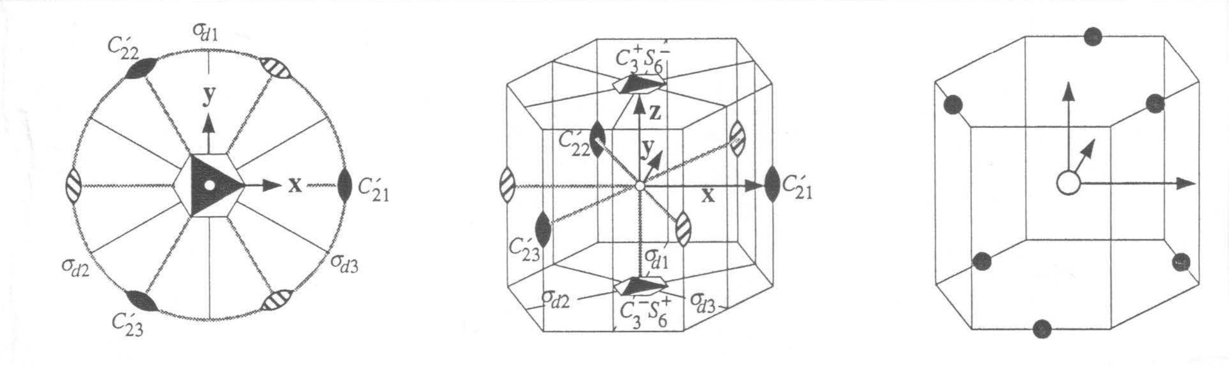
(2) Group chains: $\mathbf{I}_h \supset (\mathbf{D}_{3d}) \supset (\mathbf{C}_{2h})$, $\mathbf{I}_h \supset (\mathbf{D}_{3d}) \supset (\underline{\mathbf{C}_{3v}})$, $\mathbf{I}_h \supset (\mathbf{D}_{3d}) \supset \underline{\mathbf{D}_3}$, $\mathbf{I}_h \supset (\mathbf{D}_{3d}) \supset \underline{\mathbf{S}_6}$,
$\mathbf{O}_h \supset (\mathbf{D}_{3d}) \supset (\mathbf{C}_{2h})$, $\mathbf{O}_h \supset (\mathbf{D}_{3d}) \supset (\underline{\mathbf{C}_{3v}})$, $\mathbf{O}_h \supset (\mathbf{D}_{3d}) \supset \underline{\mathbf{D}_3}$, $\mathbf{O}_h \supset (\mathbf{D}_{3d}) \supset \underline{\mathbf{S}_6}$,
$\mathbf{D}_{9d} \supset (\mathbf{D}_{3d}) \supset (\mathbf{C}_{2h})$, $\mathbf{D}_{9d} \supset (\mathbf{D}_{3d}) \supset (\underline{\mathbf{C}_{3v}})$, $\mathbf{D}_{9d} \supset (\mathbf{D}_{3d}) \supset \underline{\mathbf{D}_3}$,
$\mathbf{D}_{9d} \supset (\mathbf{D}_{3d}) \supset \underline{\mathbf{S}_6}$,
$\mathbf{D}_{6h} \supset (\underline{\mathbf{D}_{3d}}) \supset (\mathbf{C}_{2h})$, $\mathbf{D}_{6h} \supset (\underline{\mathbf{D}_{3d}}) \supset (\underline{\mathbf{C}_{3v}})$, $\mathbf{D}_{6h} \supset (\underline{\mathbf{D}_{3d}}) \supset \underline{\mathbf{D}_3}$,
$\mathbf{D}_{6h} \supset (\underline{\mathbf{D}_{3d}}) \supset \underline{\mathbf{S}_6}$.

(3) Operations of G: E, (C_3^+, C_3^-), $(C'_{21}, C'_{22}, C'_{23})$, i, (S_6^-, S_6^+), $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3})$.

(4) Operations of $\widetilde{G}$: E, (C_3^+, C_3^-), $(C'_{21}, C'_{22}, C'_{23})$, i, (S_6^-, S_6^+), $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3})$,
$\widetilde{E}$, $(\widetilde{C}_3^+, \widetilde{C}_3^-)$, $(\widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23})$, $\widetilde{i}$, $(\widetilde{S}_6^-, \widetilde{S}_6^+)$, $(\widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3})$.

(5) Classes and representations: $|r| = 6$, $|i| = 0$, $|I| = 6$, $|\widetilde{I}| = 6$.

F **42** See Chapter **15**, p. 65



Examples: Cyclohexane C_6H_{12}, staggered C_2H_6.

T **42**.1 Parameters
Use T **35**.1. § **16**–1, p. 68

T **42**.2 Multiplication table
Use T **35**.2. § **16**–2, p. 69

T **42**.3 Factor table
Use T **35**.3. § **16**–3, p. 70

T **42**.4 Character table § **16**–4, p. 71

$\mathbf{D}_{3d}$	E	$2C_3$	$3C_2'$	i	$2S_6$	$3\sigma_d$	τ
A_{1g}	1	1	1	1	1	1	a
A_{2g}	1	1	−1	1	1	−1	a
E_g	2	−1	0	2	−1	0	a
A_{1u}	1	1	1	−1	−1	−1	a
A_{2u}	1	1	−1	−1	−1	1	a
E_u	2	−1	0	−2	1	0	a
$E_{1/2,g}$	2	1	0	2	1	0	c
${}^1E_{3/2,g}$	1	−1	i	1	−1	i	b
${}^2E_{3/2,g}$	1	−1	−i	1	−1	−i	b
$E_{1/2,u}$	2	1	0	−2	−1	0	c
${}^1E_{3/2,u}$	1	−1	i	−1	1	−i	b
${}^2E_{3/2,u}$	1	−1	−i	−1	1	i	b

T **42**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions
§ **16**–5, p. 72

$\mathbf{D}_{3d}$	0	1	2	3
A_{1g}	$^\square 1$		$x^2+y^2, {}^\square z^2$	
A_{2g}		R_z		
E_g		(R_x, R_y)	$^\square(xy, x^2-y^2), {}^\square(zx, yz)$	
A_{1u}				$^\square x(x^2-3y^2)$
A_{2u}		$^\square z$		$^\square y(3x^2-y^2), (x^2+y^2)z, {}^\square z^3$
E_u		$^\square(x, y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2),$ $^\square\{xyz, z(x^2-y^2)\}$

T **42**.6 Symmetrized bases § **16**–6, p. 74

$\mathbf{D}_{3d}$	$\langle\lvert j\,m\rangle\rvert$		ι	μ
A_{1g}	$\lvert 0\,0\rangle_+$		2	3
A_{2g}	$\lvert 4\,3\rangle_-$		2	3
E_g	$\langle\lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle\rvert$		2	±3
A_{1u}	$\lvert 3\,3\rangle_-$		2	3
A_{2u}	$\lvert 1\,0\rangle_+$		2	3
E_u	$\langle\lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle\rvert$		2	±3
$E_{1/2,g}$	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	2	±3
${}^1E_{3/2,g}$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle_+$	$\lvert \frac{5}{2}\,\frac{3}{2}\rangle_-$	2	3
${}^2E_{3/2,g}$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle_-$	$\lvert \frac{5}{2}\,\frac{3}{2}\rangle_+$	2	3
$E_{1/2,u}$	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert^\bullet$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert^\bullet$	2	±3
${}^1E_{3/2,u}$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle_+^\bullet$	$\lvert \frac{5}{2}\,\frac{3}{2}\rangle_-^\bullet$	2	3
${}^2E_{3/2,u}$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle_-^\bullet$	$\lvert \frac{5}{2}\,\frac{3}{2}\rangle_+^\bullet$	2	3

T **42**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_{3d}$	E_g	E_u	$E_{1/2,g}$	$E_{1/2,u}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_3^+	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$
C_3^-	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$
i	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$
S_6^-	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
S_6^+	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
σ_{d1}	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$
σ_{d2}	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$
σ_{d3}	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$

$\epsilon = \exp(2\pi \mathrm{i}/3)$

T **42**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{D}_{3d}$	A_{1g}	A_{2g}	E_g	A_{1u}	A_{2u}	E_u
A_{1g}	A_{1g}	A_{2g}	E_g	A_{1u}	A_{2u}	E_u
A_{2g}		A_{1g}	E_g	A_{2u}	A_{1u}	E_u
E_g			$A_{1g} \oplus \{A_{2g}\} \oplus E_g$	E_u	E_u	$A_{1u} \oplus A_{2u} \oplus E_u$
A_{1u}				A_{1g}	A_{2g}	E_g
A_{2u}					A_{1g}	E_g
E_u						$A_{1g} \oplus \{A_{2g}\} \oplus E_g$

$\twoheadrightarrow$

T **42**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{3d}$	$E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	$E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$
A_{1g}	$E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	$E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$
A_{2g}	$E_{1/2,g}$	${}^2E_{3/2,g}$	${}^1E_{3/2,g}$	$E_{1/2,u}$	${}^2E_{3/2,u}$	${}^1E_{3/2,u}$
E_g	$E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$	$E_{1/2,g}$	$E_{1/2,g}$	$E_{1/2,u} \oplus {}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$	$E_{1/2,u}$	$E_{1/2,u}$
A_{1u}	$E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	$E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$
A_{2u}	$E_{1/2,u}$	${}^2E_{3/2,u}$	${}^1E_{3/2,u}$	$E_{1/2,g}$	${}^2E_{3/2,g}$	${}^1E_{3/2,g}$
E_u	$E_{1/2,u} \oplus {}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$	$E_{1/2,u}$	$E_{1/2,u}$	$E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$	$E_{1/2,g}$	$E_{1/2,g}$
$E_{1/2,g}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_g$	E_g	E_g	$A_{1u} \oplus A_{2u} \oplus E_u$	E_u	E_u
${}^1E_{3/2,g}$		A_{2g}	A_{1g}	E_u	A_{2u}	A_{1u}
${}^2E_{3/2,g}$			A_{2g}	E_u	A_{1u}	A_{2u}
$E_{1/2,u}$				$\{A_{1g}\} \oplus A_{2g} \oplus E_g$	E_g	E_g
${}^1E_{3/2,u}$					A_{2g}	A_{1g}
${}^2E_{3/2,u}$						A_{2g}

T **42**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_{3d}$	$(\mathbf{C}_{2h})$	$(\mathbf{C}_{3v})$	$\mathbf{D}_3$	$\mathbf{S}_6$
A_{1g}	A_g	A_1	A_1	A_g
A_{2g}	B_g	A_2	A_2	A_g
E_g	$A_g \oplus B_g$	E	E	${}^1E_g \oplus {}^2E_g$
A_{1u}	A_u	A_2	A_1	A_u
A_{2u}	B_u	A_1	A_2	A_u
E_u	$A_u \oplus B_u$	E	E	${}^1E_u \oplus {}^2E_u$
$E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$
${}^1E_{3/2,g}$	${}^1E_{1/2,g}$	${}^1E_{3/2}$	${}^1E_{3/2}$	$A_{3/2,g}$
${}^2E_{3/2,g}$	${}^2E_{1/2,g}$	${}^2E_{3/2}$	${}^2E_{3/2}$	$A_{3/2,g}$
$E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$
${}^1E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2}$	${}^1E_{3/2}$	$A_{3/2,u}$
${}^2E_{3/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2}$	${}^2E_{3/2}$	$A_{3/2,u}$

↠

T **42**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{3d}$	$(\mathbf{C}_s)$	$\mathbf{C}_i$	$\mathbf{C}_3$	$(\mathbf{C}_2)$
A_{1g}	A'	A_g	A	A
A_{2g}	A''	A_g	A	B
E_g	$A' \oplus A''$	$2A_g$	${}^1E \oplus {}^2E$	$A \oplus B$
A_{1u}	A''	A_u	A	A
A_{2u}	A'	A_u	A	B
E_u	$A' \oplus A''$	$2A_u$	${}^1E \oplus {}^2E$	$A \oplus B$
$E_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1E_{3/2,g}$	${}^1E_{1/2}$	$A_{1/2,g}$	$A_{3/2}$	${}^1E_{1/2}$
${}^2E_{3/2,g}$	${}^2E_{1/2}$	$A_{1/2,g}$	$A_{3/2}$	${}^2E_{1/2}$
$E_{1/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1E_{3/2,u}$	${}^2E_{1/2}$	$A_{1/2,u}$	$A_{3/2}$	${}^1E_{1/2}$
${}^2E_{3/2,u}$	${}^1E_{1/2}$	$A_{1/2,u}$	$A_{3/2}$	${}^2E_{1/2}$

T **42**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{3d}$
$6n$	$(2n+1)\,A_{1g} \oplus 2n\,(A_{2g} \oplus 2E_g)$
$6n+1$	$2n\,(A_{1u} \oplus E_u) \oplus (2n+1)(A_{2u} \oplus E_u)$
$6n+2$	$(2n+1)(A_{1g} \oplus 2E_g) \oplus 2n\,A_{2g}$
$6n+3$	$(2n+1)(A_{1u} \oplus 2E_u) \oplus (2n+2)\,A_{2u}$
$6n+4$	$(2n+2)(A_{1g} \oplus E_g) \oplus (2n+1)(A_{2g} \oplus E_g)$
$6n+5$	$(2n+1)\,A_{1u} \oplus (2n+2)(A_{2u} \oplus 2E_u)$
$3n+\frac{1}{2}$	$(2n+1)\,E_{1/2,g} \oplus n\,({}^1E_{3/2,g} \oplus {}^2E_{3/2,g})$
$3n+\frac{3}{2}$	$(2n+1)\,E_{1/2,g} \oplus (n+1)({}^1E_{3/2,g} \oplus {}^2E_{3/2,g})$
$3n+\frac{5}{2}$	$(n+1)(2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g})$

$n = 0, 1, 2, \ldots$

T **42**.11 Clebsch–Gordan coefficients

Use T **23**.11 ▪. § **16**–11, p. 83

$\bar{8}2m$	$\lvert G\rvert = 16$	$\lvert C\rvert = 7$	$\lvert\widetilde{C}\rvert = 11$	T **43**	p. 365	$\mathbf{D}_{4d}$

(1) Product forms: $\mathbf{D}_4 \otimes \mathbf{C}_s$.

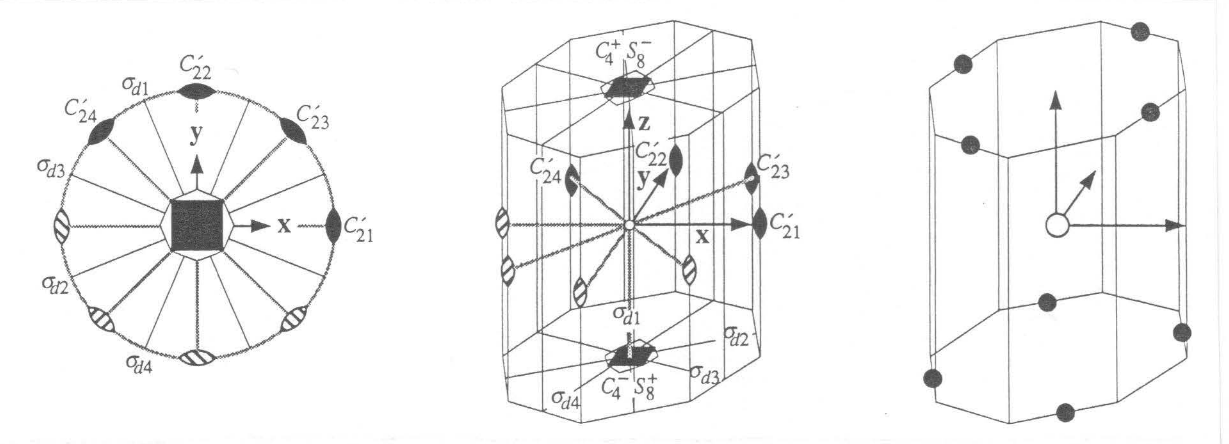
(2) Group chains: $\mathbf{D}_{8h} \supset \underline{\mathbf{D}_{4d}} \supset (\underline{\mathbf{C}_{4v}})$, $\mathbf{D}_{8h} \supset \underline{\mathbf{D}_{4d}} \supset (\underline{\mathbf{C}_{2v}})$, $\mathbf{D}_{8h} \supset \underline{\mathbf{D}_{4d}} \supset \underline{\mathbf{S}_8}$.

(3) Operations of G: E, (C_4^+, C_4^-), C_2, $(C'_{21}, C'_{22}, C'_{23}, C'_{24})$, (S_8^{3-}, S_8^{3+}), (S_8^-, S_8^+), $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_4^+, C_4^-), $(\widetilde{C}_4^+, \widetilde{C}_4^-)$, $(C_2, \widetilde{C}_2)$, $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, \widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24})$, (S_8^{3-}, S_8^{3+}), $(\widetilde{S}_8^{3-}, \widetilde{S}_8^{3+})$, (S_8^-, S_8^+), $(\widetilde{S}_8^-, \widetilde{S}_8^+)$, $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3}, \widetilde{\sigma}_{d4})$.

(5) Classes and representations: $\lvert r\rvert = 4$, $\lvert i\rvert = 3$, $\lvert I\rvert = 7$, $\lvert\widetilde{I}\rvert = 4$.

F **43** See Chapter **15**, p. 65



Examples: Puckered S_8 ring, $B_{10}H_{10}^{2-}$ (*closo*-borane).

T **43**.1 Parameters
Use T **37**.1. § **16**–1, p. 68

T **43**.2 Multiplication table
Use T **37**.2. § **16**–2, p. 69

T **43**.3 Factor table
Use T **37**.3. § **16**–3, p. 70

T **43**.4 Character table
§ **16**–4, p. 71

$\mathbf{D}_{4d}$	E	$2C_4$	C_2	$4C'_2$	$2S_8^3$	$2S_8$	$4\sigma_d$	τ
A_1	1	1	1	1	1	1	1	a
A_2	1	1	1	-1	1	1	-1	a
B_1	1	1	1	1	-1	-1	-1	a
B_2	1	1	1	-1	-1	-1	1	a
E_1	2	0	-2	0	$-\sqrt{2}$	$\sqrt{2}$	0	a
E_2	2	-2	2	0	0	0	0	a
E_3	2	0	-2	0	$\sqrt{2}$	$-\sqrt{2}$	0	a
$E_{1/2}$	2	$\sqrt{2}$	0	0	$2c_8$	$2c_8^3$	0	c
$E_{3/2}$	2	$-\sqrt{2}$	0	0	$2c_8^3$	$-2c_8$	0	c
$E_{5/2}$	2	$-\sqrt{2}$	0	0	$-2c_8^3$	$2c_8$	0	c
$E_{7/2}$	2	$\sqrt{2}$	0	0	$-2c_8$	$-2c_8^3$	0	c

$c_n^m = \cos \frac{m}{n}\pi$

T **43**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{4d}$	0	1	2	3
A_1	$^\square 1$		$x^2+y^2, {}^\square z^2$	
A_2		R_z		
B_1				
B_2		$^\square z$		$(x^2+y^2)z, {}^\square z^3$
E_1		$^\square(x,y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
E_2			$^\square(xy, x^2-y^2)$	$^\square\{xyz, z(x^2-y^2)\}$
E_3		(R_x, R_y)	$^\square(zx, yz)$	$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$

T **43**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{4d}$	$\langle \lvert j\, m\rangle \rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$	$\lvert 5\,4\rangle_-$	2	8
A_2	$\lvert 5\,4\rangle_+$	$\lvert 8\,8\rangle_-$	2	8
B_1	$\lvert 4\,4\rangle_+$	$\lvert 9\,8\rangle_-$	2	8
B_2	$\lvert 1\,0\rangle_+$	$\lvert 4\,4\rangle_-$	2	8
E_1	$\langle \lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle \rvert$	$\langle \lvert 4\,\overline{3}\rangle, -\lvert 4\,3\rangle \rvert$	2	± 8
E_2	$\langle \lvert 2\,2\rangle, \lvert 2\,\overline{2}\rangle \rvert$	$\langle \lvert 3\,\overline{2}\rangle, -\lvert 3\,2\rangle \rvert$	2	± 8
E_3	$\langle \lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle \rvert$	$\langle \lvert 3\,\overline{3}\rangle, \lvert 3\,3\rangle \rvert$	2	± 8
$E_{1/2}$	$\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \rvert$	$\langle \lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle \rvert$	2	± 8
	$\langle \lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle, -\lvert \frac{7}{2}\,\frac{7}{2}\rangle \rvert^\bullet$	$\langle \lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle, \lvert \frac{9}{2}\,\frac{7}{2}\rangle \rvert^\bullet$	2	± 8
$E_{3/2}$	$\langle \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle, -\lvert \frac{3}{2}\,\frac{3}{2}\rangle \rvert$	$\langle \lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle, \lvert \frac{5}{2}\,\frac{3}{2}\rangle \rvert$	2	± 8
	$\langle \lvert \frac{5}{2}\,\frac{5}{2}\rangle, \lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle \rvert^\bullet$	$\langle \lvert \frac{7}{2}\,\frac{5}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle \rvert^\bullet$	2	± 8
$E_{5/2}$	$\langle \lvert \frac{5}{2}\,\frac{5}{2}\rangle, \lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle \rvert$	$\langle \lvert \frac{7}{2}\,\frac{5}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle \rvert$	2	± 8
	$\langle \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle, -\lvert \frac{3}{2}\,\frac{3}{2}\rangle \rvert^\bullet$	$\langle \lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle, \lvert \frac{5}{2}\,\frac{3}{2}\rangle \rvert^\bullet$	2	± 8
$E_{7/2}$	$\langle \lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle, -\lvert \frac{7}{2}\,\frac{7}{2}\rangle \rvert$	$\langle \lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle, \lvert \frac{9}{2}\,\frac{7}{2}\rangle \rvert$	2	± 8
	$\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \rvert^\bullet$	$\langle \lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle \rvert^\bullet$	2	± 8

T **43**.7 Matrix representations

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$\mathbf{D}_{4d}$	E_1	E_2	E_3	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_4^+	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
C_4^-	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
C_2	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$
S_8^{3-}	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\delta}^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$
S_8^{3+}	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$
S_8^-	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\bar{\delta}^* \end{bmatrix}$
S_8^+	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\delta}^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$
σ_{d1}	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta}^* \\ \mathrm{i}\bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$
σ_{d2}	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta}^* \\ \mathrm{i}\bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \bar{\delta} & 0 \end{bmatrix}$
σ_{d3}	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta} \\ \mathrm{i}\bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \bar{\delta}^* & 0 \end{bmatrix}$
σ_{d4}	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\delta} \\ \mathrm{i}\bar{\delta}^* & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/16)$, $\epsilon = \exp(2\pi\mathrm{i}/8)$

T **43**.8 Direct products of representations

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$\mathbf{D}_{4d}$	A_1	A_2	B_1	B_2	E_1	E_2	E_3
A_1	A_1	A_2	B_1	B_2	E_1	E_2	E_3
A_2		A_1	B_2	B_1	E_1	E_2	E_3
B_1			A_1	A_2	E_3	E_2	E_1
B_2				A_1	E_3	E_2	E_1
E_1					$A_1 \oplus \{A_2\} \oplus E_2$	$E_1 \oplus E_3$	$B_1 \oplus B_2 \oplus E_2$
E_2						$A_1 \oplus \{A_2\} \oplus B_1 \oplus B_2$	$E_1 \oplus E_3$
E_3							$A_1 \oplus \{A_2\} \oplus E_2$

$\twoheadrightarrow$

T **43**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{4d}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$
A_1	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$
A_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$
B_1	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
B_2	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
E_1	$E_{5/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$
E_2	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{5/2}$
E_3	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{5/2} \oplus E_{7/2}$
$E_{1/2}$	$\{A_1\} \oplus A_2 \oplus E_3$	$E_2 \oplus E_3$	$E_1 \oplus E_2$	$B_1 \oplus B_2 \oplus E_1$
$E_{3/2}$		$\{A_1\} \oplus A_2 \oplus E_1$	$B_1 \oplus B_2 \oplus E_3$	$E_1 \oplus E_2$
$E_{5/2}$			$\{A_1\} \oplus A_2 \oplus E_1$	$E_2 \oplus E_3$
$E_{7/2}$				$\{A_1\} \oplus A_2 \oplus E_3$

T **43**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_{4d}$	$(\mathbf{C}_{4v})$	$(\mathbf{C}_{2v})$	$(\mathbf{D}_4)$	$(\mathbf{D}_2)$	$\mathbf{S}_8$
A_1	A_1	A_1	A_1	A	A
A_2	A_2	A_2	A_2	B_1	A
B_1	A_2	A_2	A_1	A	B
B_2	A_1	A_1	A_2	B_1	B
E_1	E	$B_1 \oplus B_2$	E	$B_2 \oplus B_3$	${}^1E_1 \oplus {}^2E_1$
E_2	$B_1 \oplus B_2$	$A_1 \oplus A_2$	$B_1 \oplus B_2$	$A \oplus B_1$	${}^1E_2 \oplus {}^2E_2$
E_3	E	$B_1 \oplus B_2$	E	$B_2 \oplus B_3$	${}^1E_3 \oplus {}^2E_3$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	$E_{3/2}$	$E_{1/2}$	$E_{3/2}$	$E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$
$E_{5/2}$	$E_{3/2}$	$E_{1/2}$	$E_{3/2}$	$E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$
$E_{7/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$

$\twoheadrightarrow$

T **43**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{4d}$	$(\mathbf{C}_s)$	$\mathbf{C}_4$	$\mathbf{C}_2$	$(\mathbf{C}_2)$
	σ_d		C_2	C_2'
A_1	A'	A	A	A
A_2	A''	A	A	B
B_1	A''	A	A	A
B_2	A'	A	A	B
E_1	$A' \oplus A''$	${}^1E \oplus {}^2E$	$2B$	$A \oplus B$
E_2	$A' \oplus A''$	$2B$	$2A$	$A \oplus B$
E_3	$A' \oplus A''$	${}^1E \oplus {}^2E$	$2B$	$A \oplus B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **43**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{4d}$
$8n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3)$
$8n+1$	$n\,(A_1 \oplus A_2 \oplus B_1 \oplus E_1 \oplus 2E_2 \oplus 2E_3) \oplus (n+1)(B_2 \oplus E_1)$
$8n+2$	$(n+1)(A_1 \oplus E_2 \oplus E_3) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus E_3)$
$8n+3$	$n\,(A_1 \oplus A_2 \oplus B_1 \oplus E_1 \oplus E_2 \oplus E_3) \oplus (n+1)(B_2 \oplus E_1 \oplus E_2 \oplus E_3)$
$8n+4$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3) \oplus n\,(A_2 \oplus E_1 \oplus E_2 \oplus E_3)$
$8n+5$	$(n+1)(A_1 \oplus A_2 \oplus B_2 \oplus E_1 \oplus E_2 \oplus 2E_3) \oplus n\,(B_1 \oplus E_1 \oplus E_2)$
$8n+6$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus E_3) \oplus n\,(A_2 \oplus E_3)$
$8n+7$	$(n+1)(A_1 \oplus A_2 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3) \oplus n\,B_1$
$8n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,(E_{3/2} \oplus E_{5/2} \oplus E_{7/2})$
$8n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus 2n\,(E_{5/2} \oplus E_{7/2})$
$8n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus 2n\,E_{7/2}$
$8n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2})$
$8n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus (2n+2)\,E_{7/2}$
$8n+\frac{11}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (2n+2)(E_{5/2} \oplus E_{7/2})$
$8n+\frac{13}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)(E_{3/2} \oplus E_{5/2} \oplus E_{7/2})$
$8n+\frac{15}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2})$

$n = 0, 1, 2, \ldots$

T **43**.11 Clebsch–Gordan coefficients

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$\mathbf{D}_{4d}$

a_2	e_1	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_2	E_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_3	E_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{5/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{7/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_1	e_1	E_3	
		1	2
1	1	1	0
1	2	0	1

b_1	e_2	E_2	
		1	2
1	1	0	1
1	2	1	0

b_1	e_3	E_1	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{1/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{3/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{5/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{7/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	1

b_2	e_1	E_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_2	E_2	
		1	2
1	1	0	$\bar{1}$
1	2	1	0

↠

T **43**.11 Clebsch–Gordan coefficients *(cont.)*

b_2	e_3	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{1/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{3/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{5/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{7/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e_1	e_1	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_1	e_2	E_1		E_3	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	e_3	B_1	B_2	E_2	
		1	1	1	2
1	1	0	0	0	1
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_1	$e_{1/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_1	$e_{3/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_1	$e_{5/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_1	$e_{7/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_2	e_2	A_1	A_2	B_1	B_2
		1	1	1	1
1	1	0	0	u	u
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	u	$\bar{u}$

e_2	e_3	E_1		E_3	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	1	0	0	0

e_2	$e_{1/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_2	$e_{3/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_2	$e_{5/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_2	$e_{7/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_3	e_3	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

$\mathrm{u} = 2^{-1/2}$

↠

T **43**.11 Clebsch–Gordan coefficients *(cont.)*

e_3	$e_{1/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_3	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_3	$e_{5/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_3	$e_{7/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{1/2}$	$e_{1/2}$	A_1	A_2	E_3	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{3/2}$	E_2		E_3	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{1/2}$	$e_{5/2}$	E_1		E_2	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{1/2}$	$e_{7/2}$	B_1	B_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{3/2}$	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{5/2}$	B_1	B_2	E_3	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{7/2}$	E_1		E_2	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{5/2}$	$e_{5/2}$	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{5/2}$	$e_{7/2}$	E_2		E_3	
		1	2	1	2
1	1	0	0	0	1
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{7/2}$	$e_{7/2}$	A_1	A_2	E_3	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

(1) Product forms: $\mathbf{D}_5 \otimes \mathbf{C}_i$.

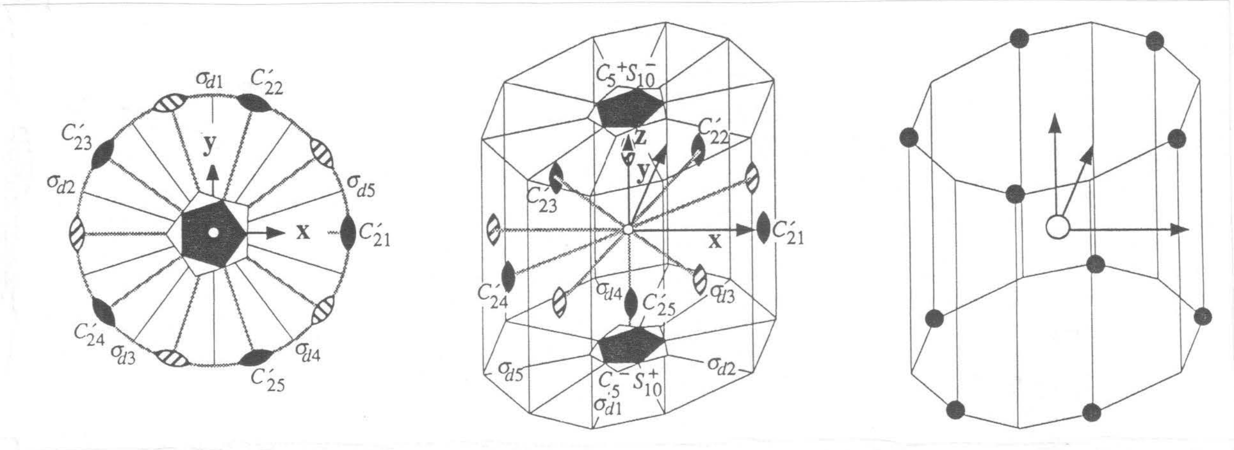
(2) Group chains: $\mathbf{I}_h \supset (\mathbf{D}_{5d}) \supset (\mathbf{C}_{2h})$, $\mathbf{I}_h \supset (\mathbf{D}_{5d}) \supset (\underline{\mathbf{C}_{5v}})$, $\mathbf{I}_h \supset (\mathbf{D}_{5d}) \supset \underline{\mathbf{D}_5}$, $\mathbf{I}_h \supset (\mathbf{D}_{5d}) \supset \underline{\mathbf{S}_{10}}$,
$\mathbf{D}_{10h} \supset (\underline{\mathbf{D}_{5d}}) \supset (\mathbf{C}_{2h})$, $\mathbf{D}_{10h} \supset (\underline{\mathbf{D}_{5d}}) \supset (\underline{\mathbf{C}_{5v}})$, $\mathbf{D}_{10h} \supset (\underline{\mathbf{D}_{5d}}) \supset \underline{\mathbf{D}_5}$,
$\mathbf{D}_{10h} \supset (\underline{\mathbf{D}_{5d}}) \supset \underline{\mathbf{S}_{10}}$.

(3) Operations of G: E, (C_5^+, C_5^-), (C_5^{2+}, C_5^{2-}), $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25})$,
i, $(S_{10}^{3-}, S_{10}^{3+})$, (S_{10}^-, S_{10}^+), $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5})$.

(4) Operations of $\widetilde{G}$: E, (C_5^+, C_5^-), (C_5^{2+}, C_5^{2-}), $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25})$,
i, $(S_{10}^{3-}, S_{10}^{3+})$, (S_{10}^-, S_{10}^+), $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5})$,
$\widetilde{E}$, $(\widetilde{C}_5^+, \widetilde{C}_5^-)$, $(\widetilde{C}_5^{2+}, \widetilde{C}_5^{2-})$, $(\widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24}, \widetilde{C}'_{25})$,
$\widetilde{\imath}$, $(\widetilde{S}_{10}^{3-}, \widetilde{S}_{10}^{3+})$, $(\widetilde{S}_{10}^-, \widetilde{S}_{10}^+)$, $(\widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3}, \widetilde{\sigma}_{d4}, \widetilde{\sigma}_{d5})$.

(5) Classes and representations: $|r| = 8$, $|i| = 0$, $|I| = 8$, $|\widetilde{I}| = 8$.

F **44** See Chapter **15**, p. 65



Examples: Ferrocene $Fe(C_5H_5)_2$ (pentagonal antiprism, cyclopentadienes staggered).

T **44**.1 Parameters
Use T **39**.1. § **16**–1, p. 68

T **44**.2 Multiplication table
Use T **39**.2. § **16**–2, p. 69

T **44**.3 Factor table
Use T **39**.3. § **16**–3, p. 70

T **44**.4 Character table

§ **16**–4, p. 71

$\mathbf{D}_{5d}$	E	$2C_5$	$2C_5^2$	$5C_2'$	i	$2S_{10}^3$	$2S_{10}$	$5\sigma_d$	τ
A_{1g}	1	1	1	1	1	1	1	1	a
A_{2g}	1	1	1	-1	1	1	1	-1	a
E_{1g}	2	$2c_5^2$	$2c_5^4$	0	2	$2c_5^2$	$2c_5^4$	0	a
E_{2g}	2	$2c_5^4$	$2c_5^2$	0	2	$2c_5^4$	$2c_5^2$	0	a
A_{1u}	1	1	1	1	-1	-1	-1	-1	a
A_{2u}	1	1	1	-1	-1	-1	-1	1	a
E_{1u}	2	$2c_5^2$	$2c_5^4$	0	-2	$-2c_5^2$	$-2c_5^4$	0	a
E_{2u}	2	$2c_5^4$	$2c_5^2$	0	-2	$-2c_5^4$	$-2c_5^2$	0	a
$E_{1/2,g}$	2	$-2c_5^4$	$2c_5^2$	0	2	$-2c_5^4$	$2c_5^2$	0	c
$E_{3/2,g}$	2	$-2c_5^2$	$2c_5^4$	0	2	$-2c_5^2$	$2c_5^4$	0	c
${}^1E_{5/2,g}$	1	-1	1	i	1	-1	1	i	b
${}^2E_{5/2,g}$	1	-1	1	$-$i	1	-1	1	$-$i	b
$E_{1/2,u}$	2	$-2c_5^4$	$2c_5^2$	0	-2	$2c_5^4$	$-2c_5^2$	0	c
$E_{3/2,u}$	2	$-2c_5^2$	$2c_5^4$	0	-2	$2c_5^2$	$-2c_5^4$	0	c
${}^1E_{5/2,u}$	1	-1	1	i	-1	1	-1	$-$i	b
${}^2E_{5/2,u}$	1	-1	1	$-$i	-1	1	-1	i	b

$c_n^m = \cos \frac{m}{n}\pi$

T **44**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{5d}$	0	1	2	3
A_{1g}	$^\square 1$		$x^2+y^2, {}^\square z^2$	
A_{2g}		R_z		
E_{1g}		(R_x, R_y)	$^\square(zx, yz)$	
E_{2g}			$^\square(xy, x^2-y^2)$	
A_{1u}				
A_{2u}		$^\square z$		$(x^2+y^2)z, {}^\square z^3$
E_{1u}		$^\square(x, y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
E_{2u}				$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}, {}^\square\{xyz, z(x^2-y^2)\}$

T **44**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{5d}$	$\langle \lvert j\, m\rangle \rvert$		ι	μ
A_{1g}	$\lvert 0\,0\rangle_+$		2	5
A_{2g}	$\lvert 6\,5\rangle_-$		2	5
E_{1g}	$\langle \lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle \rvert$		2	±5
E_{2g}	$\langle \lvert 2\,2\rangle, \lvert 2\,\overline{2}\rangle \rvert$		2	±5
A_{1u}	$\lvert 5\,5\rangle_-$		2	5
A_{2u}	$\lvert 1\,0\rangle_+$		2	5
E_{1u}	$\langle \lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle \rvert$		2	±5
E_{2u}	$\langle \lvert 3\,2\rangle, -\lvert 3\,\overline{2}\rangle \rvert$		2	±5
$E_{1/2,g}$	$\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\frac{\overline{1}}{2}\rangle \rvert$	$\langle \lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\frac{\overline{1}}{2}\rangle \rvert$	2	±5
$E_{3/2,g}$	$\langle \lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\frac{\overline{3}}{2}\rangle \rvert$	$\langle \lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\frac{\overline{3}}{2}\rangle \rvert$	2	±5
${}^1E_{5/2,g}$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle_-$	$\lvert \frac{7}{2}\,\frac{5}{2}\rangle_+$	2	5
${}^2E_{5/2,g}$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle_+$	$\lvert \frac{7}{2}\,\frac{5}{2}\rangle_-$	2	5
$E_{1/2,u}$	$\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\frac{\overline{1}}{2}\rangle \rvert^\bullet$	$\langle \lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\frac{\overline{1}}{2}\rangle \rvert^\bullet$	2	±5
$E_{3/2,u}$	$\langle \lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\frac{\overline{3}}{2}\rangle \rvert^\bullet$	$\langle \lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\frac{\overline{3}}{2}\rangle \rvert^\bullet$	2	±5
${}^1E_{5/2,u}$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle_-^\bullet$	$\lvert \frac{7}{2}\,\frac{5}{2}\rangle_+^\bullet$	2	5
${}^2E_{5/2,u}$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle_+^\bullet$	$\lvert \frac{7}{2}\,\frac{5}{2}\rangle_-^\bullet$	2	5

T **44**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_{5d}$	E_{1g}	E_{2g}	E_{1u}	E_{2u}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{1/2,u}$	$E_{3/2,u}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_5^+	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon & 0 \\ 0 & \bar\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar\delta & 0 \\ 0 & \bar\delta^* \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon & 0 \\ 0 & \bar\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar\delta & 0 \\ 0 & \bar\delta^* \end{bmatrix}$
C_5^-	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon^* & 0 \\ 0 & \bar\epsilon \end{bmatrix}$	$\begin{bmatrix} \bar\delta^* & 0 \\ 0 & \bar\delta \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon^* & 0 \\ 0 & \bar\epsilon \end{bmatrix}$	$\begin{bmatrix} \bar\delta^* & 0 \\ 0 & \bar\delta \end{bmatrix}$
C_5^{2+}	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
C_5^{2-}	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \bar1 \\ \bar1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar1 \\ \bar1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \bar\epsilon^* \\ \bar\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\epsilon^* \\ \bar\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta^* \\ \mathrm{i}\bar\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta^* \\ \mathrm{i}\bar\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \bar\delta \\ \bar\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\delta \\ \bar\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon^* \\ \mathrm{i}\bar\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon^* \\ \mathrm{i}\bar\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & \bar\delta^* \\ \bar\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\delta^* \\ \bar\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon \\ \mathrm{i}\bar\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon \\ \mathrm{i}\bar\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$
C'_{25}	$\begin{bmatrix} 0 & \bar\epsilon \\ \bar\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\epsilon \\ \bar\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta \\ \mathrm{i}\bar\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta \\ \mathrm{i}\bar\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$
i	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar1 & 0 \\ 0 & \bar1 \end{bmatrix}$	$\begin{bmatrix} \bar1 & 0 \\ 0 & \bar1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar1 & 0 \\ 0 & \bar1 \end{bmatrix}$	$\begin{bmatrix} \bar1 & 0 \\ 0 & \bar1 \end{bmatrix}$
S_{10}^{3-}	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar\delta^* & 0 \\ 0 & \bar\delta \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon^* & 0 \\ 0 & \bar\epsilon \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon & 0 \\ 0 & \bar\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar\delta & 0 \\ 0 & \bar\delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$
S_{10}^{3+}	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar\delta & 0 \\ 0 & \bar\delta^* \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon & 0 \\ 0 & \bar\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon^* & 0 \\ 0 & \bar\epsilon \end{bmatrix}$	$\begin{bmatrix} \bar\delta^* & 0 \\ 0 & \bar\delta \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$
S_{10}^-	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon^* & 0 \\ 0 & \bar\epsilon \end{bmatrix}$	$\begin{bmatrix} \bar\delta & 0 \\ 0 & \bar\delta^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar\delta^* & 0 \\ 0 & \bar\delta \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon & 0 \\ 0 & \bar\epsilon^* \end{bmatrix}$
S_{10}^+	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon & 0 \\ 0 & \bar\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar\delta^* & 0 \\ 0 & \bar\delta \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar\delta & 0 \\ 0 & \bar\delta^* \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon^* & 0 \\ 0 & \bar\epsilon \end{bmatrix}$
σ_{d1}	$\begin{bmatrix} 0 & \bar1 \\ \bar1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar1 \\ \bar1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
σ_{d2}	$\begin{bmatrix} 0 & \bar\epsilon^* \\ \bar\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\delta \\ \bar\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta^* \\ \mathrm{i}\bar\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon \\ \mathrm{i}\bar\epsilon^* & 0 \end{bmatrix}$
σ_{d3}	$\begin{bmatrix} 0 & \bar\delta \\ \bar\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\epsilon \\ \bar\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon^* \\ \mathrm{i}\bar\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta^* \\ \mathrm{i}\bar\delta & 0 \end{bmatrix}$
σ_{d4}	$\begin{bmatrix} 0 & \bar\delta^* \\ \bar\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\epsilon^* \\ \bar\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon \\ \mathrm{i}\bar\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta \\ \mathrm{i}\bar\delta^* & 0 \end{bmatrix}$
σ_{d5}	$\begin{bmatrix} 0 & \bar\epsilon \\ \bar\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\delta^* \\ \bar\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta \\ \mathrm{i}\bar\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon^* \\ \mathrm{i}\bar\epsilon & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/5)$, $\epsilon = \exp(4\pi\mathrm{i}/5)$

T **44**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{D}_{5d}$	A_{1g}	A_{2g}	E_{1g}	E_{2g}
A_{1g}	A_{1g}	A_{2g}	E_{1g}	E_{2g}
A_{2g}		A_{1g}	E_{1g}	E_{2g}
E_{1g}			$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$	$E_{1g} \oplus E_{2g}$
E_{2g}				$A_{1g} \oplus \{A_{2g}\} \oplus E_{1g}$

$\twoheadrightarrow$

T **44**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{5d}$	A_{1u}	A_{2u}	E_{1u}	E_{2u}
A_{1g}	A_{1u}	A_{2u}	E_{1u}	E_{2u}
A_{2g}	A_{2u}	A_{1u}	E_{1u}	E_{2u}
E_{1g}	E_{1u}	E_{1u}	$A_{1u} \oplus A_{2u} \oplus E_{2u}$	$E_{1u} \oplus E_{2u}$
E_{2g}	E_{2u}	E_{2u}	$E_{1u} \oplus E_{2u}$	$A_{1u} \oplus A_{2u} \oplus E_{1u}$
A_{1u}	A_{1g}	A_{2g}	E_{1g}	E_{2g}
A_{2u}		A_{1g}	E_{1g}	E_{2g}
E_{1u}			$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$	$E_{1g} \oplus E_{2g}$
E_{2u}				$A_{1g} \oplus \{A_{2g}\} \oplus E_{1g}$

$\twoheadrightarrow$

T **44**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{5d}$	$E_{1/2,g}$	$E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$
A_{1g}	$E_{1/2,g}$	$E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$
A_{2g}	$E_{1/2,g}$	$E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{5/2,g}$
E_{1g}	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g}$	$E_{3/2,g}$	$E_{3/2,g}$
E_{2g}	$E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g}$	$E_{1/2,g}$
A_{1u}	$E_{1/2,u}$	$E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{5/2,u}$
A_{2u}	$E_{1/2,u}$	$E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{5/2,u}$
E_{1u}	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus {}^1E_{5/2,u} \oplus {}^2E_{5/2,u}$	$E_{3/2,u}$	$E_{3/2,u}$
E_{2u}	$E_{3/2,u} \oplus {}^1E_{5/2,u} \oplus {}^2E_{5/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u}$	$E_{1/2,u}$
$E_{1/2,g}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$	$E_{1g} \oplus E_{2g}$	E_{2g}	E_{2g}
$E_{3/2,g}$		$\{A_{1g}\} \oplus A_{2g} \oplus E_{2g}$	E_{1g}	E_{1g}
${}^1E_{5/2,g}$			A_{2g}	A_{1g}
${}^2E_{5/2,g}$				A_{2g}

$\twoheadrightarrow$

T **44**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{5d}$	$E_{1/2,u}$	$E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{5/2,u}$
A_{1g}	$E_{1/2,u}$	$E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{5/2,u}$
A_{2g}	$E_{1/2,u}$	$E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{5/2,u}$
E_{1g}	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus {}^1E_{5/2,u} \oplus {}^2E_{5/2,u}$	$E_{3/2,u}$	$E_{3/2,u}$
E_{2g}	$E_{3/2,u} \oplus {}^1E_{5/2,u} \oplus {}^2E_{5/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u}$	$E_{1/2,u}$
A_{1u}	$E_{1/2,g}$	$E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$
A_{2u}	$E_{1/2,g}$	$E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{5/2,g}$
E_{1u}	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g}$	$E_{3/2,g}$	$E_{3/2,g}$
E_{2u}	$E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g}$	$E_{1/2,g}$
$E_{1/2,g}$	$A_{1u} \oplus A_{2u} \oplus E_{1u}$	$E_{1u} \oplus E_{2u}$	E_{2u}	E_{2u}
$E_{3/2,g}$	$E_{1u} \oplus E_{2u}$	$A_{1u} \oplus A_{2u} \oplus E_{2u}$	E_{1u}	E_{1u}
${}^1E_{5/2,g}$	E_{2u}	E_{1u}	A_{2u}	A_{1u}
${}^2E_{5/2,g}$	E_{2u}	E_{1u}	A_{1u}	A_{2u}
$E_{1/2,u}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$	$E_{1g} \oplus E_{2g}$	E_{2g}	E_{2g}
$E_{3/2,u}$		$\{A_{1g}\} \oplus A_{2g} \oplus E_{2g}$	E_{1g}	E_{1g}
${}^1E_{5/2,u}$			A_{2g}	A_{1g}
${}^2E_{5/2,u}$				A_{2g}

T **44**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_{5d}$	$(\mathbf{C}_{2h})$	$(\mathbf{C}_{5v})$	$\mathbf{D}_5$	$\mathbf{S}_{10}$
A_{1g}	A_g	A_1	A_1	A_g
A_{2g}	B_g	A_2	A_2	A_g
E_{1g}	$A_g \oplus B_g$	E_1	E_1	${}^1E_{1g} \oplus {}^2E_{1g}$
E_{2g}	$A_g \oplus B_g$	E_2	E_2	${}^1E_{2g} \oplus {}^2E_{2g}$
A_{1u}	A_u	A_2	A_1	A_u
A_{2u}	B_u	A_1	A_2	A_u
E_{1u}	$A_u \oplus B_u$	E_1	E_1	${}^1E_{1u} \oplus {}^2E_{1u}$
E_{2u}	$A_u \oplus B_u$	E_2	E_2	${}^1E_{2u} \oplus {}^2E_{2u}$
$E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$
$E_{3/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{3/2}$	$E_{3/2}$	${}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$
${}^1E_{5/2,g}$	${}^1E_{1/2,g}$	${}^1E_{5/2}$	${}^1E_{5/2}$	$A_{5/2,g}$
${}^2E_{5/2,g}$	${}^2E_{1/2,g}$	${}^2E_{5/2}$	${}^2E_{5/2}$	$A_{5/2,g}$
$E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$
$E_{3/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{3/2}$	$E_{3/2}$	${}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$
${}^1E_{5/2,u}$	${}^1E_{1/2,u}$	${}^2E_{5/2}$	${}^1E_{5/2}$	$A_{5/2,u}$
${}^2E_{5/2,u}$	${}^2E_{1/2,u}$	${}^1E_{5/2}$	${}^2E_{5/2}$	$A_{5/2,u}$

$\twoheadrightarrow$

T **44**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{5d}$	$(\mathbf{C}_s)$	$\mathbf{C}_i$	$\mathbf{C}_5$	$(\mathbf{C}_2)$
A_{1g}	A'	A_g	A	A
A_{2g}	A''	A_g	A	B
E_{1g}	$A' \oplus A''$	$2A_g$	${}^1E_1 \oplus {}^2E_1$	$A \oplus B$
E_{2g}	$A' \oplus A''$	$2A_g$	${}^1E_2 \oplus {}^2E_2$	$A \oplus B$
A_{1u}	A''	A_u	A	A
A_{2u}	A'	A_u	A	B
E_{1u}	$A' \oplus A''$	$2A_u$	${}^1E_1 \oplus {}^2E_1$	$A \oplus B$
E_{2u}	$A' \oplus A''$	$2A_u$	${}^1E_2 \oplus {}^2E_2$	$A \oplus B$
$E_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,g}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1E_{5/2,g}$	${}^1E_{1/2}$	$A_{1/2,g}$	$A_{5/2}$	${}^1E_{1/2}$
${}^2E_{5/2,g}$	${}^2E_{1/2}$	$A_{1/2,g}$	$A_{5/2}$	${}^2E_{1/2}$
$E_{1/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,u}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1E_{5/2,u}$	${}^2E_{1/2}$	$A_{1/2,u}$	$A_{5/2}$	${}^1E_{1/2}$
${}^2E_{5/2,u}$	${}^1E_{1/2}$	$A_{1/2,u}$	$A_{5/2}$	${}^2E_{1/2}$

T **44**.10 ♣ Subduction from O(3) § **16**–10, p. 82

j	$\mathbf{D}_{5d}$
$10n$	$(2n+1)\,A_{1g} \oplus 2n\,(A_{2g} \oplus 2E_{1g} \oplus 2E_{2g})$
$10n+1$	$2n\,(A_{1u} \oplus E_{1u} \oplus 2E_{2u}) \oplus (2n+1)(A_{2u} \oplus E_{1u})$
$10n+2$	$(2n+1)(A_{1g} \oplus E_{1g} \oplus E_{2g}) \oplus 2n\,(A_{2g} \oplus E_{1g} \oplus E_{2g})$
$10n+3$	$2n\,(A_{1u} \oplus E_{1u}) \oplus (2n+1)(A_{2u} \oplus E_{1u} \oplus 2E_{2u})$
$10n+4$	$(2n+1)(A_{1g} \oplus 2E_{1g} \oplus 2E_{2g}) \oplus 2n\,A_{2g}$
$10n+5$	$(2n+1)(A_{1u} \oplus 2E_{1u} \oplus 2E_{2u}) \oplus (2n+2)\,A_{2u}$
$10n+6$	$(2n+2)(A_{1g} \oplus E_{1g}) \oplus (2n+1)(A_{2g} \oplus E_{1g} \oplus 2E_{2g})$
$10n+7$	$(2n+1)(A_{1u} \oplus E_{1u} \oplus E_{2u}) \oplus (2n+2)(A_{2u} \oplus E_{1u} \oplus E_{2u})$
$10n+8$	$(2n+2)(A_{1g} \oplus E_{1g} \oplus 2E_{2g}) \oplus (2n+1)(A_{2g} \oplus E_{1g})$
$10n+9$	$(2n+1)\,A_{1u} \oplus (2n+2)(A_{2u} \oplus 2E_{1u} \oplus 2E_{2u})$
$5n+\frac{1}{2}$	$(2n+1)\,E_{1/2,g} \oplus n\,(2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g})$
$5n+\frac{3}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g}) \oplus n\,({}^1E_{5/2,g} \oplus {}^2E_{5/2,g})$
$5n+\frac{5}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g}) \oplus (n+1)({}^1E_{5/2,g} \oplus {}^2E_{5/2,g})$
$5n+\frac{7}{2}$	$(2n+1)\,E_{1/2,g} \oplus (n+1)(2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g})$
$5n+\frac{9}{2}$	$(n+1)(2E_{1/2,g} \oplus 2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g})$

$n = 0, 1, 2, \ldots$

T **44**.11 Clebsch–Gordan coefficients

Use T **25**.11 ▪. § **16**–11, p. 83

(1) Product forms: $\mathbf{D}_6 \otimes \mathbf{C}_s$.

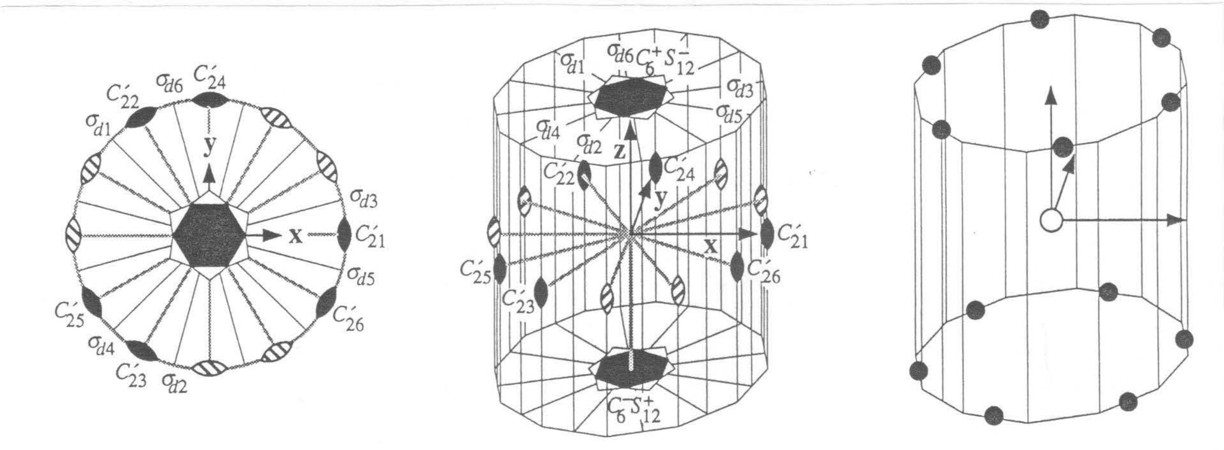
(2) Group chains: $\mathbf{D}_{6d} \supset (\underline{\mathbf{C}_{6v}})$, $\mathbf{D}_{6d} \supset (\mathbf{D}_{2d})$, $\mathbf{D}_{6d} \supset (\underline{\mathbf{D}_6})$, $\mathbf{D}_{6d} \supset \underline{\mathbf{S}_{12}}$.

(3) Operations of G: E, (C_6^+, C_6^-), (C_3^+, C_3^-), C_2, $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26})$,
$(S_{12}^{5-}, S_{12}^{5+})$, (S_4^-, S_4^+), (S_{12}^-, S_{12}^+), $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \sigma_{d6})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_6^+, C_6^-), $(\widetilde{C}_6^+, \widetilde{C}_6^-)$, (C_3^+, C_3^-), $(\widetilde{C}_3^+, \widetilde{C}_3^-)$, $(C_2, \widetilde{C}_2)$,
$(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, \widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24}, \widetilde{C}'_{25}, \widetilde{C}'_{26})$,
$(S_{12}^{5-}, S_{12}^{5+})$, $(\widetilde{S}_{12}^{5-}, \widetilde{S}_{12}^{5+})$, (S_4^-, S_4^+), $(\widetilde{S}_4^-, \widetilde{S}_4^+)$, (S_{12}^-, S_{12}^+), $(\widetilde{S}_{12}^-, \widetilde{S}_{12}^+)$,
$(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \sigma_{d6}, \widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3}, \widetilde{\sigma}_{d4}, \widetilde{\sigma}_{d5}, \widetilde{\sigma}_{d6})$.

(5) Classes and representations: $|r| = 6$, $|\text{i}| = 3$, $|I| = 9$, $|\widetilde{I}| = 6$.

F **45** See Chapter **15**, p. 65



Examples: $Cr(C_6H_6)_2$ (benzene rings staggered).

T **45**.0 Subgroup elements

§ **16**–0, p. 68

$\mathbf{D}_{6d}$	$\mathbf{D}_{2d}$	$\mathbf{D}_{6}$	$\mathbf{D}_{3}$	$\mathbf{D}_{2}$	$\mathbf{S}_{12}$	$\mathbf{S}_{4}$	$\mathbf{C}_{6}$	$\mathbf{C}_{3}$	$\mathbf{C}_{2}$
E	E	E	E	E	E	E	E	E	E
C_6^+		C_6^+			C_6^+		C_6^+		
C_6^-		C_6^-			C_6^-		C_6^-		
C_3^+		C_3^+	C_3^+		C_3^+		C_3^+	C_3^+	
C_3^-		C_3^-	C_3^-		C_3^-		C_3^-	C_3^-	
C_2	C_2	C_2		C_{2z}	C_2	C_2	C_2		C_2
C'_{21}	C'_{21}	C'_{21}	C'_{21}	C_{2x}					
C'_{22}		C'_{22}	C'_{22}						
C'_{23}		C'_{23}	C'_{23}						
C'_{24}	C'_{22}	C''_{21}		C_{2y}					
C'_{25}		C''_{22}							
C'_{26}		C''_{23}							
S_{12}^{5-}					S_{12}^{5-}				
S_{12}^{5+}					S_{12}^{5+}				
S_4^-	S_4^-				S_4^-	S_4^-			
S_4^+	S_4^+				S_4^+	S_4^+			
S_{12}^-					S_{12}^-				
S_{12}^+					S_{12}^+				
σ_{d1}	σ_{d1}								
σ_{d2}									
σ_{d3}									
σ_{d4}	σ_{d2}								
σ_{d5}									
σ_{d6}									

T **45**.1 Parameters

§ **16**–1, p. 68

$\mathbf{D}_{6d}$	α	β	γ	ϕ	$\mathbf{n}$	λ	$\mathbf{\Lambda}$
E	0	0	0	0	$(0\ \ 0\ \ 0)$	$[\![\,1,$	$(0\ \ 0\ \ 0)]\!]$
C_6^+	0	0	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$(0\ \ 0\ \ 1)$	$[\![\frac{\sqrt{3}}{2},$	$(0\ \ 0\ \ \frac{1}{2})]\!]$
C_6^-	0	0	$-\frac{\pi}{3}$	$\frac{\pi}{3}$	$(0\ \ 0\ -1)$	$[\![\frac{\sqrt{3}}{2},$	$(0\ \ 0\ -\frac{1}{2})]\!]$
C_3^+	0	0	$\frac{2\pi}{3}$	$\frac{2\pi}{3}$	$(0\ \ 0\ \ 1)$	$[\![\,\frac{1}{2},$	$(0\ \ 0\ \ \frac{\sqrt{3}}{2})]\!]$
C_3^-	0	0	$-\frac{2\pi}{3}$	$\frac{2\pi}{3}$	$(0\ \ 0\ -1)$	$[\![\,\frac{1}{2},$	$(0\ \ 0\ -\frac{\sqrt{3}}{2})]\!]$
C_2	0	0	π	π	$(0\ \ 0\ \ 1)$	$[\![\,0,$	$(0\ \ 0\ \ 1)]\!]$
C'_{21}	0	π	π	π	$(1\ \ 0\ \ 0)$	$[\![\,0,$	$(1\ \ 0\ \ 0)]\!]$
C'_{22}	0	π	$-\frac{\pi}{3}$	π	$(-\frac{1}{2}\ \ \frac{\sqrt{3}}{2}\ \ 0)$	$[\![\,0,$	$(-\frac{1}{2}\ \ \frac{\sqrt{3}}{2}\ \ 0)]\!]$
C'_{23}	0	π	$\frac{\pi}{3}$	π	$(-\frac{1}{2}\ -\frac{\sqrt{3}}{2}\ \ 0)$	$[\![\,0,$	$(-\frac{1}{2}\ -\frac{\sqrt{3}}{2}\ \ 0)]\!]$
C'_{24}	0	π	0	π	$(0\ \ 1\ \ 0)$	$[\![\,0,$	$(0\ \ 1\ \ 0)]\!]$
C'_{25}	0	π	$\frac{2\pi}{3}$	π	$(-\frac{\sqrt{3}}{2}\ -\frac{1}{2}\ \ 0)$	$[\![\,0,$	$(-\frac{\sqrt{3}}{2}\ -\frac{1}{2}\ \ 0)]\!]$
C'_{26}	0	π	$-\frac{2\pi}{3}$	π	$(\frac{\sqrt{3}}{2}\ -\frac{1}{2}\ \ 0)$	$[\![\,0,$	$(\frac{\sqrt{3}}{2}\ -\frac{1}{2}\ \ 0)]\!]$
S_{12}^{5-}	0	0	$\frac{\pi}{6}$	$\frac{\pi}{6}$	$(0\ \ 0\ \ 1)$	$[\![c_{12},$	$(0\ \ 0\ \ s_{12})]\!]$
S_{12}^{5+}	0	0	$-\frac{\pi}{6}$	$\frac{\pi}{6}$	$(0\ \ 0\ -1)$	$[\![c_{12},$	$(0\ \ 0\ -s_{12})]\!]$
S_4^-	0	0	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$(0\ \ 0\ \ 1)$	$[\![\frac{1}{\sqrt{2}},$	$(0\ \ 0\ \ \frac{1}{\sqrt{2}})]\!]$
S_4^+	0	0	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	$(0\ \ 0\ -1)$	$[\![\frac{1}{\sqrt{2}},$	$(0\ \ 0\ -\frac{1}{\sqrt{2}})]\!]$
S_{12}^-	0	0	$\frac{5\pi}{6}$	$\frac{5\pi}{6}$	$(0\ \ 0\ \ 1)$	$[\![s_{12},$	$(0\ \ 0\ \ c_{12})]\!]$
S_{12}^+	0	0	$-\frac{5\pi}{6}$	$\frac{5\pi}{6}$	$(0\ \ 0\ -1)$	$[\![s_{12},$	$(0\ \ 0\ -c_{12})]\!]$
σ_{d1}	0	π	$\frac{\pi}{2}$	π	$(\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)$	$[\![\,0,$	$(\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)]\!]$
σ_{d2}	0	π	$-\frac{5\pi}{6}$	π	$(-c_{12}\ \ s_{12}\ \ 0)$	$[\![\,0,$	$(-c_{12}\ \ s_{12}\ \ 0)]\!]$
σ_{d3}	0	π	$-\frac{\pi}{6}$	π	$(s_{12}\ -c_{12}\ \ 0)$	$[\![\,0,$	$(s_{12}\ -c_{12}\ \ 0)]\!]$
σ_{d4}	0	π	$-\frac{\pi}{2}$	π	$(-\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)$	$[\![\,0,$	$(-\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)]\!]$
σ_{d5}	0	π	$\frac{\pi}{6}$	π	$(-s_{12}\ -c_{12}\ \ 0)$	$[\![\,0,$	$(-s_{12}\ -c_{12}\ \ 0)]\!]$
σ_{d6}	0	π	$\frac{5\pi}{6}$	π	$(c_{12}\ \ s_{12}\ \ 0)$	$[\![\,0,$	$(c_{12}\ \ s_{12}\ \ 0)]\!]$

$c_n = \cos\frac{\pi}{n}$, $s_n = \sin\frac{\pi}{n}$

T **45**.2 Multiplication table

§ **16**–2, p. 69

$\mathbf{D}_{6d}$	E	C_6^+	C_6^-	C_3^+	C_3^-	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	S_{12}^{5-}	S_{12}^{5+}	S_4^-	S_4^+	S_{12}^-	S_{12}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}
E	E	C_6^+	C_6^-	C_3^+	C_3^-	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	S_{12}^{5-}	S_{12}^{5+}	S_4^-	S_4^+	S_{12}^-	S_{12}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}
C_6^+	C_6^+	C_3^+	E	C_2	C_6^-	C_3^-	C'_{25}	C'_{26}	C'_{24}	C'_{22}	C'_{23}	C'_{21}	S_4^-	S_{12}^{5-}	S_{12}^-	S_{12}^{5+}	S_{12}^+	S_4^+	σ_{d5}	σ_{d6}	σ_{d4}	σ_{d2}	σ_{d3}	σ_{d1}
C_6^-	C_6^-	E	C_3^-	C_6^+	C_2	C_3^+	C'_{26}	C'_{24}	C'_{25}	C'_{23}	C'_{21}	C'_{22}	S_{12}^{5+}	S_4^+	S_{12}^{5-}	S_{12}^+	S_4^-	S_{12}^-	σ_{d6}	σ_{d4}	σ_{d5}	σ_{d3}	σ_{d1}	σ_{d2}
C_3^+	C_3^+	C_2	C_6^+	C_3^-	E	C_6^-	C'_{23}	C'_{21}	C'_{22}	C'_{26}	C'_{24}	C'_{25}	S_{12}^-	S_4^-	S_{12}^+	S_{12}^{5-}	S_4^+	S_{12}^{5+}	σ_{d3}	σ_{d1}	σ_{d2}	σ_{d6}	σ_{d4}	σ_{d5}
C_3^-	C_3^-	C_6^-	C_2	E	C_3^+	C_6^+	C'_{22}	C'_{23}	C'_{21}	C'_{25}	C'_{26}	C'_{24}	S_4^+	S_{12}^+	S_{12}^{5+}	S_{12}^-	S_{12}^{5-}	S_4^-	σ_{d2}	σ_{d3}	σ_{d1}	σ_{d5}	σ_{d6}	σ_{d4}
C_2	C_2	C_3^-	C_3^+	C_6^-	C_6^+	E	C'_{24}	C'_{25}	C'_{26}	C'_{21}	C'_{22}	C'_{23}	S_{12}^+	S_{12}^-	S_4^+	S_4^-	S_{12}^{5+}	S_{12}^{5-}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d1}	σ_{d2}	σ_{d3}
C'_{21}	C'_{21}	C'_{26}	C'_{25}	C'_{22}	C'_{23}	C'_{24}	E	C_3^+	C_3^-	C_2	C_6^-	C_6^+	σ_{d2}	σ_{d6}	σ_{d4}	σ_{d1}	σ_{d3}	σ_{d5}	S_4^+	S_{12}^{5-}	S_{12}^-	S_4^-	S_{12}^+	S_{12}^{5+}
C'_{22}	C'_{22}	C'_{24}	C'_{26}	C'_{23}	C'_{21}	C'_{25}	C_3^-	E	C_3^+	C_6^+	C_2	C_6^-	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d2}	σ_{d1}	σ_{d6}	S_{12}^-	S_4^+	S_{12}^{5-}	S_{12}^{5+}	S_4^-	S_{12}^+
C'_{23}	C'_{23}	C'_{25}	C'_{24}	C'_{21}	C'_{22}	C'_{26}	C_3^+	C_3^-	E	C_6^-	C_6^+	C_2	σ_{d1}	σ_{d5}	σ_{d6}	σ_{d3}	σ_{d2}	σ_{d4}	S_{12}^{5-}	S_{12}^-	S_4^+	S_{12}^+	S_{12}^{5+}	S_4^-
C'_{24}	C'_{24}	C'_{23}	C'_{22}	C'_{25}	C'_{26}	C'_{21}	C_2	C_6^-	C_6^+	E	C_3^+	C_3^-	σ_{d5}	σ_{d3}	σ_{d1}	σ_{d4}	σ_{d6}	σ_{d2}	S_4^-	S_{12}^+	S_{12}^{5+}	S_4^+	S_{12}^{5-}	S_{12}^-
C'_{25}	C'_{25}	C'_{21}	C'_{23}	C'_{26}	C'_{24}	C'_{22}	C_6^+	C_2	C_6^-	C_3^-	E	C_3^+	σ_{d6}	σ_{d1}	σ_{d2}	σ_{d5}	σ_{d4}	σ_{d3}	S_{12}^{5+}	S_4^-	S_{12}^+	S_{12}^-	S_4^+	S_{12}^{5-}
C'_{26}	C'_{26}	C'_{22}	C'_{21}	C'_{24}	C'_{25}	C'_{23}	C_6^-	C_6^+	C_2	C_3^+	C_3^-	E	σ_{d4}	σ_{d2}	σ_{d3}	σ_{d6}	σ_{d5}	σ_{d1}	S_{12}^+	S_{12}^{5+}	S_4^-	S_{12}^{5-}	S_{12}^-	S_4^+
S_{12}^{5-}	S_{12}^{5-}	S_4^-	S_{12}^{5+}	S_{12}^-	S_4^+	S_{12}^+	σ_{d6}	σ_{d4}	σ_{d5}	σ_{d3}	σ_{d1}	σ_{d2}	C_6^+	E	C_3^+	C_6^-	C_2	C_3^-	C'_{23}	C'_{21}	C'_{22}	C'_{26}	C'_{24}	C'_{25}
S_{12}^{5+}	S_{12}^{5+}	S_{12}^{5-}	S_4^+	S_4^-	S_{12}^+	S_{12}^-	σ_{d2}	σ_{d3}	σ_{d1}	σ_{d5}	σ_{d6}	σ_{d4}	E	C_6^-	C_6^+	C_3^-	C_3^+	C_2	C'_{25}	C'_{26}	C'_{24}	C'_{22}	C'_{23}	C'_{21}
S_4^-	S_4^-	S_{12}^-	S_{12}^{5-}	S_{12}^+	S_{12}^{5+}	S_4^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	C_3^+	C_6^+	C_2	E	C_3^-	C_6^-	C'_{24}	C'_{25}	C'_{26}	C'_{21}	C'_{22}	C'_{23}
S_4^+	S_4^+	S_{12}^{5+}	S_{12}^+	S_{12}^{5-}	S_{12}^-	S_4^-	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d1}	σ_{d2}	σ_{d3}	C_6^-	C_3^-	E	C_2	C_6^+	C_3^+	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}
S_{12}^-	S_{12}^-	S_{12}^+	S_4^-	S_4^+	S_{12}^{5-}	S_{12}^{5+}	σ_{d5}	σ_{d6}	σ_{d4}	σ_{d2}	σ_{d3}	σ_{d1}	C_2	C_3^+	C_3^-	C_6^+	C_6^-	E	C'_{22}	C'_{23}	C'_{21}	C'_{25}	C'_{26}	C'_{24}
S_{12}^+	S_{12}^+	S_4^+	S_{12}^-	S_{12}^{5+}	S_4^-	S_{12}^{5-}	σ_{d3}	σ_{d1}	σ_{d2}	σ_{d6}	σ_{d4}	σ_{d5}	C_3^-	C_2	C_6^-	C_3^+	E	C_6^+	C'_{26}	C'_{24}	C'_{25}	C'_{23}	C'_{21}	C'_{22}
σ_{d1}	σ_{d1}	σ_{d6}	σ_{d5}	σ_{d2}	σ_{d3}	σ_{d4}	S_4^-	S_{12}^+	S_{12}^{5+}	S_4^+	S_{12}^{5-}	S_{12}^-	C'_{25}	C'_{23}	C'_{21}	C'_{24}	C'_{26}	C'_{22}	E	C_3^+	C_3^-	C_2	C_6^-	C_6^+
σ_{d2}	σ_{d2}	σ_{d4}	σ_{d6}	σ_{d3}	σ_{d1}	σ_{d5}	S_{12}^{5+}	S_4^-	S_{12}^+	S_{12}^-	S_4^+	S_{12}^{5-}	C'_{26}	C'_{21}	C'_{22}	C'_{25}	C'_{24}	C'_{23}	C_3^-	E	C_3^+	C_6^+	C_2	C_6^-
σ_{d3}	σ_{d3}	σ_{d5}	σ_{d4}	σ_{d1}	σ_{d2}	σ_{d6}	S_{12}^+	S_{12}^{5+}	S_4^-	S_{12}^{5-}	S_{12}^-	S_4^+	C'_{24}	C'_{22}	C'_{23}	C'_{26}	C'_{25}	C'_{21}	C_3^+	C_3^-	E	C_6^-	C_6^+	C_2
σ_{d4}	σ_{d4}	σ_{d3}	σ_{d2}	σ_{d5}	σ_{d6}	σ_{d1}	S_4^+	S_{12}^{5-}	S_{12}^-	S_4^-	S_{12}^+	S_{12}^{5+}	C'_{22}	C'_{26}	C'_{24}	C'_{21}	C'_{23}	C'_{25}	C_2	C_6^-	C_6^+	E	C_3^+	C_3^-
σ_{d5}	σ_{d5}	σ_{d1}	σ_{d3}	σ_{d6}	σ_{d4}	σ_{d2}	S_{12}^-	S_4^+	S_{12}^{5-}	S_{12}^{5+}	S_4^-	S_{12}^+	C'_{23}	C'_{24}	C'_{25}	C'_{22}	C'_{21}	C'_{26}	C_6^+	C_2	C_6^-	C_3^-	E	C_3^+
σ_{d6}	σ_{d6}	σ_{d2}	σ_{d1}	σ_{d4}	σ_{d5}	σ_{d3}	S_{12}^{5-}	S_{12}^-	S_4^+	S_{12}^+	S_{12}^{5+}	S_4^-	C'_{21}	C'_{25}	C'_{26}	C'_{23}	C'_{22}	C'_{24}	C_6^-	C_6^+	C_2	C_3^+	C_3^-	E

T **45**.3 Factor table

§ **16**–3, p. 70

$\mathbf{D}_{6d}$	E	C_6^+	C_6^-	C_3^+	C_3^-	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	S_{12}^{5-}	S_{12}^{5+}	S_4^-	S_4^+	S_{12}^-	S_{12}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_6^+	1	1	1	1	1	−1	−1	−1	−1	1	1	1	1	1	1	1	−1	1	−1	−1	−1	1	1	1
C_6^-	1	1	1	1	−1	1	1	1	1	−1	−1	−1	1	1	1	1	1	−1	1	1	1	−1	−1	−1
C_3^+	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1
C_3^-	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1
C_2	1	−1	1	−1	1	−1	1	1	1	−1	−1	−1	−1	1	−1	1	−1	1	1	1	1	−1	−1	−1
C'_{21}	1	1	−1	−1	−1	−1	−1	1	1	1	1	−1	−1	1	−1	1	1	−1	−1	1	−1	1	1	−1
C'_{22}	1	1	−1	−1	−1	−1	1	−1	1	−1	1	1	−1	1	−1	1	1	−1	−1	−1	1	−1	1	1
C'_{23}	1	1	−1	−1	−1	−1	1	1	−1	1	−1	1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	1
C'_{24}	1	−1	1	−1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	1	1	−1	−1	1	−1	1	−1
C'_{25}	1	−1	1	−1	−1	1	1	−1	−1	1	−1	1	−1	−1	1	1	1	1	1	−1	−1	−1	−1	1
C'_{26}	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1	−1	−1	1	1	1	1	−1	1	−1	1	−1	−1
S_{12}^{5-}	1	1	1	1	1	−1	1	1	1	−1	−1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1
S_{12}^{5+}	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	−1	−1	−1	−1	1	1	1
S_4^-	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	−1	−1	−1
S_4^+	1	1	1	1	−1	1	−1	−1	−1	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1
S_{12}^-	1	−1	1	−1	1	−1	−1	−1	−1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1
S_{12}^+	1	1	−1	1	−1	1	1	1	1	1	1	1	1	−1	1	−1	1	−1	1	1	1	−1	−1	−1
σ_{d1}	1	1	−1	−1	−1	−1	−1	−1	1	−1	1	−1	−1	−1	1	1	1	1	−1	1	1	1	1	−1
σ_{d2}	1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	1	1	1	1	1	−1	1	−1	1	1
σ_{d3}	1	1	−1	−1	−1	−1	−1	1	−1	1	−1	−1	−1	−1	1	1	1	1	1	1	−1	1	−1	1
σ_{d4}	1	−1	1	−1	−1	1	1	−1	1	−1	−1	1	1	−1	1	−1	−1	1	−1	−1	1	−1	1	1
σ_{d5}	1	−1	1	−1	−1	1	1	1	−1	1	−1	−1	1	−1	1	−1	−1	1	1	−1	−1	1	−1	1
σ_{d6}	1	−1	1	−1	−1	1	−1	1	1	−1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1

T **45**.4 Character table

§ **16**–4, p. 71

$\mathbf{D}_{6d}$	E	$2C_6$	$2C_3$	C_2	$6C_2'$	$2S_{12}^5$	$2S_4$	$2S_{12}$	$6\sigma_d$	τ
A_1	1	1	1	1	1	1	1	1	1	a
A_2	1	1	1	1	-1	1	1	1	-1	a
B_1	1	1	1	1	1	-1	-1	-1	-1	a
B_2	1	1	1	1	-1	-1	-1	-1	1	a
E_1	2	1	-1	-2	0	$-\sqrt{3}$	0	$\sqrt{3}$	0	a
E_2	2	-1	-1	2	0	1	-2	1	0	a
E_3	2	-2	2	-2	0	0	0	0	0	a
E_4	2	-1	-1	2	0	-1	2	-1	0	a
E_5	2	1	-1	-2	0	$\sqrt{3}$	0	$-\sqrt{3}$	0	a
$E_{1/2}$	2	$\sqrt{3}$	1	0	0	$2c_{12}$	$\sqrt{2}$	$2c_{12}^5$	0	c
$E_{3/2}$	2	0	-2	0	0	$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	0	c
$E_{5/2}$	2	$-\sqrt{3}$	1	0	0	$2c_{12}^5$	$-\sqrt{2}$	$2c_{12}$	0	c
$E_{7/2}$	2	$-\sqrt{3}$	1	0	0	$-2c_{12}^5$	$\sqrt{2}$	$-2c_{12}$	0	c
$E_{9/2}$	2	0	-2	0	0	$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	0	c
$E_{11/2}$	2	$\sqrt{3}$	1	0	0	$-2c_{12}$	$-\sqrt{2}$	$-2c_{12}^5$	0	c

$c_n^m = \cos \frac{m}{n}\pi$

T **45**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{6d}$	0	1	2	3
A_1	$^{\square}1$		$x^2+y^2, {}^{\square}z^2$	
A_2		R_z		
B_1				
B_2		$^{\square}z$		$(x^2+y^2)z, {}^{\square}z^3$
E_1		$^{\square}(x,y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^{\square}(xz^2, yz^2)$
E_2			$^{\square}(xy, x^2-y^2)$	
E_3				$^{\square}\{x(x^2-3y^2), y(3x^2-y^2)\}$
E_4				$^{\square}\{xyz, z(x^2-y^2)\}$
E_5		(R_x, R_y)	$^{\square}(zx, yz)$	

T **45**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{6d}$	$\langle \lvert j\,m\rangle\rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$	$\lvert 7\,6\rangle_-$	2	12
A_2	$\lvert 7\,6\rangle_+$	$\lvert 12\,12\rangle_-$	2	12
B_1	$\lvert 6\,6\rangle_+$	$\lvert 13\,12\rangle_-$	2	12
B_2	$\lvert 1\,0\rangle_+$	$\lvert 6\,6\rangle_-$	2	12
E_1	$\langle \lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle \rvert$	$\langle \lvert 6\,\overline{5}\rangle, -\lvert 6\,5\rangle \rvert$	2	±12
E_2	$\langle \lvert 2\,\overline{2}\rangle, -\lvert 2\,2\rangle \rvert$	$\langle \lvert 5\,4\rangle, \lvert 5\,\overline{4}\rangle \rvert$	2	±12
E_3	$\langle \lvert 3\,3\rangle, \lvert 3\,\overline{3}\rangle \rvert$	$\langle \lvert 4\,\overline{3}\rangle, -\lvert 4\,3\rangle \rvert$	2	±12
E_4	$\langle \lvert 3\,\overline{2}\rangle, \lvert 3\,2\rangle \rvert$	$\langle \lvert 4\,4\rangle, -\lvert 4\,\overline{4}\rangle \rvert$	2	±12
E_5	$\langle \lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle \rvert$	$\langle \lvert 5\,\overline{5}\rangle, \lvert 5\,5\rangle \rvert$	2	±12
$E_{1/2}$	$\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \rvert$	$\langle \lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle \rvert$	2	±12
	$\langle \lvert \frac{11}{2}\,\overline{\frac{11}{2}}\rangle, -\lvert \frac{11}{2}\,\frac{11}{2}\rangle \rvert^\bullet$	$\langle \lvert \frac{13}{2}\,\overline{\frac{11}{2}}\rangle, \lvert \frac{13}{2}\,\frac{11}{2}\rangle \rvert^\bullet$	2	±12
$E_{3/2}$	$\langle \lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle \rvert$	$\langle \lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle \rvert$	2	±12
	$\langle \lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle, -\lvert \frac{9}{2}\,\frac{9}{2}\rangle \rvert^\bullet$	$\langle \lvert \frac{11}{2}\,\overline{\frac{9}{2}}\rangle, \lvert \frac{11}{2}\,\frac{9}{2}\rangle \rvert^\bullet$	2	±12
$E_{5/2}$	$\langle \lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle, \lvert \frac{5}{2}\,\frac{5}{2}\rangle \rvert$	$\langle \lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle, -\lvert \frac{7}{2}\,\frac{5}{2}\rangle \rvert$	2	±12
	$\langle \lvert \frac{7}{2}\,\frac{7}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle \rvert^\bullet$	$\langle \lvert \frac{9}{2}\,\frac{7}{2}\rangle, \lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle \rvert^\bullet$	2	±12
$E_{7/2}$	$\langle \lvert \frac{7}{2}\,\frac{7}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle \rvert$	$\langle \lvert \frac{9}{2}\,\frac{7}{2}\rangle, \lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle \rvert$	2	±12
	$\langle \lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle, \lvert \frac{5}{2}\,\frac{5}{2}\rangle \rvert^\bullet$	$\langle \lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle, -\lvert \frac{7}{2}\,\frac{5}{2}\rangle \rvert^\bullet$	2	±12
$E_{9/2}$	$\langle \lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle, -\lvert \frac{9}{2}\,\frac{9}{2}\rangle \rvert$	$\langle \lvert \frac{11}{2}\,\overline{\frac{9}{2}}\rangle, \lvert \frac{11}{2}\,\frac{9}{2}\rangle \rvert$	2	±12
	$\langle \lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle \rvert^\bullet$	$\langle \lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle \rvert^\bullet$	2	±12
$E_{11/2}$	$\langle \lvert \frac{11}{2}\,\overline{\frac{11}{2}}\rangle, -\lvert \frac{11}{2}\,\frac{11}{2}\rangle \rvert$	$\langle \lvert \frac{13}{2}\,\overline{\frac{11}{2}}\rangle, \lvert \frac{13}{2}\,\frac{11}{2}\rangle \rvert$	2	±12
	$\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \rvert^\bullet$	$\langle \lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle \rvert^\bullet$	2	±12

T **45**.7 Matrix representations

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$\mathbf{D}_{6d}$	E_1	E_2	E_3	E_4	E_5
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_6^+	$\begin{bmatrix} \bar\eta & 0 \\ 0 & \bar\eta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \bar1 & 0 \\ 0 & \bar1 \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \bar\eta & 0 \\ 0 & \bar\eta^* \end{bmatrix}$
C_6^-	$\begin{bmatrix} \bar\eta^* & 0 \\ 0 & \bar\eta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \bar1 & 0 \\ 0 & \bar1 \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \bar\eta^* & 0 \\ 0 & \bar\eta \end{bmatrix}$
C_3^+	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$
C_3^-	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$
C_2	$\begin{bmatrix} \bar1 & 0 \\ 0 & \bar1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar1 & 0 \\ 0 & \bar1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar1 & 0 \\ 0 & \bar1 \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \bar1 \\ \bar1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar1 \\ \bar1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar1 \\ \bar1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar1 \\ \bar1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar1 \\ \bar1 & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \bar\eta \\ \bar\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta \\ \bar\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar1 \\ \bar1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta \\ \bar\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta \\ \bar\eta^* & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \bar\eta^* \\ \bar\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta^* \\ \bar\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar1 \\ \bar1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta^* \\ \bar\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta^* \\ \bar\eta & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar1 \\ \bar1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar1 \\ \bar1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
C'_{25}	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta \\ \bar\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta \\ \bar\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$
C'_{26}	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta^* \\ \bar\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta^* \\ \bar\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$
S_{12}^{5-}	$\begin{bmatrix} \mathrm{i}\bar\eta^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \bar\eta^* & 0 \\ 0 & \bar\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\bar\eta \end{bmatrix}$
S_{12}^{5+}	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\bar\eta^* \end{bmatrix}$	$\begin{bmatrix} \bar\eta & 0 \\ 0 & \bar\eta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\eta & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$
S_4^-	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar1 & 0 \\ 0 & \bar1 \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$
S_4^+	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar1 & 0 \\ 0 & \bar1 \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$
S_{12}^-	$\begin{bmatrix} \mathrm{i}\bar\eta & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \bar\eta & 0 \\ 0 & \bar\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\bar\eta^* \end{bmatrix}$
S_{12}^+	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\bar\eta \end{bmatrix}$	$\begin{bmatrix} \bar\eta^* & 0 \\ 0 & \bar\eta \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\eta^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$
σ_{d1}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar1 \\ \bar1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
σ_{d2}	$\begin{bmatrix} 0 & \mathrm{i}\bar\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta \\ \bar\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\bar\eta^* & 0 \end{bmatrix}$
σ_{d3}	$\begin{bmatrix} 0 & \mathrm{i}\bar\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta^* \\ \bar\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\bar\eta & 0 \end{bmatrix}$
σ_{d4}	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar1 \\ \bar1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$
σ_{d5}	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\bar\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta \\ \bar\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$
σ_{d6}	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\bar\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta^* \\ \bar\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/24)$, $\epsilon = \exp(2\pi\mathrm{i}/8)$, $\eta = \exp(2\pi\mathrm{i}/3)$ ⇛

T **45**.7 Matrix representations *(cont.)*

$\mathbf{D}_{6d}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$	$E_{11/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_6^+	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\overline{\eta} \end{bmatrix}$
C_6^-	$\begin{bmatrix} \mathrm{i}\overline{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$
C_3^+	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$
C_3^-	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$
C_2	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$
C'_{25}	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \eta & 0 \end{bmatrix}$
C'_{26}	$\begin{bmatrix} 0 & \overline{\eta} \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \eta^* & 0 \end{bmatrix}$
S_{12}^{5-}	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta}^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \overline{\delta}^* & 0 \\ 0 & \overline{\delta} \end{bmatrix}$
S_{12}^{5+}	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\overline{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\delta} & 0 \\ 0 & \overline{\delta}^* \end{bmatrix}$
S_4^-	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$
S_4^+	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$
S_{12}^-	$\begin{bmatrix} \mathrm{i}\overline{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon}^* & 0 \\ 0 & \overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\delta} & 0 \\ 0 & \overline{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\overline{\delta}^* \end{bmatrix}$
S_{12}^+	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \overline{\delta}^* & 0 \\ 0 & \overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta}^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$
σ_{d1}	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \overline{\epsilon}^* & 0 \end{bmatrix}$
σ_{d2}	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$
σ_{d3}	$\begin{bmatrix} 0 & \delta^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \delta & 0 \end{bmatrix}$
σ_{d4}	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \overline{\epsilon} & 0 \end{bmatrix}$
σ_{d5}	$\begin{bmatrix} 0 & \delta \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \delta^* & 0 \end{bmatrix}$
σ_{d6}	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/24)$, $\epsilon = \exp(2\pi\mathrm{i}/8)$, $\eta = \exp(2\pi\mathrm{i}/3)$

T **45**.8 Direct products of representations

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$\mathbf{D}_{6d}$	A_1	A_2	B_1	B_2	E_1	E_2	E_3	E_4	E_5
A_1	A_1	A_2	B_1	B_2	E_1	E_2	E_3	E_4	E_5
A_2		A_1	B_2	B_1	E_1	E_2	E_3	E_4	E_5
B_1			A_1	A_2	E_5	E_4	E_3	E_2	E_1
B_2				A_1	E_5	E_4	E_3	E_2	E_1
E_1					$A_1 \oplus \{A_2\} \oplus E_2$	$E_1 \oplus E_3$	$E_2 \oplus E_4$	$E_3 \oplus E_5$	$B_1 \oplus B_2 \oplus E_4$
E_2						$A_1 \oplus \{A_2\} \oplus E_4$	$E_1 \oplus E_5$	$\oplus E_2 \oplus E_2$	$E_3 \oplus E_5$
E_3							$A_1 \oplus \{A_2\} \oplus B_1 \oplus B_2$	$E_1 \oplus E_5$	$E_2 \oplus E_4$
E_4								$A_1 \oplus \{A_2\} \oplus E_4$	$E_1 \oplus E_3$
E_5									$A_1 \oplus \{A_2\} \oplus E_2$

$\twoheadrightarrow$

T **45**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{6d}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$	$E_{11/2}$
A_1	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$	$E_{11/2}$
A_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$	$E_{11/2}$
B_1	$E_{11/2}$	$E_{9/2}$	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
B_2	$E_{11/2}$	$E_{9/2}$	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
E_1	$E_{9/2} \oplus E_{11/2}$	$E_{7/2} \oplus E_{11/2}$	$E_{5/2} \oplus E_{9/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$
E_2	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{9/2}$	$E_{3/2} \oplus E_{11/2}$	$E_{5/2} \oplus E_{11/2}$	$E_{7/2} \oplus E_{9/2}$
E_3	$E_{5/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{9/2}$	$E_{1/2} \oplus E_{11/2}$	$E_{1/2} \oplus E_{11/2}$	$E_{3/2} \oplus E_{9/2}$	$E_{5/2} \oplus E_{7/2}$
E_4	$E_{7/2} \oplus E_{9/2}$	$E_{5/2} \oplus E_{11/2}$	$E_{3/2} \oplus E_{11/2}$	$E_{1/2} \oplus E_{9/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{5/2}$
E_5	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{5/2} \oplus E_{9/2}$	$E_{7/2} \oplus E_{11/2}$	$E_{9/2} \oplus E_{11/2}$
$E_{1/2}$	$\{A_1\} \oplus A_2 \oplus E_5$	$E_2 \oplus E_5$	$E_2 \oplus E_3$	$E_3 \oplus E_4$	$E_1 \oplus E_4$	$B_1 \oplus B_2 \oplus E_1$
$E_{3/2}$		$\{A_1\} \oplus A_2 \oplus E_3$	$E_4 \oplus E_5$	$E_1 \oplus E_2$	$B_1 \oplus B_2 \oplus E_3$	$E_1 \oplus E_4$
$E_{5/2}$			$\{A_1\} \oplus A_2 \oplus E_1$	$B_1 \oplus B_2 \oplus E_5$	$E_1 \oplus E_2$	$E_3 \oplus E_4$
$E_{7/2}$				$\{A_1\} \oplus A_2 \oplus E_1$	$E_4 \oplus E_5$	$E_2 \oplus E_3$
$E_{9/2}$					$\{A_1\} \oplus A_2 \oplus E_3$	$E_2 \oplus E_5$
$E_{11/2}$						$\{A_1\} \oplus A_2 \oplus E_5$

T **45**.9 Subduction (descent of symmetry)

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$\mathbf{D}_{6d}$	$(\mathbf{C}_{6v})$	$(\mathbf{C}_{3v})$	$(\mathbf{C}_{2v})$	$(\mathbf{D}_{2d})$	$(\mathbf{D}_6)$
A_1	A_1	A_1	A_1	A_1	A_1
A_2	A_2	A_2	A_2	A_2	A_2
B_1	A_2	A_2	A_2	B_1	A_1
B_2	A_1	A_1	A_1	B_2	A_2
E_1	E_1	E	$B_1 \oplus B_2$	E	E_1
E_2	E_2	E	$A_1 \oplus A_2$	$B_1 \oplus B_2$	E_2
E_3	$B_1 \oplus B_2$	$A_1 \oplus A_2$	$B_1 \oplus B_2$	E	$B_1 \oplus B_2$
E_4	E_2	E	$A_1 \oplus A_2$	$A_1 \oplus A_2$	E_2
E_5	E_1	E	$B_1 \oplus B_2$	E	E_1
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{3/2}$	$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{1/2}$	$E_{3/2}$	$E_{3/2}$
$E_{5/2}$	$E_{5/2}$	$E_{1/2}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
$E_{7/2}$	$E_{5/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{5/2}$
$E_{9/2}$	$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{1/2}$	$E_{1/2}$	$E_{3/2}$
$E_{11/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{3/2}$	$E_{1/2}$

$\twoheadrightarrow$

T **45**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{6d}$	$(\mathbf{D}_3)$	$(\mathbf{D}_2)$	$\mathbf{S}_{12}$	$\mathbf{S}_4$	$(\mathbf{C}_s)$
A_1	A_1	A	A	A	A'
A_2	A_2	B_1	A	A	A''
B_1	A_1	A	B	B	A''
B_2	A_2	B_1	B	B	A'
E_1	E	$B_2 \oplus B_3$	${}^1E_1 \oplus {}^2E_1$	${}^1E \oplus {}^2E$	$A' \oplus A''$
E_2	E	$A \oplus B_1$	${}^1E_2 \oplus {}^2E_2$	$2B$	$A' \oplus A''$
E_3	$A_1 \oplus A_2$	$B_2 \oplus B_3$	${}^1E_3 \oplus {}^2E_3$	${}^1E \oplus {}^2E$	$A' \oplus A''$
E_4	E	$A \oplus B_1$	${}^1E_4 \oplus {}^2E_4$	$2A$	$A' \oplus A''$
E_5	E	$B_2 \oplus B_3$	${}^1E_5 \oplus {}^2E_5$	${}^1E \oplus {}^2E$	$A' \oplus A''$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{9/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{1/2}$	${}^1E_{9/2} \oplus {}^2E_{9/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{11/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{11/2} \oplus {}^2E_{11/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

$\twoheadrightarrow$

T 45.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{6d}$	$\mathbf{C}_6$	$\mathbf{C}_3$	$\mathbf{C}_2$	$(\mathbf{C}_2)$
			C_2	C_2'
A_1	A	A	A	A
A_2	A	A	A	B
B_1	A	A	A	A
B_2	A	A	A	B
E_1	${}^1E_1 \oplus {}^2E_1$	${}^1E \oplus {}^2E$	$2B$	$A \oplus B$
E_2	${}^1E_2 \oplus {}^2E_2$	${}^1E \oplus {}^2E$	$2A$	$A \oplus B$
E_3	$2B$	$2A$	$2B$	$A \oplus B$
E_4	${}^1E_2 \oplus {}^2E_2$	${}^1E \oplus {}^2E$	$2A$	$A \oplus B$
E_5	${}^1E_1 \oplus {}^2E_1$	${}^1E \oplus {}^2E$	$2B$	$A \oplus B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$2A_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{9/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$2A_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{11/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T 45.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{6d}$
$12n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5)$
$12n+1$	$n\,(A_1 \oplus A_2 \oplus B_1 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5) \oplus (n+1)(B_2 \oplus E_1)$
$12n+2$	$(n+1)(A_1 \oplus E_2 \oplus E_5) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus 2E_3 \oplus 2E_4 \oplus E_5)$
$12n+3$	$n\,(A_1 \oplus A_2 \oplus B_1 \oplus E_1 \oplus 2E_2 \oplus E_3 \oplus E_4 \oplus 2E_5) \oplus (n+1)(B_2 \oplus E_1 \oplus E_3 \oplus E_4)$
$12n+4$	$(n+1)(A_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5)$
$12n+5$	$n\,(A_1 \oplus A_2 \oplus B_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5) \oplus (n+1)(B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5)$
$12n+6$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5) \oplus n\,(A_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5)$
$12n+7$	$(n+1)(A_1 \oplus A_2 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus 2E_5) \oplus n\,(B_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4)$
$12n+8$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus E_3 \oplus 2E_4 \oplus E_5) \oplus n\,(A_2 \oplus E_2 \oplus E_3 \oplus E_5)$
$12n+9$	$(n+1)(A_1 \oplus A_2 \oplus B_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus E_4 \oplus 2E_5) \oplus n\,(B_1 \oplus E_1 \oplus E_4)$
$12n+10$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4 \oplus E_5) \oplus n\,(A_2 \oplus E_5)$
$12n+11$	$(n+1)(A_1 \oplus A_2 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5) \oplus n\,B_1$
$12n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2})$
$12n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus 2n\,(E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2})$
$12n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus 2n\,(E_{7/2} \oplus E_{9/2} \oplus E_{11/2})$
$12n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus 2n\,(E_{9/2} \oplus E_{11/2})$
$12n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2}) \oplus 2n\,E_{11/2}$
$12n+\frac{11}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2})$
$12n+\frac{13}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2}) \oplus (2n+2)\,E_{11/2}$
$12n+\frac{15}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus (2n+2)(E_{9/2} \oplus E_{11/2})$
$12n+\frac{17}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus (2n+2)(E_{7/2} \oplus E_{9/2} \oplus E_{11/2})$
$12n+\frac{19}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (2n+2)(E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2})$
$12n+\frac{21}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2})$
$12n+\frac{23}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2})$

$n = 0, 1, 2, \ldots$

T **45**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{D}_{6d}$

a_2	e_1	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_2	E_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_3	E_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_4	E_4	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_5	E_5	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{5/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{7/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{9/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{11/2}$	$E_{11/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_1	e_1	E_5	
		1	2
1	1	1	0
1	2	0	1

b_1	e_2	E_4	
		1	2
1	1	1	0
1	2	0	1

b_1	e_3	E_3	
		1	2
1	1	0	1
1	2	1	0

b_1	e_4	E_2	
		1	2
1	1	1	0
1	2	0	1

b_1	e_5	E_1	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{1/2}$	$E_{11/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{3/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{5/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{7/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{9/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{11/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	1

b_2	e_1	E_5	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_2	E_4	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_3	E_3	
		1	2
1	1	0	$\bar{1}$
1	2	1	0

b_2	e_4	E_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_5	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{1/2}$	$E_{11/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{3/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{5/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{7/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{9/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

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T **45**.11 Clebsch–Gordan coefficients *(cont.)*

b_2	$e_{11/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e_1	e_1	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_1	e_2	E_1		E_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_1	e_3	E_2		E_4	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	$\bar{1}$

e_1	e_4	E_3		E_5	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	e_5	B_1	B_2	E_4	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_1	$e_{1/2}$	$E_{9/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_1	$e_{3/2}$	$E_{7/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_1	$e_{5/2}$	$E_{5/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_1	$e_{7/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_1	$e_{9/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	$e_{11/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_2	e_2	A_1	A_2	E_4	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_2	e_3	E_1		E_5	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	$\bar{1}$	0	0

e_2	e_4	B_1	B_2	E_2	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_2	e_5	E_3		E_5	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_2	$e_{1/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_2	$e_{3/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

$u = 2^{-1/2}$

↠

T **45**.11 Clebsch–Gordan coefficients *(cont.)*

e_2	$e_{5/2}$	$E_{1/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_2	$e_{7/2}$	$E_{3/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_2	$e_{9/2}$	$E_{5/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_2	$e_{11/2}$	$E_{7/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_3	e_3	A_1	A_2	B_1	B_2
		1	1	1	1
1	1	0	0	u	u
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	u	$\bar{u}$

e_3	e_4	E_1		E_5	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e_3	e_5	E_2		E_4	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e_3	$e_{1/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e_3	$e_{3/2}$	$E_{3/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_3	$e_{5/2}$	$E_{1/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e_3	$e_{7/2}$	$E_{1/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e_3	$e_{9/2}$	$E_{3/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_3	$e_{11/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e_4	e_4	A_1	A_2	E_4	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_4	e_5	E_1		E_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

e_4	$e_{1/2}$	$E_{7/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_4	$e_{3/2}$	$E_{5/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_4	$e_{5/2}$	$E_{3/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

$u = 2^{-1/2}$

↠

T **45**.11 Clebsch–Gordan coefficients *(cont.)*

e_4	$e_{7/2}$	$E_{1/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_4	$e_{9/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_4	$e_{11/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_5	e_5	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{\mathrm{u}}$	0	0
2	2	0	0	1	0

e_5	$e_{1/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_5	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_5	$e_{5/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_5	$e_{7/2}$	$E_{5/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_5	$e_{9/2}$	$E_{7/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_5	$e_{11/2}$	$E_{9/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

$e_{1/2}$	$e_{1/2}$	A_1	A_2	E_5	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\mathrm{u}}$	u	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{3/2}$	E_2		E_5	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{1/2}$	$e_{5/2}$	E_2		E_3	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

$e_{1/2}$	$e_{7/2}$	E_3		E_4	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{9/2}$	E_1		E_4	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{1/2}$	$e_{11/2}$	B_1	B_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\mathrm{u}}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{3/2}$	A_1	A_2	E_3	
		1	1	1	2
1	1	0	0	0	1
1	2	u	u	0	0
2	1	$\bar{\mathrm{u}}$	u	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{5/2}$	E_4		E_5	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$\mathrm{u} = 2^{-1/2}$

↠

T **45**.11 Clebsch–Gordan coefficients *(cont.)*

$e_{3/2}$	$e_{7/2}$	E_1		E_2	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{3/2}$	$e_{9/2}$	B_1	B_2	E_3	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{11/2}$	E_1		E_4	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{5/2}$	$e_{5/2}$	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{5/2}$	$e_{7/2}$	B_1	B_2	E_5	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{5/2}$	$e_{9/2}$	E_1		E_2	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{5/2}$	$e_{11/2}$	E_3		E_4	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{7/2}$	$e_{7/2}$	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{7/2}$	$e_{9/2}$	E_4		E_5	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{7/2}$	$e_{11/2}$	E_2		E_3	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	0	1	0	0

$e_{9/2}$	$e_{9/2}$	A_1	A_2	E_3	
		1	1	1	2
1	1	0	0	0	1
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{9/2}$	$e_{11/2}$	E_2		E_5	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{11/2}$	$e_{11/2}$	A_1	A_2	E_5	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

(1) Product forms: $\mathbf{D}_7 \otimes \mathbf{C}_i$.

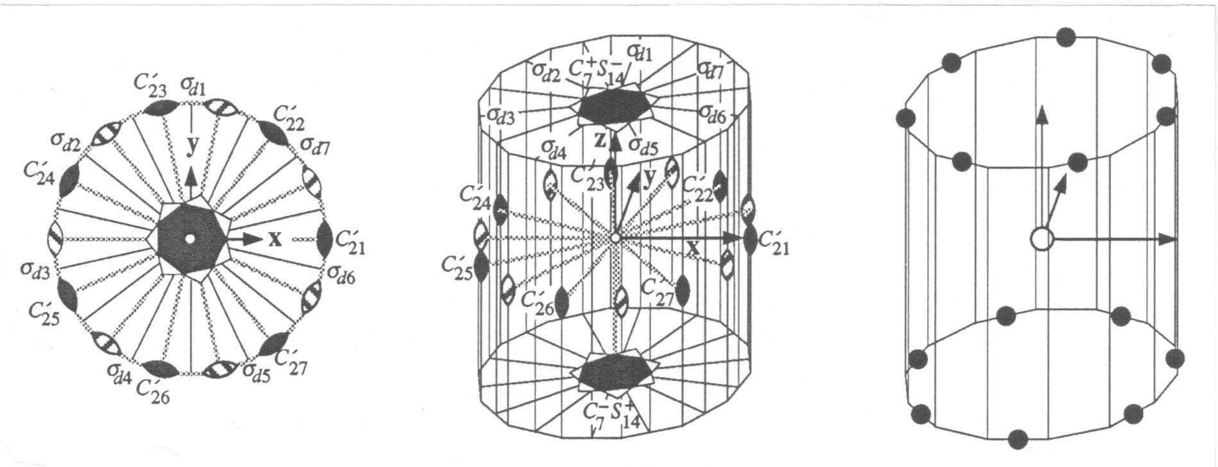
(2) Group chains: $\mathbf{D}_{7d} \supset (\mathbf{C}_{2h})$, $\mathbf{D}_{7d} \supset (\underline{\mathbf{C}_{7v}})$, $\mathbf{D}_{7d} \supset \underline{\mathbf{D}_7}$, $\mathbf{D}_{7d} \supset \underline{\mathbf{S}_{14}}$.

(3) Operations of G: E, (C_7^+, C_7^-), (C_7^{2+}, C_7^{2-}), (C_7^{3+}, C_7^{3-}), $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27})$,
i, $(S_{14}^{5-}, S_{14}^{5+})$, $(S_{14}^{3-}, S_{14}^{3+})$, (S_{14}^-, S_{14}^+), $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \sigma_{d6}, \sigma_{d7})$.

(4) Operations of $\widetilde{G}$: E, (C_7^+, C_7^-), (C_7^{2+}, C_7^{2-}), (C_7^{3+}, C_7^{3-}), $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27})$,
i, $(S_{14}^{5-}, S_{14}^{5+})$, $(S_{14}^{3-}, S_{14}^{3+})$, (S_{14}^-, S_{14}^+), $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \sigma_{d6}, \sigma_{d7})$,
$\widetilde{E}$, $(\widetilde{C}_7^+, \widetilde{C}_7^-)$, $(\widetilde{C}_7^{2+}, \widetilde{C}_7^{2-})$, $(\widetilde{C}_7^{3+}, \widetilde{C}_7^{3-})$, $(\widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24}, \widetilde{C}'_{25}, \widetilde{C}'_{26}, \widetilde{C}'_{27})$,
$\widetilde{i}$, $(\widetilde{S}_{14}^{5-}, \widetilde{S}_{14}^{5+})$, $(\widetilde{S}_{14}^{3-}, \widetilde{S}_{14}^{3+})$, $(\widetilde{S}_{14}^-, \widetilde{S}_{14}^+)$, $(\widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3}, \widetilde{\sigma}_{d4}, \widetilde{\sigma}_{d5}, \widetilde{\sigma}_{d6}, \widetilde{\sigma}_{d7})$.

(5) Classes and representations: $|r| = 10$, $|i| = 0$, $|I| = 10$, $|\widetilde{I}| = 10$.

F **46** See Chapter **15**, p. 65



Examples:

T **46**.1 Parameters

§ **16**–1, p. 68

$\mathbf{D}_{7d}$		α	β	γ	ϕ	$\mathbf{n}$	λ	$\mathbf{\Lambda}$
E	i	0	0	0	0	(0 0 0)	⟦ 1,	(0 0 0)⟧
C_7^+	S_{14}^{5-}	0	0	$\frac{2\pi}{7}$	$\frac{2\pi}{7}$	(0 0 1)	⟦c_7,	(0 0 s_7)⟧
C_7^-	S_{14}^{5+}	0	0	$-\frac{2\pi}{7}$	$\frac{2\pi}{7}$	(0 0 −1)	⟦c_7,	(0 0 $-s_7$)⟧
C_7^{2+}	S_{14}^{3-}	0	0	$\frac{4\pi}{7}$	$\frac{4\pi}{7}$	(0 0 1)	⟦c_7^2,	(0 0 s_7^2)⟧
C_7^{2-}	S_{14}^{3+}	0	0	$-\frac{4\pi}{7}$	$\frac{4\pi}{7}$	(0 0 −1)	⟦c_7^2,	(0 0 $-s_7^2$)⟧
C_7^{3+}	S_{14}^-	0	0	$\frac{6\pi}{7}$	$\frac{6\pi}{7}$	(0 0 1)	⟦c_7^3,	(0 0 s_7^3)⟧
C_7^{3-}	S_{14}^+	0	0	$-\frac{6\pi}{7}$	$\frac{6\pi}{7}$	(0 0 −1)	⟦c_7^3,	(0 0 $-s_7^3$)⟧
C'_{21}	σ_{d1}	0	π	π	π	(1 0 0)	⟦ 0,	(1 0 0)⟧
C'_{22}	σ_{d2}	0	π	$\frac{3\pi}{7}$	π	(c_7^2 s_7^2 0)	⟦ 0,	(c_7^2 s_7^2 0)⟧
C'_{23}	σ_{d3}	0	π	$-\frac{\pi}{7}$	π	($-c_7^3$ s_7^3 0)	⟦ 0,	($-c_7^3$ s_7^3 0)⟧
C'_{24}	σ_{d4}	0	π	$-\frac{5\pi}{7}$	π	($-c_7$ s_7 0)	⟦ 0,	($-c_7$ s_7 0)⟧
C'_{25}	σ_{d5}	0	π	$\frac{5\pi}{7}$	π	($-c_7$ $-s_7$ 0)	⟦ 0,	($-c_7$ $-s_7$ 0)⟧
C'_{26}	σ_{d6}	0	π	$\frac{\pi}{7}$	π	($-c_7^3$ $-s_7^3$ 0)	⟦ 0,	($-c_7^3$ $-s_7^3$ 0)⟧
C'_{27}	σ_{d7}	0	π	$-\frac{3\pi}{7}$	π	(c_7^2 $-s_7^2$ 0)	⟦ 0,	(c_7^2 $-s_7^2$ 0)⟧

$c_n^m = \cos \frac{m}{n}\pi$, $s_n^m = \sin \frac{m}{n}\pi$

T **46**.2 Multiplication table

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$\mathbf{D}_{7d}$	E	C_7^+	C_7^-	C_7^{2+}	C_7^{2-}	C_7^{3+}	C_7^{3-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	i	S_{14}^{5-}	S_{14}^{5+}	S_{14}^{3-}	S_{14}^{3+}	S_{14}^-	S_{14}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}
E	E	C_7^+	C_7^-	C_7^{2+}	C_7^{2-}	C_7^{3+}	C_7^{3-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	i	S_{14}^{5-}	S_{14}^{5+}	S_{14}^{3-}	S_{14}^{3+}	S_{14}^-	S_{14}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}
C_7^+	C_7^+	C_7^{2+}	E	C_7^{3+}	C_7^-	C_7^{3-}	C_7^{2-}	C'_{25}	C'_{26}	C'_{27}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	S_{14}^{5-}	S_{14}^{3-}	i	S_{14}^-	S_{14}^{5+}	S_{14}^+	S_{14}^{3+}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}
C_7^-	C_7^-	E	C_7^{2-}	C_7^+	C_7^{3-}	C_7^{2+}	C_7^{3+}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{21}	C'_{22}	C'_{23}	S_{14}^{5+}	i	S_{14}^{3+}	S_{14}^{5-}	S_{14}^+	S_{14}^{3-}	S_{14}^-	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d1}	σ_{d2}	σ_{d3}
C_7^{2+}	C_7^{2+}	C_7^{3+}	C_7^+	C_7^{3-}	E	C_7^{2-}	C_7^-	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{21}	S_{14}^{3-}	S_{14}^-	S_{14}^{5-}	S_{14}^+	i	S_{14}^{3+}	S_{14}^{5+}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d1}
C_7^{2-}	C_7^{2-}	C_7^-	C_7^{3-}	E	C_7^{3+}	C_7^+	C_7^{2+}	C'_{27}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	S_{14}^{3+}	S_{14}^{5+}	S_{14}^+	i	S_{14}^-	S_{14}^{5-}	S_{14}^{3-}	σ_{d7}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}
C_7^{3+}	C_7^{3+}	C_7^{3-}	C_7^{2+}	C_7^{2-}	C_7^+	C_7^-	E	C'_{26}	C'_{27}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	S_{14}^-	S_{14}^+	S_{14}^{3-}	S_{14}^{3+}	S_{14}^{5-}	S_{14}^{5+}	i	σ_{d6}	σ_{d7}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}
C_7^{3-}	C_7^{3-}	C_7^{2-}	C_7^{3+}	C_7^-	C_7^{2+}	E	C_7^+	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{21}	C'_{22}	S_{14}^+	S_{14}^{3+}	S_{14}^-	S_{14}^{5+}	S_{14}^{3-}	i	S_{14}^{5-}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d1}	σ_{d2}
C'_{21}	C'_{21}	C'_{24}	C'_{25}	C'_{27}	C'_{22}	C'_{23}	C'_{26}	E	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}	σ_{d1}	σ_{d4}	σ_{d5}	σ_{d7}	σ_{d2}	σ_{d3}	σ_{d6}	i	S_{14}^{3+}	S_{14}^-	S_{14}^{5-}	S_{14}^{5+}	S_{14}^+	S_{14}^{3-}
C'_{22}	C'_{22}	C'_{25}	C'_{26}	C'_{21}	C'_{23}	C'_{24}	C'_{27}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}	σ_{d2}	σ_{d5}	σ_{d6}	σ_{d1}	σ_{d3}	σ_{d4}	σ_{d7}	S_{14}^{3-}	i	S_{14}^{3+}	S_{14}^-	S_{14}^{5-}	S_{14}^{5+}	S_{14}^+
C'_{23}	C'_{23}	C'_{26}	C'_{27}	C'_{22}	C'_{24}	C'_{25}	C'_{21}	C_7^{3-}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-	σ_{d3}	σ_{d6}	σ_{d7}	σ_{d2}	σ_{d4}	σ_{d5}	σ_{d1}	S_{14}^+	S_{14}^{3-}	i	S_{14}^{3+}	S_{14}^-	S_{14}^{5-}	S_{14}^{5+}
C'_{24}	C'_{24}	C'_{27}	C'_{21}	C'_{23}	C'_{25}	C'_{26}	C'_{22}	C_7^-	C_7^{3-}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}	C_7^+	σ_{d4}	σ_{d7}	σ_{d1}	σ_{d3}	σ_{d5}	σ_{d6}	σ_{d2}	S_{14}^{5+}	S_{14}^+	S_{14}^{3-}	i	S_{14}^{3+}	S_{14}^-	S_{14}^{5-}
C'_{25}	C'_{25}	C'_{21}	C'_{22}	C'_{24}	C'_{26}	C'_{27}	C'_{23}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d4}	σ_{d6}	σ_{d7}	σ_{d3}	S_{14}^{5-}	S_{14}^{5+}	S_{14}^+	S_{14}^{3-}	i	S_{14}^{3+}	S_{14}^-
C'_{26}	C'_{26}	C'_{22}	C'_{23}	C'_{25}	C'_{27}	C'_{21}	C'_{24}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}	E	C_7^{2-}	σ_{d6}	σ_{d2}	σ_{d3}	σ_{d5}	σ_{d7}	σ_{d1}	σ_{d4}	S_{14}^-	S_{14}^{5-}	S_{14}^{5+}	S_{14}^+	S_{14}^{3-}	i	S_{14}^{3+}
C'_{27}	C'_{27}	C'_{23}	C'_{24}	C'_{26}	C'_{21}	C'_{22}	C'_{25}	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}	E	σ_{d7}	σ_{d3}	σ_{d4}	σ_{d6}	σ_{d1}	σ_{d2}	σ_{d5}	S_{14}^{3+}	S_{14}^-	S_{14}^{5-}	S_{14}^{5+}	S_{14}^+	S_{14}^{3-}	i
i	i	S_{14}^{5-}	S_{14}^{5+}	S_{14}^{3-}	S_{14}^{3+}	S_{14}^-	S_{14}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	E	C_7^+	C_7^-	C_7^{2+}	C_7^{2-}	C_7^{3+}	C_7^{3-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}
S_{14}^{5-}	S_{14}^{5-}	S_{14}^{3-}	i	S_{14}^-	S_{14}^{5+}	S_{14}^+	S_{14}^{3+}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	C_7^+	C_7^{2+}	E	C_7^{3+}	C_7^-	C_7^{3-}	C_7^{2-}	C'_{25}	C'_{26}	C'_{27}	C'_{21}	C'_{22}	C'_{23}	C'_{24}
S_{14}^{5+}	S_{14}^{5+}	i	S_{14}^{3+}	S_{14}^{5-}	S_{14}^+	S_{14}^{3-}	S_{14}^-	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d1}	σ_{d2}	σ_{d3}	C_7^-	E	C_7^{2-}	C_7^+	C_7^{3-}	C_7^{2+}	C_7^{3+}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{21}	C'_{22}	C'_{23}
S_{14}^{3-}	S_{14}^{3-}	S_{14}^-	S_{14}^{5-}	S_{14}^+	i	S_{14}^{3+}	S_{14}^{5+}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d1}	C_7^{2+}	C_7^{3+}	C_7^+	C_7^{3-}	E	C_7^{2-}	C_7^-	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{21}
S_{14}^{3+}	S_{14}^{3+}	S_{14}^{5+}	S_{14}^+	i	S_{14}^-	S_{14}^{5-}	S_{14}^{3-}	σ_{d7}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	C_7^{2-}	C_7^-	C_7^{3-}	E	C_7^{3+}	C_7^+	C_7^{2+}	C'_{27}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}
S_{14}^-	S_{14}^-	S_{14}^+	S_{14}^{3-}	S_{14}^{3+}	S_{14}^{5-}	S_{14}^{5+}	i	σ_{d6}	σ_{d7}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	C_7^{3+}	C_7^{3-}	C_7^{2+}	C_7^{2-}	C_7^+	C_7^-	E	C'_{26}	C'_{27}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}
S_{14}^+	S_{14}^+	S_{14}^{3+}	S_{14}^-	S_{14}^{5+}	S_{14}^{3-}	i	S_{14}^{5-}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d1}	σ_{d2}	C_7^{3-}	C_7^{2-}	C_7^{3+}	C_7^-	C_7^{2+}	E	C_7^+	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{21}	C'_{22}
σ_{d1}	σ_{d1}	σ_{d4}	σ_{d5}	σ_{d7}	σ_{d2}	σ_{d3}	σ_{d6}	i	S_{14}^{3+}	S_{14}^-	S_{14}^{5-}	S_{14}^{5+}	S_{14}^+	S_{14}^{3-}	C'_{21}	C'_{24}	C'_{25}	C'_{27}	C'_{22}	C'_{23}	C'_{26}	E	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}
σ_{d2}	σ_{d2}	σ_{d5}	σ_{d6}	σ_{d1}	σ_{d3}	σ_{d4}	σ_{d7}	S_{14}^{3-}	i	S_{14}^{3+}	S_{14}^-	S_{14}^{5-}	S_{14}^{5+}	S_{14}^+	C'_{22}	C'_{25}	C'_{26}	C'_{21}	C'_{23}	C'_{24}	C'_{27}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}
σ_{d3}	σ_{d3}	σ_{d6}	σ_{d7}	σ_{d2}	σ_{d4}	σ_{d5}	σ_{d1}	S_{14}^+	S_{14}^{3-}	i	S_{14}^{3+}	S_{14}^-	S_{14}^{5-}	S_{14}^{5+}	C'_{23}	C'_{26}	C'_{27}	C'_{22}	C'_{24}	C'_{25}	C'_{21}	C_7^{3-}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-
σ_{d4}	σ_{d4}	σ_{d7}	σ_{d1}	σ_{d3}	σ_{d5}	σ_{d6}	σ_{d2}	S_{14}^{5+}	S_{14}^+	S_{14}^{3-}	i	S_{14}^{3+}	S_{14}^-	S_{14}^{5-}	C'_{24}	C'_{27}	C'_{21}	C'_{23}	C'_{25}	C'_{26}	C'_{22}	C_7^-	C_7^{3-}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}	C_7^+
σ_{d5}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d4}	σ_{d6}	σ_{d7}	σ_{d3}	S_{14}^{5-}	S_{14}^{5+}	S_{14}^+	S_{14}^{3-}	i	S_{14}^{3+}	S_{14}^-	C'_{25}	C'_{21}	C'_{22}	C'_{24}	C'_{26}	C'_{27}	C'_{23}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}	E	C_7^{2-}	C_7^{3+}
σ_{d6}	σ_{d6}	σ_{d2}	σ_{d3}	σ_{d5}	σ_{d7}	σ_{d1}	σ_{d4}	S_{14}^-	S_{14}^{5-}	S_{14}^{5+}	S_{14}^+	S_{14}^{3-}	i	S_{14}^{3+}	C'_{26}	C'_{22}	C'_{23}	C'_{25}	C'_{27}	C'_{21}	C'_{24}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}	E	C_7^{2-}
σ_{d7}	σ_{d7}	σ_{d3}	σ_{d4}	σ_{d6}	σ_{d1}	σ_{d2}	σ_{d5}	S_{14}^{3+}	S_{14}^-	S_{14}^{5-}	S_{14}^{5+}	S_{14}^+	S_{14}^{3-}	i	C'_{27}	C'_{23}	C'_{24}	C'_{26}	C'_{21}	C'_{22}	C'_{25}	C_7^{2-}	C_7^{3+}	C_7^+	C_7^-	C_7^{3-}	C_7^{2+}	E

T **46**.3 Factor table

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$\mathbf{D}_{7d}$	E	C_7^+	C_7^-	C_7^{2+}	C_7^{2-}	C_7^{3+}	C_7^{3-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	i	S_{14}^{5-}	S_{14}^{5+}	S_{14}^{3-}	S_{14}^{3+}	S_{14}^-	S_{14}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_7^+	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1
C_7^-	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1
C_7^{2+}	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1
C_7^{2-}	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1
C_7^{3+}	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1
C_7^{3-}	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1
C'_{21}	1	−1	−1	1	1	−1	−1	−1	−1	1	1	1	1	−1	1	−1	−1	1	1	−1	−1	−1	−1	1	1	1	1	−1
C'_{22}	1	−1	−1	1	1	−1	−1	−1	−1	−1	1	1	1	1	1	−1	−1	1	1	−1	−1	−1	−1	−1	1	1	1	1
C'_{23}	1	−1	−1	1	1	−1	−1	1	−1	−1	−1	1	1	1	1	−1	−1	1	1	−1	−1	1	−1	−1	−1	1	1	1
C'_{24}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	−1	1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1	−1	1	1
C'_{25}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	−1	1
C'_{26}	1	−1	−1	1	1	−1	−1	1	1	1	1	−1	−1	−1	1	−1	−1	1	1	−1	−1	1	1	1	1	−1	−1	−1
C'_{27}	1	−1	−1	1	1	−1	−1	−1	1	1	1	1	−1	−1	1	−1	−1	1	1	−1	−1	−1	1	1	1	1	−1	−1
i	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_{14}^{5-}	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1
S_{14}^{5+}	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1
S_{14}^{3-}	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1
S_{14}^{3+}	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1
S_{14}^-	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1
S_{14}^+	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1
σ_{d1}	1	−1	−1	1	1	−1	−1	−1	−1	1	1	1	1	−1	1	−1	−1	1	1	−1	−1	−1	−1	1	1	1	1	−1
σ_{d2}	1	−1	−1	1	1	−1	−1	−1	−1	−1	1	1	1	1	1	−1	−1	1	1	−1	−1	−1	−1	−1	1	1	1	1
σ_{d3}	1	−1	−1	1	1	−1	−1	1	−1	−1	−1	1	1	1	1	−1	−1	1	1	−1	−1	1	−1	−1	−1	1	1	1
σ_{d4}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	−1	1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1	−1	1	1
σ_{d5}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	−1	1
σ_{d6}	1	−1	−1	1	1	−1	−1	1	1	1	1	−1	−1	−1	1	−1	−1	1	1	−1	−1	1	1	1	1	−1	−1	−1
σ_{d7}	1	−1	−1	1	1	−1	−1	−1	1	1	1	1	−1	−1	1	−1	−1	1	1	−1	−1	−1	1	1	1	1	−1	−1

T **46**.4 Character table

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$\mathbf{D}_{7d}$	E	$2C_7$	$2C_7^2$	$2C_7^3$	$7C_2'$	i	$2S_{14}^5$	$2S_{14}^3$	$2S_{14}$	$7\sigma_d$	τ
A_{1g}	1	1	1	1	1	1	1	1	1	1	a
A_{2g}	1	1	1	1	−1	1	1	1	1	−1	a
E_{1g}	2	$2c_7^2$	$2c_7^4$	$2c_7^6$	0	2	$2c_7^2$	$2c_7^4$	$2c_7^6$	0	a
E_{2g}	2	$2c_7^4$	$2c_7^6$	$2c_7^2$	0	2	$2c_7^4$	$2c_7^6$	$2c_7^2$	0	a
E_{3g}	2	$2c_7^6$	$2c_7^2$	$2c_7^4$	0	2	$2c_7^6$	$2c_7^2$	$2c_7^4$	0	a
A_{1u}	1	1	1	1	1	−1	−1	−1	−1	−1	a
A_{2u}	1	1	1	1	−1	−1	−1	−1	−1	1	a
E_{1u}	2	$2c_7^2$	$2c_7^4$	$2c_7^6$	0	−2	$-2c_7^2$	$-2c_7^4$	$-2c_7^6$	0	a
E_{2u}	2	$2c_7^4$	$2c_7^6$	$2c_7^2$	0	−2	$-2c_7^4$	$-2c_7^6$	$-2c_7^2$	0	a
E_{3u}	2	$2c_7^6$	$2c_7^2$	$2c_7^4$	0	−2	$-2c_7^6$	$-2c_7^2$	$-2c_7^4$	0	a
$E_{1/2,g}$	2	$-2c_7^6$	$2c_7^2$	$-2c_7^4$	0	2	$-2c_7^6$	$2c_7^2$	$-2c_7^4$	0	c
$E_{3/2,g}$	2	$-2c_7^4$	$2c_7^6$	$-2c_7^2$	0	2	$-2c_7^4$	$2c_7^6$	$-2c_7^2$	0	c
$E_{5/2,g}$	2	$-2c_7^2$	$2c_7^4$	$-2c_7^6$	0	2	$-2c_7^2$	$2c_7^4$	$-2c_7^6$	0	c
${}^1E_{7/2,g}$	1	−1	1	−1	i	1	−1	1	−1	i	b
${}^2E_{7/2,g}$	1	−1	1	−1	−i	1	−1	1	−1	−i	b
$E_{1/2,u}$	2	$-2c_7^6$	$2c_7^2$	$-2c_7^4$	0	−2	$2c_7^6$	$-2c_7^2$	$2c_7^4$	0	c
$E_{3/2,u}$	2	$-2c_7^4$	$2c_7^6$	$-2c_7^2$	0	−2	$2c_7^4$	$-2c_7^6$	$2c_7^2$	0	c
$E_{5/2,u}$	2	$-2c_7^2$	$2c_7^4$	$-2c_7^6$	0	−2	$2c_7^2$	$-2c_7^4$	$2c_7^6$	0	c
${}^1E_{7/2,u}$	1	−1	1	−1	i	−1	1	−1	1	−i	b
${}^2E_{7/2,u}$	1	−1	1	−1	−i	−1	1	−1	1	i	b

$c_n^m = \cos \frac{m}{n}\pi$

T **46**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{7d}$	0	1	2	3
A_{1g}	$\square 1$		$x^2+y^2, \square z^2$	
A_{2g}		R_z		
E_{1g}		(R_x, R_y)	$\square(zx, yz)$	
E_{2g}			$\square(xy, x^2-y^2)$	
E_{3g}				
A_{1u}				
A_{2u}		$\square z$		$(x^2+y^2)z, \square z^3$
E_{1u}		$\square(x, y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, \square(xz^2, yz^2)$
E_{2u}				$\square\{xyz, z(x^2-y^2)\}$
E_{3u}				$\square\{x(x^2-3y^2), y(3x^2-y^2)\}$

T **46**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{7d}$	$\langle\lvert j\,m\rangle\rvert$		ι	μ
A_{1g}	$\lvert 0\,0\rangle_+$		2	7
A_{2g}	$\lvert 8\,7\rangle_-$		2	7
E_{1g}	$\langle\lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle\rvert$		2	± 7
E_{2g}	$\langle\lvert 2\,2\rangle, \lvert 2\,\overline{2}\rangle\rvert$		2	± 7
E_{3g}	$\langle\lvert 4\,3\rangle, -\lvert 4\,\overline{3}\rangle\rvert$		2	± 7
A_{1u}	$\lvert 7\,7\rangle_-$		2	7
A_{2u}	$\lvert 1\,0\rangle_+$		2	7
E_{1u}	$\langle\lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle\rvert$		2	± 7
E_{2u}	$\langle\lvert 3\,2\rangle, -\lvert 3\,\overline{2}\rangle\rvert$		2	± 7
E_{3u}	$\langle\lvert 3\,3\rangle, \lvert 3\,\overline{3}\rangle\rvert$		2	± 7
$E_{1/2,g}$	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\frac{\overline{1}}{2}\rangle\rvert$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\frac{\overline{1}}{2}\rangle\rvert$	2	± 7
$E_{3/2,g}$	$\langle\lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\frac{\overline{3}}{2}\rangle\rvert$	$\langle\lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\frac{\overline{3}}{2}\rangle\rvert$	2	± 7
$E_{5/2,g}$	$\langle\lvert \frac{5}{2}\,\frac{5}{2}\rangle, \lvert \frac{5}{2}\,\frac{\overline{5}}{2}\rangle\rvert$	$\langle\lvert \frac{7}{2}\,\frac{5}{2}\rangle, -\lvert \frac{7}{2}\,\frac{\overline{5}}{2}\rangle\rvert$	2	± 7
${}^1E_{7/2,g}$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle_+$	$\lvert \frac{9}{2}\,\frac{7}{2}\rangle_-$	2	7
${}^2E_{7/2,g}$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle_-$	$\lvert \frac{9}{2}\,\frac{7}{2}\rangle_+$	2	7
$E_{1/2,u}$	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\frac{\overline{1}}{2}\rangle\rvert^\bullet$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\frac{\overline{1}}{2}\rangle\rvert^\bullet$	2	± 7
$E_{3/2,u}$	$\langle\lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\frac{\overline{3}}{2}\rangle\rvert^\bullet$	$\langle\lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\frac{\overline{3}}{2}\rangle\rvert^\bullet$	2	± 7
$E_{5/2,u}$	$\langle\lvert \frac{5}{2}\,\frac{5}{2}\rangle, \lvert \frac{5}{2}\,\frac{\overline{5}}{2}\rangle\rvert^\bullet$	$\langle\lvert \frac{7}{2}\,\frac{5}{2}\rangle, -\lvert \frac{7}{2}\,\frac{\overline{5}}{2}\rangle\rvert^\bullet$	2	± 7
${}^1E_{7/2,u}$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle_+^\bullet$	$\lvert \frac{9}{2}\,\frac{7}{2}\rangle_-^\bullet$	2	7
${}^2E_{7/2,u}$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle_-^\bullet$	$\lvert \frac{9}{2}\,\frac{7}{2}\rangle_+^\bullet$	2	7

T **46**.7 Matrix representations

Use T **27**.7 ▪. § **16**–7, p. 77

T **46**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{D}_{7d}$	A_{1g}	A_{2g}	E_{1g}	E_{2g}	E_{3g}
A_{1g}	A_{1g}	A_{2g}	E_{1g}	E_{2g}	E_{3g}
A_{2g}		A_{1g}	E_{1g}	E_{2g}	E_{3g}
E_{1g}			$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$	$E_{1g} \oplus E_{3g}$	$E_{2g} \oplus E_{3g}$
E_{2g}				$A_{1g} \oplus \{A_{2g}\} \oplus E_{3g}$	$E_{1g} \oplus E_{2g}$
E_{3g}					$A_{1g} \oplus \{A_{2g}\} \oplus E_{1g}$

⇒

T **46**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{7d}$	A_{1u}	A_{2u}	E_{1u}	E_{2u}	E_{3u}
A_{1g}	A_{1u}	A_{2u}	E_{1u}	E_{2u}	E_{3u}
A_{2g}	A_{2u}	A_{1u}	E_{1u}	E_{2u}	E_{3u}
E_{1g}	E_{1u}	E_{1u}	$A_{1u} \oplus A_{2u} \oplus E_{2u}$	$E_{1u} \oplus E_{3u}$	$E_{2u} \oplus E_{3u}$
E_{2g}	E_{2u}	E_{2u}	$E_{1u} \oplus E_{3u}$	$A_{1u} \oplus A_{2u} \oplus E_{3u}$	$E_{1u} \oplus E_{2u}$
E_{3g}	E_{3u}	E_{3u}	$E_{2u} \oplus E_{3u}$	$E_{1u} \oplus E_{2u}$	$A_{1u} \oplus A_{2u} \oplus E_{1u}$
A_{1u}	A_{1g}	A_{2g}	E_{1g}	E_{2g}	E_{3g}
A_{2u}		A_{1g}	E_{1g}	E_{2g}	E_{3g}
E_{1u}			$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$	$E_{1g} \oplus E_{3g}$	$E_{2g} \oplus E_{3g}$
E_{2u}				$A_{1g} \oplus \{A_{2g}\} \oplus E_{3g}$	$E_{1g} \oplus E_{2g}$
E_{3u}					$A_{1g} \oplus \{A_{2g}\} \oplus E_{1g}$

$\twoheadrightarrow$

T **46**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{7d}$	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	${}^1E_{7/2,g}$	${}^2E_{7/2,g}$
A_{1g}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	${}^1E_{7/2,g}$	${}^2E_{7/2,g}$
A_{2g}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	${}^2E_{7/2,g}$	${}^1E_{7/2,g}$
E_{1g}	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{3/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g}$	$E_{5/2,g}$	$E_{5/2,g}$
E_{2g}	$E_{3/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{3/2,g}$	$E_{3/2,g}$
E_{3g}	$E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g}$	$E_{3/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g}$	$E_{1/2,g}$
A_{1u}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	${}^1E_{7/2,u}$	${}^2E_{7/2,u}$
A_{2u}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	${}^2E_{7/2,u}$	${}^1E_{7/2,u}$
E_{1u}	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{3/2,u} \oplus {}^1E_{7/2,u} \oplus {}^2E_{7/2,u}$	$E_{5/2,u}$	$E_{5/2,u}$
E_{2u}	$E_{3/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus {}^1E_{7/2,u} \oplus {}^2E_{7/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{3/2,u}$	$E_{3/2,u}$
E_{3u}	$E_{5/2,u} \oplus {}^1E_{7/2,u} \oplus {}^2E_{7/2,u}$	$E_{3/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u}$	$E_{1/2,u}$
$E_{1/2,g}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$	$E_{1g} \oplus E_{2g}$	$E_{2g} \oplus E_{3g}$	E_{3g}	E_{3g}
$E_{3/2,g}$		$\{A_{1g}\} \oplus A_{2g} \oplus E_{3g}$	$E_{1g} \oplus E_{3g}$	E_{2g}	E_{2g}
$E_{5/2,g}$			$\{A_{1g}\} \oplus A_{2g} \oplus E_{2g}$	E_{1g}	E_{1g}
${}^1E_{7/2,g}$				A_{2g}	A_{1g}
${}^2E_{7/2,g}$					A_{2g}

$\twoheadrightarrow$

T **46**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{7d}$	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	${}^1E_{7/2,u}$	${}^2E_{7/2,u}$
A_{1g}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	${}^1E_{7/2,u}$	${}^2E_{7/2,u}$
A_{2g}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	${}^2E_{7/2,u}$	${}^1E_{7/2,u}$
E_{1g}	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{3/2,u} \oplus {}^1E_{7/2,u} \oplus {}^2E_{7/2,u}$	$E_{5/2,u}$	$E_{5/2,u}$
E_{2g}	$E_{3/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus {}^1E_{7/2,u} \oplus {}^2E_{7/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{3/2,u}$	$E_{3/2,u}$
E_{3g}	$E_{5/2,u} \oplus {}^1E_{7/2,u} \oplus {}^2E_{7/2,u}$	$E_{3/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u}$	$E_{1/2,u}$
A_{1u}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	${}^1E_{7/2,g}$	${}^2E_{7/2,g}$
A_{2u}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	${}^2E_{7/2,g}$	${}^1E_{7/2,g}$
E_{1u}	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{3/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g}$	$E_{5/2,g}$	$E_{5/2,g}$
E_{2u}	$E_{3/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{3/2,g}$	$E_{3/2,g}$
E_{3u}	$E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g}$	$E_{3/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g}$	$E_{1/2,g}$
$E_{1/2,g}$	$A_{1u} \oplus A_{2u} \oplus E_{1u}$	$E_{1u} \oplus E_{2u}$	$E_{2u} \oplus E_{3u}$	E_{3u}	E_{3u}
$E_{3/2,g}$	$E_{1u} \oplus E_{2u}$	$A_{1u} \oplus A_{2u} \oplus E_{3u}$	$E_{1u} \oplus E_{3u}$	E_{2u}	E_{2u}
$E_{5/2,g}$	$E_{2u} \oplus E_{3u}$	$E_{1u} \oplus E_{3u}$	$A_{1u} \oplus A_{2u} \oplus E_{2u}$	E_{1u}	E_{1u}
${}^1E_{7/2,g}$	E_{3u}	E_{2u}	E_{1u}	A_{2u}	A_{1u}
${}^2E_{7/2,g}$	E_{3u}	E_{2u}	E_{1u}	A_{1u}	A_{2u}
$E_{1/2,u}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$	$E_{1g} \oplus E_{2g}$	$E_{2g} \oplus E_{3g}$	E_{3g}	E_{3g}
$E_{3/2,u}$		$\{A_{1g}\} \oplus A_{2g} \oplus E_{3g}$	$E_{1g} \oplus E_{3g}$	E_{2g}	E_{2g}
$E_{5/2,u}$			$\{A_{1g}\} \oplus A_{2g} \oplus E_{2g}$	E_{1g}	E_{1g}
${}^1E_{7/2,u}$				A_{2g}	A_{1g}
${}^2E_{7/2,u}$					A_{2g}

T **46**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_{7d}$	$(\mathbf{C}_{2h})$	$(\mathbf{C}_{7v})$	$\mathbf{D}_7$	$\mathbf{S}_{14}$
A_{1g}	A_g	A_1	A_1	A_g
A_{2g}	B_g	A_2	A_2	A_g
E_{1g}	$A_g \oplus B_g$	E_1	E_1	${}^1E_{1g} \oplus {}^2E_{1g}$
E_{2g}	$A_g \oplus B_g$	E_2	E_2	${}^1E_{2g} \oplus {}^2E_{2g}$
E_{3g}	$A_g \oplus B_g$	E_3	E_3	${}^1E_{3g} \oplus {}^2E_{3g}$
A_{1u}	A_u	A_2	A_1	A_u
A_{2u}	B_u	A_1	A_2	A_u
E_{1u}	$A_u \oplus B_u$	E_1	E_1	${}^1E_{1u} \oplus {}^2E_{1u}$
E_{2u}	$A_u \oplus B_u$	E_2	E_2	${}^1E_{2u} \oplus {}^2E_{2u}$
E_{3u}	$A_u \oplus B_u$	E_3	E_3	${}^1E_{3u} \oplus {}^2E_{3u}$
$E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$
$E_{3/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{3/2}$	$E_{3/2}$	${}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$
$E_{5/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{5/2}$	$E_{5/2}$	${}^1E_{5/2,g} \oplus {}^2E_{5/2,g}$
${}^1E_{7/2,g}$	${}^1E_{1/2,g}$	${}^1E_{7/2}$	${}^1E_{7/2}$	$A_{7/2,g}$
${}^2E_{7/2,g}$	${}^2E_{1/2,g}$	${}^2E_{7/2}$	${}^2E_{7/2}$	$A_{7/2,g}$
$E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$
$E_{3/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{3/2}$	$E_{3/2}$	${}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$
$E_{5/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{5/2}$	$E_{5/2}$	${}^1E_{5/2,u} \oplus {}^2E_{5/2,u}$
${}^1E_{7/2,u}$	${}^1E_{1/2,u}$	${}^2E_{7/2}$	${}^1E_{7/2}$	$A_{7/2,u}$
${}^2E_{7/2,u}$	${}^2E_{1/2,u}$	${}^1E_{7/2}$	${}^2E_{7/2}$	$A_{7/2,u}$

$\twoheadrightarrow$

T **46**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{7d}$	$(\mathbf{C}_s)$	$\mathbf{C}_i$	$\mathbf{C}_7$	$(\mathbf{C}_2)$
A_{1g}	A'	A_g	A	A
A_{2g}	A''	A_g	A	B
E_{1g}	$A' \oplus A''$	$2A_g$	${}^1E_1 \oplus {}^2E_1$	$A \oplus B$
E_{2g}	$A' \oplus A''$	$2A_g$	${}^1E_2 \oplus {}^2E_2$	$A \oplus B$
E_{3g}	$A' \oplus A''$	$2A_g$	${}^1E_3 \oplus {}^2E_3$	$A \oplus B$
A_{1u}	A''	A_u	A	A
A_{2u}	A'	A_u	A	B
E_{1u}	$A' \oplus A''$	$2A_u$	${}^1E_1 \oplus {}^2E_1$	$A \oplus B$
E_{2u}	$A' \oplus A''$	$2A_u$	${}^1E_2 \oplus {}^2E_2$	$A \oplus B$
E_{3u}	$A' \oplus A''$	$2A_u$	${}^1E_3 \oplus {}^2E_3$	$A \oplus B$
$E_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,g}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,g}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1E_{7/2,g}$	${}^1E_{1/2}$	$A_{1/2,g}$	$A_{7/2}$	${}^1E_{1/2}$
${}^2E_{7/2,g}$	${}^2E_{1/2}$	$A_{1/2,g}$	$A_{7/2}$	${}^2E_{1/2}$
$E_{1/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,u}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,u}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1E_{7/2,u}$	${}^2E_{1/2}$	$A_{1/2,u}$	$A_{7/2}$	${}^1E_{1/2}$
${}^2E_{7/2,u}$	${}^1E_{1/2}$	$A_{1/2,u}$	$A_{7/2}$	${}^2E_{1/2}$

T **46**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{7d}$
$14n$	$(2n+1)\,A_{1g} \oplus 2n\,(A_{2g} \oplus 2E_{1g} \oplus 2E_{2g} \oplus 2E_{3g})$
$14n+1$	$2n\,(A_{1u} \oplus E_{1u} \oplus 2E_{2u} \oplus 2E_{3u}) \oplus (2n+1)(A_{2u} \oplus E_{1u})$
$14n+2$	$(2n+1)(A_{1g} \oplus E_{1g} \oplus E_{2g}) \oplus 2n\,(A_{2g} \oplus E_{1g} \oplus E_{2g} \oplus 2E_{3g})$
$14n+3$	$2n\,(A_{1u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u}) \oplus (2n+1)(A_{2u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u})$
$14n+4$	$(2n+1)(A_{1g} \oplus E_{1g} \oplus E_{2g} \oplus 2E_{3g}) \oplus 2n\,(A_{2g} \oplus E_{1g} \oplus E_{2g})$
$14n+5$	$2n\,(A_{1u} \oplus E_{1u}) \oplus (2n+1)(A_{2u} \oplus E_{1u} \oplus 2E_{2u} \oplus 2E_{3u})$
$14n+6$	$(2n+1)(A_{1g} \oplus 2E_{1g} \oplus 2E_{2g} \oplus 2E_{3g}) \oplus 2n\,A_{2g}$
$14n+7$	$(2n+1)(A_{1u} \oplus 2E_{1u} \oplus 2E_{2u} \oplus 2E_{3u}) \oplus (2n+2)\,A_{2u}$
$14n+8$	$(2n+2)(A_{1g} \oplus E_{1g}) \oplus (2n+1)(A_{2g} \oplus E_{1g} \oplus 2E_{2g} \oplus 2E_{3g})$
$14n+9$	$(2n+1)(A_{1u} \oplus E_{1u} \oplus E_{2u} \oplus 2E_{3u}) \oplus (2n+2)(A_{2u} \oplus E_{1u} \oplus E_{2u})$
$14n+10$	$(2n+2)(A_{1g} \oplus E_{1g} \oplus E_{2g} \oplus E_{3g}) \oplus (2n+1)(A_{2g} \oplus E_{1g} \oplus E_{2g} \oplus E_{3g})$
$14n+11$	$(2n+1)(A_{1u} \oplus E_{1u} \oplus E_{2u}) \oplus (2n+2)(A_{2u} \oplus E_{1u} \oplus E_{2u} \oplus 2E_{3u})$
$14n+12$	$(2n+2)(A_{1g} \oplus E_{1g} \oplus 2E_{2g} \oplus 2E_{3g}) \oplus (2n+1)(A_{2g} \oplus E_{1g})$
$14n+13$	$(2n+1)\,A_{1u} \oplus (2n+2)(A_{2u} \oplus 2E_{1u} \oplus 2E_{2u} \oplus 2E_{3u})$
$7n+\frac{1}{2}$	$(2n+1)\,E_{1/2,g} \oplus n\,(2E_{3/2,g} \oplus 2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g})$
$7n+\frac{3}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g}) \oplus n\,(2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g})$
$7n+\frac{5}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g}) \oplus n\,({}^1E_{7/2,g} \oplus {}^2E_{7/2,g})$
$7n+\frac{7}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g}) \oplus (n+1)({}^1E_{7/2,g} \oplus {}^2E_{7/2,g})$
$7n+\frac{9}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g}) \oplus (n+1)(2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g})$
$7n+\frac{11}{2}$	$(2n+1)\,E_{1/2,g} \oplus (n+1)(2E_{3/2,g} \oplus 2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g})$
$7n+\frac{13}{2}$	$(n+1)(2E_{1/2,g} \oplus 2E_{3/2,g} \oplus 2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g})$

$n = 0, 1, 2, \ldots$

T **46**.11 Clebsch–Gordan coefficients

Use T **27**.11 ▪. § **16**–11, p. 83

(1) Product forms: $\mathbf{D}_8 \otimes \mathbf{C}_s$.

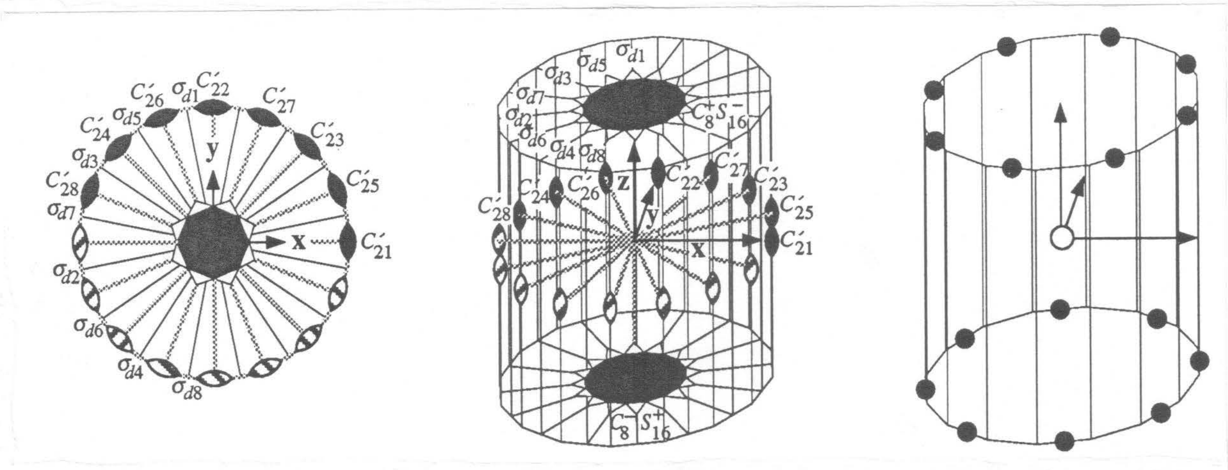
(2) Group chains: $\mathbf{D}_{8d} \supset (\underline{\mathbf{C}_{8v}})$, $\mathbf{D}_{8d} \supset (\underline{\mathbf{D}_8})$, $\mathbf{D}_{8d} \supset \underline{\mathbf{S}_{16}}$.

(3) Operations of G: E, (C_8^+, C_8^-), (C_4^+, C_4^-), (C_8^{3+}, C_8^{3-}), C_2, $(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27}, C'_{28})$,
$(S_{16}^{7-}, S_{16}^{7+})$, $(S_{16}^{5-}, S_{16}^{5+})$, $(S_{16}^{3-}, S_{16}^{3+})$, (S_{16}^-, S_{16}^+),
$(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \sigma_{d6}, \sigma_{d7}, \sigma_{d8})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_8^+, C_8^-), $(\widetilde{C}_8^+, \widetilde{C}_8^-)$, (C_4^+, C_4^-),
$(\widetilde{C}_4^+, \widetilde{C}_4^-)$, (C_8^{3+}, C_8^{3-}), $(\widetilde{C}_8^{3+}, \widetilde{C}_8^{3-})$, $(C_2, \widetilde{C}_2)$,
$(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27}, C'_{28}, \widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24}, \widetilde{C}'_{25}, \widetilde{C}'_{26}, \widetilde{C}'_{27}, \widetilde{C}'_{28})$,
$(S_{16}^{7-}, S_{16}^{7+})$, $(\widetilde{S}_{16}^{7-}, \widetilde{S}_{16}^{7+})$, $(S_{16}^{5-}, S_{16}^{5+})$, $(\widetilde{S}_{16}^{5-}, \widetilde{S}_{16}^{5+})$,
$(S_{16}^{3-}, S_{16}^{3+})$, $(\widetilde{S}_{16}^{3-}, \widetilde{S}_{16}^{3+})$, (S_{16}^-, S_{16}^+), $(\widetilde{S}_{16}^-, \widetilde{S}_{16}^+)$,
$(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \sigma_{d6}, \sigma_{d7}, \sigma_{d8}, \widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3}, \widetilde{\sigma}_{d4}, \widetilde{\sigma}_{d5}, \widetilde{\sigma}_{d6}, \widetilde{\sigma}_{d7}, \widetilde{\sigma}_{d8})$.

(5) Classes and representations: $|r| = 8$, $|i| = 3$, $|I| = 11$, $|\widetilde{I}| = 8$.

F **47** See Chapter **15**, p. 65



Examples:

T **47**.0 Subgroup elements

§ **16**–0, p. 68

$\mathbf{D}_{8d}$	$\mathbf{D}_8$	$\mathbf{D}_4$	$\mathbf{D}_2$	$\mathbf{S}_{16}$	$\mathbf{C}_8$	$\mathbf{C}_4$	$\mathbf{C}_2$
E	E	E	E	E	E	E	E
C_8^+	C_8^+			C_8^+	C_8^+		
C_8^-	C_8^-			C_8^-	C_8^-		
C_4^+	C_4^+	C_4^+		C_4^+	C_4^+	C_4^+	
C_4^-	C_4^-	C_4^-		C_4^-	C_4^-	C_4^-	
C_8^{3+}	C_8^{3+}			C_8^{3+}	C_8^{3+}		
C_8^{3-}	C_8^{3-}			C_8^{3-}	C_8^{3-}		
C_2	C_2	C_2	C_{2z}	C_2	C_2	C_2	C_2
C'_{21}	C'_{21}	C'_{21}	C_{2x}				
C'_{22}	C'_{22}	C'_{22}	C_{2y}				
C'_{23}	C'_{23}	C''_{21}					
C'_{24}	C'_{24}	C''_{22}					
C'_{25}	C''_{21}						
C'_{26}	C''_{22}						
C'_{27}	C''_{23}						
C'_{28}	C''_{24}						
S_{16}^{7-}				S_{16}^{7-}			
S_{16}^{7+}				S_{16}^{7+}			
S_{16}^{5-}				S_{16}^{5-}			
S_{16}^{5+}				S_{16}^{5+}			
S_{16}^{3-}				S_{16}^{3-}			
S_{16}^{3+}				S_{16}^{3+}			
S_{16}^{-}				S_{16}^{-}			
S_{16}^{+}				S_{16}^{+}			
σ_{d1}							
σ_{d2}							
σ_{d3}							
σ_{d4}							
σ_{d5}							
σ_{d6}							
σ_{d7}							
σ_{d8}							

T **47**.1 Parameters

§ **16**–1, p. 68

$\mathbf{D}_{8d}$	α	β	γ	ϕ	$\mathbf{n}$	λ	$\mathbf{\Lambda}$
E	0	0	0	0	$(0\ \ 0\ \ 0)$	$\llbracket\ 1,$	$(0\ \ 0\ \ 0)\rrbracket$
C_8^+	0	0	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$(0\ \ 0\ \ 1)$	$\llbracket\ c_8,$	$(0\ \ 0\ \ s_8)\rrbracket$
C_8^-	0	0	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$(0\ \ 0\ -1)$	$\llbracket\ c_8,$	$(0\ \ 0\ -s_8)\rrbracket$
C_4^+	0	0	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$(0\ \ 0\ \ 1)$	$\llbracket\frac{1}{\sqrt{2}},$	$(0\ \ 0\ \ \frac{1}{\sqrt{2}})\rrbracket$
C_4^-	0	0	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	$(0\ \ 0\ -1)$	$\llbracket\frac{1}{\sqrt{2}},$	$(0\ \ 0\ -\frac{1}{\sqrt{2}})\rrbracket$
C_8^{3+}	0	0	$\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$(0\ \ 0\ \ 1)$	$\llbracket\ s_8,$	$(0\ \ 0\ \ c_8)\rrbracket$
C_8^{3-}	0	0	$-\frac{3\pi}{4}$	$\frac{3\pi}{4}$	$(0\ \ 0\ -1)$	$\llbracket\ s_8,$	$(0\ \ 0\ -c_8)\rrbracket$
C_2	0	0	π	π	$(0\ \ 0\ \ 1)$	$\llbracket\ 0,$	$(0\ \ 0\ \ 1)\rrbracket$
C'_{21}	0	π	π	π	$(1\ \ 0\ \ 0)$	$\llbracket\ 0,$	$(1\ \ 0\ \ 0)\rrbracket$
C'_{22}	0	π	0	π	$(0\ \ 1\ \ 0)$	$\llbracket\ 0,$	$(0\ \ 1\ \ 0)\rrbracket$
C'_{23}	0	π	$\frac{\pi}{2}$	π	$(\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)$	$\llbracket\ 0,$	$(\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)\rrbracket$
C'_{24}	0	π	$-\frac{\pi}{2}$	π	$(-\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)$	$\llbracket\ 0,$	$(-\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)\rrbracket$
C'_{25}	0	π	$\frac{3\pi}{4}$	π	$(c_8\ \ s_8\ \ 0)$	$\llbracket\ 0,$	$(c_8\ \ s_8\ \ 0)\rrbracket$
C'_{26}	0	π	$-\frac{\pi}{4}$	π	$(-s_8\ \ c_8\ \ 0)$	$\llbracket\ 0,$	$(-s_8\ \ c_8\ \ 0)\rrbracket$
C'_{27}	0	π	$\frac{\pi}{4}$	π	$(s_8\ \ c_8\ \ 0)$	$\llbracket\ 0,$	$(s_8\ \ c_8\ \ 0)\rrbracket$
C'_{28}	0	π	$-\frac{3\pi}{4}$	π	$(-c_8\ \ s_8\ \ 0)$	$\llbracket\ 0,$	$(-c_8\ \ s_8\ \ 0)\rrbracket$
S_{16}^{7-}	0	0	$\frac{\pi}{8}$	$\frac{\pi}{8}$	$(0\ \ 0\ \ 1)$	$\llbracket c_{16},$	$(0\ \ 0\ \ s_{16})\rrbracket$
S_{16}^{7+}	0	0	$-\frac{\pi}{8}$	$\frac{\pi}{8}$	$(0\ \ 0\ -1)$	$\llbracket c_{16},$	$(0\ \ 0\ -s_{16})\rrbracket$
S_{16}^{5-}	0	0	$\frac{3\pi}{8}$	$\frac{3\pi}{8}$	$(0\ \ 0\ \ 1)$	$\llbracket c_{16}^3,$	$(0\ \ 0\ \ s_{16}^3)\rrbracket$
S_{16}^{5+}	0	0	$-\frac{3\pi}{8}$	$\frac{3\pi}{8}$	$(0\ \ 0\ -1)$	$\llbracket c_{16}^3,$	$(0\ \ 0\ -s_{16}^3)\rrbracket$
S_{16}^{3-}	0	0	$\frac{5\pi}{8}$	$\frac{5\pi}{8}$	$(0\ \ 0\ \ 1)$	$\llbracket s_{16}^3,$	$(0\ \ 0\ \ c_{16}^3)\rrbracket$
S_{16}^{3+}	0	0	$-\frac{5\pi}{8}$	$\frac{5\pi}{8}$	$(0\ \ 0\ -1)$	$\llbracket s_{16}^3,$	$(0\ \ 0\ -c_{16}^3)\rrbracket$
S_{16}^-	0	0	$\frac{7\pi}{8}$	$\frac{7\pi}{8}$	$(0\ \ 0\ \ 1)$	$\llbracket s_{16},$	$(0\ \ 0\ \ c_{16})\rrbracket$
S_{16}^+	0	0	$-\frac{7\pi}{8}$	$\frac{7\pi}{8}$	$(0\ \ 0\ -1)$	$\llbracket s_{16},$	$(0\ \ 0\ -c_{16})\rrbracket$
σ_{d1}	0	π	$\frac{7\pi}{8}$	π	$(c_{16}\ s_{16}\ \ 0)$	$\llbracket\ 0,$	$(c_{16}\ s_{16}\ \ 0)\rrbracket$
σ_{d2}	0	π	$-\frac{\pi}{8}$	π	$(-s_{16}\ c_{16}\ \ 0)$	$\llbracket\ 0,$	$(-s_{16}\ c_{16}\ \ 0)\rrbracket$
σ_{d3}	0	π	$\frac{3\pi}{8}$	π	$(s_{16}^3\ c_{16}^3\ \ 0)$	$\llbracket\ 0,$	$(s_{16}^3\ c_{16}^3\ \ 0)\rrbracket$
σ_{d4}	0	π	$-\frac{5\pi}{8}$	π	$(-c_{16}^3\ s_{16}^3\ \ 0)$	$\llbracket\ 0,$	$(-c_{16}^3\ s_{16}^3\ \ 0)\rrbracket$
σ_{d5}	0	π	$\frac{5\pi}{8}$	π	$(c_{16}^3\ s_{16}^3\ \ 0)$	$\llbracket\ 0,$	$(c_{16}^3\ s_{16}^3\ \ 0)\rrbracket$
σ_{d6}	0	π	$-\frac{3\pi}{8}$	π	$(-s_{16}^3\ c_{16}^3\ \ 0)$	$\llbracket\ 0,$	$(-s_{16}^3\ c_{16}^3\ \ 0)\rrbracket$
σ_{d7}	0	π	$\frac{\pi}{8}$	π	$(s_{16}\ c_{16}\ \ 0)$	$\llbracket\ 0,$	$(s_{16}\ c_{16}\ \ 0)\rrbracket$
σ_{d8}	0	π	$-\frac{7\pi}{8}$	π	$(-c_{16}\ s_{16}\ \ 0)$	$\llbracket\ 0,$	$(-c_{16}\ s_{16}\ \ 0)\rrbracket$

$c_n^m = \cos \frac{m}{n}\pi$, $s_n^m = \sin \frac{m}{n}\pi$

T **47**.2 Multiplication table

§ **16**–2, p. 69

$\mathbf{D}_{8d}$	E	C_8^+	C_8^-	C_4^+	C_4^-	C_8^{3+}	C_8^{3-}	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	S_{16}^{7-}	S_{16}^{7+}	S_{16}^{5-}	S_{16}^{5+}	S_{16}^{3-}	S_{16}^{3+}	S_{16}^-	S_{16}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}
E	E	C_8^+	C_8^-	C_4^+	C_4^-	C_8^{3+}	C_8^{3-}	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	S_{16}^{7-}	S_{16}^{7+}	S_{16}^{5-}	S_{16}^{5+}	S_{16}^{3-}	S_{16}^{3+}	S_{16}^-	S_{16}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}
C_8^+	C_8^+	C_4^+	E	C_8^{3+}	C_8^-	C_2	C_4^-	C_8^{3-}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{23}	C'_{24}	C'_{22}	C'_{21}	S_{16}^{5-}	S_{16}^{7-}	S_{16}^{3-}	S_{16}^{7+}	S_{16}^-	S_{16}^{5+}	S_{16}^+	S_{16}^{3+}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d3}	σ_{d4}	σ_{d2}	σ_{d1}
C_8^-	C_8^-	E	C_4^-	C_8^+	C_8^{3-}	C_4^+	C_2	C_8^{3+}	C'_{28}	C'_{27}	C'_{25}	C'_{26}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	S_{16}^{7+}	S_{16}^{5+}	S_{16}^{7-}	S_{16}^{3+}	S_{16}^{5-}	S_{16}^+	S_{16}^{3-}	S_{16}^-	σ_{d8}	σ_{d7}	σ_{d5}	σ_{d6}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}
C_4^+	C_4^+	C_8^{3+}	C_8^+	C_2	E	C_8^{3-}	C_8^-	C_4^-	C'_{23}	C'_{24}	C'_{22}	C'_{21}	C'_{27}	C'_{28}	C'_{26}	C'_{25}	S_{16}^{3-}	S_{16}^{5-}	S_{16}^-	S_{16}^{7-}	S_{16}^+	S_{16}^{7+}	S_{16}^{3+}	S_{16}^{5+}	σ_{d3}	σ_{d4}	σ_{d2}	σ_{d1}	σ_{d7}	σ_{d8}	σ_{d6}	σ_{d5}
C_4^-	C_4^-	C_8^-	C_8^{3-}	E	C_2	C_8^+	C_8^{3+}	C_4^+	C'_{24}	C'_{23}	C'_{21}	C'_{22}	C'_{28}	C'_{27}	C'_{25}	C'_{26}	S_{16}^{5+}	S_{16}^{3+}	S_{16}^{7+}	S_{16}^+	S_{16}^{7-}	S_{16}^-	S_{16}^{5-}	S_{16}^{3-}	σ_{d4}	σ_{d3}	σ_{d1}	σ_{d2}	σ_{d8}	σ_{d7}	σ_{d5}	σ_{d6}
C_8^{3+}	C_8^{3+}	C_2	C_4^+	C_8^{3-}	C_8^+	C_4^-	E	C_8^-	C'_{27}	C'_{28}	C'_{26}	C'_{25}	C'_{22}	C'_{21}	C'_{24}	C'_{23}	S_{16}^-	S_{16}^{3-}	S_{16}^+	S_{16}^{5-}	S_{16}^{3+}	S_{16}^{7-}	S_{16}^{5+}	S_{16}^{7+}	σ_{d7}	σ_{d8}	σ_{d6}	σ_{d5}	σ_{d2}	σ_{d1}	σ_{d4}	σ_{d3}
C_8^{3-}	C_8^{3-}	C_4^-	C_2	C_8^-	C_8^{3+}	E	C_4^+	C_8^+	C'_{26}	C'_{25}	C'_{28}	C'_{27}	C'_{24}	C'_{23}	C'_{21}	C'_{22}	S_{16}^{3+}	S_{16}^+	S_{16}^{5+}	S_{16}^-	S_{16}^{7+}	S_{16}^{3-}	S_{16}^{7-}	S_{16}^{5-}	σ_{d6}	σ_{d5}	σ_{d8}	σ_{d7}	σ_{d4}	σ_{d3}	σ_{d1}	σ_{d2}
C_2	C_2	C_8^{3-}	C_8^{3+}	C_4^-	C_4^+	C_8^-	C_8^+	E	C'_{22}	C'_{21}	C'_{24}	C'_{23}	C'_{26}	C'_{25}	C'_{28}	C'_{27}	S_{16}^+	S_{16}^-	S_{16}^{3+}	S_{16}^{3-}	S_{16}^{5+}	S_{16}^{5-}	S_{16}^{7+}	S_{16}^{7-}	σ_{d2}	σ_{d1}	σ_{d4}	σ_{d3}	σ_{d6}	σ_{d5}	σ_{d8}	σ_{d7}
C'_{21}	C'_{21}	C'_{28}	C'_{25}	C'_{24}	C'_{23}	C'_{26}	C'_{27}	C'_{22}	E	C_2	C_4^-	C_4^+	C_8^-	C_8^{3+}	C_8^{3-}	C_8^+	σ_{d8}	σ_{d1}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d3}	σ_{d2}	σ_{d7}	S_{16}^{7+}	S_{16}^-	S_{16}^{3+}	S_{16}^{5-}	S_{16}^{5+}	S_{16}^{3-}	S_{16}^+	S_{16}^{7-}
C'_{22}	C'_{22}	C'_{27}	C'_{26}	C'_{23}	C'_{24}	C'_{25}	C'_{28}	C'_{21}	C_2	E	C_4^+	C_4^-	C_8^{3+}	C_8^-	C_8^+	C_8^{3-}	σ_{d7}	σ_{d2}	σ_{d3}	σ_{d6}	σ_{d5}	σ_{d4}	σ_{d1}	σ_{d8}	S_{16}^-	S_{16}^{7+}	S_{16}^{5-}	S_{16}^{3+}	S_{16}^{3-}	S_{16}^{5+}	S_{16}^{7-}	S_{16}^+
C'_{23}	C'_{23}	C'_{25}	C'_{27}	C'_{21}	C'_{22}	C'_{28}	C'_{26}	C'_{24}	C_4^+	C_4^-	E	C_2	C_8^+	C_8^{3-}	C_8^-	C_8^{3+}	σ_{d5}	σ_{d3}	σ_{d1}	σ_{d7}	σ_{d8}	σ_{d2}	σ_{d4}	σ_{d6}	S_{16}^{5-}	S_{16}^{3+}	S_{16}^{7+}	S_{16}^-	S_{16}^{7-}	S_{16}^+	S_{16}^{5+}	S_{16}^{3-}
C'_{24}	C'_{24}	C'_{26}	C'_{28}	C'_{22}	C'_{21}	C'_{27}	C'_{25}	C'_{23}	C_4^-	C_4^+	C_2	E	C_8^{3-}	C_8^+	C_8^{3+}	C_8^-	σ_{d6}	σ_{d4}	σ_{d2}	σ_{d8}	σ_{d7}	σ_{d1}	σ_{d3}	σ_{d5}	S_{16}^{3+}	S_{16}^{5-}	S_{16}^-	S_{16}^{7+}	S_{16}^+	S_{16}^{7-}	S_{16}^{3-}	S_{16}^{5+}
C'_{25}	C'_{25}	C'_{21}	C'_{23}	C'_{28}	C'_{27}	C'_{24}	C'_{22}	C'_{26}	C_8^+	C_8^{3-}	C_8^-	C_8^{3+}	E	C_2	C_4^-	C_4^+	σ_{d1}	σ_{d5}	σ_{d8}	σ_{d3}	σ_{d4}	σ_{d7}	σ_{d6}	σ_{d2}	S_{16}^{7-}	S_{16}^+	S_{16}^{5+}	S_{16}^{3-}	S_{16}^{7+}	S_{16}^-	S_{16}^{3+}	S_{16}^{5-}
C'_{26}	C'_{26}	C'_{22}	C'_{24}	C'_{27}	C'_{28}	C'_{23}	C'_{21}	C'_{25}	C_8^{3-}	C_8^+	C_8^{3+}	C_8^-	C_2	E	C_4^+	C_4^-	σ_{d2}	σ_{d6}	σ_{d7}	σ_{d4}	σ_{d3}	σ_{d8}	σ_{d5}	σ_{d1}	S_{16}^+	S_{16}^{7-}	S_{16}^{3-}	S_{16}^{5+}	S_{16}^-	S_{16}^{7+}	S_{16}^{5-}	S_{16}^{3+}
C'_{27}	C'_{27}	C'_{23}	C'_{22}	C'_{25}	C'_{26}	C'_{21}	C'_{24}	C'_{28}	C_8^{3+}	C_8^-	C_8^+	C_8^{3-}	C_4^+	C_4^-	E	C_2	σ_{d3}	σ_{d7}	σ_{d5}	σ_{d2}	σ_{d1}	σ_{d6}	σ_{d8}	σ_{d4}	S_{16}^{3-}	S_{16}^{5+}	S_{16}^{7-}	S_{16}^+	S_{16}^{5-}	S_{16}^{3+}	S_{16}^{7+}	S_{16}^-
C'_{28}	C'_{28}	C'_{24}	C'_{21}	C'_{26}	C'_{25}	C'_{22}	C'_{23}	C'_{27}	C_8^-	C_8^{3+}	C_8^{3-}	C_8^+	C_4^-	C_4^+	C_2	E	σ_{d4}	σ_{d8}	σ_{d6}	σ_{d1}	σ_{d2}	σ_{d5}	σ_{d7}	σ_{d3}	S_{16}^{5+}	S_{16}^{3-}	S_{16}^+	S_{16}^{7-}	S_{16}^{3+}	S_{16}^{5-}	S_{16}^-	S_{16}^{7+}
S_{16}^{7-}	S_{16}^{7-}	S_{16}^{5-}	S_{16}^{7+}	S_{16}^{3-}	S_{16}^{5+}	S_{16}^-	S_{16}^{3+}	S_{16}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}	C_8^+	E	C_4^+	C_8^-	C_8^{3+}	C_4^-	C_2	C_8^{3-}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{23}	C'_{24}	C'_{22}	C'_{21}
S_{16}^{7+}	S_{16}^{7+}	S_{16}^{7-}	S_{16}^{5+}	S_{16}^{5-}	S_{16}^{3+}	S_{16}^{3-}	S_{16}^+	S_{16}^-	σ_{d8}	σ_{d7}	σ_{d5}	σ_{d6}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	E	C_8^-	C_8^+	C_4^-	C_4^+	C_8^{3-}	C_8^{3+}	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}
S_{16}^{5-}	S_{16}^{5-}	S_{16}^{3-}	S_{16}^{7-}	S_{16}^-	S_{16}^{7+}	S_{16}^+	S_{16}^{5+}	S_{16}^{3+}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d3}	σ_{d4}	σ_{d2}	σ_{d1}	C_4^+	C_8^+	C_8^{3+}	E	C_2	C_8^-	C_8^{3-}	C_4^-	C'_{23}	C'_{24}	C'_{22}	C'_{21}	C'_{27}	C'_{28}	C'_{26}	C'_{25}
S_{16}^{5+}	S_{16}^{5+}	S_{16}^{7+}	S_{16}^{3+}	S_{16}^{7-}	S_{16}^+	S_{16}^{5-}	S_{16}^-	S_{16}^{3-}	σ_{d4}	σ_{d3}	σ_{d1}	σ_{d2}	σ_{d8}	σ_{d7}	σ_{d5}	σ_{d6}	C_8^-	C_4^-	E	C_8^{3-}	C_8^+	C_2	C_4^+	C_8^{3+}	C'_{28}	C'_{27}	C'_{25}	C'_{26}	C'_{21}	C'_{22}	C'_{23}	C'_{24}
S_{16}^{3-}	S_{16}^{3-}	S_{16}^-	S_{16}^{5-}	S_{16}^+	S_{16}^{7-}	S_{16}^{3+}	S_{16}^{7+}	S_{16}^{5+}	σ_{d3}	σ_{d4}	σ_{d2}	σ_{d1}	σ_{d7}	σ_{d8}	σ_{d6}	σ_{d5}	C_8^{3+}	C_4^+	C_2	C_8^+	C_8^{3-}	E	C_4^-	C_8^-	C'_{27}	C'_{28}	C'_{26}	C'_{25}	C'_{22}	C'_{21}	C'_{24}	C'_{23}
S_{16}^{3+}	S_{16}^{3+}	S_{16}^{5+}	S_{16}^+	S_{16}^{7+}	S_{16}^-	S_{16}^{7-}	S_{16}^{3-}	S_{16}^{5-}	σ_{d6}	σ_{d5}	σ_{d8}	σ_{d7}	σ_{d4}	σ_{d3}	σ_{d1}	σ_{d2}	C_4^-	C_8^{3-}	C_8^-	C_2	E	C_8^{3+}	C_8^+	C_4^+	C'_{24}	C'_{23}	C'_{21}	C'_{22}	C'_{28}	C'_{27}	C'_{25}	C'_{26}
S_{16}^-	S_{16}^-	S_{16}^+	S_{16}^{3-}	S_{16}^{3+}	S_{16}^{5-}	S_{16}^{5+}	S_{16}^{7-}	S_{16}^{7+}	σ_{d7}	σ_{d8}	σ_{d6}	σ_{d5}	σ_{d2}	σ_{d1}	σ_{d4}	σ_{d3}	C_2	C_8^{3+}	C_8^{3-}	C_4^+	C_4^-	C_8^+	C_8^-	E	C'_{22}	C'_{21}	C'_{24}	C'_{23}	C'_{26}	C'_{25}	C'_{28}	C'_{27}
S_{16}^+	S_{16}^+	S_{16}^{3+}	S_{16}^-	S_{16}^{5+}	S_{16}^{3-}	S_{16}^{7+}	S_{16}^{5-}	S_{16}^{7-}	σ_{d2}	σ_{d1}	σ_{d4}	σ_{d3}	σ_{d6}	σ_{d5}	σ_{d8}	σ_{d7}	C_8^{3-}	C_2	C_4^-	C_8^{3+}	C_8^-	C_4^+	E	C_8^+	C'_{26}	C'_{25}	C'_{28}	C'_{27}	C'_{24}	C'_{23}	C'_{21}	C'_{22}
σ_{d1}	σ_{d1}	σ_{d8}	σ_{d5}	σ_{d4}	σ_{d3}	σ_{d6}	σ_{d7}	σ_{d2}	S_{16}^{7-}	S_{16}^+	S_{16}^{5+}	S_{16}^{3-}	S_{16}^{7+}	S_{16}^-	S_{16}^{3+}	S_{16}^{5-}	C'_{21}	C'_{25}	C'_{28}	C'_{23}	C'_{24}	C'_{27}	C'_{26}	C'_{22}	E	C_2	C_4^-	C_4^+	C_8^-	C_8^{3+}	C_8^{3-}	C_8^+
σ_{d2}	σ_{d2}	σ_{d7}	σ_{d6}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d8}	σ_{d1}	S_{16}^+	S_{16}^{7-}	S_{16}^{3-}	S_{16}^{5+}	S_{16}^-	S_{16}^{7+}	S_{16}^{5-}	S_{16}^{3+}	C'_{22}	C'_{26}	C'_{27}	C'_{24}	C'_{23}	C'_{28}	C'_{25}	C'_{21}	C_2	E	C_4^+	C_4^-	C_8^{3+}	C_8^-	C_8^+	C_8^{3-}
σ_{d3}	σ_{d3}	σ_{d5}	σ_{d7}	σ_{d1}	σ_{d2}	σ_{d8}	σ_{d6}	σ_{d4}	S_{16}^{3-}	S_{16}^{5+}	S_{16}^{7-}	S_{16}^+	S_{16}^{5-}	S_{16}^{3+}	S_{16}^{7+}	S_{16}^-	C'_{23}	C'_{27}	C'_{25}	C'_{22}	C'_{21}	C'_{26}	C'_{28}	C'_{24}	C_4^+	C_4^-	E	C_2	C_8^+	C_8^{3-}	C_8^-	C_8^{3+}
σ_{d4}	σ_{d4}	σ_{d6}	σ_{d8}	σ_{d2}	σ_{d1}	σ_{d7}	σ_{d5}	σ_{d3}	S_{16}^{5+}	S_{16}^{3-}	S_{16}^+	S_{16}^{7-}	S_{16}^{3+}	S_{16}^{5-}	S_{16}^-	S_{16}^{7+}	C'_{24}	C'_{28}	C'_{26}	C'_{21}	C'_{22}	C'_{25}	C'_{27}	C'_{23}	C_4^-	C_4^+	C_2	E	C_8^{3-}	C_8^+	C_8^{3+}	C_8^-
σ_{d5}	σ_{d5}	σ_{d1}	σ_{d3}	σ_{d8}	σ_{d7}	σ_{d4}	σ_{d2}	σ_{d6}	S_{16}^{5-}	S_{16}^{3+}	S_{16}^{7+}	S_{16}^-	S_{16}^{7-}	S_{16}^+	S_{16}^{5+}	S_{16}^{3-}	C'_{25}	C'_{23}	C'_{21}	C'_{27}	C'_{28}	C'_{22}	C'_{24}	C'_{26}	C_8^+	C_8^{3-}	C_8^-	C_8^{3+}	E	C_2	C_4^-	C_4^+
σ_{d6}	σ_{d6}	σ_{d2}	σ_{d4}	σ_{d7}	σ_{d8}	σ_{d3}	σ_{d1}	σ_{d5}	S_{16}^{3+}	S_{16}^{5-}	S_{16}^-	S_{16}^{7+}	S_{16}^+	S_{16}^{7-}	S_{16}^{3-}	S_{16}^{5+}	C'_{26}	C'_{24}	C'_{22}	C'_{28}	C'_{27}	C'_{21}	C'_{23}	C'_{25}	C_8^{3-}	C_8^+	C_8^{3+}	C_8^-	C_2	E	C_4^+	C_4^-
σ_{d7}	σ_{d7}	σ_{d3}	σ_{d2}	σ_{d5}	σ_{d6}	σ_{d1}	σ_{d4}	σ_{d8}	S_{16}^-	S_{16}^{7+}	S_{16}^{5-}	S_{16}^{3+}	S_{16}^{3-}	S_{16}^{5+}	S_{16}^{7-}	S_{16}^+	C'_{27}	C'_{22}	C'_{23}	C'_{26}	C'_{25}	C'_{24}	C'_{21}	C'_{28}	C_8^{3+}	C_8^-	C_8^+	C_8^{3-}	C_4^+	C_4^-	E	C_2
σ_{d8}	σ_{d8}	σ_{d4}	σ_{d1}	σ_{d6}	σ_{d5}	σ_{d2}	σ_{d3}	σ_{d7}	S_{16}^{7+}	S_{16}^-	S_{16}^{3+}	S_{16}^{5-}	S_{16}^{5+}	S_{16}^{3-}	S_{16}^+	S_{16}^{7-}	C'_{28}	C'_{21}	C'_{24}	C'_{25}	C'_{26}	C'_{23}	C'_{22}	C'_{27}	C_8^-	C_8^{3+}	C_8^{3-}	C_8^+	C_4^-	C_4^+	C_2	E

T **47**.3 Factor table

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$\mathbf{D}_{8d}$	E	C_8^+	C_8^-	C_4^+	C_4^-	C_8^{3+}	C_8^{3-}	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	S_{16}^{7-}	S_{16}^{7+}	S_{16}^{5-}	S_{16}^{5+}	S_{16}^{3-}	S_{16}^{3+}	S_{16}^-	S_{16}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_8^+	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	1	−1
C_8^-	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	1	1	1	1	1	1	1
C_4^+	1	1	1	1	1	−1	1	−1	1	1	1	−1	1	1	1	−1	1	1	1	1	−1	1	−1	1	1	1	1	−1	1	1	1	−1
C_4^-	1	1	1	1	−1	1	−1	1	−1	1	1	1	−1	1	1	1	1	1	1	1	1	−1	1	−1	−1	1	1	1	−1	1	1	1
C_8^{3+}	1	1	1	−1	1	−1	1	−1	1	1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	1	1	1	1	−1	1	−1	1	−1
C_8^{3-}	1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	1
C_2	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	1	−1
C'_{21}	1	−1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1
C'_{22}	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1
C'_{23}	1	1	1	1	1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	1	−1	−1	−1	1
C'_{24}	1	1	1	1	−1	1	−1	1	1	−1	−1	−1	1	−1	−1	−1	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	1	−1	−1	−1
C'_{25}	1	1	1	−1	1	−1	1	−1	−1	−1	−1	1	−1	1	−1	1	1	1	−1	1	−1	1	−1	1	−1	−1	−1	1	−1	1	−1	1
C'_{26}	1	1	1	1	1	1	−1	1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1
C'_{27}	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	1
C'_{28}	1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	−1	−1	1	1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	−1	−1
S_{16}^{7-}	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1
S_{16}^{7+}	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	1
S_{16}^{5-}	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	−1	1	1	1	1	−1	1	1	1	−1
S_{16}^{5+}	1	1	1	1	1	1	−1	1	−1	1	1	1	−1	1	1	1	1	1	1	1	1	−1	1	−1	−1	1	1	1	1	1	1	1
S_{16}^{3-}	1	1	1	−1	1	−1	1	−1	1	1	1	−1	1	1	1	−1	1	1	1	1	−1	1	−1	1	1	1	1	−1	1	−1	1	−1
S_{16}^{3+}	1	1	1	1	−1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	−1	1	−1	1	−1	−1	1	1	1	−1	1	1	1
S_{16}^-	1	−1	1	−1	1	−1	1	−1	1	1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	1	−1
S_{16}^+	1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	1
σ_{d1}	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	1	−1	1	−1	1	1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1
σ_{d2}	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
σ_{d3}	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	−1	1	−1	−1	−1	1	−1	−1	−1	1
σ_{d4}	1	1	1	1	−1	1	−1	1	1	−1	1	−1	1	−1	−1	−1	1	1	1	−1	1	−1	1	−1	1	−1	−1	−1	1	−1	−1	−1
σ_{d5}	1	1	1	−1	1	−1	1	−1	−1	−1	−1	1	−1	−1	−1	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	1	−1	1	−1	1
σ_{d6}	1	1	1	1	1	1	−1	1	1	−1	−1	−1	1	−1	−1	−1	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1
σ_{d7}	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	1
σ_{d8}	1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	−1	−1

T **47**.4 Character table

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$\mathbf{D}_{8d}$	E	$2C_8$	$2C_4$	$2C_8^3$	C_2	$8C_2'$	$2S_{16}^7$	$2S_{16}^5$	$2S_{16}^3$	$2S_{16}$	$8\sigma_d$	τ
A_1	1	1	1	1	1	1	1	1	1	1	1	a
A_2	1	1	1	1	1	-1	1	1	1	1	-1	a
B_1	1	1	1	1	1	1	-1	-1	-1	-1	-1	a
B_2	1	1	1	1	1	-1	-1	-1	-1	-1	1	a
E_1	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	0	$-2c_8$	$-2c_8^3$	$2c_8^3$	$2c_8$	0	a
E_2	2	0	-2	0	2	0	$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	0	a
E_3	2	$-\sqrt{2}$	0	$\sqrt{2}$	-2	0	$-2c_8^3$	$2c_8$	$-2c_8$	$2c_8^3$	0	a
E_4	2	-2	2	-2	2	0	0	0	0	0	0	a
E_5	2	$-\sqrt{2}$	0	$\sqrt{2}$	-2	0	$2c_8^3$	$-2c_8$	$2c_8$	$-2c_8^3$	0	a
E_6	2	0	-2	0	2	0	$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	0	a
E_7	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	0	$2c_8$	$2c_8^3$	$-2c_8^3$	$-2c_8$	0	a
$E_{1/2}$	2	$2c_8$	$\sqrt{2}$	$2c_8^3$	0	0	$2c_{16}$	$2c_{16}^3$	$2c_{16}^5$	$2c_{16}^7$	0	c
$E_{3/2}$	2	$2c_8^3$	$-\sqrt{2}$	$-2c_8$	0	0	$2c_{16}^3$	$-2c_{16}^7$	$-2c_{16}$	$-2c_{16}^5$	0	c
$E_{5/2}$	2	$-2c_8^3$	$-\sqrt{2}$	$2c_8$	0	0	$2c_{16}^5$	$-2c_{16}$	$2c_{16}^7$	$2c_{16}^3$	0	c
$E_{7/2}$	2	$-2c_8$	$\sqrt{2}$	$-2c_8^3$	0	0	$2c_{16}^7$	$-2c_{16}^5$	$2c_{16}^3$	$-2c_{16}$	0	c
$E_{9/2}$	2	$-2c_8$	$\sqrt{2}$	$-2c_8^3$	0	0	$-2c_{16}^7$	$2c_{16}^5$	$-2c_{16}^3$	$2c_{16}$	0	c
$E_{11/2}$	2	$-2c_8^3$	$-\sqrt{2}$	$2c_8$	0	0	$-2c_{16}^5$	$2c_{16}$	$-2c_{16}^7$	$-2c_{16}^3$	0	c
$E_{13/2}$	2	$2c_8^3$	$-\sqrt{2}$	$-2c_8$	0	0	$-2c_{16}^3$	$2c_{16}^7$	$2c_{16}$	$2c_{16}^5$	0	c
$E_{15/2}$	2	$2c_8$	$\sqrt{2}$	$2c_8^3$	0	0	$-2c_{16}$	$-2c_{16}^3$	$-2c_{16}^5$	$-2c_{16}^7$	0	c

$c_n^m = \cos \frac{m}{n}\pi$

T **47**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

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$\mathbf{D}_{8d}$	0	1	2	3
A_1	$^\square 1$		$x^2+y^2, {}^\square z^2$	
A_2		R_z		
B_1				
B_2		$^\square z$		$(x^2+y^2)z, {}^\square z^3$
E_1		$^\square(x,y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
E_2			$^\square(xy, x^2-y^2)$	
E_3				$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$
E_4				
E_5				
E_6				$^\square\{xyz, z(x^2-y^2)\}$
E_7		(R_x, R_y)	$^\square(zx, yz)$	

T **47**.6 Symmetrized bases

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$\mathbf{D}_{8d}$	$\bigl\langle \lvert j\, m\rangle \bigr\rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$	$\lvert 9\,8\rangle_-$	2	16
A_2	$\lvert 9\,8\rangle_+$	$\lvert 16\,16\rangle_-$	2	16
B_1	$\lvert 8\,8\rangle_+$	$\lvert 17\,16\rangle_-$	2	16
B_2	$\lvert 1\,0\rangle_+$	$\lvert 8\,8\rangle_-$	2	16
E_1	$\bigl\langle \lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle \bigr\rvert$	$\bigl\langle \lvert 8\,\overline{7}\rangle, -\lvert 8\,7\rangle \bigr\rvert$	2	±16
E_2	$\bigl\langle \lvert 2\,2\rangle, \lvert 2\,\overline{2}\rangle \bigr\rvert$	$\bigl\langle \lvert 7\,\overline{6}\rangle, -\lvert 7\,6\rangle \bigr\rvert$	2	±16
E_3	$\bigl\langle \lvert 3\,\overline{3}\rangle, \lvert 3\,3\rangle \bigr\rvert$	$\bigl\langle \lvert 6\,5\rangle, -\lvert 6\,\overline{5}\rangle \bigr\rvert$	2	±16
E_4	$\bigl\langle \lvert 4\,4\rangle, \lvert 4\,\overline{4}\rangle \bigr\rvert$	$\bigl\langle \lvert 5\,\overline{4}\rangle, -\lvert 5\,4\rangle \bigr\rvert$	2	±16
E_5	$\bigl\langle \lvert 4\,\overline{3}\rangle, -\lvert 4\,3\rangle \bigr\rvert$	$\bigl\langle \lvert 5\,5\rangle, \lvert 5\,\overline{5}\rangle \bigr\rvert$	2	±16
E_6	$\bigl\langle \lvert 3\,2\rangle, -\lvert 3\,\overline{2}\rangle \bigr\rvert$	$\bigl\langle \lvert 6\,\overline{6}\rangle, \lvert 6\,6\rangle \bigr\rvert$	2	±16
E_7	$\bigl\langle \lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle \bigr\rvert$	$\bigl\langle \lvert 7\,\overline{7}\rangle, \lvert 7\,7\rangle \bigr\rvert$	2	±16
$E_{1/2}$	$\bigl\langle \lvert \tfrac{1}{2}\,\tfrac{1}{2}\rangle, \lvert \tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle \bigr\rvert$	$\bigl\langle \lvert \tfrac{3}{2}\,\tfrac{1}{2}\rangle, -\lvert \tfrac{3}{2}\,\overline{\tfrac{1}{2}}\rangle \bigr\rvert$	2	±16
	$\bigl\langle \lvert \tfrac{15}{2}\,\overline{\tfrac{15}{2}}\rangle, -\lvert \tfrac{15}{2}\,\tfrac{15}{2}\rangle \bigr\rvert^{\bullet}$	$\bigl\langle \lvert \tfrac{17}{2}\,\overline{\tfrac{15}{2}}\rangle, \lvert \tfrac{17}{2}\,\tfrac{15}{2}\rangle \bigr\rvert^{\bullet}$	2	±16
$E_{3/2}$	$\bigl\langle \lvert \tfrac{3}{2}\,\overline{\tfrac{3}{2}}\rangle, -\lvert \tfrac{3}{2}\,\tfrac{3}{2}\rangle \bigr\rvert$	$\bigl\langle \lvert \tfrac{5}{2}\,\overline{\tfrac{3}{2}}\rangle, \lvert \tfrac{5}{2}\,\tfrac{3}{2}\rangle \bigr\rvert$	2	±16
	$\bigl\langle \lvert \tfrac{13}{2}\,\tfrac{13}{2}\rangle, \lvert \tfrac{13}{2}\,\overline{\tfrac{13}{2}}\rangle \bigr\rvert^{\bullet}$	$\bigl\langle \lvert \tfrac{15}{2}\,\tfrac{13}{2}\rangle, -\lvert \tfrac{15}{2}\,\overline{\tfrac{13}{2}}\rangle \bigr\rvert^{\bullet}$	2	±16
$E_{5/2}$	$\bigl\langle \lvert \tfrac{5}{2}\,\tfrac{5}{2}\rangle, \lvert \tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle \bigr\rvert$	$\bigl\langle \lvert \tfrac{7}{2}\,\tfrac{5}{2}\rangle, -\lvert \tfrac{7}{2}\,\overline{\tfrac{5}{2}}\rangle \bigr\rvert$	2	±16
	$\bigl\langle \lvert \tfrac{11}{2}\,\overline{\tfrac{11}{2}}\rangle, -\lvert \tfrac{11}{2}\,\tfrac{11}{2}\rangle \bigr\rvert^{\bullet}$	$\bigl\langle \lvert \tfrac{13}{2}\,\overline{\tfrac{11}{2}}\rangle, \lvert \tfrac{13}{2}\,\tfrac{11}{2}\rangle \bigr\rvert^{\bullet}$	2	±16
$E_{7/2}$	$\bigl\langle \lvert \tfrac{7}{2}\,\overline{\tfrac{7}{2}}\rangle, -\lvert \tfrac{7}{2}\,\tfrac{7}{2}\rangle \bigr\rvert$	$\bigl\langle \lvert \tfrac{9}{2}\,\overline{\tfrac{7}{2}}\rangle, \lvert \tfrac{9}{2}\,\tfrac{7}{2}\rangle \bigr\rvert$	2	±16
	$\bigl\langle \lvert \tfrac{9}{2}\,\tfrac{9}{2}\rangle, \lvert \tfrac{9}{2}\,\overline{\tfrac{9}{2}}\rangle \bigr\rvert^{\bullet}$	$\bigl\langle \lvert \tfrac{11}{2}\,\tfrac{9}{2}\rangle, -\lvert \tfrac{11}{2}\,\overline{\tfrac{9}{2}}\rangle \bigr\rvert^{\bullet}$	2	±16
$E_{9/2}$	$\bigl\langle \lvert \tfrac{9}{2}\,\tfrac{9}{2}\rangle, \lvert \tfrac{9}{2}\,\overline{\tfrac{9}{2}}\rangle \bigr\rvert$	$\bigl\langle \lvert \tfrac{11}{2}\,\tfrac{9}{2}\rangle, -\lvert \tfrac{11}{2}\,\overline{\tfrac{9}{2}}\rangle \bigr\rvert$	2	±16
	$\bigl\langle \lvert \tfrac{7}{2}\,\overline{\tfrac{7}{2}}\rangle, -\lvert \tfrac{7}{2}\,\tfrac{7}{2}\rangle \bigr\rvert^{\bullet}$	$\bigl\langle \lvert \tfrac{9}{2}\,\overline{\tfrac{7}{2}}\rangle, \lvert \tfrac{9}{2}\,\tfrac{7}{2}\rangle \bigr\rvert^{\bullet}$	2	±16
$E_{11/2}$	$\bigl\langle \lvert \tfrac{11}{2}\,\overline{\tfrac{11}{2}}\rangle, -\lvert \tfrac{11}{2}\,\tfrac{11}{2}\rangle \bigr\rvert$	$\bigl\langle \lvert \tfrac{13}{2}\,\overline{\tfrac{11}{2}}\rangle, \lvert \tfrac{13}{2}\,\tfrac{11}{2}\rangle \bigr\rvert$	2	±16
	$\bigl\langle \lvert \tfrac{5}{2}\,\tfrac{5}{2}\rangle, \lvert \tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle \bigr\rvert^{\bullet}$	$\bigl\langle \lvert \tfrac{7}{2}\,\tfrac{5}{2}\rangle, -\lvert \tfrac{7}{2}\,\overline{\tfrac{5}{2}}\rangle \bigr\rvert^{\bullet}$	2	±16
$E_{13/2}$	$\bigl\langle \lvert \tfrac{13}{2}\,\tfrac{13}{2}\rangle, \lvert \tfrac{13}{2}\,\overline{\tfrac{13}{2}}\rangle \bigr\rvert$	$\bigl\langle \lvert \tfrac{15}{2}\,\tfrac{13}{2}\rangle, -\lvert \tfrac{15}{2}\,\overline{\tfrac{13}{2}}\rangle \bigr\rvert$	2	±16
	$\bigl\langle \lvert \tfrac{3}{2}\,\overline{\tfrac{3}{2}}\rangle, -\lvert \tfrac{3}{2}\,\tfrac{3}{2}\rangle \bigr\rvert^{\bullet}$	$\bigl\langle \lvert \tfrac{5}{2}\,\overline{\tfrac{3}{2}}\rangle, \lvert \tfrac{5}{2}\,\tfrac{3}{2}\rangle \bigr\rvert^{\bullet}$	2	±16
$E_{15/2}$	$\bigl\langle \lvert \tfrac{15}{2}\,\overline{\tfrac{15}{2}}\rangle, -\lvert \tfrac{15}{2}\,\tfrac{15}{2}\rangle \bigr\rvert$	$\bigl\langle \lvert \tfrac{17}{2}\,\overline{\tfrac{15}{2}}\rangle, \lvert \tfrac{17}{2}\,\tfrac{15}{2}\rangle \bigr\rvert$	2	±16
	$\bigl\langle \lvert \tfrac{1}{2}\,\tfrac{1}{2}\rangle, \lvert \tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle \bigr\rvert^{\bullet}$	$\bigl\langle \lvert \tfrac{3}{2}\,\tfrac{1}{2}\rangle, -\lvert \tfrac{3}{2}\,\overline{\tfrac{1}{2}}\rangle \bigr\rvert^{\bullet}$	2	±16

T **47**.7 Matrix representations

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$\mathbf{D}_{8d}$	E_1	E_2	E_3	E_4	E_5	E_6	E_7
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_8^+	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\theta}^* & 0 \\ 0 & \bar{\theta} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\theta}^* & 0 \\ 0 & \bar{\theta} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$
C_8^-	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{\theta} & 0 \\ 0 & \bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\theta} & 0 \\ 0 & \bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$
C_4^+	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$
C_4^-	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$
C_8^{3+}	$\begin{bmatrix} \bar{\theta} & 0 \\ 0 & \bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{\theta} & 0 \\ 0 & \bar{\theta}^* \end{bmatrix}$
C_8^{3-}	$\begin{bmatrix} \bar{\theta}^* & 0 \\ 0 & \bar{\theta} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\theta}^* & 0 \\ 0 & \bar{\theta} \end{bmatrix}$
C_2	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$
C'_{25}	$\begin{bmatrix} 0 & \bar{\theta}^* \\ \bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta}^* \\ \bar{\theta} & 0 \end{bmatrix}$
C'_{26}	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta}^* \\ \bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta}^* \\ \bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$
C'_{27}	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta} \\ \bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta} \\ \bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$
C'_{28}	$\begin{bmatrix} 0 & \bar{\theta} \\ \bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta} \\ \bar{\theta}^* & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/32)$, $\epsilon = \exp(4\pi\mathrm{i}/32)$, $\eta = \exp(6\pi\mathrm{i}/32)$, $\theta = \exp(8\pi\mathrm{i}/32)$ $\twoheadrightarrow$

T **47**.7 Matrix representations *(cont.)*

$\mathbf{D}_{8d}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$	$E_{11/2}$	$E_{13/2}$	$E_{15/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_8^+	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
C_8^-	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
C_4^+	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \bar{\theta}^* & 0 \\ 0 & \bar{\theta} \end{bmatrix}$	$\begin{bmatrix} \bar{\theta}^* & 0 \\ 0 & \bar{\theta} \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \bar{\theta}^* & 0 \\ 0 & \bar{\theta} \end{bmatrix}$	$\begin{bmatrix} \bar{\theta}^* & 0 \\ 0 & \bar{\theta} \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$
C_4^-	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\theta} & 0 \\ 0 & \bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\theta} & 0 \\ 0 & \bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\theta} & 0 \\ 0 & \bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\theta} & 0 \\ 0 & \bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$
C_8^{3+}	$\begin{bmatrix} \mathrm{i}\bar{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$
C_8^{3-}	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar{\epsilon} \end{bmatrix}$
C_2	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \bar{\theta} \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta} \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta} \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta} \\ \theta^* & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & \bar{\theta}^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta}^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta}^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta}^* \\ \theta & 0 \end{bmatrix}$
C'_{25}	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$
C'_{26}	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$
C'_{27}	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$
C'_{28}	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/32)$, $\epsilon = \exp(4\pi\mathrm{i}/32)$, $\eta = \exp(6\pi\mathrm{i}/32)$, $\theta = \exp(8\pi\mathrm{i}/32)$ $\twoheadrightarrow$

T **47**.7 Matrix representations *(cont.)*

$\mathbf{D}_{8d}$	E_1	E_2	E_3	E_4	E_5	E_6	E_7
S_{16}^{7-}	$\begin{bmatrix} \bar\epsilon^* & 0 \\ 0 & \bar\epsilon \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\epsilon^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{1}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar\epsilon \end{bmatrix}$	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
S_{16}^{7+}	$\begin{bmatrix} \bar\epsilon & 0 \\ 0 & \bar\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{1}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\epsilon & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
S_{16}^{5-}	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{1}} \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon & 0 \\ 0 & \bar\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\epsilon & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$
S_{16}^{5+}	$\begin{bmatrix} \mathrm{i}\bar\epsilon^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{1}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon^* & 0 \\ 0 & \bar\epsilon \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar\epsilon \end{bmatrix}$
S_{16}^{3-}	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar\epsilon \end{bmatrix}$	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon^* & 0 \\ 0 & \bar\epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{1}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\epsilon^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$
S_{16}^{3+}	$\begin{bmatrix} \mathrm{i}\bar\epsilon & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon & 0 \\ 0 & \bar\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{1}} \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar\epsilon^* \end{bmatrix}$
S_{16}^{-}	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\epsilon & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{1}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\bar\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon & 0 \\ 0 & \bar\epsilon^* \end{bmatrix}$
S_{16}^{+}	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\bar\epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{1}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\epsilon^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \bar\epsilon^* & 0 \\ 0 & \bar\epsilon \end{bmatrix}$
σ_{d1}	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\bar\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{1}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\epsilon^* \\ \bar\epsilon & 0 \end{bmatrix}$
σ_{d2}	$\begin{bmatrix} 0 & \bar\epsilon^* \\ \bar\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{1}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\bar\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$
σ_{d3}	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{1}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\epsilon^* \\ \bar\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\bar\epsilon & 0 \end{bmatrix}$
σ_{d4}	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\bar\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\epsilon^* \\ \bar\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{1}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$
σ_{d5}	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\epsilon \\ \bar\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{1}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\bar\epsilon^* & 0 \end{bmatrix}$
σ_{d6}	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\bar\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{1}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\epsilon \\ \bar\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$
σ_{d7}	$\begin{bmatrix} 0 & \bar\epsilon \\ \bar\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\bar\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{1}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$
σ_{d8}	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{1}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\bar\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\epsilon \\ \bar\epsilon^* & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/32)$, $\epsilon = \exp(4\pi\mathrm{i}/32)$, $\eta = \exp(6\pi\mathrm{i}/32)$, $\theta = \exp(8\pi\mathrm{i}/32)$ $\ggg$

T 47.7 Matrix representations *(cont.)*

$\mathbf{D}_{8d}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$	$E_{11/2}$	$E_{13/2}$	$E_{15/2}$
S_{16}^{7-}	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\eta & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\bar\delta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\delta^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\bar\eta^* \end{bmatrix}$	$\begin{bmatrix} \bar\eta & 0 \\ 0 & \bar\eta^* \end{bmatrix}$	$\begin{bmatrix} \bar\delta^* & 0 \\ 0 & \bar\delta \end{bmatrix}$
S_{16}^{7+}	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\bar\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\delta & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\bar\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\eta^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \bar\eta^* & 0 \\ 0 & \bar\eta \end{bmatrix}$	$\begin{bmatrix} \bar\delta & 0 \\ 0 & \bar\delta^* \end{bmatrix}$
S_{16}^{5-}	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\bar\delta^* \end{bmatrix}$	$\begin{bmatrix} \bar\delta & 0 \\ 0 & \bar\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\eta^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\bar\eta \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\delta & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \bar\eta^* & 0 \\ 0 & \bar\eta \end{bmatrix}$
S_{16}^{5+}	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\delta^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$	$\begin{bmatrix} \bar\delta^* & 0 \\ 0 & \bar\delta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\bar\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\eta & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\bar\delta \end{bmatrix}$	$\begin{bmatrix} \bar\eta & 0 \\ 0 & \bar\eta^* \end{bmatrix}$
S_{16}^{3-}	$\begin{bmatrix} \mathrm{i}\bar\eta & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \bar\delta^* & 0 \\ 0 & \bar\delta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\bar\delta \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \bar\eta & 0 \\ 0 & \bar\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\delta^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\bar\eta^* \end{bmatrix}$
S_{16}^{3+}	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\bar\eta \end{bmatrix}$	$\begin{bmatrix} \bar\delta & 0 \\ 0 & \bar\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\delta & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \bar\eta^* & 0 \\ 0 & \bar\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\bar\delta^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\eta^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$
S_{16}^{-}	$\begin{bmatrix} \mathrm{i}\bar\delta & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\eta^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \bar\delta & 0 \\ 0 & \bar\delta^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \bar\eta^* & 0 \\ 0 & \bar\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\bar\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta & 0 \\ 0 & \mathrm{i}\bar\delta^* \end{bmatrix}$
S_{16}^{+}	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\bar\delta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\bar\eta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \bar\delta^* & 0 \\ 0 & \bar\delta \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \bar\eta & 0 \\ 0 & \bar\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\eta & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar\delta^* & 0 \\ 0 & \mathrm{i}\delta \end{bmatrix}$
σ_{d1}	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta^* \\ \mathrm{i}\bar\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\eta \\ \mathrm{i}\bar\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \bar\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \bar\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$
σ_{d2}	$\begin{bmatrix} 0 & \bar\delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta^* \\ \mathrm{i}\bar\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\eta \\ \mathrm{i}\bar\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \bar\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \bar\delta & 0 \end{bmatrix}$
σ_{d3}	$\begin{bmatrix} 0 & \bar\eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \bar\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\eta \\ \mathrm{i}\bar\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta^* \\ \mathrm{i}\bar\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \bar\eta^* & 0 \end{bmatrix}$
σ_{d4}	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \bar\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta^* \\ \mathrm{i}\bar\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \bar\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\eta \\ \mathrm{i}\bar\eta^* & 0 \end{bmatrix}$
σ_{d5}	$\begin{bmatrix} 0 & \mathrm{i}\bar\eta^* \\ \mathrm{i}\bar\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \bar\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \bar\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta \\ \mathrm{i}\bar\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$
σ_{d6}	$\begin{bmatrix} 0 & \bar\eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta \\ \mathrm{i}\bar\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \bar\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\eta^* \\ \mathrm{i}\bar\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \bar\eta & 0 \end{bmatrix}$
σ_{d7}	$\begin{bmatrix} 0 & \bar\delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\eta^* \\ \mathrm{i}\bar\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta \\ \mathrm{i}\bar\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \bar\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \bar\delta^* & 0 \end{bmatrix}$
σ_{d8}	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \bar\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar\delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \bar\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\eta^* \\ \mathrm{i}\bar\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar\delta \\ \mathrm{i}\bar\delta^* & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/32)$, $\epsilon = \exp(4\pi\mathrm{i}/32)$, $\eta = \exp(6\pi\mathrm{i}/32)$, $\theta = \exp(8\pi\mathrm{i}/32)$

T 47.8 Direct products of representations

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$\mathbf{D}_{8d}$	A_1	A_2	B_1	B_2	E_1	E_2	E_3
A_1	A_1	A_2	B_1	B_2	E_1	E_2	E_3
A_2		A_1	B_2	B_1	E_1	E_2	E_3
B_1			A_1	A_2	E_7	E_6	E_5
B_2				A_1	E_7	E_6	E_5
E_1					$A_1 \oplus \{A_2\} \oplus E_2$	$E_1 \oplus E_3$	$E_2 \oplus E_4$
E_2						$A_1 \oplus \{A_2\} \oplus E_4$	$E_1 \oplus E_5$
E_3							$A_1 \oplus \{A_2\} \oplus E_6$

$\twoheadrightarrow$

T **47**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{8d}$	E_4	E_5	E_6	E_7
A_1	E_4	E_5	E_6	E_7
A_2	E_4	E_5	E_6	E_7
B_1	E_4	E_3	E_2	E_1
B_2	E_4	E_3	E_2	E_1
E_1	$E_3 \oplus E_5$	$E_4 \oplus E_6$	$E_5 \oplus E_7$	$B_1 \oplus B_2 \oplus E_6$
E_2	$E_2 \oplus E_6$	$E_3 \oplus E_7$	$B_1 \oplus B_2 \oplus E_4$	$E_5 \oplus E_7$
E_3	$E_1 \oplus E_7$	$B_1 \oplus B_2 \oplus E_2$	$E_3 \oplus E_7$	$E_4 \oplus E_6$
E_4	$A_1 \oplus \{A_2\} \oplus B_1 \oplus B_2$	$E_1 \oplus E_7$	$E_2 \oplus E_6$	$E_3 \oplus E_5$
E_5		$A_1 \oplus \{A_2\} \oplus E_6$	$E_1 \oplus E_5$	$E_2 \oplus E_4$
E_6			$A_1 \oplus \{A_2\} \oplus E_4$	$E_1 \oplus E_3$
E_7				$A_1 \oplus \{A_2\} \oplus E_2$

⟹

T **47**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{8d}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$
A_1	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$
A_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$
B_1	$E_{15/2}$	$E_{13/2}$	$E_{11/2}$	$E_{9/2}$
B_2	$E_{15/2}$	$E_{13/2}$	$E_{11/2}$	$E_{9/2}$
E_1	$E_{13/2} \oplus E_{15/2}$	$E_{11/2} \oplus E_{15/2}$	$E_{9/2} \oplus E_{13/2}$	$E_{7/2} \oplus E_{11/2}$
E_2	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{9/2}$	$E_{3/2} \oplus E_{11/2}$
E_3	$E_{9/2} \oplus E_{11/2}$	$E_{7/2} \oplus E_{13/2}$	$E_{5/2} \oplus E_{15/2}$	$E_{3/2} \oplus E_{15/2}$
E_4	$E_{7/2} \oplus E_{9/2}$	$E_{5/2} \oplus E_{11/2}$	$E_{3/2} \oplus E_{13/2}$	$E_{1/2} \oplus E_{15/2}$
E_5	$E_{5/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{9/2}$	$E_{1/2} \oplus E_{11/2}$	$E_{1/2} \oplus E_{13/2}$
E_6	$E_{11/2} \oplus E_{13/2}$	$E_{9/2} \oplus E_{15/2}$	$E_{7/2} \oplus E_{15/2}$	$E_{5/2} \oplus E_{13/2}$
E_7	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{5/2} \oplus E_{9/2}$
$E_{1/2}$	$\{A_1\} \oplus A_2 \oplus E_7$	$E_2 \oplus E_7$	$E_2 \oplus E_5$	$E_4 \oplus E_5$
$E_{3/2}$		$\{A_1\} \oplus A_2 \oplus E_5$	$E_4 \oplus E_7$	$E_2 \oplus E_3$
$E_{5/2}$			$\{A_1\} \oplus A_2 \oplus E_3$	$E_6 \oplus E_7$
$E_{7/2}$				$\{A_1\} \oplus A_2 \oplus E_1$

⟹

T **47**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{8d}$	$E_{9/2}$	$E_{11/2}$	$E_{13/2}$	$E_{15/2}$
A_1	$E_{9/2}$	$E_{11/2}$	$E_{13/2}$	$E_{15/2}$
A_2	$E_{9/2}$	$E_{11/2}$	$E_{13/2}$	$E_{15/2}$
B_1	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
B_2	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
E_1	$E_{5/2} \oplus E_{9/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$
E_2	$E_{5/2} \oplus E_{13/2}$	$E_{7/2} \oplus E_{15/2}$	$E_{9/2} \oplus E_{15/2}$	$E_{11/2} \oplus E_{13/2}$
E_3	$E_{1/2} \oplus E_{13/2}$	$E_{1/2} \oplus E_{11/2}$	$E_{3/2} \oplus E_{9/2}$	$E_{5/2} \oplus E_{7/2}$
E_4	$E_{1/2} \oplus E_{15/2}$	$E_{3/2} \oplus E_{13/2}$	$E_{5/2} \oplus E_{11/2}$	$E_{7/2} \oplus E_{9/2}$
E_5	$E_{3/2} \oplus E_{15/2}$	$E_{5/2} \oplus E_{15/2}$	$E_{7/2} \oplus E_{13/2}$	$E_{9/2} \oplus E_{11/2}$
E_6	$E_{3/2} \oplus E_{11/2}$	$E_{1/2} \oplus E_{9/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{5/2}$
E_7	$E_{7/2} \oplus E_{11/2}$	$E_{9/2} \oplus E_{13/2}$	$E_{11/2} \oplus E_{15/2}$	$E_{13/2} \oplus E_{15/2}$
$E_{1/2}$	$E_3 \oplus E_4$	$E_3 \oplus E_6$	$E_1 \oplus E_6$	$B_1 \oplus B_2 \oplus E_1$
$E_{3/2}$	$E_5 \oplus E_6$	$E_1 \oplus E_4$	$B_1 \oplus B_2 \oplus E_3$	$E_1 \oplus E_6$
$E_{5/2}$	$E_1 \oplus E_2$	$B_1 \oplus B_2 \oplus E_5$	$E_1 \oplus E_4$	$E_3 \oplus E_6$
$E_{7/2}$	$B_1 \oplus B_2 \oplus E_7$	$E_1 \oplus E_2$	$E_5 \oplus E_6$	$E_3 \oplus E_4$
$E_{9/2}$	$\{A_1\} \oplus A_2 \oplus E_1$	$E_6 \oplus E_7$	$E_2 \oplus E_3$	$E_4 \oplus E_5$
$E_{11/2}$		$\{A_1\} \oplus A_2 \oplus E_3$	$E_4 \oplus E_7$	$E_2 \oplus E_5$
$E_{13/2}$			$\{A_1\} \oplus A_2 \oplus E_5$	$E_2 \oplus E_7$
$E_{15/2}$				$\{A_1\} \oplus A_2 \oplus E_7$

T **47**.9 Subduction (descent of symmetry) § **16**–9, p. 82

$\mathbf{D}_{8d}$	$(\mathbf{C}_{8v})$	$(\mathbf{C}_{4v})$	$(\mathbf{C}_{2v})$	$(\mathbf{D}_8)$	$(\mathbf{D}_4)$	$(\mathbf{D}_2)$
A_1	A_1	A_1	A_1	A_1	A_1	A
A_2	A_2	A_2	A_2	A_2	A_2	B_1
B_1	A_2	A_2	A_2	A_1	A_1	A
B_2	A_1	A_1	A_1	A_2	A_2	B_1
E_1	E_1	E	$B_1 \oplus B_2$	E_1	E	$B_2 \oplus B_3$
E_2	E_2	$B_1 \oplus B_2$	$A_1 \oplus A_2$	E_2	$B_1 \oplus B_2$	$A \oplus B_1$
E_3	E_3	E	$B_1 \oplus B_2$	E_3	E	$B_2 \oplus B_3$
E_4	$B_1 \oplus B_2$	$A_1 \oplus A_2$	$A_1 \oplus A_2$	$B_1 \oplus B_2$	$A_1 \oplus A_2$	$A \oplus B_1$
E_5	E_3	E	$B_1 \oplus B_2$	E_3	E	$B_2 \oplus B_3$
E_6	E_2	$B_1 \oplus B_2$	$A_1 \oplus A_2$	E_2	$B_1 \oplus B_2$	$A \oplus B_1$
E_7	E_1	E	$B_1 \oplus B_2$	E_1	E	$B_2 \oplus B_3$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2}$
$E_{5/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
$E_{7/2}$	$E_{7/2}$	$E_{1/2}$	$E_{1/2}$	$E_{7/2}$	$E_{1/2}$	$E_{1/2}$
$E_{9/2}$	$E_{7/2}$	$E_{1/2}$	$E_{1/2}$	$E_{7/2}$	$E_{1/2}$	$E_{1/2}$
$E_{11/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
$E_{13/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2}$
$E_{15/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$

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T **47**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{8d}$	$\mathbf{S}_{16}$	$(\mathbf{C}_s)$	$\mathbf{C}_8$	$\mathbf{C}_4$	$\mathbf{C}_2$	$(\mathbf{C}_2)$
					C_2	C_2'
A_1	A	A'	A	A	A	A
A_2	A	A''	A	A	A	B
B_1	B	A''	A	A	A	A
B_2	B	A'	A	A	A	B
E_1	${}^1E_1 \oplus {}^2E_1$	$A' \oplus A''$	${}^1E_1 \oplus {}^2E_1$	${}^1E \oplus {}^2E$	$2B$	$A \oplus B$
E_2	${}^1E_2 \oplus {}^2E_2$	$A' \oplus A''$	${}^1E_2 \oplus {}^2E_2$	$2B$	$2A$	$A \oplus B$
E_3	${}^1E_3 \oplus {}^2E_3$	$A' \oplus A''$	${}^1E_3 \oplus {}^2E_3$	${}^1E \oplus {}^2E$	$2B$	$A \oplus B$
E_4	${}^1E_4 \oplus {}^2E_4$	$A' \oplus A''$	$2B$	$2A$	$2A$	$A \oplus B$
E_5	${}^1E_5 \oplus {}^2E_5$	$A' \oplus A''$	${}^1E_3 \oplus {}^2E_3$	${}^1E \oplus {}^2E$	$2B$	$A \oplus B$
E_6	${}^1E_6 \oplus {}^2E_6$	$A' \oplus A''$	${}^1E_2 \oplus {}^2E_2$	$2B$	$2A$	$A \oplus B$
E_7	${}^1E_7 \oplus {}^2E_7$	$A' \oplus A''$	${}^1E_1 \oplus {}^2E_1$	${}^1E \oplus {}^2E$	$2B$	$A \oplus B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{9/2}$	${}^1E_{9/2} \oplus {}^2E_{9/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{11/2}$	${}^1E_{11/2} \oplus {}^2E_{11/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{13/2}$	${}^1E_{13/2} \oplus {}^2E_{13/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{15/2}$	${}^1E_{15/2} \oplus {}^2E_{15/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **47**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{8d}$
$16n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5 \oplus 2E_6 \oplus 2E_7)$
$16n+1$	$n\,(A_1 \oplus A_2 \oplus B_1 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5 \oplus 2E_6 \oplus 2E_7) \oplus (n+1)(B_2 \oplus E_1)$
$16n+2$	$(n+1)(A_1 \oplus E_2 \oplus E_7) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5 \oplus 2E_6 \oplus E_7)$
$16n+3$	$n\,(A_1 \oplus A_2 \oplus B_1 \oplus E_1 \oplus 2E_2 \oplus E_3 \oplus 2E_4 \oplus 2E_5 \oplus E_6 \oplus 2E_7) \oplus$ $(n+1)(B_2 \oplus E_1 \oplus E_3 \oplus E_6)$
$16n+4$	$(n+1)(A_1 \oplus E_2 \oplus E_4 \oplus E_5 \oplus E_7) \oplus$ $n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus 2E_3 \oplus E_4 \oplus E_5 \oplus 2E_6 \oplus E_7)$
$16n+5$	$n\,(A_1 \oplus A_2 \oplus B_1 \oplus E_1 \oplus 2E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus 2E_7) \oplus$ $(n+1)(B_2 \oplus E_1 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6)$
$16n+6$	$(n+1)(A_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7) \oplus$ $n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7)$
$16n+7$	$n\,(A_1 \oplus A_2 \oplus B_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7) \oplus$ $(n+1)(B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7)$
$16n+8$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7) \oplus$ $n\,(A_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7)$
$16n+9$	$(n+1)(A_1 \oplus A_2 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus 2E_7) \oplus$ $n\,(B_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6)$
$16n+10$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus 2E_6 \oplus E_7) \oplus$ $n\,(A_2 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_7)$
$16n+11$	$(n+1)(A_1 \oplus A_2 \oplus B_2 \oplus E_1 \oplus 2E_2 \oplus E_3 \oplus E_4 \oplus 2E_5 \oplus E_6 \oplus 2E_7) \oplus$ $n\,(B_1 \oplus E_1 \oplus E_3 \oplus E_4 \oplus E_6)$
$16n+12$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus 2E_3 \oplus 2E_4 \oplus E_5 \oplus 2E_6 \oplus E_7) \oplus$ $n\,(A_2 \oplus E_2 \oplus E_5 \oplus E_7)$
$16n+13$	$(n+1)(A_1 \oplus A_2 \oplus B_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5 \oplus E_6 \oplus 2E_7) \oplus n\,(B_1 \oplus E_1 \oplus E_6)$
$16n+14$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5 \oplus 2E_6 \oplus E_7) \oplus n\,(A_2 \oplus E_7)$
$16n+15$	$(n+1)(A_1 \oplus A_2 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5 \oplus 2E_6 \oplus 2E_7) \oplus n\,B_1$
$16n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2})$
$16n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus 2n\,(E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2})$
$16n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus 2n\,(E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2})$
$16n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus 2n\,(E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2})$
$16n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2}) \oplus 2n\,(E_{11/2} \oplus E_{13/2} \oplus E_{15/2})$
$16n+\frac{11}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2}) \oplus 2n\,(E_{13/2} \oplus E_{15/2})$
$16n+\frac{13}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2}) \oplus 2n\,E_{15/2}$
$16n+\frac{15}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2})$
$16n+\frac{17}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2}) \oplus (2n+2)\,E_{15/2}$
$16n+\frac{19}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2}) \oplus (2n+2)(E_{13/2} \oplus E_{15/2})$
$16n+\frac{21}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2}) \oplus (2n+2)(E_{11/2} \oplus E_{13/2} \oplus E_{15/2})$
$16n+\frac{23}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus (2n+2)(E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2})$
$16n+\frac{25}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus (2n+2)(E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2})$
$16n+\frac{27}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (2n+2)(E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2})$
$16n+\frac{29}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2})$
$16n+\frac{31}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2})$

$n = 0, 1, 2, \ldots$

T **47**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{D}_{8d}$

a_2	e_1	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_2	E_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_3	E_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_4	E_4	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_5	E_5	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_6	E_6	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_7	E_7	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{5/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{7/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{9/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{11/2}$	$E_{11/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{13/2}$	$E_{13/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{15/2}$	$E_{15/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_1	e_1	E_7	
		1	2
1	1	1	0
1	2	0	1

b_1	e_2	E_6	
		1	2
1	1	1	0
1	2	0	1

b_1	e_3	E_5	
		1	2
1	1	1	0
1	2	0	1

b_1	e_4	E_4	
		1	2
1	1	0	1
1	2	1	0

b_1	e_5	E_3	
		1	2
1	1	1	0
1	2	0	1

b_1	e_6	E_2	
		1	2
1	1	1	0
1	2	0	1

b_1	e_7	E_1	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{1/2}$	$E_{15/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{3/2}$	$E_{13/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{5/2}$	$E_{11/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{7/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{9/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{11/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	1

↠

T 47.11 Clebsch–Gordan coefficients *(cont.)*

b_1	$e_{13/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{15/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	1

b_2	e_1	E_7	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_2	E_6	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_3	E_5	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_4	E_4	
		1	2
1	1	0	$\bar{1}$
1	2	1	0

b_2	e_5	E_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_6	E_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_7	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{1/2}$	$E_{15/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{3/2}$	$E_{13/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{5/2}$	$E_{11/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{7/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{9/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{11/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{13/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{15/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e_1	e_1	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_1	e_2	E_1		E_3	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	e_3	E_2		E_4	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	1	0	0	0

e_1	e_4	E_3		E_5	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	0	1

e_1	e_5	E_4		E_6	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	e_6	E_5		E_7	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_1	e_7	B_1	B_2	E_6	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_1	$e_{1/2}$	$E_{13/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$u = 2^{-1/2}$

↠

T **47**.11 Clebsch–Gordan coefficients *(cont.)*

e_1	$e_{3/2}$	$E_{11/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_1	$e_{5/2}$	$E_{9/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_1	$e_{7/2}$	$E_{7/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_1	$e_{9/2}$	$E_{5/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_1	$e_{11/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_1	$e_{13/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_1	$e_{15/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_2	e_2	A_1	A_2	E_4	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_2	e_3	E_1		E_5	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	1	0	0	0

e_2	e_4	E_2		E_6	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_2	e_5	E_3		E_7	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

e_2	e_6	B_1	B_2	E_4	
		1	1	1	2
1	1	0	0	0	1
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_2	e_7	E_5		E_7	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	1	0	0	0

e_2	$e_{1/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_2	$e_{3/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_2	$e_{5/2}$	$E_{1/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_2	$e_{7/2}$	$E_{3/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_2	$e_{9/2}$	$E_{5/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

↠

T **47**.11 Clebsch–Gordan coefficients *(cont.)*

e_2	$e_{11/2}$	$E_{7/2}$		$E_{15/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_2	$e_{13/2}$	$E_{9/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_2	$e_{15/2}$	$E_{11/2}$		$E_{13/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_3	e_3	A_1	A_2	E_6	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_3	e_4	E_1		E_7	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	0	1	0	0

e_3	e_5	B_1	B_2	E_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_3	e_6	E_3		E_7	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_3	e_7	E_4		E_6	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

e_3	$e_{1/2}$	$E_{9/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_3	$e_{3/2}$	$E_{7/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_3	$e_{5/2}$	$E_{5/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_3	$e_{7/2}$	$E_{3/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_3	$e_{9/2}$	$E_{1/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_3	$e_{11/2}$	$E_{1/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_3	$e_{13/2}$	$E_{3/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_3	$e_{15/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_4	e_4	A_1	A_2	B_1	B_2
		1	1	1	1
1	1	0	0	u	u
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	u	$\bar{u}$

e_4	e_5	E_1		E_7	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

↠

T **47**.11 Clebsch–Gordan coefficients *(cont.)*

e_4	e_6	E_2		E_6	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	1	0	0	0

e_4	e_7	E_3		E_5	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_4	$e_{1/2}$	$E_{7/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_4	$e_{3/2}$	$E_{5/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_4	$e_{5/2}$	$E_{3/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_4	$e_{7/2}$	$E_{1/2}$		$E_{15/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_4	$e_{9/2}$	$E_{1/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_4	$e_{11/2}$	$E_{3/2}$		$E_{13/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_4	$e_{13/2}$	$E_{5/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_4	$e_{15/2}$	$E_{7/2}$		$E_{9/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_5	e_5	A_1	A_2	E_6	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	0	1

e_5	e_6	E_1		E_5	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_5	e_7	E_2		E_4	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_5	$e_{1/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_5	$e_{3/2}$	$E_{3/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_5	$e_{5/2}$	$E_{1/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_5	$e_{7/2}$	$E_{1/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_5	$e_{9/2}$	$E_{3/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$\text{u} = 2^{-1/2}$

↠

T **47**.11 Clebsch–Gordan coefficients *(cont.)*

e_5	$e_{11/2}$	$E_{5/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_5	$e_{13/2}$	$E_{7/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_5	$e_{15/2}$	$E_{9/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_6	e_6	A_1	A_2	E_4	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_6	e_7	E_1		E_3	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

e_6	$e_{1/2}$	$E_{11/2}$		$E_{13/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_6	$e_{3/2}$	$E_{9/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_6	$e_{5/2}$	$E_{7/2}$		$E_{15/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_6	$e_{7/2}$	$E_{5/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_6	$e_{9/2}$	$E_{3/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_6	$e_{11/2}$	$E_{1/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_6	$e_{13/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_6	$e_{15/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_7	e_7	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_7	$e_{1/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_7	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_7	$e_{5/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_7	$e_{7/2}$	$E_{5/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$u = 2^{-1/2}$

↠

T **47**.11 Clebsch–Gordan coefficients *(cont.)*

e_7	$e_{9/2}$	$E_{7/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_7	$e_{11/2}$	$E_{9/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_7	$e_{13/2}$	$E_{11/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_7	$e_{15/2}$	$E_{13/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{1/2}$	$e_{1/2}$	A_1	A_2	E_7	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{3/2}$	E_2		E_7	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{1/2}$	$e_{5/2}$	E_2		E_5	
		1	2	1	2
1	1	0	0	0	1
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{1/2}$	$e_{7/2}$	E_4		E_5	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{9/2}$	E_3		E_4	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	1	0	0

$e_{1/2}$	$e_{11/2}$	E_3		E_6	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{1/2}$	$e_{13/2}$	E_1		E_6	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{1/2}$	$e_{15/2}$	B_1	B_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{3/2}$	A_1	A_2	E_5	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{5/2}$	E_4		E_7	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{7/2}$	E_2		E_3	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{9/2}$	E_5		E_6	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{3/2}$	$e_{11/2}$	E_1		E_4	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

$e_{3/2}$	$e_{13/2}$	B_1	B_2	E_3	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

↠

T **47**.11 Clebsch–Gordan coefficients *(cont.)*

$e_{3/2}$	$e_{15/2}$	E_1		E_6	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{5/2}$	$e_{5/2}$	A_1	A_2	E_3	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\mathrm{u}}$	u	0	0
2	2	0	0	0	1

$e_{5/2}$	$e_{7/2}$	E_6		E_7	
		1	2	1	2
1	1	0	0	0	1
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{5/2}$	$e_{9/2}$	E_1		E_2	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{5/2}$	$e_{11/2}$	B_1	B_2	E_5	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\mathrm{u}}$	u	0	0
2	2	0	0	0	1

$e_{5/2}$	$e_{13/2}$	E_1		E_4	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	1	0	0

$e_{5/2}$	$e_{15/2}$	E_3		E_6	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{7/2}$	$e_{7/2}$	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\mathrm{u}}$	u	0	0
2	2	0	0	0	1

$e_{7/2}$	$e_{9/2}$	B_1	B_2	E_7	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\mathrm{u}}$	u	0	0
2	2	0	0	0	1

$e_{7/2}$	$e_{11/2}$	E_1		E_2	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{7/2}$	$e_{13/2}$	E_5		E_6	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{7/2}$	$e_{15/2}$	E_3		E_4	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

$e_{9/2}$	$e_{9/2}$	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\mathrm{u}}$	u	0	0
2	2	0	0	0	1

$e_{9/2}$	$e_{11/2}$	E_6		E_7	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{9/2}$	$e_{13/2}$	E_2		E_3	
		1	2	1	2
1	1	0	0	0	1
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{9/2}$	$e_{15/2}$	E_4		E_5	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{11/2}$	$e_{11/2}$	A_1	A_2	E_3	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\mathrm{u}}$	u	0	0
2	2	0	0	0	1

$e_{11/2}$	$e_{13/2}$	E_4		E_7	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$\mathrm{u} = 2^{-1/2}$

↠

T **47**.11 Clebsch–Gordan coefficients *(cont.)*

$e_{11/2}$	$e_{15/2}$	E_2		E_5	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{13/2}$	$e_{13/2}$	A_1	A_2	E_5	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{13/2}$	$e_{15/2}$	E_2		E_7	
		1	2	1	2
1	1	0	0	0	1
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{15/2}$	$e_{15/2}$	A_1	A_2	E_7	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

(1) Product forms: $\mathbf{D}_9 \otimes \mathbf{C}_i$.

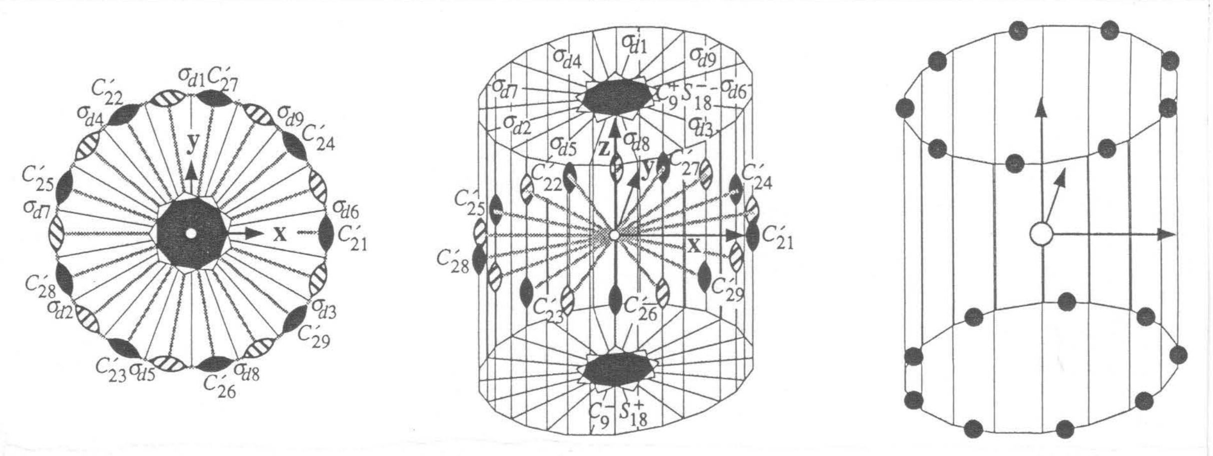
(2) Group chains: $\mathbf{D}_{9d} \supset (\mathbf{D}_{3d})$, $\mathbf{D}_{9d} \supset (\underline{\mathbf{C}_{9v}})$, $\mathbf{D}_{9d} \supset \underline{\mathbf{D}_9}$, $\mathbf{D}_{9d} \supset \underline{\mathbf{S}_{18}}$.

(3) Operations of G: E, (C_9^+, C_9^-), (C_9^{2+}, C_9^{2-}), (C_3^+, C_3^-), (C_9^{4+}, C_9^{4-}),
$(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27}, C'_{28}, C'_{29})$,
i, $(S_{18}^{7-}, S_{18}^{7+})$, $(S_{18}^{5-}, S_{18}^{5+})$, (S_6^-, S_6^+), (S_{18}^-, S_{18}^+),
$(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \sigma_{d6}, \sigma_{d7}, \sigma_{d8}, \sigma_{d9})$,

(4) Operations of $\widetilde{G}$: E, (C_9^+, C_9^-), (C_9^{2+}, C_9^{2-}), (C_3^+, C_3^-), (C_9^{4+}, C_9^{4-}),
$(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27}, C'_{28}, C'_{29})$,
i, $(S_{18}^{7-}, S_{18}^{7+})$, $(S_{18}^{5-}, S_{18}^{5+})$, (S_6^-, S_6^+), (S_{18}^-, S_{18}^+),
$(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \sigma_{d6}, \sigma_{d7}, \sigma_{d8}, \sigma_{d9})$,
$\widetilde{E}$, $(\widetilde{C}_9^+, \widetilde{C}_9^-)$, $(\widetilde{C}_9^{2+}, \widetilde{C}_9^{2-})$, $(\widetilde{C}_3^+, \widetilde{C}_3^-)$, $(\widetilde{C}_9^{4+}, \widetilde{C}_9^{4-})$,
$(\widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24}, \widetilde{C}'_{25}, \widetilde{C}'_{26}, \widetilde{C}'_{27}, \widetilde{C}'_{28}, \widetilde{C}'_{29})$,
$\widetilde{i}$, $(\widetilde{S}_{18}^{7-}, \widetilde{S}_{18}^{7+})$, $(\widetilde{S}_{18}^{5-}, \widetilde{S}_{18}^{5+})$, $(\widetilde{S}_6^-, \widetilde{S}_6^+)$, $(\widetilde{S}_{18}^-, \widetilde{S}_{18}^+)$,
$(\widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3}, \widetilde{\sigma}_{d4}, \widetilde{\sigma}_{d5}, \widetilde{\sigma}_{d6}, \widetilde{\sigma}_{d7}, \widetilde{\sigma}_{d8}, \widetilde{\sigma}_{d9})$.

(5) Classes and representations: $|r| = 12$, $|i| = 0$, $|I| = 12$, $|\widetilde{I}| = 12$.

F **48** See Chapter **15**, p. 65



Examples:

T **48**.1 Parameters

§ **16**–1, p. 68

$\mathbf{D}_{9d}$		α	β	γ	ϕ	$\mathbf{n}$			λ	$\boldsymbol{\Lambda}$		
E	i	0	0	0	0	(0	0	0)	⟦ 1,	(0	0	0)⟧
C_9^+	S_{18}^{7-}	0	0	$\frac{2\pi}{9}$	$\frac{2\pi}{9}$	(0	0	1)	⟦c_9,	(0	0	s_9)⟧
C_9^-	S_{18}^{7+}	0	0	$-\frac{2\pi}{9}$	$\frac{2\pi}{9}$	(0	0	−1)	⟦c_9,	(0	0	$-s_9$)⟧
C_9^{2+}	S_{18}^{5-}	0	0	$\frac{4\pi}{9}$	$\frac{4\pi}{9}$	(0	0	1)	⟦c_9^2,	(0	0	s_9^2)⟧
C_9^{2-}	S_{18}^{5+}	0	0	$-\frac{4\pi}{9}$	$\frac{4\pi}{9}$	(0	0	−1)	⟦c_9^2,	(0	0	$-s_9^2$)⟧
C_3^+	S_6^-	0	0	$\frac{2\pi}{3}$	$\frac{2\pi}{3}$	(0	0	1)	⟦ $\frac{1}{2}$,	(0	0	$\frac{\sqrt{3}}{2}$)⟧
C_3^-	S_6^+	0	0	$-\frac{2\pi}{3}$	$\frac{2\pi}{3}$	(0	0	−1)	⟦ $\frac{1}{2}$,	(0	0	$-\frac{\sqrt{3}}{2}$)⟧
C_9^{4+}	S_{18}^-	0	0	$\frac{8\pi}{9}$	$\frac{8\pi}{9}$	(0	0	1)	⟦c_9^4,	(0	0	s_9^4)⟧
C_9^{4-}	S_{18}^+	0	0	$-\frac{8\pi}{9}$	$\frac{8\pi}{9}$	(0	0	−1)	⟦c_9^4,	(0	0	$-s_9^4$)⟧
C'_{21}	σ_{d1}	0	π	π	π	(1	0	0)	⟦ 0,	(1	0	0)⟧
C'_{22}	σ_{d2}	0	π	$-\frac{\pi}{3}$	π	($-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0)	⟦ 0,	($-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0)⟧
C'_{23}	σ_{d3}	0	π	$\frac{\pi}{3}$	π	($-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0)	⟦ 0,	($-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	0)⟧
C'_{24}	σ_{d4}	0	π	$\frac{5\pi}{9}$	π	(c_9^2	s_9^2	0)	⟦ 0,	(c_9^2	s_9^2	0)⟧
C'_{25}	σ_{d5}	0	π	$-\frac{7\pi}{9}$	π	($-c_9$	s_9	0)	⟦ 0,	($-c_9$	s_9	0)⟧
C'_{26}	σ_{d6}	0	π	$-\frac{\pi}{9}$	π	(c_9^4	$-s_9^4$	0)	⟦ 0,	(c_9^4	$-s_9^4$	0)⟧
C'_{27}	σ_{d7}	0	π	$\frac{\pi}{9}$	π	(c_9^4	s_9^4	0)	⟦ 0,	(c_9^4	s_9^4	0)⟧
C'_{28}	σ_{d8}	0	π	$\frac{7\pi}{9}$	π	($-c_9$	$-s_9$	0)	⟦ 0,	($-c_9$	$-s_9$	0)⟧
C'_{29}	σ_{d9}	0	π	$-\frac{5\pi}{9}$	π	(c_9^2	$-s_9^2$	0)	⟦ 0,	(c_9^2	$-s_9^2$	0)⟧

$c_n^m = \cos\frac{m}{n}\pi$, $s_n^m = \sin\frac{m}{n}\pi$

T **48**.2 Multiplication table

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$\mathbf{D}_{9d}$	E	C_9^+	C_9^-	C_9^{2+}	C_9^{2-}	C_3^+	C_3^-	C_9^{4+}	C_9^{4-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}
E	E	C_9^+	C_9^-	C_9^{2+}	C_9^{2-}	C_3^+	C_3^-	C_9^{4+}	C_9^{4-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}
C_9^+	C_9^+	C_9^{2+}	E	C_3^+	C_9^-	C_9^{4+}	C_9^{2-}	C_9^{4-}	C_3^-	C'_{28}	C'_{29}	C'_{27}	C'_{23}	C'_{21}	C'_{22}	C'_{26}	C'_{24}	C'_{25}
C_9^-	C_9^-	E	C_9^{2-}	C_9^+	C_3^-	C_9^{2+}	C_9^{4-}	C_3^+	C_9^{4+}	C'_{25}	C'_{26}	C'_{24}	C'_{28}	C'_{29}	C'_{27}	C'_{23}	C'_{21}	C'_{22}
C_9^{2+}	C_9^{2+}	C_3^+	C_9^+	C_9^{4+}	E	C_9^{4-}	C_9^-	C_3^-	C_9^{2-}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}	C'_{22}	C'_{23}	C'_{21}
C_9^{2-}	C_9^{2-}	C_9^-	C_3^-	E	C_9^{4-}	C_9^+	C_9^{4+}	C_9^{2+}	C_3^+	C'_{29}	C'_{27}	C'_{28}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}
C_3^+	C_3^+	C_9^{4+}	C_9^{2+}	C_9^{4-}	C_9^+	C_3^-	E	C_9^{2-}	C_9^-	C'_{23}	C'_{21}	C'_{22}	C'_{26}	C'_{24}	C'_{25}	C'_{29}	C'_{27}	C'_{28}
C_3^-	C_3^-	C_9^{2-}	C_9^{4-}	C_9^-	C_9^{4+}	E	C_3^+	C_9^+	C_9^{2+}	C'_{22}	C'_{23}	C'_{21}	C'_{25}	C'_{26}	C'_{24}	C'_{28}	C'_{29}	C'_{27}
C_9^{4+}	C_9^{4+}	C_9^{4-}	C_3^+	C_3^-	C_9^{2+}	C_9^{2-}	C_9^+	C_9^-	E	C'_{27}	C'_{28}	C'_{29}	C'_{22}	C'_{23}	C'_{21}	C'_{25}	C'_{26}	C'_{24}
C_9^{4-}	C_9^{4-}	C_3^-	C_9^{4+}	C_9^{2-}	C_3^+	C_9^-	C_9^{2+}	E	C_9^+	C'_{26}	C'_{24}	C'_{25}	C'_{29}	C'_{27}	C'_{28}	C'_{21}	C'_{22}	C'_{23}
C'_{21}	C'_{21}	C'_{25}	C'_{28}	C'_{29}	C'_{24}	C'_{22}	C'_{23}	C'_{26}	C'_{27}	E	C_3^+	C_3^-	C_9^{2-}	C_9^+	C_9^{4+}	C_9^{4-}	C_9^-	C_9^{2+}
C'_{22}	C'_{22}	C'_{26}	C'_{29}	C'_{27}	C'_{25}	C'_{23}	C'_{21}	C'_{24}	C'_{28}	C_3^-	E	C_3^+	C_9^{4+}	C_9^{2-}	C_9^+	C_9^{2+}	C_9^{4-}	C_9^-
C'_{23}	C'_{23}	C'_{24}	C'_{27}	C'_{28}	C'_{26}	C'_{21}	C'_{22}	C'_{25}	C'_{29}	C_3^+	C_3^-	E	C_9^+	C_9^{4+}	C_9^{2-}	C_9^-	C_9^{2+}	C_9^{4-}
C'_{24}	C'_{24}	C'_{28}	C'_{23}	C'_{21}	C'_{27}	C'_{25}	C'_{26}	C'_{29}	C'_{22}	C_9^{2+}	C_9^{4-}	C_9^-	E	C_3^+	C_3^-	C_9^{2-}	C_9^+	C_9^{4+}
C'_{25}	C'_{25}	C'_{29}	C'_{21}	C'_{22}	C'_{28}	C'_{26}	C'_{24}	C'_{27}	C'_{23}	C_9^-	C_9^{2+}	C_9^{4-}	C_3^-	E	C_3^+	C_9^{4+}	C_9^{2-}	C_9^+
C'_{26}	C'_{26}	C'_{27}	C'_{22}	C'_{23}	C'_{29}	C'_{24}	C'_{25}	C'_{28}	C'_{21}	C_9^{4-}	C_9^-	C_9^{2+}	C_3^+	C_3^-	E	C_9^+	C_9^{4+}	C_9^{2-}
C'_{27}	C'_{27}	C'_{23}	C'_{26}	C'_{24}	C'_{22}	C'_{28}	C'_{29}	C'_{21}	C'_{25}	C_9^{4+}	C_9^{2-}	C_9^+	C_9^{2+}	C_9^{4-}	C_9^-	E	C_3^+	C_3^-
C'_{28}	C'_{28}	C'_{21}	C'_{24}	C'_{25}	C'_{23}	C'_{29}	C'_{27}	C'_{22}	C'_{26}	C_9^+	C_9^{4+}	C_9^{2-}	C_9^-	C_9^{2+}	C_9^{4-}	C_3^-	E	C_3^+
C'_{29}	C'_{29}	C'_{22}	C'_{25}	C'_{26}	C'_{21}	C'_{27}	C'_{28}	C'_{23}	C'_{24}	C_9^{2-}	C_9^+	C_9^{4+}	C_9^{4-}	C_9^-	C_9^{2+}	C_3^+	C_3^-	E
i	i	S_{18}^{7-}	S_{18}^{7+}	S_{18}^{5-}	S_{18}^{5+}	S_6^-	S_6^+	S_{18}^-	S_{18}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}
S_{18}^{7-}	S_{18}^{7-}	S_{18}^{5-}	i	S_6^-	S_{18}^{7+}	S_{18}^-	S_{18}^{5+}	S_{18}^+	S_6^+	σ_{d8}	σ_{d9}	σ_{d7}	σ_{d3}	σ_{d1}	σ_{d2}	σ_{d6}	σ_{d4}	σ_{d5}
S_{18}^{7+}	S_{18}^{7+}	i	S_{18}^{5+}	S_{18}^{7-}	S_6^+	S_{18}^{5-}	S_{18}^+	S_6^-	S_{18}^-	σ_{d5}	σ_{d6}	σ_{d4}	σ_{d8}	σ_{d9}	σ_{d7}	σ_{d3}	σ_{d1}	σ_{d2}
S_{18}^{5-}	S_{18}^{5-}	S_6^-	S_{18}^{7-}	S_{18}^-	i	S_{18}^+	S_{18}^{7+}	S_6^+	S_{18}^{5+}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d2}	σ_{d3}	σ_{d1}
S_{18}^{5+}	S_{18}^{5+}	S_{18}^{7+}	S_6^+	i	S_{18}^+	S_{18}^{7-}	S_{18}^-	S_{18}^{5-}	S_6^-	σ_{d9}	σ_{d7}	σ_{d8}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}
S_6^-	S_6^-	S_{18}^-	S_{18}^{5-}	S_{18}^+	S_{18}^{7-}	S_6^+	i	S_{18}^{5+}	S_{18}^{7+}	σ_{d3}	σ_{d1}	σ_{d2}	σ_{d6}	σ_{d4}	σ_{d5}	σ_{d9}	σ_{d7}	σ_{d8}
S_6^+	S_6^+	S_{18}^{5+}	S_{18}^+	S_{18}^{7+}	S_{18}^-	i	S_6^-	S_{18}^{7-}	S_{18}^{5-}	σ_{d2}	σ_{d3}	σ_{d1}	σ_{d5}	σ_{d6}	σ_{d4}	σ_{d8}	σ_{d9}	σ_{d7}
S_{18}^-	S_{18}^-	S_{18}^+	S_6^-	S_6^+	S_{18}^{5-}	S_{18}^{5+}	S_{18}^{7-}	S_{18}^{7+}	i	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d2}	σ_{d3}	σ_{d1}	σ_{d5}	σ_{d6}	σ_{d4}
S_{18}^+	S_{18}^+	S_6^+	S_{18}^-	S_{18}^{5+}	S_6^-	S_{18}^{7+}	S_{18}^{5-}	i	S_{18}^{7-}	σ_{d6}	σ_{d4}	σ_{d5}	σ_{d9}	σ_{d7}	σ_{d8}	σ_{d1}	σ_{d2}	σ_{d3}
σ_{d1}	σ_{d1}	σ_{d5}	σ_{d8}	σ_{d9}	σ_{d4}	σ_{d2}	σ_{d3}	σ_{d6}	σ_{d7}	i	S_6^-	S_6^+	S_{18}^{5+}	S_{18}^{7-}	S_{18}^-	S_{18}^+	S_{18}^{7+}	S_{18}^{5-}
σ_{d2}	σ_{d2}	σ_{d6}	σ_{d9}	σ_{d7}	σ_{d5}	σ_{d3}	σ_{d1}	σ_{d4}	σ_{d8}	S_6^+	i	S_6^-	S_{18}^-	S_{18}^{5+}	S_{18}^{7-}	S_{18}^{5-}	S_{18}^+	S_{18}^{7+}
σ_{d3}	σ_{d3}	σ_{d4}	σ_{d7}	σ_{d8}	σ_{d6}	σ_{d1}	σ_{d2}	σ_{d5}	σ_{d9}	S_6^-	S_6^+	i	S_{18}^{7-}	S_{18}^-	S_{18}^{5+}	S_{18}^{7+}	S_{18}^{5-}	S_{18}^+
σ_{d4}	σ_{d4}	σ_{d8}	σ_{d3}	σ_{d1}	σ_{d7}	σ_{d5}	σ_{d6}	σ_{d9}	σ_{d2}	S_{18}^{5-}	S_{18}^+	S_{18}^{7+}	i	S_6^-	S_6^+	S_{18}^{5+}	S_{18}^{7-}	S_{18}^-
σ_{d5}	σ_{d5}	σ_{d9}	σ_{d1}	σ_{d2}	σ_{d8}	σ_{d6}	σ_{d4}	σ_{d7}	σ_{d3}	S_{18}^{7+}	S_{18}^{5-}	S_{18}^+	S_6^+	i	S_6^-	S_{18}^-	S_{18}^{5+}	S_{18}^{7-}
σ_{d6}	σ_{d6}	σ_{d7}	σ_{d2}	σ_{d3}	σ_{d9}	σ_{d4}	σ_{d5}	σ_{d8}	σ_{d1}	S_{18}^+	S_{18}^{7+}	S_{18}^{5-}	S_6^-	S_6^+	i	S_{18}^{7-}	S_{18}^-	S_{18}^{5+}
σ_{d7}	σ_{d7}	σ_{d3}	σ_{d6}	σ_{d4}	σ_{d2}	σ_{d8}	σ_{d9}	σ_{d1}	σ_{d5}	S_{18}^-	S_{18}^{5+}	S_{18}^{7-}	S_{18}^{5-}	S_{18}^+	S_{18}^{7+}	i	S_6^-	S_6^+
σ_{d8}	σ_{d8}	σ_{d1}	σ_{d4}	σ_{d5}	σ_{d3}	σ_{d9}	σ_{d7}	σ_{d2}	σ_{d6}	S_{18}^{7-}	S_{18}^-	S_{18}^{5+}	S_{18}^{7+}	S_{18}^{5-}	S_{18}^+	S_6^+	i	S_6^-
σ_{d9}	σ_{d9}	σ_{d2}	σ_{d5}	σ_{d6}	σ_{d1}	σ_{d7}	σ_{d8}	σ_{d3}	σ_{d4}	S_{18}^{5+}	S_{18}^{7-}	S_{18}^-	S_{18}^+	S_{18}^{7+}	S_{18}^{5-}	S_6^-	S_6^+	i

$\twoheadrightarrow$

T **48**.2 Multiplication table *(cont.)*

$\mathbf{D}_{9d}$	i	S_{18}^{7-}	S_{18}^{7+}	S_{18}^{5-}	S_{18}^{5+}	S_6^-	S_6^+	S_{18}^-	S_{18}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}
E	i	S_{18}^{7-}	S_{18}^{7+}	S_{18}^{5-}	S_{18}^{5+}	S_6^-	S_6^+	S_{18}^-	S_{18}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}
C_9^+	S_{18}^{7-}	S_{18}^{5-}	i	S_6^-	S_{18}^{7+}	S_{18}^-	S_{18}^{5+}	S_{18}^+	S_6^+	σ_{d8}	σ_{d9}	σ_{d7}	σ_{d3}	σ_{d1}	σ_{d2}	σ_{d6}	σ_{d4}	σ_{d5}
C_9^-	S_{18}^{7+}	i	S_{18}^{5+}	S_{18}^{7-}	S_6^+	S_{18}^{5-}	S_{18}^+	S_6^-	S_{18}^-	σ_{d5}	σ_{d6}	σ_{d4}	σ_{d8}	σ_{d9}	σ_{d7}	σ_{d3}	σ_{d1}	σ_{d2}
C_9^{2+}	S_{18}^{5-}	S_6^-	S_{18}^{7-}	S_{18}^-	i	S_{18}^+	S_{18}^{7+}	S_6^+	S_{18}^{5+}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d2}	σ_{d3}	σ_{d1}
C_9^{2-}	S_{18}^{5+}	S_{18}^{7+}	S_6^+	i	S_{18}^+	S_{18}^{7-}	S_{18}^-	S_{18}^{5-}	S_6^-	σ_{d9}	σ_{d7}	σ_{d8}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}
C_3^+	S_6^-	S_{18}^-	S_{18}^{5-}	S_{18}^+	S_{18}^{7-}	S_6^+	i	S_{18}^{5+}	S_{18}^{7+}	σ_{d3}	σ_{d1}	σ_{d2}	σ_{d6}	σ_{d4}	σ_{d5}	σ_{d9}	σ_{d7}	σ_{d8}
C_3^-	S_6^+	S_{18}^{5+}	S_{18}^+	S_{18}^{7+}	S_{18}^-	i	S_6^-	S_{18}^{7-}	S_{18}^{5-}	σ_{d2}	σ_{d3}	σ_{d1}	σ_{d5}	σ_{d6}	σ_{d4}	σ_{d8}	σ_{d9}	σ_{d7}
C_9^{4+}	S_{18}^-	S_{18}^+	S_6^-	S_6^+	S_{18}^{5-}	S_{18}^{5+}	S_{18}^{7-}	S_{18}^{7+}	i	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d2}	σ_{d3}	σ_{d1}	σ_{d5}	σ_{d6}	σ_{d4}
C_9^{4-}	S_{18}^+	S_6^+	S_{18}^-	S_{18}^{5+}	S_6^-	S_{18}^{7+}	S_{18}^{5-}	i	S_{18}^{7-}	σ_{d6}	σ_{d4}	σ_{d5}	σ_{d9}	σ_{d7}	σ_{d8}	σ_{d1}	σ_{d2}	σ_{d3}
C'_{21}	σ_{d1}	σ_{d5}	σ_{d8}	σ_{d9}	σ_{d4}	σ_{d2}	σ_{d3}	σ_{d6}	σ_{d7}	i	S_6^-	S_6^+	S_{18}^{5+}	S_{18}^{7-}	S_{18}^-	S_{18}^+	S_{18}^{7+}	S_{18}^{5-}
C'_{22}	σ_{d2}	σ_{d6}	σ_{d9}	σ_{d7}	σ_{d5}	σ_{d3}	σ_{d1}	σ_{d4}	σ_{d8}	S_6^+	i	S_6^-	S_{18}^-	S_{18}^{5+}	S_{18}^{7-}	S_{18}^{5-}	S_{18}^+	S_{18}^{7+}
C'_{23}	σ_{d3}	σ_{d4}	σ_{d7}	σ_{d8}	σ_{d6}	σ_{d1}	σ_{d2}	σ_{d5}	σ_{d9}	S_6^-	S_6^+	i	S_{18}^{7-}	S_{18}^-	S_{18}^{5+}	S_{18}^{7+}	S_{18}^{5-}	S_{18}^+
C'_{24}	σ_{d4}	σ_{d8}	σ_{d3}	σ_{d1}	σ_{d7}	σ_{d5}	σ_{d6}	σ_{d9}	σ_{d2}	S_{18}^{5-}	S_{18}^+	S_{18}^{7+}	i	S_6^-	S_6^+	S_{18}^{5+}	S_{18}^{7-}	S_{18}^-
C'_{25}	σ_{d5}	σ_{d9}	σ_{d1}	σ_{d2}	σ_{d8}	σ_{d6}	σ_{d4}	σ_{d7}	σ_{d3}	S_{18}^{7+}	S_{18}^{5-}	S_{18}^+	S_6^+	i	S_6^-	S_{18}^-	S_{18}^{5+}	S_{18}^{7-}
C'_{26}	σ_{d6}	σ_{d7}	σ_{d2}	σ_{d3}	σ_{d9}	σ_{d4}	σ_{d5}	σ_{d8}	σ_{d1}	S_{18}^+	S_{18}^{7+}	S_{18}^{5-}	S_6^-	S_6^+	i	S_{18}^{7-}	S_{18}^-	S_{18}^{5+}
C'_{27}	σ_{d7}	σ_{d3}	σ_{d6}	σ_{d4}	σ_{d2}	σ_{d8}	σ_{d9}	σ_{d1}	σ_{d5}	S_{18}^-	S_{18}^{5+}	S_{18}^{7-}	S_{18}^{5-}	S_{18}^+	S_{18}^{7+}	i	S_6^-	S_6^+
C'_{28}	σ_{d8}	σ_{d1}	σ_{d4}	σ_{d5}	σ_{d3}	σ_{d9}	σ_{d7}	σ_{d2}	σ_{d6}	S_{18}^{7-}	S_{18}^-	S_{18}^{5+}	S_{18}^{7+}	S_{18}^{5-}	S_{18}^+	S_6^+	i	S_6^-
C'_{29}	σ_{d9}	σ_{d2}	σ_{d5}	σ_{d6}	σ_{d1}	σ_{d7}	σ_{d8}	σ_{d3}	σ_{d4}	S_{18}^{5+}	S_{18}^{7-}	S_{18}^-	S_{18}^+	S_{18}^{7+}	S_{18}^{5-}	S_6^-	S_6^+	i
i	E	C_9^+	C_9^-	C_9^{2+}	C_9^{2-}	C_3^+	C_3^-	C_9^{4+}	C_9^{4-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}
S_{18}^{7-}	C_9^+	C_9^{2+}	E	C_3^+	C_9^-	C_9^{4+}	C_9^{2-}	C_9^{4-}	C_3^-	C'_{28}	C'_{29}	C'_{27}	C'_{23}	C'_{21}	C'_{22}	C'_{26}	C'_{24}	C'_{25}
S_{18}^{7+}	C_9^-	E	C_9^{2-}	C_9^+	C_3^-	C_9^{2+}	C_9^{4-}	C_3^+	C_9^{4+}	C'_{25}	C'_{26}	C'_{24}	C'_{28}	C'_{29}	C'_{27}	C'_{23}	C'_{21}	C'_{22}
S_{18}^{5-}	C_9^{2+}	C_3^+	C_9^+	C_9^{4+}	E	C_9^{4-}	C_9^-	C_3^-	C_9^{2-}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}	C'_{22}	C'_{23}	C'_{21}
S_{18}^{5+}	C_9^{2-}	C_9^-	C_3^-	E	C_9^{4-}	C_9^+	C_9^{4+}	C_9^{2+}	C_3^+	C'_{29}	C'_{27}	C'_{28}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}
S_6^-	C_3^+	C_9^{4+}	C_9^{2+}	C_9^{4-}	C_9^+	C_3^-	E	C_9^{2-}	C_9^-	C'_{23}	C'_{21}	C'_{22}	C'_{26}	C'_{24}	C'_{25}	C'_{29}	C'_{27}	C'_{28}
S_6^+	C_3^-	C_9^{2-}	C_9^{4-}	C_9^-	C_9^{4+}	E	C_3^+	C_9^+	C_9^{2+}	C'_{22}	C'_{23}	C'_{21}	C'_{25}	C'_{26}	C'_{24}	C'_{28}	C'_{29}	C'_{27}
S_{18}^-	C_9^{4+}	C_9^{4-}	C_3^+	C_3^-	C_9^{2+}	C_9^{2-}	C_9^+	C_9^-	E	C'_{27}	C'_{28}	C'_{29}	C'_{22}	C'_{23}	C'_{21}	C'_{25}	C'_{26}	C'_{24}
S_{18}^+	C_9^{4-}	C_3^-	C_9^{4+}	C_9^{2-}	C_3^+	C_9^-	C_9^{2+}	E	C_9^+	C'_{26}	C'_{24}	C'_{25}	C'_{29}	C'_{27}	C'_{28}	C'_{21}	C'_{22}	C'_{23}
σ_{d1}	C'_{21}	C'_{25}	C'_{28}	C'_{29}	C'_{24}	C'_{22}	C'_{23}	C'_{26}	C'_{27}	E	C_3^+	C_3^-	C_9^{2-}	C_9^+	C_9^{4+}	C_9^{4-}	C_9^-	C_9^{2+}
σ_{d2}	C'_{22}	C'_{26}	C'_{29}	C'_{27}	C'_{25}	C'_{23}	C'_{21}	C'_{24}	C'_{28}	C_3^-	E	C_3^+	C_9^{4+}	C_9^{2-}	C_9^+	C_9^{2+}	C_9^{4-}	C_9^-
σ_{d3}	C'_{23}	C'_{24}	C'_{27}	C'_{28}	C'_{26}	C'_{21}	C'_{22}	C'_{25}	C'_{29}	C_3^+	C_3^-	E	C_9^+	C_9^{4+}	C_9^{2-}	C_9^-	C_9^{2+}	C_9^{4-}
σ_{d4}	C'_{24}	C'_{28}	C'_{23}	C'_{21}	C'_{27}	C'_{25}	C'_{26}	C'_{29}	C'_{22}	C_9^{2+}	C_9^{4-}	C_9^-	E	C_3^+	C_3^-	C_9^{2-}	C_9^+	C_9^{4+}
σ_{d5}	C'_{25}	C'_{29}	C'_{21}	C'_{22}	C'_{28}	C'_{26}	C'_{24}	C'_{27}	C'_{23}	C_9^-	C_9^{2+}	C_9^{4-}	C_3^-	E	C_3^+	C_9^{4+}	C_9^{2-}	C_9^+
σ_{d6}	C'_{26}	C'_{27}	C'_{22}	C'_{23}	C'_{29}	C'_{24}	C'_{25}	C'_{28}	C'_{21}	C_9^{4-}	C_9^-	C_9^{2+}	C_3^+	C_3^-	E	C_9^+	C_9^{4+}	C_9^{2-}
σ_{d7}	C'_{27}	C'_{23}	C'_{26}	C'_{24}	C'_{22}	C'_{28}	C'_{29}	C'_{21}	C'_{25}	C_9^{4+}	C_9^{2-}	C_9^+	C_9^{2+}	C_9^{4-}	C_9^-	E	C_3^+	C_3^-
σ_{d8}	C'_{28}	C'_{21}	C'_{24}	C'_{25}	C'_{23}	C'_{29}	C'_{27}	C'_{22}	C'_{26}	C_9^+	C_9^{4+}	C_9^{2-}	C_9^-	C_9^{2+}	C_9^{4-}	C_3^-	E	C_3^+
σ_{d9}	C'_{29}	C'_{22}	C'_{25}	C'_{26}	C'_{21}	C'_{27}	C'_{28}	C'_{23}	C'_{24}	C_9^{2-}	C_9^+	C_9^{4+}	C_9^{4-}	C_9^-	C_9^{2+}	C_3^+	C_3^-	E

T **48**.3 Factor table

§ **16**–3, p. 70

$\mathbf{D}_{9d}$	E	C_9^+	C_9^-	C_9^{2+}	C_9^{2-}	C_3^+	C_3^-	C_9^{4+}	C_9^{4-}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_9^+	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_9^-	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_9^{2+}	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
C_9^{2-}	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1
C_3^+	1	1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_3^-	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_9^{4+}	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
C_9^{4-}	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1
C'_{21}	1	−1	−1	1	1	−1	−1	1	1	−1	1	1	−1	1	−1	−1	1	−1
C'_{22}	1	−1	−1	1	1	−1	−1	1	1	1	−1	1	−1	−1	1	−1	−1	1
C'_{23}	1	−1	−1	1	1	−1	−1	1	1	1	1	−1	1	−1	−1	1	−1	−1
C'_{24}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	1	1	−1	1	−1
C'_{25}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	1	−1	1	−1	−1	1
C'_{26}	1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	1	1	−1	1	−1	−1
C'_{27}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	−1	1	−1	1	1
C'_{28}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	1	−1	−1	1	−1	1
C'_{29}	1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	−1	1	−1	1	1	−1
i	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_{18}^{7-}	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_{18}^{7+}	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_{18}^{5-}	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
S_{18}^{5+}	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1
S_6^-	1	1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_6^+	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_{18}^-	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
S_{18}^+	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1
σ_{d1}	1	−1	−1	1	1	−1	−1	1	1	−1	1	1	−1	1	−1	−1	1	−1
σ_{d2}	1	−1	−1	1	1	−1	−1	1	1	1	−1	1	−1	−1	1	−1	−1	1
σ_{d3}	1	−1	−1	1	1	−1	−1	1	1	1	1	−1	1	−1	−1	1	−1	−1
σ_{d4}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	1	1	−1	1	−1
σ_{d5}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	1	−1	1	−1	−1	1
σ_{d6}	1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	1	1	−1	1	−1	−1
σ_{d7}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	−1	1	−1	1	1
σ_{d8}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	1	−1	−1	1	−1	1
σ_{d9}	1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	−1	1	−1	1	1	−1

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T **48**.3 Factor table *(cont.)*

$\mathbf{D}_{9d}$	i	S_{18}^{7-}	S_{18}^{7+}	S_{18}^{5-}	S_{18}^{5+}	S_6^-	S_6^+	S_{18}^-	S_{18}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_9^+	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_9^-	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_9^{2+}	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
C_9^{2-}	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1
C_3^+	1	1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_3^-	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_9^{4+}	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
C_9^{4-}	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1
C'_{21}	1	−1	−1	1	1	−1	−1	1	1	−1	1	1	−1	1	−1	−1	1	−1
C'_{22}	1	−1	−1	1	1	−1	−1	1	1	1	−1	1	−1	−1	1	−1	−1	1
C'_{23}	1	−1	−1	1	1	−1	−1	1	1	1	1	−1	1	−1	−1	1	−1	−1
C'_{24}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	1	1	−1	1	−1
C'_{25}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	1	−1	1	−1	−1	1
C'_{26}	1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	1	1	−1	1	−1	−1
C'_{27}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	−1	1	−1	1	1
C'_{28}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	1	−1	−1	1	−1	1
C'_{29}	1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	−1	1	−1	1	1	−1
i	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S_{18}^{7-}	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_{18}^{7+}	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_{18}^{5-}	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
S_{18}^{5+}	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1
S_6^-	1	1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_6^+	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_{18}^-	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
S_{18}^+	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1
σ_{d1}	1	−1	−1	1	1	−1	−1	1	1	−1	1	1	−1	1	−1	−1	1	−1
σ_{d2}	1	−1	−1	1	1	−1	−1	1	1	1	−1	1	−1	−1	1	−1	−1	1
σ_{d3}	1	−1	−1	1	1	−1	−1	1	1	1	1	−1	1	−1	−1	1	−1	−1
σ_{d4}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	1	1	−1	1	−1
σ_{d5}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	1	−1	1	−1	−1	1
σ_{d6}	1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	1	1	−1	1	−1	−1
σ_{d7}	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	−1	1	−1	1	1
σ_{d8}	1	−1	−1	1	1	−1	−1	1	1	1	−1	−1	1	−1	−1	1	−1	1
σ_{d9}	1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	−1	1	−1	1	1	−1

T **48**.4 Character table

§ **16**–4, p. 71

$\mathbf{D}_{9d}$	E	$2C_9$	$2C_9^2$	$2C_3$	$2C_9^4$	$9C_2'$	i	$2S_{18}^7$	$2S_{18}^5$	$2S_6$	$2S_{18}$	$9\sigma_d$	τ
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1	a
A_{2g}	1	1	1	1	1	-1	1	1	1	1	1	-1	a
E_{1g}	2	$2c_9^2$	$2c_9^4$	-1	$2c_9^8$	0	2	$2c_9^2$	$2c_9^4$	-1	$2c_9^8$	0	a
E_{2g}	2	$2c_9^4$	$2c_9^8$	-1	$2c_9^2$	0	2	$2c_9^4$	$2c_9^8$	-1	$2c_9^2$	0	a
E_{3g}	2	-1	-1	2	-1	0	2	-1	-1	2	-1	0	a
E_{4g}	2	$2c_9^8$	$2c_9^2$	-1	$2c_9^4$	0	2	$2c_9^8$	$2c_9^2$	-1	$2c_9^4$	0	a
A_{1u}	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	a
A_{2u}	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	a
E_{1u}	2	$2c_9^2$	$2c_9^4$	-1	$2c_9^8$	0	-2	$-2c_9^2$	$-2c_9^4$	1	$-2c_9^8$	0	a
E_{2u}	2	$2c_9^4$	$2c_9^8$	-1	$2c_9^2$	0	-2	$-2c_9^4$	$-2c_9^8$	1	$-2c_9^2$	0	a
E_{3u}	2	-1	-1	2	-1	0	-2	1	1	-2	1	0	a
E_{4u}	2	$2c_9^8$	$2c_9^2$	-1	$2c_9^4$	0	-2	$-2c_9^8$	$-2c_9^2$	1	$-2c_9^4$	0	a
$E_{1/2,g}$	2	$-2c_9^8$	$2c_9^2$	1	$2c_9^4$	0	2	$-2c_9^8$	$2c_9^2$	1	$2c_9^4$	0	c
$E_{3/2,g}$	2	1	-1	-2	-1	0	2	1	-1	-2	-1	0	c
$E_{5/2,g}$	2	$-2c_9^4$	$2c_9^8$	1	$2c_9^2$	0	2	$-2c_9^4$	$2c_9^8$	1	$2c_9^2$	0	c
$E_{7/2,g}$	2	$-2c_9^2$	$2c_9^4$	1	$2c_9^8$	0	2	$-2c_9^2$	$2c_9^4$	1	$2c_9^8$	0	c
${}^1E_{9/2,g}$	1	-1	1	-1	1	i	1	-1	1	-1	1	i	b
${}^2E_{9/2,g}$	1	-1	1	-1	1	$-$i	1	-1	1	-1	1	$-$i	b
$E_{1/2,u}$	2	$-2c_9^8$	$2c_9^2$	1	$2c_9^4$	0	-2	$2c_9^8$	$-2c_9^2$	-1	$-2c_9^4$	0	c
$E_{3/2,u}$	2	1	-1	-2	-1	0	-2	-1	1	2	1	0	c
$E_{5/2,u}$	2	$-2c_9^4$	$2c_9^8$	1	$2c_9^2$	0	-2	$2c_9^4$	$-2c_9^8$	-1	$-2c_9^2$	0	c
$E_{7/2,u}$	2	$-2c_9^2$	$2c_9^4$	1	$2c_9^8$	0	-2	$2c_9^2$	$-2c_9^4$	-1	$-2c_9^8$	0	c
${}^1E_{9/2,u}$	1	-1	1	-1	1	i	-1	1	-1	1	-1	$-$i	b
${}^2E_{9/2,u}$	1	-1	1	-1	1	$-$i	-1	1	-1	1	-1	i	b

$c_n^m = \cos \frac{m}{n}\pi$

T **48**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{9d}$	0	1	2	3
A_{1g}	$\square 1$		$x^2+y^2, \square z^2$	
A_{2g}		R_z		
E_{1g}		(R_x, R_y)	$\square(zx, yz)$	
E_{2g}			$\square(xy, x^2-y^2)$	
E_{3g}				
E_{4g}				
A_{1u}				
A_{2u}		$\square z$		$(x^2+y^2)z, \square z^3$
E_{1u}		$\square(x, y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, \square(xz^2, yz^2)$
E_{2u}				$\square\{xyz, (x^2-y^2)z\}$
E_{3u}				$\square\{x(x^2-3y^2), y(3x^2-y^2)\}$
E_{4u}				

T **48**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{9d}$	$\langle\lvert j\,m\rangle\rvert$		ι	μ
A_{1g}	$\lvert 0\,0\rangle_+$		2	9
A_{2g}	$\lvert 10\,9\rangle_-$		2	9
E_{1g}	$\langle\lvert 2\,1\rangle, -\lvert 2\,\overline{1}\rangle\rvert$		2	± 9
E_{2g}	$\langle\lvert 2\,\overline{2}\rangle, -\lvert 2\,2\rangle\rvert$		2	± 9
E_{3g}	$\langle\lvert 4\,3\rangle, -\lvert 4\,\overline{3}\rangle\rvert$		2	± 9
E_{4g}	$\langle\lvert 4\,4\rangle, -\lvert 4\,\overline{4}\rangle\rvert$		2	± 9
A_{1u}	$\lvert 9\,9\rangle_-$		2	9
A_{2u}	$\lvert 1\,0\rangle_+$		2	9
E_{1u}	$\langle\lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle\rvert$		2	± 9
E_{2u}	$\langle\lvert 3\,\overline{2}\rangle, \lvert 3\,2\rangle\rvert$		2	± 9
E_{3u}	$\langle\lvert 3\,3\rangle, \lvert 3\,\overline{3}\rangle\rvert$		2	± 9
E_{4u}	$\langle\lvert 5\,4\rangle, \lvert 5\,\overline{4}\rangle\rvert$		2	± 9
$E_{1/2,g}$	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	2	± 9
$E_{3/2,g}$	$\langle\lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle\rvert$	$\langle\lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle\rvert$	2	± 9
$E_{5/2,g}$	$\langle\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle, \lvert \frac{5}{2}\,\frac{5}{2}\rangle\rvert$	$\langle\lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle, -\lvert \frac{7}{2}\,\frac{5}{2}\rangle\rvert$	2	± 9
$E_{7/2,g}$	$\langle\lvert \frac{7}{2}\,\frac{7}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle\rvert$	$\langle\lvert \frac{9}{2}\,\frac{7}{2}\rangle, \lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle\rvert$	2	± 9
${}^1E_{9/2,g}$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle_-$	$\lvert \frac{11}{2}\,\frac{9}{2}\rangle_+$	2	9
${}^2E_{9/2,g}$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle_+$	$\lvert \frac{11}{2}\,\frac{9}{2}\rangle_-$	2	9
$E_{1/2,u}$	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert^\bullet$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert^\bullet$	2	± 9
$E_{3/2,u}$	$\langle\lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle\rvert^\bullet$	$\langle\lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle\rvert^\bullet$	2	± 9
$E_{5/2,u}$	$\langle\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle, \lvert \frac{5}{2}\,\frac{5}{2}\rangle\rvert^\bullet$	$\langle\lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle, -\lvert \frac{7}{2}\,\frac{5}{2}\rangle\rvert^\bullet$	2	± 9
$E_{7/2,u}$	$\langle\lvert \frac{7}{2}\,\frac{7}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle\rvert^\bullet$	$\langle\lvert \frac{9}{2}\,\frac{7}{2}\rangle, \lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle\rvert^\bullet$	2	± 9
${}^1E_{9/2,u}$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle^\bullet_-$	$\lvert \frac{11}{2}\,\frac{9}{2}\rangle^\bullet_+$	2	9
${}^2E_{9/2,u}$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle^\bullet_+$	$\lvert \frac{11}{2}\,\frac{9}{2}\rangle^\bullet_-$	2	9

T **48**.7 Matrix representations

Use T **29**.7 ▪. § **16**–7, p. 77

T **48**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{D}_{9d}$	A_{1g}	A_{2g}	E_{1g}	E_{2g}	E_{3g}	E_{4g}
A_{1g}	A_{1g}	A_{2g}	E_{1g}	E_{2g}	E_{3g}	E_{4g}
A_{2g}		A_{1g}	E_{1g}	E_{2g}	E_{3g}	E_{4g}
E_{1g}			$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$	$E_{1g} \oplus E_{3g}$	$E_{2g} \oplus E_{4g}$	$E_{3g} \oplus E_{4g}$
E_{2g}				$A_{1g} \oplus \{A_{2g}\} \oplus E_{4g}$	$E_{1g} \oplus E_{4g}$	$E_{2g} \oplus E_{3g}$
E_{3g}					$A_{1g} \oplus \{A_{2g}\} \oplus E_{3g}$	$E_{1g} \oplus E_{2g}$
E_{4g}						$A_{1g} \oplus \{A_{2g}\} \oplus E_{1g}$

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T **48**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{9d}$	A_{1u}	A_{2u}	E_{1u}	E_{2u}	E_{3u}	E_{4u}
A_{1g}	A_{1u}	A_{2u}	E_{1u}	E_{2u}	E_{3u}	E_{4u}
A_{2g}	A_{2u}	A_{1u}	E_{1u}	E_{2u}	E_{3u}	E_{4u}
E_{1g}	E_{1u}	E_{1u}	$A_{1u} \oplus A_{2u} \oplus E_{2u}$	$E_{1u} \oplus E_{3u}$	$E_{2u} \oplus E_{4u}$	$E_{3u} \oplus E_{4u}$
E_{2g}	E_{2u}	E_{2u}	$E_{1u} \oplus E_{3u}$	$A_{1u} \oplus A_{2u} \oplus E_{4u}$	$E_{1u} \oplus E_{4u}$	$E_{2u} \oplus E_{3u}$
E_{3g}	E_{3u}	E_{3u}	$E_{2u} \oplus E_{4u}$	$E_{1u} \oplus E_{4u}$	$A_{1u} \oplus A_{2u} \oplus E_{3u}$	$E_{1u} \oplus E_{2u}$
E_{4g}	E_{4u}	E_{4u}	$E_{3u} \oplus E_{4u}$	$E_{2u} \oplus E_{3u}$	$E_{1u} \oplus E_{2u}$	$A_{1u} \oplus A_{2u} \oplus E_{1u}$
A_{1u}	A_{1g}	A_{2g}	E_{1g}	E_{2g}	E_{3g}	E_{4g}
A_{2u}		A_{1g}	E_{1g}	E_{2g}	E_{3g}	E_{4g}
E_{1u}			$A_{1g} \oplus \{A_{2g}\} \oplus E_{2g}$	$E_{1g} \oplus E_{3g}$	$E_{2g} \oplus E_{4g}$	$E_{3g} \oplus E_{4g}$
E_{2u}				$A_{1g} \oplus \{A_{2g}\} \oplus E_{4g}$	$E_{1g} \oplus E_{4g}$	$E_{2g} \oplus E_{3g}$
E_{3u}					$A_{1g} \oplus \{A_{2g}\} \oplus E_{3g}$	$E_{1g} \oplus E_{2g}$
E_{4u}						$A_{1g} \oplus \{A_{2g}\} \oplus E_{1g}$

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T **48**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{9d}$	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{7/2,g}$	$^{1}E_{9/2,g}$	$^{2}E_{9/2,g}$
A_{1g}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{7/2,g}$	$^{1}E_{9/2,g}$	$^{2}E_{9/2,g}$
A_{2g}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{7/2,g}$	$^{2}E_{9/2,g}$	$^{1}E_{9/2,g}$
E_{1g}	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{3/2,g} \oplus E_{7/2,g}$	$E_{5/2,g} \oplus {}^{1}E_{9/2,g} \oplus {}^{2}E_{9/2,g}$	$E_{7/2,g}$	$E_{7/2,g}$
E_{2g}	$E_{3/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{7/2,g}$	$E_{1/2,g} \oplus {}^{1}E_{9/2,g} \oplus {}^{2}E_{9/2,g}$	$E_{3/2,g} \oplus E_{7/2,g}$	$E_{5/2,g}$	$E_{5/2,g}$
E_{3g}	$E_{5/2,g} \oplus E_{7/2,g}$	$E_{3/2,g} \oplus {}^{1}E_{9/2,g} \oplus {}^{2}E_{9/2,g}$	$E_{1/2,g} \oplus E_{7/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{3/2,g}$	$E_{3/2,g}$
E_{4g}	$E_{7/2,g} \oplus {}^{1}E_{9/2,g} \oplus {}^{2}E_{9/2,g}$	$E_{5/2,g} \oplus E_{7/2,g}$	$E_{3/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g}$	$E_{1/2,g}$
A_{1u}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	$E_{7/2,u}$	$^{1}E_{9/2,u}$	$^{2}E_{9/2,u}$
A_{2u}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	$E_{7/2,u}$	$^{2}E_{9/2,u}$	$^{1}E_{9/2,u}$
E_{1u}	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{3/2,u} \oplus E_{7/2,u}$	$E_{5/2,u} \oplus {}^{1}E_{9/2,u} \oplus {}^{2}E_{9/2,u}$	$E_{7/2,u}$	$E_{7/2,u}$
E_{2u}	$E_{3/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{7/2,u}$	$E_{1/2,u} \oplus {}^{1}E_{9/2,u} \oplus {}^{2}E_{9/2,u}$	$E_{3/2,u} \oplus E_{7/2,u}$	$E_{5/2,u}$	$E_{5/2,u}$
E_{3u}	$E_{5/2,u} \oplus E_{7/2,u}$	$E_{3/2,u} \oplus {}^{1}E_{9/2,u} \oplus {}^{2}E_{9/2,u}$	$E_{1/2,u} \oplus E_{7/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{3/2,u}$	$E_{3/2,u}$
E_{4u}	$E_{7/2,u} \oplus {}^{1}E_{9/2,u} \oplus {}^{2}E_{9/2,u}$	$E_{5/2,u} \oplus E_{7/2,u}$	$E_{3/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u}$	$E_{1/2,u}$
$E_{1/2,g}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$	$E_{1g} \oplus E_{2g}$	$E_{2g} \oplus E_{3g}$	$E_{3g} \oplus E_{4g}$	E_{4g}	E_{4g}
$E_{3/2,g}$		$\{A_{1g}\} \oplus A_{2g} \oplus E_{3g}$	$E_{1g} \oplus E_{4g}$	$E_{2g} \oplus E_{4g}$	E_{3g}	E_{3g}
$E_{5/2,g}$			$\{A_{1g}\} \oplus A_{2g} \oplus E_{4g}$	$E_{1g} \oplus E_{3g}$	E_{2g}	E_{2g}
$E_{7/2,g}$				$\{A_{1g}\} \oplus A_{2g} \oplus E_{2g}$	E_{1g}	E_{1g}
$^{1}E_{9/2,g}$					A_{2g}	A_{1g}
$^{2}E_{9/2,g}$						A_{2g}

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T **48**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{9d}$	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	$E_{7/2,u}$	${}^1E_{9/2,u}$	${}^2E_{9/2,u}$
A_{1g}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	$E_{7/2,u}$	${}^1E_{9/2,u}$	${}^2E_{9/2,u}$
A_{2g}	$E_{1/2,u}$	$E_{3/2,u}$	$E_{5/2,u}$	$E_{7/2,u}$	${}^2E_{9/2,u}$	${}^1E_{9/2,u}$
E_{1g}	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{3/2,u} \oplus E_{7/2,u}$	$E_{5/2,u} \oplus {}^1E_{9/2,u} \oplus {}^2E_{9/2,u}$	$E_{7/2,u}$	$E_{7/2,u}$
E_{2g}	$E_{3/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{7/2,u}$	$E_{1/2,u} \oplus {}^1E_{9/2,u} \oplus {}^2E_{9/2,u}$	$E_{3/2,u} \oplus E_{7/2,u}$	$E_{5/2,u}$	$E_{5/2,u}$
E_{3g}	$E_{5/2,u} \oplus E_{7/2,u}$	$E_{3/2,u} \oplus {}^1E_{9/2,u} \oplus {}^2E_{9/2,u}$	$E_{1/2,u} \oplus E_{7/2,u}$	$E_{1/2,u} \oplus E_{5/2,u}$	$E_{3/2,u}$	$E_{3/2,u}$
E_{4g}	$E_{7/2,u} \oplus {}^1E_{9/2,u} \oplus {}^2E_{9/2,u}$	$E_{5/2,u} \oplus E_{7/2,u}$	$E_{3/2,u} \oplus E_{5/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u}$	$E_{1/2,u}$
A_{1u}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{7/2,g}$	${}^1E_{9/2,g}$	${}^2E_{9/2,g}$
A_{2u}	$E_{1/2,g}$	$E_{3/2,g}$	$E_{5/2,g}$	$E_{7/2,g}$	${}^2E_{9/2,g}$	${}^1E_{9/2,g}$
E_{1u}	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{3/2,g} \oplus E_{7/2,g}$	$E_{5/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g}$	$E_{7/2,g}$	$E_{7/2,g}$
E_{2u}	$E_{3/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{7/2,g}$	$E_{1/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g}$	$E_{3/2,g} \oplus E_{7/2,g}$	$E_{5/2,g}$	$E_{5/2,g}$
E_{3u}	$E_{5/2,g} \oplus E_{7/2,g}$	$E_{3/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g}$	$E_{1/2,g} \oplus E_{7/2,g}$	$E_{1/2,g} \oplus E_{5/2,g}$	$E_{3/2,g}$	$E_{3/2,g}$
E_{4u}	$E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g}$	$E_{5/2,g} \oplus E_{7/2,g}$	$E_{3/2,g} \oplus E_{5/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g}$	$E_{1/2,g}$
$E_{1/2,g}$	$A_{1u} \oplus A_{2u} \oplus E_{1u}$	$E_{1u} \oplus E_{2u}$	$E_{2u} \oplus E_{3u}$	$E_{3u} \oplus E_{4u}$	E_{4u}	E_{4u}
$E_{3/2,g}$	$E_{1u} \oplus E_{2u}$	$A_{1u} \oplus A_{2u} \oplus E_{3u}$	$E_{1u} \oplus E_{4u}$	$E_{2u} \oplus E_{4u}$	E_{3u}	E_{3u}
$E_{5/2,g}$	$E_{2u} \oplus E_{3u}$	$E_{1u} \oplus E_{4u}$	$A_{1u} \oplus A_{2u} \oplus E_{4u}$	$E_{1u} \oplus E_{3u}$	E_{2u}	E_{2u}
$E_{7/2,g}$	$E_{3u} \oplus E_{4u}$	$E_{2u} \oplus E_{4u}$	$E_{1u} \oplus E_{3u}$	$A_{1u} \oplus A_{2u} \oplus E_{2u}$	E_{1u}	E_{1u}
${}^1E_{9/2,g}$	E_{4u}	E_{3u}	E_{2u}	E_{1u}	A_{2u}	A_{1u}
${}^2E_{9/2,g}$	E_{4u}	E_{3u}	E_{2u}	E_{1u}	A_{1u}	A_{2u}
$E_{1/2,u}$	$\{A_{1g}\} \oplus A_{2g} \oplus E_{1g}$	$E_{1g} \oplus E_{2g}$	$E_{2g} \oplus E_{3g}$	$E_{3g} \oplus E_{4g}$	E_{4g}	E_{4g}
$E_{3/2,u}$		$\{A_{1g}\} \oplus A_{2g} \oplus E_{3g}$	$E_{1g} \oplus E_{4g}$	$E_{2g} \oplus E_{4g}$	E_{3g}	E_{3g}
$E_{5/2,u}$			$\{A_{1g}\} \oplus A_{2g} \oplus E_{4g}$	$E_{1g} \oplus E_{3g}$	E_{2g}	E_{2g}
$E_{7/2,u}$				$\{A_{1g}\} \oplus A_{2g} \oplus E_{2g}$	E_{1g}	E_{1g}
${}^1E_{9/2,u}$					A_{2g}	A_{1g}
${}^2E_{9/2,u}$						A_{2g}

T 48.9 Subduction (descent of symmetry)

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$\mathbf{D}_{9d}$	$(\mathbf{C}_{2h})$	$(\mathbf{C}_{9v})$	$(\mathbf{C}_{3v})$	$\mathbf{D}_9$	$(\mathbf{D}_3)$	$\mathbf{S}_{18}$
A_{1g}	A_g	A_1	A_1	A_1	A_1	A_g
A_{2g}	B_g	A_2	A_2	A_2	A_2	A_g
E_{1g}	$A_g \oplus B_g$	E_1	E	E_1	E	${}^1E_{1g} \oplus {}^2E_{1g}$
E_{2g}	$A_g \oplus B_g$	E_2	E	E_2	E	${}^1E_{2g} \oplus {}^2E_{2g}$
E_{3g}	$A_g \oplus B_g$	E_3	$A_1 \oplus A_2$	E_3	$A_1 \oplus A_2$	${}^1E_{3g} \oplus {}^2E_{3g}$
E_{4g}	$A_g \oplus B_g$	E_4	E	E_4	E	${}^1E_{4g} \oplus {}^2E_{4g}$
A_{1u}	A_u	A_2	A_2	A_1	A_1	A_u
A_{2u}	B_u	A_1	A_1	A_2	A_2	A_u
E_{1u}	$A_u \oplus B_u$	E_1	E	E_1	E	${}^1E_{1u} \oplus {}^2E_{1u}$
E_{2u}	$A_u \oplus B_u$	E_2	E	E_2	E	${}^1E_{2u} \oplus {}^2E_{2u}$
E_{3u}	$A_u \oplus B_u$	E_3	$A_1 \oplus A_2$	E_3	$A_1 \oplus A_2$	${}^1E_{3u} \oplus {}^2E_{3u}$
E_{4u}	$A_u \oplus B_u$	E_4	E	E_4	E	${}^1E_{4u} \oplus {}^2E_{4u}$
$E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$
$E_{3/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$
$E_{5/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{5/2}$	$E_{1/2}$	$E_{5/2}$	$E_{1/2}$	${}^1E_{5/2,g} \oplus {}^2E_{5/2,g}$
$E_{7/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{7/2}$	$E_{1/2}$	$E_{7/2}$	$E_{1/2}$	${}^1E_{7/2,g} \oplus {}^2E_{7/2,g}$
${}^1E_{9/2,g}$	${}^1E_{1/2,g}$	${}^1E_{9/2}$	${}^1E_{3/2}$	${}^1E_{9/2}$	${}^1E_{3/2}$	$A_{9/2,g}$
${}^2E_{9/2,g}$	${}^2E_{1/2,g}$	${}^2E_{9/2}$	${}^2E_{3/2}$	${}^2E_{9/2}$	${}^2E_{3/2}$	$A_{9/2,g}$
$E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$
$E_{3/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$
$E_{5/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{5/2}$	$E_{1/2}$	$E_{5/2}$	$E_{1/2}$	${}^1E_{5/2,u} \oplus {}^2E_{5/2,u}$
$E_{7/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{7/2}$	$E_{1/2}$	$E_{7/2}$	$E_{1/2}$	${}^1E_{7/2,u} \oplus {}^2E_{7/2,u}$
${}^1E_{9/2,u}$	${}^1E_{1/2,u}$	${}^2E_{9/2}$	${}^2E_{3/2}$	${}^1E_{9/2}$	${}^1E_{3/2}$	$A_{9/2,u}$
${}^2E_{9/2,u}$	${}^2E_{1/2,u}$	${}^1E_{9/2}$	${}^1E_{3/2}$	${}^2E_{9/2}$	${}^2E_{3/2}$	$A_{9/2,u}$

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T 48.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{9d}$	$\mathbf{S}_6$	$(\mathbf{C}_s)$	$\mathbf{C}_i$	$\mathbf{C}_9$	$\mathbf{C}_3$	$(\mathbf{C}_2)$
A_{1g}	A_g	A'	A_g	A	A	A
A_{2g}	A_g	A''	A_g	A	A	B
E_{1g}	${}^1E_g \oplus {}^2E_g$	$A' \oplus A''$	$2A_g$	${}^1E_1 \oplus {}^2E_1$	${}^1E \oplus {}^2E$	$A \oplus B$
E_{2g}	${}^1E_g \oplus {}^2E_g$	$A' \oplus A''$	$2A_g$	${}^1E_2 \oplus {}^2E_2$	${}^1E \oplus {}^2E$	$A \oplus B$
E_{3g}	$2A_g$	$A' \oplus A''$	$2A_g$	${}^1E_3 \oplus {}^2E_3$	$2A$	$A \oplus B$
E_{4g}	${}^1E_g \oplus {}^2E_g$	$A' \oplus A''$	$2A_g$	${}^1E_4 \oplus {}^2E_4$	${}^1E \oplus {}^2E$	$A \oplus B$
A_{1u}	A_u	A''	A_u	A	A	A
A_{2u}	A_u	A'	A_u	A	A	B
E_{1u}	${}^1E_u \oplus {}^2E_u$	$A' \oplus A''$	$2A_u$	${}^1E_1 \oplus {}^2E_1$	${}^1E \oplus {}^2E$	$A \oplus B$
E_{2u}	${}^1E_u \oplus {}^2E_u$	$A' \oplus A''$	$2A_u$	${}^1E_2 \oplus {}^2E_2$	${}^1E \oplus {}^2E$	$A \oplus B$
E_{3u}	$2A_u$	$A' \oplus A''$	$2A_u$	${}^1E_3 \oplus {}^2E_3$	$2A$	$A \oplus B$
E_{4u}	${}^1E_u \oplus {}^2E_u$	$A' \oplus A''$	$2A_u$	${}^1E_4 \oplus {}^2E_4$	${}^1E \oplus {}^2E$	$A \oplus B$
$E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2,g}$	$2A_{3/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,g}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$2A_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,g}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,g}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1E_{9/2,g}$	$A_{3/2,g}$	${}^1E_{1/2}$	$A_{1/2,g}$	$A_{9/2}$	$A_{3/2}$	${}^1E_{1/2}$
${}^2E_{9/2,g}$	$A_{3/2,g}$	${}^2E_{1/2}$	$A_{1/2,g}$	$A_{9/2}$	$A_{3/2}$	${}^2E_{1/2}$
$E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2,u}$	$2A_{3/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,u}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$2A_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,u}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	$2A_{1/2,u}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1E_{9/2,u}$	$A_{3/2,u}$	${}^2E_{1/2}$	$A_{1/2,u}$	$A_{9/2}$	$A_{3/2}$	${}^1E_{1/2}$
${}^2E_{9/2,u}$	$A_{3/2,u}$	${}^1E_{1/2}$	$A_{1/2,u}$	$A_{9/2}$	$A_{3/2}$	${}^2E_{1/2}$

T **48**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{9d}$
$18n$	$(2n+1)\,A_{1g} \oplus 2n\,(A_{2g} \oplus 2E_{1g} \oplus 2E_{2g} \oplus 2E_{3g} \oplus 2E_{4g})$
$18n+1$	$2n\,(A_{1u} \oplus E_{1u} \oplus 2E_{2u} \oplus 2E_{3u} \oplus 2E_{4u}) \oplus (2n+1)(A_{2u} \oplus E_{1u})$
$18n+2$	$(2n+1)(A_{1g} \oplus E_{1g} \oplus E_{2g}) \oplus 2n\,(A_{2g} \oplus E_{1g} \oplus E_{2g} \oplus 2E_{3g} \oplus 2E_{4g})$
$18n+3$	$2n\,(A_{1u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u} \oplus 2E_{4u}) \oplus (2n+1)(A_{2u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u})$
$18n+4$	$(2n+1)(A_{1g} \oplus E_{1g} \oplus E_{2g} \oplus E_{3g} \oplus E_{4g}) \oplus 2n\,(A_{2g} \oplus E_{1g} \oplus E_{2g} \oplus E_{3g} \oplus E_{4g})$
$18n+5$	$2n\,(A_{1u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u}) \oplus (2n+1)(A_{2u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u} \oplus 2E_{4u})$
$18n+6$	$(2n+1)(A_{1g} \oplus E_{1g} \oplus E_{2g} \oplus 2E_{3g} \oplus 2E_{4g}) \oplus 2n\,(A_{2g} \oplus E_{1g} \oplus E_{2g})$
$18n+7$	$2n\,(A_{1u} \oplus E_{1u}) \oplus (2n+1)(A_{2u} \oplus E_{1u} \oplus 2E_{2u} \oplus 2E_{3u} \oplus 2E_{4u})$
$18n+8$	$(2n+1)(A_{1g} \oplus 2E_{1g} \oplus 2E_{2g} \oplus 2E_{3g} \oplus 2E_{4g}) \oplus 2n\,A_{2g}$
$18n+9$	$(2n+1)(A_{1u} \oplus 2E_{1u} \oplus 2E_{2u} \oplus 2E_{3u} \oplus 2E_{4u}) \oplus (2n+2)\,A_{2u}$
$18n+10$	$(2n+2)(A_{1g} \oplus E_{1g}) \oplus (2n+1)(A_{2g} \oplus E_{1g} \oplus 2E_{2g} \oplus 2E_{3g} \oplus 2E_{4g})$
$18n+11$	$(2n+1)(A_{1u} \oplus E_{1u} \oplus E_{2u} \oplus 2E_{3u} \oplus 2E_{4u}) \oplus (2n+2)(A_{2u} \oplus E_{1u} \oplus E_{2u})$
$18n+12$	$(2n+2)(A_{1g} \oplus E_{1g} \oplus E_{2g} \oplus E_{3g}) \oplus (2n+1)(A_{2g} \oplus E_{1g} \oplus E_{2g} \oplus E_{3g} \oplus 2E_{4g})$
$18n+13$	$(2n+1)(A_{1u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u} \oplus E_{4u}) \oplus (2n+2)(A_{2u} \oplus E_{1u} \oplus E_{2u} \oplus E_{3u} \oplus E_{4u})$
$18n+14$	$(2n+2)(A_{1g} \oplus E_{1g} \oplus E_{2g} \oplus E_{3g} \oplus 2E_{4g}) \oplus (2n+1)(A_{2g} \oplus E_{1g} \oplus E_{2g} \oplus E_{3g})$
$18n+15$	$(2n+1)(A_{1u} \oplus E_{1u} \oplus E_{2u}) \oplus (2n+2)(A_{2u} \oplus E_{1u} \oplus E_{2u} \oplus 2E_{3u} \oplus 2E_{4u})$
$18n+16$	$(2n+2)(A_{1g} \oplus E_{1g} \oplus 2E_{2g} \oplus 2E_{3g} \oplus 2E_{4g}) \oplus (2n+1)(A_{2g} \oplus E_{1g})$
$18n+17$	$(2n+1)\,A_{1u} \oplus (2n+2)(A_{2u} \oplus 2E_{1u} \oplus 2E_{2u} \oplus 2E_{3u} \oplus 2E_{4u})$
$9n+\frac{1}{2}$	$(2n+1)\,E_{1/2,g} \oplus n\,(2E_{3/2,g} \oplus 2E_{5/2,g} \oplus 2E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$9n+\frac{3}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g}) \oplus n\,(2E_{5/2,g} \oplus 2E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$9n+\frac{5}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g}) \oplus n\,(2E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$9n+\frac{7}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g} \oplus E_{7/2,g}) \oplus n\,({}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$9n+\frac{9}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g} \oplus E_{7/2,g}) \oplus (n+1)({}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$9n+\frac{11}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g} \oplus E_{5/2,g}) \oplus (n+1)(2E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$9n+\frac{13}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{3/2,g}) \oplus (n+1)(2E_{5/2,g} \oplus 2E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$9n+\frac{15}{2}$	$(2n+1)\,E_{1/2,g} \oplus (n+1)(2E_{3/2,g} \oplus 2E_{5/2,g} \oplus 2E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$9n+\frac{17}{2}$	$(n+1)(2E_{1/2,g} \oplus 2E_{3/2,g} \oplus 2E_{5/2,g} \oplus 2E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$

$n = 0, 1, 2, \ldots$

T **48**.11 Clebsch–Gordan coefficients

Use T **29**.11 ▪. § **16**–11, p. 83

(1) Product forms: $\mathbf{D}_{10} \otimes \mathbf{C}_s$.

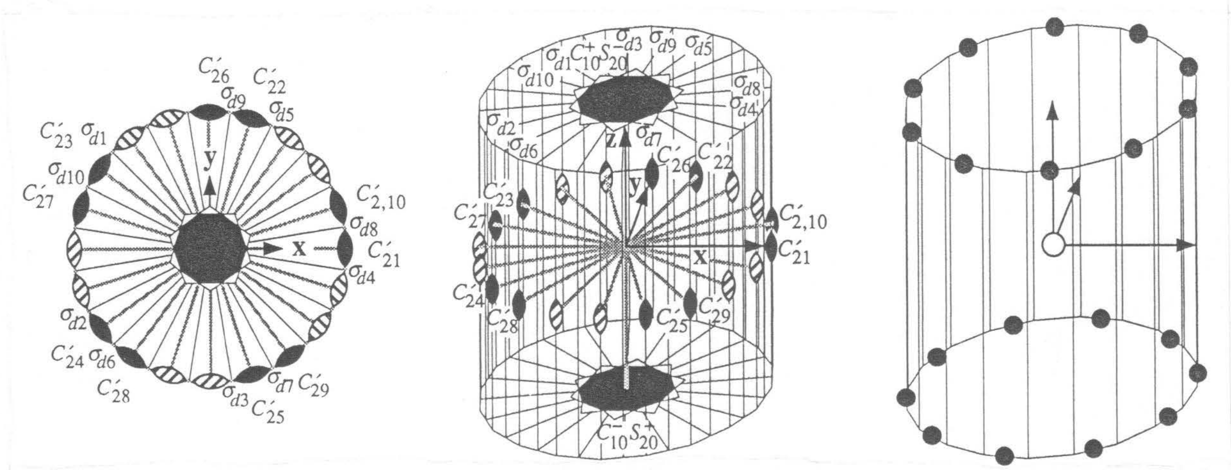
(2) Group chains: $\mathbf{D}_{10d} \supset (\underline{\mathbf{C}_{10v}})$, $\mathbf{D}_{10d} \supset (\mathbf{D}_{2d})$, $\mathbf{D}_{10d} \supset (\underline{\mathbf{D}_{10}})$, $\mathbf{D}_{10d} \supset \underline{\mathbf{S}_{20}}$.

(3) Operations of G: E, (C_{10}^{+}, C_{10}^{-}), (C_5^{+}, C_5^{-}), $(C_{10}^{3+}, C_{10}^{3-})$, (C_5^{2+}, C_5^{2-}), C_2,
$(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27}, C'_{28}, C'_{29}, C'_{2,10})$,
$(S_{20}^{9-}, S_{20}^{9+})$, $(S_{20}^{7-}, S_{20}^{7+})$, (S_4^{-}, S_4^{+}), $(S_{20}^{3-}, S_{20}^{3+})$, (S_{20}^{-}, S_{20}^{+}),
$(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \sigma_{d6}, \sigma_{d7}, \sigma_{d8}, \sigma_{d9}, \sigma_{d10})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_{10}^{+}, C_{10}^{-}), $(\widetilde{C}_{10}^{+}, \widetilde{C}_{10}^{-})$, (C_5^{+}, C_5^{-}), $(\widetilde{C}_5^{+}, \widetilde{C}_5^{-})$,
$(C_{10}^{3+}, C_{10}^{3-})$, $(\widetilde{C}_{10}^{3+}, \widetilde{C}_{10}^{3-})$, (C_5^{2+}, C_5^{2-}), $(\widetilde{C}_5^{2+}, \widetilde{C}_5^{2-})$, $(C_2, \widetilde{C}_2)$,
$(C'_{21}, C'_{22}, C'_{23}, C'_{24}, C'_{25}, C'_{26}, C'_{27}, C'_{28}, C'_{29}, C'_{2,10},$
$\widetilde{C}'_{21}, \widetilde{C}'_{22}, \widetilde{C}'_{23}, \widetilde{C}'_{24}, \widetilde{C}'_{25}, \widetilde{C}'_{26}, \widetilde{C}'_{27}, \widetilde{C}'_{28}, \widetilde{C}'_{29}, \widetilde{C}'_{2,10})$,
$(S_{20}^{9-}, S_{20}^{9+})$, $(\widetilde{S}_{20}^{9-}, \widetilde{S}_{20}^{9+})$, $(S_{20}^{7-}, S_{20}^{7+})$, $(\widetilde{S}_{20}^{7-}, \widetilde{S}_{20}^{7+})$, (S_4^{-}, S_4^{+}),
$(\widetilde{S}_4^{-}, \widetilde{S}_4^{+})$, $(S_{20}^{3-}, S_{20}^{3+})$, $(\widetilde{S}_{20}^{3-}, \widetilde{S}_{20}^{3+})$, (S_{20}^{-}, S_{20}^{+}), $(\widetilde{S}_{20}^{3-}, \widetilde{S}_{20}^{3+})$,
$(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \sigma_{d6}, \sigma_{d7}, \sigma_{d8}, \sigma_{d9}, \sigma_{d10},$
$\widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3}, \widetilde{\sigma}_{d4}, \widetilde{\sigma}_{d5}, \widetilde{\sigma}_{d6}, \widetilde{\sigma}_{d7}, \widetilde{\sigma}_{d8}, \widetilde{\sigma}_{d9}, \widetilde{\sigma}_{d10})$.

(5) Classes and representations: $|r| = 10$, $|\mathrm{i}| = 3$, $|I| = 13$, $|\widetilde{I}| = 10$.

F **49** See Chapter **15**, p. 65



Examples:

T **49**.0 Subgroup elements

§ **16**–0, p. 68

$\mathbf{D}_{10d}$	$\mathbf{D}_{2d}$	$\mathbf{D}_{10}$	$\mathbf{D}_{5}$	$\mathbf{D}_{2}$	$\mathbf{S}_{20}$	$\mathbf{S}_{4}$	$\mathbf{C}_{10}$	$\mathbf{C}_{5}$	$\mathbf{C}_{2}$
E	E	E	E	E	E	E	E	E	E
C_{10}^{+}		C_{10}^{+}			C_{10}^{+}		C_{10}^{+}		
C_{10}^{-}		C_{10}^{-}			C_{10}^{-}		C_{10}^{-}		
C_{5}^{+}		C_{5}^{+}	C_{5}^{+}		C_{5}^{+}		C_{5}^{+}	C_{5}^{+}	
C_{5}^{-}		C_{5}^{-}	C_{5}^{-}		C_{5}^{-}		C_{5}^{-}	C_{5}^{-}	
C_{10}^{3+}		C_{10}^{3+}			C_{10}^{3+}		C_{10}^{3+}		
C_{10}^{3-}		C_{10}^{3-}			C_{10}^{3-}		C_{10}^{3-}		
C_{5}^{2+}		C_{5}^{2+}	C_{5}^{2+}		C_{5}^{2+}		C_{5}^{2+}	C_{5}^{2+}	
C_{5}^{2-}		C_{5}^{2-}	C_{5}^{2-}		C_{5}^{2-}		C_{5}^{2-}	C_{5}^{2-}	
C_{2}	C_{2}	C_{2}		C_{2z}	C_{2}	C_{2}	C_{2}		C_{2}
C'_{21}	C'_{21}	C'_{21}	C'_{21}	C_{2x}					
C'_{22}		C'_{22}	C'_{22}						
C'_{23}		C'_{23}	C'_{23}						
C'_{24}		C'_{24}	C'_{24}						
C'_{25}		C'_{25}	C'_{25}						
C'_{26}	C'_{22}	C''_{21}		C_{2y}					
C'_{27}		C''_{22}							
C'_{28}		C''_{23}							
C'_{29}		C''_{24}							
$C'_{2,10}$		C''_{25}							
S_{20}^{9-}					S_{20}^{9-}				
S_{20}^{9+}					S_{20}^{9+}				
S_{20}^{7-}					S_{20}^{7-}				
S_{20}^{7+}					S_{20}^{7+}				
S_{4}^{-}	S_{4}^{-}				S_{4}^{-}	S_{4}^{-}			
S_{4}^{+}	S_{4}^{+}				S_{4}^{+}	S_{4}^{+}			
S_{20}^{3-}					S_{20}^{3-}				
S_{20}^{3+}					S_{20}^{3+}				
S_{20}^{-}					S_{20}^{-}				
S_{20}^{+}					S_{20}^{+}				
σ_{d1}	σ_{d1}								
σ_{d2}									
σ_{d3}									
σ_{d4}									
σ_{d5}									
σ_{d6}	σ_{d2}								
σ_{d7}									
σ_{d8}									
σ_{d9}									
σ_{d10}									

T **49**.1 Parameters

§ **16**–1, p. 68

$\mathbf{D}_{10d}$	α	β	γ	ϕ	$\mathbf{n}$	λ	$\boldsymbol{\Lambda}$
E	0	0	0	0	$(0\ \ 0\ \ 0)$	$[\![\ 1,$	$(0\ \ 0\ \ 0)]\!]$
C_{10}^{+}	0	0	$\frac{\pi}{5}$	$\frac{\pi}{5}$	$(0\ \ 0\ \ 1)$	$[\![c_{10},$	$(0\ \ 0\ \ s_{10})]\!]$
C_{10}^{-}	0	0	$-\frac{\pi}{5}$	$\frac{\pi}{5}$	$(0\ \ 0\ {-1})$	$[\![c_{10},$	$(0\ \ 0\ {-s_{10}})]\!]$
C_{5}^{+}	0	0	$\frac{2\pi}{5}$	$\frac{2\pi}{5}$	$(0\ \ 0\ \ 1)$	$[\![\ c_{5},$	$(0\ \ 0\ \ s_{5})]\!]$
C_{5}^{-}	0	0	$-\frac{2\pi}{5}$	$\frac{2\pi}{5}$	$(0\ \ 0\ {-1})$	$[\![\ c_{5},$	$(0\ \ 0\ {-s_{5}})]\!]$
C_{10}^{3+}	0	0	$\frac{3\pi}{5}$	$\frac{3\pi}{5}$	$(0\ \ 0\ \ 1)$	$[\![\ s_{5},$	$(0\ \ 0\ \ c_{5})]\!]$
C_{10}^{3-}	0	0	$-\frac{3\pi}{5}$	$\frac{3\pi}{5}$	$(0\ \ 0\ {-1})$	$[\![\ s_{5},$	$(0\ \ 0\ {-c_{5}})]\!]$
C_{5}^{2+}	0	0	$\frac{4\pi}{5}$	$\frac{4\pi}{5}$	$(0\ \ 0\ \ 1)$	$[\![s_{10},$	$(0\ \ 0\ \ c_{10})]\!]$
C_{5}^{2-}	0	0	$-\frac{4\pi}{5}$	$\frac{4\pi}{5}$	$(0\ \ 0\ {-1})$	$[\![s_{10},$	$(0\ \ 0\ {-c_{10}})]\!]$
C_{2}	0	0	π	π	$(0\ \ 0\ \ 1)$	$[\![\ 0,$	$(0\ \ 0\ \ 1)]\!]$
C_{21}'	0	π	π	π	$(1\ \ 0\ \ 0)$	$[\![\ 0,$	$(1\ \ 0\ \ 0)]\!]$
C_{22}'	0	π	$\frac{\pi}{5}$	π	$(s_{10}\ \ c_{10}\ \ 0)$	$[\![\ 0,$	$(s_{10}\ \ c_{10}\ \ 0)]\!]$
C_{23}'	0	π	$-\frac{3\pi}{5}$	π	$(-c_{5}\ \ s_{5}\ \ 0)$	$[\![\ 0,$	$(-c_{5}\ \ s_{5}\ \ 0)]\!]$
C_{24}'	0	π	$\frac{3\pi}{5}$	π	$(-c_{5}\ {-s_{5}}\ \ 0)$	$[\![\ 0,$	$(-c_{5}\ {-s_{5}}\ \ 0)]\!]$
C_{25}'	0	π	$-\frac{\pi}{5}$	π	$(s_{10}\ {-c_{10}}\ \ 0)$	$[\![\ 0,$	$(s_{10}\ {-c_{10}}\ \ 0)]\!]$
C_{26}'	0	π	0	π	$(0\ \ 1\ \ 0)$	$[\![\ 0,$	$(0\ \ 1\ \ 0)]\!]$
C_{27}'	0	π	$-\frac{4\pi}{5}$	π	$(-c_{10}\ \ s_{10}\ \ 0)$	$[\![\ 0,$	$(-c_{10}\ \ s_{10}\ \ 0)]\!]$
C_{28}'	0	π	$\frac{2\pi}{5}$	π	$(-s_{5}\ {-c_{5}}\ \ 0)$	$[\![\ 0,$	$(-s_{5}\ {-c_{5}}\ \ 0)]\!]$
C_{29}'	0	π	$-\frac{2\pi}{5}$	π	$(s_{5}\ {-c_{5}}\ \ 0)$	$[\![\ 0,$	$(s_{5}\ {-c_{5}}\ \ 0)]\!]$
$C_{2,10}'$	0	π	$\frac{4\pi}{5}$	π	$(c_{10}\ \ s_{10}\ \ 0)$	$[\![\ 0,$	$(c_{10}\ \ s_{10}\ \ 0)]\!]$
S_{20}^{9-}	0	0	$\frac{\pi}{10}$	$\frac{\pi}{10}$	$(0\ \ 0\ \ 1)$	$[\![c_{20},$	$(0\ \ 0\ \ s_{20})]\!]$
S_{20}^{9+}	0	0	$-\frac{\pi}{10}$	$\frac{\pi}{10}$	$(0\ \ 0\ {-1})$	$[\![c_{20},$	$(0\ \ 0\ {-s_{20}})]\!]$
S_{20}^{7-}	0	0	$\frac{3\pi}{10}$	$\frac{3\pi}{10}$	$(0\ \ 0\ \ 1)$	$[\![c_{20}^{3},$	$(0\ \ 0\ \ s_{20}^{3})]\!]$
S_{20}^{7+}	0	0	$-\frac{3\pi}{10}$	$\frac{3\pi}{10}$	$(0\ \ 0\ {-1})$	$[\![c_{20}^{3},$	$(0\ \ 0\ {-s_{20}^{3}})]\!]$
S_{4}^{-}	0	0	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$(0\ \ 0\ \ 1)$	$[\![\frac{1}{\sqrt{2}},$	$(0\ \ 0\ \ \frac{1}{\sqrt{2}})]\!]$
S_{4}^{+}	0	0	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	$(0\ \ 0\ {-1})$	$[\![\frac{1}{\sqrt{2}},$	$(0\ \ 0\ {-\frac{1}{\sqrt{2}}})]\!]$
S_{20}^{3-}	0	0	$\frac{7\pi}{10}$	$\frac{7\pi}{10}$	$(0\ \ 0\ \ 1)$	$[\![s_{20}^{3},$	$(0\ \ 0\ \ c_{20}^{3})]\!]$
S_{20}^{3+}	0	0	$-\frac{7\pi}{10}$	$\frac{7\pi}{10}$	$(0\ \ 0\ {-1})$	$[\![s_{20}^{3},$	$(0\ \ 0\ {-c_{20}^{3}})]\!]$
S_{20}^{-}	0	0	$\frac{9\pi}{10}$	$\frac{9\pi}{10}$	$(0\ \ 0\ \ 1)$	$[\![s_{20},$	$(0\ \ 0\ \ c_{20})]\!]$
S_{20}^{+}	0	0	$-\frac{9\pi}{10}$	$\frac{9\pi}{10}$	$(0\ \ 0\ {-1})$	$[\![s_{20},$	$(0\ \ 0\ {-c_{20}})]\!]$
σ_{d1}	0	π	$\frac{\pi}{2}$	π	$(\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)$	$[\![\ 0,$	$(\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)]\!]$
σ_{d2}	0	π	$-\frac{3\pi}{10}$	π	$(-s_{20}^{3}\ \ c_{20}^{3}\ \ 0)$	$[\![\ 0,$	$(-s_{20}^{3}\ \ c_{20}^{3}\ \ 0)]\!]$
σ_{d3}	0	π	$\frac{9\pi}{10}$	π	$(-c_{20}\ {-s_{20}}\ \ 0)$	$[\![\ 0,$	$(-c_{20}\ {-s_{20}}\ \ 0)]\!]$
σ_{d4}	0	π	$\frac{\pi}{10}$	π	$(-s_{20}\ {-c_{20}}\ \ 0)$	$[\![\ 0,$	$(-s_{20}\ {-c_{20}}\ \ 0)]\!]$
σ_{d5}	0	π	$-\frac{7\pi}{10}$	π	$(c_{20}^{3}\ {-s_{20}^{3}}\ \ 0)$	$[\![\ 0,$	$(c_{20}^{3}\ {-s_{20}^{3}}\ \ 0)]\!]$
σ_{d6}	0	π	$-\frac{\pi}{2}$	π	$(-\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)$	$[\![\ 0,$	$(-\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)]\!]$
σ_{d7}	0	π	$\frac{7\pi}{10}$	π	$(-c_{20}^{3}\ {-s_{20}^{3}}\ \ 0)$	$[\![\ 0,$	$(-c_{20}^{3}\ {-s_{20}^{3}}\ \ 0)]\!]$
σ_{d8}	0	π	$-\frac{\pi}{10}$	π	$(s_{20}\ {-c_{20}}\ \ 0)$	$[\![\ 0,$	$(s_{20}\ {-c_{20}}\ \ 0)]\!]$
σ_{d9}	0	π	$-\frac{9\pi}{10}$	π	$(c_{20}\ {-s_{20}}\ \ 0)$	$[\![\ 0,$	$(c_{20}\ {-s_{20}}\ \ 0)]\!]$
σ_{d10}	0	π	$\frac{3\pi}{10}$	π	$(s_{20}^{3}\ \ c_{20}^{3}\ \ 0)$	$[\![\ 0,$	$(s_{20}^{3}\ \ c_{20}^{3}\ \ 0)]\!]$

$c_n^m = \cos\frac{m}{n}\pi$, $s_n^m = \sin\frac{m}{n}\pi$

T **49**.2 Multiplication table

§ **16**–2, p. 69

$\mathbf{D}_{10d}$	E	C_{10}^{+}	C_{10}^{-}	C_{5}^{+}	C_{5}^{-}	C_{10}^{3+}	C_{10}^{3-}	C_{5}^{2+}	C_{5}^{2-}	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}	C'_{20}
E	E	C_{10}^{+}	C_{10}^{-}	C_{5}^{+}	C_{5}^{-}	C_{10}^{3+}	C_{10}^{3-}	C_{5}^{2+}	C_{5}^{2-}	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}	C'_{20}
C_{10}^{+}	C_{10}^{+}	C_{5}^{+}	E	C_{10}^{3+}	C_{10}^{-}	C_{5}^{2+}	C_{5}^{-}	C_2	C_{10}^{3-}	C_{5}^{2-}	C'_{20}	C'_{26}	C'_{27}	C'_{28}	C'_{29}	C'_{25}	C'_{21}	C'_{22}	C'_{23}	C'_{24}
C_{10}^{-}	C_{10}^{-}	E	C_{5}^{-}	C_{10}^{+}	C_{10}^{3-}	C_{5}^{+}	C_{5}^{2-}	C_{10}^{3+}	C_2	C_{5}^{2+}	C'_{27}	C'_{28}	C'_{29}	C'_{20}	C'_{26}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{21}
C_{5}^{+}	C_{5}^{+}	C_{10}^{3+}	C_{10}^{+}	C_{5}^{2+}	E	C_2	C_{10}^{-}	C_{5}^{2-}	C_{5}^{-}	C_{10}^{3-}	C'_{24}	C'_{25}	C'_{21}	C'_{22}	C'_{23}	C'_{29}	C'_{20}	C'_{26}	C'_{27}	C'_{28}
C_{5}^{-}	C_{5}^{-}	C_{10}^{-}	C_{10}^{3-}	E	C_{5}^{2-}	C_{10}^{+}	C_2	C_{5}^{+}	C_{5}^{2+}	C_{10}^{3+}	C'_{23}	C'_{24}	C'_{25}	C'_{21}	C'_{22}	C'_{28}	C'_{29}	C'_{20}	C'_{26}	C'_{27}
C_{10}^{3+}	C_{10}^{3+}	C_{5}^{2+}	C_{5}^{+}	C_2	C_{10}^{+}	C_{5}^{2-}	E	C_{10}^{3-}	C_{10}^{-}	C_{5}^{-}	C'_{28}	C'_{29}	C'_{20}	C'_{26}	C'_{27}	C'_{23}	C'_{24}	C'_{25}	C'_{21}	C'_{22}
C_{10}^{3-}	C_{10}^{3-}	C_{5}^{-}	C_{5}^{2-}	C_{10}^{-}	C_2	E	C_{5}^{2+}	C_{10}^{+}	C_{10}^{3+}	C_{5}^{+}	C'_{29}	C'_{20}	C'_{26}	C'_{27}	C'_{28}	C'_{24}	C'_{25}	C'_{21}	C'_{22}	C'_{23}
C_{5}^{2+}	C_{5}^{2+}	C_2	C_{10}^{3+}	C_{5}^{2-}	C_{5}^{+}	C_{10}^{3-}	C_{10}^{+}	C_{5}^{-}	E	C_{10}^{-}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{21}	C'_{27}	C'_{28}	C'_{29}	C'_{20}	C'_{26}
C_{5}^{2-}	C_{5}^{2-}	C_{10}^{3-}	C_2	C_{5}^{-}	C_{5}^{2+}	C_{10}^{-}	C_{10}^{3+}	E	C_{5}^{+}	C_{10}^{+}	C'_{25}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{20}	C'_{26}	C'_{27}	C'_{28}	C'_{29}
C_2	C_2	C_{5}^{2-}	C_{5}^{2+}	C_{10}^{3-}	C_{10}^{3+}	C_{5}^{-}	C_{5}^{+}	C_{10}^{-}	C_{10}^{+}	E	C'_{26}	C'_{27}	C'_{28}	C'_{29}	C'_{20}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}
C'_{21}	C'_{21}	C'_{27}	C'_{20}	C'_{23}	C'_{24}	C'_{29}	C'_{28}	C'_{25}	C'_{22}	C'_{26}	E	C_{5}^{2-}	C_{5}^{+}	C_{5}^{-}	C_{5}^{2+}	C_2	C_{10}^{+}	C_{10}^{3-}	C_{10}^{3+}	C_{10}^{-}
C'_{22}	C'_{22}	C'_{28}	C'_{26}	C'_{24}	C'_{25}	C'_{20}	C'_{29}	C'_{21}	C'_{23}	C'_{27}	C_{5}^{2+}	E	C_{5}^{2-}	C_{5}^{+}	C_{5}^{-}	C_{10}^{-}	C_2	C_{10}^{+}	C_{10}^{3-}	C_{10}^{3+}
C'_{23}	C'_{23}	C'_{29}	C'_{27}	C'_{25}	C'_{21}	C'_{26}	C'_{20}	C'_{22}	C'_{24}	C'_{28}	C_{5}^{-}	C_{5}^{2+}	E	C_{5}^{2-}	C_{5}^{+}	C_{10}^{3+}	C_{10}^{-}	C_2	C_{10}^{+}	C_{10}^{3-}
C'_{24}	C'_{24}	C'_{20}	C'_{28}	C'_{21}	C'_{22}	C'_{27}	C'_{26}	C'_{23}	C'_{25}	C'_{29}	C_{5}^{+}	C_{5}^{-}	C_{5}^{2+}	E	C_{5}^{2-}	C_{10}^{3-}	C_{10}^{3+}	C_{10}^{-}	C_2	C_{10}^{+}
C'_{25}	C'_{25}	C'_{26}	C'_{29}	C'_{22}	C'_{23}	C'_{28}	C'_{27}	C'_{24}	C'_{21}	C'_{20}	C_{5}^{2-}	C_{5}^{+}	C_{5}^{-}	C_{5}^{2+}	E	C_{10}^{+}	C_{10}^{3-}	C_{10}^{3+}	C_{10}^{-}	C_2
C'_{26}	C'_{26}	C'_{22}	C'_{25}	C'_{28}	C'_{29}	C'_{24}	C'_{23}	C'_{20}	C'_{27}	C'_{21}	C_2	C_{10}^{+}	C_{10}^{3-}	C_{10}^{3+}	C_{10}^{-}	E	C_{5}^{2-}	C_{5}^{+}	C_{5}^{-}	C_{5}^{2+}
C'_{27}	C'_{27}	C'_{23}	C'_{21}	C'_{29}	C'_{20}	C'_{25}	C'_{24}	C'_{26}	C'_{28}	C'_{22}	C_{10}^{-}	C_2	C_{10}^{+}	C_{10}^{3-}	C_{10}^{3+}	C_{5}^{2+}	E	C_{5}^{2-}	C_{5}^{+}	C_{5}^{-}
C'_{28}	C'_{28}	C'_{24}	C'_{22}	C'_{20}	C'_{26}	C'_{21}	C'_{25}	C'_{27}	C'_{29}	C'_{23}	C_{10}^{3+}	C_{10}^{-}	C_2	C_{10}^{+}	C_{10}^{3-}	C_{5}^{-}	C_{5}^{2+}	E	C_{5}^{2-}	C_{5}^{+}
C'_{29}	C'_{29}	C'_{25}	C'_{23}	C'_{26}	C'_{27}	C'_{22}	C'_{21}	C'_{28}	C'_{20}	C'_{24}	C_{10}^{3-}	C_{10}^{3+}	C_{10}^{-}	C_2	C_{10}^{+}	C_{5}^{+}	C_{5}^{-}	C_{5}^{2+}	E	C_{5}^{2-}
C'_{20}	C'_{20}	C'_{21}	C'_{24}	C'_{27}	C'_{28}	C'_{23}	C'_{22}	C'_{29}	C'_{26}	C'_{25}	C_{10}^{+}	C_{10}^{3-}	C_{10}^{3+}	C_{10}^{-}	C_2	C_{5}^{2-}	C_{5}^{+}	C_{5}^{-}	C_{5}^{2+}	E
S_{20}^{9-}	S_{20}^{9-}	S_{20}^{7-}	S_{20}^{9+}	S_{4}^{-}	S_{20}^{7+}	S_{20}^{3-}	S_{4}^{+}	S_{20}^{-}	S_{20}^{3+}	S_{20}^{+}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d8}	σ_{d9}	σ_{d10}	σ_{d6}	σ_{d7}
S_{20}^{9+}	S_{20}^{9+}	S_{20}^{9-}	S_{20}^{7+}	S_{20}^{7-}	S_{4}^{+}	S_{4}^{-}	S_{20}^{3+}	S_{20}^{3-}	S_{20}^{+}	S_{20}^{-}	σ_{d9}	σ_{d10}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}
S_{20}^{7-}	S_{20}^{7-}	S_{4}^{-}	S_{20}^{9-}	S_{20}^{3-}	S_{20}^{9+}	S_{20}^{-}	S_{20}^{7+}	S_{20}^{+}	S_{4}^{+}	S_{20}^{3+}	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d10}	σ_{d6}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}
S_{20}^{7+}	S_{20}^{7+}	S_{20}^{9+}	S_{4}^{+}	S_{20}^{9-}	S_{20}^{3+}	S_{20}^{7-}	S_{20}^{+}	S_{4}^{-}	S_{20}^{-}	S_{20}^{3-}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d10}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}
S_{4}^{-}	S_{4}^{-}	S_{20}^{3-}	S_{20}^{7-}	S_{20}^{-}	S_{20}^{9-}	S_{20}^{+}	S_{20}^{9+}	S_{20}^{3+}	S_{20}^{7+}	S_{4}^{+}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d10}
S_{4}^{+}	S_{4}^{+}	S_{20}^{7+}	S_{20}^{3+}	S_{20}^{9+}	S_{20}^{+}	S_{20}^{9-}	S_{20}^{-}	S_{20}^{7-}	S_{20}^{3-}	S_{4}^{-}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d10}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}
S_{20}^{3-}	S_{20}^{3-}	S_{20}^{-}	S_{4}^{-}	S_{20}^{+}	S_{20}^{7-}	S_{20}^{3+}	S_{20}^{9-}	S_{4}^{+}	S_{20}^{9+}	S_{20}^{7+}	σ_{d10}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}
S_{20}^{3+}	S_{20}^{3+}	S_{4}^{+}	S_{20}^{+}	S_{20}^{7+}	S_{20}^{-}	S_{20}^{9+}	S_{20}^{3-}	S_{20}^{9-}	S_{4}^{-}	S_{20}^{7-}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d10}	σ_{d6}
S_{20}^{-}	S_{20}^{-}	S_{20}^{+}	S_{20}^{3-}	S_{20}^{3+}	S_{4}^{-}	S_{4}^{+}	S_{20}^{7-}	S_{20}^{7+}	S_{20}^{9-}	S_{20}^{9+}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d9}	σ_{d10}	σ_{d6}	σ_{d7}	σ_{d8}
S_{20}^{+}	S_{20}^{+}	S_{20}^{3+}	S_{20}^{-}	S_{4}^{+}	S_{20}^{3-}	S_{20}^{7+}	S_{4}^{-}	S_{20}^{9+}	S_{20}^{7-}	S_{20}^{9-}	σ_{d8}	σ_{d9}	σ_{d10}	σ_{d6}	σ_{d7}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}
σ_{d1}	σ_{d1}	σ_{d7}	σ_{d10}	σ_{d3}	σ_{d4}	σ_{d9}	σ_{d8}	σ_{d5}	σ_{d2}	σ_{d6}	S_{4}^{-}	S_{20}^{7+}	S_{20}^{-}	S_{20}^{9-}	S_{20}^{3+}	S_{4}^{+}	S_{20}^{3-}	S_{20}^{9+}	S_{20}^{+}	S_{20}^{7-}
σ_{d2}	σ_{d2}	σ_{d8}	σ_{d6}	σ_{d4}	σ_{d5}	σ_{d10}	σ_{d9}	σ_{d1}	σ_{d3}	σ_{d7}	S_{20}^{3+}	S_{4}^{-}	S_{20}^{7+}	S_{20}^{-}	S_{20}^{9-}	S_{20}^{7-}	S_{4}^{+}	S_{20}^{3-}	S_{20}^{9+}	S_{20}^{+}
σ_{d3}	σ_{d3}	σ_{d9}	σ_{d7}	σ_{d5}	σ_{d1}	σ_{d6}	σ_{d10}	σ_{d2}	σ_{d4}	σ_{d8}	S_{20}^{9-}	S_{20}^{3+}	S_{4}^{-}	S_{20}^{7+}	S_{20}^{-}	S_{20}^{+}	S_{20}^{7-}	S_{4}^{+}	S_{20}^{3-}	S_{20}^{9+}
σ_{d4}	σ_{d4}	σ_{d10}	σ_{d8}	σ_{d1}	σ_{d2}	σ_{d7}	σ_{d6}	σ_{d3}	σ_{d5}	σ_{d9}	S_{20}^{-}	S_{20}^{9-}	S_{20}^{3+}	S_{4}^{-}	S_{20}^{7+}	S_{20}^{9+}	S_{20}^{+}	S_{20}^{7-}	S_{4}^{+}	S_{20}^{3-}
σ_{d5}	σ_{d5}	σ_{d6}	σ_{d9}	σ_{d2}	σ_{d3}	σ_{d8}	σ_{d7}	σ_{d4}	σ_{d1}	σ_{d10}	S_{20}^{7+}	S_{20}^{-}	S_{20}^{9-}	S_{20}^{3+}	S_{4}^{-}	S_{20}^{3-}	S_{20}^{9+}	S_{20}^{+}	S_{20}^{7-}	S_{4}^{+}
σ_{d6}	σ_{d6}	σ_{d2}	σ_{d5}	σ_{d8}	σ_{d9}	σ_{d4}	σ_{d3}	σ_{d10}	σ_{d7}	σ_{d1}	S_{4}^{+}	S_{20}^{3-}	S_{20}^{9+}	S_{20}^{+}	S_{20}^{7-}	S_{4}^{-}	S_{20}^{7+}	S_{20}^{-}	S_{20}^{9-}	S_{20}^{3+}
σ_{d7}	σ_{d7}	σ_{d3}	σ_{d1}	σ_{d9}	σ_{d10}	σ_{d5}	σ_{d4}	σ_{d6}	σ_{d8}	σ_{d2}	S_{20}^{7-}	S_{4}^{+}	S_{20}^{3-}	S_{20}^{9+}	S_{20}^{+}	S_{20}^{3+}	S_{4}^{-}	S_{20}^{7+}	S_{20}^{-}	S_{20}^{9-}
σ_{d8}	σ_{d8}	σ_{d4}	σ_{d2}	σ_{d10}	σ_{d6}	σ_{d1}	σ_{d5}	σ_{d7}	σ_{d9}	σ_{d3}	S_{20}^{+}	S_{20}^{7-}	S_{4}^{+}	S_{20}^{3-}	S_{20}^{9+}	S_{20}^{9-}	S_{20}^{3+}	S_{4}^{-}	S_{20}^{7+}	S_{20}^{-}
σ_{d9}	σ_{d9}	σ_{d5}	σ_{d3}	σ_{d6}	σ_{d7}	σ_{d2}	σ_{d1}	σ_{d8}	σ_{d10}	σ_{d4}	S_{20}^{9+}	S_{20}^{+}	S_{20}^{7-}	S_{4}^{+}	S_{20}^{3-}	S_{20}^{-}	S_{20}^{9-}	S_{20}^{3+}	S_{4}^{-}	S_{20}^{7+}
σ_{d10}	σ_{d10}	σ_{d1}	σ_{d4}	σ_{d7}	σ_{d8}	σ_{d3}	σ_{d2}	σ_{d9}	σ_{d6}	σ_{d5}	S_{20}^{3-}	S_{20}^{9+}	S_{20}^{+}	S_{20}^{7-}	S_{4}^{+}	S_{20}^{7+}	S_{20}^{-}	S_{20}^{9-}	S_{20}^{3+}	S_{4}^{-}

$C'_{20} \equiv C'_{2,10}$ $\twoheadrightarrow$

T **49**.2 Multiplication table *(cont.)*

$\mathbf{D}_{10d}$	S_{20}^{9-}	S_{20}^{9+}	S_{20}^{7-}	S_{20}^{7+}	S_4^-	S_4^+	S_{20}^{3-}	S_{20}^{3+}	S_{20}^-	S_{20}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d10}
E	S_{20}^{9-}	S_{20}^{9+}	S_{20}^{7-}	S_{20}^{7+}	S_4^-	S_4^+	S_{20}^{3-}	S_{20}^{3+}	S_{20}^-	S_{20}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d10}
C_{10}^+	S_{20}^{7-}	S_{20}^{9-}	S_4^-	S_{20}^{9+}	S_{20}^{3-}	S_{20}^{7+}	S_{20}^-	S_4^+	S_{20}^+	S_{20}^{3+}	σ_{d10}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}
C_{10}^-	S_{20}^{9+}	S_{20}^{7+}	S_{20}^{9-}	S_4^+	S_{20}^{7-}	S_{20}^{3+}	S_4^-	S_{20}^+	S_{20}^{3-}	S_{20}^-	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d10}	σ_{d6}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}
C_5^+	S_4^-	S_{20}^{7-}	S_{20}^{3-}	S_{20}^{9-}	S_{20}^-	S_{20}^{9+}	S_{20}^+	S_{20}^{7+}	S_{20}^{3+}	S_4^+	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d9}	σ_{d10}	σ_{d6}	σ_{d7}	σ_{d8}
C_5^-	S_{20}^{7+}	S_4^+	S_{20}^{9+}	S_{20}^{3+}	S_{20}^{9-}	S_{20}^+	S_{20}^{7-}	S_{20}^-	S_4^-	S_{20}^{3-}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d8}	σ_{d9}	σ_{d10}	σ_{d6}	σ_{d7}
C_{10}^{3+}	S_{20}^{3-}	S_4^-	S_{20}^-	S_{20}^{7-}	S_{20}^+	S_{20}^{9-}	S_{20}^{3+}	S_{20}^{9+}	S_4^+	S_{20}^{7+}	σ_{d8}	σ_{d9}	σ_{d10}	σ_{d6}	σ_{d7}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}
C_{10}^{3-}	S_4^+	S_{20}^{3+}	S_{20}^{7+}	S_{20}^+	S_{20}^{9+}	S_{20}^-	S_{20}^{9-}	S_{20}^{3-}	S_{20}^{7-}	S_4^-	σ_{d9}	σ_{d10}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}
C_5^{2+}	S_{20}^-	S_{20}^{3-}	S_{20}^+	S_4^-	S_{20}^{3+}	S_{20}^{7-}	S_4^+	S_{20}^{9-}	S_{20}^{7+}	S_{20}^{9+}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d1}	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d10}	σ_{d6}
C_5^{2-}	S_{20}^{3+}	S_{20}^+	S_4^+	S_{20}^-	S_{20}^{7+}	S_{20}^{3-}	S_{20}^{9+}	S_4^-	S_{20}^{9-}	S_{20}^{7-}	σ_{d5}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d10}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}
C_2	S_{20}^+	S_{20}^-	S_{20}^{3+}	S_{20}^{3-}	S_4^+	S_4^-	S_{20}^{7+}	S_{20}^{7-}	S_{20}^{9+}	S_{20}^{9-}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d10}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}
C'_{21}	σ_{d9}	σ_{d3}	σ_{d5}	σ_{d7}	σ_{d6}	σ_{d1}	σ_{d2}	σ_{d10}	σ_{d8}	σ_{d4}	S_4^+	S_{20}^{3-}	S_{20}^{9+}	S_{20}^+	S_{20}^{7-}	S_4^-	S_{20}^{7+}	S_{20}^-	S_{20}^{9-}	S_{20}^{3+}
C'_{22}	σ_{d10}	σ_{d4}	σ_{d1}	σ_{d8}	σ_{d7}	σ_{d2}	σ_{d3}	σ_{d6}	σ_{d9}	σ_{d5}	S_{20}^{7-}	S_4^+	S_{20}^{3-}	S_{20}^{9+}	S_{20}^+	S_{20}^{3+}	S_4^-	S_{20}^{7+}	S_{20}^-	S_{20}^{9-}
C'_{23}	σ_{d6}	σ_{d5}	σ_{d2}	σ_{d9}	σ_{d8}	σ_{d3}	σ_{d4}	σ_{d7}	σ_{d10}	σ_{d1}	S_{20}^+	S_{20}^{7-}	S_4^+	S_{20}^{3-}	S_{20}^{9+}	S_{20}^{9-}	S_{20}^{3+}	S_4^-	S_{20}^{7+}	S_{20}^-
C'_{24}	σ_{d7}	σ_{d1}	σ_{d3}	σ_{d10}	σ_{d9}	σ_{d4}	σ_{d5}	σ_{d8}	σ_{d6}	σ_{d2}	S_{20}^{9+}	S_{20}^+	S_{20}^{7-}	S_4^+	S_{20}^{3-}	S_{20}^-	S_{20}^{9-}	S_{20}^{3+}	S_4^-	S_{20}^{7+}
C'_{25}	σ_{d8}	σ_{d2}	σ_{d4}	σ_{d6}	σ_{d10}	σ_{d5}	σ_{d1}	σ_{d9}	σ_{d7}	σ_{d3}	S_{20}^{3-}	S_{20}^{9+}	S_{20}^+	S_{20}^{7-}	S_4^+	S_{20}^{7+}	S_{20}^-	S_{20}^{9-}	S_{20}^{3+}	S_4^-
C'_{26}	σ_{d4}	σ_{d8}	σ_{d10}	σ_{d2}	σ_{d1}	σ_{d6}	σ_{d7}	σ_{d5}	σ_{d3}	σ_{d9}	S_4^-	S_{20}^{7+}	S_{20}^-	S_{20}^{9-}	S_{20}^{3+}	S_4^+	S_{20}^{3-}	S_{20}^{9+}	S_{20}^+	S_{20}^{7-}
C'_{27}	σ_{d5}	σ_{d9}	σ_{d6}	σ_{d3}	σ_{d2}	σ_{d7}	σ_{d8}	σ_{d1}	σ_{d4}	σ_{d10}	S_{20}^{3+}	S_4^-	S_{20}^{7+}	S_{20}^-	S_{20}^{9-}	S_{20}^{7-}	S_4^+	S_{20}^{3-}	S_{20}^{9+}	S_{20}^+
C'_{28}	σ_{d1}	σ_{d10}	σ_{d7}	σ_{d4}	σ_{d3}	σ_{d8}	σ_{d9}	σ_{d2}	σ_{d5}	σ_{d6}	S_{20}^{9-}	S_{20}^{3+}	S_4^-	S_{20}^{7+}	S_{20}^-	S_{20}^+	S_{20}^{7-}	S_4^+	S_{20}^{3-}	S_{20}^{9+}
C'_{29}	σ_{d2}	σ_{d6}	σ_{d8}	σ_{d5}	σ_{d4}	σ_{d9}	σ_{d10}	σ_{d3}	σ_{d1}	σ_{d7}	S_{20}^-	S_{20}^{9-}	S_{20}^{3+}	S_4^-	S_{20}^{7+}	S_{20}^{9+}	S_{20}^+	S_{20}^{7-}	S_4^+	S_{20}^{3-}
C'_{20}	σ_{d3}	σ_{d7}	σ_{d9}	σ_{d1}	σ_{d5}	σ_{d10}	σ_{d6}	σ_{d4}	σ_{d2}	σ_{d8}	S_{20}^{7+}	S_{20}^-	S_{20}^{9-}	S_{20}^{3+}	S_4^-	S_{20}^{3-}	S_{20}^{9+}	S_{20}^+	S_{20}^{7-}	S_4^+
S_{20}^{9-}	C_{10}^+	E	C_5^+	C_{10}^-	C_{10}^{3+}	C_5^-	C_5^{2+}	C_{10}^{3-}	C_2	C_5^{2-}	C'_{28}	C'_{29}	C'_{20}	C'_{26}	C'_{27}	C'_{23}	C'_{24}	C'_{25}	C'_{21}	C'_{22}
S_{20}^{9+}	E	C_{10}^-	C_{10}^+	C_5^-	C_5^+	C_{10}^{3-}	C_{10}^{3+}	C_5^{2-}	C_5^{2+}	C_2	C'_{24}	C'_{25}	C'_{21}	C'_{22}	C'_{23}	C'_{29}	C'_{20}	C'_{26}	C'_{27}	C'_{28}
S_{20}^{7-}	C_5^+	C_{10}^+	C_{10}^{3+}	E	C_5^{2+}	C_{10}^-	C_2	C_5^-	C_5^{2-}	C_{10}^{3-}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{21}	C'_{27}	C'_{28}	C'_{29}	C'_{20}	C'_{26}
S_{20}^{7+}	C_{10}^-	C_5^-	E	C_{10}^{3-}	C_{10}^+	C_5^{2-}	C_5^+	C_2	C_{10}^{3+}	C_5^{2+}	C'_{20}	C'_{26}	C'_{27}	C'_{28}	C'_{29}	C'_{25}	C'_{21}	C'_{22}	C'_{23}	C'_{24}
S_4^-	C_{10}^{3+}	C_5^+	C_5^{2+}	C_{10}^+	C_2	E	C_5^{2-}	C_{10}^-	C_{10}^{3-}	C_5^-	C'_{26}	C'_{27}	C'_{28}	C'_{29}	C'_{20}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}
S_4^+	C_5^-	C_{10}^{3-}	C_{10}^-	C_5^{2-}	E	C_2	C_{10}^+	C_5^{2+}	C_5^+	C_{10}^{3+}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}	C'_{20}
S_{20}^{3-}	C_5^{2+}	C_{10}^{3+}	C_2	C_5^+	C_5^{2-}	C_{10}^+	C_{10}^{3-}	E	C_5^-	C_{10}^-	C'_{25}	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{20}	C'_{26}	C'_{27}	C'_{28}	C'_{29}
S_{20}^{3+}	C_{10}^{3-}	C_5^{2-}	C_5^-	C_2	C_{10}^-	C_5^{2+}	E	C_{10}^{3+}	C_{10}^+	C_5^+	C'_{27}	C'_{28}	C'_{29}	C'_{20}	C'_{26}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{21}
S_{20}^-	C_2	C_5^{2+}	C_5^{2-}	C_{10}^{3+}	C_{10}^{3-}	C_5^+	C_5^-	C_{10}^+	C_{10}^-	E	C'_{29}	C'_{20}	C'_{26}	C'_{27}	C'_{28}	C'_{24}	C'_{25}	C'_{21}	C'_{22}	C'_{23}
S_{20}^+	C_5^{2-}	C_2	C_{10}^{3-}	C_5^{2+}	C_5^-	C_{10}^{3+}	C_{10}^-	C_5^+	E	C_{10}^+	C'_{23}	C'_{24}	C'_{25}	C'_{21}	C'_{22}	C'_{28}	C'_{29}	C'_{20}	C'_{26}	C'_{27}
σ_{d1}	C'_{24}	C'_{28}	C'_{20}	C'_{22}	C'_{21}	C'_{26}	C'_{27}	C'_{25}	C'_{23}	C'_{29}	E	C_5^{2-}	C_5^+	C_5^-	C_5^{2+}	C_2	C_{10}^+	C_{10}^{3-}	C_{10}^{3+}	C_{10}^-
σ_{d2}	C'_{25}	C'_{29}	C'_{26}	C'_{23}	C'_{22}	C'_{27}	C'_{28}	C'_{21}	C'_{24}	C'_{20}	C_5^{2+}	E	C_5^{2-}	C_5^+	C_5^-	C_{10}^-	C_2	C_{10}^+	C_{10}^{3-}	C_{10}^{3+}
σ_{d3}	C'_{21}	C'_{20}	C'_{27}	C'_{24}	C'_{23}	C'_{28}	C'_{29}	C'_{22}	C'_{25}	C'_{26}	C_5^-	C_5^{2+}	E	C_5^{2-}	C_5^+	C_{10}^{3+}	C_{10}^-	C_2	C_{10}^+	C_{10}^{3-}
σ_{d4}	C'_{22}	C'_{26}	C'_{28}	C'_{25}	C'_{24}	C'_{29}	C'_{20}	C'_{23}	C'_{21}	C'_{27}	C_5^+	C_5^-	C_5^{2+}	E	C_5^{2-}	C_{10}^{3-}	C_{10}^{3+}	C_{10}^-	C_2	C_{10}^+
σ_{d5}	C'_{23}	C'_{27}	C'_{29}	C'_{21}	C'_{25}	C'_{20}	C'_{26}	C'_{24}	C'_{22}	C'_{28}	C_5^{2-}	C_5^+	C_5^-	C_5^{2+}	E	C_{10}^+	C_{10}^{3-}	C_{10}^{3+}	C_{10}^-	C_2
σ_{d6}	C'_{29}	C'_{23}	C'_{25}	C'_{27}	C'_{26}	C'_{21}	C'_{22}	C'_{20}	C'_{28}	C'_{24}	C_2	C_{10}^+	C_{10}^{3-}	C_{10}^{3+}	C_{10}^-	E	C_5^{2-}	C_5^+	C_5^-	C_5^{2+}
σ_{d7}	C'_{20}	C'_{24}	C'_{21}	C'_{28}	C'_{27}	C'_{22}	C'_{23}	C'_{26}	C'_{29}	C'_{25}	C_{10}^-	C_2	C_{10}^+	C_{10}^{3-}	C_{10}^{3+}	C_5^{2+}	E	C_5^{2-}	C_5^+	C_5^-
σ_{d8}	C'_{26}	C'_{25}	C'_{22}	C'_{29}	C'_{28}	C'_{23}	C'_{24}	C'_{27}	C'_{20}	C'_{21}	C_{10}^{3+}	C_{10}^-	C_2	C_{10}^+	C_{10}^{3-}	C_5^-	C_5^{2+}	E	C_5^{2-}	C_5^+
σ_{d9}	C'_{27}	C'_{21}	C'_{23}	C'_{20}	C'_{29}	C'_{24}	C'_{25}	C'_{28}	C'_{26}	C'_{22}	C_{10}^{3-}	C_{10}^{3+}	C_{10}^-	C_2	C_{10}^+	C_5^+	C_5^-	C_5^{2+}	E	C_5^{2-}
σ_{d10}	C'_{28}	C'_{22}	C'_{24}	C'_{26}	C'_{20}	C'_{25}	C'_{21}	C'_{29}	C'_{27}	C'_{23}	C_{10}^+	C_{10}^{3-}	C_{10}^{3+}	C_{10}^-	C_2	C_5^{2-}	C_5^+	C_5^-	C_5^{2+}	E

$C'_{20} \equiv C'_{2,10}$

T **49**.3 Factor table

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$\mathbf{D}_{10d}$	E	C_{10}^{+}	C_{10}^{-}	C_5^{+}	C_5^{-}	C_{10}^{3+}	C_{10}^{3-}	C_5^{2+}	C_5^{2-}	C_2	C'_{21}	C'_{22}	C'_{23}	C'_{24}	C'_{25}	C'_{26}	C'_{27}	C'_{28}	C'_{29}	C'_{20}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_{10}^{+}	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	−1	−1	−1	−1	−1
C_{10}^{-}	1	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	1	1	1	1	1
C_5^{+}	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_5^{-}	1	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_{10}^{3+}	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	1
C_{10}^{3-}	1	1	1	1	−1	1	−1	1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1
C_5^{2+}	1	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
C_5^{2-}	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1
C_2	1	−1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	−1	−1	−1	−1	−1
C'_{21}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	−1	1	1	−1	1	1	1	−1	−1
C'_{22}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	−1	−1	1	1	−1	1	1	1	−1
C'_{23}	1	−1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	−1	1	−1	−1	1	1	1
C'_{24}	1	−1	1	−1	−1	1	−1	1	1	−1	1	1	−1	−1	−1	1	−1	−1	1	1
C'_{25}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1
C'_{26}	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	1	1	−1	−1	1	1	−1
C'_{27}	1	1	−1	−1	−1	−1	1	1	1	1	1	−1	−1	−1	1	−1	−1	−1	1	1
C'_{28}	1	1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	−1	1	−1	−1	−1	1
C'_{29}	1	1	−1	−1	−1	−1	1	1	1	1	−1	1	1	−1	−1	1	1	−1	−1	−1
C'_{20}	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1
S_{20}^{9-}	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_{20}^{9+}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1
S_{20}^{7-}	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	1
S_{20}^{7+}	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	1	1	1	1
S_4^{-}	1	1	1	1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
S_4^{+}	1	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	1	1	1	1	1
S_{20}^{3-}	1	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	−1	−1	−1	−1	−1
S_{20}^{3+}	1	1	1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_{20}^{-}	1	−1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_{20}^{+}	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1
σ_{d1}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	−1	1	1	1	−1	1	1	1	−1
σ_{d2}	1	−1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	1	−1	−1	1	1	1
σ_{d3}	1	−1	1	−1	−1	1	−1	1	1	−1	1	1	−1	−1	1	1	−1	−1	1	1
σ_{d4}	1	−1	1	−1	−1	1	−1	1	1	−1	1	1	1	−1	−1	1	1	−1	−1	1
σ_{d5}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	1	−1	1	1	1	−1	−1
σ_{d6}	1	1	−1	−1	−1	−1	1	1	1	1	1	−1	−1	−1	1	−1	−1	1	1	1
σ_{d7}	1	1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	−1	1	−1	−1	1	1
σ_{d8}	1	1	−1	−1	−1	−1	1	1	1	1	−1	1	1	−1	−1	1	1	−1	−1	1
σ_{d9}	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	1	1	−1	1	1	1	−1	−1
σ_{d10}	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	1	1	−1	1	1	1	−1

$C'_{20} \equiv C'_{2,10}$

$\twoheadrightarrow$

T **49**.3 Factor table *(cont.)*

$\mathbf{D}_{10d}$	S_{20}^{9-}	S_{20}^{9+}	S_{20}^{7-}	S_{20}^{7+}	S_4^-	S_4^+	S_{20}^{3-}	S_{20}^{3+}	S_{20}^-	S_{20}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}	σ_{d7}	σ_{d8}	σ_{d9}	σ_{d10}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_{10}^+	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1
C_{10}^-	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	1
C_5^+	1	1	1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_5^-	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
C_{10}^{3+}	1	1	1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	1	1	1	1	1
C_{10}^{3-}	1	1	1	1	1	−1	1	−1	1	−1	1	1	1	1	1	−1	−1	−1	−1	−1
C_5^{2+}	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1
C_5^{2-}	1	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
C_2	−1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1
C'_{21}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	1	−1	1	1	−1	−1	−1
C'_{22}	1	−1	1	−1	−1	1	−1	1	1	−1	−1	−1	1	1	1	−1	1	1	−1	−1
C'_{23}	1	−1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	1	−1	−1	1	1	−1
C'_{24}	1	−1	1	−1	−1	1	−1	1	1	−1	1	1	−1	−1	1	−1	−1	−1	1	1
C'_{25}	1	−1	1	−1	−1	1	−1	1	1	−1	1	1	1	−1	−1	1	−1	−1	−1	1
C'_{26}	−1	−1	1	1	1	1	−1	−1	−1	−1	−1	−1	1	1	1	−1	1	1	1	−1
C'_{27}	−1	−1	1	1	1	1	−1	−1	−1	−1	1	−1	−1	1	1	−1	−1	1	1	1
C'_{28}	−1	−1	1	1	1	1	−1	−1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	1
C'_{29}	−1	−1	1	1	1	1	−1	−1	−1	−1	1	1	1	−1	−1	1	1	−1	−1	1
C'_{20}	−1	−1	1	1	1	1	−1	−1	−1	−1	−1	1	1	1	−1	1	1	1	−1	−1
S_{20}^{9-}	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	1	1	1	1	1
S_{20}^{9+}	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_{20}^{7-}	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	1	1	1	1
S_{20}^{7+}	1	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	−1	−1	−1	−1	−1
S_4^-	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1
S_4^+	1	1	1	1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1
S_{20}^{3-}	1	1	1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
S_{20}^{3+}	1	1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	1
S_{20}^-	1	1	−1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	1	1	1	1	1
S_{20}^+	1	−1	1	−1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
σ_{d1}	−1	−1	1	1	1	1	−1	−1	−1	−1	−1	−1	1	1	−1	1	1	1	−1	−1
σ_{d2}	−1	−1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	1	1	−1	1	1	1	−1
σ_{d3}	−1	−1	1	1	1	1	−1	−1	−1	−1	1	−1	−1	−1	1	−1	−1	1	1	1
σ_{d4}	−1	−1	1	1	1	1	−1	−1	−1	−1	1	1	−1	−1	−1	1	−1	−1	1	1
σ_{d5}	−1	−1	1	1	1	1	−1	−1	−1	−1	−1	1	1	−1	−1	1	1	−1	−1	1
σ_{d6}	−1	1	−1	1	1	−1	1	−1	−1	1	−1	−1	−1	1	1	−1	−1	1	1	−1
σ_{d7}	−1	1	−1	1	1	−1	1	−1	−1	1	1	−1	−1	−1	1	−1	−1	−1	1	1
σ_{d8}	−1	1	−1	1	1	−1	1	−1	−1	1	1	1	−1	−1	−1	1	−1	−1	−1	1
σ_{d9}	−1	1	−1	1	1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	−1	−1	−1
σ_{d10}	−1	1	−1	1	1	−1	1	−1	−1	1	−1	−1	1	1	−1	−1	1	1	−1	−1

$C'_{20} \equiv C'_{2,10}$

T **49**.4 Character table

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$\mathbf{D}_{10d}$	E	$2C_{10}$	$2C_5$	$2C_{10}^3$	$2C_5^2$	C_2	$10C_2'$	$2S_{20}^9$	$2S_{20}^7$	$2S_4$	$2S_{20}^3$	$2S_{20}$	$10\sigma_d$	τ
A_1	1	1	1	1	1	1	1	1	1	1	1	1	1	a
A_2	1	1	1	1	1	1	-1	1	1	1	1	1	-1	a
B_1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	a
B_2	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	a
E_1	2	$2c_5$	$2c_5^2$	$-2c_5^2$	$-2c_5$	-2	0	$-2c_{10}$	$-2c_{10}^3$	0	$2c_{10}^3$	$2c_{10}$	0	a
E_2	2	$2c_5^2$	$-2c_5$	$-2c_5$	$2c_5^2$	2	0	$2c_5$	$-2c_5^2$	-2	$-2c_5^2$	$2c_5$	0	a
E_3	2	$-2c_5^2$	$-2c_5$	$2c_5$	$2c_5^2$	-2	0	$-2c_{10}^3$	$2c_{10}$	0	$-2c_{10}$	$2c_{10}^3$	0	a
E_4	2	$-2c_5$	$2c_5^2$	$2c_5^2$	$-2c_5$	2	0	$2c_5^2$	$-2c_5$	2	$-2c_5$	$2c_5^2$	0	a
E_5	2	-2	2	-2	2	-2	0	0	0	0	0	0	0	a
E_6	2	$-2c_5$	$2c_5^2$	$2c_5^2$	$-2c_5$	2	0	$-2c_5^2$	$2c_5$	-2	$2c_5$	$-2c_5^2$	0	a
E_7	2	$-2c_5^2$	$-2c_5$	$2c_5$	$2c_5^2$	-2	0	$2c_{10}^3$	$-2c_{10}$	0	$2c_{10}$	$-2c_{10}^3$	0	a
E_8	2	$2c_5^2$	$-2c_5$	$-2c_5$	$2c_5^2$	2	0	$-2c_5$	$2c_5^2$	2	$2c_5^2$	$-2c_5$	0	a
E_9	2	$2c_5$	$2c_5^2$	$-2c_5^2$	$-2c_5$	-2	0	$2c_{10}$	$2c_{10}^3$	0	$-2c_{10}^3$	$-2c_{10}$	0	a
$E_{1/2}$	2	$2c_{10}$	$2c_5$	$2c_{10}^3$	$2c_5^2$	0	0	$2c_{20}$	$2c_{20}^3$	$\sqrt{2}$	$2c_{20}^7$	$2c_{20}^9$	0	c
$E_{3/2}$	2	$2c_{10}^3$	$-2c_5^2$	$-2c_{10}$	$-2c_5$	0	0	$2c_{20}^3$	$2c_{20}^9$	$-\sqrt{2}$	$-2c_{20}$	$-2c_{20}^7$	0	c
$E_{5/2}$	2	0	-2	0	2	0	0	$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	0	c
$E_{7/2}$	2	$-2c_{10}^3$	$-2c_5^2$	$2c_{10}$	$-2c_5$	0	0	$2c_{20}^7$	$-2c_{20}$	$\sqrt{2}$	$2c_{20}^9$	$-2c_{20}^3$	0	c
$E_{9/2}$	2	$-2c_{10}$	$2c_5$	$-2c_{10}^3$	$2c_5^2$	0	0	$2c_{20}^9$	$-2c_{20}^7$	$\sqrt{2}$	$-2c_{20}^3$	$2c_{20}$	0	c
$E_{11/2}$	2	$-2c_{10}$	$2c_5$	$-2c_{10}^3$	$2c_5^2$	0	0	$-2c_{20}^9$	$2c_{20}^7$	$-\sqrt{2}$	$2c_{20}^3$	$-2c_{20}$	0	c
$E_{13/2}$	2	$-2c_{10}^3$	$-2c_5^2$	$2c_{10}$	$-2c_5$	0	0	$-2c_{20}^7$	$2c_{20}$	$-\sqrt{2}$	$-2c_{20}^9$	$2c_{20}^3$	0	c
$E_{15/2}$	2	0	-2	0	2	0	0	$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	$-\sqrt{2}$	0	c
$E_{17/2}$	2	$2c_{10}^3$	$-2c_5^2$	$-2c_{10}$	$-2c_5$	0	0	$-2c_{20}^3$	$-2c_{20}^9$	$\sqrt{2}$	$2c_{20}$	$2c_{20}^7$	0	c
$E_{19/2}$	2	$2c_{10}$	$2c_5$	$2c_{10}^3$	$2c_5^2$	0	0	$-2c_{20}$	$-2c_{20}^3$	$-\sqrt{2}$	$-2c_{20}^7$	$-2c_{20}^9$	0	c

$c_n^m = \cos \frac{m}{n}\pi$

T **49**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{D}_{10d}$	0	1	2	3
A_1	$^\square 1$		$x^2+y^2, {}^\square z^2$	
A_2		R_z		
B_1				
B_2		$^\square z$		$(x^2+y^2)z, {}^\square z^3$
E_1		$^\square(x,y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
E_2			$^\square(xy, x^2-y^2)$	
E_3				$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$
E_4				
E_5				
E_6				
E_7				
E_8				$^\square\{xyz, z(x^2-y^2)\}$
E_9		(R_x, R_y)	$^\square(zx, yz)$	

T **49**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{D}_{10d}$	$\big\langle\vert j\,m\rangle\big\vert$		ι	μ
A_1	$\vert 0\,0\rangle_+$	$\vert 11\,10\rangle_-$	2	20
A_2	$\vert 11\,10\rangle_+$	$\vert 20\,20\rangle_-$	2	20
B_1	$\vert 10\,10\rangle_+$	$\vert 21\,20\rangle_-$	2	20
B_2	$\vert 1\,0\rangle_+$	$\vert 10\,10\rangle_-$	2	20
E_1	$\big\langle\vert 1\,1\rangle, \vert 1\,\overline{1}\rangle\big\vert$	$\big\langle\vert 10\,\overline{9}\rangle, -\vert 10\,9\rangle\big\vert$	2	± 20
E_2	$\big\langle\vert 2\,2\rangle, \vert 2\,\overline{2}\rangle\big\vert$	$\big\langle\vert 9\,\overline{8}\rangle, -\vert 9\,8\rangle\big\vert$	2	± 20
E_3	$\big\langle\vert 3\,\overline{3}\rangle, -\vert 3\,3\rangle\big\vert$	$\big\langle\vert 8\,7\rangle, \vert 8\,\overline{7}\rangle\big\vert$	2	± 20
E_4	$\big\langle\vert 4\,\overline{4}\rangle, -\vert 4\,4\rangle\big\vert$	$\big\langle\vert 7\,6\rangle, \vert 7\,\overline{6}\rangle\big\vert$	2	± 20
E_5	$\big\langle\vert 5\,5\rangle, \vert 5\,\overline{5}\rangle\big\vert$	$\big\langle\vert 6\,\overline{5}\rangle, -\vert 6\,5\rangle\big\vert$	2	± 20
E_6	$\big\langle\vert 5\,\overline{4}\rangle, \vert 5\,4\rangle\big\vert$	$\big\langle\vert 6\,6\rangle, -\vert 6\,\overline{6}\rangle\big\vert$	2	± 20
E_7	$\big\langle\vert 4\,\overline{3}\rangle, \vert 4\,3\rangle\big\vert$	$\big\langle\vert 7\,7\rangle, -\vert 7\,\overline{7}\rangle\big\vert$	2	± 20
E_8	$\big\langle\vert 3\,2\rangle, -\vert 3\,\overline{2}\rangle\big\vert$	$\big\langle\vert 8\,\overline{8}\rangle, \vert 8\,8\rangle\big\vert$	2	± 20
E_9	$\big\langle\vert 2\,1\rangle, -\vert 2\,\overline{1}\rangle\big\vert$	$\big\langle\vert 9\,\overline{9}\rangle, \vert 9\,9\rangle\big\vert$	2	± 20
$E_{1/2}$	$\big\langle\vert\tfrac{1}{2}\,\tfrac{1}{2}\rangle, \vert\tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle\big\vert$	$\big\langle\vert\tfrac{3}{2}\,\tfrac{1}{2}\rangle, -\vert\tfrac{3}{2}\,\overline{\tfrac{1}{2}}\rangle\big\vert$	2	± 20
	$\big\langle\vert\tfrac{19}{2}\,\overline{\tfrac{19}{2}}\rangle, -\vert\tfrac{19}{2}\,\tfrac{19}{2}\rangle\big\vert^{\bullet}$	$\big\langle\vert\tfrac{21}{2}\,\overline{\tfrac{19}{2}}\rangle, \vert\tfrac{21}{2}\,\tfrac{19}{2}\rangle\big\vert^{\bullet}$	2	± 20
$E_{3/2}$	$\big\langle\vert\tfrac{3}{2}\,\tfrac{3}{2}\rangle, \vert\tfrac{3}{2}\,\overline{\tfrac{3}{2}}\rangle\big\vert$	$\big\langle\vert\tfrac{5}{2}\,\tfrac{3}{2}\rangle, -\vert\tfrac{5}{2}\,\overline{\tfrac{3}{2}}\rangle\big\vert$	2	± 20
	$\big\langle\vert\tfrac{17}{2}\,\overline{\tfrac{17}{2}}\rangle, -\vert\tfrac{17}{2}\,\tfrac{17}{2}\rangle\big\vert^{\bullet}$	$\big\langle\vert\tfrac{19}{2}\,\overline{\tfrac{17}{2}}\rangle, \vert\tfrac{19}{2}\,\tfrac{17}{2}\rangle\big\vert^{\bullet}$	2	± 20
$E_{5/2}$	$\big\langle\vert\tfrac{5}{2}\,\tfrac{5}{2}\rangle, \vert\tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle\big\vert$	$\big\langle\vert\tfrac{7}{2}\,\tfrac{5}{2}\rangle, -\vert\tfrac{7}{2}\,\overline{\tfrac{5}{2}}\rangle\big\vert$	2	± 20
	$\big\langle\vert\tfrac{15}{2}\,\overline{\tfrac{15}{2}}\rangle, -\vert\tfrac{15}{2}\,\tfrac{15}{2}\rangle\big\vert^{\bullet}$	$\big\langle\vert\tfrac{17}{2}\,\overline{\tfrac{15}{2}}\rangle, \vert\tfrac{17}{2}\,\tfrac{15}{2}\rangle\big\vert^{\bullet}$	2	± 20
$E_{7/2}$	$\big\langle\vert\tfrac{7}{2}\,\overline{\tfrac{7}{2}}\rangle, \vert\tfrac{7}{2}\,\tfrac{7}{2}\rangle\big\vert$	$\big\langle\vert\tfrac{9}{2}\,\overline{\tfrac{7}{2}}\rangle, -\vert\tfrac{9}{2}\,\tfrac{7}{2}\rangle\big\vert$	2	± 20
	$\big\langle\vert\tfrac{13}{2}\,\tfrac{13}{2}\rangle, -\vert\tfrac{13}{2}\,\overline{\tfrac{13}{2}}\rangle\big\vert^{\bullet}$	$\big\langle\vert\tfrac{15}{2}\,\tfrac{13}{2}\rangle, \vert\tfrac{15}{2}\,\overline{\tfrac{13}{2}}\rangle\big\vert^{\bullet}$	2	± 20
$E_{9/2}$	$\big\langle\vert\tfrac{9}{2}\,\overline{\tfrac{9}{2}}\rangle, \vert\tfrac{9}{2}\,\tfrac{9}{2}\rangle\big\vert$	$\big\langle\vert\tfrac{11}{2}\,\overline{\tfrac{9}{2}}\rangle, -\vert\tfrac{11}{2}\,\tfrac{9}{2}\rangle\big\vert$	2	± 20
	$\big\langle\vert\tfrac{11}{2}\,\tfrac{11}{2}\rangle, -\vert\tfrac{11}{2}\,\overline{\tfrac{11}{2}}\rangle\big\vert^{\bullet}$	$\big\langle\vert\tfrac{13}{2}\,\tfrac{11}{2}\rangle, \vert\tfrac{13}{2}\,\overline{\tfrac{11}{2}}\rangle\big\vert^{\bullet}$	2	± 20
$E_{11/2}$	$\big\langle\vert\tfrac{11}{2}\,\tfrac{11}{2}\rangle, -\vert\tfrac{11}{2}\,\overline{\tfrac{11}{2}}\rangle\big\vert$	$\big\langle\vert\tfrac{13}{2}\,\tfrac{11}{2}\rangle, \vert\tfrac{13}{2}\,\overline{\tfrac{11}{2}}\rangle\big\vert$	2	± 20
	$\big\langle\vert\tfrac{9}{2}\,\overline{\tfrac{9}{2}}\rangle, \vert\tfrac{9}{2}\,\tfrac{9}{2}\rangle\big\vert^{\bullet}$	$\big\langle\vert\tfrac{11}{2}\,\overline{\tfrac{9}{2}}\rangle, -\vert\tfrac{11}{2}\,\tfrac{9}{2}\rangle\big\vert^{\bullet}$	2	± 20
$E_{13/2}$	$\big\langle\vert\tfrac{13}{2}\,\tfrac{13}{2}\rangle, -\vert\tfrac{13}{2}\,\overline{\tfrac{13}{2}}\rangle\big\vert$	$\big\langle\vert\tfrac{15}{2}\,\tfrac{13}{2}\rangle, \vert\tfrac{15}{2}\,\overline{\tfrac{13}{2}}\rangle\big\vert$	2	± 20
	$\big\langle\vert\tfrac{7}{2}\,\overline{\tfrac{7}{2}}\rangle, \vert\tfrac{7}{2}\,\tfrac{7}{2}\rangle\big\vert^{\bullet}$	$\big\langle\vert\tfrac{9}{2}\,\overline{\tfrac{7}{2}}\rangle, -\vert\tfrac{9}{2}\,\tfrac{7}{2}\rangle\big\vert^{\bullet}$	2	± 20
$E_{15/2}$	$\big\langle\vert\tfrac{15}{2}\,\overline{\tfrac{15}{2}}\rangle, -\vert\tfrac{15}{2}\,\tfrac{15}{2}\rangle\big\vert$	$\big\langle\vert\tfrac{17}{2}\,\overline{\tfrac{15}{2}}\rangle, \vert\tfrac{17}{2}\,\tfrac{15}{2}\rangle\big\vert$	2	± 20
	$\big\langle\vert\tfrac{5}{2}\,\tfrac{5}{2}\rangle, \vert\tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle\big\vert^{\bullet}$	$\big\langle\vert\tfrac{7}{2}\,\tfrac{5}{2}\rangle, -\vert\tfrac{7}{2}\,\overline{\tfrac{5}{2}}\rangle\big\vert^{\bullet}$	2	± 20
$E_{17/2}$	$\big\langle\vert\tfrac{17}{2}\,\overline{\tfrac{17}{2}}\rangle, -\vert\tfrac{17}{2}\,\tfrac{17}{2}\rangle\big\vert$	$\big\langle\vert\tfrac{19}{2}\,\overline{\tfrac{17}{2}}\rangle, \vert\tfrac{19}{2}\,\tfrac{17}{2}\rangle\big\vert$	2	± 20
	$\big\langle\vert\tfrac{3}{2}\,\tfrac{3}{2}\rangle, \vert\tfrac{3}{2}\,\overline{\tfrac{3}{2}}\rangle\big\vert^{\bullet}$	$\big\langle\vert\tfrac{5}{2}\,\tfrac{3}{2}\rangle, -\vert\tfrac{5}{2}\,\overline{\tfrac{3}{2}}\rangle\big\vert^{\bullet}$	2	± 20
$E_{19/2}$	$\big\langle\vert\tfrac{19}{2}\,\overline{\tfrac{19}{2}}\rangle, -\vert\tfrac{19}{2}\,\tfrac{19}{2}\rangle\big\vert$	$\big\langle\vert\tfrac{21}{2}\,\overline{\tfrac{19}{2}}\rangle, \vert\tfrac{21}{2}\,\tfrac{19}{2}\rangle\big\vert$	2	± 20
	$\big\langle\vert\tfrac{1}{2}\,\tfrac{1}{2}\rangle, \vert\tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle\big\vert^{\bullet}$	$\big\langle\vert\tfrac{3}{2}\,\tfrac{1}{2}\rangle, -\vert\tfrac{3}{2}\,\overline{\tfrac{1}{2}}\rangle\big\vert^{\bullet}$	2	± 20

T **49**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{D}_{10d}$	E_1	E_2	E_3	E_4	E_5	E_6
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_{10}^{+}	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$
C_{10}^{-}	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$
C_{5}^{+}	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$
C_{5}^{-}	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$
C_{10}^{3+}	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$
C_{10}^{3-}	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$
C_{5}^{2+}	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$
C_{5}^{2-}	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$
C_2	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \overline{1} & 0 \\ 0 & \overline{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$
C'_{25}	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$
C'_{26}	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{1} \\ \overline{1} & 0 \end{bmatrix}$
C'_{27}	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$
C'_{28}	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$
C'_{29}	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$
$C'_{2,10}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$

$\eta = \exp(2\pi i/5)$, $\theta = \exp(4\pi i/5)$ ↠

T **49**.7 Matrix representations *(cont.)*

$\mathbf{D}_{10d}$	E_7	E_8	E_9	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_{10}^{+}	$\begin{bmatrix} \bar{\eta}^* & 0 \\ 0 & \bar{\eta} \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \bar{\theta} & 0 \\ 0 & \bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta^* & 0 \\ 0 & \mathrm{i}\bar{\theta} \end{bmatrix}$	$\begin{bmatrix} \bar{\imath} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$
C_{10}^{-}	$\begin{bmatrix} \bar{\eta} & 0 \\ 0 & \bar{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\theta}^* & 0 \\ 0 & \bar{\theta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\bar{\eta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\theta} & 0 \\ 0 & \mathrm{i}\theta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\imath} \end{bmatrix}$
C_{5}^{+}	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \bar{\theta} & 0 \\ 0 & \bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\eta} & 0 \\ 0 & \bar{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$
C_{5}^{-}	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\theta}^* & 0 \\ 0 & \bar{\theta} \end{bmatrix}$	$\begin{bmatrix} \bar{\eta}^* & 0 \\ 0 & \bar{\eta} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$
C_{10}^{3+}	$\begin{bmatrix} \bar{\theta} & 0 \\ 0 & \bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\eta} & 0 \\ 0 & \bar{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta^* & 0 \\ 0 & \mathrm{i}\bar{\theta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\imath} \end{bmatrix}$
C_{10}^{3-}	$\begin{bmatrix} \bar{\theta}^* & 0 \\ 0 & \bar{\theta} \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \bar{\eta}^* & 0 \\ 0 & \bar{\eta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\theta} & 0 \\ 0 & \mathrm{i}\theta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\bar{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\imath} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$
C_{5}^{2+}	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_{5}^{2-}	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_{2}	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\imath} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\imath} \end{bmatrix}$	$\begin{bmatrix} \bar{\imath} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\imath} \\ \bar{\imath} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\imath} \\ \bar{\imath} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta}^* \\ \bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta}^* \\ \mathrm{i}\bar{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\theta \\ \mathrm{i}\theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\imath} \\ \bar{\imath} & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta} \\ \bar{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\theta}^* \\ \mathrm{i}\bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\imath} \\ \bar{\imath} & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta}^* \\ \bar{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\theta} \\ \mathrm{i}\bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\imath} \\ \bar{\imath} & 0 \end{bmatrix}$
C'_{25}	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta} \\ \bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta} \\ \mathrm{i}\bar{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\theta^* \\ \mathrm{i}\theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\imath} \\ \bar{\imath} & 0 \end{bmatrix}$
C'_{26}	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$
C'_{27}	$\begin{bmatrix} 0 & \bar{\eta} \\ \bar{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta}^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta} \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$
C'_{28}	$\begin{bmatrix} 0 & \bar{\theta} \\ \bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta}^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta}^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$
C'_{29}	$\begin{bmatrix} 0 & \bar{\theta}^* \\ \bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta} \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta} \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$
$C'_{2,10}$	$\begin{bmatrix} 0 & \bar{\eta}^* \\ \bar{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta} \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta}^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$

$\eta = \exp(2\pi\mathrm{i}/5)$, $\theta = \exp(4\pi\mathrm{i}/5)$ $\twoheadrightarrow$

T **49**.7 Matrix representations *(cont.)*

$\mathbf{D}_{10d}$	$E_{7/2}$	$E_{9/2}$	$E_{11/2}$	$E_{13/2}$	$E_{15/2}$	$E_{17/2}$	$E_{19/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_{10}^{+}	$\begin{bmatrix} \mathrm{i}\bar{\theta}^* & 0 \\ 0 & \mathrm{i}\theta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\bar{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\bar{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\theta}^* & 0 \\ 0 & \mathrm{i}\theta \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta^* & 0 \\ 0 & \mathrm{i}\bar{\theta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$
C_{10}^{-}	$\begin{bmatrix} \mathrm{i}\theta & 0 \\ 0 & \mathrm{i}\bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta & 0 \\ 0 & \mathrm{i}\bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\theta} & 0 \\ 0 & \mathrm{i}\theta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\bar{\eta} \end{bmatrix}$
C_{5}^{+}	$\begin{bmatrix} \bar{\eta} & 0 \\ 0 & \bar{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\theta} & 0 \\ 0 & \bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\theta} & 0 \\ 0 & \bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\eta} & 0 \\ 0 & \bar{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\eta} & 0 \\ 0 & \bar{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\theta} & 0 \\ 0 & \bar{\theta}^* \end{bmatrix}$
C_{5}^{-}	$\begin{bmatrix} \bar{\eta}^* & 0 \\ 0 & \bar{\eta} \end{bmatrix}$	$\begin{bmatrix} \bar{\theta}^* & 0 \\ 0 & \bar{\theta} \end{bmatrix}$	$\begin{bmatrix} \bar{\theta}^* & 0 \\ 0 & \bar{\theta} \end{bmatrix}$	$\begin{bmatrix} \bar{\eta}^* & 0 \\ 0 & \bar{\eta} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\eta}^* & 0 \\ 0 & \bar{\eta} \end{bmatrix}$	$\begin{bmatrix} \bar{\theta}^* & 0 \\ 0 & \bar{\theta} \end{bmatrix}$
C_{10}^{3+}	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\bar{\eta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\theta}^* & 0 \\ 0 & \mathrm{i}\theta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\theta}^* & 0 \\ 0 & \mathrm{i}\theta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\bar{\eta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta^* & 0 \\ 0 & \mathrm{i}\bar{\theta} \end{bmatrix}$
C_{10}^{3-}	$\begin{bmatrix} \mathrm{i}\bar{\eta} & 0 \\ 0 & \mathrm{i}\bar{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta & 0 \\ 0 & \mathrm{i}\bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta & 0 \\ 0 & \mathrm{i}\bar{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\bar{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\theta} & 0 \\ 0 & \mathrm{i}\theta^* \end{bmatrix}$
C_{5}^{2+}	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$
C_{5}^{2-}	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$
C_{2}	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$
C'_{21}	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C'_{22}	$\begin{bmatrix} 0 & \mathrm{i}\theta \\ \mathrm{i}\theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta}^* \\ \mathrm{i}\bar{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta}^* \\ \mathrm{i}\bar{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\theta \\ \mathrm{i}\theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\theta \\ \mathrm{i}\theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta}^* \\ \mathrm{i}\bar{\eta} & 0 \end{bmatrix}$
C'_{23}	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\theta}^* \\ \mathrm{i}\bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\theta}^* \\ \mathrm{i}\bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\theta}^* \\ \mathrm{i}\bar{\theta} & 0 \end{bmatrix}$
C'_{24}	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\theta} \\ \mathrm{i}\bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\theta} \\ \mathrm{i}\bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\theta} \\ \mathrm{i}\bar{\theta}^* & 0 \end{bmatrix}$
C'_{25}	$\begin{bmatrix} 0 & \mathrm{i}\theta^* \\ \mathrm{i}\theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta} \\ \mathrm{i}\bar{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta} \\ \mathrm{i}\bar{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\theta^* \\ \mathrm{i}\theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\theta^* \\ \mathrm{i}\theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\eta} \\ \mathrm{i}\bar{\eta}^* & 0 \end{bmatrix}$
C'_{26}	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$
C'_{27}	$\begin{bmatrix} 0 & \theta \\ \bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \bar{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \bar{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta} \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta}^* \\ \eta & 0 \end{bmatrix}$
C'_{28}	$\begin{bmatrix} 0 & \eta^* \\ \bar{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \bar{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta}^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta}^* \\ \theta & 0 \end{bmatrix}$
C'_{29}	$\begin{bmatrix} 0 & \eta \\ \bar{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \bar{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \bar{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta} \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta} \\ \theta^* & 0 \end{bmatrix}$
$C'_{2,10}$	$\begin{bmatrix} 0 & \theta^* \\ \bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \bar{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \bar{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \bar{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\theta}^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta} \\ \eta^* & 0 \end{bmatrix}$

$\eta = \exp(2\pi\mathrm{i}/5)$, $\theta = \exp(4\pi\mathrm{i}/5)$ $\twoheadrightarrow$

T 49.7 Matrix representations *(cont.)*

$\mathbf{D}_{10d}$	E_1	E_2	E_3	E_4	E_5	E_6
S_{20}^{9-}	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta & 0 \\ 0 & \mathrm{i}\overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$
S_{20}^{9+}	$\begin{bmatrix} \mathrm{i}\overline{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\theta}^* & 0 \\ 0 & \mathrm{i}\theta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$
S_{20}^{7-}	$\begin{bmatrix} \mathrm{i}\overline{\theta}^* & 0 \\ 0 & \mathrm{i}\theta \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$
S_{20}^{7+}	$\begin{bmatrix} \mathrm{i}\theta & 0 \\ 0 & \mathrm{i}\overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$
S_4^-	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$
S_4^+	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$
S_{20}^{3-}	$\begin{bmatrix} \mathrm{i}\overline{\theta} & 0 \\ 0 & \mathrm{i}\theta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$
S_{20}^{3+}	$\begin{bmatrix} \mathrm{i}\theta^* & 0 \\ 0 & \mathrm{i}\overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$
S_{20}^-	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \overline{\theta}^* & 0 \\ 0 & \overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta^* & 0 \\ 0 & \mathrm{i}\overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\mathrm{i}} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta}^* & 0 \\ 0 & \overline{\eta} \end{bmatrix}$
S_{20}^+	$\begin{bmatrix} \mathrm{i}\overline{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\theta} & 0 \\ 0 & \overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\theta} & 0 \\ 0 & \mathrm{i}\theta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \overline{\eta} & 0 \\ 0 & \overline{\eta}^* \end{bmatrix}$
σ_{d1}	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
σ_{d2}	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta}^* \\ \mathrm{i}\theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$
σ_{d3}	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\theta \\ \mathrm{i}\overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$
σ_{d4}	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\theta^* \\ \mathrm{i}\overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$
σ_{d5}	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta} \\ \mathrm{i}\theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$
σ_{d6}	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
σ_{d7}	$\begin{bmatrix} 0 & \mathrm{i}\theta^* \\ \mathrm{i}\overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$
σ_{d8}	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta} \\ \mathrm{i}\theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta} \\ \overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$
σ_{d9}	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta}^* \\ \overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta}^* \\ \mathrm{i}\theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$
σ_{d10}	$\begin{bmatrix} 0 & \mathrm{i}\theta \\ \mathrm{i}\overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\eta}^* \\ \overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\theta} \\ \overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$

$\eta = \exp(2\pi\mathrm{i}/5)$, $\theta = \exp(4\pi\mathrm{i}/5)$ $\twoheadrightarrow$

T **49**.7 Matrix representations *(cont.)*

$\mathbf{D}_{10d}$	E_7	E_8	E_9	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
S_{20}^{9-}	$\begin{bmatrix} \mathrm{i}\overline{\theta} & 0 \\ 0 & \mathrm{i}\theta^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \zeta^* & 0 \\ 0 & \zeta \end{bmatrix}$
S_{20}^{9+}	$\begin{bmatrix} \mathrm{i}\theta^* & 0 \\ 0 & \mathrm{i}\overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \zeta & 0 \\ 0 & \zeta^* \end{bmatrix}$
S_{20}^{7-}	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta^* & 0 \\ 0 & \mathrm{i}\overline{\theta} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\zeta} & 0 \\ 0 & \overline{\zeta}^* \end{bmatrix}$
S_{20}^{7+}	$\begin{bmatrix} \mathrm{i}\overline{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\theta} & 0 \\ 0 & \mathrm{i}\theta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \overline{\zeta}^* & 0 \\ 0 & \overline{\zeta} \end{bmatrix}$
S_4^-	$\begin{bmatrix} \bar{\imath} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{\imath} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \zeta^* & 0 \\ 0 & \zeta \end{bmatrix}$	$\begin{bmatrix} \overline{\zeta} & 0 \\ 0 & \overline{\zeta}^* \end{bmatrix}$	$\begin{bmatrix} \overline{\zeta}^* & 0 \\ 0 & \overline{\zeta} \end{bmatrix}$
S_4^+	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\imath} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \bar{\imath} \end{bmatrix}$	$\begin{bmatrix} \zeta & 0 \\ 0 & \zeta^* \end{bmatrix}$	$\begin{bmatrix} \overline{\zeta}^* & 0 \\ 0 & \overline{\zeta} \end{bmatrix}$	$\begin{bmatrix} \overline{\zeta} & 0 \\ 0 & \overline{\zeta}^* \end{bmatrix}$
S_{20}^{3-}	$\begin{bmatrix} \mathrm{i}\eta^* & 0 \\ 0 & \mathrm{i}\overline{\eta} \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\theta & 0 \\ 0 & \mathrm{i}\overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon} & 0 \\ 0 & \mathrm{i}\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \overline{\delta}^* & 0 \\ 0 & \overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \zeta & 0 \\ 0 & \zeta^* \end{bmatrix}$
S_{20}^{3+}	$\begin{bmatrix} \mathrm{i}\overline{\eta} & 0 \\ 0 & \mathrm{i}\eta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\theta}^* & 0 \\ 0 & \mathrm{i}\theta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon^* & 0 \\ 0 & \mathrm{i}\overline{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \overline{\delta} & 0 \\ 0 & \overline{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \zeta^* & 0 \\ 0 & \zeta \end{bmatrix}$
S_{20}^-	$\begin{bmatrix} \mathrm{i}\overline{\theta}^* & 0 \\ 0 & \mathrm{i}\theta \end{bmatrix}$	$\begin{bmatrix} \theta^* & 0 \\ 0 & \theta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\eta}^* & 0 \\ 0 & \mathrm{i}\eta \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\delta} & 0 \\ 0 & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\epsilon & 0 \\ 0 & \mathrm{i}\overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \zeta^* & 0 \\ 0 & \zeta \end{bmatrix}$
S_{20}^+	$\begin{bmatrix} \mathrm{i}\theta & 0 \\ 0 & \mathrm{i}\overline{\theta}^* \end{bmatrix}$	$\begin{bmatrix} \theta & 0 \\ 0 & \theta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\eta & 0 \\ 0 & \mathrm{i}\overline{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & 0 \\ 0 & \mathrm{i}\overline{\delta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\overline{\epsilon}^* & 0 \\ 0 & \mathrm{i}\epsilon \end{bmatrix}$	$\begin{bmatrix} \zeta & 0 \\ 0 & \zeta^* \end{bmatrix}$
σ_{d1}	$\begin{bmatrix} 0 & \bar{\imath} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\imath} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\zeta} \\ \zeta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta^* \\ \overline{\zeta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta \\ \overline{\zeta}^* & 0 \end{bmatrix}$
σ_{d2}	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\theta^* \\ \mathrm{i}\overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta \\ \mathrm{i}\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta \\ \overline{\zeta}^* & 0 \end{bmatrix}$
σ_{d3}	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta} \\ \mathrm{i}\theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\delta^* \\ \mathrm{i}\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon}^* \\ \mathrm{i}\overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta \\ \overline{\zeta}^* & 0 \end{bmatrix}$
σ_{d4}	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta}^* \\ \mathrm{i}\theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta \\ \overline{\zeta}^* & 0 \end{bmatrix}$
σ_{d5}	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\theta \\ \mathrm{i}\overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\epsilon} \\ \mathrm{i}\overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta \\ \overline{\zeta}^* & 0 \end{bmatrix}$
σ_{d6}	$\begin{bmatrix} 0 & \mathrm{i} \\ \bar{\imath} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\imath} \\ \mathrm{i} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\zeta}^* \\ \zeta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta \\ \overline{\zeta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta^* \\ \overline{\zeta} & 0 \end{bmatrix}$
σ_{d7}	$\begin{bmatrix} 0 & \mathrm{i}\eta \\ \mathrm{i}\overline{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta}^* \\ \mathrm{i}\theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon^* \\ \mathrm{i}\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta^* \\ \overline{\zeta} & 0 \end{bmatrix}$
σ_{d8}	$\begin{bmatrix} 0 & \mathrm{i}\theta \\ \mathrm{i}\overline{\theta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta \\ \theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta} \\ \mathrm{i}\eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta^* \\ \overline{\zeta} & 0 \end{bmatrix}$
σ_{d9}	$\begin{bmatrix} 0 & \mathrm{i}\theta^* \\ \mathrm{i}\overline{\theta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \theta^* \\ \theta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta} \\ \mathrm{i}\overline{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\epsilon \\ \mathrm{i}\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta^* \\ \overline{\zeta} & 0 \end{bmatrix}$
σ_{d10}	$\begin{bmatrix} 0 & \mathrm{i}\eta^* \\ \mathrm{i}\overline{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\theta} \\ \mathrm{i}\theta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\overline{\delta}^* \\ \mathrm{i}\overline{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta^* \\ \overline{\zeta} & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/40)$, $\epsilon = \exp(6\pi\mathrm{i}/40)$, $\zeta = \exp(2\pi\mathrm{i}/8)$, $\eta = \exp(2\pi\mathrm{i}/5)$, $\theta = \exp(4\pi\mathrm{i}/5)$ $\twoheadrightarrow$

T 49.7 Matrix representations *(cont.)*

$\mathbf{D}_{10d}$	$E_{7/2}$	$E_{9/2}$	$E_{11/2}$	$E_{13/2}$	$E_{15/2}$	$E_{17/2}$	$E_{19/2}$
S_{20}^{9-}	$\begin{bmatrix} i\epsilon^* & 0 \\ 0 & i\bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} i\delta^* & 0 \\ 0 & i\bar{\delta} \end{bmatrix}$	$\begin{bmatrix} i\bar{\delta}^* & 0 \\ 0 & i\delta \end{bmatrix}$	$\begin{bmatrix} i\bar{\epsilon}^* & 0 \\ 0 & i\epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\zeta}^* & 0 \\ 0 & \bar{\zeta} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$
S_{20}^{9+}	$\begin{bmatrix} i\bar{\epsilon} & 0 \\ 0 & i\epsilon^* \end{bmatrix}$	$\begin{bmatrix} i\bar{\delta} & 0 \\ 0 & i\delta^* \end{bmatrix}$	$\begin{bmatrix} i\delta & 0 \\ 0 & i\bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} i\epsilon & 0 \\ 0 & i\bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\zeta} & 0 \\ 0 & \bar{\zeta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$
S_{20}^{7-}	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} i\bar{\epsilon}^* & 0 \\ 0 & i\epsilon \end{bmatrix}$	$\begin{bmatrix} i\epsilon^* & 0 \\ 0 & i\bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \zeta & 0 \\ 0 & \zeta^* \end{bmatrix}$	$\begin{bmatrix} i\delta & 0 \\ 0 & i\bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$
S_{20}^{7+}	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$	$\begin{bmatrix} i\epsilon & 0 \\ 0 & i\bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} i\bar{\epsilon} & 0 \\ 0 & i\epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \zeta^* & 0 \\ 0 & \zeta \end{bmatrix}$	$\begin{bmatrix} i\bar{\delta}^* & 0 \\ 0 & i\delta \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$
S_4^-	$\begin{bmatrix} \zeta^* & 0 \\ 0 & \zeta \end{bmatrix}$	$\begin{bmatrix} \zeta & 0 \\ 0 & \zeta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\zeta} & 0 \\ 0 & \bar{\zeta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\zeta}^* & 0 \\ 0 & \bar{\zeta} \end{bmatrix}$	$\begin{bmatrix} \zeta^* & 0 \\ 0 & \zeta \end{bmatrix}$	$\begin{bmatrix} \zeta & 0 \\ 0 & \zeta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\zeta}^* & 0 \\ 0 & \bar{\zeta} \end{bmatrix}$
S_4^+	$\begin{bmatrix} \zeta & 0 \\ 0 & \zeta^* \end{bmatrix}$	$\begin{bmatrix} \zeta^* & 0 \\ 0 & \zeta \end{bmatrix}$	$\begin{bmatrix} \bar{\zeta}^* & 0 \\ 0 & \bar{\zeta} \end{bmatrix}$	$\begin{bmatrix} \bar{\zeta} & 0 \\ 0 & \bar{\zeta}^* \end{bmatrix}$	$\begin{bmatrix} \zeta & 0 \\ 0 & \zeta^* \end{bmatrix}$	$\begin{bmatrix} \zeta^* & 0 \\ 0 & \zeta \end{bmatrix}$	$\begin{bmatrix} \bar{\zeta} & 0 \\ 0 & \bar{\zeta}^* \end{bmatrix}$
S_{20}^{3-}	$\begin{bmatrix} i\delta^* & 0 \\ 0 & i\bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} i\bar{\delta}^* & 0 \\ 0 & i\delta \end{bmatrix}$	$\begin{bmatrix} \bar{\zeta} & 0 \\ 0 & \bar{\zeta}^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} i\epsilon & 0 \\ 0 & i\bar{\epsilon}^* \end{bmatrix}$
S_{20}^{3+}	$\begin{bmatrix} i\bar{\delta} & 0 \\ 0 & i\delta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} i\delta & 0 \\ 0 & i\bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\zeta}^* & 0 \\ 0 & \bar{\zeta} \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} i\bar{\epsilon}^* & 0 \\ 0 & i\epsilon \end{bmatrix}$
S_{20}^-	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\zeta}^* & 0 \\ 0 & \bar{\zeta} \end{bmatrix}$	$\begin{bmatrix} i\bar{\epsilon} & 0 \\ 0 & i\epsilon^* \end{bmatrix}$	$\begin{bmatrix} i\delta & 0 \\ 0 & i\bar{\delta}^* \end{bmatrix}$
S_{20}^+	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\zeta} & 0 \\ 0 & \bar{\zeta}^* \end{bmatrix}$	$\begin{bmatrix} i\epsilon^* & 0 \\ 0 & i\bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} i\bar{\delta}^* & 0 \\ 0 & i\delta \end{bmatrix}$
σ_{d1}	$\begin{bmatrix} 0 & \zeta \\ \bar{\zeta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta^* \\ \bar{\zeta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta}^* \\ \zeta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta} \\ \zeta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta} \\ \zeta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta}^* \\ \zeta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta \\ \bar{\zeta}^* & 0 \end{bmatrix}$
σ_{d2}	$\begin{bmatrix} 0 & \bar{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\bar{\epsilon}^* \\ i\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\epsilon^* \\ i\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta} \\ \zeta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\bar{\delta} \\ i\bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$
σ_{d3}	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta} \\ \zeta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\epsilon^* \\ i\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\bar{\delta}^* \\ i\bar{\delta} & 0 \end{bmatrix}$
σ_{d4}	$\begin{bmatrix} 0 & i\epsilon \\ i\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\delta \\ i\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\bar{\delta} \\ i\bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\bar{\epsilon} \\ i\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta} \\ \zeta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta} \\ \delta^* & 0 \end{bmatrix}$
σ_{d5}	$\begin{bmatrix} 0 & i\bar{\delta}^* \\ i\bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\delta^* \\ i\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta} \\ \zeta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\epsilon \\ i\epsilon^* & 0 \end{bmatrix}$
σ_{d6}	$\begin{bmatrix} 0 & \zeta^* \\ \bar{\zeta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta \\ \bar{\zeta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta} \\ \zeta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta}^* \\ \zeta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta}^* \\ \zeta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta} \\ \zeta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \zeta^* \\ \bar{\zeta} & 0 \end{bmatrix}$
σ_{d7}	$\begin{bmatrix} 0 & i\delta \\ i\delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\bar{\delta} \\ i\bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta}^* \\ \zeta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\bar{\epsilon}^* \\ i\bar{\epsilon} & 0 \end{bmatrix}$
σ_{d8}	$\begin{bmatrix} 0 & i\bar{\epsilon}^* \\ i\bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\bar{\delta}^* \\ i\bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\delta^* \\ i\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\epsilon^* \\ i\epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta}^* \\ \zeta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \delta & 0 \end{bmatrix}$
σ_{d9}	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta}^* \\ \zeta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\bar{\epsilon} \\ i\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\delta \\ i\delta^* & 0 \end{bmatrix}$
σ_{d10}	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\epsilon \\ i\epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\bar{\epsilon} \\ i\bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\zeta}^* \\ \zeta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & i\delta^* \\ i\delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \bar{\epsilon}^* & 0 \end{bmatrix}$

$\delta = \exp(2\pi i/40)$, $\epsilon = \exp(6\pi i/40)$, $\zeta = \exp(2\pi i/8)$, $\eta = \exp(2\pi i/5)$, $\theta = \exp(4\pi i/5)$

T **49**.8 Direct products of representations § **16**–8, p. 81

$\mathbf{D}_{10d}$	A_1	A_2	B_1	B_2	E_1	E_2	E_3	E_4
A_1	A_1	A_2	B_1	B_2	E_1	E_2	E_3	E_4
A_2		A_1	B_2	B_1	E_1	E_2	E_3	E_4
B_1			A_1	A_2	E_9	E_8	E_7	E_6
B_2				A_1	E_9	E_8	E_7	E_6
E_1					$A_1 \oplus \{A_2\} \oplus E_2$	$E_1 \oplus E_3$	$E_2 \oplus E_4$	$E_3 \oplus E_5$
E_2						$A_1 \oplus \{A_2\} \oplus E_4$	$E_1 \oplus E_5$	$E_2 \oplus E_6$
E_3							$A_1 \oplus \{A_2\} \oplus E_6$	$E_1 \oplus E_7$
E_4								$A_1 \oplus \{A_2\} \oplus E_8$

$\twoheadrightarrow$

T **49**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{10d}$	E_5	E_6	E_7	E_8	E_9
A_1	E_5	E_6	E_7	E_8	E_9
A_2	E_5	E_6	E_7	E_8	E_9
B_1	E_5	E_4	E_3	E_2	E_1
B_2	E_5	E_4	E_3	E_2	E_1
E_1	$E_4 \oplus E_6$	$E_5 \oplus E_7$	$E_6 \oplus E_8$	$E_7 \oplus E_9$	$B_1 \oplus B_2 \oplus E_8$
E_2	$E_3 \oplus E_7$	$E_4 \oplus E_8$	$E_5 \oplus E_9$	$B_1 \oplus B_2 \oplus E_6$	$E_7 \oplus E_9$
E_3	$E_2 \oplus E_8$	$E_3 \oplus E_9$	$B_1 \oplus B_2 \oplus E_4$	$E_5 \oplus E_9$	$E_6 \oplus E_8$
E_4	$E_1 \oplus E_9$	$B_1 \oplus B_2 \oplus E_2$	$E_3 \oplus E_9$	$E_4 \oplus E_8$	$E_5 \oplus E_7$
E_5	$A_1 \oplus \{A_2\} \oplus B_1 \oplus B_2$	$E_1 \oplus E_9$	$E_2 \oplus E_8$	$E_3 \oplus E_7$	$E_4 \oplus E_6$
E_6		$A_1 \oplus \{A_2\} \oplus E_8$	$E_1 \oplus E_7$	$E_2 \oplus E_6$	$E_3 \oplus E_5$
E_7			$A_1 \oplus \{A_2\} \oplus E_6$	$E_1 \oplus E_5$	$E_2 \oplus E_4$
E_8				$A_1 \oplus \{A_2\} \oplus E_4$	$E_1 \oplus E_3$
E_9					$A_1 \oplus \{A_2\} \oplus E_2$

$\twoheadrightarrow$

T **49**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{10d}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$
A_1	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$
A_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	$E_{9/2}$
B_1	$E_{19/2}$	$E_{17/2}$	$E_{15/2}$	$E_{13/2}$	$E_{11/2}$
B_2	$E_{19/2}$	$E_{17/2}$	$E_{15/2}$	$E_{13/2}$	$E_{11/2}$
E_1	$E_{17/2} \oplus E_{19/2}$	$E_{15/2} \oplus E_{19/2}$	$E_{13/2} \oplus E_{17/2}$	$E_{11/2} \oplus E_{15/2}$	$E_{9/2} \oplus E_{13/2}$
E_2	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{9/2}$	$E_{3/2} \oplus E_{11/2}$	$E_{5/2} \oplus E_{13/2}$
E_3	$E_{13/2} \oplus E_{15/2}$	$E_{11/2} \oplus E_{17/2}$	$E_{9/2} \oplus E_{19/2}$	$E_{7/2} \oplus E_{19/2}$	$E_{5/2} \oplus E_{17/2}$
E_4	$E_{7/2} \oplus E_{9/2}$	$E_{5/2} \oplus E_{11/2}$	$E_{3/2} \oplus E_{13/2}$	$E_{1/2} \oplus E_{15/2}$	$E_{1/2} \oplus E_{17/2}$
E_5	$E_{9/2} \oplus E_{11/2}$	$E_{7/2} \oplus E_{13/2}$	$E_{5/2} \oplus E_{15/2}$	$E_{3/2} \oplus E_{17/2}$	$E_{1/2} \oplus E_{19/2}$
E_6	$E_{11/2} \oplus E_{13/2}$	$E_{9/2} \oplus E_{15/2}$	$E_{7/2} \oplus E_{17/2}$	$E_{5/2} \oplus E_{19/2}$	$E_{3/2} \oplus E_{19/2}$
E_7	$E_{5/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{9/2}$	$E_{1/2} \oplus E_{11/2}$	$E_{1/2} \oplus E_{13/2}$	$E_{3/2} \oplus E_{15/2}$
E_8	$E_{15/2} \oplus E_{17/2}$	$E_{13/2} \oplus E_{19/2}$	$E_{11/2} \oplus E_{19/2}$	$E_{9/2} \oplus E_{17/2}$	$E_{7/2} \oplus E_{15/2}$
E_9	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{5/2} \oplus E_{9/2}$	$E_{7/2} \oplus E_{11/2}$
$E_{1/2}$	$\{A_1\} \oplus A_2 \oplus E_9$	$E_2 \oplus E_9$	$E_2 \oplus E_7$	$E_4 \oplus E_7$	$E_4 \oplus E_5$
$E_{3/2}$		$\{A_1\} \oplus A_2 \oplus E_7$	$E_4 \oplus E_9$	$E_2 \oplus E_5$	$E_6 \oplus E_7$
$E_{5/2}$			$\{A_1\} \oplus A_2 \oplus E_5$	$E_6 \oplus E_9$	$E_2 \oplus E_3$
$E_{7/2}$				$\{A_1\} \oplus A_2 \oplus E_3$	$E_8 \oplus E_9$
$E_{9/2}$					$\{A_1\} \oplus A_2 \oplus E_1$

$\twoheadrightarrow$

T **49**.8 Direct products of representations *(cont.)*

$\mathbf{D}_{10d}$	$E_{11/2}$	$E_{13/2}$	$E_{15/2}$	$E_{17/2}$	$E_{19/2}$
A_1	$E_{11/2}$	$E_{13/2}$	$E_{15/2}$	$E_{17/2}$	$E_{19/2}$
A_2	$E_{11/2}$	$E_{13/2}$	$E_{15/2}$	$E_{17/2}$	$E_{19/2}$
B_1	$E_{9/2}$	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
B_2	$E_{9/2}$	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
E_1	$E_{7/2} \oplus E_{11/2}$	$E_{5/2} \oplus E_{9/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$
E_2	$E_{7/2} \oplus E_{15/2}$	$E_{9/2} \oplus E_{17/2}$	$E_{11/2} \oplus E_{19/2}$	$E_{13/2} \oplus E_{19/2}$	$E_{15/2} \oplus E_{17/2}$
E_3	$E_{3/2} \oplus E_{15/2}$	$E_{1/2} \oplus E_{13/2}$	$E_{1/2} \oplus E_{11/2}$	$E_{3/2} \oplus E_{9/2}$	$E_{5/2} \oplus E_{7/2}$
E_4	$E_{3/2} \oplus E_{19/2}$	$E_{5/2} \oplus E_{19/2}$	$E_{7/2} \oplus E_{17/2}$	$E_{9/2} \oplus E_{15/2}$	$E_{11/2} \oplus E_{13/2}$
E_5	$E_{1/2} \oplus E_{19/2}$	$E_{3/2} \oplus E_{17/2}$	$E_{5/2} \oplus E_{15/2}$	$E_{7/2} \oplus E_{13/2}$	$E_{9/2} \oplus E_{11/2}$
E_6	$E_{1/2} \oplus E_{17/2}$	$E_{1/2} \oplus E_{15/2}$	$E_{3/2} \oplus E_{13/2}$	$E_{5/2} \oplus E_{11/2}$	$E_{7/2} \oplus E_{9/2}$
E_7	$E_{5/2} \oplus E_{17/2}$	$E_{7/2} \oplus E_{19/2}$	$E_{9/2} \oplus E_{19/2}$	$E_{11/2} \oplus E_{17/2}$	$E_{13/2} \oplus E_{15/2}$
E_8	$E_{5/2} \oplus E_{13/2}$	$E_{3/2} \oplus E_{11/2}$	$E_{1/2} \oplus E_{9/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{5/2}$
E_9	$E_{9/2} \oplus E_{13/2}$	$E_{11/2} \oplus E_{15/2}$	$E_{13/2} \oplus E_{17/2}$	$E_{15/2} \oplus E_{19/2}$	$E_{17/2} \oplus E_{19/2}$
$E_{1/2}$	$E_5 \oplus E_6$	$E_3 \oplus E_6$	$E_3 \oplus E_8$	$E_1 \oplus E_8$	$B_1 \oplus B_2 \oplus E_1$
$E_{3/2}$	$E_3 \oplus E_4$	$E_5 \oplus E_8$	$E_1 \oplus E_6$	$B_1 \oplus B_2 \oplus E_3$	$E_1 \oplus E_8$
$E_{5/2}$	$E_7 \oplus E_8$	$E_1 \oplus E_4$	$B_1 \oplus B_2 \oplus E_5$	$E_1 \oplus E_6$	$E_3 \oplus E_8$
$E_{7/2}$	$E_1 \oplus E_2$	$B_1 \oplus B_2 \oplus E_7$	$E_1 \oplus E_4$	$E_5 \oplus E_8$	$E_3 \oplus E_6$
$E_{9/2}$	$B_1 \oplus B_2 \oplus E_9$	$E_1 \oplus E_2$	$E_7 \oplus E_8$	$E_3 \oplus E_4$	$E_5 \oplus E_6$
$E_{11/2}$	$\{A_1\} \oplus A_2 \oplus E_1$	$E_8 \oplus E_9$	$E_2 \oplus E_3$	$E_6 \oplus E_7$	$E_4 \oplus E_5$
$E_{13/2}$		$\{A_1\} \oplus A_2 \oplus E_3$	$E_6 \oplus E_9$	$E_2 \oplus E_5$	$E_4 \oplus E_7$
$E_{15/2}$			$\{A_1\} \oplus A_2 \oplus E_5$	$E_4 \oplus E_9$	$E_2 \oplus E_7$
$E_{17/2}$				$\{A_1\} \oplus A_2 \oplus E_7$	$E_2 \oplus E_9$
$E_{19/2}$					$\{A_1\} \oplus A_2 \oplus E_9$

T **49**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{D}_{10d}$	$(\mathbf{C}_{10v})$	$(\mathbf{C}_{5v})$	$(\mathbf{C}_{2v})$	$(\mathbf{D}_{2d})$	$(\mathbf{D}_{10})$
A_1	A_1	A_1	A_1	A_1	A_1
A_2	A_2	A_2	A_2	A_2	A_2
B_1	A_2	A_2	A_2	B_1	A_1
B_2	A_1	A_1	A_1	B_2	A_2
E_1	E_1	E_1	$B_1 \oplus B_2$	E	E_1
E_2	E_2	E_2	$A_1 \oplus A_2$	$B_1 \oplus B_2$	E_2
E_3	E_3	E_2	$B_1 \oplus B_2$	E	E_3
E_4	E_4	E_1	$A_1 \oplus A_2$	$A_1 \oplus A_2$	E_4
E_5	$B_1 \oplus B_2$	$A_1 \oplus A_2$	$B_1 \oplus B_2$	E	$B_1 \oplus B_2$
E_6	E_4	E_1	$A_1 \oplus A_2$	$B_1 \oplus B_2$	E_4
E_7	E_3	E_2	$B_1 \oplus B_2$	E	E_3
E_8	E_2	E_2	$A_1 \oplus A_2$	$A_1 \oplus A_2$	E_2
E_9	E_1	E_1	$B_1 \oplus B_2$	E	E_1
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2}$	$E_{3/2}$	$E_{3/2}$
$E_{5/2}$	$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
$E_{7/2}$	$E_{7/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	$E_{7/2}$
$E_{9/2}$	$E_{9/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{9/2}$
$E_{11/2}$	$E_{9/2}$	$E_{1/2}$	$E_{1/2}$	$E_{3/2}$	$E_{9/2}$
$E_{13/2}$	$E_{7/2}$	$E_{3/2}$	$E_{1/2}$	$E_{3/2}$	$E_{7/2}$
$E_{15/2}$	$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
$E_{17/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	$E_{3/2}$
$E_{19/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{3/2}$	$E_{1/2}$

$\twoheadrightarrow$

T **49**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{10d}$	$(\mathbf{D}_5)$	$(\mathbf{D}_2)$	$\mathbf{S}_{20}$	$\mathbf{S}_4$	$(\mathbf{C}_s)$
A_1	A_1	A	A	A	A'
A_2	A_2	B_1	A	A	A''
B_1	A_1	A	B	B	A''
B_2	A_2	B_1	B	B	A'
E_1	E_1	$B_2 \oplus B_3$	${}^1E_1 \oplus {}^2E_1$	${}^1E \oplus {}^2E$	$A' \oplus A''$
E_2	E_2	$A \oplus B_1$	${}^1E_2 \oplus {}^2E_2$	$2B$	$A' \oplus A''$
E_3	E_2	$B_2 \oplus B_3$	${}^1E_3 \oplus {}^2E_3$	${}^1E \oplus {}^2E$	$A' \oplus A''$
E_4	E_1	$A \oplus B_1$	${}^1E_4 \oplus {}^2E_4$	$2A$	$A' \oplus A''$
E_5	$A_1 \oplus A_2$	$B_2 \oplus B_3$	${}^1E_5 \oplus {}^2E_5$	${}^1E \oplus {}^2E$	$A' \oplus A''$
E_6	E_1	$A \oplus B_1$	${}^1E_6 \oplus {}^2E_6$	$2B$	$A' \oplus A''$
E_7	E_2	$B_2 \oplus B_3$	${}^1E_7 \oplus {}^2E_7$	${}^1E \oplus {}^2E$	$A' \oplus A''$
E_8	E_2	$A \oplus B_1$	${}^1E_8 \oplus {}^2E_8$	$2A$	$A' \oplus A''$
E_9	E_1	$B_2 \oplus B_3$	${}^1E_9 \oplus {}^2E_9$	${}^1E \oplus {}^2E$	$A' \oplus A''$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	$E_{3/2}$	$E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	$E_{3/2}$	$E_{1/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{9/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{9/2} \oplus {}^2E_{9/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{11/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{11/2} \oplus {}^2E_{11/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{13/2}$	$E_{3/2}$	$E_{1/2}$	${}^1E_{13/2} \oplus {}^2E_{13/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{15/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{1/2}$	${}^1E_{15/2} \oplus {}^2E_{15/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{17/2}$	$E_{3/2}$	$E_{1/2}$	${}^1E_{17/2} \oplus {}^2E_{17/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{19/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{19/2} \oplus {}^2E_{19/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

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T **49**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{D}_{10d}$	$\mathbf{C}_{10}$	$\mathbf{C}_5$	$\mathbf{C}_2$	$(\mathbf{C}_2)$
			C_2	C_2'
A_1	A	A	A	A
A_2	A	A	A	B
B_1	A	A	A	A
B_2	A	A	A	B
E_1	${}^1E_1 \oplus {}^2E_1$	${}^1E_1 \oplus {}^2E_1$	$2B$	$A \oplus B$
E_2	${}^1E_2 \oplus {}^2E_2$	${}^1E_2 \oplus {}^2E_2$	$2A$	$A \oplus B$
E_3	${}^1E_3 \oplus {}^2E_3$	${}^1E_2 \oplus {}^2E_2$	$2B$	$A \oplus B$
E_4	${}^1E_4 \oplus {}^2E_4$	${}^1E_1 \oplus {}^2E_1$	$2A$	$A \oplus B$
E_5	$2B$	$2A$	$2B$	$A \oplus B$
E_6	${}^1E_4 \oplus {}^2E_4$	${}^1E_1 \oplus {}^2E_1$	$2A$	$A \oplus B$
E_7	${}^1E_3 \oplus {}^2E_3$	${}^1E_2 \oplus {}^2E_2$	$2B$	$A \oplus B$
E_8	${}^1E_2 \oplus {}^2E_2$	${}^1E_2 \oplus {}^2E_2$	$2A$	$A \oplus B$
E_9	${}^1E_1 \oplus {}^2E_1$	${}^1E_1 \oplus {}^2E_1$	$2B$	$A \oplus B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$2A_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{9/2}$	${}^1E_{9/2} \oplus {}^2E_{9/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{11/2}$	${}^1E_{9/2} \oplus {}^2E_{9/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{13/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{15/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$2A_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{17/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{19/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **49**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{D}_{10d}$
$20n$	$(n+1)\,A_1 \oplus$ $n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5 \oplus 2E_6 \oplus 2E_7 \oplus 2E_8 \oplus 2E_9)$
$20n+1$	$n\,(A_1 \oplus A_2 \oplus B_1 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5 \oplus 2E_6 \oplus 2E_7 \oplus 2E_8 \oplus 2E_9) \oplus$ $(n+1)(B_2 \oplus E_1)$
$20n+2$	$(n+1)(A_1 \oplus E_2 \oplus E_9) \oplus$ $n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5 \oplus 2E_6 \oplus 2E_7 \oplus 2E_8 \oplus E_9)$
$20n+3$	$n\,(A_1 \oplus A_2 \oplus B_1 \oplus E_1 \oplus 2E_2 \oplus E_3 \oplus 2E_4 \oplus 2E_5 \oplus 2E_6 \oplus 2E_7 \oplus E_8 \oplus 2E_9) \oplus$ $(n+1)(B_2 \oplus E_1 \oplus E_3 \oplus E_8)$
$20n+4$	$(n+1)(A_1 \oplus E_2 \oplus E_4 \oplus E_7 \oplus E_9) \oplus$ $n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus 2E_3 \oplus E_4 \oplus 2E_5 \oplus 2E_6 \oplus E_7 \oplus 2E_8 \oplus E_9)$
$20n+5$	$n\,(A_1 \oplus A_2 \oplus B_1 \oplus E_1 \oplus 2E_2 \oplus E_3 \oplus 2E_4 \oplus E_5 \oplus E_6 \oplus 2E_7 \oplus E_8 \oplus 2E_9) \oplus$ $(n+1)(B_2 \oplus E_1 \oplus E_3 \oplus E_5 \oplus E_6 \oplus E_8)$
$20n+6$	$(n+1)(A_1 \oplus E_2 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7 \oplus E_9) \oplus$ $n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus 2E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7 \oplus 2E_8 \oplus E_9)$
$20n+7$	$n\,(A_1 \oplus A_2 \oplus B_1 \oplus E_1 \oplus 2E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7 \oplus E_8 \oplus 2E_9) \oplus$ $(n+1)(B_2 \oplus E_1 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7 \oplus E_8)$
$20n+8$	$(n+1)(A_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7 \oplus E_8 \oplus E_9) \oplus$ $n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7 \oplus E_8 \oplus E_9)$
$20n+9$	$n\,(A_1 \oplus A_2 \oplus B_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7 \oplus E_8 \oplus E_9) \oplus$ $(n+1)(B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7 \oplus E_8 \oplus E_9)$
$20n+10$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7 \oplus E_8 \oplus E_9) \oplus$ $n\,(A_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7 \oplus E_8 \oplus E_9)$
$20n+11$	$(n+1)(A_1 \oplus A_2 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7 \oplus E_8 \oplus 2E_9) \oplus$ $n\,(B_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7 \oplus E_8)$
$20n+12$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7 \oplus 2E_8 \oplus E_9) \oplus$ $n\,(A_2 \oplus E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_7 \oplus E_9)$
$20n+13$	$(n+1)(A_1 \oplus A_2 \oplus B_2 \oplus E_1 \oplus 2E_2 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus 2E_7 \oplus E_8 \oplus 2E_9) \oplus$ $n\,(B_1 \oplus E_1 \oplus E_3 \oplus E_4 \oplus E_5 \oplus E_6 \oplus E_8)$
$20n+14$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus 2E_3 \oplus E_4 \oplus E_5 \oplus 2E_6 \oplus E_7 \oplus 2E_8 \oplus E_9) \oplus$ $n\,(A_2 \oplus E_2 \oplus E_4 \oplus E_5 \oplus E_7 \oplus E_9)$
$20n+15$	$(n+1)(A_1 \oplus A_2 \oplus B_2 \oplus E_1 \oplus 2E_2 \oplus E_3 \oplus 2E_4 \oplus 2E_5 \oplus E_6 \oplus 2E_7 \oplus E_8 \oplus 2E_9) \oplus$ $n\,(B_1 \oplus E_1 \oplus E_3 \oplus E_6 \oplus E_8)$
$20n+16$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5 \oplus 2E_6 \oplus E_7 \oplus 2E_8 \oplus E_9) \oplus$ $n\,(A_2 \oplus E_2 \oplus E_7 \oplus E_9)$
$20n+17$	$(n+1)(A_1 \oplus A_2 \oplus B_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5 \oplus 2E_6 \oplus 2E_7 \oplus E_8 \oplus 2E_9) \oplus$ $n\,(B_1 \oplus E_1 \oplus E_8)$
$20n+18$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5 \oplus 2E_6 \oplus 2E_7 \oplus 2E_8 \oplus E_9) \oplus$ $n\,(A_2 \oplus E_9)$
$20n+19$	$(n+1)(A_1 \oplus A_2 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4 \oplus 2E_5 \oplus 2E_6 \oplus 2E_7 \oplus 2E_8 \oplus 2E_9) \oplus$ $n\,B_1$

$n = 0, 1, 2, \ldots$ ↠

T **49**.10 Subduction from O(3) *(cont.)*

j	$\mathbf{D}_{10d}$
$20n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus$ $2n\,(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$
$20n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus$ $2n\,(E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$
$20n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus$ $2n\,(E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$
$20n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus$ $2n\,(E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$
$20n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2}) \oplus$ $2n\,(E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$
$20n+\frac{11}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2}) \oplus$ $2n\,(E_{13/2} \oplus E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$
$20n+\frac{13}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2}) \oplus$ $2n\,(E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$
$20n+\frac{15}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2}) \oplus$ $2n\,(E_{17/2} \oplus E_{19/2})$
$20n+\frac{17}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2}) \oplus$ $2n\,E_{19/2}$
$20n+\frac{19}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$
$20n+\frac{21}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2}) \oplus$ $(2n+2)\,E_{19/2}$
$20n+\frac{23}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2}) \oplus$ $(2n+2)(E_{17/2} \oplus E_{19/2})$
$20n+\frac{25}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2}) \oplus$ $(2n+2)(E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$
$20n+\frac{27}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2}) \oplus$ $(2n+2)(E_{13/2} \oplus E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$
$20n+\frac{29}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2}) \oplus$ $(2n+2)(E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$
$20n+\frac{31}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus$ $(2n+2)(E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$
$20n+\frac{33}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus$ $(2n+2)(E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$
$20n+\frac{35}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus$ $(2n+2)(E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$
$20n+\frac{37}{2}$	$(2n+1)\,E_{1/2} \oplus$ $(2n+2)(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$
$20n+\frac{39}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2} \oplus E_{11/2} \oplus E_{13/2} \oplus E_{15/2} \oplus E_{17/2} \oplus E_{19/2})$

$n = 0, 1, 2, \ldots$

T **49**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{D}_{10d}$

a_2	e_1	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_2	E_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_3	E_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_4	E_4	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_5	E_5	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_6	E_6	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_7	E_7	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_8	E_8	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_9	E_9	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{5/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{7/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{9/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{11/2}$	$E_{11/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{13/2}$	$E_{13/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{15/2}$	$E_{15/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{17/2}$	$E_{17/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{19/2}$	$E_{19/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_1	e_1	E_9	
		1	2
1	1	1	0
1	2	0	1

b_1	e_2	E_8	
		1	2
1	1	1	0
1	2	0	1

b_1	e_3	E_7	
		1	2
1	1	1	0
1	2	0	1

b_1	e_4	E_6	
		1	2
1	1	1	0
1	2	0	1

b_1	e_5	E_5	
		1	2
1	1	0	1
1	2	1	0

b_1	e_6	E_4	
		1	2
1	1	1	0
1	2	0	1

b_1	e_7	E_3	
		1	2
1	1	1	0
1	2	0	1

b_1	e_8	E_2	
		1	2
1	1	1	0
1	2	0	1

b_1	e_9	E_1	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{1/2}$	$E_{19/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{3/2}$	$E_{17/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{5/2}$	$E_{15/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{7/2}$	$E_{13/2}$	
		1	2
1	1	1	0
1	2	0	1

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T **49**.11 Clebsch–Gordan coefficients *(cont.)*

b_1	$e_{9/2}$	$E_{11/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{11/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{13/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{15/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{17/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	1

b_1	$e_{19/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	1

b_2	e_1	E_9	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_2	E_8	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_3	E_7	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_4	E_6	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_5	E_5	
		1	2
1	1	0	$\bar{1}$
1	2	1	0

b_2	e_6	E_4	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_7	E_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_8	E_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	e_9	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{1/2}$	$E_{19/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{3/2}$	$E_{17/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{5/2}$	$E_{15/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{7/2}$	$E_{13/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{9/2}$	$E_{11/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{11/2}$	$E_{9/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{13/2}$	$E_{7/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{15/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{17/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

b_2	$e_{19/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e_1	e_1	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{\mathrm{u}}$	0	0
2	2	0	0	0	1

e_1	e_2	E_1		E_3	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	e_3	E_2		E_4	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_1	e_4	E_3		E_5	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_1	e_5	E_4		E_6	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	$\bar{1}$

$\mathrm{u} = 2^{-1/2}$

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T **49**.11 Clebsch–Gordan coefficients *(cont.)*

e_1	e_6	E_5		E_7	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_1	e_7	E_6		E_8	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	e_8	E_7		E_9	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_1	e_9	B_1	B_2	E_8	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_1	$e_{1/2}$	$E_{17/2}$		$E_{19/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_1	$e_{3/2}$	$E_{15/2}$		$E_{19/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_1	$e_{5/2}$	$E_{13/2}$		$E_{17/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_1	$e_{7/2}$	$E_{11/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	$e_{9/2}$	$E_{9/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_1	$e_{11/2}$	$E_{7/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_1	$e_{13/2}$	$E_{5/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_1	$e_{15/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	$e_{17/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_1	$e_{19/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_2	e_2	A_1	A_2	E_4	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_2	e_3	E_1		E_5	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_2	e_4	E_2		E_6	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	1	0	0	0

e_2	e_5	E_3		E_7	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	$\bar{1}$

$u = 2^{-1/2}$

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T **49**.11 Clebsch–Gordan coefficients *(cont.)*

e_2	e_6	E_4		E_8	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

e_2	e_7	E_5		E_9	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_2	e_8	B_1	B_2	E_6	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_2	e_9	E_7		E_9	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	1	0	0	0

e_2	$e_{1/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_2	$e_{3/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_2	$e_{5/2}$	$E_{1/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_2	$e_{7/2}$	$E_{3/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_2	$e_{9/2}$	$E_{5/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_2	$e_{11/2}$	$E_{7/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_2	$e_{13/2}$	$E_{9/2}$		$E_{17/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_2	$e_{15/2}$	$E_{11/2}$		$E_{19/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_2	$e_{17/2}$	$E_{13/2}$		$E_{19/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_2	$e_{19/2}$	$E_{15/2}$		$E_{17/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_3	e_3	A_1	A_2	E_6	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_3	e_4	E_1		E_7	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

e_3	e_5	E_2		E_8	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	$\bar{1}$	0	0

e_3	e_6	E_3		E_9	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	1	0	0	0

$u = 2^{-1/2}$

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T **49**.11 Clebsch–Gordan coefficients *(cont.)*

e_3	e_7	B_1	B_2	E_4	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_3	e_8	E_5		E_9	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_3	e_9	E_6		E_8	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

e_3	$e_{1/2}$	$E_{13/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_3	$e_{3/2}$	$E_{11/2}$		$E_{17/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_3	$e_{5/2}$	$E_{9/2}$		$E_{19/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_3	$e_{7/2}$	$E_{7/2}$		$E_{19/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_3	$e_{9/2}$	$E_{5/2}$		$E_{17/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_3	$e_{11/2}$	$E_{3/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_3	$e_{13/2}$	$E_{1/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_3	$e_{15/2}$	$E_{1/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_3	$e_{17/2}$	$E_{3/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_3	$e_{19/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_4	e_4	A_1	A_2	E_8	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_4	e_5	E_1		E_9	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	$\bar{1}$	0	0

e_4	e_6	B_1	B_2	E_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_4	e_7	E_3		E_9	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_4	e_8	E_4		E_8	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$u = 2^{-1/2}$

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T **49**.11 Clebsch–Gordan coefficients *(cont.)*

e_4	e_9	E_5 1	2	E_7 1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_4	$e_{1/2}$	$E_{7/2}$ 1	2	$E_{9/2}$ 1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_4	$e_{3/2}$	$E_{5/2}$ 1	2	$E_{11/2}$ 1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_4	$e_{5/2}$	$E_{3/2}$ 1	2	$E_{13/2}$ 1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_4	$e_{7/2}$	$E_{1/2}$ 1	2	$E_{15/2}$ 1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_4	$e_{9/2}$	$E_{1/2}$ 1	2	$E_{17/2}$ 1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_4	$e_{11/2}$	$E_{3/2}$ 1	2	$E_{19/2}$ 1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_4	$e_{13/2}$	$E_{5/2}$ 1	2	$E_{19/2}$ 1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_4	$e_{15/2}$	$E_{7/2}$ 1	2	$E_{17/2}$ 1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_4	$e_{17/2}$	$E_{9/2}$ 1	2	$E_{15/2}$ 1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_4	$e_{19/2}$	$E_{11/2}$ 1	2	$E_{13/2}$ 1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_5	e_5	A_1 1	A_2 1	B_1 1	B_2 1
1	1	0	0	u	u
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	u	$\bar{u}$

e_5	e_6	E_1 1	2	E_9 1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e_5	e_7	E_2 1	2	E_8 1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e_5	e_8	E_3 1	2	E_7 1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e_5	e_9	E_4 1	2	E_6 1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e_5	$e_{1/2}$	$E_{9/2}$ 1	2	$E_{11/2}$ 1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e_5	$e_{3/2}$	$E_{7/2}$ 1	2	$E_{13/2}$ 1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

$u = 2^{-1/2}$

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T **49**.11 Clebsch–Gordan coefficients *(cont.)*

e_5	$e_{5/2}$	$E_{5/2}$ 1	2	$E_{15/2}$ 1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_5	$e_{7/2}$	$E_{3/2}$ 1	2	$E_{17/2}$ 1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e_5	$e_{9/2}$	$E_{1/2}$ 1	2	$E_{19/2}$ 1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e_5	$e_{11/2}$	$E_{1/2}$ 1	2	$E_{19/2}$ 1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e_5	$e_{13/2}$	$E_{3/2}$ 1	2	$E_{17/2}$ 1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	$\bar{1}$	0	0

e_5	$e_{15/2}$	$E_{5/2}$ 1	2	$E_{15/2}$ 1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_5	$e_{17/2}$	$E_{7/2}$ 1	2	$E_{13/2}$ 1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e_5	$e_{19/2}$	$E_{9/2}$ 1	2	$E_{11/2}$ 1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e_6	e_6	A_1 1	A_2 1	E_8 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_6	e_7	E_1 1	2	E_7 1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_6	e_8	E_2 1	2	E_6 1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_6	e_9	E_3 1	2	E_5 1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	1	0	0

e_6	$e_{1/2}$	$E_{11/2}$ 1	2	$E_{13/2}$ 1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_6	$e_{3/2}$	$E_{9/2}$ 1	2	$E_{15/2}$ 1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_6	$e_{5/2}$	$E_{7/2}$ 1	2	$E_{17/2}$ 1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_6	$e_{7/2}$	$E_{5/2}$ 1	2	$E_{19/2}$ 1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_6	$e_{9/2}$	$E_{3/2}$ 1	2	$E_{19/2}$ 1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_6	$e_{11/2}$	$E_{1/2}$ 1	2	$E_{17/2}$ 1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

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T **49**.11 Clebsch–Gordan coefficients *(cont.)*

e_6	$e_{13/2}$	$E_{1/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_6	$e_{15/2}$	$E_{3/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_6	$e_{17/2}$	$E_{5/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_6	$e_{19/2}$	$E_{7/2}$		$E_{9/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_7	e_7	A_1	A_2	E_6	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

e_7	e_8	E_1		E_5	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

e_7	e_9	E_2		E_4	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	1	0	0	0

e_7	$e_{1/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_7	$e_{3/2}$	$E_{3/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_7	$e_{5/2}$	$E_{1/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_7	$e_{7/2}$	$E_{1/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_7	$e_{9/2}$	$E_{3/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_7	$e_{11/2}$	$E_{5/2}$		$E_{17/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_7	$e_{13/2}$	$E_{7/2}$		$E_{19/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_7	$e_{15/2}$	$E_{9/2}$		$E_{19/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_7	$e_{17/2}$	$E_{11/2}$		$E_{17/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_7	$e_{19/2}$	$E_{13/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_8	e_8	A_1	A_2	E_4	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	1	0

$u = 2^{-1/2}$

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T **49**.11 Clebsch–Gordan coefficients *(cont.)*

e_8	e_9	E_1		E_3	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

e_8	$e_{1/2}$	$E_{15/2}$		$E_{17/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_8	$e_{3/2}$	$E_{13/2}$		$E_{19/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_8	$e_{5/2}$	$E_{11/2}$		$E_{19/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_8	$e_{7/2}$	$E_{9/2}$		$E_{17/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_8	$e_{9/2}$	$E_{7/2}$		$E_{15/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_8	$e_{11/2}$	$E_{5/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_8	$e_{13/2}$	$E_{3/2}$		$E_{11/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_8	$e_{15/2}$	$E_{1/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_8	$e_{17/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_8	$e_{19/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_9	e_9	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{u}$	0	0
2	2	0	0	0	1

e_9	$e_{1/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_9	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_9	$e_{5/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_9	$e_{7/2}$	$E_{5/2}$		$E_{9/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_9	$e_{9/2}$	$E_{7/2}$		$E_{11/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

e_9	$e_{11/2}$	$E_{9/2}$		$E_{13/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

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T **49**.11 Clebsch–Gordan coefficients *(cont.)*

e_9	$e_{13/2}$	$E_{11/2}$ 1	2	$E_{15/2}$ 1	2
1	1	0	0	0	1
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_9	$e_{15/2}$	$E_{13/2}$ 1	2	$E_{17/2}$ 1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_9	$e_{17/2}$	$E_{15/2}$ 1	2	$E_{19/2}$ 1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

e_9	$e_{19/2}$	$E_{17/2}$ 1	2	$E_{19/2}$ 1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

$e_{1/2}$	$e_{1/2}$	A_1 1	A_2 1	E_9 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{3/2}$	E_2 1	2	E_9 1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	1	0	0

$e_{1/2}$	$e_{5/2}$	E_2 1	2	E_7 1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{1/2}$	$e_{7/2}$	E_4 1	2	E_7 1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{9/2}$	E_4 1	2	E_5 1	2
1	1	1	0	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	0	1	0	0

$e_{1/2}$	$e_{11/2}$	E_5 1	2	E_6 1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{13/2}$	E_3 1	2	E_6 1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	1	0	0

$e_{1/2}$	$e_{15/2}$	E_3 1	2	E_8 1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{1/2}$	$e_{17/2}$	E_1 1	2	E_8 1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{19/2}$	B_1 1	B_2 1	E_1 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{3/2}$	$e_{3/2}$	A_1 1	A_2 1	E_7 1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{5/2}$	E_4 1	2	E_9 1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{3/2}$	$e_{7/2}$	E_2 1	2	E_5 1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

$e_{3/2}$	$e_{9/2}$	E_6 1	2	E_7 1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

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T **49**.11 Clebsch–Gordan coefficients *(cont.)*

$e_{3/2}$	$e_{11/2}$	E_3		E_4	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

$e_{3/2}$	$e_{13/2}$	E_5		E_8	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{15/2}$	E_1		E_6	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{17/2}$	B_1	B_2	E_3	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{3/2}$	$e_{19/2}$	E_1		E_8	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$e_{5/2}$	$e_{5/2}$	A_1	A_2	E_5	
		1	1	1	2
1	1	0	0	0	1
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{5/2}$	$e_{7/2}$	E_6		E_9	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{5/2}$	$e_{9/2}$	E_2		E_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{5/2}$	$e_{11/2}$	E_7		E_8	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{5/2}$	$e_{13/2}$	E_1		E_4	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{5/2}$	$e_{15/2}$	B_1	B_2	E_5	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{5/2}$	$e_{17/2}$	E_1		E_6	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{5/2}$	$e_{19/2}$	E_3		E_8	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{7/2}$	$e_{7/2}$	A_1	A_2	E_3	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{7/2}$	$e_{9/2}$	E_8		E_9	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

$e_{7/2}$	$e_{11/2}$	E_1		E_2	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$e_{7/2}$	$e_{13/2}$	B_1	B_2	E_7	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{7/2}$	$e_{15/2}$	E_1		E_4	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$\mathrm{u} = 2^{-1/2}$

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T **49**.11 Clebsch–Gordan coefficients *(cont.)*

$e_{7/2}$	$e_{17/2}$	E_5		E_8	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{7/2}$	$e_{19/2}$	E_3		E_6	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

$e_{9/2}$	$e_{9/2}$	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{9/2}$	$e_{11/2}$	B_1	B_2	E_9	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{9/2}$	$e_{13/2}$	E_1		E_2	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{9/2}$	$e_{15/2}$	E_7		E_8	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{9/2}$	$e_{17/2}$	E_3		E_4	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	1	0	0

$e_{9/2}$	$e_{19/2}$	E_5		E_6	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{11/2}$	$e_{11/2}$	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$e_{11/2}$	$e_{13/2}$	E_8		E_9	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	1	0	0

$e_{11/2}$	$e_{15/2}$	E_2		E_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	1	0	0	0

$e_{11/2}$	$e_{17/2}$	E_6		E_7	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{11/2}$	$e_{19/2}$	E_4		E_5	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	0	1	0	0

$e_{13/2}$	$e_{13/2}$	A_1	A_2	E_3	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{13/2}$	$e_{15/2}$	E_6		E_9	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{13/2}$	$e_{17/2}$	E_2		E_5	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	1	0	0	0

$e_{13/2}$	$e_{19/2}$	E_4		E_7	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

$e_{15/2}$	$e_{15/2}$	A_1	A_2	E_5	
		1	1	1	2
1	1	0	0	0	1
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$\mathrm{u} = 2^{-1/2}$

↠

T **49**.11 Clebsch–Gordan coefficients *(cont.)*

$e_{15/2}$	$e_{17/2}$	E_4 1	2	E_9 1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

$e_{15/2}$	$e_{19/2}$	E_2 1	2	E_7 1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

$e_{17/2}$	$e_{17/2}$	A_1 1	A_2 1	E_7 1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	1	0

$e_{17/2}$	$e_{19/2}$	E_2 1	2	E_9 1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

$e_{19/2}$	$e_{19/2}$	A_1 1	A_2 1	E_9 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{u}$	u	0	0
2	2	0	0	0	1

$u = 2^{-1/2}$

The groups $\mathbf{C}_{nv}$

Notation for headers

Items in header read from left to right

1		Hermann–Mauguin symbol for the point group.
2		$\lvert G\rvert$ order of the group.
3		$\lvert C\rvert$ number of classes in the group.
4		$\lvert\widetilde{C}\rvert$ number of classes in the double group.
5		Number of the table.
6		Page reference for the notation of the header, of the first five subsections below it, and of the footers.
7	□	This symbol indicates a crystallographic point group.
8		Schönflies notation for the point group.

Notation for the first five subsections below the header

(1) Product forms — Direct and semidirect product forms. ⊗ Direct product. ⓢ Semidirect product.

(2) Group chains *(See pp.* 41, 67*)* — Groups underlined: invariant.
Groups in brackets: when subducing one or more of the representations in the tables to these subgroups in the setting used for them in the tables a change of bases (similarity transformation) is required.
For $\mathbf{C}_{3v}$ two settings A and B are used (see **15**.2) and this group is labelled with superscripts A and B, respectively, in the corresponding group chains.

(3) Operations of G — Lists all the operations of G, enclosing in brackets all the operations of the same class.

(4) Operations of $\widetilde{G}$ — Lists all the operations of $\widetilde{G}$, enclosing in brackets all the operations of the same class.

(5) Classes and representations — $\lvert r\rvert$ number of regular classes in G (p. 51).
$\lvert \mathrm{i}\rvert$ number of irregular classes in G (p. 51).
$\lvert I\rvert$ number of irreducible representations in G.
$\lvert\widetilde{I}\rvert$ number of spinor representations, also called the number of double-group representations.

Use of the footers

Finding your way about the tables — Each page of the tables contains a footer giving the ordering of the group types listed in the tables. The current group type is recognized because it carries no page number under its name. On opening the tables at any page you should then see at a glance whether to move backwards or forwards to find the group type desired. More precisely, the page number for the group type desired gives the page, like the present one, which carries the contents for the group type wanted.

(1) Product forms: $\mathbf{C}_2 \otimes \mathbf{C}_s$.

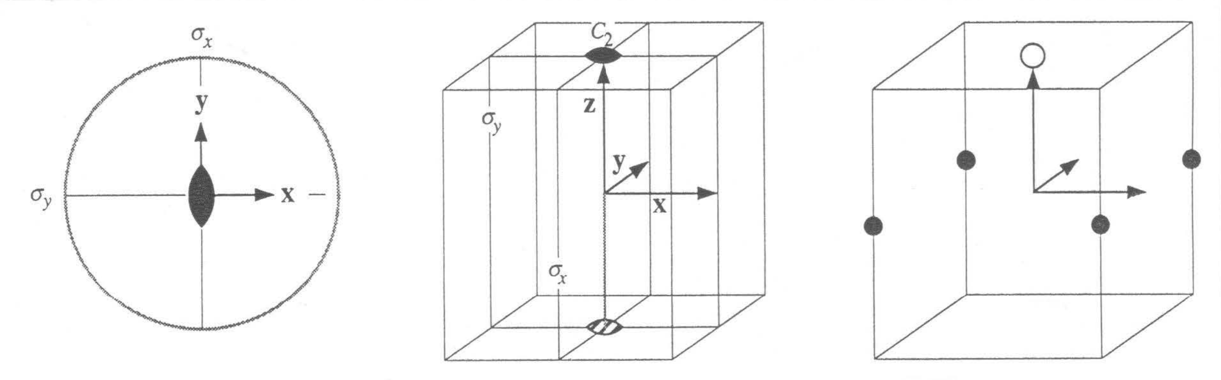
(2) Group chains: $\mathbf{C}_{10v} \supset (\mathbf{C}_{2v}) \supset (\underline{\mathbf{C}_s})$, $\mathbf{C}_{10v} \supset (\mathbf{C}_{2v}) \supset \underline{\mathbf{C}_2}$, $\mathbf{C}_{6v} \supset (\mathbf{C}_{2v}) \supset (\underline{\mathbf{C}_s})$, $\mathbf{C}_{6v} \supset (\mathbf{C}_{2v}) \supset \underline{\mathbf{C}_2}$,
$\mathbf{C}_{4v} \supset (\underline{\mathbf{C}_{2v}}) \supset (\underline{\mathbf{C}_s})$, $\mathbf{C}_{4v} \supset (\underline{\mathbf{C}_{2v}}) \supset \underline{\mathbf{C}_2}$, $\mathbf{D}_{2d} \supset (\underline{\mathbf{C}_{2v}}) \supset (\underline{\mathbf{C}_s})$, $\mathbf{D}_{2d} \supset (\underline{\mathbf{C}_{2v}}) \supset \underline{\mathbf{C}_2}$,
$\mathbf{D}_{7h} \supset (\mathbf{C}_{2v}) \supset (\underline{\mathbf{C}_s})$, $\mathbf{D}_{7h} \supset (\mathbf{C}_{2v}) \supset \underline{\mathbf{C}_2}$, $\mathbf{D}_{5h} \supset (\mathbf{C}_{2v}) \supset (\underline{\mathbf{C}_s})$, $\mathbf{D}_{5h} \supset (\mathbf{C}_{2v}) \supset \underline{\mathbf{C}_2}$,
$\mathbf{D}_{3h} \supset (\mathbf{C}_{2v}) \supset (\underline{\mathbf{C}_s})$, $\mathbf{D}_{3h} \supset (\mathbf{C}_{2v}) \supset \underline{\mathbf{C}_2}$, $\mathbf{D}_{2h} \supset (\underline{\mathbf{C}_{2v}}) \supset (\underline{\mathbf{C}_s})$, $\mathbf{D}_{2h} \supset (\underline{\mathbf{C}_{2v}}) \supset \underline{\mathbf{C}_2}$.

(3) Operations of G: E, C_2, σ_x, σ_y.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, $(C_2, \widetilde{C}_2)$, $(\sigma_x, \widetilde{\sigma}_x)$, $(\sigma_y, \widetilde{\sigma}_y)$.

(5) Classes and representations: $|r| = 1$, $|i| = 3$, $|I| = 4$, $|\widetilde{I}| = 1$.

F **50** See Chapter **15**, p. 65



Examples: H_2O, H_2CO, CH_2Cl_2, 1,2,3-trichlorobenzene $C_6H_3Cl_3$, propane $(CH_3)_2CH_2$.

T **50**.1 Parameters
Use T **31**.1 ◇. § **16**–1, p. 68

T **50**.2 Multiplication table
Use T **31**.2 ◇. § **16**–2, p. 69

T **50**.3 Factor table
Use T **31**.3 ◇. § **16**–3, p. 70

T **50**.4 Character table
§ **16**–4, p. 71

$\mathbf{C}_{2v}$	E	C_2	σ_x	σ_y	τ
A_1	1	1	1	1	a
A_2	1	1	−1	−1	a
B_1	1	−1	−1	1	a
B_2	1	−1	1	−1	a
$E_{1/2}$	2	0	0	0	c

T **50**.5 Cartesian tensors and *s, p, d,* and *f* functions
§ **16**–5, p. 72

$\mathbf{C}_{2v}$	0	1	2	3
A_1	$^\square 1$	$^\square z$	$^\square x^2, y^2, {}^\square z^2$	$^\square x^2 z, y^2 z, {}^\square z^3$
A_2		R_z	$^\square xy$	$^\square xyz$
B_1		$^\square x, R_y$	$^\square zx$	$^\square x^3, xy^2, {}^\square xz^2$
B_2		$^\square y, R_x$	$^\square yz$	$^\square x^2 y, y^3, {}^\square yz^2$

T **50**.6 Symmetrized bases § **16**–6, p. 74

$\mathbf{C}_{2v}$	$\langle \lvert j\,m\rangle \rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$		1	2
A_2	$\lvert 2\,2\rangle_-$		1	2
B_1	$\lvert 1\,1\rangle_-$		1	2
B_2	$\lvert 1\,1\rangle_+$		1	2
$E_{1/2}$	$\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \rvert$	$\langle \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle, -\lvert \frac{3}{2}\,\frac{3}{2}\rangle \rvert$	2	± 2
	$\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, -\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \rvert^\bullet$	$\langle \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle, \lvert \frac{3}{2}\,\frac{3}{2}\rangle \rvert^\bullet$	2	± 2

T **50**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{C}_{2v}$	$E_{1/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_2	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$
σ_x	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
σ_y	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$

T **50**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_{2v}$	A_1	A_2	B_1	B_2	$E_{1/2}$
A_1	A_1	A_2	B_1	B_2	$E_{1/2}$
A_2		A_1	B_2	B_1	$E_{1/2}$
B_1			A_1	A_2	$E_{1/2}$
B_2				A_1	$E_{1/2}$
$E_{1/2}$					$\{A_1\} \oplus A_2 \oplus B_1 \oplus B_2$

T **50**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{2v}$	$(\mathbf{C}_s)$ σ_x	$(\mathbf{C}_s)$ σ_y	$\mathbf{C}_2$
A_1	A'	A'	A
A_2	A''	A''	A
B_1	A''	A'	B
B_2	A'	A''	B
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **50**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{2v}$
$2n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus B_1 \oplus B_2)$
$2n+1$	$n\,A_2 \oplus (n+1)(A_1 \oplus B_1 \oplus B_2)$
$n+\frac{1}{2}$	$(n+1)\,E_{1/2}$

$n = 0, 1, 2, \ldots$

T **50**.11 Clebsch–Gordan coefficients § **16**–11, p. 83 $\mathbf{C}_{2v}$

a_2	$e_{1/2}$	$E_{1/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

b_1	$e_{1/2}$	$E_{1/2}$ 1	2
1	1	0	$\bar{1}$
1	2	1	0

b_2	$e_{1/2}$	$E_{1/2}$ 1	2
1	1	0	1
1	2	1	0

$e_{1/2}$	$e_{1/2}$	A_1 1	A_2 1	B_1 1	B_2 1
1	1	0	0	u	u
1	2	u	u	0	0
2	1	$\bar{\mathrm{u}}$	u	0	0
2	2	0	0	u	$\bar{\mathrm{u}}$

$\mathrm{u} = 2^{-1/2}$

(1) Product forms: $\mathbf{C}_3 \circledotwedge \mathbf{C}_s$.

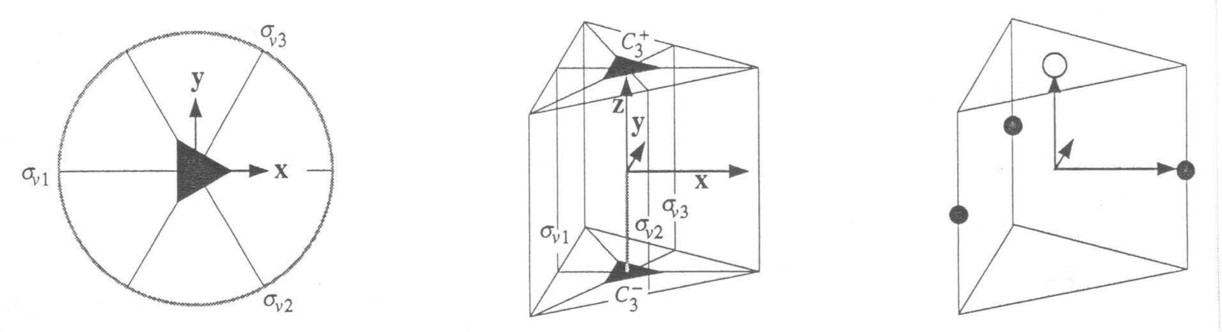
(2) Group chains: $\mathbf{C}_{9v} \supset (\mathbf{C}_{3v}) \supset (\mathbf{C}_s)$, $\mathbf{C}_{9v} \supset (\mathbf{C}_{3v}) \supset \underline{\mathbf{C}_3}$,
$\mathbf{C}_{6v} \supset (\underline{\mathbf{C}_{3v}}) \supset (\mathbf{C}_s)$, $\mathbf{C}_{6v} \supset (\underline{\mathbf{C}_{3v}}) \supset \underline{\mathbf{C}_3}$,
$\mathbf{D}_{3d} \supset (\underline{\mathbf{C}_{3v}}) \supset (\mathbf{C}_s)$, $\mathbf{D}_{3d} \supset (\underline{\mathbf{C}_{3v}}) \supset \underline{\mathbf{C}_3}$,
$\mathbf{D}_{3h} \supset (\underline{\mathbf{C}_{3v}}) \supset (\mathbf{C}_s)$, $\mathbf{D}_{3h} \supset (\underline{\mathbf{C}_{3v}}) \supset \underline{\mathbf{C}_3}$.

(3) Operations of G: E, (C_3^+, C_3^-), $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3})$.

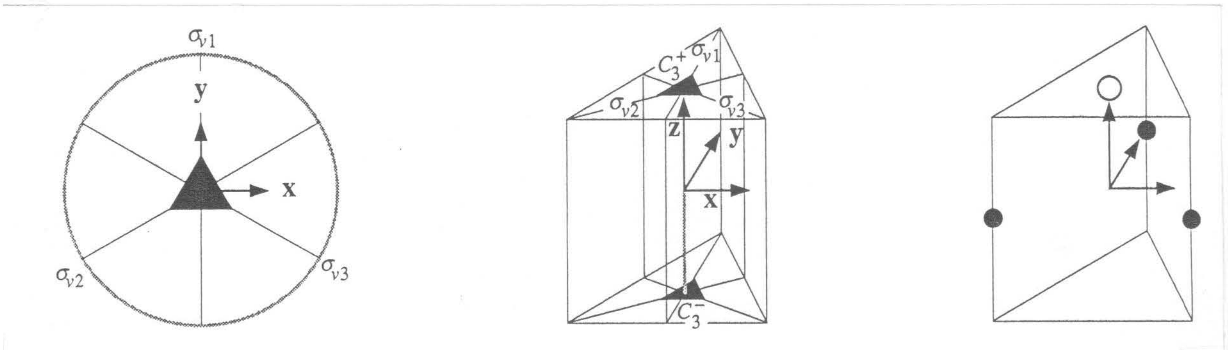
(4) Operations of $\widetilde{G}$: E, (C_3^+, C_3^-), $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3})$,
$\widetilde{E}$, $(\widetilde{C}_3^+, \widetilde{C}_3^-)$, $(\widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2}, \widetilde{\sigma}_{v3})$.

(5) Classes and representations: $|r| = 3$, $|\mathrm{i}| = 0$, $|I| = 3$, $|\widetilde{I}| = 3$.

F **51**A See Chapter **15**, p. 65



F **51**B See Chapter **15**, p. 65



Examples: NH_3, chloromethane CH_3Cl, isobutane $(CH_3)_3CH$.

T **51**.1A Parameters

§ **16**–1, p. 68

$\mathbf{C}_{3v}^{A}$	α	β	γ	ϕ	$\mathbf{n}$	λ	$\boldsymbol{\Lambda}$
E	0	0	0	0	$(0\ \ 0\ \ 0)$	$[\![1,$	$(0\ \ 0\ \ 0)]\!]$
C_3^+	0	0	$\frac{2\pi}{3}$	$\frac{2\pi}{3}$	$(0\ \ 0\ \ 1)$	$[\![\frac{1}{2},$	$(0\ \ 0\ \ \frac{\sqrt{3}}{2})]\!]$
C_3^-	0	0	$-\frac{2\pi}{3}$	$\frac{2\pi}{3}$	$(0\ \ 0\ -1)$	$[\![\frac{1}{2},$	$(0\ \ 0\ -\frac{\sqrt{3}}{2})]\!]$
σ_{v1}	0	π	0	π	$(0\ \ 1\ \ 0)$	$[\![0,$	$(0\ \ 1\ \ 0)]\!]$
σ_{v2}	0	π	$\frac{2\pi}{3}$	π	$(-\frac{\sqrt{3}}{2}\ -\frac{1}{2}\ \ 0)$	$[\![0,$	$(-\frac{\sqrt{3}}{2}\ -\frac{1}{2}\ \ 0)]\!]$
σ_{v3}	0	π	$-\frac{2\pi}{3}$	π	$(\frac{\sqrt{3}}{2}\ -\frac{1}{2}\ \ 0)$	$[\![0,$	$(\frac{\sqrt{3}}{2}\ -\frac{1}{2}\ \ 0)]\!]$

T **51**.1B Parameters

§ **16**–1, p. 68

$\mathbf{C}_{3v}^{B}$	α	β	γ	ϕ	$\mathbf{n}$	λ	$\boldsymbol{\Lambda}$
E	0	0	0	0	$(0\ \ 0\ \ 0)$	$[\![1,$	$(0\ \ 0\ \ 0)]\!]$
C_3^+	0	0	$\frac{2\pi}{3}$	$\frac{2\pi}{3}$	$(0\ \ 0\ \ 1)$	$[\![\frac{1}{2},$	$(0\ \ 0\ \ \frac{\sqrt{3}}{2})]\!]$
C_3^-	0	0	$-\frac{2\pi}{3}$	$\frac{2\pi}{3}$	$(0\ \ 0\ -1)$	$[\![\frac{1}{2},$	$(0\ \ 0\ -\frac{\sqrt{3}}{2})]\!]$
σ_{v1}	0	π	π	π	$(1\ \ 0\ \ 0)$	$[\![0,$	$(1\ \ 0\ \ 0)]\!]$
σ_{v2}	0	π	$-\frac{\pi}{3}$	π	$(-\frac{1}{2}\ \ \frac{\sqrt{3}}{2}\ \ 0)$	$[\![0,$	$(-\frac{1}{2}\ \ \frac{\sqrt{3}}{2}\ \ 0)]\!]$
σ_{v3}	0	π	$\frac{\pi}{3}$	π	$(-\frac{1}{2}\ -\frac{\sqrt{3}}{2}\ \ 0)$	$[\![0,$	$(-\frac{1}{2}\ -\frac{\sqrt{3}}{2}\ \ 0)]\!]$

T **51**.2 Multiplication table

Use T **35**.2 ◇. § **16**–2, p. 69

T **51**.3 Factor table

Use T **35**.3 ◇. § **16**–3, p. 70

T **51**.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_{3v}$	E	$2C_3$	$3\sigma_v$	τ
A_1	1	1	1	a
A_2	1	1	-1	a
E	2	-1	0	a
$E_{1/2}$	2	1	0	c
${}^1E_{3/2}$	1	-1	i	b
${}^2E_{3/2}$	1	-1	$-$i	b

T **51**.5A Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{3v}^{A}$	0	1	2	3
A_1	$^{\square}1$	$^{\square}z$	$x^2+y^2, {}^{\square}z^2$	$^{\square}x(x^2-3y^2), (x^2+y^2)z, {}^{\square}z^3$
A_2		R_z		$^{\square}y(3x^2-y^2)$
E		$^{\square}(x,y), (R_x,R_y)$	$^{\square}(xy, x^2-y^2), {}^{\square}(zx, yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, {}^{\square}(xz^2, yz^2),$ $^{\square}\{xyz, z(x^2-y^2)\}$

T **51**.5B Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{3v}^{B}$	0	1	2	3
A_1	$^{\square}1$	$^{\square}z$	$x^2+y^2, {}^{\square}z^2$	$^{\square}y(3x^2-y^2), (x^2+y^2)z, {}^{\square}z^3$
A_2		R_z		$^{\square}x(x^2-3y^2)$
E		$^{\square}(x,y), (R_x,R_y)$	$^{\square}(xy, x^2-y^2), {}^{\square}(zx, yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, {}^{\square}(xz^2, yz^2),$ $^{\square}\{xyz, z(x^2-y^2)\}$

T **51**.6*A* Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{3v}^{A}$	$\langle\vert j\,m\rangle\vert$		ι	μ
A_1	$\vert 0\,0\rangle_+$	$\vert 3\,3\rangle_-$	1	6
A_2	$\vert 3\,3\rangle_+$	$\vert 6\,6\rangle_-$	1	6
E	$\langle\vert 1\,1\rangle, \vert 1\,\bar{1}\rangle\vert$	$\langle\vert 2\,\bar{2}\rangle, -\vert 2\,2\rangle\vert$	1	± 6
$E_{1/2}$	$\langle\vert \frac{1}{2}\,\frac{1}{2}\rangle, \vert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\vert$	$\langle\vert \frac{3}{2}\,\frac{1}{2}\rangle, -\vert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\vert$	2	± 6
	$\langle\vert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle, -\vert \frac{5}{2}\,\frac{5}{2}\rangle\vert$	$\langle\vert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle, \vert \frac{7}{2}\,\frac{5}{2}\rangle\vert$	2	± 6
	$\langle\vert \frac{1}{2}\,\frac{1}{2}\rangle, -\vert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\vert^{\bullet}$	$\langle\vert \frac{3}{2}\,\frac{1}{2}\rangle, \vert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\vert^{\bullet}$	2	± 6
	$\langle\vert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle, \vert \frac{5}{2}\,\frac{5}{2}\rangle\vert^{\bullet}$	$\langle\vert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle, -\vert \frac{7}{2}\,\frac{5}{2}\rangle\vert^{\bullet}$	2	± 6
${}^{1}E_{3/2}$	$\frac{1}{\sqrt{2}}\vert \frac{3}{2}\,\frac{3}{2}\rangle - \frac{\mathrm{i}}{\sqrt{2}}\vert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	$\frac{1}{\sqrt{2}}\vert \frac{5}{2}\,\frac{3}{2}\rangle + \frac{\mathrm{i}}{\sqrt{2}}\vert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle$	2	± 6
	$\frac{1}{\sqrt{2}}\vert \frac{3}{2}\,\frac{3}{2}\rangle^{\bullet} + \frac{\mathrm{i}}{\sqrt{2}}\vert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle^{\bullet}$	$\frac{1}{\sqrt{2}}\vert \frac{5}{2}\,\frac{3}{2}\rangle^{\bullet} - \frac{\mathrm{i}}{\sqrt{2}}\vert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle^{\bullet}$	2	± 6
${}^{2}E_{3/2}$	$\frac{1}{\sqrt{2}}\vert \frac{3}{2}\,\frac{3}{2}\rangle + \frac{\mathrm{i}}{\sqrt{2}}\vert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	$\frac{1}{\sqrt{2}}\vert \frac{5}{2}\,\frac{3}{2}\rangle - \frac{\mathrm{i}}{\sqrt{2}}\vert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle$	2	± 6
	$\frac{1}{\sqrt{2}}\vert \frac{3}{2}\,\frac{3}{2}\rangle^{\bullet} - \frac{\mathrm{i}}{\sqrt{2}}\vert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle^{\bullet}$	$\frac{1}{\sqrt{2}}\vert \frac{5}{2}\,\frac{3}{2}\rangle^{\bullet} + \frac{\mathrm{i}}{\sqrt{2}}\vert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle^{\bullet}$	2	± 6

T **51**.6*B* Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{3v}^{B}$	$\langle\vert j\,m\rangle\vert$		ι	μ
A_1	$\vert 0\,0\rangle_+$		1	3
A_2	$\vert 3\,3\rangle_-$		1	3
E	$\langle\vert 1\,1\rangle, -\vert 1\,\bar{1}\rangle\vert$		1	± 3
$E_{1/2}$	$\langle\vert \frac{1}{2}\,\frac{1}{2}\rangle, \vert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\vert$	$\langle\vert \frac{3}{2}\,\frac{1}{2}\rangle, -\vert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\vert$	2	± 3
	$\langle\vert \frac{1}{2}\,\frac{1}{2}\rangle, -\vert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\vert^{\bullet}$	$\langle\vert \frac{3}{2}\,\frac{1}{2}\rangle, \vert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\vert^{\bullet}$	2	± 3
${}^{1}E_{3/2}$	$\vert \frac{3}{2}\,\frac{3}{2}\rangle_+$	$\vert \frac{5}{2}\,\frac{3}{2}\rangle_-$	2	3
	$\vert \frac{3}{2}\,\frac{3}{2}\rangle_-^{\bullet}$	$\vert \frac{5}{2}\,\frac{3}{2}\rangle_+^{\bullet}$	2	3
${}^{2}E_{3/2}$	$\vert \frac{3}{2}\,\frac{3}{2}\rangle_-$	$\vert \frac{5}{2}\,\frac{3}{2}\rangle_+$	2	3
	$\vert \frac{3}{2}\,\frac{3}{2}\rangle_+^{\bullet}$	$\vert \frac{5}{2}\,\frac{3}{2}\rangle_-^{\bullet}$	2	3

T **51**.7*A* Matrix representations

§ **16**–7, p. 77

$\mathbf{C}_{3v}^{A}$	E	$E_{1/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_3^+	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$
C_3^-	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$
σ_{v1}	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$
σ_{v2}	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$
σ_{v3}	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$

$\epsilon = \exp(2\pi\mathrm{i}/3)$

T **51**.7*B* Matrix representations

§ **16**–7, p. 77

$\mathbf{C}_{3v}^{B}$	E	$E_{1/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_3^+	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$
C_3^-	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$
σ_{v1}	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
σ_{v2}	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon}^* \\ \mathrm{i}\bar{\epsilon} & 0 \end{bmatrix}$
σ_{v3}	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i}\bar{\epsilon} \\ \mathrm{i}\bar{\epsilon}^* & 0 \end{bmatrix}$

$\epsilon = \exp(2\pi\mathrm{i}/3)$

T **51**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_{3v}$	A_1	A_2	E	$E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$
A_1	A_1	A_2	E	$E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$
A_2		A_1	E	$E_{1/2}$	${}^2E_{3/2}$	${}^1E_{3/2}$
E			$A_1 \oplus \{A_2\} \oplus E$	$E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{1/2}$	$E_{1/2}$
$E_{1/2}$				$\{A_1\} \oplus A_2 \oplus E$	E	E
${}^1E_{3/2}$					A_2	A_1
${}^2E_{3/2}$						A_2

T **51**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{3v}$	$(\mathbf{C}_s)$	$\mathbf{C}_3$
A_1	A'	A
A_2	A''	A
E	$A' \oplus A''$	${}^1E \oplus {}^2E$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1E_{3/2}$	${}^1E_{1/2}$	$A_{3/2}$
${}^2E_{3/2}$	${}^2E_{1/2}$	$A_{3/2}$

T **51**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{3v}$
$3n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus 2E)$
$3n+1$	$(n+1)(A_1 \oplus E) \oplus n\,(A_2 \oplus E)$
$3n+2$	$(n+1)(A_1 \oplus 2E) \oplus n\,A_2$
$3n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus n\,({}^1E_{3/2} \oplus {}^2E_{3/2})$
$3n+\frac{3}{2}$	$(2n+1)\,E_{1/2} \oplus (n+1)({}^1E_{3/2} \oplus {}^2E_{3/2})$
$3n+\frac{5}{2}$	$(n+1)(2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2})$

$n = 0, 1, 2, \ldots$

T **51**.11A Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{C}_{3v}^A$

a_2	e	E 1	E 2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$ 1	$E_{1/2}$ 2
1	1	1	0
1	2	0	$\bar{1}$

e	e	A_1 1	A_2 1	E 1	E 2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	1	0

e	$e_{1/2}$	$E_{1/2}$ 1	$E_{1/2}$ 2	${}^1E_{3/2}$ 1	${}^2E_{3/2}$ 1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	iu	i$\bar{\text{u}}$

e	${}^1e_{3/2}$	$E_{1/2}$ 1	$E_{1/2}$ 2
1	1	0	$\bar{\text{i}}$
2	1	1	0

e	${}^2e_{3/2}$	$E_{1/2}$ 1	$E_{1/2}$ 2
1	1	0	i
2	1	1	0

$e_{1/2}$	$e_{1/2}$	A_1 1	A_2 1	E 1	E 2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\text{u}}$	u	0	0
2	2	0	0	0	$\bar{1}$

$e_{1/2}$	${}^1e_{3/2}$	E 1	E 2
1	1	0	i
2	1	1	0

$e_{1/2}$	${}^2e_{3/2}$	E 1	E 2
1	1	0	$\bar{\text{i}}$
2	1	1	0

$\text{u} = 2^{-1/2}$

T **51**.11*B* Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{C}_{3v}^{B}$

a_2	e	E 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

e	e	A_1 1	A_2 1	E 1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	1	0

e	$e_{1/2}$	$E_{1/2}$ 1	2	${}^1E_{3/2}$ 1	${}^2E_{3/2}$ 1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	u	$\bar{\text{u}}$

e	${}^1e_{3/2}$	$E_{1/2}$ 1	2
1	1	0	1
2	1	1	0

e	${}^2e_{3/2}$	$E_{1/2}$ 1	2
1	1	0	$\bar{1}$
2	1	1	0

$e_{1/2}$	$e_{1/2}$	A_1 1	A_2 1	E 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\text{u}}$	u	0	0
2	2	0	0	0	1

$e_{1/2}$	${}^1e_{3/2}$	E 1	2
1	1	0	$\bar{1}$
2	1	1	0

$e_{1/2}$	${}^2e_{3/2}$	E 1	2
1	1	0	1
2	1	1	0

$\text{u} = 2^{-1/2}$

(1) Product forms: $\mathbf{C}_4 \otimes \mathbf{C}_s$.

(2) Group chains: $\mathbf{C}_{8v} \supset (\underline{\mathbf{C}_{4v}}) \supset (\underline{\mathbf{C}_{2v}})$, $\mathbf{C}_{8v} \supset (\underline{\mathbf{C}_{4v}}) \supset \underline{\mathbf{C}_4}$,
$\mathbf{D}_{4d} \supset (\underline{\mathbf{C}_{4v}}) \supset (\underline{\mathbf{C}_{2v}})$, $\mathbf{D}_{4d} \supset (\underline{\mathbf{C}_{4v}}) \supset \underline{\mathbf{C}_4}$,
$\mathbf{D}_{4h} \supset (\underline{\mathbf{C}_{4v}}) \supset (\underline{\mathbf{C}_{2v}})$, $\mathbf{D}_{4h} \supset (\underline{\mathbf{C}_{4v}}) \supset \underline{\mathbf{C}_4}$.

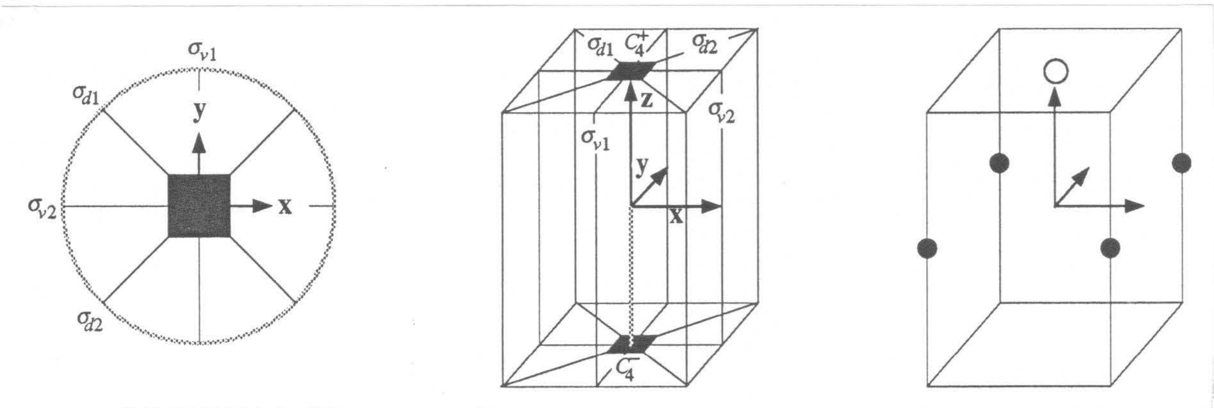
(3) Operations of G: E, (C_4^+, C_4^-), C_2, $(\sigma_{v1}, \sigma_{v2})$, $(\sigma_{d1}, \sigma_{d2})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_4^+, C_4^-), $(\widetilde{C}_4^+, \widetilde{C}_4^-)$, $(C_2, \widetilde{C}_2)$, $(\sigma_{v1}, \sigma_{v2}, \widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2})$, $(\sigma_{d1}, \sigma_{d2}, \widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2})$.

(5) Classes and representations: $|r| = 2$, $|i| = 3$, $|I| = 5$, $|\widetilde{I}| = 2$.

F **52**

See Chapter **15**, p. 65



Examples: Non-planar $(PtCl_4)^{2-}$, SF_5Cl, $XeOF_4$.

T 52.1 Parameters
Use T **33**.1. § **16**–1, p. 68

T 52.2 Multiplication table
Use T **33**.2. § **16**–2, p. 69

T 52.3 Factor table
Use T **33**.3. § **16**–3, p. 70

T 52.4 Character table
§ **16**–4, p. 71

$\mathbf{C}_{4v}$	E	$2C_4$	C_2	$2\sigma_v$	$2\sigma_d$	τ
A_1	1	1	1	1	1	a
A_2	1	1	1	−1	−1	a
B_1	1	−1	1	1	−1	a
B_2	1	−1	1	−1	1	a
E	2	0	−2	0	0	a
$E_{1/2}$	2	$\sqrt{2}$	0	0	0	c
$E_{3/2}$	2	$-\sqrt{2}$	0	0	0	c

T **52**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{4v}$	0	1	2	3
A_1	$\square 1$	$\square z$	$x^2+y^2, \square z^2$	$(x^2+y^2)z, \square z^3$
A_2		R_z		
B_1			$\square x^2-y^2$	$\square z(x^2-y^2)$
B_2			$\square xy$	$\square xyz$
E		$\square(x,y),(R_x,R_y)$	$\square(zx,yz)$	$\{x(x^2+y^2),y(x^2+y^2)\}, \square(xz^2,yz^2),$ $\square\{x(x^2-3y^2),y(3x^2-y^2)\}$

T **52**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{4v}$	$\langle\|j\,m\rangle\|$		ι	μ
A_1	$\|0\,0\rangle_+$		1	4
A_2	$\|4\,4\rangle_-$		1	4
B_1	$\|2\,2\rangle_+$		1	4
B_2	$\|2\,2\rangle_-$		1	4
E	$\langle\|1\,1\rangle,-\|1\,\bar{1}\rangle\|$		1	± 4
$E_{1/2}$	$\langle\|\frac{1}{2}\,\frac{1}{2}\rangle,\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle\|$	$\langle\|\frac{3}{2}\,\frac{1}{2}\rangle,-\|\frac{3}{2}\,\overline{\frac{1}{2}}\rangle\|$	2	± 4
	$\langle\|\frac{1}{2}\,\frac{1}{2}\rangle,-\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle\|^\bullet$	$\langle\|\frac{3}{2}\,\frac{1}{2}\rangle,\|\frac{3}{2}\,\overline{\frac{1}{2}}\rangle\|^\bullet$	2	± 4
$E_{3/2}$	$\langle\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle,-\|\frac{3}{2}\,\frac{3}{2}\rangle\|$	$\langle\|\frac{5}{2}\,\overline{\frac{3}{2}}\rangle,\|\frac{5}{2}\,\frac{3}{2}\rangle\|$	2	± 4
	$\langle\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle,\|\frac{3}{2}\,\frac{3}{2}\rangle\|^\bullet$	$\langle\|\frac{5}{2}\,\overline{\frac{3}{2}}\rangle,-\|\frac{5}{2}\,\frac{3}{2}\rangle\|^\bullet$	2	± 4

T **52**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{C}_{4v}$	E	$E_{1/2}$	$E_{3/2}$
E	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
C_4^+	$\begin{bmatrix}\bar{\mathrm{i}} & 0\\0 & \mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon}^* & 0\\0 & \bar{\epsilon}\end{bmatrix}$
C_4^-	$\begin{bmatrix}\mathrm{i} & 0\\0 & \bar{\mathrm{i}}\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\bar{\epsilon} & 0\\0 & \bar{\epsilon}^*\end{bmatrix}$
C_2	$\begin{bmatrix}\bar{1} & 0\\0 & \bar{1}\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}} & 0\\0 & \mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}} & 0\\0 & \mathrm{i}\end{bmatrix}$
σ_{v1}	$\begin{bmatrix}0 & \bar{1}\\\bar{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\mathrm{i}}\\\bar{\mathrm{i}} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\mathrm{i}}\\\bar{\mathrm{i}} & 0\end{bmatrix}$
σ_{v2}	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{1}\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{1}\\1 & 0\end{bmatrix}$
σ_{d1}	$\begin{bmatrix}0 & \mathrm{i}\\\bar{\mathrm{i}} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}\\\epsilon^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon\\\bar{\epsilon}^* & 0\end{bmatrix}$
σ_{d2}	$\begin{bmatrix}0 & \bar{\mathrm{i}}\\\mathrm{i} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\epsilon}^*\\\epsilon & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon^*\\\bar{\epsilon} & 0\end{bmatrix}$

$\epsilon = \exp(2\pi\mathrm{i}/8)$

T **52**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_{4v}$	A_1	A_2	B_1	B_2	E	$E_{1/2}$	$E_{3/2}$
A_1	A_1	A_2	B_1	B_2	E	$E_{1/2}$	$E_{3/2}$
A_2		A_1	B_2	B_1	E	$E_{1/2}$	$E_{3/2}$
B_1			A_1	A_2	E	$E_{3/2}$	$E_{1/2}$
B_2				A_1	E	$E_{3/2}$	$E_{1/2}$
E					$A_1 \oplus \{A_2\} \oplus B_1 \oplus B_2$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{3/2}$
$E_{1/2}$						$\{A_1\} \oplus A_2 \oplus E$	$B_1 \oplus B_2 \oplus E$
$E_{3/2}$							$\{A_1\} \oplus A_2 \oplus E$

T **52**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{4v}$	$(\mathbf{C}_{2v})$ σ_v	$(\mathbf{C}_{2v})$ σ_d	$(\mathbf{C}_s)$ σ_v	$(\mathbf{C}_s)$ σ_d	$\mathbf{C}_4$	$\mathbf{C}_2$
A_1	A_1	A_1	A'	A'	A	A
A_2	A_2	A_2	A''	A''	A	A
B_1	A_1	A_2	A'	A''	B	A
B_2	A_2	A_1	A''	A'	B	A
E	$B_1 \oplus B_2$	$B_1 \oplus B_2$	$A' \oplus A''$	$A' \oplus A''$	${}^1E \oplus {}^2E$	$2B$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **52**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{4v}$
$4n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E)$
$4n+1$	$(n+1)(A_1 \oplus E) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus E)$
$4n+2$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E) \oplus n\,(A_2 \oplus E)$
$4n+3$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus 2E) \oplus n\,A_2$
$4n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,E_{3/2}$
$4n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2})$
$4n+\frac{5}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)\,E_{3/2}$
$4n+\frac{7}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2})$

$n = 0, 1, 2, \ldots$

T **52**.11 Clebsch–Gordan coefficients

Use T **24**.11 •. § **16**–11, p. 83

(1) Product forms: $\mathbf{C}_5 \otimes \mathbf{C}_s$.

(2) Group chains: $\mathbf{C}_{10v} \supset (\underline{\mathbf{C}_{5v}}) \supset (\mathbf{C}_s)$, $\mathbf{C}_{10v} \supset (\underline{\mathbf{C}_{5v}}) \supset \underline{\mathbf{C}_5}$,
$\mathbf{D}_{5d} \supset (\underline{\mathbf{C}_{5v}}) \supset (\mathbf{C}_s)$, $\mathbf{D}_{5d} \supset (\underline{\mathbf{C}_{5v}}) \supset \underline{\mathbf{C}_5}$,
$\mathbf{D}_{5h} \supset (\underline{\mathbf{C}_{5v}}) \supset (\mathbf{C}_s)$, $\mathbf{D}_{5h} \supset (\underline{\mathbf{C}_{5v}}) \supset \underline{\mathbf{C}_5}$.

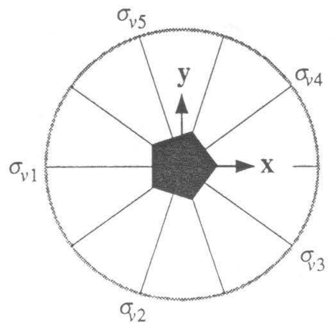
(3) Operations of G: E, (C_5^+, C_5^-), (C_5^{2+}, C_5^{2-}), $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5})$.

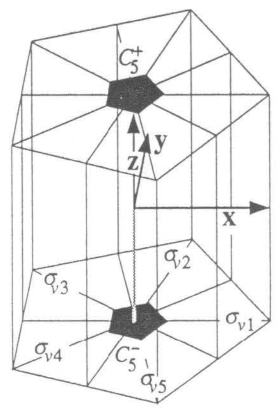
(4) Operations of $\widetilde{G}$: E, (C_5^+, C_5^-), (C_5^{2+}, C_5^{2-}), $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5})$,
$\widetilde{E}$, $(\widetilde{C}_5^+, \widetilde{C}_5^-)$, $(\widetilde{C}_5^{2+}, \widetilde{C}_5^{2-})$, $(\widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2}, \widetilde{\sigma}_{v3}, \widetilde{\sigma}_{v4}, \widetilde{\sigma}_{v5})$.

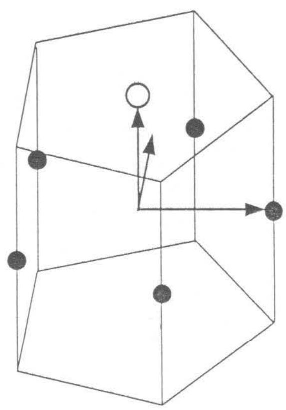
(5) Classes and representations: $|r| = 4$, $|\mathrm{i}| = 0$, $|I| = 4$, $|\widetilde{I}| = 4$.

F **53**

See Chapter **15**, p. 65







Examples:

T **53**.1 Parameters
Use T **39**.1. § **16**–1, p. 68

T **53**.2 Multiplication table
Use T **39**.2. § **16**–2, p. 69

T **53**.3 Factor table
Use T **39**.3. § **16**–3, p. 70

T **53**.4 Character table
§ **16**–4, p. 71

$\mathbf{C}_{5v}$	E	$2C_5$	$2C_5^2$	$5\sigma_v$	τ
A_1	1	1	1	1	a
A_2	1	1	1	−1	a
E_1	2	$2c_5^2$	$2c_5^4$	0	a
E_2	2	$2c_5^4$	$2c_5^2$	0	a
$E_{1/2}$	2	$-2c_5^4$	$2c_5^2$	0	c
$E_{3/2}$	2	$-2c_5^2$	$2c_5^4$	0	c
${}^1E_{5/2}$	1	−1	1	i	b
${}^2E_{5/2}$	1	−1	1	−i	b

$c_n^m = \cos \frac{m}{n}\pi$

T **53**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions § **16**–5, p. 72

$\mathbf{C}_{5v}$	0	1	2	3
A_1	$^{\square}1$	$^{\square}z$	$x^2+y^2, {}^{\square}z^2$	$(x^2+y^2)z, {}^{\square}z^3$
A_2		R_z		
E_1		$^{\square}(x,y), (R_x,R_y)$	$^{\square}(zx,yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, {}^{\square}(xz^2,yz^2)$
E_2			$^{\square}(xy, x^2-y^2)$	$^{\square}\{x(x^2-3y^2), y(3x^2-y^2)\}, {}^{\square}\{xyz, z(x^2-y^2)\}$

T **53**.6 Symmetrized bases § **16**–6, p. 74

$\mathbf{C}_{5v}$	$\langle\|j\,m\rangle\|$		ι	μ
A_1	$\|0\,0\rangle_+$	$\|5\,5\rangle_-$	1	10
A_2	$\|5\,5\rangle_+$	$\|10\,10\rangle_-$	1	10
E_1	$\langle\|1\,1\rangle, \|1\,\overline{1}\rangle\|$	$\langle\|4\,\overline{4}\rangle, -\|4\,4\rangle\|$	1	±10
E_2	$\langle\|2\,2\rangle, \|2\,\overline{2}\rangle\|$	$\langle\|3\,\overline{3}\rangle, -\|3\,3\rangle\|$	1	±10
$E_{1/2}$	$\langle\|\frac12\,\frac12\rangle, \|\frac12\,\overline{\frac12}\rangle\|$	$\langle\|\frac32\,\frac12\rangle, -\|\frac32\,\overline{\frac12}\rangle\|$	2	±10
	$\langle\|\frac92\,\overline{\frac92}\rangle, -\|\frac92\,\frac92\rangle\|$	$\langle\|\frac{11}2\,\overline{\frac92}\rangle, \|\frac{11}2\,\frac92\rangle\|$	2	±10
	$\langle\|\frac12\,\frac12\rangle, -\|\frac12\,\overline{\frac12}\rangle\|^\bullet$	$\langle\|\frac32\,\frac12\rangle, \|\frac32\,\overline{\frac12}\rangle\|^\bullet$	2	±10
	$\langle\|\frac92\,\overline{\frac92}\rangle, \|\frac92\,\frac92\rangle\|^\bullet$	$\langle\|\frac{11}2\,\overline{\frac92}\rangle, -\|\frac{11}2\,\frac92\rangle\|^\bullet$	2	±10
$E_{3/2}$	$\langle\|\frac32\,\frac32\rangle, \|\frac32\,\overline{\frac32}\rangle\|$	$\langle\|\frac52\,\frac32\rangle, -\|\frac52\,\overline{\frac32}\rangle\|$	2	±10
	$\langle\|\frac72\,\overline{\frac72}\rangle, -\|\frac72\,\frac72\rangle\|$	$\langle\|\frac92\,\overline{\frac72}\rangle, \|\frac92\,\frac72\rangle\|$	2	±10
	$\langle\|\frac32\,\frac32\rangle, -\|\frac32\,\overline{\frac32}\rangle\|^\bullet$	$\langle\|\frac52\,\frac32\rangle, \|\frac52\,\overline{\frac32}\rangle\|^\bullet$	2	±10
	$\langle\|\frac72\,\overline{\frac72}\rangle, \|\frac72\,\frac72\rangle\|^\bullet$	$\langle\|\frac92\,\overline{\frac72}\rangle, -\|\frac92\,\frac72\rangle\|^\bullet$	2	±10
$^1E_{5/2}$	$\frac{1}{\sqrt2}\|\frac52\,\frac52\rangle - \frac{\mathrm{i}}{\sqrt2}\|\frac52\,\overline{\frac52}\rangle$	$\frac{1}{\sqrt2}\|\frac72\,\frac52\rangle + \frac{\mathrm{i}}{\sqrt2}\|\frac72\,\overline{\frac52}\rangle$	2	±10
	$\frac{1}{\sqrt2}\|\frac52\,\frac52\rangle^\bullet + \frac{\mathrm{i}}{\sqrt2}\|\frac52\,\overline{\frac52}\rangle^\bullet$	$\frac{1}{\sqrt2}\|\frac72\,\frac52\rangle^\bullet - \frac{\mathrm{i}}{\sqrt2}\|\frac72\,\overline{\frac52}\rangle^\bullet$	2	±10
$^2E_{5/2}$	$\frac{1}{\sqrt2}\|\frac52\,\frac52\rangle + \frac{\mathrm{i}}{\sqrt2}\|\frac52\,\overline{\frac52}\rangle$	$\frac{1}{\sqrt2}\|\frac72\,\frac52\rangle - \frac{\mathrm{i}}{\sqrt2}\|\frac72\,\overline{\frac52}\rangle$	2	±10
	$\frac{1}{\sqrt2}\|\frac52\,\frac52\rangle^\bullet - \frac{\mathrm{i}}{\sqrt2}\|\frac52\,\overline{\frac52}\rangle^\bullet$	$\frac{1}{\sqrt2}\|\frac72\,\frac52\rangle^\bullet + \frac{\mathrm{i}}{\sqrt2}\|\frac72\,\overline{\frac52}\rangle^\bullet$	2	±10

T **53**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{C}_{5v}$	E_1	E_2	$E_{1/2}$	$E_{3/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_5^+	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$
C_5^-	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$
C_5^{2+}	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
C_5^{2-}	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
σ_{v1}	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$
σ_{v2}	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$
σ_{v3}	$\begin{bmatrix} 0 & \bar{\delta} \\ \bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \delta & 0 \end{bmatrix}$
σ_{v4}	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta} \\ \delta^* & 0 \end{bmatrix}$
σ_{v5}	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/5)$, $\epsilon = \exp(4\pi\mathrm{i}/5)$

T **53**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_{5v}$	A_1	A_2	E_1	E_2	$E_{1/2}$	$E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$
A_1	A_1	A_2	E_1	E_2	$E_{1/2}$	$E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$
A_2		A_1	E_1	E_2	$E_{1/2}$	$E_{3/2}$	${}^2E_{5/2}$	${}^1E_{5/2}$
E_1			$A_1 \oplus \{A_2\} \oplus E_2$	$E_1 \oplus E_2$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{3/2}$	$E_{3/2}$
E_2				$A_1 \oplus \{A_2\} \oplus E_1$	$E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2}$	$E_{1/2}$
$E_{1/2}$					$\{A_1\} \oplus A_2 \oplus E_1$	$E_1 \oplus E_2$	E_2	E_2
$E_{3/2}$						$\{A_1\} \oplus A_2 \oplus E_2$	E_1	E_1
${}^1E_{5/2}$							A_2	A_1
${}^2E_{5/2}$								A_2

T **53**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{5v}$	$(\mathbf{C}_s)$	$\mathbf{C}_5$
A_1	A'	A
A_2	A''	A
E_1	$A' \oplus A''$	${}^1E_1 \oplus {}^2E_1$
E_2	$A' \oplus A''$	${}^1E_2 \oplus {}^2E_2$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$
${}^1E_{5/2}$	${}^1E_{1/2}$	$A_{5/2}$
${}^2E_{5/2}$	${}^2E_{1/2}$	$A_{5/2}$

T **53**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{5v}$
$5n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus 2E_1 \oplus 2E_2)$
$5n+1$	$(n+1)(A_1 \oplus E_1) \oplus n\,(A_2 \oplus E_1 \oplus 2E_2)$
$5n+2$	$(n+1)(A_1 \oplus E_1 \oplus E_2) \oplus n\,(A_2 \oplus E_1 \oplus E_2)$
$5n+3$	$(n+1)(A_1 \oplus E_1 \oplus 2E_2) \oplus n\,(A_2 \oplus E_1)$
$5n+4$	$(n+1)(A_1 \oplus 2E_1 \oplus 2E_2) \oplus n\,A_2$
$5n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus n\,(2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2})$
$5n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus n\,({}^1E_{5/2} \oplus {}^2E_{5/2})$
$5n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (n+1)({}^1E_{5/2} \oplus {}^2E_{5/2})$
$5n+\frac{7}{2}$	$(2n+1)\,E_{1/2} \oplus (n+1)(2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2})$
$5n+\frac{9}{2}$	$(n+1)(2E_{1/2} \oplus 2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2})$

$n = 0, 1, 2, \ldots$

T **53**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{C}_{5v}$

a_2	e_1	E_1 1	E_1 2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_2	E_2 1	E_2 2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$ 1	$E_{1/2}$ 2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{3/2}$	$E_{3/2}$ 1	$E_{3/2}$ 2
1	1	1	0
1	2	0	$\bar{1}$

e_1	e_1	A_1 1	A_2 1	E_2 1	E_2 2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	0	1

e_1	e_2	E_1 1	E_1 2	E_2 1	E_2 2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	$e_{1/2}$	$E_{1/2}$ 1	$E_{1/2}$ 2	$E_{3/2}$ 1	$E_{3/2}$ 2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	0	$\bar{1}$

$\text{u} = 2^{-1/2}$ ↠

T **53**.11 Clebsch–Gordan coefficients *(cont.)*

e_1	$e_{3/2}$	$E_{1/2}$ 1	$E_{1/2}$ 2	$^1E_{5/2}$ 1	$^2E_{5/2}$ 1
1	1	0	0	u	u
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	iu	i$\bar{\text{u}}$

e_1	$^1e_{5/2}$	$E_{3/2}$ 1	$E_{3/2}$ 2
1	1	0	$\bar{\text{i}}$
2	1	1	0

e_1	$^2e_{5/2}$	$E_{3/2}$ 1	$E_{3/2}$ 2
1	1	0	i
2	1	1	0

e_2	e_2	A_1 1	A_2 1	E_1 1	E_1 2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	1	0

e_2	$e_{1/2}$	$E_{3/2}$ 1	$E_{3/2}$ 2	$^1E_{5/2}$ 1	$^2E_{5/2}$ 1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	i$\bar{\text{u}}$	iu

e_2	$e_{3/2}$	$E_{1/2}$ 1	$E_{1/2}$ 2	$E_{3/2}$ 1	$E_{3/2}$ 2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_2	$^1e_{5/2}$	$E_{1/2}$ 1	$E_{1/2}$ 2
1	1	0	i
2	1	1	0

e_2	$^2e_{5/2}$	$E_{1/2}$ 1	$E_{1/2}$ 2
1	1	0	$\bar{\text{i}}$
2	1	1	0

$e_{1/2}$	$e_{1/2}$	A_1 1	A_2 1	E_1 1	E_1 2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\text{u}}$	u	0	0
2	2	0	0	0	$\bar{1}$

$e_{1/2}$	$e_{3/2}$	E_1 1	E_1 2	E_2 1	E_2 2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{1/2}$	$^1e_{5/2}$	E_2 1	E_2 2
1	1	0	$\bar{\text{i}}$
2	1	1	0

$e_{1/2}$	$^2e_{5/2}$	E_2 1	E_2 2
1	1	0	i
2	1	1	0

$e_{3/2}$	$e_{3/2}$	A_1 1	A_2 1	E_2 1	E_2 2
1	1	0	0	0	1
1	2	u	u	0	0
2	1	$\bar{\text{u}}$	u	0	0
2	2	0	0	1	0

$e_{3/2}$	$^1e_{5/2}$	E_1 1	E_1 2
1	1	0	$\bar{\text{i}}$
2	1	1	0

$e_{3/2}$	$^2e_{5/2}$	E_1 1	E_1 2
1	1	0	i
2	1	1	0

$\text{u} = 2^{-1/2}$

$6mm$	$\lvert G\rvert = 12$	$\lvert C\rvert = 6$	$\lvert\widetilde{C}\rvert = 9$	T **54**	p. 481	□	$\mathbf{C}_{6v}$

(1) Product forms: $\mathbf{C}_6 \otimes \mathbf{C}_s$.

(2) Group chains: $\mathbf{D}_{6d} \supset (\underline{\mathbf{C}_{6v}}) \supset (\underline{\mathbf{C}_{3v}})$, $\mathbf{D}_{6d} \supset (\underline{\mathbf{C}_{6v}}) \supset (\mathbf{C}_{2v})$, $\mathbf{D}_{6d} \supset (\underline{\mathbf{C}_{6v}}) \supset \underline{\mathbf{C}_6}$,
$\mathbf{D}_{6h} \supset (\underline{\mathbf{C}_{6v}}) \supset (\underline{\mathbf{C}_{3v}})$, $\mathbf{D}_{6h} \supset (\underline{\mathbf{C}_{6v}}) \supset (\mathbf{C}_{2v})$, $\mathbf{D}_{6h} \supset (\underline{\mathbf{C}_{6v}}) \supset \underline{\mathbf{C}_6}$.

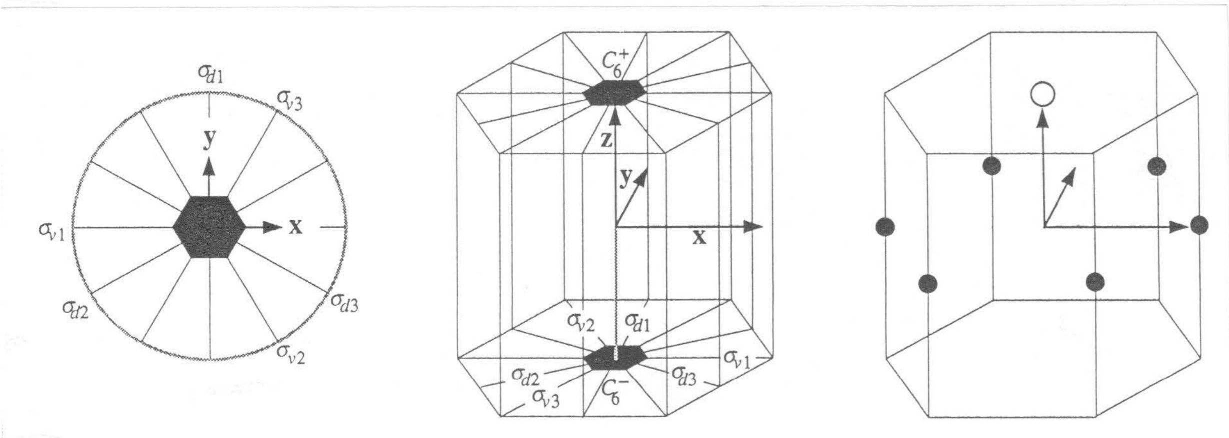
(3) Operations of G: E, (C_6^+, C_6^-), (C_3^+, C_3^-), C_2, $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3})$, $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_6^+, C_6^-), $(\widetilde{C}_6^+, \widetilde{C}_6^-)$, (C_3^+, C_3^-), $(\widetilde{C}_3^+, \widetilde{C}_3^-)$, $(C_2, \widetilde{C}_2)$,
$(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3})$, $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2}, \widetilde{\sigma}_{v3})$.

(5) Classes and representations: $\lvert r\rvert = 3$, $\lvert i\rvert = 3$, $\lvert I\rvert = 6$, $\lvert\widetilde{I}\rvert = 3$.

F **54**

See Chapter **15**, p. 65



Examples: Possible excited state of C_6H_6 with the six H and the six C in different planes.

T **54**.1 Parameters
Use T **35**.1. § **16**–1, p. 68

T **54**.2 Multiplication table
Use T **35**.2. § **16**–2, p. 69

T **54**.3 Factor table
Use T **35**.3. § **16**–3, p. 70

T **54**.4 Character table
§ **16**–4, p. 71

$\mathbf{C}_{6v}$	E	$2C_6$	$2C_3$	C_2	$3\sigma_d$	$3\sigma_v$	τ
A_1	1	1	1	1	1	1	a
A_2	1	1	1	1	−1	−1	a
B_1	1	−1	1	−1	−1	1	a
B_2	1	−1	1	−1	1	−1	a
E_1	2	1	−1	−2	0	0	a
E_2	2	−1	−1	2	0	0	a
$E_{1/2}$	2	$\sqrt{3}$	1	0	0	0	c
$E_{3/2}$	2	0	−2	0	0	0	c
$E_{5/2}$	2	$-\sqrt{3}$	1	0	0	0	c

T **54**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{6v}$	0	1	2	3
A_1	$^\square 1$	$^\square z$	$x^2+y^2, {}^\square z^2$	$(x^2+y^2)z, {}^\square z^3$
A_2		R_z		
B_1				$^\square x(x^2-3y^2)$
B_2				$^\square y(3x^2-y^2)$
E_1		$^\square(x,y),(R_x,R_y)$	$^\square(zx,yz)$	$\{x(x^2+y^2),y(x^2+y^2)\},{}^\square(xz^2,yz^2)$
E_2			$^\square(xy,x^2-y^2)$	$^\square\{xyz,(x^2-y^2)z\}$

T **54**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{6v}$	$\langle \lvert j\,m\rangle \rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$		1	6
A_2	$\lvert 6\,6\rangle_-$		1	6
B_1	$\lvert 3\,3\rangle_-$		1	6
B_2	$\lvert 3\,3\rangle_+$		1	6
E_1	$\langle \lvert 1\,1\rangle, -\lvert 1\,\overline{1}\rangle \rvert$		1	± 6
E_2	$\langle \lvert 2\,\overline{2}\rangle, -\lvert 2\,2\rangle \rvert$		1	± 6
$E_{1/2}$	$\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \rvert$	$\langle \lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle \rvert$	2	± 6
	$\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle, -\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \rvert^\bullet$	$\langle \lvert \frac{3}{2}\,\frac{1}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle \rvert^\bullet$	2	± 6
$E_{3/2}$	$\langle \lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle \rvert$	$\langle \lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle \rvert$	2	± 6
	$\langle \lvert \frac{3}{2}\,\frac{3}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle \rvert^\bullet$	$\langle \lvert \frac{5}{2}\,\frac{3}{2}\rangle, \lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle \rvert^\bullet$	2	± 6
$E_{5/2}$	$\langle \lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle, \lvert \frac{5}{2}\,\frac{5}{2}\rangle \rvert$	$\langle \lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle, -\lvert \frac{7}{2}\,\frac{5}{2}\rangle \rvert$	2	± 6
	$\langle \lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle, -\lvert \frac{5}{2}\,\frac{5}{2}\rangle \rvert^\bullet$	$\langle \lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle, \lvert \frac{7}{2}\,\frac{5}{2}\rangle \rvert^\bullet$	2	± 6

T **54**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{C}_{6v}$	E_1	E_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
E	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
C_6^+	$\begin{bmatrix}\overline{\epsilon} & 0\\0 & \overline{\epsilon}^*\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\mathrm{i}\epsilon^* & 0\\0 & \mathrm{i}\overline{\epsilon}\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}} & 0\\0 & \mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\mathrm{i}\overline{\epsilon}^* & 0\\0 & \mathrm{i}\epsilon\end{bmatrix}$
C_6^-	$\begin{bmatrix}\overline{\epsilon}^* & 0\\0 & \overline{\epsilon}\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\mathrm{i}\overline{\epsilon} & 0\\0 & \mathrm{i}\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\mathrm{i} & 0\\0 & \bar{\mathrm{i}}\end{bmatrix}$	$\begin{bmatrix}\mathrm{i}\epsilon & 0\\0 & \mathrm{i}\overline{\epsilon}^*\end{bmatrix}$
C_3^+	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\epsilon^* & 0\\0 & \epsilon\end{bmatrix}$	$\begin{bmatrix}\overline{\epsilon} & 0\\0 & \overline{\epsilon}^*\end{bmatrix}$	$\begin{bmatrix}\overline{1} & 0\\0 & \overline{1}\end{bmatrix}$	$\begin{bmatrix}\overline{\epsilon} & 0\\0 & \overline{\epsilon}^*\end{bmatrix}$
C_3^-	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\epsilon & 0\\0 & \epsilon^*\end{bmatrix}$	$\begin{bmatrix}\overline{\epsilon}^* & 0\\0 & \overline{\epsilon}\end{bmatrix}$	$\begin{bmatrix}\overline{1} & 0\\0 & \overline{1}\end{bmatrix}$	$\begin{bmatrix}\overline{\epsilon}^* & 0\\0 & \overline{\epsilon}\end{bmatrix}$
C_2	$\begin{bmatrix}\overline{1} & 0\\0 & \overline{1}\end{bmatrix}$	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}\bar{\mathrm{i}} & 0\\0 & \mathrm{i}\end{bmatrix}$	$\begin{bmatrix}\mathrm{i} & 0\\0 & \bar{\mathrm{i}}\end{bmatrix}$	$\begin{bmatrix}\mathrm{i} & 0\\0 & \bar{\mathrm{i}}\end{bmatrix}$
σ_{d1}	$\begin{bmatrix}0 & \overline{1}\\\overline{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{1}\\\overline{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\mathrm{i}}\\\bar{\mathrm{i}} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \mathrm{i}\\\mathrm{i} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \bar{\mathrm{i}}\\\bar{\mathrm{i}} & 0\end{bmatrix}$
σ_{d2}	$\begin{bmatrix}0 & \overline{\epsilon}\\\overline{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\epsilon}\\\overline{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \mathrm{i}\overline{\epsilon}^*\\\mathrm{i}\overline{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \mathrm{i}\\\mathrm{i} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \mathrm{i}\overline{\epsilon}^*\\\mathrm{i}\overline{\epsilon} & 0\end{bmatrix}$
σ_{d3}	$\begin{bmatrix}0 & \overline{\epsilon}^*\\\overline{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\epsilon}^*\\\overline{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \mathrm{i}\overline{\epsilon}\\\mathrm{i}\overline{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \mathrm{i}\\\mathrm{i} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \mathrm{i}\overline{\epsilon}\\\mathrm{i}\overline{\epsilon}^* & 0\end{bmatrix}$
σ_{v1}	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{1}\\\overline{1} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{1}\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{1}\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 1\\\overline{1} & 0\end{bmatrix}$
σ_{v2}	$\begin{bmatrix}0 & \epsilon\\\epsilon^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\epsilon}\\\overline{\epsilon}^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\epsilon}^*\\\epsilon & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{1}\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon^*\\\overline{\epsilon} & 0\end{bmatrix}$
σ_{v3}	$\begin{bmatrix}0 & \epsilon^*\\\epsilon & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\epsilon}^*\\\overline{\epsilon} & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{\epsilon}\\\epsilon^* & 0\end{bmatrix}$	$\begin{bmatrix}0 & \overline{1}\\1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & \epsilon\\\overline{\epsilon}^* & 0\end{bmatrix}$

$\epsilon = \exp(2\pi\mathrm{i}/3)$

T **54**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_{6v}$	A_1	A_2	B_1	B_2	E_1	E_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
A_1	A_1	A_2	B_1	B_2	E_1	E_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
A_2		A_1	B_2	B_1	E_1	E_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
B_1			A_1	A_2	E_2	E_1	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
B_2				A_1	E_2	E_1	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$
E_1					$A_1 \oplus \{A_2\} \oplus E_2$	$B_1 \oplus B_2 \oplus E_1$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus E_{5/2}$
E_2						$A_1 \oplus \{A_2\} \oplus E_2$	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$
$E_{1/2}$							$\{A_1\} \oplus A_2 \oplus E_1$	$E_1 \oplus E_2$	$B_1 \oplus B_2 \oplus E_2$
$E_{3/2}$								$\{A_1\} \oplus A_2 \oplus B_1 \oplus B_2$	$E_1 \oplus E_2$
$E_{5/2}$									$\{A_1\} \oplus A_2 \oplus E_1$

T 54.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{6v}$	$(\mathbf{C}_{3v})$ σ_v	$(\mathbf{C}_{3v})$ σ_d	$(\mathbf{C}_{2v})$	$(\mathbf{C}_s)$ σ_v
A_1	A_1	A_1	A_1	A'
A_2	A_2	A_2	A_2	A''
B_1	A_1	A_2	B_1	A'
B_2	A_2	A_1	B_2	A''
E_1	E	E	$B_1 \oplus B_2$	$A' \oplus A''$
E_2	E	E	$A_1 \oplus A_2$	$A' \oplus A''$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

$\twoheadrightarrow$

T 54.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{C}_{6v}$	$(\mathbf{C}_s)$ σ_d	$\mathbf{C}_6$	$\mathbf{C}_3$	$\mathbf{C}_2$
A_1	A'	A	A	A
A_2	A''	A	A	A
B_1	A''	B	A	B
B_2	A'	B	A	B
E_1	$A' \oplus A''$	${}^1E_1 \oplus {}^2E_1$	${}^1E \oplus {}^2E$	$2B$
E_2	$A' \oplus A''$	${}^1E_2 \oplus {}^2E_2$	${}^1E \oplus {}^2E$	$2A$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$2A_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T 54.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{6v}$
$6n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2)$
$6n+1$	$(n+1)(A_1 \oplus E_1) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus 2E_2)$
$6n+2$	$(n+1)(A_1 \oplus E_1 \oplus E_2) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2)$
$6n+3$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2) \oplus n\,(A_2 \oplus E_1 \oplus E_2)$
$6n+4$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus 2E_2) \oplus n\,(A_2 \oplus E_1)$
$6n+5$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2) \oplus n\,A_2$
$6n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,(E_{3/2} \oplus E_{5/2})$
$6n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus 2n\,E_{5/2}$
$6n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2})$
$6n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (2n+2)\,E_{5/2}$
$6n+\frac{9}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)(E_{3/2} \oplus E_{5/2})$
$6n+\frac{11}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2})$

$n = 0, 1, 2, \ldots$

T 54.11 Clebsch–Gordan coefficients

Use T **26**.11 •. § **16**–11, p. 83

$7m$	$\|G\| = 14$	$\|C\| = 5$	$\|\widetilde{C}\| = 10$	T **55**	p. 481	$\mathbf{C}_{7v}$

(1) Product forms: $\mathbf{C}_7 \otimes \mathbf{C}_s$.

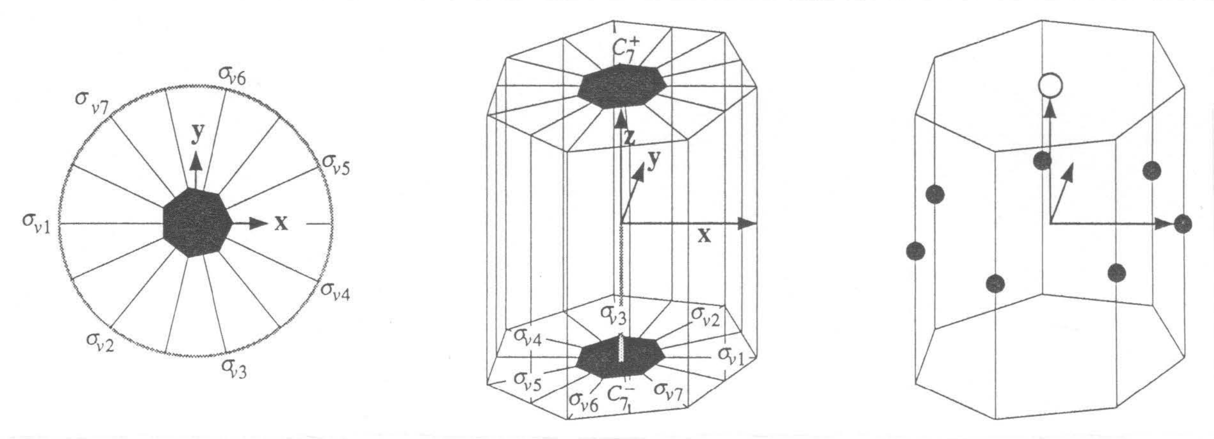
(2) Group chains: $\mathbf{D}_{7d} \supset (\underline{\mathbf{C}_{7v}}) \supset (\mathbf{C}_s)$, $\mathbf{D}_{7d} \supset (\underline{\mathbf{C}_{7v}}) \supset \underline{\mathbf{C}_7}$,
$\mathbf{D}_{7h} \supset (\underline{\mathbf{C}_{7v}}) \supset (\mathbf{C}_s)$, $\mathbf{D}_{7h} \supset (\underline{\mathbf{C}_{7v}}) \supset \underline{\mathbf{C}_7}$.

(3) Operations of G: E, (C_7^+, C_7^-), (C_7^{2+}, C_7^{2-}), (C_7^{3+}, C_7^{3-}), $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5}, \sigma_{v6}, \sigma_{v7})$.

(4) Operations of $\widetilde{G}$: E, (C_7^+, C_7^-), (C_7^{2+}, C_7^{2-}), (C_7^{3+}, C_7^{3-}), $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5}, \sigma_{v6}, \sigma_{v7})$,
$\widetilde{E}$, $(\widetilde{C}_7^+, \widetilde{C}_7^-)$, $(\widetilde{C}_7^{2+}, \widetilde{C}_7^{2-})$, $(\widetilde{C}_7^{3+}, \widetilde{C}_7^{3-})$, $(\widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2}, \widetilde{\sigma}_{v3}, \widetilde{\sigma}_{v4}, \widetilde{\sigma}_{v4}, \widetilde{\sigma}_{v6}, \widetilde{\sigma}_{v7})$.

(5) Classes and representations: $|r| = 5$, $|i| = 0$, $|I| = 5$, $|\widetilde{I}| = 5$.

F **55** See Chapter **15**, p. 65



Examples:

T **55**.1 Parameters
Use T **36**.1. § **16**–1, p. 68

T **55**.2 Multiplication table
Use T **36**.2. § **16**–2, p. 69

T **55**.3 Factor table
Use T **36**.3. § **16**–3, p. 70

T **55**.4 Character table
§ **16**–4, p. 71

$\mathbf{C}_{7v}$	E	$2C_7$	$2C_7^2$	$2C_7^3$	$7\sigma_v$	τ
A_1	1	1	1	1	1	a
A_2	1	1	1	1	-1	a
E_1	2	$2c_7^2$	$2c_7^4$	$2c_7^6$	0	a
E_2	2	$2c_7^4$	$2c_7^6$	$2c_7^2$	0	a
E_3	2	$2c_7^6$	$2c_7^2$	$2c_7^4$	0	a
$E_{1/2}$	2	$-2c_7^6$	$2c_7^2$	$-2c_7^4$	0	c
$E_{3/2}$	2	$-2c_7^4$	$2c_7^6$	$-2c_7^2$	0	c
$E_{5/2}$	2	$-2c_7^2$	$2c_7^4$	$-2c_7^6$	0	c
${}^1E_{7/2}$	1	-1	1	-1	i	b
${}^2E_{7/2}$	1	-1	1	-1	$-$i	b

$c_n^m = \cos \frac{m}{n}\pi$

T **55**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{7v}$	0	1	2	3
A_1	$^\square 1$	$^\square z$	$x^2+y^2, {}^\square z^2$	$(x^2+y^2)z, {}^\square z^3$
A_2		R_z		
E_1		$^\square(x,y), (R_x,R_y)$	$^\square(zx,yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2,yz^2)$
E_2			$^\square(xy, x^2-y^2)$	$^\square\{xyz, z(x^2-y^2)\}$
E_3				$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$

T **55**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{7v}$	$\langle\lvert j\,m\rangle\rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$	$\lvert 7\,7\rangle_-$	1	14
A_2	$\lvert 7\,7\rangle_+$	$\lvert 14\,14\rangle_-$	1	14
E_1	$\langle\lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle\rvert$	$\langle\lvert 6\,\overline{6}\rangle, -\lvert 6\,6\rangle\rvert$	1	± 14
E_2	$\langle\lvert 2\,2\rangle, \lvert 2\,\overline{2}\rangle\rvert$	$\langle\lvert 5\,\overline{5}\rangle, -\lvert 5\,5\rangle\rvert$	1	± 14
E_3	$\langle\lvert 3\,3\rangle, \lvert 3\,\overline{3}\rangle\rvert$	$\langle\lvert 4\,\overline{4}\rangle, -\lvert 4\,4\rangle\rvert$	1	± 14
$E_{1/2}$	$\langle\lvert \tfrac{1}{2}\,\tfrac{1}{2}\rangle, \lvert \tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle\rvert$	$\langle\lvert \tfrac{3}{2}\,\tfrac{1}{2}\rangle, -\lvert \tfrac{3}{2}\,\overline{\tfrac{1}{2}}\rangle\rvert$	2	± 14
	$\langle\lvert \tfrac{13}{2}\,\overline{\tfrac{13}{2}}\rangle, -\lvert \tfrac{13}{2}\,\tfrac{13}{2}\rangle\rvert$	$\langle\lvert \tfrac{15}{2}\,\overline{\tfrac{13}{2}}\rangle, \lvert \tfrac{15}{2}\,\tfrac{13}{2}\rangle\rvert$	2	± 14
	$\langle\lvert \tfrac{1}{2}\,\tfrac{1}{2}\rangle, -\lvert \tfrac{1}{2}\,\overline{\tfrac{1}{2}}\rangle\rvert^\bullet$	$\langle\lvert \tfrac{3}{2}\,\tfrac{1}{2}\rangle, \lvert \tfrac{3}{2}\,\overline{\tfrac{1}{2}}\rangle\rvert^\bullet$	2	± 14
	$\langle\lvert \tfrac{13}{2}\,\overline{\tfrac{13}{2}}\rangle, \lvert \tfrac{13}{2}\,\tfrac{13}{2}\rangle\rvert^\bullet$	$\langle\lvert \tfrac{15}{2}\,\overline{\tfrac{13}{2}}\rangle, -\lvert \tfrac{15}{2}\,\tfrac{13}{2}\rangle\rvert^\bullet$	2	± 14
$E_{3/2}$	$\langle\lvert \tfrac{3}{2}\,\tfrac{3}{2}\rangle, \lvert \tfrac{3}{2}\,\overline{\tfrac{3}{2}}\rangle\rvert$	$\langle\lvert \tfrac{5}{2}\,\tfrac{3}{2}\rangle, -\lvert \tfrac{5}{2}\,\overline{\tfrac{3}{2}}\rangle\rvert$	2	± 14
	$\langle\lvert \tfrac{11}{2}\,\overline{\tfrac{11}{2}}\rangle, -\lvert \tfrac{11}{2}\,\tfrac{11}{2}\rangle\rvert$	$\langle\lvert \tfrac{13}{2}\,\overline{\tfrac{11}{2}}\rangle, \lvert \tfrac{13}{2}\,\tfrac{11}{2}\rangle\rvert$	2	± 14
	$\langle\lvert \tfrac{3}{2}\,\tfrac{3}{2}\rangle, -\lvert \tfrac{3}{2}\,\overline{\tfrac{3}{2}}\rangle\rvert^\bullet$	$\langle\lvert \tfrac{5}{2}\,\tfrac{3}{2}\rangle, \lvert \tfrac{5}{2}\,\overline{\tfrac{3}{2}}\rangle\rvert^\bullet$	2	± 14
	$\langle\lvert \tfrac{11}{2}\,\overline{\tfrac{11}{2}}\rangle, \lvert \tfrac{11}{2}\,\tfrac{11}{2}\rangle\rvert^\bullet$	$\langle\lvert \tfrac{13}{2}\,\overline{\tfrac{11}{2}}\rangle, -\lvert \tfrac{13}{2}\,\tfrac{11}{2}\rangle\rvert^\bullet$	2	± 14
$E_{5/2}$	$\langle\lvert \tfrac{5}{2}\,\tfrac{5}{2}\rangle, \lvert \tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle\rvert$	$\langle\lvert \tfrac{7}{2}\,\tfrac{5}{2}\rangle, -\lvert \tfrac{7}{2}\,\overline{\tfrac{5}{2}}\rangle\rvert$	2	± 14
	$\langle\lvert \tfrac{9}{2}\,\overline{\tfrac{9}{2}}\rangle, -\lvert \tfrac{9}{2}\,\tfrac{9}{2}\rangle\rvert$	$\langle\lvert \tfrac{11}{2}\,\overline{\tfrac{9}{2}}\rangle, \lvert \tfrac{11}{2}\,\tfrac{9}{2}\rangle\rvert$	2	± 14
	$\langle\lvert \tfrac{5}{2}\,\tfrac{5}{2}\rangle, -\lvert \tfrac{5}{2}\,\overline{\tfrac{5}{2}}\rangle\rvert^\bullet$	$\langle\lvert \tfrac{7}{2}\,\tfrac{5}{2}\rangle, \lvert \tfrac{7}{2}\,\overline{\tfrac{5}{2}}\rangle\rvert^\bullet$	2	± 14
	$\langle\lvert \tfrac{9}{2}\,\overline{\tfrac{9}{2}}\rangle, \lvert \tfrac{9}{2}\,\tfrac{9}{2}\rangle\rvert^\bullet$	$\langle\lvert \tfrac{11}{2}\,\overline{\tfrac{9}{2}}\rangle, -\lvert \tfrac{11}{2}\,\tfrac{9}{2}\rangle\rvert^\bullet$	2	± 14
$^1E_{7/2}$	$\tfrac{1}{\sqrt{2}}\lvert \tfrac{7}{2}\,\tfrac{7}{2}\rangle - \tfrac{\mathrm{i}}{\sqrt{2}}\lvert \tfrac{7}{2}\,\overline{\tfrac{7}{2}}\rangle$	$\tfrac{1}{\sqrt{2}}\lvert \tfrac{9}{2}\,\tfrac{7}{2}\rangle + \tfrac{\mathrm{i}}{\sqrt{2}}\lvert \tfrac{9}{2}\,\overline{\tfrac{7}{2}}\rangle$	2	± 14
	$\tfrac{1}{\sqrt{2}}\lvert \tfrac{7}{2}\,\tfrac{7}{2}\rangle^\bullet + \tfrac{\mathrm{i}}{\sqrt{2}}\lvert \tfrac{7}{2}\,\overline{\tfrac{7}{2}}\rangle^\bullet$	$\tfrac{1}{\sqrt{2}}\lvert \tfrac{9}{2}\,\tfrac{7}{2}\rangle^\bullet - \tfrac{\mathrm{i}}{\sqrt{2}}\lvert \tfrac{9}{2}\,\overline{\tfrac{7}{2}}\rangle^\bullet$	2	± 14
$^2E_{7/2}$	$\tfrac{1}{\sqrt{2}}\lvert \tfrac{7}{2}\,\tfrac{7}{2}\rangle + \tfrac{\mathrm{i}}{\sqrt{2}}\lvert \tfrac{7}{2}\,\overline{\tfrac{7}{2}}\rangle$	$\tfrac{1}{\sqrt{2}}\lvert \tfrac{9}{2}\,\tfrac{7}{2}\rangle - \tfrac{\mathrm{i}}{\sqrt{2}}\lvert \tfrac{9}{2}\,\overline{\tfrac{7}{2}}\rangle$	2	± 14
	$\tfrac{1}{\sqrt{2}}\lvert \tfrac{7}{2}\,\tfrac{7}{2}\rangle^\bullet - \tfrac{\mathrm{i}}{\sqrt{2}}\lvert \tfrac{7}{2}\,\overline{\tfrac{7}{2}}\rangle^\bullet$	$\tfrac{1}{\sqrt{2}}\lvert \tfrac{9}{2}\,\tfrac{7}{2}\rangle^\bullet + \tfrac{\mathrm{i}}{\sqrt{2}}\lvert \tfrac{9}{2}\,\overline{\tfrac{7}{2}}\rangle^\bullet$	2	± 14

T **55**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{C}_{7v}$	E_1	E_2	E_3	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_7^+	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \bar{\eta} & 0 \\ 0 & \bar{\eta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$
C_7^-	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \bar{\eta}^* & 0 \\ 0 & \bar{\eta} \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$
C_7^{2+}	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$
C_7^{2-}	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$
C_7^{3+}	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} \delta & 0 \\ 0 & \delta^* \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon} & 0 \\ 0 & \bar{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & 0 \\ 0 & \bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \bar{\eta} & 0 \\ 0 & \bar{\eta}^* \end{bmatrix}$
C_7^{3-}	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} \delta^* & 0 \\ 0 & \delta \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & 0 \\ 0 & \bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\eta}^* & 0 \\ 0 & \bar{\eta} \end{bmatrix}$
σ_{v1}	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$
σ_{v2}	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta} \\ \bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta}^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$
σ_{v3}	$\begin{bmatrix} 0 & \bar{\eta} \\ \bar{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta}^* \\ \eta & 0 \end{bmatrix}$
σ_{v4}	$\begin{bmatrix} 0 & \bar{\delta} \\ \bar{\delta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta} \\ \bar{\eta}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta}^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \delta & 0 \end{bmatrix}$
σ_{v5}	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta}^* \\ \bar{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta} \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta} \\ \delta^* & 0 \end{bmatrix}$
σ_{v6}	$\begin{bmatrix} 0 & \bar{\eta}^* \\ \bar{\eta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \bar{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \delta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta} \\ \eta^* & 0 \end{bmatrix}$
σ_{v7}	$\begin{bmatrix} 0 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta}^* \\ \bar{\delta} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\delta} \\ \delta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\eta} \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$

$\delta = \exp(2\pi i/7)$, $\epsilon = \exp(4\pi i/7)$, $\eta = \exp(6\pi i/7)$

T **55**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_{7v}$	A_1	A_2	E_1	E_2	E_3
A_1	A_1	A_2	E_1	E_2	E_3
A_2		A_1	E_1	E_2	E_3
E_1			$A_1 \oplus \{A_2\} \oplus E_2$	$E_1 \oplus E_3$	$E_2 \oplus E_3$
E_2				$A_1 \oplus \{A_2\} \oplus E_3$	$E_1 \oplus E_2$
E_3					$A_1 \oplus \{A_2\} \oplus E_1$

⇛

T **55**.8 Direct products of representations *(cont.)*

$\mathbf{C}_{7v}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$
A_1	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$
A_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	${}^2E_{7/2}$	${}^1E_{7/2}$
E_1	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}$	$E_{5/2}$	$E_{5/2}$
E_2	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2}$	$E_{3/2}$
E_3	$E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}$	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2}$	$E_{1/2}$
$E_{1/2}$	$\{A_1\} \oplus A_2 \oplus E_1$	$E_1 \oplus E_2$	$E_2 \oplus E_3$	E_3	E_3
$E_{3/2}$		$\{A_1\} \oplus A_2 \oplus E_3$	$E_1 \oplus E_3$	E_2	E_2
$E_{5/2}$			$\{A_1\} \oplus A_2 \oplus E_2$	E_1	E_1
${}^1E_{7/2}$				A_2	A_1
${}^2E_{7/2}$					A_2

T **55**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{7v}$	$(\mathbf{C}_s)$	$\mathbf{C}_7$
A_1	A'	A
A_2	A''	A
E_1	$A' \oplus A''$	${}^1E_1 \oplus {}^2E_1$
E_2	$A' \oplus A''$	${}^1E_2 \oplus {}^2E_2$
E_3	$A' \oplus A''$	${}^1E_3 \oplus {}^2E_3$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$
$E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$
${}^1E_{7/2}$	${}^1E_{1/2}$	$A_{7/2}$
${}^2E_{7/2}$	${}^2E_{1/2}$	$A_{7/2}$

T **55**.10 Subduction from O(3) § **16**–10, p. 82

j	$\mathbf{C}_{7v}$
$7n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3)$
$7n+1$	$(n+1)(A_1 \oplus E_1) \oplus n\,(A_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3)$
$7n+2$	$(n+1)(A_1 \oplus E_1 \oplus E_2) \oplus n\,(A_2 \oplus E_1 \oplus E_2 \oplus 2E_3)$
$7n+3$	$(n+1)(A_1 \oplus E_1 \oplus E_2 \oplus E_3) \oplus n\,(A_2 \oplus E_1 \oplus E_2 \oplus E_3)$
$7n+4$	$(n+1)(A_1 \oplus E_1 \oplus E_2 \oplus 2E_3) \oplus n\,(A_2 \oplus E_1 \oplus E_2)$
$7n+5$	$(n+1)(A_1 \oplus E_1 \oplus 2E_2 \oplus 2E_3) \oplus n\,(A_2 \oplus E_1)$
$7n+6$	$(n+1)(A_1 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3) \oplus n\,A_2$
$7n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus n\,(2E_{3/2} \oplus 2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$7n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus n\,(2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$7n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus n\,({}^1E_{7/2} \oplus {}^2E_{7/2})$
$7n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus (n+1)({}^1E_{7/2} \oplus {}^2E_{7/2})$
$7n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (n+1)(2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$7n+\frac{11}{2}$	$(2n+1)\,E_{1/2} \oplus (n+1)(2E_{3/2} \oplus 2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$
$7n+\frac{13}{2}$	$(n+1)(2E_{1/2} \oplus 2E_{3/2} \oplus 2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2})$

$n = 0, 1, 2, \ldots$

T **55**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{C}_{7v}$

a_2	e_1	E_1	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_2	E_2	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_3	E_3	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{3/2}$	$E_{3/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{5/2}$	$E_{5/2}$	
		1	2
1	1	1	0
1	2	0	$\bar{1}$

e_1	e_1	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	0	1

e_1	e_2	E_1		E_3	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_1	e_3	E_2		E_3	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	$e_{1/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	0	$\bar{1}$

e_1	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e_1	$e_{5/2}$	$E_{3/2}$		${}^1E_{7/2}$	${}^2E_{7/2}$
		1	2	1	1
1	1	0	0	u	u
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	iu	i$\bar{\text{u}}$

e_1	${}^1e_{7/2}$	$E_{5/2}$	
		1	2
1	1	0	$\bar{\text{i}}$
2	1	1	0

e_1	${}^2e_{7/2}$	$E_{5/2}$	
		1	2
1	1	0	i
2	1	1	0

e_2	e_2	A_1	A_2	E_3	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	1	0

e_2	e_3	E_1		E_2	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_2	$e_{1/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_2	$e_{3/2}$	$E_{1/2}$		${}^1E_{7/2}$	${}^2E_{7/2}$
		1	2	1	1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	i$\bar{\text{u}}$	iu

e_2	$e_{5/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$\text{u} = 2^{-1/2}$

↠

T **55**.11 Clebsch–Gordan coefficients *(cont.)*

e_2	$^1e_{7/2}$	$E_{3/2}$	
		1	2
1	1	0	i
2	1	1	0

e_2	$^2e_{7/2}$	$E_{3/2}$	
		1	2
1	1	0	ī
2	1	1	0

e_3	e_3	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	ū	0	0
2	2	0	0	1	0

e_3	$e_{1/2}$	$E_{5/2}$		$^1E_{7/2}$	$^2E_{7/2}$
		1	2	1	1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	iu	iū

e_3	$e_{3/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

e_3	$e_{5/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	1	0

e_3	$^1e_{7/2}$	$E_{1/2}$	
		1	2
1	1	0	ī
2	1	1	0

e_3	$^2e_{7/2}$	$E_{1/2}$	
		1	2
1	1	0	i
2	1	1	0

$e_{1/2}$	$e_{1/2}$	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	ū	u	0	0
2	2	0	0	0	$\bar{1}$

$e_{1/2}$	$e_{3/2}$	E_1		E_2	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{1/2}$	$e_{5/2}$	E_2		E_3	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

$e_{1/2}$	$^1e_{7/2}$	E_3	
		1	2
1	1	0	i
2	1	1	0

$e_{1/2}$	$^2e_{7/2}$	E_3	
		1	2
1	1	0	ī
2	1	1	0

$e_{3/2}$	$e_{3/2}$	A_1	A_2	E_3	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	ū	u	0	0
2	2	0	0	0	$\bar{1}$

$e_{3/2}$	$e_{5/2}$	E_1		E_3	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{3/2}$	$^1e_{7/2}$	E_2	
		1	2
1	1	0	ī
2	1	1	0

$e_{3/2}$	$^2e_{7/2}$	E_2	
		1	2
1	1	0	i
2	1	1	0

$e_{5/2}$	$e_{5/2}$	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	0	1
1	2	u	u	0	0
2	1	ū	u	0	0
2	2	0	0	1	0

$e_{5/2}$	$^1e_{7/2}$	E_1	
		1	2
1	1	0	i
2	1	1	0

$e_{5/2}$	$^2e_{7/2}$	E_1	
		1	2
1	1	0	ī
2	1	1	0

$u = 2^{-1/2}$

(1) Product forms: $\mathbf{C}_8 \otimes \mathbf{C}_s$.

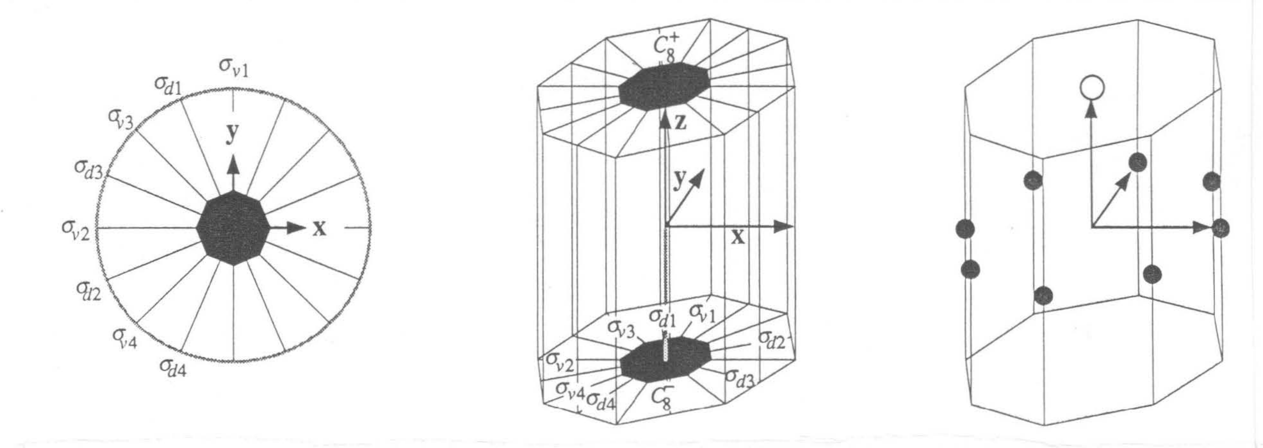
(2) Group chains: $\mathbf{D}_{8d} \supset (\underline{\mathbf{C}_{8v}}) \supset (\underline{\mathbf{C}_{4v}})$, $\mathbf{D}_{8d} \supset (\underline{\mathbf{C}_{8v}}) \supset \underline{\mathbf{C}_8}$,
$\mathbf{D}_{8h} \supset (\underline{\mathbf{C}_{8v}}) \supset (\underline{\mathbf{C}_{4v}})$, $\mathbf{D}_{8h} \supset (\underline{\mathbf{C}_{8v}}) \supset \underline{\mathbf{C}_8}$.

(3) Operations of G: E, (C_8^+, C_8^-), (C_4^+, C_4^-), (C_8^{3+}, C_8^{3-}), C_2, $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4})$, $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_8^+, C_8^-), $(\widetilde{C}_8^+, \widetilde{C}_8^-)$, (C_4^+, C_4^-), $(\widetilde{C}_4^+, \widetilde{C}_4^-)$, (C_8^{3+}, C_8^{3-}), $(\widetilde{C}_8^{3+}, \widetilde{C}_8^{3-})$,
$(C_2, \widetilde{C}_2)$, $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2}, \widetilde{\sigma}_{v3}, \widetilde{\sigma}_{v4})$, $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3}, \widetilde{\sigma}_{d4})$.

(5) Classes and representations: $|r| = 4$, $|i| = 3$, $|I| = 7$, $|\widetilde{I}| = 4$.

F **56** See Chapter **15**, p. 65



Examples:

T 56.1 Parameters
Use T **37**.1. § **16**–1, p. 68

T 56.2 Multiplication table
Use T **37**.2. § **16**–2, p. 69

T 56.3 Factor table
Use T **37**.3. § **16**–3, p. 70

T 56.4 Character table
§ **16**–4, p. 71

$\mathbf{C}_{8v}$	E	$2C_8$	$2C_4$	$2C_8^3$	C_2	$4\sigma_v$	$4\sigma_d$	τ
A_1	1	1	1	1	1	1	1	a
A_2	1	1	1	1	1	−1	−1	a
B_1	1	−1	1	−1	1	1	−1	a
B_2	1	−1	1	−1	1	−1	1	a
E_1	2	$\sqrt{2}$	0	$-\sqrt{2}$	−2	0	0	a
E_2	2	0	−2	0	2	0	0	a
E_3	2	$-\sqrt{2}$	0	$\sqrt{2}$	−2	0	0	a
$E_{1/2}$	2	$2c_8$	$\sqrt{2}$	$2c_8^3$	0	0	0	c
$E_{3/2}$	2	$2c_8^3$	$-\sqrt{2}$	$-2c_8$	0	0	0	c
$E_{5/2}$	2	$-2c_8^3$	$-\sqrt{2}$	$2c_8$	0	0	0	c
$E_{7/2}$	2	$-2c_8$	$\sqrt{2}$	$-2c_8^3$	0	0	0	c

$c_n^m = \cos \frac{m}{n}\pi$

T **56**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{8v}$	0	1	2	3
A_1	$^{\square}1$	$^{\square}z$	$x^2+y^2, {}^{\square}z^2$	$(x^2+y^2)z, {}^{\square}z^3$
A_2		R_z		
B_1				
B_2				
E_1		$^{\square}(x,y),(R_x,R_y)$	$^{\square}(zx,yz)$	$\{x(x^2+y^2),y(x^2+y^2)\}, {}^{\square}(xz^2,yz^2)$
E_2			$^{\square}(xy,x^2-y^2)$	$^{\square}\{xyz,z(x^2-y^2)\}$
E_3				$^{\square}\{x(x^2-3y^2),y(3x^2-y^2)\}$

T **56**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{8v}$	$\langle\lvert j\,m\rangle\rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$		1	8
A_2	$\lvert 8\,8\rangle_-$		1	8
B_1	$\lvert 4\,4\rangle_+$		1	8
B_2	$\lvert 4\,4\rangle_-$		1	8
E_1	$\langle\lvert 1\,1\rangle, -\lvert 1\,\overline{1}\rangle\rvert$		1	± 8
E_2	$\langle\lvert 2\,2\rangle, \lvert 2\,\overline{2}\rangle\rvert$		1	± 8
E_3	$\langle\lvert 3\,\overline{3}\rangle, -\lvert 3\,3\rangle\rvert$		1	± 8
$E_{1/2}$	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	2	± 8
	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, -\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert^\bullet$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert^\bullet$	2	± 8
$E_{3/2}$	$\langle\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle, -\lvert \frac{3}{2}\,\frac{3}{2}\rangle\rvert$	$\langle\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle, \lvert \frac{5}{2}\,\frac{3}{2}\rangle\rvert$	2	± 8
	$\langle\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle, \lvert \frac{3}{2}\,\frac{3}{2}\rangle\rvert^\bullet$	$\langle\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle, -\lvert \frac{5}{2}\,\frac{3}{2}\rangle\rvert^\bullet$	2	± 8
$E_{5/2}$	$\langle\lvert \frac{5}{2}\,\frac{5}{2}\rangle, \lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle\rvert$	$\langle\lvert \frac{7}{2}\,\frac{5}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle\rvert$	2	± 8
	$\langle\lvert \frac{5}{2}\,\frac{5}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle\rvert^\bullet$	$\langle\lvert \frac{7}{2}\,\frac{5}{2}\rangle, \lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle\rvert^\bullet$	2	± 8
$E_{7/2}$	$\langle\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle, -\lvert \frac{7}{2}\,\frac{7}{2}\rangle\rvert$	$\langle\lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle, \lvert \frac{9}{2}\,\frac{7}{2}\rangle\rvert$	2	± 8
	$\langle\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle, \lvert \frac{7}{2}\,\frac{7}{2}\rangle\rvert^\bullet$	$\langle\lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle, -\lvert \frac{9}{2}\,\frac{7}{2}\rangle\rvert^\bullet$	2	± 8

T **56**.7 Matrix representations

Use T **28**.7 •. § **16**–7, p. 77

T **56**.8 Direct products of representations

Use T **28**.8 •. § **16**–8, p. 81

T **56**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{8v}$	$(\mathbf{C}_{4v})$	$(\mathbf{C}_{4v})$	$(\mathbf{C}_{2v})$	$(\mathbf{C}_{2v})$	$(\mathbf{C}_s)$
	σ_v	σ_d	σ_v	σ_d	σ_v
A_1	A_1	A_1	A_1	A_1	A'
A_2	A_2	A_2	A_2	A_2	A''
B_1	A_1	A_2	A_1	A_2	A'
B_2	A_2	A_1	A_2	A_1	A''
E_1	E	E	$B_1 \oplus B_2$	$B_1 \oplus B_2$	$A' \oplus A''$
E_2	$B_1 \oplus B_2$	$B_1 \oplus B_2$	$A_1 \oplus A_2$	$A_1 \oplus A_2$	$A' \oplus A''$
E_3	E	E	$B_1 \oplus B_2$	$B_1 \oplus B_2$	$A' \oplus A''$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

$\twoheadrightarrow$

T **56**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{C}_{8v}$	$(\mathbf{C}_s)$ σ_d	$\mathbf{C}_8$	$\mathbf{C}_4$	$\mathbf{C}_2$
A_1	A'	A	A	A
A_2	A''	A	A	A
B_1	A''	B	A	A
B_2	A'	B	A	A
E_1	$A' \oplus A''$	${}^1E_1 \oplus {}^2E_1$	${}^1E \oplus {}^2E$	$2B$
E_2	$A' \oplus A''$	${}^1E_2 \oplus {}^2E_2$	$2B$	$2A$
E_3	$A' \oplus A''$	${}^1E_3 \oplus {}^2E_3$	${}^1E \oplus {}^2E$	$2B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **56**.10 Subduction from O(3) § **16**–10, p. 82

j	$\mathbf{C}_{8v}$
$8n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3)$
$8n+1$	$(n+1)(A_1 \oplus E_1) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3)$
$8n+2$	$(n+1)(A_1 \oplus E_1 \oplus E_2) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus 2E_3)$
$8n+3$	$(n+1)(A_1 \oplus E_1 \oplus E_2 \oplus E_3) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3)$
$8n+4$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3) \oplus n\,(A_2 \oplus E_1 \oplus E_2 \oplus E_3)$
$8n+5$	$(n+1)\,(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus 2E_3) \oplus n\,(A_2 \oplus E_1 \oplus E_2)$
$8n+6$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3) \oplus n\,(A_2 \oplus E_1)$
$8n+7$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3) \oplus n\,A_2$
$8n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,(E_{3/2} \oplus E_{5/2} \oplus E_{7/2})$
$8n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus 2n\,(E_{5/2} \oplus E_{7/2})$
$8n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus 2n\,E_{7/2}$
$8n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2})$
$8n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus (2n+2)\,E_{7/2}$
$8n+\frac{11}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (2n+2)(E_{5/2} \oplus E_{7/2})$
$8n+\frac{13}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)(E_{3/2} \oplus E_{5/2} \oplus E_{7/2})$
$8n+\frac{15}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2})$

$n = 0, 1, 2, \ldots$

T **56**.11 Clebsch–Gordan coefficients

Use T **28**.11 •. § **16**–11, p. 83

(1) Product forms: $\mathbf{C}_9 \otimes \mathbf{C}_s$.

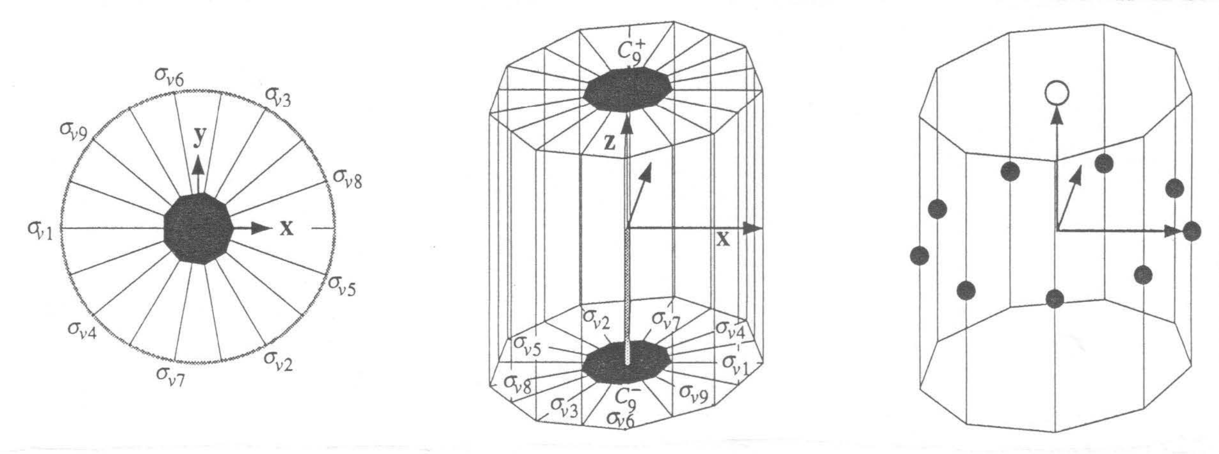
(2) Group chains: $\mathbf{D}_{9d} \supset (\underline{\mathbf{C}_{9v}}) \supset (\mathbf{C}_{3v})$, $\mathbf{D}_{9d} \supset (\underline{\mathbf{C}_{9v}}) \supset \underline{\mathbf{C}_9}$,
$\mathbf{D}_{9h} \supset (\underline{\mathbf{C}_{9v}}) \supset (\mathbf{C}_{3v})$, $\mathbf{D}_{9h} \supset (\underline{\mathbf{C}_{9v}}) \supset \underline{\mathbf{C}_9}$.

(3) Operations of G: E, (C_9^+, C_9^-), (C_9^{2+}, C_9^{2-}), (C_3^+, C_3^-), (C_9^{4+}, C_9^{4-}),
$(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5}, \sigma_{v6}, \sigma_{v7}, \sigma_{v8}, \sigma_{v9})$.

(4) Operations of $\widetilde{G}$: E, (C_9^+, C_9^-), (C_9^{2+}, C_9^{2-}), (C_3^+, C_3^-), (C_9^{4+}, C_9^{4-}),
$(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5}, \sigma_{v6}, \sigma_{v7}, \sigma_{v8}, \sigma_{v9})$,
$\widetilde{E}$, $(\widetilde{C}_9^+, \widetilde{C}_9^-)$, $(\widetilde{C}_9^{2+}, \widetilde{C}_9^{2-})$, $(\widetilde{C}_3^+, \widetilde{C}_3^-)$, $(\widetilde{C}_9^{4+}, \widetilde{C}_9^{4-})$,
$(\widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2}, \widetilde{\sigma}_{v3}, \widetilde{\sigma}_{v4}, \widetilde{\sigma}_{v5}, \widetilde{\sigma}_{v6}, \widetilde{\sigma}_{v7}, \widetilde{\sigma}_{v8}, \widetilde{\sigma}_{v9})$.

(5) Classes and representations: $|r| = 6$, $|i| = 0$, $|I| = 6$, $|\widetilde{I}| = 6$.

F **57** See Chapter **15**, p. 65



Examples:

T **57**.1 Parameters
Use T **38**.1. § **16**–1, p. 68

T **57**.2 Multiplication table
Use T **38**.2. § **16**–2, p. 69

T **57**.3 Factor table
Use T **38**.3. § **16**–3, p. 70

T **57**.4 Character table § **16**–4, p. 71

$\mathbf{C}_{9v}$	E	$2C_9$	$2C_9^2$	$2C_3$	$2C_9^4$	$9\sigma_v$	τ
A_1	1	1	1	1	1	1	a
A_2	1	1	1	1	1	-1	a
E_1	2	$2c_9^2$	$2c_9^4$	-1	$2c_9^8$	0	a
E_2	2	$2c_9^4$	$2c_9^8$	-1	$2c_9^2$	0	a
E_3	2	-1	-1	2	-1	0	a
E_4	2	$2c_9^8$	$2c_9^2$	-1	$2c_9^4$	0	a
$E_{1/2}$	2	$-2c_9^8$	$2c_9^2$	1	$2c_9^4$	0	c
$E_{3/2}$	2	1	-1	-2	-1	0	c
$E_{5/2}$	2	$-2c_9^4$	$2c_9^8$	1	$2c_9^2$	0	c
$E_{7/2}$	2	$-2c_9^2$	$2c_9^4$	1	$2c_9^8$	0	c
${}^1E_{9/2}$	1	-1	1	-1	1	i	b
${}^2E_{9/2}$	1	-1	1	-1	1	$-$i	b

$c_n^m = \cos \frac{m}{n}\pi$

T **57**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions § **16**–5, p. 72

$\mathbf{C}_{9v}$	0	1	2	3
A_1	$^\square 1$	$^\square z$	$x^2+y^2, {}^\square z^2$	$(x^2+y^2)z, {}^\square z^3$
A_2		R_z		
E_1		$^\square(x,y), (R_x,R_y)$	$^\square(zx,yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2,yz^2)$
E_2			$^\square(xy, x^2-y^2)$	$^\square\{xyz, z(x^2-y^2)\}$
E_3				$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$
E_4				

T **57**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{9v}$	$\langle\lvert j\,m\rangle\rvert$		ι	μ
A_1	$\lvert 0\,0\rangle_+$	$\lvert 9\,9\rangle_-$	1	18
A_2	$\lvert 9\,9\rangle_+$	$\lvert 18\,18\rangle_-$	1	18
E_1	$\langle\lvert 1\,1\rangle, \lvert 1\,\overline{1}\rangle\rvert$	$\langle\lvert 8\,\overline{8}\rangle, -\lvert 8\,8\rangle\rvert$	1	± 18
E_2	$\langle\lvert 2\,\overline{2}\rangle, -\lvert 2\,2\rangle\rvert$	$\langle\lvert 7\,7\rangle, \lvert 7\,\overline{7}\rangle\rvert$	1	± 18
E_3	$\langle\lvert 3\,3\rangle, \lvert 3\,\overline{3}\rangle\rvert$	$\langle\lvert 6\,\overline{6}\rangle, -\lvert 6\,6\rangle\rvert$	1	± 18
E_4	$\langle\lvert 4\,4\rangle, -\lvert 4\,\overline{4}\rangle\rvert$	$\langle\lvert 5\,\overline{5}\rangle, \lvert 5\,5\rangle\rvert$	1	± 18
$E_{1/2}$	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	2	± 18
	$\langle\lvert \frac{17}{2}\,\overline{\frac{17}{2}}\rangle, -\lvert \frac{17}{2}\,\frac{17}{2}\rangle\rvert$	$\langle\lvert \frac{19}{2}\,\overline{\frac{17}{2}}\rangle, \lvert \frac{19}{2}\,\frac{17}{2}\rangle\rvert$	2	± 18
	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, -\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert^\bullet$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert^\bullet$	2	± 18
	$\langle\lvert \frac{17}{2}\,\overline{\frac{17}{2}}\rangle, \lvert \frac{17}{2}\,\frac{17}{2}\rangle\rvert^\bullet$	$\langle\lvert \frac{19}{2}\,\overline{\frac{17}{2}}\rangle, -\lvert \frac{19}{2}\,\frac{17}{2}\rangle\rvert^\bullet$	2	± 18
$E_{3/2}$	$\langle\lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle\rvert$	$\langle\lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle\rvert$	2	± 18
	$\langle\lvert \frac{15}{2}\,\overline{\frac{15}{2}}\rangle, -\lvert \frac{15}{2}\,\frac{15}{2}\rangle\rvert$	$\langle\lvert \frac{17}{2}\,\overline{\frac{15}{2}}\rangle, \lvert \frac{17}{2}\,\frac{15}{2}\rangle\rvert$	2	± 18
	$\langle\lvert \frac{3}{2}\,\frac{3}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle\rvert^\bullet$	$\langle\lvert \frac{5}{2}\,\frac{3}{2}\rangle, \lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle\rvert^\bullet$	2	± 18
	$\langle\lvert \frac{15}{2}\,\overline{\frac{15}{2}}\rangle, \lvert \frac{15}{2}\,\frac{15}{2}\rangle\rvert^\bullet$	$\langle\lvert \frac{17}{2}\,\overline{\frac{15}{2}}\rangle, -\lvert \frac{17}{2}\,\frac{15}{2}\rangle\rvert^\bullet$	2	± 18
$E_{5/2}$	$\langle\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle, \lvert \frac{5}{2}\,\frac{5}{2}\rangle\rvert$	$\langle\lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle, -\lvert \frac{7}{2}\,\frac{5}{2}\rangle\rvert$	2	± 18
	$\langle\lvert \frac{13}{2}\,\overline{\frac{5}{2}}\rangle, \lvert \frac{13}{2}\,\frac{5}{2}\rangle\rvert$	$\langle\lvert \frac{15}{2}\,\overline{\frac{5}{2}}\rangle, -\lvert \frac{15}{2}\,\frac{5}{2}\rangle\rvert$	2	± 18
	$\langle\lvert \frac{13}{2}\,\overline{\frac{5}{2}}\rangle, \lvert \frac{13}{2}\,\frac{5}{2}\rangle\rvert$	$\langle\lvert \frac{15}{2}\,\overline{\frac{5}{2}}\rangle, -\lvert \frac{15}{2}\,\frac{5}{2}\rangle\rvert$	2	± 18
	$\langle\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle, -\lvert \frac{5}{2}\,\frac{5}{2}\rangle\rvert^\bullet$	$\langle\lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle, \lvert \frac{7}{2}\,\frac{5}{2}\rangle\rvert^\bullet$	2	± 18
	$\langle\lvert \frac{13}{2}\,\overline{\frac{5}{2}}\rangle, -\lvert \frac{13}{2}\,\frac{5}{2}\rangle\rvert^\bullet$	$\langle\lvert \frac{15}{2}\,\overline{\frac{5}{2}}\rangle, \lvert \frac{15}{2}\,\frac{5}{2}\rangle\rvert^\bullet$	2	± 18
$E_{7/2}$	$\langle\lvert \frac{7}{2}\,\frac{7}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle\rvert$	$\langle\lvert \frac{9}{2}\,\frac{7}{2}\rangle, \lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle\rvert$	2	± 18
	$\langle\lvert \frac{11}{2}\,\overline{\frac{11}{2}}\rangle, \lvert \frac{11}{2}\,\frac{11}{2}\rangle\rvert$	$\langle\lvert \frac{13}{2}\,\overline{\frac{11}{2}}\rangle, -\lvert \frac{13}{2}\,\frac{11}{2}\rangle\rvert$	2	± 18
	$\langle\lvert \frac{7}{2}\,\frac{7}{2}\rangle, \lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle\rvert^\bullet$	$\langle\lvert \frac{9}{2}\,\frac{7}{2}\rangle, -\lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle\rvert^\bullet$	2	± 18
	$\langle\lvert \frac{11}{2}\,\overline{\frac{11}{2}}\rangle, -\lvert \frac{11}{2}\,\frac{11}{2}\rangle\rvert^\bullet$	$\langle\lvert \frac{13}{2}\,\overline{\frac{11}{2}}\rangle, \lvert \frac{13}{2}\,\frac{11}{2}\rangle\rvert^\bullet$	2	± 18
${}^1E_{9/2}$	$\frac{1}{\sqrt{2}}\lvert \frac{9}{2}\,\frac{9}{2}\rangle - \frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle$	$\frac{1}{\sqrt{2}}\lvert \frac{11}{2}\,\frac{9}{2}\rangle + \frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{11}{2}\,\overline{\frac{9}{2}}\rangle$	2	± 18
	$\frac{1}{\sqrt{2}}\lvert \frac{9}{2}\,\frac{9}{2}\rangle^\bullet - \frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle^\bullet$	$\frac{1}{\sqrt{2}}\lvert \frac{11}{2}\,\frac{9}{2}\rangle^\bullet + \frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{11}{2}\,\overline{\frac{9}{2}}\rangle^\bullet$	2	± 18
${}^2E_{9/2}$	$\frac{1}{\sqrt{2}}\lvert \frac{9}{2}\,\frac{9}{2}\rangle + \frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle$	$\frac{1}{\sqrt{2}}\lvert \frac{11}{2}\,\frac{9}{2}\rangle - \frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{11}{2}\,\overline{\frac{9}{2}}\rangle$	2	± 18
	$\frac{1}{\sqrt{2}}\lvert \frac{9}{2}\,\frac{9}{2}\rangle^\bullet + \frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle^\bullet$	$\frac{1}{\sqrt{2}}\lvert \frac{11}{2}\,\frac{9}{2}\rangle^\bullet - \frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{11}{2}\,\overline{\frac{9}{2}}\rangle^\bullet$	2	± 18

T **57**.7 Matrix representations

§ **16**–7, p. 77

$\mathbf{C}_{9v}$	E_1	E_2	E_3	E_4	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$
E	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$
C_9^+	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\theta^*&0\\0&\theta\end{bmatrix}$	$\begin{bmatrix}\bar\theta&0\\0&\bar\theta^*\end{bmatrix}$	$\begin{bmatrix}\bar\eta&0\\0&\bar\eta^*\end{bmatrix}$	$\begin{bmatrix}\bar\epsilon^*&0\\0&\bar\epsilon\end{bmatrix}$	$\begin{bmatrix}\bar\delta&0\\0&\bar\delta^*\end{bmatrix}$
C_9^-	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\theta&0\\0&\theta^*\end{bmatrix}$	$\begin{bmatrix}\bar\theta^*&0\\0&\bar\theta\end{bmatrix}$	$\begin{bmatrix}\bar\eta^*&0\\0&\bar\eta\end{bmatrix}$	$\begin{bmatrix}\bar\epsilon&0\\0&\bar\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\bar\delta^*&0\\0&\bar\delta\end{bmatrix}$
C_9^{2+}	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\theta&0\\0&\theta^*\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\theta^*&0\\0&\theta\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$
C_9^{2-}	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\theta^*&0\\0&\theta\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\theta&0\\0&\theta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$
C_3^+	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\bar\eta&0\\0&\bar\eta^*\end{bmatrix}$	$\begin{bmatrix}\bar1&0\\0&\bar1\end{bmatrix}$	$\begin{bmatrix}\bar\eta&0\\0&\bar\eta^*\end{bmatrix}$	$\begin{bmatrix}\bar\eta&0\\0&\bar\eta^*\end{bmatrix}$
C_3^-	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\bar\eta^*&0\\0&\bar\eta\end{bmatrix}$	$\begin{bmatrix}\bar1&0\\0&\bar1\end{bmatrix}$	$\begin{bmatrix}\bar\eta^*&0\\0&\bar\eta\end{bmatrix}$	$\begin{bmatrix}\bar\eta^*&0\\0&\bar\eta\end{bmatrix}$
C_9^{4+}	$\begin{bmatrix}\theta^*&0\\0&\theta\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\theta&0\\0&\theta^*\end{bmatrix}$
C_9^{4-}	$\begin{bmatrix}\theta&0\\0&\theta^*\end{bmatrix}$	$\begin{bmatrix}\delta&0\\0&\delta^*\end{bmatrix}$	$\begin{bmatrix}\eta&0\\0&\eta^*\end{bmatrix}$	$\begin{bmatrix}\epsilon^*&0\\0&\epsilon\end{bmatrix}$	$\begin{bmatrix}\epsilon&0\\0&\epsilon^*\end{bmatrix}$	$\begin{bmatrix}\eta^*&0\\0&\eta\end{bmatrix}$	$\begin{bmatrix}\delta^*&0\\0&\delta\end{bmatrix}$	$\begin{bmatrix}\theta^*&0\\0&\theta\end{bmatrix}$
σ_{v1}	$\begin{bmatrix}0&\bar1\\\bar1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\\bar1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\\bar1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\\bar1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&1\\\bar1&0\end{bmatrix}$	$\begin{bmatrix}0&1\\\bar1&0\end{bmatrix}$
σ_{v2}	$\begin{bmatrix}0&\bar\eta\\\bar\eta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta\\\bar\eta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\\bar1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta\\\bar\eta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta^*\\\eta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&\eta^*\\\bar\eta&0\end{bmatrix}$	$\begin{bmatrix}0&\eta^*\\\bar\eta&0\end{bmatrix}$
σ_{v3}	$\begin{bmatrix}0&\bar\eta^*\\\bar\eta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta^*\\\bar\eta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\\bar1&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta^*\\\bar\eta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta\\\eta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar1\\1&0\end{bmatrix}$	$\begin{bmatrix}0&\eta\\\bar\eta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\eta\\\bar\eta^*&0\end{bmatrix}$
σ_{v4}	$\begin{bmatrix}0&\bar\epsilon^*\\\bar\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\theta\\\bar\theta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta\\\bar\eta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\delta\\\bar\delta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\delta^*\\\delta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta^*\\\eta&0\end{bmatrix}$	$\begin{bmatrix}0&\theta^*\\\bar\theta&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon\\\bar\epsilon^*&0\end{bmatrix}$
σ_{v5}	$\begin{bmatrix}0&\bar\delta\\\bar\delta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\epsilon^*\\\bar\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta\\\bar\eta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\theta\\\bar\theta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\theta^*\\\theta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta^*\\\eta&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon\\\bar\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\delta^*\\\bar\delta&0\end{bmatrix}$
σ_{v6}	$\begin{bmatrix}0&\bar\theta\\\bar\theta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\delta\\\bar\delta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta\\\bar\eta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\epsilon^*\\\bar\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\epsilon\\\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta^*\\\eta&0\end{bmatrix}$	$\begin{bmatrix}0&\delta^*\\\bar\delta&0\end{bmatrix}$	$\begin{bmatrix}0&\theta^*\\\bar\theta&0\end{bmatrix}$
σ_{v7}	$\begin{bmatrix}0&\bar\theta^*\\\bar\theta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\delta^*\\\bar\delta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta^*\\\bar\eta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\epsilon\\\bar\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\epsilon^*\\\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta\\\eta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\delta\\\bar\delta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\theta\\\bar\theta^*&0\end{bmatrix}$
σ_{v8}	$\begin{bmatrix}0&\bar\delta^*\\\bar\delta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\epsilon\\\bar\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta^*\\\bar\eta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\theta^*\\\bar\theta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\theta\\\theta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta\\\eta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon^*\\\bar\epsilon&0\end{bmatrix}$	$\begin{bmatrix}0&\delta\\\bar\delta^*&0\end{bmatrix}$
σ_{v9}	$\begin{bmatrix}0&\bar\epsilon\\\bar\epsilon^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\theta^*\\\bar\theta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta^*\\\bar\eta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\delta^*\\\bar\delta&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\delta\\\delta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\bar\eta\\\eta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\theta\\\bar\theta^*&0\end{bmatrix}$	$\begin{bmatrix}0&\epsilon^*\\\bar\epsilon&0\end{bmatrix}$

$\delta = \exp(2\pi i/9)$, $\epsilon = \exp(4\pi i/9)$, $\eta = \exp(6\pi i/9)$, $\theta = \exp(8\pi i/9)$

T **57**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_{9v}$	A_1	A_2	E_1	E_2	E_3	E_4
A_1	A_1	A_2	E_1	E_2	E_3	E_4
A_2		A_1	E_1	E_2	E_3	E_4
E_1			$A_1 \oplus \{A_2\} \oplus E_2$	$E_1 \oplus E_3$	$E_2 \oplus E_4$	$E_3 \oplus E_4$
E_2				$A_1 \oplus \{A_2\} \oplus E_4$	$E_1 \oplus E_4$	$E_2 \oplus E_3$
E_3					$A_1 \oplus \{A_2\} \oplus E_3$	$E_1 \oplus E_2$
E_4						$A_1 \oplus \{A_2\} \oplus E_1$

$\twoheadrightarrow$

T **57**.8 Direct products of representations *(cont.)*

$\mathbf{C}_{9v}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$
A_1	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$
A_2	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{7/2}$	${}^2E_{9/2}$	${}^1E_{9/2}$
E_1	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{5/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2}$	$E_{7/2}$	$E_{7/2}$
E_2	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{1/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2}$	$E_{3/2} \oplus E_{7/2}$	$E_{5/2}$	$E_{5/2}$
E_3	$E_{5/2} \oplus E_{7/2}$	$E_{3/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2}$	$E_{1/2} \oplus E_{7/2}$	$E_{1/2} \oplus E_{5/2}$	$E_{3/2}$	$E_{3/2}$
E_4	$E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2}$	$E_{5/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2}$	$E_{1/2}$
$E_{1/2}$	$\{A_1\} \oplus A_2 \oplus E_1$	$E_1 \oplus E_2$	$E_2 \oplus E_3$	$E_3 \oplus E_4$	E_4	E_4
$E_{3/2}$		$\{A_1\} \oplus A_2 \oplus E_3$	$E_1 \oplus E_4$	$E_2 \oplus E_4$	E_3	E_3
$E_{5/2}$			$\{A_1\} \oplus A_2 \oplus E_4$	$E_1 \oplus E_3$	E_2	E_2
$E_{7/2}$				$\{A_1\} \oplus A_2 \oplus E_2$	E_1	E_1
${}^1E_{9/2}$					A_2	A_1
${}^2E_{9/2}$						A_2

T **57**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{9v}$	$(\mathbf{C}_{3v})$	$(\mathbf{C}_s)$	$\mathbf{C}_9$	$\mathbf{C}_3$
A_1	A_1	A'	A	A
A_2	A_2	A''	A	A
E_1	E	$A' \oplus A''$	${}^1E_1 \oplus {}^2E_1$	${}^1E \oplus {}^2E$
E_2	E	$A' \oplus A''$	${}^1E_2 \oplus {}^2E_2$	${}^1E \oplus {}^2E$
E_3	$A_1 \oplus A_2$	$A' \oplus A''$	${}^1E_3 \oplus {}^2E_3$	$2A$
E_4	E	$A' \oplus A''$	${}^1E_4 \oplus {}^2E_4$	${}^1E \oplus {}^2E$
$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$2A_{3/2}$
$E_{5/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1E_{9/2}$	${}^1E_{3/2}$	${}^1E_{1/2}$	$A_{9/2}$	$A_{3/2}$
${}^2E_{9/2}$	${}^2E_{3/2}$	${}^2E_{1/2}$	$A_{9/2}$	$A_{3/2}$

T **57**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{9v}$
$9n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4)$
$9n+1$	$(n+1)(A_1 \oplus E_1) \oplus n\,(A_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4)$
$9n+2$	$(n+1)(A_1 \oplus E_1 \oplus E_2) \oplus n\,(A_2 \oplus E_1 \oplus E_2 \oplus 2E_3 \oplus 2E_4)$
$9n+3$	$(n+1)(A_1 \oplus E_1 \oplus E_2 \oplus E_3) \oplus n\,(A_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus 2E_4)$
$9n+4$	$(n+1)(A_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4) \oplus n\,(A_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4)$
$9n+5$	$(n+1)(A_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus 2E_4) \oplus n\,(A_2 \oplus E_1 \oplus E_2 \oplus E_3)$
$9n+6$	$(n+1)(A_1 \oplus E_1 \oplus E_2 \oplus 2E_3 \oplus 2E_4) \oplus n\,(A_2 \oplus E_1 \oplus E_2)$
$9n+7$	$(n+1)(A_1 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4) \oplus n\,(A_2 \oplus E_1)$
$9n+8$	$(n+1)(A_1 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4) \oplus n\,A_2$
$9n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus n\,(2E_{3/2} \oplus 2E_{5/2} \oplus 2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus n\,(2E_{5/2} \oplus 2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus n\,(2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus n\,({}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus (n+1)({}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{11}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus (n+1)(2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{13}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (n+1)(2E_{5/2} \oplus 2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{15}{2}$	$(2n+1)\,E_{1/2} \oplus (n+1)(2E_{3/2} \oplus 2E_{5/2} \oplus 2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$9n+\frac{17}{2}$	$(n+1)(2E_{1/2} \oplus 2E_{3/2} \oplus 2E_{5/2} \oplus 2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$

$n = 0, 1, 2, \ldots$

T **57**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{C}_{9v}$

a_2	e_1	E_1 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_2	E_2 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_3	E_3 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	e_4	E_4 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{1/2}$	$E_{1/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{3/2}$	$E_{3/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{5/2}$	$E_{5/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{7/2}$	$E_{7/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

e_1	e_1	A_1 1	A_2 1	E_2 1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	1	0

e_1	e_2	E_1 1	2	E_3 1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_1	e_3	E_2 1	2	E_4 1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	$\bar{1}$

$\text{u} = 2^{-1/2}$ $\twoheadrightarrow$

T **57**.11 Clebsch–Gordan coefficients *(cont.)*

e_1	e_4	E_3		E_4	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	$e_{1/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	0	$\bar{1}$

e_1	$e_{3/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_1	$e_{5/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	1	0	0	0

e_1	$e_{7/2}$	$E_{5/2}$		${}^1E_{9/2}$	${}^2E_{9/2}$
		1	2	1	1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	iū	iu

e_1	${}^1e_{9/2}$	$E_{7/2}$	
		1	2
1	1	0	i
2	1	1	0

e_1	${}^2e_{9/2}$	$E_{7/2}$	
		1	2
1	1	0	ī
2	1	1	0

e_2	e_2	A_1	A_2	E_4	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	ū	0	0
2	2	0	0	1	0

e_2	e_3	E_1		E_4	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	$\bar{1}$	0	0

e_2	e_4	E_2		E_3	
		1	2	1	2
1	1	0	$\bar{1}$	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_2	$e_{1/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	1	0	0	0

e_2	$e_{3/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	0	1
2	1	0	0	1	0
2	2	1	0	0	0

e_2	$e_{5/2}$	$E_{1/2}$		${}^1E_{9/2}$	${}^2E_{9/2}$
		1	2	1	1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	iū	iu

e_2	$e_{7/2}$	$E_{3/2}$		$E_{7/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	0	1	0	0

e_2	${}^1e_{9/2}$	$E_{5/2}$	
		1	2
1	1	0	i
2	1	1	0

e_2	${}^2e_{9/2}$	$E_{5/2}$	
		1	2
1	1	0	ī
2	1	1	0

e_3	e_3	A_1	A_2	E_3	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	ū	0	0
2	2	0	0	1	0

e_3	e_4	E_1		E_2	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

$u = 2^{-1/2}$

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T **57**.11 Clebsch–Gordan coefficients *(cont.)*

e_3	$e_{1/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_3	$e_{3/2}$	$E_{3/2}$		${}^1E_{9/2}$	${}^2E_{9/2}$
		1	2	1	1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	1	0	0
2	2	0	0	iu	i$\bar{\mathrm{u}}$

e_3	$e_{5/2}$	$E_{1/2}$		$E_{7/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	0	$\bar{1}$
2	1	0	0	1	0
2	2	0	1	0	0

e_3	$e_{7/2}$	$E_{1/2}$		$E_{5/2}$	
		1	2	1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	$\bar{1}$

e_3	${}^1e_{9/2}$	$E_{3/2}$	
		1	2
1	1	0	$\bar{\mathrm{i}}$
2	1	1	0

e_3	${}^2e_{9/2}$	$E_{3/2}$	
		1	2
1	1	0	i
2	1	1	0

e_4	e_4	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	0	$\bar{1}$
1	2	u	u	0	0
2	1	u	$\bar{\mathrm{u}}$	0	0
2	2	0	0	1	0

e_4	$e_{1/2}$	$E_{7/2}$		${}^1E_{9/2}$	${}^2E_{9/2}$
		1	2	1	1
1	1	0	0	u	u
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	iu	i$\bar{\mathrm{u}}$

e_4	$e_{3/2}$	$E_{5/2}$		$E_{7/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

e_4	$e_{5/2}$	$E_{3/2}$		$E_{5/2}$	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	0	1	0	0

e_4	$e_{7/2}$	$E_{1/2}$		$E_{3/2}$	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	1	0

e_4	${}^1e_{9/2}$	$E_{1/2}$	
		1	2
1	1	0	$\bar{\mathrm{i}}$
2	1	1	0

e_4	${}^2e_{9/2}$	$E_{1/2}$	
		1	2
1	1	0	i
2	1	1	0

$e_{1/2}$	$e_{1/2}$	A_1	A_2	E_1	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\mathrm{u}}$	u	0	0
2	2	0	0	0	$\bar{1}$

$e_{1/2}$	$e_{3/2}$	E_1		E_2	
		1	2	1	2
1	1	0	0	0	$\bar{1}$
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	1	0

$e_{1/2}$	$e_{5/2}$	E_2		E_3	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	$\bar{1}$
2	2	0	1	0	0

$e_{1/2}$	$e_{7/2}$	E_3		E_4	
		1	2	1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{1/2}$	${}^1e_{9/2}$	E_4	
		1	2
1	1	0	i
2	1	1	0

$\mathrm{u} = 2^{-1/2}$

↠

T **57**.11 Clebsch–Gordan coefficients *(cont.)*

$e_{1/2}$	$^2e_{9/2}$	E_4	
		1	2
1	1	0	ī
2	1	1	0

$e_{3/2}$	$e_{3/2}$	A_1	A_2	E_3	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	ū	u	0	0
2	2	0	0	0	1̄

$e_{3/2}$	$e_{5/2}$	E_1		E_4	
		1	2	1	2
1	1	0	1	0	0
1	2	0	0	1	0
2	1	0	0	0	1̄
2	2	1	0	0	0

$e_{3/2}$	$e_{7/2}$	E_2		E_4	
		1	2	1	2
1	1	0	0	0	1
1	2	1	0	0	0
2	1	0	1̄	0	0
2	2	0	0	1	0

$e_{3/2}$	$^1e_{9/2}$	E_3	
		1	2
1	1	0	i
2	1	1	0

$e_{3/2}$	$^2e_{9/2}$	E_3	
		1	2
1	1	0	ī
2	1	1	0

$e_{5/2}$	$e_{5/2}$	A_1	A_2	E_4	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	ū	u	0	0
2	2	0	0	0	1̄

$e_{5/2}$	$e_{7/2}$	E_1		E_3	
		1	2	1	2
1	1	1	0	0	0
1	2	0	0	1	0
2	1	0	0	0	1
2	2	0	1̄	0	0

$e_{5/2}$	$^1e_{9/2}$	E_2	
		1	2
1	1	0	ī
2	1	1	0

$e_{5/2}$	$^2e_{9/2}$	E_2	
		1	2
1	1	0	i
2	1	1	0

$e_{7/2}$	$e_{7/2}$	A_1	A_2	E_2	
		1	1	1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	ū	u	0	0
2	2	0	0	0	1̄

$e_{7/2}$	$^1e_{9/2}$	E_1	
		1	2
1	1	0	ī
2	1	1	0

$e_{7/2}$	$^2e_{9/2}$	E_1	
		1	2
1	1	0	i
2	1	1	0

$u = 2^{-1/2}$

(1) Product forms: $\mathbf{C}_{10} \otimes \mathbf{C}_s$.

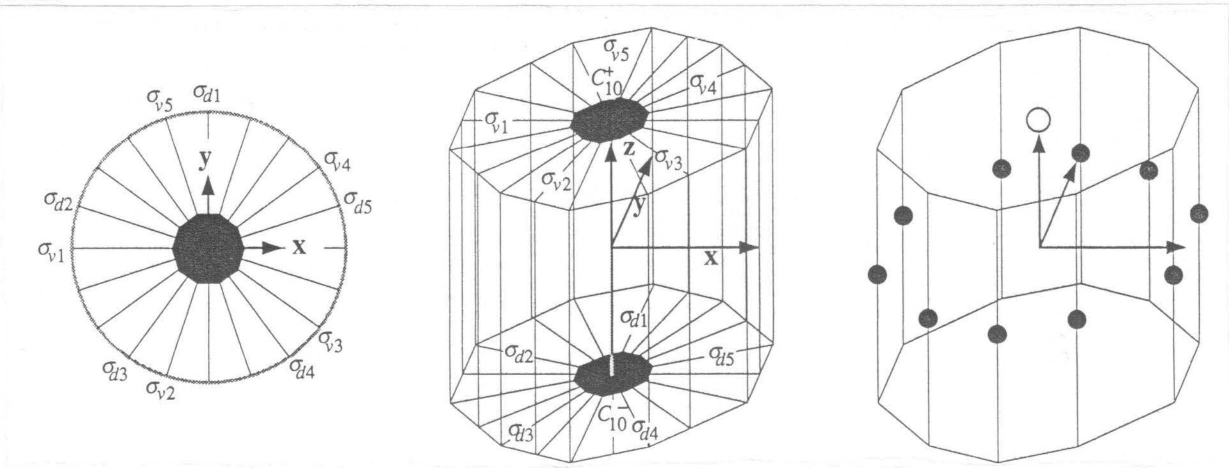
(2) Group chains: $\mathbf{D}_{10d} \supset (\underline{\mathbf{C}_{10v}}) \supset (\underline{\mathbf{C}_{5v}})$, $\mathbf{D}_{10d} \supset (\underline{\mathbf{C}_{10v}}) \supset (\mathbf{C}_{2v})$, $\mathbf{D}_{10d} \supset (\underline{\mathbf{C}_{10v}}) \supset \underline{\mathbf{C}_{10}}$,
$\mathbf{D}_{10h} \supset (\underline{\mathbf{C}_{10v}}) \supset (\underline{\mathbf{C}_{5v}})$, $\mathbf{D}_{10h} \supset (\underline{\mathbf{C}_{10v}}) \supset (\mathbf{C}_{2v})$, $\mathbf{D}_{10h} \supset (\underline{\mathbf{C}_{10v}}) \supset \underline{\mathbf{C}_{10}}$.

(3) Operations of G: E, (C_{10}^{+}, C_{10}^{-}), (C_5^{+}, C_5^{-}), $(C_{10}^{3+}, C_{10}^{3-})$, (C_5^{2+}, C_5^{2-}), C_2,
$(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5})$, $(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, (C_{10}^{+}, C_{10}^{-}), $(\widetilde{C}_{10}^{+}, \widetilde{C}_{10}^{-})$, (C_5^{+}, C_5^{-}), $(\widetilde{C}_5^{+}, \widetilde{C}_5^{-})$, $(C_{10}^{3+}, C_{10}^{3-})$, $(\widetilde{C}_{10}^{3+}, \widetilde{C}_{10}^{3-})$,
(C_5^{2+}, C_5^{2-}), $(\widetilde{C}_5^{2+}, \widetilde{C}_5^{2-})$, $(C_2, \widetilde{C}_2)$, $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3}, \widetilde{\sigma}_{d4}, \widetilde{\sigma}_{d5})$,
$(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}, \sigma_{v4}, \sigma_{v5}, \widetilde{\sigma}_{v1}, \widetilde{\sigma}_{v2}, \widetilde{\sigma}_{v3}, \widetilde{\sigma}_{v4}, \widetilde{\sigma}_{v5})$.

(5) Classes and representations: $|r| = 4$, $|i| = 3$, $|I| = 7$, $|\widetilde{I}| = 4$.

F **58** See Chapter **15**, p. 65



Examples:

T **58**.1 Parameters
Use T **39**.1. § **16**–1, p. 68

T **58**.2 Multiplication table
Use T **39**.2. § **16**–2, p. 69

T **58**.3 Factor table
Use T **39**.3. § **16**–3, p. 70

T **58**.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_{10v}$	E	$2C_{10}$	$2C_5$	$2C_{10}^3$	$2C_5^2$	C_2	$5\sigma_d$	$5\sigma_v$	τ
A_1	1	1	1	1	1	1	1	1	a
A_2	1	1	1	1	1	1	-1	-1	a
B_1	1	-1	1	-1	1	-1	-1	1	a
B_2	1	-1	1	-1	1	-1	1	-1	a
E_1	2	$2c_5$	$2c_5^2$	$-2c_5^2$	$-2c_5$	-2	0	0	a
E_2	2	$2c_5^2$	$-2c_5$	$-2c_5$	$2c_5^2$	2	0	0	a
E_3	2	$-2c_5^2$	$-2c_5$	$2c_5$	$2c_5^2$	-2	0	0	a
E_4	2	$-2c_5$	$2c_5^2$	$2c_5^2$	$-2c_5$	2	0	0	a
$E_{1/2}$	2	$2c_{10}$	$2c_5$	$2c_{10}^3$	$2c_5^2$	0	0	0	c
$E_{3/2}$	2	$2c_{10}^3$	$-2c_5^2$	$-2c_{10}$	$-2c_5$	0	0	0	c
$E_{5/2}$	2	0	-2	0	2	0	0	0	c
$E_{7/2}$	2	$-2c_{10}^3$	$-2c_5^2$	$2c_{10}$	$-2c_5$	0	0	0	c
$E_{9/2}$	2	$-2c_{10}$	$2c_5$	$-2c_{10}^3$	$2c_5^2$	0	0	0	c

$c_n^m = \cos \frac{m}{n}\pi$

T **58**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{10v}$	0	1	2	3
A_1	$^{\square}1$	$^{\square}z$	$x^2+y^2, {}^{\square}z^2$	$(x^2+y^2)z, {}^{\square}z^3$
A_2		R_z		
B_1				
B_2				
E_1		$^{\square}(x,y),(R_x,R_y)$	$^{\square}(zx,yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, {}^{\square}(xz^2,yz^2)$
E_2			$^{\square}(xy, x^2-y^2)$	$^{\square}\{xyz, z(x^2-y^2)\}$
E_3				$^{\square}\{x(x^2-3y^2), y(3x^2-y^2)\}$
E_4				

T **58**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{10v}$	$\langle\|j\,m\rangle\|$		ι	μ
A_1	$\|0\,0\rangle_+$		1	10
A_2	$\|10\,10\rangle_-$		1	10
B_1	$\|5\,5\rangle_-$		1	10
B_2	$\|5\,5\rangle_+$		1	10
E_1	$\langle\|1\,1\rangle, -\|1\,\overline{1}\rangle\|$		1	±10
E_2	$\langle\|2\,2\rangle, \|2\,\overline{2}\rangle\|$		1	±10
E_3	$\langle\|3\,\overline{3}\rangle, \|3\,3\rangle\|$		1	±10
E_4	$\langle\|4\,\overline{4}\rangle, -\|4\,4\rangle\|$		1	±10
$E_{1/2}$	$\langle\|\frac{1}{2}\,\frac{1}{2}\rangle, \|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle\|$	$\langle\|\frac{3}{2}\,\frac{1}{2}\rangle, -\|\frac{3}{2}\,\overline{\frac{1}{2}}\rangle\|$	2	±10
	$\langle\|\frac{1}{2}\,\frac{1}{2}\rangle, -\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle\|^{\bullet}$	$\langle\|\frac{3}{2}\,\frac{1}{2}\rangle, \|\frac{3}{2}\,\overline{\frac{1}{2}}\rangle\|^{\bullet}$	2	±10
$E_{3/2}$	$\langle\|\frac{3}{2}\,\frac{3}{2}\rangle, \|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle\|$	$\langle\|\frac{5}{2}\,\frac{3}{2}\rangle, -\|\frac{5}{2}\,\overline{\frac{3}{2}}\rangle\|$	2	±10
	$\langle\|\frac{3}{2}\,\frac{3}{2}\rangle, -\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle\|^{\bullet}$	$\langle\|\frac{5}{2}\,\frac{3}{2}\rangle, \|\frac{5}{2}\,\overline{\frac{3}{2}}\rangle\|^{\bullet}$	2	±10
$E_{5/2}$	$\langle\|\frac{5}{2}\,\frac{5}{2}\rangle, \|\frac{5}{2}\,\overline{\frac{5}{2}}\rangle\|$	$\langle\|\frac{7}{2}\,\frac{5}{2}\rangle, -\|\frac{7}{2}\,\overline{\frac{5}{2}}\rangle\|$	2	±10
	$\langle\|\frac{5}{2}\,\frac{5}{2}\rangle, -\|\frac{5}{2}\,\overline{\frac{5}{2}}\rangle\|^{\bullet}$	$\langle\|\frac{7}{2}\,\frac{5}{2}\rangle, \|\frac{7}{2}\,\overline{\frac{5}{2}}\rangle\|^{\bullet}$	2	±10
$E_{7/2}$	$\langle\|\frac{7}{2}\,\overline{\frac{7}{2}}\rangle, \|\frac{7}{2}\,\frac{7}{2}\rangle\|$	$\langle\|\frac{9}{2}\,\overline{\frac{7}{2}}\rangle, -\|\frac{9}{2}\,\frac{7}{2}\rangle\|$	2	±10
	$\langle\|\frac{7}{2}\,\overline{\frac{7}{2}}\rangle, -\|\frac{7}{2}\,\frac{7}{2}\rangle\|^{\bullet}$	$\langle\|\frac{9}{2}\,\overline{\frac{7}{2}}\rangle, \|\frac{9}{2}\,\frac{7}{2}\rangle\|^{\bullet}$	2	±10
$E_{9/2}$	$\langle\|\frac{9}{2}\,\overline{\frac{9}{2}}\rangle, \|\frac{9}{2}\,\frac{9}{2}\rangle\|$	$\langle\|\frac{11}{2}\,\overline{\frac{9}{2}}\rangle, -\|\frac{11}{2}\,\frac{9}{2}\rangle\|$	2	±10
	$\langle\|\frac{9}{2}\,\overline{\frac{9}{2}}\rangle, -\|\frac{9}{2}\,\frac{9}{2}\rangle\|^{\bullet}$	$\langle\|\frac{11}{2}\,\overline{\frac{9}{2}}\rangle, \|\frac{11}{2}\,\frac{9}{2}\rangle\|^{\bullet}$	2	±10

T **58**.7 Matrix representations

Use T **30**.7 •. § **16**–7, p. 77

T **58**.8 Direct products of representations

Use T **30**.8 •. § **16**–8, p. 81

T **58**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{10v}$	$(\mathbf{C}_{5v})$	$(\mathbf{C}_{5v})$	$(\mathbf{C}_{2v})$	$(\mathbf{C}_s)$
	σ_v	σ_d		σ_v
A_1	A_1	A_1	A_1	A'
A_2	A_2	A_2	A_2	A''
B_1	A_1	A_2	B_1	A'
B_2	A_2	A_1	B_2	A''
E_1	E_1	E_1	$B_1 \oplus B_2$	$A' \oplus A''$
E_2	E_2	E_2	$A_1 \oplus A_2$	$A' \oplus A''$
E_3	E_2	E_2	$B_1 \oplus B_2$	$A' \oplus A''$
E_4	E_1	E_1	$A_1 \oplus A_2$	$A' \oplus A''$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{9/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

$\twoheadrightarrow$

T **58**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{C}_{10v}$	$(\mathbf{C}_s)$ σ_d	$\mathbf{C}_{10}$	$\mathbf{C}_5$	$\mathbf{C}_2$
A_1	A'	A	A	A
A_2	A''	A	A	A
B_1	A''	B	A	B
B_2	A'	B	A	B
E_1	$A' \oplus A''$	${}^1E_1 \oplus {}^2E_1$	${}^1E_1 \oplus {}^2E_1$	$2B$
E_2	$A' \oplus A''$	${}^1E_2 \oplus {}^2E_2$	${}^1E_2 \oplus {}^2E_2$	$2A$
E_3	$A' \oplus A''$	${}^1E_3 \oplus {}^2E_3$	${}^1E_2 \oplus {}^2E_2$	$2B$
E_4	$A' \oplus A''$	${}^1E_4 \oplus {}^2E_4$	${}^1E_1 \oplus {}^2E_1$	$2A$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$2A_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{9/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{9/2} \oplus {}^2E_{9/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **58**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{10v}$
$10n$	$(n+1)\,A_1 \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4)$
$10n+1$	$(n+1)(A_1 \oplus E_1) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4)$
$10n+2$	$(n+1)(A_1 \oplus E_1 \oplus E_2) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus 2E_3 \oplus 2E_4)$
$10n+3$	$(n+1)(A_1 \oplus E_1 \oplus E_2 \oplus E_3) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus 2E_4)$
$10n+4$	$(n+1)(A_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4) \oplus n\,(A_2 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4)$
$10n+5$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4) \oplus n\,(A_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus E_4)$
$10n+6$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus E_3 \oplus 2E_4) \oplus n\,(A_2 \oplus E_1 \oplus E_2 \oplus E_3)$
$10n+7$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus E_2 \oplus 2E_3 \oplus 2E_4) \oplus n\,(A_2 \oplus E_1 \oplus E_2)$
$10n+8$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4) \oplus n\,(A_2 \oplus E_1)$
$10n+9$	$(n+1)(A_1 \oplus B_1 \oplus B_2 \oplus 2E_1 \oplus 2E_2 \oplus 2E_3 \oplus 2E_4) \oplus n\,A_2$
$10n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$
$10n+\frac{3}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus 2n\,(E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$
$10n+\frac{5}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus 2n\,(E_{7/2} \oplus E_{9/2})$
$10n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus 2n\,E_{9/2}$
$10n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$
$10n+\frac{11}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}) \oplus (2n+2)\,E_{9/2}$
$10n+\frac{13}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2}) \oplus (2n+2)(E_{7/2} \oplus E_{9/2})$
$10n+\frac{15}{2}$	$(2n+1)(E_{1/2} \oplus E_{3/2}) \oplus (2n+2)(E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$
$10n+\frac{17}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)(E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$
$10n+\frac{19}{2}$	$(2n+2)(E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus E_{9/2})$

$n = 0, 1, 2, \ldots$

T **58**.11 Clebsch–Gordan coefficients

Use T **30**.11 •. § **16**–11, p. 83

(1) Product forms: $\mathbf{C}_\infty \otimes \mathbf{C}_s$.

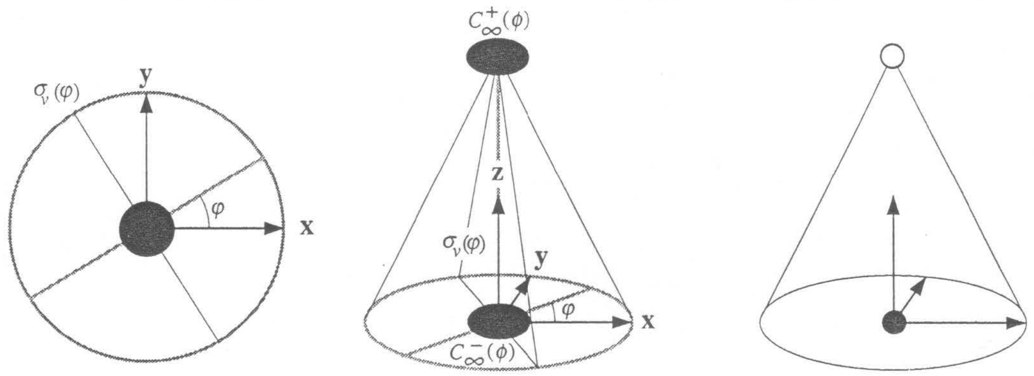
(2) Group chains: $\mathbf{C}_{\infty v} \supset (\mathbf{C}_{nv}) \supset (\mathbf{C}_s)$, $\quad \mathbf{C}_{\infty v} \supset (\mathbf{C}_{nv}) \supset \underline{\mathbf{C}_n}$; $\quad (n = 2, 3, \ldots, 10)$.

(3) Operations of G: E, $(C_\infty^+(\phi), C_\infty^-(\phi))$, C_2, $(\sigma_v(\varphi))$; $\quad 0 < \phi < \pi$; $\quad 0 \le \varphi < \pi$.

(4) Operations of $\widetilde{G}$: E, $(C_\infty^+(\phi), C_\infty^-(\phi))$, C_2, $(\sigma_v(\varphi))$,
$\widetilde{E}$, $(\widetilde{C}_\infty^+(\phi), \widetilde{C}_\infty^-(\phi))$, $\widetilde{C}_2$, $(\widetilde{\sigma}_v(\varphi))$; $\quad 0 < \phi < \pi$; $\quad 0 \le \varphi < \pi$.

(5) Classes and representations: $\lvert r\rvert = \infty$, $\quad \lvert i\rvert = 0$, $\quad \lvert I\rvert = \infty$, $\quad \lvert\widetilde{I}\rvert = \infty$.

F **59** See Chapter **15**, p. 65



Examples: HCl, HCN, COS.

T 59.1 Parameters

Use T **40**.1. § **16**–1, p. 68

T **59**.2 Multiplication table

§ **16**–2, p. 69

$\mathbf{C}_{\infty v}$	E	$C_\infty^+(\phi)$	$C_\infty^+(\phi')$	$C_\infty^-(\phi)$	$C_\infty^-(\phi')$	C_2	$\sigma_v(\varphi)$	$\sigma_v(\varphi')$
E	E	$C_\infty^+(\phi)$	$C_\infty^+(\phi')$	$C_\infty^-(\phi)$	$C_\infty^-(\phi')$	C_2	$\sigma_v(\varphi)$	$\sigma_v(\varphi')$
$C_\infty^+(\phi)$	$C_\infty^+(\phi)$		$C_\infty^+(\phi+\phi')^a$ $C_2{}^b$ $C_\infty^-(2\pi-\phi-\phi')^c$	E	$C_\infty^+(\phi-\phi')^d$ $C_\infty^-(\phi'-\phi)^e$	$C_\infty^-(\pi-\phi)$	$\sigma_v(\varphi+\frac{\phi}{2})^f$ $\sigma_v(\varphi+\frac{\phi}{2}-\pi)^g$	
$C_\infty^+(\phi')$	$C_\infty^+(\phi')$	$C_\infty^+(\phi'+\phi)^a$ $C_2{}^b$ $C_\infty^-(2\pi-\phi'-\phi)^c$			E			
$C_\infty^-(\phi)$	$C_\infty^-(\phi)$	E	$C_\infty^+(\phi'-\phi)^e$ $C_\infty^-(\phi-\phi')^d$		$C_\infty^-(\phi+\phi')^a$ $C_2{}^b$ $C_\infty^+(2\pi-\phi-\phi')^c$	$C_\infty^+(\pi-\phi)$	$\sigma_v(\varphi-\frac{\phi}{2})^h$ $\sigma_v(\varphi-\frac{\phi}{2}+\pi)^i$	
$C_\infty^-(\phi')$	$C_\infty^-(\phi')$		E	$C_\infty^-(\phi+\phi')^a$ $C_2{}^b$ $C_\infty^+(2\pi-\phi-\phi')^c$				
C_2	C_2	$C_\infty^-(\pi-\phi)$		$C_\infty^+(\pi-\phi)$		E	$\sigma_v(\varphi+\frac{\pi}{2})^j$ $\sigma_v(\varphi-\frac{\pi}{2})^k$	
$\sigma_v(\varphi)$	$\sigma_v(\varphi)$	$\sigma_v(\varphi-\frac{\phi}{2})^h$ $\sigma_v(\varphi-\frac{\phi}{2}+\pi)^i$		$\sigma_v(\varphi+\frac{\phi}{2})^f$ $\sigma_v(\varphi+\frac{\phi}{2}-\pi)^g$		$\sigma_v(\varphi+\frac{\pi}{2})^j$ $\sigma_v(\varphi-\frac{\pi}{2})^k$	E	$C_\infty^-(2(\varphi'-\varphi))^l$ $C_2{}^m$ $C_\infty^+(2(\pi-\varphi'+\varphi))^n$
$\sigma_v(\varphi')$	$\sigma_v(\varphi')$						$C_\infty^-(2(\varphi-\varphi'))^p$ $C_2{}^q$ $C_\infty^+(2(\pi-\varphi+\varphi'))^r$	

$0<\phi<\pi;\quad 0\le\varphi<\pi.$

[a] $0<\phi+\phi'<\pi$; [b] $\phi+\phi'=\pi$; [c] $\pi<\phi+\phi'<2\pi$.

[d] $\phi>\phi'$; [e] $\phi<\phi'$.

[f] $0<\varphi+\frac{\phi}{2}<\pi$; [g] $\varphi+\frac{\phi}{2}\ge\pi$.

[h] $0\le\varphi-\frac{\phi}{2}<\pi$; [i] $-\frac{\pi}{2}<\varphi-\frac{\phi}{2}<0$.

[j] $0\le\varphi<\frac{\pi}{2}$; [k] $\frac{\pi}{2}\le\varphi<\pi$.

[l] $0<2(\varphi'-\varphi)<\pi$; [m] $2(\varphi'-\varphi)=\pi$; [n] $2(\varphi'-\varphi)>\pi$.

[p] $0<2(\varphi-\varphi')<\pi$; [q] $2(\varphi-\varphi')=\pi$; [r] $2(\varphi-\varphi')>\pi$.

T **59**.3 Factor table

§ **16**–3, p. 70

$\mathbf{C}_{\infty v}$	E	$C_\infty^+(\phi)$	$C_\infty^+(\phi')$	$C_\infty^-(\phi)$	$C_\infty^-(\phi')$	C_2	$\sigma_v(\varphi)$	$\sigma_v(\varphi')$
E	1	1	1	1	1	1	1	1
$C_\infty^+(\phi)$	1		1^a 1^b -1^c	1	1^d 1^e	-1	1^f -1^g	
$C_\infty^+(\phi')$	1	1^a 1^b -1^c			1			
$C_\infty^-(\phi)$	1	1	1^e 1^d		1^a -1^b -1^c	1	1^h -1^i	
$C_\infty^-(\phi')$	1		1	1^a -1^b -1^c				
C_2	1	-1		1		-1	1^j -1^k	
$\sigma_v(\varphi)$	1	1^h -1^i		1^f -1^g		-1^j 1^k	-1	-1^l 1^m 1^n
$\sigma_v(\varphi')$	1						-1^p 1^q 1^r	

$0 < \phi < \pi$; $\quad 0 \le \varphi < \pi$.

[a] $0 < \phi + \phi' < \pi$;
[b] $\phi + \phi' = \pi$;
[c] $\pi < \phi + \phi' < 2\pi$.
[d] $\phi > \phi'$;
[e] $\phi < \phi'$.
[f] $0 < \varphi + \frac{\phi}{2} < \pi$;
[g] $\varphi + \frac{\phi}{2} \ge \pi$.
[h] $0 \le \varphi - \frac{\phi}{2} < \pi$;
[i] $-\frac{\pi}{2} < \varphi - \frac{\phi}{2} < 0$.
[j] $0 \le \varphi < \frac{\pi}{2}$;
[k] $\frac{\pi}{2} \le \varphi < \pi$.
[l] $0 < 2(\varphi' - \varphi) < \pi$;
[m] $2(\varphi' - \varphi) = \pi$;
[n] $2(\varphi' - \varphi) > \pi$.
[p] $0 < 2(\varphi - \varphi') < \pi$;
[q] $2(\varphi - \varphi') = \pi$;
[r] $2(\varphi - \varphi') > \pi$.

$\mathbf{C}_n$ 107 $\mathbf{C}_i$ 137 $\mathbf{S}_n$ 143 $\mathbf{D}_n$ 193 $\mathbf{D}_{nh}$ 245 $\mathbf{D}_{nd}$ 365 $\mathbf{C}_{nv}$ $\mathbf{C}_{nh}$ 531 **O** 579 **I** 641

T **59**.4 Character table
§ **16**–4, p. 71

$\mathbf{C}_{\infty v}$	E	$2C_\infty(\phi)$	C_2	$\infty\sigma_v(\varphi)$	τ
A_1 (Σ^+)	1	1	1	1	a
A_2 (Σ^-)	1	1	1	-1	a
E_1 (Π)	2	$2\cos\phi$	-2	0	a
E_2 (Δ)	2	$2\cos 2\phi$	2	0	a
E_3 (Φ)	2	$2\cos 3\phi$	-2	0	a
E_n	2	$2\cos n\phi$	$2(-1)^n$	0	a
$E_{1/2}$	2	$2\cos\frac{1}{2}\phi$	0	0	c
$E_{3/2}$	2	$2\cos\frac{3}{2}\phi$	0	0	c
$E_{5/2}$	2	$2\cos\frac{5}{2}\phi$	0	0	c
$E_{7/2}$	2	$2\cos\frac{7}{2}\phi$	0	0	c
$E_{n+1/2}$	2	$2\cos(n+\frac{1}{2})\phi$	0	0	c

$0 < \phi < \pi, \qquad 0 \le \varphi < \pi, \qquad n = 4, 5, 6, \ldots$

T **59**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions
§ **16**–5, p. 72

$\mathbf{C}_{\infty v}$	0	1	2	3
A_1 (Σ^+)	$\square 1$	$\square z$	$x^2+y^2, \square z^2$	$(x^2+y^2)z, \square z^3$
A_2 (Σ^-)		R_z		
E_1 (Π)		$\square(x, y), (R_x, R_y)$	$\square(zx, yz)$	$\{x(x^2+y^2), y(x^2+y^2)\}, \square(xz^2, yz^2)$
E_2 (Δ)			$\square(xy, x^2-y^2)$	$\square\{xyz, z(x^2-y^2)\}$
E_3 (Φ)				$\square\{x(x^2-3y^2), y(3x^2-y^2)\}$

T **59**.6 Symmetrized bases
§ **16**–6, p. 74

$\mathbf{C}_{\infty v}$	$\langle\lvert j\,m\rangle\rvert$		ι
A_1 (Σ^+)	$\lvert 0\,0\rangle$		1
A_2 (Σ^-)			
E_1 (Π)	$\langle\lvert 1\,1\rangle, -\lvert 1\,\bar{1}\rangle\rvert$		1
E_2 (Δ)	$\langle\lvert 2\,2\rangle, \lvert 2\,\bar{2}\rangle\rvert$		1
E_3 (Φ)	$\langle\lvert 3\,3\rangle, -\lvert 3\,\bar{3}\rangle\rvert$		1
E_n	$\langle\lvert n\,n\rangle, (-1)^n\,\lvert n\,\bar{n}\rangle\rvert$		1
$E_{1/2}$	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert$	2
	$\langle\lvert \frac{1}{2}\,\frac{1}{2}\rangle, -\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle\rvert^\bullet$	$\langle\lvert \frac{3}{2}\,\frac{1}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle\rvert^\bullet$	2
$E_{3/2}$	$\langle\lvert \frac{3}{2}\,\frac{3}{2}\rangle, \lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle\rvert$	$\langle\lvert \frac{5}{2}\,\frac{3}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle\rvert$	2
	$\langle\lvert \frac{3}{2}\,\frac{3}{2}\rangle, -\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle\rvert^\bullet$	$\langle\lvert \frac{5}{2}\,\frac{3}{2}\rangle, \lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle\rvert^\bullet$	2
$E_{5/2}$	$\langle\lvert \frac{5}{2}\,\frac{5}{2}\rangle, \lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle\rvert$	$\langle\lvert \frac{7}{2}\,\frac{5}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle\rvert$	2
	$\langle\lvert \frac{5}{2}\,\frac{5}{2}\rangle, -\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle\rvert^\bullet$	$\langle\lvert \frac{7}{2}\,\frac{5}{2}\rangle, \lvert \frac{7}{2}\,\overline{\frac{5}{2}}\rangle\rvert^\bullet$	2
$E_{7/2}$	$\langle\lvert \frac{7}{2}\,\frac{7}{2}\rangle, \lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle\rvert$	$\langle\lvert \frac{9}{2}\,\frac{7}{2}\rangle, -\lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle\rvert$	2
	$\langle\lvert \frac{7}{2}\,\frac{7}{2}\rangle, -\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle\rvert^\bullet$	$\langle\lvert \frac{9}{2}\,\frac{7}{2}\rangle, \lvert \frac{9}{2}\,\overline{\frac{7}{2}}\rangle\rvert^\bullet$	2
$E_{n+1/2}$	$\langle\lvert n+\frac{1}{2}, n+\frac{1}{2}\rangle, \lvert n+\frac{1}{2}, -n-\frac{1}{2}\rangle\rvert$	$\langle\lvert n+\frac{3}{2}, n+\frac{1}{2}\rangle, -\lvert n+\frac{3}{2}, -n-\frac{1}{2}\rangle\rvert$	2
	$\langle\lvert n+\frac{1}{2}, n+\frac{1}{2}\rangle, -\lvert n+\frac{1}{2}, -n-\frac{1}{2}\rangle\rvert^\bullet$	$\langle\lvert n+\frac{3}{2}, n+\frac{1}{2}\rangle, \lvert n+\frac{3}{2}, -n-\frac{1}{2}\rangle\rvert^\bullet$	2

The μ column mentioned on p. 74 is not relevant here.
$n = 4, 5, 6, \ldots$

T 59.7 Matrix representations

§ 16–7, p. 77

$\mathbf{C}_{\infty v}$	E_n $(n = 1,3,5,\ldots)$	E_p $(p = 2,4,6,\ldots)$	$E_{n/2}$ $(n = 1,3,5,\ldots)$
E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$C_\infty^+(\phi)$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}n\phi} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}n\phi} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}p\phi} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}p\phi} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}n\phi/2} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}n\phi/2} \end{bmatrix}$
$C_\infty^-(\phi)$	$\begin{bmatrix} \mathrm{e}^{\mathrm{i}n\phi} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}n\phi} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{\mathrm{i}p\phi} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}p\phi} \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{\mathrm{i}n\phi/2} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}n\phi/2} \end{bmatrix}$
C_2	$\begin{bmatrix} \bar{1} & 0 \\ 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \mathrm{e}^{-\mathrm{i}n\pi/2} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}n\pi/2} \end{bmatrix}$
$\sigma_v(\varphi)$	$\begin{bmatrix} 0 & -\mathrm{e}^{-2\mathrm{i}n\varphi} \\ -\mathrm{e}^{2\mathrm{i}n\varphi} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{e}^{-2\mathrm{i}p\varphi} \\ \mathrm{e}^{2\mathrm{i}p\varphi} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{e}^{-\mathrm{i}n(\frac{\pi}{2}+\varphi)} \\ \mathrm{e}^{-\mathrm{i}n(\frac{\pi}{2}-\varphi)} & 0 \end{bmatrix}$

T 59.8 Direct products of representations

§ 16–8, p. 81

$\mathbf{C}_{\infty v}$	A_1	A_2	E_1	E_2	E_3
A_1	A_1	A_2	E_1	E_2	E_3
A_2		A_1	E_1	E_2	E_3
E_1			$A_1 \oplus \{A_2\} \oplus E_2$	$E_1 \oplus E_3$	$E_2 \oplus E_4$
E_2				$A_1 \oplus \{A_2\} \oplus E_4$	$E_1 \oplus E_5$
E_3					$A_1 \oplus \{A_2\} \oplus E_6$

$\twoheadrightarrow$

T 59.8 Direct products of representations *(cont.)*

$\mathbf{C}_{\infty v}$	E_n	E_p	$E_{1/2}$	$E_{3/2}$
A_1	E_n	E_p	$E_{1/2}$	$E_{3/2}$
A_2	E_n	E_p	$E_{1/2}$	$E_{3/2}$
E_1	$E_{n-1} \oplus E_{n+1}$	$E_{p-1} \oplus E_{p+1}$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus E_{5/2}$
E_2	$E_{n-2} \oplus E_{n+2}$	$E_{p-2} \oplus E_{p+2}$	$E_{3/2} \oplus E_{5/2}$	$E_{1/2} \oplus E_{7/2}$
E_3	$E_{n-3} \oplus E_{n+3}$	$E_{p-3} \oplus E_{p+3}$	$E_{5/2} \oplus E_{7/2}$	$E_{3/2} \oplus E_{9/2}$
E_n	$A_1 \oplus \{A_2\} \oplus E_{2n}$	$E_{p-n} \oplus E_{p+n}$	$E_{n-1/2} \oplus E_{n+1/2}$	$E_{n-3/2} \oplus E_{n+3/2}$
$E_{1/2}$			$\{A_1\} \oplus A_2 \oplus E_1$	$E_1 \oplus E_2$
$E_{3/2}$				$\{A_1\} \oplus A_2 \oplus E_3$

$n = 4,5,6,\ldots,\quad p = 5,6,7,\ldots,\quad p > n$ $\twoheadrightarrow$

T 59.8 Direct products of representations *(cont.)*

$\mathbf{C}_{\infty v}$	$E_{5/2}$	$E_{7/2}$	$E_{n+1/2}$	$E_{p+1/2}$
A_1	$E_{5/2}$	$E_{7/2}$	$E_{n+1/2}$	$E_{p+1/2}$
A_2	$E_{5/2}$	$E_{7/2}$	$E_{n+1/2}$	$E_{p+1/2}$
E_1	$E_{3/2} \oplus E_{7/2}$	$E_{5/2} \oplus E_{9/2}$	$E_{n-1/2} \oplus E_{n+3/2}$	$E_{p-1/2} \oplus E_{p+3/2}$
E_2	$E_{1/2} \oplus E_{9/2}$	$E_{3/2} \oplus E_{11/2}$	$E_{n-3/2} \oplus E_{n+5/2}$	$E_{p-3/2} \oplus E_{p+5/2}$
E_3	$E_{1/2} \oplus E_{11/2}$	$E_{1/2} \oplus E_{13/2}$	$E_{n-5/2} \oplus E_{n+7/2}$	$E_{p-5/2} \oplus E_{p+7/2}$
E_n	$E_{n-5/2} \oplus E_{n+5/2}$	$E_{n-7/2} \oplus E_{n+7/2}$	$E_{1/2} \oplus E_{2n+1/2}$	$E_{p-n+1/2} \oplus E_{p+n+1/2}$
$E_{1/2}$	$E_2 \oplus E_3$	$E_3 \oplus E_4$	$E_n \oplus E_{n+1}$	$E_p \oplus E_{p+1}$
$E_{3/2}$	$E_1 \oplus E_4$	$E_2 \oplus E_5$	$E_{n-1} \oplus E_{n+2}$	$E_{p-1} \oplus E_{p+2}$
$E_{5/2}$	$\{A_1\} \oplus A_2 \oplus E_5$	$E_1 \oplus E_6$	$E_{n-2} \oplus E_{n+3}$	$E_{p-2} \oplus E_{p+3}$
$E_{7/2}$		$\{A_1\} \oplus A_2 \oplus E_7$	$E_{n-3} \oplus E_{n+4}$	$E_{p-3} \oplus E_{p+4}$
$E_{n+1/2}$			$\{A_1\} \oplus A_2 \oplus E_{2n+1}$	$E_{p-n} \oplus E_{p+n+1}$

$n = 4,5,6,\ldots,\quad p = 5,6,7,\ldots,\quad p > n$

T **59**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{\infty v}$	$(\mathbf{C}_{10v})$	$(\mathbf{C}_{9v})$	$(\mathbf{C}_{8v})$	$(\mathbf{C}_{7v})$	$(\mathbf{C}_{6v})$	$(\mathbf{C}_{5v})$	$(\mathbf{C}_{4v})$	$(\mathbf{C}_{3v})$	$(\mathbf{C}_{2v})$
A_1	A_1	A_1	A_1	A_1	A_1	A_1	A_1	A_1	A_1
A_2	A_2	A_2	A_2	A_2	A_2	A_2	A_2	A_2	A_2
E_1	E_1	E_1	E_1	E_1	E_1	E_1	E	E	$B_1 \oplus B_2$
E_2	E_2	E_2	E_2	E_2	E_2	E_2	$B_1 \oplus B_2$	E	$A_1 \oplus A_2$
E_3	E_3	E_3	E_3	E_3	$B_1 \oplus B_2$	E_2	E	$A_1 \oplus A_2$	$B_1 \oplus B_2$
E_4	E_4	E_4	$B_1 \oplus B_2$	E_3	E_2	E_1	$A_1 \oplus A_2$	E	$A_1 \oplus A_2$
E_5	$B_1 \oplus B_2$	E_4	E_3	E_2	E_1	$A_1 \oplus A_2$	E	E	$B_1 \oplus B_2$
E_6	E_4	E_3	E_2	E_1	$A_1 \oplus A_2$	E_1	$B_1 \oplus B_2$	$A_1 \oplus A_2$	$B_1 \oplus B_2$
E_7	E_3	E_2	E_1	$A_1 \oplus A_2$	E_1	E_2	E	E	$B_1 \oplus B_2$
E_8	E_2	E_1	$A_1 \oplus A_2$	E_1	E_2	E_2	$A_1 \oplus A_2$	E	$A_1 \oplus A_2$
E_9	E_1	$A_1 \oplus A_2$	E_1	E_2	$B_1 \oplus B_2$	E_1	E	$A_1 \oplus A_2$	$B_1 \oplus B_2$
E_{10}	$A_1 \oplus A_2$	E_1	E_2	E_3	E_2	$A_1 \oplus A_2$	$B_1 \oplus B_2$	E	$A_1 \oplus A_2$
$\vdots$									
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	$E_{3/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{1/2}$
$E_{5/2}$	$E_{5/2}$	$E_{5/2}$	$E_{5/2}$	$E_{5/2}$	$E_{5/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$
$E_{7/2}$	$E_{7/2}$	$E_{7/2}$	$E_{7/2}$	${}^1E_{7/2} \oplus {}^2E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{9/2}$	$E_{9/2}$	${}^1E_{9/2} \oplus {}^2E_{9/2}$	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{1/2}$
$E_{11/2}$	$E_{9/2}$	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$
$E_{13/2}$	$E_{7/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	$E_{3/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$
$E_{15/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	$E_{3/2}$	${}^1E_{5/2} \oplus {}^2E_{5/2}$	$E_{1/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	$E_{1/2}$
$E_{17/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{19/2}$	$E_{1/2}$	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$	$E_{5/2}$	$E_{1/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$
$\vdots$									

Other subgroups: $\mathbf{C}_n$, $\mathbf{C}_s$ (see $\mathbf{C}_{nv}$; $n = 2, 3, \ldots, 10$).

T **59**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{\infty v}$
0	A_1
1	$A_1 \oplus E_1$
2	$A_1 \oplus E_1 \oplus E_2$
3	$A_1 \oplus E_1 \oplus E_2 \oplus E_3$
n	$A_1 \oplus E_1 \oplus E_2 \oplus E_3 \oplus \cdots \oplus E_n$
$\frac{1}{2}$	$E_{1/2}$
$\frac{3}{2}$	$E_{1/2} \oplus E_{3/2}$
$\frac{5}{2}$	$E_{1/2} \oplus E_{3/2} \oplus E_{5/2}$
$\frac{7}{2}$	$E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2}$
$n + \frac{1}{2}$	$E_{1/2} \oplus E_{3/2} \oplus E_{5/2} \oplus E_{7/2} \oplus \cdots \oplus E_{n+1/2}$

$n = 4, 5, 6, \ldots$

T **59**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 $\mathbf{C}_{\infty v}$

a_2	e_n	E_n 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	$e_{n-1/2}$	$E_{n-1/2}$ 1	2
1	1	1	0
1	2	0	$\bar{1}$

e_n	e_n	A_1 1	A_2 1	E_{2n} 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	0	1

e_n	e_p	E_{p-n} 1	2	E_{p+n} 1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

e_n	$e_{n-1/2}$	$E_{1/2}$ 1	2	$E_{2n-1/2}$ 1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_p	$e_{n-1/2}$	$E_{p-n+1/2}$ 1	2	$E_{p+n-1/2}$ 1	2
1	1	0	0	1	0
1	2	1	0	0	0
2	1	0	$\bar{1}$	0	0
2	2	0	0	0	1

e_n	$e_{p-1/2}$	$E_{p-n-1/2}$ 1	2	$E_{p+n-1/2}$ 1	2
1	1	0	0	1	0
1	2	0	1	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$e_{n-1/2}$	$e_{n-1/2}$	A_1 1	A_2 1	E_{2n-1} 1	2
1	1	0	0	1	0
1	2	u	u	0	0
2	1	$\bar{\text{u}}$	u	0	0
2	2	0	0	0	1

$e_{n-1/2}$	$e_{p-1/2}$	E_{p-n} 1	2	E_{p+n-1} 1	2
1	1	0	0	1	0
1	2	0	$\bar{1}$	0	0
2	1	1	0	0	0
2	2	0	0	0	1

$n = 1, 2, 3, \ldots,$ $p = 2, 3, 4, \ldots,$ $p > n,$ $\text{u} = 2^{-1/2}$

The groups $\mathbf{C}_{nh}$

Notation for headers

Items in header read from left to right

1		Hermann–Mauguin symbol for the point group.
2		$\|G\|$ order of the group.
3		$\|C\|$ number of classes in the group.
4		$\|\widetilde{C}\|$ number of classes in the double group.
5		Number of the table.
6		Page reference for the notation of the header, of the first five subsections below it, and of the footers.
7	☐	This symbol indicates a crystallographic point group.
8		Schönflies notation for the point group.

Notation for the first five subsections below the header

(1) Product forms — Direct and semidirect product forms. ⊗ Direct product. ⓢ Semidirect product.

(2) Group chains *(See pp.* 41, 67*)* — Groups underlined: invariant. Groups in brackets: when subducing one or more of the representations in the tables to these subgroups in the setting used for them in the tables a change of bases (similarity transformation) is required.

(3) Operations of G — Lists all the operations of G, enclosing in brackets all the operations of the same class.

(4) Operations of $\widetilde{G}$ — Lists all the operations of $\widetilde{G}$, enclosing in brackets all the operations of the same class.

(5) Classes and representations — $|r|$ number of regular classes in G (p. 51).
$|\mathsf{i}|$ number of irregular classes in G (p. 51).
$|I|$ number of irreducible representations in G.
$|\widetilde{I}|$ number of spinor representations, also called the number of double-group representations.

Use of the footers

Finding your way about the tables — Each page of the tables contains a footer giving the ordering of the group types listed in the tables. The current group type is recognized because it carries no page number under its name. On opening the tables at any page you should then see at a glance whether to move backwards or forwards to find the group type desired. More precisely, the page number for the group type desired gives the page, like the present one, which carries the contents for the group type wanted.

(1) Product forms: $\mathbf{C}_2 \otimes \mathbf{C}_i$, $\mathbf{C}_2 \otimes \mathbf{C}_s$.

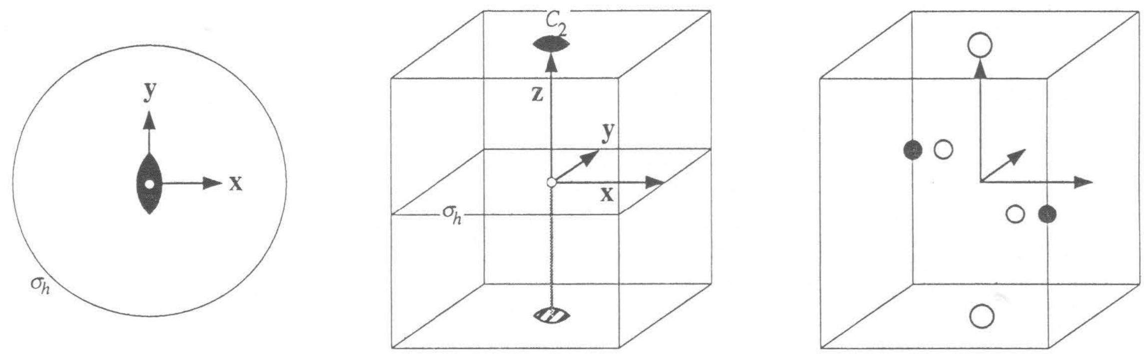
(2) Group chains: $\mathbf{D}_{7d} \supset (\mathbf{C}_{2h}) \supset \underline{\mathbf{C}_s}$, $\mathbf{D}_{7d} \supset (\mathbf{C}_{2h}) \supset \underline{\mathbf{C}_i}$, $\mathbf{D}_{7d} \supset (\mathbf{C}_{2h}) \supset \underline{\mathbf{C}_2}$,
$\mathbf{D}_{5d} \supset (\mathbf{C}_{2h}) \supset \underline{\mathbf{C}_s}$, $\mathbf{D}_{5d} \supset (\mathbf{C}_{2h}) \supset \underline{\mathbf{C}_i}$, $\mathbf{D}_{5d} \supset (\mathbf{C}_{2h}) \supset \underline{\mathbf{C}_2}$,
$\mathbf{D}_{3d} \supset (\mathbf{C}_{2h}) \supset \underline{\mathbf{C}_s}$, $\mathbf{D}_{3d} \supset (\mathbf{C}_{2h}) \supset \underline{\mathbf{C}_i}$, $\mathbf{D}_{3d} \supset (\mathbf{C}_{2h}) \supset \underline{\mathbf{C}_2}$,
$\mathbf{D}_{2h} \supset \underline{\mathbf{C}_{2h}} \supset \underline{\mathbf{C}_s}$, $\mathbf{D}_{2h} \supset \underline{\mathbf{C}_{2h}} \supset \underline{\mathbf{C}_i}$, $\mathbf{D}_{2h} \supset \underline{\mathbf{C}_{2h}} \supset \underline{\mathbf{C}_2}$,
$\mathbf{C}_{10h} \supset \underline{\mathbf{C}_{2h}} \supset \underline{\mathbf{C}_s}$, $\mathbf{C}_{10h} \supset \underline{\mathbf{C}_{2h}} \supset \underline{\mathbf{C}_i}$, $\mathbf{C}_{10h} \supset \underline{\mathbf{C}_{2h}} \supset \underline{\mathbf{C}_2}$,
$\mathbf{C}_{6h} \supset \underline{\mathbf{C}_{2h}} \supset \underline{\mathbf{C}_s}$, $\mathbf{C}_{6h} \supset \underline{\mathbf{C}_{2h}} \supset \underline{\mathbf{C}_i}$, $\mathbf{C}_{6h} \supset \underline{\mathbf{C}_{2h}} \supset \underline{\mathbf{C}_2}$,
$\mathbf{C}_{4h} \supset \underline{\mathbf{C}_{2h}} \supset \underline{\mathbf{C}_s}$, $\mathbf{C}_{4h} \supset \underline{\mathbf{C}_{2h}} \supset \underline{\mathbf{C}_i}$, $\mathbf{C}_{4h} \supset \underline{\mathbf{C}_{2h}} \supset \underline{\mathbf{C}_2}$.

(3) Operations of G: E, C_2, i, σ_h.

(4) Operations of $\widetilde{G}$: E, C_2, i, σ_h,
$\widetilde{E}$, $\widetilde{C}_2$, $\widetilde{\imath}$, $\widetilde{\sigma}_h$.

(5) Classes and representations: $|r| = 4$, $|\mathrm{i}| = 0$, $|I| = 4$, $|\widetilde{I}| = 4$.

F **60** See Chapter **15**, p. 65



Examples: Planar *trans*-1,2-dichloroethylene $C_2H_2Cl_2$, 1,4-dibromo-2,5-dichlorobenzene $C_6H_2Cl_2Br_2$.

T **60**.1 Parameters

Use T **31**.1 ◇. § **16**–1, p. 68

T **60**.2 Multiplication table

Use T **31**.2 ◇. § **16**–2, p. 69

T **60**.3 Factor table

Use T **31**.3 ◇. § **16**–3, p. 70

T **60**.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_{2h}$	E	C_2	i	σ_h	τ
A_g	1	1	1	1	a
B_g	1	−1	1	−1	a
A_u	1	1	−1	−1	a
B_u	1	−1	−1	1	a
${}^1E_{1/2,g}$	1	i	1	i	b
${}^2E_{1/2,g}$	1	−i	1	−i	b
${}^1E_{1/2,u}$	1	i	−1	−i	b
${}^2E_{1/2,u}$	1	−i	−1	i	b

T **60**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{2h}$	0	1	2	3
A_g	$\square 1$	R_z	$\square x^2, y^2, \square z^2, \square xy$	
B_g		R_x, R_y	$\square zx, \square yz$	
A_u		$\square z$		$\square x^2 z, y^2 z, \square z^3, \square xyz$
B_u		$\square x, \square y$		$\square x^3, xy^2, \square xz^2, \square x^2 y, y^3, \square yz^2$

T **60**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{2h}$	$\lvert j\, m\rangle$	ι	μ
A_g	$\lvert 0\, 0\rangle$	2	± 2
B_g	$\lvert 2\, 1\rangle$	2	± 2
A_u	$\lvert 1\, 0\rangle$	2	± 2
B_u	$\lvert 1\, 1\rangle$	2	± 2
${}^1E_{1/2,g}$	$\lvert \frac{1}{2}\, \overline{\frac{1}{2}}\rangle$	1	± 2
${}^2E_{1/2,g}$	$\lvert \frac{1}{2}\, \frac{1}{2}\rangle$	1	± 2
${}^1E_{1/2,u}$	$\lvert \frac{1}{2}\, \overline{\frac{1}{2}}\rangle^{\bullet}$	1	± 2
${}^2E_{1/2,u}$	$\lvert \frac{1}{2}\, \frac{1}{2}\rangle^{\bullet}$	1	± 2

T **60**.7 Matrix representations

Use T **60**.4 ♠. § **16**–7, p. 77

T **60**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_{2h}$	A_g	B_g	A_u	B_u	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$
A_g	A_g	B_g	A_u	B_u	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$
B_g		A_g	B_u	A_u	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$
A_u			A_g	B_g	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$
B_u				A_g	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$
${}^1E_{1/2,g}$					B_g	A_g	B_u	A_u
${}^2E_{1/2,g}$						B_g	A_u	B_u
${}^1E_{1/2,u}$							B_g	A_g
${}^2E_{1/2,u}$								B_g

T **60**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{2h}$	$\mathbf{C}_s$	$\mathbf{C}_i$	$\mathbf{C}_2$
A_g	A'	A_g	A
B_g	A''	A_g	B
A_u	A''	A_u	A
B_u	A'	A_u	B
${}^1E_{1/2,g}$	${}^1E_{1/2}$	$A_{1/2,g}$	${}^1E_{1/2}$
${}^2E_{1/2,g}$	${}^2E_{1/2}$	$A_{1/2,g}$	${}^2E_{1/2}$
${}^1E_{1/2,u}$	${}^2E_{1/2}$	$A_{1/2,u}$	${}^1E_{1/2}$
${}^2E_{1/2,u}$	${}^1E_{1/2}$	$A_{1/2,u}$	${}^2E_{1/2}$

T **60**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{2h}$
$2n$	$(2n+1)\, A_g \oplus 2n\, B_g$
$2n+1$	$(2n+1)\, A_u \oplus (2n+2)\, B_u$
$n+\frac{1}{2}$	$(n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g})$

$n = 0, 1, 2, \ldots$

T **60**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: $\mathbf{C}_3 \otimes \mathbf{C}_s$.

(2) Group chains: $\mathbf{C}_{9h} \supset \underline{\mathbf{C}_{3h}} \supset \underline{\mathbf{C}_s}$, $\mathbf{C}_{9h} \supset \underline{\mathbf{C}_{3h}} \supset \underline{\mathbf{C}_3}$, $\mathbf{C}_{6h} \supset \underline{\mathbf{C}_{3h}} \supset \underline{\mathbf{C}_s}$, $\mathbf{C}_{6h} \supset \underline{\mathbf{C}_{3h}} \supset \underline{\mathbf{C}_3}$, $\mathbf{D}_{3h} \supset \underline{\mathbf{C}_{3h}} \supset \underline{\mathbf{C}_s}$, $\mathbf{D}_{3h} \supset \underline{\mathbf{C}_{3h}} \supset \underline{\mathbf{C}_3}$.

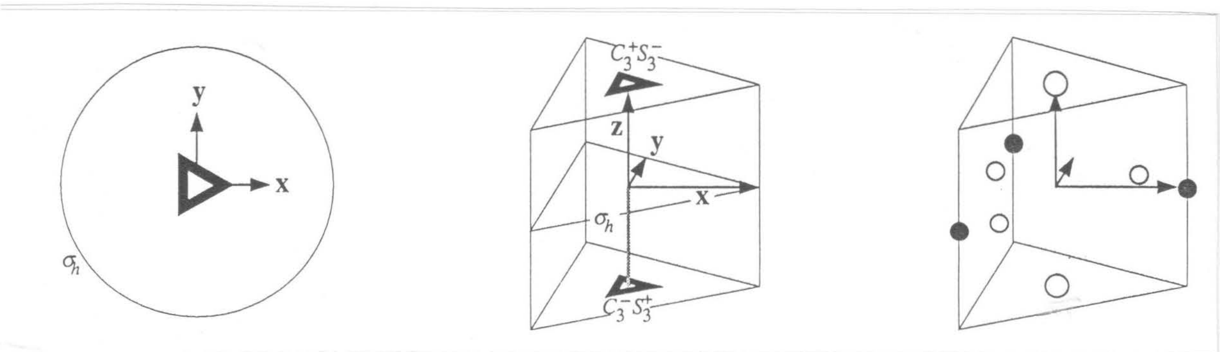
(3) Operations of G: E, C_3^+, C_3^-, σ_h, S_3^+, S_3^-.

(4) Operations of $\widetilde{G}$: E, C_3^+, C_3^-, σ_h, S_3^+, S_3^-,
$\widetilde{E}$, $\widetilde{C}_3^+$, $\widetilde{C}_3^-$, $\widetilde{\sigma}_h$, $\widetilde{S}_3^+$, $\widetilde{S}_3^-$.

(5) Classes and representations: $|r| = 6$, $|\text{i}| = 0$, $|I| = 6$, $|\widetilde{I}| = 6$.

F **61**

See Chapter **15**, p. 65



Examples: $C^+(NH_2)_3$, planar $B(OH)_3$.

T **61**.1 Parameters

Use T **35**.1. § **16**–1, p. 68

T **61**.2 Multiplication table

Use T **35**.2. § **16**–2, p. 69

T **61**.3 Factor table

Use T **35**.3. § **16**–3, p. 70

T **61**.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_{3h}$	E	C_3^+	C_3^-	σ_h	S_3^+	S_3^-	τ
A'	1	1	1	1	1	1	a
${}^1E'$	1	ϵ^*	ϵ	1	ϵ^*	ϵ	b
${}^2E'$	1	ϵ	ϵ^*	1	ϵ	ϵ^*	b
A''	1	1	1	-1	-1	-1	a
${}^1E''$	1	ϵ^*	ϵ	-1	$-\epsilon^*$	$-\epsilon$	b
${}^2E''$	1	ϵ	ϵ^*	-1	$-\epsilon$	$-\epsilon^*$	b
${}^1E_{1/2}$	1	$-\epsilon^*$	$-\epsilon$	i	iϵ^*	$-$iϵ	b
${}^2E_{1/2}$	1	$-\epsilon$	$-\epsilon^*$	$-$i	$-$iϵ	iϵ^*	b
${}^1E_{3/2}$	1	-1	-1	i	i	$-$i	b
${}^2E_{3/2}$	1	-1	-1	$-$i	$-$i	i	b
${}^1E_{5/2}$	1	$-\epsilon$	$-\epsilon^*$	i	iϵ	$-$iϵ^*	b
${}^2E_{5/2}$	1	$-\epsilon^*$	$-\epsilon$	$-$i	$-$iϵ^*	iϵ	b

$\epsilon = \exp(2\pi\text{i}/3)$

T **61**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{3h}$	0	1	2	3
A'	$^\square 1$	R_z	$x^2+y^2, {}^\square z^2$	$^\square x(x^2-3y^2), {}^\square y(3x^2-y^2)$
$^1E' \oplus {}^2E'$		$^\square(x,y)$	$^\square(xy, x^2-y^2)$	$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
A''		$^\square z$		$(x^2+y^2)z, {}^\square z^3$
$^1E'' \oplus {}^2E''$		(R_x, R_y)	$^\square(zx, yz)$	$^\square\{xyz, z(x^2-y^2)\}$

T **61**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{3h}$	$\lvert j\, m\rangle$		ι	μ
A'	$\lvert 0\,0\rangle$	$\lvert 3\,3\rangle$	2	± 6
$^1E'$	$\lvert 1\,1\rangle$	$\lvert 2\,\overline{2}\rangle$	2	± 6
$^2E'$	$\lvert 1\,\overline{1}\rangle$	$\lvert 2\,2\rangle$	2	± 6
A''	$\lvert 1\,0\rangle$	$\lvert 4\,3\rangle$	2	± 6
$^1E''$	$\lvert 2\,1\rangle$	$\lvert 3\,\overline{2}\rangle$	2	± 6
$^2E''$	$\lvert 2\,\overline{1}\rangle$	$\lvert 3\,2\rangle$	2	± 6
$^1E_{1/2}$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle^\bullet$	1	± 6
$^2E_{1/2}$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle$	$\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle^\bullet$	1	± 6
$^1E_{3/2}$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle$	$\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle^\bullet$	1	± 6
$^2E_{3/2}$	$\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle^\bullet$	1	± 6
$^1E_{5/2}$	$\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle^\bullet$	1	± 6
$^2E_{5/2}$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle^\bullet$	1	± 6

T **61**.7 Matrix representations

Use T **61**.4 ♠. § **16**–7, p. 77

T **61**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_{3h}$	A'	$^1E'$	$^2E'$	A''	$^1E''$	$^2E''$	$^1E_{1/2}$	$^2E_{1/2}$	$^1E_{3/2}$	$^2E_{3/2}$	$^1E_{5/2}$	$^2E_{5/2}$
A'	A'	$^1E'$	$^2E'$	A''	$^1E''$	$^2E''$	$^1E_{1/2}$	$^2E_{1/2}$	$^1E_{3/2}$	$^2E_{3/2}$	$^1E_{5/2}$	$^2E_{5/2}$
$^1E'$		$^2E'$	A'	$^1E''$	$^2E''$	A''	$^1E_{5/2}$	$^2E_{3/2}$	$^1E_{1/2}$	$^2E_{5/2}$	$^1E_{3/2}$	$^2E_{1/2}$
$^2E'$			$^1E'$	$^2E''$	A''	$^1E''$	$^1E_{3/2}$	$^2E_{5/2}$	$^1E_{5/2}$	$^2E_{1/2}$	$^1E_{1/2}$	$^2E_{3/2}$
A''				A'	$^1E'$	$^2E'$	$^2E_{5/2}$	$^1E_{5/2}$	$^2E_{3/2}$	$^1E_{3/2}$	$^2E_{1/2}$	$^1E_{1/2}$
$^1E''$					$^2E'$	A'	$^2E_{1/2}$	$^1E_{3/2}$	$^2E_{5/2}$	$^1E_{1/2}$	$^2E_{3/2}$	$^1E_{5/2}$
$^2E''$						$^1E'$	$^2E_{3/2}$	$^1E_{1/2}$	$^2E_{1/2}$	$^1E_{5/2}$	$^2E_{5/2}$	$^1E_{3/2}$
$^1E_{1/2}$							$^2E''$	A'	$^1E''$	$^1E'$	A''	$^2E'$
$^2E_{1/2}$								$^1E''$	$^2E'$	$^2E''$	$^1E'$	A''
$^1E_{3/2}$									A''	A'	$^2E''$	$^1E'$
$^2E_{3/2}$										A''	$^2E'$	$^1E''$
$^1E_{5/2}$											$^1E''$	A'
$^2E_{5/2}$												$^2E''$

T **61**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{3h}$	$\mathbf{C}_s$	$\mathbf{C}_3$
A'	A'	A
${}^1E'$	A'	${}^1E$
${}^2E'$	A'	${}^2E$
A''	A''	A
${}^1E''$	A''	${}^1E$
${}^2E''$	A''	${}^2E$
${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$
${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$
${}^1E_{3/2}$	${}^1E_{1/2}$	$A_{3/2}$
${}^2E_{3/2}$	${}^2E_{1/2}$	$A_{3/2}$
${}^1E_{5/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^2E_{5/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$

T **61**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{3h}$
$3n$	$(n+1)\,A' \oplus n\,({}^1E' \oplus {}^2E' \oplus A'' \oplus {}^1E'' \oplus {}^2E'')$
$3n+1$	$n\,(A' \oplus {}^1E'' \oplus {}^2E'') \oplus (n+1)({}^1E' \oplus {}^2E' \oplus A'')$
$3n+2$	$(n+1)(A' \oplus {}^1E' \oplus {}^2E' \oplus {}^1E'' \oplus {}^2E'') \oplus n\,A''$
$6n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus 2n\,({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2})$
$6n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus 2n\,({}^1E_{5/2} \oplus {}^2E_{5/2})$
$6n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2})$
$6n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus (2n+2)({}^1E_{5/2} \oplus {}^2E_{5/2})$
$6n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus (2n+2)({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2})$
$6n+\frac{11}{2}$	$(2n+2)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2})$

$n = 0, 1, 2, \ldots$

T **61**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: $\mathbf{C}_4 \otimes \mathbf{C}_i$, $\mathbf{C}_4 \otimes \mathbf{C}_s$.

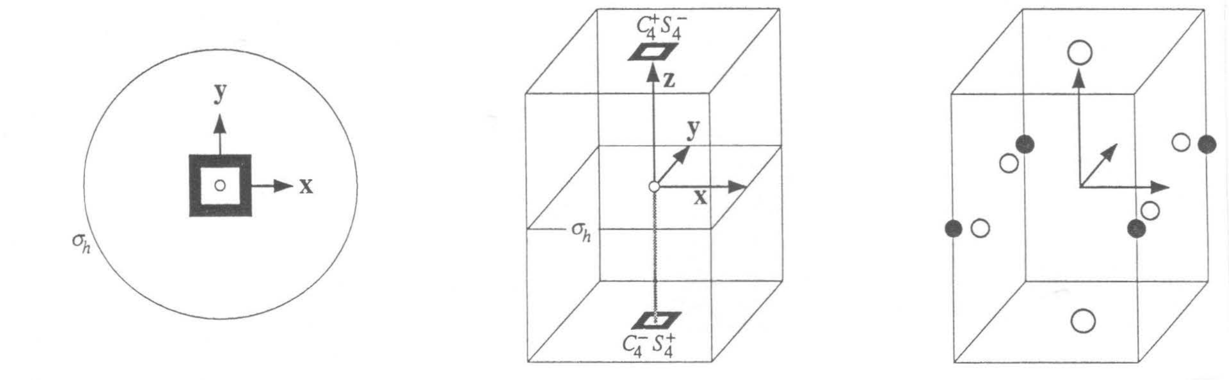
(2) Group chains: $\mathbf{C}_{8h} \supset \underline{\mathbf{C}_{4h}} \supset \underline{\mathbf{C}_{2h}}$, $\mathbf{C}_{8h} \supset \underline{\mathbf{C}_{4h}} \supset \underline{\mathbf{S}_4}$, $\mathbf{C}_{8h} \supset \underline{\mathbf{C}_{4h}} \supset \underline{\mathbf{C}_4}$,
$\mathbf{D}_{4h} \supset \underline{\mathbf{C}_{4h}} \supset \underline{\mathbf{C}_{2h}}$, $\mathbf{D}_{4h} \supset \underline{\mathbf{C}_{4h}} \supset \underline{\mathbf{S}_4}$, $\mathbf{D}_{4h} \supset \underline{\mathbf{C}_{4h}} \supset \underline{\mathbf{C}_4}$.

(3) Operations of G: E, C_4^+, C_2, C_4^-, i, S_4^-, σ_h, S_4^+.

(4) Operations of $\widetilde{G}$: E, C_4^+, C_2, C_4^-, i, S_4^-, σ_h, S_4^+,
$\widetilde{E}$, $\widetilde{C}_4^+$, $\widetilde{C}_2$, $\widetilde{C}_4^-$, $\widetilde{\imath}$, $\widetilde{S}_4^-$, $\widetilde{\sigma}_h$, $\widetilde{S}_4^+$.

(5) Classes and representations: $\lvert r\rvert = 8$, $\lvert \mathrm{i}\rvert = 0$, $\lvert I\rvert = 8$, $\lvert\widetilde{I}\rvert = 8$.

F **62** See Chapter **15**, p. 65



Examples:

T **62**.1 Parameters
Use T **33**.1. § **16**–1, p. 68

T **62**.2 Multiplication table
Use T **33**.2. § **16**–2, p. 69

T **62**.3 Factor table
Use T **33**.3. § **16**–3, p. 70

T **62**.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_{4h}$	E	C_4^+	C_2	C_4^-	i	S_4^-	σ_h	S_4^+	τ
A_g	1	1	1	1	1	1	1	1	a
B_g	1	−1	1	−1	1	−1	1	−1	a
^1E_g	1	−i	−1	i	1	−i	−1	i	b
^2E_g	1	i	−1	−i	1	i	−1	−i	b
A_u	1	1	1	1	−1	−1	−1	−1	a
B_u	1	−1	1	−1	−1	1	−1	1	a
^1E_u	1	−i	−1	i	−1	i	1	−i	b
^2E_u	1	i	−1	−i	−1	−i	1	i	b
$^1E_{1/2,g}$	1	ϵ^*	−i	ϵ	1	ϵ^*	−i	ϵ	b
$^2E_{1/2,g}$	1	ϵ	i	ϵ^*	1	ϵ	i	ϵ^*	b
$^1E_{3/2,g}$	1	$-\epsilon^*$	−i	$-\epsilon$	1	$-\epsilon^*$	−i	$-\epsilon$	b
$^2E_{3/2,g}$	1	$-\epsilon$	i	$-\epsilon^*$	1	$-\epsilon$	i	$-\epsilon^*$	b
$^1E_{1/2,u}$	1	ϵ^*	−i	ϵ	−1	$-\epsilon^*$	i	$-\epsilon$	b
$^2E_{1/2,u}$	1	ϵ	i	ϵ^*	−1	$-\epsilon$	−i	$-\epsilon^*$	b
$^1E_{3/2,u}$	1	$-\epsilon^*$	−i	$-\epsilon$	−1	ϵ^*	i	ϵ	b
$^2E_{3/2,u}$	1	$-\epsilon$	i	$-\epsilon^*$	−1	ϵ	−i	ϵ^*	b

$\epsilon = \exp(2\pi\mathrm{i}/8)$

T **62**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{4h}$	0	1	2	3
A_g	$^\square 1$	R_z	$x^2+y^2, {}^\square z^2$	
B_g			$^\square x^2-y^2, {}^\square xy$	
$^1E_g \oplus {}^2E_g$		(R_x, R_y)	$^\square(zx, yz)$	
A_u		$^\square z$		$(x^2+y^2)z, {}^\square z^3$
B_u				$^\square z(x^2-y^2), {}^\square xyz$
$^1E_u \oplus {}^2E_u$		$^\square(x, y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2),$ $^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$

T **62**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{4h}$	$\|j\,m\rangle$	ι	μ	$\mathbf{C}_{4h}$	$\|j\,m\rangle$	ι	μ
A_g	$\|0\,0\rangle$	2	±4	$^1E_{1/2,g}$	$\|\frac{1}{2}\,\frac{1}{2}\rangle$	1	±4
B_g	$\|2\,2\rangle$	2	±4	$^2E_{1/2,g}$	$\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	1	±4
^1E_g	$\|2\,1\rangle$	2	±4	$^1E_{3/2,g}$	$\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	1	±4
^2E_g	$\|2\,\overline{1}\rangle$	2	±4	$^2E_{3/2,g}$	$\|\frac{3}{2}\,\frac{3}{2}\rangle$	1	±4
A_u	$\|1\,0\rangle$	2	±4	$^1E_{1/2,u}$	$\|\frac{1}{2}\,\frac{1}{2}\rangle^\bullet$	1	±4
B_u	$\|3\,2\rangle$	2	±4	$^2E_{1/2,u}$	$\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle^\bullet$	1	±4
^1E_u	$\|1\,1\rangle$	2	±4	$^1E_{3/2,u}$	$\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle^\bullet$	1	±4
^2E_u	$\|1\,\overline{1}\rangle$	2	±4	$^2E_{3/2,u}$	$\|\frac{3}{2}\,\frac{3}{2}\rangle^\bullet$	1	±4

T **62**.7 Matrix representations

Use T **62**.4 ♠. § **16**–7, p. 77

T **62**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_{4h}$	A_g	B_g	${}^1E_g$	${}^2E_g$	A_u	B_u	${}^1E_u$	${}^2E_u$
A_g	A_g	B_g	${}^1E_g$	${}^2E_g$	A_u	B_u	${}^1E_u$	${}^2E_u$
B_g		A_g	${}^2E_g$	${}^1E_g$	B_u	A_u	${}^2E_u$	${}^1E_u$
${}^1E_g$			B_g	A_g	${}^1E_u$	${}^2E_u$	B_u	A_u
${}^2E_g$				B_g	${}^2E_u$	${}^1E_u$	A_u	B_u
A_u					A_g	B_g	${}^1E_g$	${}^2E_g$
B_u						A_g	${}^2E_g$	${}^1E_g$
${}^1E_u$							B_g	A_g
${}^2E_u$								B_g

$\twoheadrightarrow$

T **62**.8 Direct products of representations *(cont.)*

$\mathbf{C}_{4h}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$
A_g	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$
B_g	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$
${}^1E_g$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$
${}^2E_g$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$
A_u	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$
B_u	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$
${}^1E_u$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$
${}^2E_u$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$
${}^1E_{1/2,g}$	${}^1E_g$	A_g	${}^2E_g$	B_g	${}^1E_u$	A_u	${}^2E_u$	B_u
${}^2E_{1/2,g}$		${}^2E_g$	B_g	${}^1E_g$	A_u	${}^2E_u$	B_u	${}^1E_u$
${}^1E_{3/2,g}$			${}^1E_g$	A_g	${}^2E_u$	B_u	${}^1E_u$	A_u
${}^2E_{3/2,g}$				${}^2E_g$	B_u	${}^1E_u$	A_u	${}^2E_u$
${}^1E_{1/2,u}$					${}^1E_g$	A_g	${}^2E_g$	B_g
${}^2E_{1/2,u}$						${}^2E_g$	B_g	${}^1E_g$
${}^1E_{3/2,u}$							${}^1E_g$	A_g
${}^2E_{3/2,u}$								${}^2E_g$

T **62**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{4h}$	$\mathbf{C}_{2h}$	$\mathbf{S}_4$	$\mathbf{C}_s$	$\mathbf{C}_i$	$\mathbf{C}_4$	$\mathbf{C}_2$
A_g	A_g	A	A'	A_g	A	A
B_g	A_g	B	A'	A_g	B	A
${}^1E_g$	B_g	${}^1E$	A''	A_g	${}^1E$	B
${}^2E_g$	B_g	${}^2E$	A''	A_g	${}^2E$	B
A_u	A_u	B	A''	A_u	A	A
B_u	A_u	A	A''	A_u	B	A
${}^1E_u$	B_u	${}^2E$	A'	A_u	${}^1E$	B
${}^2E_u$	B_u	${}^1E$	A'	A_u	${}^2E$	B
${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2}$	${}^2E_{1/2}$	$A_{1/2,g}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2}$	${}^1E_{1/2}$	$A_{1/2,g}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^1E_{3/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2}$	${}^2E_{1/2}$	$A_{1/2,g}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2}$	${}^1E_{1/2}$	$A_{1/2,g}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2}$	${}^1E_{1/2}$	$A_{1/2,u}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2}$	${}^2E_{1/2}$	$A_{1/2,u}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^1E_{3/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2}$	${}^1E_{1/2}$	$A_{1/2,u}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2}$	${}^2E_{1/2}$	$A_{1/2,u}$	${}^2E_{3/2}$	${}^1E_{1/2}$

T **62**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_4$
$4n$	$(2n+1)\,A_g \oplus 2n\,(B_g \oplus {}^1E_g \oplus {}^2E_g)$
$4n+1$	$(2n+1)(A_u \oplus {}^1E_u \oplus {}^2E_u) \oplus 2n\,B_u$
$4n+2$	$(2n+1)(A_g \oplus {}^1E_g \oplus {}^2E_g) \oplus (2n+2)\,B_g$
$4n+3$	$(2n+1)\,A_u \oplus (2n+2)(B_u \oplus {}^1E_u \oplus {}^2E_u)$
$4n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus 2n\,({}^1E_{3/2,g} \oplus {}^2E_{3/2,g})$
$4n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g})$
$4n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus (2n+2)({}^1E_{3/2,g} \oplus {}^2E_{3/2,g})$
$4n+\frac{7}{2}$	$(2n+2)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g})$

$n = 0, 1, 2, \ldots$

T **62**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: $\mathbf{C}_5 \otimes \mathbf{C}_s$.

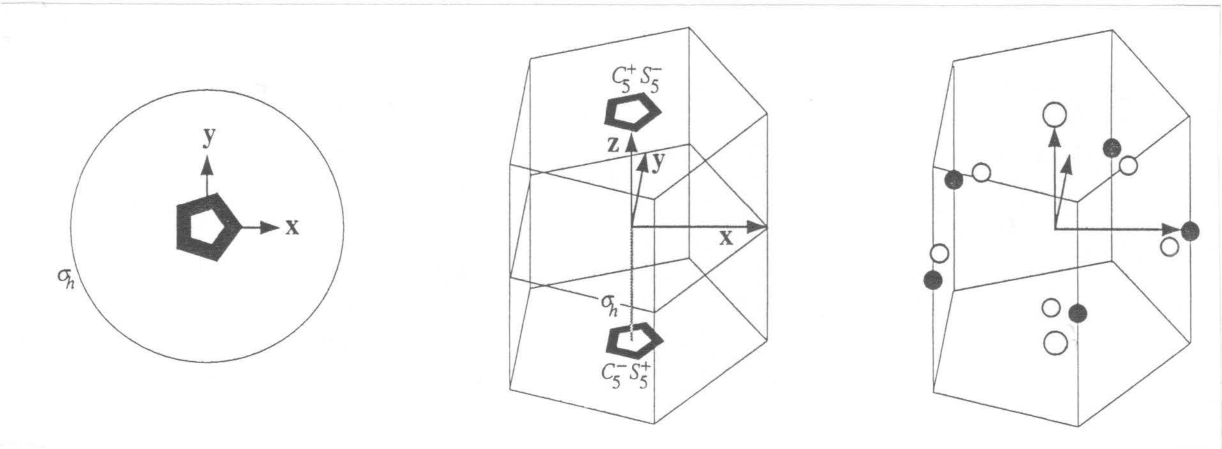
(2) Group chains: $\mathbf{C}_{10h} \supset \underline{\mathbf{C}_{5h}} \supset \underline{\mathbf{C}_s}$, $\mathbf{C}_{10h} \supset \underline{\mathbf{C}_{5h}} \supset \underline{\mathbf{C}_5}$, $\mathbf{D}_{5h} \supset \underline{\mathbf{C}_{5h}} \supset \underline{\mathbf{C}_s}$, $\mathbf{D}_{5h} \supset \underline{\mathbf{C}_{5h}} \supset \underline{\mathbf{C}_5}$.

(3) Operations of G: E, C_5^+, C_5^{2+}, C_5^{2-}, C_5^-, σ_h, S_5^+, S_5^{2+}, S_5^{2-}, S_5^-.

(4) Operations of $\widetilde{G}$: E, C_5^+, C_5^{2+}, C_5^{2-}, C_5^-, σ_h, S_5^+, S_5^{2+}, S_5^{2-}, S_5^-,
$\widetilde{E}$, $\widetilde{C}_5^+$, $\widetilde{C}_5^{2+}$, $\widetilde{C}_5^{2-}$, $\widetilde{C}_5^-$, $\widetilde{\sigma}_h$, $\widetilde{S}_5^+$, $\widetilde{S}_5^{2+}$, $\widetilde{S}_5^{2-}$, $\widetilde{S}_5^-$.

(5) Classes and representations: $|r| = 10$, $|i| = 0$, $|I| = 10$, $|\widetilde{I}| = 10$.

F **63** See Chapter **15**, p. 65



Examples:

T **63**.1 Parameters
Use T **39**.1. § **16**–1, p. 68

T **63**.2 Multiplication table
Use T **39**.2. § **16**–2, p. 69

T **63**.3 Factor table
Use T **39**.3. § **16**–3, p. 70

T **63**.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_{5h}$	E	C_5^+	C_5^{2+}	C_5^{2-}	C_5^-	σ_h	S_5^+	S_5^{2+}	S_5^{2-}	S_5^-	τ
A'	1	1	1	1	1	1	1	1	1	1	a
${}^1E_1'$	1	δ^*	ϵ^*	ϵ	δ	1	δ^*	ϵ^*	ϵ	δ	b
${}^2E_1'$	1	δ	ϵ	ϵ^*	δ^*	1	δ	ϵ	ϵ^*	δ^*	b
${}^1E_2'$	1	ϵ^*	δ	δ^*	ϵ	1	ϵ^*	δ	δ^*	ϵ	b
${}^2E_2'$	1	ϵ	δ^*	δ	ϵ^*	1	ϵ	δ^*	δ	ϵ^*	b
A''	1	1	1	1	1	-1	-1	-1	-1	-1	a
${}^1E_1''$	1	δ^*	ϵ^*	ϵ	δ	-1	$-\delta^*$	$-\epsilon^*$	$-\epsilon$	$-\delta$	b
${}^2E_1''$	1	δ	ϵ	ϵ^*	δ^*	-1	$-\delta$	$-\epsilon$	$-\epsilon^*$	$-\delta^*$	b
${}^1E_2''$	1	ϵ^*	δ	δ^*	ϵ	-1	$-\epsilon^*$	$-\delta$	$-\delta^*$	$-\epsilon$	b
${}^2E_2''$	1	ϵ	δ^*	δ	ϵ^*	-1	$-\epsilon$	$-\delta^*$	$-\delta$	$-\epsilon^*$	b
${}^1E_{1/2}$	1	$-\epsilon^*$	δ	δ^*	$-\epsilon$	i	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\delta$	$\mathrm{i}\delta^*$	$-\mathrm{i}\epsilon$	b
${}^2E_{1/2}$	1	$-\epsilon$	δ^*	δ	$-\epsilon^*$	$-\mathrm{i}$	$-\mathrm{i}\epsilon$	$\mathrm{i}\delta^*$	$-\mathrm{i}\delta$	$\mathrm{i}\epsilon^*$	b
${}^1E_{3/2}$	1	$-\delta$	ϵ	ϵ^*	$-\delta^*$	i	$\mathrm{i}\delta$	$-\mathrm{i}\epsilon$	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\delta^*$	b
${}^2E_{3/2}$	1	$-\delta^*$	ϵ^*	ϵ	$-\delta$	$-\mathrm{i}$	$-\mathrm{i}\delta^*$	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\epsilon$	$\mathrm{i}\delta$	b
${}^1E_{5/2}$	1	-1	1	1	-1	i	i	$-\mathrm{i}$	i	$-\mathrm{i}$	b
${}^2E_{5/2}$	1	-1	1	1	-1	$-\mathrm{i}$	$-\mathrm{i}$	i	$-\mathrm{i}$	i	b
${}^1E_{7/2}$	1	$-\delta^*$	ϵ^*	ϵ	$-\delta$	i	$\mathrm{i}\delta^*$	$-\mathrm{i}\epsilon^*$	$\mathrm{i}\epsilon$	$-\mathrm{i}\delta$	b
${}^2E_{7/2}$	1	$-\delta$	ϵ	ϵ^*	$-\delta^*$	$-\mathrm{i}$	$-\mathrm{i}\delta$	$\mathrm{i}\epsilon$	$-\mathrm{i}\epsilon^*$	$\mathrm{i}\delta^*$	b
${}^1E_{9/2}$	1	$-\epsilon$	δ^*	δ	$-\epsilon^*$	i	$\mathrm{i}\epsilon$	$-\mathrm{i}\delta^*$	$\mathrm{i}\delta$	$-\mathrm{i}\epsilon^*$	b
${}^2E_{9/2}$	1	$-\epsilon^*$	δ	δ^*	$-\epsilon$	$-\mathrm{i}$	$-\mathrm{i}\epsilon^*$	$\mathrm{i}\delta$	$-\mathrm{i}\delta^*$	$\mathrm{i}\epsilon$	b

$\delta = \exp(2\pi\mathrm{i}/5)$, $\epsilon = \exp(4\pi\mathrm{i}/5)$

T **63**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{5h}$	0	1	2	3
A'	$^\square 1$	R_z	$x^2+y^2, {}^\square z^2$	
${}^1E_1' \oplus {}^2E_1'$		$^\square(x,y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
${}^1E_2' \oplus {}^2E_2'$			$^\square(xy, x^2-y^2)$	$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$
A''		$^\square z$		$(x^2+y^2)z, {}^\square z^3$
${}^1E_1'' \oplus {}^2E_1''$		(R_x, R_y)	$^\square(zx, yz)$	
${}^1E_2'' \oplus {}^2E_2''$				$^\square\{xyz, z(x^2-y^2)\}$

T **63**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{5h}$	$\lvert j\, m\rangle$		ι	μ	$\mathbf{C}_{5h}$	$\lvert j\, m\rangle$		ι	μ
A'	$\lvert 0\,0\rangle$	$\lvert 5\,5\rangle$	2	± 10	${}^1E_{1/2}$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle^\bullet$	1	± 10
${}^1E_1'$	$\lvert 1\,1\rangle$	$\lvert 4\,\overline{4}\rangle$	2	± 10	${}^2E_{1/2}$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle$	$\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle^\bullet$	1	± 10
${}^2E_1'$	$\lvert 1\,\overline{1}\rangle$	$\lvert 4\,4\rangle$	2	± 10	${}^1E_{3/2}$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle$	$\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle^\bullet$	1	± 10
${}^1E_2'$	$\lvert 2\,2\rangle$	$\lvert 3\,\overline{3}\rangle$	2	± 10	${}^2E_{3/2}$	$\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle^\bullet$	1	± 10
${}^2E_2'$	$\lvert 2\,\overline{2}\rangle$	$\lvert 3\,3\rangle$	2	± 10	${}^1E_{5/2}$	$\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle^\bullet$	1	± 10
A''	$\lvert 1\,0\rangle$	$\lvert 6\,5\rangle$	2	± 10	${}^2E_{5/2}$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle$	$\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle^\bullet$	1	± 10
${}^1E_1''$	$\lvert 2\,1\rangle$	$\lvert 5\,\overline{4}\rangle$	2	± 10	${}^1E_{7/2}$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle$	$\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle^\bullet$	1	± 10
${}^2E_1''$	$\lvert 2\,\overline{1}\rangle$	$\lvert 5\,4\rangle$	2	± 10	${}^2E_{7/2}$	$\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle^\bullet$	1	± 10
${}^1E_2''$	$\lvert 3\,2\rangle$	$\lvert 4\,\overline{3}\rangle$	2	± 10	${}^1E_{9/2}$	$\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle^\bullet$	1	± 10
${}^2E_2''$	$\lvert 3\,\overline{2}\rangle$	$\lvert 4\,3\rangle$	2	± 10	${}^2E_{9/2}$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle^\bullet$	1	± 10

T **63**.7 Matrix representations

Use T **63**.4 ♠. § **16**–7, p. 77

T **63**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_{5h}$	A'	${}^1E_1'$	${}^2E_1'$	${}^1E_2'$	${}^2E_2'$	A''	${}^1E_1''$	${}^2E_1''$	${}^1E_2''$	${}^2E_2''$
A'	A'	${}^1E_1'$	${}^2E_1'$	${}^1E_2'$	${}^2E_2'$	A''	${}^1E_1''$	${}^2E_1''$	${}^1E_2''$	${}^2E_2''$
${}^1E_1'$		${}^1E_2'$	A'	${}^2E_2'$	${}^2E_1'$	${}^1E_1''$	${}^1E_2''$	A''	${}^2E_2''$	${}^2E_1''$
${}^2E_1'$			${}^2E_2'$	${}^1E_1'$	${}^1E_2'$	${}^2E_1''$	A''	${}^2E_2''$	${}^1E_1''$	${}^1E_2''$
${}^1E_2'$				${}^2E_1'$	A'	${}^1E_2''$	${}^2E_2''$	${}^1E_1''$	${}^2E_1''$	A''
${}^2E_2'$					${}^1E_1'$	${}^2E_2''$	${}^2E_1''$	${}^1E_2''$	A''	${}^1E_1''$
A''						A'	${}^1E_1'$	${}^2E_1'$	${}^1E_2'$	${}^2E_2'$
${}^1E_1''$							${}^1E_2'$	A'	${}^2E_2'$	${}^2E_1'$
${}^2E_1''$								${}^2E_2'$	${}^1E_1'$	${}^1E_2'$
${}^1E_2''$									${}^2E_1'$	A'
${}^2E_2''$										${}^1E_1'$

$\twoheadrightarrow$

T **63**.8 Direct products of representations *(cont.)*

$\mathbf{C}_{5h}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$
A'	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$
${}^1E_1'$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^2E_1'$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{9/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$
${}^1E_2'$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{9/2}$	${}^1E_{9/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$
${}^2E_2'$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$
A''	${}^2E_{9/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^1E_1''$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$
${}^2E_1''$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$
${}^1E_2''$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{9/2}$	${}^2E_{9/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$
${}^2E_2''$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$
${}^1E_{1/2}$	${}^2E_1''$	A'	${}^1E_1''$	${}^2E_2'$	${}^1E_2''$	${}^1E_2'$	${}^2E_2''$	${}^1E_1'$	A''	${}^2E_1'$
${}^2E_{1/2}$		${}^1E_1''$	${}^1E_2'$	${}^2E_1''$	${}^2E_2'$	${}^2E_2''$	${}^2E_1'$	${}^1E_2''$	${}^1E_1'$	A''
${}^1E_{3/2}$			${}^2E_2''$	A'	${}^2E_1''$	${}^2E_1'$	A''	${}^2E_2'$	${}^1E_2''$	${}^1E_1'$
${}^2E_{3/2}$				${}^1E_2''$	${}^1E_1'$	${}^1E_1''$	${}^1E_2'$	A''	${}^2E_1'$	${}^2E_2''$
${}^1E_{5/2}$					A''	A'	${}^1E_1''$	${}^2E_1'$	${}^2E_2''$	${}^1E_2'$
${}^2E_{5/2}$						A''	${}^1E_1'$	${}^2E_1''$	${}^2E_2'$	${}^1E_2''$
${}^1E_{7/2}$							${}^1E_2''$	A'	${}^2E_1''$	${}^2E_2'$
${}^2E_{7/2}$								${}^2E_2''$	${}^1E_2'$	${}^1E_1''$
${}^1E_{9/2}$									${}^1E_1''$	A'
${}^2E_{9/2}$										${}^2E_1''$

T **63**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{5h}$	$\mathbf{C}_s$	$\mathbf{C}_5$	$\mathbf{C}_{5h}$	$\mathbf{C}_s$	$\mathbf{C}_5$
A'	A'	A	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$
${}^1E_1'$	A'	${}^1E_1$	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$
${}^2E_1'$	A'	${}^2E_1$	${}^1E_{3/2}$	${}^1E_{1/2}$	${}^1E_{3/2}$
${}^1E_2'$	A'	${}^1E_2$	${}^2E_{3/2}$	${}^2E_{1/2}$	${}^2E_{3/2}$
${}^2E_2'$	A'	${}^2E_2$	${}^1E_{5/2}$	${}^1E_{1/2}$	$A_{5/2}$
A''	A''	A	${}^2E_{5/2}$	${}^2E_{1/2}$	$A_{5/2}$
${}^1E_1''$	A''	${}^1E_1$	${}^1E_{7/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$
${}^2E_1''$	A''	${}^2E_1$	${}^2E_{7/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$
${}^1E_2''$	A''	${}^1E_2$	${}^1E_{9/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^2E_2''$	A''	${}^2E_2$	${}^2E_{9/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$

T **63**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{5h}$
$5n$	$(n+1)\,A' \oplus n\,({}^1E_1' \oplus {}^2E_1' \oplus {}^1E_2' \oplus {}^2E_2' \oplus A'' \oplus {}^1E_1'' \oplus {}^2E_1'' \oplus {}^1E_2'' \oplus {}^2E_2'')$
$5n+1$	$n\,(A' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_1'' \oplus {}^2E_1'' \oplus {}^1E_2'' \oplus {}^2E_2'') \oplus (n+1)({}^1E_1' \oplus {}^2E_1' \oplus A'')$
$5n+2$	$(n+1)(A' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_1'' \oplus {}^2E_1'') \oplus n\,({}^1E_1' \oplus {}^2E_1' \oplus A'' \oplus {}^1E_2'' \oplus {}^2E_2'')$
$5n+3$	$n\,(A' \oplus {}^1E_1'' \oplus {}^2E_1'') \oplus (n+1)({}^1E_1' \oplus {}^2E_1' \oplus {}^1E_2' \oplus {}^2E_2' \oplus A'' \oplus {}^1E_2'' \oplus {}^2E_2'')$
$5n+4$	$(n+1)(A' \oplus {}^1E_1' \oplus {}^2E_1' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_1'' \oplus {}^2E_1'' \oplus {}^1E_2'' \oplus {}^2E_2'') \oplus n\,A''$
$10n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus$ $2n\,({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus$ $2n\,({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus$ $2n\,({}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}) \oplus$ $2n\,({}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{11}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2}) \oplus$ $(2n+2)({}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{13}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}) \oplus$ $(2n+2)({}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{15}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}) \oplus$ $(2n+2)({}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{17}{2}$	$(2n+1)({}^1E_{1/2} \oplus {}^2E_{1/2}) \oplus$ $(2n+2)({}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$
$10n+\frac{19}{2}$	$(2n+2)({}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2} \oplus {}^1E_{7/2} \oplus {}^2E_{7/2} \oplus {}^1E_{9/2} \oplus {}^2E_{9/2})$

$n = 0, 1, 2, \ldots$

T **63**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

6/m $|G| = 12$ $|C| = 12$ $|\widetilde{C}| = 24$ T **64** p. 531 □ $\mathbf{C}_{6h}$

(1) Product forms: $\mathbf{C}_6 \otimes \mathbf{C}_i$, $\mathbf{C}_6 \otimes \mathbf{C}_s$.

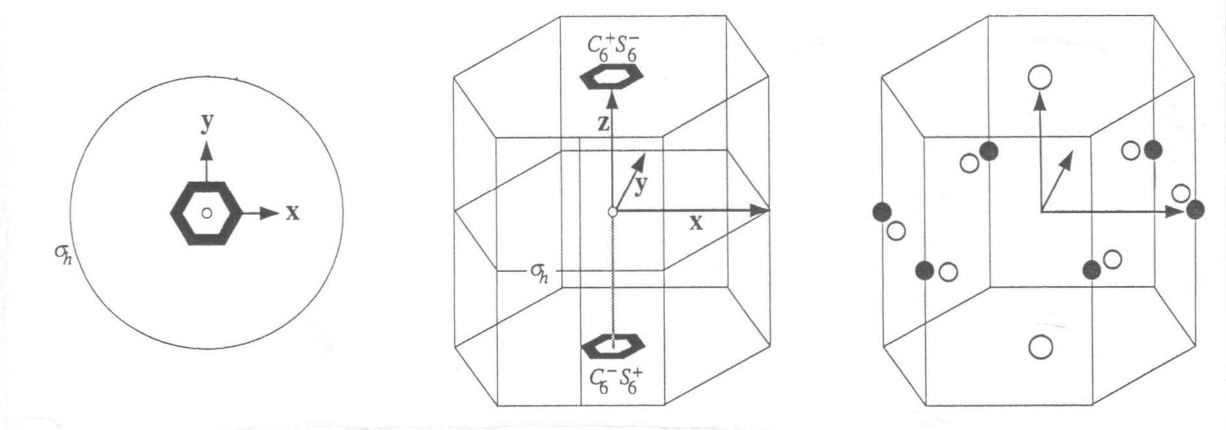
(2) Group chains: $\mathbf{D}_{6h} \supset \underline{\mathbf{C}_{6h}} \supset \underline{\mathbf{C}_{3h}}$, $\mathbf{D}_{6h} \supset \underline{\mathbf{C}_{6h}} \supset \underline{\mathbf{C}_{2h}}$, $\mathbf{D}_{6h} \supset \underline{\mathbf{C}_{6h}} \supset \underline{\mathbf{S}_6}$, $\mathbf{D}_{6h} \supset \underline{\mathbf{C}_{6h}} \supset \underline{\mathbf{C}_6}$.

(3) Operations of G: $E,\ C_6^+,\ C_3^+,\ C_2,\ C_3^-,\ C_6^-,\ i,\ S_3^-,\ S_6^-,\ \sigma_h,\ S_6^+,\ S_3^+$.

(4) Operations of $\widetilde{G}$: $E,\ C_6^+,\ C_3^+,\ C_2,\ C_3^-,\ C_6^-,\ i,\ S_3^-,\ S_6^-,\ \sigma_h,\ S_6^+,\ S_3^+,$
$\widetilde{E},\ \widetilde{C}_6^+,\ \widetilde{C}_3^+,\ \widetilde{C}_2,\ \widetilde{C}_3^-,\ \widetilde{C}_6^-,\ \widetilde{i},\ \widetilde{S}_3^-,\ \widetilde{S}_6^-,\ \widetilde{\sigma}_h,\ \widetilde{S}_6^+,\ \widetilde{S}_3^+$.

(5) Classes and representations: $|r| = 12$, $|\text{i}| = 0$, $|I| = 12$, $|\widetilde{I}| = 12$.

F **64** See Chapter **15**, p. 65



Examples:

T **64**.1 Parameters
Use T **35**.1. § **16**–1, p. 68

T **64**.2 Multiplication table
Use T **35**.2. § **16**–2, p. 69

T **64**.3 Factor table
Use T **35**.3. § **16**–3, p. 70

T **64**.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_{6h}$	E	C_6^+	C_3^+	C_2	C_3^-	C_6^-	i	S_3^-	S_6^-	σ_h	S_6^+	S_3^+	τ
A_g	1	1	1	1	1	1	1	1	1	1	1	1	a
B_g	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	a
${}^1E_{1g}$	1	$-\epsilon$	ϵ^*	-1	ϵ	$-\epsilon^*$	1	$-\epsilon$	ϵ^*	-1	ϵ	$-\epsilon^*$	b
${}^2E_{1g}$	1	$-\epsilon^*$	ϵ	-1	ϵ^*	$-\epsilon$	1	$-\epsilon^*$	ϵ	-1	ϵ^*	$-\epsilon$	b
${}^1E_{2g}$	1	ϵ	ϵ^*	1	ϵ	ϵ^*	1	ϵ	ϵ^*	1	ϵ	ϵ^*	b
${}^2E_{2g}$	1	ϵ^*	ϵ	1	ϵ^*	ϵ	1	ϵ^*	ϵ	1	ϵ^*	ϵ	b
A_u	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	a
B_u	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	a
${}^1E_{1u}$	1	$-\epsilon$	ϵ^*	-1	ϵ	$-\epsilon^*$	-1	ϵ	$-\epsilon^*$	1	$-\epsilon$	ϵ^*	b
${}^2E_{1u}$	1	$-\epsilon^*$	ϵ	-1	ϵ^*	$-\epsilon$	-1	ϵ^*	$-\epsilon$	1	$-\epsilon^*$	ϵ	b
${}^1E_{2u}$	1	ϵ	ϵ^*	1	ϵ	ϵ^*	-1	$-\epsilon$	$-\epsilon^*$	-1	$-\epsilon$	$-\epsilon^*$	b
${}^2E_{2u}$	1	ϵ^*	ϵ	1	ϵ^*	ϵ	-1	$-\epsilon^*$	$-\epsilon$	-1	$-\epsilon^*$	$-\epsilon$	b
${}^1E_{1/2,g}$	1	$-\mathrm{i}\epsilon$	$-\epsilon^*$	i	$-\epsilon$	$\mathrm{i}\epsilon^*$	1	$-\mathrm{i}\epsilon$	$-\epsilon^*$	i	$-\epsilon$	$\mathrm{i}\epsilon^*$	b
${}^2E_{1/2,g}$	1	$\mathrm{i}\epsilon^*$	$-\epsilon$	$-\mathrm{i}$	$-\epsilon^*$	$-\mathrm{i}\epsilon$	1	$\mathrm{i}\epsilon^*$	$-\epsilon$	$-\mathrm{i}$	$-\epsilon^*$	$-\mathrm{i}\epsilon$	b
${}^1E_{3/2,g}$	1	$-\mathrm{i}$	-1	i	-1	i	1	$-\mathrm{i}$	-1	i	-1	i	b
${}^2E_{3/2,g}$	1	i	-1	$-\mathrm{i}$	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	-1	$-\mathrm{i}$	b
${}^1E_{5/2,g}$	1	$-\mathrm{i}\epsilon^*$	$-\epsilon$	i	$-\epsilon^*$	$\mathrm{i}\epsilon$	1	$-\mathrm{i}\epsilon^*$	$-\epsilon$	i	$-\epsilon^*$	$\mathrm{i}\epsilon$	b
${}^2E_{5/2,g}$	1	$\mathrm{i}\epsilon$	$-\epsilon^*$	$-\mathrm{i}$	$-\epsilon$	$-\mathrm{i}\epsilon^*$	1	$\mathrm{i}\epsilon$	$-\epsilon^*$	$-\mathrm{i}$	$-\epsilon$	$-\mathrm{i}\epsilon^*$	b
${}^1E_{1/2,u}$	1	$-\mathrm{i}\epsilon$	$-\epsilon^*$	i	$-\epsilon$	$\mathrm{i}\epsilon^*$	-1	$\mathrm{i}\epsilon$	ϵ^*	$-\mathrm{i}$	ϵ	$-\mathrm{i}\epsilon^*$	b
${}^2E_{1/2,u}$	1	$\mathrm{i}\epsilon^*$	$-\epsilon$	$-\mathrm{i}$	$-\epsilon^*$	$-\mathrm{i}\epsilon$	-1	$-\mathrm{i}\epsilon^*$	ϵ	i	ϵ^*	$\mathrm{i}\epsilon$	b
${}^1E_{3/2,u}$	1	$-\mathrm{i}$	-1	i	-1	i	-1	i	1	$-\mathrm{i}$	1	$-\mathrm{i}$	b
${}^2E_{3/2,u}$	1	i	-1	$-\mathrm{i}$	-1	$-\mathrm{i}$	-1	$-\mathrm{i}$	1	i	1	i	b
${}^1E_{5/2,u}$	1	$-\mathrm{i}\epsilon^*$	$-\epsilon$	i	$-\epsilon^*$	$\mathrm{i}\epsilon$	-1	$\mathrm{i}\epsilon^*$	ϵ	$-\mathrm{i}$	ϵ^*	$-\mathrm{i}\epsilon$	b
${}^2E_{5/2,u}$	1	$\mathrm{i}\epsilon$	$-\epsilon^*$	$-\mathrm{i}$	$-\epsilon$	$-\mathrm{i}\epsilon^*$	-1	$-\mathrm{i}\epsilon$	ϵ^*	i	ϵ	$\mathrm{i}\epsilon^*$	b

$\epsilon = \exp(2\pi\mathrm{i}/3)$

T **64**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{6h}$	0	1	2	3
A_g	$^\square 1$	R_z	$x^2+y^2, {}^\square z^2$	
B_g				
${}^1E_{1g} \oplus {}^2E_{1g}$		(R_x, R_y)	$^\square(zx, yz)$	
${}^1E_{2g} \oplus {}^2E_{2g}$			$^\square(xy, x^2-y^2)$	
A_u		$^\square z$		$(x^2+y^2)z, {}^\square z^3$
B_u				$^\square x(x^2-3y^2), {}^\square y(3x^2-y^2)$
${}^1E_{1u} \oplus {}^2E_{1u}$		$^\square(x, y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
${}^1E_{2u} \oplus {}^2E_{2u}$				$^\square\{xyz, z(x^2-y^2)\}$

T **64**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{6h}$	$\lvert j\, m\rangle$	ι	μ	$\mathbf{C}_{6h}$	$\lvert j\, m\rangle$	ι	μ
A_g	$\lvert 0\,0\rangle$	2	± 6	${}^1E_{1/2,g}$	$\lvert\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	1	± 6
B_g	$\lvert 4\,3\rangle$	2	± 6	${}^2E_{1/2,g}$	$\lvert\frac{1}{2}\,\frac{1}{2}\rangle$	1	± 6
${}^1E_{1g}$	$\lvert 2\,1\rangle$	2	± 6	${}^1E_{3/2,g}$	$\lvert\frac{3}{2}\,\frac{3}{2}\rangle$	1	± 6
${}^2E_{1g}$	$\lvert 2\,\overline{1}\rangle$	2	± 6	${}^2E_{3/2,g}$	$\lvert\frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	1	± 6
${}^1E_{2g}$	$\lvert 2\,\overline{2}\rangle$	2	± 6	${}^1E_{5/2,g}$	$\lvert\frac{5}{2}\,\overline{\frac{5}{2}}\rangle$	1	± 6
${}^2E_{2g}$	$\lvert 2\,2\rangle$	2	± 6	${}^2E_{5/2,g}$	$\lvert\frac{5}{2}\,\frac{5}{2}\rangle$	1	± 6
A_u	$\lvert 1\,0\rangle$	2	± 6	${}^1E_{1/2,u}$	$\lvert\frac{1}{2}\,\overline{\frac{1}{2}}\rangle^\bullet$	1	± 6
B_u	$\lvert 3\,3\rangle$	2	± 6	${}^2E_{1/2,u}$	$\lvert\frac{1}{2}\,\frac{1}{2}\rangle^\bullet$	1	± 6
${}^1E_{1u}$	$\lvert 1\,1\rangle$	2	± 6	${}^1E_{3/2,u}$	$\lvert\frac{3}{2}\,\frac{3}{2}\rangle^\bullet$	1	± 6
${}^2E_{1u}$	$\lvert 1\,\overline{1}\rangle$	2	± 6	${}^2E_{3/2,u}$	$\lvert\frac{3}{2}\,\overline{\frac{3}{2}}\rangle^\bullet$	1	± 6
${}^1E_{2u}$	$\lvert 3\,\overline{2}\rangle$	2	± 6	${}^1E_{5/2,u}$	$\lvert\frac{5}{2}\,\overline{\frac{5}{2}}\rangle^\bullet$	1	± 6
${}^2E_{2u}$	$\lvert 3\,2\rangle$	2	± 6	${}^2E_{5/2,u}$	$\lvert\frac{5}{2}\,\frac{5}{2}\rangle^\bullet$	1	± 6

T **64**.7 Matrix representations

Use T **64**.4 ♠. § **16**–7, p. 77

T **64**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_{6h}$	A_g	B_g	${}^1E_{1g}$	${}^2E_{1g}$	${}^1E_{2g}$	${}^2E_{2g}$	A_u	B_u	${}^1E_{1u}$	${}^2E_{1u}$	${}^1E_{2u}$	${}^2E_{2u}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$
A_g	A_g	B_g	${}^1E_{1g}$	${}^2E_{1g}$	${}^1E_{2g}$	${}^2E_{2g}$	A_u	B_u	${}^1E_{1u}$	${}^2E_{1u}$	${}^1E_{2u}$	${}^2E_{2u}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$
B_g		A_g	${}^1E_{2g}$	${}^2E_{2g}$	${}^1E_{1g}$	${}^2E_{1g}$	B_u	A_u	${}^1E_{2u}$	${}^2E_{2u}$	${}^1E_{1u}$	${}^2E_{1u}$	${}^2E_{5/2,g}$	${}^1E_{5/2,g}$
${}^1E_{1g}$			${}^2E_{2g}$	A_g	${}^2E_{1g}$	B_g	${}^1E_{1u}$	${}^1E_{2u}$	${}^2E_{2u}$	A_u	${}^2E_{1u}$	B_u	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$
${}^2E_{1g}$				${}^1E_{2g}$	B_g	${}^1E_{1g}$	${}^2E_{1u}$	${}^2E_{2u}$	A_u	${}^1E_{2u}$	B_u	${}^1E_{1u}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$
${}^1E_{2g}$					${}^2E_{2g}$	A_g	${}^1E_{2u}$	${}^1E_{1u}$	${}^2E_{1u}$	B_u	${}^2E_{2u}$	A_u	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$
${}^2E_{2g}$						${}^1E_{2g}$	${}^2E_{2u}$	${}^2E_{1u}$	B_u	${}^1E_{1u}$	A_u	${}^1E_{2u}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$
A_u							A_g	B_g	${}^1E_{1g}$	${}^2E_{1g}$	${}^1E_{2g}$	${}^2E_{2g}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$
B_u								A_g	${}^1E_{2g}$	${}^2E_{2g}$	${}^1E_{1g}$	${}^2E_{1g}$	${}^2E_{5/2,u}$	${}^1E_{5/2,u}$
${}^1E_{1u}$									${}^2E_{2g}$	A_g	${}^2E_{1g}$	B_g	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$
${}^2E_{1u}$										${}^1E_{2g}$	B_g	${}^1E_{1g}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$
${}^1E_{2u}$											${}^2E_{2g}$	A_g	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$
${}^2E_{2u}$												${}^1E_{2g}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$
${}^1E_{1/2,g}$													${}^2E_{1g}$	A_g
${}^2E_{1/2,g}$														${}^1E_{1g}$

$\twoheadrightarrow$

T **64**.8 Direct products of representations *(cont.)*

$\mathbf{C}_{6h}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{5/2,u}$
A_g	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{5/2,u}$
B_g	${}^2E_{3/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^2E_{5/2,u}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$
${}^1E_{1g}$	${}^2E_{5/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$
${}^2E_{1g}$	${}^2E_{1/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{5/2,u}$	${}^2E_{5/2,u}$	${}^1E_{3/2,u}$
${}^1E_{2g}$	${}^1E_{1/2,g}$	${}^2E_{5/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{5/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$
${}^2E_{2g}$	${}^1E_{5/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{5/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$
A_u	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{5/2,u}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$
B_u	${}^2E_{3/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^2E_{5/2,g}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$
${}^1E_{1u}$	${}^2E_{5/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$
${}^2E_{1u}$	${}^2E_{1/2,u}$	${}^1E_{5/2,u}$	${}^2E_{5/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$	${}^1E_{3/2,g}$
${}^1E_{2u}$	${}^1E_{1/2,u}$	${}^2E_{5/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{5/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$
${}^2E_{2u}$	${}^1E_{5/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{5/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$
${}^1E_{1/2,g}$	${}^1E_{1g}$	${}^1E_{2g}$	B_g	${}^2E_{2g}$	${}^2E_{1u}$	A_u	${}^1E_{1u}$	${}^1E_{2u}$	B_u	${}^2E_{2u}$
${}^2E_{1/2,g}$	${}^2E_{2g}$	${}^2E_{1g}$	${}^1E_{2g}$	B_g	A_u	${}^1E_{1u}$	${}^2E_{2u}$	${}^2E_{1u}$	${}^1E_{2u}$	B_u
${}^1E_{3/2,g}$	B_g	A_g	${}^2E_{1g}$	${}^1E_{2g}$	${}^1E_{1u}$	${}^2E_{2u}$	B_u	A_u	${}^2E_{1u}$	${}^1E_{2u}$
${}^2E_{3/2,g}$		B_g	${}^2E_{2g}$	${}^1E_{1g}$	${}^1E_{2u}$	${}^2E_{1u}$	A_u	B_u	${}^2E_{2u}$	${}^1E_{1u}$
${}^1E_{5/2,g}$			${}^1E_{1g}$	A_g	B_u	${}^1E_{2u}$	${}^2E_{1u}$	${}^2E_{2u}$	${}^1E_{1u}$	A_u
${}^2E_{5/2,g}$				${}^2E_{1g}$	${}^2E_{2u}$	B_u	${}^1E_{2u}$	${}^1E_{1u}$	A_u	${}^2E_{1u}$
${}^1E_{1/2,u}$					${}^2E_{1g}$	A_g	${}^1E_{1g}$	${}^1E_{2g}$	B_g	${}^2E_{2g}$
${}^2E_{1/2,u}$						${}^1E_{1g}$	${}^2E_{2g}$	${}^2E_{1g}$	${}^1E_{2g}$	B_g
${}^1E_{3/2,u}$							B_g	A_g	${}^2E_{1g}$	${}^1E_{2g}$
${}^2E_{3/2,u}$								B_g	${}^2E_{2g}$	${}^1E_{1g}$
${}^1E_{5/2,u}$									${}^1E_{1g}$	A_g
${}^2E_{5/2,u}$										${}^2E_{1g}$

T **64**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{6h}$	$\mathbf{C}_{3h}$	$\mathbf{C}_{2h}$	$\mathbf{S}_6$	$\mathbf{C}_s$	$\mathbf{C}_i$	$\mathbf{C}_6$	$\mathbf{C}_3$	$\mathbf{C}_2$
A_g	A'	A_g	A_g	A'	A_g	A	A	A
B_g	A''	B_g	A_g	A''	A_g	B	A	B
${}^1E_{1g}$	${}^1E''$	B_g	${}^1E_g$	A''	A_g	${}^1E_1$	${}^1E$	B
${}^2E_{1g}$	${}^2E''$	B_g	${}^2E_g$	A''	A_g	${}^2E_1$	${}^2E$	B
${}^1E_{2g}$	${}^1E'$	A_g	${}^1E_g$	A'	A_g	${}^1E_2$	${}^1E$	A
${}^2E_{2g}$	${}^2E'$	A_g	${}^2E_g$	A'	A_g	${}^2E_2$	${}^2E$	A
A_u	A''	A_u	A_u	A''	A_u	A	A	A
B_u	A'	B_u	A_u	A'	A_u	B	A	B
${}^1E_{1u}$	${}^1E'$	B_u	${}^1E_u$	A'	A_u	${}^1E_1$	${}^1E$	B
${}^2E_{1u}$	${}^2E'$	B_u	${}^2E_u$	A'	A_u	${}^2E_1$	${}^2E$	B
${}^1E_{2u}$	${}^1E''$	A_u	${}^1E_u$	A''	A_u	${}^1E_2$	${}^1E$	A
${}^2E_{2u}$	${}^2E''$	A_u	${}^2E_u$	A''	A_u	${}^2E_2$	${}^2E$	A
${}^1E_{1/2,g}$	${}^1E_{1/2}$	${}^1E_{1/2,g}$	${}^1E_{1/2,g}$	${}^1E_{1/2}$	$A_{1/2,g}$	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$
${}^2E_{1/2,g}$	${}^2E_{1/2}$	${}^2E_{1/2,g}$	${}^2E_{1/2,g}$	${}^2E_{1/2}$	$A_{1/2,g}$	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$
${}^1E_{3/2,g}$	${}^1E_{3/2}$	${}^1E_{1/2,g}$	$A_{3/2,g}$	${}^1E_{1/2}$	$A_{1/2,g}$	${}^1E_{3/2}$	$A_{3/2}$	${}^1E_{1/2}$
${}^2E_{3/2,g}$	${}^2E_{3/2}$	${}^2E_{1/2,g}$	$A_{3/2,g}$	${}^2E_{1/2}$	$A_{1/2,g}$	${}^2E_{3/2}$	$A_{3/2}$	${}^2E_{1/2}$
${}^1E_{5/2,g}$	${}^1E_{5/2}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2}$	$A_{1/2,g}$	${}^1E_{5/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_{5/2,g}$	${}^2E_{5/2}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2}$	$A_{1/2,g}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_{1/2,u}$	${}^2E_{5/2}$	${}^1E_{1/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2}$	$A_{1/2,u}$	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$
${}^2E_{1/2,u}$	${}^1E_{5/2}$	${}^2E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2}$	$A_{1/2,u}$	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$
${}^1E_{3/2,u}$	${}^2E_{3/2}$	${}^1E_{1/2,u}$	$A_{3/2,u}$	${}^2E_{1/2}$	$A_{1/2,u}$	${}^1E_{3/2}$	$A_{3/2}$	${}^1E_{1/2}$
${}^2E_{3/2,u}$	${}^1E_{3/2}$	${}^2E_{1/2,u}$	$A_{3/2,u}$	${}^1E_{1/2}$	$A_{1/2,u}$	${}^2E_{3/2}$	$A_{3/2}$	${}^2E_{1/2}$
${}^1E_{5/2,u}$	${}^2E_{1/2}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^2E_{1/2}$	$A_{1/2,u}$	${}^1E_{5/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_{5/2,u}$	${}^1E_{1/2}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^1E_{1/2}$	$A_{1/2,u}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$

T **64**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{6h}$
$6n$	$(2n+1)\,A_g \oplus 2n\,(B_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g})$
$6n+1$	$(2n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u}) \oplus 2n\,(B_u \oplus {}^1E_{2u} \oplus {}^2E_{2u})$
$6n+2$	$(2n+1)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g}) \oplus 2n\,B_g$
$6n+3$	$(2n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u}) \oplus (2n+2)\,B_u$
$6n+4$	$(2n+1)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g}) \oplus (2n+2)(B_g \oplus {}^1E_{2g} \oplus {}^2E_{2g})$
$6n+5$	$(2n+1)\,A_u \oplus (2n+2)(B_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u})$
$6n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus 2n\,({}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g})$
$6n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}) \oplus 2n\,({}^1E_{5/2,g} \oplus {}^2E_{5/2,g})$
$6n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g})$
$6n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}) \oplus (2n+2)({}^1E_{5/2,g} \oplus {}^2E_{5/2,g})$
$6n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus (2n+2)({}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g})$
$6n+\frac{11}{2}$	$(2n+2)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g})$

$n = 0, 1, 2, \ldots$

T **64**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: $\mathbf{C}_7 \otimes \mathbf{C}_s$.

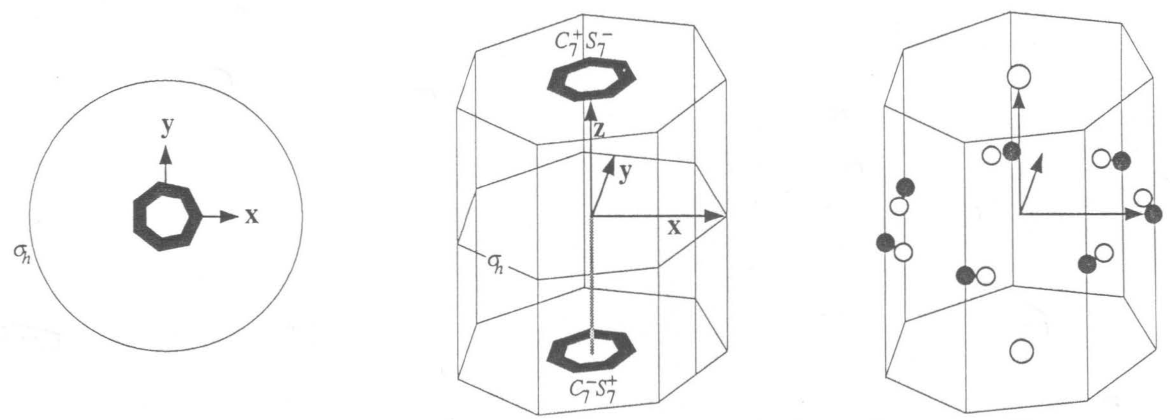
(2) Group chains: $\mathbf{D}_{7h} \supset \underline{\mathbf{C}_{7h}} \supset \underline{\mathbf{C}_s}$, $\quad \mathbf{D}_{7h} \supset \underline{\mathbf{C}_{7h}} \supset \underline{\mathbf{C}_7}$.

(3) Operations of G: $E,\ C_7^+,\ C_7^{2+},\ C_7^{3+},\ C_7^{3-},\ C_7^{2-},\ C_7^-,\ \sigma_h,\ S_7^+,\ S_7^{2+},\ S_7^{3+},\ S_7^{3-},\ S_7^{2-},\ S_7^-$.

(4) Operations of $\widetilde{G}$: $E,\ C_7^+,\ C_7^{2+},\ C_7^{3+},\ C_7^{3-},\ C_7^{2-},\ C_7^-,\ \sigma_h,\ S_7^+,\ S_7^{2+},\ S_7^{3+},\ S_7^{3-},\ S_7^{2-},\ S_7^-,$
$\widetilde{E},\ \widetilde{C}_7^+,\ \widetilde{C}_7^{2+},\ \widetilde{C}_7^{3+},\ \widetilde{C}_7^{3-},\ \widetilde{C}_7^{2-},\ \widetilde{C}_7^-,\ \widetilde{\sigma}_h,\ \widetilde{S}_7^+,\ \widetilde{S}_7^{2+},\ \widetilde{S}_7^{3+},\ \widetilde{S}_7^{3-},\ \widetilde{S}_7^{2-},\ \widetilde{S}_7^-$.

(5) Classes and representations: $|r| = 14$, $\quad |\text{i}| = 0$, $\quad |I| = 14$, $\quad |\widetilde{I}| = 14$.

F **65** See Chapter **15**, p. 65



Examples:

T 65.1 Parameters
Use T **36**.1. § **16**–1, p. 68

T 65.2 Multiplication table
Use T **36**.2. § **16**–2, p. 69

T 65.3 Factor table
Use T **36**.3. § **16**–3, p. 70

T **65**.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_{7h}$	E	C_7^+	C_7^{2+}	C_7^{3+}	C_7^{3-}	C_7^{2-}	C_7^-	σ_h	S_7^+	S_7^{2+}	S_7^{3+}	S_7^{3-}	S_7^{2-}	S_7^-	τ
A'	1	1	1	1	1	1	1	1	1	1	1	1	1	1	a
${}^1E_1'$	1	δ^*	ϵ^*	η^*	η	ϵ	δ	1	δ^*	ϵ^*	η^*	η	ϵ	δ	b
${}^2E_1'$	1	δ	ϵ	η	η^*	ϵ^*	δ^*	1	δ	ϵ	η	η^*	ϵ^*	δ^*	b
${}^1E_2'$	1	ϵ^*	η	δ	δ^*	η^*	ϵ	1	ϵ^*	η	δ	δ^*	η^*	ϵ	b
${}^2E_2'$	1	ϵ	η^*	δ^*	δ	η	ϵ^*	1	ϵ	η^*	δ^*	δ	η	ϵ^*	b
${}^1E_3'$	1	η^*	δ	ϵ^*	ϵ	δ^*	η	1	η^*	δ	ϵ^*	ϵ	δ^*	η	b
${}^2E_3'$	1	η	δ^*	ϵ	ϵ^*	δ	η^*	1	η	δ^*	ϵ	ϵ^*	δ	η^*	b
A''	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	a
${}^1E_1''$	1	δ^*	ϵ^*	η^*	η	ϵ	δ	-1	$-\delta^*$	$-\epsilon^*$	$-\eta^*$	$-\eta$	$-\epsilon$	$-\delta$	b
${}^2E_1''$	1	δ	ϵ	η	η^*	ϵ^*	δ^*	-1	$-\delta$	$-\epsilon$	$-\eta$	$-\eta^*$	$-\epsilon^*$	$-\delta^*$	b
${}^1E_2''$	1	ϵ^*	η	δ	δ^*	η^*	ϵ	-1	$-\epsilon^*$	$-\eta$	$-\delta$	$-\delta^*$	$-\eta^*$	$-\epsilon$	b
${}^2E_2''$	1	ϵ	η^*	δ^*	δ	η	ϵ^*	-1	$-\epsilon$	$-\eta^*$	$-\delta^*$	$-\delta$	$-\eta$	$-\epsilon^*$	b
${}^1E_3''$	1	η^*	δ	ϵ^*	ϵ	δ^*	η	-1	$-\eta^*$	$-\delta$	$-\epsilon^*$	$-\epsilon$	$-\delta^*$	$-\eta$	b
${}^2E_3''$	1	η	δ^*	ϵ	ϵ^*	δ	η^*	-1	$-\eta$	$-\delta^*$	$-\epsilon$	$-\epsilon^*$	$-\delta$	$-\eta^*$	b
${}^1E_{1/2}$	1	$-\eta^*$	δ	$-\epsilon^*$	$-\epsilon$	δ^*	$-\eta$	i	$\mathrm{i}\eta^*$	$-\mathrm{i}\delta$	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\epsilon$	$\mathrm{i}\delta^*$	$-\mathrm{i}\eta$	b
${}^2E_{1/2}$	1	$-\eta$	δ^*	$-\epsilon$	$-\epsilon^*$	δ	$-\eta^*$	$-$i	$-\mathrm{i}\eta$	$\mathrm{i}\delta^*$	$-\mathrm{i}\epsilon$	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\delta$	$\mathrm{i}\eta^*$	b
${}^1E_{3/2}$	1	$-\epsilon$	η^*	$-\delta^*$	$-\delta$	η	$-\epsilon^*$	i	$\mathrm{i}\epsilon$	$-\mathrm{i}\eta^*$	$\mathrm{i}\delta^*$	$-\mathrm{i}\delta$	$\mathrm{i}\eta$	$-\mathrm{i}\epsilon^*$	b
${}^2E_{3/2}$	1	$-\epsilon^*$	η	$-\delta$	$-\delta^*$	η^*	$-\epsilon$	$-$i	$-\mathrm{i}\epsilon^*$	$\mathrm{i}\eta$	$-\mathrm{i}\delta$	$\mathrm{i}\delta^*$	$-\mathrm{i}\eta^*$	$\mathrm{i}\epsilon$	b
${}^1E_{5/2}$	1	$-\delta^*$	ϵ^*	$-\eta^*$	$-\eta$	ϵ	$-\delta$	i	$\mathrm{i}\delta^*$	$-\mathrm{i}\epsilon^*$	$\mathrm{i}\eta^*$	$-\mathrm{i}\eta$	$\mathrm{i}\epsilon$	$-\mathrm{i}\delta$	b
${}^2E_{5/2}$	1	$-\delta$	ϵ	$-\eta$	$-\eta^*$	ϵ^*	$-\delta^*$	$-$i	$-\mathrm{i}\delta$	$\mathrm{i}\epsilon$	$-\mathrm{i}\eta$	$\mathrm{i}\eta^*$	$-\mathrm{i}\epsilon^*$	$\mathrm{i}\delta^*$	b
${}^1E_{7/2}$	1	-1	1	-1	-1	1	-1	i	i	$-$i	i	$-$i	i	$-$i	b
${}^2E_{7/2}$	1	-1	1	-1	-1	1	-1	$-$i	$-$i	i	$-$i	i	$-$i	i	b
${}^1E_{9/2}$	1	$-\delta$	ϵ	$-\eta$	$-\eta^*$	ϵ^*	$-\delta^*$	i	$\mathrm{i}\delta$	$-\mathrm{i}\epsilon$	$\mathrm{i}\eta$	$-\mathrm{i}\eta^*$	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\delta^*$	b
${}^2E_{9/2}$	1	$-\delta^*$	ϵ^*	$-\eta^*$	$-\eta$	ϵ	$-\delta$	$-$i	$-\mathrm{i}\delta^*$	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\eta^*$	$\mathrm{i}\eta$	$-\mathrm{i}\epsilon$	$\mathrm{i}\delta$	b
${}^1E_{11/2}$	1	$-\epsilon^*$	η	$-\delta$	$-\delta^*$	η^*	$-\epsilon$	i	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\eta$	$\mathrm{i}\delta$	$-\mathrm{i}\delta^*$	$\mathrm{i}\eta^*$	$-\mathrm{i}\epsilon$	b
${}^2E_{11/2}$	1	$-\epsilon$	η^*	$-\delta^*$	$-\delta$	η	$-\epsilon^*$	$-$i	$-\mathrm{i}\epsilon$	$\mathrm{i}\eta^*$	$-\mathrm{i}\delta^*$	$\mathrm{i}\delta$	$-\mathrm{i}\eta$	$\mathrm{i}\epsilon^*$	b
${}^1E_{13/2}$	1	$-\eta$	δ^*	$-\epsilon$	$-\epsilon^*$	δ	$-\eta^*$	i	$\mathrm{i}\eta$	$-\mathrm{i}\delta^*$	$\mathrm{i}\epsilon$	$-\mathrm{i}\epsilon^*$	$\mathrm{i}\delta$	$-\mathrm{i}\eta^*$	b
${}^2E_{13/2}$	1	$-\eta^*$	δ	$-\epsilon^*$	$-\epsilon$	δ^*	$-\eta$	$-$i	$-\mathrm{i}\eta^*$	$\mathrm{i}\delta$	$-\mathrm{i}\epsilon^*$	$\mathrm{i}\epsilon$	$-\mathrm{i}\delta^*$	$\mathrm{i}\eta$	b

$\delta = \exp(2\pi\mathrm{i}/7)$, $\epsilon = \exp(4\pi\mathrm{i}/7)$, $\eta = \exp(6\pi\mathrm{i}/7)$

T **65**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{7h}$	0	1	2	3
A'	$^{\square}1$	R_z	$x^2+y^2, {}^{\square}z^2$	
${}^1E_1' \oplus {}^2E_1'$		$^{\square}(x,y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^{\square}(xz^2, yz^2)$
${}^1E_2' \oplus {}^2E_2'$			$^{\square}(xy, x^2-y^2)$	
${}^1E_3' \oplus {}^2E_3'$				$^{\square}\{x(x^2-3y^2), y(3x^2-y^2)\}$
A''		$^{\square}z$		$(x^2+y^2)z, {}^{\square}z^3$
${}^1E_1'' \oplus {}^2E_1''$		(R_x, R_y)	$^{\square}(zx, yz)$	
${}^1E_2'' \oplus {}^2E_2''$				$^{\square}\{xyz, (x^2-y^2)z\}$
${}^1E_3'' \oplus {}^2E_3''$				

T **65**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{7h}$	$\lvert j\,m\rangle$		ι	μ	$\mathbf{C}_{7h}$	$\lvert j\,m\rangle$		ι	μ
A'	$\lvert 0\,0\rangle$	$\lvert 7\,7\rangle$	2	± 14	${}^1E_{1/2}$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	$\lvert \frac{13}{2}\,\frac{13}{2}\rangle^\bullet$	1	± 14
${}^1E'_1$	$\lvert 1\,1\rangle$	$\lvert 6\,\overline{6}\rangle$	2	± 14	${}^2E_{1/2}$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle$	$\lvert \frac{13}{2}\,\overline{\frac{13}{2}}\rangle^\bullet$	1	± 14
${}^2E'_1$	$\lvert 1\,\overline{1}\rangle$	$\lvert 6\,6\rangle$	2	± 14	${}^1E_{3/2}$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle$	$\lvert \frac{11}{2}\,\overline{\frac{11}{2}}\rangle^\bullet$	1	± 14
${}^1E'_2$	$\lvert 2\,2\rangle$	$\lvert 5\,\overline{5}\rangle$	2	± 14	${}^2E_{3/2}$	$\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	$\lvert \frac{11}{2}\,\frac{11}{2}\rangle^\bullet$	1	± 14
${}^2E'_2$	$\lvert 2\,\overline{2}\rangle$	$\lvert 5\,5\rangle$	2	± 14	${}^1E_{5/2}$	$\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle^\bullet$	1	± 14
${}^1E'_3$	$\lvert 3\,3\rangle$	$\lvert 4\,\overline{4}\rangle$	2	± 14	${}^2E_{5/2}$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle$	$\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle^\bullet$	1	± 14
${}^2E'_3$	$\lvert 3\,\overline{3}\rangle$	$\lvert 4\,4\rangle$	2	± 14	${}^1E_{7/2}$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle$	$\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle^\bullet$	1	± 14
A''	$\lvert 1\,0\rangle$	$\lvert 8\,7\rangle$	2	± 14	${}^2E_{7/2}$	$\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle^\bullet$	1	± 14
${}^1E''_1$	$\lvert 2\,1\rangle$	$\lvert 7\,\overline{6}\rangle$	2	± 14	${}^1E_{9/2}$	$\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle^\bullet$	1	± 14
${}^2E''_1$	$\lvert 2\,\overline{1}\rangle$	$\lvert 7\,6\rangle$	2	± 14	${}^2E_{9/2}$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle$	$\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle^\bullet$	1	± 14
${}^1E''_2$	$\lvert 3\,2\rangle$	$\lvert 6\,\overline{5}\rangle$	2	± 14	${}^1E_{11/2}$	$\lvert \frac{11}{2}\,\frac{11}{2}\rangle$	$\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle^\bullet$	1	± 14
${}^2E''_2$	$\lvert 3\,\overline{2}\rangle$	$\lvert 6\,5\rangle$	2	± 14	${}^2E_{11/2}$	$\lvert \frac{11}{2}\,\overline{\frac{11}{2}}\rangle$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle^\bullet$	1	± 14
${}^1E''_3$	$\lvert 4\,3\rangle$	$\lvert 5\,\overline{4}\rangle$	2	± 14	${}^1E_{13/2}$	$\lvert \frac{13}{2}\,\overline{\frac{13}{2}}\rangle$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle^\bullet$	1	± 14
${}^2E''_3$	$\lvert 4\,\overline{3}\rangle$	$\lvert 5\,4\rangle$	2	± 14	${}^2E_{13/2}$	$\lvert \frac{13}{2}\,\frac{13}{2}\rangle$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle^\bullet$	1	± 14

T **65**.7 Matrix representations

Use T **65**.4 ♠. § **16**–7, p. 77

T **65**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_{7h}$	A'	${}^1E'_1$	${}^2E'_1$	${}^1E'_2$	${}^2E'_2$	${}^1E'_3$	${}^2E'_3$	A''	${}^1E''_1$	${}^2E''_1$	${}^1E''_2$	${}^2E''_2$	${}^1E''_3$	${}^2E''_3$
A'	A'	${}^1E'_1$	${}^2E'_1$	${}^1E'_2$	${}^2E'_2$	${}^1E'_3$	${}^2E'_3$	A''	${}^1E''_1$	${}^2E''_1$	${}^1E''_2$	${}^2E''_2$	${}^1E''_3$	${}^2E''_3$
${}^1E'_1$		${}^1E'_2$	A'	${}^1E'_3$	${}^2E'_1$	${}^2E'_3$	${}^2E'_2$	${}^1E''_1$	${}^1E''_2$	A''	${}^1E''_3$	${}^2E''_1$	${}^2E''_3$	${}^2E''_2$
${}^2E'_1$			${}^2E'_2$	${}^1E'_1$	${}^2E'_3$	${}^1E'_2$	${}^1E'_3$	${}^2E''_1$	A''	${}^2E''_2$	${}^1E''_1$	${}^2E''_3$	${}^1E''_2$	${}^1E''_3$
${}^1E'_2$				${}^2E'_3$	A'	${}^2E'_2$	${}^2E'_1$	${}^1E''_2$	${}^1E''_3$	${}^1E''_1$	${}^2E''_3$	A''	${}^2E''_2$	${}^2E''_1$
${}^2E'_2$					${}^1E'_3$	${}^1E'_1$	${}^1E'_2$	${}^2E''_2$	${}^2E''_1$	${}^2E''_3$	A''	${}^1E''_3$	${}^1E''_1$	${}^1E''_2$
${}^1E'_3$						${}^2E'_1$	A'	${}^1E''_3$	${}^2E''_3$	${}^1E''_2$	${}^2E''_2$	${}^1E''_1$	${}^2E''_1$	A''
${}^2E'_3$							${}^1E'_1$	${}^2E''_3$	${}^2E''_2$	${}^1E''_3$	${}^2E''_1$	${}^1E''_2$	A''	${}^1E''_1$
A''								A'	${}^1E'_1$	${}^2E'_1$	${}^1E'_2$	${}^2E'_2$	${}^1E'_3$	${}^2E'_3$
${}^1E''_1$									${}^1E'_2$	A'	${}^1E'_3$	${}^2E'_1$	${}^2E'_3$	${}^2E'_2$
${}^2E''_1$										${}^2E'_2$	${}^1E'_1$	${}^2E'_3$	${}^1E'_2$	${}^1E'_3$
${}^1E''_2$											${}^2E'_3$	A'	${}^2E'_2$	${}^2E'_1$
${}^2E''_2$												${}^1E'_3$	${}^1E'_1$	${}^1E'_2$
${}^1E''_3$													${}^2E'_1$	A'
${}^2E''_3$														${}^1E'_1$

⇛

T **65**.8 Direct products of representations *(cont.)*

$\mathbf{C}_{7h}$	$^1E_{1/2}$	$^2E_{1/2}$	$^1E_{3/2}$	$^2E_{3/2}$	$^1E_{5/2}$	$^2E_{5/2}$
A'	$^2E_{1/2}$	$^1E_{3/2}$	$^2E_{3/2}$	$^1E_{5/2}$	$^2E_{5/2}$	$^1E_{7/2}$
$^1E'_1$	$^1E_{13/2}$	$^2E_{11/2}$	$^1E_{9/2}$	$^2E_{13/2}$	$^1E_{11/2}$	$^2E_{7/2}$
$^2E'_1$	$^1E_{11/2}$	$^2E_{13/2}$	$^1E_{13/2}$	$^2E_{9/2}$	$^1E_{7/2}$	$^2E_{11/2}$
$^1E'_2$	$^1E_{3/2}$	$^2E_{5/2}$	$^1E_{7/2}$	$^2E_{1/2}$	$^1E_{1/2}$	$^2E_{9/2}$
$^2E'_2$	$^1E_{5/2}$	$^2E_{3/2}$	$^1E_{1/2}$	$^2E_{7/2}$	$^1E_{9/2}$	$^2E_{1/2}$
$^1E'_3$	$^1E_{9/2}$	$^2E_{7/2}$	$^1E_{5/2}$	$^2E_{11/2}$	$^1E_{13/2}$	$^2E_{3/2}$
$^2E'_3$	$^1E_{7/2}$	$^2E_{9/2}$	$^1E_{11/2}$	$^2E_{5/2}$	$^1E_{3/2}$	$^2E_{13/2}$
A''	$^2E_{13/2}$	$^1E_{13/2}$	$^2E_{11/2}$	$^1E_{11/2}$	$^2E_{9/2}$	$^1E_{9/2}$
$^1E''_1$	$^2E_{1/2}$	$^1E_{3/2}$	$^2E_{5/2}$	$^1E_{1/2}$	$^2E_{3/2}$	$^1E_{7/2}$
$^2E''_1$	$^2E_{3/2}$	$^1E_{1/2}$	$^2E_{1/2}$	$^1E_{5/2}$	$^2E_{7/2}$	$^1E_{3/2}$
$^1E''_2$	$^2E_{11/2}$	$^1E_{9/2}$	$^2E_{7/2}$	$^1E_{13/2}$	$^2E_{13/2}$	$^1E_{5/2}$
$^2E''_2$	$^2E_{9/2}$	$^1E_{11/2}$	$^2E_{13/2}$	$^1E_{7/2}$	$^2E_{5/2}$	$^1E_{13/2}$
$^1E''_3$	$^2E_{5/2}$	$^1E_{7/2}$	$^2E_{9/2}$	$^1E_{3/2}$	$^2E_{1/2}$	$^1E_{11/2}$
$^2E''_3$	$^2E_{7/2}$	$^1E_{5/2}$	$^2E_{3/2}$	$^1E_{9/2}$	$^2E_{11/2}$	$^1E_{1/2}$
$^1E_{1/2}$	$^2E''_1$	A'	$^1E''_1$	$^2E'_2$	$^2E''_3$	$^1E'_2$
$^2E_{1/2}$		$^1E''_1$	$^1E'_2$	$^2E''_1$	$^2E'_2$	$^1E''_3$
$^1E_{3/2}$			$^1E''_3$	A'	$^2E''_1$	$^2E'_3$
$^2E_{3/2}$				$^2E''_3$	$^1E'_3$	$^1E''_1$
$^1E_{5/2}$					$^1E''_2$	A'
$^2E_{5/2}$						$^2E''_2$

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T **65**.8 Direct products of representations *(cont.)*

$\mathbf{C}_{7h}$	$^1E_{7/2}$	$^2E_{7/2}$	$^1E_{9/2}$	$^2E_{9/2}$	$^1E_{11/2}$	$^2E_{11/2}$	$^1E_{13/2}$	$^2E_{13/2}$
A'	$^1E_{1/2}$	$^2E_{7/2}$	$^1E_{9/2}$	$^2E_{9/2}$	$^1E_{11/2}$	$^2E_{11/2}$	$^1E_{13/2}$	$^2E_{13/2}$
$^1E'_1$	$^1E_{5/2}$	$^2E_{9/2}$	$^1E_{7/2}$	$^2E_{3/2}$	$^1E_{1/2}$	$^2E_{5/2}$	$^1E_{3/2}$	$^2E_{1/2}$
$^2E'_1$	$^1E_{9/2}$	$^2E_{5/2}$	$^1E_{3/2}$	$^2E_{7/2}$	$^1E_{5/2}$	$^2E_{1/2}$	$^1E_{1/2}$	$^2E_{3/2}$
$^1E'_2$	$^1E_{11/2}$	$^2E_{3/2}$	$^1E_{5/2}$	$^2E_{13/2}$	$^1E_{13/2}$	$^2E_{7/2}$	$^1E_{9/2}$	$^2E_{11/2}$
$^2E'_2$	$^1E_{3/2}$	$^2E_{11/2}$	$^1E_{13/2}$	$^2E_{5/2}$	$^1E_{7/2}$	$^2E_{13/2}$	$^1E_{11/2}$	$^2E_{9/2}$
$^1E'_3$	$^1E_{1/2}$	$^2E_{13/2}$	$^1E_{11/2}$	$^2E_{1/2}$	$^1E_{3/2}$	$^2E_{9/2}$	$^1E_{7/2}$	$^2E_{5/2}$
$^2E'_3$	$^1E_{13/2}$	$^2E_{1/2}$	$^1E_{1/2}$	$^2E_{11/2}$	$^1E_{9/2}$	$^2E_{3/2}$	$^1E_{5/2}$	$^2E_{7/2}$
A''	$^2E_{7/2}$	$^1E_{7/2}$	$^2E_{5/2}$	$^1E_{5/2}$	$^2E_{3/2}$	$^1E_{3/2}$	$^2E_{1/2}$	$^1E_{1/2}$
$^1E''_1$	$^2E_{9/2}$	$^1E_{5/2}$	$^2E_{7/2}$	$^1E_{11/2}$	$^2E_{13/2}$	$^1E_{9/2}$	$^2E_{11/2}$	$^1E_{13/2}$
$^2E''_1$	$^2E_{5/2}$	$^1E_{9/2}$	$^2E_{11/2}$	$^1E_{7/2}$	$^2E_{9/2}$	$^1E_{13/2}$	$^2E_{13/2}$	$^1E_{11/2}$
$^1E''_2$	$^2E_{3/2}$	$^1E_{11/2}$	$^2E_{9/2}$	$^1E_{1/2}$	$^2E_{1/2}$	$^1E_{7/2}$	$^2E_{5/2}$	$^1E_{3/2}$
$^2E''_2$	$^2E_{11/2}$	$^1E_{3/2}$	$^2E_{1/2}$	$^1E_{9/2}$	$^2E_{7/2}$	$^1E_{1/2}$	$^2E_{3/2}$	$^1E_{5/2}$
$^1E''_3$	$^2E_{13/2}$	$^1E_{1/2}$	$^2E_{3/2}$	$^1E_{13/2}$	$^2E_{11/2}$	$^1E_{5/2}$	$^2E_{7/2}$	$^1E_{9/2}$
$^2E''_3$	$^2E_{1/2}$	$^1E_{13/2}$	$^2E_{13/2}$	$^1E_{3/2}$	$^2E_{5/2}$	$^1E_{11/2}$	$^2E_{9/2}$	$^1E_{7/2}$
$^1E_{1/2}$	$^1E''_3$	$^1E'_3$	$^1E''_2$	$^2E'_3$	$^2E''_2$	$^1E'_1$	A''	$^2E'_1$
$^2E_{1/2}$	$^2E'_3$	$^2E''_3$	$^1E'_3$	$^2E''_2$	$^2E'_1$	$^1E''_2$	$^1E'_1$	A''
$^1E_{3/2}$	$^2E''_2$	$^2E'_2$	$^2E''_3$	$^2E'_1$	A''	$^1E'_3$	$^1E''_2$	$^1E'_1$
$^2E_{3/2}$	$^1E'_2$	$^1E''_2$	$^1E'_1$	$^1E''_3$	$^2E'_3$	A''	$^2E'_1$	$^2E''_2$
$^1E_{5/2}$	$^1E''_1$	$^1E'_1$	A''	$^1E'_2$	$^1E''_3$	$^2E'_1$	$^2E''_2$	$^2E'_3$
$^2E_{5/2}$	$^2E'_1$	$^2E''_1$	$^2E'_2$	A''	$^1E'_1$	$^2E''_3$	$^1E'_3$	$^1E''_2$
$^1E_{7/2}$	A''	A'	$^2E''_1$	$^1E'_1$	$^1E''_2$	$^2E'_2$	$^2E''_3$	$^1E'_3$
$^2E_{7/2}$		A''	$^2E'_1$	$^1E''_1$	$^1E'_2$	$^2E''_2$	$^2E'_3$	$^1E''_3$
$^1E_{9/2}$			$^2E''_2$	A'	$^1E''_1$	$^2E'_3$	$^1E''_3$	$^1E'_2$
$^2E_{9/2}$				$^1E''_2$	$^1E'_3$	$^2E''_1$	$^2E'_2$	$^2E''_3$
$^1E_{11/2}$					$^2E''_3$	A'	$^2E''_1$	$^2E'_2$
$^2E_{11/2}$						$^1E''_3$	$^1E'_2$	$^1E''_1$
$^1E_{13/2}$							$^1E''_1$	A'
$^2E_{13/2}$								$^2E''_1$

T **65**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{7h}$	$\mathbf{C}_s$	$\mathbf{C}_7$
A'	A'	A
${}^1E_1'$	A'	${}^1E_1$
${}^2E_1'$	A'	${}^2E_1$
${}^1E_2'$	A'	${}^1E_2$
${}^2E_2'$	A'	${}^2E_2$
${}^1E_3'$	A'	${}^1E_3$
${}^2E_3'$	A'	${}^2E_3$
A''	A''	A
${}^1E_1''$	A''	${}^1E_1$
${}^2E_1''$	A''	${}^2E_1$
${}^1E_2''$	A''	${}^1E_2$
${}^2E_2''$	A''	${}^2E_2$
${}^1E_3''$	A''	${}^1E_3$
${}^2E_3''$	A''	${}^2E_3$
${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$
${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$
${}^1E_{3/2}$	${}^1E_{1/2}$	${}^1E_{3/2}$
${}^2E_{3/2}$	${}^2E_{1/2}$	${}^2E_{3/2}$
${}^1E_{5/2}$	${}^1E_{1/2}$	${}^1E_{5/2}$
${}^2E_{5/2}$	${}^2E_{1/2}$	${}^2E_{5/2}$
${}^1E_{7/2}$	${}^1E_{1/2}$	$A_{7/2}$
${}^2E_{7/2}$	${}^2E_{1/2}$	$A_{7/2}$
${}^1E_{9/2}$	${}^1E_{1/2}$	${}^2E_{5/2}$
${}^2E_{9/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$
${}^1E_{11/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$
${}^2E_{11/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$
${}^1E_{13/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^2E_{13/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$

T **65**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{7h}$
$7n$	$(n+1)\,A' \oplus n\,({}^1E_1' \oplus {}^2E_1' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_3' \oplus {}^2E_3' \oplus A'' \oplus {}^1E_1'' \oplus {}^2E_1'' \oplus {}^1E_2'' \oplus {}^2E_2'' \oplus {}^1E_3'' \oplus {}^2E_3'')$
$7n+1$	$n\,(A' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_3' \oplus {}^2E_3' \oplus {}^1E_1'' \oplus {}^2E_1'' \oplus {}^1E_2'' \oplus {}^2E_2'' \oplus {}^1E_3'' \oplus {}^2E_3'') \oplus (n+1)({}^1E_1' \oplus {}^2E_1' \oplus A'')$
$7n+2$	$(n+1)(A' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_1'' \oplus {}^2E_1'') \oplus n\,({}^1E_1' \oplus {}^2E_1' \oplus {}^1E_3' \oplus {}^2E_3' \oplus A'' \oplus {}^1E_2'' \oplus {}^2E_2'' \oplus {}^1E_3'' \oplus {}^2E_3'')$
$7n+3$	$n\,(A' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_1'' \oplus {}^2E_1'' \oplus {}^1E_3'' \oplus {}^2E_3'') \oplus (n+1)({}^1E_1' \oplus {}^2E_1' \oplus {}^1E_3' \oplus {}^2E_3' \oplus A'' \oplus {}^1E_2'' \oplus {}^2E_2'')$
$7n+4$	$(n+1)(A' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_3' \oplus {}^2E_3' \oplus {}^1E_1'' \oplus {}^2E_1'' \oplus {}^1E_3'' \oplus {}^2E_3'') \oplus n\,({}^1E_1' \oplus {}^2E_1' \oplus A'' \oplus {}^1E_2'' \oplus {}^2E_2'')$
$7n+5$	$n\,(A' \oplus {}^1E_1'' \oplus {}^2E_1'') \oplus (n+1)({}^1E_1' \oplus {}^2E_1' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_3' \oplus {}^2E_3' \oplus A'' \oplus {}^1E_2'' \oplus {}^2E_2'' \oplus {}^1E_3'' \oplus {}^2E_3'')$
$7n+6$	$(n+1)(A' \oplus {}^1E_1' \oplus {}^2E_1' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_3' \oplus {}^2E_3' \oplus {}^1E_1'' \oplus {}^2E_1'' \oplus {}^1E_2'' \oplus {}^2E_2'' \oplus {}^1E_3'' \oplus {}^2E_3'') \oplus n\,A''$

$n = 0, 1, 2, \ldots$ $\twoheadrightarrow$

T **65**.10 Subduction from O(3) *(cont.)*

j	$\mathbf{C}_{7h}$
$14n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2})\oplus 2n\,({}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2})$
$14n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2})\oplus 2n\,({}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2})$
$14n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2})\oplus 2n\,({}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2})$
$14n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2})\oplus 2n\,({}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2})$
$14n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2})\oplus 2n\,({}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2})$
$14n+\frac{11}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2})\oplus 2n\,({}^1E_{13/2}\oplus{}^2E_{13/2})$
$14n+\frac{13}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2})$
$14n+\frac{15}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2})\oplus(2n+2)({}^1E_{13/2}\oplus{}^2E_{13/2})$
$14n+\frac{17}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2})\oplus(2n+2)({}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2})$
$14n+\frac{19}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2})\oplus(2n+2)({}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2})$
$14n+\frac{21}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2})\oplus(2n+2)({}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2})$
$14n+\frac{23}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2})\oplus(2n+2)({}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2})$
$14n+\frac{25}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2})\oplus(2n+2)({}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2})$
$14n+\frac{27}{2}$	$(2n+2)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2})$

$n = 0, 1, 2, \ldots$

T **65**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: $\mathbf{C}_8 \otimes \mathbf{C}_i$, $\mathbf{C}_8 \otimes \mathbf{C}_s$.

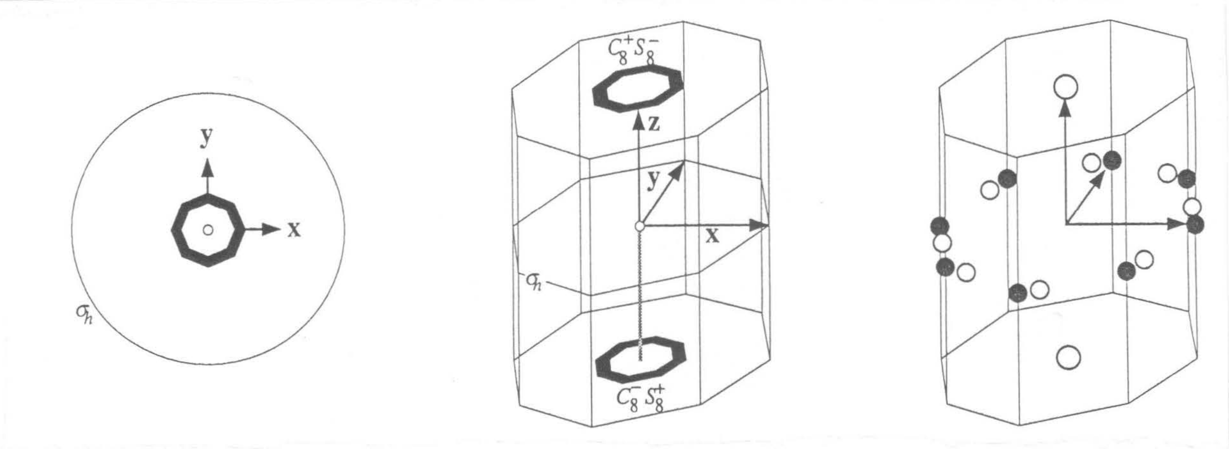
(2) Group chains: $\mathbf{D}_{8h} \supset \underline{\mathbf{C}_{8h}} \supset \underline{\mathbf{C}_{4h}}$, $\mathbf{D}_{8h} \supset \underline{\mathbf{C}_{8h}} \supset \underline{\mathbf{S}_8}$, $\mathbf{D}_{8h} \supset \underline{\mathbf{C}_{8h}} \supset \underline{\mathbf{C}_8}$.

(3) Operations of G: E, C_8^+, C_4^+, C_8^{3+}, C_2, C_8^{3-}, C_4^-, C_8^-, i, S_8^{3-}, S_4^-, S_8^-, σ_h, S_8^+, S_4^+, S_8^{3+}.

(4) Operations of $\widetilde{G}$: E, C_8^+, C_4^+, C_8^{3+}, C_2, C_8^{3-}, C_4^-, C_8^-, i, S_8^{3-}, S_4^-, S_8^-, σ_h, S_8^+, S_4^+, S_8^{3+},
$\widetilde{E}$, $\widetilde{C}_8^+$, $\widetilde{C}_4^+$, $\widetilde{C}_8^{3+}$, $\widetilde{C}_2$, $\widetilde{C}_8^{3-}$, $\widetilde{C}_4^-$, $\widetilde{C}_8^-$, $\widetilde{\imath}$, $\widetilde{S}_8^{3-}$, $\widetilde{S}_4^-$, $\widetilde{S}_8^-$, $\widetilde{\sigma}_h$, $\widetilde{S}_8^+$, $\widetilde{S}_4^+$, $\widetilde{S}_8^{3+}$.

(5) Classes and representations: $|r| = 16$, $|i| = 0$, $|I| = 16$, $|\widetilde{I}| = 16$.

F **66** See Chapter **15**, p. 65



Examples:

T **66**.1 Parameters
Use T **37**.1. § **16**–1, p. 68

T **66**.2 Multiplication table
Use T **37**.2. § **16**–2, p. 69

T **66**.3 Factor table
Use T **37**.3. § **16**–3, p. 70

T **66**.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_{8h}$	E	C_8^+	C_4^+	C_8^{3+}	C_2	C_8^{3-}	C_4^-	C_8^-	i	S_8^{3-}	S_4^-	S_8^-	σ_h	S_8^+	S_4^+	S_8^{3+}	τ
A_g	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	a
B_g	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	a
${}^1E_{1g}$	1	ϵ^*	$-\mathrm{i}$	$-\epsilon$	-1	$-\epsilon^*$	i	ϵ	1	ϵ^*	$-\mathrm{i}$	$-\epsilon$	-1	$-\epsilon^*$	i	ϵ	b
${}^2E_{1g}$	1	ϵ	i	$-\epsilon^*$	-1	$-\epsilon$	$-\mathrm{i}$	ϵ^*	1	ϵ	i	$-\epsilon^*$	-1	$-\epsilon$	$-\mathrm{i}$	ϵ^*	b
${}^1E_{2g}$	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	-1	i	b
${}^2E_{2g}$	1	i	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	b
${}^1E_{3g}$	1	$-\epsilon^*$	$-\mathrm{i}$	ϵ	-1	ϵ^*	i	$-\epsilon$	1	$-\epsilon^*$	$-\mathrm{i}$	ϵ	-1	ϵ^*	i	$-\epsilon$	b
${}^2E_{3g}$	1	$-\epsilon$	i	ϵ^*	-1	ϵ	$-\mathrm{i}$	$-\epsilon^*$	1	$-\epsilon$	i	ϵ^*	-1	ϵ	$-\mathrm{i}$	$-\epsilon^*$	b
A_u	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	a
B_u	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	a
${}^1E_{1u}$	1	ϵ^*	$-\mathrm{i}$	$-\epsilon$	-1	$-\epsilon^*$	i	ϵ	-1	$-\epsilon^*$	i	ϵ	1	ϵ^*	$-\mathrm{i}$	$-\epsilon$	b
${}^2E_{1u}$	1	ϵ	i	$-\epsilon^*$	-1	$-\epsilon$	$-\mathrm{i}$	ϵ^*	-1	$-\epsilon$	$-\mathrm{i}$	ϵ^*	1	ϵ	i	$-\epsilon^*$	b
${}^1E_{2u}$	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	-1	i	-1	i	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	b
${}^2E_{2u}$	1	i	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	1	i	b
${}^1E_{3u}$	1	$-\epsilon^*$	$-\mathrm{i}$	ϵ	-1	ϵ^*	i	$-\epsilon$	-1	ϵ^*	i	$-\epsilon$	1	$-\epsilon^*$	$-\mathrm{i}$	ϵ	b
${}^2E_{3u}$	1	$-\epsilon$	i	ϵ^*	-1	ϵ	$-\mathrm{i}$	$-\epsilon^*$	-1	ϵ	$-\mathrm{i}$	$-\epsilon^*$	1	$-\epsilon$	i	ϵ^*	b
${}^1E_{1/2,g}$	1	δ	ϵ	$\mathrm{i}\delta^*$	i	$-\mathrm{i}\delta$	ϵ^*	δ^*	1	δ	ϵ	$\mathrm{i}\delta^*$	i	$-\mathrm{i}\delta$	ϵ^*	δ^*	b
${}^2E_{1/2,g}$	1	δ^*	ϵ^*	$-\mathrm{i}\delta$	$-\mathrm{i}$	$\mathrm{i}\delta^*$	ϵ	δ	1	δ^*	ϵ^*	$-\mathrm{i}\delta$	$-\mathrm{i}$	$\mathrm{i}\delta^*$	ϵ	δ	b
${}^1E_{3/2,g}$	1	$-\mathrm{i}\delta$	$-\epsilon$	$-\delta^*$	i	$-\delta$	$-\epsilon^*$	$\mathrm{i}\delta^*$	1	$-\mathrm{i}\delta$	$-\epsilon$	$-\delta^*$	i	$-\delta$	$-\epsilon^*$	$\mathrm{i}\delta^*$	b
${}^2E_{3/2,g}$	1	$\mathrm{i}\delta^*$	$-\epsilon^*$	$-\delta$	$-\mathrm{i}$	$-\delta^*$	$-\epsilon$	$-\mathrm{i}\delta$	1	$\mathrm{i}\delta^*$	$-\epsilon^*$	$-\delta$	$-\mathrm{i}$	$-\delta^*$	$-\epsilon$	$-\mathrm{i}\delta$	b
${}^1E_{5/2,g}$	1	$\mathrm{i}\delta$	$-\epsilon$	δ^*	i	δ	$-\epsilon^*$	$-\mathrm{i}\delta^*$	1	$\mathrm{i}\delta$	$-\epsilon$	δ^*	i	δ	$-\epsilon^*$	$-\mathrm{i}\delta^*$	b
${}^2E_{5/2,g}$	1	$-\mathrm{i}\delta^*$	$-\epsilon^*$	δ	$-\mathrm{i}$	δ^*	$-\epsilon$	$\mathrm{i}\delta$	1	$-\mathrm{i}\delta^*$	$-\epsilon^*$	δ	$-\mathrm{i}$	δ^*	$-\epsilon$	$\mathrm{i}\delta$	b
${}^1E_{7/2,g}$	1	$-\delta$	ϵ	$-\mathrm{i}\delta^*$	i	$\mathrm{i}\delta$	ϵ^*	$-\delta^*$	1	$-\delta$	ϵ	$-\mathrm{i}\delta^*$	i	$\mathrm{i}\delta$	ϵ^*	$-\delta^*$	b
${}^2E_{7/2,g}$	1	$-\delta^*$	ϵ^*	$\mathrm{i}\delta$	$-\mathrm{i}$	$-\mathrm{i}\delta^*$	ϵ	$-\delta$	1	$-\delta^*$	ϵ^*	$\mathrm{i}\delta$	$-\mathrm{i}$	$-\mathrm{i}\delta^*$	ϵ	$-\delta$	b
${}^1E_{1/2,u}$	1	δ	ϵ	$\mathrm{i}\delta^*$	i	$-\mathrm{i}\delta$	ϵ^*	δ^*	-1	$-\delta$	$-\epsilon$	$-\mathrm{i}\delta^*$	$-\mathrm{i}$	$\mathrm{i}\delta$	$-\epsilon^*$	$-\delta^*$	b
${}^2E_{1/2,u}$	1	δ^*	ϵ^*	$-\mathrm{i}\delta$	$-\mathrm{i}$	$\mathrm{i}\delta^*$	ϵ	δ	-1	$-\delta^*$	$-\epsilon^*$	$\mathrm{i}\delta$	i	$-\mathrm{i}\delta^*$	$-\epsilon$	$-\delta$	b
${}^1E_{3/2,u}$	1	$-\mathrm{i}\delta$	$-\epsilon$	$-\delta^*$	i	$-\delta$	$-\epsilon^*$	$\mathrm{i}\delta^*$	-1	$\mathrm{i}\delta$	ϵ	δ^*	$-\mathrm{i}$	δ	ϵ^*	$-\mathrm{i}\delta^*$	b
${}^2E_{3/2,u}$	1	$\mathrm{i}\delta^*$	$-\epsilon^*$	$-\delta$	$-\mathrm{i}$	$-\delta^*$	$-\epsilon$	$-\mathrm{i}\delta$	-1	$-\mathrm{i}\delta^*$	ϵ^*	δ	i	δ^*	ϵ	$\mathrm{i}\delta$	b
${}^1E_{5/2,u}$	1	$\mathrm{i}\delta$	$-\epsilon$	δ^*	i	δ	$-\epsilon^*$	$-\mathrm{i}\delta^*$	-1	$-\mathrm{i}\delta$	ϵ	$-\delta^*$	$-\mathrm{i}$	$-\delta$	ϵ^*	$\mathrm{i}\delta^*$	b
${}^2E_{5/2,u}$	1	$-\mathrm{i}\delta^*$	$-\epsilon^*$	δ	$-\mathrm{i}$	δ^*	$-\epsilon$	$\mathrm{i}\delta$	-1	$\mathrm{i}\delta^*$	ϵ^*	$-\delta$	i	$-\delta^*$	ϵ	$-\mathrm{i}\delta$	b
${}^1E_{7/2,u}$	1	$-\delta$	ϵ	$-\mathrm{i}\delta^*$	i	$\mathrm{i}\delta$	ϵ^*	$-\delta^*$	-1	δ	$-\epsilon$	$\mathrm{i}\delta^*$	$-\mathrm{i}$	$-\mathrm{i}\delta$	$-\epsilon^*$	δ^*	b
${}^2E_{7/2,u}$	1	$-\delta^*$	ϵ^*	$\mathrm{i}\delta$	$-\mathrm{i}$	$-\mathrm{i}\delta^*$	ϵ	$-\delta$	-1	δ^*	$-\epsilon^*$	$-\mathrm{i}\delta$	i	$\mathrm{i}\delta^*$	$-\epsilon$	δ	b

$\delta = \exp(2\pi\mathrm{i}/16)$, $\epsilon = \exp(2\pi\mathrm{i}/8)$

T **66**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{8h}$	0	1	2	3
A_g	$^\square 1$	R_z	$x^2+y^2, {}^\square z^2$	
B_g				
${}^1E_{1g} \oplus {}^2E_{1g}$		(R_x, R_y)	$^\square(zx, yz)$	
${}^1E_{2g} \oplus {}^2E_{2g}$			$^\square(xy, x^2-y^2)$	
${}^1E_{3g} \oplus {}^2E_{3g}$				
A_u		$^\square z$		$(x^2+y^2)z, {}^\square z^3$
B_u				
${}^1E_{1u} \oplus {}^2E_{1u}$		$^\square(x, y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^\square(xz^2, yz^2)$
${}^1E_{2u} \oplus {}^2E_{2u}$				$^\square\{xyz, z(x^2-y^2)\}$
${}^1E_{3u} \oplus {}^2E_{3u}$				$^\square\{x(x^2-3y^2), y(3x^2-y^2)\}$

T **66**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{8h}$	$\|j\,m\rangle$	ι	μ	$\mathbf{C}_{8h}$	$\|j\,m\rangle$	ι	μ
A_g	$\|0\,0\rangle$	2	± 8	${}^1E_{1/2,g}$	$\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	1	± 8
B_g	$\|4\,4\rangle$	2	± 8	${}^2E_{1/2,g}$	$\|\frac{1}{2}\,\frac{1}{2}\rangle$	1	± 8
${}^1E_{1g}$	$\|2\,1\rangle$	2	± 8	${}^1E_{3/2,g}$	$\|\frac{3}{2}\,\frac{3}{2}\rangle$	1	± 8
${}^2E_{1g}$	$\|2\,\overline{1}\rangle$	2	± 8	${}^2E_{3/2,g}$	$\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	1	± 8
${}^1E_{2g}$	$\|2\,2\rangle$	2	± 8	${}^1E_{5/2,g}$	$\|\frac{5}{2}\,\overline{\frac{5}{2}}\rangle$	1	± 8
${}^2E_{2g}$	$\|2\,\overline{2}\rangle$	2	± 8	${}^2E_{5/2,g}$	$\|\frac{5}{2}\,\frac{5}{2}\rangle$	1	± 8
${}^1E_{3g}$	$\|4\,\overline{3}\rangle$	2	± 8	${}^1E_{7/2,g}$	$\|\frac{7}{2}\,\frac{7}{2}\rangle$	1	± 8
${}^2E_{3g}$	$\|4\,3\rangle$	2	± 8	${}^2E_{7/2,g}$	$\|\frac{7}{2}\,\overline{\frac{7}{2}}\rangle$	1	± 8
A_u	$\|1\,0\rangle$	2	± 8	${}^1E_{1/2,u}$	$\|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle^\bullet$	1	± 8
B_u	$\|5\,4\rangle$	2	± 8	${}^2E_{1/2,u}$	$\|\frac{1}{2}\,\frac{1}{2}\rangle^\bullet$	1	± 8
${}^1E_{1u}$	$\|1\,1\rangle$	2	± 8	${}^1E_{3/2,u}$	$\|\frac{3}{2}\,\frac{3}{2}\rangle^\bullet$	1	± 8
${}^2E_{1u}$	$\|1\,\overline{1}\rangle$	2	± 8	${}^2E_{3/2,u}$	$\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle^\bullet$	1	± 8
${}^1E_{2u}$	$\|3\,2\rangle$	2	± 8	${}^1E_{5/2,u}$	$\|\frac{5}{2}\,\overline{\frac{5}{2}}\rangle^\bullet$	1	± 8
${}^2E_{2u}$	$\|3\,\overline{2}\rangle$	2	± 8	${}^2E_{5/2,u}$	$\|\frac{5}{2}\,\frac{5}{2}\rangle^\bullet$	1	± 8
${}^1E_{3u}$	$\|3\,\overline{3}\rangle$	2	± 8	${}^1E_{7/2,u}$	$\|\frac{7}{2}\,\frac{7}{2}\rangle^\bullet$	1	± 8
${}^2E_{3u}$	$\|3\,3\rangle$	2	± 8	${}^2E_{7/2,u}$	$\|\frac{7}{2}\,\overline{\frac{7}{2}}\rangle^\bullet$	1	± 8

T **66**.7 Matrix representations

Use T **66**.4 ♠. § **16**–7, p. 77

T **66**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_{8h}$	A_g	B_g	${}^1E_{1g}$	${}^2E_{1g}$	${}^1E_{2g}$	${}^2E_{2g}$	${}^1E_{3g}$	${}^2E_{3g}$	A_u	B_u	${}^1E_{1u}$	${}^2E_{1u}$	${}^1E_{2u}$	${}^2E_{2u}$	${}^1E_{3u}$	${}^2E_{3u}$
A_g	A_g	B_g	${}^1E_{1g}$	${}^2E_{1g}$	${}^1E_{2g}$	${}^2E_{2g}$	${}^1E_{3g}$	${}^2E_{3g}$	A_u	B_u	${}^1E_{1u}$	${}^2E_{1u}$	${}^1E_{2u}$	${}^2E_{2u}$	${}^1E_{3u}$	${}^2E_{3u}$
B_g		A_g	${}^1E_{3g}$	${}^2E_{3g}$	${}^2E_{2g}$	${}^1E_{2g}$	${}^1E_{1g}$	${}^2E_{1g}$	B_u	A_u	${}^1E_{3u}$	${}^2E_{3u}$	${}^2E_{2u}$	${}^1E_{2u}$	${}^1E_{1u}$	${}^2E_{1u}$
${}^1E_{1g}$			${}^1E_{2g}$	A_g	${}^2E_{3g}$	${}^2E_{1g}$	${}^2E_{2g}$	B_g	${}^1E_{1u}$	${}^1E_{3u}$	${}^1E_{2u}$	A_u	${}^2E_{3u}$	${}^2E_{1u}$	${}^2E_{2u}$	B_u
${}^2E_{1g}$				${}^2E_{2g}$	${}^1E_{1g}$	${}^1E_{3g}$	B_g	${}^1E_{2g}$	${}^2E_{1u}$	${}^2E_{3u}$	A_u	${}^2E_{2u}$	${}^1E_{1u}$	${}^1E_{3u}$	B_u	${}^1E_{2u}$
${}^1E_{2g}$					B_g	A_g	${}^2E_{1g}$	${}^1E_{3g}$	${}^1E_{2u}$	${}^2E_{2u}$	${}^2E_{3u}$	${}^1E_{1u}$	B_u	A_u	${}^2E_{1u}$	${}^1E_{3u}$
${}^2E_{2g}$						B_g	${}^2E_{3g}$	${}^1E_{1g}$	${}^2E_{2u}$	${}^1E_{2u}$	${}^2E_{1u}$	${}^1E_{3u}$	A_u	B_u	${}^2E_{3u}$	${}^1E_{1u}$
${}^1E_{3g}$							${}^1E_{2g}$	A_g	${}^1E_{3u}$	${}^1E_{1u}$	${}^2E_{2u}$	B_u	${}^2E_{1u}$	${}^2E_{3u}$	${}^1E_{2u}$	A_u
${}^2E_{3g}$								${}^2E_{2g}$	${}^2E_{3u}$	${}^2E_{1u}$	B_u	${}^1E_{2u}$	${}^1E_{3u}$	${}^1E_{1u}$	A_u	${}^2E_{2u}$
A_u									A_g	B_g	${}^1E_{1g}$	${}^2E_{1g}$	${}^1E_{2g}$	${}^2E_{2g}$	${}^1E_{3g}$	${}^2E_{3g}$
B_u										A_g	${}^1E_{3g}$	${}^2E_{3g}$	${}^2E_{2g}$	${}^1E_{2g}$	${}^1E_{1g}$	${}^2E_{1g}$
${}^1E_{1u}$											${}^1E_{2g}$	A_g	${}^2E_{3g}$	${}^2E_{1g}$	${}^2E_{2g}$	B_g
${}^2E_{1u}$												${}^2E_{2g}$	${}^1E_{1g}$	${}^1E_{3g}$	B_g	${}^1E_{2g}$
${}^1E_{2u}$													B_g	A_g	${}^2E_{1g}$	${}^1E_{3g}$
${}^2E_{2u}$														B_g	${}^2E_{3g}$	${}^1E_{1g}$
${}^1E_{3u}$															${}^1E_{2g}$	A_g
${}^2E_{3u}$																${}^2E_{2g}$

$\Rrightarrow$

T **66**.8 Direct products of representations *(cont.)*

$\mathbf{C}_{8h}$	$^1E_{1/2,g}$	$^2E_{1/2,g}$	$^1E_{3/2,g}$	$^2E_{3/2,g}$	$^1E_{5/2,g}$	$^2E_{5/2,g}$	$^1E_{7/2,g}$	$^2E_{7/2,g}$	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$^1E_{3/2,u}$	$^2E_{3/2,u}$	$^1E_{5/2,u}$	$^2E_{5/2,u}$	$^1E_{7/2,u}$	$^2E_{7/2,u}$
A_g	$^1E_{1/2,g}$	$^2E_{1/2,g}$	$^1E_{3/2,g}$	$^2E_{3/2,g}$	$^1E_{5/2,g}$	$^2E_{5/2,g}$	$^1E_{7/2,g}$	$^2E_{7/2,g}$	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$^1E_{3/2,u}$	$^2E_{3/2,u}$	$^1E_{5/2,u}$	$^2E_{5/2,u}$	$^1E_{7/2,u}$	$^2E_{7/2,u}$
B_g	$^1E_{7/2,g}$	$^2E_{7/2,g}$	$^1E_{5/2,g}$	$^2E_{5/2,g}$	$^1E_{3/2,g}$	$^2E_{3/2,g}$	$^1E_{1/2,g}$	$^2E_{1/2,g}$	$^1E_{7/2,u}$	$^2E_{7/2,u}$	$^1E_{5/2,u}$	$^2E_{5/2,u}$	$^1E_{3/2,u}$	$^2E_{3/2,u}$	$^1E_{1/2,u}$	$^2E_{1/2,u}$
$^1E_{1g}$	$^2E_{1/2,g}$	$^1E_{3/2,g}$	$^2E_{5/2,g}$	$^1E_{1/2,g}$	$^2E_{3/2,g}$	$^1E_{7/2,g}$	$^2E_{7/2,g}$	$^1E_{5/2,g}$	$^2E_{1/2,u}$	$^1E_{3/2,u}$	$^2E_{5/2,u}$	$^1E_{1/2,u}$	$^2E_{3/2,u}$	$^1E_{7/2,u}$	$^2E_{7/2,u}$	$^1E_{5/2,u}$
$^2E_{1g}$	$^2E_{3/2,g}$	$^1E_{1/2,g}$	$^2E_{1/2,g}$	$^1E_{5/2,g}$	$^2E_{7/2,g}$	$^1E_{3/2,g}$	$^2E_{5/2,g}$	$^1E_{7/2,g}$	$^2E_{3/2,u}$	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$^1E_{5/2,u}$	$^2E_{7/2,u}$	$^1E_{3/2,u}$	$^2E_{5/2,u}$	$^1E_{7/2,u}$
$^1E_{2g}$	$^1E_{3/2,g}$	$^2E_{5/2,g}$	$^1E_{7/2,g}$	$^2E_{1/2,g}$	$^1E_{1/2,g}$	$^2E_{7/2,g}$	$^1E_{5/2,g}$	$^2E_{3/2,g}$	$^1E_{3/2,u}$	$^2E_{5/2,u}$	$^1E_{7/2,u}$	$^2E_{1/2,u}$	$^1E_{1/2,u}$	$^2E_{7/2,u}$	$^1E_{5/2,u}$	$^2E_{3/2,u}$
$^2E_{2g}$	$^1E_{5/2,g}$	$^2E_{3/2,g}$	$^1E_{1/2,g}$	$^2E_{7/2,g}$	$^1E_{7/2,g}$	$^2E_{1/2,g}$	$^1E_{3/2,g}$	$^2E_{5/2,g}$	$^1E_{5/2,u}$	$^2E_{3/2,u}$	$^1E_{1/2,u}$	$^2E_{7/2,u}$	$^1E_{7/2,u}$	$^2E_{1/2,u}$	$^1E_{3/2,u}$	$^2E_{5/2,u}$
$^1E_{3g}$	$^2E_{7/2,g}$	$^1E_{5/2,g}$	$^2E_{3/2,g}$	$^1E_{7/2,g}$	$^2E_{5/2,g}$	$^1E_{1/2,g}$	$^2E_{1/2,g}$	$^1E_{3/2,g}$	$^2E_{7/2,u}$	$^1E_{5/2,u}$	$^2E_{3/2,u}$	$^1E_{7/2,u}$	$^2E_{5/2,u}$	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$^1E_{3/2,u}$
$^2E_{3g}$	$^2E_{5/2,g}$	$^1E_{7/2,g}$	$^2E_{7/2,g}$	$^1E_{3/2,g}$	$^2E_{1/2,g}$	$^1E_{5/2,g}$	$^2E_{3/2,g}$	$^1E_{1/2,g}$	$^2E_{5/2,u}$	$^1E_{7/2,u}$	$^2E_{7/2,u}$	$^1E_{3/2,u}$	$^2E_{1/2,u}$	$^1E_{5/2,u}$	$^2E_{3/2,u}$	$^1E_{1/2,u}$
A_u	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$^1E_{3/2,u}$	$^2E_{3/2,u}$	$^1E_{5/2,u}$	$^2E_{5/2,u}$	$^1E_{7/2,u}$	$^2E_{7/2,u}$	$^1E_{1/2,g}$	$^2E_{1/2,g}$	$^1E_{3/2,g}$	$^2E_{3/2,g}$	$^1E_{5/2,g}$	$^2E_{5/2,g}$	$^1E_{7/2,g}$	$^2E_{7/2,g}$
B_u	$^1E_{7/2,u}$	$^2E_{7/2,u}$	$^1E_{5/2,u}$	$^2E_{5/2,u}$	$^1E_{3/2,u}$	$^2E_{3/2,u}$	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$^1E_{7/2,g}$	$^2E_{7/2,g}$	$^1E_{5/2,g}$	$^2E_{5/2,g}$	$^1E_{3/2,g}$	$^2E_{3/2,g}$	$^1E_{1/2,g}$	$^2E_{1/2,g}$
$^1E_{1u}$	$^2E_{1/2,u}$	$^1E_{3/2,u}$	$^2E_{5/2,u}$	$^1E_{1/2,u}$	$^2E_{3/2,u}$	$^1E_{7/2,u}$	$^2E_{7/2,u}$	$^1E_{5/2,u}$	$^2E_{1/2,g}$	$^1E_{3/2,g}$	$^2E_{5/2,g}$	$^1E_{1/2,g}$	$^2E_{3/2,g}$	$^1E_{7/2,g}$	$^2E_{7/2,g}$	$^1E_{5/2,g}$
$^2E_{1u}$	$^2E_{3/2,u}$	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$^1E_{5/2,u}$	$^2E_{7/2,u}$	$^1E_{3/2,u}$	$^2E_{5/2,u}$	$^1E_{7/2,u}$	$^2E_{3/2,g}$	$^1E_{1/2,g}$	$^2E_{1/2,g}$	$^1E_{5/2,g}$	$^2E_{7/2,g}$	$^1E_{3/2,g}$	$^2E_{5/2,g}$	$^1E_{7/2,g}$
$^1E_{2u}$	$^1E_{3/2,u}$	$^2E_{5/2,u}$	$^1E_{7/2,u}$	$^2E_{1/2,u}$	$^1E_{1/2,u}$	$^2E_{7/2,u}$	$^1E_{5/2,u}$	$^2E_{3/2,u}$	$^1E_{3/2,g}$	$^2E_{5/2,g}$	$^1E_{7/2,g}$	$^2E_{1/2,g}$	$^1E_{1/2,g}$	$^2E_{7/2,g}$	$^1E_{5/2,g}$	$^2E_{3/2,g}$
$^2E_{2u}$	$^1E_{5/2,u}$	$^2E_{3/2,u}$	$^1E_{1/2,u}$	$^2E_{7/2,u}$	$^1E_{7/2,u}$	$^2E_{1/2,u}$	$^1E_{3/2,u}$	$^2E_{5/2,u}$	$^1E_{5/2,g}$	$^2E_{3/2,g}$	$^1E_{1/2,g}$	$^2E_{7/2,g}$	$^1E_{7/2,g}$	$^2E_{1/2,g}$	$^1E_{3/2,g}$	$^2E_{5/2,g}$
$^1E_{3u}$	$^2E_{7/2,u}$	$^1E_{5/2,u}$	$^2E_{3/2,u}$	$^1E_{7/2,u}$	$^2E_{5/2,u}$	$^1E_{1/2,u}$	$^2E_{1/2,u}$	$^1E_{3/2,u}$	$^2E_{7/2,g}$	$^1E_{5/2,g}$	$^2E_{3/2,g}$	$^1E_{7/2,g}$	$^2E_{5/2,g}$	$^1E_{1/2,g}$	$^2E_{1/2,g}$	$^1E_{3/2,g}$
$^2E_{3u}$	$^2E_{5/2,u}$	$^1E_{7/2,u}$	$^2E_{7/2,u}$	$^1E_{3/2,u}$	$^2E_{1/2,u}$	$^1E_{5/2,u}$	$^2E_{3/2,u}$	$^1E_{1/2,u}$	$^2E_{5/2,g}$	$^1E_{7/2,g}$	$^2E_{7/2,g}$	$^1E_{3/2,g}$	$^2E_{1/2,g}$	$^1E_{5/2,g}$	$^2E_{3/2,g}$	$^1E_{1/2,g}$
$^1E_{1/2,g}$	$^2E_{1g}$	A_g	$^1E_{1g}$	$^2E_{2g}$	$^1E_{3g}$	$^1E_{2g}$	$^2E_{3g}$	B_g	$^2E_{1u}$	A_u	$^1E_{1u}$	$^2E_{2u}$	$^1E_{3u}$	$^1E_{2u}$	$^2E_{3u}$	B_u
$^2E_{1/2,g}$		$^1E_{1g}$	$^1E_{2g}$	$^2E_{1g}$	$^2E_{2g}$	$^2E_{3g}$	B_g	$^1E_{3g}$	A_u	$^1E_{1u}$	$^1E_{2u}$	$^2E_{1u}$	$^2E_{2u}$	$^2E_{3u}$	B_u	$^1E_{3u}$
$^1E_{3/2,g}$			$^2E_{3g}$	A_g	$^2E_{1g}$	B_g	$^1E_{3g}$	$^2E_{2g}$	$^1E_{1u}$	$^1E_{2u}$	$^2E_{3u}$	A_u	$^2E_{1u}$	B_u	$^1E_{3u}$	$^2E_{2u}$
$^2E_{3/2,g}$				$^1E_{3g}$	B_g	$^1E_{1g}$	$^1E_{2g}$	$^2E_{3g}$	$^2E_{2u}$	$^2E_{1u}$	A_u	$^1E_{3u}$	B_u	$^1E_{1u}$	$^1E_{2u}$	$^2E_{3u}$
$^1E_{5/2,g}$					$^2E_{3g}$	A_g	$^1E_{1g}$	$^1E_{2g}$	$^1E_{3u}$	$^2E_{2u}$	$^2E_{1u}$	B_u	$^2E_{3u}$	A_u	$^1E_{1u}$	$^1E_{2u}$
$^2E_{5/2,g}$						$^1E_{3g}$	$^2E_{2g}$	$^2E_{1g}$	$^1E_{2u}$	$^2E_{3u}$	B_u	$^1E_{1u}$	A_u	$^1E_{3u}$	$^2E_{2u}$	$^2E_{1u}$
$^1E_{7/2,g}$							$^2E_{1g}$	A_g	$^2E_{3u}$	B_u	$^1E_{3u}$	$^1E_{2u}$	$^1E_{1u}$	$^2E_{2u}$	$^2E_{1u}$	A_u
$^2E_{7/2,g}$								$^1E_{1g}$	B_u	$^1E_{3u}$	$^2E_{2u}$	$^2E_{3u}$	$^1E_{2u}$	$^2E_{1u}$	A_u	$^1E_{1u}$
$^1E_{1/2,u}$									$^2E_{1g}$	A_g	$^1E_{1g}$	$^2E_{2g}$	$^1E_{3g}$	$^1E_{2g}$	$^2E_{3g}$	B_g
$^2E_{1/2,u}$										$^1E_{1g}$	$^1E_{2g}$	$^2E_{1g}$	$^2E_{2g}$	$^2E_{3g}$	B_g	$^1E_{3g}$
$^1E_{3/2,u}$											$^2E_{3g}$	A_g	$^2E_{1g}$	B_g	$^1E_{3g}$	$^2E_{2g}$
$^2E_{3/2,u}$												$^1E_{3g}$	B_g	$^1E_{1g}$	$^1E_{2g}$	$^2E_{3g}$
$^1E_{5/2,u}$													$^2E_{3g}$	A_g	$^1E_{1g}$	$^1E_{2g}$
$^2E_{5/2,u}$														$^1E_{3g}$	$^2E_{2g}$	$^2E_{1g}$
$^1E_{7/2,u}$															$^2E_{1g}$	A_g
$^2E_{7/2,u}$																$^1E_{1g}$

T **66**.9 Subduction (descent of symmetry)

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$\mathbf{C}_{8h}$	$\mathbf{C}_{4h}$	$\mathbf{C}_{2h}$	$\mathbf{S}_8$	$\mathbf{S}_4$	$\mathbf{C}_s$	$\mathbf{C}_i$	$\mathbf{C}_8$	$\mathbf{C}_4$	$\mathbf{C}_2$
A_g	A_g	A_g	A	A	A'	A_g	A	A	A
B_g	A_g	A_g	B	A	A'	A_g	B	A	A
${}^1E_{1g}$	${}^1E_g$	B_g	${}^1E_3$	${}^1E$	A''	A_g	${}^1E_1$	${}^1E$	B
${}^2E_{1g}$	${}^2E_g$	B_g	${}^2E_3$	${}^2E$	A''	A_g	${}^2E_1$	${}^2E$	B
${}^1E_{2g}$	B_g	A_g	${}^1E_2$	B	A'	A_g	${}^1E_2$	B	A
${}^2E_{2g}$	B_g	A_g	${}^2E_2$	B	A'	A_g	${}^2E_2$	B	A
${}^1E_{3g}$	${}^1E_g$	B_g	${}^1E_1$	${}^1E$	A''	A_g	${}^1E_3$	${}^1E$	B
${}^2E_{3g}$	${}^2E_g$	B_g	${}^2E_1$	${}^2E$	A''	A_g	${}^2E_3$	${}^2E$	B
A_u	A_u	A_u	B	B	A''	A_u	A	A	A
B_u	A_u	A_u	A	B	A''	A_u	B	A	A
${}^1E_{1u}$	${}^1E_u$	B_u	${}^1E_1$	${}^2E$	A'	A_u	${}^1E_1$	${}^1E$	B
${}^2E_{1u}$	${}^2E_u$	B_u	${}^2E_1$	${}^1E$	A'	A_u	${}^2E_1$	${}^2E$	B
${}^1E_{2u}$	B_u	A_u	${}^2E_2$	A	A''	A_u	${}^1E_2$	B	A
${}^2E_{2u}$	B_u	A_u	${}^1E_2$	A	A''	A_u	${}^2E_2$	B	A
${}^1E_{3u}$	${}^1E_u$	B_u	${}^1E_3$	${}^2E$	A'	A_u	${}^1E_3$	${}^1E$	B
${}^2E_{3u}$	${}^2E_u$	B_u	${}^2E_3$	${}^1E$	A'	A_u	${}^2E_3$	${}^2E$	B
${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	$A_{1/2,g}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	$A_{1/2,g}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	$A_{1/2,g}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_{3/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$	${}^2E_{3/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	$A_{1/2,g}$	${}^2E_{3/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	$A_{1/2,g}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_{5/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	$A_{1/2,g}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_{7/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	$A_{1/2,g}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_{7/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^2E_{7/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	$A_{1/2,g}$	${}^2E_{7/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^1E_{7/2}$	${}^2E_{3/2}$	${}^2E_{1/2}$	$A_{1/2,u}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^1E_{1/2}$	$A_{1/2,u}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^1E_{5/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$	$A_{1/2,u}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_{3/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$	$A_{1/2,u}$	${}^2E_{3/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$	$A_{1/2,u}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_{5/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$	$A_{1/2,u}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_{7/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^2E_{1/2}$	$A_{1/2,u}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_{7/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^1E_{1/2}$	$A_{1/2,u}$	${}^2E_{7/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$

T **66**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{8h}$
$8n$	$(2n+1)\,A_g \oplus 2n\,(B_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g})$
$8n+1$	$(2n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u}) \oplus 2n\,(B_u \oplus {}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u})$
$8n+2$	$(2n+1)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g}) \oplus 2n\,(B_g \oplus {}^1E_{3g} \oplus {}^2E_{3g})$
$8n+3$	$(2n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u}) \oplus 2n\,B_u$
$8n+4$	$(2n+1)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g}) \oplus (2n+2)\,B_g$
$8n+5$	$(2n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u}) \oplus (2n+2)(B_u \oplus {}^1E_{3u} \oplus {}^2E_{3u})$
$8n+6$	$(2n+1)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g}) \oplus (2n+2)(B_g \oplus {}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g})$
$8n+7$	$(2n+1)\,A_u \oplus (2n+2)(B_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u})$
$8n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus 2n\,({}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g})$
$8n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}) \oplus 2n\,({}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g})$
$8n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g}) \oplus 2n\,({}^1E_{7/2,g} \oplus {}^2E_{7/2,g})$
$8n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g})$
$8n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g}) \oplus$ $(2n+2)({}^1E_{7/2,g} \oplus {}^2E_{7/2,g})$
$8n+\frac{11}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}) \oplus$ $(2n+2)({}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g})$
$8n+\frac{13}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus$ $(2n+2)({}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g})$
$8n+\frac{15}{2}$	$(2n+2)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g})$

$n = 0, 1, 2, \ldots$

T **66**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

$\overline{18}$	$|G| = 18$	$|C| = 18$	$|\widetilde{C}| = 36$	T **67**	p. 531	$\mathbf{C}_{9h}$

(1) Product forms: $\mathbf{C}_9 \otimes \mathbf{C}_s$.

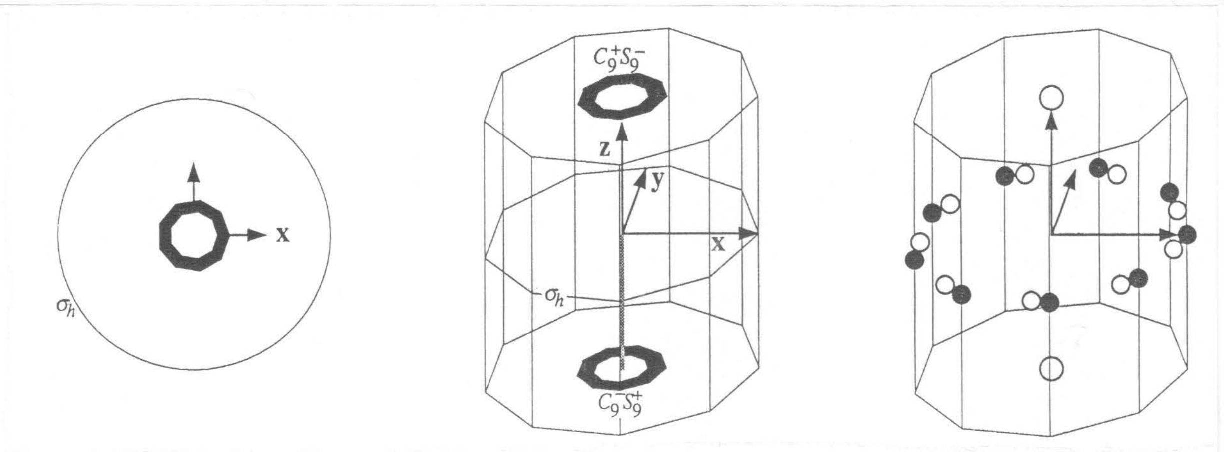
(2) Group chains: $\mathbf{D}_{9h} \supset \underline{\mathbf{C}_{9h}} \supset \underline{\mathbf{C}_{3h}}$, $\mathbf{D}_{9h} \supset \underline{\mathbf{C}_{9h}} \supset \underline{\mathbf{C}_9}$.

(3) Operations of G: $E,\ C_9^+,\ C_9^{2+},\ C_3^+,\ C_9^{4+},\ C_9^{4-},\ C_3^-,\ C_9^{2-},\ C_9^-,$
$\sigma_h,\ S_9^+,\ S_9^{2+},\ S_3^+,\ S_9^{4+},\ S_9^{4-},\ S_3^-,\ S_9^{2-},\ S_9^-.$

(4) Operations of $\widetilde{G}$: $E,\ C_9^+,\ C_9^{2+},\ C_3^+,\ C_9^{4+},\ C_9^{4-},\ C_3^-,\ C_9^{2-},\ C_9^-,$
$\sigma_h,\ S_9^+,\ S_9^{2+},\ S_3^+,\ S_9^{4+},\ S_9^{4-},\ S_3^-,\ S_9^{2-},\ S_9^-,$
$\widetilde{E},\ \widetilde{C}_9^+,\ \widetilde{C}_9^{2+},\ \widetilde{C}_3^+,\ \widetilde{C}_9^{4+},\ \widetilde{C}_9^{4-},\ \widetilde{C}_3^-,\ \widetilde{C}_9^{2-},\ \widetilde{C}_9^-,$
$\widetilde{\sigma}_h,\ \widetilde{S}_9^+,\ \widetilde{S}_9^{2+},\ \widetilde{S}_3^+,\ \widetilde{S}_9^{4+},\ \widetilde{S}_9^{4-},\ \widetilde{S}_3^-,\ \widetilde{S}_9^{2-},\ \widetilde{S}_9^-.$

(5) Classes and representations: $|r| = 18$, $|i| = 0$, $|I| = 18$, $|\widetilde{I}| = 18$.

F **67** See Chapter **15**, p. 65



Examples:

T **67**.1 Parameters
Use T **38**.1. § **16**–1, p. 68

T **67**.2 Multiplication table
Use T **38**.2. § **16**–2, p. 69

T **67**.3 Factor table
Use T **38**.3. § **16**–3, p. 70

T **67**.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_{9h}$	E	C_9^+	C_9^{2+}	C_3^+	C_9^{4+}	C_9^{4-}	C_3^-	C_9^{2-}	C_9^-	τ
A'	1	1	1	1	1	1	1	1	1	a
${}^1E_1'$	1	δ^*	ϵ^*	η^*	θ^*	θ	η	ϵ	δ	b
${}^2E_1'$	1	δ	ϵ	η	θ	θ^*	η^*	ϵ^*	δ^*	b
${}^1E_2'$	1	ϵ^*	θ^*	η	δ	δ^*	η^*	θ	ϵ	b
${}^2E_2'$	1	ϵ	θ	η^*	δ^*	δ	η	θ^*	ϵ^*	b
${}^1E_3'$	1	η^*	η	1	η^*	η	1	η^*	η	b
${}^2E_3'$	1	η	η^*	1	η	η^*	1	η	η^*	b
${}^1E_4'$	1	θ^*	δ	η^*	ϵ	ϵ^*	η	δ^*	θ	b
${}^2E_4'$	1	θ	δ^*	η	ϵ^*	ϵ	η^*	δ	θ^*	b
A''	1	1	1	1	1	1	1	1	1	a
${}^1E_1''$	1	δ^*	ϵ^*	η^*	θ^*	θ	η	ϵ	δ	b
${}^2E_1''$	1	δ	ϵ	η	θ	θ^*	η^*	ϵ^*	δ^*	b
${}^1E_2''$	1	ϵ^*	θ^*	η	δ	δ^*	η^*	θ	ϵ	b
${}^2E_2''$	1	ϵ	θ	η^*	δ^*	δ	η	θ^*	ϵ^*	b
${}^1E_3''$	1	η^*	η	1	η^*	η	1	η^*	η	b
${}^2E_3''$	1	η	η^*	1	η	η^*	1	η	η^*	b
${}^1E_4''$	1	θ^*	δ	η^*	ϵ	ϵ^*	η	δ^*	θ	b
${}^2E_4''$	1	θ	δ^*	η	ϵ^*	ϵ	η^*	δ	θ^*	b
${}^1E_{1/2}$	1	$-\theta^*$	δ	$-\eta^*$	ϵ	ϵ^*	$-\eta$	δ^*	$-\theta$	b
${}^2E_{1/2}$	1	$-\theta$	δ^*	$-\eta$	ϵ^*	ϵ	$-\eta^*$	δ	$-\theta^*$	b
${}^1E_{3/2}$	1	$-\eta$	η^*	-1	η	η^*	-1	η	$-\eta^*$	b
${}^2E_{3/2}$	1	$-\eta^*$	η	-1	η^*	η	-1	η^*	$-\eta$	b
${}^1E_{5/2}$	1	$-\epsilon^*$	θ^*	$-\eta$	δ	δ^*	$-\eta^*$	θ	$-\epsilon$	b
${}^2E_{5/2}$	1	$-\epsilon$	θ	$-\eta^*$	δ^*	δ	$-\eta$	θ^*	$-\epsilon^*$	b
${}^1E_{7/2}$	1	$-\delta$	ϵ	$-\eta$	θ	θ^*	$-\eta^*$	ϵ^*	$-\delta^*$	b
${}^2E_{7/2}$	1	$-\delta^*$	ϵ^*	$-\eta^*$	θ^*	θ	$-\eta$	ϵ	$-\delta$	b
${}^1E_{9/2}$	1	-1	1	-1	1	1	-1	1	-1	b
${}^2E_{9/2}$	1	-1	1	-1	1	1	-1	1	-1	b
${}^1E_{11/2}$	1	$-\delta^*$	ϵ^*	$-\eta^*$	θ^*	θ	$-\eta$	ϵ	$-\delta$	b
${}^2E_{11/2}$	1	$-\delta$	ϵ	$-\eta$	θ	θ^*	$-\eta^*$	ϵ^*	$-\delta^*$	b
${}^1E_{13/2}$	1	$-\epsilon$	θ	$-\eta^*$	δ^*	δ	$-\eta$	θ^*	$-\epsilon^*$	b
${}^2E_{13/2}$	1	$-\epsilon^*$	θ^*	$-\eta$	δ	δ^*	$-\eta^*$	θ	$-\epsilon$	b
${}^1E_{15/2}$	1	$-\eta^*$	η	-1	η^*	η	-1	η^*	$-\eta$	b
${}^2E_{15/2}$	1	$-\eta$	η^*	-1	η	η^*	-1	η	$-\eta^*$	b
${}^1E_{17/2}$	1	$-\theta$	δ^*	$-\eta$	ϵ^*	ϵ	$-\eta^*$	δ	$-\theta^*$	b
${}^2E_{17/2}$	1	$-\theta^*$	δ	$-\eta^*$	ϵ	ϵ^*	$-\eta$	δ^*	$-\theta$	b

$\delta = \exp(2\pi i/9)$, $\epsilon = \exp(4\pi i/9)$, $\eta = \exp(6\pi i/9)$, $\theta = \exp(8\pi i/9)$ ⇛

T **67**.4 Character table *(cont.)*

$\mathbf{C}_{9h}$	σ_h	S_9^+	S_9^{2+}	S_3^+	S_9^{4+}	S_9^{4-}	S_3^-	S_9^{2-}	S_9^-	τ
A'	1	1	1	1	1	1	1	1	1	a
${}^1E_1'$	1	δ^*	ϵ^*	η^*	θ^*	θ	η	ϵ	δ	b
${}^2E_1'$	1	δ	ϵ	η	θ	θ^*	η^*	ϵ^*	δ^*	b
${}^1E_2'$	1	ϵ^*	θ^*	η	δ	δ^*	η^*	θ	ϵ	b
${}^2E_2'$	1	ϵ	θ	η^*	δ^*	δ	η	θ^*	ϵ^*	b
${}^1E_3'$	1	η^*	η	1	η^*	η	1	η^*	η	b
${}^2E_3'$	1	η	η^*	1	η	η^*	1	η	η^*	b
${}^1E_4'$	1	θ^*	δ	η^*	ϵ	ϵ^*	η	δ^*	θ	b
${}^2E_4'$	1	θ	δ^*	η	ϵ^*	ϵ	η^*	δ	θ^*	b
A''	-1	-1	-1	-1	-1	-1	-1	-1	-1	a
${}^1E_1''$	-1	$-\delta^*$	$-\epsilon^*$	$-\eta^*$	$-\theta^*$	$-\theta$	$-\eta$	$-\epsilon$	$-\delta$	b
${}^2E_1''$	-1	$-\delta$	$-\epsilon$	$-\eta$	$-\theta$	$-\theta^*$	$-\eta^*$	$-\epsilon^*$	$-\delta^*$	b
${}^1E_2''$	-1	$-\epsilon^*$	$-\theta^*$	$-\eta$	$-\delta$	$-\delta^*$	$-\eta^*$	$-\theta$	$-\epsilon$	b
${}^2E_2''$	-1	$-\epsilon$	$-\theta$	$-\eta^*$	$-\delta^*$	$-\delta$	$-\eta$	$-\theta^*$	$-\epsilon^*$	b
${}^1E_3''$	-1	$-\eta^*$	$-\eta$	-1	$-\eta^*$	$-\eta$	-1	$-\eta^*$	$-\eta$	b
${}^2E_3''$	-1	$-\eta$	$-\eta^*$	-1	$-\eta$	$-\eta^*$	-1	$-\eta$	$-\eta^*$	b
${}^1E_4''$	-1	$-\theta^*$	$-\delta$	$-\eta^*$	$-\epsilon$	$-\epsilon^*$	$-\eta$	$-\delta^*$	$-\theta$	b
${}^2E_4''$	-1	$-\theta$	$-\delta^*$	$-\eta$	$-\epsilon^*$	$-\epsilon$	$-\eta^*$	$-\delta$	$-\theta^*$	b
${}^1E_{1/2}$	i	$\mathrm{i}\theta^*$	$-\mathrm{i}\delta$	$\mathrm{i}\eta^*$	$-\mathrm{i}\epsilon$	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\eta$	$\mathrm{i}\delta^*$	$-\mathrm{i}\theta$	b
${}^2E_{1/2}$	$-\mathrm{i}$	$-\mathrm{i}\theta$	$\mathrm{i}\delta^*$	$-\mathrm{i}\eta$	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\epsilon$	$\mathrm{i}\eta^*$	$-\mathrm{i}\delta$	$\mathrm{i}\theta^*$	b
${}^1E_{3/2}$	i	$\mathrm{i}\eta$	$-\mathrm{i}\eta^*$	i	$-\mathrm{i}\eta$	$\mathrm{i}\eta^*$	$-\mathrm{i}$	$\mathrm{i}\eta$	$-\mathrm{i}\eta^*$	b
${}^2E_{3/2}$	$-\mathrm{i}$	$-\mathrm{i}\eta^*$	$\mathrm{i}\eta$	$-\mathrm{i}$	$\mathrm{i}\eta^*$	$-\mathrm{i}\eta$	i	$-\mathrm{i}\eta^*$	$\mathrm{i}\eta$	b
${}^1E_{5/2}$	i	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\theta^*$	$\mathrm{i}\eta$	$-\mathrm{i}\delta$	$\mathrm{i}\delta^*$	$-\mathrm{i}\eta^*$	$\mathrm{i}\theta$	$-\mathrm{i}\epsilon$	b
${}^2E_{5/2}$	$-\mathrm{i}$	$-\mathrm{i}\epsilon$	$\mathrm{i}\theta$	$-\mathrm{i}\eta^*$	$\mathrm{i}\delta^*$	$-\mathrm{i}\delta$	$\mathrm{i}\eta$	$-\mathrm{i}\theta^*$	$\mathrm{i}\epsilon^*$	b
${}^1E_{7/2}$	i	$\mathrm{i}\delta$	$-\mathrm{i}\epsilon$	$\mathrm{i}\eta$	$-\mathrm{i}\theta$	$\mathrm{i}\theta^*$	$-\mathrm{i}\eta^*$	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\delta^*$	b
${}^2E_{7/2}$	$-\mathrm{i}$	$-\mathrm{i}\delta^*$	$\mathrm{i}\epsilon^*$	$-\mathrm{i}\eta^*$	$\mathrm{i}\theta^*$	$-\mathrm{i}\theta$	$\mathrm{i}\eta$	$-\mathrm{i}\epsilon$	$\mathrm{i}\delta$	b
${}^1E_{9/2}$	i	i	$-\mathrm{i}$	i	$-\mathrm{i}$	i	$-\mathrm{i}$	i	$-\mathrm{i}$	b
${}^2E_{9/2}$	$-\mathrm{i}$	$-\mathrm{i}$	i	$-\mathrm{i}$	i	$-\mathrm{i}$	i	$-\mathrm{i}$	i	b
${}^1E_{11/2}$	i	$\mathrm{i}\delta^*$	$-\mathrm{i}\epsilon^*$	$\mathrm{i}\eta^*$	$-\mathrm{i}\theta^*$	$\mathrm{i}\theta$	$-\mathrm{i}\eta$	$\mathrm{i}\epsilon$	$-\mathrm{i}\delta$	b
${}^2E_{11/2}$	$-\mathrm{i}$	$-\mathrm{i}\delta$	$\mathrm{i}\epsilon$	$-\mathrm{i}\eta$	$\mathrm{i}\theta$	$-\mathrm{i}\theta^*$	$\mathrm{i}\eta^*$	$-\mathrm{i}\epsilon^*$	$\mathrm{i}\delta^*$	b
${}^1E_{13/2}$	i	$\mathrm{i}\epsilon$	$-\mathrm{i}\theta$	$\mathrm{i}\eta^*$	$-\mathrm{i}\delta^*$	$\mathrm{i}\delta$	$-\mathrm{i}\eta$	$\mathrm{i}\theta^*$	$-\mathrm{i}\epsilon^*$	b
${}^2E_{13/2}$	$-\mathrm{i}$	$-\mathrm{i}\epsilon^*$	$\mathrm{i}\theta^*$	$-\mathrm{i}\eta$	$\mathrm{i}\delta$	$-\mathrm{i}\delta^*$	$\mathrm{i}\eta^*$	$-\mathrm{i}\theta$	$\mathrm{i}\epsilon$	b
${}^1E_{15/2}$	i	$\mathrm{i}\eta^*$	$-\mathrm{i}\eta$	i	$-\mathrm{i}\eta^*$	$\mathrm{i}\eta$	$-\mathrm{i}$	$\mathrm{i}\eta^*$	$-\mathrm{i}\eta$	b
${}^2E_{15/2}$	$-\mathrm{i}$	$-\mathrm{i}\eta$	$\mathrm{i}\eta^*$	$-\mathrm{i}$	$\mathrm{i}\eta$	$-\mathrm{i}\eta^*$	i	$-\mathrm{i}\eta$	$\mathrm{i}\eta^*$	b
${}^1E_{17/2}$	i	$\mathrm{i}\theta$	$-\mathrm{i}\delta^*$	$\mathrm{i}\eta$	$-\mathrm{i}\epsilon^*$	$\mathrm{i}\epsilon$	$-\mathrm{i}\eta^*$	$\mathrm{i}\delta$	$-\mathrm{i}\theta^*$	b
${}^2E_{17/2}$	$-\mathrm{i}$	$-\mathrm{i}\theta^*$	$\mathrm{i}\delta$	$-\mathrm{i}\eta^*$	$\mathrm{i}\epsilon$	$-\mathrm{i}\epsilon^*$	$\mathrm{i}\eta$	$-\mathrm{i}\delta^*$	$\mathrm{i}\theta$	b

$\delta = \exp(2\pi\mathrm{i}/9)$, $\epsilon = \exp(4\pi\mathrm{i}/9)$, $\eta = \exp(6\pi\mathrm{i}/9)$, $\theta = \exp(8\pi\mathrm{i}/9)$

T **67**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{C}_{9h}$	0	1	2	3
A'	$\square 1$	R_z	$x^2+y^2, \square z^2$	
${}^1E_1' \oplus {}^2E_1'$		$\square(x,y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, \square(xz^2, yz^2)$
${}^1E_2' \oplus {}^2E_2'$			$\square(xy, x^2-y^2)$	
${}^1E_3' \oplus {}^2E_3'$				$\square\{x(x^2-3y^2), y(3x^2-y^2)\}$
${}^1E_4' \oplus {}^2E_4'$				
A''		$\square z$		$(x^2+y^2)z, \square z^3$
${}^1E_1'' \oplus {}^2E_1''$		(R_x, R_y)	$\square(zx, yz)$	
${}^1E_2'' \oplus {}^2E_2''$				$\square\{xyz, (x^2-y^2)z\}$
${}^1E_3'' \oplus {}^2E_3''$				
${}^1E_4'' \oplus {}^2E_4''$				

T **67**.6 Symmetrized bases

§ **16**–6, p. 74

$\mathbf{C}_{9h}$	$\lvert j\,m\rangle$		ι	μ	$\mathbf{C}_{9h}$	$\lvert j\,m\rangle$		ι	μ
A'	$\lvert 0\,0\rangle$	$\lvert 9\,9\rangle$	2	±18	${}^1E_{1/2}$	$\lvert \tfrac12\,\overline{\tfrac12}\rangle$	$\lvert \tfrac{17}{2}\,\tfrac{17}{2}\rangle^\bullet$	1	±18
${}^1E_1'$	$\lvert 1\,1\rangle$	$\lvert 8\,\bar{8}\rangle$	2	±18	${}^2E_{1/2}$	$\lvert \tfrac12\,\tfrac12\rangle$	$\lvert \tfrac{17}{2}\,\overline{\tfrac{17}{2}}\rangle^\bullet$	1	±18
${}^2E_1'$	$\lvert 1\,\bar{1}\rangle$	$\lvert 8\,8\rangle$	2	±18	${}^1E_{3/2}$	$\lvert \tfrac32\,\tfrac32\rangle$	$\lvert \tfrac{15}{2}\,\overline{\tfrac{15}{2}}\rangle^\bullet$	1	±18
${}^1E_2'$	$\lvert 2\,2\rangle$	$\lvert 7\,\bar{7}\rangle$	2	±18	${}^2E_{3/2}$	$\lvert \tfrac32\,\overline{\tfrac32}\rangle$	$\lvert \tfrac{15}{2}\,\tfrac{15}{2}\rangle^\bullet$	1	±18
${}^2E_2'$	$\lvert 2\,\bar{2}\rangle$	$\lvert 7\,7\rangle$	2	±18	${}^1E_{5/2}$	$\lvert \tfrac52\,\overline{\tfrac52}\rangle$	$\lvert \tfrac{13}{2}\,\tfrac{13}{2}\rangle^\bullet$	1	±18
${}^1E_3'$	$\lvert 3\,3\rangle$	$\lvert 6\,\bar{6}\rangle$	2	±18	${}^2E_{5/2}$	$\lvert \tfrac52\,\tfrac52\rangle$	$\lvert \tfrac{13}{2}\,\overline{\tfrac{13}{2}}\rangle^\bullet$	1	±18
${}^2E_3'$	$\lvert 3\,\bar{3}\rangle$	$\lvert 6\,6\rangle$	2	±18	${}^1E_{7/2}$	$\lvert \tfrac72\,\tfrac72\rangle$	$\lvert \tfrac{11}{2}\,\overline{\tfrac{11}{2}}\rangle^\bullet$	1	±18
${}^1E_4'$	$\lvert 4\,4\rangle$	$\lvert 5\,\bar{5}\rangle$	2	±18	${}^2E_{7/2}$	$\lvert \tfrac72\,\overline{\tfrac72}\rangle$	$\lvert \tfrac{11}{2}\,\tfrac{11}{2}\rangle^\bullet$	1	±18
${}^2E_4'$	$\lvert 4\,\bar{4}\rangle$	$\lvert 5\,5\rangle$	2	±18	${}^1E_{9/2}$	$\lvert \tfrac92\,\overline{\tfrac92}\rangle$	$\lvert \tfrac92\,\tfrac92\rangle^\bullet$	1	±18
A''	$\lvert 1\,0\rangle$	$\lvert 10\,9\rangle$	2	±18	${}^2E_{9/2}$	$\lvert \tfrac92\,\tfrac92\rangle$	$\lvert \tfrac92\,\overline{\tfrac92}\rangle^\bullet$	1	±18
${}^1E_1''$	$\lvert 2\,1\rangle$	$\lvert 9\,\bar{8}\rangle$	2	±18	${}^1E_{11/2}$	$\lvert \tfrac{11}{2}\,\tfrac{11}{2}\rangle$	$\lvert \tfrac72\,\overline{\tfrac72}\rangle^\bullet$	1	±18
${}^2E_1''$	$\lvert 2\,\bar{1}\rangle$	$\lvert 9\,8\rangle$	2	±18	${}^2E_{11/2}$	$\lvert \tfrac{11}{2}\,\overline{\tfrac{11}{2}}\rangle$	$\lvert \tfrac72\,\tfrac72\rangle^\bullet$	1	±18
${}^1E_2''$	$\lvert 3\,2\rangle$	$\lvert 8\,\bar{7}\rangle$	2	±18	${}^1E_{13/2}$	$\lvert \tfrac{13}{2}\,\overline{\tfrac{13}{2}}\rangle$	$\lvert \tfrac52\,\tfrac52\rangle^\bullet$	1	±18
${}^2E_2''$	$\lvert 3\,\bar{2}\rangle$	$\lvert 8\,7\rangle$	2	±18	${}^2E_{13/2}$	$\lvert \tfrac{13}{2}\,\tfrac{13}{2}\rangle$	$\lvert \tfrac52\,\overline{\tfrac52}\rangle^\bullet$	1	±18
${}^1E_3''$	$\lvert 4\,3\rangle$	$\lvert 7\,\bar{6}\rangle$	2	±18	${}^1E_{15/2}$	$\lvert \tfrac{15}{2}\,\tfrac{15}{2}\rangle$	$\lvert \tfrac32\,\overline{\tfrac32}\rangle^\bullet$	1	±18
${}^2E_3''$	$\lvert 4\,\bar{3}\rangle$	$\lvert 7\,6\rangle$	2	±18	${}^2E_{15/2}$	$\lvert \tfrac{15}{2}\,\overline{\tfrac{15}{2}}\rangle$	$\lvert \tfrac32\,\tfrac32\rangle^\bullet$	1	±18
${}^1E_4''$	$\lvert 5\,4\rangle$	$\lvert 6\,\bar{5}\rangle$	2	±18	${}^1E_{17/2}$	$\lvert \tfrac{17}{2}\,\overline{\tfrac{17}{2}}\rangle$	$\lvert \tfrac12\,\tfrac12\rangle^\bullet$	1	±18
${}^2E_4''$	$\lvert 5\,\bar{4}\rangle$	$\lvert 6\,5\rangle$	2	±18	${}^2E_{17/2}$	$\lvert \tfrac{17}{2}\,\tfrac{17}{2}\rangle$	$\lvert \tfrac12\,\overline{\tfrac12}\rangle^\bullet$	1	±18

T **67**.7 Matrix representations

Use T **67**.4 ♠. § **16**–7, p. 77

T **67**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{C}_{9h}$	A'	${}^1E_1'$	${}^2E_1'$	${}^1E_2'$	${}^2E_2'$	${}^1E_3'$	${}^2E_3'$	${}^1E_4'$	${}^2E_4'$
A'	A'	${}^1E_1'$	${}^2E_1'$	${}^1E_2'$	${}^2E_2'$	${}^1E_3'$	${}^2E_3'$	${}^1E_4'$	${}^2E_4'$
${}^1E_1'$		${}^1E_2'$	A'	${}^1E_3'$	${}^2E_1'$	${}^1E_4'$	${}^2E_2'$	${}^2E_4'$	${}^2E_3'$
${}^2E_1'$			${}^2E_2'$	${}^1E_1'$	${}^2E_3'$	${}^1E_2'$	${}^2E_4'$	${}^1E_3'$	${}^1E_4'$
${}^1E_2'$				${}^1E_4'$	A'	${}^2E_4'$	${}^2E_1'$	${}^2E_3'$	${}^2E_2'$
${}^2E_2'$					${}^2E_4'$	${}^1E_1'$	${}^1E_4'$	${}^1E_2'$	${}^1E_3'$
${}^1E_3'$						${}^2E_3'$	A'	${}^2E_2'$	${}^2E_1'$
${}^2E_3'$							${}^1E_3'$	${}^1E_1'$	${}^1E_2'$
${}^1E_4'$								${}^2E_1'$	A'
${}^2E_4'$									${}^1E_1'$

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T **67**.8 Direct products of representations *(cont.)*

$\mathbf{C}_{9h}$	A''	${}^1E''_1$	${}^2E''_1$	${}^1E''_2$	${}^2E''_2$	${}^1E''_3$	${}^2E''_3$	${}^1E''_4$	${}^2E''_4$
A'	A''	${}^1E''_1$	${}^2E''_1$	${}^1E''_2$	${}^2E''_2$	${}^1E''_3$	${}^2E''_3$	${}^1E''_4$	${}^2E''_4$
${}^1E'_1$	${}^1E''_1$	${}^1E''_2$	A''	${}^1E''_3$	${}^2E''_1$	${}^1E''_4$	${}^2E''_2$	${}^2E''_4$	${}^2E''_3$
${}^2E'_1$	${}^2E''_1$	A''	${}^2E''_2$	${}^1E''_1$	${}^2E''_3$	${}^1E''_2$	${}^2E''_4$	${}^1E''_3$	${}^1E''_4$
${}^1E'_2$	${}^1E''_2$	${}^1E''_3$	${}^1E''_1$	${}^1E''_4$	A''	${}^2E''_4$	${}^2E''_1$	${}^2E''_3$	${}^2E''_2$
${}^2E'_2$	${}^2E''_2$	${}^2E''_1$	${}^2E''_3$	A''	${}^2E''_4$	${}^1E''_1$	${}^1E''_4$	${}^1E''_2$	${}^1E''_3$
${}^1E'_3$	${}^1E''_3$	${}^1E''_4$	${}^1E''_2$	${}^2E''_4$	${}^1E''_1$	${}^2E''_3$	A''	${}^2E''_2$	${}^2E''_1$
${}^2E'_3$	${}^2E''_3$	${}^2E''_2$	${}^2E''_4$	${}^2E''_1$	${}^1E''_4$	A''	${}^1E''_3$	${}^1E''_1$	${}^1E''_2$
${}^1E'_4$	${}^1E''_4$	${}^2E''_4$	${}^1E''_3$	${}^2E''_3$	${}^1E''_2$	${}^2E''_2$	${}^1E''_1$	${}^2E''_1$	A''
${}^2E'_4$	${}^2E''_4$	${}^2E''_3$	${}^1E''_4$	${}^2E''_2$	${}^1E''_3$	${}^2E''_1$	${}^1E''_2$	A''	${}^1E''_1$
A''	A'	${}^1E'_1$	${}^2E'_1$	${}^1E'_2$	${}^2E'_2$	${}^1E'_3$	${}^2E'_3$	${}^1E'_4$	${}^2E'_4$
${}^1E''_1$		${}^1E'_2$	A'	${}^1E'_3$	${}^2E'_1$	${}^1E'_4$	${}^2E'_2$	${}^2E'_4$	${}^2E'_3$
${}^2E''_1$			${}^2E'_2$	${}^1E'_1$	${}^2E'_3$	${}^1E'_2$	${}^2E'_4$	${}^1E'_3$	${}^1E'_4$
${}^1E''_2$				${}^1E'_4$	A'	${}^2E'_4$	${}^2E'_1$	${}^2E'_3$	${}^2E'_2$
${}^2E''_2$					${}^2E'_4$	${}^1E'_1$	${}^1E'_4$	${}^1E'_2$	${}^1E'_3$
${}^1E''_3$						${}^2E'_3$	A'	${}^2E'_2$	${}^2E'_1$
${}^2E''_3$							${}^1E'_3$	${}^1E'_1$	${}^1E'_2$
${}^1E''_4$								${}^2E'_1$	A'
${}^2E''_4$									${}^1E'_1$

$\twoheadrightarrow$

T **67**.8 Direct products of representations *(cont.)*

$\mathbf{C}_{9h}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$
A'	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{7/2}$
${}^1E'_1$	${}^1E_{17/2}$	${}^2E_{15/2}$	${}^1E_{13/2}$	${}^2E_{17/2}$	${}^1E_{15/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{13/2}$
${}^2E'_1$	${}^1E_{15/2}$	${}^2E_{17/2}$	${}^1E_{17/2}$	${}^2E_{13/2}$	${}^1E_{11/2}$	${}^2E_{15/2}$	${}^1E_{13/2}$	${}^2E_{9/2}$
${}^1E'_2$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{3/2}$
${}^2E'_2$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{11/2}$
${}^1E'_3$	${}^1E_{13/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{15/2}$	${}^1E_{17/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{17/2}$
${}^2E'_3$	${}^1E_{11/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$	${}^2E_{17/2}$	${}^1E_{17/2}$	${}^2E_{5/2}$
${}^1E'_4$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{1/2}$
${}^2E'_4$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{11/2}$	${}^1E_{13/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{15/2}$
A''	${}^2E_{17/2}$	${}^1E_{17/2}$	${}^2E_{15/2}$	${}^1E_{15/2}$	${}^2E_{13/2}$	${}^1E_{13/2}$	${}^2E_{11/2}$	${}^1E_{11/2}$
${}^1E''_1$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{5/2}$
${}^2E''_1$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{9/2}$
${}^1E''_2$	${}^2E_{15/2}$	${}^1E_{13/2}$	${}^2E_{11/2}$	${}^1E_{17/2}$	${}^2E_{17/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{15/2}$
${}^2E''_2$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{17/2}$	${}^1E_{11/2}$	${}^2E_{9/2}$	${}^1E_{17/2}$	${}^2E_{15/2}$	${}^1E_{7/2}$
${}^1E''_3$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{11/2}$	${}^2E_{13/2}$	${}^1E_{1/2}$
${}^2E''_3$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{9/2}$	${}^2E_{11/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{13/2}$
${}^1E''_4$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{13/2}$	${}^2E_{15/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{17/2}$
${}^2E''_4$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{13/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{15/2}$	${}^2E_{17/2}$	${}^1E_{3/2}$
${}^1E_{1/2}$	${}^2E''_1$	A'	${}^1E''_1$	${}^2E'_2$	${}^2E''_3$	${}^1E'_2$	${}^1E''_3$	${}^2E'_4$
${}^2E_{1/2}$		${}^1E''_1$	${}^1E'_2$	${}^2E''_1$	${}^2E'_2$	${}^1E''_3$	${}^1E'_4$	${}^2E''_3$
${}^1E_{3/2}$			${}^1E''_3$	A'	${}^2E''_1$	${}^1E'_4$	${}^2E''_4$	${}^2E'_2$
${}^2E_{3/2}$				${}^2E''_3$	${}^2E'_4$	${}^1E''_1$	${}^1E'_2$	${}^1E''_4$
${}^1E_{5/2}$					${}^1E''_4$	A'	${}^1E''_1$	${}^1E'_3$
${}^2E_{5/2}$						${}^2E''_4$	${}^2E'_3$	${}^2E''_1$
${}^1E_{7/2}$							${}^2E''_2$	A'
${}^2E_{7/2}$								${}^1E''_2$

$\twoheadrightarrow$

T **67**.8 Direct products of representations *(cont.)*

$\mathbf{C}_{9h}$	${}^1E_{9/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{11/2}$	${}^1E_{13/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{15/2}$	${}^1E_{17/2}$	${}^2E_{17/2}$
A'	${}^1E_{9/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{11/2}$	${}^1E_{13/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{15/2}$	${}^1E_{17/2}$	${}^2E_{17/2}$
${}^1E'_1$	${}^1E_{11/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^2E'_1$	${}^1E_{7/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$	${}^2E_{7/2}$	${}^1E_{5/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$
${}^1E'_2$	${}^1E_{5/2}$	${}^2E_{13/2}$	${}^1E_{15/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{17/2}$	${}^1E_{17/2}$	${}^2E_{11/2}$	${}^1E_{13/2}$	${}^2E_{15/2}$
${}^2E'_2$	${}^1E_{13/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{15/2}$	${}^1E_{17/2}$	${}^2E_{9/2}$	${}^1E_{11/2}$	${}^2E_{17/2}$	${}^1E_{15/2}$	${}^2E_{13/2}$
${}^1E'_3$	${}^1E_{15/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$	${}^2E_{13/2}$	${}^1E_{11/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$
${}^2E'_3$	${}^1E_{3/2}$	${}^2E_{15/2}$	${}^1E_{13/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^2E_{11/2}$	${}^1E_{9/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$
${}^1E'_4$	${}^1E_{1/2}$	${}^2E_{17/2}$	${}^1E_{17/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$	${}^2E_{15/2}$	${}^1E_{13/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$	${}^2E_{11/2}$
${}^2E'_4$	${}^1E_{17/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	${}^2E_{17/2}$	${}^1E_{15/2}$	${}^2E_{5/2}$	${}^1E_{7/2}$	${}^2E_{13/2}$	${}^1E_{11/2}$	${}^2E_{9/2}$
A''	${}^2E_{9/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^1E''_1$	${}^2E_{7/2}$	${}^1E_{11/2}$	${}^2E_{13/2}$	${}^1E_{9/2}$	${}^2E_{11/2}$	${}^1E_{15/2}$	${}^2E_{17/2}$	${}^1E_{13/2}$	${}^2E_{15/2}$	${}^1E_{17/2}$
${}^2E''_1$	${}^2E_{11/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{13/2}$	${}^2E_{15/2}$	${}^1E_{11/2}$	${}^2E_{13/2}$	${}^1E_{17/2}$	${}^2E_{17/2}$	${}^1E_{15/2}$
${}^1E''_2$	${}^2E_{13/2}$	${}^1E_{5/2}$	${}^2E_{3/2}$	${}^1E_{11/2}$	${}^2E_{9/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{7/2}$	${}^2E_{5/2}$	${}^1E_{3/2}$
${}^2E''_2$	${}^2E_{5/2}$	${}^1E_{13/2}$	${}^2E_{11/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$	${}^1E_{9/2}$	${}^2E_{7/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{5/2}$
${}^1E''_3$	${}^2E_{3/2}$	${}^1E_{15/2}$	${}^2E_{17/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{17/2}$	${}^2E_{15/2}$	${}^1E_{9/2}$	${}^2E_{11/2}$	${}^1E_{13/2}$
${}^2E''_3$	${}^2E_{15/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{17/2}$	${}^2E_{17/2}$	${}^1E_{7/2}$	${}^2E_{9/2}$	${}^1E_{15/2}$	${}^2E_{13/2}$	${}^1E_{11/2}$
${}^1E''_4$	${}^2E_{17/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^1E_{15/2}$	${}^2E_{13/2}$	${}^1E_{3/2}$	${}^2E_{5/2}$	${}^1E_{11/2}$	${}^2E_{9/2}$	${}^1E_{7/2}$
${}^2E''_4$	${}^2E_{1/2}$	${}^1E_{17/2}$	${}^2E_{15/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	${}^1E_{13/2}$	${}^2E_{11/2}$	${}^1E_{5/2}$	${}^2E_{7/2}$	${}^1E_{9/2}$
${}^1E_{1/2}$	${}^1E''_4$	${}^1E'_4$	${}^2E''_4$	${}^1E'_3$	${}^1E''_2$	${}^2E'_3$	${}^2E''_2$	${}^1E'_1$	A''	${}^2E'_1$
${}^2E_{1/2}$	${}^2E'_4$	${}^2E''_4$	${}^2E'_3$	${}^1E''_4$	${}^1E'_3$	${}^2E''_2$	${}^2E'_1$	${}^1E''_2$	${}^1E'_1$	A''
${}^1E_{3/2}$	${}^2E''_3$	${}^2E'_3$	${}^2E''_2$	${}^2E'_4$	${}^1E''_4$	${}^2E'_1$	A''	${}^1E'_3$	${}^1E''_2$	${}^1E'_1$
${}^2E_{3/2}$	${}^1E'_3$	${}^1E''_3$	${}^1E'_4$	${}^1E''_2$	${}^1E'_1$	${}^2E''_4$	${}^2E'_3$	A''	${}^2E'_1$	${}^2E''_2$
${}^1E_{5/2}$	${}^1E''_2$	${}^1E'_2$	${}^1E''_3$	${}^1E'_1$	A''	${}^1E'_4$	${}^2E''_4$	${}^2E'_1$	${}^2E''_2$	${}^2E'_3$
${}^2E_{5/2}$	${}^2E'_2$	${}^2E''_2$	${}^2E'_1$	${}^2E''_3$	${}^2E'_4$	A''	${}^1E'_1$	${}^1E''_4$	${}^1E'_3$	${}^1E''_2$
${}^1E_{7/2}$	${}^2E''_1$	${}^2E'_1$	A''	${}^2E'_2$	${}^2E''_3$	${}^1E'_1$	${}^1E''_2$	${}^2E'_4$	${}^1E''_4$	${}^1E'_3$
${}^2E_{7/2}$	${}^1E'_1$	${}^1E''_1$	${}^1E'_2$	A''	${}^2E'_1$	${}^1E''_3$	${}^1E'_4$	${}^2E''_2$	${}^2E'_3$	${}^2E''_4$
${}^1E_{9/2}$	A''	A'	${}^1E''_1$	${}^2E'_1$	${}^2E''_2$	${}^1E'_2$	${}^1E''_3$	${}^2E'_3$	${}^2E''_4$	${}^1E'_4$
${}^2E_{9/2}$		A''	${}^1E'_1$	${}^2E''_1$	${}^2E'_2$	${}^1E''_2$	${}^1E'_3$	${}^2E''_3$	${}^2E'_4$	${}^1E''_4$
${}^1E_{11/2}$			${}^1E''_2$	A'	${}^2E''_1$	${}^1E'_3$	${}^1E''_4$	${}^2E'_2$	${}^2E''_3$	${}^2E'_4$
${}^2E_{11/2}$				${}^2E''_2$	${}^2E'_3$	${}^1E''_1$	${}^1E'_2$	${}^2E''_4$	${}^1E'_4$	${}^1E''_3$
${}^1E_{13/2}$					${}^2E''_4$	A'	${}^1E''_1$	${}^1E'_4$	${}^1E''_3$	${}^1E'_2$
${}^2E_{13/2}$						${}^1E''_4$	${}^2E'_4$	${}^2E''_1$	${}^2E'_2$	${}^2E''_3$
${}^1E_{15/2}$							${}^2E''_3$	A'	${}^2E''_1$	${}^2E'_2$
${}^2E_{15/2}$								${}^1E''_3$	${}^1E'_2$	${}^1E''_1$
${}^1E_{17/2}$									${}^1E''_1$	A'
${}^2E_{17/2}$										${}^2E''_1$

T **67**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{C}_{9h}$	$\mathbf{C}_{3h}$	$\mathbf{C}_s$	$\mathbf{C}_9$	$\mathbf{C}_3$	$\mathbf{C}_{9h}$	$\mathbf{C}_{3h}$	$\mathbf{C}_s$	$\mathbf{C}_9$	$\mathbf{C}_3$
A'	A'	A'	A	A	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$
${}^1E_1'$	${}^1E'$	A'	${}^1E_1$	${}^1E$	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$
${}^2E_1'$	${}^2E'$	A'	${}^2E_1$	${}^2E$	${}^1E_{3/2}$	${}^1E_{3/2}$	${}^1E_{1/2}$	${}^1E_{3/2}$	$A_{3/2}$
${}^1E_2'$	${}^2E'$	A'	${}^1E_2$	${}^2E$	${}^2E_{3/2}$	${}^2E_{3/2}$	${}^2E_{1/2}$	${}^2E_{3/2}$	$A_{3/2}$
${}^2E_2'$	${}^1E'$	A'	${}^2E_2$	${}^1E$	${}^1E_{5/2}$	${}^1E_{5/2}$	${}^1E_{1/2}$	${}^1E_{5/2}$	${}^2E_{1/2}$
${}^1E_3'$	A'	A'	${}^1E_3$	A	${}^2E_{5/2}$	${}^2E_{5/2}$	${}^2E_{1/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$
${}^2E_3'$	A'	A'	${}^2E_3$	A	${}^1E_{7/2}$	${}^1E_{5/2}$	${}^1E_{1/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$
${}^1E_4'$	${}^1E'$	A'	${}^1E_4$	${}^1E$	${}^2E_{7/2}$	${}^2E_{5/2}$	${}^2E_{1/2}$	${}^2E_{7/2}$	${}^1E_{1/2}$
${}^2E_4'$	${}^2E'$	A'	${}^2E_4$	${}^2E$	${}^1E_{9/2}$	${}^1E_{3/2}$	${}^1E_{1/2}$	$A_{9/2}$	$A_{3/2}$
A''	A''	A''	A	A	${}^2E_{9/2}$	${}^2E_{3/2}$	${}^2E_{1/2}$	$A_{9/2}$	$A_{3/2}$
${}^1E_1''$	${}^1E''$	A''	${}^1E_1$	${}^1E$	${}^1E_{11/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^2E_{7/2}$	${}^1E_{1/2}$
${}^2E_1''$	${}^2E''$	A''	${}^2E_1$	${}^2E$	${}^2E_{11/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^1E_{7/2}$	${}^2E_{1/2}$
${}^1E_2''$	${}^2E''$	A''	${}^1E_2$	${}^2E$	${}^1E_{13/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^2E_{5/2}$	${}^1E_{1/2}$
${}^2E_2''$	${}^1E''$	A''	${}^2E_2$	${}^1E$	${}^2E_{13/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^1E_{5/2}$	${}^2E_{1/2}$
${}^1E_3''$	A''	A''	${}^1E_3$	A	${}^1E_{15/2}$	${}^1E_{3/2}$	${}^1E_{1/2}$	${}^2E_{3/2}$	$A_{3/2}$
${}^2E_3''$	A''	A''	${}^2E_3$	A	${}^2E_{15/2}$	${}^2E_{3/2}$	${}^2E_{1/2}$	${}^1E_{3/2}$	$A_{3/2}$
${}^1E_4''$	${}^1E''$	A''	${}^1E_4$	${}^1E$	${}^1E_{17/2}$	${}^1E_{5/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$
${}^2E_4''$	${}^2E''$	A''	${}^2E_4$	${}^2E$	${}^2E_{17/2}$	${}^2E_{5/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$

T **67**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{9h}$
$9n$	$(n+1)\,A' \oplus n\,({}^1E_1' \oplus {}^2E_1' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_3' \oplus {}^2E_3' \oplus {}^1E_4' \oplus {}^2E_4' \oplus A'' \oplus {}^1E_1'' \oplus {}^2E_1'' \oplus {}^1E_2'' \oplus {}^2E_2'' \oplus {}^1E_3'' \oplus {}^2E_3'' \oplus {}^1E_4'' \oplus {}^2E_4'')$
$9n+1$	$n\,(A' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_3' \oplus {}^2E_3' \oplus {}^1E_4' \oplus {}^2E_4' \oplus {}^1E_1'' \oplus {}^2E_1'' \oplus {}^1E_2'' \oplus {}^2E_2'' \oplus {}^1E_3'' \oplus {}^2E_3'' \oplus {}^1E_4'' \oplus {}^2E_4'') \oplus (n+1)({}^1E_1' \oplus {}^2E_1' \oplus A'')$
$9n+2$	$(n+1)(A' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_1'' \oplus {}^2E_1'') \oplus n\,({}^1E_1' \oplus {}^2E_1' \oplus {}^1E_3' \oplus {}^2E_3' \oplus {}^1E_4' \oplus {}^2E_4' \oplus A'' \oplus {}^1E_2'' \oplus {}^2E_2'' \oplus {}^1E_3'' \oplus {}^2E_3'' \oplus {}^1E_4'' \oplus {}^2E_4'')$
$9n+3$	$n\,(A' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_4' \oplus {}^2E_4' \oplus {}^1E_1'' \oplus {}^2E_1'' \oplus {}^1E_3'' \oplus {}^2E_3'' \oplus {}^1E_4'' \oplus {}^2E_4'') \oplus (n+1)({}^1E_1' \oplus {}^2E_1' \oplus {}^1E_3' \oplus {}^2E_3' \oplus A'' \oplus {}^1E_2'' \oplus {}^2E_2'')$
$9n+4$	$(n+1)(A' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_4' \oplus {}^2E_4' \oplus {}^1E_1'' \oplus {}^2E_1'' \oplus {}^1E_3'' \oplus {}^2E_3'') \oplus n\,({}^1E_1' \oplus {}^2E_1' \oplus {}^1E_3' \oplus {}^2E_3' \oplus A'' \oplus {}^1E_2'' \oplus {}^2E_2'' \oplus {}^1E_4'' \oplus {}^2E_4'')$
$9n+5$	$n\,(A' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_1'' \oplus {}^2E_1'' \oplus {}^1E_3'' \oplus {}^2E_3'') \oplus (n+1)({}^1E_1' \oplus {}^2E_1' \oplus {}^1E_3' \oplus {}^2E_3' \oplus {}^1E_4' \oplus {}^2E_4' \oplus A'' \oplus {}^1E_2'' \oplus {}^2E_2'' \oplus {}^1E_4'' \oplus {}^2E_4'')$
$9n+6$	$(n+1)(A' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_3' \oplus {}^2E_3' \oplus {}^1E_4' \oplus {}^2E_4' \oplus {}^1E_1'' \oplus {}^2E_1'' \oplus {}^1E_3'' \oplus {}^2E_3'' \oplus {}^1E_4'' \oplus {}^2E_4'') \oplus n\,({}^1E_1' \oplus {}^2E_1' \oplus A'' \oplus {}^1E_2'' \oplus {}^2E_2'')$
$9n+7$	$n\,(A' \oplus {}^1E_1'' \oplus {}^2E_1'') \oplus (n+1)({}^1E_1' \oplus {}^2E_1' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_3' \oplus {}^2E_3' \oplus {}^1E_4' \oplus {}^2E_4' \oplus A'' \oplus {}^1E_2'' \oplus {}^2E_2'' \oplus {}^1E_3'' \oplus {}^2E_3'' \oplus {}^1E_4'' \oplus {}^2E_4'')$
$9n+8$	$(n+1)(A' \oplus {}^1E_1' \oplus {}^2E_1' \oplus {}^1E_2' \oplus {}^2E_2' \oplus {}^1E_3' \oplus {}^2E_3' \oplus {}^1E_4' \oplus {}^2E_4' \oplus {}^1E_1'' \oplus {}^2E_1'' \oplus {}^1E_2'' \oplus {}^2E_2'' \oplus {}^1E_3'' \oplus {}^2E_3'' \oplus {}^1E_4'' \oplus {}^2E_4'') \oplus n\,A''$

$n = 0, 1, 2, \ldots$ ↠

T **67**.10 Subduction from O(3) *(cont.)*

j	$\mathbf{C}_{9h}$
$18n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2})\oplus 2n\,({}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus$ ${}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2})\oplus 2n\,({}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus$ ${}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2})\oplus 2n\,({}^1E_{7/2}\oplus{}^2E_{7/2}\oplus$ ${}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2})\oplus$ $2n\,({}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus$ ${}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2})\oplus$ $2n\,({}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{11}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus$ ${}^1E_{11/2}\oplus{}^2E_{11/2})\oplus 2n\,({}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{13}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus$ ${}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2})\oplus 2n\,({}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{15}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus$ ${}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2})\oplus 2n\,({}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{17}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus$ ${}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{19}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus$ ${}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2})\oplus(2n+2)({}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{21}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus$ ${}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2})\oplus(2n+2)({}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{23}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus$ ${}^1E_{11/2}\oplus{}^2E_{11/2})\oplus(2n+2)({}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{25}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2})\oplus$ $(2n+2)({}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{27}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2})\oplus$ $(2n+2)({}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus$ ${}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{29}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2})\oplus(2n+2)({}^1E_{7/2}\oplus{}^2E_{7/2}\oplus$ ${}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{31}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2})\oplus(2n+2)({}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus$ ${}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{33}{2}$	$(2n+1)({}^1E_{1/2}\oplus{}^2E_{1/2})\oplus(2n+2)({}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus$ ${}^1E_{9/2}\oplus{}^2E_{9/2}\oplus{}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2})$
$18n+\frac{35}{2}$	$(2n+2)({}^1E_{1/2}\oplus{}^2E_{1/2}\oplus{}^1E_{3/2}\oplus{}^2E_{3/2}\oplus{}^1E_{5/2}\oplus{}^2E_{5/2}\oplus{}^1E_{7/2}\oplus{}^2E_{7/2}\oplus{}^1E_{9/2}\oplus{}^2E_{9/2}\oplus$ ${}^1E_{11/2}\oplus{}^2E_{11/2}\oplus{}^1E_{13/2}\oplus{}^2E_{13/2}\oplus{}^1E_{15/2}\oplus{}^2E_{15/2}\oplus{}^1E_{17/2}\oplus{}^2E_{17/2})$

$n=0,1,2,\ldots$

T **67**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

(1) Product forms: $\mathbf{C}_{10} \otimes \mathbf{C}_i$, $\mathbf{C}_{10} \otimes \mathbf{C}_s$.

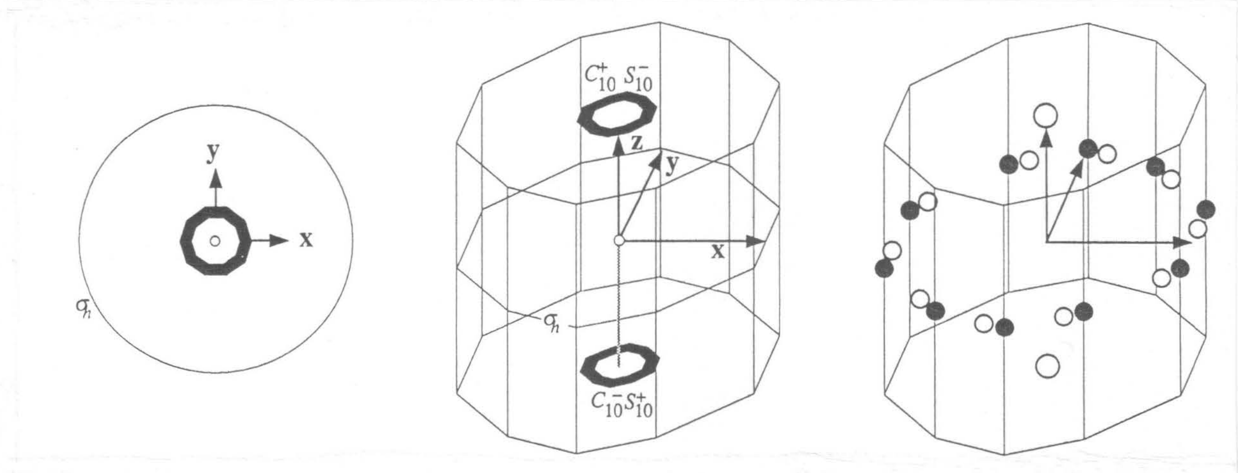
(2) Group chains: $\mathbf{D}_{10h} \supset \underline{\mathbf{C}_{10h}} \supset \underline{\mathbf{C}_{5h}}$, $\mathbf{D}_{10h} \supset \underline{\mathbf{C}_{10h}} \supset \underline{\mathbf{S}_{10}}$, $\mathbf{D}_{10h} \supset \underline{\mathbf{C}_{10h}} \supset \underline{\mathbf{C}_{10}}$.

(3) Operations of G: E, C_{10}^{+}, C_5^{+}, C_{10}^{3+}, C_5^{2+}, C_2, C_5^{2-}, C_{10}^{3-}, C_5^{-}, C_{10}^{-},
i, S_5^{2-}, S_{10}^{3-}, S_5^{-}, S_{10}^{-}, σ_h, S_{10}^{+}, S_5^{+}, S_{10}^{3+}, S_5^{2+}.

(4) Operations of $\widetilde{G}$: E, C_{10}^{+}, C_5^{+}, C_{10}^{3+}, C_5^{2+}, C_2, C_5^{2-}, C_{10}^{3-}, C_5^{-}, C_{10}^{-},
i, S_5^{2-}, S_{10}^{3-}, S_5^{-}, S_{10}^{-}, σ_h, S_{10}^{+}, S_5^{+}, S_{10}^{3+}, S_5^{2+},
$\widetilde{E}$, $\widetilde{C}_{10}^{+}$, $\widetilde{C}_5^{+}$, $\widetilde{C}_{10}^{3+}$, $\widetilde{C}_5^{2+}$, $\widetilde{C}_2$, $\widetilde{C}_5^{2-}$, $\widetilde{C}_{10}^{3-}$, $\widetilde{C}_5^{-}$, $\widetilde{C}_{10}^{-}$,
$\widetilde{\imath}$, $\widetilde{S}_5^{2-}$, $\widetilde{S}_{10}^{3-}$, $\widetilde{S}_5^{-}$, $\widetilde{S}_{10}^{-}$, $\widetilde{\sigma}_h$, $\widetilde{S}_{10}^{+}$, $\widetilde{S}_5^{+}$, $\widetilde{S}_{10}^{3+}$, $\widetilde{S}_5^{2+}$.

(5) Classes and representations: $|r| = 20$, $|\mathrm{i}| = 0$, $|I| = 20$, $|\widetilde{I}| = 20$.

F **68** See Chapter **15**, p. 65



Examples:

T **68**.1 Parameters
Use T **39**.1. § **16**–1, p. 68

T **68**.2 Multiplication table
Use T **39**.2. § **16**–2, p. 69

T **68**.3 Factor table
Use T **39**.3. § **16**–3, p. 70

T **68**.4 Character table

§ **16**–4, p. 71

$\mathbf{C}_{10h}$	E	C_{10}^{+}	C_5^{+}	C_{10}^{3+}	C_5^{2+}	C_2	C_5^{2-}	C_{10}^{3-}	C_5^{-}	C_{10}^{-}	τ
A_g	1	1	1	1	1	1	1	1	1	1	a
B_g	1	-1	1	-1	1	-1	1	-1	1	-1	a
${}^1E_{1g}$	1	$-\epsilon$	δ^*	$-\delta$	ϵ^*	-1	ϵ	$-\delta^*$	δ	$-\epsilon^*$	b
${}^2E_{1g}$	1	$-\epsilon^*$	δ	$-\delta^*$	ϵ	-1	ϵ^*	$-\delta$	δ^*	$-\epsilon$	b
${}^1E_{2g}$	1	δ^*	ϵ^*	ϵ	δ	1	δ^*	ϵ^*	ϵ	δ	b
${}^2E_{2g}$	1	δ	ϵ	ϵ^*	δ^*	1	δ	ϵ	ϵ^*	δ^*	b
${}^1E_{3g}$	1	$-\delta^*$	ϵ^*	$-\epsilon$	δ	-1	δ^*	$-\epsilon^*$	ϵ	$-\delta$	b
${}^2E_{3g}$	1	$-\delta$	ϵ	$-\epsilon^*$	δ^*	-1	δ	$-\epsilon$	ϵ^*	$-\delta^*$	b
${}^1E_{4g}$	1	ϵ	δ^*	δ	ϵ^*	1	ϵ	δ^*	δ	ϵ^*	b
${}^2E_{4g}$	1	ϵ^*	δ	δ^*	ϵ	1	ϵ^*	δ	δ^*	ϵ	b
A_u	1	1	1	1	1	1	1	1	1	1	a
B_u	1	-1	1	-1	1	-1	1	-1	1	-1	a
${}^1E_{1u}$	1	$-\epsilon$	δ^*	$-\delta$	ϵ^*	-1	ϵ	$-\delta^*$	δ	$-\epsilon^*$	b
${}^2E_{1u}$	1	$-\epsilon^*$	δ	$-\delta^*$	ϵ	-1	ϵ^*	$-\delta$	δ^*	$-\epsilon$	b
${}^1E_{2u}$	1	δ^*	ϵ^*	ϵ	δ	1	δ^*	ϵ^*	ϵ	δ	b
${}^2E_{2u}$	1	δ	ϵ	ϵ^*	δ^*	1	δ	ϵ	ϵ^*	δ^*	b
${}^1E_{3u}$	1	$-\delta^*$	ϵ^*	$-\epsilon$	δ	-1	δ^*	$-\epsilon^*$	ϵ	$-\delta$	b
${}^2E_{3u}$	1	$-\delta$	ϵ	$-\epsilon^*$	δ^*	-1	δ	$-\epsilon$	ϵ^*	$-\delta^*$	b
${}^1E_{4u}$	1	ϵ	δ^*	δ	ϵ^*	1	ϵ	δ^*	δ	ϵ^*	b
${}^2E_{4u}$	1	ϵ^*	δ	δ^*	ϵ	1	ϵ^*	δ	δ^*	ϵ	b
${}^1E_{1/2,g}$	1	$\mathrm{i}\delta^*$	$-\epsilon^*$	$-\mathrm{i}\epsilon$	δ	i	δ^*	$\mathrm{i}\epsilon^*$	$-\epsilon$	$-\mathrm{i}\delta$	b
${}^2E_{1/2,g}$	1	$-\mathrm{i}\delta$	$-\epsilon$	$\mathrm{i}\epsilon^*$	δ^*	$-$i	δ	$-\mathrm{i}\epsilon$	$-\epsilon^*$	$\mathrm{i}\delta^*$	b
${}^1E_{3/2,g}$	1	$\mathrm{i}\epsilon^*$	$-\delta$	$-\mathrm{i}\delta^*$	ϵ	i	ϵ^*	$\mathrm{i}\delta$	$-\delta^*$	$-\mathrm{i}\epsilon$	b
${}^2E_{3/2,g}$	1	$-\mathrm{i}\epsilon$	$-\delta^*$	$\mathrm{i}\delta$	ϵ^*	$-$i	ϵ	$-\mathrm{i}\delta^*$	$-\delta$	$\mathrm{i}\epsilon^*$	b
${}^1E_{5/2,g}$	1	i	-1	$-$i	1	i	1	i	-1	$-$i	b
${}^2E_{5/2,g}$	1	$-$i	-1	i	1	$-$i	1	$-$i	-1	i	b
${}^1E_{7/2,g}$	1	$\mathrm{i}\epsilon$	$-\delta^*$	$-\mathrm{i}\delta$	ϵ^*	i	ϵ	$\mathrm{i}\delta^*$	$-\delta$	$-\mathrm{i}\epsilon^*$	b
${}^2E_{7/2,g}$	1	$-\mathrm{i}\epsilon^*$	$-\delta$	$\mathrm{i}\delta^*$	ϵ	$-$i	ϵ^*	$-\mathrm{i}\delta$	$-\delta^*$	$\mathrm{i}\epsilon$	b
${}^1E_{9/2,g}$	1	$\mathrm{i}\delta$	$-\epsilon$	$-\mathrm{i}\epsilon^*$	δ^*	i	δ	$\mathrm{i}\epsilon$	$-\epsilon^*$	$-\mathrm{i}\delta^*$	b
${}^2E_{9/2,g}$	1	$-\mathrm{i}\delta^*$	$-\epsilon^*$	$\mathrm{i}\epsilon$	δ	$-$i	δ^*	$-\mathrm{i}\epsilon^*$	$-\epsilon$	$\mathrm{i}\delta$	b
${}^1E_{1/2,u}$	1	$\mathrm{i}\delta^*$	$-\epsilon^*$	$-\mathrm{i}\epsilon$	δ	i	δ^*	$\mathrm{i}\epsilon^*$	$-\epsilon$	$-\mathrm{i}\delta$	b
${}^2E_{1/2,u}$	1	$-\mathrm{i}\delta$	$-\epsilon$	$\mathrm{i}\epsilon^*$	δ^*	$-$i	δ	$-\mathrm{i}\epsilon$	$-\epsilon^*$	$\mathrm{i}\delta^*$	b
${}^1E_{3/2,u}$	1	$\mathrm{i}\epsilon^*$	$-\delta$	$-\mathrm{i}\delta^*$	ϵ	i	ϵ^*	$\mathrm{i}\delta$	$-\delta^*$	$-\mathrm{i}\epsilon$	b
${}^2E_{3/2,u}$	1	$-\mathrm{i}\epsilon$	$-\delta^*$	$\mathrm{i}\delta$	ϵ^*	$-$i	ϵ	$-\mathrm{i}\delta^*$	$-\delta$	$\mathrm{i}\epsilon^*$	b
${}^1E_{5/2,u}$	1	i	-1	$-$i	1	i	1	i	-1	$-$i	b
${}^2E_{5/2,u}$	1	$-$i	-1	i	1	$-$i	1	$-$i	-1	i	b
${}^1E_{7/2,u}$	1	$\mathrm{i}\epsilon$	$-\delta^*$	$-\mathrm{i}\delta$	ϵ^*	i	ϵ	$\mathrm{i}\delta^*$	$-\delta$	$-\mathrm{i}\epsilon^*$	b
${}^2E_{7/2,u}$	1	$-\mathrm{i}\epsilon^*$	$-\delta$	$\mathrm{i}\delta^*$	ϵ	$-$i	ϵ^*	$-\mathrm{i}\delta$	$-\delta^*$	$\mathrm{i}\epsilon$	b
${}^1E_{9/2,u}$	1	$\mathrm{i}\delta$	$-\epsilon$	$-\mathrm{i}\epsilon^*$	δ^*	i	δ	$\mathrm{i}\epsilon$	$-\epsilon^*$	$-\mathrm{i}\delta^*$	b
${}^2E_{9/2,u}$	1	$-\mathrm{i}\delta^*$	$-\epsilon^*$	$\mathrm{i}\epsilon$	δ	$-$i	δ^*	$-\mathrm{i}\epsilon^*$	$-\epsilon$	$\mathrm{i}\delta$	b

$\delta = \exp(2\pi\mathrm{i}/5)$, $\epsilon = \exp(4\pi\mathrm{i}/5)$ $\twoheadrightarrow$

T **68**.4 Character table *(cont.)*

$\mathbf{C}_{10h}$	i	S_5^{2-}	S_{10}^{3-}	S_5^-	S_{10}^-	σ_h	S_{10}^+	S_5^+	S_{10}^{3+}	S_5^{2+}	τ
A_g	1	1	1	1	1	1	1	1	1	1	a
B_g	1	-1	1	-1	1	-1	1	-1	1	-1	a
${}^1E_{1g}$	1	$-\epsilon$	δ^*	$-\delta$	ϵ^*	-1	ϵ	$-\delta^*$	δ	$-\epsilon^*$	b
${}^2E_{1g}$	1	$-\epsilon^*$	δ	$-\delta^*$	ϵ	-1	ϵ^*	$-\delta$	δ^*	$-\epsilon$	b
${}^1E_{2g}$	1	δ^*	ϵ^*	ϵ	δ	1	δ^*	ϵ^*	ϵ	δ	b
${}^2E_{2g}$	1	δ	ϵ	ϵ^*	δ^*	1	δ	ϵ	ϵ^*	δ^*	b
${}^1E_{3g}$	1	$-\delta^*$	ϵ^*	$-\epsilon$	δ	-1	δ^*	$-\epsilon^*$	ϵ	$-\delta$	b
${}^2E_{3g}$	1	$-\delta$	ϵ	$-\epsilon^*$	δ^*	-1	δ	$-\epsilon$	ϵ^*	$-\delta^*$	b
${}^1E_{4g}$	1	ϵ	δ^*	δ	ϵ^*	1	ϵ	δ^*	δ	ϵ^*	b
${}^2E_{4g}$	1	ϵ^*	δ	δ^*	ϵ	1	ϵ^*	δ	δ^*	ϵ	b
A_u	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	a
B_u	-1	1	-1	1	-1	1	-1	1	-1	1	a
${}^1E_{1u}$	-1	ϵ	$-\delta^*$	δ	$-\epsilon^*$	1	$-\epsilon$	δ^*	$-\delta$	ϵ^*	b
${}^2E_{1u}$	-1	ϵ^*	$-\delta$	δ^*	$-\epsilon$	1	$-\epsilon^*$	δ	$-\delta^*$	ϵ	b
${}^1E_{2u}$	-1	$-\delta^*$	$-\epsilon^*$	$-\epsilon$	$-\delta$	-1	$-\delta^*$	$-\epsilon^*$	$-\epsilon$	$-\delta$	b
${}^2E_{2u}$	-1	$-\delta$	$-\epsilon$	$-\epsilon^*$	$-\delta^*$	-1	$-\delta$	$-\epsilon$	$-\epsilon^*$	$-\delta^*$	b
${}^1E_{3u}$	-1	δ^*	$-\epsilon^*$	ϵ	$-\delta$	1	$-\delta^*$	ϵ^*	$-\epsilon$	δ	b
${}^2E_{3u}$	-1	δ	$-\epsilon$	ϵ^*	$-\delta^*$	1	$-\delta$	ϵ	$-\epsilon^*$	δ^*	b
${}^1E_{4u}$	-1	$-\epsilon$	$-\delta^*$	$-\delta$	$-\epsilon^*$	-1	$-\epsilon$	$-\delta^*$	$-\delta$	$-\epsilon^*$	b
${}^2E_{4u}$	-1	$-\epsilon^*$	$-\delta$	$-\delta^*$	$-\epsilon$	-1	$-\epsilon^*$	$-\delta$	$-\delta^*$	$-\epsilon$	b
${}^1E_{1/2,g}$	1	$\mathrm{i}\delta^*$	$-\epsilon^*$	$-\mathrm{i}\epsilon$	δ	i	δ^*	$\mathrm{i}\epsilon^*$	$-\epsilon$	$-\mathrm{i}\delta$	b
${}^2E_{1/2,g}$	1	$-\mathrm{i}\delta$	$-\epsilon$	$\mathrm{i}\epsilon^*$	δ^*	$-\mathrm{i}$	δ	$-\mathrm{i}\epsilon$	$-\epsilon^*$	$\mathrm{i}\delta^*$	b
${}^1E_{3/2,g}$	1	$\mathrm{i}\epsilon^*$	$-\delta$	$-\mathrm{i}\delta^*$	ϵ	i	ϵ^*	$\mathrm{i}\delta$	$-\delta^*$	$-\mathrm{i}\epsilon$	b
${}^2E_{3/2,g}$	1	$-\mathrm{i}\epsilon$	$-\delta^*$	$\mathrm{i}\delta$	ϵ^*	$-\mathrm{i}$	ϵ	$-\mathrm{i}\delta^*$	$-\delta$	$\mathrm{i}\epsilon^*$	b
${}^1E_{5/2,g}$	1	i	-1	$-\mathrm{i}$	1	i	1	i	-1	$-\mathrm{i}$	b
${}^2E_{5/2,g}$	1	$-\mathrm{i}$	-1	i	1	$-\mathrm{i}$	1	$-\mathrm{i}$	-1	i	b
${}^1E_{7/2,g}$	1	$\mathrm{i}\epsilon$	$-\delta^*$	$-\mathrm{i}\delta$	ϵ^*	i	ϵ	$\mathrm{i}\delta^*$	$-\delta$	$-\mathrm{i}\epsilon^*$	b
${}^2E_{7/2,g}$	1	$-\mathrm{i}\epsilon^*$	$-\delta$	$\mathrm{i}\delta^*$	ϵ	$-\mathrm{i}$	ϵ^*	$-\mathrm{i}\delta$	$-\delta^*$	$\mathrm{i}\epsilon$	b
${}^1E_{9/2,g}$	1	$\mathrm{i}\delta$	$-\epsilon$	$-\mathrm{i}\epsilon^*$	δ^*	i	δ	$\mathrm{i}\epsilon$	$-\epsilon^*$	$-\mathrm{i}\delta^*$	b
${}^2E_{9/2,g}$	1	$-\mathrm{i}\delta^*$	$-\epsilon^*$	$\mathrm{i}\epsilon$	δ	$-\mathrm{i}$	δ^*	$-\mathrm{i}\epsilon^*$	$-\epsilon$	$\mathrm{i}\delta$	b
${}^1E_{1/2,u}$	-1	$-\mathrm{i}\delta^*$	ϵ^*	$\mathrm{i}\epsilon$	$-\delta$	$-\mathrm{i}$	$-\delta^*$	$-\mathrm{i}\epsilon^*$	ϵ	$\mathrm{i}\delta$	b
${}^2E_{1/2,u}$	-1	$\mathrm{i}\delta$	ϵ	$-\mathrm{i}\epsilon^*$	$-\delta^*$	i	$-\delta$	$\mathrm{i}\epsilon$	ϵ^*	$-\mathrm{i}\delta^*$	b
${}^1E_{3/2,u}$	-1	$-\mathrm{i}\epsilon^*$	δ	$\mathrm{i}\delta^*$	$-\epsilon$	$-\mathrm{i}$	$-\epsilon^*$	$-\mathrm{i}\delta$	δ^*	$\mathrm{i}\epsilon$	b
${}^2E_{3/2,u}$	-1	$\mathrm{i}\epsilon$	δ^*	$-\mathrm{i}\delta$	$-\epsilon^*$	i	$-\epsilon$	$\mathrm{i}\delta^*$	δ	$-\mathrm{i}\epsilon^*$	b
${}^1E_{5/2,u}$	-1	$-\mathrm{i}$	1	i	-1	$-\mathrm{i}$	-1	$-\mathrm{i}$	1	i	b
${}^2E_{5/2,u}$	-1	i	1	$-\mathrm{i}$	-1	i	-1	i	1	$-\mathrm{i}$	b
${}^1E_{7/2,u}$	-1	$-\mathrm{i}\epsilon$	δ^*	$\mathrm{i}\delta$	$-\epsilon^*$	$-\mathrm{i}$	$-\epsilon$	$-\mathrm{i}\delta^*$	δ	$\mathrm{i}\epsilon^*$	b
${}^2E_{7/2,u}$	-1	$\mathrm{i}\epsilon^*$	δ	$-\mathrm{i}\delta^*$	$-\epsilon$	i	$-\epsilon^*$	$\mathrm{i}\delta$	δ^*	$-\mathrm{i}\epsilon$	b
${}^1E_{9/2,u}$	-1	$-\mathrm{i}\delta$	ϵ	$\mathrm{i}\epsilon^*$	$-\delta^*$	$-\mathrm{i}$	$-\delta$	$-\mathrm{i}\epsilon$	ϵ^*	$\mathrm{i}\delta^*$	b
${}^2E_{9/2,u}$	-1	$\mathrm{i}\delta^*$	ϵ^*	$-\mathrm{i}\epsilon$	$-\delta$	i	$-\delta^*$	$\mathrm{i}\epsilon^*$	ϵ	$-\mathrm{i}\delta$	b

$\delta = \exp(2\pi\mathrm{i}/5)$, $\epsilon = \exp(4\pi\mathrm{i}/5)$

T **68**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions § **16**–5, p. 72

$\mathbf{C}_{10h}$	0	1	2	3
A_g	$^{\square}1$	R_z	$x^2+y^2, {}^{\square}z^2$	
B_g				
${}^1E_{1g} \oplus {}^2E_{1g}$		(R_x, R_y)	$^{\square}(zx, yz)$	
${}^1E_{2g} \oplus {}^2E_{2g}$			$^{\square}(xy, x^2-y^2)$	
${}^1E_{3g} \oplus {}^2E_{3g}$				
${}^1E_{4g} \oplus {}^2E_{4g}$				
A_u		$^{\square}z$		$(x^2+y^2)z, {}^{\square}z^3$
B_u				
${}^1E_{1u} \oplus {}^2E_{1u}$		$^{\square}(x, y)$		$\{x(x^2+y^2), y(x^2+y^2)\}, {}^{\square}(xz^2, yz^2)$
${}^1E_{2u} \oplus {}^2E_{2u}$				$^{\square}\{xyz, (x^2-y^2)z\}$
${}^1E_{3u} \oplus {}^2E_{3u}$				$^{\square}\{x(x^2-3y^2), y(3x^2-y^2)\}$
${}^1E_{4u} \oplus {}^2E_{4u}$				

T **68**.6 Symmetrized bases § **16**–6, p. 74

$\mathbf{C}_{10h}$	$\lvert j\,m\rangle$	ι	μ	$\mathbf{C}_{10h}$	$\lvert j\,m\rangle$	ι	μ
A_g	$\lvert 0\,0\rangle$	2	± 10	${}^1E_{1/2,g}$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle$	1	± 10
B_g	$\lvert 6\,5\rangle$	2	± 10	${}^2E_{1/2,g}$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle$	1	± 10
${}^1E_{1g}$	$\lvert 2\,1\rangle$	2	± 10	${}^1E_{3/2,g}$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle$	1	± 10
${}^2E_{1g}$	$\lvert 2\,\overline{1}\rangle$	2	± 10	${}^2E_{3/2,g}$	$\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle$	1	± 10
${}^1E_{2g}$	$\lvert 2\,2\rangle$	2	± 10	${}^1E_{5/2,g}$	$\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle$	1	± 10
${}^2E_{2g}$	$\lvert 2\,\overline{2}\rangle$	2	± 10	${}^2E_{5/2,g}$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle$	1	± 10
${}^1E_{3g}$	$\lvert 4\,\overline{3}\rangle$	2	± 10	${}^1E_{7/2,g}$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle$	1	± 10
${}^2E_{3g}$	$\lvert 4\,3\rangle$	2	± 10	${}^2E_{7/2,g}$	$\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle$	1	± 10
${}^1E_{4g}$	$\lvert 4\,\overline{4}\rangle$	2	± 10	${}^1E_{9/2,g}$	$\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle$	1	± 10
${}^2E_{4g}$	$\lvert 4\,4\rangle$	2	± 10	${}^2E_{9/2,g}$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle$	1	± 10
A_u	$\lvert 1\,0\rangle$	2	± 10	${}^1E_{1/2,u}$	$\lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle^{\bullet}$	1	± 10
B_u	$\lvert 5\,5\rangle$	2	± 10	${}^2E_{1/2,u}$	$\lvert \frac{1}{2}\,\frac{1}{2}\rangle^{\bullet}$	1	± 10
${}^1E_{1u}$	$\lvert 1\,1\rangle$	2	± 10	${}^1E_{3/2,u}$	$\lvert \frac{3}{2}\,\frac{3}{2}\rangle^{\bullet}$	1	± 10
${}^2E_{1u}$	$\lvert 1\,\overline{1}\rangle$	2	± 10	${}^2E_{3/2,u}$	$\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle^{\bullet}$	1	± 10
${}^1E_{2u}$	$\lvert 3\,2\rangle$	2	± 10	${}^1E_{5/2,u}$	$\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle^{\bullet}$	1	± 10
${}^2E_{2u}$	$\lvert 3\,\overline{2}\rangle$	2	± 10	${}^2E_{5/2,u}$	$\lvert \frac{5}{2}\,\frac{5}{2}\rangle^{\bullet}$	1	± 10
${}^1E_{3u}$	$\lvert 3\,\overline{3}\rangle$	2	± 10	${}^1E_{7/2,u}$	$\lvert \frac{7}{2}\,\frac{7}{2}\rangle^{\bullet}$	1	± 10
${}^2E_{3u}$	$\lvert 3\,3\rangle$	2	± 10	${}^2E_{7/2,u}$	$\lvert \frac{7}{2}\,\overline{\frac{7}{2}}\rangle^{\bullet}$	1	± 10
${}^1E_{4u}$	$\lvert 5\,\overline{4}\rangle$	2	± 10	${}^1E_{9/2,u}$	$\lvert \frac{9}{2}\,\overline{\frac{9}{2}}\rangle^{\bullet}$	1	± 10
${}^2E_{4u}$	$\lvert 5\,4\rangle$	2	± 10	${}^2E_{9/2,u}$	$\lvert \frac{9}{2}\,\frac{9}{2}\rangle^{\bullet}$	1	± 10

T **68**.7 Matrix representations

Use T **68**.4 ♠. § **16**–7, p. 77

T **68**.8 Direct products of representations § **16**–8, p. 81

$\mathbf{C}_{10h}$	A_g	B_g	${}^1E_{1g}$	${}^2E_{1g}$	${}^1E_{2g}$	${}^2E_{2g}$	${}^1E_{3g}$	${}^2E_{3g}$	${}^1E_{4g}$	${}^2E_{4g}$
A_g	A_g	B_g	${}^1E_{1g}$	${}^2E_{1g}$	${}^1E_{2g}$	${}^2E_{2g}$	${}^1E_{3g}$	${}^2E_{3g}$	${}^1E_{4g}$	${}^2E_{4g}$
B_g		A_g	${}^1E_{4g}$	${}^2E_{4g}$	${}^1E_{3g}$	${}^2E_{3g}$	${}^1E_{2g}$	${}^2E_{2g}$	${}^1E_{1g}$	${}^2E_{1g}$
${}^1E_{1g}$			${}^1E_{2g}$	A_g	${}^2E_{3g}$	${}^2E_{1g}$	${}^2E_{2g}$	${}^2E_{4g}$	${}^1E_{3g}$	B_g
${}^2E_{1g}$				${}^2E_{2g}$	${}^1E_{1g}$	${}^1E_{3g}$	${}^1E_{4g}$	${}^1E_{2g}$	B_g	${}^2E_{3g}$
${}^1E_{2g}$					${}^2E_{4g}$	A_g	${}^2E_{1g}$	B_g	${}^2E_{2g}$	${}^1E_{4g}$
${}^2E_{2g}$						${}^1E_{4g}$	B_g	${}^1E_{1g}$	${}^2E_{4g}$	${}^1E_{2g}$
${}^1E_{3g}$							${}^2E_{4g}$	A_g	${}^2E_{3g}$	${}^1E_{1g}$
${}^2E_{3g}$								${}^1E_{4g}$	${}^2E_{1g}$	${}^1E_{3g}$
${}^1E_{4g}$									${}^1E_{2g}$	A_g
${}^2E_{4g}$										${}^2E_{2g}$

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T **68**.8 Direct products of representations *(cont.)*

$\mathbf{C}_{10h}$	A_u	B_u	${}^1E_{1u}$	${}^2E_{1u}$	${}^1E_{2u}$	${}^2E_{2u}$	${}^1E_{3u}$	${}^2E_{3u}$	${}^1E_{4u}$	${}^2E_{4u}$
A_g	A_u	B_u	${}^1E_{1u}$	${}^2E_{1u}$	${}^1E_{2u}$	${}^2E_{2u}$	${}^1E_{3u}$	${}^2E_{3u}$	${}^1E_{4u}$	${}^2E_{4u}$
B_g	B_u	A_u	${}^1E_{4u}$	${}^2E_{4u}$	${}^1E_{3u}$	${}^2E_{3u}$	${}^1E_{2u}$	${}^2E_{2u}$	${}^1E_{1u}$	${}^2E_{1u}$
${}^1E_{1g}$	${}^1E_{1u}$	${}^1E_{4u}$	${}^1E_{2u}$	A_u	${}^2E_{3u}$	${}^2E_{1u}$	${}^2E_{2u}$	${}^2E_{4u}$	${}^1E_{3u}$	B_u
${}^2E_{1g}$	${}^2E_{1u}$	${}^2E_{4u}$	A_u	${}^2E_{2u}$	${}^1E_{1u}$	${}^1E_{3u}$	${}^1E_{4u}$	${}^1E_{2u}$	B_u	${}^2E_{3u}$
${}^1E_{2g}$	${}^1E_{2u}$	${}^1E_{3u}$	${}^2E_{3u}$	${}^1E_{1u}$	${}^2E_{4u}$	A_u	${}^2E_{1u}$	B_u	${}^2E_{2u}$	${}^1E_{4u}$
${}^2E_{2g}$	${}^2E_{2u}$	${}^2E_{3u}$	${}^2E_{1u}$	${}^1E_{3u}$	A_u	${}^1E_{4u}$	B_u	${}^1E_{1u}$	${}^2E_{4u}$	${}^1E_{2u}$
${}^1E_{3g}$	${}^1E_{3u}$	${}^1E_{2u}$	${}^2E_{2u}$	${}^1E_{4u}$	${}^2E_{1u}$	B_u	${}^2E_{4u}$	A_u	${}^2E_{3u}$	${}^1E_{1u}$
${}^2E_{3g}$	${}^2E_{3u}$	${}^2E_{2u}$	${}^2E_{4u}$	${}^1E_{2u}$	B_u	${}^1E_{1u}$	A_u	${}^1E_{4u}$	${}^2E_{1u}$	${}^1E_{3u}$
${}^1E_{4g}$	${}^1E_{4u}$	${}^1E_{1u}$	${}^1E_{3u}$	B_u	${}^2E_{2u}$	${}^2E_{4u}$	${}^2E_{3u}$	${}^2E_{1u}$	${}^1E_{2u}$	A_u
${}^2E_{4g}$	${}^2E_{4u}$	${}^2E_{1u}$	B_u	${}^2E_{3u}$	${}^1E_{4u}$	${}^1E_{2u}$	${}^1E_{1u}$	${}^1E_{3u}$	A_u	${}^2E_{2u}$
A_u	A_g	B_g	${}^1E_{1g}$	${}^2E_{1g}$	${}^1E_{2g}$	${}^2E_{2g}$	${}^1E_{3g}$	${}^2E_{3g}$	${}^1E_{4g}$	${}^2E_{4g}$
B_u		A_g	${}^1E_{4g}$	${}^2E_{4g}$	${}^1E_{3g}$	${}^2E_{3g}$	${}^1E_{2g}$	${}^2E_{2g}$	${}^1E_{1g}$	${}^2E_{1g}$
${}^1E_{1u}$			${}^1E_{2g}$	A_g	${}^2E_{3g}$	${}^2E_{1g}$	${}^2E_{2g}$	${}^2E_{4g}$	${}^1E_{3g}$	B_g
${}^2E_{1u}$				${}^2E_{2g}$	${}^1E_{1g}$	${}^1E_{3g}$	${}^1E_{4g}$	${}^1E_{2g}$	B_g	${}^2E_{3g}$
${}^1E_{2u}$					${}^2E_{4g}$	A_g	${}^2E_{1g}$	B_g	${}^2E_{2g}$	${}^1E_{4g}$
${}^2E_{2u}$						${}^1E_{4g}$	B_g	${}^1E_{1g}$	${}^2E_{4g}$	${}^1E_{2g}$
${}^1E_{3u}$							${}^2E_{4g}$	A_g	${}^2E_{3g}$	${}^1E_{1g}$
${}^2E_{3u}$								${}^1E_{4g}$	${}^2E_{1g}$	${}^1E_{3g}$
${}^1E_{4u}$									${}^1E_{2g}$	A_g
${}^2E_{4u}$										${}^2E_{2g}$

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T **68**.8 Direct products of representations *(cont.)*

$\mathbf{C}_{10h}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$	${}^1E_{7/2,g}$	${}^2E_{7/2,g}$	${}^1E_{9/2,g}$	${}^2E_{9/2,g}$
A_g	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$	${}^1E_{7/2,g}$	${}^2E_{7/2,g}$	${}^1E_{9/2,g}$	${}^2E_{9/2,g}$
B_g	${}^2E_{9/2,g}$	${}^1E_{9/2,g}$	${}^2E_{7/2,g}$	${}^1E_{7/2,g}$	${}^2E_{5/2,g}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$
${}^1E_{1g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$	${}^1E_{7/2,g}$	${}^2E_{9/2,g}$	${}^1E_{5/2,g}$	${}^2E_{7/2,g}$	${}^1E_{9/2,g}$
${}^2E_{1g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{5/2,g}$	${}^2E_{7/2,g}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{9/2,g}$	${}^2E_{9/2,g}$	${}^1E_{7/2,g}$
${}^1E_{2g}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{7/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^2E_{9/2,g}$	${}^1E_{9/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{7/2,g}$
${}^2E_{2g}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{7/2,g}$	${}^1E_{9/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{9/2,g}$	${}^1E_{7/2,g}$	${}^2E_{5/2,g}$
${}^1E_{3g}$	${}^2E_{7/2,g}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	${}^1E_{9/2,g}$	${}^2E_{9/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{7/2,g}$	${}^2E_{5/2,g}$	${}^1E_{3/2,g}$
${}^2E_{3g}$	${}^2E_{5/2,g}$	${}^1E_{7/2,g}$	${}^2E_{9/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$	${}^1E_{9/2,g}$	${}^2E_{7/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$
${}^1E_{4g}$	${}^1E_{9/2,g}$	${}^2E_{7/2,g}$	${}^1E_{5/2,g}$	${}^2E_{9/2,g}$	${}^1E_{7/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{5/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$
${}^2E_{4g}$	${}^1E_{7/2,g}$	${}^2E_{9/2,g}$	${}^1E_{9/2,g}$	${}^2E_{5/2,g}$	${}^1E_{3/2,g}$	${}^2E_{7/2,g}$	${}^1E_{5/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$
A_u	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{5/2,u}$	${}^1E_{7/2,u}$	${}^2E_{7/2,u}$	${}^1E_{9/2,u}$	${}^2E_{9/2,u}$
B_u	${}^2E_{9/2,u}$	${}^1E_{9/2,u}$	${}^2E_{7/2,u}$	${}^1E_{7/2,u}$	${}^2E_{5/2,u}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$
${}^1E_{1u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$	${}^1E_{7/2,u}$	${}^2E_{9/2,u}$	${}^1E_{5/2,u}$	${}^2E_{7/2,u}$	${}^1E_{9/2,u}$
${}^2E_{1u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{5/2,u}$	${}^2E_{7/2,u}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{9/2,u}$	${}^2E_{9/2,u}$	${}^1E_{7/2,u}$
${}^1E_{2u}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{7/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^2E_{9/2,u}$	${}^1E_{9/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{7/2,u}$
${}^2E_{2u}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{7/2,u}$	${}^1E_{9/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{9/2,u}$	${}^1E_{7/2,u}$	${}^2E_{5/2,u}$
${}^1E_{3u}$	${}^2E_{7/2,u}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	${}^1E_{9/2,u}$	${}^2E_{9/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{7/2,u}$	${}^2E_{5/2,u}$	${}^1E_{3/2,u}$
${}^2E_{3u}$	${}^2E_{5/2,u}$	${}^1E_{7/2,u}$	${}^2E_{9/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$	${}^1E_{9/2,u}$	${}^2E_{7/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$
${}^1E_{4u}$	${}^1E_{9/2,u}$	${}^2E_{7/2,u}$	${}^1E_{5/2,u}$	${}^2E_{9/2,u}$	${}^1E_{7/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{5/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$
${}^2E_{4u}$	${}^1E_{7/2,u}$	${}^2E_{9/2,u}$	${}^1E_{9/2,u}$	${}^2E_{5/2,u}$	${}^1E_{3/2,u}$	${}^2E_{7/2,u}$	${}^1E_{5/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$
${}^1E_{1/2,g}$	${}^2E_{1g}$	A_g	${}^1E_{1g}$	${}^2E_{2g}$	${}^1E_{3g}$	${}^1E_{2g}$	${}^2E_{3g}$	${}^1E_{4g}$	B_g	${}^2E_{4g}$
${}^2E_{1/2,g}$		${}^1E_{1g}$	${}^1E_{2g}$	${}^2E_{1g}$	${}^2E_{2g}$	${}^2E_{3g}$	${}^2E_{4g}$	${}^1E_{3g}$	${}^1E_{4g}$	B_g
${}^1E_{3/2,g}$			${}^2E_{3g}$	A_g	${}^2E_{1g}$	${}^2E_{4g}$	B_g	${}^2E_{2g}$	${}^1E_{3g}$	${}^1E_{4g}$
${}^2E_{3/2,g}$				${}^1E_{3g}$	${}^1E_{4g}$	${}^1E_{1g}$	${}^1E_{2g}$	B_g	${}^2E_{4g}$	${}^2E_{3g}$
${}^1E_{5/2,g}$					B_g	A_g	${}^1E_{1g}$	${}^2E_{4g}$	${}^2E_{3g}$	${}^1E_{2g}$
${}^2E_{5/2,g}$						B_g	${}^1E_{4g}$	${}^2E_{1g}$	${}^2E_{2g}$	${}^1E_{3g}$
${}^1E_{7/2,g}$							${}^1E_{3g}$	A_g	${}^2E_{1g}$	${}^2E_{2g}$
${}^2E_{7/2,g}$								${}^2E_{3g}$	${}^1E_{2g}$	${}^1E_{1g}$
${}^1E_{9/2,g}$									${}^1E_{1g}$	A_g
${}^2E_{9/2,g}$										${}^2E_{1g}$

$\Rrightarrow$

T **68**.8 Direct products of representations *(cont.)*

$\mathbf{C}_{10h}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{5/2,u}$	${}^1E_{7/2,u}$	${}^2E_{7/2,u}$	${}^1E_{9/2,u}$	${}^2E_{9/2,u}$
A_g	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{5/2,u}$	${}^1E_{7/2,u}$	${}^2E_{7/2,u}$	${}^1E_{9/2,u}$	${}^2E_{9/2,u}$
B_g	${}^2E_{9/2,u}$	${}^1E_{9/2,u}$	${}^2E_{7/2,u}$	${}^1E_{7/2,u}$	${}^2E_{5/2,u}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$
${}^1E_{1g}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$	${}^1E_{7/2,u}$	${}^2E_{9/2,u}$	${}^1E_{5/2,u}$	${}^2E_{7/2,u}$	${}^1E_{9/2,u}$
${}^2E_{1g}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{5/2,u}$	${}^2E_{7/2,u}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{9/2,u}$	${}^2E_{9/2,u}$	${}^1E_{7/2,u}$
${}^1E_{2g}$	${}^1E_{3/2,u}$	${}^2E_{5/2,u}$	${}^1E_{7/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^2E_{9/2,u}$	${}^1E_{9/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$	${}^2E_{7/2,u}$
${}^2E_{2g}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{7/2,u}$	${}^1E_{9/2,u}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{9/2,u}$	${}^1E_{7/2,u}$	${}^2E_{5/2,u}$
${}^1E_{3g}$	${}^2E_{7/2,u}$	${}^1E_{5/2,u}$	${}^2E_{3/2,u}$	${}^1E_{9/2,u}$	${}^2E_{9/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{7/2,u}$	${}^2E_{5/2,u}$	${}^1E_{3/2,u}$
${}^2E_{3g}$	${}^2E_{5/2,u}$	${}^1E_{7/2,u}$	${}^2E_{9/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$	${}^1E_{9/2,u}$	${}^2E_{7/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$	${}^1E_{5/2,u}$
${}^1E_{4g}$	${}^1E_{9/2,u}$	${}^2E_{7/2,u}$	${}^1E_{5/2,u}$	${}^2E_{9/2,u}$	${}^1E_{7/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2,u}$	${}^2E_{5/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2,u}$
${}^2E_{4g}$	${}^1E_{7/2,u}$	${}^2E_{9/2,u}$	${}^1E_{9/2,u}$	${}^2E_{5/2,u}$	${}^1E_{3/2,u}$	${}^2E_{7/2,u}$	${}^1E_{5/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$
A_u	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{5/2,g}$	${}^1E_{7/2,g}$	${}^2E_{7/2,g}$	${}^1E_{9/2,g}$	${}^2E_{9/2,g}$
B_u	${}^2E_{9/2,g}$	${}^1E_{9/2,g}$	${}^2E_{7/2,g}$	${}^1E_{7/2,g}$	${}^2E_{5/2,g}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$
${}^1E_{1u}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$	${}^1E_{7/2,g}$	${}^2E_{9/2,g}$	${}^1E_{5/2,g}$	${}^2E_{7/2,g}$	${}^1E_{9/2,g}$
${}^2E_{1u}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{5/2,g}$	${}^2E_{7/2,g}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{9/2,g}$	${}^2E_{9/2,g}$	${}^1E_{7/2,g}$
${}^1E_{2u}$	${}^1E_{3/2,g}$	${}^2E_{5/2,g}$	${}^1E_{7/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^2E_{9/2,g}$	${}^1E_{9/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$	${}^2E_{7/2,g}$
${}^2E_{2u}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{7/2,g}$	${}^1E_{9/2,g}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{9/2,g}$	${}^1E_{7/2,g}$	${}^2E_{5/2,g}$
${}^1E_{3u}$	${}^2E_{7/2,g}$	${}^1E_{5/2,g}$	${}^2E_{3/2,g}$	${}^1E_{9/2,g}$	${}^2E_{9/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{7/2,g}$	${}^2E_{5/2,g}$	${}^1E_{3/2,g}$
${}^2E_{3u}$	${}^2E_{5/2,g}$	${}^1E_{7/2,g}$	${}^2E_{9/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$	${}^1E_{9/2,g}$	${}^2E_{7/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$	${}^1E_{5/2,g}$
${}^1E_{4u}$	${}^1E_{9/2,g}$	${}^2E_{7/2,g}$	${}^1E_{5/2,g}$	${}^2E_{9/2,g}$	${}^1E_{7/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2,g}$	${}^2E_{5/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2,g}$
${}^2E_{4u}$	${}^1E_{7/2,g}$	${}^2E_{9/2,g}$	${}^1E_{9/2,g}$	${}^2E_{5/2,g}$	${}^1E_{3/2,g}$	${}^2E_{7/2,g}$	${}^1E_{5/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$
${}^1E_{1/2,g}$	${}^2E_{1u}$	A_u	${}^1E_{1u}$	${}^2E_{2u}$	${}^1E_{3u}$	${}^1E_{2u}$	${}^2E_{3u}$	${}^1E_{4u}$	B_u	${}^2E_{4u}$
${}^2E_{1/2,g}$	A_u	${}^1E_{1u}$	${}^1E_{2u}$	${}^2E_{1u}$	${}^2E_{2u}$	${}^2E_{3u}$	${}^2E_{4u}$	${}^1E_{3u}$	${}^1E_{4u}$	B_u
${}^1E_{3/2,g}$	${}^1E_{1u}$	${}^1E_{2u}$	${}^2E_{3u}$	A_u	${}^2E_{1u}$	${}^2E_{4u}$	B_u	${}^2E_{2u}$	${}^1E_{3u}$	${}^1E_{4u}$
${}^2E_{3/2,g}$	${}^2E_{2u}$	${}^2E_{1u}$	A_u	${}^1E_{3u}$	${}^1E_{4u}$	${}^1E_{1u}$	${}^1E_{2u}$	B_u	${}^2E_{4u}$	${}^2E_{3u}$
${}^1E_{5/2,g}$	${}^1E_{3u}$	${}^2E_{2u}$	${}^2E_{1u}$	${}^1E_{4u}$	B_u	A_u	${}^1E_{1u}$	${}^2E_{4u}$	${}^2E_{3u}$	${}^1E_{2u}$
${}^2E_{5/2,g}$	${}^1E_{2u}$	${}^2E_{3u}$	${}^2E_{4u}$	${}^1E_{1u}$	A_u	B_u	${}^1E_{4u}$	${}^2E_{1u}$	${}^2E_{2u}$	${}^1E_{3u}$
${}^1E_{7/2,g}$	${}^2E_{3u}$	${}^2E_{4u}$	B_u	${}^1E_{2u}$	${}^1E_{1u}$	${}^1E_{4u}$	${}^1E_{3u}$	A_u	${}^2E_{1u}$	${}^2E_{2u}$
${}^2E_{7/2,g}$	${}^1E_{4u}$	${}^1E_{3u}$	${}^2E_{2u}$	B_u	${}^2E_{4u}$	${}^2E_{1u}$	A_u	${}^2E_{3u}$	${}^1E_{2u}$	${}^1E_{1u}$
${}^1E_{9/2,g}$	B_u	${}^1E_{4u}$	${}^1E_{3u}$	${}^2E_{4u}$	${}^2E_{3u}$	${}^2E_{2u}$	${}^2E_{1u}$	${}^1E_{2u}$	${}^1E_{1u}$	A_u
${}^2E_{9/2,g}$	${}^2E_{4u}$	B_u	${}^1E_{4u}$	${}^2E_{3u}$	${}^1E_{2u}$	${}^1E_{3u}$	${}^2E_{2u}$	${}^1E_{1u}$	A_u	${}^2E_{1u}$
${}^1E_{1/2,u}$	${}^2E_{1g}$	A_g	${}^1E_{1g}$	${}^2E_{2g}$	${}^1E_{3g}$	${}^1E_{2g}$	${}^2E_{3g}$	${}^1E_{4g}$	B_g	${}^2E_{4g}$
${}^2E_{1/2,u}$		${}^1E_{1g}$	${}^1E_{2g}$	${}^2E_{1g}$	${}^2E_{2g}$	${}^2E_{3g}$	${}^2E_{4g}$	${}^1E_{3g}$	${}^1E_{4g}$	B_g
${}^1E_{3/2,u}$			${}^2E_{3g}$	A_g	${}^2E_{1g}$	${}^2E_{4g}$	B_g	${}^2E_{2g}$	${}^1E_{3g}$	${}^1E_{4g}$
${}^2E_{3/2,u}$				${}^1E_{3g}$	${}^1E_{4g}$	${}^1E_{1g}$	${}^1E_{2g}$	B_g	${}^2E_{4g}$	${}^2E_{3g}$
${}^1E_{5/2,u}$					B_g	A_g	${}^1E_{1g}$	${}^2E_{4g}$	${}^2E_{3g}$	${}^1E_{2g}$
${}^2E_{5/2,u}$						B_g	${}^1E_{4g}$	${}^2E_{1g}$	${}^2E_{2g}$	${}^1E_{3g}$
${}^1E_{7/2,u}$							${}^1E_{3g}$	A_g	${}^2E_{1g}$	${}^2E_{2g}$
${}^2E_{7/2,u}$								${}^2E_{3g}$	${}^1E_{2g}$	${}^1E_{1g}$
${}^1E_{9/2,u}$									${}^1E_{1g}$	A_g
${}^2E_{9/2,u}$										${}^2E_{1g}$

T **68**.9 Subduction (descent of symmetry) § **16**–9, p. 82

$\mathbf{C}_{10h}$	$\mathbf{C}_{5h}$	$\mathbf{C}_{2h}$	$\mathbf{S}_{10}$	$\mathbf{C}_s$	$\mathbf{C}_i$	$\mathbf{C}_{10}$	$\mathbf{C}_5$	$\mathbf{C}_2$
A_g	A'	A_g	A_g	A'	A_g	A	A	A
B_g	A''	B_g	A_g	A''	A_g	B	A	B
${}^1E_{1g}$	${}^1E_1''$	B_g	${}^1E_{1g}$	A''	A_g	${}^1E_1$	${}^1E_1$	B
${}^2E_{1g}$	${}^2E_1''$	B_g	${}^2E_{1g}$	A''	A_g	${}^2E_1$	${}^2E_1$	B
${}^1E_{2g}$	${}^1E_2'$	A_g	${}^1E_{2g}$	A'	A_g	${}^1E_2$	${}^1E_2$	A
${}^2E_{2g}$	${}^2E_2'$	A_g	${}^2E_{2g}$	A'	A_g	${}^2E_2$	${}^2E_2$	A
${}^1E_{3g}$	${}^1E_2''$	B_g	${}^1E_{2g}$	A''	A_g	${}^1E_3$	${}^1E_2$	B
${}^2E_{3g}$	${}^2E_2''$	B_g	${}^2E_{2g}$	A''	A_g	${}^2E_3$	${}^2E_2$	B
${}^1E_{4g}$	${}^1E_1'$	A_g	${}^1E_{1g}$	A'	A_g	${}^1E_4$	${}^1E_1$	A
${}^2E_{4g}$	${}^2E_1'$	A_g	${}^2E_{1g}$	A'	A_g	${}^2E_4$	${}^2E_1$	A
A_u	A''	A_u	A_u	A''	A_u	A	A	A
B_u	A'	B_u	A_u	A'	A_u	B	A	B
${}^1E_{1u}$	${}^1E_1'$	B_u	${}^1E_{1u}$	A'	A_u	${}^1E_1$	${}^1E_1$	B
${}^2E_{1u}$	${}^2E_1'$	B_u	${}^2E_{1u}$	A'	A_u	${}^2E_1$	${}^2E_1$	B
${}^1E_{2u}$	${}^1E_2''$	A_u	${}^1E_{2u}$	A''	A_u	${}^1E_2$	${}^1E_2$	A
${}^2E_{2u}$	${}^2E_2''$	A_u	${}^2E_{2u}$	A''	A_u	${}^2E_2$	${}^2E_2$	A
${}^1E_{3u}$	${}^1E_2'$	B_u	${}^1E_{2u}$	A'	A_u	${}^1E_3$	${}^1E_2$	B
${}^2E_{3u}$	${}^2E_2'$	B_u	${}^2E_{2u}$	A'	A_u	${}^2E_3$	${}^2E_2$	B
${}^1E_{4u}$	${}^1E_1''$	A_u	${}^1E_{1u}$	A''	A_u	${}^1E_4$	${}^1E_1$	A
${}^2E_{4u}$	${}^2E_1''$	A_u	${}^2E_{1u}$	A''	A_u	${}^2E_4$	${}^2E_1$	A
${}^1E_{1/2,g}$	${}^1E_{1/2}$	${}^1E_{1/2,g}$	${}^1E_{1/2,g}$	${}^1E_{1/2}$	$A_{1/2,g}$	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$
${}^2E_{1/2,g}$	${}^2E_{1/2}$	${}^2E_{1/2,g}$	${}^2E_{1/2,g}$	${}^2E_{1/2}$	$A_{1/2,g}$	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$
${}^1E_{3/2,g}$	${}^1E_{3/2}$	${}^1E_{1/2,g}$	${}^1E_{3/2,g}$	${}^1E_{1/2}$	$A_{1/2,g}$	${}^1E_{3/2}$	${}^1E_{3/2}$	${}^1E_{1/2}$
${}^2E_{3/2,g}$	${}^2E_{3/2}$	${}^2E_{1/2,g}$	${}^2E_{3/2,g}$	${}^2E_{1/2}$	$A_{1/2,g}$	${}^2E_{3/2}$	${}^2E_{3/2}$	${}^2E_{1/2}$
${}^1E_{5/2,g}$	${}^1E_{5/2}$	${}^1E_{1/2,g}$	$A_{5/2,g}$	${}^1E_{1/2}$	$A_{1/2,g}$	${}^1E_{5/2}$	$A_{5/2}$	${}^1E_{1/2}$
${}^2E_{5/2,g}$	${}^2E_{5/2}$	${}^2E_{1/2,g}$	$A_{5/2,g}$	${}^2E_{1/2}$	$A_{1/2,g}$	${}^2E_{5/2}$	$A_{5/2}$	${}^2E_{1/2}$
${}^1E_{7/2,g}$	${}^1E_{7/2}$	${}^1E_{1/2,g}$	${}^2E_{3/2,g}$	${}^1E_{1/2}$	$A_{1/2,g}$	${}^1E_{7/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_{7/2,g}$	${}^2E_{7/2}$	${}^2E_{1/2,g}$	${}^1E_{3/2,g}$	${}^2E_{1/2}$	$A_{1/2,g}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_{9/2,g}$	${}^1E_{9/2}$	${}^1E_{1/2,g}$	${}^2E_{1/2,g}$	${}^1E_{1/2}$	$A_{1/2,g}$	${}^1E_{9/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_{9/2,g}$	${}^2E_{9/2}$	${}^2E_{1/2,g}$	${}^1E_{1/2,g}$	${}^2E_{1/2}$	$A_{1/2,g}$	${}^2E_{9/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$
${}^1E_{1/2,u}$	${}^2E_{9/2}$	${}^1E_{1/2,u}$	${}^1E_{1/2,u}$	${}^2E_{1/2}$	$A_{1/2,u}$	${}^1E_{1/2}$	${}^1E_{1/2}$	${}^1E_{1/2}$
${}^2E_{1/2,u}$	${}^1E_{9/2}$	${}^2E_{1/2,u}$	${}^2E_{1/2,u}$	${}^1E_{1/2}$	$A_{1/2,u}$	${}^2E_{1/2}$	${}^2E_{1/2}$	${}^2E_{1/2}$
${}^1E_{3/2,u}$	${}^2E_{7/2}$	${}^1E_{1/2,u}$	${}^1E_{3/2,u}$	${}^2E_{1/2}$	$A_{1/2,u}$	${}^1E_{3/2}$	${}^1E_{3/2}$	${}^1E_{1/2}$
${}^2E_{3/2,u}$	${}^1E_{7/2}$	${}^2E_{1/2,u}$	${}^2E_{3/2,u}$	${}^1E_{1/2}$	$A_{1/2,u}$	${}^2E_{3/2}$	${}^2E_{3/2}$	${}^2E_{1/2}$
${}^1E_{5/2,u}$	${}^2E_{5/2}$	${}^1E_{1/2,u}$	$A_{5/2,u}$	${}^2E_{1/2}$	$A_{1/2,u}$	${}^1E_{5/2}$	$A_{5/2}$	${}^1E_{1/2}$
${}^2E_{5/2,u}$	${}^1E_{5/2}$	${}^2E_{1/2,u}$	$A_{5/2,u}$	${}^1E_{1/2}$	$A_{1/2,u}$	${}^2E_{5/2}$	$A_{5/2}$	${}^2E_{1/2}$
${}^1E_{7/2,u}$	${}^2E_{3/2}$	${}^1E_{1/2,u}$	${}^2E_{3/2,u}$	${}^2E_{1/2}$	$A_{1/2,u}$	${}^1E_{7/2}$	${}^2E_{3/2}$	${}^1E_{1/2}$
${}^2E_{7/2,u}$	${}^1E_{3/2}$	${}^2E_{1/2,u}$	${}^1E_{3/2,u}$	${}^1E_{1/2}$	$A_{1/2,u}$	${}^2E_{7/2}$	${}^1E_{3/2}$	${}^2E_{1/2}$
${}^1E_{9/2,u}$	${}^2E_{1/2}$	${}^1E_{1/2,u}$	${}^2E_{1/2,u}$	${}^2E_{1/2}$	$A_{1/2,u}$	${}^1E_{9/2}$	${}^2E_{1/2}$	${}^1E_{1/2}$
${}^2E_{9/2,u}$	${}^1E_{1/2}$	${}^2E_{1/2,u}$	${}^1E_{1/2,u}$	${}^1E_{1/2}$	$A_{1/2,u}$	${}^2E_{9/2}$	${}^1E_{1/2}$	${}^2E_{1/2}$

T **68**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{C}_{10h}$
$10n$	$(2n+1)\,A_g \oplus 2n\,(B_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g} \oplus {}^1E_{4g} \oplus {}^2E_{4g})$
$10n+1$	$(2n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u}) \oplus 2n\,(B_u \oplus {}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u} \oplus {}^1E_{4u} \oplus {}^2E_{4u})$
$10n+2$	$(2n+1)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g}) \oplus 2n\,(B_g \oplus {}^1E_{3g} \oplus {}^2E_{3g} \oplus {}^1E_{4g} \oplus {}^2E_{4g})$
$10n+3$	$(2n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u}) \oplus 2n\,(B_u \oplus {}^1E_{4u} \oplus {}^2E_{4u})$
$10n+4$	$(2n+1)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g} \oplus {}^1E_{4g} \oplus {}^2E_{4g}) \oplus 2n\,B_g$
$10n+5$	$(2n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u} \oplus {}^1E_{4u} \oplus {}^2E_{4u}) \oplus (2n+2)\,B_u$
$10n+6$	$(2n+1)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g} \oplus {}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g}) \oplus (2n+2)(B_g \oplus {}^1E_{4g} \oplus {}^2E_{4g})$
$10n+7$	$(2n+1)(A_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u}) \oplus (2n+2)(B_u \oplus {}^1E_{3u} \oplus {}^2E_{3u} \oplus {}^1E_{4u} \oplus {}^2E_{4u})$
$10n+8$	$(2n+1)(A_g \oplus {}^1E_{1g} \oplus {}^2E_{1g}) \oplus (2n+2)(B_g \oplus {}^1E_{2g} \oplus {}^2E_{2g} \oplus {}^1E_{3g} \oplus {}^2E_{3g} \oplus {}^1E_{4g} \oplus {}^2E_{4g})$
$10n+9$	$(2n+1)\,A_u \oplus (2n+2)(B_u \oplus {}^1E_{1u} \oplus {}^2E_{1u} \oplus {}^1E_{2u} \oplus {}^2E_{2u} \oplus {}^1E_{3u} \oplus {}^2E_{3u} \oplus {}^1E_{4u} \oplus {}^2E_{4u})$
$10n+\frac{1}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus 2n\,({}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$10n+\frac{3}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}) \oplus 2n\,({}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$10n+\frac{5}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g}) \oplus 2n\,({}^1E_{7/2,g} \oplus {}^2E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$10n+\frac{7}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g}) \oplus 2n\,({}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$10n+\frac{9}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$10n+\frac{11}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g}) \oplus (2n+2)({}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$10n+\frac{13}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g}) \oplus (2n+2)({}^1E_{7/2,g} \oplus {}^2E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$10n+\frac{15}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}) \oplus (2n+2)({}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$10n+\frac{17}{2}$	$(2n+1)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g}) \oplus (2n+2)({}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$
$10n+\frac{19}{2}$	$(2n+2)({}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g} \oplus {}^1E_{7/2,g} \oplus {}^2E_{7/2,g} \oplus {}^1E_{9/2,g} \oplus {}^2E_{9/2,g})$

$n = 0, 1, 2, \ldots$

T **68**.11 Clebsch–Gordan coefficients

§ **16**–11 ♠, p. 83

The cubic groups

Notation for headers

Items in header read from left to right

1		Hermann–Mauguin symbol for the point group.
2		$\lvert G\rvert$ order of the group.
3		$\lvert C\rvert$ number of classes in the group.
4		$\lvert\widetilde{C}\rvert$ number of classes in the double group.
5		Number of the table.
6		Page reference for the notation of the header, of the first six subsections below it, and of the footers.
7	□	This symbol indicates a crystallographic point group.
8		Schönflies notation for the point group.

Notation for the first six subsections below the header

(1) Product forms	Direct and semidirect product forms (p. 37, note on p. 39).
(2) Group chains *(See pp.* 41, 67*)*	Groups underlined: invariant. Groups in brackets: when subducing one or more of the representations in the tables to these subgroups in the setting used for them in the tables a change of bases (similarity transformation) is required.
(3) Operations of G	Lists all the operations of G, enclosing in brackets all the operations of the same class.
(4) Operations of $\widetilde{G}$	Lists all the operations of $\widetilde{G}$, enclosing in brackets all the operations of the same class.
(5) Classes and representations	$\lvert r\rvert$ number of regular classes in G (p. 51). $\lvert\mathrm{i}\rvert$ number of irregular classes in G (p. 51). $\lvert I\rvert$ number of irreducible representations in G. $\lvert\widetilde{I}\rvert$ number of spinor representations, also called the number of double-group representations.
(6) Subduction *(See p.* 41*)*	When subducing spinor representations to the subgroups $\mathbf{D}_3$, $\mathbf{D}_{3d}$, or $\mathbf{C}_{3v}$ of which there are four isomorphs in different settings, it is mathematically impossible to ensure that in more than two of these settings the character remains a class function on subduction. The two subgroups for which subduction does not suffer from this difficulty are listed in this subsection.

Use of the footers

Finding your way about the tables — Each page of the tables contains a footer giving the ordering of the group types listed in the tables. The current group type is recognized because it carries no page number under its name. On opening the tables at any page you should then see at a glance whether to move backwards or forwards to find the group type desired. More precisely, the page number for the group type desired gives the page, like the present one, which carries the contents for the group type wanted.

(1) Product forms: $\mathbf{T} \otimes \mathbf{C}_2'$.

(2) Group chains: $\mathbf{O}_h \supset \underline{\mathbf{O}} \supset \underline{\mathbf{T}}$, $\mathbf{O}_h \supset \underline{\mathbf{O}} \supset (\mathbf{D}_4)$, $\mathbf{O}_h \supset \underline{\mathbf{O}} \supset (\mathbf{D}_3)$.

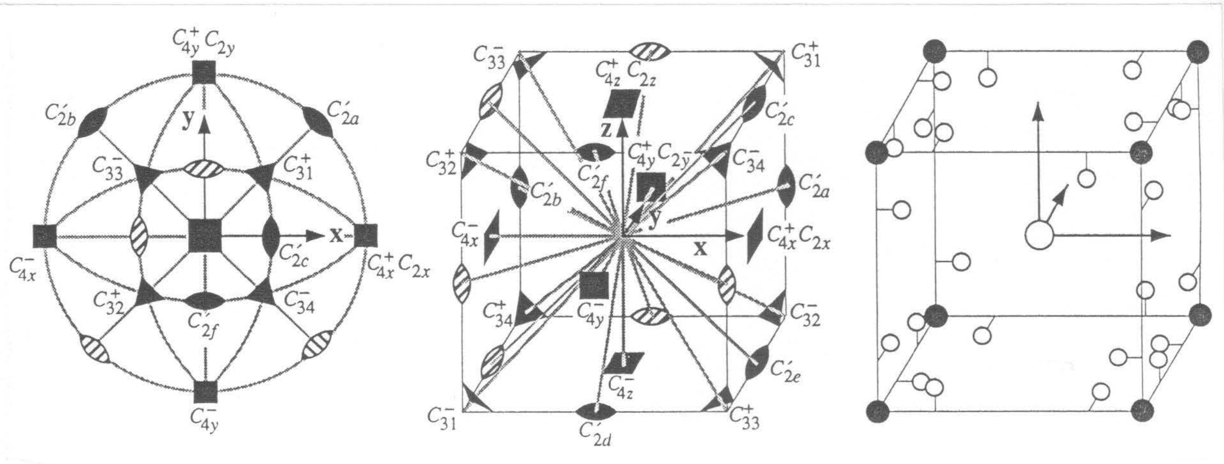
(3) Operations of G: E, (C_{2x}, C_{2y}, C_{2z}), $(C_{31}^+, C_{32}^+, C_{33}^+, C_{34}^+, C_{31}^-, C_{32}^-, C_{33}^-, C_{34}^-)$,
$(C_{4x}^+, C_{4y}^+, C_{4z}^+, C_{4x}^-, C_{4y}^-, C_{4z}^-)$, $(C_{2a}', C_{2b}', C_{2c}', C_{2d}', C_{2e}', C_{2f}')$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, $(C_{2x}, C_{2y}, C_{2z}, \widetilde{C}_{2x}, \widetilde{C}_{2y}, \widetilde{C}_{2z})$,
$(C_{31}^+, C_{32}^+, C_{33}^+, C_{34}^+, C_{31}^-, C_{32}^-, C_{33}^-, C_{34}^-)$, $(\widetilde{C}_{31}^+, \widetilde{C}_{32}^+, \widetilde{C}_{33}^+, \widetilde{C}_{34}^+, \widetilde{C}_{31}^-, \widetilde{C}_{32}^-, \widetilde{C}_{33}^-, \widetilde{C}_{34}^-)$,
$(C_{4x}^+, C_{4y}^+, C_{4z}^+, C_{4x}^-, C_{4y}^-, C_{4z}^-)$, $(\widetilde{C}_{4x}^+, \widetilde{C}_{4y}^+, \widetilde{C}_{4z}^+, \widetilde{C}_{4x}^-, \widetilde{C}_{4y}^-, \widetilde{C}_{4z}^-)$,
$(C_{2a}', C_{2b}', C_{2c}', C_{2d}', C_{2e}', C_{2f}', \widetilde{C}_{2a}', \widetilde{C}_{2b}', \widetilde{C}_{2c}', \widetilde{C}_{2d}', \widetilde{C}_{2e}', \widetilde{C}_{2f}')$.

(5) Classes and representations: $|r| = 3$, $|\mathrm{i}| = 2$, $|I| = 5$, $|\widetilde{I}| = 3$.

(6) Subduction: $\mathbf{D}_3\ (E, C_{31}^+, C_{31}^-, C_{2b}, C_{2f}, C_{2e})$, $\mathbf{D}_3\ (E, C_{32}^+, C_{32}^-, C_{2b}, C_{2d}, C_{2c})$.

F **69** See Chapter **15**, p. 65



Examples: $(NEt_4)U(NCS)_8$, Na_3PaF_8.

T **69**.1 Parameters
Use T **71**.1. § **16**–1, p. 68

T **69**.2 Multiplication table
Use T **71**.2. § **16**–2, p. 69

T **69**.3 Factor table
Use T **71**.3. § **16**–3, p. 70

T **69**.4 Character table § **16**–4, p. 71

O	E	$3C_2$	$8C_3$	$6C_4$	$6C_2'$	τ
A_1	1	1	1	1	1	a
A_2	1	1	1	−1	−1	a
E	2	2	−1	0	0	a
T_1	3	−1	0	1	−1	a
T_2	3	−1	0	−1	1	a
$E_{1/2}$	2	0	1	$\sqrt{2}$	0	c
$E_{5/2}$	2	0	1	$-\sqrt{2}$	0	c
$F_{3/2}$	4	0	−1	0	0	c

T **69**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

O	0	1	2	3
A_1	$\square 1$		$x^2+y^2+z^2$	
A_2				xyz^a
E			$\square(x^2-y^2, 2z^2-x^2-y^2)$	
T_1		$\square(x,y,z),(R_x,R_y,R_z)$		(x^3,y^3,z^3), $\{x(y^2+z^2), y(z^2+x^2), z(x^2+y^2)\}$ [b]
T_2			$\square(zx,yz,xy)$	$\{x(y^2-z^2), y(z^2-x^2), z(x^2-y^2)\}$ [b]

[a] f function: f_{xyz}; [b] f functions: f_{xz^2}, f_{yz^2}, $f_{z(x^2-y^2)}$, $f_{x(x^2-y^2)}$, $f_{y(x^2-y^2)}$, f_{z^3}.

T **69**.6*a* Bases of irreducible representations

§ **16**–6, pp. 74, 75

O	$\langle\lvert j\,m\rangle\rvert$
A_1	$\lvert 0\,0\rangle$
A_2	$\lvert 3\,2\rangle_-$
E	$\langle \frac{1}{\sqrt2}\lvert 2\,0\rangle - \frac{\mathrm{i}}{\sqrt2}\lvert 2\,2\rangle_+, \frac{1}{\sqrt2}\lvert 2\,0\rangle + \frac{\mathrm{i}}{\sqrt2}\lvert 2\,2\rangle_+ \rvert$
T_1	$\langle \lvert 1\,1\rangle_+, \lvert 1\,0\rangle, \lvert 1\,1\rangle_- \rvert$
T_2	$\langle \lvert 2\,1\rangle_-, -\lvert 2\,2\rangle_-, -\lvert 2\,1\rangle_+ \rvert$
$E_{1/2}$	$\langle \lvert \frac12\,\frac12\rangle, \lvert \frac12\,\overline{\frac12}\rangle \rvert$
$E_{5/2}$	$\langle \frac{1}{\sqrt6}\lvert \frac52\,\frac52\rangle - \sqrt{\frac56}\,\lvert \frac52\,\overline{\frac32}\rangle, -\sqrt{\frac56}\,\lvert \frac52\,\frac32\rangle + \frac{1}{\sqrt6}\lvert \frac52\,\overline{\frac52}\rangle \rvert$
$F_{3/2}$	$\langle \frac{1}{\sqrt2}\lvert \frac32\,\frac12\rangle - \frac{\mathrm{i}}{\sqrt2}\lvert \frac32\,\overline{\frac32}\rangle, \frac{\mathrm{i}}{\sqrt2}\lvert \frac32\,\frac32\rangle - \frac{1}{\sqrt2}\lvert \frac32\,\overline{\frac12}\rangle, \frac{1}{\sqrt2}\lvert \frac32\,\frac12\rangle + \frac{\mathrm{i}}{\sqrt2}\lvert \frac32\,\overline{\frac32}\rangle, -\frac{\mathrm{i}}{\sqrt2}\lvert \frac32\,\frac32\rangle - \frac{1}{\sqrt2}\lvert \frac32\,\overline{\frac12}\rangle \rvert$

T **69**.6*b* Symmetrized harmonics

Use T **71**.6*b*. § **16**–6, pp. 74, 75

T **69**.6*c* Spin harmonics

§ **16**–6, pp. 74, 75

O	$\langle \quad \rvert$
$E_{1/2}$	$\langle a_1\,\alpha, a_1\,\beta \rvert$
	$\langle \frac{1}{\sqrt3}\,(t_1^{(1)}\,\beta - t_1^{(2)}\,\alpha + t_1^{(3)}\,\beta), \frac{1}{\sqrt3}\,(-t_1^{(1)}\,\alpha + t_1^{(2)}\,\beta + t_1^{(3)}\,\alpha) \rvert$
$E_{5/2}$	$\langle a_2\,\alpha, a_2\,\beta \rvert$
	$\langle \frac{1}{\sqrt3}\,(t_2^{(1)}\,\beta - t_2^{(2)}\,\alpha + t_2^{(3)}\,\beta), \frac{1}{\sqrt3}\,(-t_2^{(1)}\,\alpha + t_2^{(2)}\,\beta + t_2^{(3)}\,\alpha) \rvert$
$F_{3/2}$	$\langle e^{(1)}\,\alpha, e^{(1)}\,\beta, e^{(2)}\,\alpha, e^{(2)}\,\beta \rvert$
	$\langle \frac{1}{\sqrt3}\,(t_1^{(1)}\,\beta - \omega^*\,t_1^{(2)}\,\alpha + \omega\,t_1^{(3)}\,\beta), \frac{1}{\sqrt3}\,(-t_1^{(1)}\,\alpha + \omega^*\,t_1^{(2)}\,\beta + \omega\,t_1^{(3)}\,\alpha),$
	$\frac{1}{\sqrt3}\,(\omega\,t_1^{(1)}\,\beta - \omega^*\,t_1^{(2)}\,\alpha + t_1^{(3)}\,\beta), \frac{1}{\sqrt3}\,(-\omega\,t_1^{(1)}\,\alpha + \omega^*\,t_1^{(2)}\,\beta + t_1^{(3)}\,\alpha) \rvert$
	$\langle \frac{1}{\sqrt3}\,(t_2^{(1)}\,\beta - \omega^*\,t_2^{(2)}\,\alpha + \omega\,t_2^{(3)}\,\beta), \frac{1}{\sqrt3}\,(-t_2^{(1)}\,\alpha + \omega^*\,t_2^{(2)}\,\beta + \omega\,t_2^{(3)}\,\alpha),$
	$\frac{1}{\sqrt3}\,(-\omega\,t_2^{(1)}\,\beta + \omega^*\,t_2^{(2)}\,\alpha - t_2^{(3)}\,\beta), \frac{1}{\sqrt3}\,(\omega\,t_2^{(1)}\,\alpha - \omega^*\,t_2^{(2)}\,\beta - t_2^{(3)}\,\alpha) \rvert$

$\alpha = \lvert \frac12\,\frac12\rangle$, $\beta = \lvert \frac12\,\overline{\frac12}\rangle$; $\omega = \exp(2\pi\mathrm{i}/3)$

T **69**.7 Matrix representations

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O	$\mathbf{T}_d$	E	T_1	T_2	$E_{1/2}$	$E_{5/2}$	$F_{3/2}$
E	E	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
C_{2x}	C_{2x}	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 & 0 \\ 0 & \bar{1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 & 0 \\ 0 & \bar{1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\text{i}} \\ \bar{\text{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\text{i}} \\ \bar{\text{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\text{i}} & 0 & 0 \\ \bar{\text{i}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{\text{i}} \\ 0 & 0 & \bar{\text{i}} & 0 \end{bmatrix}$
C_{2y}	C_{2y}	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \bar{1} & 0 \\ 0 & 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \bar{1} & 0 \\ 0 & 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{1} \\ 0 & 0 & 1 & 0 \end{bmatrix}$
C_{2z}	C_{2z}	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\text{i}} & 0 \\ 0 & \text{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\text{i}} & 0 \\ 0 & \text{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\text{i}} & 0 & 0 & 0 \\ 0 & \text{i} & 0 & 0 \\ 0 & 0 & \bar{\text{i}} & 0 \\ 0 & 0 & 0 & \text{i} \end{bmatrix}$
C_{31}^+	C_{31}^+	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \bar{\text{i}} \\ \bar{\text{i}} & 0 & 0 \\ 0 & \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \bar{\text{i}} \\ \bar{\text{i}} & 0 & 0 \\ 0 & \bar{1} & 0 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon^* & \overline{\epsilon} \\ \epsilon^* & \epsilon \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon^* & \overline{\epsilon} \\ \epsilon^* & \epsilon \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \text{i}\delta^* & \delta^* & 0 & 0 \\ \text{i}\delta^* & \overline{\delta}^* & 0 & 0 \\ 0 & 0 & \overline{\delta} & \text{i}\delta \\ 0 & 0 & \overline{\delta} & \text{i}\overline{\delta} \end{bmatrix}$
C_{32}^+	C_{32}^+	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \bar{\text{i}} \\ \text{i} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \bar{\text{i}} \\ \text{i} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon^* & \epsilon \\ \overline{\epsilon}^* & \epsilon \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon^* & \epsilon \\ \overline{\epsilon}^* & \epsilon \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \text{i}\delta^* & \overline{\delta}^* & 0 & 0 \\ \text{i}\overline{\delta}^* & \overline{\delta}^* & 0 & 0 \\ 0 & 0 & \overline{\delta} & \text{i}\overline{\delta} \\ 0 & 0 & \delta & \text{i}\overline{\delta} \end{bmatrix}$
C_{33}^+	C_{33}^+	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \text{i} \\ \bar{\text{i}} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \text{i} \\ \bar{\text{i}} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon & \epsilon^* \\ \overline{\epsilon} & \epsilon^* \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon & \epsilon^* \\ \overline{\epsilon} & \epsilon^* \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \overline{\delta}^* & \text{i}\delta^* & 0 & 0 \\ \delta^* & \text{i}\delta^* & 0 & 0 \\ 0 & 0 & \text{i}\overline{\delta} & \overline{\delta} \\ 0 & 0 & \text{i}\delta & \overline{\delta} \end{bmatrix}$
C_{34}^+	C_{34}^+	$\begin{bmatrix} \eta & 0 \\ 0 & \eta^* \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \text{i} \\ \text{i} & 0 & 0 \\ 0 & \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \text{i} \\ \text{i} & 0 & 0 \\ 0 & \bar{1} & 0 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon & \overline{\epsilon}^* \\ \epsilon & \epsilon^* \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon & \overline{\epsilon}^* \\ \epsilon & \epsilon^* \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \overline{\delta}^* & \text{i}\overline{\delta}^* & 0 & 0 \\ \overline{\delta}^* & \text{i}\delta^* & 0 & 0 \\ 0 & 0 & \text{i}\overline{\delta} & \delta \\ 0 & 0 & \text{i}\overline{\delta} & \overline{\delta} \end{bmatrix}$

$\delta = \exp(2\pi\text{i}/24)$, $\epsilon = \exp(2\pi\text{i}/8)$, $\eta = \exp(2\pi\text{i}/3)$

$\twoheadrightarrow$

T **69**.7 Matrix representations *(cont.)*

O	$\mathbf{T}_d$	E	T_1	T_2	$E_{1/2}$	$E_{5/2}$	$F_{3/2}$
C_{31}^-	C_{31}^-	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} & 0 \\ 0 & 0 & \bar{1} \\ \mathrm{i} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} & 0 \\ 0 & 0 & \bar{1} \\ \mathrm{i} & 0 & 0 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon & \epsilon \\ \bar{\epsilon}^* & \epsilon^* \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon & \epsilon \\ \bar{\epsilon}^* & \epsilon^* \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \mathrm{i}\bar{\delta} & \mathrm{i}\bar{\delta} & 0 & 0 \\ \delta & \bar{\delta} & 0 & 0 \\ 0 & 0 & \bar{\delta}^* & \bar{\delta}^* \\ 0 & 0 & \mathrm{i}\bar{\delta}^* & \mathrm{i}\delta^* \end{bmatrix}$
C_{32}^-	C_{32}^-	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} & 0 \\ 0 & 0 & 1 \\ \mathrm{i} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} & 0 \\ 0 & 0 & 1 \\ \mathrm{i} & 0 & 0 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon & \bar{\epsilon} \\ \epsilon^* & \epsilon^* \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon & \bar{\epsilon} \\ \epsilon^* & \epsilon^* \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \mathrm{i}\bar{\delta} & \mathrm{i}\delta & 0 & 0 \\ \bar{\delta} & \bar{\delta} & 0 & 0 \\ 0 & 0 & \bar{\delta}^* & \delta^* \\ 0 & 0 & \mathrm{i}\delta^* & \mathrm{i}\delta^* \end{bmatrix}$
C_{33}^-	C_{33}^-	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} & 0 \\ 0 & 0 & 1 \\ \bar{\mathrm{i}} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} & 0 \\ 0 & 0 & 1 \\ \bar{\mathrm{i}} & 0 & 0 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon^* & \bar{\epsilon}^* \\ \epsilon & \epsilon \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon^* & \bar{\epsilon}^* \\ \epsilon & \epsilon \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \bar{\delta} & \delta & 0 & 0 \\ \mathrm{i}\bar{\delta} & \mathrm{i}\bar{\delta} & 0 & 0 \\ 0 & 0 & \mathrm{i}\delta^* & \mathrm{i}\bar{\delta}^* \\ 0 & 0 & \bar{\delta}^* & \bar{\delta}^* \end{bmatrix}$
C_{34}^-	C_{34}^-	$\begin{bmatrix} \eta^* & 0 \\ 0 & \eta \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} & 0 \\ 0 & 0 & \bar{1} \\ \bar{\mathrm{i}} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} & 0 \\ 0 & 0 & \bar{1} \\ \bar{\mathrm{i}} & 0 & 0 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon^* & \epsilon^* \\ \bar{\epsilon} & \epsilon \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \epsilon^* & \epsilon^* \\ \bar{\epsilon} & \epsilon \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \bar{\delta} & \bar{\delta} & 0 & 0 \\ \mathrm{i}\delta & \mathrm{i}\bar{\delta} & 0 & 0 \\ 0 & 0 & \mathrm{i}\delta^* & \mathrm{i}\delta^* \\ 0 & 0 & \delta^* & \bar{\delta}^* \end{bmatrix}$
C_{4x}^+	S_{4x}^-	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} & 0 \\ \bar{\mathrm{i}} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & \bar{1} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 1 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \bar{1} & \mathrm{i} \\ \mathrm{i} & \bar{1} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & \eta^* & \mathrm{i}\bar{\eta}^* \\ 0 & 0 & \mathrm{i}\bar{\eta}^* & \eta^* \\ \eta & \mathrm{i}\bar{\eta} & 0 & 0 \\ \mathrm{i}\bar{\eta} & \eta & 0 & 0 \end{bmatrix}$
C_{4y}^+	S_{4y}^-	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 & 0 \\ 0 & 0 & \bar{1} \\ 0 & 1 & 0 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & \bar{1} \\ 1 & 1 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \bar{1} & 1 \\ \bar{1} & \bar{1} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & \eta & \bar{\eta} \\ 0 & 0 & \eta & \eta \\ \eta^* & \bar{\eta}^* & 0 & 0 \\ \eta^* & \eta^* & 0 & 0 \end{bmatrix}$
C_{4z}^+	S_{4z}^-	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \bar{\mathrm{i}} \\ 0 & 1 & 0 \\ \bar{\mathrm{i}} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \mathrm{i} \\ 0 & \bar{1} & 0 \\ \mathrm{i} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & 0 \\ 0 & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\epsilon}^* & 0 \\ 0 & \bar{\epsilon} \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \epsilon^* & 0 \\ 0 & 0 & 0 & \epsilon \\ \epsilon^* & 0 & 0 & 0 \\ 0 & \epsilon & 0 & 0 \end{bmatrix}$
C_{4x}^-	S_{4x}^+	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} & 0 \\ \bar{\mathrm{i}} & 0 & 0 \\ 0 & 0 & \bar{1} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & \mathrm{i} \\ \mathrm{i} & 1 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \bar{1} & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & \bar{1} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & \eta^* & \mathrm{i}\eta^* \\ 0 & 0 & \mathrm{i}\eta^* & \eta^* \\ \eta & \mathrm{i}\eta & 0 & 0 \\ \mathrm{i}\eta & \eta & 0 & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/24)$, $\epsilon = \exp(2\pi\mathrm{i}/8)$, $\eta = \exp(2\pi\mathrm{i}/3)$

$\twoheadrightarrow$

T **69**.7 Matrix representations *(cont.)*

O	$\mathbf{T}_d$	E	T_1	T_2	$E_{1/2}$	$E_{5/2}$	$F_{3/2}$
C_{4y}^-	S_{4y}^+	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & \bar{1} \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 & 0 \\ 0 & 0 & 1 \\ 0 & \bar{1} & 0 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ \bar{1} & 1 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \bar{1} & \bar{1} \\ 1 & \bar{1} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & \eta & \eta \\ 0 & 0 & \overline{\eta} & \eta \\ \eta^* & \eta^* & 0 & 0 \\ \overline{\eta}^* & \eta^* & 0 & 0 \end{bmatrix}$
C_{4z}^-	S_{4z}^+	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \mathrm{i} \\ 0 & 1 & 0 \\ \mathrm{i} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \bar{\mathrm{i}} \\ 0 & \bar{1} & 0 \\ \bar{\mathrm{i}} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \overline{\epsilon} & 0 \\ 0 & \overline{\epsilon}^* \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \epsilon & 0 \\ 0 & 0 & 0 & \epsilon^* \\ \epsilon & 0 & 0 & 0 \\ 0 & \epsilon^* & 0 & 0 \end{bmatrix}$
C_{2a}'	σ_{d1}	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \bar{\mathrm{i}} \\ 0 & \bar{1} & 0 \\ \mathrm{i} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \mathrm{i} \\ 0 & 1 & 0 \\ \bar{\mathrm{i}} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon} \\ \epsilon^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon \\ \overline{\epsilon}^* & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & \overline{\epsilon} \\ 0 & 0 & \epsilon^* & 0 \\ 0 & \overline{\epsilon} & 0 & 0 \\ \epsilon^* & 0 & 0 & 0 \end{bmatrix}$
C_{2b}'	σ_{d2}	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \mathrm{i} \\ 0 & \bar{1} & 0 \\ \bar{\mathrm{i}} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \bar{\mathrm{i}} \\ 0 & 1 & 0 \\ \mathrm{i} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \overline{\epsilon}^* \\ \epsilon & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \epsilon^* \\ \overline{\epsilon} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & \overline{\epsilon}^* \\ 0 & 0 & \epsilon & 0 \\ 0 & \overline{\epsilon}^* & 0 & 0 \\ \epsilon & 0 & 0 & 0 \end{bmatrix}$
C_{2c}'	σ_{d3}	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 & 0 \\ 0 & 0 & \bar{1} \\ 0 & \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \bar{\mathrm{i}} & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & \mathrm{i} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \mathrm{i} & \mathrm{i} \\ \mathrm{i} & \bar{\mathrm{i}} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & \mathrm{i}\overline{\eta} & \mathrm{i}\overline{\eta} \\ 0 & 0 & \mathrm{i}\overline{\eta} & \mathrm{i}\eta \\ \mathrm{i}\overline{\eta}^* & \mathrm{i}\overline{\eta}^* & 0 & 0 \\ \mathrm{i}\overline{\eta}^* & \mathrm{i}\eta^* & 0 & 0 \end{bmatrix}$
C_{2d}'	σ_{d4}	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} & 0 \\ \bar{\mathrm{i}} & 0 & 0 \\ 0 & 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \mathrm{i} & 1 \\ \bar{1} & \bar{\mathrm{i}} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \bar{\mathrm{i}} & \bar{1} \\ 1 & \mathrm{i} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & \mathrm{i}\eta^* & \eta^* \\ 0 & 0 & \overline{\eta}^* & \mathrm{i}\overline{\eta}^* \\ \mathrm{i}\eta & \eta & 0 & 0 \\ \overline{\eta} & \mathrm{i}\overline{\eta} & 0 & 0 \end{bmatrix}$
C_{2e}'	σ_{d5}	$\begin{bmatrix} 0 & \eta \\ \eta^* & 0 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & \bar{1} \\ 0 & \bar{1} & 0 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \mathrm{i} & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & \bar{\mathrm{i}} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \bar{\mathrm{i}} & \mathrm{i} \\ \mathrm{i} & \mathrm{i} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & \mathrm{i}\eta & \mathrm{i}\overline{\eta} \\ 0 & 0 & \mathrm{i}\overline{\eta} & \mathrm{i}\overline{\eta} \\ \mathrm{i}\eta^* & \mathrm{i}\overline{\eta}^* & 0 & 0 \\ \mathrm{i}\overline{\eta}^* & \mathrm{i}\overline{\eta}^* & 0 & 0 \end{bmatrix}$
C_{2f}'	σ_{d6}	$\begin{bmatrix} 0 & \eta^* \\ \eta & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 0 & \mathrm{i} & 0 \\ \bar{\mathrm{i}} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \bar{\mathrm{i}} & 1 \\ \bar{1} & \mathrm{i} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \mathrm{i} & \bar{1} \\ 1 & \bar{\mathrm{i}} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & \mathrm{i}\overline{\eta}^* & \eta^* \\ 0 & 0 & \overline{\eta}^* & \mathrm{i}\eta^* \\ \mathrm{i}\overline{\eta} & \eta & 0 & 0 \\ \overline{\eta} & \mathrm{i}\eta & 0 & 0 \end{bmatrix}$

$\epsilon = \exp(2\pi\mathrm{i}/8)$, $\eta = \exp(2\pi\mathrm{i}/3)$

T **69**.8 Direct products of representations § **16**–8, p. 81

$\mathbf{O}$, $\mathbf{T}_d$	A_1	A_2	E	T_1	T_2
A_1	A_1	A_2	E	T_1	T_2
A_2		A_1	E	T_2	T_1
E			$A_1 \oplus \{A_2\} \oplus E$	$T_1 \oplus T_2$	$T_1 \oplus T_2$
T_1				$A_1 \oplus E \oplus \{T_1\} \oplus T_2$	$A_2 \oplus E \oplus T_1 \oplus T_2$
T_2					$A_1 \oplus E \oplus \{T_1\} \oplus T_2$

$\twoheadrightarrow$

T **69**.8 Direct products of representations *(cont.)*

$\mathbf{O}$, $\mathbf{T}_d$	$E_{1/2}$	$E_{5/2}$	$F_{3/2}$
A_1	$E_{1/2}$	$E_{5/2}$	$F_{3/2}$
A_2	$E_{5/2}$	$E_{1/2}$	$F_{3/2}$
E	$F_{3/2}$	$F_{3/2}$	$E_{1/2} \oplus E_{5/2} \oplus F_{3/2}$
T_1	$E_{1/2} \oplus F_{3/2}$	$E_{5/2} \oplus F_{3/2}$	$E_{1/2} \oplus E_{5/2} \oplus 2F_{3/2}$
T_2	$E_{5/2} \oplus F_{3/2}$	$E_{1/2} \oplus F_{3/2}$	$E_{1/2} \oplus E_{5/2} \oplus 2F_{3/2}$
$E_{1/2}$	$\{A_1\} \oplus T_1$	$A_2 \oplus T_2$	$E \oplus T_1 \oplus T_2$
$E_{5/2}$		$\{A_1\} \oplus T_1$	$E \oplus T_1 \oplus T_2$
$F_{3/2}$			$\{A_1\} \oplus A_2 \oplus \{E\} \oplus 2T_1 \oplus T_2 \oplus \{T_2\}$

T **69**.9 Subduction (descent of symmetry) § **16**–9, p. 82

$\mathbf{O}$	$\mathbf{T}$	$(\mathbf{D}_4)$	$(\mathbf{D}_3)$	$\mathbf{D}_2$	$(\mathbf{D}_2)$
				C_{2z}, C_{2x}, C_{2y}	C_{2z}, C'_{2a}, C'_{2b}
A_1	A	A_1	A_1	A	A
A_2	A	B_1	A_2	A	B_1
E	${}^1E \oplus {}^2E$	$A_1 \oplus B_1$	E	$2A$	$A \oplus B_1$
T_1	T	$A_2 \oplus E$	$A_2 \oplus E$	$B_1 \oplus B_2 \oplus B_3$	$B_1 \oplus B_2 \oplus B_3$
T_2	T	$B_2 \oplus E$	$A_1 \oplus E$	$B_1 \oplus B_2 \oplus B_3$	$A \oplus B_2 \oplus B_3$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{5/2}$	$E_{1/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$F_{3/2}$	${}^1F_{3/2} \oplus {}^2F_{3/2}$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}$	$2E_{1/2}$	$2E_{1/2}$

$\twoheadrightarrow$

T **69**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{O}$	$(\mathbf{C}_4)$	$(\mathbf{C}_3)$	$\mathbf{C}_2$	$(\mathbf{C}_2)$
			C_2	C'_2
A_1	A	A	A	A
A_2	B	A	A	B
E	$A \oplus B$	${}^1E \oplus {}^2E$	$2A$	$A \oplus B$
T_1	$A \oplus {}^1E \oplus {}^2E$	$A \oplus {}^1E \oplus {}^2E$	$A \oplus 2B$	$A \oplus 2B$
T_2	$B \oplus {}^1E \oplus {}^2E$	$A \oplus {}^1E \oplus {}^2E$	$A \oplus 2B$	$2A \oplus B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$F_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2} \oplus 2A_{3/2}$	$2\,{}^1E_{1/2} \oplus 2\,{}^2E_{1/2}$	$2\,{}^1E_{1/2} \oplus 2\,{}^2E_{1/2}$

T **69**.10 Subduction from O(3)

§ **16**–10, p. 82

j	**O**
$12n$	$(n+1)\,A_1 \oplus n(A_2 \oplus 2E \oplus 3T_1 \oplus 3T_2)$
$12n+1$	$n\,(A_1 \oplus A_2 \oplus 2E \oplus 2T_1 \oplus 3T_2) \oplus (n+1)\,T_1$
$12n+2$	$n\,(A_1 \oplus A_2 \oplus E \oplus 3T_1 \oplus 2T_2) \oplus (n+1)(E \oplus T_2)$
$12n+3$	$n\,(A_1 \oplus 2E \oplus 2T_1 \oplus 2T_2) \oplus (n+1)(A_2 \oplus T_1 \oplus T_2)$
$12n+4$	$(n+1)(A_1 \oplus E \oplus T_1 \oplus T_2) \oplus n\,(A_2 \oplus E \oplus 2T_1 \oplus 2T_2)$
$12n+5$	$n\,(A_1 \oplus A_2 \oplus E \oplus T_1 \oplus 2T_2) \oplus (n+1)(E \oplus 2T_1 \oplus T_2)$
$12n+6$	$(n+1)(A_1 \oplus A_2 \oplus E \oplus T_1 \oplus 2T_2) \oplus n\,(E \oplus 2T_1 \oplus T_2)$
$12n+7$	$n\,(A_1 \oplus E \oplus T_1 \oplus T_2) \oplus (n+1)(A_2 \oplus E \oplus 2T_1 \oplus 2T_2)$
$12n+8$	$(n+1)(A_1 \oplus 2E \oplus 2T_1 \oplus 2T_2) \oplus n\,(A_2 \oplus T_1 \oplus T_2)$
$12n+9$	$(n+1)(A_1 \oplus A_2 \oplus E \oplus 3T_1 \oplus 2T_2) \oplus n\,(E \oplus T_2)$
$12n+10$	$(n+1)(A_1 \oplus A_2 \oplus 2E \oplus 2T_1 \oplus 3T_2) \oplus n\,T_1$
$12n+11$	$n\,A_1 \oplus (n+1)(A_2 \oplus 2E \oplus 3T_1 \oplus 3T_2)$
$12n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n\,(E_{5/2} \oplus 2F_{3/2})$
$12n+\frac{3}{2}$	$2n\,(E_{1/2} \oplus E_{5/2} \oplus F_{3/2}) \oplus (2n+1)\,F_{3/2}$
$12n+\frac{5}{2}$	$2n\,(E_{1/2} \oplus F_{3/2}) \oplus (2n+1)(E_{5/2} \oplus F_{3/2})$
$12n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{5/2} \oplus F_{3/2}) \oplus 2n\,F_{3/2}$
$12n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus 2F_{3/2}) \oplus 2n\,E_{5/2}$
$12n+\frac{11}{2}$	$(2n+1)(E_{1/2} \oplus E_{5/2} \oplus 2F_{3/2})$
$12n+\frac{13}{2}$	$(2n+1)(E_{1/2} \oplus 2F_{3/2}) \oplus (2n+2)\,E_{5/2}$
$12n+\frac{15}{2}$	$(2n+1)(E_{1/2} \oplus E_{5/2} \oplus F_{3/2}) \oplus (2n+2)\,F_{3/2}$
$12n+\frac{17}{2}$	$(2n+2)(E_{1/2} \oplus F_{3/2}) \oplus (2n+1)(E_{5/2} \oplus F_{3/2})$
$12n+\frac{19}{2}$	$(2n+2)(E_{1/2} \oplus E_{5/2} \oplus F_{3/2}) \oplus (2n+1)\,F_{3/2}$
$12n+\frac{21}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)(E_{5/2} \oplus 2F_{3/2})$
$12n+\frac{23}{2}$	$(2n+2)(E_{1/2} \oplus E_{5/2} \oplus 2F_{3/2})$

$n = 0, 1, 2, \ldots$

T **69**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83

O, $\mathbf{T}_d$

a_2	e	E 1	2
1	1	1	0
1	2	0	$\bar{1}$

a_2	t_1	T_2 1	2	3
1	1	1	0	0
1	2	0	1	0
1	3	0	0	1

a_2	t_2	T_1 1	2	3
1	1	1	0	0
1	2	0	1	0
1	3	0	0	1

a_2	$e_{1/2}$	$E_{5/2}$ 1	2
1	1	1	0
1	2	0	1

a_2	$e_{5/2}$	$E_{1/2}$ 1	2
1	1	1	0
1	2	0	1

a_2	$f_{3/2}$	$F_{3/2}$ 1	2	3	4
1	1	1	0	0	0
1	2	0	1	0	0
1	3	0	0	$\bar{1}$	0
1	4	0	0	0	$\bar{1}$

e	e	A_1 1	A_2 1	E 1	2
1	1	0	0	0	1
1	2	u	u	0	0
2	1	u	$\bar{\text{u}}$	0	0
2	2	0	0	1	0

$\text{u} = 2^{-1/2}$

↠

T **69**.11 Clebsch–Gordan coefficients *(cont.)* **O**, $\mathbf{T}_d$

e	t_1	T_1 1	2	3	T_2 1	2	3
1	1	u	0	0	u	0	0
1	2	0	$u\omega$	0	0	$u\omega$	0
1	3	0	0	$u\omega^*$	0	0	$u\omega^*$
2	1	$u\omega^*$	0	0	$\bar{u}\omega^*$	0	0
2	2	0	$u\omega$	0	0	$\bar{u}\omega$	0
2	3	0	0	u	0	0	$\bar{u}$

e	t_2	T_1 1	2	3	T_2 1	2	3
1	1	u	0	0	u	0	0
1	2	0	$u\omega$	0	0	$u\omega$	0
1	3	0	0	$u\omega^*$	0	0	$u\omega^*$
2	1	$\bar{u}\omega^*$	0	0	$u\omega^*$	0	0
2	2	0	$\bar{u}\omega$	0	0	$u\omega$	0
2	3	0	0	$\bar{u}$	0	0	u

e	$f_{3/2}$	$E_{1/2}$ 1	2	$E_{5/2}$ 1	2	$F_{3/2}$ 1	2	3	4
1	1	0	0	0	0	0	0	1	0
1	2	0	0	0	0	0	0	0	1
1	3	u	0	u	0	0	0	0	0
1	4	0	u	0	u	0	0	0	0
2	1	u	0	$\bar{u}$	0	0	0	0	0
2	2	0	u	0	$\bar{u}$	0	0	0	0
2	3	0	0	0	0	1	0	0	0
2	4	0	0	0	0	0	1	0	0

e	$e_{1/2}$	$F_{3/2}$ 1	2	3	4
1	1	1	0	0	0
1	2	0	1	0	0
2	1	0	0	1	0
2	2	0	0	0	1

e	$e_{5/2}$	$F_{3/2}$ 1	2	3	4
1	1	1	0	0	0
1	2	0	1	0	0
2	1	0	0	$\bar{1}$	0
2	2	0	0	0	$\bar{1}$

t_1	t_1	A_1 1	E 1	2	T_1 1	2	3	T_2 1	2	3
1	1	v	v	$v\omega$	0	0	0	0	0	0
1	2	0	0	0	0	0	$\bar{u}$	0	0	$\bar{u}$
1	3	0	0	0	0	u	0	0	$\bar{u}$	0
2	1	0	0	0	0	0	u	0	0	$\bar{u}$
2	2	$\bar{v}$	$\bar{v}\omega^*$	$\bar{v}\omega^*$	0	0	0	0	0	0
2	3	0	0	0	u	0	0	u	0	0
3	1	0	0	0	0	$\bar{u}$	0	0	$\bar{u}$	0
3	2	0	0	0	$\bar{u}$	0	0	u	0	0
3	3	$\bar{v}$	$\bar{v}\omega$	$\bar{v}$	0	0	0	0	0	0

t_1	t_2	A_2 1	E 1	2	T_1 1	2	3	T_2 1	2	3
1	1	v	v	$\bar{v}\omega$	0	0	0	0	0	0
1	2	0	0	0	0	0	$\bar{u}$	0	0	$\bar{u}$
1	3	0	0	0	0	$\bar{u}$	0	0	u	0
2	1	0	0	0	0	0	$\bar{u}$	0	0	u
2	2	$\bar{v}$	$\bar{v}\omega^*$	$v\omega^*$	0	0	0	0	0	0
2	3	0	0	0	u	0	0	u	0	0
3	1	0	0	0	0	$\bar{u}$	0	0	$\bar{u}$	0
3	2	0	0	0	u	0	0	$\bar{u}$	0	0
3	3	$\bar{v}$	$\bar{v}\omega$	v	0	0	0	0	0	0

t_1	$e_{1/2}$	$E_{1/2}$ 1	2	$F_{3/2}$ 1	2	3	4
1	1	0	$\bar{v}$	0	$\bar{v}$	0	$\bar{v}\omega$
1	2	v	0	v	0	$v\omega$	0
2	1	$\bar{v}$	0	$\bar{v}\omega^*$	0	$\bar{v}\omega^*$	0
2	2	0	v	0	$v\omega^*$	0	$v\omega^*$
3	1	0	v	0	$v\omega$	0	v
3	2	v	0	$v\omega$	0	v	0

t_1	$e_{5/2}$	$E_{5/2}$ 1	2	$F_{3/2}$ 1	2	3	4
1	1	0	$\bar{v}$	0	$\bar{v}$	0	$v\omega$
1	2	v	0	v	0	$\bar{v}\omega$	0
2	1	$\bar{v}$	0	$\bar{v}\omega^*$	0	$v\omega^*$	0
2	2	0	v	0	$v\omega^*$	0	$\bar{v}\omega^*$
3	1	0	v	0	$v\omega$	0	$\bar{v}$
3	2	v	0	$v\omega$	0	$\bar{v}$	0

$u = 2^{-1/2}$, $v = 3^{-1/2}$, $\omega = \exp(2\pi i/3)$ ⇛

T **69**.11 Clebsch–Gordan coefficients *(cont.)* **O**, $\mathbf{T}_d$

t_1	$f_{3/2}$	$E_{1/2}$ 1	2	$E_{5/2}$ 1	2	$F_{3/2}$ 1	2	3	4	$F_{3/2}$ 1	2	3	4
1	1	0	$\bar{x}$	0	$\bar{x}$	0	$\bar{v}$	0	0	0	0	0	$\bar{v}\omega^*$
1	2	x	0	x	0	v	0	0	0	0	0	$v\omega^*$	0
1	3	0	$\bar{x}\omega^*$	0	$x\omega^*$	0	0	0	$\bar{v}$	0	$\bar{v}$	0	0
1	4	$x\omega^*$	0	$\bar{x}\omega^*$	0	0	0	v	0	v	0	0	0
2	1	$\bar{x}\omega$	0	$\bar{x}\omega$	0	$\bar{v}$	0	0	0	0	0	$\bar{v}\omega$	0
2	2	0	$x\omega$	0	$x\omega$	0	v	0	0	0	0	0	$v\omega$
2	3	$\bar{x}\omega$	0	$x\omega$	0	0	0	$\bar{v}$	0	$\bar{v}\omega$	0	0	0
2	4	0	$x\omega$	0	$\bar{x}\omega$	0	0	0	v	0	$v\omega$	0	0
3	1	0	$x\omega^*$	0	$x\omega^*$	0	v	0	0	0	0	0	v
3	2	$x\omega^*$	0	$x\omega^*$	0	v	0	0	0	0	0	v	0
3	3	0	x	0	$\bar{x}$	0	0	0	v	0	$v\omega^*$	0	0
3	4	x	0	$\bar{x}$	0	0	0	v	0	$v\omega^*$	0	0	0

t_2	t_2	A_1 1	E 1	2	T_1 1	2	3	T_2 1	2	3
1	1	v	v	$v\omega$	0	0	0	0	0	0
1	2	0	0	0	0	0	$\bar{u}$	0	0	$\bar{u}$
1	3	0	0	0	0	u	0	0	$\bar{u}$	0
2	1	0	0	0	0	0	u	0	0	$\bar{u}$
2	2	$\bar{v}$	$\bar{v}\omega^*$	$\bar{v}\omega^*$	0	0	0	0	0	0
2	3	0	0	0	u	0	0	u	0	0
3	1	0	0	0	0	$\bar{u}$	0	0	$\bar{u}$	0
3	2	0	0	0	$\bar{u}$	0	0	u	0	0
3	3	$\bar{v}$	$\bar{v}\omega$	$\bar{v}$	0	0	0	0	0	0

t_2	$e_{1/2}$	$E_{5/2}$ 1	2	$F_{3/2}$ 1	2	3	4
1	1	0	$\bar{v}$	0	$\bar{v}$	0	$v\omega$
1	2	v	0	v	0	$\bar{v}\omega$	0
2	1	$\bar{v}$	0	$\bar{v}\omega^*$	0	$v\omega^*$	0
2	2	0	v	0	$v\omega^*$	0	$\bar{v}\omega^*$
3	1	0	v	0	$v\omega$	0	$\bar{v}$
3	2	v	0	$v\omega$	0	$\bar{v}$	0

t_2	$e_{5/2}$	$E_{1/2}$ 1	2	$F_{3/2}$ 1	2	3	4
1	1	0	$\bar{v}$	0	$\bar{v}$	0	$\bar{v}\omega$
1	2	v	0	v	0	$v\omega$	0
2	1	$\bar{v}$	0	$\bar{v}\omega^*$	0	$\bar{v}\omega^*$	0
2	2	0	v	0	$v\omega^*$	0	$v\omega^*$
3	1	0	v	0	$v\omega$	0	v
3	2	v	0	$v\omega$	0	v	0

t_2	$f_{3/2}$	$E_{1/2}$ 1	2	$E_{5/2}$ 1	2	$F_{3/2}$ 1	2	3	4	$F_{3/2}$ 1	2	3	4
1	1	0	$\bar{x}$	0	$\bar{x}$	0	$\bar{v}$	0	0	0	0	0	$v\omega^*$
1	2	x	0	x	0	v	0	0	0	0	0	$\bar{v}\omega^*$	0
1	3	0	$x\omega^*$	0	$\bar{x}\omega^*$	0	0	0	v	0	$\bar{v}$	0	0
1	4	$\bar{x}\omega^*$	0	$x\omega^*$	0	0	0	$\bar{v}$	0	v	0	0	0
2	1	$\bar{x}\omega$	0	$\bar{x}\omega$	0	$\bar{v}$	0	0	0	0	0	$v\omega$	0
2	2	0	$x\omega$	0	$x\omega$	0	v	0	0	0	0	0	$\bar{v}\omega$
2	3	$x\omega$	0	$\bar{x}\omega$	0	0	0	v	0	$\bar{v}\omega$	0	0	0
2	4	0	$\bar{x}\omega$	0	$x\omega$	0	0	0	$\bar{v}$	0	$v\omega$	0	0
3	1	0	$x\omega^*$	0	$x\omega^*$	0	v	0	0	0	0	0	$\bar{v}$
3	2	$x\omega^*$	0	$x\omega^*$	0	v	0	0	0	0	0	$\bar{v}$	0
3	3	0	$\bar{x}$	0	x	0	0	0	$\bar{v}$	0	$v\omega^*$	0	0
3	4	$\bar{x}$	0	x	0	0	0	$\bar{v}$	0	$v\omega^*$	0	0	0

$u = 2^{-1/2}$, $v = 3^{-1/2}$, $x = 6^{-1/2}$, $\omega = \exp(2\pi i/3)$ ↠

T **69**.11 Clebsch–Gordan coefficients *(cont.)* $\mathbf{O}$, $\mathbf{T}_d$

$e_{1/2}$	$e_{1/2}$	A_1 1	T_1 1	2	3
1	1	0	u	0	u
1	2	u	0	u	0
2	1	$\bar{u}$	0	u	0
2	2	0	u	0	$\bar{u}$

$e_{1/2}$	$e_{5/2}$	A_2 1	T_2 1	2	3
1	1	0	u	0	u
1	2	u	0	u	0
2	1	$\bar{u}$	0	u	0
2	2	0	u	0	$\bar{u}$

$e_{1/2}$	$f_{3/2}$	E 1	2	T_1 1	2	3	T_2 1	2	3
1	1	0	0	w	0	$w\omega^*$	w	0	$w\omega^*$
1	2	u	0	0	$w\omega$	0	0	$w\omega$	0
1	3	0	0	$w\omega^*$	0	w	$\bar{w}\omega^*$	0	$\bar{w}$
1	4	0	u	0	$w\omega$	0	0	$\bar{w}\omega$	0
2	1	$\bar{u}$	0	0	$w\omega$	0	0	$w\omega$	0
2	2	0	0	w	0	$\bar{w}\omega^*$	w	0	$\bar{w}\omega^*$
2	3	0	$\bar{u}$	0	$w\omega$	0	0	$\bar{w}\omega$	0
2	4	0	0	$w\omega^*$	0	$\bar{w}$	$\bar{w}\omega^*$	0	w

$e_{5/2}$	$e_{5/2}$	A_1 1	T_1 1	2	3
1	1	0	u	0	u
1	2	u	0	u	0
2	1	$\bar{u}$	0	u	0
2	2	0	u	0	$\bar{u}$

$e_{5/2}$	$f_{3/2}$	E 1	2	T_1 1	2	3	T_2 1	2	3
1	1	0	0	w	0	$w\omega^*$	w	0	$w\omega^*$
1	2	u	0	0	$w\omega$	0	0	$w\omega$	0
1	3	0	0	$\bar{w}\omega^*$	0	$\bar{w}$	$w\omega^*$	0	w
1	4	0	$\bar{u}$	0	$\bar{w}\omega$	0	0	$w\omega$	0
2	1	$\bar{u}$	0	0	$w\omega$	0	0	$w\omega$	0
2	2	0	0	w	0	$\bar{w}\omega^*$	w	0	$\bar{w}\omega^*$
2	3	0	u	0	$\bar{w}\omega$	0	0	$w\omega$	0
2	4	0	0	$\bar{w}\omega^*$	0	w	$w\omega^*$	0	$\bar{w}$

$f_{3/2}$	$f_{3/2}$	A_1 1	A_2 1	E 1	2	T_1 1	2	3	T_1 1	2	3	T_2 1	2	3	T_2 1	2	3
1	1	0	0	0	0	w	0	$w\omega$	0	0	0	w	0	$w\omega$	0	0	0
1	2	0	0	0	u	0	$w\omega^*$	0	0	0	0	0	$w\omega^*$	0	0	0	0
1	3	0	0	0	0	0	0	0	w	0	w	0	0	0	w	0	w
1	4	w	w	0	0	0	0	0	0	w	0	0	0	0	0	w	0
2	1	0	0	0	$\bar{u}$	0	$w\omega^*$	0	0	0	0	0	$w\omega^*$	0	0	0	0
2	2	0	0	0	0	w	0	$\bar{w}\omega$	0	0	0	w	0	$\bar{w}\omega$	0	0	0
2	3	$\bar{w}$	$\bar{w}$	0	0	0	0	0	0	w	0	0	0	0	0	w	0
2	4	0	0	0	0	0	0	0	w	0	$\bar{w}$	0	0	0	w	0	$\bar{w}$
3	1	0	0	0	0	0	0	0	w	0	w	0	0	0	$\bar{w}$	0	$\bar{w}$
3	2	w	$\bar{w}$	0	0	0	0	0	0	w	0	0	0	0	0	$\bar{w}$	0
3	3	0	0	0	0	$w\omega$	0	w	0	0	0	$\bar{w}\omega$	0	$\bar{w}$	0	0	0
3	4	0	0	u	0	0	$w\omega^*$	0	0	0	0	0	$\bar{w}\omega^*$	0	0	0	0
4	1	$\bar{w}$	w	0	0	0	0	0	0	w	0	0	0	0	0	$\bar{w}$	0
4	2	0	0	0	0	0	0	0	w	0	$\bar{w}$	0	0	0	$\bar{w}$	0	w
4	3	0	0	$\bar{u}$	0	0	$w\omega^*$	0	0	0	0	0	$\bar{w}\omega^*$	0	0	0	0
4	4	0	0	0	0	$w\omega$	0	$\bar{w}$	0	0	0	$\bar{w}\omega$	0	w	0	0	0

$u = 2^{-1/2}$, $w = 4^{-1/2}$, $\omega = \exp(2\pi i/3)$

(1) Product forms: $\mathbf{D}_2 \circledtimes \mathbf{C}'_3$.

(2) Group chains: $\mathbf{I} \supset \mathbf{T} \supset \underline{\mathbf{D}_2}$, $\mathbf{I} \supset \mathbf{T} \supset (\mathbf{C}_3)$, $\mathbf{T}_d \supset \underline{\mathbf{T}} \supset \underline{\mathbf{D}_2}$, $\mathbf{T}_d \supset \underline{\mathbf{T}} \supset (\mathbf{C}_3)$,
$\mathbf{T}_h \supset \underline{\mathbf{T}} \supset \underline{\mathbf{D}_2}$, $\mathbf{T}_h \supset \underline{\mathbf{T}} \supset (\mathbf{C}_3)$, $\mathbf{O} \supset \underline{\mathbf{T}} \supset \underline{\mathbf{D}_2}$, $\mathbf{O} \supset \underline{\mathbf{T}} \supset (\mathbf{C}_3)$.

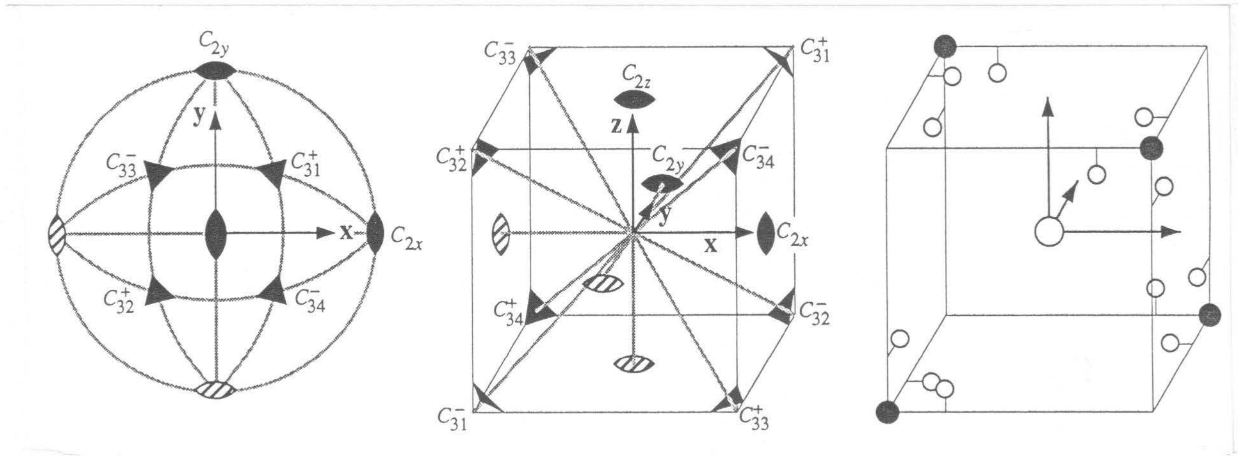
(3) Operations of G: E, (C_{2x}, C_{2y}, C_{2z}), $(C_{31}^+, C_{32}^+, C_{33}^+, C_{34}^+)$, $(C_{31}^-, C_{32}^-, C_{33}^-, C_{34}^-)$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, $(C_{2x}, C_{2y}, C_{2z}, \widetilde{C}_{2x}, \widetilde{C}_{2y}, \widetilde{C}_{2z})$, $(C_{31}^+, C_{32}^+, C_{33}^+, C_{34}^+)$,
$(\widetilde{C}_{31}^+, \widetilde{C}_{32}^+, \widetilde{C}_{33}^+, \widetilde{C}_{34}^+)$, $(C_{31}^-, C_{32}^-, C_{33}^-, C_{34}^-)$, $(\widetilde{C}_{31}^-, \widetilde{C}_{32}^-, \widetilde{C}_{33}^-, \widetilde{C}_{34}^-)$.

(5) Classes and representations: $|r| = 3$, $|\mathrm{i}| = 1$, $|I| = 4$, $|\widetilde{I}| = 3$.

(6) Subduction: no failure in subduction.

F **70** See Chapter **15**, p. 65



Examples: Possible excited state of $C(CH_3)_4$ partly rotated.

T **70**.1 Parameters
Use T **71**.1. § **16**–1, p. 68

T **70**.2 Multiplication table
Use T **71**.2. § **16**–2, p. 69

T **70**.3 Factor table
Use T **71**.3. § **16**–3, p. 70

T **70**.4 Character table
§ **16**–4, p. 71

T	E	$3C_2$	$4C_3^+$	$4C_3^-$	τ
A	1	1	1	1	a
${}^1E$	1	1	ϵ	ϵ^*	b
${}^2E$	1	1	ϵ^*	ϵ	b
T	3	-1	0	0	a
$E_{1/2}$	2	0	1	1	c
${}^1F_{3/2}$	2	0	ϵ	ϵ^*	b
${}^2F_{3/2}$	2	0	ϵ^*	ϵ	b

$\epsilon = \exp(2\pi \mathrm{i}/3)$

T **70**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

T	0	1	2	3
A	$^{\square}1$		$x^2+y^2+z^2$	xyz^a
$^1E \oplus {}^2E$			$^{\square}(x^2-y^2, 2z^2-x^2-y^2)$	
T		$^{\square}(x,y,z),(R_x,R_y,R_z)$	$^{\square}(zx,yz,xy)$	$(x^3,y^3,z^3),(xy^2,yz^2,zx^2),$ $(xz^2,yx^2,zy^2)^b$

^{a} f function: f_{xyz}; ^{b} f functions: f_{xz^2}, f_{yz^2}, $f_{z(x^2-y^2)}$, $f_{x(x^2-y^2)}$, $f_{y(x^2-y^2)}$, f_{z^3}.

T **70**.6a Bases of irreducible representations

§ **16**–6, pp. 74, 75

T	$\langle\,|j\,m\rangle\,|$
A	$|0\,0\rangle$
^1E	$\frac{1}{\sqrt{2}}|2\,0\rangle - \frac{\mathrm{i}}{\sqrt{2}}|2\,2\rangle_+$
^2E	$\frac{1}{\sqrt{2}}|2\,0\rangle + \frac{\mathrm{i}}{\sqrt{2}}|2\,2\rangle_+$
T	$\langle\,|1\,1\rangle_+, |1\,0\rangle, |1\,1\rangle_-\,|$
$E_{1/2}$	$\langle\,|\frac{1}{2}\,\frac{1}{2}\rangle, |\frac{1}{2}\,\overline{\frac{1}{2}}\rangle\,|$
$^1F_{3/2}$	$\langle\,\frac{1}{\sqrt{2}}|\frac{3}{2}\,\frac{1}{2}\rangle - \frac{\mathrm{i}}{\sqrt{2}}|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle, \quad \frac{\mathrm{i}}{\sqrt{2}}|\frac{3}{2}\,\frac{3}{2}\rangle - \frac{1}{\sqrt{2}}|\frac{3}{2}\,\overline{\frac{1}{2}}\rangle\,|$
$^2F_{3/2}$	$\langle\,\frac{1}{\sqrt{2}}|\frac{3}{2}\,\frac{1}{2}\rangle + \frac{\mathrm{i}}{\sqrt{2}}|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle, -\frac{\mathrm{i}}{\sqrt{2}}|\frac{3}{2}\,\frac{3}{2}\rangle - \frac{1}{\sqrt{2}}|\frac{3}{2}\,\overline{\frac{1}{2}}\rangle\,|$

T **70**.6b Symmetrized harmonics

Use T **71**.6b. § **16**–6, pp. 74, 75

T **70**.6c Spin harmonics

§ **16**–6, pp. 74, 75

T	$\langle\quad|$
$E_{1/2}$	$\langle\,a\,\alpha, a\,\beta\,|$
	$\langle\,\frac{1}{\sqrt{3}}\,(t^{(1)}\,\beta - t^{(2)}\,\alpha + t^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t^{(1)}\,\alpha + t^{(2)}\,\beta + t^{(3)}\,\alpha)\,|$
$^1F_{3/2}$	$\langle\,{}^1e\,\alpha, {}^1e\,\beta\,|$
	$\langle\,\frac{1}{\sqrt{3}}\,(t^{(1)}\,\beta - \omega^*\,t^{(2)}\,\alpha + \omega\,t^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t^{(1)}\,\alpha + \omega^*\,t^{(2)}\,\beta + \omega\,t^{(3)}\,\alpha)\,|$
$^2F_{3/2}$	$\langle\,{}^2e\,\alpha, {}^2e\,\beta\,|$
	$\langle\,\frac{1}{\sqrt{3}}\,(t^{(1)}\,\beta - \omega\,t^{(2)}\,\alpha + \omega^*\,t^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t^{(1)}\,\alpha + \omega\,t^{(2)}\,\beta + \omega^*\,t^{(3)}\,\alpha)\,|$

$\alpha = |\frac{1}{2}\,\frac{1}{2}\rangle$, $\beta = |\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$; $\omega = \exp(2\pi\mathrm{i}/3)$

T **70**.7 Matrix representations

§ **16**–7, p. 77

T	T	$E_{1/2}$	${}^1F_{3/2}$	${}^2F_{3/2}$
E	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
C_{2x}	$\begin{bmatrix} \bar{1} & 0 & 0 \\ 0 & \bar{1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
C_{2y}	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \bar{1} & 0 \\ 0 & 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{1} \\ 1 & 0 \end{bmatrix}$
C_{2z}	$\begin{bmatrix} \bar{1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \bar{1} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$	$\begin{bmatrix} \bar{\mathrm{i}} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$
C_{31}^+	$\begin{bmatrix} 0 & 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 & 0 \\ 0 & \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & \bar{\epsilon} \\ \epsilon^* & \epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & \delta^* \\ \mathrm{i}\delta^* & \bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & \mathrm{i}\delta \\ \bar{\delta} & \mathrm{i}\bar{\delta} \end{bmatrix}$
C_{32}^+	$\begin{bmatrix} 0 & 0 & \bar{\mathrm{i}} \\ \mathrm{i} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & \epsilon \\ \bar{\epsilon}^* & \epsilon \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & \bar{\delta}^* \\ \mathrm{i}\bar{\delta}^* & \bar{\delta}^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & \mathrm{i}\bar{\delta} \\ \delta & \mathrm{i}\bar{\delta} \end{bmatrix}$
C_{33}^+	$\begin{bmatrix} 0 & 0 & \mathrm{i} \\ \bar{\mathrm{i}} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} \epsilon & \epsilon^* \\ \bar{\epsilon} & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & \mathrm{i}\delta^* \\ \delta^* & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\delta} & \bar{\delta} \\ \mathrm{i}\delta & \bar{\delta} \end{bmatrix}$
C_{34}^+	$\begin{bmatrix} 0 & 0 & \mathrm{i} \\ \mathrm{i} & 0 & 0 \\ 0 & \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} \epsilon & \bar{\epsilon}^* \\ \epsilon & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & \mathrm{i}\bar{\delta}^* \\ \bar{\delta}^* & \mathrm{i}\delta^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\delta} & \delta \\ \mathrm{i}\bar{\delta} & \bar{\delta} \end{bmatrix}$
C_{31}^-	$\begin{bmatrix} 0 & \mathrm{i} & 0 \\ 0 & 0 & \bar{1} \\ \mathrm{i} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \epsilon & \epsilon \\ \bar{\epsilon}^* & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\delta} & \mathrm{i}\bar{\delta} \\ \delta & \bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & \bar{\delta}^* \\ \mathrm{i}\bar{\delta}^* & \mathrm{i}\delta^* \end{bmatrix}$
C_{32}^-	$\begin{bmatrix} 0 & \bar{\mathrm{i}} & 0 \\ 0 & 0 & 1 \\ \mathrm{i} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \epsilon & \bar{\epsilon} \\ \epsilon^* & \epsilon^* \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\bar{\delta} & \mathrm{i}\delta \\ \bar{\delta} & \bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \bar{\delta}^* & \delta^* \\ \mathrm{i}\delta^* & \mathrm{i}\delta^* \end{bmatrix}$
C_{33}^-	$\begin{bmatrix} 0 & \mathrm{i} & 0 \\ 0 & 0 & 1 \\ \bar{\mathrm{i}} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & \bar{\epsilon}^* \\ \epsilon & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & \delta \\ \mathrm{i}\bar{\delta} & \mathrm{i}\bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & \mathrm{i}\bar{\delta}^* \\ \bar{\delta}^* & \bar{\delta}^* \end{bmatrix}$
C_{34}^-	$\begin{bmatrix} 0 & \bar{\mathrm{i}} & 0 \\ 0 & 0 & \bar{1} \\ \bar{\mathrm{i}} & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \epsilon^* & \epsilon^* \\ \bar{\epsilon} & \epsilon \end{bmatrix}$	$\begin{bmatrix} \bar{\delta} & \bar{\delta} \\ \mathrm{i}\delta & \mathrm{i}\bar{\delta} \end{bmatrix}$	$\begin{bmatrix} \mathrm{i}\delta^* & \mathrm{i}\delta^* \\ \delta^* & \bar{\delta}^* \end{bmatrix}$

$\delta = 2^{-1/2}\exp(2\pi\mathrm{i}/24)$, $\epsilon = 2^{-1/2}\exp(2\pi\mathrm{i}/8)$

T **70**.8 Direct products of representations

§ **16**–8, p. 81

T	A	^1E	^2E	T	$E_{1/2}$	$^1F_{3/2}$	$^2F_{3/2}$
A	A	^1E	^2E	T	$E_{1/2}$	$^1F_{3/2}$	$^2F_{3/2}$
^1E		^2E	A	T	$^1F_{3/2}$	$^2F_{3/2}$	$E_{1/2}$
^2E			^1E	T	$^2F_{3/2}$	$E_{1/2}$	$^1F_{3/2}$
T				$A \oplus {}^1E \oplus {}^2E$ $\oplus T \oplus \{T\}$	$E_{1/2}$ $\oplus {}^1F_{3/2} \oplus {}^2F_{3/2}$	$E_{1/2}$ $\oplus {}^1F_{3/2} \oplus {}^2F_{3/2}$	$E_{1/2}$ $\oplus {}^1F_{3/2} \oplus {}^2F_{3/2}$
$E_{1/2}$					$\{A\} \oplus T$	$^1E \oplus T$	$^2E \oplus T$
$^1F_{3/2}$						$\{^2E\} \oplus T$	$A \oplus T$
$^2F_{3/2}$							$\{^1E\} \oplus T$

T **70**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

T	$\mathbf{D}_2$	$(\mathbf{C}_3)$	$\mathbf{C}_2$
A	A	A	A
^1E	A	^2E	A
^2E	A	^1E	A
T	$B_1 \oplus B_2 \oplus B_3$	$A \oplus {}^1E \oplus {}^2E$	$A \oplus 2B$
$E_{1/2}$	$E_{1/2}$	$^1E_{1/2} \oplus {}^2E_{1/2}$	$^1E_{1/2} \oplus {}^2E_{1/2}$
$^1F_{3/2}$	$E_{1/2}$	$^1E_{1/2} \oplus A_{3/2}$	$^1E_{1/2} \oplus {}^2E_{1/2}$
$^2F_{3/2}$	$E_{1/2}$	$^2E_{1/2} \oplus A_{3/2}$	$^1E_{1/2} \oplus {}^2E_{1/2}$

T **70**.10 Subduction from O(3)

§ **16**–10, p. 82

j	**T**
$6n$	$(n+1)\,A \oplus n\,(^1E \oplus {}^2E \oplus 3T)$
$6n+1$	$n\,(A \oplus {}^1E \oplus {}^2E \oplus 2T) \oplus (n+1)\,T$
$6n+2$	$n\,(A \oplus 2T) \oplus (n+1)(^1E \oplus {}^2E \oplus T)$
$6n+3$	$(n+1)(A \oplus 2T) \oplus n\,(^1E \oplus {}^2E \oplus T)$
$6n+4$	$(n+1)(A \oplus {}^1E \oplus {}^2E \oplus 2T) \oplus n\,T$
$6n+5$	$n\,A \oplus (n+1)(^1E \oplus {}^2E \oplus 3T)$
$3n+\frac{1}{2}$	$(n+1)\,E_{1/2} \oplus n\,(^1F_{3/2} \oplus {}^2F_{3/2})$
$3n+\frac{3}{2}$	$n\,E_{1/2} \oplus (n+1)(^1F_{3/2} \oplus {}^2F_{3/2})$
$3n+\frac{5}{2}$	$(n+1)(E_{1/2} \oplus {}^1F_{3/2} \oplus {}^2F_{3/2})$

$n = 0, 1, 2, \ldots$

T **70**.11 Clebsch–Gordan coefficients

§ **16**–11, p. 83 **T**

^1e	t	T 1	2	3
1	1	1	0	0
1	2	0	ω	0
1	3	0	0	ω^*

^1e	$e_{1/2}$	$^1F_{3/2}$ 1	2
1	1	1	0
1	2	0	1

^1e	$^1f_{3/2}$	$^2F_{3/2}$ 1	2
1	1	1	0
1	2	0	1

^1e	$^2f_{3/2}$	$E_{1/2}$ 1	2
1	1	1	0
1	2	0	1

$\omega = \exp(2\pi i/3)$

↠

T **70**.11 Clebsch–Gordan coefficients *(cont.)*

^2e	t	T 1	2	3
1	1	1	0	0
1	2	0	ω^*	0
1	3	0	0	ω

^2e	$e_{1/2}$	$^2F_{3/2}$ 1	2
1	1	1	0
1	2	0	1

^2e	$^1f_{3/2}$	$E_{1/2}$ 1	2
1	1	1	0
1	2	0	1

^2e	$^2f_{3/2}$	$^1F_{3/2}$ 1	2
1	1	1	0
1	2	0	1

t	t	A 1	^1E 1	^2E 1	T 1	2	3	T 1	2	3
1	1	v	v	v	0	0	0	0	0	0
1	2	0	0	0	0	0	$\bar{1}$	0	0	0
1	3	0	0	0	0	0	0	0	$\bar{1}$	0
2	1	0	0	0	0	0	0	0	0	$\bar{1}$
2	2	$\bar{\mathrm{v}}$	$\bar{\mathrm{v}}\omega^*$	$\bar{\mathrm{v}}\omega$	0	0	0	0	0	0
2	3	0	0	0	1	0	0	0	0	0
3	1	0	0	0	0	$\bar{1}$	0	0	0	0
3	2	0	0	0	0	0	0	1	0	0
3	3	$\bar{\mathrm{v}}$	$\bar{\mathrm{v}}\omega$	$\bar{\mathrm{v}}\omega^*$	0	0	0	0	0	0

t	$e_{1/2}$	$E_{1/2}$ 1	2	$^1F_{1/2}$ 1	2	$^2F_{1/2}$ 1	2
1	1	0	$\bar{\mathrm{v}}$	0	$\bar{\mathrm{v}}$	0	$\bar{\mathrm{v}}$
1	2	v	0	v	0	v	0
2	1	$\bar{\mathrm{v}}$	0	$\bar{\mathrm{v}}\omega^*$	0	$\bar{\mathrm{v}}\omega$	0
2	2	0	v	0	$\mathrm{v}\omega^*$	0	$\mathrm{v}\omega$
3	1	0	v	0	$\mathrm{v}\omega$	0	$\mathrm{v}\omega^*$
3	2	v	0	$\mathrm{v}\omega$	0	$\mathrm{v}\omega^*$	0

t	$^1f_{1/2}$	$E_{1/2}$ 1	2	$^1F_{1/2}$ 1	2	$^2F_{1/2}$ 1	2
1	1	0	$\bar{\mathrm{v}}$	0	$\bar{\mathrm{v}}$	0	$\bar{\mathrm{v}}$
1	2	v	0	v	0	v	0
2	1	$\bar{\mathrm{v}}\omega$	0	$\bar{\mathrm{v}}$	0	$\bar{\mathrm{v}}\omega^*$	0
2	2	0	$\mathrm{v}\omega$	0	v	0	$\mathrm{v}\omega^*$
3	1	0	$\mathrm{v}\omega^*$	0	v	0	$\mathrm{v}\omega$
3	2	$\mathrm{v}\omega^*$	0	v	0	$\mathrm{v}\omega$	0

t	$^2f_{1/2}$	$E_{1/2}$ 1	2	$^1F_{1/2}$ 1	2	$^2F_{1/2}$ 1	2
1	1	0	$\bar{\mathrm{v}}$	0	$\bar{\mathrm{v}}$	0	$\bar{\mathrm{v}}$
1	2	v	0	v	0	v	0
2	1	$\bar{\mathrm{v}}\omega^*$	0	$\bar{\mathrm{v}}\omega$	0	$\bar{\mathrm{v}}$	0
2	2	0	$\mathrm{v}\omega^*$	0	$\mathrm{v}\omega$	0	v
3	1	0	$\mathrm{v}\omega$	0	$\mathrm{v}\omega^*$	0	v
3	2	$\mathrm{v}\omega$	0	$\mathrm{v}\omega^*$	0	v	0

$e_{1/2}$	$e_{1/2}$	A 1	T 1	2	3
1	1	0	u	0	u
1	2	u	0	u	0
2	1	$\bar{\mathrm{u}}$	0	u	0
2	2	0	u	0	$\bar{\mathrm{u}}$

$e_{1/2}$	$^1f_{3/2}$	^1E 1	T 1	2	3
1	1	0	u	0	$\mathrm{u}\omega^*$
1	2	u	0	$\mathrm{u}\omega$	0
2	1	$\bar{\mathrm{u}}$	0	$\mathrm{u}\omega$	0
2	2	0	u	0	$\bar{\mathrm{u}}\omega^*$

$e_{1/2}$	$^2f_{3/2}$	^2E 1	T 1	2	3
1	1	0	u	0	$\mathrm{u}\omega$
1	2	u	0	$\mathrm{u}\omega^*$	0
2	1	$\bar{\mathrm{u}}$	0	$\mathrm{u}\omega^*$	0
2	2	0	u	0	$\bar{\mathrm{u}}\omega$

$^1f_{3/2}$	$^1f_{3/2}$	^2E 1	T 1	2	3
1	1	0	u	0	$\mathrm{u}\omega$
1	2	u	0	$\mathrm{u}\omega^*$	0
2	1	$\bar{\mathrm{u}}$	0	$\mathrm{u}\omega^*$	0
2	2	0	u	0	$\bar{\mathrm{u}}\omega$

$^1f_{3/2}$	$^2f_{3/2}$	A 1	T 1	2	3
1	1	0	u	0	u
1	2	u	0	u	0
2	1	$\bar{\mathrm{u}}$	0	u	0
2	2	0	u	0	$\bar{\mathrm{u}}$

$^2f_{3/2}$	$^2f_{3/2}$	^1E 1	T 1	2	3
1	1	0	u	0	$\mathrm{u}\omega^*$
1	2	u	0	$\mathrm{u}\omega$	0
2	1	$\bar{\mathrm{u}}$	0	$\mathrm{u}\omega$	0
2	2	0	u	0	$\bar{\mathrm{u}}\omega^*$

$\mathrm{u} = 2^{-1/2}$, $\mathrm{v} = 3^{-1/2}$, $\omega = \exp(2\pi\mathrm{i}/3)$

(1) Product forms: $\mathbf{O} \otimes \mathbf{C}_i$.

(2) Group chains: $\mathbf{O}_h \supset (\underline{\mathbf{T}_d})$, $\mathbf{O}_h \supset \underline{\mathbf{T}_h}$, $\mathbf{O}_h \supset \underline{\mathbf{O}}$, $\mathbf{O}_h \supset (\underline{\mathbf{D}_{3d}})$, $\mathbf{O}_h \supset (\underline{\mathbf{D}_{4h}})$.

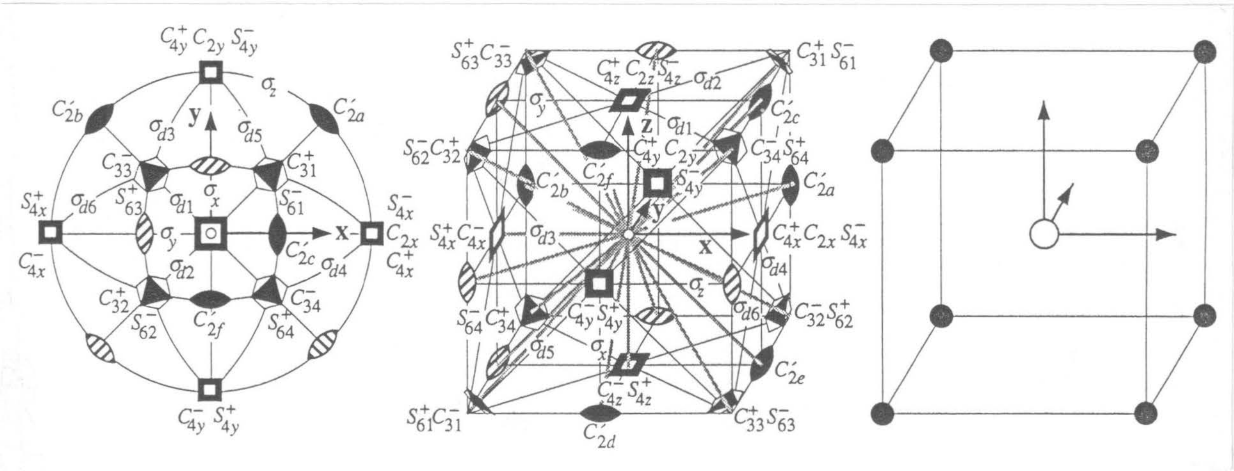
(3) Operations of G: E, (C_{2x}, C_{2y}, C_{2z}), $(C_{31}^+, C_{32}^+, C_{33}^+, C_{34}^+, C_{31}^-, C_{32}^-, C_{33}^-, C_{34}^-)$,
$(C_{4x}^+, C_{4y}^+, C_{4z}^+, C_{4x}^-, C_{4y}^-, C_{4z}^-)$, $(C'_{2a}, C'_{2b}, C'_{2c}, C'_{2d}, C'_{2e}, C'_{2f})$,
i, $(\sigma_x, \sigma_y, \sigma_z)$, $(S_{61}^-, S_{62}^-, S_{63}^-, S_{64}^-, S_{61}^+, S_{62}^+, S_{63}^+, S_{64}^+)$,
$(S_{4x}^-, S_{4y}^-, S_{4z}^-, S_{4x}^+, S_{4y}^+, S_{4z}^+)$, $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \sigma_{d6})$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, $(C_{2x}, C_{2y}, C_{2z}, \widetilde{C}_{2x}, \widetilde{C}_{2y}, \widetilde{C}_{2z})$,
$(C_{31}^+, C_{32}^+, C_{33}^+, C_{34}^+, C_{31}^-, C_{32}^-, C_{33}^-, C_{34}^-)$, $(\widetilde{C}_{31}^+, \widetilde{C}_{32}^+, \widetilde{C}_{33}^+, \widetilde{C}_{34}^+, \widetilde{C}_{31}^-, \widetilde{C}_{32}^-, \widetilde{C}_{33}^-, \widetilde{C}_{34}^-)$,
$(C_{4x}^+, C_{4y}^+, C_{4z}^+, C_{4x}^-, C_{4y}^-, C_{4z}^-)$, $(\widetilde{C}_{4x}^+, \widetilde{C}_{4y}^+, \widetilde{C}_{4z}^+, \widetilde{C}_{4x}^-, \widetilde{C}_{4y}^-, \widetilde{C}_{4z}^-)$,
$(C'_{2a}, C'_{2b}, C'_{2c}, C'_{2d}, C'_{2e}, C'_{2f}, \widetilde{C}'_{2a}, \widetilde{C}'_{2b}, \widetilde{C}'_{2c}, \widetilde{C}'_{2d}, \widetilde{C}'_{2e}, \widetilde{C}'_{2f})$,
i, $\widetilde{i}$, $(\sigma_x, \sigma_y, \sigma_z, \widetilde{\sigma}_x, \widetilde{\sigma}_y, \widetilde{\sigma}_z)$,
$(S_{61}^-, S_{62}^-, S_{63}^-, S_{64}^-, S_{61}^+, S_{62}^+, S_{63}^+, S_{64}^+)$, $(\widetilde{S}_{61}^-, \widetilde{S}_{62}^-, \widetilde{S}_{63}^-, \widetilde{S}_{64}^-, \widetilde{S}_{61}^+, \widetilde{S}_{62}^+, \widetilde{S}_{63}^+, \widetilde{S}_{64}^+)$,
$(S_{4x}^-, S_{4y}^-, S_{4z}^-, S_{4x}^+, S_{4y}^+, S_{4z}^+)$, $(\widetilde{S}_{4x}^-, \widetilde{S}_{4y}^-, \widetilde{S}_{4z}^-, \widetilde{S}_{4x}^+, \widetilde{S}_{4y}^+, \widetilde{S}_{4z}^+)$,
$(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \sigma_{d6}, \widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3}, \widetilde{\sigma}_{d4}, \widetilde{\sigma}_{d5}, \widetilde{\sigma}_{d6})$.

(5) Classes and representations: $|r| = 6$, $|\mathrm{i}| = 4$, $|I| = 10$, $|\widetilde{I}| = 6$.

(6) Subduction: $\mathbf{D}_3\ (E, C_{31}^+, C_{31}^-, C_{2b}, C_{2f}, C_{2e})$, $\mathbf{D}_3\ (E, C_{32}^+, C_{32}^-, C_{2b}, C_{2d}, C_{2c})$,
$\mathbf{D}_{3d}\ (E, C_{31}^+, C_{31}^-, C_{2b}, C_{2f}, C_{2e}, i, S_{61}^-, S_{61}^+, \sigma_{d2}, \sigma_{d6}, \sigma_{d5})$,
$\mathbf{D}_{3d}\ (E, C_{32}^+, C_{32}^-, C_{2b}, C_{2d}, C_{2c}, i, S_{62}^-, S_{62}^+, \sigma_{d2}, \sigma_{d4}, \sigma_{d3})$,
$\mathbf{C}_{3v}\ (E, C_{31}^+, C_{31}^-, \sigma_{d2}, \sigma_{d6}, \sigma_{d5})$, $\mathbf{C}_{3v}\ (E, C_{32}^+, C_{32}^-, \sigma_{d2}, \sigma_{d4}, \sigma_{d3})$.

F **71** See Chapter **15**, p. 65



Examples: Sulphur hexafluoride SF_6, $PtCl_6^{2-}$, FeF_6^{3-}, $Fe(CN)_6^{3-}$, $B_6H_6^{2-}$ (*closo*-borane).

T **71**.0 Subgroup elements § **16**–0, p. 68

$\mathbf{O}_h$	$\mathbf{T}_d$	$\mathbf{T}_h$	$\mathbf{T}$	$\mathbf{O}$	$\mathbf{C}_{4h}$	$\mathbf{C}_{2h}$	$\mathbf{C}_{4v}$	$\mathbf{C}_{2v}$	$\mathbf{D}_{2d}$	$\mathbf{D}_{4h}$	$\mathbf{D}_{2h}$	$\mathbf{D}_4$	$\mathbf{D}_2$	$\mathbf{S}_4$	$\mathbf{C}_s$	$\mathbf{C}_i$	$\mathbf{C}_4$	$\mathbf{C}_2$
E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E	E
C_{2x}	C_{2x}	C_{2x}	C_{2x}	C_{2x}					C'_{21}	C'_{21}	C_{2x}	C'_{21}	C_{2x}					
C_{2y}	C_{2y}	C_{2y}	C_{2y}	C_{2y}					C'_{22}	C'_{22}	C_{2y}	C'_{22}	C_{2y}					
C_{2z}	C_{2z}	C_{2z}	C_{2z}	C_{2z}	C_2	C_2	C_2	C_2	C_2	C_2	C_{2z}	C_2	C_{2z}	C_2			C_2	C_2
C_{31}^+	C_{31}^+	C_{31}^+	C_{31}^+	C_{31}^+														
C_{32}^+	C_{32}^+	C_{32}^+	C_{32}^+	C_{32}^+														
C_{33}^+	C_{33}^+	C_{33}^+	C_{33}^+	C_{33}^+														
C_{34}^+	C_{34}^+	C_{34}^+	C_{34}^+	C_{34}^+														
C_{31}^-	C_{31}^-	C_{31}^-	C_{31}^-	C_{31}^-														
C_{32}^-	C_{32}^-	C_{32}^-	C_{32}^-	C_{32}^-														
C_{33}^-	C_{33}^-	C_{33}^-	C_{33}^-	C_{33}^-														
C_{34}^-	C_{34}^-	C_{34}^-	C_{34}^-	C_{34}^-														
C_{4x}^+				C_{4x}^+														
C_{4y}^+				C_{4y}^+														
C_{4z}^+				C_{4z}^+	C_4^+		C_4^+			C_4^+		C_4^+					C_4^+	
C_{4x}^-				C_{4x}^-														
C_{4y}^-				C_{4y}^-														
C_{4z}^-				C_{4z}^-	C_4^-		C_4^-			C_4^-		C_4^-					C_4^-	
C'_{2a}				C'_{2a}						C''_{21}		C''_{21}						
C'_{2b}				C'_{2b}						C''_{22}		C''_{22}						
C'_{2c}				C'_{2c}														
C'_{2d}				C'_{2d}														
C'_{2e}				C'_{2e}														
C'_{2f}				C'_{2f}														
i		i			i	i				i	i					i		
σ_x		σ_x					σ_{v1}	σ_x		σ_{v1}	σ_x							
σ_y		σ_y					σ_{v2}	σ_y		σ_{v2}	σ_y							
σ_z		σ_z			σ_h	σ_h				σ_h	σ_z				σ_h			
S_{61}^-		S_{61}^-																
S_{62}^-		S_{62}^-																
S_{63}^-		S_{63}^-																
S_{64}^-		S_{64}^-																
S_{61}^+		S_{61}^+																
S_{62}^+		S_{62}^+																
S_{63}^+		S_{63}^+																
S_{64}^+		S_{64}^+																
S_{4x}^-	S_{4x}^-																	
S_{4y}^-	S_{4y}^-																	
S_{4z}^-	S_{4z}^-				S_4^-				S_4^-	S_4^-				S_4^-				
S_{4x}^+	S_{4x}^+																	
S_{4y}^+	S_{4y}^+																	
S_{4z}^+	S_{4z}^+				S_4^+				S_4^+	S_4^+				S_4^+				
σ_{d1}	σ_{d1}						σ_{d1}		σ_{d1}	σ_{d1}								
σ_{d2}	σ_{d2}						σ_{d2}		σ_{d2}	σ_{d2}								
σ_{d3}	σ_{d3}																	
σ_{d4}	σ_{d4}																	
σ_{d5}	σ_{d5}																	
σ_{d6}	σ_{d6}																	

T **71**.1 Parameters

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$\mathbf{O}_h$		α	β	γ	ϕ	$\mathbf{n}$	λ	$\mathbf{\Lambda}$
E	i	0	0	0	0	$(0\ \ 0\ \ 0)$	$[\![\ 1,$	$(0\ \ 0\ \ 0)]\!]$
C_{2x}	σ_x	0	π	π	π	$(1\ \ 0\ \ 0)$	$[\![\ 0,$	$(1\ \ 0\ \ 0)]\!]$
C_{2y}	σ_y	0	π	0	π	$(0\ \ 1\ \ 0)$	$[\![\ 0,$	$(0\ \ 1\ \ 0)]\!]$
C_{2z}	σ_z	0	0	π	π	$(0\ \ 0\ \ 1)$	$[\![\ 0,$	$(0\ \ 0\ \ 1)]\!]$
C_{31}^+	S_{61}^-	0	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$(\frac{1}{\sqrt{3}}\ \ \frac{1}{\sqrt{3}}\ \ \frac{1}{\sqrt{3}})$	$[\![\ \frac{1}{2},$	$(\frac{1}{2}\ \ \frac{1}{2}\ \ \frac{1}{2})]\!]$
C_{32}^+	S_{62}^-	π	$\frac{\pi}{2}$	$-\frac{\pi}{2}$	$\frac{2\pi}{3}$	$(-\frac{1}{\sqrt{3}}\ -\frac{1}{\sqrt{3}}\ \ \frac{1}{\sqrt{3}})$	$[\![\ \frac{1}{2},$	$(-\frac{1}{2}\ -\frac{1}{2}\ \ \frac{1}{2})]\!]$
C_{33}^+	S_{63}^-	π	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$(\frac{1}{\sqrt{3}}\ -\frac{1}{\sqrt{3}}\ -\frac{1}{\sqrt{3}})$	$[\![\ \frac{1}{2},$	$(\frac{1}{2}\ -\frac{1}{2}\ -\frac{1}{2})]\!]$
C_{34}^+	S_{64}^-	0	$\frac{\pi}{2}$	$-\frac{\pi}{2}$	$\frac{2\pi}{3}$	$(-\frac{1}{\sqrt{3}}\ \ \frac{1}{\sqrt{3}}\ -\frac{1}{\sqrt{3}})$	$[\![\ \frac{1}{2},$	$(-\frac{1}{2}\ \ \frac{1}{2}\ -\frac{1}{2})]\!]$
C_{31}^-	S_{61}^+	$\frac{\pi}{2}$	$\frac{\pi}{2}$	π	$\frac{2\pi}{3}$	$(-\frac{1}{\sqrt{3}}\ -\frac{1}{\sqrt{3}}\ -\frac{1}{\sqrt{3}})$	$[\![\ \frac{1}{2},$	$(-\frac{1}{2}\ -\frac{1}{2}\ -\frac{1}{2})]\!]$
C_{32}^-	S_{62}^+	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	0	$\frac{2\pi}{3}$	$(\frac{1}{\sqrt{3}}\ \ \frac{1}{\sqrt{3}}\ -\frac{1}{\sqrt{3}})$	$[\![\ \frac{1}{2},$	$(\frac{1}{2}\ \ \frac{1}{2}\ -\frac{1}{2})]\!]$
C_{33}^-	S_{63}^+	$\frac{\pi}{2}$	$\frac{\pi}{2}$	0	$\frac{2\pi}{3}$	$(-\frac{1}{\sqrt{3}}\ \ \frac{1}{\sqrt{3}}\ \ \frac{1}{\sqrt{3}})$	$[\![\ \frac{1}{2},$	$(-\frac{1}{2}\ \ \frac{1}{2}\ \ \frac{1}{2})]\!]$
C_{34}^-	S_{64}^+	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	π	$\frac{2\pi}{3}$	$(\frac{1}{\sqrt{3}}\ -\frac{1}{\sqrt{3}}\ \ \frac{1}{\sqrt{3}})$	$[\![\ \frac{1}{2},$	$(\frac{1}{2}\ -\frac{1}{2}\ \ \frac{1}{2})]\!]$
C_{4x}^+	S_{4x}^-	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$(1\ \ 0\ \ 0)$	$[\![\frac{1}{\sqrt{2}},$	$(\frac{1}{\sqrt{2}}\ \ 0\ \ 0)]\!]$
C_{4y}^+	S_{4y}^-	0	$\frac{\pi}{2}$	0	$\frac{\pi}{2}$	$(0\ \ 1\ \ 0)$	$[\![\frac{1}{\sqrt{2}},$	$(0\ \ \frac{1}{\sqrt{2}}\ \ 0)]\!]$
C_{4z}^+	S_{4z}^-	0	0	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$(0\ \ 0\ \ 1)$	$[\![\frac{1}{\sqrt{2}},$	$(0\ \ 0\ \ \frac{1}{\sqrt{2}})]\!]$
C_{4x}^-	S_{4x}^+	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	$(-1\ \ 0\ \ 0)$	$[\![\frac{1}{\sqrt{2}},$	$(-\frac{1}{\sqrt{2}}\ \ 0\ \ 0)]\!]$
C_{4y}^-	S_{4y}^+	π	$\frac{\pi}{2}$	π	$\frac{\pi}{2}$	$(0\ -1\ \ 0)$	$[\![\frac{1}{\sqrt{2}},$	$(0\ -\frac{1}{\sqrt{2}}\ \ 0)]\!]$
C_{4z}^-	S_{4z}^+	0	0	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	$(0\ \ 0\ -1)$	$[\![\frac{1}{\sqrt{2}},$	$(0\ \ 0\ -\frac{1}{\sqrt{2}})]\!]$
C_{2a}'	σ_{d1}	0	π	$\frac{\pi}{2}$	π	$(\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)$	$[\![\ 0,$	$(\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)]\!]$
C_{2b}'	σ_{d2}	0	π	$-\frac{\pi}{2}$	π	$(-\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)$	$[\![\ 0,$	$(-\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\ \ 0)]\!]$
C_{2c}'	σ_{d3}	0	$\frac{\pi}{2}$	π	π	$(\frac{1}{\sqrt{2}}\ \ 0\ \ \frac{1}{\sqrt{2}})$	$[\![\ 0,$	$(\frac{1}{\sqrt{2}}\ \ 0\ \ \frac{1}{\sqrt{2}})]\!]$
C_{2d}'	σ_{d4}	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{2}$	π	$(0\ -\frac{1}{\sqrt{2}}\ -\frac{1}{\sqrt{2}})$	$[\![\ 0,$	$(0\ -\frac{1}{\sqrt{2}}\ -\frac{1}{\sqrt{2}})]\!]$
C_{2e}'	σ_{d5}	π	$\frac{\pi}{2}$	0	π	$(\frac{1}{\sqrt{2}}\ \ 0\ -\frac{1}{\sqrt{2}})$	$[\![\ 0,$	$(\frac{1}{\sqrt{2}}\ \ 0\ -\frac{1}{\sqrt{2}})]\!]$
C_{2f}'	σ_{d6}	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	$-\frac{\pi}{2}$	π	$(0\ -\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}})$	$[\![\ 0,$	$(0\ -\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}})]\!]$

T **71**.2 Multiplication table

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$\mathbf{O}_h$	E	C_{2x}	C_{2y}	C_{2z}	C_{31}^+	C_{32}^+	C_{33}^+	C_{34}^+	C_{31}^-	C_{32}^-	C_{33}^-	C_{34}^-	C_{4x}^+	C_{4y}^+	C_{4z}^+	C_{4x}^-	C_{4y}^-	C_{4z}^-	C'_{2a}	C'_{2b}	C'_{2c}	C'_{2d}	C'_{2e}	C'_{2f}
E	E	C_{2x}	C_{2y}	C_{2z}	C_{31}^+	C_{32}^+	C_{33}^+	C_{34}^+	C_{31}^-	C_{32}^-	C_{33}^-	C_{34}^-	C_{4x}^+	C_{4y}^+	C_{4z}^+	C_{4x}^-	C_{4y}^-	C_{4z}^-	C'_{2a}	C'_{2b}	C'_{2c}	C'_{2d}	C'_{2e}	C'_{2f}
C_{2x}	C_{2x}	E	C_{2z}	C_{2y}	C_{34}^+	C_{33}^+	C_{32}^+	C_{31}^+	C_{32}^-	C_{31}^-	C_{34}^-	C_{33}^-	C_{4x}^-	C'_{2c}	C'_{2b}	C_{4x}^+	C'_{2e}	C'_{2a}	C_{4z}^-	C_{4z}^+	C_{4y}^+	C'_{2f}	C_{4y}^-	C'_{2d}
C_{2y}	C_{2y}	C_{2z}	E	C_{2x}	C_{32}^+	C_{31}^+	C_{34}^+	C_{33}^+	C_{33}^-	C_{34}^-	C_{31}^-	C_{32}^-	C'_{2f}	C_{4y}^-	C'_{2a}	C'_{2d}	C_{4y}^+	C'_{2b}	C_{4z}^+	C_{4z}^-	C'_{2e}	C_{4x}^-	C'_{2c}	C_{4x}^+
C_{2z}	C_{2z}	C_{2y}	C_{2x}	E	C_{33}^+	C_{34}^+	C_{31}^+	C_{32}^+	C_{34}^-	C_{33}^-	C_{32}^-	C_{31}^-	C'_{2d}	C'_{2e}	C_{4z}^-	C'_{2f}	C'_{2c}	C_{4z}^+	C'_{2b}	C'_{2a}	C_{4y}^-	C_{4x}^+	C_{4y}^+	C_{4x}^-
C_{31}^+	C_{31}^+	C_{32}^+	C_{33}^+	C_{34}^+	C_{31}^-	C_{34}^-	C_{32}^-	C_{33}^-	E	C_{2y}	C_{2z}	C_{2x}	C'_{2a}	C'_{2d}	C'_{2c}	C_{4z}^+	C_{4x}^+	C_{4y}^+	C_{4y}^-	C'_{2e}	C_{4x}^-	C_{4z}^-	C'_{2f}	C'_{2b}
C_{32}^+	C_{32}^+	C_{31}^+	C_{34}^+	C_{33}^+	C_{33}^-	C_{32}^-	C_{34}^-	C_{31}^-	C_{2y}	E	C_{2x}	C_{2z}	C_{4z}^+	C_{4x}^-	C'_{2e}	C'_{2a}	C'_{2f}	C_{4y}^-	C_{4y}^+	C'_{2c}	C'_{2d}	C'_{2b}	C_{4x}^+	C_{4z}^-
C_{33}^+	C_{33}^+	C_{34}^+	C_{31}^+	C_{32}^+	C_{34}^-	C_{31}^-	C_{33}^-	C_{32}^-	C_{2z}	C_{2x}	E	C_{2y}	C'_{2b}	C_{4x}^+	C_{4y}^-	C_{4z}^-	C'_{2d}	C'_{2e}	C'_{2c}	C_{4y}^+	C'_{2f}	C_{4z}^+	C_{4x}^-	C'_{2a}
C_{34}^+	C_{34}^+	C_{33}^+	C_{32}^+	C_{31}^+	C_{32}^-	C_{33}^-	C_{31}^-	C_{34}^-	C_{2x}	C_{2z}	C_{2y}	E	C_{4z}^-	C'_{2f}	C_{4y}^+	C'_{2b}	C_{4x}^-	C'_{2c}	C'_{2e}	C_{4y}^-	C_{4x}^+	C'_{2a}	C'_{2d}	C_{4z}^+
C_{31}^-	C_{31}^-	C_{34}^-	C_{32}^-	C_{33}^-	E	C_{2x}	C_{2y}	C_{2z}	C_{31}^+	C_{33}^+	C_{34}^+	C_{32}^+	C_{4y}^-	C_{4z}^-	C_{4x}^-	C'_{2c}	C'_{2a}	C'_{2d}	C_{4x}^+	C'_{2f}	C_{4z}^+	C_{4y}^+	C'_{2b}	C'_{2e}
C_{32}^-	C_{32}^-	C_{33}^-	C_{31}^-	C_{34}^-	C_{2x}	E	C_{2z}	C_{2y}	C_{34}^+	C_{32}^+	C_{31}^+	C_{33}^+	C'_{2e}	C'_{2a}	C_{4x}^+	C_{4y}^+	C_{4z}^-	C'_{2f}	C_{4x}^-	C'_{2d}	C'_{2b}	C'_{2c}	C_{4z}^+	C_{4y}^-
C_{33}^-	C_{33}^-	C_{32}^-	C_{34}^-	C_{31}^-	C_{2y}	C_{2z}	E	C_{2x}	C_{32}^+	C_{34}^+	C_{33}^+	C_{31}^+	C_{4y}^+	C'_{2b}	C'_{2d}	C'_{2e}	C_{4z}^+	C_{4x}^-	C'_{2f}	C_{4x}^+	C'_{2a}	C_{4y}^-	C_{4z}^-	C'_{2c}
C_{34}^-	C_{34}^-	C_{31}^-	C_{33}^-	C_{32}^-	C_{2z}	C_{2y}	C_{2x}	E	C_{33}^+	C_{31}^+	C_{32}^+	C_{34}^+	C'_{2c}	C_{4z}^+	C'_{2f}	C_{4y}^-	C'_{2b}	C_{4x}^+	C'_{2d}	C_{4x}^-	C_{4z}^-	C'_{2e}	C'_{2a}	C_{4y}^+
C_{4x}^+	C_{4x}^+	C_{4x}^-	C'_{2d}	C'_{2f}	C'_{2c}	C_{4y}^-	C'_{2e}	C_{4y}^+	C_{4z}^-	C'_{2a}	C_{4z}^+	C'_{2b}	C_{2x}	C_{31}^+	C_{34}^-	E	C_{33}^+	C_{32}^-	C_{31}^-	C_{33}^-	C_{34}^+	C_{2z}	C_{32}^+	C_{2y}
C_{4y}^+	C_{4y}^+	C'_{2e}	C_{4y}^-	C'_{2c}	C'_{2a}	C_{4z}^+	C_{4z}^-	C'_{2b}	C_{4x}^-	C'_{2f}	C'_{2d}	C_{4x}^+	C_{32}^-	C_{2y}	C_{31}^+	C_{33}^-	E	C_{34}^+	C_{32}^+	C_{33}^+	C_{2x}	C_{31}^-	C_{2z}	C_{34}^-
C_{4z}^+	C_{4z}^+	C'_{2a}	C'_{2b}	C_{4z}^-	C'_{2d}	C'_{2f}	C_{4x}^+	C_{4x}^-	C_{4y}^-	C_{4y}^+	C'_{2e}	C'_{2c}	C_{31}^+	C_{33}^-	C_{2z}	C_{32}^+	C_{34}^-	E	C_{2y}	C_{2x}	C_{31}^-	C_{33}^+	C_{32}^-	C_{34}^+
C_{4x}^-	C_{4x}^-	C_{4x}^+	C'_{2f}	C'_{2d}	C_{4y}^+	C'_{2e}	C_{4y}^-	C'_{2c}	C'_{2a}	C_{4z}^-	C'_{2b}	C_{4z}^+	E	C_{34}^+	C_{33}^-	C_{2x}	C_{32}^+	C_{31}^-	C_{32}^-	C_{34}^-	C_{31}^+	C_{2y}	C_{33}^+	C_{2z}
C_{4y}^-	C_{4y}^-	C'_{2c}	C_{4y}^+	C'_{2e}	C_{4z}^+	C'_{2a}	C'_{2b}	C_{4z}^-	C'_{2d}	C_{4x}^+	C_{4x}^-	C'_{2f}	C_{34}^-	E	C_{32}^+	C_{31}^-	C_{2y}	C_{33}^+	C_{31}^+	C_{34}^+	C_{2z}	C_{33}^-	C_{2x}	C_{32}^-
C_{4z}^-	C_{4z}^-	C'_{2b}	C'_{2a}	C_{4z}^+	C_{4x}^+	C_{4x}^-	C'_{2d}	C'_{2f}	C'_{2c}	C'_{2e}	C_{4y}^+	C_{4y}^-	C_{33}^+	C_{32}^-	E	C_{34}^+	C_{31}^-	C_{2z}	C_{2x}	C_{2y}	C_{34}^-	C_{31}^+	C_{33}^-	C_{32}^+
C'_{2a}	C'_{2a}	C_{4z}^+	C_{4z}^-	C'_{2b}	C_{4x}^-	C_{4x}^+	C'_{2f}	C'_{2d}	C_{4y}^+	C_{4y}^-	C'_{2c}	C'_{2e}	C_{32}^+	C_{31}^-	C_{2x}	C_{31}^+	C_{32}^-	C_{2y}	E	C_{2z}	C_{33}^-	C_{34}^+	C_{34}^-	C_{33}^+
C'_{2b}	C'_{2b}	C_{4z}^-	C_{4z}^+	C'_{2a}	C'_{2f}	C'_{2d}	C_{4x}^-	C_{4x}^+	C'_{2e}	C'_{2c}	C_{4y}^-	C_{4y}^+	C_{34}^+	C_{34}^-	C_{2y}	C_{33}^+	C_{33}^-	C_{2x}	C_{2z}	E	C_{32}^-	C_{32}^+	C_{31}^-	C_{31}^+
C'_{2c}	C'_{2c}	C_{4y}^-	C'_{2e}	C_{4y}^+	C_{4z}^-	C'_{2b}	C'_{2a}	C_{4z}^+	C_{4x}^+	C'_{2d}	C'_{2f}	C_{4x}^-	C_{31}^-	C_{2z}	C_{34}^+	C_{34}^-	C_{2x}	C_{31}^+	C_{33}^+	C_{32}^+	E	C_{32}^-	C_{2y}	C_{33}^-
C'_{2d}	C'_{2d}	C'_{2f}	C_{4x}^+	C_{4x}^-	C_{4y}^-	C'_{2c}	C_{4y}^+	C'_{2e}	C_{4z}^+	C'_{2b}	C_{4z}^-	C'_{2a}	C_{2y}	C_{33}^+	C_{31}^-	C_{2z}	C_{31}^+	C_{33}^-	C_{34}^-	C_{32}^-	C_{32}^+	E	C_{34}^+	C_{2x}
C'_{2e}	C'_{2e}	C_{4y}^+	C'_{2c}	C_{4y}^-	C'_{2b}	C_{4z}^-	C_{4z}^+	C'_{2a}	C'_{2f}	C_{4x}^-	C_{4x}^+	C'_{2d}	C_{33}^-	C_{2x}	C_{33}^+	C_{32}^-	C_{2z}	C_{32}^+	C_{34}^+	C_{31}^+	C_{2y}	C_{34}^-	E	C_{31}^-
C'_{2f}	C'_{2f}	C'_{2d}	C_{4x}^-	C_{4x}^+	C'_{2e}	C_{4y}^+	C'_{2c}	C_{4y}^-	C'_{2b}	C_{4z}^+	C'_{2a}	C_{4z}^-	C_{2z}	C_{32}^+	C_{32}^-	C_{2y}	C_{34}^+	C_{34}^-	C_{33}^-	C_{31}^-	C_{33}^+	C_{2x}	C_{31}^+	E

$\Rrightarrow$

T **71**.2 Multiplication table *(cont.)*

$\mathbf{O}_h$	E	C_{2x}	C_{2y}	C_{2z}	C_{31}^+	C_{32}^+	C_{33}^+	C_{34}^+	C_{31}^-	C_{32}^-	C_{33}^-	C_{34}^-	C_{4x}^+	C_{4y}^+	C_{4z}^+	C_{4x}^-	C_{4y}^-	C_{4z}^-	C'_{2a}	C'_{2b}	C'_{2c}	C'_{2d}	C'_{2e}	C'_{2f}
i	i	σ_x	σ_y	σ_z	S_{61}^-	S_{62}^-	S_{63}^-	S_{64}^-	S_{61}^+	S_{62}^+	S_{63}^+	S_{64}^+	S_{4x}^-	S_{4y}^-	S_{4z}^-	S_{4x}^+	S_{4y}^+	S_{4z}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}
σ_x	σ_x	i	σ_z	σ_y	S_{64}^-	S_{63}^-	S_{62}^-	S_{61}^-	S_{62}^+	S_{61}^+	S_{64}^+	S_{63}^+	S_{4x}^+	σ_{d3}	σ_{d2}	S_{4x}^-	σ_{d5}	σ_{d1}	S_{4z}^+	S_{4z}^-	S_{4y}^-	σ_{d6}	S_{4y}^+	σ_{d4}
σ_y	σ_y	σ_z	i	σ_x	S_{62}^-	S_{61}^-	S_{64}^-	S_{63}^-	S_{63}^+	S_{64}^+	S_{61}^+	S_{62}^+	σ_{d6}	S_{4y}^+	σ_{d1}	σ_{d4}	S_{4y}^-	σ_{d2}	S_{4z}^-	S_{4z}^+	σ_{d5}	S_{4x}^+	σ_{d3}	S_{4x}^-
σ_z	σ_z	σ_y	σ_x	i	S_{63}^-	S_{64}^-	S_{61}^-	S_{62}^-	S_{64}^+	S_{63}^+	S_{62}^+	S_{61}^+	σ_{d4}	σ_{d5}	S_{4z}^+	σ_{d6}	σ_{d3}	S_{4z}^-	σ_{d2}	σ_{d1}	S_{4y}^+	S_{4x}^-	S_{4y}^-	S_{4x}^+
S_{61}^-	S_{61}^-	S_{62}^-	S_{63}^-	S_{64}^-	S_{61}^+	S_{64}^+	S_{62}^+	S_{63}^+	i	σ_y	σ_z	σ_x	σ_{d1}	σ_{d4}	σ_{d3}	S_{4z}^-	S_{4x}^-	S_{4y}^-	S_{4y}^+	σ_{d5}	S_{4x}^+	S_{4z}^+	σ_{d6}	σ_{d2}
S_{62}^-	S_{62}^-	S_{61}^-	S_{64}^-	S_{63}^-	S_{63}^+	S_{62}^+	S_{64}^+	S_{61}^+	σ_y	i	σ_x	σ_z	S_{4z}^-	S_{4x}^+	σ_{d5}	σ_{d1}	σ_{d6}	S_{4y}^+	S_{4y}^-	σ_{d3}	σ_{d4}	σ_{d2}	S_{4x}^-	S_{4z}^+
S_{63}^-	S_{63}^-	S_{64}^-	S_{61}^-	S_{62}^-	S_{64}^+	S_{61}^+	S_{63}^+	S_{62}^+	σ_z	σ_x	i	σ_y	σ_{d2}	S_{4x}^-	S_{4y}^+	S_{4z}^+	σ_{d4}	σ_{d5}	σ_{d3}	S_{4y}^-	σ_{d6}	S_{4z}^-	S_{4x}^+	σ_{d1}
S_{64}^-	S_{64}^-	S_{63}^-	S_{62}^-	S_{61}^-	S_{62}^+	S_{63}^+	S_{61}^+	S_{64}^+	σ_x	σ_z	σ_y	i	S_{4z}^+	σ_{d6}	S_{4y}^-	σ_{d2}	S_{4x}^+	σ_{d3}	σ_{d5}	S_{4y}^+	S_{4x}^-	σ_{d1}	σ_{d4}	S_{4z}^-
S_{61}^+	S_{61}^+	S_{64}^+	S_{62}^+	S_{63}^+	i	σ_x	σ_y	σ_z	S_{61}^-	S_{63}^-	S_{64}^-	S_{62}^-	S_{4y}^+	S_{4z}^+	S_{4x}^+	σ_{d3}	σ_{d1}	σ_{d4}	S_{4x}^-	σ_{d6}	S_{4z}^-	S_{4y}^-	σ_{d2}	σ_{d5}
S_{62}^+	S_{62}^+	S_{63}^+	S_{61}^+	S_{64}^+	σ_x	i	σ_z	σ_y	S_{64}^-	S_{62}^-	S_{61}^-	S_{63}^-	σ_{d5}	σ_{d1}	S_{4x}^-	S_{4y}^-	S_{4z}^+	σ_{d6}	S_{4x}^+	σ_{d4}	σ_{d2}	σ_{d3}	S_{4z}^-	S_{4y}^+
S_{63}^+	S_{63}^+	S_{62}^+	S_{64}^+	S_{61}^+	σ_y	σ_z	i	σ_x	S_{62}^-	S_{64}^-	S_{63}^-	S_{61}^-	S_{4y}^-	σ_{d2}	σ_{d4}	σ_{d5}	S_{4z}^-	S_{4x}^+	σ_{d6}	S_{4x}^-	σ_{d1}	S_{4y}^+	S_{4z}^+	σ_{d3}
S_{64}^+	S_{64}^+	S_{61}^+	S_{63}^+	S_{62}^+	σ_z	σ_y	σ_x	i	S_{63}^-	S_{61}^-	S_{62}^-	S_{64}^-	σ_{d3}	S_{4z}^-	σ_{d6}	S_{4y}^+	σ_{d2}	S_{4x}^-	σ_{d4}	S_{4x}^+	S_{4z}^+	σ_{d5}	σ_{d1}	S_{4y}^-
S_{4x}^-	S_{4x}^-	S_{4x}^+	σ_{d4}	σ_{d6}	σ_{d3}	S_{4y}^+	σ_{d5}	S_{4y}^-	S_{4z}^+	σ_{d1}	S_{4z}^-	σ_{d2}	σ_x	S_{61}^-	S_{64}^+	i	S_{63}^-	S_{62}^+	S_{61}^+	S_{63}^+	S_{64}^-	σ_z	S_{62}^-	σ_y
S_{4y}^-	S_{4y}^-	σ_{d5}	S_{4y}^+	σ_{d3}	σ_{d1}	S_{4z}^-	S_{4z}^+	σ_{d2}	S_{4x}^+	σ_{d6}	σ_{d4}	S_{4x}^-	S_{62}^+	σ_y	S_{61}^-	S_{63}^+	i	S_{64}^-	S_{62}^-	S_{63}^-	σ_x	S_{61}^+	σ_z	S_{64}^+
S_{4z}^-	S_{4z}^-	σ_{d1}	σ_{d2}	S_{4z}^+	σ_{d4}	σ_{d6}	S_{4x}^-	S_{4x}^+	S_{4y}^+	S_{4y}^-	σ_{d5}	σ_{d3}	S_{61}^-	S_{63}^+	σ_z	S_{62}^-	S_{64}^+	i	σ_y	σ_x	S_{61}^+	S_{63}^-	S_{62}^+	S_{64}^-
S_{4x}^+	S_{4x}^+	S_{4x}^-	σ_{d6}	σ_{d4}	S_{4y}^-	σ_{d5}	S_{4y}^+	σ_{d3}	σ_{d1}	S_{4z}^+	σ_{d2}	S_{4z}^-	i	S_{64}^-	S_{63}^+	σ_x	S_{62}^-	S_{61}^+	S_{62}^+	S_{64}^+	S_{61}^-	σ_y	S_{63}^-	σ_z
S_{4y}^+	S_{4y}^+	σ_{d3}	S_{4y}^-	σ_{d5}	S_{4z}^-	σ_{d1}	σ_{d2}	S_{4z}^+	σ_{d4}	S_{4x}^-	S_{4x}^+	σ_{d6}	S_{64}^+	i	S_{62}^-	S_{61}^+	σ_y	S_{63}^-	S_{61}^-	S_{64}^-	σ_z	S_{63}^+	σ_x	S_{62}^+
S_{4z}^+	S_{4z}^+	σ_{d2}	σ_{d1}	S_{4z}^-	S_{4x}^-	S_{4x}^+	σ_{d4}	σ_{d6}	σ_{d3}	σ_{d5}	S_{4y}^-	S_{4y}^+	S_{63}^-	S_{62}^+	i	S_{64}^-	S_{61}^+	σ_z	σ_x	σ_y	S_{64}^+	S_{61}^-	S_{63}^+	S_{62}^-
σ_{d1}	σ_{d1}	S_{4z}^-	S_{4z}^+	σ_{d2}	S_{4x}^+	S_{4x}^-	σ_{d6}	σ_{d4}	S_{4y}^-	S_{4y}^+	σ_{d3}	σ_{d5}	S_{62}^-	S_{61}^+	σ_x	S_{61}^-	S_{62}^+	σ_y	i	σ_z	S_{63}^+	S_{64}^-	S_{64}^+	S_{63}^-
σ_{d2}	σ_{d2}	S_{4z}^+	S_{4z}^-	σ_{d1}	σ_{d6}	σ_{d4}	S_{4x}^+	S_{4x}^-	σ_{d5}	σ_{d3}	S_{4y}^+	S_{4y}^-	S_{64}^-	S_{64}^+	σ_y	S_{63}^-	S_{63}^+	σ_x	σ_z	i	S_{62}^+	S_{62}^-	S_{61}^+	S_{61}^-
σ_{d3}	σ_{d3}	S_{4y}^+	σ_{d5}	S_{4y}^-	S_{4z}^+	σ_{d2}	σ_{d1}	S_{4z}^-	S_{4x}^-	σ_{d4}	σ_{d6}	S_{4x}^+	S_{61}^+	σ_z	S_{64}^-	S_{64}^+	σ_x	S_{61}^-	S_{63}^-	S_{62}^-	i	S_{62}^+	σ_y	S_{63}^+
σ_{d4}	σ_{d4}	σ_{d6}	S_{4x}^-	S_{4x}^+	S_{4y}^+	σ_{d3}	S_{4y}^-	σ_{d5}	S_{4z}^-	σ_{d2}	S_{4z}^+	σ_{d1}	σ_y	S_{63}^-	S_{61}^+	σ_z	S_{61}^-	S_{63}^+	S_{64}^+	S_{62}^+	S_{62}^-	i	S_{64}^-	σ_x
σ_{d5}	σ_{d5}	S_{4y}^-	σ_{d3}	S_{4y}^+	σ_{d2}	S_{4z}^+	S_{4z}^-	σ_{d1}	σ_{d6}	S_{4x}^+	S_{4x}^-	σ_{d4}	S_{63}^+	σ_x	S_{63}^-	S_{62}^+	σ_z	S_{62}^-	S_{64}^-	S_{61}^-	σ_y	S_{64}^+	i	S_{61}^+
σ_{d6}	σ_{d6}	σ_{d4}	S_{4x}^+	S_{4x}^-	σ_{d5}	S_{4y}^-	σ_{d3}	S_{4y}^+	σ_{d2}	S_{4z}^-	σ_{d1}	S_{4z}^+	σ_z	S_{62}^-	S_{62}^+	σ_y	S_{64}^-	S_{64}^+	S_{63}^+	S_{61}^+	S_{63}^-	σ_x	S_{61}^-	i

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T **71**.2 Multiplication table *(cont.)*

$\mathbf{O}_h$	i	σ_x	σ_y	σ_z	S_{61}^-	S_{62}^-	S_{63}^-	S_{64}^-	S_{61}^+	S_{62}^+	S_{63}^+	S_{64}^+	S_{4x}^-	S_{4y}^-	S_{4z}^-	S_{4x}^+	S_{4y}^+	S_{4z}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}
E	i	σ_x	σ_y	σ_z	S_{61}^-	S_{62}^-	S_{63}^-	S_{64}^-	S_{61}^+	S_{62}^+	S_{63}^+	S_{64}^+	S_{4x}^-	S_{4y}^-	S_{4z}^-	S_{4x}^+	S_{4y}^+	S_{4z}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}
C_{2x}	σ_x	i	σ_z	σ_y	S_{64}^-	S_{63}^-	S_{62}^-	S_{61}^-	S_{62}^+	S_{61}^+	S_{64}^+	S_{63}^+	S_{4x}^+	σ_{d3}	σ_{d2}	S_{4x}^-	σ_{d5}	σ_{d1}	S_{4z}^+	S_{4z}^-	S_{4y}^-	σ_{d6}	S_{4y}^+	σ_{d4}
C_{2y}	σ_y	σ_z	i	σ_x	S_{62}^-	S_{61}^-	S_{64}^-	S_{63}^-	S_{63}^+	S_{64}^+	S_{61}^+	S_{62}^+	σ_{d6}	S_{4y}^+	σ_{d1}	σ_{d4}	S_{4y}^-	σ_{d2}	S_{4z}^-	S_{4z}^+	σ_{d5}	S_{4x}^+	σ_{d3}	S_{4x}^-
C_{2z}	σ_z	σ_y	σ_x	i	S_{63}^-	S_{64}^-	S_{61}^-	S_{62}^-	S_{64}^+	S_{63}^+	S_{62}^+	S_{61}^+	σ_{d4}	σ_{d5}	S_{4z}^+	σ_{d6}	σ_{d3}	S_{4z}^-	σ_{d2}	σ_{d1}	S_{4y}^+	S_{4x}^-	S_{4y}^-	S_{4x}^+
C_{31}^+	S_{61}^-	S_{62}^-	S_{63}^-	S_{64}^-	S_{61}^+	S_{64}^+	S_{62}^+	S_{63}^+	i	σ_y	σ_z	σ_x	σ_{d1}	σ_{d4}	σ_{d3}	S_{4z}^-	S_{4x}^-	S_{4y}^-	S_{4y}^+	σ_{d5}	S_{4x}^+	S_{4z}^+	σ_{d6}	σ_{d2}
C_{32}^+	S_{62}^-	S_{61}^-	S_{64}^-	S_{63}^-	S_{63}^+	S_{62}^+	S_{64}^+	S_{61}^+	σ_y	i	σ_x	σ_z	S_{4z}^-	S_{4x}^+	σ_{d5}	σ_{d1}	σ_{d6}	S_{4y}^+	S_{4y}^-	σ_{d3}	σ_{d4}	σ_{d2}	S_{4x}^-	S_{4z}^+
C_{33}^+	S_{63}^-	S_{64}^-	S_{61}^-	S_{62}^-	S_{64}^+	S_{61}^+	S_{63}^+	S_{62}^+	σ_z	σ_x	i	σ_y	σ_{d2}	S_{4x}^-	S_{4y}^+	S_{4z}^+	σ_{d4}	σ_{d5}	σ_{d3}	S_{4y}^-	σ_{d6}	S_{4z}^-	S_{4x}^+	σ_{d1}
C_{34}^+	S_{64}^-	S_{63}^-	S_{62}^-	S_{61}^-	S_{62}^+	S_{63}^+	S_{61}^+	S_{64}^+	σ_x	σ_z	σ_y	i	S_{4z}^+	σ_{d6}	S_{4y}^-	σ_{d2}	S_{4x}^+	σ_{d3}	σ_{d5}	S_{4y}^+	S_{4x}^-	σ_{d1}	σ_{d4}	S_{4z}^-
C_{31}^-	S_{61}^+	S_{64}^+	S_{62}^+	S_{63}^+	i	σ_x	σ_y	σ_z	S_{61}^-	S_{63}^-	S_{64}^-	S_{62}^-	S_{4y}^+	S_{4z}^+	S_{4x}^+	σ_{d3}	σ_{d1}	σ_{d4}	S_{4x}^-	σ_{d6}	S_{4z}^-	S_{4y}^-	σ_{d2}	σ_{d5}
C_{32}^-	S_{62}^+	S_{63}^+	S_{61}^+	S_{64}^+	σ_x	i	σ_z	σ_y	S_{64}^-	S_{62}^-	S_{61}^-	S_{63}^-	σ_{d5}	σ_{d1}	S_{4x}^-	S_{4y}^-	S_{4z}^+	σ_{d6}	S_{4x}^+	σ_{d4}	σ_{d2}	σ_{d3}	S_{4z}^-	S_{4y}^+
C_{33}^-	S_{63}^+	S_{62}^+	S_{64}^+	S_{61}^+	σ_y	σ_z	i	σ_x	S_{62}^-	S_{64}^-	S_{63}^-	S_{61}^-	S_{4y}^-	σ_{d2}	σ_{d4}	σ_{d5}	S_{4z}^-	S_{4x}^+	σ_{d6}	S_{4x}^-	σ_{d1}	S_{4y}^+	S_{4z}^+	σ_{d3}
C_{34}^-	S_{64}^+	S_{61}^+	S_{63}^+	S_{62}^+	σ_z	σ_y	σ_x	i	S_{63}^-	S_{61}^-	S_{62}^-	S_{64}^-	σ_{d3}	S_{4z}^-	σ_{d6}	S_{4y}^+	σ_{d2}	S_{4x}^-	σ_{d4}	S_{4x}^+	S_{4z}^+	σ_{d5}	σ_{d1}	S_{4y}^-
C_{4x}^+	S_{4x}^-	S_{4x}^+	σ_{d4}	σ_{d6}	σ_{d3}	S_{4y}^+	σ_{d5}	S_{4y}^-	S_{4z}^+	σ_{d1}	S_{4z}^-	σ_{d2}	σ_x	S_{61}^-	S_{64}^+	i	S_{63}^-	S_{62}^+	S_{61}^+	S_{63}^+	S_{64}^-	σ_z	S_{62}^-	σ_y
C_{4y}^+	S_{4y}^-	σ_{d5}	S_{4y}^+	σ_{d3}	σ_{d1}	S_{4z}^-	S_{4z}^+	σ_{d2}	S_{4x}^+	σ_{d6}	σ_{d4}	S_{4x}^-	S_{62}^+	σ_y	S_{61}^-	S_{63}^+	i	S_{64}^-	S_{62}^-	S_{63}^-	σ_x	S_{61}^+	σ_z	S_{64}^+
C_{4z}^+	S_{4z}^-	σ_{d1}	σ_{d2}	S_{4z}^+	σ_{d4}	σ_{d6}	S_{4x}^-	S_{4x}^+	S_{4y}^+	S_{4y}^-	σ_{d5}	σ_{d3}	S_{61}^-	S_{63}^+	σ_z	S_{62}^-	S_{64}^+	i	σ_y	σ_x	S_{61}^+	S_{63}^-	S_{62}^+	S_{64}^-
C_{4x}^-	S_{4x}^+	S_{4x}^-	σ_{d6}	σ_{d4}	S_{4y}^-	σ_{d5}	S_{4y}^+	σ_{d3}	σ_{d1}	S_{4z}^+	σ_{d2}	S_{4z}^-	i	S_{64}^-	S_{63}^+	σ_x	S_{62}^-	S_{61}^+	S_{62}^+	S_{64}^+	S_{61}^-	σ_y	S_{63}^-	σ_z
C_{4y}^-	S_{4y}^+	σ_{d3}	S_{4y}^-	σ_{d5}	S_{4z}^-	σ_{d1}	σ_{d2}	S_{4z}^+	σ_{d4}	S_{4x}^-	S_{4x}^+	σ_{d6}	S_{64}^+	i	S_{62}^-	S_{61}^+	σ_y	S_{63}^-	S_{61}^-	S_{64}^-	σ_z	S_{63}^+	σ_x	S_{62}^+
C_{4z}^-	S_{4z}^+	σ_{d2}	σ_{d1}	S_{4z}^-	S_{4x}^-	S_{4x}^+	σ_{d4}	σ_{d6}	σ_{d3}	σ_{d5}	S_{4y}^-	S_{4y}^+	S_{63}^-	S_{62}^+	i	S_{64}^-	S_{61}^+	σ_z	σ_x	σ_y	S_{64}^+	S_{61}^-	S_{63}^+	S_{62}^-
C_{2a}'	σ_{d1}	S_{4z}^-	S_{4z}^+	σ_{d2}	S_{4x}^+	S_{4x}^-	σ_{d6}	σ_{d4}	S_{4y}^-	S_{4y}^+	σ_{d3}	σ_{d5}	S_{62}^-	S_{61}^+	σ_x	S_{61}^-	S_{62}^+	σ_y	i	σ_z	S_{63}^+	S_{64}^-	S_{64}^+	S_{63}^-
C_{2b}'	σ_{d2}	S_{4z}^+	S_{4z}^-	σ_{d1}	σ_{d6}	σ_{d4}	S_{4x}^+	S_{4x}^-	σ_{d5}	σ_{d3}	S_{4y}^+	S_{4y}^-	S_{64}^-	S_{64}^+	σ_y	S_{63}^-	S_{63}^+	σ_x	σ_z	i	S_{62}^+	S_{62}^-	S_{61}^+	S_{61}^-
C_{2c}'	σ_{d3}	S_{4y}^+	σ_{d5}	S_{4y}^-	S_{4z}^+	σ_{d2}	σ_{d1}	S_{4z}^-	S_{4x}^-	σ_{d4}	σ_{d6}	S_{4x}^+	S_{61}^+	σ_z	S_{64}^-	S_{64}^+	σ_x	S_{61}^-	S_{63}^-	S_{62}^-	i	S_{62}^+	σ_y	S_{63}^+
C_{2d}'	σ_{d4}	σ_{d6}	S_{4x}^-	S_{4x}^+	S_{4y}^+	σ_{d3}	S_{4y}^-	σ_{d5}	S_{4z}^-	σ_{d2}	S_{4z}^+	σ_{d1}	σ_y	S_{63}^-	S_{61}^+	σ_z	S_{61}^-	S_{63}^+	S_{64}^+	S_{62}^+	S_{62}^-	i	S_{64}^-	σ_x
C_{2e}'	σ_{d5}	S_{4y}^-	σ_{d3}	S_{4y}^+	σ_{d2}	S_{4z}^+	S_{4z}^-	σ_{d1}	σ_{d6}	S_{4x}^+	S_{4x}^-	σ_{d4}	S_{63}^+	σ_x	S_{63}^-	S_{62}^+	σ_z	S_{62}^-	S_{64}^-	S_{61}^-	σ_y	S_{64}^+	i	S_{61}^+
C_{2f}'	σ_{d6}	σ_{d4}	S_{4x}^+	S_{4x}^-	σ_{d5}	S_{4y}^-	σ_{d3}	S_{4y}^+	σ_{d2}	S_{4z}^-	σ_{d1}	S_{4z}^+	σ_z	S_{62}^-	S_{62}^+	σ_y	S_{64}^-	S_{64}^+	S_{63}^+	S_{61}^+	S_{63}^-	σ_x	S_{61}^-	i

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T **71**.2 Multiplication table *(cont.)*

$\mathbf{O}_h$	i	σ_x	σ_y	σ_z	S_{61}^{-}	S_{62}^{-}	S_{63}^{-}	S_{64}^{-}	S_{61}^{+}	S_{62}^{+}	S_{63}^{+}	S_{64}^{+}	S_{4x}^{-}	S_{4y}^{-}	S_{4z}^{-}	S_{4x}^{+}	S_{4y}^{+}	S_{4z}^{+}	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}
i	E	C_{2x}	C_{2y}	C_{2z}	C_{31}^{+}	C_{32}^{+}	C_{33}^{+}	C_{34}^{+}	C_{31}^{-}	C_{32}^{-}	C_{33}^{-}	C_{34}^{-}	C_{4x}^{+}	C_{4y}^{+}	C_{4z}^{+}	C_{4x}^{-}	C_{4y}^{-}	C_{4z}^{-}	C'_{2a}	C'_{2b}	C'_{2c}	C'_{2d}	C'_{2e}	C'_{2f}
σ_x	C_{2x}	E	C_{2z}	C_{2y}	C_{34}^{+}	C_{33}^{+}	C_{32}^{+}	C_{31}^{+}	C_{32}^{-}	C_{31}^{-}	C_{34}^{-}	C_{33}^{-}	C_{4x}^{-}	C'_{2c}	C'_{2b}	C_{4x}^{+}	C'_{2e}	C'_{2a}	C_{4z}^{-}	C_{4z}^{+}	C_{4y}^{+}	C'_{2f}	C_{4y}^{-}	C'_{2d}
σ_y	C_{2y}	C_{2z}	E	C_{2x}	C_{32}^{+}	C_{31}^{+}	C_{34}^{+}	C_{33}^{+}	C_{33}^{-}	C_{34}^{-}	C_{31}^{-}	C_{32}^{-}	C'_{2f}	C_{4y}^{-}	C'_{2a}	C'_{2d}	C_{4y}^{+}	C'_{2b}	C_{4z}^{+}	C_{4z}^{-}	C'_{2e}	C_{4x}^{-}	C'_{2c}	C_{4x}^{+}
σ_z	C_{2z}	C_{2y}	C_{2x}	E	C_{33}^{+}	C_{34}^{+}	C_{31}^{+}	C_{32}^{+}	C_{34}^{-}	C_{33}^{-}	C_{32}^{-}	C_{31}^{-}	C'_{2d}	C'_{2e}	C_{4z}^{-}	C'_{2f}	C'_{2c}	C_{4z}^{+}	C'_{2b}	C'_{2a}	C_{4y}^{-}	C_{4x}^{+}	C_{4y}^{+}	C_{4x}^{-}
S_{61}^{-}	C_{31}^{+}	C_{32}^{+}	C_{33}^{+}	C_{34}^{+}	C_{31}^{-}	C_{34}^{-}	C_{32}^{-}	C_{33}^{-}	E	C_{2y}	C_{2z}	C_{2x}	C'_{2a}	C'_{2d}	C'_{2c}	C_{4z}^{+}	C_{4x}^{+}	C_{4y}^{+}	C_{4y}^{-}	C'_{2e}	C_{4x}^{-}	C_{4z}^{-}	C'_{2f}	C'_{2b}
S_{62}^{-}	C_{32}^{+}	C_{31}^{+}	C_{34}^{+}	C_{33}^{+}	C_{33}^{-}	C_{32}^{-}	C_{34}^{-}	C_{31}^{-}	C_{2y}	E	C_{2x}	C_{2z}	C_{4z}^{+}	C_{4x}^{-}	C'_{2e}	C'_{2a}	C'_{2f}	C_{4y}^{-}	C_{4y}^{+}	C'_{2c}	C'_{2d}	C'_{2b}	C_{4x}^{+}	C_{4z}^{-}
S_{63}^{-}	C_{33}^{+}	C_{34}^{+}	C_{31}^{+}	C_{32}^{+}	C_{34}^{-}	C_{31}^{-}	C_{33}^{-}	C_{32}^{-}	C_{2z}	C_{2x}	E	C_{2y}	C'_{2b}	C_{4x}^{+}	C_{4y}^{-}	C_{4z}^{-}	C'_{2d}	C'_{2e}	C'_{2c}	C_{4y}^{+}	C'_{2f}	C_{4z}^{+}	C_{4x}^{-}	C'_{2a}
S_{64}^{-}	C_{34}^{+}	C_{33}^{+}	C_{32}^{+}	C_{31}^{+}	C_{32}^{-}	C_{33}^{-}	C_{31}^{-}	C_{34}^{-}	C_{2x}	C_{2z}	C_{2y}	E	C_{4z}^{-}	C'_{2f}	C_{4y}^{+}	C'_{2b}	C_{4x}^{-}	C'_{2c}	C'_{2e}	C_{4y}^{-}	C_{4x}^{+}	C'_{2a}	C'_{2d}	C_{4z}^{+}
S_{61}^{+}	C_{31}^{-}	C_{34}^{-}	C_{32}^{-}	C_{33}^{-}	E	C_{2x}	C_{2y}	C_{2z}	C_{31}^{+}	C_{33}^{+}	C_{34}^{+}	C_{32}^{+}	C_{4y}^{-}	C_{4z}^{-}	C_{4x}^{-}	C'_{2c}	C'_{2a}	C'_{2d}	C_{4x}^{+}	C'_{2f}	C_{4z}^{+}	C_{4y}^{+}	C'_{2b}	C'_{2e}
S_{62}^{+}	C_{32}^{-}	C_{33}^{-}	C_{31}^{-}	C_{34}^{-}	C_{2x}	E	C_{2z}	C_{2y}	C_{34}^{+}	C_{32}^{+}	C_{31}^{+}	C_{33}^{+}	C'_{2e}	C'_{2a}	C_{4x}^{+}	C_{4y}^{+}	C_{4z}^{-}	C'_{2f}	C_{4x}^{-}	C'_{2d}	C'_{2b}	C'_{2c}	C_{4z}^{+}	C_{4y}^{-}
S_{63}^{+}	C_{33}^{-}	C_{32}^{-}	C_{34}^{-}	C_{31}^{-}	C_{2y}	C_{2z}	E	C_{2x}	C_{32}^{+}	C_{34}^{+}	C_{33}^{+}	C_{31}^{+}	C_{4y}^{+}	C'_{2b}	C'_{2d}	C'_{2e}	C_{4z}^{+}	C_{4x}^{-}	C'_{2f}	C_{4x}^{+}	C'_{2a}	C_{4y}^{-}	C_{4z}^{-}	C'_{2c}
S_{64}^{+}	C_{34}^{-}	C_{31}^{-}	C_{33}^{-}	C_{32}^{-}	C_{2z}	C_{2y}	C_{2x}	E	C_{33}^{+}	C_{31}^{+}	C_{32}^{+}	C_{34}^{+}	C'_{2c}	C_{4z}^{+}	C'_{2f}	C_{4y}^{-}	C'_{2b}	C_{4x}^{+}	C'_{2d}	C_{4x}^{-}	C_{4z}^{-}	C'_{2e}	C'_{2a}	C_{4y}^{+}
S_{4x}^{-}	C_{4x}^{+}	C_{4x}^{-}	C'_{2d}	C'_{2f}	C'_{2c}	C_{4y}^{-}	C'_{2e}	C_{4y}^{+}	C_{4z}^{-}	C'_{2a}	C_{4z}^{+}	C'_{2b}	C_{2x}	C_{31}^{+}	C_{34}^{-}	E	C_{33}^{+}	C_{32}^{-}	C_{31}^{-}	C_{33}^{-}	C_{34}^{+}	C_{2z}	C_{32}^{+}	C_{2y}
S_{4y}^{-}	C_{4y}^{+}	C'_{2e}	C_{4y}^{-}	C'_{2c}	C'_{2a}	C_{4z}^{+}	C_{4z}^{-}	C'_{2b}	C_{4x}^{-}	C'_{2f}	C'_{2d}	C_{4x}^{+}	C_{32}^{-}	C_{2y}	C_{31}^{+}	C_{33}^{-}	E	C_{34}^{+}	C_{32}^{+}	C_{33}^{+}	C_{2x}	C_{31}^{-}	C_{2z}	C_{34}^{-}
S_{4z}^{-}	C_{4z}^{+}	C'_{2a}	C'_{2b}	C_{4z}^{-}	C'_{2d}	C'_{2f}	C_{4x}^{+}	C_{4x}^{-}	C_{4y}^{-}	C_{4y}^{+}	C'_{2e}	C'_{2c}	C_{31}^{+}	C_{33}^{-}	C_{2z}	C_{32}^{+}	C_{34}^{-}	E	C_{2y}	C_{2x}	C_{31}^{-}	C_{33}^{+}	C_{32}^{-}	C_{34}^{+}
S_{4x}^{+}	C_{4x}^{-}	C_{4x}^{+}	C'_{2f}	C'_{2d}	C_{4y}^{+}	C'_{2e}	C_{4y}^{-}	C'_{2c}	C'_{2a}	C_{4z}^{-}	C'_{2b}	C_{4z}^{+}	E	C_{34}^{+}	C_{33}^{-}	C_{2x}	C_{32}^{+}	C_{31}^{-}	C_{32}^{-}	C_{34}^{-}	C_{31}^{+}	C_{2y}	C_{33}^{+}	C_{2z}
S_{4y}^{+}	C_{4y}^{-}	C'_{2c}	C_{4y}^{+}	C'_{2e}	C_{4z}^{+}	C'_{2a}	C'_{2b}	C_{4z}^{-}	C'_{2d}	C_{4x}^{+}	C_{4x}^{-}	C'_{2f}	C_{34}^{-}	E	C_{32}^{+}	C_{31}^{-}	C_{2y}	C_{33}^{+}	C_{31}^{+}	C_{34}^{+}	C_{2z}	C_{33}^{-}	C_{2x}	C_{32}^{-}
S_{4z}^{+}	C_{4z}^{-}	C'_{2b}	C'_{2a}	C_{4z}^{+}	C_{4x}^{+}	C_{4x}^{-}	C'_{2d}	C'_{2f}	C'_{2c}	C'_{2e}	C_{4y}^{+}	C_{4y}^{-}	C_{33}^{+}	C_{32}^{-}	E	C_{34}^{+}	C_{31}^{-}	C_{2z}	C_{2x}	C_{2y}	C_{34}^{-}	C_{31}^{+}	C_{33}^{-}	C_{32}^{+}
σ_{d1}	C'_{2a}	C_{4z}^{+}	C_{4z}^{-}	C'_{2b}	C_{4x}^{-}	C_{4x}^{+}	C'_{2f}	C'_{2d}	C_{4y}^{+}	C_{4y}^{-}	C'_{2c}	C'_{2e}	C_{32}^{+}	C_{31}^{-}	C_{2x}	C_{31}^{+}	C_{32}^{-}	C_{2y}	E	C_{2z}	C_{33}^{-}	C_{34}^{+}	C_{34}^{-}	C_{33}^{+}
σ_{d2}	C'_{2b}	C_{4z}^{-}	C_{4z}^{+}	C'_{2a}	C'_{2f}	C'_{2d}	C_{4x}^{-}	C_{4x}^{+}	C'_{2e}	C'_{2c}	C_{4y}^{-}	C_{4y}^{+}	C_{34}^{+}	C_{34}^{-}	C_{2y}	C_{33}^{+}	C_{33}^{-}	C_{2x}	C_{2z}	E	C_{32}^{-}	C_{32}^{+}	C_{31}^{-}	C_{31}^{+}
σ_{d3}	C'_{2c}	C_{4y}^{-}	C'_{2e}	C_{4y}^{+}	C_{4z}^{-}	C'_{2b}	C'_{2a}	C_{4z}^{+}	C_{4x}^{+}	C'_{2d}	C'_{2f}	C_{4x}^{-}	C_{31}^{-}	C_{2z}	C_{34}^{+}	C_{34}^{-}	C_{2x}	C_{31}^{+}	C_{33}^{+}	C_{32}^{+}	E	C_{32}^{-}	C_{2y}	C_{33}^{-}
σ_{d4}	C'_{2d}	C'_{2f}	C_{4x}^{+}	C_{4x}^{-}	C_{4y}^{-}	C'_{2c}	C_{4y}^{+}	C'_{2e}	C_{4z}^{+}	C'_{2b}	C_{4z}^{-}	C'_{2a}	C_{2y}	C_{33}^{+}	C_{31}^{-}	C_{2z}	C_{31}^{+}	C_{33}^{-}	C_{34}^{-}	C_{32}^{-}	C_{32}^{+}	E	C_{34}^{+}	C_{2x}
σ_{d5}	C'_{2e}	C_{4y}^{+}	C'_{2c}	C_{4y}^{-}	C'_{2b}	C_{4z}^{-}	C_{4z}^{+}	C'_{2a}	C'_{2f}	C_{4x}^{-}	C_{4x}^{+}	C'_{2d}	C_{33}^{-}	C_{2x}	C_{33}^{+}	C_{32}^{-}	C_{2z}	C_{32}^{+}	C_{34}^{+}	C_{31}^{+}	C_{2y}	C_{34}^{-}	E	C_{31}^{-}
σ_{d6}	C'_{2f}	C'_{2d}	C_{4x}^{-}	C_{4x}^{+}	C'_{2e}	C_{4y}^{+}	C'_{2c}	C_{4y}^{-}	C'_{2b}	C_{4z}^{+}	C'_{2a}	C_{4z}^{-}	C_{2z}	C_{32}^{+}	C_{32}^{-}	C_{2y}	C_{34}^{+}	C_{34}^{-}	C_{33}^{-}	C_{31}^{-}	C_{33}^{+}	C_{2x}	C_{31}^{+}	E

T **71**.3 Factor table

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$\mathbf{O}_h$	E	C_{2x}	C_{2y}	C_{2z}	C_{31}^+	C_{32}^+	C_{33}^+	C_{34}^+	C_{31}^-	C_{32}^-	C_{33}^-	C_{34}^-	C_{4x}^+	C_{4y}^+	C_{4z}^+	C_{4x}^-	C_{4y}^-	C_{4z}^-	C'_{2a}	C'_{2b}	C'_{2c}	C'_{2d}	C'_{2e}	C'_{2f}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_{2x}	1	−1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	1	1	1	−1	1	−1	−1	−1	1
C_{2y}	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	−1	1
C_{2z}	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	−1	−1	1	1	1	1	−1	−1	1	1	−1
C_{31}^+	1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	−1	1	1	1	1	−1	−1	−1	1	−1	−1
C_{32}^+	1	1	1	−1	1	−1	1	1	−1	1	−1	1	1	1	−1	−1	1	1	1	−1	−1	−1	1	−1
C_{33}^+	1	−1	1	1	1	1	−1	1	−1	1	1	−1	−1	1	1	1	1	1	1	1	1	−1	−1	−1
C_{34}^+	1	1	−1	1	1	1	1	−1	−1	−1	1	1	1	−1	1	1	1	−1	1	−1	1	−1	1	1
C_{31}^-	1	1	1	1	1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	1	1	−1	1	−1	−1	−1
C_{32}^-	1	−1	−1	1	1	1	−1	1	1	−1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	1
C_{33}^-	1	1	−1	−1	1	1	1	−1	1	1	−1	1	1	1	−1	−1	1	1	−1	−1	1	1	1	1
C_{34}^-	1	−1	1	−1	1	−1	1	1	1	1	1	−1	1	1	1	1	−1	1	−1	1	−1	1	1	−1
C_{4x}^+	1	−1	−1	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1
C_{4y}^+	1	1	−1	1	1	1	1	1	1	−1	−1	1	1	1	1	1	1	1	−1	−1	1	1	−1	1
C_{4z}^+	1	1	1	−1	−1	1	1	1	1	1	−1	1	1	1	1	1	1	1	1	−1	−1	1	1	−1
C_{4x}^-	1	1	−1	−1	1	−1	1	−1	−1	1	1	1	1	1	1	−1	1	1	1	−1	1	−1	1	1
C_{4y}^-	1	1	1	−1	1	−1	−1	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	−1	1	−1
C_{4z}^-	1	−1	1	1	1	1	1	−1	−1	1	1	1	1	1	1	1	1	−1	1	1	1	−1	−1	1
C'_{2a}	1	−1	−1	−1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	1	1	−1	1	−1	1	−1	1
C'_{2b}	1	1	−1	1	−1	−1	1	−1	−1	−1	−1	1	1	−1	1	−1	1	−1	−1	−1	1	1	1	1
C'_{2c}	1	−1	−1	−1	−1	−1	1	1	1	−1	1	−1	−1	1	−1	1	1	1	−1	1	−1	1	1	−1
C'_{2d}	1	1	1	1	1	−1	−1	1	−1	−1	1	−1	−1	1	1	−1	−1	−1	1	1	1	−1	−1	−1
C'_{2e}	1	−1	1	1	−1	1	−1	1	−1	−1	1	1	−1	1	1	1	−1	−1	−1	1	−1	−1	−1	1
C'_{2f}	1	−1	1	−1	−1	−1	1	1	−1	1	−1	−1	1	1	−1	−1	−1	1	1	1	−1	1	1	−1

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T **71**.3 Factor table *(cont.)*

$\mathbf{O}_h$	E	C_{2x}	C_{2y}	C_{2z}	C_{31}^+	C_{32}^+	C_{33}^+	C_{34}^+	C_{31}^-	C_{32}^-	C_{33}^-	C_{34}^-	C_{4x}^+	C_{4y}^+	C_{4z}^+	C_{4x}^-	C_{4y}^-	C_{4z}^-	C'_{2a}	C'_{2b}	C'_{2c}	C'_{2d}	C'_{2e}	C'_{2f}
i	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
σ_x	1	−1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	1	1	1	−1	1	−1	−1	−1	1
σ_y	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	−1	1
σ_z	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	−1	−1	1	1	1	1	−1	−1	1	1	−1
S_{61}^-	1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	−1	1	1	1	1	−1	−1	−1	1	−1	−1
S_{62}^-	1	1	1	−1	1	−1	1	1	−1	1	−1	1	1	1	−1	−1	1	1	1	−1	−1	−1	1	−1
S_{63}^-	1	−1	1	1	1	1	−1	1	−1	1	1	−1	−1	1	1	1	1	1	1	1	1	−1	−1	−1
S_{64}^-	1	1	−1	1	1	1	1	−1	−1	−1	1	1	1	−1	1	1	1	−1	1	−1	1	−1	1	1
S_{61}^+	1	1	1	1	1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	1	1	−1	1	−1	−1	−1
S_{62}^+	1	−1	−1	1	1	1	−1	1	1	−1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	1
S_{63}^+	1	1	−1	−1	1	1	1	−1	1	1	−1	1	1	1	−1	−1	1	1	−1	−1	1	1	1	1
S_{64}^+	1	−1	1	−1	1	−1	1	1	1	1	1	−1	1	1	1	1	−1	1	−1	1	−1	1	1	−1
S_{4x}^-	1	−1	−1	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1
S_{4y}^-	1	1	−1	1	1	1	1	1	1	−1	−1	1	1	1	1	1	1	1	−1	−1	1	1	−1	1
S_{4z}^-	1	1	1	−1	−1	1	1	1	1	1	−1	1	1	1	1	1	1	1	1	−1	−1	1	1	−1
S_{4x}^+	1	1	−1	−1	1	−1	1	−1	−1	1	1	1	1	1	1	−1	1	1	1	−1	1	−1	1	1
S_{4y}^+	1	1	1	−1	1	−1	−1	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	−1	1	−1
S_{4z}^+	1	−1	1	1	1	1	1	−1	−1	1	1	1	1	1	1	1	1	−1	1	1	1	−1	−1	1
σ_{d1}	1	−1	−1	−1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	1	1	−1	1	−1	1	−1	1
σ_{d2}	1	1	−1	1	−1	−1	1	−1	−1	−1	−1	1	1	−1	1	−1	1	−1	−1	−1	1	1	1	1
σ_{d3}	1	−1	−1	−1	−1	−1	1	1	1	−1	1	−1	−1	1	−1	1	1	1	−1	1	−1	1	1	−1
σ_{d4}	1	1	1	1	1	−1	−1	1	−1	−1	1	−1	−1	1	1	−1	−1	−1	1	1	1	−1	−1	−1
σ_{d5}	1	−1	1	1	−1	1	−1	1	−1	−1	1	1	−1	1	1	1	−1	−1	−1	1	−1	−1	−1	1
σ_{d6}	1	−1	1	−1	−1	−1	1	1	−1	1	−1	−1	1	1	−1	−1	−1	1	1	1	−1	1	1	−1

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T **71**.3 Factor table *(cont.)*

$\mathbf{O}_h$	i	σ_x	σ_y	σ_z	S_{61}^-	S_{62}^-	S_{63}^-	S_{64}^-	S_{61}^+	S_{62}^+	S_{63}^+	S_{64}^+	S_{4x}^-	S_{4y}^-	S_{4z}^-	S_{4x}^+	S_{4y}^+	S_{4z}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_{2x}	1	−1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	1	1	1	−1	1	−1	−1	−1	1
C_{2y}	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	−1	1
C_{2z}	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	−1	−1	1	1	1	1	−1	−1	1	1	−1
C_{31}^+	1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	−1	1	1	1	1	−1	−1	−1	1	−1	−1
C_{32}^+	1	1	1	−1	1	−1	1	1	−1	1	−1	1	1	1	−1	−1	1	1	1	−1	−1	−1	1	−1
C_{33}^+	1	−1	1	1	1	1	−1	1	−1	1	1	−1	−1	1	1	1	1	1	1	1	1	−1	−1	−1
C_{34}^+	1	1	−1	1	1	1	1	−1	−1	−1	1	1	1	−1	1	1	1	−1	1	−1	1	−1	1	1
C_{31}^-	1	1	1	1	1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	1	1	−1	1	−1	−1	−1
C_{32}^-	1	−1	−1	1	1	1	−1	1	1	−1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	1
C_{33}^-	1	1	−1	−1	1	1	1	−1	1	1	−1	1	1	1	−1	−1	1	1	−1	−1	1	1	1	1
C_{34}^-	1	−1	1	−1	1	−1	1	1	1	1	1	−1	1	1	1	1	−1	1	−1	1	−1	1	1	−1
C_{4x}^+	1	−1	−1	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1
C_{4y}^+	1	1	−1	1	1	1	1	1	1	−1	−1	1	1	1	1	1	1	1	−1	−1	1	1	−1	1
C_{4z}^+	1	1	1	−1	−1	1	1	1	1	1	−1	1	1	1	1	1	1	1	1	−1	−1	1	1	−1
C_{4x}^-	1	1	−1	−1	1	−1	1	−1	−1	1	1	1	1	1	1	−1	1	1	1	−1	1	−1	1	1
C_{4y}^-	1	1	1	−1	1	−1	−1	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	−1	1	−1
C_{4z}^-	1	−1	1	1	1	1	1	−1	−1	1	1	1	1	1	1	1	1	−1	1	1	1	−1	−1	1
C'_{2a}	1	−1	−1	−1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	1	1	−1	1	−1	1	−1	1
C'_{2b}	1	1	−1	1	−1	−1	1	−1	−1	−1	−1	1	1	−1	1	−1	1	−1	−1	−1	1	1	1	1
C'_{2c}	1	−1	−1	−1	−1	−1	1	1	1	−1	1	−1	−1	1	−1	1	1	1	−1	1	−1	1	1	−1
C'_{2d}	1	1	1	1	1	−1	−1	1	−1	−1	1	−1	−1	1	1	−1	−1	−1	1	1	1	−1	−1	−1
C'_{2e}	1	−1	1	1	−1	1	−1	1	−1	−1	1	1	−1	1	1	1	−1	−1	−1	1	−1	−1	−1	1
C'_{2f}	1	−1	1	−1	−1	−1	1	1	−1	1	−1	−1	1	1	−1	−1	−1	1	1	1	−1	1	1	−1

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T **71**.3 Factor table *(cont.)*

$\mathbf{O}_h$	i	σ_x	σ_y	σ_z	S_{61}^-	S_{62}^-	S_{63}^-	S_{64}^-	S_{61}^+	S_{62}^+	S_{63}^+	S_{64}^+	S_{4x}^-	S_{4y}^-	S_{4z}^-	S_{4x}^+	S_{4y}^+	S_{4z}^+	σ_{d1}	σ_{d2}	σ_{d3}	σ_{d4}	σ_{d5}	σ_{d6}
i	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
σ_x	1	−1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	1	1	1	−1	1	−1	−1	−1	1
σ_y	1	−1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	−1	1
σ_z	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	−1	−1	1	1	1	1	−1	−1	1	1	−1
S_{61}^-	1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	−1	1	1	1	1	−1	−1	−1	1	−1	−1
S_{62}^-	1	1	1	−1	1	−1	1	1	−1	1	−1	1	1	1	−1	−1	1	1	1	−1	−1	−1	1	−1
S_{63}^-	1	−1	1	1	1	1	−1	1	−1	1	1	−1	−1	1	1	1	1	1	1	1	1	−1	−1	−1
S_{64}^-	1	1	−1	1	1	1	1	−1	−1	−1	1	1	1	−1	1	1	1	−1	1	−1	1	−1	1	1
S_{61}^+	1	1	1	1	1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	1	1	−1	1	−1	−1	−1
S_{62}^+	1	−1	−1	1	1	1	−1	1	1	−1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	1
S_{63}^+	1	1	−1	−1	1	1	1	−1	1	1	−1	1	1	1	−1	−1	1	1	−1	−1	1	1	1	1
S_{64}^+	1	−1	1	−1	1	−1	1	1	1	1	1	−1	1	1	1	1	−1	1	−1	1	−1	1	1	−1
S_{4x}^-	1	−1	−1	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1
S_{4y}^-	1	1	−1	1	1	1	1	1	1	−1	−1	1	1	1	1	1	1	1	−1	−1	1	1	−1	1
S_{4z}^-	1	1	1	−1	−1	1	1	1	1	1	−1	1	1	1	1	1	1	1	1	−1	−1	1	1	−1
S_{4x}^+	1	1	−1	−1	1	−1	1	−1	−1	1	1	1	1	1	1	−1	1	1	1	−1	1	−1	1	1
S_{4y}^+	1	1	1	−1	1	−1	−1	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	−1	1	−1
S_{4z}^+	1	−1	1	1	1	1	1	−1	−1	1	1	1	1	1	1	1	1	−1	1	1	1	−1	−1	1
σ_{d1}	1	−1	−1	−1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1	1	1	−1	1	−1	1	−1	1
σ_{d2}	1	1	−1	1	−1	−1	1	−1	−1	−1	−1	1	1	−1	1	−1	1	−1	−1	−1	1	1	1	1
σ_{d3}	1	−1	−1	−1	−1	−1	1	1	1	−1	1	−1	−1	1	−1	1	1	1	−1	1	−1	1	1	−1
σ_{d4}	1	1	1	1	1	−1	−1	1	−1	−1	1	−1	−1	1	1	−1	−1	−1	1	1	1	−1	−1	−1
σ_{d5}	1	−1	1	1	−1	1	−1	1	−1	−1	1	1	−1	1	1	1	−1	−1	−1	1	−1	−1	−1	1
σ_{d6}	1	−1	1	−1	−1	−1	1	1	−1	1	−1	−1	1	1	−1	−1	−1	1	1	1	−1	1	1	−1

T **71**.4 Character table

§ **16**–4, p. 71

$\mathbf{O}_h$	E	$3C_2$	$8C_3$	$6C_4$	$6C_2'$	i	3σ	$8S_6$	$6S_4$	$6\sigma_d$	τ
A_{1g}	1	1	1	1	1	1	1	1	1	1	a
A_{2g}	1	1	1	−1	−1	1	1	1	−1	−1	a
E_g	2	2	−1	0	0	2	2	−1	0	0	a
T_{1g}	3	−1	0	1	−1	3	−1	0	1	−1	a
T_{2g}	3	−1	0	−1	1	3	−1	0	−1	1	a
A_{1u}	1	1	1	1	1	−1	−1	−1	−1	−1	a
A_{2u}	1	1	1	−1	−1	−1	−1	−1	1	1	a
E_u	2	2	−1	0	0	−2	−2	1	0	0	a
T_{1u}	3	−1	0	1	−1	−3	1	0	−1	1	a
T_{2u}	3	−1	0	−1	1	−3	1	0	1	−1	a
$E_{1/2,g}$	2	0	1	$\sqrt{2}$	0	2	0	1	$\sqrt{2}$	0	c
$E_{5/2,g}$	2	0	1	$-\sqrt{2}$	0	2	0	1	$-\sqrt{2}$	0	c
$F_{3/2,g}$	4	0	−1	0	0	4	0	−1	0	0	c
$E_{1/2,u}$	2	0	1	$\sqrt{2}$	0	−2	0	−1	$-\sqrt{2}$	0	c
$E_{5/2,u}$	2	0	1	$-\sqrt{2}$	0	−2	0	−1	$\sqrt{2}$	0	c
$F_{3/2,u}$	4	0	−1	0	0	−4	0	1	0	0	c

T **71**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{O}_h$	0	1	2	3
A_{1g}	$\square 1$		$x^2+y^2+z^2$	
A_{2g}				
E_g			$\square(x^2-y^2, 2z^2-x^2-y^2)$	
T_{1g}		(R_x, R_y, R_z)		
T_{2g}			$\square(xy, yz, zx)$	
A_{1u}				
A_{2u}				xyz^a
E_u				
T_{1u}		$\square(x, y, z)$		(x^3, y^3, z^3), $\{x(y^2+z^2), y(z^2+x^2), z(x^2+y^2)\}$ [b]
T_{2u}				$\{x(z^2-y^2), y(x^2-z^2), z(y^2-x^2)\}$ [b]

[a] f function: f_{xyz}; [b] f functions: f_{xz^2}, f_{yz^2}, $f_{z(x^2-y^2)}$, $f_{x(x^2-y^2)}$, $f_{y(x^2-y^2)}$, f_{z^3}.

T **71**.6a Bases of irreducible representations — § **16**–6, pp. 74, 75

$\mathbf{O}_h$	$\langle \lvert j\, m\rangle \rvert$
A_{1g}	$\lvert 0\,0\rangle$
A_{2g}	$\sqrt{\frac{11}{16}}\,\lvert 6\,2\rangle_+ - \sqrt{\frac{5}{16}}\,\lvert 6\,6\rangle_+$
E_g	$\big\langle \frac{1}{\sqrt{2}}\lvert 2\,0\rangle - \frac{\mathrm{i}}{\sqrt{2}}\lvert 2\,2\rangle_+,\ \frac{1}{\sqrt{2}}\lvert 2\,0\rangle + \frac{\mathrm{i}}{\sqrt{2}}\lvert 2\,2\rangle_+ \big\rvert$
T_{1g}	$\big\langle \sqrt{\frac{7}{8}}\,\lvert 4\,1\rangle_- - \sqrt{\frac{1}{8}}\,\lvert 4\,3\rangle_-,\ -\lvert 4\,4\rangle_-,\ \sqrt{\frac{7}{8}}\,\lvert 4\,1\rangle_+ + \sqrt{\frac{1}{8}}\,\lvert 4\,3\rangle_+ \big\rvert$
T_{2g}	$\big\langle \lvert 2\,1\rangle_-,\ -\lvert 2\,2\rangle_-,\ -\lvert 2\,1\rangle_+ \big\rvert$
A_{1u}	$\sqrt{\frac{17}{24}}\,\lvert 9\,4\rangle_- - \sqrt{\frac{7}{24}}\,\lvert 9\,8\rangle_-$
A_{2u}	$\lvert 3\,2\rangle_-$
E_u	$\big\langle \frac{1}{\sqrt{2}}\,\lvert 5\,2\rangle_- - \frac{\mathrm{i}}{\sqrt{2}}\,\lvert 5\,4\rangle_-,\ -\frac{1}{\sqrt{2}}\,\lvert 5\,2\rangle_- - \frac{\mathrm{i}}{\sqrt{2}}\,\lvert 5\,4\rangle_- \big\rvert$
T_{1u}	$\big\langle \lvert 1\,1\rangle_+,\ \lvert 1\,0\rangle,\ \lvert 1\,1\rangle_- \big\rvert$
T_{2u}	$\big\langle \sqrt{\frac{5}{8}}\,\lvert 3\,1\rangle_+ - \sqrt{\frac{3}{8}}\,\lvert 3\,3\rangle_+,\ \lvert 3\,2\rangle_+,\ -\sqrt{\frac{5}{8}}\,\lvert 3\,1\rangle_- - \sqrt{\frac{3}{8}}\,\lvert 3\,3\rangle_- \big\rvert$
$E_{1/2,g}$	$\big\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle,\ \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \big\rvert$
$E_{5/2,g}$	$\big\langle \frac{1}{\sqrt{6}}\lvert \frac{5}{2}\,\frac{5}{2}\rangle - \sqrt{\frac{5}{6}}\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle,\ -\sqrt{\frac{5}{6}}\lvert \frac{5}{2}\,\frac{3}{2}\rangle + \frac{1}{\sqrt{6}}\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle \big\rvert$
$F_{3/2,g}$	$\big\langle \frac{1}{\sqrt{2}}\lvert \frac{3}{2}\,\frac{1}{2}\rangle - \frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle,\ \frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{3}{2}\,\frac{3}{2}\rangle - \frac{1}{\sqrt{2}}\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle,\ \frac{1}{\sqrt{2}}\lvert \frac{3}{2}\,\frac{1}{2}\rangle + \frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle,\ -\frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{3}{2}\,\frac{3}{2}\rangle - \frac{1}{\sqrt{2}}\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle \big\rvert$
$E_{1/2,u}$	$\big\langle \lvert \frac{1}{2}\,\frac{1}{2}\rangle,\ \lvert \frac{1}{2}\,\overline{\frac{1}{2}}\rangle \big\rvert^{\bullet}$
$E_{5/2,u}$	$\big\langle \frac{1}{\sqrt{6}}\lvert \frac{5}{2}\,\frac{5}{2}\rangle - \sqrt{\frac{5}{6}}\lvert \frac{5}{2}\,\overline{\frac{3}{2}}\rangle,\ -\sqrt{\frac{5}{6}}\lvert \frac{5}{2}\,\frac{3}{2}\rangle + \frac{1}{\sqrt{6}}\lvert \frac{5}{2}\,\overline{\frac{5}{2}}\rangle \big\rvert^{\bullet}$
$F_{3/2,u}$	$\big\langle \frac{1}{\sqrt{2}}\lvert \frac{3}{2}\,\frac{1}{2}\rangle - \frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle,\ \frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{3}{2}\,\frac{3}{2}\rangle - \frac{1}{\sqrt{2}}\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle,\ \frac{1}{\sqrt{2}}\lvert \frac{3}{2}\,\frac{1}{2}\rangle + \frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{3}{2}\,\overline{\frac{3}{2}}\rangle,\ -\frac{\mathrm{i}}{\sqrt{2}}\lvert \frac{3}{2}\,\frac{3}{2}\rangle - \frac{1}{\sqrt{2}}\lvert \frac{3}{2}\,\overline{\frac{1}{2}}\rangle \big\rvert^{\bullet}$

T **71**.6b Symmetrized harmonics — § **16**–6, pp. 74, 75

One-dimensional representations

$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	Column of the basis 1 Coefficient	$\pm$
A_{1g}	A_1	A	A_g	A_1	0	0	1	$+$
A_{2u}	A_2	A	A_u	A_1	3	2	1	$-$
A_{1g}	A_1	A	A_g	A_1	4	0	0.763762615826	$+$
						4	0.645497224368	$+$
A_{1g}	A_1	A	A_g	A_1	6	0	0.353553390593	$+$
						4	−0.935414346693	$+$
A_{2g}	A_2	A	A_g	A_2	6	2	0.829156197589	$+$
						6	−0.559016994375	$+$
A_{2u}	A_2	A	A_u	A_1	7	2	0.735980072194	$-$
						6	0.677003200386	$-$
A_{1g}	A_1	A	A_g	A_1	8	0	0.718070330817	$+$
						4	0.381881307913	$+$
						8	0.581843335157	$+$
A_{1u}	A_1	A	A_u	A_2	9	4	0.841625411530	$-$
						8	−0.540061724867	$-$
A_{2u}	A_2	A	A_u	A_1	9	2	0.433012701892	$-$
						6	−0.901387818866	$-$
A_{1g}	A_1	A	A_g	A_1	10	0	0.411425367878	$+$
						4	−0.586301969978	$+$

⇛

T **71**.6*b* Symmetrized harmonics *(cont.)*

One-dimensional representations								
							Column of the basis 1	
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	Coefficient	±
A_{1g}	A_1	A	A_g	A_1	10	8	−0.697838926019	+
A_{2g}	A_2	A	A_g	A_2	10	2	0.802015689788	+
						6	0.157288217401	+
						10	−0.576221528581	+
A_{2u}	A_2	A	A_u	A_1	11	2	0.665363309278	−
						6	0.459279326772	−
						10	0.588518620493	−
A_{1g}	A_1	A	A_g	A_1	12	0	0.695502665943	+
						4	0.314125566803	+
						8	0.348449537593	+
						12	0.544227975850	+
A_{1g}	A_1	A	A_g	A_1	12	4	0.558979374200	+
						8	−0.806267508183	+
						12	0.193583998482	+
A_{2g}	A_2	A	A_g	A_2	12	2	0.210406352883	+
						6	−0.826797284708	+
						10	0.521666001065	+
A_{1u}	A_1	A	A_u	A_2	13	4	0.786441087007	−
						8	0.228217732294	−
						12	−0.573957388081	−
A_{2u}	A_2	A	A_u	A_1	13	2	0.497389016096	−
						6	−0.493446636764	−
						10	−0.713522657898	−
A_{1g}	A_1	A	A_g	A_1	14	0	0.440096461964	+
						4	−0.457681828621	+
						8	−0.491132301422	+
						12	−0.596348480686	+
A_{2g}	A_2	A	A_g	A_2	14	2	0.777543289926	+
						6	0.248530839384	+
						10	0.020171788261	+
						14	−0.577279787560	+
A_{1u}	A_1	A	A_u	A_2	15	4	0.299739470207	−
						8	−0.810092587301	−
						12	0.503891109269	−
A_{2u}	A_2	A	A_u	A_1	15	2	0.634946889183	−
						6	0.318161585237	−
						10	0.483018535151	−
						14	0.512160861739	−
A_{2u}	A_2	A	A_u	A_1	15	6	0.648649259562	−
						10	−0.712102682285	−
						14	0.268633408111	−
A_{1g}	A_1	A	A_g	A_1	16	0	0.681361683315	+
						4	0.275868022760	+
						8	0.290489869847	+
						12	0.327569746984	+
						16	0.517645425853	+
A_{1g}	A_1	A	A_g	A_1	16	4	0.637048214634	+
						8	−0.329990348811	+
						12	−0.647980748513	+

⇛

T **71**.6b Symmetrized harmonics *(cont.)*

One-dimensional representations								
							Column of the basis	
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	1	
							Coefficient	±
A_{1g}	A_1	A	A_g	A_1	16	16	0.255728159340	+
A_{2g}	A_2	A	A_g	A_2	16	2	0.278048912813	+
						6	−0.719994393786	+
						10	−0.280380600702	+
						14	0.570686948992	+
A_{1u}	A_1	A	A_u	A_2	17	4	0.736753683115	−
						8	0.350316077517	−
						12	0.038273277231	−
						16	−0.577068291019	−
A_{2u}	A_2	A	A_u	A_1	17	2	0.524544072862	−
						6	−0.323629924644	−
						10	−0.504294706537	−
						14	−0.604817357934	−
A_{1g}	A_1	A	A_g	A_1	18	0	0.457915144210	+
						4	−0.386455976323	+
						8	−0.402094634007	+
						12	−0.437465928526	+
						16	−0.536571491741	+
A_{1g}	A_1	A	A_g	A_1	18	4	0.148727527118	+
						8	−0.637746007755	+
						12	0.723341669444	+
						16	−0.218945156413	+
A_{2g}	A_2	A	A_g	A_2	18	2	0.759120166861	+
						6	0.265186489844	+
						10	0.147854721474	+
						14	−0.058999411988	+
						18	−0.572774605402	+
A_{2g}	A_2	A	A_g	A_2	18	6	0.386921004710	+
						10	−0.782089226470	+
						14	0.483084960929	+
						18	−0.072508609678	+

↠

T **71**.6b Symmetrized harmonics *(cont.)*

Two-dimensional representations										
							Column of the basis			
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	1		2	
							Coefficient	±	Coefficient	±
E_g	E	${}^{1,2}E$	${}^{1,2}E_g$	E	2	0	0.707106781187	+	0.707106781187	+
						2	−0.707106781187 i	+	0.707106781187 i	+
E_g	E	${}^{1,2}E$	${}^{1,2}E_g$	E	4	0	0.456435464588	+	0.456435464588	+
						2	0.707106781187 i	+	−0.707106781187 i	+
						4	−0.540061724867	+	−0.540061724867	+
E_u	E	${}^{1,2}E$	${}^{1,2}E_u$	$E^{\triangle}$	5	2	0.707106781187	−	−0.707106781187	−
						4	−0.707106781187 i	−	−0.707106781187 i	−
E_g	E	${}^{1,2}E$	${}^{1,2}E_g$	E	6	0	0.661437827766	+	0.661437827766	+
						2	−0.395284707521 i	+	0.395284707521 i	+
						4	0.25	+	0.25	+

↠

T **71**.6*b* Symmetrized harmonics *(cont.)*

Two-dimensional representations										
							Column of the basis			
							1		2	
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	Coefficient	±	Coefficient	±
E_g	E	$^{1,2}E$	$^{1,2}E_g$	E	6	6	−0.586301969978 i	+	0.586301969978 i	+
E_u	E	$^{1,2}E$	$^{1,2}E_u$	$E^{\triangle}$	7	2	0.478713553878	−	−0.478713553878	−
						4	0.707106781187 i	−	0.707106781187 i	−
						6	−0.520416499867	−	0.520416499867	−
E_g	E	$^{1,2}E$	$^{1,2}E_g$	E	8	0	0.492125492126	+	0.492125492126	+
						2	0.460101671793 i	+	−0.460101671793 i	+
						4	−0.278605397905	+	−0.278605397905	+
						6	0.536941758120 i	+	−0.536941758120 i	+
						8	−0.424489731629	+	−0.424489731629	+
E_g	E	$^{1,2}E$	$^{1,2}E_g$	E	8	2	0.536941758120 i	+	−0.536941758120 i	+
						4	0.591153419673	+	0.591153419673	+
						6	−0.460101671793 i	+	0.460101671793 i	+
						8	−0.387991796832	+	−0.387991796832	+
E_u	E	$^{1,2}E$	$^{1,2}E_u$	$E^{\triangle}$	9	2	0.637377439199	−	−0.637377439199	−
						4	−0.381881307913 i	−	−0.381881307913 i	−
						6	0.306186217848	−	−0.306186217848	−
						8	−0.595119035712 i	−	−0.595119035712 i	−
E_g	E	$^{1,2}E$	$^{1,2}E_g$	E	10	0	0.644487845761	+	0.644487845761	+
						2	−0.314647791600 i	+	0.314647791600 i	+
						4	0.187140459880	+	0.187140459880	+
						6	−0.343798977702 i	+	0.343798977702 i	+
						8	0.222741700053	+	0.222741700053	+
						10	−0.531788520159 i	+	0.531788520159 i	+
E_g	E	$^{1,2}E$	$^{1,2}E_g$	E	10	2	0.281748440826 i	+	−0.281748440826 i	+
						4	−0.541390292004	+	−0.541390292004	+
						6	−0.607809568257 i	+	0.607809568257 i	+
						8	0.454858826147	+	0.454858826147	+
						10	0.226241784000 i	+	−0.226241784000 i	+
E_u	E	$^{1,2}E$	$^{1,2}E_u$	$E^{\triangle}$	11	2	0.527869144138	−	−0.527869144138	−
						4	0.350233566926 i	−	0.350233566926 i	−
						6	−0.289453945298	−	0.289453945298	−
						8	0.614277175710 i	−	0.614277175710 i	−
						10	−0.370905082492	−	0.370905082492	−
E_u	E	$^{1,2}E$	$^{1,2}E_u$	$E^{\triangle}$	11	4	0.614277175710 i	−	0.614277175710 i	−
						6	0.557447453624	−	−0.557447453624	−
						8	−0.350233566926 i	−	−0.350233566926 i	−
						10	−0.435031420071	−	0.435031420071	−
E_g	E	$^{1,2}E$	$^{1,2}E_g$	E	12	0	0.508072849927	+	0.508072849927	+
						2	0.364873518395 i	+	−0.364873518395 i	+
						4	−0.215003782611	+	−0.215003782611	+
						6	0.386893033360 i	+	−0.386893033360 i	+
						8	−0.238496883249	+	−0.238496883249	+
						10	0.466026926595 i	+	−0.466026926595 i	+
						12	−0.372497770879	+	−0.372497770879	+
E_g	E	$^{1,2}E$	$^{1,2}E_g$	E	12	2	0.587138739062 i	+	−0.587138739062 i	+
						4	0.498203740666	+	0.498203740666	+
						6	−0.092287083266 i	+	0.092287083266 i	+
						8	0.239535068790	+	0.239535068790	+
						10	−0.383081186376 i	+	0.383081186376 i	+

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T **71**.6*b* Symmetrized harmonics *(cont.)*

Two-dimensional representations										
							Column of the basis			
							1		2	
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	Coefficient	±	Coefficient	±
E_g	E	${}^{1,2}E$	${}^{1,2}E_g$	E	12	12	−0.440926279106	+	−0.440926279106	+
E_u	E	${}^{1,2}E$	${}^{1,2}E_u$	$E^\triangle$	13	2	0.613434661014	−	−0.613434661014	−
						4	−0.238475165189 i	−	−0.238475165189 i	−
						6	0.200049779344	−	−0.200049779344	−
						8	−0.438287107083 i	−	−0.438287107083 i	−
						10	0.289271503005	−	−0.289271503005	−
						12	−0.501032940387 i	−	−0.501032940387 i	−
E_u	E	${}^{1,2}E$	${}^{1,2}E_u$	$E^\triangle$	13	4	0.365902724671 i	−	0.365902724671 i	−
						6	−0.581579998038	−	0.581579998038	−
						8	−0.530907473199 i	−	−0.530907473199 i	−
						10	0.402199833270	−	−0.402199833270	−
						12	0.290262727508 i	−	0.290262727508 i	−
E_g	E	${}^{1,2}E$	${}^{1,2}E_g$	E	14	0	0.634946889183	+	0.634946889183	+
						2	−0.270658886297 i	+	0.270658886297 i	+
						4	0.158614962064	+	0.158614962064	+
						6	−0.282257393651 i	+	0.282257393651 i	+
						8	0.170207612553	+	0.170207612553	+
						10	−0.316146608760 i	+	0.316146608760 i	+
						12	0.206671503490	+	0.206671503490	+
						14	−0.497117544216 i	+	0.497117544216 i	+
E_g	E	${}^{1,2}E$	${}^{1,2}E_g$	E	14	2	0.352784613347 i	+	−0.352784613347 i	+
						4	−0.473870433650	+	−0.473870433650	+
						6	−0.490432996567 i	+	0.490432996567 i	+
						8	−0.163827091707	+	−0.163827091707	+
						10	−0.264779428592 i	+	0.264779428592 i	+
						12	0.498605551649	+	0.498605551649	+
						14	0.254775090343 i	+	−0.254775090343 i	+
E_g	E	${}^{1,2}E$	${}^{1,2}E_g$	E	14	4	0.381512487078	+	0.381512487078	+
						6	−0.385904716926 i	+	0.385904716926 i	+
						8	−0.568844955943	+	−0.568844955943	+
						10	0.574229680044 i	+	−0.574229680044 i	+
						12	0.175680500630	+	0.175680500630	+
						14	−0.146074720643 i	+	0.146074720643 i	+
E_u	E	${}^{1,2}E$	${}^{1,2}E_u$	$E^\triangle$	15	2	0.546279437613	−	−0.546279437613	−
						4	0.218530144883 i	−	0.218530144883 i	−
						6	−0.184901439533	−	0.184901439533	−
						8	0.411638486446 i	−	0.411638486446 i	−
						10	−0.280709006413	−	0.280709006413	−
						12	0.531787863959 i	−	0.531787863959 i	−
						14	−0.297645237521	−	0.297645237521	−
E_u	E	${}^{1,2}E$	${}^{1,2}E_u$	$E^\triangle$	15	4	0.638218380162 i	−	0.638218380162 i	−
						6	0.452576103582	−	−0.452576103582	−
						8	0.049282415492 i	−	0.049282415492 i	−
						10	0.225840399420	−	−0.225840399420	−
						12	−0.300413952316 i	−	−0.300413952316 i	−
						14	−0.494136605056	−	0.494136605056	−
E_g	E	${}^{1,2}E$	${}^{1,2}E_g$	E	16	0	0.517564612638	+	0.517564612638	+
						2	0.310950301297 i	+	−0.310950301297 i	+
						4	−0.181586893472	+	−0.181586893472	+

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T **71**.6*b* Symmetrized harmonics *(cont.)*

Two-dimensional representations										
							Column of the basis			
							1		2	
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	Coefficient	±	Coefficient	±
E_g	E	${}^{1,2}E$	${}^{1,2}E_g$	E	16	6	0.320950246588 i	+	−0.320950246588 i	+
						8	−0.191211552984	+	−0.191211552984	+
						10	0.347179489292 i	+	−0.347179489292 i	+
						12	−0.215618947623	+	−0.215618947623	+
						14	0.423989683310 i	+	−0.423989683310 i	+
						16	−0.340734036009	+	−0.340734036009	+
E_g	E	${}^{1,2}E$	${}^{1,2}E_g$	E	16	2	0.603866136793 i	+	−0.603866136793 i	+
						4	0.431125588060	+	0.431125588060	+
						6	0.000492250378 i	+	−0.000492250378 i	+
						8	0.311770353381	+	0.311770353381	+
						10	−0.114223669779 i	+	0.114223669779 i	+
						12	0.084302827144	+	0.084302827144	+
						14	−0.349711881106 i	+	0.349711881106 i	+
						16	−0.458064414038	+	−0.458064414038	+
E_g	E	${}^{1,2}E$	${}^{1,2}E_g$	E	16	4	0.200474762911	+	0.200474762911	+
						6	0.371207130000 i	+	−0.371207130000 i	+
						8	−0.519228650367	+	−0.519228650367	+
						10	−0.571937684414 i	+	0.571937684414 i	+
						12	0.427564039612	+	0.427564039612	+
						14	0.187330061082 i	+	−0.187330061082 i	+
						16	−0.086025985053	+	−0.086025985053	+
E_u	E	${}^{1,2}E$	${}^{1,2}E_u$	$E^{\triangle}$	17	2	0.602018901541	−	−0.602018901541	−
						4	−0.165725133424 i	−	−0.165725133424 i	−
						6	0.140990721669	−	−0.140990721669	−
						8	−0.316853083859 i	−	−0.316853083859 i	−
						10	0.219698084739	−	−0.219698084739	−
						12	−0.430243911042 i	−	−0.430243911042 i	−
						14	0.263491195589	−	−0.263491195589	−
						16	−0.432469051392 i	−	−0.432469051392 i	−
E_u	E	${}^{1,2}E$	${}^{1,2}E_u$	$E^{\triangle}$	17	4	0.448477630836 i	−	0.448477630836 i	−
						6	−0.455643064157	−	0.455643064157	−
						8	−0.404833680205 i	−	−0.404833680205 i	−
						10	−0.269690109484	−	0.269690109484	−
						12	−0.190424700527 i	−	−0.190424700527 i	−
						14	0.468675413193	−	−0.468675413193	−
						16	0.314190928324 i	−	0.314190928324 i	−
E_u	E	${}^{1,2}E$	${}^{1,2}E_u$	$E^{\triangle}$	17	6	0.469193830337	−	−0.469193830337	−
						8	−0.417556148964 i	−	−0.417556148964 i	−
						10	−0.501840035158	−	0.501840035158	−
						12	0.527158599063 i	−	0.527158599063 i	−
						14	0.167373022575	−	−0.167373022575	−
						16	−0.218519275801 i	−	−0.218519275801 i	−
E_g	E	${}^{1,2}E$	${}^{1,2}E_g$	E	18	0	0.628615033507	+	0.628615033507	+
						2	−0.241611852083 i	+	0.241611852083 i	+
						4	0.140757088755	+	0.140757088755	+
						6	−0.247674640934 i	+	0.247674640934 i	+
						8	0.146453085357	+	0.146453085357	+
						10	−0.262567376315 i	+	0.262567376315 i	+
						12	0.159336209818	+	0.159336209818	+

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T **71**.6*b* Symmetrized harmonics *(cont.)*

Two-dimensional representations										
							Column of the basis			
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	1		2	
							Coefficient	±	Coefficient	±
E_g	E	$^{1,2}E$	$^{1,2}E_g$	E	18	14	−0.297557972185 i	+	0.297557972185 i	+
						16	0.195432974812	+	0.195432974812	+
						18	−0.472015477771 i	+	0.472015477771 i	+
E_g	E	$^{1,2}E$	$^{1,2}E_g$	E	18	2	0.391780549627 i	+	−0.391780549627 i	+
						4	−0.416112583699	+	−0.416112583699	+
						6	−0.409656209510 i	+	0.409656209510 i	+
						8	−0.270906676008	+	−0.270906676008	+
						10	−0.305168647653 i	+	0.305168647653 i	+
						12	−0.000905425882	+	−0.000905425882	+
						14	−0.126345504065 i	+	0.126345504065 i	+
						16	0.503447187682	+	0.503447187682	+
						18	0.263815657299 i	+	−0.263815657299 i	+
E_g	E	$^{1,2}E$	$^{1,2}E_g$	E	18	4	0.470429282861	+	0.470429282861	+
						6	−0.401027671249 i	+	0.401027671249 i	+
						8	−0.347794528611	+	−0.347794528611	+
						10	0.145492889768 i	+	−0.145492889768 i	+
						12	−0.342507441900	+	−0.342507441900	+
						14	0.526374450127 i	+	−0.526374450127 i	+
						16	0.201056976845	+	0.201056976845	+
						18	−0.202332805480 i	+	0.202332805480 i	+

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T **71**.6*b* Symmetrized harmonics *(cont.)*

Three-dimensional representations												
							Column of the basis					
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	1		2		3	
							Coefficient	±	Coefficient	±	Coefficient	±
T_{1u}	T_1	T	T_u	T_2	1	0	0	+	1	+	0	−
						1	1	+	0	+	1	−
T_{2g}	T_2	T	T_g	T_2	2	1	1	−	0	−	−1	+
						2	0	−	−1	−	0	+
T_{1u}	T_1	T	T_u	T_2	3	0	0	+	−1	+	0	−
						1	0.612372435696	+	0	+	0.612372435696	−
						3	0.790569415042	+	0	+	−0.790569415042	−
T_{2u}	T_2	T	T_u	T_1	3	1	0.790569415042	+	0	+	−0.790569415042	−
						2	0	+	1	+	0	−
						3	−0.612372435696	+	0	+	−0.612372435696	−
T_{1g}	T_1	T	T_g	T_1	4	1	0.935414346693	−	0	−	0.935414346693	+
						3	−0.353553390593	−	0	−	0.353553390593	+
						4	0	−	−1	−	0	+
T_{2g}	T_2	T	T_g	T_2	4	1	0.353553390593	−	0	−	−0.353553390593	+
						2	0	−	1	−	0	+
						3	0.935414346693	−	0	−	0.935414346693	+
T_{1u}	T_1	T	T_u	T_2	5	0	0	+	1	+	0	−
						1	0.484122918276	+	0	+	0.484122918276	−
						3	0.522912516584	+	0	+	−0.522912516584	−
						5	0.701560760020	+	0	+	0.701560760020	−

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T **71**.6*b* Symmetrized harmonics *(cont.)*

Three-dimensional representations													
							Column of the basis						
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±
T_{1u}	T_1	T	T_u	T_2	5	1	0.572821961869	+	0	+	0.572821961869	−
						3	−0.795495128835	+	0	+	0.795495128835	−
						4	0	+	1	+	0	−
						5	0.197642353761	+	0	+	0.197642353761	−
T_{2u}	T_2	T	T_u	T_1	5	1	0.661437827766	+	0	+	−0.661437827766	−
						2	0	+	−1	+	0	−
						3	0.306186217848	+	0	+	0.306186217848	−
						5	−0.684653196881	+	0	+	0.684653196881	−
T_{1g}	T_1	T	T_g	T_1	6	1	0.433012701892	−	0	−	0.433012701892	+
						3	0.684653196881	−	0	−	−0.684653196881	+
						4	0	−	1	−	0	+
						5	−0.586301969978	−	0	−	−0.586301969978	+
T_{2g}	T_2	T	T_g	T_2	6	1	0.197642353761	−	0	−	−0.197642353761	+
						2	0	−	−1	−	0	+
						3	0.5625	−	0	−	0.5625	+
						5	0.802827036167	−	0	−	−0.802827036167	+
T_{2g}	T_2	T	T_g	T_2	6	1	0.879452954967	−	0	−	−0.879452954967	+
						3	−0.463512405443	−	0	−	−0.463512405443	+
						5	0.108253175473	−	0	−	−0.108253175473	+
						6	0	−	−1	−	0	+
T_{1u}	T_1	T	T_u	T_2	7	0	0	+	−1	+	0	−
						1	0.413398642354	+	0	+	0.413398642354	−
						3	0.429616471402	+	0	+	−0.429616471402	−
						5	0.474958879799	+	0	+	0.474958879799	−
						7	0.647259849288	+	0	+	−0.647259849288	−
T_{1u}	T_1	T	T_u	T_2	7	1	0.538552748113	+	0	+	0.538552748113	−
						3	−0.103644524699	+	0	+	0.103644524699	−
						4	0	+	−1	+	0	−
						5	−0.78125	+	0	+	−0.78125	−
						7	0.298106000443	+	0	+	−0.298106000443	−
T_{2u}	T_2	T	T_u	T_1	7	1	0.574099158465	+	0	+	−0.574099158465	−
						2	0	+	1	+	0	−
						3	0.419844651330	+	0	+	0.419844651330	−
						5	0.073287746247	+	0	+	−0.073287746247	−
						7	−0.699120541287	+	0	+	−0.699120541287	−
T_{2u}	T_2	T	T_u	T_1	7	1	0.457681828621	+	0	+	−0.457681828621	−
						3	−0.792728180873	+	0	+	−0.792728180873	−
						5	0.398360899499	+	0	+	−0.398360899499	−
						6	0	+	1	+	0	−
						7	−0.058463396668	+	0	+	−0.058463396668	−
T_{1g}	T_1	T	T_g	T_1	8	1	0.274217637106	−	0	−	0.274217637106	+
						3	0.605153647845	−	0	−	−0.605153647845	+
						4	0	−	−1	−	0	+
						5	0.338020432075	−	0	−	0.338020432075	+
						7	−0.666585281491	−	0	−	0.666585281491	+
T_{1g}	T_1	T	T_g	T_1	8	1	0.835608872320	−	0	−	0.835608872320	+
						3	−0.516334738808	−	0	−	0.516334738808	+
						5	0.184877493222	−	0	−	0.184877493222	+
						7	−0.03125	−	0	−	0.03125	+

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T **71**.6*b* Symmetrized harmonics *(cont.)*

Three-dimensional representations													
							Column of the basis						
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	1		2		3	
							Coefficient	±	Coefficient	±	Coefficient	±
T_{1g}	T_1	T	T_g	T_1	8	8	0	−	−1	−	0	+
T_{2g}	T_2	T	T_g	T_2	8	1	0.130728129146	−	0	−	−0.130728129146	+
						2	0	−	1	−	0	+
						3	0.380814300217	−	0	−	0.380814300217	+
						5	0.590864700037	−	0	−	−0.590864700037	+
						7	0.699120541287	−	0	−	0.699120541287	+
T_{2g}	T_2	T	T_g	T_2	8	1	0.457681828621	−	0	−	−0.457681828621	+
						3	0.471346972781	−	0	−	0.471346972781	+
						5	−0.708831013888	−	0	−	0.708831013888	+
						6	0	−	1	−	0	+
						7	0.256744948831	−	0	−	0.256744948831	+
T_{1u}	T_1	T	T_u	T_2	9	0	0	+	1	+	0	−
						1	0.366854902559	+	0	+	0.366854902559	−
						3	0.375487963772	+	0	+	−0.375487963772	−
						5	0.396364090436	+	0	+	0.396364090436	−
						7	0.443148525028	+	0	+	−0.443148525028	−
						9	0.609049392176	+	0	+	0.609049392176	−
T_{1u}	T_1	T	T_u	T_2	9	1	0.494352875611	+	0	+	0.494352875611	−
						3	0.137996263536	+	0	+	−0.137996263536	−
						4	0	+	1	+	0	−
						5	−0.392184387438	+	0	+	−0.392184387438	−
						7	−0.672329061686	+	0	+	0.672329061686	−
						9	0.361576139544	+	0	+	0.361576139544	−
T_{1u}	T_1	T	T_u	T_2	9	1	0.385196657363	+	0	+	0.385196657363	−
						3	−0.752680755907	+	0	+	0.752680755907	−
						5	0.509312687906	+	0	+	0.509312687906	−
						7	−0.159440090875	+	0	+	0.159440090875	−
						8	0	+	1	+	0	−
						9	0.016572815184	+	0	+	0.016572815184	−
T_{2u}	T_2	T	T_u	T_1	9	1	0.513014223731	+	0	+	−0.513014223731	−
						2	0	+	−1	+	0	−
						3	0.429616471402	+	0	+	0.429616471402	−
						5	0.251945554634	+	0	+	−0.251945554634	−
						7	−0.056336738679	+	0	+	−0.056336738679	−
						9	−0.696846972531	+	0	+	0.696846972531	−
T_{2u}	T_2	T	T_u	T_1	9	1	0.457681828621	+	0	+	−0.457681828621	−
						3	−0.298106000443	+	0	+	−0.298106000443	−
						5	−0.605153647845	+	0	+	0.605153647845	−
						6	0	+	−1	+	0	−
						7	0.568329171234	+	0	+	0.568329171234	−
						9	−0.111584819196	+	0	+	0.111584819196	−
T_{1g}	T_1	T	T_g	T_1	10	1	0.195156187450	−	0	−	0.195156187450	+
						3	0.486135912066	−	0	−	−0.486135912066	+
						4	0	−	1	−	0	+
						5	0.494105884401	−	0	−	0.494105884401	+
						7	0.091108623357	−	0	−	−0.091108623357	+
						9	−0.687855021970	−	0	−	−0.687855021970	+
T_{1g}	T_1	T	T_g	T_1	10	1	0.464564648354	−	0	−	0.464564648354	+
						3	0.315609529324	−	0	−	−0.315609529324	+

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T **71**.6*b* Symmetrized harmonics *(cont.)*

Three-dimensional representations												
							Column of the basis					
							1		2		3	
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	Coefficient	±	Coefficient	±	Coefficient	±
T_{1g}	T_1	T	T_g	T_1	10	5	−0.705724361915	−	0	−	−0.705724361915	+
						7	0.421006049541	−	0	−	−0.421006049541	+
						8	0	−	1	−	0	+
						9	−0.096318968796	−	0	−	−0.096318968796	+
T_{2g}	T_2	T	T_g	T_2	10	1	0.094721528539	−	0	−	−0.094721528539	+
						2	0	−	−1	−	0	+
						3	0.278852629650	−	0	−	0.278852629650	+
						5	0.445381025429	−	0	−	−0.445381025429	+
						7	0.574869423013	−	0	−	0.574869423013	+
						9	0.620024137950	−	0	−	−0.620024137950	+
T_{2g}	T_2	T	T_g	T_2	10	1	0.310491592957	−	0	−	−0.310491592957	+
						3	0.539062500000	−	0	−	0.539062500000	+
						5	−0.017469281074	−	0	−	0.017469281074	+
						6	0	−	−1	−	0	+
						7	−0.692552898053	−	0	−	−0.692552898053	+
						9	0.364790212881	−	0	−	−0.364790212881	+
T_{2g}	T_2	T	T_g	T_2	10	1	0.800447720176	−	0	−	−0.800447720176	+
						3	−0.543797142353	−	0	−	−0.543797142353	+
						5	0.243193475254	−	0	−	−0.243193475254	+
						7	−0.065945089907	−	0	−	−0.065945089907	+
						9	0.008734640537	−	0	−	−0.008734640537	+
						10	0	−	−1	−	0	+
T_{1u}	T_1	T	T_u	T_2	11	0	0	+	−1	+	0	−
						1	0.333212512690	+	0	+	0.333212512690	−
						3	0.338460276675	+	0	+	−0.338460276675	−
						5	0.350339670208	+	0	+	0.350339670208	−
						7	0.372965059746	+	0	+	−0.372965059746	−
						9	0.419758325709	+	0	+	0.419758325709	−
						11	0.579979473935	+	0	+	−0.579979473935	−
T_{1u}	T_1	T	T_u	T_2	11	1	0.456379743967	+	0	+	0.456379743967	−
						3	0.235349536428	+	0	+	−0.235349536428	−
						4	0	+	−1	+	0	−
						5	−0.134354558765	+	0	+	−0.134354558765	−
						7	−0.495108519710	+	0	+	0.495108519710	−
						9	−0.557226254436	+	0	+	−0.557226254436	−
						11	0.403290754440	+	0	+	−0.403290754441	−
T_{1u}	T_1	T	T_u	T_2	11	1	0.400223860088	+	0	+	0.400223860088	−
						3	−0.394018463167	+	0	+	0.394018463167	−
						5	−0.407847856765	+	0	+	−0.407847856765	−
						7	0.655537536431	+	0	+	−0.655537536431	−
						8	0	+	−1	+	0	−
						9	−0.295012403324	+	0	+	−0.295012403324	−
						11	0.038323079825	+	0	+	−0.038323079825	−
T_{2u}	T_2	T	T_u	T_1	11	1	0.467650076701	+	0	+	−0.467650076701	−
						2	0	+	1	+	0	−
						3	0.416551701262	+	0	+	0.416551701262	−
						5	0.310141244521	+	0	+	−0.310141244521	−
						7	0.136899991476	+	0	+	0.136899991476	−
						9	−0.135949285588	+	0	+	0.135949285588	−

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T **71**.6*b* Symmetrized harmonics *(cont.)*

Three-dimensional representations												
							Column of the basis					
							1		2		3	
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	Coefficient	±	Coefficient	±	Coefficient	±
T_{2u}	T_2	T	T_u	T_1	11	11	−0.688750084186	+	0	+	−0.688750084186	−
T_{2u}	T_2	T	T_u	T_1	11	1	0.435529357832	+	0	+	−0.435529357832	−
						3	−0.047641839522	+	0	+	−0.047641839522	−
						5	−0.522728282943	+	0	+	0.522728282943	−
						6	0	+	1	+	0	−
						7	−0.324256986638	+	0	+	−0.324256986638	−
						9	0.636036888060	+	0	+	−0.636036888060	−
						11	−0.158474160191	+	0	+	−0.158474160191	−
T_{2u}	T_2	T	T_u	T_1	11	1	0.334851305401	+	0	+	−0.334851305401	−
						3	−0.706413209675	+	0	+	−0.706413209675	−
						5	0.568716664438	+	0	+	−0.568716664438	−
						7	−0.249300933011	+	0	+	−0.249300933011	−
						9	0.056959635045	+	0	+	−0.056959635045	−
						10	0	+	1	+	0	−
						11	−0.004580484140	+	0	+	−0.004580484140	−
T_{1g}	T_1	T	T_g	T_1	12	1	0.148424649937	−	0	−	0.148424649937	+
						3	0.392578125000	−	0	−	−0.392578125000	+
						4	0	−	−1	−	0	+
						5	0.484014561584	−	0	−	0.484014561584	+
						7	0.341229998663	−	0	−	−0.341229998663	+
						9	−0.074462490393	−	0	−	−0.074462490393	+
						11	−0.683812743940	−	0	−	0.683812743940	+
T_{1g}	T_1	T	T_g	T_1	12	1	0.329285522120	−	0	−	0.329285522120	+
						3	0.454973331550	−	0	−	−0.454973331550	+
						5	−0.234081847915	−	0	−	−0.234081847915	+
						7	−0.542968750000	−	0	−	0.542968750000	+
						8	0	−	−1	−	0	+
						9	0.554921275561	−	0	−	0.554921275561	+
						11	−0.164387698535	−	0	−	0.164387698535	+
T_{1g}	T_1	T	T_g	T_1	12	1	0.771444817024	−	0	−	0.771444817024	+
						3	−0.558330767422	−	0	−	0.558330767422	+
						5	0.287258809963	−	0	−	0.287258809963	+
						7	−0.100666495350	−	0	−	0.100666495350	+
						9	0.021967230233	−	0	−	0.021967230233	+
						11	−0.002392079827	−	0	−	0.002392079827	+
						12	0	−	−1	−	0	+
T_{2g}	T_2	T	T_g	T_2	12	1	0.072712931519	−	0	−	−0.072712931519	+
						2	0	−	1	−	0	+
						3	0.215287184424	−	0	−	0.215287184424	+
						5	0.348702559987	−	0	−	−0.348702559987	+
						7	0.464355212030	−	0	−	0.464355212030	+
						9	0.547185319322	−	0	−	−0.547185319322	+
						11	0.558330767422	−	0	−	0.558330767422	+
T_{2g}	T_2	T	T_g	T_2	12	1	0.231303110983	−	0	−	−0.231303110983	+
						3	0.490039802025	−	0	−	0.490039802025	+
						5	0.274047171883	−	0	−	−0.274047171883	+
						6	0	−	1	−	0	+
						7	−0.302378192684	−	0	−	−0.302378192684	+
						9	−0.589308337814	−	0	−	0.589308337814	+

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T **71**.6b Symmetrized harmonics *(cont.)*

Three-dimensional representations												
							Column of the basis					
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±
T_{2g}	T_2	T	T_g	T_2	12	11	0.438795080233	−	0	−	0.438795080233	+
T_{2g}	T_2	T	T_g	T_2	12	1	0.464355212030	−	0	−	−0.464355212030	+
						3	0.201645377220	−	0	−	0.201645377220	+
						5	−0.657056040982	−	0	−	0.657056040982	+
						7	0.521109342556	−	0	−	0.521109342556	+
						9	−0.198340781363	−	0	−	0.198340781363	+
						10	0	−	1	−	0	+
						11	0.033116845621	−	0	−	0.033116845621	+
T_{1u}	T_1	T	T_u	T_2	13	0	0	+	1	+	0	−
						1	0.307421812957	+	0	+	0.307421812957	−
						3	0.310895616020	+	0	+	−0.310895616020	−
						5	0.318479550386	+	0	+	0.318479550386	−
						7	0.331848094015	+	0	+	−0.331848094015	−
						9	0.355163443107	+	0	+	0.355163443107	−
						11	0.401472611516	+	0	+	−0.401472611516	−
						13	0.556742340967	+	0	+	0.556742340967	−
T_{1u}	T_1	T	T_u	T_2	13	1	0.424930948370	+	0	+	0.424930948370	−
						3	0.276938774049	+	0	+	−0.276938774049	−
						4	0	+	1	+	0	−
						5	0.014846480269	+	0	+	0.014846480269	−
						7	−0.289606767616	+	0	+	0.289606767616	−
						9	−0.515307010508	+	0	+	−0.515307010508	−
						11	−0.454826449108	+	0	+	0.454826449108	−
						13	0.431552653604	+	0	+	0.431552653604	−
T_{1u}	T_1	T	T_u	T_2	13	1	0.390484593907	+	0	+	0.390484593907	−
						3	−0.166734284354	+	0	+	0.166734284354	−
						5	−0.509231931101	+	0	+	−0.509231931101	−
						7	−0.012788868021	+	0	+	0.012788868021	−
						8	0	+	1	+	0	−
						9	0.629899845337	+	0	+	0.629899845337	−
						11	−0.399432268171	+	0	+	0.399432268171	−
						13	0.062616240172	+	0	+	0.062616240172	−
T_{1u}	T_1	T	T_u	T_2	13	1	0.297591298944	+	0	+	0.297591298944	−
						3	−0.662098842301	+	0	+	0.662098842301	−
						5	0.598455854090	+	0	+	0.598455854090	−
						7	−0.321634310016	+	0	+	0.321634310016	−
						9	0.105377157495	+	0	+	0.105377157495	−
						11	−0.018989684065	+	0	+	0.018989684065	−
						12	0	+	1	+	0	−
						13	0.001244877811	+	0	+	0.001244877811	−
T_{2u}	T_2	T	T_u	T_1	13	1	0.432364706587	+	0	+	−0.432364706587	−
						2	0	+	−1	+	0	−
						3	0.398383641810	+	0	+	0.398383641810	−
						5	0.328472135480	+	0	+	−0.328472135480	−
						7	0.217801900466	+	0	+	0.217801900466	−
						9	0.055501063431	+	0	+	−0.055501063431	−
						11	−0.188213263302	+	0	+	−0.188213263302	−
						13	−0.678612571559	+	0	+	0.678612571559	−
T_{2u}	T_2	T	T_u	T_1	13	1	0.411606902651	+	0	+	−0.411606902651	−

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T **71**.6*b* Symmetrized harmonics *(cont.)*

Three-dimensional representations													
							Column of the basis						
							1		2		3		
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	Coefficient	±	Coefficient	±	Coefficient	±	
T_{2u}	T_2	T	T_u	T_1	13	3	0.083251595147	+	0	+	0.083251595147	−	
						5	−0.340126358562	+	0	+	0.340126358562	−	
						6	0	+	−1	+	0	−	
						7	−0.491080841371	+	0	+	−0.491080841371	−	
						9	−0.087450067714	+	0	+	0.087450067714	−	
						11	0.648033291868	+	0	+	0.648033291868	−	
						13	−0.198009937461	+	0	+	0.198009937461	−	
T_{2u}	T_2	T	T_u	T_1	13	1	0.357109558733	+	0	+	−0.357109558733	−	
						3	−0.441399228201	+	0	+	−0.441399228201	−	
						5	−0.239382341636	+	0	+	0.239382341636	−	
						7	0.643268620031	+	0	+	0.643268620031	−	
						9	−0.435558917645	+	0	+	0.435558917645	−	
						10	0	+	−1	+	0	−	
						11	0.129129851642	+	0	+	0.129129851642	−	
						13	−0.012448778109	+	0	+	0.012448778109	−	
T_{1g}	T_1	T	T_g	T_1	14	1	0.117941907646	−	0	−	0.117941907646	+	
						3	0.322924131902	−	0	−	−0.322924131902	+	
						4	0	−	1	−	0	+	
						5	0.437482016058	−	0	−	0.437482016058	+	
						7	0.409643233852	−	0	−	−0.409643233852	+	
						9	0.203122652480	−	0	−	0.203122652480	+	
						11	−0.185424764506	−	0	−	0.185424764506	+	
						13	−0.668558496168	−	0	−	−0.668558496168	+	
T_{1g}	T_1	T	T_g	T_1	14	1	0.253123794364	−	0	−	0.253123794364	+	
						3	0.457118881823	−	0	−	−0.457118881823	+	
						5	0.075265034641	−	0	−	0.075265034641	+	
						7	−0.460515232957	−	0	−	0.460515232957	+	
						8	0	−	1	−	0	+	
						9	−0.298272307848	−	0	−	−0.298272307848	+	
						11	0.607403034274	−	0	−	−0.607403034274	+	
						13	−0.226554081201	−	0	−	−0.226554081201	+	
T_{1g}	T_1	T	T_g	T_1	14	1	0.461026457915	−	0	−	0.461026457915	+	
						3	0.116381095882	−	0	−	−0.116381095882	+	
						5	−0.594029812383	−	0	−	−0.594029812383	+	
						7	0.573887320647	−	0	−	−0.573887320647	+	
						9	−0.291312047546	−	0	−	−0.291312047546	+	
						11	0.081935281475	−	0	−	−0.081935281475	+	
						12	0	−	1	−	0	+	
						13	−0.010764359221	−	0	−	−0.010764359221	+	
T_{2g}	T_2	T	T_g	T_2	14	1	0.058097261763	−	0	−	−0.058097261763	+	
						2	0	−	−1	−	0	+	
						3	0.172607767194	−	0	−	0.172607767194	+	
						5	0.281807981758	−	0	−	−0.281807981758	+	
						7	0.381153325816	−	0	−	0.381153325816	+	
						9	0.463898924696	−	0	−	−0.463898924696	+	
						11	0.517586474081	−	0	−	0.517586474081	+	
						13	0.508959428669	−	0	−	−0.508959428669	+	
T_{2g}	T_2	T	T_g	T_2	14	1	0.181760686757	−	0	−	−0.181760686757	+	
						3	0.427069508709	−	0	−	0.427069508709	+	

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T **71**.6*b* Symmetrized harmonics *(cont.)*

Three-dimensional representations													
							Column of the basis						
$\mathbf{O}_h$	**O**	**T**	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±
T_{2g}	T_2	T	T_g	T_2	14	5	0.375043628062	−	0	−	−0.375043628062	+
						6	0	−	−1	−	0	+
						7	0.011075484592	−	0	−	0.011075484592	+
						9	−0.433852797098	−	0	−	0.433852797098	+
						11	−0.466237966571	−	0	−	−0.466237966571	+
						13	0.488045292109	−	0	−	−0.488045292109	+
T_{2g}	T_2	T	T_g	T_2	14	1	0.339306285779	−	0	−	−0.339306285779	+
						3	0.375560579917	−	0	−	0.375560579917	+
						5	−0.356481346504	−	0	−	0.356481346504	+
						7	−0.357389688345	−	0	−	−0.357389688345	+
						9	0.616455319848	−	0	−	−0.616455319848	+
						10	0	−	−1	−	0	+
						11	−0.323486328125	−	0	−	−0.323486328125	+
						13	0.066019615640	−	0	−	−0.066019615640	+
T_{2g}	T_2	T	T_g	T_2	14	1	0.746948232213	−	0	−	−0.746948232213	+
						3	−0.565676778457	−	0	−	−0.565676778457	+
						5	0.320812765098	−	0	−	−0.320812765098	+
						7	−0.132829104115	−	0	−	−0.132829104115	+
						9	0.038268604915	−	0	−	−0.038268604915	+
						11	−0.006986859385	−	0	−	−0.006986859385	+
						13	0.000645935379	−	0	−	−0.000645935379	+
						14	0	−	−1	−	0	+
T_{1u}	T_1	T	T_u	T_2	15	0	0	+	−1	+	0	−
						1	0.286832247522	+	0	+	0.286832247522	−
						3	0.289273417050	+	0	+	−0.289273417050	−
						5	0.294485970009	+	0	+	0.294485970009	−
						7	0.303278526282	+	0	+	−0.303278526282	−
						9	0.317391967641	+	0	+	0.317391967641	−
						11	0.340933663393	+	0	+	−0.340933663393	−
						13	0.386582437246	+	0	+	0.386582437246	−
						15	0.537521065809	+	0	+	−0.537521065809	−
T_{1u}	T_1	T	T_u	T_2	15	1	0.398735888330	+	0	+	0.398735888330	−
						3	0.293993796979	+	0	+	−0.293993796979	−
						4	0	+	−1	+	0	−
						5	0.102298636358	+	0	+	0.102298636358	−
						7	−0.139289715337	+	0	+	0.139289715337	−
						9	−0.370576337190	+	0	+	−0.370576337190	−
						11	−0.498679040364	+	0	+	0.498679040364	−
						13	−0.367696267579	+	0	+	−0.367696267579	−
						15	0.451112569899	+	0	+	−0.451112569899	−
T_{1u}	T_1	T	T_u	T_2	15	1	0.375543240616	+	0	+	0.375543240616	−
						3	−0.028644157129	+	0	+	0.028644157129	−
						5	−0.422057112488	+	0	+	−0.422057112488	−
						7	−0.328684692786	+	0	+	0.328684692786	−
						8	0	+	−1	+	0	−
						9	0.253945039574	+	0	+	0.253945039574	−
						11	0.526582946154	+	0	+	−0.526582946154	−
						13	−0.471825523814	+	0	+	−0.471825523814	−
						15	0.087085854130	+	0	+	−0.087085854130	−

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T **71**.6*b* Symmetrized harmonics *(cont.)*

Three-dimensional representations													
							Column of the basis						
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±
T_{1u}	T_1	T	T_u	T_2	15	1	0.323438072204	+	0	+	0.323438072204	−
						3	−0.463245738503	+	0	+	0.463245738503	−
						5	−0.105010627141	+	0	+	−0.105010627141	−
						7	0.583988189685	+	0	+	−0.583988189685	−
						9	−0.522590193402	+	0	+	−0.522590193402	−
						11	0.230249429864	+	0	+	−0.230249429864	−
						12	0	+	−1	+	0	−
						13	−0.050905076690	+	0	+	−0.050905076690	−
						15	0.003889045977	+	0	+	−0.003889045977	−
T_{2u}	T_2	T	T_u	T_1	15	1	0.403948343397	+	0	+	−0.403948343397	−
						2	0	+	1	+	0	−
						3	0.379998951788	+	0	+	0.379998951788	−
						5	0.331084697342	+	0	+	−0.331084697342	−
						7	0.254830205173	+	0	+	0.254830205173	−
						9	0.146491165004	+	0	+	−0.146491165004	−
						11	−0.004034788545	+	0	+	−0.004034788545	−
						13	−0.224175988749	+	0	+	0.224175988749	−
						15	−0.667937278006	+	0	+	−0.667937278006	−
T_{2u}	T_2	T	T_u	T_1	15	1	0.389568320621	+	0	+	−0.389568320621	−
						3	0.155172615187	+	0	+	0.155172615187	−
						5	−0.190163940456	+	0	+	0.190163940456	−
						6	0	+	1	+	0	−
						7	−0.437149955154	+	0	+	−0.437149955154	−
						9	−0.371207701605	+	0	+	0.371207701605	−
						11	0.087858194090	+	0	+	0.087858194090	−
						13	0.630938244546	+	0	+	−0.630938244546	−
						15	−0.230864217872	+	0	+	−0.230864217872	−
T_{2u}	T_2	T	T_u	T_1	15	1	0.354855007579	+	0	+	−0.354855007579	−
						3	−0.243595659470	+	0	+	−0.243595659470	−
						5	−0.441345843680	+	0	+	0.441345843680	−
						7	0.202618793476	+	0	+	0.202618793476	−
						9	0.491123889468	+	0	+	−0.491123889468	−
						10	0	+	1	+	0	−
						11	−0.543504032594	+	0	+	−0.543504032594	−
						13	0.204373101479	+	0	+	−0.204373101479	−
						15	−0.023040751274	+	0	+	−0.023040751274	−
T_{2u}	T_2	T	T_u	T_1	15	1	0.268760532905	+	0	+	−0.268760532905	−
						3	−0.621815765617	+	0	+	−0.621815765617	−
						5	0.610809307113	+	0	+	−0.610809307113	−
						7	−0.377427854702	+	0	+	−0.377427854702	−
						9	0.154562058304	+	0	+	−0.154562058304	−
						11	−0.040584202602	+	0	+	−0.040584202602	−
						13	0.006042788719	+	0	+	−0.006042788719	−
						14	0	+	1	+	0	−
						15	−0.000334303319	+	0	+	−0.000334303319	−
T_{1g}	T_1	T	T_g	T_1	16	1	0.096707657901	−	0	−	0.096707657901	+
						3	0.270644638767	−	0	−	−0.270644638767	+
						4	0	−	−1	−	0	+
						5	0.387238259517	−	0	−	0.387238259517	+

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T **71**.6*b* Symmetrized harmonics *(cont.)*

Three-dimensional representations													
							Column of the basis						
							1		2		3		
$\mathbf{O}_h$	**O**	**T**	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	Coefficient	±	Coefficient	±	Coefficient	±	
T_{1g}	T_1	T	T_g	T_1	16	7	0.412149671419	−	0	−	−0.412149671419	+	
						9	0.317939504830	−	0	−	0.317939504830	+	
						11	0.089925728590	−	0	−	−0.089925728590	+	
						13	−0.260449963666	−	0	−	−0.260449963666	+	
						15	−0.648515179589	−	0	−	0.648515179589	+	
T_{1g}	T_1	T	T_g	T_1	16	1	0.203666917794	−	0	−	0.203666917794	+	
						3	0.424064570276	−	0	−	−0.424064570276	+	
						5	0.234076309612	−	0	−	0.234076309612	+	
						7	−0.223958473797	−	0	−	0.223958473797	+	
						8	0	−	−1	−	0	+	
						9	−0.470426264339	−	0	−	−0.470426264339	+	
						11	−0.074106834759	−	0	−	0.074106834759	+	
						13	0.607107712927	−	0	−	0.607107712927	+	
						15	−0.279941854485	−	0	−	0.279941854485	+	
T_{1g}	T_1	T	T_g	T_1	16	1	0.344496285358	−	0	−	0.344496285358	+	
						3	0.305941787666	−	0	−	−0.305941787666	+	
						5	−0.420440087931	−	0	−	−0.420440087931	+	
						7	−0.185803486910	−	0	−	0.185803486911	+	
						9	0.593449880643	−	0	−	0.593449880643	+	
						11	−0.445583645773	−	0	−	0.445583645773	+	
						12	0	−	−1	−	0	+	
						13	0.158425868960	−	0	−	0.158425868960	+	
						15	−0.024552524842	−	0	−	0.024552524842	+	
T_{1g}	T_1	T	T_g	T_1	16	1	0.725858347137	−	0	−	0.725858347137	+	
						3	−0.568785438901	−	0	−	0.568785438901	+	
						5	0.346645944592	−	0	−	0.346645944592	+	
						7	−0.161624505044	−	0	−	0.161624505044	+	
						9	0.055988370897	−	0	−	0.055988370897	+	
						11	−0.013694744057	−	0	−	0.013694744057	+	
						13	0.002149269636	−	0	−	0.002149269636	+	
						15	−0.000172633492	−	0	−	0.000172633492	+	
						16	0	−	−1	−	0	+	
T_{2g}	T_2	T	T_g	T_2	16	1	0.047805374587	−	0	−	−0.047805374587	+	
						2	0	−	1	−	0	+	
						3	0.142349818087	−	0	−	0.142349818087	+	
						5	0.233571184146	−	0	−	−0.233571184146	+	
						7	0.318789151069	−	0	−	0.318789151069	+	
						9	0.394399290395	−	0	−	−0.394399290395	+	
						11	0.454787075308	−	0	−	0.454787075308	+	
						13	0.489242028079	−	0	−	−0.489242028079	+	
						15	0.468539548361	−	0	−	0.468539548361	+	
T_{2g}	T_2	T	T_g	T_2	16	1	0.148028286494	−	0	−	−0.148028286494	+	
						3	0.370081021598	−	0	−	0.370081021598	+	
						5	0.397315980689	−	0	−	−0.397315980689	+	
						6	0	−	1	−	0	+	
						7	0.187905726946	−	0	−	0.187905726946	+	
						9	−0.177962125801	−	0	−	0.177962125801	+	
						11	−0.474118602994	−	0	−	−0.474118602994	+	
						13	−0.348040903727	−	0	−	0.348040903727	+	

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T **71**.6b Symmetrized harmonics *(cont.)*

Three-dimensional representations													
							Column of the basis						
							1		2		3		
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	Coefficient	±	Coefficient	±	Coefficient	±
T_{2g}	T_2	T	T_g	T_2	16	15	0.519968916888	−	0	−	0.519968916888	+
T_{2g}	T_2	T	T_g	T_2	16	1	0.266876177082	−	0	−	−0.266876177082	+
						3	0.412275425511	−	0	−	0.412275425511	+
						5	−0.076470742418	−	0	−	0.076470742418	+
						7	−0.477772544465	−	0	−	−0.477772544465	+
						9	−0.018938413327	−	0	−	0.018938413327	+
						10	0	−	1	−	0	+
						11	0.580307481679	−	0	−	0.580307481679	+
						13	−0.420745529758	−	0	−	0.420745529758	+
						15	0.102710580427	−	0	−	0.102710580427	+
T_{2g}	T_2	T	T_g	T_2	16	1	0.456288026832	−	0	−	−0.456288026832	+
						3	0.051078440271	−	0	−	0.051078440271	+
						5	−0.529205321386	−	0	−	0.529205321386	+
						7	0.595086669891	−	0	−	0.595086669891	+
						9	−0.367037001671	−	0	−	0.367037001671	+
						11	0.138970148303	−	0	−	0.138970148303	+
						13	−0.031074627886	−	0	−	0.031074627886	+
						14	0	−	1	−	0	+
						15	0.003364139106	−	0	−	0.003364139106	+
T_{1u}	T_1	T	T_u	T_2	17	0	0	+	1	+	0	−
						1	0.269899339812	+	0	+	0.269899339812	−
						3	0.271692710636	+	0	+	−0.271692710636	−
						5	0.275466401246	+	0	+	0.275466401246	−
						7	0.281657430462	+	0	+	−0.281657430462	−
						9	0.291127538604	+	0	+	0.291127538604	−
						11	0.305620251279	+	0	+	−0.305620251279	−
						13	0.329163084307	+	0	+	0.329163084307	−
						15	0.374098829310	+	0	+	−0.374098829310	−
						17	0.521217343455	+	0	+	0.521217343455	−
T_{1u}	T_1	T	T_u	T_2	17	1	0.376622217833	+	0	+	0.376622217833	−
						3	0.299308988345	+	0	+	−0.299308988345	−
						4	0	+	1	+	0	−
						5	0.154829722827	+	0	+	0.154829722827	−
						7	−0.035883481435	+	0	+	0.035883481435	−
						9	−0.239557662224	+	0	+	−0.239557662224	−
						11	−0.407228648228	+	0	+	0.407228648228	−
						13	−0.466226940096	+	0	+	−0.466226940096	−
						15	−0.294374431531	+	0	+	0.294374431531	−
						17	0.464825834202	+	0	+	0.464825834202	−
T_{1u}	T_1	T	T_u	T_2	17	1	0.360035646689	+	0	+	0.360035646689	−
						3	0.057225463648	+	0	+	−0.057225463648	−
						5	−0.308257511306	+	0	+	−0.308257511306	−
						7	−0.410918526520	+	0	+	0.410918526520	−
						8	0	+	1	+	0	−
						9	−0.094299494178	+	0	+	−0.094299494178	−
						11	0.394124019186	+	0	+	−0.394124019186	−
						13	0.398210853740	+	0	+	0.398210853740	−
						15	−0.517892430798	+	0	+	0.517892430798	−
						17	0.110509093268	+	0	+	0.110509093268	−

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T **71**.6*b* Symmetrized harmonics *(cont.)*

Three-dimensional representations													
							Column of the basis						
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±
T_{1u}	T_1	T	T_u	T_2	17	1	0.325920019166	+	0	+	0.325920019166	−
						3	−0.293550295616	+	0	+	0.293550295616	−
						5	−0.357653308421	+	0	+	−0.357653308421	−
						7	0.333892207227	+	0	+	−0.333892207227	−
						9	0.313564894165	+	0	+	0.313564894165	−
						11	−0.584509138532	+	0	+	0.584509138532	−
						12	0	+	1	+	0	−
						13	0.345954544971	+	0	+	0.345954544971	−
						15	−0.092093758687	+	0	+	0.092093758687	−
						17	0.008049007615	+	0	+	0.008049007615	−
T_{1u}	T_1	T	T_u	T_2	17	1	0.245704212020	+	0	+	0.245704212020	−
						3	−0.585797722357	+	0	+	0.585797722357	−
						5	0.612789245229	+	0	+	0.612789245229	−
						7	−0.419523793426	+	0	+	0.419523793426	−
						9	0.200708443433	+	0	+	0.200708443433	−
						11	−0.066765012448	+	0	+	0.066765012448	−
						13	0.014652124577	+	0	+	0.014652124577	−
						15	−0.001859444825	+	0	+	0.001859444825	−
						16	0	+	1	+	0	−
						17	0.000088973265	+	0	+	0.000088973265	−
T_{2u}	T_2	T	T_u	T_1	17	1	0.380445891679	+	0	+	−0.380445891679	−
						2	0	+	−1	+	0	−
						3	0.362817283421	+	0	+	0.362817283421	−
						5	0.326983692194	+	0	+	−0.326983692194	−
						7	0.271645203300	+	0	+	0.271645203300	−
						9	0.194385234772	+	0	+	−0.194385234772	−
						11	0.090694216899	+	0	+	0.090694216899	−
						13	−0.048840330506	+	0	+	0.048840330506	−
						15	−0.249784988092	+	0	+	−0.249784988092	−
						17	−0.657362998869	+	0	+	0.657362998869	−
T_{2u}	T_2	T	T_u	T_1	17	1	0.369979329466	+	0	+	−0.369979329466	−
						3	0.196019837329	+	0	+	0.196019837329	−
						5	−0.081119373661	+	0	+	0.081119373661	−
						6	0	+	−1	+	0	−
						7	−0.336187624411	+	0	+	−0.336187624411	−
						9	−0.423754742857	+	0	+	0.423754742857	−
						11	−0.235014995340	+	0	+	−0.235014995340	−
						13	0.210932413676	+	0	+	−0.210932413676	−
						15	0.599319704894	+	0	+	0.599319704894	−
						17	−0.258093637288	+	0	+	0.258093637288	−
T_{2u}	T_2	T	T_u	T_1	17	1	0.345911060450	+	0	+	−0.345911060450	−
						3	−0.109960894136	+	0	+	−0.109960894136	−
						5	−0.433818037430	+	0	+	0.433818037430	−
						7	−0.142231035380	+	0	+	−0.142231035380	−
						9	0.417118919822	+	0	+	−0.417118919822	−
						10	0	+	−1	+	0	−
						11	0.268554687500	+	0	+	0.268554687500	−
						13	−0.581113975770	+	0	+	0.581113975770	−
						15	0.273442091222	+	0	+	0.273442091222	−

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T **71**.6b Symmetrized harmonics *(cont.)*

Three-dimensional representations												
							Column of the basis					
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±
T_{2u}	T_2	T	T_u	T_1	17	17	−0.035391234167	+	0	+	0.035391234167	−
T_{2u}	T_2	T	T_u	T_1	17	1	0.296330429338	+	0	+	−0.296330429338	−
						3	−0.470999090449	+	0	+	−0.470999090449	−
						7	0.505964732153	+	0	+	0.505964732153	−
						9	−0.564814811377	+	0	+	0.564814811377	−
						11	0.322086538782	+	0	+	0.322086538782	−
						13	−0.106026762769	+	0	+	0.106026762769	−
						14	0	+	−1	+	0	−
						15	0.018688123648	+	0	+	0.018688123648	−
						17	−0.001180363745	+	0	+	0.001180363745	−
T_{1g}	T_1	T	T_g	T_1	18	1	0.081196672105	−	0	−	0.081196672105	+
						3	0.230621804316	−	0	−	−0.230621804316	+
						4	0	−	1	−	0	+
						5	0.341713857106	−	0	−	0.341713857106	+
						7	0.390924339042	−	0	−	−0.390924339042	+
						9	0.358127335546	−	0	−	0.358127335546	+
						11	0.229081455919	−	0	−	−0.229081455919	+
						15	−0.311536615266	−	0	−	0.311536615266	+
						17	−0.626771026976	−	0	−	−0.626771026976	+
T_{1g}	T_1	T	T_g	T_1	18	1	0.168964892008	−	0	−	0.168964892008	+
						3	0.383926949228	−	0	−	−0.383926949228	+
						5	0.307694682699	−	0	−	0.307694682699	+
						7	−0.028901548261	−	0	−	0.028901548261	+
						8	0	−	1	−	0	+
						9	−0.373082503769	−	0	−	−0.373082503769	+
						11	−0.377217412614	−	0	−	0.377217412614	+
						13	0.101430954473	−	0	−	0.101430954473	+
						15	0.575807747302	−	0	−	−0.575807747302	+
						17	−0.324366019020	−	0	−	−0.324366019020	+
T_{1g}	T_1	T	T_g	T_1	18	1	0.275742488655	−	0	−	0.275742488655	+
						3	0.365487566819	−	0	−	−0.365487566819	+
						5	−0.184999964946	−	0	−	−0.184999964946	+
						7	−0.424493294073	−	0	−	0.424493294073	+
						9	0.188402388609	−	0	−	0.188402388609	+
						11	0.445779655767	−	0	−	−0.445779655767	+
						12	0	−	1	−	0	+
						13	−0.532741472539	−	0	−	−0.532741472539	+
						15	0.236902752109	−	0	−	−0.236902752109	+
						17	−0.042591344898	−	0	−	−0.042591344898	+
T_{1g}	T_1	T	T_g	T_1	18	1	0.450947296924	−	0	−	0.450947296924	+
						5	−0.467573597057	−	0	−	−0.467573597057	+
						7	0.596041723858	−	0	−	−0.596041723858	+
						9	−0.424694810009	−	0	−	−0.424694810009	+
						11	0.196720995538	−	0	−	−0.196720995538	+
						13	−0.059722206451	−	0	−	−0.059722206451	+
						15	0.011095647515	−	0	−	−0.011095647515	+
						16	0	−	1	−	0	+
						17	−0.001021313509	−	0	−	−0.001021313509	+
T_{2g}	T_2	T	T_g	T_2	18	1	0.040234218060	−	0	−	−0.040234218060	+

↠

T **71**.6b Symmetrized harmonics *(cont.)*

Three-dimensional representations												
							Column of the basis					
							1		2		3	
$\mathbf{O}_h$	$\mathbf{O}$	$\mathbf{T}$	$\mathbf{T}_h$	$\mathbf{T}_d$	j	m	Coefficient	±	Coefficient	±	Coefficient	±
T_{2g}	T_2	T	T_g	T_2	18	2	0	−	−1	−	0	+
						3	0.119990537911	−	0	−	0.119990537911	+
						5	0.197545306541	−	0	−	−0.197545306541	+
						7	0.271192754235	−	0	−	0.271192754235	+
						9	0.338782849659	−	0	−	−0.338782849659	+
						11	0.397297026369	−	0	−	0.397297026369	+
						13	0.441888608945	−	0	−	−0.441888608945	+
						15	0.463113762697	−	0	−	0.463113762697	+
						17	0.434804754026	−	0	−	−0.434804754026	+
T_{2g}	T_2	T	T_g	T_2	18	1	0.123731457195	−	0	−	−0.123731457195	+
						3	0.322146711663	−	0	−	0.322146711663	+
						5	0.387575443292	−	0	−	−0.387575443292	+
						6	0	−	−1	−	0	+
						7	0.277504478141	−	0	−	0.277504478141	+
						9	0.014688437209	−	0	−	−0.014688437209	+
						11	−0.295963808902	−	0	−	−0.295963808902	+
						13	−0.464231047222	−	0	−	0.464231047222	+
						15	−0.242773241778	−	0	−	−0.242773241778	+
						17	0.539841573555	−	0	−	−0.539841573555	+
T_{2g}	T_2	T	T_g	T_2	18	1	0.218619098836	−	0	−	−0.218619098836	+
						3	0.403611556003	−	0	−	0.403611556003	+
						5	0.102672490094	−	0	−	−0.102672490094	+
						7	−0.351512349048	−	0	−	−0.351512349048	+
						9	−0.348320652952	−	0	−	0.348320652952	+
						10	0	−	−1	−	0	+
						11	0.228231481627	−	0	−	0.228231481627	+
						13	0.474646869010	−	0	−	−0.474646869010	+
						15	−0.486747741639	−	0	−	−0.486747741639	+
						17	0.139896136467	−	0	−	−0.139896136467	+
T_{2g}	T_2	T	T_g	T_2	18	1	0.346854202913	−	0	−	−0.346854202913	+
						3	0.246291319695	−	0	−	0.246291319695	+
						5	−0.448797723174	−	0	−	0.448797723174	+
						7	−0.042378302743	−	0	−	−0.042378302743	+
						9	0.524305644096	−	0	−	−0.524305644096	+
						11	−0.519417988829	−	0	−	−0.519417988829	+
						13	0.257378983144	−	0	−	−0.257378983144	+
						14	0	−	−1	−	0	+
						15	−0.069335160117	−	0	−	−0.069335160117	+
						17	0.008641177673	−	0	−	−0.008641177673	+
T_{2g}	T_2	T	T_g	T_2	18	1	0.707417957644	−	0	−	−0.707417957644	+
						3	−0.569293056529	−	0	−	−0.569293056529	+
						5	0.366750129488	−	0	−	−0.366750129488	+
						7	−0.187006606687	−	0	−	−0.187006606687	+
						9	0.074026077294	−	0	−	−0.074026077294	+
						11	−0.022043115180	−	0	−	−0.022043115180	+
						13	0.004684423391	−	0	−	−0.004684423391	+
						15	−0.000644672552	−	0	−	−0.000644672552	+
						17	0.000045776367	−	0	−	−0.000045776367	+
						18	0	−	−1	−	0	+

T **71**.6c Spin harmonics

§ **16**–6, pp. 74, 75

$\mathbf{O}_h$	$\langle \quad \vert$
$E_{1/2,g}$	$\langle a_{1g}\,\alpha, a_{1g}\,\beta \vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_{1g}^{(1)}\,\beta - t_{1g}^{(2)}\,\alpha + t_{1g}^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_{1g}^{(1)}\,\alpha + t_{1g}^{(2)}\,\beta + t_{1g}^{(3)}\,\alpha) \vert$
$E_{5/2,g}$	$\langle a_{2g}\,\alpha, a_{2g}\,\beta \vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_{2g}^{(1)}\,\beta - t_{2g}^{(2)}\,\alpha + t_{2g}^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_{2g}^{(1)}\,\alpha + t_{2g}^{(2)}\,\beta + t_{2g}^{(3)}\,\alpha) \vert$
$F_{3/2,g}$	$\langle e_g^{(1)}\,\alpha, e_g^{(1)}\,\beta, e_g^{(2)}\,\alpha, e_g^{(2)}\,\beta \vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_{1g}^{(1)}\,\beta - \omega^*\,t_{1g}^{(2)}\,\alpha + \omega\,t_{1g}^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_{1g}^{(1)}\,\alpha + \omega^*\,t_{1g}^{(2)}\,\beta + \omega\,t_{1g}^{(3)}\,\alpha),$
	$\frac{1}{\sqrt{3}}\,(\omega\,t_{1g}^{(1)}\,\beta - \omega^*\,t_{1g}^{(2)}\,\alpha + t_{1g}^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-\omega\,t_{1g}^{(1)}\,\alpha + \omega^*\,t_{1g}^{(2)}\,\beta + t_{1g}^{(3)}\,\alpha) \vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_{2g}^{(1)}\,\beta - \omega^*\,t_{2g}^{(2)}\,\alpha + \omega\,t_{2g}^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_{2g}^{(1)}\,\alpha + \omega^*\,t_{2g}^{(2)}\,\beta + \omega\,t_{2g}^{(3)}\,\alpha),$
	$\frac{1}{\sqrt{3}}\,(-\omega\,t_{2g}^{(1)}\,\beta + \omega^*\,t_{2g}^{(2)}\,\alpha - t_{2g}^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(\omega\,t_{2g}^{(1)}\,\alpha - \omega^*\,t_{2g}^{(2)}\,\beta - t_{2g}^{(3)}\,\alpha) \vert$
$E_{1/2,u}$	$\langle a_{1u}\,\alpha, a_{1u}\,\beta \vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_{1u}^{(1)}\,\beta - t_{1u}^{(2)}\,\alpha + t_{1u}^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_{1u}^{(1)}\,\alpha + t_{1u}^{(2)}\,\beta + t_{1u}^{(3)}\,\alpha) \vert$
$E_{5/2,u}$	$\langle a_{2u}\,\alpha, a_{2u}\,\beta \vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_{2u}^{(1)}\,\beta - t_{2u}^{(2)}\,\alpha + t_{2u}^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_{2u}^{(1)}\,\alpha + t_{2u}^{(2)}\,\beta + t_{2u}^{(3)}\,\alpha) \vert$
$F_{3/2,u}$	$\langle e_u^{(1)}\,\alpha, e_u^{(1)}\,\beta, e_u^{(2)}\,\alpha, e_u^{(2)}\,\beta \vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_{1u}^{(1)}\,\beta - \omega^*\,t_{1u}^{(2)}\,\alpha + \omega\,t_{1u}^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_{1u}^{(1)}\,\alpha + \omega^*\,t_{1u}^{(2)}\,\beta + \omega\,t_{1u}^{(3)}\,\alpha),$
	$\frac{1}{\sqrt{3}}\,(\omega\,t_{1u}^{(1)}\,\beta - \omega^*\,t_{1u}^{(2)}\,\alpha + t_{1u}^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-\omega\,t_{1u}^{(1)}\,\alpha + \omega^*\,t_{1u}^{(2)}\,\beta + t_{1u}^{(3)}\,\alpha) \vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_{2u}^{(1)}\,\beta - \omega^*\,t_{2u}^{(2)}\,\alpha + \omega\,t_{2u}^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_{2u}^{(1)}\,\alpha + \omega^*\,t_{2u}^{(2)}\,\beta + \omega\,t_{2u}^{(3)}\,\alpha),$
	$\frac{1}{\sqrt{3}}\,(-\omega\,t_{2u}^{(1)}\,\beta + \omega^*\,t_{2u}^{(2)}\,\alpha - t_{2u}^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(\omega\,t_{2u}^{(1)}\,\alpha - \omega^*\,t_{2u}^{(2)}\,\beta - t_{2u}^{(3)}\,\alpha) \vert$

$\alpha = \vert\frac{1}{2}\,\frac{1}{2}\rangle$, $\beta = \vert\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$; $\omega = \exp(2\pi i/3)$

T **71**.7 Matrix representations

Use T **69**.7 ▪. § **16**–7, p. 77

T **71**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{O}_h$	A_{1g}	A_{2g}	E_g	T_{1g}	T_{2g}
A_{1g}	A_{1g}	A_{2g}	E_g	T_{1g}	T_{2g}
A_{2g}		A_{1g}	E_g	T_{2g}	T_{1g}
E_g			$A_{1g} \oplus \{A_{2g}\} \oplus E_g$	$T_{1g} \oplus T_{2g}$	$T_{1g} \oplus T_{2g}$
T_{1g}				$A_{1g} \oplus E_g \oplus \{T_{1g}\} \oplus T_{2g}$	$A_{2g} \oplus E_g \oplus T_{1g} \oplus T_{2g}$
T_{2g}					$A_{1g} \oplus E_g \oplus \{T_{1g}\} \oplus T_{2g}$

$\twoheadrightarrow$

T **71**.8 Direct products of representations *(cont.)*

$\mathbf{O}_h$	A_{1u}	A_{2u}	E_u	T_{1u}	T_{2u}
A_{1g}	A_{1u}	A_{2u}	E_u	T_{1u}	T_{2u}
A_{2g}	A_{2u}	A_{1u}	E_u	T_{2u}	T_{1u}
E_g	E_u	E_u	$A_{1u} \oplus A_{2u} \oplus E_u$	$T_{1u} \oplus T_{2u}$	$T_{1u} \oplus T_{2u}$
T_{1g}	T_{1u}	T_{2u}	$T_{1u} \oplus T_{2u}$	$A_{1u} \oplus E_u \oplus T_{1u} \oplus T_{2u}$	$A_{2u} \oplus E_u \oplus T_{1u} \oplus T_{2u}$
T_{2g}	T_{2u}	T_{1u}	$T_{1u} \oplus T_{2u}$	$A_{2u} \oplus E_u \oplus T_{1u} \oplus T_{2u}$	$A_{1u} \oplus E_u \oplus T_{1u} \oplus T_{2u}$
A_{1u}	A_{1g}	A_{2g}	E_g	T_{1g}	T_{2g}
A_{2u}		A_{1g}	E_g	T_{2g}	T_{1g}
E_u			$A_{1g} \oplus \{A_{2g}\} \oplus E_g$	$T_{1g} \oplus T_{2g}$	$T_{1g} \oplus T_{2g}$
T_{1u}				$A_{1g} \oplus E_g \oplus \{T_{1g}\} \oplus T_{2g}$	$A_{2g} \oplus E_g \oplus T_{1g} \oplus T_{2g}$
T_{2u}					$A_{1g} \oplus E_g \oplus \{T_{1g}\} \oplus T_{2g}$

$\twoheadrightarrow$

T **71**.8 Direct products of representations *(cont.)*

$\mathbf{O}_h$	$E_{1/2,g}$	$E_{5/2,g}$	$F_{3/2,g}$
A_{1g}	$E_{1/2,g}$	$E_{5/2,g}$	$F_{3/2,g}$
A_{2g}	$E_{5/2,g}$	$E_{1/2,g}$	$F_{3/2,g}$
E_g	$F_{3/2,g}$	$F_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g} \oplus F_{3/2,g}$
T_{1g}	$E_{1/2,g} \oplus F_{3/2,g}$	$E_{5/2,g} \oplus F_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g} \oplus 2F_{3/2,g}$
T_{2g}	$E_{5/2,g} \oplus F_{3/2,g}$	$E_{1/2,g} \oplus F_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g} \oplus 2F_{3/2,g}$
A_{1u}	$E_{1/2,u}$	$E_{5/2,u}$	$F_{3/2,u}$
A_{2u}	$E_{5/2,u}$	$E_{1/2,u}$	$F_{3/2,u}$
E_u	$F_{3/2,u}$	$F_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u} \oplus F_{3/2,u}$
T_{1u}	$E_{1/2,u} \oplus F_{3/2,u}$	$E_{5/2,u} \oplus F_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u} \oplus 2F_{3/2,u}$
T_{2u}	$E_{5/2,u} \oplus F_{3/2,u}$	$E_{1/2,u} \oplus F_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u} \oplus 2F_{3/2,u}$
$E_{1/2,g}$	$\{A_{1g}\} \oplus T_{1g}$	$A_{2g} \oplus T_{2g}$	$E_g \oplus T_{1g} \oplus T_{2g}$
$E_{5/2,g}$		$\{A_{1g}\} \oplus T_{1g}$	$E_g \oplus T_{1g} \oplus T_{2g}$
$F_{3/2,g}$			$\{A_{1g}\} \oplus A_{2g} \oplus \{E_g\} \oplus 2T_{1g} \oplus T_{2g} \oplus \{T_{2g}\}$

$\twoheadrightarrow$

T **71**.8 Direct products of representations *(cont.)*

$\mathbf{O}_h$	$E_{1/2,u}$	$E_{5/2,u}$	$F_{3/2,u}$
A_{1g}	$E_{1/2,u}$	$E_{5/2,u}$	$F_{3/2,u}$
A_{2g}	$E_{5/2,u}$	$E_{1/2,u}$	$F_{3/2,u}$
E_g	$F_{3/2,u}$	$F_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u} \oplus F_{3/2,u}$
T_{1g}	$E_{1/2,u} \oplus F_{3/2,u}$	$E_{5/2,u} \oplus F_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u} \oplus 2F_{3/2,u}$
T_{2g}	$E_{5/2,u} \oplus F_{3/2,u}$	$E_{1/2,u} \oplus F_{3/2,u}$	$E_{1/2,u} \oplus E_{5/2,u} \oplus 2F_{3/2,u}$
A_{1u}	$E_{1/2,g}$	$E_{5/2,g}$	$F_{3/2,g}$
A_{2u}	$E_{5/2,g}$	$E_{1/2,g}$	$F_{3/2,g}$
E_u	$F_{3/2,g}$	$F_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g} \oplus F_{3/2,g}$
T_{1u}	$E_{1/2,g} \oplus F_{3/2,g}$	$E_{5/2,g} \oplus F_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g} \oplus 2F_{3/2,g}$
T_{2u}	$E_{5/2,g} \oplus F_{3/2,g}$	$E_{1/2,g} \oplus F_{3/2,g}$	$E_{1/2,g} \oplus E_{5/2,g} \oplus 2F_{3/2,g}$
$E_{1/2,g}$	$A_{1u} \oplus T_{1u}$	$A_{2u} \oplus T_{2u}$	$E_u \oplus T_{1u} \oplus T_{2u}$
$E_{5/2,g}$	$A_{2u} \oplus T_{2u}$	$A_{1u} \oplus T_{1u}$	$E_u \oplus T_{1u} \oplus T_{2u}$
$F_{3/2,g}$	$E_u \oplus T_{1u} \oplus T_{2u}$	$E_u \oplus T_{1u} \oplus T_{2u}$	$A_{1u} \oplus A_{2u} \oplus E_u \oplus 2T_{1u} \oplus 2T_{2u}$
$E_{1/2,u}$	$\{A_{1g}\} \oplus T_{1g}$	$A_{2g} \oplus T_{2g}$	$E_g \oplus T_{1g} \oplus T_{2g}$
$E_{5/2,u}$		$\{A_{1g}\} \oplus T_{1g}$	$E_g \oplus T_{1g} \oplus T_{2g}$
$F_{3/2,u}$			$\{A_{1g}\} \oplus A_{2g} \oplus \{E_g\} \oplus 2T_{1g} \oplus T_{2g} \oplus \{T_{2g}\}$

T **71**.9 Subduction (descent of symmetry) § **16**–9, p. 82

$\mathbf{O}_h$	$(\mathbf{T}_d)$	$\mathbf{T}_h$	$\mathbf{T}$	$\mathbf{O}$	$(\mathbf{C}_{4h})$
A_{1g}	A_1	A_g	A	A_1	A_g
A_{2g}	A_2	A_g	A	A_2	B_g
E_g	E	${}^1E_g \oplus {}^2E_g$	${}^1E \oplus {}^2E$	E	$A_g \oplus B_g$
T_{1g}	T_1	T_g	T	T_1	$A_g \oplus {}^1E_g \oplus {}^2E_g$
T_{2g}	T_2	T_g	T	T_2	$B_g \oplus {}^1E_g \oplus {}^2E_g$
A_{1u}	A_2	A_u	A	A_1	A_u
A_{2u}	A_1	A_u	A	A_2	B_u
E_u	E	${}^1E_u \oplus {}^2E_u$	${}^1E \oplus {}^2E$	E	$A_u \oplus B_u$
T_{1u}	T_2	T_u	T	T_1	$A_u \oplus {}^1E_u \oplus {}^2E_u$
T_{2u}	T_1	T_u	T	T_2	$B_u \oplus {}^1E_u \oplus {}^2E_u$
$E_{1/2,g}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$
$E_{5/2,g}$	$E_{5/2}$	$E_{1/2,g}$	$E_{1/2}$	$E_{5/2}$	${}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$
$F_{3/2,g}$	$F_{3/2}$	${}^1F_{3/2,g} \oplus {}^2F_{3/2,g}$	${}^1F_{3/2} \oplus {}^2F_{3/2}$	$F_{3/2}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$
$E_{1/2,u}$	$E_{5/2}$	$E_{1/2,u}$	$E_{1/2}$	$E_{1/2}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$
$E_{5/2,u}$	$E_{1/2}$	$E_{1/2,u}$	$E_{1/2}$	$E_{5/2}$	${}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$
$F_{3/2,u}$	$F_{3/2}$	${}^1F_{3/2,u} \oplus {}^2F_{3/2,u}$	${}^1F_{3/2} \oplus {}^2F_{3/2}$	$F_{3/2}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u} \oplus {}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$

$\twoheadrightarrow$

T **71**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{O}_h$	$\mathbf{C}_{2h}$	$(\mathbf{C}_{2h})$	$(\mathbf{C}_{4v})$	$(\mathbf{C}_{2v})$	$(\mathbf{C}_{2v})$
	C_2	C_2'		$C_{2z}, \sigma_x, \sigma_y$	$C_{2z}, \sigma_{d1}, \sigma_{d2}$
A_{1g}	A_g	A_g	A_1	A_1	A_1
A_{2g}	A_g	B_g	B_1	A_1	A_2
E_g	$2A_g$	$A_g \oplus B_g$	$A_1 \oplus B_1$	$2A_1$	$A_1 \oplus A_2$
T_{1g}	$A_g \oplus 2B_g$	$A_g \oplus 2B_g$	$A_2 \oplus E$	$A_2 \oplus B_1 \oplus B_2$	$A_2 \oplus B_1 \oplus B_2$
T_{2g}	$A_g \oplus 2B_g$	$2A_g \oplus B_g$	$B_2 \oplus E$	$A_2 \oplus B_1 \oplus B_2$	$A_1 \oplus B_1 \oplus B_2$
A_{1u}	A_u	A_u	A_2	A_2	A_2
A_{2u}	A_u	B_u	B_2	A_2	A_1
E_u	$2A_u$	$A_u \oplus B_u$	$A_2 \oplus B_2$	$2A_2$	$A_1 \oplus A_2$
T_{1u}	$A_u \oplus 2B_u$	$A_u \oplus 2B_u$	$A_1 \oplus E$	$A_1 \oplus B_1 \oplus B_2$	$A_1 \oplus B_1 \oplus B_2$
T_{2u}	$A_u \oplus 2B_u$	$2A_u \oplus B_u$	$B_1 \oplus E$	$A_1 \oplus B_1 \oplus B_2$	$A_2 \oplus B_1 \oplus B_2$
$E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{5/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$
$F_{3/2,g}$	$2\,{}^1E_{1/2,g} \oplus 2\,{}^2E_{1/2,g}$	$2\,{}^1E_{1/2,g} \oplus 2\,{}^2E_{1/2,g}$	$E_{1/2} \oplus E_{3/2}$	$2E_{1/2}$	$2E_{1/2}$
$E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{5/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$
$F_{3/2,u}$	$2\,{}^1E_{1/2,u} \oplus 2\,{}^2E_{1/2,u}$	$2\,{}^1E_{1/2,u} \oplus 2\,{}^2E_{1/2,u}$	$E_{1/2} \oplus E_{3/2}$	$2E_{1/2}$	$2E_{1/2}$

$\twoheadrightarrow$

T **71**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{O}_h$	$(\mathbf{C}_{2v})$	$(\mathbf{D}_{3d})$	$(\mathbf{D}_{4h})$	$\mathbf{D}_{2h}$	$(\mathbf{D}_{2h})$
	$C'_{2a}, \sigma_{d2}, \sigma_z$			C_2	C'_2
A_{1g}	A_1	A_{1g}	A_{1g}	A_g	A_g
A_{2g}	B_1	A_{2g}	B_{1g}	A_g	B_{1g}
E_g	$A_1 \oplus B_1$	E_g	$A_{1g} \oplus B_{1g}$	$2A_g$	$A_g \oplus B_{1g}$
T_{1g}	$A_2 \oplus B_1 \oplus B_2$	$A_{2g} \oplus E_g$	$A_{2g} \oplus E_g$	$B_{1g} \oplus B_{2g} \oplus B_{3g}$	$B_{1g} \oplus B_{2g} \oplus B_{3g}$
T_{2g}	$A_1 \oplus A_2 \oplus B_2$	$A_{1g} \oplus E_g$	$B_{2g} \oplus E_g$	$B_{1g} \oplus B_{2g} \oplus B_{3g}$	$A_g \oplus B_{2g} \oplus B_{3g}$
A_{1u}	A_2	A_{1u}	A_{1u}	A_u	A_u
A_{2u}	B_2	A_{2u}	B_{1u}	A_u	B_{1u}
E_u	$A_2 \oplus B_2$	E_u	$A_{1u} \oplus B_{1u}$	$2A_u$	$A_u \oplus B_{1u}$
T_{1u}	$A_1 \oplus B_1 \oplus B_2$	$A_{2u} \oplus E_u$	$A_{2u} \oplus E_u$	$B_{1u} \oplus B_{2u} \oplus B_{3u}$	$B_{1u} \oplus B_{2u} \oplus B_{3u}$
T_{2u}	$A_1 \oplus A_2 \oplus B_1$	$A_{1u} \oplus E_u$	$B_{2u} \oplus E_u$	$B_{1u} \oplus B_{2u} \oplus B_{3u}$	$A_u \oplus B_{2u} \oplus B_{3u}$
$E_{1/2,g}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2,g}$	$E_{1/2,g}$	$E_{1/2,g}$
$E_{5/2,g}$	$E_{1/2}$	$E_{1/2,g}$	$E_{3/2,g}$	$E_{1/2,g}$	$E_{1/2,g}$
$F_{3/2,g}$	$2E_{1/2}$	$E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$	$2E_{1/2,g}$	$2E_{1/2,g}$
$E_{1/2,u}$	$E_{1/2}$	$E_{1/2,u}$	$E_{1/2,u}$	$E_{1/2,u}$	$E_{1/2,u}$
$E_{5/2,u}$	$E_{1/2}$	$E_{1/2,u}$	$E_{3/2,u}$	$E_{1/2,u}$	$E_{1/2,u}$
$F_{3/2,u}$	$2E_{1/2}$	$E_{1/2,u} \oplus {}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$	$2E_{1/2,u}$	$2E_{1/2,u}$

Other subgroups: $\mathbf{C}_{3v}$, $\mathbf{D}_{2d}$, $\mathbf{S}_4$, $\mathbf{C}_s$ (see $\mathbf{T}_d$); $\mathbf{S}_6$, $\mathbf{C}_i$ (see $\mathbf{T}_h$); $\mathbf{D}_4$, $\mathbf{D}_3$, $2\mathbf{D}_2$, $\mathbf{C}_4$, $\mathbf{C}_3$, $2\mathbf{C}_2$ (see $\mathbf{O}$)

T **71**.10 ♣ Subduction from O(3) § **16**–10, p. 82

j	$\mathbf{O}_h$
$12n$	$(n+1)\,A_{1g} \oplus n\,(A_{2g} \oplus 2E_g \oplus 3T_{1g} \oplus 3T_{2g})$
$12n+1$	$n\,(A_{1u} \oplus A_{2u} \oplus 2E_u \oplus 2T_{1u} \oplus 3T_{2u}) \oplus (n+1)\,T_{1u}$
$12n+2$	$n\,(A_{1g} \oplus A_{2g} \oplus E_g \oplus 3T_{1g} \oplus 2T_{2g}) \oplus (n+1)(E_g \oplus T_{2g})$
$12n+3$	$n\,(A_{1u} \oplus 2E_u \oplus 2T_{1u} \oplus 2T_{2u}) \oplus (n+1)(A_{2u} \oplus T_{1u} \oplus T_{2u})$
$12n+4$	$(n+1)(A_{1g} \oplus E_g \oplus T_{1g} \oplus T_{2g}) \oplus n\,(A_{2g} \oplus E_g \oplus 2T_{1g} \oplus 2T_{2g})$
$12n+5$	$n\,(A_{1u} \oplus A_{2u} \oplus E_u \oplus T_{1u} \oplus 2T_{2u}) \oplus (n+1)(E_u \oplus 2T_{1u} \oplus T_{2u})$
$12n+6$	$(n+1)(A_{1g} \oplus A_{2g} \oplus E_g \oplus T_{1g} \oplus 2T_{2g}) \oplus n\,(E_g \oplus 2T_{1g} \oplus T_{2g})$
$12n+7$	$n\,(A_{1u} \oplus E_u \oplus T_{1u} \oplus T_{2u}) \oplus (n+1)(A_{2u} \oplus E_u \oplus 2T_{1u} \oplus 2T_{2u})$
$12n+8$	$(n+1)(A_{1g} \oplus 2E_g \oplus 2T_{1g} \oplus 2T_{2g}) \oplus n\,(A_{2g} \oplus T_{1g} \oplus T_{2g})$
$12n+9$	$(n+1)(A_{1u} \oplus A_{2u} \oplus E_u \oplus 3T_{1u} \oplus 2T_{2u}) \oplus n\,(E_u \oplus T_{2u})$
$12n+10$	$(n+1)(A_{1g} \oplus A_{2g} \oplus 2E_g \oplus 2T_{1g} \oplus 3T_{2g}) \oplus n\,T_{1g}$
$12n+11$	$n\,A_{1u} \oplus (n+1)(A_{2u} \oplus 2E_u \oplus 3T_{1u} \oplus 3T_{2u})$

$n = 0, 1, 2, \ldots$ $\twoheadrightarrow$

T **71**.10 ♣ Subduction from O(3) *(cont.)*

j	$\mathbf{O}_h$
$12n+\frac{1}{2}$	$(2n+1)\,E_{1/2,g} \oplus 2n\,(E_{5/2,g} \oplus 2F_{3/2,g})$
$12n+\frac{3}{2}$	$2n\,(E_{1/2,g} \oplus E_{5/2,g} \oplus F_{3/2,g}) \oplus (2n+1)\,F_{3/2,g}$
$12n+\frac{5}{2}$	$2n\,(E_{1/2,g} \oplus F_{3/2,g}) \oplus (2n+1)(E_{5/2,g} \oplus F_{3/2,g})$
$12n+\frac{7}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{5/2,g} \oplus F_{3/2,g}) \oplus 2n\,F_{3/2,g}$
$12n+\frac{9}{2}$	$(2n+1)(E_{1/2,g} \oplus 2F_{3/2,g}) \oplus 2n\,E_{5/2,g}$
$12n+\frac{11}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{5/2,g} \oplus 2F_{3/2,g})$
$12n+\frac{13}{2}$	$(2n+1)(E_{1/2,g} \oplus 2F_{3/2,g}) \oplus (2n+2)\,E_{5/2,g}$
$12n+\frac{15}{2}$	$(2n+1)(E_{1/2,g} \oplus E_{5/2,g} \oplus F_{3/2,g}) \oplus (2n+2)\,F_{3/2,g}$
$12n+\frac{17}{2}$	$(2n+2)(E_{1/2,g} \oplus F_{3/2,g}) \oplus (2n+1)(E_{5/2,g} \oplus F_{3/2,g})$
$12n+\frac{19}{2}$	$(2n+2)(E_{1/2,g} \oplus E_{5/2,g} \oplus F_{3/2,g}) \oplus (2n+1)\,F_{3/2,g}$
$12n+\frac{21}{2}$	$(2n+1)\,E_{1/2,g} \oplus (2n+2)(E_{5/2,g} \oplus 2F_{3/2,g})$
$12n+\frac{23}{2}$	$(2n+2)(E_{1/2,g} \oplus E_{5/2,g} \oplus 2F_{3/2,g})$

$n = 0, 1, 2, \ldots$

T **71**.11 Clebsch–Gordan coefficients

Use T **69**.11 ▪. § **16**–11, p. 83

(1) Product forms: $\mathbf{T} \otimes \mathbf{C}_i$.

(2) Group chains: $\mathbf{I}_h \supset \mathbf{T}_h \supset \underline{\mathbf{T}}$, $\mathbf{I}_h \supset \mathbf{T}_h \supset \underline{\mathbf{D}_{2h}}$, $\mathbf{I}_h \supset \mathbf{T}_h \supset (\mathbf{S}_6)$,
$\mathbf{O}_h \supset \underline{\mathbf{T}_h} \supset \underline{\mathbf{T}}$, $\mathbf{O}_h \supset \underline{\mathbf{T}_h} \supset \underline{\mathbf{D}_{2h}}$, $\mathbf{O}_h \supset \underline{\mathbf{T}_h} \supset (\mathbf{S}_6)$.

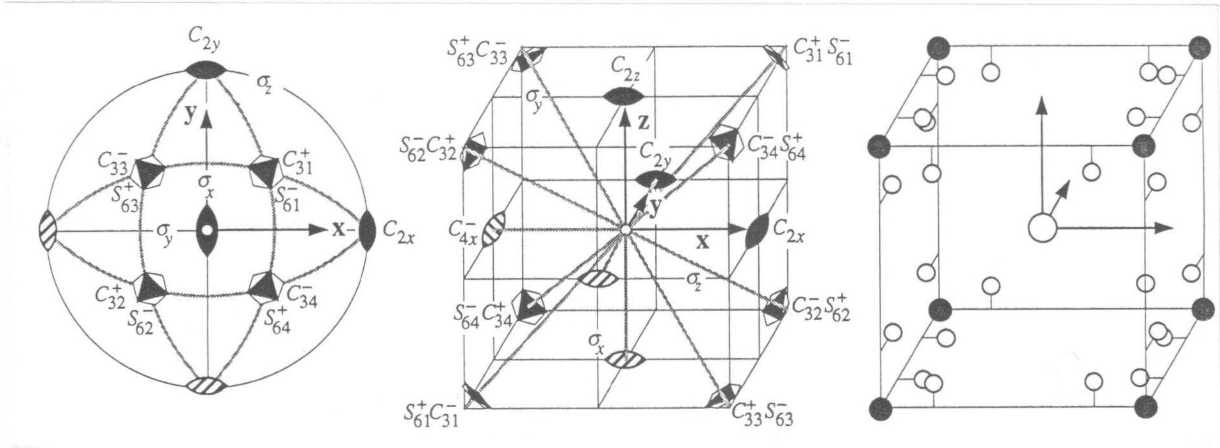
(3) Operations of G: E, (C_{2x}, C_{2y}, C_{2z}), $(C_{31}^+, C_{32}^+, C_{33}^+, C_{34}^+)$, $(C_{31}^-, C_{32}^-, C_{33}^-, C_{34}^-)$,
i, $(\sigma_x, \sigma_y, \sigma_z)$, $(S_{61}^-, S_{62}^-, S_{63}^-, S_{64}^-)$, $(S_{61}^+, S_{62}^+, S_{63}^+, S_{64}^+)$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, $(C_{2x}, C_{2y}, C_{2z}, \widetilde{C}_{2x}, \widetilde{C}_{2y}, \widetilde{C}_{2z})$, $(C_{31}^+, C_{32}^+, C_{33}^+, C_{34}^+)$,
$(\widetilde{C}_{31}^+, \widetilde{C}_{32}^+, \widetilde{C}_{33}^+, \widetilde{C}_{34}^+)$, $(C_{31}^-, C_{32}^-, C_{33}^-, C_{34}^-)$, $(\widetilde{C}_{31}^-, \widetilde{C}_{32}^-, \widetilde{C}_{33}^-, \widetilde{C}_{34}^-)$,
i, $\widetilde{i}$, $(\sigma_x, \sigma_y, \sigma_z)$, $(\widetilde{\sigma}_x, \widetilde{\sigma}_y, \widetilde{\sigma}_z)$, $(S_{61}^-, S_{62}^-, S_{63}^-, S_{64}^-)$,
$(\widetilde{S}_{61}^-, \widetilde{S}_{62}^-, \widetilde{S}_{63}^-, \widetilde{S}_{64}^-)$, $(S_{61}^+, S_{62}^+, S_{63}^+, S_{64}^+)$, $(\widetilde{S}_{61}^+, \widetilde{S}_{62}^+, \widetilde{S}_{63}^+, \widetilde{S}_{64}^+)$.

(5) Classes and representations: $|r| = 6$, $|i| = 2$, $|I| = 8$, $|\widetilde{I}| = 6$.

(6) Subduction: no failure in subduction.

F **72** See Chapter **15**, p. 65



Examples: CH_4, $C(CH_3)_4$ symmetrical.

T **72**.1 Parameters
Use T **71**.1. § **16**–1, p. 68

T **72**.2 Multiplication table
Use T **71**.2. § **16**–2, p. 69

T **72**.3 Factor table
Use T **71**.3. § **16**–3, p. 70

T **72**.4 Character table

§ **16**–4, p. 71

$\mathbf{T}_h$	E	$3C_2$	$4C_3^+$	$4C_3^-$	i	3σ	$4S_6^-$	$4S_6^+$	τ
A_g	1	1	1	1	1	1	1	1	a
${}^1E_g$	1	1	ϵ	ϵ^*	1	1	ϵ	ϵ^*	b
${}^2E_g$	1	1	ϵ^*	ϵ	1	1	ϵ^*	ϵ	b
T_g	3	-1	0	0	3	-1	0	0	a
A_u	1	1	1	1	-1	-1	-1	-1	a
${}^1E_u$	1	1	ϵ	ϵ^*	-1	-1	$-\epsilon$	$-\epsilon^*$	b
${}^2E_u$	1	1	ϵ^*	ϵ	-1	-1	$-\epsilon^*$	$-\epsilon$	b
T_u	3	-1	0	0	-3	1	0	0	a
$E_{1/2,g}$	2	0	1	1	2	0	1	1	c
${}^1F_{3/2,g}$	2	0	ϵ	ϵ^*	2	0	ϵ	ϵ^*	b
${}^2F_{3/2,g}$	2	0	ϵ^*	ϵ	2	0	ϵ^*	ϵ	b
$E_{1/2,u}$	2	0	1	1	-2	0	-1	-1	c
${}^1F_{3/2,u}$	2	0	ϵ	ϵ^*	-2	0	$-\epsilon$	$-\epsilon^*$	b
${}^2F_{3/2,u}$	2	0	ϵ^*	ϵ	-2	0	$-\epsilon^*$	$-\epsilon$	b

$\epsilon = \exp(2\pi i/3)$

T **72**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{T}_h$	0	1	2	3
A_g	$^\square 1$		$x^2+y^2+z^2$	
${}^1E_g \oplus {}^2E_g$			$^\square(x^2-y^2, 2z^2-x^2-y^2)$	
T_g		(R_x, R_y, R_z)	$^\square(zx, yz, xy)$	
A_u				xyz^a
${}^1E_u \oplus {}^2E_u$				
T_u		$^\square(x, y, z)$		$(x^3, y^3, z^3), (xy^2, yz^2, zx^2), (xz^2, yx^2, zy^2)^b$

[a] f function: f_{xyz}; [b] f functions: f_{xz^2}, f_{yz^2}, $f_{z(x^2-y^2)}$, $f_{x(x^2-y^2)}$, $f_{y(x^2-y^2)}$, f_{z^3}.

T **72**.6a Bases of irreducible representations

§ **16**–6, pp. 74, 75

$\mathbf{T}_h$	$\langle \lvert j\, m\rangle \rvert$
A_g	$\lvert 0\,0\rangle$
${}^1E_g$	$\frac{1}{\sqrt2}\lvert 2\,0\rangle - \frac{i}{\sqrt2}\lvert 2\,2\rangle_+$
${}^2E_g$	$\frac{1}{\sqrt2}\lvert 2\,0\rangle + \frac{i}{\sqrt2}\lvert 2\,2\rangle_+$
T_g	$\langle \lvert 2\,1\rangle_-, -\lvert 2\,2\rangle_-, -\lvert 2\,1\rangle_+ \rvert$
A_u	$\lvert 3\,2\rangle_-$
${}^1E_u$	$\frac{1}{\sqrt2}\lvert 5\,2\rangle_- - \frac{i}{\sqrt2}\lvert 5\,4\rangle_-$
${}^2E_u$	$-\frac{1}{\sqrt2}\lvert 5\,2\rangle_- - \frac{i}{\sqrt2}\lvert 5\,4\rangle_-$
T_u	$\langle \lvert 1\,1\rangle_+, \lvert 1\,0\rangle, \lvert 1\,1\rangle_- \rvert$
$E_{1/2,g}$	$\langle \lvert \tfrac12\,\tfrac12\rangle, \lvert \tfrac12\,\overline{\tfrac12}\rangle \rvert$
${}^1F_{3/2,g}$	$\langle \frac{1}{\sqrt2}\lvert \tfrac32\,\tfrac12\rangle - \frac{i}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac32}\rangle,\ \ \frac{i}{\sqrt2}\lvert \tfrac32\,\tfrac32\rangle - \frac{1}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac12}\rangle \rvert$
${}^2F_{3/2,g}$	$\langle \frac{1}{\sqrt2}\lvert \tfrac32\,\tfrac12\rangle + \frac{i}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac32}\rangle, -\frac{i}{\sqrt2}\lvert \tfrac32\,\tfrac32\rangle - \frac{1}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac12}\rangle \rvert$
$E_{1/2,u}$	$\langle \lvert \tfrac12\,\tfrac12\rangle, \lvert \tfrac12\,\overline{\tfrac12}\rangle \rvert^\bullet$
${}^1F_{3/2,u}$	$\langle \frac{1}{\sqrt2}\lvert \tfrac32\,\tfrac12\rangle - \frac{i}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac32}\rangle,\ \ \frac{i}{\sqrt2}\lvert \tfrac32\,\tfrac32\rangle - \frac{1}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac12}\rangle \rvert^\bullet$
${}^2F_{3/2,u}$	$\langle \frac{1}{\sqrt2}\lvert \tfrac32\,\tfrac12\rangle + \frac{i}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac32}\rangle, -\frac{i}{\sqrt2}\lvert \tfrac32\,\tfrac32\rangle - \frac{1}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac12}\rangle \rvert^\bullet$

T **72**.6b Symmetrized harmonics

Use T **71**.6b. § **16**–6, pp. 74, 75

T **72**.6c Spin harmonics

§ **16**–6, pp. 74, 75

$\mathbf{T}_h$	$\langle \quad \vert$
$E_{1/2,g}$	$\langle a_g\,\alpha, a_g\,\beta\vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_g^{(1)}\,\beta - t_g^{(2)}\,\alpha + t_g^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_g^{(1)}\,\alpha + t_g^{(2)}\,\beta + t_g^{(3)}\,\alpha)\vert$
${}^1F_{3/2,g}$	$\langle {}^1e_g\,\alpha, {}^1e_g\,\beta\vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_g^{(1)}\,\beta - \omega^*\,t_g^{(2)}\,\alpha + \omega\,t_g^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_g^{(1)}\,\alpha + \omega^*\,t_g^{(2)}\,\beta + \omega\,t_g^{(3)}\,\alpha)\vert$
${}^2F_{3/2,g}$	$\langle {}^2e_g\,\alpha, {}^2e_g\,\beta\vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_g^{(1)}\,\beta - \omega\,t_g^{(2)}\,\alpha + \omega^*\,t_g^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_g^{(1)}\,\alpha + \omega\,t_g^{(2)}\,\beta + \omega^*\,t_g^{(3)}\,\alpha)\vert$
$E_{1/2,u}$	$\langle a_u\,\alpha, a_u\,\beta\vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_u^{(1)}\,\beta - t_u^{(2)}\,\alpha + t_u^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_u^{(1)}\,\alpha + t_u^{(2)}\,\beta + t_u^{(3)}\,\alpha)\vert$
${}^1F_{3/2,u}$	$\langle {}^1e_u\,\alpha, {}^1e_u\,\beta\vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_u^{(1)}\,\beta - \omega^*\,t_u^{(2)}\,\alpha + \omega\,t_u^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_u^{(1)}\,\alpha + \omega^*\,t_u^{(2)}\,\beta + \omega\,t_u^{(3)}\,\alpha)\vert$
${}^2F_{3/2,u}$	$\langle {}^2e_u\,\alpha, {}^2e_u\,\beta\vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_u^{(1)}\,\beta - \omega\,t_u^{(2)}\,\alpha + \omega^*\,t_u^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_u^{(1)}\,\alpha + \omega\,t_u^{(2)}\,\beta + \omega^*\,t_u^{(3)}\,\alpha)\vert$

$\alpha = \vert\frac{1}{2}\,\frac{1}{2}\rangle$, $\beta = \vert\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$; $\omega = \exp(2\pi\mathrm{i}/3)$

T **72**.7 Matrix representations

Use T **70**.7 ▪. § **16**–7, p. 77

T **72**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{T}_h$	A_g	${}^1E_g$	${}^2E_g$	T_g	A_u	${}^1E_u$	${}^2E_u$	T_u
A_g	A_g	${}^1E_g$	${}^2E_g$	T_g	A_u	${}^1E_u$	${}^2E_u$	T_u
${}^1E_g$		${}^2E_g$	A_g	T_g	${}^1E_u$	${}^2E_u$	A_u	T_u
${}^2E_g$			${}^1E_g$	T_g	${}^2E_u$	A_u	${}^1E_u$	T_u
T_g				$A_g \oplus {}^1E_g \oplus {}^2E_g \oplus T_g \oplus \{T_g\}$	T_u	T_u	T_u	$A_u \oplus {}^1E_u \oplus {}^2E_u \oplus 2T_u$
A_u					A_g	${}^1E_g$	${}^2E_g$	T_g
${}^1E_u$						${}^2E_g$	A_g	T_g
${}^2E_u$							${}^1E_g$	T_g
T_u								$A_g \oplus {}^1E_g \oplus {}^2E_g \oplus T_g \oplus \{T_g\}$

↠

T **72**.8 Direct products of representations *(cont.)*

$\mathbf{T}_h$	$E_{1/2,g}$	${}^1F_{3/2,g}$	${}^2F_{3/2,g}$
A_g	$E_{1/2,g}$	${}^1F_{3/2,g}$	${}^2F_{3/2,g}$
${}^1E_g$	${}^1F_{3/2,g}$	${}^2F_{3/2,g}$	$E_{1/2,g}$
${}^2E_g$	${}^2F_{3/2,g}$	$E_{1/2,g}$	${}^1F_{3/2,g}$
T_g	$E_{1/2,g} \oplus {}^1F_{3/2,g} \oplus {}^2F_{3/2,g}$	$E_{1/2,g} \oplus {}^1F_{3/2,g} \oplus {}^2F_{3/2,g}$	$E_{1/2,g} \oplus {}^1F_{3/2,g} \oplus {}^2F_{3/2,g}$
A_u	$E_{1/2,u}$	${}^1F_{3/2,u}$	${}^2F_{3/2,u}$
${}^1E_u$	${}^1F_{3/2,u}$	${}^2F_{3/2,u}$	$E_{1/2,u}$
${}^2E_u$	${}^2F_{3/2,u}$	$E_{1/2,u}$	${}^1F_{3/2,u}$
T_u	$E_{1/2,u} \oplus {}^1F_{3/2,u} \oplus {}^2F_{3/2,u}$	$E_{1/2,u} \oplus {}^1F_{3/2,u} \oplus {}^2F_{3/2,u}$	$E_{1/2,u} \oplus {}^1F_{3/2,u} \oplus {}^2F_{3/2,u}$
$E_{1/2,g}$	$\{A_g\} \oplus T_g$	${}^1E_g \oplus T_g$	${}^2E_g \oplus T_g$
${}^1F_{3/2,g}$		$\{{}^2E_g\} \oplus T_g$	$A_g \oplus T_g$
${}^2F_{3/2,g}$			$\{{}^1E_g\} \oplus T_g$

⇒

T **72**.8 Direct products of representations *(cont.)*

$\mathbf{T}_h$	$E_{1/2,u}$	${}^1F_{3/2,u}$	${}^2F_{3/2,u}$
A_g	$E_{1/2,u}$	${}^1F_{3/2,u}$	${}^2F_{3/2,u}$
${}^1E_g$	${}^1F_{3/2,u}$	${}^2F_{3/2,u}$	$E_{1/2,u}$
${}^2E_g$	${}^2F_{3/2,u}$	$E_{1/2,u}$	${}^1F_{3/2,u}$
T_g	$E_{1/2,u} \oplus {}^1F_{3/2,u} \oplus {}^2F_{3/2,u}$	$E_{1/2,u} \oplus {}^1F_{3/2,u} \oplus {}^2F_{3/2,u}$	$E_{1/2,u} \oplus {}^1F_{3/2,u} \oplus {}^2F_{3/2,u}$
A_u	$E_{1/2,g}$	${}^1F_{3/2,g}$	${}^2F_{3/2,g}$
${}^1E_u$	${}^1F_{3/2,g}$	${}^2F_{3/2,g}$	$E_{1/2,g}$
${}^2E_u$	${}^2F_{3/2,g}$	$E_{1/2,g}$	${}^1F_{3/2,g}$
T_u	$E_{1/2,g} \oplus {}^1F_{3/2,g} \oplus {}^2F_{3/2,g}$	$E_{1/2,g} \oplus {}^1F_{3/2,g} \oplus {}^2F_{3/2,g}$	$E_{1/2,g} \oplus {}^1F_{3/2,g} \oplus {}^2F_{3/2,g}$
$E_{1/2,g}$	$A_u \oplus T_u$	${}^1E_u \oplus T_u$	${}^2E_u \oplus T_u$
${}^1F_{3/2,g}$	${}^1E_u \oplus T_u$	${}^2E_u \oplus T_u$	$A_u \oplus T_u$
${}^2F_{3/2,g}$	${}^2E_u \oplus T_u$	$A_u \oplus T_u$	${}^1E_u \oplus T_u$
$E_{1/2,u}$	$\{A_g\} \oplus T_g$	${}^1E_g \oplus T_g$	${}^2E_g \oplus T_g$
${}^1F_{3/2,u}$		$\{{}^2E_g\} \oplus T_g$	$A_g \oplus T_g$
${}^2F_{3/2,u}$			$\{{}^1E_g\} \oplus T_g$

T **72**.9 Subduction (descent of symmetry) § **16**–9, p. 82

$\mathbf{T}_h$	$\mathbf{T}$	$\mathbf{C}_{2h}$	$(\mathbf{C}_{2v})$	$\mathbf{D}_{2h}$	$\mathbf{D}_2$
A_g	A	A_g	A_1	A_g	A
${}^1E_g$	${}^1E$	A_g	A_1	A_g	A
${}^2E_g$	${}^2E$	A_g	A_1	A_g	A
T_g	T	$A_g \oplus 2B_g$	$A_2 \oplus B_1 \oplus B_2$	$B_{1g} \oplus B_{2g} \oplus B_{3g}$	$B_1 \oplus B_2 \oplus B_3$
A_u	A	A_u	A_2	A_u	A
${}^1E_u$	${}^1E$	A_u	A_2	A_u	A
${}^2E_u$	${}^2E$	A_u	A_2	A_u	A
T_u	T	$A_u \oplus 2B_u$	$A_1 \oplus B_1 \oplus B_2$	$B_{1u} \oplus B_{2u} \oplus B_{3u}$	$B_1 \oplus B_2 \oplus B_3$
$E_{1/2,g}$	$E_{1/2}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2}$
${}^1F_{3/2,g}$	${}^1F_{3/2}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2}$
${}^2F_{3/2,g}$	${}^2F_{3/2}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2}$
$E_{1/2,u}$	$E_{1/2}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{1/2}$	$E_{1/2,u}$	$E_{1/2}$
${}^1F_{3/2,u}$	${}^1F_{3/2}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{1/2}$	$E_{1/2,u}$	$E_{1/2}$
${}^2F_{3/2,u}$	${}^2F_{3/2}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$E_{1/2}$	$E_{1/2,u}$	$E_{1/2}$

⇒

T **72**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{T}_h$	$(\mathbf{S}_6)$	$\mathbf{C}_i$	$(\mathbf{C}_3)$	$\mathbf{C}_2$
A_g	A_g	A_g	A	A
${}^1E_g$	${}^2E_g$	A_g	${}^2E$	A
${}^2E_g$	${}^1E_g$	A_g	${}^1E$	A
T_g	$A_g \oplus {}^1E_g \oplus {}^2E_g$	$3A_g$	$A \oplus {}^1E \oplus {}^2E$	$A \oplus 2B$
A_u	A_u	A_u	A	A
${}^1E_u$	${}^2E_u$	A_u	${}^2E$	A
${}^2E_u$	${}^1E_u$	A_u	${}^1E$	A
T_u	$A_u \oplus {}^1E_u \oplus {}^2E_u$	$3A_u$	$A \oplus {}^1E \oplus {}^2E$	$A \oplus 2B$
$E_{1/2,g}$	${}^1E_{1/2,g} \oplus {}^2E_{1/2,g}$	$2A_{1/2,g}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1F_{3/2,g}$	$A_{3/2,g} \oplus {}^1E_{1/2,g}$	$2A_{1/2,g}$	$A_{3/2} \oplus {}^1E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^2F_{3/2,g}$	$A_{3/2,g} \oplus {}^2E_{1/2,g}$	$2A_{1/2,g}$	$A_{3/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{1/2,u}$	${}^1E_{1/2,u} \oplus {}^2E_{1/2,u}$	$2A_{1/2,u}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^1F_{3/2,u}$	$A_{3/2,u} \oplus {}^1E_{1/2,u}$	$2A_{1/2,u}$	$A_{3/2} \oplus {}^1E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
${}^2F_{3/2,u}$	$A_{3/2,u} \oplus {}^2E_{1/2,u}$	$2A_{1/2,u}$	$A_{3/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$

T **72**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{T}_h$
$6n$	$(n+1)\,A_g \oplus n\,({}^1E_g \oplus {}^2E_g \oplus 3T_g)$
$6n+1$	$n\,(A_u \oplus {}^1E_u \oplus {}^2E_u \oplus 2T_u) \oplus (n+1)\,T_u$
$6n+2$	$n\,(A_g \oplus 2T_g) \oplus (n+1)({}^1E_g \oplus {}^2E_g \oplus T_g)$
$6n+3$	$(n+1)(A_u \oplus 2T_u) \oplus n\,({}^1E_u \oplus {}^2E_u \oplus T_u)$
$6n+4$	$(n+1)(A_g \oplus {}^1E_g \oplus {}^2E_g \oplus 2T_g) \oplus n\,T_g$
$6n+5$	$n\,A_u \oplus (n+1)({}^1E_u \oplus {}^2E_u \oplus 3T_u)$
$3n+\frac{1}{2}$	$(n+1)\,E_{1/2,g} \oplus n\,({}^1F_{3/2,g} \oplus {}^2F_{3/2,g})$
$3n+\frac{3}{2}$	$n\,E_{1/2,g} \oplus (n+1)({}^1F_{3/2,g} \oplus {}^2F_{3/2,g})$
$3n+\frac{5}{2}$	$(n+1)(E_{1/2,g} \oplus {}^1F_{3/2,g} \oplus {}^2F_{3/2,g})$

$n = 0, 1, 2, \ldots$

T **72**.11 Clebsch–Gordan coefficients

Use T **70**.11 ▪. § **16**–11, p. 83

(1) Product forms: $\mathbf{T} \otimes \mathbf{C}'_s$.

(2) Group chains: $\mathbf{O}_h \supset (\underline{\mathbf{T}_d}) \supset \underline{\mathbf{T}}$, $\mathbf{O}_h \supset (\underline{\mathbf{T}_d}) \supset (\mathbf{C}_{3v})$, $\mathbf{O}_h \supset (\underline{\mathbf{T}_d}) \supset (\mathbf{D}_{2d})$.

(3) Operations of G: E, (C_{2x}, C_{2y}, C_{2z}), $(C_{31}^+, C_{32}^+, C_{33}^+, C_{34}^+, C_{31}^-, C_{32}^-, C_{33}^-, C_{34}^-)$,
$(S_{4x}^-, S_{4y}^-, S_{4z}^-, S_{4x}^+, S_{4y}^+, S_{4z}^+)$, $(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \sigma_{d6})$.

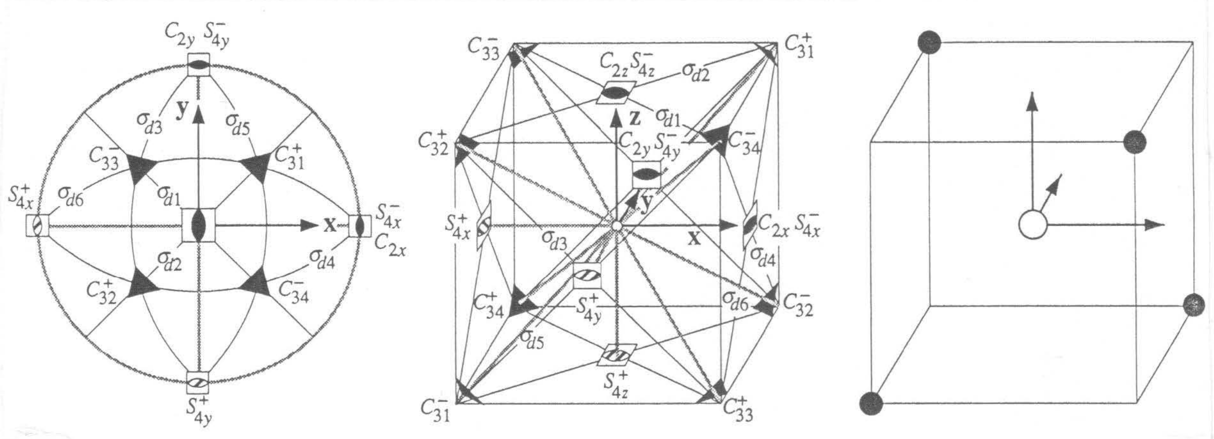
(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, $(C_{2x}, C_{2y}, C_{2z}, \widetilde{C}_{2x}, \widetilde{C}_{2y}, \widetilde{C}_{2z})$,
$(C_{31}^+, C_{32}^+, C_{33}^+, C_{34}^+, C_{31}^-, C_{32}^-, C_{33}^-, C_{34}^-)$, $(\widetilde{C}_{31}^+, \widetilde{C}_{32}^+, \widetilde{C}_{33}^+, \widetilde{C}_{34}^+, \widetilde{C}_{31}^-, \widetilde{C}_{32}^-, \widetilde{C}_{33}^-, \widetilde{C}_{34}^-)$,
$(S_{4x}^-, S_{4y}^-, S_{4z}^-, S_{4x}^+, S_{4y}^+, S_{4z}^+)$, $(\widetilde{S}_{4x}^-, \widetilde{S}_{4y}^-, \widetilde{S}_{4z}^-, \widetilde{S}_{4x}^+, \widetilde{S}_{4y}^+, \widetilde{S}_{4z}^+)$,
$(\sigma_{d1}, \sigma_{d2}, \sigma_{d3}, \sigma_{d4}, \sigma_{d5}, \sigma_{d6}, \widetilde{\sigma}_{d1}, \widetilde{\sigma}_{d2}, \widetilde{\sigma}_{d3}, \widetilde{\sigma}_{d4}, \widetilde{\sigma}_{d5}, \widetilde{\sigma}_{d6})$.

(5) Classes and representations: $|r| = 3$, $|i| = 2$, $|I| = 5$, $|\widetilde{I}| = 3$.

(6) Subduction: $\mathbf{C}_{3v}$ $(E, C_{31}^+, C_{31}^-, \sigma_{d2}, \sigma_{d6}, \sigma_{d5})$, $\mathbf{C}_{3v}$ $(E, C_{32}^+, C_{32}^-, \sigma_{d2}, \sigma_{d4}, \sigma_{d3})$.

F **73**

See Chapter **15**, p. 65



Examples: Adamantane $C_{10}H_{16}$, CCl_4, $SnCl_4$.

T **73**.1 Parameters

Use T **71**.1. § **16**–1, p. 68

T **73**.2 Multiplication table

Use T **71**.2. § **16**–2, p. 69

T **73**.3 Factor table

Use T **71**.3. § **16**–3, p. 70

T **73**.4 Character table

§ **16**–4, p. 71

$\mathbf{T}_d$	E	$3C_2$	$8C_3$	$6S_4$	$6\sigma_d$	τ
A_1	1	1	1	1	1	a
A_2	1	1	1	−1	−1	a
E	2	2	−1	0	0	a
T_1	3	−1	0	1	−1	a
T_2	3	−1	0	−1	1	a
$E_{1/2}$	2	0	1	$\sqrt{2}$	0	c
$E_{5/2}$	2	0	1	$-\sqrt{2}$	0	c
$F_{3/2}$	4	0	−1	0	0	c

T **73**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{T}_d$	0	1	2	3	
A_1	$^{\square}1$		$x^2+y^2+z^2$	xyz^a	
A_2					
E			$^{\square}(x^2-y^2, 2z^2-x^2-y^2)$		
T_1		(R_x, R_y, R_z)		$(x(z^2-y^2), y(x^2-z^2), z(y^2-x^2))$	⌉
T_2		$^{\square}(x, y, z)$	$^{\square}(zx, yz, xy)$	(x^3, y^3, z^3),	b
				$\{x(y^2+z^2), y(z^2+x^2), z(x^2+y^2)\}$	⌋

[a] f function: f_{xyz}; [b] f functions: f_{xz^2}, f_{yz^2}, $f_{z(x^2-y^2)}$, $f_{x(x^2-y^2)}$, $f_{y(x^2-y^2)}$, f_{z^3}.

T **73**.6a Bases of irreducible representations

§ **16**–6, pp. 74, 75

$\mathbf{T}_d$	$\langle\|j\,m\rangle\|$
A_1	$\|0\,0\rangle$
A_2	$\sqrt{\frac{11}{16}}\,\|6\,2\rangle_+ - \sqrt{\frac{5}{16}}\,\|6\,6\rangle_+$
E	$\langle \frac{1}{\sqrt{2}}\|2\,0\rangle - \frac{\mathrm{i}}{\sqrt{2}}\|2\,2\rangle_+, \frac{1}{\sqrt{2}}\|2\,0\rangle + \frac{\mathrm{i}}{\sqrt{2}}\|2\,2\rangle_+ \|$
T_1	$\langle \sqrt{\frac{5}{8}}\,\|3\,1\rangle_+ - \sqrt{\frac{3}{8}}\,\|3\,3\rangle_+, \|3\,2\rangle_+, -\sqrt{\frac{5}{8}}\,\|3\,1\rangle_- - \sqrt{\frac{3}{8}}\,\|3\,3\rangle_- \|$
T_2	$\langle \|1\,1\rangle_+, \|1\,0\rangle, \|1\,1\rangle_- \|$
$E_{1/2}$	$\langle \|\frac{1}{2}\,\frac{1}{2}\rangle, \|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle \|$
	$\langle \frac{1}{\sqrt{6}}\|\frac{5}{2}\,\frac{5}{2}\rangle - \sqrt{\frac{5}{6}}\,\|\frac{5}{2}\,\overline{\frac{3}{2}}\rangle, -\sqrt{\frac{5}{6}}\,\|\frac{5}{2}\,\frac{3}{2}\rangle + \frac{1}{\sqrt{6}}\|\frac{5}{2}\,\overline{\frac{5}{2}}\rangle \|^{\bullet}$
$E_{5/2}$	$\langle \frac{1}{\sqrt{6}}\|\frac{5}{2}\,\frac{5}{2}\rangle - \sqrt{\frac{5}{6}}\,\|\frac{5}{2}\,\overline{\frac{3}{2}}\rangle, -\sqrt{\frac{5}{6}}\,\|\frac{5}{2}\,\frac{3}{2}\rangle + \frac{1}{\sqrt{6}}\|\frac{5}{2}\,\overline{\frac{5}{2}}\rangle \|$
	$\langle \|\frac{1}{2}\,\frac{1}{2}\rangle, \|\frac{1}{2}\,\overline{\frac{1}{2}}\rangle \|^{\bullet}$
$F_{3/2}$	$\langle \frac{1}{\sqrt{2}}\|\frac{3}{2}\,\frac{1}{2}\rangle - \frac{\mathrm{i}}{\sqrt{2}}\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle, \frac{\mathrm{i}}{\sqrt{2}}\|\frac{3}{2}\,\frac{3}{2}\rangle - \frac{1}{\sqrt{2}}\|\frac{3}{2}\,\overline{\frac{1}{2}}\rangle, \quad \frac{1}{\sqrt{2}}\|\frac{3}{2}\,\frac{1}{2}\rangle + \frac{\mathrm{i}}{\sqrt{2}}\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle, -\frac{\mathrm{i}}{\sqrt{2}}\|\frac{3}{2}\,\frac{3}{2}\rangle - \frac{1}{\sqrt{2}}\|\frac{3}{2}\,\overline{\frac{1}{2}}\rangle \|$
	$\langle \frac{1}{\sqrt{2}}\|\frac{3}{2}\,\frac{1}{2}\rangle - \frac{\mathrm{i}}{\sqrt{2}}\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle, \frac{\mathrm{i}}{\sqrt{2}}\|\frac{3}{2}\,\frac{3}{2}\rangle - \frac{1}{\sqrt{2}}\|\frac{3}{2}\,\overline{\frac{1}{2}}\rangle, -\frac{1}{\sqrt{2}}\|\frac{3}{2}\,\frac{1}{2}\rangle - \frac{\mathrm{i}}{\sqrt{2}}\|\frac{3}{2}\,\overline{\frac{3}{2}}\rangle, \quad \frac{\mathrm{i}}{\sqrt{2}}\|\frac{3}{2}\,\frac{3}{2}\rangle + \frac{1}{\sqrt{2}}\|\frac{3}{2}\,\overline{\frac{1}{2}}\rangle \|^{\bullet}$

T **73**.6b Symmetrized harmonics

Use T **71**.6b. § **16**–6, pp. 74, 75

T **73**.6*c* Spin harmonics — § **16**–6, pp. 74, 75

$\mathbf{T}_d$	$\langle\quad\vert$
$E_{1/2}$	$\langle a_1\,\alpha, a_1\,\beta\vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_1^{(1)}\,\beta - t_1^{(2)}\,\alpha + t_1^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_1^{(1)}\,\alpha + t_1^{(2)}\,\beta + t_1^{(3)}\,\alpha)\vert$
$E_{5/2}$	$\langle a_2\,\alpha, a_2\,\beta\vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_2^{(1)}\,\beta - t_2^{(2)}\,\alpha + t_2^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_2^{(1)}\,\alpha + t_2^{(2)}\,\beta + t_2^{(3)}\,\alpha)\vert$
$F_{3/2}$	$\langle e^{(1)}\,\alpha, e^{(1)}\,\beta, e^{(2)}\,\alpha, e^{(2)}\,\beta\vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_1^{(1)}\,\beta - \omega^*\,t_1^{(2)}\,\alpha + \omega\,t_1^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_1^{(1)}\,\alpha + \omega^*\,t_1^{(2)}\,\beta + \omega\,t_1^{(3)}\,\alpha),$
	$\frac{1}{\sqrt{3}}\,(\omega\,t_1^{(1)}\,\beta - \omega^*\,t_1^{(2)}\,\alpha + t_1^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-\omega\,t_1^{(1)}\,\alpha + \omega^*\,t_1^{(2)}\,\beta + t_1^{(3)}\,\alpha)\vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_2^{(1)}\,\beta - \omega^*\,t_2^{(2)}\,\alpha + \omega\,t_2^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_2^{(1)}\,\alpha + \omega^*\,t_2^{(2)}\,\beta + \omega\,t_2^{(3)}\,\alpha),$
	$\frac{1}{\sqrt{3}}\,(-\omega\,t_2^{(1)}\,\beta + \omega^*\,t_2^{(2)}\,\alpha - t_2^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(\omega\,t_2^{(1)}\,\alpha - \omega^*\,t_2^{(2)}\,\beta - t_2^{(3)}\,\alpha)\vert$

$\alpha = \vert\frac{1}{2}\,\frac{1}{2}\rangle$, $\beta = \vert\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$; $\quad \omega = \exp(2\pi\mathrm{i}/3)$

T **73**.7 Matrix representations

Use T **69**.7 •. § **16**–7, p. 77

T **73**.8 Direct products of representations

Use T **69**.8 •. § **16**–8, p. 81

T **73**.9 Subduction (descent of symmetry) — § **16**–9, p. 82

$\mathbf{T}_d$	$\mathbf{T}$	$(\mathbf{C}_{3v})$	$(\mathbf{C}_{2v})$	$(\mathbf{D}_{2d})$	$\mathbf{D}_2$
A_1	A	A_1	A_1	A_1	A
A_2	A	A_2	A_1	B_1	A
E	${}^1E \oplus {}^2E$	E	$A_1 \oplus A_2$	$A_1 \oplus B_1$	$2A$
T_1	T	$A_2 \oplus E$	$A_2 \oplus B_1 \oplus B_2$	$A_2 \oplus E$	$B_1 \oplus B_2 \oplus B_3$
T_2	T	$A_1 \oplus E$	$A_1 \oplus B_1 \oplus B_2$	$B_2 \oplus E$	$B_1 \oplus B_2 \oplus B_3$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{5/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{3/2}$	$E_{1/2}$
$F_{3/2}$	${}^1F_{3/2} \oplus {}^2F_{3/2}$	$E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}$	$2E_{1/2}$	$E_{1/2} \oplus E_{3/2}$	$2E_{1/2}$

$\twoheadrightarrow$

T **73**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{T}_d$	$(\mathbf{S}_4)$	$(\mathbf{C}_s)$	$(\mathbf{C}_3)$	$\mathbf{C}_2$
A_1	A	A'	A	A
A_2	A	A''	A	A
E	$A \oplus B$	$A' \oplus A''$	${}^1E \oplus {}^2E$	$2A$
T_1	$A \oplus {}^1E \oplus {}^2E$	$A' \oplus 2A''$	$A \oplus {}^1E \oplus {}^2E$	$A \oplus 2B$
T_2	$B \oplus {}^1E \oplus {}^2E$	$2A' \oplus A''$	$A \oplus {}^1E \oplus {}^2E$	$A \oplus 2B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{5/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$F_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}$	$2\,{}^1E_{1/2} \oplus 2\,{}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2} \oplus 2A_{3/2}$	$2\,{}^1E_{1/2} \oplus 2\,{}^2E_{1/2}$

T **73**.10 Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{T}_d$
$12n$	$(n+1)\,A_1 \oplus n(A_2 \oplus 2E \oplus 3T_1 \oplus 3T_2)$
$12n+1$	$n\,(A_1 \oplus A_2 \oplus 2E \oplus 3T_1 \oplus 2T_2) \oplus (n+1)\,T_2$
$12n+2$	$n\,(A_1 \oplus A_2 \oplus E \oplus 3T_1 \oplus 2T_2) \oplus (n+1)(E \oplus T_2)$
$12n+3$	$(n+1)(A_1 \oplus T_1 \oplus T_2) \oplus n\,(A_2 \oplus 2E \oplus 2T_1 \oplus 2T_2)$
$12n+4$	$(n+1)(A_1 \oplus E \oplus T_1 \oplus T_2) \oplus n\,(A_2 \oplus E \oplus 2T_1 \oplus 2T_2)$
$12n+5$	$n\,(A_1 \oplus A_2 \oplus E \oplus 2T_1 \oplus T_2) \oplus (n+1)(E \oplus T_1 \oplus 2T_2)$
$12n+6$	$(n+1)(A_1 \oplus A_2 \oplus E \oplus T_1 \oplus 2T_2) \oplus n\,(E \oplus 2T_1 \oplus T_2)$
$12n+7$	$(n+1)(A_1 \oplus E \oplus 2T_1 \oplus 2T_2) \oplus n\,(A_2 \oplus E \oplus T_1 \oplus T_2)$
$12n+8$	$(n+1)(A_1 \oplus 2E \oplus 2T_1 \oplus 2T_2) \oplus n\,(A_2 \oplus T_1 \oplus T_2)$
$12n+9$	$(n+1)(A_1 \oplus A_2 \oplus E \oplus 2T_1 \oplus 3T_2) \oplus n\,(E \oplus T_1)$
$12n+10$	$(n+1)(A_1 \oplus A_2 \oplus 2E \oplus 2T_1 \oplus 3T_2) \oplus n\,T_1$
$12n+11$	$(n+1)(A_1 \oplus 2E \oplus 3T_1 \oplus 3T_2) \oplus n\,A_2$
$12n+\frac{1}{2}$	$(2n+1)\,E_{1/2} \oplus 2n(E_{5/2} \oplus 2F_{3/2})$
$12n+\frac{3}{2}$	$2n\,(E_{1/2} \oplus E_{5/2} \oplus F_{3/2}) \oplus (2n+1)\,F_{3/2}$
$12n+\frac{5}{2}$	$2n\,(E_{1/2} \oplus F_{3/2}) \oplus (2n+1)(E_{5/2} \oplus F_{3/2})$
$12n+\frac{7}{2}$	$(2n+1)(E_{1/2} \oplus E_{5/2} \oplus F_{3/2}) \oplus 2n\,F_{3/2}$
$12n+\frac{9}{2}$	$(2n+1)(E_{1/2} \oplus 2F_{3/2}) \oplus 2n\,E_{5/2}$
$12n+\frac{11}{2}$	$(2n+1)(E_{1/2} \oplus E_{5/2} \oplus 2F_{3/2})$
$12n+\frac{13}{2}$	$(2n+1)(E_{1/2} \oplus 2F_{3/2}) \oplus (2n+2)\,E_{5/2}$
$12n+\frac{15}{2}$	$(2n+1)(E_{1/2} \oplus E_{5/2} \oplus F_{3/2}) \oplus (2n+2)\,F_{3/2}$
$12n+\frac{17}{2}$	$(2n+2)(E_{1/2} \oplus F_{3/2}) \oplus (2n+1)(E_{5/2} \oplus F_{3/2})$
$12n+\frac{19}{2}$	$(2n+2)(E_{1/2} \oplus E_{5/2} \oplus F_{3/2}) \oplus (2n+1)\,F_{3/2}$
$12n+\frac{21}{2}$	$(2n+1)\,E_{1/2} \oplus (2n+2)(E_{5/2} \oplus 2F_{3/2})$
$12n+\frac{23}{2}$	$(2n+2)(E_{1/2} \oplus E_{5/2} \oplus 2F_{3/2})$

$n = 0, 1, 2, \ldots$

T **73**.11 Clebsch–Gordan coefficients

Use T **69**.11 •. § **16**–11, p. 83

The icosahedral groups

Notation for headers

Items in header read from left to right

1		Hermann–Mauguin symbol for the point group.
2		$\lvert G\rvert$ order of the group.
3		$\lvert C\rvert$ number of classes in the group.
4		$\lvert\widetilde{C}\rvert$ number of classes in the double group.
5		Number of the table.
6		Page reference for the notation of the header, of the first six subsections below it, and of the footers.
7	□	This symbol indicates a crystallographic point group.
8		Schönflies notation for the point group.

Notation for the first six subsections below the header

(1) Product forms	Direct and semidirect product forms. ⊗ Direct product. ⓢ Semidirect product.
(2) Group chains *(See pp. 41, 67)*	Groups underlined: invariant. Groups in brackets: when subducing one or more of the representations in the tables to these subgroups in the setting used for them in the tables a change of bases (similarity transformation) is required.
(3) Operations of G	Lists all the operations of G, enclosing in brackets all the operations of the same class.
(4) Operations of $\widetilde{G}$	Lists all the operations of $\widetilde{G}$, enclosing in brackets all the operations of the same class.
(5) Classes and representations	$\lvert r\rvert$ number of regular classes in G (p. 51). $\lvert \mathrm{i}\rvert$ number of irregular classes in G (p. 51). $\lvert I\rvert$ number of irreducible representations in G. $\lvert\widetilde{I}\rvert$ number of spinor representations, also called the number of double-group representations.
(6) Subduction *(See p. 41)*	When subducing spinor representations to certain subgroups of which there are several isomorphs in different settings, it is mathematically impossible to ensure that in more than a few of these settings the character remains a class function on subduction. The isomorphs for which subduction does not suffer from this difficulty are listed in this subsection.

Use of the footers

Finding your way about the tables	Each page of the tables contains a footer giving the ordering of the group types listed in the tables. The current group type is recognized because it carries no page number under its name. On opening the tables at any page you should then see at a glance whether to move backwards or forwards to find the group type desired. More precisely, the page number for the group type desired gives the page, like the present one, which carries the contents for the group type wanted.

(1) Product forms: none.

(2) Group chains: $\mathbf{I}_h \supset \underline{\mathbf{I}} \supset \mathbf{T}$, $\mathbf{I}_h \supset \underline{\mathbf{I}} \supset (\mathbf{D}_5)$, $\mathbf{I}_h \supset \underline{\mathbf{I}} \supset (\mathbf{D}_3)$.

(3) Operations of G: E, $(C_{51}^+, C_{52}^+, C_{53}^+, C_{54}^+, C_{55}^+, C_{56}^+, C_{51}^-, C_{52}^-, C_{53}^-, C_{54}^-, C_{55}^-, C_{56}^-)$,
$(C_{51}^{2+}, C_{52}^{2+}, C_{53}^{2+}, C_{54}^{2+}, C_{55}^{2+}, C_{56}^{2+}, C_{51}^{2-}, C_{52}^{2-}, C_{53}^{2-}, C_{54}^{2-}, C_{55}^{2-}, C_{56}^{2-})$,
$(C_{31}^+, C_{32}^+, C_{33}^+, C_{34}^+, C_{35}^+, C_{36}^+, C_{37}^+, C_{38}^+, C_{39}^+, C_{3,10}^+,$
$C_{31}^-, C_{32}^-, C_{33}^-, C_{34}^-, C_{35}^-, C_{36}^-, C_{37}^-, C_{38}^-, C_{39}^-, C_{3,10}^-)$,
$(C_{2a}, C_{2b}, C_{2c}, C_{2d}, C_{2e}, C_{2f}, C_{2g}, C_{2h}, C_{2i}, C_{2j}, C_{2k}, C_{2l}, C_{2m}, C_{2n}, C_{2o})$.

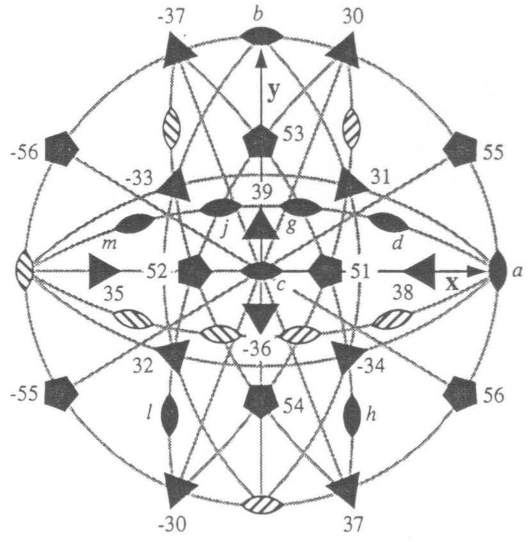
(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, $(C_{51}^+, C_{52}^+, C_{53}^+, C_{54}^+, C_{55}^+, C_{56}^+, C_{51}^-, C_{52}^-, C_{53}^-, C_{54}^-, C_{55}^-, C_{56}^-)$,
$(\widetilde{C}_{51}^+, \widetilde{C}_{52}^+, \widetilde{C}_{53}^+, \widetilde{C}_{54}^+, \widetilde{C}_{55}^+, \widetilde{C}_{56}^+, \widetilde{C}_{51}^-, \widetilde{C}_{52}^-, \widetilde{C}_{53}^-, \widetilde{C}_{54}^-, \widetilde{C}_{55}^-, \widetilde{C}_{56}^-)$,
$(C_{51}^{2+}, C_{52}^{2+}, C_{53}^{2+}, C_{54}^{2+}, C_{55}^{2+}, C_{56}^{2+}, C_{51}^{2-}, C_{52}^{2-}, C_{53}^{2-}, C_{54}^{2-}, C_{55}^{2-}, C_{56}^{2-})$,
$(\widetilde{C}_{51}^{2+}, \widetilde{C}_{52}^{2+}, \widetilde{C}_{53}^{2+}, \widetilde{C}_{54}^{2+}, \widetilde{C}_{55}^{2+}, \widetilde{C}_{56}^{2+}, \widetilde{C}_{51}^{2-}, \widetilde{C}_{52}^{2-}, \widetilde{C}_{53}^{2-}, \widetilde{C}_{54}^{2-}, \widetilde{C}_{55}^{2-}, \widetilde{C}_{56}^{2-})$,
$(C_{31}^+, C_{32}^+, C_{33}^+, C_{34}^+, C_{35}^+, C_{36}^+, C_{37}^+, C_{38}^+, C_{39}^+, C_{3,10}^+,$
$C_{31}^-, C_{32}^-, C_{33}^-, C_{34}^-, C_{35}^-, C_{36}^-, C_{37}^-, C_{38}^-, C_{39}^-, C_{3,10}^-)$,
$(\widetilde{C}_{31}^+, \widetilde{C}_{32}^+, \widetilde{C}_{33}^+, \widetilde{C}_{34}^+, \widetilde{C}_{35}^+, \widetilde{C}_{36}^+, \widetilde{C}_{37}^+, \widetilde{C}_{38}^+, \widetilde{C}_{39}^+, \widetilde{C}_{3,10}^+,$
$\widetilde{C}_{31}^-, \widetilde{C}_{32}^-, \widetilde{C}_{33}^-, \widetilde{C}_{34}^-, \widetilde{C}_{35}^-, \widetilde{C}_{36}^-, \widetilde{C}_{37}^-, \widetilde{C}_{38}^-, \widetilde{C}_{39}^-, \widetilde{C}_{3,10}^-)$,
$(C_{2a}, C_{2b}, C_{2c}, C_{2d}, C_{2e}, C_{2f}, C_{2g}, C_{2h}, C_{2i}, C_{2j}, C_{2k}, C_{2l}, C_{2m}, C_{2n}, C_{2o},$
$\widetilde{C}_{2a}, \widetilde{C}_{2b}, \widetilde{C}_{2c}, \widetilde{C}_{2d}, \widetilde{C}_{2e}, \widetilde{C}_{2f}, \widetilde{C}_{2g}, \widetilde{C}_{2h}, \widetilde{C}_{2i}, \widetilde{C}_{2j}, \widetilde{C}_{2k}, \widetilde{C}_{2l}, \widetilde{C}_{2m}, \widetilde{C}_{2n}, \widetilde{C}_{2o})$.

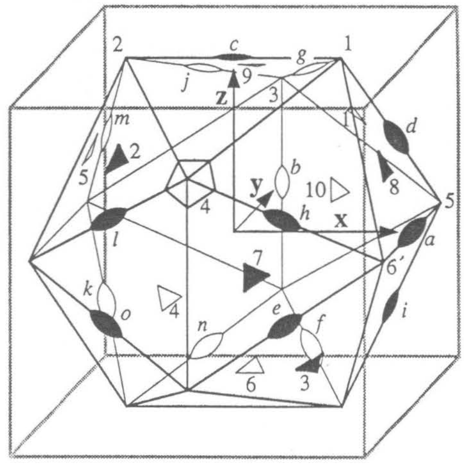
(5) Classes and representations: $\lvert r\rvert = 4$, $\lvert\mathrm{i}\rvert = 1$, $\lvert I\rvert = 5$, $\lvert\widetilde{I}\rvert = 4$.

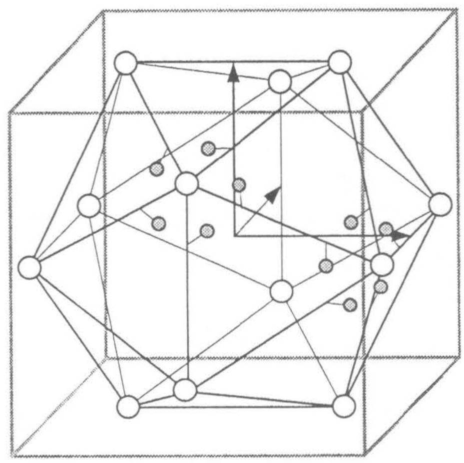
(6) Subduction: $\mathbf{D}_3$ $(E, C_{31}^+, C_{31}^-, C_{2f}, C_{2m}, C_{2h})$,
$\mathbf{D}_3$ $(E, C_{32}^+, C_{32}^-, C_{2e}, C_{2g}, C_{2k})$,
$\mathbf{D}_3$ $(E, C_{33}^+, C_{33}^-, C_{2d}, C_{2l}, C_{2n})$,
$\mathbf{D}_3$ $(E, C_{34}^+, C_{34}^-, C_{2i}, C_{2o}, C_{2j})$,
$\mathbf{D}_5$ $(E, C_{51}^+, C_{51}^-, C_{51}^{2+}, C_{51}^{2-}, C_{2b}, C_{2m}, C_{2l}, C_{2e}, C_{2i})$,
$\mathbf{D}_5$ $(E, C_{52}^+, C_{52}^-, C_{52}^{2+}, C_{52}^{2-}, C_{2b}, C_{2k}, C_{2o}, C_{2h}, C_{2d})$.

F 74

See Chapter **15**, p. 65







$5i = C_{5i}^{+}$, $\quad -5i = C_{5i}^{-}$, $\quad i = 1, 2, \ldots, 6$. $\qquad$ $3i = C_{3i}^{+}$, $\quad -3i = C_{3i}^{-}$, $\quad i = 1, 2, \ldots, 10$.

$r = C_{2r}$, $\quad r = a, b, \ldots, o$.

The poles of the operations C_{5i}^{2+}, C_{5i}^{2-} coincide respectively with those of C_{5i}^{+}, C_{5i}^{-} and are not identified in the figures.

In the first three-dimensional figure the symmetry elements are identified as above, except that in all cases the order of the rotation axis is left implicit, to be read from the position of the symbol. For simplicity of the figure, only one five-fold axis is identified by a pentagon which has been left open. No antipoles are shown and the open digons and triangles correspond to symmetry elements in the back of the icosahedron. In the second three-dimensional figure only a few of the ornaments required around each vertex are shown.

Examples:

T 74.1 Parameters

Use T **75**.1. § **16**–1, p. 68

T **74**.2 Multiplication table

§ **16**–2, p. 69

I	E	C_{51}^{+}	C_{52}^{+}	C_{53}^{+}	C_{54}^{+}	C_{55}^{+}	C_{56}^{+}	C_{51}^{-}	C_{52}^{-}	C_{53}^{-}	C_{54}^{-}	C_{55}^{-}	C_{56}^{-}	C_{51}^{2+}	C_{52}^{2+}	C_{53}^{2+}	C_{54}^{2+}	C_{55}^{2+}	C_{56}^{2+}	C_{51}^{2-}	C_{52}^{2-}	C_{53}^{2-}	C_{54}^{2-}	C_{55}^{2-}	C_{56}^{2-}	C_{31}^{+}	C_{32}^{+}	C_{33}^{+}	C_{34}^{+}	C_{35}^{+}
E	E	C_{51}^{+}	C_{52}^{+}	C_{53}^{+}	C_{54}^{+}	C_{55}^{+}	C_{56}^{+}	C_{51}^{-}	C_{52}^{-}	C_{53}^{-}	C_{54}^{-}	C_{55}^{-}	C_{56}^{-}	C_{51}^{2+}	C_{52}^{2+}	C_{53}^{2+}	C_{54}^{2+}	C_{55}^{2+}	C_{56}^{2+}	C_{51}^{2-}	C_{52}^{2-}	C_{53}^{2-}	C_{54}^{2-}	C_{55}^{2-}	C_{56}^{2-}	C_{31}^{+}	C_{32}^{+}	C_{33}^{+}	C_{34}^{+}	C_{35}^{+}
C_{51}^{+}	C_{51}^{+}	C_{51}^{2+}	C_{36}^{-}	C_{39}^{+}	C_{34}^{-}	C_{31}^{+}	C_{38}^{+}	E	C_{55}^{+}	C_{56}^{+}	C_{53}^{+}	C_{54}^{+}	C_{52}^{+}	C_{51}^{2-}	C_{2f}	C_{2j}	C_{2h}	C_{2o}	C_{2a}	C_{51}^{-}	C_{32}^{-}	C_{33}^{+}	C_{37}^{-}	C_{30}^{-}	C_{35}^{+}	C_{2g}	C_{54}^{2+}	C_{35}^{-}	C_{56}^{-}	C_{32}^{+}
C_{52}^{+}	C_{52}^{+}	C_{39}^{+}	C_{52}^{2+}	C_{33}^{-}	C_{36}^{-}	C_{53}^{+}	C_{51}^{+}	C_{55}^{-}	E	C_{54}^{+}	C_{56}^{-}	C_{32}^{+}	C_{35}^{+}	C_{2g}	C_{52}^{2-}	C_{2e}	C_{2n}	C_{30}^{+}	C_{38}^{+}	C_{31}^{-}	C_{52}^{-}	C_{37}^{+}	C_{34}^{+}	C_{2l}	C_{2a}	C_{53}^{2+}	C_{2f}	C_{56}^{+}	C_{38}^{-}	C_{2i}
C_{53}^{+}	C_{53}^{+}	C_{31}^{+}	C_{39}^{+}	C_{53}^{2+}	C_{51}^{+}	C_{30}^{+}	C_{55}^{+}	C_{56}^{-}	C_{54}^{-}	E	C_{37}^{-}	C_{52}^{+}	C_{33}^{-}	C_{2d}	C_{2c}	C_{53}^{2-}	C_{34}^{-}	C_{2l}	C_{35}^{-}	C_{38}^{-}	C_{36}^{+}	C_{53}^{-}	C_{2h}	C_{32}^{+}	C_{2m}	C_{2o}	C_{36}^{-}	C_{52}^{-}	C_{56}^{2-}	C_{52}^{2+}
C_{54}^{+}	C_{54}^{+}	C_{36}^{-}	C_{32}^{+}	C_{52}^{+}	C_{54}^{2+}	C_{51}^{+}	C_{34}^{-}	C_{53}^{-}	C_{56}^{+}	C_{37}^{+}	E	C_{30}^{-}	C_{55}^{-}	C_{2c}	C_{2i}	C_{33}^{-}	C_{54}^{2-}	C_{31}^{+}	C_{2k}	C_{39}^{-}	C_{35}^{-}	C_{2e}	C_{54}^{-}	C_{2o}	C_{38}^{-}	C_{39}^{+}	C_{2l}	C_{56}^{2+}	C_{51}^{-}	C_{55}^{2-}
C_{55}^{+}	C_{55}^{+}	C_{38}^{+}	C_{51}^{+}	C_{31}^{+}	C_{56}^{+}	C_{55}^{2+}	C_{35}^{-}	C_{54}^{-}	C_{32}^{-}	C_{52}^{-}	C_{30}^{+}	E	C_{53}^{+}	C_{2k}	C_{36}^{-}	C_{2g}	C_{37}^{+}	C_{55}^{2-}	C_{2m}	C_{34}^{+}	C_{2f}	C_{39}^{-}	C_{2b}	C_{55}^{-}	C_{33}^{-}	C_{2d}	C_{54}^{+}	C_{52}^{2-}	C_{37}^{-}	C_{52}^{+}
C_{56}^{+}	C_{56}^{+}	C_{34}^{-}	C_{54}^{+}	C_{51}^{+}	C_{37}^{+}	C_{38}^{+}	C_{56}^{2+}	C_{52}^{-}	C_{35}^{-}	C_{33}^{+}	C_{55}^{+}	C_{53}^{-}	E	C_{2n}	C_{32}^{+}	C_{39}^{+}	C_{2b}	C_{2d}	C_{56}^{2-}	C_{36}^{+}	C_{2i}	C_{2j}	C_{30}^{+}	C_{31}^{-}	C_{56}^{-}	C_{51}^{2+}	C_{30}^{-}	C_{2m}	C_{54}^{-}	C_{55}^{-}
C_{51}^{-}	C_{51}^{-}	E	C_{56}^{-}	C_{54}^{-}	C_{55}^{-}	C_{52}^{-}	C_{53}^{-}	C_{51}^{2-}	C_{39}^{-}	C_{31}^{-}	C_{36}^{+}	C_{38}^{-}	C_{34}^{+}	C_{51}^{+}	C_{33}^{-}	C_{30}^{+}	C_{32}^{+}	C_{35}^{-}	C_{37}^{+}	C_{51}^{2+}	C_{2j}	C_{2o}	C_{2f}	C_{2a}	C_{2h}	C_{55}^{+}	C_{35}^{+}	C_{53}^{2-}	C_{2n}	C_{56}^{2-}
C_{52}^{-}	C_{52}^{-}	C_{56}^{+}	E	C_{55}^{+}	C_{53}^{-}	C_{35}^{-}	C_{33}^{+}	C_{36}^{+}	C_{52}^{2-}	C_{39}^{-}	C_{32}^{-}	C_{51}^{-}	C_{54}^{-}	C_{34}^{-}	C_{52}^{+}	C_{31}^{+}	C_{30}^{-}	C_{2a}	C_{2e}	C_{2n}	C_{52}^{2+}	C_{2g}	C_{2l}	C_{38}^{-}	C_{37}^{-}	C_{38}^{+}	C_{55}^{-}	C_{2j}	C_{54}^{2-}	C_{56}^{-}
C_{53}^{-}	C_{53}^{-}	C_{54}^{+}	C_{55}^{-}	E	C_{30}^{-}	C_{56}^{+}	C_{37}^{+}	C_{39}^{-}	C_{33}^{+}	C_{53}^{2-}	C_{52}^{-}	C_{31}^{-}	C_{51}^{-}	C_{36}^{-}	C_{35}^{+}	C_{53}^{+}	C_{2l}	C_{38}^{+}	C_{2h}	C_{2c}	C_{2m}	C_{53}^{2+}	C_{32}^{-}	C_{2d}	C_{34}^{+}	C_{51}^{+}	C_{55}^{2-}	C_{2e}	C_{36}^{+}	C_{38}^{-}
C_{54}^{-}	C_{54}^{-}	C_{55}^{+}	C_{53}^{+}	C_{30}^{+}	E	C_{32}^{-}	C_{52}^{-}	C_{34}^{+}	C_{36}^{+}	C_{51}^{-}	C_{54}^{2-}	C_{56}^{-}	C_{37}^{-}	C_{38}^{+}	C_{39}^{+}	C_{2o}	C_{54}^{+}	C_{2i}	C_{33}^{+}	C_{2k}	C_{2c}	C_{31}^{-}	C_{54}^{2+}	C_{35}^{+}	C_{2e}	C_{55}^{2+}	C_{52}^{+}	C_{39}^{-}	C_{2h}	C_{33}^{-}
C_{55}^{-}	C_{55}^{-}	C_{52}^{+}	C_{35}^{+}	C_{56}^{-}	C_{32}^{+}	E	C_{54}^{+}	C_{31}^{-}	C_{53}^{-}	C_{30}^{-}	C_{51}^{-}	C_{55}^{2-}	C_{38}^{-}	C_{39}^{+}	C_{2m}	C_{37}^{-}	C_{2f}	C_{55}^{+}	C_{34}^{-}	C_{2g}	C_{33}^{+}	C_{2b}	C_{36}^{+}	C_{55}^{2+}	C_{2k}	C_{53}^{+}	C_{2i}	C_{37}^{+}	C_{51}^{2-}	C_{2a}
C_{56}^{-}	C_{56}^{-}	C_{53}^{+}	C_{33}^{-}	C_{37}^{-}	C_{52}^{+}	C_{54}^{-}	E	C_{38}^{-}	C_{51}^{-}	C_{55}^{-}	C_{34}^{+}	C_{35}^{+}	C_{56}^{2-}	C_{31}^{+}	C_{2j}	C_{2b}	C_{36}^{-}	C_{32}^{-}	C_{56}^{+}	C_{2d}	C_{39}^{-}	C_{30}^{-}	C_{2n}	C_{2i}	C_{56}^{2+}	C_{30}^{+}	C_{52}^{2+}	C_{53}^{-}	C_{2k}	C_{2m}
C_{51}^{2+}	C_{51}^{2+}	C_{51}^{2-}	C_{2n}	C_{2c}	C_{2k}	C_{2g}	C_{2d}	C_{51}^{+}	C_{31}^{+}	C_{38}^{+}	C_{39}^{+}	C_{34}^{-}	C_{36}^{-}	C_{51}^{-}	C_{54}^{2-}	C_{52}^{2-}	C_{56}^{2-}	C_{53}^{2-}	C_{55}^{2-}	E	C_{30}^{+}	C_{35}^{-}	C_{33}^{-}	C_{37}^{+}	C_{32}^{+}	C_{39}^{-}	C_{2h}	C_{55}^{2+}	C_{52}^{+}	C_{54}^{2+}
C_{52}^{2+}	C_{52}^{2+}	C_{2j}	C_{52}^{2-}	C_{2m}	C_{2c}	C_{33}^{-}	C_{39}^{+}	C_{32}^{+}	C_{52}^{+}	C_{36}^{-}	C_{35}^{+}	C_{2f}	C_{2i}	C_{53}^{2-}	C_{52}^{-}	C_{56}^{2+}	C_{51}^{2-}	C_{37}^{-}	C_{31}^{+}	C_{30}^{-}	E	C_{34}^{-}	C_{38}^{-}	C_{54}^{2-}	C_{55}^{2+}	C_{2e}	C_{36}^{+}	C_{51}^{+}	C_{55}^{2-}	C_{32}^{-}
C_{53}^{2+}	C_{53}^{2+}	C_{2o}	C_{2g}	C_{53}^{2-}	C_{31}^{+}	C_{2b}	C_{30}^{+}	C_{33}^{-}	C_{37}^{-}	C_{53}^{+}	C_{2e}	C_{39}^{+}	C_{2j}	C_{55}^{2-}	C_{51}^{2-}	C_{53}^{-}	C_{38}^{+}	C_{54}^{2+}	C_{32}^{-}	C_{35}^{+}	C_{34}^{+}	E	C_{56}^{2+}	C_{36}^{-}	C_{52}^{2-}	C_{30}^{-}	C_{51}^{2+}	C_{54}^{-}	C_{2m}	C_{2c}
C_{54}^{2+}	C_{54}^{2+}	C_{2f}	C_{2l}	C_{32}^{+}	C_{54}^{2-}	C_{36}^{-}	C_{2n}	C_{37}^{+}	C_{34}^{-}	C_{2h}	C_{54}^{+}	C_{2b}	C_{30}^{-}	C_{52}^{2-}	C_{55}^{2+}	C_{35}^{+}	C_{54}^{-}	C_{39}^{+}	C_{51}^{2-}	C_{33}^{+}	C_{38}^{+}	C_{56}^{2-}	E	C_{53}^{2+}	C_{31}^{-}	C_{52}^{2+}	C_{30}^{+}	C_{2k}	C_{53}^{-}	C_{2o}
C_{55}^{2+}	C_{55}^{2+}	C_{2a}	C_{38}^{+}	C_{2d}	C_{35}^{-}	C_{55}^{2-}	C_{2i}	C_{30}^{+}	C_{2l}	C_{32}^{-}	C_{2o}	C_{55}^{+}	C_{31}^{+}	C_{56}^{2-}	C_{34}^{-}	C_{51}^{2-}	C_{33}^{+}	C_{55}^{-}	C_{52}^{2+}	C_{37}^{-}	C_{54}^{2+}	C_{36}^{+}	C_{53}^{2-}	E	C_{39}^{+}	C_{38}^{-}	C_{56}^{+}	C_{2f}	C_{53}^{2+}	C_{51}^{+}
C_{56}^{2+}	C_{56}^{2+}	C_{2h}	C_{37}^{+}	C_{34}^{-}	C_{2e}	C_{2k}	C_{56}^{2-}	C_{35}^{-}	C_{2a}	C_{2m}	C_{38}^{+}	C_{33}^{+}	C_{56}^{+}	C_{54}^{2-}	C_{30}^{-}	C_{36}^{-}	C_{53}^{2+}	C_{51}^{2-}	C_{56}^{-}	C_{32}^{-}	C_{55}^{2-}	C_{52}^{2+}	C_{31}^{+}	C_{39}^{-}	E	C_{2n}	C_{53}^{2-}	C_{35}^{+}	C_{55}^{+}	C_{53}^{-}
C_{51}^{2-}	C_{51}^{2-}	C_{51}^{-}	C_{34}^{+}	C_{36}^{+}	C_{38}^{-}	C_{39}^{-}	C_{31}^{-}	C_{51}^{2+}	C_{2g}	C_{2d}	C_{2c}	C_{2k}	C_{2n}	E	C_{37}^{-}	C_{32}^{-}	C_{35}^{+}	C_{33}^{+}	C_{30}^{-}	C_{51}^{+}	C_{53}^{2+}	C_{55}^{2+}	C_{52}^{2+}	C_{56}^{2+}	C_{54}^{2+}	C_{52}^{-}	C_{56}^{2-}	C_{2o}	C_{36}^{-}	C_{2h}
C_{52}^{2-}	C_{52}^{2-}	C_{33}^{+}	C_{52}^{-}	C_{35}^{-}	C_{39}^{-}	C_{2m}	C_{2j}	C_{2f}	C_{52}^{2+}	C_{2c}	C_{2i}	C_{36}^{+}	C_{32}^{-}	C_{37}^{+}	E	C_{38}^{+}	C_{31}^{-}	C_{56}^{2-}	C_{53}^{2+}	C_{54}^{2+}	C_{52}^{+}	C_{51}^{2+}	C_{55}^{2-}	C_{34}^{+}	C_{30}^{+}	C_{56}^{2+}	C_{51}^{-}	C_{39}^{+}	C_{2l}	C_{54}^{-}
C_{53}^{2-}	C_{53}^{2-}	C_{30}^{-}	C_{31}^{-}	C_{53}^{-}	C_{2o}	C_{37}^{+}	C_{2b}	C_{2j}	C_{2e}	C_{53}^{2+}	C_{33}^{+}	C_{2g}	C_{39}^{-}	C_{32}^{+}	C_{38}^{-}	E	C_{55}^{2+}	C_{34}^{-}	C_{54}^{2-}	C_{52}^{2+}	C_{56}^{2-}	C_{53}^{+}	C_{35}^{-}	C_{51}^{2+}	C_{36}^{+}	C_{54}^{+}	C_{2d}	C_{37}^{-}	C_{52}^{2-}	C_{51}^{2-}
C_{54}^{2-}	C_{54}^{2-}	C_{32}^{-}	C_{30}^{+}	C_{2l}	C_{54}^{-}	C_{2f}	C_{36}^{+}	C_{2h}	C_{2n}	C_{34}^{+}	C_{54}^{2+}	C_{37}^{-}	C_{2b}	C_{35}^{-}	C_{31}^{+}	C_{55}^{2-}	E	C_{52}^{2+}	C_{39}^{-}	C_{56}^{2+}	C_{51}^{2+}	C_{38}^{-}	C_{54}^{+}	C_{33}^{-}	C_{53}^{2-}	C_{2i}	C_{53}^{+}	C_{51}^{2-}	C_{37}^{+}	C_{53}^{2+}
C_{55}^{2-}	C_{55}^{2-}	C_{35}^{+}	C_{2a}	C_{38}^{-}	C_{2i}	C_{55}^{-}	C_{32}^{+}	C_{2o}	C_{30}^{-}	C_{2l}	C_{31}^{-}	C_{55}^{2+}	C_{2d}	C_{33}^{-}	C_{56}^{2+}	C_{34}^{+}	C_{52}^{2-}	E	C_{36}^{-}	C_{53}^{2+}	C_{37}^{+}	C_{54}^{2-}	C_{39}^{-}	C_{55}^{+}	C_{51}^{2+}	C_{56}^{-}	C_{35}^{-}	C_{54}^{2+}	C_{2g}	C_{38}^{+}
C_{56}^{2-}	C_{56}^{2-}	C_{37}^{-}	C_{2e}	C_{2h}	C_{33}^{-}	C_{34}^{+}	C_{56}^{-}	C_{2a}	C_{38}^{-}	C_{35}^{+}	C_{2k}	C_{2m}	C_{56}^{2+}	C_{30}^{+}	C_{53}^{2-}	C_{54}^{2+}	C_{39}^{+}	C_{36}^{+}	E	C_{55}^{2+}	C_{31}^{-}	C_{32}^{+}	C_{51}^{2+}	C_{52}^{2-}	C_{56}^{+}	C_{54}^{2-}	C_{2j}	C_{55}^{-}	C_{38}^{+}	C_{33}^{+}
C_{31}^{+}	C_{31}^{+}	C_{2d}	C_{51}^{2+}	C_{2g}	C_{38}^{+}	C_{2o}	C_{55}^{2+}	C_{53}^{+}	C_{30}^{+}	C_{55}^{+}	C_{53}^{2+}	C_{51}^{+}	C_{39}^{+}	C_{38}^{-}	C_{2n}	C_{39}^{-}	C_{56}^{2+}	C_{30}^{-}	C_{2i}	C_{56}^{-}	C_{54}^{2-}	C_{52}^{-}	C_{2e}	C_{54}^{+}	C_{52}^{2+}	C_{31}^{-}	C_{34}^{-}	C_{32}^{-}	C_{33}^{-}	C_{36}^{-}
C_{32}^{+}	C_{32}^{+}	C_{52}^{2+}	C_{2i}	C_{35}^{+}	C_{2f}	C_{52}^{+}	C_{36}^{-}	C_{30}^{-}	C_{54}^{+}	C_{54}^{2+}	C_{55}^{-}	C_{2l}	C_{55}^{2-}	C_{2j}	C_{35}^{-}	C_{56}^{2-}	C_{36}^{+}	C_{53}^{+}	C_{51}^{2+}	C_{53}^{2-}	C_{56}^{+}	C_{2h}	C_{51}^{-}	C_{30}^{+}	C_{2d}	C_{33}^{-}	C_{32}^{-}	C_{34}^{-}	C_{31}^{-}	C_{55}^{2+}
C_{33}^{+}	C_{33}^{+}	C_{37}^{+}	C_{53}^{-}	C_{56}^{+}	C_{53}^{2-}	C_{56}^{2+}	C_{2e}	C_{52}^{2-}	C_{2m}	C_{2j}	C_{35}^{-}	C_{39}^{-}	C_{52}^{-}	C_{54}^{2+}	C_{55}^{-}	C_{51}^{+}	C_{2o}	C_{2k}	C_{37}^{-}	C_{2f}	C_{35}^{+}	C_{39}^{+}	C_{55}^{2+}	C_{51}^{2-}	C_{54}^{-}	C_{34}^{-}	C_{31}^{-}	C_{33}^{-}	C_{32}^{-}	C_{51}^{-}
C_{34}^{+}	C_{34}^{+}	C_{54}^{-}	C_{37}^{-}	C_{54}^{2-}	C_{56}^{-}	C_{36}^{+}	C_{51}^{-}	C_{2k}	C_{51}^{2-}	C_{38}^{-}	C_{2n}	C_{56}^{2-}	C_{2h}	C_{55}^{+}	C_{53}^{2+}	C_{2l}	C_{52}^{+}	C_{52}^{2-}	C_{53}^{-}	C_{38}^{+}	C_{2g}	C_{55}^{2-}	C_{36}^{-}	C_{2m}	C_{37}^{+}	C_{32}^{-}	C_{33}^{-}	C_{31}^{-}	C_{34}^{-}	C_{2e}
C_{35}^{+}	C_{35}^{+}	C_{33}^{-}	C_{2m}	C_{56}^{2-}	C_{52}^{2+}	C_{56}^{-}	C_{52}^{+}	C_{55}^{2-}	C_{55}^{-}	C_{32}^{+}	C_{38}^{-}	C_{2i}	C_{2a}	C_{53}^{2+}	C_{33}^{+}	C_{2h}	C_{2c}	C_{54}^{-}	C_{51}^{+}	C_{2o}	C_{53}^{-}	C_{54}^{2+}	C_{51}^{2-}	C_{32}^{-}	C_{38}^{+}	C_{37}^{-}	C_{52}^{2-}	C_{54}^{+}	C_{2d}	C_{35}^{-}

$C_{30}^{+} \equiv C_{3,10}^{+}$, $C_{30}^{-} \equiv C_{3,10}^{-}$

$\twoheadrightarrow$

C$_n$ 107 **C**$_i$ 137 **S**$_n$ 143 **D**$_n$ 193 **D**$_{nh}$ 245 **D**$_{nd}$ 365 **C**$_{nv}$ 481 **C**$_{nh}$ 531 **O** 579 **I**

T **74**.2 Multiplication table *(cont.)*

I	E	C_{51}^{+}	C_{52}^{+}	C_{53}^{+}	C_{54}^{+}	C_{55}^{+}	C_{56}^{+}	C_{51}^{-}	C_{52}^{-}	C_{53}^{-}	C_{54}^{-}	C_{55}^{-}	C_{56}^{-}	C_{51}^{2+}	C_{52}^{2+}	C_{53}^{2+}	C_{54}^{2+}	C_{55}^{2+}	C_{56}^{2+}	C_{51}^{2-}	C_{52}^{2-}	C_{53}^{2-}	C_{54}^{2-}	C_{55}^{2-}	C_{56}^{2-}	C_{31}^{+}	C_{32}^{+}	C_{33}^{+}	C_{34}^{+}	C_{35}^{+}
C_{36}^{+}	C_{36}^{+}	C_{52}^{-}	C_{54}^{-}	C_{32}^{-}	C_{51}^{-}	C_{52}^{2-}	C_{39}^{-}	C_{2n}	C_{2c}	C_{51}^{2-}	C_{2f}	C_{34}^{+}	C_{54}^{2-}	C_{56}^{+}	C_{53}^{+}	C_{55}^{2+}	C_{55}^{-}	C_{2m}	C_{53}^{2-}	C_{34}^{-}	C_{39}^{+}	C_{2d}	C_{32}^{+}	C_{56}^{2-}	C_{2b}	C_{35}^{-}	C_{56}^{-}	C_{2g}	C_{54}^{2+}	C_{37}^{-}
C_{37}^{+}	C_{37}^{+}	C_{54}^{2+}	C_{30}^{-}	C_{54}^{+}	C_{2b}	C_{34}^{-}	C_{2h}	C_{33}^{+}	C_{56}^{2+}	C_{2e}	C_{56}^{+}	C_{53}^{2-}	C_{53}^{-}	C_{2f}	C_{55}^{2-}	C_{52}^{+}	C_{30}^{+}	C_{51}^{2+}	C_{34}^{+}	C_{52}^{2-}	C_{2a}	C_{33}^{-}	C_{55}^{+}	C_{2g}	C_{51}^{-}	C_{36}^{-}	C_{2o}	C_{56}^{2-}	C_{52}^{-}	C_{31}^{-}
C_{38}^{+}	C_{38}^{+}	C_{2k}	C_{34}^{-}	C_{51}^{2+}	C_{56}^{2+}	C_{2d}	C_{2a}	C_{55}^{+}	C_{55}^{2+}	C_{35}^{-}	C_{31}^{+}	C_{56}^{+}	C_{51}^{+}	C_{34}^{+}	C_{54}^{2+}	C_{2c}	C_{2e}	C_{31}^{-}	C_{35}^{+}	C_{54}^{-}	C_{2l}	C_{52}^{2-}	C_{53}^{2+}	C_{53}^{-}	C_{52}^{+}	C_{51}^{2-}	C_{37}^{+}	C_{2i}	C_{53}^{+}	C_{54}^{+}
C_{39}^{+}	C_{39}^{+}	C_{2g}	C_{2c}	C_{2j}	C_{51}^{2+}	C_{53}^{2+}	C_{31}^{+}	C_{52}^{+}	C_{53}^{+}	C_{51}^{+}	C_{33}^{-}	C_{36}^{-}	C_{52}^{2+}	C_{31}^{-}	C_{36}^{+}	C_{33}^{+}	C_{2k}	C_{2b}	C_{55}^{2+}	C_{55}^{-}	C_{54}^{-}	C_{56}^{+}	C_{56}^{2-}	C_{54}^{2+}	C_{2i}	C_{53}^{2-}	C_{2n}	C_{55}^{+}	C_{35}^{+}	C_{2f}
C_{30}^{+}	C_{30}^{+}	C_{55}^{2+}	C_{31}^{+}	C_{2o}	C_{55}^{+}	C_{2l}	C_{32}^{-}	C_{37}^{-}	C_{54}^{2-}	C_{54}^{-}	C_{2b}	C_{53}^{+}	C_{53}^{2+}	C_{2a}	C_{51}^{2+}	C_{31}^{-}	C_{56}^{+}	C_{32}^{+}	C_{52}^{2-}	C_{56}^{2-}	C_{2n}	C_{51}^{-}	C_{37}^{+}	C_{52}^{+}	C_{2j}	C_{55}^{2-}	C_{51}^{+}	C_{36}^{+}	C_{2e}	C_{39}^{+}
C_{31}^{-}	C_{31}^{-}	C_{55}^{-}	C_{38}^{-}	C_{51}^{-}	C_{55}^{2-}	C_{53}^{-}	C_{30}^{-}	C_{2g}	C_{53}^{2-}	C_{2o}	C_{39}^{-}	C_{2d}	C_{51}^{2-}	C_{52}^{+}	C_{56}^{2-}	C_{54}^{-}	C_{2i}	C_{56}^{+}	C_{54}^{2+}	C_{39}^{+}	C_{2e}	C_{30}^{+}	C_{52}^{2-}	C_{38}^{+}	C_{2n}	E	C_{2a}	C_{2b}	C_{2c}	C_{2k}
C_{32}^{-}	C_{32}^{-}	C_{35}^{-}	C_{55}^{+}	C_{55}^{2+}	C_{52}^{-}	C_{2i}	C_{52}^{2-}	C_{54}^{2-}	C_{2f}	C_{36}^{+}	C_{2l}	C_{54}^{-}	C_{30}^{+}	C_{56}^{2+}	C_{51}^{+}	C_{2d}	C_{53}^{-}	C_{35}^{+}	C_{2j}	C_{2h}	C_{36}^{-}	C_{51}^{2-}	C_{30}^{-}	C_{56}^{-}	C_{53}^{2+}	C_{2a}	E	C_{2c}	C_{2b}	C_{53}^{+}
C_{33}^{-}	C_{33}^{-}	C_{53}^{2+}	C_{2j}	C_{2e}	C_{39}^{+}	C_{37}^{-}	C_{53}^{+}	C_{35}^{+}	C_{56}^{-}	C_{52}^{+}	C_{56}^{2-}	C_{52}^{2+}	C_{2m}	C_{2o}	C_{39}^{-}	C_{37}^{+}	C_{51}^{2+}	C_{54}^{2-}	C_{55}^{+}	C_{55}^{2-}	C_{51}^{-}	C_{54}^{+}	C_{2k}	C_{2f}	C_{35}^{-}	C_{2b}	C_{2c}	E	C_{2a}	C_{52}^{2-}
C_{34}^{-}	C_{34}^{-}	C_{2n}	C_{54}^{2+}	C_{36}^{-}	C_{2h}	C_{51}^{2+}	C_{2k}	C_{56}^{+}	C_{38}^{+}	C_{56}^{2+}	C_{51}^{+}	C_{37}^{+}	C_{54}^{+}	C_{36}^{+}	C_{2l}	C_{52}^{2+}	C_{37}^{-}	C_{2g}	C_{38}^{-}	C_{52}^{-}	C_{55}^{2+}	C_{2m}	C_{53}^{+}	C_{53}^{2-}	C_{55}^{-}	C_{2c}	C_{2b}	C_{2a}	E	C_{30}^{-}
C_{35}^{-}	C_{35}^{-}	C_{56}^{2+}	C_{56}^{+}	C_{38}^{+}	C_{33}^{+}	C_{2a}	C_{2m}	C_{32}^{-}	C_{2i}	C_{52}^{2-}	C_{55}^{2+}	C_{52}^{-}	C_{55}^{+}	C_{2h}	C_{54}^{+}	C_{51}^{2+}	C_{53}^{2-}	C_{38}^{-}	C_{33}^{-}	C_{54}^{2-}	C_{32}^{+}	C_{2c}	C_{2o}	C_{51}^{-}	C_{53}^{+}	C_{2k}	C_{53}^{-}	C_{52}^{2+}	C_{30}^{+}	E
C_{36}^{-}	C_{36}^{-}	C_{2c}	C_{2f}	C_{52}^{2+}	C_{2n}	C_{39}^{+}	C_{51}^{2+}	C_{54}^{+}	C_{51}^{+}	C_{34}^{-}	C_{52}^{+}	C_{54}^{2+}	C_{32}^{+}	C_{39}^{-}	C_{32}^{-}	C_{2m}	C_{34}^{+}	C_{53}^{2+}	C_{2d}	C_{53}^{-}	C_{55}^{+}	C_{56}^{2+}	C_{56}^{-}	C_{2b}	C_{55}^{2-}	C_{2j}	C_{54}^{2-}	C_{38}^{+}	C_{55}^{-}	C_{2l}
C_{37}^{-}	C_{37}^{-}	C_{30}^{+}	C_{53}^{2+}	C_{2b}	C_{53}^{+}	C_{54}^{2-}	C_{54}^{-}	C_{56}^{2-}	C_{34}^{+}	C_{56}^{-}	C_{2h}	C_{33}^{-}	C_{2e}	C_{55}^{2+}	C_{2g}	C_{30}^{-}	C_{51}^{+}	C_{2f}	C_{52}^{-}	C_{2a}	C_{51}^{2-}	C_{55}^{-}	C_{34}^{-}	C_{52}^{2+}	C_{33}^{+}	C_{2l}	C_{39}^{+}	C_{51}^{-}	C_{56}^{2+}	C_{2j}
C_{38}^{-}	C_{38}^{-}	C_{56}^{-}	C_{56}^{2-}	C_{34}^{+}	C_{35}^{+}	C_{51}^{-}	C_{55}^{-}	C_{2d}	C_{31}^{-}	C_{55}^{2-}	C_{51}^{2-}	C_{2a}	C_{2k}	C_{53}^{+}	C_{2e}	C_{54}^{2-}	C_{52}^{2+}	C_{52}^{-}	C_{54}^{+}	C_{31}^{+}	C_{53}^{2-}	C_{2l}	C_{2c}	C_{35}^{-}	C_{34}^{-}	C_{54}^{-}	C_{2m}	C_{30}^{-}	C_{51}^{2+}	C_{56}^{2+}
C_{39}^{-}	C_{39}^{-}	C_{53}^{-}	C_{51}^{-}	C_{52}^{-}	C_{31}^{-}	C_{33}^{+}	C_{53}^{2-}	C_{2c}	C_{2j}	C_{2g}	C_{52}^{2-}	C_{51}^{2-}	C_{36}^{+}	C_{54}^{+}	C_{56}^{-}	C_{55}^{+}	C_{55}^{2-}	C_{56}^{2+}	C_{2b}	C_{36}^{-}	C_{33}^{-}	C_{31}^{+}	C_{2i}	C_{2k}	C_{54}^{2-}	C_{56}^{+}	C_{38}^{-}	C_{53}^{2+}	C_{2f}	C_{34}^{+}
C_{30}^{-}	C_{30}^{-}	C_{32}^{+}	C_{55}^{2-}	C_{55}^{-}	C_{2l}	C_{54}^{+}	C_{54}^{2+}	C_{53}^{2-}	C_{37}^{+}	C_{2b}	C_{53}^{-}	C_{2o}	C_{31}^{-}	C_{52}^{2+}	C_{2a}	C_{56}^{-}	C_{32}^{-}	C_{51}^{+}	C_{2n}	C_{2j}	C_{56}^{2+}	C_{37}^{-}	C_{52}^{-}	C_{31}^{+}	C_{51}^{2-}	C_{52}^{+}	C_{55}^{2+}	C_{2h}	C_{39}^{-}	C_{2d}
C_{2a}	C_{2a}	C_{56}^{2-}	C_{56}^{2+}	C_{2k}	C_{2m}	C_{38}^{-}	C_{35}^{+}	C_{55}^{2+}	C_{55}^{2-}	C_{2i}	C_{2d}	C_{35}^{-}	C_{38}^{+}	C_{37}^{-}	C_{37}^{+}	C_{2n}	C_{2j}	C_{51}^{-}	C_{52}^{+}	C_{30}^{+}	C_{30}^{-}	C_{2f}	C_{2g}	C_{52}^{-}	C_{51}^{+}	C_{34}^{+}	C_{33}^{+}	C_{32}^{+}	C_{31}^{+}	C_{56}^{+}
C_{2b}	C_{2b}	C_{2l}	C_{2o}	C_{30}^{-}	C_{30}^{+}	C_{54}^{2+}	C_{54}^{2-}	C_{2e}	C_{2h}	C_{37}^{-}	C_{37}^{+}	C_{53}^{2+}	C_{53}^{2-}	C_{2i}	C_{2d}	C_{55}^{-}	C_{55}^{+}	C_{36}^{-}	C_{36}^{+}	C_{2m}	C_{2k}	C_{56}^{-}	C_{56}^{+}	C_{39}^{+}	C_{39}^{-}	C_{32}^{+}	C_{31}^{+}	C_{34}^{+}	C_{33}^{+}	C_{2g}
C_{2c}	C_{2c}	C_{39}^{-}	C_{36}^{+}	C_{52}^{2-}	C_{51}^{2-}	C_{2j}	C_{2g}	C_{36}^{-}	C_{39}^{+}	C_{51}^{2+}	C_{52}^{2+}	C_{2n}	C_{2f}	C_{53}^{-}	C_{54}^{-}	C_{35}^{-}	C_{38}^{-}	C_{2e}	C_{2o}	C_{54}^{+}	C_{53}^{+}	C_{38}^{+}	C_{35}^{+}	C_{2h}	C_{2l}	C_{33}^{+}	C_{34}^{+}	C_{31}^{+}	C_{32}^{+}	C_{54}^{2-}
C_{2d}	C_{2d}	C_{38}^{-}	C_{2k}	C_{51}^{2-}	C_{2a}	C_{31}^{-}	C_{55}^{2-}	C_{31}^{+}	C_{2o}	C_{55}^{2+}	C_{2g}	C_{38}^{+}	C_{51}^{2+}	C_{56}^{-}	C_{2h}	C_{36}^{+}	C_{2m}	C_{53}^{-}	C_{32}^{+}	C_{53}^{+}	C_{2b}	C_{32}^{-}	C_{2j}	C_{56}^{+}	C_{36}^{-}	C_{51}^{-}	C_{56}^{2+}	C_{2l}	C_{39}^{+}	C_{34}^{-}
C_{2e}	C_{2e}	C_{2b}	C_{53}^{2-}	C_{37}^{+}	C_{53}^{2+}	C_{2h}	C_{37}^{-}	C_{2m}	C_{56}^{2-}	C_{33}^{-}	C_{56}^{2+}	C_{2j}	C_{33}^{+}	C_{2l}	C_{31}^{-}	C_{54}^{+}	C_{31}^{+}	C_{2n}	C_{54}^{-}	C_{2i}	C_{38}^{-}	C_{52}^{+}	C_{38}^{+}	C_{2c}	C_{52}^{-}	C_{54}^{2+}	C_{2g}	C_{56}^{-}	C_{35}^{-}	C_{39}^{-}
C_{2f}	C_{2f}	C_{52}^{2-}	C_{32}^{-}	C_{2i}	C_{36}^{+}	C_{52}^{2+}	C_{2c}	C_{54}^{2+}	C_{36}^{-}	C_{2n}	C_{32}^{+}	C_{54}^{2-}	C_{2l}	C_{33}^{+}	C_{55}^{+}	C_{2a}	C_{51}^{-}	C_{33}^{-}	C_{2g}	C_{37}^{+}	C_{51}^{+}	C_{2k}	C_{55}^{-}	C_{37}^{-}	C_{2o}	C_{2m}	C_{54}^{-}	C_{51}^{2+}	C_{30}^{-}	C_{30}^{+}
C_{2g}	C_{2g}	C_{31}^{-}	C_{51}^{2-}	C_{39}^{-}	C_{2d}	C_{53}^{2-}	C_{2o}	C_{39}^{+}	C_{53}^{2+}	C_{31}^{+}	C_{2j}	C_{51}^{2+}	C_{2c}	C_{55}^{-}	C_{34}^{+}	C_{52}^{-}	C_{2a}	C_{37}^{+}	C_{2l}	C_{52}^{+}	C_{37}^{-}	C_{55}^{+}	C_{2m}	C_{34}^{-}	C_{2f}	C_{53}^{-}	C_{2k}	C_{30}^{+}	C_{52}^{2+}	C_{2n}
C_{2h}	C_{2h}	C_{54}^{2-}	C_{2b}	C_{54}^{2+}	C_{37}^{-}	C_{2n}	C_{34}^{+}	C_{56}^{2+}	C_{2k}	C_{56}^{2-}	C_{34}^{-}	C_{2e}	C_{37}^{+}	C_{32}^{-}	C_{2o}	C_{32}^{+}	C_{53}^{+}	C_{2c}	C_{51}^{-}	C_{35}^{-}	C_{2d}	C_{35}^{+}	C_{51}^{+}	C_{2j}	C_{53}^{-}	C_{2f}	C_{53}^{2+}	C_{38}^{-}	C_{56}^{+}	C_{53}^{2-}
C_{2i}	C_{2i}	C_{2m}	C_{35}^{-}	C_{2a}	C_{52}^{2-}	C_{35}^{+}	C_{52}^{2+}	C_{2l}	C_{32}^{+}	C_{2f}	C_{55}^{2-}	C_{32}^{-}	C_{55}^{2+}	C_{2e}	C_{56}^{+}	C_{2k}	C_{39}^{-}	C_{56}^{-}	C_{39}^{+}	C_{2b}	C_{54}^{+}	C_{2n}	C_{31}^{-}	C_{54}^{-}	C_{31}^{+}	C_{56}^{2-}	C_{52}^{-}	C_{36}^{-}	C_{2o}	C_{55}^{+}
C_{2j}	C_{2j}	C_{53}^{2-}	C_{39}^{-}	C_{33}^{+}	C_{2g}	C_{2e}	C_{53}^{2+}	C_{52}^{2+}	C_{33}^{-}	C_{39}^{+}	C_{2m}	C_{2c}	C_{52}^{2-}	C_{30}^{-}	C_{51}^{-}	C_{56}^{+}	C_{2d}	C_{2h}	C_{30}^{+}	C_{32}^{+}	C_{56}^{-}	C_{51}^{+}	C_{2a}	C_{2n}	C_{32}^{-}	C_{37}^{+}	C_{51}^{2-}	C_{53}^{+}	C_{2i}	C_{36}^{+}
C_{2k}	C_{2k}	C_{34}^{+}	C_{2h}	C_{2n}	C_{56}^{2-}	C_{51}^{2-}	C_{38}^{-}	C_{38}^{+}	C_{2d}	C_{2a}	C_{51}^{2+}	C_{56}^{2+}	C_{34}^{-}	C_{54}^{-}	C_{2b}	C_{2f}	C_{33}^{-}	C_{39}^{-}	C_{55}^{-}	C_{55}^{+}	C_{2o}	C_{2i}	C_{39}^{+}	C_{33}^{+}	C_{54}^{+}	C_{36}^{+}	C_{2e}	C_{55}^{2-}	C_{51}^{+}	C_{37}^{+}
C_{2l}	C_{2l}	C_{2i}	C_{55}^{2+}	C_{55}^{2-}	C_{32}^{-}	C_{32}^{+}	C_{2f}	C_{2b}	C_{54}^{2+}	C_{54}^{2-}	C_{30}^{-}	C_{30}^{+}	C_{2o}	C_{2m}	C_{38}^{+}	C_{38}^{-}	C_{52}^{-}	C_{52}^{+}	C_{2c}	C_{2e}	C_{34}^{-}	C_{34}^{+}	C_{53}^{-}	C_{53}^{+}	C_{2g}	C_{35}^{+}	C_{55}^{+}	C_{2n}	C_{53}^{2-}	C_{31}^{+}
C_{2m}	C_{2m}	C_{2e}	C_{33}^{+}	C_{56}^{2+}	C_{2j}	C_{56}^{2-}	C_{33}^{-}	C_{2i}	C_{35}^{+}	C_{52}^{2+}	C_{2a}	C_{52}^{2-}	C_{35}^{-}	C_{2b}	C_{53}^{-}	C_{34}^{-}	C_{2g}	C_{34}^{+}	C_{53}^{+}	C_{2l}	C_{55}^{-}	C_{36}^{-}	C_{2d}	C_{36}^{+}	C_{55}^{+}	C_{2h}	C_{39}^{-}	C_{52}^{+}	C_{55}^{2+}	C_{52}^{-}
C_{2n}	C_{2n}	C_{36}^{+}	C_{54}^{2-}	C_{2f}	C_{34}^{+}	C_{2c}	C_{51}^{2-}	C_{34}^{-}	C_{51}^{2+}	C_{2k}	C_{36}^{-}	C_{2h}	C_{54}^{2+}	C_{52}^{-}	C_{30}^{+}	C_{2i}	C_{56}^{-}	C_{2j}	C_{31}^{-}	C_{56}^{+}	C_{31}^{+}	C_{2a}	C_{52}^{+}	C_{2e}	C_{30}^{-}	C_{52}^{2-}	C_{37}^{-}	C_{2d}	C_{54}^{+}	C_{2b}
C_{2o}	C_{2o}	C_{55}^{2-}	C_{2d}	C_{31}^{-}	C_{55}^{2+}	C_{30}^{-}	C_{2l}	C_{53}^{2+}	C_{2b}	C_{30}^{+}	C_{53}^{2-}	C_{31}^{+}	C_{2g}	C_{35}^{+}	C_{2k}	C_{51}^{-}	C_{35}^{-}	C_{54}^{+}	C_{2f}	C_{33}^{-}	C_{2h}	C_{54}^{-}	C_{33}^{+}	C_{51}^{+}	C_{2c}	C_{55}^{-}	C_{38}^{+}	C_{54}^{2-}	C_{2j}	C_{51}^{2+}

$C_{30}^{+} \equiv C_{3,10}^{+}$, $C_{30}^{-} \equiv C_{3,10}^{-}$

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C$_n$ 107 **C**$_i$ 137 **S**$_n$ 143 **D**$_n$ 193 **D**$_{nh}$ 245 **D**$_{nd}$ 365 **C**$_{nv}$ 481 **C**$_{nh}$ 531 **O** 579 **I**

T **74**.2 Multiplication table *(cont.)*

I	C_{36}^{+}	C_{37}^{+}	C_{38}^{+}	C_{39}^{+}	C_{30}^{+}	C_{31}^{-}	C_{32}^{-}	C_{33}^{-}	C_{34}^{-}	C_{35}^{-}	C_{36}^{-}	C_{37}^{-}	C_{38}^{-}	C_{39}^{-}	C_{30}^{-}	C_{2a}	C_{2b}	C_{2c}	C_{2d}	C_{2e}	C_{2f}	C_{2g}	C_{2h}	C_{2i}	C_{2j}	C_{2k}	C_{2l}	C_{2m}	C_{2n}	C_{2o}
E	C_{36}^{+}	C_{37}^{+}	C_{38}^{+}	C_{39}^{+}	C_{30}^{+}	C_{31}^{-}	C_{32}^{-}	C_{33}^{-}	C_{34}^{-}	C_{35}^{-}	C_{36}^{-}	C_{37}^{-}	C_{38}^{-}	C_{39}^{-}	C_{30}^{-}	C_{2a}	C_{2b}	C_{2c}	C_{2d}	C_{2e}	C_{2f}	C_{2g}	C_{2h}	C_{2i}	C_{2j}	C_{2k}	C_{2l}	C_{2m}	C_{2n}	C_{2o}
C_{51}^{+}	C_{54}^{-}	C_{56}^{2+}	C_{2d}	C_{2c}	C_{53}^{2+}	C_{53}^{-}	C_{30}^{+}	C_{52}^{2+}	C_{2k}	C_{55}^{2+}	C_{2n}	C_{33}^{-}	C_{55}^{-}	C_{52}^{-}	C_{37}^{+}	C_{55}^{2-}	C_{2e}	C_{36}^{+}	C_{31}^{-}	C_{2m}	C_{54}^{2-}	C_{39}^{-}	C_{56}^{2-}	C_{2l}	C_{52}^{2-}	C_{38}^{-}	C_{2b}	C_{2i}	C_{34}^{+}	C_{53}^{2-}
C_{52}^{+}	C_{51}^{-}	C_{34}^{-}	C_{31}^{+}	C_{2j}	C_{37}^{-}	C_{30}^{-}	C_{54}^{-}	C_{2m}	C_{51}^{2+}	C_{55}^{+}	C_{2c}	C_{56}^{2-}	C_{55}^{2-}	C_{53}^{-}	C_{54}^{2+}	C_{55}^{2+}	C_{2h}	C_{39}^{-}	C_{2o}	C_{56}^{2+}	C_{36}^{+}	C_{53}^{2-}	C_{2k}	C_{32}^{-}	C_{33}^{+}	C_{2d}	C_{54}^{2-}	C_{35}^{-}	C_{51}^{2-}	C_{2b}
C_{53}^{+}	C_{34}^{+}	C_{56}^{+}	C_{55}^{2+}	C_{2g}	C_{2b}	C_{55}^{-}	C_{54}^{2-}	C_{2j}	C_{38}^{+}	C_{32}^{-}	C_{51}^{2+}	C_{2e}	C_{35}^{+}	C_{51}^{-}	C_{54}^{+}	C_{2i}	C_{37}^{+}	C_{51}^{2-}	C_{55}^{2-}	C_{33}^{+}	C_{2n}	C_{31}^{-}	C_{56}^{2+}	C_{2f}	C_{39}^{-}	C_{2a}	C_{54}^{2+}	C_{52}^{2-}	C_{2k}	C_{30}^{-}
C_{54}^{+}	C_{52}^{-}	C_{2h}	C_{51}^{2+}	C_{52}^{2+}	C_{53}^{+}	C_{53}^{2-}	C_{55}^{+}	C_{35}^{+}	C_{2n}	C_{38}^{+}	C_{2f}	C_{56}^{-}	C_{31}^{-}	C_{33}^{+}	C_{2b}	C_{2d}	C_{37}^{-}	C_{52}^{2-}	C_{2g}	C_{56}^{2-}	C_{32}^{-}	C_{2j}	C_{34}^{+}	C_{55}^{2+}	C_{2m}	C_{51}^{2-}	C_{30}^{+}	C_{2a}	C_{36}^{+}	C_{53}^{2+}
C_{55}^{+}	C_{54}^{2-}	C_{33}^{+}	C_{2a}	C_{51}^{2+}	C_{2o}	C_{51}^{-}	C_{2l}	C_{39}^{+}	C_{56}^{2+}	C_{2i}	C_{34}^{-}	C_{53}^{2+}	C_{56}^{-}	C_{36}^{+}	C_{53}^{-}	C_{35}^{+}	C_{53}^{2-}	C_{2n}	C_{38}^{-}	C_{2j}	C_{54}^{2+}	C_{51}^{2-}	C_{2e}	C_{32}^{+}	C_{2c}	C_{56}^{2-}	C_{30}^{-}	C_{52}^{2+}	C_{2h}	C_{31}^{-}
C_{56}^{+}	C_{32}^{-}	C_{2e}	C_{2k}	C_{36}^{-}	C_{31}^{+}	C_{39}^{-}	C_{55}^{2+}	C_{52}^{+}	C_{2h}	C_{2a}	C_{54}^{2+}	C_{53}^{+}	C_{51}^{-}	C_{52}^{2-}	C_{53}^{2-}	C_{38}^{-}	C_{53}^{2+}	C_{2f}	C_{51}^{2-}	C_{33}^{-}	C_{2l}	C_{2c}	C_{37}^{-}	C_{55}^{2-}	C_{52}^{2+}	C_{34}^{+}	C_{2o}	C_{35}^{+}	C_{54}^{2-}	C_{2g}
C_{51}^{-}	C_{2c}	C_{30}^{-}	C_{56}^{+}	C_{53}^{+}	C_{32}^{-}	C_{2d}	C_{52}^{2-}	C_{37}^{-}	C_{54}^{+}	C_{33}^{+}	C_{52}^{+}	C_{54}^{2-}	C_{2k}	C_{2g}	C_{55}^{2-}	C_{56}^{2+}	C_{2l}	C_{39}^{+}	C_{38}^{+}	C_{2b}	C_{52}^{2+}	C_{31}^{+}	C_{54}^{2+}	C_{2m}	C_{53}^{2+}	C_{34}^{-}	C_{2i}	C_{2e}	C_{36}^{-}	C_{55}^{2+}
C_{52}^{-}	C_{2f}	C_{53}^{2-}	C_{56}^{2+}	C_{51}^{+}	C_{55}^{2+}	C_{51}^{2-}	C_{2i}	C_{53}^{+}	C_{37}^{+}	C_{2m}	C_{54}^{+}	C_{30}^{+}	C_{34}^{+}	C_{2c}	C_{31}^{-}	C_{56}^{2-}	C_{2o}	C_{36}^{-}	C_{2k}	C_{53}^{2+}	C_{32}^{+}	C_{51}^{2+}	C_{2b}	C_{35}^{+}	C_{39}^{+}	C_{2h}	C_{55}^{2-}	C_{33}^{-}	C_{54}^{2+}	C_{2d}
C_{53}^{-}	C_{52}^{2-}	C_{2b}	C_{34}^{-}	C_{52}^{+}	C_{55}^{+}	C_{2g}	C_{35}^{-}	C_{56}^{-}	C_{54}^{2+}	C_{56}^{2+}	C_{32}^{+}	C_{54}^{-}	C_{51}^{2-}	C_{2j}	C_{2o}	C_{2k}	C_{30}^{+}	C_{52}^{2+}	C_{51}^{2+}	C_{37}^{-}	C_{2i}	C_{39}^{+}	C_{54}^{2-}	C_{2a}	C_{33}^{-}	C_{2n}	C_{55}^{2+}	C_{56}^{2-}	C_{2f}	C_{31}^{+}
C_{54}^{-}	C_{2n}	C_{53}^{-}	C_{35}^{-}	C_{31}^{+}	C_{2l}	C_{38}^{-}	C_{2f}	C_{53}^{2+}	C_{56}^{+}	C_{52}^{2-}	C_{51}^{+}	C_{2b}	C_{56}^{2-}	C_{51}^{2-}	C_{55}^{-}	C_{2m}	C_{30}^{-}	C_{51}^{2+}	C_{2a}	C_{53}^{2-}	C_{36}^{-}	C_{2d}	C_{37}^{+}	C_{52}^{2+}	C_{2g}	C_{56}^{2+}	C_{32}^{+}	C_{2j}	C_{34}^{-}	C_{55}^{2-}
C_{55}^{-}	C_{39}^{-}	C_{54}^{2+}	C_{51}^{+}	C_{33}^{-}	C_{54}^{-}	C_{2o}	C_{52}^{-}	C_{56}^{2-}	C_{36}^{-}	C_{56}^{+}	C_{52}^{2+}	C_{34}^{+}	C_{2d}	C_{53}^{2-}	C_{2l}	C_{38}^{+}	C_{54}^{2-}	C_{2j}	C_{31}^{+}	C_{2h}	C_{52}^{2-}	C_{53}^{2+}	C_{2n}	C_{35}^{-}	C_{2e}	C_{51}^{2+}	C_{32}^{-}	C_{56}^{2+}	C_{2c}	C_{30}^{+}
C_{56}^{-}	C_{51}^{2-}	C_{54}^{+}	C_{55}^{+}	C_{53}^{2+}	C_{54}^{2-}	C_{55}^{2-}	C_{36}^{+}	C_{2e}	C_{51}^{+}	C_{52}^{-}	C_{39}^{+}	C_{2h}	C_{2a}	C_{31}^{-}	C_{32}^{+}	C_{35}^{-}	C_{54}^{2+}	C_{2g}	C_{55}^{2+}	C_{37}^{+}	C_{2c}	C_{2o}	C_{34}^{-}	C_{52}^{2-}	C_{53}^{2-}	C_{38}^{+}	C_{2f}	C_{33}^{+}	C_{51}^{2+}	C_{2l}
C_{51}^{2+}	C_{53}^{+}	C_{2a}	C_{31}^{-}	C_{36}^{+}	C_{2j}	C_{56}^{+}	C_{53}^{2+}	C_{2f}	C_{38}^{-}	C_{2o}	C_{34}^{+}	C_{52}^{2+}	C_{54}^{+}	C_{55}^{+}	C_{56}^{2+}	C_{30}^{-}	C_{2m}	C_{54}^{-}	C_{53}^{-}	C_{2i}	C_{37}^{-}	C_{52}^{-}	C_{35}^{+}	C_{2b}	C_{32}^{-}	C_{55}^{-}	C_{2e}	C_{2l}	C_{56}^{-}	C_{33}^{+}
C_{52}^{2+}	C_{55}^{-}	C_{51}^{2+}	C_{53}^{2+}	C_{33}^{+}	C_{56}^{2-}	C_{54}^{2+}	C_{56}^{-}	C_{35}^{-}	C_{2g}	C_{53}^{+}	C_{39}^{-}	C_{2a}	C_{2l}	C_{54}^{+}	C_{2n}	C_{30}^{+}	C_{2k}	C_{53}^{-}	C_{2b}	C_{38}^{+}	C_{51}^{-}	C_{37}^{+}	C_{2d}	C_{54}^{-}	C_{56}^{+}	C_{2o}	C_{34}^{+}	C_{55}^{+}	C_{31}^{-}	C_{2h}
C_{53}^{2+}	C_{56}^{2-}	C_{55}^{+}	C_{2l}	C_{31}^{-}	C_{37}^{+}	C_{52}^{+}	C_{2h}	C_{39}^{-}	C_{55}^{2+}	C_{54}^{2-}	C_{2d}	C_{33}^{+}	C_{52}^{2+}	C_{56}^{-}	C_{51}^{+}	C_{2f}	C_{56}^{+}	C_{38}^{-}	C_{32}^{+}	C_{52}^{-}	C_{2k}	C_{55}^{-}	C_{35}^{-}	C_{2n}	C_{51}^{-}	C_{2i}	C_{34}^{-}	C_{36}^{+}	C_{2a}	C_{54}^{+}
C_{54}^{2+}	C_{56}^{+}	C_{34}^{+}	C_{2c}	C_{2i}	C_{52}^{+}	C_{2e}	C_{51}^{+}	C_{55}^{2-}	C_{36}^{+}	C_{51}^{2+}	C_{32}^{-}	C_{55}^{-}	C_{53}^{2-}	C_{56}^{2+}	C_{37}^{-}	C_{2g}	C_{56}^{-}	C_{35}^{-}	C_{2j}	C_{38}^{-}	C_{55}^{+}	C_{2m}	C_{51}^{-}	C_{31}^{+}	C_{2a}	C_{39}^{-}	C_{53}^{+}	C_{2d}	C_{52}^{-}	C_{33}^{-}
C_{55}^{2+}	C_{2b}	C_{52}^{2-}	C_{35}^{+}	C_{2k}	C_{31}^{-}	C_{54}^{-}	C_{30}^{-}	C_{51}^{2+}	C_{2m}	C_{32}^{+}	C_{56}^{2+}	C_{2g}	C_{53}^{+}	C_{54}^{2-}	C_{52}^{-}	C_{52}^{+}	C_{39}^{-}	C_{2h}	C_{56}^{-}	C_{2c}	C_{37}^{+}	C_{34}^{+}	C_{2j}	C_{54}^{+}	C_{2n}	C_{33}^{-}	C_{53}^{-}	C_{36}^{-}	C_{2e}	C_{51}^{-}
C_{56}^{2+}	C_{55}^{2+}	C_{33}^{-}	C_{34}^{+}	C_{54}^{2+}	C_{51}^{2+}	C_{52}^{2-}	C_{2d}	C_{54}^{+}	C_{37}^{-}	C_{38}^{-}	C_{2b}	C_{51}^{+}	C_{52}^{-}	C_{2i}	C_{2j}	C_{51}^{-}	C_{39}^{+}	C_{2l}	C_{36}^{+}	C_{52}^{+}	C_{2o}	C_{2f}	C_{53}^{+}	C_{31}^{-}	C_{32}^{+}	C_{54}^{-}	C_{2g}	C_{55}^{-}	C_{30}^{+}	C_{2c}
C_{51}^{2-}	C_{39}^{+}	C_{55}^{2-}	C_{53}^{-}	C_{54}^{-}	C_{52}^{2-}	C_{38}^{+}	C_{2j}	C_{54}^{2-}	C_{55}^{-}	C_{53}^{2-}	C_{56}^{-}	C_{2f}	C_{34}^{-}	C_{31}^{+}	C_{2a}	C_{37}^{+}	C_{2i}	C_{53}^{+}	C_{56}^{+}	C_{2l}	C_{33}^{-}	C_{55}^{+}	C_{32}^{+}	C_{2e}	C_{30}^{+}	C_{54}^{+}	C_{2m}	C_{2b}	C_{52}^{+}	C_{35}^{-}
C_{52}^{2-}	C_{32}^{+}	C_{2g}	C_{2e}	C_{56}^{+}	C_{2a}	C_{2n}	C_{35}^{+}	C_{55}^{+}	C_{53}^{2-}	C_{33}^{-}	C_{53}^{-}	C_{55}^{2+}	C_{54}^{2-}	C_{36}^{-}	C_{51}^{2-}	C_{37}^{-}	C_{2d}	C_{54}^{+}	C_{2h}	C_{31}^{+}	C_{55}^{-}	C_{34}^{-}	C_{2o}	C_{56}^{-}	C_{51}^{+}	C_{2b}	C_{38}^{-}	C_{53}^{+}	C_{30}^{-}	C_{2k}
C_{53}^{2-}	C_{2m}	C_{30}^{+}	C_{54}^{2+}	C_{55}^{-}	C_{56}^{+}	C_{39}^{+}	C_{56}^{2+}	C_{51}^{-}	C_{2l}	C_{2h}	C_{55}^{2-}	C_{52}^{-}	C_{2c}	C_{33}^{-}	C_{31}^{+}	C_{2n}	C_{55}^{+}	C_{35}^{+}	C_{36}^{-}	C_{54}^{-}	C_{2a}	C_{52}^{+}	C_{32}^{-}	C_{2k}	C_{56}^{-}	C_{2f}	C_{38}^{+}	C_{34}^{+}	C_{2i}	C_{51}^{+}
C_{54}^{2-}	C_{34}^{-}	C_{51}^{-}	C_{52}^{2-}	C_{55}^{2+}	C_{32}^{+}	C_{56}^{2-}	C_{36}^{-}	C_{2o}	C_{52}^{-}	C_{2c}	C_{55}^{+}	C_{30}^{-}	C_{2e}	C_{2k}	C_{56}^{-}	C_{2j}	C_{55}^{-}	C_{38}^{+}	C_{2m}	C_{31}^{-}	C_{51}^{+}	C_{2a}	C_{53}^{-}	C_{39}^{+}	C_{2d}	C_{33}^{+}	C_{52}^{+}	C_{2g}	C_{56}^{+}	C_{35}^{+}
C_{55}^{2-}	C_{53}^{2-}	C_{2f}	C_{52}^{+}	C_{56}^{2-}	C_{51}^{-}	C_{30}^{+}	C_{53}^{-}	C_{2k}	C_{52}^{2+}	C_{54}^{+}	C_{2m}	C_{51}^{2-}	C_{31}^{+}	C_{2b}	C_{32}^{-}	C_{51}^{+}	C_{36}^{+}	C_{2e}	C_{53}^{+}	C_{2n}	C_{33}^{+}	C_{37}^{-}	C_{2c}	C_{56}^{+}	C_{2h}	C_{39}^{+}	C_{52}^{-}	C_{34}^{-}	C_{2j}	C_{54}^{-}
C_{56}^{2-}	C_{2d}	C_{52}^{+}	C_{54}^{-}	C_{2b}	C_{2n}	C_{2i}	C_{51}^{2-}	C_{37}^{+}	C_{53}^{+}	C_{51}^{-}	C_{53}^{2+}	C_{34}^{-}	C_{35}^{-}	C_{55}^{2-}	C_{52}^{2+}	C_{52}^{-}	C_{36}^{-}	C_{2o}	C_{32}^{-}	C_{54}^{+}	C_{2g}	C_{2l}	C_{51}^{+}	C_{39}^{-}	C_{30}^{-}	C_{55}^{+}	C_{2c}	C_{53}^{-}	C_{31}^{+}	C_{2f}
C_{31}^{+}	C_{37}^{-}	C_{35}^{-}	C_{55}^{2-}	C_{51}^{2-}	C_{53}^{2-}	E	C_{2b}	C_{2c}	C_{2a}	C_{2l}	C_{2k}	C_{2j}	C_{52}^{+}	C_{54}^{-}	C_{56}^{+}	C_{32}^{+}	C_{33}^{+}	C_{34}^{+}	C_{55}^{-}	C_{52}^{2-}	C_{2h}	C_{51}^{-}	C_{2m}	C_{54}^{2+}	C_{36}^{+}	C_{35}^{+}	C_{37}^{+}	C_{2f}	C_{56}^{2-}	C_{53}^{-}
C_{32}^{+}	C_{53}^{-}	C_{2n}	C_{39}^{+}	C_{2m}	C_{56}^{-}	C_{2b}	E	C_{2a}	C_{2c}	C_{51}^{+}	C_{52}^{2-}	C_{38}^{-}	C_{2o}	C_{37}^{+}	C_{54}^{2-}	C_{31}^{+}	C_{34}^{+}	C_{33}^{+}	C_{53}^{2+}	C_{2k}	C_{52}^{-}	C_{2e}	C_{51}^{2-}	C_{55}^{+}	C_{56}^{2+}	C_{2g}	C_{54}^{-}	C_{38}^{+}	C_{39}^{-}	C_{37}^{-}
C_{33}^{+}	C_{2i}	C_{53}^{2+}	C_{2h}	C_{54}^{+}	C_{38}^{+}	C_{2c}	C_{2a}	E	C_{2b}	C_{56}^{2-}	C_{30}^{-}	C_{55}^{+}	C_{36}^{+}	C_{52}^{2+}	C_{2g}	C_{34}^{+}	C_{31}^{+}	C_{32}^{+}	C_{2n}	C_{53}^{+}	C_{55}^{2-}	C_{36}^{-}	C_{30}^{+}	C_{38}^{-}	C_{52}^{+}	C_{54}^{2-}	C_{2d}	C_{56}^{-}	C_{2l}	C_{51}^{2+}
C_{34}^{+}	C_{51}^{2+}	C_{55}^{-}	C_{52}^{-}	C_{30}^{+}	C_{2f}	C_{2a}	C_{2c}	C_{2b}	E	C_{39}^{-}	C_{53}^{+}	C_{54}^{2+}	C_{56}^{2+}	C_{2d}	C_{35}^{+}	C_{33}^{+}	C_{32}^{+}	C_{31}^{+}	C_{35}^{-}	C_{30}^{-}	C_{39}^{+}	C_{55}^{2+}	C_{54}^{+}	C_{2j}	C_{2o}	C_{56}^{+}	C_{52}^{2+}	C_{53}^{2-}	C_{51}^{+}	C_{2i}
C_{35}^{+}	C_{31}^{-}	C_{36}^{-}	C_{53}^{+}	C_{2e}	C_{34}^{+}	C_{2l}	C_{51}^{-}	C_{56}^{2+}	C_{39}^{+}	E	C_{2j}	C_{2k}	C_{55}^{2+}	C_{30}^{-}	C_{2f}	C_{55}^{+}	C_{2n}	C_{53}^{2-}	C_{30}^{+}	C_{34}^{-}	C_{39}^{-}	C_{2b}	C_{51}^{2+}	C_{52}^{-}	C_{37}^{+}	C_{31}^{+}	C_{36}^{+}	C_{56}^{+}	C_{2g}	C_{54}^{2-}

$C_{30}^{+} \equiv C_{3,10}^{+}$, $C_{30}^{-} \equiv C_{3,10}^{-}$ ⇛

C$_n$ 107 **C**$_i$ 137 **S**$_n$ 143 **D**$_n$ 193 **D**$_{nh}$ 245 **D**$_{nd}$ 365 **C**$_{nv}$ 481 **C**$_{nh}$ 531 **O** 579 **I**

T **74**.2 Multiplication table *(cont.)*

I	C^+_{36}	C^+_{37}	C^+_{38}	C^+_{39}	C^+_{30}	C^-_{31}	C^-_{32}	C^-_{33}	C^-_{34}	C^-_{35}	C^-_{36}	C^-_{37}	C^-_{38}	C^-_{39}	C^-_{30}	C_{2a}	C_{2b}	C_{2c}	C_{2d}	C_{2e}	C_{2f}	C_{2g}	C_{2h}	C_{2i}	C_{2j}	C_{2k}	C_{2l}	C_{2m}	C_{2n}	C_{2o}
C^+_{36}	C^-_{36}	C^-_{31}	C^+_{33}	C^+_{55}	C_{2i}	C_{2k}	C^{2+}_{52}	C^+_{30}	C^-_{53}	C_{2j}	E	C_{2l}	C_{2h}	C^{2+}_{51}	C^-_{38}	C_{2e}	C^{2-}_{55}	C^+_{51}	C^{2+}_{56}	C_{2o}	C^+_{52}	C^+_{38}	C^-_{30}	C^-_{33}	C^+_{31}	C^+_{37}	C^+_{35}	C^{2+}_{53}	C^+_{54}	C_{2a}
C^+_{37}	C^-_{35}	C^-_{37}	C_{2n}	C^+_{32}	C^+_{51}	C_{2j}	C^+_{38}	C^-_{55}	C^{2-}_{54}	C_{2k}	C_{2l}	E	C^-_{39}	C_{2m}	C^{2+}_{53}	C^{2-}_{51}	C^+_{53}	C_{2i}	C_{2c}	C^-_{56}	C^{2+}_{55}	C^{2+}_{52}	C^-_{54}	C_{2d}	C^+_{35}	C^+_{36}	C^+_{31}	C^-_{38}	C^-_{32}	C^+_{39}
C^+_{38}	C^+_{30}	C_{2m}	C^-_{38}	C_{2n}	C_{2g}	C^-_{52}	C_{2o}	C^-_{36}	C^{2-}_{56}	C^{2-}_{55}	C_{2h}	C^+_{39}	E	C^-_{32}	C^+_{33}	C^-_{55}	C_{2j}	C^{2-}_{54}	C^-_{51}	C^{2+}_{52}	C_{2b}	C^+_{36}	C^-_{33}	C^-_{30}	C_{2f}	C^-_{56}	C^{2-}_{53}	C^+_{32}	C^-_{37}	C^-_{39}
C^+_{39}	C^-_{56}	C^+_{38}	C_{2o}	C^-_{39}	C_{2e}	C^+_{54}	C^-_{37}	C^{2-}_{52}	C_{2d}	C^+_{30}	C^{2-}_{51}	C_{2m}	C^+_{32}	E	C^-_{34}	C_{2l}	C^{2+}_{56}	C^-_{51}	C^-_{30}	C^-_{35}	C^+_{34}	C^-_{53}	C_{2a}	C^{2-}_{54}	C^-_{52}	C^{2-}_{55}	C_{2h}	C^-_{32}	C^-_{38}	C^+_{37}
C^+_{30}	C_{2h}	C^-_{52}	C_{2i}	C_{2d}	C^-_{30}	C^-_{56}	C^{2+}_{54}	C_{2g}	C^-_{35}	C_{2f}	C^+_{38}	C^{2-}_{53}	C^-_{33}	C^+_{34}	E	C^{2+}_{52}	C^-_{53}	C_{2k}	C^+_{35}	C^-_{39}	C^-_{34}	C^-_{38}	C^+_{33}	C^-_{36}	C^{2-}_{51}	C_{2m}	C^+_{54}	C_{2c}	C^{2+}_{56}	C^-_{55}
C^-_{31}	C_{2j}	C_{2l}	C^+_{54}	C^-_{56}	C^-_{52}	C^+_{31}	C^+_{33}	C^+_{34}	C^+_{32}	C^+_{37}	C^+_{35}	C^+_{36}	C^{2+}_{51}	C^{2+}_{53}	C^{2+}_{55}	C^-_{34}	C^-_{32}	C^-_{33}	C^+_{51}	C^{2-}_{54}	C_{2m}	C^+_{53}	C_{2f}	C^{2+}_{56}	C^-_{37}	C^-_{36}	C^-_{35}	C_{2h}	C^{2+}_{52}	C^+_{55}
C^-_{32}	C^{2+}_{54}	C^-_{39}	C_{2m}	C^+_{38}	C^{2-}_{55}	C^+_{34}	C^+_{32}	C^+_{31}	C^+_{33}	C^{2+}_{52}	C^+_{56}	C_{2o}	C^-_{37}	C_{2n}	C^-_{51}	C^-_{33}	C^-_{31}	C^-_{34}	C^{2-}_{56}	C_{2g}	C^+_{54}	C_{2k}	C^{2-}_{53}	C^+_{52}	C^{2+}_{51}	C_{2e}	C^-_{55}	C^+_{39}	C^+_{37}	C^-_{38}
C^-_{33}	C^-_{38}	C^+_{51}	C^+_{30}	C^{2-}_{53}	C_{2h}	C^+_{32}	C^+_{34}	C^+_{33}	C^+_{31}	C^-_{54}	C_{2g}	C^{2+}_{56}	C_{2i}	C^-_{55}	C^-_{36}	C^-_{32}	C^-_{34}	C^-_{31}	C_{2l}	C^+_{56}	C^{2-}_{51}	C^-_{30}	C^+_{38}	C^+_{36}	C^-_{53}	C^{2+}_{55}	C_{2n}	C^-_{52}	C_{2d}	C^{2+}_{54}
C^-_{34}	C^+_{55}	C^{2-}_{56}	C^{2-}_{51}	C_{2f}	C^+_{39}	C^+_{33}	C^+_{31}	C^+_{32}	C^+_{34}	C_{2d}	C^{2-}_{54}	C^+_{52}	C^-_{53}	C^-_{35}	C_{2e}	C^-_{31}	C^-_{33}	C^-_{32}	C^-_{39}	C^+_{35}	C^+_{30}	C^{2-}_{52}	C^-_{56}	C_{2o}	C_{2i}	C^-_{51}	C^{2+}_{53}	C^{2-}_{55}	C^-_{54}	C_{2j}
C^-_{35}	C_{2l}	C_{2j}	C^{2-}_{56}	C^-_{34}	C_{2d}	C^+_{36}	C^{2-}_{55}	C^+_{51}	C_{2e}	C^+_{35}	C^+_{37}	C^+_{31}	C^-_{54}	C_{2f}	C^-_{39}	C^-_{56}	C_{2g}	C^{2+}_{54}	C^+_{34}	C^+_{39}	C^-_{30}	C_{2n}	C^{2+}_{53}	C^-_{55}	C^-_{36}	C^-_{37}	C^-_{31}	C^+_{52}	C_{2b}	C^{2-}_{51}
C^-_{36}	E	C_{2k}	C_{2g}	C^{2-}_{52}	C^-_{33}	C^+_{37}	C^+_{53}	C_{2i}	C^{2-}_{51}	C^+_{31}	C^+_{36}	C^+_{35}	C^-_{30}	C^+_{56}	C_{2h}	C_{2o}	C^{2-}_{56}	C^-_{52}	C^{2-}_{53}	C_{2a}	C^-_{54}	C^+_{33}	C^-_{38}	C^+_{30}	C^-_{35}	C^-_{31}	C^-_{37}	C^{2+}_{55}	C^-_{51}	C_{2e}
C^-_{37}	C_{2k}	E	C^-_{32}	C_{2o}	C^{2+}_{54}	C^+_{35}	C_{2n}	C^{2-}_{53}	C^+_{55}	C^+_{36}	C^+_{31}	C^+_{37}	C_{2m}	C^-_{38}	C^+_{52}	C^{2-}_{52}	C^+_{54}	C_{2d}	C_{2i}	C^-_{53}	C^{2+}_{51}	C^{2-}_{55}	C^+_{56}	C_{2c}	C^-_{31}	C^-_{35}	C^-_{36}	C^-_{39}	C^+_{38}	C^+_{32}
C^-_{38}	C_{2g}	C^+_{32}	E	C^-_{37}	C^+_{36}	C^{2+}_{55}	C^-_{39}	C_{2h}	C^+_{52}	C^-_{53}	C^-_{33}	C_{2n}	C^+_{38}	C_{2o}	C_{2i}	C^+_{56}	C_{2f}	C^{2+}_{53}	C^+_{55}	C^{2+}_{54}	C_{2j}	C^+_{30}	C^-_{36}	C^+_{33}	C_{2b}	C^+_{51}	C^{2-}_{52}	C^+_{37}	C^+_{39}	C^-_{32}
C^-_{39}	C^{2+}_{52}	C_{2o}	C^+_{37}	E	C^-_{35}	C^{2+}_{51}	C_{2m}	C^-_{54}	C^-_{30}	C_{2e}	C^-_{55}	C^-_{32}	C_{2n}	C^+_{39}	C_{2d}	C_{2h}	C^{2+}_{55}	C^+_{52}	C^-_{34}	C^+_{30}	C^+_{35}	C^+_{51}	C_{2l}	C^{2-}_{56}	C^+_{53}	C^{2+}_{54}	C_{2a}	C^-_{37}	C^+_{32}	C^+_{38}
C^-_{30}	C^+_{33}	C^{2-}_{54}	C^-_{36}	C^+_{35}	E	C^{2+}_{53}	C^+_{56}	C^-_{38}	C_{2f}	C^-_{34}	C_{2i}	C^-_{51}	C_{2g}	C_{2e}	C^+_{30}	C^{2+}_{51}	C^-_{54}	C_{2m}	C^+_{39}	C^+_{34}	C^-_{35}	C^-_{33}	C^+_{36}	C^+_{38}	C^{2-}_{56}	C_{2c}	C^+_{55}	C_{2k}	C^{2-}_{52}	C^+_{53}
C_{2a}	C_{2o}	C^{2+}_{52}	C^-_{56}	C_{2h}	C^{2-}_{51}	C^-_{32}	C^-_{31}	C^-_{34}	C^-_{33}	C^-_{55}	C_{2e}	C^{2+}_{51}	C^+_{55}	C_{2l}	C^{2-}_{52}	E	C_{2c}	C_{2b}	C^-_{54}	C^-_{36}	C^{2-}_{53}	C^{2-}_{54}	C^+_{39}	C^-_{53}	C^{2+}_{54}	C^+_{53}	C^-_{39}	C^+_{54}	C^{2+}_{53}	C^+_{36}
C_{2b}	C^{2+}_{56}	C^-_{54}	C_{2f}	C^{2-}_{55}	C^+_{54}	C^-_{33}	C^-_{34}	C^-_{31}	C^-_{32}	C_{2n}	C^{2+}_{55}	C^-_{53}	C_{2j}	C^{2-}_{56}	C^+_{53}	C_{2c}	E	C_{2a}	C^{2+}_{52}	C^-_{51}	C^+_{38}	C^+_{35}	C^-_{52}	C^{2+}_{51}	C^-_{38}	C^{2-}_{52}	C^+_{51}	C^{2-}_{51}	C^-_{35}	C^+_{52}
C_{2c}	C^+_{52}	C_{2d}	C^{2-}_{53}	C^-_{52}	C_{2m}	C^-_{34}	C^-_{33}	C^-_{32}	C^-_{31}	C^{2+}_{53}	C^-_{51}	C_{2i}	C^{2+}_{54}	C^+_{51}	C_{2k}	C_{2b}	C_{2a}	E	C^+_{37}	C^{2+}_{55}	C^-_{56}	C^+_{56}	C^{2-}_{55}	C^-_{37}	C^+_{55}	C^-_{30}	C^{2-}_{56}	C^+_{30}	C^-_{55}	C^{2+}_{56}
C_{2d}	C^{2+}_{53}	C_{2i}	C^-_{55}	C^+_{34}	C^-_{39}	C^+_{55}	C^{2-}_{53}	C_{2n}	C^+_{35}	C^-_{30}	C^{2-}_{56}	C_{2c}	C^+_{51}	C^+_{30}	C^-_{35}	C^+_{54}	C^{2-}_{52}	C^-_{37}	E	C_{2f}	C_{2e}	C^-_{54}	C^{2+}_{52}	C^+_{37}	C^{2-}_{54}	C^+_{52}	C^+_{33}	C^{2+}_{54}	C^-_{33}	C^-_{52}
C_{2e}	C_{2a}	C^+_{53}	C^{2-}_{54}	C^-_{30}	C^-_{34}	C^{2+}_{52}	C_{2k}	C^-_{53}	C^+_{30}	C^+_{34}	C_{2o}	C^+_{56}	C^{2-}_{52}	C^+_{35}	C^+_{39}	C^+_{36}	C^+_{51}	C^{2-}_{55}	C_{2f}	E	C_{2d}	C^+_{32}	C^+_{55}	C^{2-}_{51}	C^-_{55}	C^-_{32}	C^{2+}_{51}	C^-_{51}	C^{2+}_{55}	C^-_{36}
C_{2f}	C^+_{54}	C^{2-}_{51}	C_{2j}	C^-_{35}	C^+_{35}	C_{2h}	C^+_{52}	C^{2+}_{55}	C^-_{39}	C^+_{39}	C^-_{52}	C^{2-}_{55}	C_{2b}	C^-_{34}	C^+_{34}	C^{2+}_{53}	C^-_{38}	C^+_{56}	C_{2e}	C_{2d}	E	C^{2+}_{56}	C^-_{31}	C^+_{53}	C^+_{38}	C^{2-}_{53}	C^-_{56}	C^+_{31}	C^-_{53}	C^{2-}_{56}
C_{2g}	C^-_{33}	C^{2+}_{55}	C^-_{30}	C^-_{51}	C^+_{33}	C^+_{51}	C_{2e}	C^+_{36}	C^{2-}_{55}	C_{2b}	C^-_{38}	C^{2-}_{52}	C^-_{36}	C^+_{53}	C^+_{38}	C^{2+}_{54}	C^-_{35}	C^-_{56}	C^+_{54}	C^-_{32}	C^{2-}_{56}	E	C_{2i}	C_{2h}	C^-_{54}	C^+_{32}	C^{2+}_{56}	C^{2-}_{54}	C^+_{35}	C^+_{56}
C_{2h}	C^+_{38}	C^-_{56}	C^+_{36}	C_{2l}	C^-_{36}	C_{2m}	C^{2+}_{51}	C^-_{30}	C^-_{54}	C^{2-}_{51}	C^+_{30}	C^+_{54}	C^+_{33}	C_{2a}	C^-_{33}	C^-_{39}	C^+_{52}	C^{2+}_{55}	C^{2-}_{52}	C^-_{55}	C^+_{31}	C_{2i}	E	C_{2g}	C^{2-}_{55}	C^-_{52}	C^+_{39}	C^-_{31}	C^+_{55}	C^{2+}_{52}
C_{2i}	C^-_{30}	C_{2c}	C^-_{33}	C^{2+}_{56}	C^-_{38}	C^{2-}_{54}	C^-_{55}	C^+_{38}	C_{2j}	C^+_{52}	C^+_{33}	C_{2d}	C^+_{30}	C^{2+}_{54}	C^+_{36}	C^+_{53}	C^{2-}_{51}	C^+_{37}	C^-_{37}	C^{2+}_{51}	C^-_{53}	C_{2h}	C_{2g}	E	C^-_{34}	C^{2+}_{53}	C^-_{51}	C^+_{51}	C^{2-}_{53}	C^+_{34}
C_{2j}	C^+_{35}	C^+_{31}	C_{2b}	C^-_{53}	C^{2+}_{56}	C^-_{36}	C^{2-}_{56}	C^-_{52}	C_{2o}	C^-_{37}	C^-_{31}	C^-_{35}	C_{2f}	C^+_{52}	C^{2+}_{51}	C^{2-}_{54}	C^+_{38}	C^-_{55}	C^{2+}_{54}	C^+_{55}	C^-_{38}	C^+_{54}	C^{2+}_{55}	C^+_{34}	E	C_{2l}	C_{2k}	C^-_{54}	C^{2-}_{55}	C^-_{34}
C_{2k}	C^+_{31}	C^+_{35}	C^-_{51}	C^{2-}_{54}	C_{2c}	C^-_{35}	C_{2g}	C^{2+}_{54}	C^-_{56}	C^-_{31}	C^-_{37}	C^-_{36}	C^+_{56}	C^{2+}_{55}	C_{2m}	C^-_{53}	C^{2+}_{52}	C^+_{30}	C^-_{52}	C^+_{32}	C^{2+}_{53}	C^-_{32}	C^+_{52}	C^{2-}_{53}	C_{2l}	E	C_{2j}	C^-_{30}	C^+_{53}	C^{2-}_{52}
C_{2l}	C^+_{37}	C^+_{36}	C^{2+}_{52}	C_{2a}	C^-_{55}	C^-_{37}	C^+_{54}	C_{2d}	C^{2-}_{52}	C^-_{36}	C^-_{35}	C^-_{31}	C^{2+}_{53}	C_{2h}	C^-_{54}	C^+_{39}	C^-_{51}	C^{2+}_{56}	C^-_{33}	C^{2-}_{51}	C^+_{56}	C^{2-}_{56}	C^-_{39}	C^+_{51}	C_{2k}	C_{2j}	E	C^{2+}_{51}	C^+_{33}	C^-_{56}
C_{2m}	C^{2-}_{55}	C^+_{39}	C^-_{37}	C^+_{37}	C_{2k}	C_{2f}	C^-_{38}	C^+_{56}	C^{2+}_{53}	C^-_{56}	C^{2-}_{53}	C^+_{38}	C^-_{32}	C^+_{32}	C_{2c}	C^-_{54}	C^{2+}_{51}	C^-_{30}	C^{2-}_{54}	C^+_{51}	C^-_{31}	C^{2+}_{54}	C^+_{31}	C^-_{51}	C^+_{54}	C^+_{30}	C^{2-}_{51}	E	C_{2o}	C_{2n}
C_{2n}	C^+_{51}	C^-_{38}	C^-_{39}	C^-_{32}	C^{2+}_{52}	C^{2+}_{56}	C^+_{39}	C_{2l}	C^-_{51}	C_{2g}	C^-_{54}	C^+_{32}	C^+_{37}	C^+_{38}	C^{2-}_{56}	C^{2-}_{53}	C^+_{35}	C^+_{55}	C^+_{33}	C^{2-}_{55}	C^+_{53}	C^-_{35}	C^-_{55}	C^{2+}_{53}	C^{2+}_{55}	C^-_{53}	C^-_{33}	C_{2o}	E	C_{2m}
C_{2o}	C_{2e}	C^-_{32}	C^+_{32}	C^-_{38}	C^-_{53}	C^+_{53}	C^+_{37}	C^{2-}_{51}	C_{2i}	C^{2+}_{54}	C_{2a}	C^-_{39}	C^+_{39}	C^-_{37}	C^+_{55}	C^-_{36}	C^-_{52}	C^{2-}_{56}	C^+_{52}	C^+_{36}	C^{2+}_{56}	C^-_{56}	C^{2-}_{52}	C^-_{34}	C^+_{34}	C^{2+}_{52}	C^+_{56}	C_{2n}	C_{2m}	E

$C^+_{30} \equiv C^+_{3,10}$, $C^-_{30} \equiv C^-_{3,10}$

T **74**.3 Factor table

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I	E	C_{51}^{+}	C_{52}^{+}	C_{53}^{+}	C_{54}^{+}	C_{55}^{+}	C_{56}^{+}	C_{51}^{-}	C_{52}^{-}	C_{53}^{-}	C_{54}^{-}	C_{55}^{-}	C_{56}^{-}	C_{51}^{2+}	C_{52}^{2+}	C_{53}^{2+}	C_{54}^{2+}	C_{55}^{2+}	C_{56}^{2+}	C_{51}^{2-}	C_{52}^{2-}	C_{53}^{2-}	C_{54}^{2-}	C_{55}^{2-}	C_{56}^{2-}	C_{31}^{+}	C_{32}^{+}	C_{33}^{+}	C_{34}^{+}	C_{35}^{+}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_{51}^{+}	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	1	1	−1	1	1	1	1	1	1	1	1	1	1	1	1
C_{52}^{+}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	1	1	1	1	1	1	1	−1	1	−1	1	1	−1
C_{53}^{+}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	1	−1	1	1	−1	1	1	1	1
C_{54}^{+}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	1	1
C_{55}^{+}	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	1	1	1	−1	−1	1	1	1	1	1	1	1	1	1	1	1
C_{56}^{+}	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	1	1	−1	1	−1	1	1	−1	1	1	1	1	1	−1	1	1
C_{51}^{-}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	1	1	−1	−1	1	1	1	1	1
C_{52}^{-}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	1	1	1	1	−1	1	1
C_{53}^{-}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	1	−1	1	1	1	1	1	1
C_{54}^{-}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	−1	1	−1	1	−1	1	1	1	−1	1
C_{55}^{-}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	1	1	−1	1	−1	1	−1	1	1	−1	1	1	−1
C_{56}^{-}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	1	1	1	−1	−1	1	1	1	1	1
C_{51}^{2+}	1	−1	−1	1	−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	−1	1	1	1	1
C_{52}^{2+}	1	1	−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	−1	−1	1	1	−1
C_{53}^{2+}	1	−1	1	−1	1	1	1	1	1	1	−1	1	1	−1	−1	−1	1	−1	1	1	1	1	−1	1	−1	−1	1	1	1	1
C_{54}^{2+}	1	−1	1	1	−1	1	−1	1	1	1	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1	1	−1	1	1	−1	−1	1	1
C_{55}^{2+}	1	1	1	1	1	−1	1	1	−1	1	−1	1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1	1	1	1	1
C_{56}^{2+}	1	1	1	1	1	−1	−1	1	1	−1	1	1	1	−1	1	1	−1	−1	−1	1	−1	−1	1	1	1	−1	1	−1	1	1
C_{51}^{2-}	1	1	1	1	1	1	1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	1	1	1	−1	−1
C_{52}^{2-}	1	1	1	1	1	−1	−1	1	−1	−1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	−1	−1	1
C_{53}^{2-}	1	1	1	1	1	1	−1	−1	1	−1	1	−1	1	1	1	1	−1	1	−1	−1	−1	−1	1	−1	1	1	−1	−1	1	1
C_{54}^{2-}	1	1	1	−1	1	1	1	−1	1	1	−1	1	1	1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	1	−1	1
C_{55}^{2-}	1	1	−1	1	−1	1	1	1	1	1	1	−1	−1	1	−1	1	−1	1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	−1
C_{56}^{2-}	1	1	−1	−1	1	1	1	−1	1	1	1	1	−1	1	−1	−1	1	1	1	−1	1	1	−1	−1	−1	1	1	1	−1	−1
C_{31}^{+}	1	1	1	1	1	−1	1	1	1	1	1	1	1	−1	−1	−1	1	−1	1	1	1	1	−1	1	1	−1	1	1	1	1
C_{32}^{+}	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	1	1	1	−1	−1	1	−1	1	1	−1
C_{33}^{+}	1	1	1	1	1	1	1	1	−1	−1	1	1	1	1	1	1	1	−1	−1	1	−1	−1	1	1	1	1	1	−1	1	1
C_{34}^{+}	1	1	1	1	1	1	1	1	1	1	1	1	−1	1	1	−1	1	1	1	−1	−1	1	−1	1	−1	1	1	1	−1	−1
C_{35}^{+}	1	1	1	1	1	1	1	1	1	1	1	−1	−1	1	−1	−1	1	1	1	1	1	1	1	−1	−1	1	−1	1	−1	−1

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T **74**.3 Factor table *(cont.)*

I	E	C_{51}^{+}	C_{52}^{+}	C_{53}^{+}	C_{54}^{+}	C_{55}^{+}	C_{56}^{+}	C_{51}^{-}	C_{52}^{-}	C_{53}^{-}	C_{54}^{-}	C_{55}^{-}	C_{56}^{-}	C_{51}^{2+}	C_{52}^{2+}	C_{53}^{2+}	C_{54}^{2+}	C_{55}^{2+}	C_{56}^{2+}	C_{51}^{2-}	C_{52}^{2-}	C_{53}^{2-}	C_{54}^{2-}	C_{55}^{2-}	C_{56}^{2-}	C_{31}^{+}	C_{32}^{+}	C_{33}^{+}	C_{34}^{+}	C_{35}^{+}
C_{36}^{+}	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	1
C_{37}^{+}	1	1	1	1	−1	1	1	1	1	1	1	1	1	−1	1	1	−1	1	−1	1	1	−1	1	−1	1	1	1	−1	1	1
C_{38}^{+}	1	−1	1	1	1	1	1	1	1	1	1	1	1	−1	1	1	1	−1	−1	1	−1	1	1	1	1	−1	1	1	1	1
C_{39}^{+}	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	1	1	1	1	1	1	1	−1	−1	−1	1	1	−1
C_{30}^{+}	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	1	−1	1	1	−1	1	1	−1	1
C_{31}^{-}	1	1	1	1	1	1	1	−1	1	1	1	−1	1	1	1	1	−1	1	1	−1	1	−1	1	−1	1	1	−1	−1	−1	1
C_{32}^{-}	1	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	−1	−1	−1	−1	1	−1	1	1	1	1	−1	1	1
C_{33}^{-}	1	1	1	−1	1	1	1	1	1	1	1	1	1	−1	−1	−1	1	1	1	1	1	1	1	−1	−1	1	1	1	−1	−1
C_{34}^{-}	1	−1	1	1	1	1	−1	1	1	1	1	1	1	−1	1	1	−1	1	−1	1	1	−1	1	1	1	1	−1	1	1	1
C_{35}^{-}	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	1	1	1	−1	−1	1	−1	−1	−1	1	1	−1	1	−1	1	1
C_{36}^{-}	1	1	−1	1	−1	1	1	1	1	1	1	1	1	−1	−1	1	−1	1	1	1	1	1	1	−1	1	1	−1	1	1	1
C_{37}^{-}	1	1	1	1	1	1	1	1	1	1	−1	1	−1	1	1	−1	1	1	1	−1	1	1	−1	1	−1	−1	1	1	−1	1
C_{38}^{-}	1	1	1	1	1	1	1	−1	1	1	1	−1	1	1	−1	1	1	1	1	−1	1	1	−1	−1	−1	1	1	1	−1	−1
C_{39}^{-}	1	1	1	1	1	1	1	−1	−1	−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	1	1	1	1	1	−1	1	1
C_{30}^{-}	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1	1	1	−1
C_{2a}	1	−1	1	−1	−1	−1	−1	1	−1	1	1	1	1	−1	1	−1	−1	−1	−1	1	−1	1	1	1	1	−1	1	−1	1	1
C_{2b}	1	−1	−1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1	1	−1	1	1	1	1	−1	1	−1	−1	1	1	−1	1
C_{2c}	1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	1	1	−1
C_{2d}	1	−1	−1	−1	1	−1	−1	1	−1	1	1	1	1	−1	1	−1	−1	−1	−1	1	1	1	1	1	1	−1	1	−1	1	1
C_{2e}	1	−1	1	1	−1	1	−1	−1	−1	−1	1	−1	1	1	1	1	−1	−1	−1	1	−1	−1	1	−1	1	1	−1	−1	1	1
C_{2f}	1	1	1	1	1	−1	−1	−1	−1	1	−1	1	−1	1	1	1	1	−1	−1	−1	−1	1	−1	1	−1	−1	1	−1	−1	1
C_{2g}	1	−1	−1	−1	1	−1	−1	1	1	1	1	1	1	−1	−1	−1	1	−1	−1	1	1	1	1	1	−1	−1	−1	1	1	−1
C_{2h}	1	−1	−1	1	−1	−1	−1	1	−1	−1	1	1	1	−1	1	1	−1	1	−1	1	1	−1	1	−1	1	−1	−1	−1	1	1
C_{2i}	1	−1	1	1	1	−1	−1	−1	−1	1	−1	1	1	1	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1	1	−1	−1	1
C_{2j}	1	−1	−1	−1	1	−1	1	1	1	1	1	1	−1	−1	−1	−1	1	−1	1	1	1	1	−1	−1	−1	−1	−1	1	−1	−1
C_{2k}	1	1	−1	1	1	1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	1	−1	1	−1	−1
C_{2l}	1	−1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	1	−1	1	−1	1	1	1	1	−1	1	−1	−1	1	−1	−1	1	−1
C_{2m}	1	−1	−1	−1	1	1	1	−1	1	1	−1	−1	−1	1	−1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1	1	−1	−1
C_{2n}	1	1	1	1	1	−1	1	−1	−1	1	−1	−1	−1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1	1	1	−1	−1	1
C_{2o}	1	1	−1	1	−1	1	1	−1	−1	−1	1	−1	−1	1	1	1	−1	1	−1	−1	1	−1	1	−1	−1	1	−1	−1	−1	−1

$C_{30}^{+} \equiv C_{3,10}^{+}$, $C_{30}^{-} \equiv C_{3,10}^{-}$ $\twoheadrightarrow$

$\mathbf{C}_n$ 107 $\mathbf{C}_i$ 137 $\mathbf{S}_n$ 143 $\mathbf{D}_n$ 193 $\mathbf{D}_{nh}$ 245 $\mathbf{D}_{nd}$ 365 $\mathbf{C}_{nv}$ 481 $\mathbf{C}_{nh}$ 531 **O** 579 **I**

T **74**.3 Factor table *(cont.)*

I	C_{36}^{+}	C_{37}^{+}	C_{38}^{+}	C_{39}^{+}	C_{30}^{+}	C_{31}^{-}	C_{32}^{-}	C_{33}^{-}	C_{34}^{-}	C_{35}^{-}	C_{36}^{-}	C_{37}^{-}	C_{38}^{-}	C_{39}^{-}	C_{30}^{-}	C_{2a}	C_{2b}	C_{2c}	C_{2d}	C_{2e}	C_{2f}	C_{2g}	C_{2h}	C_{2i}	C_{2j}	C_{2k}	C_{2l}	C_{2m}	C_{2n}	C_{2o}
E	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_{51}^{+}	1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	1	−1	−1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	−1	1	1
C_{52}^{+}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	1	1	−1	−1	1	−1	−1	−1	−1	1	−1
C_{53}^{+}	1	1	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	−1	−1	−1	1	1	−1	1	1	−1	−1	1	−1	1	1
C_{54}^{+}	1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	−1	1	1	−1	1	−1	1	1	−1	1	1	1	−1	−1	1	−1
C_{55}^{+}	1	1	1	1	−1	1	−1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	1	−1	1	1	1	1	−1	1
C_{56}^{+}	1	1	−1	1	1	1	1	1	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	1	−1	−1	1	1	1	1	1	−1
C_{51}^{-}	−1	1	1	1	1	−1	1	1	1	1	1	1	1	−1	1	1	−1	1	1	−1	−1	1	1	−1	1	−1	−1	−1	−1	−1
C_{52}^{-}	1	1	1	1	1	1	1	1	1	−1	1	1	1	−1	1	−1	−1	1	−1	−1	−1	1	−1	−1	1	−1	1	1	−1	−1
C_{53}^{-}	1	−1	1	1	1	−1	1	1	1	1	1	1	1	−1	1	−1	1	1	1	−1	1	1	−1	1	1	1	−1	1	1	−1
C_{54}^{-}	1	1	1	1	−1	1	1	1	1	1	1	1	1	1	1	−1	−1	1	1	1	−1	1	1	−1	1	−1	1	1	−1	1
C_{55}^{-}	1	1	1	1	1	1	1	1	1	1	1	1	−1	1	1	1	1	1	1	1	1	1	−1	1	−1	−1	−1	−1	−1	−1
C_{56}^{-}	1	1	1	1	1	1	1	−1	1	1	1	−1	−1	1	1	1	−1	1	1	1	−1	−1	1	1	−1	−1	−1	−1	−1	1
C_{51}^{2+}	1	1	−1	−1	1	1	1	−1	−1	−1	−1	1	1	1	1	−1	1	−1	−1	1	1	−1	−1	1	−1	1	1	1	1	1
C_{52}^{2+}	1	1	1	−1	1	1	1	−1	1	1	−1	−1	1	1	−1	1	1	−1	1	1	1	−1	1	1	−1	1	−1	−1	1	1
C_{53}^{2+}	1	1	−1	−1	−1	1	−1	−1	1	1	1	−1	1	1	1	1	−1	−1	−1	1	1	−1	1	1	−1	−1	1	−1	−1	1
C_{54}^{2+}	1	−1	1	−1	1	1	1	1	−1	1	−1	1	1	1	−1	1	1	−1	1	−1	1	1	−1	1	−1	1	−1	−1	1	−1
C_{55}^{2+}	1	1	−1	−1	−1	1	−1	1	−1	−1	1	1	1	1	1	−1	−1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	−1	1
C_{56}^{2+}	1	−1	−1	1	1	1	1	1	−1	−1	−1	1	1	1	−1	−1	1	1	−1	−1	−1	−1	−1	−1	1	1	−1	1	1	−1
C_{51}^{2-}	−1	1	1	1	1	−1	−1	1	1	1	1	1	−1	−1	−1	1	1	1	1	1	−1	1	1	1	1	−1	1	1	−1	−1
C_{52}^{2-}	−1	−1	1	1	1	1	−1	1	1	−1	1	1	1	−1	1	−1	1	1	1	−1	−1	1	1	−1	1	1	1	1	−1	1
C_{53}^{2-}	−1	−1	1	1	1	−1	1	1	1	1	1	1	−1	−1	−1	−1	1	1	1	−1	1	1	−1	−1	1	1	−1	1	1	−1
C_{54}^{2-}	−1	1	1	1	−1	1	−1	−1	1	−1	1	−1	−1	1	1	−1	−1	1	−1	1	−1	1	1	−1	1	−1	1	1	−1	1
C_{55}^{2-}	1	−1	1	1	1	−1	1	1	1	1	1	1	−1	−1	−1	1	1	−1	1	−1	1	1	1	1	−1	−1	−1	−1	−1	−1
C_{56}^{2-}	−1	1	1	1	1	−1	1	−1	1	1	1	−1	−1	1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	−1	−1	−1
C_{31}^{+}	1	1	−1	−1	−1	1	1	1	1	−1	−1	1	1	1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	−1	1	1	−1	1	1
C_{32}^{+}	1	−1	1	1	1	−1	1	−1	1	1	−1	1	1	1	−1	1	1	−1	1	−1	1	−1	−1	1	−1	−1	−1	−1	1	−1
C_{33}^{+}	1	−1	1	1	1	−1	1	1	−1	−1	1	1	1	−1	−1	−1	1	1	−1	−1	−1	1	−1	−1	1	1	−1	1	−1	−1
C_{34}^{+}	−1	1	1	1	1	−1	−1	1	1	1	1	−1	−1	−1	1	1	−1	1	1	1	−1	1	1	−1	−1	−1	1	−1	−1	−1
C_{35}^{+}	1	1	1	−1	1	1	1	−1	1	1	1	1	−1	1	−1	1	1	−1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1

$C_{30}^{+} \equiv C_{3,10}^{+}$, $C_{30}^{-} \equiv C_{3,10}^{-}$

↠

T **74**.3 Factor table *(cont.)*

I	C_{36}^{+}	C_{37}^{+}	C_{38}^{+}	C_{39}^{+}	C_{30}^{+}	C_{31}^{-}	C_{32}^{-}	C_{33}^{-}	C_{34}^{-}	C_{35}^{-}	C_{36}^{-}	C_{37}^{-}	C_{38}^{-}	C_{39}^{-}	C_{30}^{-}	C_{2a}	C_{2b}	C_{2c}	C_{2d}	C_{2e}	C_{2f}	C_{2g}	C_{2h}	C_{2i}	C_{2j}	C_{2k}	C_{2l}	C_{2m}	C_{2n}	C_{2o}
C_{36}^{+}	−1	1	1	1	1	1	−1	1	1	−1	1	−1	−1	−1	1	1	−1	1	1	1	−1	1	1	−1	1	−1	1	1	−1	−1
C_{37}^{+}	1	−1	−1	1	1	−1	1	1	−1	−1	1	1	1	−1	−1	−1	1	−1	1	−1	1	1	−1	1	1	1	−1	1	1	−1
C_{38}^{+}	1	−1	−1	−1	1	1	−1	1	−1	−1	1	1	1	1	1	−1	1	−1	−1	−1	1	−1	−1	−1	−1	1	1	1	1	1
C_{39}^{+}	1	1	−1	−1	−1	1	1	−1	1	1	−1	1	1	1	1	−1	−1	−1	−1	1	1	−1	1	1	−1	1	1	−1	1	1
C_{30}^{+}	−1	1	1	1	−1	1	−1	1	1	1	1	−1	1	1	1	−1	−1	−1	−1	1	−1	−1	1	−1	−1	1	1	1	−1	1
C_{31}^{-}	−1	1	1	1	1	−1	1	1	1	1	1	1	−1	−1	−1	1	1	1	1	−1	−1	1	−1	1	1	−1	−1	−1	−1	−1
C_{32}^{-}	−1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	1	1	−1	−1	1	−1	−1	−1	−1	1	−1	1	−1	1	1	−1	1
C_{33}^{-}	1	1	1	−1	−1	1	1	−1	1	1	1	−1	−1	1	1	1	−1	−1	−1	1	1	−1	1	1	−1	−1	−1	−1	−1	1
C_{34}^{-}	1	−1	−1	−1	1	1	1	1	−1	1	−1	1	1	1	1	−1	1	−1	−1	−1	1	−1	−1	−1	−1	1	−1	1	1	−1
C_{35}^{-}	−1	−1	−1	1	1	1	−1	1	1	−1	1	1	1	1	1	−1	1	1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1
C_{36}^{-}	1	−1	1	−1	1	1	1	−1	−1	1	−1	1	1	1	1	−1	1	−1	−1	1	1	−1	−1	1	−1	1	−1	−1	1	1
C_{37}^{-}	1	1	1	−1	−1	1	1	−1	1	1	1	−1	1	1	1	1	−1	1	1	1	−1	−1	1	−1	−1	−1	1	−1	−1	1
C_{38}^{-}	−1	1	1	1	1	−1	1	−1	1	1	1	1	−1	1	−1	1	1	1	1	1	−1	1	1	1	1	−1	−1	−1	−1	−1
C_{39}^{-}	−1	1	1	1	1	−1	−1	1	1	1	1	1	1	−1	−1	1	1	1	1	−1	−1	1	1	−1	1	−1	−1	1	−1	−1
C_{30}^{-}	1	−1	1	1	1	−1	1	1	−1	1	−1	1	−1	1	−1	1	1	1	1	−1	1	1	−1	1	1	−1	−1	1	1	−1
C_{2a}	−1	−1	−1	1	−1	1	−1	1	−1	−1	1	1	1	−1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1
C_{2b}	−1	1	1	−1	−1	1	−1	−1	1	1	1	−1	1	1	1	−1	−1	1	−1	1	−1	−1	1	−1	−1	−1	1	−1	−1	1
C_{2c}	1	1	−1	−1	1	1	1	−1	−1	1	−1	−1	1	1	−1	1	−1	−1	−1	1	1	−1	−1	1	−1	1	−1	−1	1	1
C_{2d}	1	1	−1	−1	−1	1	−1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	−1	1	−1	−1	−1	−1	−1	1	1	1	1	1
C_{2e}	1	−1	−1	1	1	−1	−1	1	−1	−1	1	1	1	−1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	1	−1	1	1	−1
C_{2f}	−1	1	−1	1	−1	−1	−1	1	1	−1	1	−1	1	−1	1	−1	−1	1	1	−1	−1	1	1	−1	1	−1	1	1	−1	1
C_{2g}	1	1	−1	−1	−1	1	−1	−1	−1	1	−1	−1	1	1	1	−1	−1	−1	−1	1	1	−1	1	−1	−1	1	1	−1	1	1
C_{2h}	1	−1	−1	1	1	−1	1	1	−1	−1	−1	1	1	1	−1	−1	1	−1	−1	−1	1	−1	−1	1	1	1	−1	1	1	−1
C_{2i}	−1	−1	−1	1	−1	1	−1	1	−1	−1	1	1	1	−1	1	−1	−1	1	−1	−1	−1	1	−1	−1	1	1	1	1	−1	1
C_{2j}	1	1	1	−1	−1	1	1	−1	−1	1	−1	−1	−1	1	1	1	−1	−1	−1	1	1	−1	1	1	−1	1	−1	−1	1	1
C_{2k}	−1	1	1	1	−1	−1	−1	−1	1	1	1	−1	−1	−1	1	1	−1	1	1	1	−1	1	1	1	−1	−1	1	−1	−1	−1
C_{2l}	1	−1	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	1	−1	1	1	−1	−1	−1	1	−1
C_{2m}	1	1	1	−1	1	−1	1	−1	1	1	−1	−1	−1	1	1	1	−1	−1	1	1	1	−1	1	1	−1	−1	−1	−1	1	−1
C_{2n}	−1	1	1	1	−1	−1	−1	−1	1	−1	1	−1	−1	−1	1	1	−1	1	1	1	−1	1	1	−1	1	−1	1	−1	−1	1
C_{2o}	1	−1	1	1	1	−1	1	1	−1	1	−1	1	−1	−1	−1	1	1	1	1	−1	1	1	−1	1	1	−1	−1	1	−1	−1

$C_{30}^{+} \equiv C_{3,10}^{+}$, $C_{30}^{-} \equiv C_{3,10}^{-}$

T **74**.4 Character table § **16**–4, p. 71

I	E	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$	τ
A	1	1	1	1	1	a
T_1	3	$2c_5$	$2c_5^3$	0	-1	a
T_2	3	$2c_5^3$	$2c_5$	0	-1	a
F	4	-1	-1	1	0	a
H	5	0	0	-1	1	a
$E_{1/2}$	2	$2c_5$	$2c_5^2$	1	0	c
$E_{7/2}$	2	$2c_5^3$	$2c_5^4$	1	0	c
$F_{3/2}$	4	1	-1	-1	0	c
$I_{5/2}$	6	-1	1	0	0	c

$c_n^m = \cos \frac{m}{n}\pi$

T **74**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions § **16**–5, p. 72

I	0	1	2	3
A	$^{\square}1$		$x^2+y^2+z^2$	
T_1		$^{\square}(x,y,z)$, (R_x,R_y,R_z)		
T_2				$\left.\begin{matrix} \\ \end{matrix}\right\}$ $^{\square}\{xz^2, yz^2, z(x^2-y^2), xyz,$
F				$x(x^2-3y^2), y(3x^2-y^2), z^3\}$
H			$^{\square}(x^2-y^2, 2z^2-x^2-y^2, zx, yz, xy)$	

T **74**.6*a* Bases of irreducible representations § **16**–6, pp. 74, 75

I	$\big\langle \lvert j\,m\rangle \big\rvert$
A	$\lvert 0\,0\rangle$
T_1	$\big\langle \lvert 1\,1\rangle_+,\ \lvert 1\,0\rangle,\ \lvert 1\,1\rangle_- \big\rvert$
T_2	$\Big\langle \sqrt{\tfrac{3}{32}}\,(\sqrt5-1)\,\lvert 3\,1\rangle_+ - \tfrac{1}{\sqrt{32}}\,(\sqrt5+3)\,\lvert 3\,3\rangle_+,\ \tfrac12\,\lvert 3\,0\rangle + \sqrt{\tfrac34}\,\lvert 3\,2\rangle_+,$ $-\sqrt{\tfrac{3}{32}}\,(\sqrt5+1)\,\lvert 3\,1\rangle_- + \tfrac{1}{\sqrt{32}}\,(\sqrt5-3)\,\lvert 3\,3\rangle_- \Big\rvert$
F	$\Big\langle \lvert 3\,2\rangle_-,\ \tfrac{1}{\sqrt{32}}\,(\sqrt5+3)\,\lvert 3\,1\rangle_+ + \sqrt{\tfrac{3}{32}}\,(\sqrt5-1)\,\lvert 3\,3\rangle_+,\ -\sqrt{\tfrac34}\,\lvert 3\,0\rangle + \tfrac12\,\lvert 3\,2\rangle_+,$ $-\tfrac{1}{\sqrt{32}}\,(\sqrt5-3)\,\lvert 3\,1\rangle_- - \sqrt{\tfrac{3}{32}}\,(\sqrt5+1)\,\lvert 3\,3\rangle_- \Big\rvert$
H	$\Big\langle \tfrac{1}{\sqrt2}\,\lvert 2\,0\rangle - \tfrac{\mathrm{i}}{\sqrt2}\,\lvert 2\,2\rangle_+,\ \tfrac{1}{\sqrt2}\,\lvert 2\,0\rangle + \tfrac{\mathrm{i}}{\sqrt2}\,\lvert 2\,2\rangle_+,\ \lvert 2\,1\rangle_-,\ -\lvert 2\,2\rangle_-,\ -\lvert 2\,1\rangle_+ \Big\rvert$
$E_{1/2}$	$\big\langle \lvert \tfrac12\,\tfrac12\rangle,\ \lvert \tfrac12\,\overline{\tfrac12}\rangle \big\rvert$
$E_{7/2}$	$\Big\langle \tfrac{\sqrt{21}}{8}\,\lvert \tfrac72\,\tfrac52\rangle - \tfrac{\sqrt{21}}{8}\,\lvert \tfrac72\,\tfrac12\rangle - \tfrac{\sqrt7}{8}\,\lvert \tfrac72\,\overline{\tfrac32}\rangle - \tfrac{\sqrt{15}}{8}\,\lvert \tfrac72\,\overline{\tfrac72}\rangle,\ \tfrac{\sqrt{15}}{8}\,\lvert \tfrac72\,\tfrac72\rangle + \tfrac{\sqrt7}{8}\,\lvert \tfrac72\,\tfrac32\rangle + \tfrac{\sqrt{21}}{8}\,\lvert \tfrac72\,\overline{\tfrac12}\rangle - \tfrac{\sqrt{21}}{8}\,\lvert \tfrac72\,\overline{\tfrac52}\rangle \Big\rvert$
$F_{3/2}$	$\Big\langle \tfrac{1}{\sqrt2}\,\lvert \tfrac32\,\tfrac12\rangle - \tfrac{\mathrm{i}}{\sqrt2}\,\lvert \tfrac32\,\overline{\tfrac32}\rangle,\ \tfrac{\mathrm{i}}{\sqrt2}\,\lvert \tfrac32\,\tfrac32\rangle - \tfrac{1}{\sqrt2}\,\lvert \tfrac32\,\overline{\tfrac12}\rangle,\ \tfrac{1}{\sqrt2}\,\lvert \tfrac32\,\tfrac12\rangle + \tfrac{\mathrm{i}}{\sqrt2}\,\lvert \tfrac32\,\overline{\tfrac32}\rangle,\ -\tfrac{\mathrm{i}}{\sqrt2}\,\lvert \tfrac32\,\tfrac32\rangle - \tfrac{1}{\sqrt2}\,\lvert \tfrac32\,\overline{\tfrac12}\rangle \Big\rvert$
$I_{5/2}$	$\Big\langle \sqrt{\tfrac16}\,\lvert \tfrac52\,\tfrac52\rangle - \sqrt{\tfrac56}\,\lvert \tfrac52\,\overline{\tfrac32}\rangle,\ -\sqrt{\tfrac56}\,\lvert \tfrac52\,\tfrac32\rangle + \sqrt{\tfrac16}\,\lvert \tfrac52\,\overline{\tfrac52}\rangle,$ $\sqrt{\tfrac{5}{12}}\,\lvert \tfrac52\,\tfrac52\rangle + \tfrac{\mathrm{i}}{\sqrt2}\,\lvert \tfrac52\,\tfrac12\rangle + \sqrt{\tfrac{1}{12}}\,\lvert \tfrac52\,\overline{\tfrac32}\rangle,\ \sqrt{\tfrac{1}{12}}\,\lvert \tfrac52\,\tfrac32\rangle + \tfrac{\mathrm{i}}{\sqrt2}\,\lvert \tfrac52\,\overline{\tfrac12}\rangle + \sqrt{\tfrac{5}{12}}\,\lvert \tfrac52\,\overline{\tfrac52}\rangle,$ $\sqrt{\tfrac{5}{12}}\,\lvert \tfrac52\,\tfrac52\rangle - \tfrac{\mathrm{i}}{\sqrt2}\,\lvert \tfrac52\,\tfrac12\rangle + \sqrt{\tfrac{1}{12}}\,\lvert \tfrac52\,\overline{\tfrac32}\rangle,\ \sqrt{\tfrac{1}{12}}\,\lvert \tfrac52\,\tfrac32\rangle - \tfrac{\mathrm{i}}{\sqrt2}\,\lvert \tfrac52\,\overline{\tfrac12}\rangle + \sqrt{\tfrac{5}{12}}\,\lvert \tfrac52\,\overline{\tfrac52}\rangle \Big\rvert$

T **74**.6b Symmetrized harmonics

Use T **75**.6b. § **16**–6, pp. 74, 75

T **74**.6c Spin harmonics

§ **16**–6, pp. 74, 75

I	$\langle\quad\vert$
$E_{1/2}$	$\langle a\,\alpha, a\,\beta\vert$
	$\langle \frac{1}{\sqrt{3}}\,(t_1^{(1)}\,\beta - t_1^{(2)}\,\alpha + t_1^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_1^{(1)}\,\alpha + t_1^{(2)}\,\beta + t_1^{(3)}\,\alpha)\vert$
$E_{7/2}$	$\langle \frac{1}{2}\,(f^{(1)}\,\alpha - f^{(2)}\,\beta + f^{(3)}\,\alpha - f^{(4)}\,\beta), \frac{1}{2}\,(f^{(1)}\,\beta + f^{(2)}\,\alpha - f^{(3)}\,\beta - f^{(4)}\,\alpha)\vert$
$F_{3/2}$	$\langle \frac{1}{\sqrt{3}}\,(t_1^{(1)}\,\beta - \omega^*\,t_1^{(2)}\,\alpha + \omega\,t_1^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_1^{(1)}\,\alpha + \omega^*\,t_1^{(2)}\,\beta + \omega\,t_1^{(3)}\,\alpha),$
	$\frac{1}{\sqrt{3}}\,(\omega\,t_1^{(1)}\,\beta - \omega^*\,t_1^{(2)}\,\alpha + t_1^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-\omega\,t_1^{(1)}\,\alpha + \omega^*\,t_1^{(2)}\,\beta + t_1^{(3)}\,\alpha)\vert$
	$\langle \frac{1}{\sqrt{5}}\,(\sqrt{2}\,h^{(1)}\,\alpha + \mathrm{i}\,\omega\,h^{(3)}\,\beta - \mathrm{i}\,h^{(4)}\,\alpha + \mathrm{i}\,\omega^*\,h^{(5)}\,\beta),$
	$\frac{1}{\sqrt{5}}\,(\sqrt{2}\,h^{(1)}\,\beta - \mathrm{i}\,\omega\,h^{(3)}\,\alpha + \mathrm{i}\,h^{(4)}\,\beta + \mathrm{i}\,\omega^*\,h^{(5)}\,\alpha),$
	$\frac{1}{\sqrt{5}}\,(\sqrt{2}\,h^{(2)}\,\alpha - \mathrm{i}\,\omega^*\,h^{(3)}\,\beta + \mathrm{i}\,h^{(4)}\,\alpha - \mathrm{i}\,\omega\,h^{(5)}\,\beta),$
	$\frac{1}{\sqrt{5}}\,(\sqrt{2}\,h^{(2)}\,\beta + \mathrm{i}\,\omega^*\,h^{(3)}\,\alpha - \mathrm{i}\,h^{(4)}\,\beta - \mathrm{i}\,\omega\,h^{(5)}\,\alpha)\vert$
$I_{5/2}$	$\langle \frac{1}{\sqrt{3}}\,(t_2^{(1)}\,\beta - t_2^{(2)}\,\alpha + t_2^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,(-t_2^{(1)}\,\alpha + t_2^{(2)}\,\beta + t_2^{(3)}\,\alpha),$
	$\frac{1}{\sqrt{3}}\,\rho^*\,(t_2^{(1)}\,\beta - \omega^*\,t_2^{(2)}\,\alpha + \omega\,t_2^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,\rho^*\,(-t_2^{(1)}\,\alpha + \omega^*\,t_2^{(2)}\,\beta + \omega\,t_2^{(3)}\,\alpha),$
	$\frac{1}{\sqrt{3}}\,\rho\,(t_2^{(1)}\,\beta - \omega\,t_2^{(2)}\,\alpha + \omega^*\,t_2^{(3)}\,\beta), \frac{1}{\sqrt{3}}\,\rho\,(-t_2^{(1)}\,\alpha + \omega\,t_2^{(2)}\,\beta + \omega^*\,t_2^{(3)}\,\alpha)\vert$
	$\langle \frac{1}{\sqrt{12}}\,(3\,f^{(1)}\,\alpha + f^{(2)}\,\beta - f^{(3)}\,\alpha + f^{(4)}\,\beta), \frac{1}{\sqrt{12}}\,(3\,f^{(1)}\,\beta - f^{(2)}\,\alpha + f^{(3)}\,\beta + f^{(4)}\,\alpha),$
	$\frac{1}{\sqrt{3}}\,\sigma^*\,(-\omega\,f^{(2)}\,\beta + f^{(3)}\,\alpha - \omega^*\,f^{(4)}\,\beta), \frac{1}{\sqrt{3}}\,\sigma^*\,(\omega\,f^{(2)}\,\alpha - f^{(3)}\,\beta - \omega^*\,f^{(4)}\,\alpha),$
	$\frac{1}{\sqrt{3}}\,\sigma\,(-\omega^*\,f^{(2)}\,\beta + f^{(3)}\,\alpha - \omega\,f^{(4)}\,\beta), \frac{1}{\sqrt{3}}\,\sigma\,(\omega^*\,f^{(2)}\,\alpha - f^{(3)}\,\beta - \omega\,f^{(4)}\,\alpha)\vert$
	$\langle \frac{1}{\sqrt{3}}\,(h^{(3)}\,\beta - h^{(4)}\,\alpha + h^{(5)}\,\beta), \frac{1}{\sqrt{3}}\,(-h^{(3)}\,\alpha + h^{(4)}\,\beta + h^{(5)}\,\alpha),$
	$\frac{1}{\sqrt{15}}\,(3\mathrm{i}\,h^{(1)}\,\alpha + \sqrt{2}\,\omega\,h^{(3)}\,\beta - \sqrt{2}\,h^{(4)}\,\alpha + \sqrt{2}\,\omega^*\,h^{(5)}\,\beta),$
	$\frac{1}{\sqrt{15}}\,(3\mathrm{i}\,h^{(1)}\,\beta - \sqrt{2}\,\omega\,h^{(3)}\,\alpha + \sqrt{2}\,h^{(4)}\,\beta + \sqrt{2}\,\omega^*\,h^{(5)}\,\alpha),$
	$\frac{1}{\sqrt{15}}\,(-3\mathrm{i}\,h^{(2)}\,\alpha + \sqrt{2}\,\omega^*\,h^{(3)}\,\beta - \sqrt{2}\,h^{(4)}\,\alpha + \sqrt{2}\,\omega\,h^{(5)}\,\beta),$
	$\frac{1}{\sqrt{15}}\,(-3\mathrm{i}\,h^{(2)}\,\beta - \sqrt{2}\,\omega^*\,h^{(3)}\,\alpha + \sqrt{2}\,h^{(4)}\,\beta + \sqrt{2}\,\omega\,h^{(5)}\,\alpha)\vert$

$\alpha = \vert\frac{1}{2}\,\frac{1}{2}\rangle$, $\beta = \vert\frac{1}{2}\,\overline{\frac{1}{2}}\rangle$; $\rho = \exp(\mathrm{i}\arctan(\sqrt{15} - \sqrt{12}))$, $\sigma = \exp(\mathrm{i}\arctan\sqrt{27/5})$, $\omega = \exp(2\pi\mathrm{i}/3)$

T **74**.7a Generators § **16**–7, p. 80

I		**I**	
E	$C_{2a}\,C_{2a}$	C_{36}^{+}	$C_{31}^{+}\,C_{2a}\,C_{51}^{+}\,C_{31}^{+}\,C_{2a}$
C_{51}^{+}	C_{51}^{+}	C_{37}^{+}	$C_{2a}\,C_{31}^{+}\,C_{2a}\,C_{51}^{+}$
C_{52}^{+}	$C_{51}^{+}\,C_{51}^{+}\,C_{2a}\,C_{31}^{+}$	C_{38}^{+}	$C_{2a}\,C_{51}^{+}\,C_{2a}\,C_{31}^{+}$
C_{53}^{+}	$C_{51}^{+}\,C_{2a}\,C_{31}^{+}\,C_{51}^{+}$	C_{39}^{+}	$C_{31}^{+}\,C_{51}^{+}\,C_{2a}\,C_{31}^{+}$
C_{54}^{+}	$C_{31}^{+}\,C_{51}^{+}\,C_{2a}$	$C_{3,10}^{+}$	$C_{31}^{+}\,C_{2a}\,C_{51}^{+}\,C_{2a}$
C_{55}^{+}	$C_{51}^{+}\,C_{2a}\,C_{51}^{+}\,C_{2a}$	C_{31}^{-}	$C_{31}^{+}\,C_{31}^{+}$
C_{56}^{+}	$C_{2a}\,C_{51}^{+}\,C_{2a}\,C_{51}^{+}$	C_{32}^{-}	$C_{2a}\,C_{31}^{+}\,C_{31}^{+}$
C_{51}^{-}	$C_{31}^{+}\,C_{51}^{+}\,C_{31}^{+}$	C_{33}^{-}	$C_{31}^{+}\,C_{2a}\,C_{31}^{+}$
C_{52}^{-}	$C_{2a}\,C_{51}^{+}\,C_{2a}$	C_{34}^{-}	$C_{31}^{+}\,C_{31}^{+}\,C_{2a}$
C_{53}^{-}	$C_{51}^{+}\,C_{31}^{+}\,C_{31}^{+}$	C_{35}^{-}	$C_{51}^{+}\,C_{2a}\,C_{31}^{+}\,C_{2a}$
C_{54}^{-}	$C_{2a}\,C_{31}^{+}\,C_{51}^{+}$	C_{36}^{-}	$C_{31}^{+}\,C_{51}^{+}\,C_{2a}\,C_{51}^{+}$
C_{55}^{-}	$C_{31}^{+}\,C_{31}^{+}\,C_{51}^{+}$	C_{37}^{-}	$C_{2a}\,C_{51}^{+}\,C_{51}^{+}$
C_{56}^{-}	$C_{51}^{+}\,C_{2a}\,C_{31}^{+}$	C_{38}^{-}	$C_{31}^{+}\,C_{51}^{+}\,C_{51}^{+}$
C_{51}^{2+}	$C_{51}^{+}\,C_{51}^{+}$	C_{39}^{-}	$C_{51}^{+}\,C_{51}^{+}\,C_{31}^{+}$
C_{52}^{2+}	$C_{31}^{+}\,C_{2a}\,C_{51}^{+}$	$C_{3,10}^{-}$	$C_{51}^{+}\,C_{51}^{+}\,C_{2a}$
C_{53}^{2+}	$C_{31}^{+}\,C_{2a}\,C_{31}^{+}\,C_{51}^{+}$	C_{2a}	C_{2a}
C_{54}^{2+}	$C_{51}^{+}\,C_{31}^{+}\,C_{2a}$	C_{2b}	$C_{31}^{+}\,C_{2a}\,C_{31}^{+}\,C_{31}^{+}$
C_{55}^{2+}	$C_{2a}\,C_{31}^{+}\,C_{51}^{+}\,C_{31}^{+}$	C_{2c}	$C_{31}^{+}\,C_{31}^{+}\,C_{2a}\,C_{31}^{+}$
C_{56}^{2+}	$C_{31}^{+}\,C_{51}^{+}\,C_{31}^{+}\,C_{2a}$	C_{2d}	$C_{31}^{+}\,C_{51}^{+}$
C_{51}^{2-}	$C_{51}^{+}\,C_{51}^{+}\,C_{51}^{+}$	C_{2e}	$C_{31}^{+}\,C_{2a}\,C_{51}^{+}\,C_{31}^{+}$
C_{52}^{2-}	$C_{2a}\,C_{51}^{+}\,C_{51}^{+}\,C_{2a}$	C_{2f}	$C_{51}^{+}\,C_{31}^{+}\,C_{2a}\,C_{51}^{+}$
C_{53}^{2-}	$C_{31}^{+}\,C_{31}^{+}\,C_{2a}\,C_{51}^{+}\,C_{2a}$	C_{2g}	$C_{51}^{+}\,C_{31}^{+}$
C_{54}^{2-}	$C_{2a}\,C_{51}^{+}\,C_{31}^{+}$	C_{2h}	$C_{51}^{+}\,C_{51}^{+}\,C_{31}^{+}\,C_{2a}$
C_{55}^{2-}	$C_{51}^{+}\,C_{2a}$	C_{2i}	$C_{2a}\,C_{51}^{+}\,C_{31}^{+}\,C_{31}^{+}$
C_{56}^{2-}	$C_{2a}\,C_{51}^{+}$	C_{2j}	$C_{2a}\,C_{51}^{+}\,C_{31}^{+}\,C_{2a}$
C_{31}^{+}	C_{31}^{+}	C_{2k}	$C_{51}^{+}\,C_{31}^{+}\,C_{31}^{+}\,C_{2a}$
C_{32}^{+}	$C_{31}^{+}\,C_{2a}$	C_{2l}	$C_{2a}\,C_{51}^{+}\,C_{51}^{+}\,C_{31}^{+}$
C_{33}^{+}	$C_{2a}\,C_{31}^{+}\,C_{2a}$	C_{2m}	$C_{2a}\,C_{31}^{+}\,C_{51}^{+}\,C_{2a}$
C_{34}^{+}	$C_{2a}\,C_{31}^{+}$	C_{2n}	$C_{31}^{+}\,C_{31}^{+}\,C_{2a}\,C_{51}^{+}$
C_{35}^{+}	$C_{51}^{+}\,C_{2a}\,C_{51}^{+}$	C_{2o}	$C_{31}^{+}\,C_{51}^{+}\,C_{2a}\,C_{51}^{+}\,C_{2a}$

T **74**.7b Matrices for the generators

§ **16**–7, pp. 77, 80

I	C_{51}^{+}	C_{31}^{+}	C_{2a}
T_1	$\frac{1}{2}\begin{bmatrix} \mathrm{g}_- & \bar{\mathrm{i}} & \mathrm{i}\,\overline{\mathrm{g}}_+ \\ \bar{\mathrm{i}} & \mathrm{g}_+ & \overline{\mathrm{g}}_- \\ \mathrm{i}\overline{\mathrm{g}}_+ & \overline{\mathrm{g}}_- & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 & 0 \\ 0 & \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 & 0 \\ 0 & \bar{1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$
T_2	$\frac{1}{2}\begin{bmatrix} \overline{\mathrm{g}}_+ & \bar{\mathrm{i}} & \mathrm{i}\,\mathrm{g}_- \\ \bar{\mathrm{i}} & \overline{\mathrm{g}}_- & \mathrm{g}_+ \\ \mathrm{i}\,\mathrm{g}_- & \mathrm{g}_+ & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 & 0 \\ 0 & \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} \bar{1} & 0 & 0 \\ 0 & \bar{1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$
F	$\frac{1}{4}\begin{bmatrix} \bar{1} & \bar{\mathrm{t}} & \mathrm{i}\,\bar{\mathrm{t}} & \mathrm{i}\,\bar{\mathrm{t}} \\ \bar{\mathrm{t}} & \bar{1} & 3\mathrm{i} & \bar{\mathrm{i}} \\ \mathrm{i}\,\bar{\mathrm{t}} & 3\mathrm{i} & 1 & 1 \\ \mathrm{i}\,\bar{\mathrm{t}} & \bar{\mathrm{i}} & 1 & \bar{3} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{\mathrm{i}} \\ 0 & \bar{\mathrm{i}} & 0 & 0 \\ 0 & 0 & \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \bar{1} & 0 & 0 \\ 0 & 0 & \bar{1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
H	$\frac{1}{2}\begin{bmatrix} 0 & \lambda^2\omega^* & \overline{\lambda} & \mathrm{i}\lambda\overline{\omega}^* & \mathrm{i}\lambda\overline{\omega} \\ (\lambda^*)^2\omega & 0 & \overline{\lambda}^* & \mathrm{i}\lambda^*\overline{\omega} & \mathrm{i}\lambda^*\overline{\omega}^* \\ \overline{\lambda}^* & \overline{\lambda} & 1 & 0 & \mathrm{i} \\ \mathrm{i}\lambda^*\overline{\omega} & \mathrm{i}\lambda\overline{\omega}^* & 0 & \bar{1} & \bar{1} \\ \mathrm{i}\lambda^*\overline{\omega}^* & \mathrm{i}\lambda\overline{\omega} & \mathrm{i} & \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} \omega & 0 & 0 & 0 & 0 \\ 0 & \omega^* & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{\mathrm{i}} \\ 0 & 0 & \bar{\mathrm{i}} & 0 & 0 \\ 0 & 0 & 0 & \bar{1} & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \bar{1} & 0 & 0 \\ 0 & 0 & 0 & \bar{1} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
$E_{1/2}$	$\frac{1}{2}\begin{bmatrix} \mathrm{g}_+ - \mathrm{i} & \mathrm{i}\,\overline{\mathrm{g}}_- \\ \mathrm{i}\,\overline{\mathrm{g}}_- & \mathrm{g}_+ + \mathrm{i} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \eta^* & \overline{\eta} \\ \eta^* & \eta \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
$E_{7/2}$	$\frac{1}{2}\begin{bmatrix} \overline{\mathrm{g}}_- - \mathrm{i} & \mathrm{i}\,\mathrm{g}_+ \\ \mathrm{i}\,\mathrm{g}_+ & \overline{\mathrm{g}}_- + \mathrm{i} \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \eta^* & \overline{\eta} \\ \eta^* & \eta \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} \\ \bar{\mathrm{i}} & 0 \end{bmatrix}$
$F_{3/2}$	$\frac{1}{\sqrt{8}}\begin{bmatrix} \eta & \mathrm{i}\,\overline{\mathrm{s}}_- & \mu^* & \overline{\lambda}^* \\ \mathrm{i}\,\mathrm{s}_+ & \eta^* & \overline{\lambda}^* & \nu \\ \nu^* & \lambda & \eta & \mathrm{i}\,\mathrm{s}_+ \\ \lambda & \mu & \mathrm{i}\,\overline{\mathrm{s}}_- & \eta^* \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \delta & \mathrm{i}\overline{\delta} & 0 & 0 \\ \delta & \mathrm{i}\delta & 0 & 0 \\ 0 & 0 & \mathrm{i}\overline{\delta}^* & \overline{\delta}^* \\ 0 & 0 & \mathrm{i}\overline{\delta}^* & \delta^* \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} & 0 & 0 \\ \bar{\mathrm{i}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{\mathrm{i}} \\ 0 & 0 & \bar{\mathrm{i}} & 0 \end{bmatrix}$
$I_{5/2}$	$\frac{1}{6}\begin{bmatrix} \overline{\xi}^* & \bar{\mathrm{i}} & \mathrm{i}\overline{\sigma}^* & \mathrm{s}_+\overline{\theta} & \overline{\sigma} & \mathrm{s}_-\overline{\theta}^* \\ \bar{\mathrm{i}} & \overline{\xi} & \mathrm{s}_-\theta & \overline{\sigma}^* & \mathrm{s}_+\theta^* & \mathrm{i}\sigma \\ \overline{\sigma} & \mathrm{s}_+\theta^* & \sqrt{2}\,\overline{\eta} & \sqrt{2}\,\mathrm{i}\,\mathrm{s}_- & \overline{\zeta}^* & \overline{\rho}^* \\ \mathrm{s}_-\overline{\theta}^* & \mathrm{i}\sigma & \sqrt{2}\,\mathrm{i}\,\overline{\mathrm{s}}_+ & \sqrt{2}\,\overline{\eta}^* & \overline{\rho}^* & \overline{\epsilon}^* \\ \mathrm{i}\overline{\sigma}^* & \mathrm{s}_-\theta & \overline{\epsilon} & \rho & \sqrt{2}\,\overline{\eta} & \sqrt{2}\,\mathrm{i}\,\overline{\mathrm{s}}_+ \\ \mathrm{s}_+\overline{\theta} & \overline{\sigma}^* & \rho & \overline{\zeta} & \sqrt{2}\,\mathrm{i}\,\mathrm{s}_- & \sqrt{2}\,\overline{\eta}^* \end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix} \eta^* & \overline{\eta} & 0 & 0 & 0 & 0 \\ \eta^* & \eta & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta & \mathrm{i}\overline{\delta} & 0 & 0 \\ 0 & 0 & \delta & \mathrm{i}\delta & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathrm{i}\overline{\delta}^* & \overline{\delta}^* \\ 0 & 0 & 0 & 0 & \mathrm{i}\overline{\delta}^* & \delta^* \end{bmatrix}$	$\begin{bmatrix} 0 & \bar{\mathrm{i}} & 0 & 0 & 0 & 0 \\ \bar{\mathrm{i}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{\mathrm{i}} & 0 & 0 \\ 0 & 0 & \bar{\mathrm{i}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{\mathrm{i}} \\ 0 & 0 & 0 & 0 & \bar{\mathrm{i}} & 0 \end{bmatrix}$

$\delta = \exp(2\pi\mathrm{i}/24)$, $\epsilon = (8+3\sqrt{3})^{1/2}\ \exp[\mathrm{i}\arctan\{(-6416+13216\sqrt{3}+2352\sqrt{5}-5075\sqrt{15})/13189\}]$,
$\zeta = (8-3\sqrt{3})^{1/2}\ \exp[\mathrm{i}\arctan\{(6416+13216\sqrt{3}-2352\sqrt{5}-5075\sqrt{15})/13189\}]$, $\eta = \exp(2\pi\mathrm{i}/8)$,
$\theta = \sqrt{5}\ \exp[\mathrm{i}\arctan\{(4-\sqrt{5})/\sqrt{3}\}]$, $\lambda = \exp(\mathrm{i}\arctan\sqrt{5/3})$,
$\mu = (4+\sqrt{3})^{1/2}\exp[\mathrm{i}\arctan\{(84-16\sqrt{3}+72\sqrt{5}+5\sqrt{15})/131\}]$,
$\nu = (4-\sqrt{3})^{1/2}\exp[\mathrm{i}\arctan\{(84+16\sqrt{3}+72\sqrt{5}-5\sqrt{15})/131\}]$,
$\xi = \sqrt{5}\ \exp(\mathrm{i}\arctan 2)$, $\rho = \sqrt{7}\ \exp[\mathrm{i}\arctan\{(80\sqrt{3}+49\sqrt{15})/57\}]$,
$\sigma = \sqrt{5}\ \exp\{\mathrm{i}\arctan(28+16\sqrt{3}+12\sqrt{5}+7\sqrt{15})\}$, $\omega = \exp(2\pi\mathrm{i}/3)$;
$\mathrm{g}_\pm = (\sqrt{5}\pm 1)/2$, $\mathrm{s}_\pm = (\sqrt{3}\pm 1)/\sqrt{2}$, $\mathrm{t} = \sqrt{5}$.

T **74**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{I}$	A	T_1	T_2	F	H
A	A	T_1	T_2	F	H
T_1		$A \oplus \{T_1\} \oplus H$	$F \oplus H$	$T_2 \oplus F \oplus H$	$T_1 \oplus T_2 \oplus F \oplus H$
T_2			$A \oplus \{T_2\} \oplus H$	$T_1 \oplus F \oplus H$	$T_1 \oplus T_2 \oplus F \oplus H$
F				$A \oplus \{T_1\} \oplus \{T_2\} \oplus F \oplus H$	$T_1 \oplus T_2 \oplus F \oplus 2H$
H					$A \oplus \{T_1\} \oplus \{T_2\} \oplus F \oplus \{F\} \oplus 2H$

⇛

T **74**.8 Direct products of representations *(cont.)*

$\mathbf{I}$	$E_{1/2}$	$E_{7/2}$	$F_{3/2}$	$I_{5/2}$
A	$E_{1/2}$	$E_{7/2}$	$F_{3/2}$	$I_{5/2}$
T_1	$E_{1/2} \oplus F_{3/2}$	$I_{5/2}$	$E_{1/2} \oplus F_{3/2} \oplus I_{5/2}$	$E_{7/2} \oplus F_{3/2} \oplus 2I_{5/2}$
T_2	$I_{5/2}$	$E_{7/2} \oplus F_{3/2}$	$E_{7/2} \oplus F_{3/2} \oplus I_{5/2}$	$E_{1/2} \oplus F_{3/2} \oplus 2I_{5/2}$
F	$E_{7/2} \oplus I_{5/2}$	$E_{1/2} \oplus I_{5/2}$	$F_{3/2} \oplus 2I_{5/2}$	$E_{1/2} \oplus E_{7/2} \oplus 2F_{3/2} \oplus 2I_{5/2}$
H	$F_{3/2} \oplus I_{5/2}$	$F_{3/2} \oplus I_{5/2}$	$E_{1/2} \oplus E_{7/2} \oplus F_{3/2} \oplus 2I_{5/2}$	$E_{1/2} \oplus E_{7/2} \oplus 2F_{3/2} \oplus 3I_{5/2}$
$E_{1/2}$	$\{A\} \oplus T_1$	F	$T_1 \oplus H$	$T_2 \oplus F \oplus H$
$E_{7/2}$		$\{A\} \oplus T_2$	$T_2 \oplus H$	$T_1 \oplus F \oplus H$
$F_{3/2}$			$\{A\} \oplus T_1 \oplus T_2 \oplus F \oplus \{H\}$	$T_1 \oplus T_2 \oplus 2F \oplus 2H$
$I_{5/2}$				$\{A\} \oplus 2T_1 \oplus 2T_2 \oplus F \oplus \{F\} \oplus H \oplus 2\{H\}$

T **74**.9 Subduction (descent of symmetry)

§ **16**–9, p. 82

$\mathbf{I}$	$\mathbf{T}$	$(\mathbf{D}_5)$	$(\mathbf{D}_3)$	$\mathbf{D}_2$
A	A	A_1	A_1	A
T_1	T	$A_2 \oplus E_1$	$A_2 \oplus E$	$B_1 \oplus B_2 \oplus B_3$
T_2	T	$A_2 \oplus E_2$	$A_2 \oplus E$	$B_1 \oplus B_2 \oplus B_3$
F	$A \oplus T$	$E_1 \oplus E_2$	$A_1 \oplus A_2 \oplus E$	$A \oplus B_1 \oplus B_2 \oplus B_3$
H	${}^1E \oplus {}^2E \oplus T$	$A_1 \oplus E_1 \oplus E_2$	$A_1 \oplus 2E$	$2A \oplus B_1 \oplus B_2 \oplus B_3$
$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{7/2}$	$E_{1/2}$	$E_{3/2}$	$E_{1/2}$	$E_{1/2}$
$F_{3/2}$	${}^1F_{3/2} \oplus {}^2F_{3/2}$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}$	$2E_{1/2}$
$I_{5/2}$	$E_{1/2} \oplus {}^1F_{3/2} \oplus {}^2F_{3/2}$	$E_{1/2} \oplus E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}$	$2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}$	$3E_{1/2}$

⇛

T **74**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{I}$	$(\mathbf{C}_5)$	$(\mathbf{C}_3)$	$\mathbf{C}_2$
A	A	A	A
T_1	$A \oplus {}^1E_1 \oplus {}^2E_1$	$A \oplus {}^1E \oplus {}^2E$	$A \oplus 2B$
T_2	$A \oplus {}^1E_2 \oplus {}^2E_2$	$A \oplus {}^1E \oplus {}^2E$	$A \oplus 2B$
F	${}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2$	$2A \oplus {}^1E \oplus {}^2E$	$2A \oplus 2B$
H	$A \oplus {}^1E_1 \oplus {}^2E_1 \oplus {}^1E_2 \oplus {}^2E_2$	$A \oplus 2\,{}^1E \oplus 2\,{}^2E$	$3A \oplus 2B$
$E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$E_{7/2}$	${}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2}$
$F_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2} \oplus 2A_{3/2}$	$2\,{}^1E_{1/2} \oplus 2\,{}^2E_{1/2}$
$I_{5/2}$	${}^1E_{1/2} \oplus {}^2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2} \oplus 2A_{5/2}$	$2\,{}^1E_{1/2} \oplus 2\,{}^2E_{1/2} \oplus 2A_{3/2}$	$3\,{}^1E_{1/2} \oplus 3\,{}^2E_{1/2}$

T **74**.10 Subduction from O(3) § **16**–10, p. 82

j	**I**
$30n$	$(n+1)\,A \oplus n\,(3T_1 \oplus 3T_2 \oplus 4F \oplus 5H)$
$30n+1$	$n\,(A \oplus 2T_1 \oplus 3T_2 \oplus 4F \oplus 5H) \oplus (n+1)\,T_1$
$30n+2$	$n\,(A \oplus 3T_1 \oplus 3T_2 \oplus 4F \oplus 4H) \oplus (n+1)\,H$
$30n+3$	$n\,(A \oplus 3T_1 \oplus 2T_2 \oplus 3F \oplus 5H) \oplus (n+1)(T_2 \oplus F)$
$30n+4$	$n\,(A \oplus 3T_1 \oplus 3T_2 \oplus 3F \oplus 4H) \oplus (n+1)(F \oplus H)$
$30n+5$	$n\,(A \oplus 2T_1 \oplus 2T_2 \oplus 4F \oplus 4H) \oplus (n+1)(T_1 \oplus T_2 \oplus H)$
$30n+6$	$(n+1)(A \oplus T_1 \oplus F \oplus H) \oplus n\,(2T_1 \oplus 3T_2 \oplus 3F \oplus 4H)$
$30n+7$	$n\,(A \oplus 2T_1 \oplus 2T_2 \oplus 3F \oplus 4H) \oplus (n+1)(T_1 \oplus T_2 \oplus F \oplus H)$
$30n+8$	$n\,(A \oplus 3T_1 \oplus 2T_2 \oplus 3F \oplus 3H) \oplus (n+1)(T_2 \oplus F \oplus 2H)$
$30n+9$	$n\,(A \oplus 2T_1 \oplus 2T_2 \oplus 2F \oplus 4H) \oplus (n+1)(T_1 \oplus T_2 \oplus 2F \oplus H)$
$30n+10$	$(n+1)(A \oplus T_1 \oplus T_2 \oplus F \oplus 2H) \oplus n\,(2T_1 \oplus 2T_2 \oplus 3F \oplus 3H)$
$30n+11$	$n\,(A \oplus T_1 \oplus 2T_2 \oplus 3F \oplus 3H) \oplus (n+1)(2T_1 \oplus T_2 \oplus F \oplus 2H)$
$30n+12$	$(n+1)(A \oplus T_1 \oplus T_2 \oplus 2F \oplus 2H) \oplus n\,(2T_1 \oplus 2T_2 \oplus 2F \oplus 3H)$
$30n+13$	$n\,(A \oplus 2T_1 \oplus T_2 \oplus 2F \oplus 3H) \oplus (n+1)(T_1 \oplus 2T_2 \oplus 2F \oplus 2H)$
$30n+14$	$n\,(A \oplus 2T_1 \oplus 2T_2 \oplus 2F \oplus 2H) \oplus (n+1)(T_1 \oplus T_2 \oplus 2F \oplus 3H)$
$30n+15$	$(n+1)(A \oplus 2T_1 \oplus 2T_2 \oplus 2F \oplus 2H) \oplus n\,(T_1 \oplus T_2 \oplus 2F \oplus 3H)$
$30n+16$	$(n+1)(A \oplus 2T_1 \oplus T_2 \oplus 2F \oplus 3H) \oplus n\,(T_1 \oplus 2T_2 \oplus 2F \oplus 2H)$
$30n+17$	$n\,(A \oplus T_1 \oplus T_2 \oplus 2F \oplus 2H) \oplus (n+1)(2T_1 \oplus 2T_2 \oplus 2F \oplus 3H)$
$30n+18$	$(n+1)(A \oplus T_1 \oplus 2T_2 \oplus 3F \oplus 3H) \oplus n\,(2T_1 \oplus T_2 \oplus F \oplus 2H)$
$30n+19$	$n\,(A \oplus T_1 \oplus T_2 \oplus F \oplus 2H) \oplus (n+1)(2T_1 \oplus 2T_2 \oplus 3F \oplus 3H)$
$30n+20$	$(n+1)(A \oplus 2T_1 \oplus 2T_2 \oplus 2F \oplus 4H) \oplus n\,(T_1 \oplus T_2 \oplus 2F \oplus H)$
$30n+21$	$(n+1)(A \oplus 3T_1 \oplus 2T_2 \oplus 3F \oplus 3H) \oplus n\,(T_2 \oplus F \oplus 2H)$
$30n+22$	$(n+1)(A \oplus 2T_1 \oplus 2T_2 \oplus 3F \oplus 4H) \oplus n\,(T_1 \oplus T_2 \oplus F \oplus H)$
$30n+23$	$n\,(A \oplus T_1 \oplus F \oplus H) \oplus (n+1)(2T_1 \oplus 3T_2 \oplus 3F \oplus 4H)$
$30n+24$	$(n+1)(A \oplus 2T_1 \oplus 2T_2 \oplus 4F \oplus 4H) \oplus n\,(T_1 \oplus T_2 \oplus H)$
$30n+25$	$(n+1)(A \oplus 3T_1 \oplus 3T_2 \oplus 3F \oplus 4H) \oplus n\,(F \oplus H)$
$30n+26$	$(n+1)(A \oplus 3T_1 \oplus 2T_2 \oplus 3F \oplus 5H) \oplus n\,(T_2 \oplus F)$
$30n+27$	$(n+1)(A \oplus 3T_1 \oplus 3T_2 \oplus 4F \oplus 4H) \oplus n\,H$
$30n+28$	$(n+1)(A \oplus 2T_1 \oplus 3T_2 \oplus 4F \oplus 5H) \oplus n\,T_1$
$30n+29$	$n\,A \oplus (n+1)(3T_1 \oplus 3T_2 \oplus 4F \oplus 5H)$

$n = 0, 1, 2, \ldots$ $\twoheadrightarrow$

T **74**.10 Subduction from O(3) *(cont.)*

j	**I**
$15n+\frac{1}{2}$	$(n+1)\,E_{1/2}\oplus n\,(E_{7/2}\oplus 2F_{3/2}\oplus 3I_{5/2})$
$15n+\frac{3}{2}$	$n\,(E_{1/2}\oplus E_{7/2}\oplus F_{3/2}\oplus 3I_{5/2})\oplus(n+1)\,F_{3/2}$
$15n+\frac{5}{2}$	$n\,(E_{1/2}\oplus E_{7/2}\oplus 2F_{3/2}\oplus 2I_{5/2})\oplus(n+1)\,I_{5/2}$
$15n+\frac{7}{2}$	$n\,(E_{1/2}\oplus 2F_{3/2}\oplus 2I_{5/2})\oplus(n+1)(E_{7/2}\oplus I_{5/2})$
$15n+\frac{9}{2}$	$n\,(E_{1/2}\oplus E_{7/2}\oplus F_{3/2}\oplus 2I_{5/2})\oplus(n+1)(F_{3/2}\oplus I_{5/2})$
$15n+\frac{11}{2}$	$(n+1)(E_{1/2}\oplus F_{3/2}\oplus I_{5/2})\oplus n\,(E_{7/2}\oplus F_{3/2}\oplus 2I_{5/2})$
$15n+\frac{13}{2}$	$(n+1)(E_{1/2}\oplus E_{7/2}\oplus F_{3/2}\oplus I_{5/2})\oplus n\,(F_{3/2}\oplus 2I_{5/2})$
$15n+\frac{15}{2}$	$n\,(E_{1/2}\oplus E_{7/2}\oplus F_{3/2}\oplus I_{5/2})\oplus(n+1)(F_{3/2}\oplus 2I_{5/2})$
$15n+\frac{17}{2}$	$n\,(E_{1/2}\oplus F_{3/2}\oplus I_{5/2})\oplus(n+1)(E_{7/2}\oplus F_{3/2}\oplus 2I_{5/2})$
$15n+\frac{19}{2}$	$(n+1)(E_{1/2}\oplus E_{7/2}\oplus F_{3/2}\oplus 2I_{5/2})\oplus n\,(F_{3/2}\oplus I_{5/2})$
$15n+\frac{21}{2}$	$(n+1)(E_{1/2}\oplus 2F_{3/2}\oplus 2I_{5/2})\oplus n\,(E_{7/2}\oplus I_{5/2})$
$15n+\frac{23}{2}$	$(n+1)(E_{1/2}\oplus E_{7/2}\oplus 2F_{3/2}\oplus 2I_{5/2})\oplus n\,I_{5/2}$
$15n+\frac{25}{2}$	$(n+1)(E_{1/2}\oplus E_{7/2}\oplus F_{3/2}\oplus 3I_{5/2})\oplus n\,F_{3/2}$
$15n+\frac{27}{2}$	$n\,E_{1/2}\oplus(n+1)(E_{7/2}\oplus 2F_{3/2}\oplus 3I_{5/2})$
$15n+\frac{29}{2}$	$(n+1)(E_{1/2}\oplus E_{7/2}\oplus 2F_{3/2}\oplus 3I_{5/2})$

$n=0,1,2,\ldots$

(1) Product forms: $\mathbf{I} \otimes \mathbf{C}_i$.

(2) Group chains: $\mathbf{I}_h \supset \underline{\mathbf{I}}$, $\mathbf{I}_h \supset \mathbf{T}_h$, $\mathbf{I}_h \supset (\mathbf{D}_{5d})$, $\mathbf{I}_h \supset (\mathbf{D}_{3d})$.

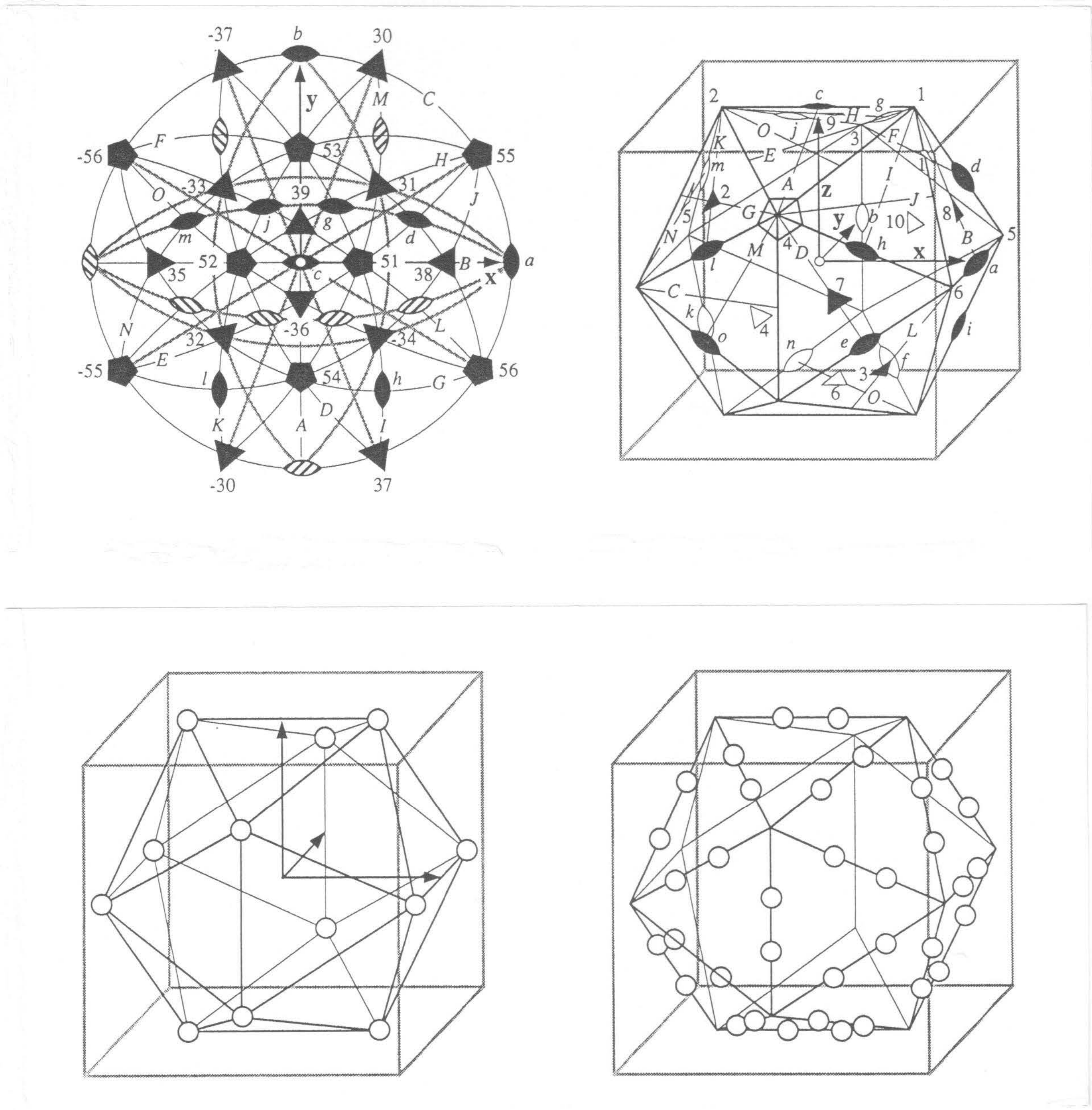
(3) Operations of G: E, $(C_{51}^+, C_{52}^+, C_{53}^+, C_{54}^+, C_{55}^+, C_{56}^+, C_{51}^-, C_{52}^-, C_{53}^-, C_{54}^-, C_{55}^-, C_{56}^-)$,
$(C_{51}^{2+}, C_{52}^{2+}, C_{53}^{2+}, C_{54}^{2+}, C_{55}^{2+}, C_{56}^{2+}, C_{51}^{2-}, C_{52}^{2-}, C_{53}^{2-}, C_{54}^{2-}, C_{55}^{2-}, C_{56}^{2-})$,
$(C_{31}^+, C_{32}^+, C_{33}^+, C_{34}^+, C_{35}^+, C_{36}^+, C_{37}^+, C_{38}^+, C_{39}^+, C_{3,10}^+,$
$C_{31}^-, C_{32}^-, C_{33}^-, C_{34}^-, C_{35}^-, C_{36}^-, C_{37}^-, C_{38}^-, C_{39}^-, C_{3,10}^-)$,
$(C_{2a}, C_{2b}, C_{2c}, C_{2d}, C_{2e}, C_{2f}, C_{2g}, C_{2h}, C_{2i}, C_{2j}, C_{2k}, C_{2l}, C_{2m}, C_{2n}, C_{2o})$,
i, $(S_{10,1}^{3-}, S_{10,2}^{3-}, S_{10,3}^{3-}, S_{10,4}^{3-}, S_{10,5}^{3-}, S_{10,6}^{3-}, S_{10,1}^{3+}, S_{10,2}^{3+}, S_{10,3}^{3+}, S_{10,4}^{3+}, S_{10,5}^{3+}, S_{10,6}^{3+})$,
$(S_{10,1}^-, S_{10,2}^-, S_{10,3}^-, S_{10,4}^-, S_{10,5}^-, S_{10,6}^-, S_{10,1}^+, S_{10,2}^+, S_{10,3}^+, S_{10,4}^+, S_{10,5}^+, S_{10,6}^+)$,
$(S_{61}^-, S_{62}^-, S_{63}^-, S_{64}^-, S_{65}^-, S_{66}^-, S_{67}^-, S_{68}^-, S_{69}^-, S_{6,10}^-,$
$S_{61}^+, S_{62}^+, S_{63}^+, S_{64}^+, S_{65}^+, S_{66}^+, S_{67}^+, S_{68}^+, S_{69}^+, S_{6,10}^+)$,
$(\sigma_a, \sigma_b, \sigma_c, \sigma_d, \sigma_e, \sigma_f, \sigma_g, \sigma_h, \sigma_i, \sigma_j, \sigma_k, \sigma_l, \sigma_m, \sigma_n, \sigma_o)$.

(4) Operations of $\widetilde{G}$: E, $\widetilde{E}$, $(C_{51}^+, C_{52}^+, C_{53}^+, C_{54}^+, C_{55}^+, C_{56}^+, C_{51}^-, C_{52}^-, C_{53}^-, C_{54}^-, C_{55}^-, C_{56}^-)$,
$(\widetilde{C}_{51}^+, \widetilde{C}_{52}^+, \widetilde{C}_{53}^+, \widetilde{C}_{54}^+, \widetilde{C}_{55}^+, \widetilde{C}_{56}^+, \widetilde{C}_{51}^-, \widetilde{C}_{52}^-, \widetilde{C}_{53}^-, \widetilde{C}_{54}^-, \widetilde{C}_{55}^-, \widetilde{C}_{56}^-)$,
$(C_{51}^{2+}, C_{52}^{2+}, C_{53}^{2+}, C_{54}^{2+}, C_{55}^{2+}, C_{56}^{2+}, C_{51}^{2-}, C_{52}^{2-}, C_{53}^{2-}, C_{54}^{2-}, C_{55}^{2-}, C_{56}^{2-})$,
$(\widetilde{C}_{51}^{2+}, \widetilde{C}_{52}^{2+}, \widetilde{C}_{53}^{2+}, \widetilde{C}_{54}^{2+}, \widetilde{C}_{55}^{2+}, \widetilde{C}_{56}^{2+}, \widetilde{C}_{51}^{2-}, \widetilde{C}_{52}^{2-}, \widetilde{C}_{53}^{2-}, \widetilde{C}_{54}^{2-}, \widetilde{C}_{55}^{2-}, \widetilde{C}_{56}^{2-})$,
$(C_{31}^+, C_{32}^+, C_{33}^+, C_{34}^+, C_{35}^+, C_{36}^+, C_{37}^+, C_{38}^+, C_{39}^+, C_{3,10}^+,$
$C_{31}^-, C_{32}^-, C_{33}^-, C_{34}^-, C_{35}^-, C_{36}^-, C_{37}^-, C_{38}^-, C_{39}^-, C_{3,10}^-)$,
$(\widetilde{C}_{31}^+, \widetilde{C}_{32}^+, \widetilde{C}_{33}^+, \widetilde{C}_{34}^+, \widetilde{C}_{35}^+, \widetilde{C}_{36}^+, \widetilde{C}_{37}^+, \widetilde{C}_{38}^+, \widetilde{C}_{39}^+, \widetilde{C}_{3,10}^+,$
$\widetilde{C}_{31}^-, \widetilde{C}_{32}^-, \widetilde{C}_{33}^-, \widetilde{C}_{34}^-, \widetilde{C}_{35}^-, \widetilde{C}_{36}^-, \widetilde{C}_{37}^-, \widetilde{C}_{38}^-, \widetilde{C}_{39}^-, \widetilde{C}_{3,10}^-)$,
$(C_{2a}, C_{2b}, C_{2c}, C_{2d}, C_{2e}, C_{2f}, C_{2g}, C_{2h}, C_{2i}, C_{2j}, C_{2k}, C_{2l}, C_{2m}, C_{2n}, C_{2o},$
$\widetilde{C}_{2a}, \widetilde{C}_{2b}, \widetilde{C}_{2c}, \widetilde{C}_{2d}, \widetilde{C}_{2e}, \widetilde{C}_{2f}, \widetilde{C}_{2g}, \widetilde{C}_{2h}, \widetilde{C}_{2i}, \widetilde{C}_{2j}, \widetilde{C}_{2k}, \widetilde{C}_{2l}, \widetilde{C}_{2m}, \widetilde{C}_{2n}, \widetilde{C}_{2o})$,
i, $\widetilde{\imath}$, $(S_{10,1}^{3-}, S_{10,2}^{3-}, S_{10,3}^{3-}, S_{10,4}^{3-}, S_{10,5}^{3-}, S_{10,6}^{3-}, S_{10,1}^{3+}, S_{10,2}^{3+}, S_{10,3}^{3+}, S_{10,4}^{3+}, S_{10,5}^{3+}, S_{10,6}^{3+})$,
$(\widetilde{S}_{10,1}^{3-}, \widetilde{S}_{10,2}^{3-}, \widetilde{S}_{10,3}^{3-}, \widetilde{S}_{10,4}^{3-}, \widetilde{S}_{10,5}^{3-}, \widetilde{S}_{10,6}^{3-}, \widetilde{S}_{10,1}^{3+}, \widetilde{S}_{10,2}^{3+}, \widetilde{S}_{10,3}^{3+}, \widetilde{S}_{10,4}^{3+}, \widetilde{S}_{10,5}^{3+}, \widetilde{S}_{10,6}^{3+})$,
$(S_{10,1}^-, S_{10,2}^-, S_{10,3}^-, S_{10,4}^-, S_{10,5}^-, S_{10,6}^-, S_{10,1}^+, S_{10,2}^+, S_{10,3}^+, S_{10,4}^+, S_{10,5}^+, S_{10,6}^+)$,
$(\widetilde{S}_{10,1}^-, \widetilde{S}_{10,2}^-, \widetilde{S}_{10,3}^-, \widetilde{S}_{10,4}^-, \widetilde{S}_{10,5}^-, \widetilde{S}_{10,6}^-, \widetilde{S}_{10,1}^+, \widetilde{S}_{10,2}^+, \widetilde{S}_{10,3}^+, \widetilde{S}_{10,4}^+, \widetilde{S}_{10,5}^+, \widetilde{S}_{10,6}^+)$,
$(S_{61}^-, S_{62}^-, S_{63}^-, S_{64}^-, S_{65}^-, S_{66}^-, S_{67}^-, S_{68}^-, S_{69}^-, S_{6,10}^-,$
$S_{61}^+, S_{62}^+, S_{63}^+, S_{64}^+, S_{65}^+, S_{66}^+, S_{67}^+, S_{68}^+, S_{69}^+, S_{6,10}^+)$,
$(\widetilde{S}_{61}^-, \widetilde{S}_{62}^-, \widetilde{S}_{63}^-, \widetilde{S}_{64}^-, \widetilde{S}_{65}^-, \widetilde{S}_{66}^-, \widetilde{S}_{67}^-, \widetilde{S}_{68}^-, \widetilde{S}_{69}^-, \widetilde{S}_{6,10}^-,$
$\widetilde{S}_{61}^+, \widetilde{S}_{62}^+, \widetilde{S}_{63}^+, \widetilde{S}_{64}^+, \widetilde{S}_{65}^+, \widetilde{S}_{66}^+, \widetilde{S}_{67}^+, \widetilde{S}_{68}^+, \widetilde{S}_{69}^+, \widetilde{S}_{6,10}^+)$,
$(\sigma_a, \sigma_b, \sigma_c, \sigma_d, \sigma_e, \sigma_f, \sigma_g, \sigma_h, \sigma_i, \sigma_j, \sigma_k, \sigma_l, \sigma_m, \sigma_n, \sigma_o,$
$\widetilde{\sigma}_a, \widetilde{\sigma}_b, \widetilde{\sigma}_c, \widetilde{\sigma}_d, \widetilde{\sigma}_e, \widetilde{\sigma}_f, \widetilde{\sigma}_g, \widetilde{\sigma}_h, \widetilde{\sigma}_i, \widetilde{\sigma}_j, \widetilde{\sigma}_k, \widetilde{\sigma}_l, \widetilde{\sigma}_m, \widetilde{\sigma}_n, \widetilde{\sigma}_o)$.

(5) Classes and representations: $|r| = 8$, $|\mathsf{i}| = 2$, $|I| = 10$, $|\widetilde{I}| = 8$.

(6) Subduction: $\mathbf{D}_{3d}$ $(E, C_{31}^+, C_{31}^-, C_{2f}, C_{2m}, C_{2h}, i, S_{61}^-, S_{61}^+, \sigma_f, \sigma_m, \sigma_h)$,
$\mathbf{D}_{3d}$ $(E, C_{32}^+, C_{32}^-, C_{2e}, C_{2g}, C_{2k}, i, S_{62}^-, S_{62}^+, \sigma_e, \sigma_g, \sigma_k)$,
$\mathbf{D}_{3d}$ $(E, C_{33}^+, C_{33}^-, C_{2d}, C_{2l}, C_{2n}, i, S_{63}^-, S_{63}^+, \sigma_d, \sigma_l, \sigma_n)$,
$\mathbf{D}_{3d}$ $(E, C_{34}^+, C_{34}^-, C_{2i}, C_{2o}, C_{2j}, i, S_{64}^-, S_{64}^+, \sigma_i, \sigma_o, \sigma_j)$,
$\mathbf{D}_{5d}$ $(E, C_{51}^+, C_{51}^-, C_{51}^{2+}, C_{51}^{2-}, C_{2b}, C_{2m}, C_{2l}, C_{2e}, C_{2i},$
$i, S_{10,1}^{3-}, S_{10,1}^{3+}, S_{10,1}^-, S_{10,1}^+, \sigma_b, \sigma_m, \sigma_l, \sigma_e, \sigma_i)$,
$\mathbf{D}_{5d}$ $(E, C_{52}^+, C_{52}^-, C_{52}^{2+}, C_{52}^{2-}, C_{2b}, C_{2k}, C_{2o}, C_{2h}, C_{2d},$
$i, S_{10,2}^{3-}, S_{10,2}^{3+}, S_{10,2}^-, S_{10,2}^+, \sigma_b, \sigma_k, \sigma_o, \sigma_h, \sigma_d)$,
as well as all their subgroups $\mathbf{D}_3$, $\mathbf{D}_5$, $\mathbf{C}_{3v}$, and $\mathbf{C}_{5v}$.

F **75** See Chapter **15**, p. 65



$5i = C_{5i}^{+}, \quad -5i = C_{5i}^{-}, \quad i = 1, 2, \ldots, 6.$
$r = C_{2r}, \quad r = a, b, \ldots, o.$

$3i = C_{3i}^{+}, \quad -3i = C_{3i}^{-}, \quad i = 1, 2, \ldots, 10.$
$R = \sigma_r, \quad r = a, b, \ldots, o.$

The poles of the operations C_{5i}^{2+}, C_{5i}^{2-} coincide respectively with those of C_{5i}^{+}, C_{5i}^{-} and are not identified in the figures. A pole of a proper operation g listed above is also the pole of an improper operation gi which may be identified from the second column of T **75**.1 and the row corresponding to g.
In the first three-dimensional figure the symmetry elements are identified as above, except that in all cases the order of the rotation axis is left implicit, to be read from the position of the symbol. For simplicity of the figure, only one five-fold axis is identified by a pentagon which has been left open. No antipoles are shown and the open digons and triangles correspond to symmetry elements in the back of the icosahedron. In the third three-dimensional figure the disposition of particles is shown only on the faces at the front of the figure.

Examples: Buckminsterfullerene C_{60} (see the third three-dimensional figure).

T **75**.0 Subgroup elements § **16**–0, p. 68

$\mathbf{I}_h$	$\mathbf{I}$	$\mathbf{T}_h$	$\mathbf{T}$	$\mathbf{C}_{2h}$	$\mathbf{C}_{2v}$	$\mathbf{D}_{2h}$	$\mathbf{D}_2$	$\mathbf{C}_s$	$\mathbf{C}_i$	$\mathbf{C}_2$
E	E	E	E	E	E	E	E	E	E	E
C_{51}^+	C_{51}^+									
C_{52}^+	C_{52}^+									
C_{53}^+	C_{53}^+									
C_{54}^+	C_{54}^+									
C_{55}^+	C_{55}^+									
C_{56}^+	C_{56}^+									
C_{51}^-	C_{51}^-									
C_{52}^-	C_{52}^-									
C_{53}^-	C_{53}^-									
C_{54}^-	C_{54}^-									
C_{55}^-	C_{55}^-									
C_{56}^-	C_{56}^-									
C_{51}^{2+}	C_{51}^{2+}									
C_{52}^{2+}	C_{52}^{2+}									
C_{53}^{2+}	C_{53}^{2+}									
C_{54}^{2+}	C_{54}^{2+}									
C_{55}^{2+}	C_{55}^{2+}									
C_{56}^{2+}	C_{56}^{2+}									
C_{51}^{2-}	C_{51}^{2-}									
C_{52}^{2-}	C_{52}^{2-}									
C_{53}^{2-}	C_{53}^{2-}									
C_{54}^{2-}	C_{54}^{2-}									
C_{55}^{2-}	C_{55}^{2-}									
C_{56}^{2-}	C_{56}^{2-}									
C_{31}^+	C_{31}^+	C_{31}^+	C_{31}^+							
C_{32}^+	C_{32}^+	C_{32}^+	C_{32}^+							
C_{33}^+	C_{33}^+	C_{33}^+	C_{33}^+							
C_{34}^+	C_{34}^+	C_{34}^+	C_{34}^+							
C_{35}^+	C_{35}^+									

$\twoheadrightarrow$

T **75**.0 Subgroup elements *(cont.)*

$\mathbf{I}_h$	$\mathbf{I}$	$\mathbf{T}_h$	$\mathbf{T}$	$\mathbf{C}_{2h}$	$\mathbf{C}_{2v}$	$\mathbf{D}_{2h}$	$\mathbf{D}_2$	$\mathbf{C}_s$	$\mathbf{C}_i$	$\mathbf{C}_2$
i		i		i		i			i	
$S_{10,1}^{3-}$										
$S_{10,2}^{3-}$										
$S_{10,3}^{3-}$										
$S_{10,4}^{3-}$										
$S_{10,5}^{3-}$										
$S_{10,6}^{3-}$										
$S_{10,1}^{3+}$										
$S_{10,2}^{3+}$										
$S_{10,3}^{3+}$										
$S_{10,4}^{3+}$										
$S_{10,5}^{3+}$										
$S_{10,6}^{3+}$										
$S_{10,1}^-$										
$S_{10,2}^-$										
$S_{10,3}^-$										
$S_{10,4}^-$										
$S_{10,5}^-$										
$S_{10,6}^-$										
$S_{10,1}^+$										
$S_{10,2}^+$										
$S_{10,3}^+$										
$S_{10,4}^+$										
$S_{10,5}^+$										
$S_{10,6}^+$										
S_{61}^-		S_{61}^-								
S_{62}^-		S_{62}^-								
S_{63}^-		S_{63}^-								
S_{64}^-		S_{64}^-								
S_{65}^-										

$\twoheadrightarrow$

T **75**.0 Subgroup elements *(cont.)*

$\mathbf{I}_h$	$\mathbf{I}$	$\mathbf{T}_h$	$\mathbf{T}$	$\mathbf{C}_{2h}$	$\mathbf{C}_{2v}$	$\mathbf{D}_{2h}$	$\mathbf{D}_2$	$\mathbf{C}_s$	$\mathbf{C}_i$	$\mathbf{C}_2$
C_{36}^+	C_{36}^+									
C_{37}^+	C_{37}^+									
C_{38}^+	C_{38}^+									
C_{39}^+	C_{39}^+									
$C_{3,10}^+$	$C_{3,10}^+$									
C_{31}^-	C_{31}^-	C_{31}^-	C_{31}^-							
C_{32}^-	C_{32}^-	C_{32}^-	C_{32}^-							
C_{33}^-	C_{33}^-	C_{33}^-	C_{33}^-							
C_{34}^-	C_{34}^-	C_{34}^-	C_{34}^-							
C_{35}^-	C_{35}^-									
C_{36}^-	C_{36}^-									
C_{37}^-	C_{37}^-									
C_{38}^-	C_{38}^-									
C_{39}^-	C_{39}^-									
$C_{3,10}^-$	$C_{3,10}^-$									
C_{2a}	C_{2a}	C_{2x}	C_{2x}			C_{2x}	C_{2x}			
C_{2b}	C_{2b}	C_{2y}	C_{2y}			C_{2y}	C_{2y}			
C_{2c}	C_{2c}			C_2	C_2	C_{2z}	C_{2z}			C_2
C_{2d}	C_{2d}									
C_{2e}	C_{2e}									
C_{2f}	C_{2f}									
C_{2g}	C_{2g}									
C_{2h}	C_{2h}									
C_{2i}	C_{2i}									
C_{2j}	C_{2j}									
C_{2k}	C_{2k}									
C_{2l}	C_{2l}									
C_{2m}	C_{2m}									
C_{2n}	C_{2n}									
C_{2o}	C_{2o}									

⇛

T **75**.0 Subgroup elements *(cont.)*

$\mathbf{I}_h$	$\mathbf{I}$	$\mathbf{T}_h$	$\mathbf{T}$	$\mathbf{C}_{2h}$	$\mathbf{C}_{2v}$	$\mathbf{D}_{2h}$	$\mathbf{D}_2$	$\mathbf{C}_s$	$\mathbf{C}_i$	$\mathbf{C}_2$
S_{66}^-										
S_{67}^-										
S_{68}^-										
S_{69}^-										
$S_{6,10}^-$										
S_{61}^+		S_{61}^+								
S_{62}^+		S_{62}^+								
S_{63}^+		S_{63}^+								
S_{64}^+		S_{64}^+								
S_{65}^+										
S_{66}^+										
S_{67}^+										
S_{68}^+										
S_{69}^+										
$S_{6,10}^+$										
σ_a		σ_x			σ_x	σ_x				
σ_b		σ_y			σ_y	σ_y				
σ_c		σ_z		σ_h		σ_z		σ_h		
σ_d										
σ_e										
σ_f										
σ_g										
σ_h										
σ_i										
σ_j										
σ_k										
σ_l										
σ_m										
σ_n										
σ_o										

T **75**.1 Parameters

§ **16**–1, p. 68

$\mathbf{I}_h$		α	β	γ	ϕ	$\mathbf{n}$	λ	$\mathbf{\Lambda}$
E	i	0	0	0	0	$(0\;\; 0\;\; 0)$	$[\![\, 1,$	$(0\;\; 0\;\; 0)]\!]$
C_{51}^{+}	$S_{10,1}^{3-}$	$-\sigma$	$\frac{\pi}{5}$	$\pi-\sigma$	$\frac{2\pi}{5}$	$(\cos\sigma\;\; 0\;\; \sin\sigma)$	$[\![\, c_5,$	$(s_{10}\;\; 0\;\; \frac{1}{2})]\!]$
C_{52}^{+}	$S_{10,2}^{3-}$	$\pi-\sigma$	$\frac{\pi}{5}$	$-\sigma$	$\frac{2\pi}{5}$	$(-\cos\sigma\;\; 0\;\; \sin\sigma)$	$[\![\, c_5,$	$(-s_{10}\;\; 0\;\; \frac{1}{2})]\!]$
C_{53}^{+}	$S_{10,3}^{3-}$	ρ	$\frac{\pi}{3}$	ρ	$\frac{2\pi}{5}$	$(0\;\; \sin\sigma\;\; \cos\sigma)$	$[\![\, c_5,$	$(0\;\; \frac{1}{2}\;\; s_{10})]\!]$
C_{54}^{+}	$S_{10,4}^{3-}$	$-\pi+\rho$	$\frac{\pi}{3}$	$-\pi+\rho$	$\frac{2\pi}{5}$	$(0\;\; -\sin\sigma\;\; \cos\sigma)$	$[\![\, c_5,$	$(0\;\; -\frac{1}{2}\;\; s_{10})]\!]$
C_{55}^{+}	$S_{10,5}^{3-}$	$-\sigma$	$\frac{2\pi}{5}$	σ	$\frac{2\pi}{5}$	$(\sin\sigma\;\; \cos\sigma\;\; 0)$	$[\![\, c_5,$	$(\frac{1}{2}\;\; s_{10}\;\; 0)]\!]$
C_{56}^{+}	$S_{10,6}^{3-}$	$-\pi+\sigma$	$\frac{2\pi}{5}$	$\pi-\sigma$	$\frac{2\pi}{5}$	$(\sin\sigma\;\; -\cos\sigma\;\; 0)$	$[\![\, c_5,$	$(\frac{1}{2}\;\; -s_{10}\;\; 0)]\!]$
C_{51}^{-}	$S_{10,1}^{3+}$	σ	$\frac{\pi}{5}$	$-\pi+\sigma$	$\frac{2\pi}{5}$	$(-\cos\sigma\;\; 0\;\; -\sin\sigma)$	$[\![\, c_5,$	$(-s_{10}\;\; 0\;\; -\frac{1}{2})]\!]$
C_{52}^{-}	$S_{10,2}^{3+}$	$-\pi+\sigma$	$\frac{\pi}{5}$	σ	$\frac{2\pi}{5}$	$(\cos\sigma\;\; 0\;\; -\sin\sigma)$	$[\![\, c_5,$	$(s_{10}\;\; 0\;\; -\frac{1}{2})]\!]$
C_{53}^{-}	$S_{10,3}^{3+}$	$\pi-\rho$	$\frac{\pi}{3}$	$\pi-\rho$	$\frac{2\pi}{5}$	$(0\;\; -\sin\sigma\;\; -\cos\sigma)$	$[\![\, c_5,$	$(0\;\; -\frac{1}{2}\;\; -s_{10})]\!]$
C_{54}^{-}	$S_{10,4}^{3+}$	$-\rho$	$\frac{\pi}{3}$	$-\rho$	$\frac{2\pi}{5}$	$(0\;\; \sin\sigma\;\; -\cos\sigma)$	$[\![\, c_5,$	$(0\;\; \frac{1}{2}\;\; -s_{10})]\!]$
C_{55}^{-}	$S_{10,5}^{3+}$	$\pi-\sigma$	$\frac{2\pi}{5}$	$-\pi+\sigma$	$\frac{2\pi}{5}$	$(-\sin\sigma\;\; -\cos\sigma\;\; 0)$	$[\![\, c_5,$	$(-\frac{1}{2}\;\; -s_{10}\;\; 0)]\!]$
C_{56}^{-}	$S_{10,6}^{3+}$	σ	$\frac{2\pi}{5}$	$-\sigma$	$\frac{2\pi}{5}$	$(-\sin\sigma\;\; \cos\sigma\;\; 0)$	$[\![\, c_5,$	$(-\frac{1}{2}\;\; s_{10}\;\; 0)]\!]$
C_{51}^{2+}	$S_{10,1}^{-}$	$-\rho$	$\frac{\pi}{3}$	$\pi-\rho$	$\frac{4\pi}{5}$	$(\cos\sigma\;\; 0\;\; \sin\sigma)$	$[\![s_{10},$	$(\frac{1}{2}\;\; 0\;\; c_5)]\!]$
C_{52}^{2+}	$S_{10,2}^{-}$	$\pi-\rho$	$\frac{\pi}{3}$	$-\rho$	$\frac{4\pi}{5}$	$(-\cos\sigma\;\; 0\;\; \sin\sigma)$	$[\![s_{10},$	$(-\frac{1}{2}\;\; 0\;\; c_5)]\!]$
C_{53}^{2+}	$S_{10,3}^{-}$	σ	$\frac{3\pi}{5}$	σ	$\frac{4\pi}{5}$	$(0\;\; \sin\sigma\;\; \cos\sigma)$	$[\![s_{10},$	$(0\;\; c_5\;\; \frac{1}{2})]\!]$
C_{54}^{2+}	$S_{10,4}^{-}$	$-\pi+\sigma$	$\frac{3\pi}{5}$	$-\pi+\sigma$	$\frac{4\pi}{5}$	$(0\;\; -\sin\sigma\;\; \cos\sigma)$	$[\![s_{10},$	$(0\;\; -c_5\;\; \frac{1}{2})]\!]$
C_{55}^{2+}	$S_{10,5}^{-}$	$-\sigma$	$\frac{4\pi}{5}$	σ	$\frac{4\pi}{5}$	$(\sin\sigma\;\; \cos\sigma\;\; 0)$	$[\![s_{10},$	$(c_5\;\; \frac{1}{2}\;\; 0)]\!]$
C_{56}^{2+}	$S_{10,6}^{-}$	$-\pi+\sigma$	$\frac{4\pi}{5}$	$\pi-\sigma$	$\frac{4\pi}{5}$	$(\sin\sigma\;\; -\cos\sigma\;\; 0)$	$[\![s_{10},$	$(c_5\;\; -\frac{1}{2}\;\; 0)]\!]$
C_{51}^{2-}	$S_{10,1}^{+}$	ρ	$\frac{\pi}{3}$	$-\pi+\rho$	$\frac{4\pi}{5}$	$(-\cos\sigma\;\; 0\;\; -\sin\sigma)$	$[\![s_{10},$	$(-\frac{1}{2}\;\; 0\;\; -c_5)]\!]$
C_{52}^{2-}	$S_{10,2}^{+}$	$-\pi+\rho$	$\frac{\pi}{3}$	ρ	$\frac{4\pi}{5}$	$(\cos\sigma\;\; 0\;\; -\sin\sigma)$	$[\![s_{10},$	$(\frac{1}{2}\;\; 0\;\; -c_5)]\!]$
C_{53}^{2-}	$S_{10,3}^{+}$	$\pi-\sigma$	$\frac{3\pi}{5}$	$\pi-\sigma$	$\frac{4\pi}{5}$	$(0\;\; -\sin\sigma\;\; -\cos\sigma)$	$[\![s_{10},$	$(0\;\; -c_5\;\; -\frac{1}{2})]\!]$
C_{54}^{2-}	$S_{10,4}^{+}$	$-\sigma$	$\frac{3\pi}{5}$	$-\sigma$	$\frac{4\pi}{5}$	$(0\;\; \sin\sigma\;\; -\cos\sigma)$	$[\![s_{10},$	$(0\;\; c_5\;\; -\frac{1}{2})]\!]$
C_{55}^{2-}	$S_{10,5}^{+}$	$\pi-\sigma$	$\frac{4\pi}{5}$	$-\pi+\sigma$	$\frac{4\pi}{5}$	$(-\sin\sigma\;\; -\cos\sigma\;\; 0)$	$[\![s_{10},$	$(-c_5\;\; -\frac{1}{2}\;\; 0)]\!]$
C_{56}^{2-}	$S_{10,6}^{+}$	σ	$\frac{4\pi}{5}$	$-\sigma$	$\frac{4\pi}{5}$	$(-\sin\sigma\;\; \cos\sigma\;\; 0)$	$[\![s_{10},$	$(-c_5\;\; \frac{1}{2}\;\; 0)]\!]$
C_{31}^{+}	S_{61}^{-}	0	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$(\frac{1}{\sqrt{3}}\;\; \frac{1}{\sqrt{3}}\;\; \frac{1}{\sqrt{3}})$	$[\![\, \frac{1}{2},$	$(\frac{1}{2}\;\; \frac{1}{2}\;\; \frac{1}{2})]\!]$
C_{32}^{+}	S_{62}^{-}	π	$\frac{\pi}{2}$	$-\frac{\pi}{2}$	$\frac{2\pi}{3}$	$(-\frac{1}{\sqrt{3}}\;\; -\frac{1}{\sqrt{3}}\;\; \frac{1}{\sqrt{3}})$	$[\![\, \frac{1}{2},$	$(-\frac{1}{2}\;\; -\frac{1}{2}\;\; \frac{1}{2})]\!]$
C_{33}^{+}	S_{63}^{-}	π	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$(\frac{1}{\sqrt{3}}\;\; -\frac{1}{\sqrt{3}}\;\; -\frac{1}{\sqrt{3}})$	$[\![\, \frac{1}{2},$	$(\frac{1}{2}\;\; -\frac{1}{2}\;\; -\frac{1}{2})]\!]$
C_{34}^{+}	S_{64}^{-}	0	$\frac{\pi}{2}$	$-\frac{\pi}{2}$	$\frac{2\pi}{3}$	$(-\frac{1}{\sqrt{3}}\;\; \frac{1}{\sqrt{3}}\;\; -\frac{1}{\sqrt{3}})$	$[\![\, \frac{1}{2},$	$(-\frac{1}{2}\;\; \frac{1}{2}\;\; -\frac{1}{2})]\!]$
C_{35}^{+}	S_{65}^{-}	$\pi-\sigma$	$\frac{3\pi}{5}$	$-\sigma$	$\frac{2\pi}{3}$	$(-\sin\rho'\;\; 0\;\; \cos\rho')$	$[\![\, \frac{1}{2},$	$(-c_5\;\; 0\;\; s_{10})]\!]$

$c_n = \cos\frac{\pi}{n}$, $s_n = \sin\frac{\pi}{n}$, $\sigma = \arctan\frac{1+\sqrt{5}}{2}$, $\rho = \arctan\frac{3-\sqrt{5}}{2}$, $\rho' = \arctan\frac{3+\sqrt{5}}{2}$ $\twoheadrightarrow$

T **75**.1 Parameters *(cont.)*

$\mathbf{I}_h$		α	β	γ	ϕ	$\mathbf{n}$	λ	$\boldsymbol{\Lambda}$
C_{36}^+	S_{66}^-	$-\sigma$	$\frac{\pi}{5}$	$-\sigma$	$\frac{2\pi}{3}$	$(0 \quad \cos\rho' \quad -\sin\rho')$	$[\![\frac{1}{2},$	$(0 \quad s_{10} \quad -c_5)]\!]$
C_{37}^+	S_{67}^-	$-\pi+\rho$	$\frac{2\pi}{3}$	$\pi-\rho$	$\frac{2\pi}{3}$	$(\cos\rho' \quad -\sin\rho' \quad 0)$	$[\![\frac{1}{2},$	$(s_{10} \quad -c_5 \quad 0)]\!]$
C_{38}^+	S_{68}^-	$-\sigma$	$\frac{3\pi}{5}$	$\pi-\sigma$	$\frac{2\pi}{3}$	$(\sin\rho' \quad 0 \quad \cos\rho')$	$[\![\frac{1}{2},$	$(c_5 \quad 0 \quad s_{10})]\!]$
C_{39}^+	S_{69}^-	σ	$\frac{\pi}{5}$	σ	$\frac{2\pi}{3}$	$(0 \quad \cos\rho' \quad \sin\rho')$	$[\![\frac{1}{2},$	$(0 \quad s_{10} \quad c_5)]\!]$
$C_{3,10}^+$	$S_{6,10}^-$	$-\rho$	$\frac{2\pi}{3}$	ρ	$\frac{2\pi}{3}$	$(\cos\rho' \quad \sin\rho' \quad 0)$	$[\![\frac{1}{2},$	$(s_{10} \quad c_5 \quad 0)]\!]$
C_{31}^-	S_{61}^+	$\frac{\pi}{2}$	$\frac{\pi}{2}$	π	$\frac{2\pi}{3}$	$(-\frac{1}{\sqrt{3}} \quad -\frac{1}{\sqrt{3}} \quad -\frac{1}{\sqrt{3}})$	$[\![\frac{1}{2},$	$(-\frac{1}{2} \quad -\frac{1}{2} \quad -\frac{1}{2})]\!]$
C_{32}^-	S_{62}^+	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	0	$\frac{2\pi}{3}$	$(\frac{1}{\sqrt{3}} \quad \frac{1}{\sqrt{3}} \quad -\frac{1}{\sqrt{3}})$	$[\![\frac{1}{2},$	$(\frac{1}{2} \quad \frac{1}{2} \quad -\frac{1}{2})]\!]$
C_{33}^-	S_{63}^+	$\frac{\pi}{2}$	$\frac{\pi}{2}$	0	$\frac{2\pi}{3}$	$(-\frac{1}{\sqrt{3}} \quad \frac{1}{\sqrt{3}} \quad \frac{1}{\sqrt{3}})$	$[\![\frac{1}{2},$	$(-\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2})]\!]$
C_{34}^-	S_{64}^+	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	π	$\frac{2\pi}{3}$	$(\frac{1}{\sqrt{3}} \quad -\frac{1}{\sqrt{3}} \quad \frac{1}{\sqrt{3}})$	$[\![\frac{1}{2},$	$(\frac{1}{2} \quad -\frac{1}{2} \quad \frac{1}{2})]\!]$
C_{35}^-	S_{65}^+	$-\pi+\sigma$	$\frac{3\pi}{5}$	σ	$\frac{2\pi}{3}$	$(\sin\rho' \quad 0 \quad -\cos\rho')$	$[\![\frac{1}{2},$	$(c_5 \quad 0 \quad -s_{10})]\!]$
C_{36}^-	S_{66}^+	$-\pi+\sigma$	$\frac{\pi}{5}$	$-\pi+\sigma$	$\frac{2\pi}{3}$	$(0 \quad -\cos\rho' \quad \sin\rho')$	$[\![\frac{1}{2},$	$(0 \quad -s_{10} \quad c_5)]\!]$
C_{37}^-	S_{67}^+	ρ	$\frac{2\pi}{3}$	$-\rho$	$\frac{2\pi}{3}$	$(-\cos\rho' \quad \sin\rho' \quad 0)$	$[\![\frac{1}{2},$	$(-s_{10} \quad c_5 \quad 0)]\!]$
C_{38}^-	S_{68}^+	σ	$\frac{3\pi}{5}$	$-\pi+\sigma$	$\frac{2\pi}{3}$	$(-\sin\rho' \quad 0 \quad -\cos\rho')$	$[\![\frac{1}{2},$	$(-c_5 \quad 0 \quad -s_{10})]\!]$
C_{39}^-	S_{69}^+	$\pi-\sigma$	$\frac{\pi}{5}$	$\pi-\sigma$	$\frac{2\pi}{3}$	$(0 \quad -\cos\rho' \quad -\sin\rho')$	$[\![\frac{1}{2},$	$(0 \quad -s_{10} \quad -c_5)]\!]$
$C_{3,10}^-$	$S_{6,10}^+$	$\pi-\rho$	$\frac{2\pi}{3}$	$-\pi+\rho$	$\frac{2\pi}{3}$	$(-\cos\rho' \quad -\sin\rho' \quad 0)$	$[\![\frac{1}{2},$	$(-s_{10} \quad -c_5 \quad 0)]\!]$
C_{2a}	σ_a	0	π	π	π	$(1 \quad 0 \quad 0)$	$[\![0,$	$(1 \quad 0 \quad 0)]\!]$
C_{2b}	σ_b	0	π	0	π	$(0 \quad 1 \quad 0)$	$[\![0,$	$(0 \quad 1 \quad 0)]\!]$
C_{2c}	σ_c	0	0	π	π	$(0 \quad 0 \quad 1)$	$[\![0,$	$(0 \quad 0 \quad 1)]\!]$
C_{2d}	σ_d	ρ	$\frac{2\pi}{3}$	$\pi-\rho$	π	$(c_5 \quad s_{10} \quad \frac{1}{2})$	$[\![0,$	$(c_5 \quad s_{10} \quad \frac{1}{2})]\!]$
C_{2e}	σ_e	$\pi-\sigma$	$\frac{4\pi}{5}$	σ	π	$(\frac{1}{2} \quad -c_5 \quad -s_{10})$	$[\![0,$	$(\frac{1}{2} \quad -c_5 \quad -s_{10})]\!]$
C_{2f}	σ_f	$-\pi+\sigma$	$\frac{2\pi}{5}$	$-\sigma$	π	$(s_{10} \quad \frac{1}{2} \quad -c_5)$	$[\![0,$	$(s_{10} \quad \frac{1}{2} \quad -c_5)]\!]$
C_{2g}	σ_g	σ	$\frac{2\pi}{5}$	$\pi-\sigma$	π	$(s_{10} \quad \frac{1}{2} \quad c_5)$	$[\![0,$	$(s_{10} \quad \frac{1}{2} \quad c_5)]\!]$
C_{2h}	σ_h	$-\sigma$	$\frac{4\pi}{5}$	$-\pi+\sigma$	π	$(\frac{1}{2} \quad -c_5 \quad s_{10})$	$[\![0,$	$(\frac{1}{2} \quad -c_5 \quad s_{10})]\!]$
C_{2i}	σ_i	$-\pi+\rho$	$\frac{2\pi}{3}$	$-\rho$	π	$(c_5 \quad s_{10} \quad -\frac{1}{2})$	$[\![0,$	$(c_5 \quad s_{10} \quad -\frac{1}{2})]\!]$
C_{2j}	σ_j	$\pi-\sigma$	$\frac{2\pi}{5}$	σ	π	$(-s_{10} \quad \frac{1}{2} \quad c_5)$	$[\![0,$	$(-s_{10} \quad \frac{1}{2} \quad c_5)]\!]$
C_{2k}	σ_k	$-\rho$	$\frac{2\pi}{3}$	$-\pi+\rho$	π	$(-c_5 \quad s_{10} \quad -\frac{1}{2})$	$[\![0,$	$(-c_5 \quad s_{10} \quad -\frac{1}{2})]\!]$
C_{2l}	σ_l	$-\pi+\sigma$	$\frac{4\pi}{5}$	$-\sigma$	π	$(-\frac{1}{2} \quad -c_5 \quad s_{10})$	$[\![0,$	$(-\frac{1}{2} \quad -c_5 \quad s_{10})]\!]$
C_{2m}	σ_m	$\pi-\rho$	$\frac{2\pi}{3}$	ρ	π	$(-c_5 \quad s_{10} \quad \frac{1}{2})$	$[\![0,$	$(-c_5 \quad s_{10} \quad \frac{1}{2})]\!]$
C_{2n}	σ_n	$-\sigma$	$\frac{2\pi}{5}$	$-\pi+\sigma$	π	$(-s_{10} \quad \frac{1}{2} \quad -c_5)$	$[\![0,$	$(-s_{10} \quad \frac{1}{2} \quad -c_5)]\!]$
C_{2o}	σ_o	σ	$\frac{4\pi}{5}$	$\pi-\sigma$	π	$(-\frac{1}{2} \quad -c_5 \quad -s_{10})$	$[\![0,$	$(-\frac{1}{2} \quad -c_5 \quad -s_{10})]\!]$

$c_n = \cos\frac{\pi}{n}$, $s_n = \sin\frac{\pi}{n}$, $\sigma = \arctan\frac{1+\sqrt{5}}{2}$, $\rho = \arctan\frac{3-\sqrt{5}}{2}$, $\rho' = \arctan\frac{3+\sqrt{5}}{2}$

T **75**.2 Multiplication table

Use T **74**.2 ▪. § **16**–2, p. 69

T **75**.3 Factor table

Use T **74**.3 ▪. § **16**–3, p. 70

T **75**.4 Character table

§ **16**–4, p. 71

$\mathbf{I}_h$	E	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$	i	$12S_{10}^3$	$12S_{10}$	$20S_6$	15σ	τ
A_g	1	1	1	1	1	1	1	1	1	1	a
T_{1g}	3	$2c_5$	$2c_5^3$	0	−1	3	$2c_5$	$2c_5^3$	0	−1	a
T_{2g}	3	$2c_5^3$	$2c_5$	0	−1	3	$2c_5^3$	$2c_5$	0	−1	a
F_g	4	−1	−1	1	0	4	−1	−1	1	0	a
H_g	5	0	0	−1	1	5	0	0	−1	1	a
A_u	1	1	1	1	1	−1	−1	−1	−1	−1	a
T_{1u}	3	$2c_5$	$2c_5^3$	0	−1	−3	$-2c_5$	$-2c_5^3$	0	1	a
T_{2u}	3	$2c_5^3$	$2c_5$	0	−1	−3	$-2c_5^3$	$-2c_5$	0	1	a
F_u	4	−1	−1	1	0	−4	1	1	−1	0	a
H_u	5	0	0	−1	1	−5	0	0	1	−1	a
$E_{1/2,g}$	2	$2c_5$	$2c_5^2$	1	0	2	$2c_5$	$2c_5^2$	1	0	c
$E_{7/2,g}$	2	$2c_5^3$	$2c_5^4$	1	0	2	$2c_5^3$	$2c_5^4$	1	0	c
$F_{3/2,g}$	4	1	−1	−1	0	4	1	−1	−1	0	c
$I_{5/2,g}$	6	−1	1	0	0	6	−1	1	0	0	c
$E_{1/2,u}$	2	$2c_5$	$2c_5^2$	1	0	−2	$-2c_5$	$-2c_5^2$	−1	0	c
$E_{7/2,u}$	2	$2c_5^3$	$2c_5^4$	1	0	−2	$-2c_5^3$	$-2c_5^4$	−1	0	c
$F_{3/2,u}$	4	1	−1	−1	0	−4	−1	1	1	0	c
$I_{5/2,u}$	6	−1	1	0	0	−6	1	−1	0	0	c

$c_n^m = \cos \frac{m}{n}\pi$

T **75**.5 Cartesian tensors and *s*, *p*, *d*, and *f* functions

§ **16**–5, p. 72

$\mathbf{I}_h$	0	1	2	3
A_g	□1		$x^2+y^2+z^2$	
T_{1g}		(R_x, R_y, R_z)		
T_{2g}				
F_g				
H_g			□$(x^2-y^2, 2z^2-x^2-y^2, zx, yz, xy)$	
A_u				
T_{1u}		□(x, y, z)		
T_{2u}				$\}$ □$\{xz^2, yz^2, z(x^2-y^2), xyz,$
F_u				$\}$ $x(x^2-3y^2), y(3x^2-y^2), z^3\}$
H_u				

T **75**.6a Bases of irreducible representations

§ **16**–6, pp. 74, 75

$\mathbf{I}_h$	$\big\langle \lvert j\, m\rangle \big\rvert$
A_g	$\lvert 0\,0\rangle$
T_{1g}	$\big\langle \frac{\sqrt{66}}{32}(\sqrt5+1)\lvert 6\,1\rangle_- - \frac{\sqrt{33}}{32}(3-\sqrt5)\lvert 6\,3\rangle_- - \frac{1}{32}(11+3\sqrt5)\lvert 6\,5\rangle_-,$
	$\frac{\sqrt{33}}{16}\lvert 6\,2\rangle_- + \frac{\sqrt{22}}{8}\lvert 6\,4\rangle_- - \frac{\sqrt{135}}{16}\lvert 6\,6\rangle_-,$
	$-\frac{\sqrt{66}}{32}(\sqrt5-1)\lvert 6\,1\rangle_+ - \frac{\sqrt{33}}{32}(3+\sqrt5)\lvert 6\,3\rangle_+ - \frac{1}{32}(11-3\sqrt5)\lvert 6\,5\rangle_+ \big\rvert$
T_{2g}	$\big\langle \frac{\sqrt{1001}}{64}\lvert 8\,1\rangle_- - \frac{\sqrt{39}}{64}(\sqrt5-2)\lvert 8\,3\rangle_- + \frac{9}{64}(2\sqrt5+1)\lvert 8\,5\rangle_- + \frac{\sqrt7}{64}(12-\sqrt5)\lvert 8\,7\rangle_-$
	$\frac{\sqrt{2574}}{64}\lvert 8\,2\rangle_- - \frac{\sqrt{13}}{32}\lvert 8\,4\rangle_- - \frac{\sqrt{210}}{64}\lvert 8\,6\rangle_- - \frac{\sqrt{315}}{32}\lvert 8\,8\rangle_-,$
	$\frac{\sqrt{1001}}{64}\lvert 8\,1\rangle_+ + \frac{\sqrt{39}}{64}(\sqrt5+2)\lvert 8\,3\rangle_+ - \frac{9}{64}(2\sqrt5-1)\lvert 8\,5\rangle_+ + \frac{\sqrt7}{64}(12+\sqrt5)\lvert 8\,7\rangle_+ \big\rvert$
F_g	$\big\langle \sqrt{\tfrac{7}{12}}\lvert 4\,0\rangle + \sqrt{\tfrac{5}{12}}\lvert 4\,4\rangle_+,\ \sqrt{\tfrac{7}{96}}(\sqrt5-1)\lvert 4\,1\rangle_- - \frac{1}{\sqrt{96}}(7+\sqrt5)\lvert 4\,3\rangle_-,$
	$-\sqrt{\tfrac{7}{12}}\lvert 4\,2\rangle_- - \sqrt{\tfrac{5}{12}}\lvert 4\,4\rangle_-,\ \sqrt{\tfrac{7}{96}}(\sqrt5+1)\lvert 4\,1\rangle_+ - \frac{1}{\sqrt{96}}(7-\sqrt5)\lvert 4\,3\rangle_+ \big\rvert$
H_g	$\big\langle \frac{1}{\sqrt2}\lvert 2\,0\rangle - \frac{\mathrm{i}}{\sqrt2}\lvert 2\,2\rangle_+,\ \frac{1}{\sqrt2}\lvert 2\,0\rangle + \frac{\mathrm{i}}{\sqrt2}\lvert 2\,2\rangle_+,\ \lvert 2\,1\rangle_-,\ -\lvert 2\,2\rangle_-,\ -\lvert 2\,1\rangle_+ \big\rvert$
A_u	$\big\langle \frac{\sqrt{5436717}}{4096}\lvert 15\,2\rangle_- + \frac{\sqrt{73370}}{2048}\lvert 15\,4\rangle_- + \frac{\sqrt{1134567}}{4096}\lvert 15\,6\rangle_- - \frac{\sqrt{33495}}{512}\lvert 15\,8\rangle_- +$
	$\frac{\sqrt{3560997}}{4096}\lvert 15\,10\rangle_- + \frac{\sqrt{207350}}{2048}\lvert 15\,12\rangle_- + \frac{\sqrt{3378375}}{4096}\lvert 15\,14\rangle_- \big\rvert$
T_{1u}	$\big\langle \lvert 1\,1\rangle_+,\ \lvert 1\,0\rangle,\ \lvert 1\,1\rangle_- \big\rvert$
T_{2u}	$\big\langle \sqrt{\tfrac{3}{32}}(\sqrt5-1)\lvert 3\,1\rangle_+ - \frac{1}{\sqrt{32}}(\sqrt5+3)\lvert 3\,3\rangle_+,\ \frac12\lvert 3\,0\rangle + \sqrt{\tfrac34}\lvert 3\,2\rangle_+,$
	$-\sqrt{\tfrac{3}{32}}(\sqrt5+1)\lvert 3\,1\rangle_- + \frac{1}{\sqrt{32}}(\sqrt5-3)\lvert 3\,3\rangle_- \big\rvert$
F_u	$\big\langle \lvert 3\,2\rangle_-,\ \frac{1}{\sqrt{32}}(\sqrt5+3)\lvert 3\,1\rangle_+ + \sqrt{\tfrac{3}{32}}(\sqrt5-1)\lvert 3\,3\rangle_+,\ -\sqrt{\tfrac34}\lvert 3\,0\rangle + \frac12\lvert 3\,2\rangle_+,$
	$-\frac{1}{\sqrt{32}}(\sqrt5-3)\lvert 3\,1\rangle_- - \sqrt{\tfrac{3}{32}}(\sqrt5+1)\lvert 3\,3\rangle_- \big\rvert$
H_u	$\big\langle \frac{1}{\sqrt2}\lvert 5\,2\rangle_- - \frac{\mathrm{i}}{\sqrt2}\lvert 5\,4\rangle_-,\ -\frac{1}{\sqrt{32}}(1-\mathrm{i}\sqrt{15})\lvert 5\,2\rangle_- + \frac{1}{\sqrt{32}}(\sqrt{15}-\mathrm{i})\lvert 5\,4\rangle_-,$
	$\frac{1}{128}(\sqrt3-\mathrm{i}\sqrt5)\{-\sqrt{28}(3-\sqrt5)\lvert 5\,1\rangle_+ + \sqrt6(\sqrt5-11)\lvert 5\,3\rangle_+ - \sqrt{150}(\sqrt5+1)\lvert 5\,5\rangle_+\},$
	$\frac{1}{32}(\sqrt3-\mathrm{i}\sqrt5)\{-\sqrt{105}\lvert 5\,0\rangle - \sqrt{20}\lvert 5\,2\rangle_+ + \sqrt3\lvert 5\,4\rangle_+\},$
	$\frac{1}{128}(\sqrt3-\mathrm{i}\sqrt5)\{-\sqrt{28}(3+\sqrt5)\lvert 5\,1\rangle_- + \sqrt6(\sqrt5+11)\lvert 5\,3\rangle_- - \sqrt{150}(\sqrt5-1)\lvert 5\,5\rangle_-\} \big\rvert$
$E_{1/2,g}$	$\big\langle \lvert \tfrac12\,\tfrac12\rangle,\ \lvert \tfrac12\,\overline{\tfrac12}\rangle \big\rvert$
$E_{7/2,g}$	$\big\langle \frac{\sqrt{21}}{8}\lvert \tfrac72\,\tfrac52\rangle - \frac{\sqrt{21}}{8}\lvert \tfrac72\,\tfrac12\rangle - \frac{\sqrt7}{8}\lvert \tfrac72\,\overline{\tfrac32}\rangle - \frac{\sqrt{15}}{8}\lvert \tfrac72\,\overline{\tfrac72}\rangle,\ \frac{\sqrt{15}}{8}\lvert \tfrac72\,\tfrac72\rangle + \frac{\sqrt7}{8}\lvert \tfrac72\,\tfrac32\rangle + \frac{\sqrt{21}}{8}\lvert \tfrac72\,\overline{\tfrac12}\rangle - \frac{\sqrt{21}}{8}\lvert \tfrac72\,\overline{\tfrac52}\rangle \big\rvert$
$F_{3/2,g}$	$\big\langle \frac{1}{\sqrt2}\lvert \tfrac32\,\tfrac12\rangle - \frac{\mathrm{i}}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac32}\rangle,\ \frac{\mathrm{i}}{\sqrt2}\lvert \tfrac32\,\tfrac32\rangle - \frac{1}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac12}\rangle,\ \frac{1}{\sqrt2}\lvert \tfrac32\,\tfrac12\rangle + \frac{\mathrm{i}}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac32}\rangle,\ -\frac{\mathrm{i}}{\sqrt2}\lvert \tfrac32\,\tfrac32\rangle - \frac{1}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac12}\rangle \big\rvert$
$I_{5/2,g}$	$\big\langle \sqrt{\tfrac16}\lvert \tfrac52\,\tfrac52\rangle - \sqrt{\tfrac56}\lvert \tfrac52\,\overline{\tfrac32}\rangle,\ -\sqrt{\tfrac56}\lvert \tfrac52\,\tfrac32\rangle + \sqrt{\tfrac16}\lvert \tfrac52\,\overline{\tfrac52}\rangle,$
	$\sqrt{\tfrac{5}{12}}\lvert \tfrac52\,\tfrac52\rangle + \frac{\mathrm{i}}{\sqrt2}\lvert \tfrac52\,\tfrac12\rangle + \sqrt{\tfrac{1}{12}}\lvert \tfrac52\,\overline{\tfrac32}\rangle,\ \sqrt{\tfrac{1}{12}}\lvert \tfrac52\,\tfrac32\rangle + \frac{\mathrm{i}}{\sqrt2}\lvert \tfrac52\,\overline{\tfrac12}\rangle + \sqrt{\tfrac{5}{12}}\lvert \tfrac52\,\overline{\tfrac52}\rangle,$
	$\sqrt{\tfrac{5}{12}}\lvert \tfrac52\,\tfrac52\rangle - \frac{\mathrm{i}}{\sqrt2}\lvert \tfrac52\,\tfrac12\rangle + \sqrt{\tfrac{1}{12}}\lvert \tfrac52\,\overline{\tfrac32}\rangle,\ \sqrt{\tfrac{1}{12}}\lvert \tfrac52\,\tfrac32\rangle - \frac{\mathrm{i}}{\sqrt2}\lvert \tfrac52\,\overline{\tfrac12}\rangle + \sqrt{\tfrac{5}{12}}\lvert \tfrac52\,\overline{\tfrac52}\rangle \big\rvert$
$E_{1/2,u}$	$\big\langle \lvert \tfrac12\,\tfrac12\rangle,\ \lvert \tfrac12\,\overline{\tfrac12}\rangle \big\rvert^\bullet$
$E_{7/2,u}$	$\big\langle \frac{\sqrt{21}}{8}\lvert \tfrac72\,\tfrac52\rangle - \frac{\sqrt{21}}{8}\lvert \tfrac72\,\tfrac12\rangle - \frac{\sqrt7}{8}\lvert \tfrac72\,\overline{\tfrac32}\rangle - \frac{\sqrt{15}}{8}\lvert \tfrac72\,\overline{\tfrac72}\rangle,\ \frac{\sqrt{15}}{8}\lvert \tfrac72\,\tfrac72\rangle + \frac{\sqrt7}{8}\lvert \tfrac72\,\tfrac32\rangle + \frac{\sqrt{21}}{8}\lvert \tfrac72\,\overline{\tfrac12}\rangle - \frac{\sqrt{21}}{8}\lvert \tfrac72\,\overline{\tfrac52}\rangle \big\rvert^\bullet$
$F_{3/2,u}$	$\big\langle \frac{1}{\sqrt2}\lvert \tfrac32\,\tfrac12\rangle - \frac{\mathrm{i}}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac32}\rangle,\ \frac{\mathrm{i}}{\sqrt2}\lvert \tfrac32\,\tfrac32\rangle - \frac{1}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac12}\rangle,\ \frac{1}{\sqrt2}\lvert \tfrac32\,\tfrac12\rangle + \frac{\mathrm{i}}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac32}\rangle,\ -\frac{\mathrm{i}}{\sqrt2}\lvert \tfrac32\,\tfrac32\rangle - \frac{1}{\sqrt2}\lvert \tfrac32\,\overline{\tfrac12}\rangle \big\rvert^\bullet$
$I_{5/2,u}$	$\big\langle \sqrt{\tfrac16}\lvert \tfrac52\,\tfrac52\rangle - \sqrt{\tfrac56}\lvert \tfrac52\,\overline{\tfrac32}\rangle,\ -\sqrt{\tfrac56}\lvert \tfrac52\,\tfrac32\rangle + \sqrt{\tfrac16}\lvert \tfrac52\,\overline{\tfrac52}\rangle,$
	$\sqrt{\tfrac{5}{12}}\lvert \tfrac52\,\tfrac52\rangle + \frac{\mathrm{i}}{\sqrt2}\lvert \tfrac52\,\tfrac12\rangle + \sqrt{\tfrac{1}{12}}\lvert \tfrac52\,\overline{\tfrac32}\rangle,\ \sqrt{\tfrac{1}{12}}\lvert \tfrac52\,\tfrac32\rangle + \frac{\mathrm{i}}{\sqrt2}\lvert \tfrac52\,\overline{\tfrac12}\rangle + \sqrt{\tfrac{5}{12}}\lvert \tfrac52\,\overline{\tfrac52}\rangle,$
	$\sqrt{\tfrac{5}{12}}\lvert \tfrac52\,\tfrac52\rangle - \frac{\mathrm{i}}{\sqrt2}\lvert \tfrac52\,\tfrac12\rangle + \sqrt{\tfrac{1}{12}}\lvert \tfrac52\,\overline{\tfrac32}\rangle,\ \sqrt{\tfrac{1}{12}}\lvert \tfrac52\,\tfrac32\rangle - \frac{\mathrm{i}}{\sqrt2}\lvert \tfrac52\,\overline{\tfrac12}\rangle + \sqrt{\tfrac{5}{12}}\lvert \tfrac52\,\overline{\tfrac52}\rangle \big\rvert^\bullet$

T **75**.6*b* Symmetrized harmonics § **16**–6, pp. 74, 75

One-dimensional representations					
				Column of the basis 1	
$\mathbf{I}_h$	**I**	j	m	Coefficient	±
A_g	A	0	0	1	+
A_g	A	6	0	0.207289049397	+
			2	−0.671693289381	+
			4	−0.548435274212	+
			6	0.452855523318	+
A_g	A	10	0	0.354444208042	+
			2	0.407225391560	+
			4	−0.505100933601	+
			6	0.079863469923	+
			8	−0.601190361084	+
			10	−0.292577864236	+
A_g	A	12	0	0.414484003842	+
			2	−0.166795045365	+
			4	0.258845991675	+
			6	0.655425507457	+
			8	0.104320463449	+
			10	−0.413539340047	+
			12	0.349143287647	+
A_u	A	15	2	0.569257046974	−
			4	0.132260240241	−
			6	0.260048962714	−
			8	−0.357453892009	−
			10	0.460708108493	−
			12	0.222342286524	−
			14	0.448739168998	−

$\twoheadrightarrow$

T **75**.6*b* Symmetrized harmonics *(cont.)*

Three-dimensional representations									
				Column of the basis 1		2		3	
$\mathbf{I}_h$	**I**	j	m	Coefficient	±	Coefficient	±	Coefficient	±
T_{1u}	T_1	1	0	0	+	1	+	0	−
			1	1	+	0	+	1	−
T_{2u}	T_2	3	0	0	+	0.5	+	0	−
			1	0.378466979034	+	0	+	−0.990839414729	−
			2	0	+	0.866025403784	+	0	−
			3	−0.925614793411	+	0	+	−0.135045378369	−
T_{1u}	T_1	5	0	0	+	0.265165042945	+	0	−
			1	0.364662582013	+	0	+	−0.706989180454	−
			2	0	+	−0.810092587301	+	0	−
			3	0.802671665078	+	0	+	−0.306593294253	−
			4	0	+	−0.522912516584	+	0	−
			5	−0.471952751195	+	0	+	0.637312208136	−
T_{2u}	T_2	5	0	0	+	0.330718913883	+	0	−
			1	0.926845403807	+	0	+	0.354023441938	−
			2	0	+	−0.433012701892	+	0	−

$\twoheadrightarrow$

T **75**.6b Symmetrized harmonics *(cont.)*

Three-dimensional representations										
				Column of the basis						
$\mathbf{I}_h$	$\mathbf{I}$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±
T_{2u}	T_2	5	3	−0.361523362929	+	0	+	0.626688405874	−
			4	0	+	0.838525491562	+	0	−
			5	0.101284033777	+	0	+	0.694211095058	−
T_{1g}	T_1	6	1	0.821560641538	−	0	−	−0.313808241248	+
			2	0	−	0.359035165409	−	0	+
			3	−0.137139230030	−	0	−	−0.939966266196	+
			4	0	−	0.586301969978	−	0	+
			5	−0.553381372891	−	0	−	−0.134118627109	+
			6	0	−	−0.726184377414	−	0	+
T_{1u}	T_1	7	0	0	+	−0.245267710879	+	0	−
			1	0.125528611918	+	0	+	−0.496338536273	−
			2	0	+	0.761628600435	+	0	−
			3	0.699118155346	+	0	+	0.377987340880	−
			4	0	+	0.532535210104	+	0	−
			5	0.478408622367	+	0	+	0.586661797840	−
			6	0	+	−0.275992527073	+	0	−
			7	−0.516334738808	+	0	+	−0.516334738808	−
T_{2u}	T_2	7	0	0	+	−0.701978231500	+	0	−
			1	0.510563028940	+	0	+	−0.062258932929	−
			2	0	+	−0.058463396668	+	0	−
			3	−0.264466360818	+	0	+	−0.893049837773	−
			4	0	+	0.122633855440	+	0	−
			5	0.703436131145	+	0	+	0.155000856933	−
			6	0	+	0.699120541287	+	0	−
			7	0.417804436160	+	0	+	−0.417804436160	−
T_{2g}	T_2	8	1	0.494352875611	−	0	−	0.494352875611	+
			2	0	−	0.792728180873	−	0	+
			3	−0.023035063234	−	0	−	0.413347438134	+
			4	0	−	−0.112673477358	−	0	+
			5	0.769519118672	−	0	−	−0.488269118672	+
			6	0	−	−0.226427761659	−	0	+
			7	0.403639624214	−	0	−	0.588517117436	+
			8	0	−	−0.554632479666	−	0	+
T_{1u}	T_1	9	0	0	+	−0.445381025429	+	0	−
			1	0.592722151408	+	0	+	0.004819197841	−
			2	0	+	−0.501365323392	+	0	−
			3	−0.236660103274	+	0	+	0.619584194152	−
			4	0	+	0.581703452156	+	0	−
			5	−0.096083758517	+	0	+	−0.251550545565	−
			6	0	+	−0.080282703617	+	0	−
			7	−0.643352129749	+	0	+	0.678115560158	−
			8	0	+	0.453259678261	+	0	−
			9	−0.411750404061	+	0	+	0.304916073349	−
T_{2u}	T_2	9	0	0	+	−0.173817152041	+	0	−
			1	0.067038010778	+	0	+	−0.662994027795	−
			2	0	+	−0.326109883705	+	0	−
			3	−0.419040298921	+	0	+	0.441683075087	−
			4	0	+	−0.745265001107	+	0	−
			5	0.221580518112	+	0	+	0.580105327661	−

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T **75**.6*b* Symmetrized harmonics *(cont.)*

Three-dimensional representations										
				Column of the basis						
$\mathbf{I}_h$	$\mathbf{I}$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±
T_{2u}	T_2	9	6	0	+	−0.431996636272	+	0	−
			7	0.595628155006	+	0	+	−0.141339509068	−
			8	0	+	0.348423486265	+	0	−
			9	−0.645011866518	+	0	+	−0.094105963071	−
T_{1g}	T_1	10	1	0.084656967375	−	0	−	0.783550484613	+
			2	0	−	−0.134502120104	−	0	+
			3	0.755796038194	−	0	−	−0.069967103164	+
			4	0	−	0.333658695376	−	0	+
			5	−0.454653620966	−	0	−	−0.336687457915	+
			6	0	−	−0.079134107888	−	0	+
			7	0.419169927117	−	0	−	−0.310410498646	+
			8	0	−	0.794266564196	−	0	+
			9	−0.197970209042	−	0	−	−0.414053282260	+
			10	0	−	0.483176440502	−	0	+
T_{2g}	T_2	10	1	0.675446422454	−	0	−	−0.533203129829	+
			2	0	−	−0.257089421609	−	0	+
			3	0.432662299876	−	0	−	0.013909384774	+
			4	0	−	0.455543116785	−	0	+
			5	0.461473376305	−	0	−	0.042720461203	+
			6	0	−	−0.727479679833	−	0	+
			7	−0.359807022686	−	0	−	−0.410588272686	+
			8	0	−	−0.038273277231	−	0	+
			9	0.118981906385	−	0	−	−0.738304262403	+
			10	0	−	−0.442373111442	−	0	+
T_{1u}	T_1	11	0	0	+	0.142357758247	+	0	−
			1	0.572903651071	+	0	+	0.304888329890	−
			2	0	+	0.520759355236	+	0	−
			3	0.140654883869	+	0	+	0.317153133394	−
			4	0	+	−0.640797738542	+	0	−
			5	−0.040582356492	+	0	+	−0.626508247537	−
			6	0	+	−0.251476714162	+	0	−
			7	0.100044456149	+	0	+	0.205625609524	−
			8	0	+	−0.484439555688	+	0	−
			9	−0.790486184544	+	0	+	−0.328995526538	−
			11	−0.124392047230	+	0	+	−0.513248930120	−
T_{1u}	T_1	11	1	0.553455779612	+	0	+	0.376570158695	−
			2	0	+	0.501476181031	+	0	−
			3	0.152374122614	+	0	+	0.205997465921	−
			4	0	+	−0.629503169473	+	0	−
			5	0.131366007588	+	0	+	−0.662731108593	−
			6	0	+	−0.387830515344	+	0	−
			7	0.156825724398	+	0	+	0.197984101936	−
			8	0	+	−0.444052670434	+	0	−
			9	−0.792741568205	+	0	+	−0.170811909372	−
			10	0	+	0.068206427339	+	0	−
			11	−0.013352791071	+	0	+	−0.555133339221	−
T_{2u}	T_2	11	0	0	+	−0.281437112237	+	0	−
			1	0.077571484083	+	0	+	−0.168901747638	−
			2	0	+	0.438518972406	+	0	−

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T **75**.6b Symmetrized harmonics *(cont.)*

Three-dimensional representations									
				Column of the basis					
$\mathbf{I}_h$	$\mathbf{I}$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±
T_{2u}	T_2	11	3	0.913450585536	+	0	+	0.091820783858	−
			4	0	+	−0.260391925908	+	0	−
			5	0.110331700111	+	0	+	0.543298148787	−
			6	0	+	0.169111754176	+	0	−
			7	−0.326039919967	+	0	+	0.567963446723	−
			8	0	+	0.645341583409	+	0	−
			9	0.184917594389	+	0	+	0.141926655516	−
			10	0	+	−0.464355212030	+	0	−
			11	−0.083192923164	+	0	+	−0.570212778235	−
T_{1g}	T_1	12	1	0.651156964009	−	0	−	0.364117351949	+
			2	0	−	0.046260622197	−	0	+
			3	−0.357016533018	−	0	−	0.073729234104	+
			4	0	−	−0.143581922374	−	0	+
			5	−0.300820574418	−	0	−	0.719430215584	+
			6	0	−	−0.545346986396	−	0	+
			7	0.412892810423	−	0	−	0.557559264287	+
			8	0	−	−0.115733163091	−	0	+
			9	0.299241502285	−	0	−	−0.166652507215	+
			10	0	−	0.573475882677	−	0	+
			11	−0.313073211553	−	0	−	−0.075877055816	+
			12	0	−	−0.581009550505	−	0	+
T_{2g}	T_2	12	1	0.328115858156	−	0	−	0.636134259705	+
			2	0	−	−0.546058262885	−	0	+
			3	0.085536518044	−	0	−	−0.267931624485	+
			4	0	−	−0.516607285235	−	0	+
			5	0.352088835423	−	0	−	0.174832653504	+
			6	0	−	0.195068206596	−	0	+
			7	−0.560907246852	−	0	−	−0.422008609634	+
			8	0	−	−0.385647980981	−	0	+
			9	−0.162189873641	−	0	−	0.529121196344	+
			10	0	−	−0.343322488064	−	0	+
			11	−0.648177788107	−	0	−	0.186865953218	+
			12	0	−	−0.360958425733	−	0	+
T_{1u}	T_1	13	0	0	+	0.498147776237	+	0	−
			1	0.276958184768	+	0	+	0.454122879619	−
			2	0	+	−0.198076150450	+	0	−
			3	−0.005932936520	+	0	+	−0.232955889284	−
			4	0	+	0.296001715939	+	0	−
			5	0.436027218361	+	0	+	−0.017417577195	−
			6	0	+	0.698805361825	+	0	−
			7	0.617010684806	+	0	+	0.564185964742	−
			8	0	+	0.098826002015	+	0	−
			9	0.037430294295	+	0	+	0.169868876390	−
			10	0	+	−0.317576563457	+	0	−
			11	−0.426296872405	+	0	+	−0.471945293860	−
			12	0	+	0.161391541807	+	0	−
			13	0.411469310501	+	0	+	0.411469310501	−
T_{2u}	T_2	13	0	0	+	0.311377429472	+	0	−
			1	0.203098369707	+	0	+	−0.085125424715	−

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T **75**.6*b* Symmetrized harmonics *(cont.)*

Three-dimensional representations									
				Column of the basis					
$\mathbf{I}_h$	**I**	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±
T_{2u}	T_2	13	2	0	+	0.546898858488	+	0	−
			3	−0.302411068415	+	0	+	−0.632184899889	−
			4	0	+	0.114302780060	+	0	−
			5	−0.246603689029	+	0	+	0.670834497482	−
			6	0	+	−0.373264365494	+	0	−
			7	0.179382342762	+	0	+	0.033340120979	−
			8	0	+	−0.218468363283	+	0	−
			9	−0.425705713987	+	0	+	0.253857049644	−
			10	0	+	−0.635472971233	+	0	−
			11	0.587164780928	+	0	+	0.266594988789	−
			13	0.498316524612	+	0	+	−0.080306525508	−
T_{2u}	T_2	13	1	0.072263387371	+	0	+	0.051706590083	−
			2	0	+	0.616785460863	+	0	−
			3	−0.541486303936	+	0	+	−0.396206418446	−
			4	0	+	0.256125292142	+	0	−
			5	−0.034657857949	+	0	+	0.767696792016	−
			6	0	+	−0.201340741692	+	0	−
			7	−0.051604290771	+	0	+	0.284386573041	−
			8	0	+	−0.352981647365	+	0	−
			9	−0.610665513151	+	0	+	−0.033552538514	−
			10	0	+	−0.543478659827	+	0	−
			11	0.335435966692	+	0	+	0.298048086681	−
			12	0	+	0.305732462564	+	0	−
			13	0.460735020212	+	0	+	−0.283115489352	−
T_{1g}	T_1	14	1	0.389833243005	−	0	−	−0.389833243005	+
			2	0	−	−0.702781873267	−	0	+
			3	0.396788764406	−	0	−	0.193666111925	+
			4	0	−	−0.158788281729	−	0	+
			5	0.278890594334	−	0	−	−0.163693467144	+
			6	0	−	−0.296894414167	−	0	+
			7	−0.242249388655	−	0	−	0.425496599419	+
			8	0	−	0.376659535928	−	0	+
			9	0.372393864337	−	0	−	−0.564774198236	+
			10	0	−	−0.427756740038	−	0	+
			11	0.330162710590	−	0	−	−0.159060014574	+
			12	0	−	−0.166180696885	−	0	+
			13	0.553569673589	−	0	−	−0.508341016648	+
			14	0	−	−0.200682257209	−	0	+
T_{2g}	T_2	14	1	0.388732104871	−	0	−	−0.262170207689	+
			2	0	−	0.374791452911	−	0	+
			3	−0.365673687016	−	0	−	−0.262453294346	+
			4	0	−	−0.605183195642	−	0	+
			5	−0.075104363880	−	0	−	−0.577192603006	+
			6	0	−	−0.292906426476	−	0	+
			7	−0.417499504847	−	0	−	0.176322769228	+
			8	0	−	0.279296875000	−	0	+
			9	−0.676322576319	−	0	−	0.183026067186	+
			10	0	−	0.240025747627	−	0	+
			11	0.036811301125	−	0	−	−0.549646800932	+

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T **75**.6*b* Symmetrized harmonics *(cont.)*

Three-dimensional representations									
				Column of the basis					
$\mathbf{I}_h$	$\mathbf{I}$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±
			12	0	−	0.138735474522	−	0	+
			13	0.276507331034	−	0	−	0.403154912570	+
			14	0	−	−0.502617007340	−	0	+
T_{1u}	T_1	15	0	0	+	−0.120686901393	+	0	−
			1	0.267064343161	+	0	+	−0.101806997440	−
			2	0	+	−0.120675194577	+	0	−
			3	0.432051982142	+	0	+	−0.511422383311	−
			4	0	+	−0.700060704030	+	0	−
			5	0.026970507060	+	0	+	0.455322689071	−
			6	0	+	0.393382948185	+	0	−
			7	−0.437560017019	+	0	+	0.123403554065	−
			8	0	+	0.196417113552	+	0	−
			9	−0.069817595906	+	0	+	0.036335949682	−
			10	0	+	0.225254481537	+	0	−
			11	−0.610791932969	+	0	+	0.436036123430	−
			12	0	+	0.486536506388	+	0	−
			13	0.227976299029	+	0	+	−0.414601270647	−
			15	0.346276002053	+	0	+	−0.377085238355	−
T_{1u}	T_1	15	1	0.490932637169	+	0	+	−0.490932637169	−
			2	0	+	0.127289745370	+	0	−
			3	−0.375149179690	+	0	+	−0.598430163596	−
			4	0	+	0.059148577580	+	0	−
			5	0.100999339140	+	0	+	−0.320328134921	−
			6	0	+	0.174446147432	+	0	−
			7	−0.475609073799	+	0	+	0.066496594220	−
			8	0	+	−0.319716480541	+	0	−
			9	0.574425230233	+	0	+	−0.056425328385	−
			10	0	+	0.515087324188	+	0	−
			11	−0.164475758480	+	0	+	−0.422814150339	−
			12	0	+	0.298303480164	+	0	−
			13	0.077127294000	+	0	+	−0.304960340320	−
			14	0	+	0.702387900232	+	0	−
			15	−0.137397748956	+	0	+	−0.137397748956	−
T_{2u}	T_2	15	0	0	+	0.120330804748	+	0	−
			1	0.553867125294	+	0	+	−0.325067152417	−
			2	0	+	0.290320964219	+	0	−
			3	−0.135326133749	+	0	+	0.487384882603	−
			4	0	+	−0.023229497441	+	0	−
			5	−0.134065872671	+	0	+	0.075496201002	−
			6	0	+	0.678811800277	+	0	−
			7	0.187400667410	+	0	+	−0.567009241726	−
			8	0	+	0.163650009684	+	0	−
			9	−0.542848121102	+	0	+	−0.283789004926	−
			10	0	+	0.162717061142	+	0	−
			11	−0.025581760546	+	0	+	−0.034359074072	−
			12	0	+	−0.621812670955	+	0	−
			13	0.386961981442	+	0	+	−0.405978457758	−
			15	−0.420413240211	+	0	+	−0.288189022361	−

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T **75**.6*b* Symmetrized harmonics *(cont.)*

Three-dimensional representations										
				Column of the basis						
$\mathbf{I}_h$	$\mathbf{I}$	j	m	1 Coefficient	$\pm$	2 Coefficient	$\pm$	3 Coefficient	$\pm$
T_{2u}	T_2	15	1	0.730269318598	+	0	+	0.098599368428	−
			2	0	+	0.194673252959	+	0	−
			3	−0.015735978758	+	0	+	0.481864364764	−
			4	0	+	−0.194819660070	+	0	−
			5	−0.475114215189	+	0	+	0.088561832375	−
			6	0	+	0.533610668966	+	0	−
			7	0.182582576570	+	0	+	−0.282341734186	−
			8	0	+	−0.346093708843	+	0	−
			9	−0.399461615555	+	0	+	−0.237354583347	−
			10	0	+	0.249596555153	+	0	−
			11	0.151683842047	+	0	+	−0.312912089933	−
			12	0	+	−0.639316279622	+	0	−
			13	0.075238186208	+	0	+	−0.610192345123	−
			14	0	+	−0.220460149602	+	0	−
			15	−0.138386529838	+	0	+	−0.379392900817	−

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T **75**.6*b* Symmetrized harmonics *(cont.)*

Four-dimensional representations												
				Column of the basis								
$\mathbf{I}_h$	$\mathbf{I}$	j	m	1 Coefficient	$\pm$	2 Coefficient	$\pm$	3 Coefficient	$\pm$	4 Coefficient	$\pm$
F_u	F	3	0	0	−	0	+	−0.866025403784	+	0	−
			1	0	−	0.925614793411	+	0	+	0.135045378369	−
			2	1	−	0	+	0.5	+	0	−
			3	0	−	0.378466979034	+	0	+	−0.990839414729	−
F_g	F	4	0	0.763762615826	+	0	−	0	−	0	+
			1	0	+	0.333776501991	−	0	−	0.873838226858	+
			2	0	+	0	−	−0.763762615826	−	0	+
			3	0	+	−0.942652240606	−	0	−	−0.486216776018	+
			4	0.645497224368	+	0	−	−0.645497224368	−	0	+
F_g	F	6	0	0.286410980935	+	0	−	0	−	0	+
			1	0	+	0.511271242969	−	0	−	−0.886271242969	+
			2	0.486135912066	+	0	−	−0.618718433538	−	0	+
			3	0	+	−0.252269356817	−	0	−	0.340657704465	+
			4	0.757772228311	+	0	−	−0.433012701892	−	0	+
			5	0	+	0.821560641538	−	0	−	−0.313808241248	+
			6	0.327752765053	+	0	−	−0.655505530106	−	0	+
F_u	F	7	0	0	−	0	+	−0.266634112596	+	0	−
			1	0	−	−0.764923830499	+	0	+	0.361810943084	−
			2	0.735980072194	−	0	+	−0.643982563170	+	0	−
			3	0	−	0.239605063030	+	0	+	−0.109500938064	−
			4	0	−	0	+	0.578926808845	+	0	−
			5	0	−	0.363173525750	+	0	+	0.794680091941	−
			6	0.677003200386	−	0	+	−0.423127000241	+	0	−
			7	0	−	0.474958879799	+	0	+	0.474958879799	−

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T **75**.6*b* Symmetrized harmonics *(cont.)*

Four-dimensional representations											
				Column of the basis							
$\mathbf{I}_h$	$\mathbf{I}$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±	4 Coefficient	±
F_g	F	8	0	0.718070330817	+	0	−	0	−	0	+
			1	0	+	−0.612518059906	−	0	−	−0.344909047850	+
			2	0	+	0	−	0.223897763127	−	0	+
			3	0	+	−0.452434714914	−	0	−	0.287075257973	+
			4	0.381881307913	+	0	−	0.859232942804	−	0	+
			5	0	+	0.040630032501	−	0	−	−0.729076350912	+
			6	0	+	0	−	−0.356304820343	−	0	+
			7	0	+	0.646895397640	−	0	−	−0.516791272674	+
			8	0.581843335157	+	0	−	0.290921667579	−	0	+
F_u	F	9	0	0	−	0	+	0.150738993836	+	0	−
			1	0	−	0.115636337034	+	0	+	0.647189631088	−
			2	0.433012701892	−	0	+	0.656284876305	+	0	−
			3	0	−	−0.796106327086	+	0	+	0.139835982349	−
			4	0	−	0	+	0.102049613106	+	0	−
			5	0	−	0.125111633855	+	0	+	0.643315854229	−
			6	0.901387818866	−	0	+	−0.154926031368	+	0	−
			7	0	−	0.009107830659	+	0	+	0.240936034511	−
			8	0	−	0	+	0.715647761220	+	0	−
			9	0	−	0.580609231778	+	0	+	−0.299476240167	−
F_u	F	9	0	0	−	0	+	0.697753735403	+	0	−
			1	0	−	0.387310907427	+	0	+	0.340793991733	−
			2	0	−	0	+	−0.436367348935	+	0	−
			3	0	−	0.303614283173	+	0	+	0.332649573695	−
			4	0.841625411530	−	0	+	−0.059176786748	+	0	−
			5	0	−	0.856538142412	+	0	+	0.106172868135	−
			6	0	−	0	+	0.438304923603	+	0	−
			7	0	−	0.018465426881	+	0	+	−0.565835401449	−
			8	0.540061724867	−	0	+	0.356525123057	+	0	−
			9	0	−	0.154305108942	+	0	+	0.664651289668	−
F_g	F	10	0	0.208902218080	+	0	−	0	−	0	+
			1	0	+	0.362781113941	−	0	−	−0.052929051206	+
			2	0.690938960499	+	0	−	−0.210918208994	−	0	+
			3	0	+	−0.297342153290	−	0	−	−0.852304673140	+
			4	0.297696232551	+	0	−	0.487139289629	−	0	+
			5	0	+	0.376218110434	−	0	−	−0.076682336799	+
			6	0.135504278549	+	0	−	0.674668671094	−	0	+
			7	0	+	0.654385912922	−	0	−	0.457129957094	+
			8	0.354329389702	+	0	−	0.128847050801	−	0	+
			9	0	+	−0.458493748698	−	0	−	−0.236489494973	+
			10	0.496416602623	+	0	−	−0.496416602623	−	0	+
F_u	F	11	0	0	−	0	+	0.516795546342	+	0	−
			1	0	−	−0.089092869006	+	0	+	−0.610651608632	−
			2	0.665363309278	−	0	+	0.207926034149	+	0	−
			3	0	−	−0.297905571335	+	0	+	−0.402283612084	−
			4	0	−	0	+	−0.076226943235	+	0	−
			5	0	−	0.500333191702	+	0	+	−0.449913807952	−
			6	0.459279326772	−	0	+	−0.105251512385	+	0	−
			7	0	−	−0.671772427144	+	0	+	0.485183385725	−
			8	0	−	0	+	0.530847733730	+	0	−

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T **75**.6b Symmetrized harmonics *(cont.)*

Four-dimensional representations											
				Column of the basis							
				1		2		3		4	
$\mathbf{I}_h$	$\mathbf{I}$	j	m	Coefficient	±	Coefficient	±	Coefficient	±	Coefficient	±
F_u	F	11	9	0	−	−0.162392543099	+	0	+	−0.043203900112	−
			10	0.588518620493	−	0	+	0.625301034274	+	0	−
			11	0	−	−0.418726517689	+	0	+	0.159939297766	−
F_g	F	12	0	0.558504224597	+	0	−	0	−	0	+
			1	0	+	0.384177172360	−	0	−	0.102315511639	+
			2	0.123783984398	+	0	−	0.696727794987	−	0	+
			3	0	+	0.651657366837	−	0	−	−0.039069132324	+
			4	0.199081119261	+	0	−	−0.175572173336	−	0	+
			5	0	+	0.238713424055	−	0	−	−0.353206241495	+
			6	0.486412414780	+	0	−	0.300562227035	−	0	+
			7	0	+	−0.025229392930	−	0	−	0.537090173259	+
			8	0.356502977419	+	0	−	−0.625458688541	−	0	+
			9	0	+	0.529579601284	−	0	−	0.139073689861	+
			10	0.306900886081	+	0	−	0.044682887629	−	0	+
			11	0	+	0.299458513482	−	0	−	0.745280581073	+
			12	0.418614022974	+	0	−	−0.014560487756	−	0	+
F_g	F	12	1	0	+	−0.551150511024	−	0	−	−0.095335145325	+
			2	0.033582307569	+	0	−	0.010495371626	−	0	+
			3	0	+	0.230240464142	−	0	−	−0.932016976476	+
			4	0.551813621659	+	0	−	−0.394317180848	−	0	+
			5	0	+	−0.093074554148	−	0	−	0.036096244140	+
			6	0.131962558791	+	0	−	−0.651564974680	−	0	+
			7	0	+	0.190575086409	−	0	−	0.038228787830	+
			8	0.795931682368	+	0	−	−0.216394806820	−	0	+
			9	0	+	0.500834810153	−	0	−	−0.345195393922	+
			10	0.083261497840	+	0	−	−0.164700589849	−	0	+
			11	0	+	−0.589410892009	−	0	−	0.018202378453	+
			12	0.191102377347	+	0	−	0.587241866725	−	0	+
F_u	F	13	0	0	−	0	+	−0.138080768048	+	0	−
			1	0	−	0.584383654804	+	0	+	0.038712663408	−
			2	0.497389016096	−	0	+	−0.296296191229	+	0	−
			3	0	−	0.440986650450	+	0	+	−0.173562475601	−
			4	0	−	0	+	0.848242025092	+	0	−
			5	0	−	−0.251280659348	+	0	+	−0.209693422040	−
			6	0.493446636764	−	0	+	−0.328643170188	+	0	−
			7	0	−	−0.305487905332	+	0	+	0.145840345156	−
			8	0	−	0	+	0.143792981826	+	0	−
			9	0	−	−0.441564406249	+	0	+	−0.393908389717	−
			10	0.713522657898	−	0	+	−0.026478379883	+	0	−
			11	0	−	−0.334055786551	+	0	+	0.655303769827	−
			12	0	−	0	+	−0.210342282699	+	0	−
			13	0	−	0.031453054901	+	0	+	0.564402169551	−
F_u	F	13	0	0	−	0	+	−0.425053885143	+	0	−
			1	0	−	0.386820143221	+	0	+	−0.357114973715	−
			2	0	−	0	+	−0.279228593354	+	0	−
			3	0	−	−0.570088482656	+	0	+	0.253030308466	−
			4	0.786441087007	−	0	+	−0.249602884060	+	0	−
			5	0	−	−0.358623002523	+	0	+	−0.196388694538	−
			6	0	−	0	+	−0.068867745862	+	0	−

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T **75**.6*b* Symmetrized harmonics *(cont.)*

Four-dimensional representations											
				Column of the basis							
$\mathbf{I}_h$	$\mathbf{I}$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±	4 Coefficient	±
F_u	F	13	7	0	−	0.291529769710	+	0	+	0.565465587306	−
			8	0.228217732294	−	0	+	0.497777641678	+	0	−
			9	0	−	0.049085407390	+	0	+	0.573849954468	−
			10	0	−	0	+	−0.624157277829	+	0	−
			11	0	−	−0.186929377859	+	0	+	0.332272185817	−
			12	0.573957388081	−	0	+	0.192253304797	+	0	−
			13	0	−	−0.523848513075	+	0	+	−0.102059458482	−
F_g	F	14	0	0.440096461964	+	0	−	0	−	0	+
			1	0	+	−0.360509541371	−	0	−	0.588005115441	+
			2	0	+	0	−	−0.455691525040	−	0	+
			3	0	+	0.118876696293	−	0	−	0.285572606407	+
			4	0.457681828621	+	0	−	−0.289626782174	−	0	+
			5	0	+	−0.086017283280	−	0	−	−0.603997724073	+
			6	0	+	0	−	−0.191397019897	−	0	+
			7	0	+	0.560896150710	−	0	−	0.322818354708	+
			8	0.491132301422	+	0	−	−0.069065479887	−	0	+
			9	0	+	−0.386308363042	−	0	−	0.100838400471	+
			10	0	+	0	−	0.575045843085	−	0	+
			11	0	+	0.376973703412	−	0	−	0.294679841312	+
			12	0.596348480686	+	0	−	0.358740882913	−	0	+
			13	0	+	0.492498134346	−	0	−	−0.081662341250	+
			14	0	+	0	−	0.455725749321	−	0	+
F_g	F	14	1	0	+	0.271945736581	−	0	−	0.315749989717	+
			2	0.777543289926	+	0	−	−0.112379303622	−	0	+
			3	0	+	−0.134824027417	−	0	−	−0.408714556148	+
			4	0	+	0	−	−0.187346315979	−	0	+
			5	0	+	−0.657633028117	−	0	−	−0.080192901305	+
			6	0.248530839384	+	0	−	0.746563341744	−	0	+
			7	0	+	0.048758705394	−	0	−	−0.120064139601	+
			8	0	+	0	−	0.299290915768	−	0	+
			9	0	+	0.205575906746	−	0	−	−0.763761148030	+
			10	0.020171788261	+	0	−	−0.172169364654	−	0	+
			11	0	+	0.609306452386	−	0	−	0.090875229222	+
			12	0	+	0	−	0.520981363877	−	0	+
			13	0	+	−0.243913059534	−	0	−	0.347589784117	+
			14	0.577279787560	+	0	−	−0.065394975934	−	0	+
F_u	F	15	0	0	−	0	+	−0.557692090532	+	0	−
			1	0	−	0.370970224370	+	0	+	0.181491811742	−
			2	0.281254273841	−	0	+	0.391941083962	+	0	−
			3	0	−	0.042213676485	+	0	+	−0.195497463750	−
			4	0.267693972303	−	0	+	−0.299433300104	+	0	−
			5	0	−	0.504093354223	+	0	+	0.046776455883	−
			6	0.191929542911	−	0	+	−0.465386032670	+	0	−
			7	0	−	0.671095556283	+	0	+	−0.106184616134	−
			8	0.723484639771	−	0	+	0.265069803364	+	0	−
			9	0	−	0.199965623650	+	0	+	−0.514073398584	−
			10	0.157970146898	−	0	+	0.206068273630	+	0	−
			11	0	−	−0.191330397796	+	0	+	−0.131073027609	−
			12	0.450019520469	−	0	+	−0.298101307862	+	0	−

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T **75**.6*b* Symmetrized harmonics *(cont.)*

Four-dimensional representations												
				Column of the basis								
				1		2		3		4		
$\mathbf{I}_h$	$\mathbf{I}$	j	m	Coefficient	±	Coefficient	±	Coefficient	±	Coefficient	±
F_u	F	15	13	0	−	−0.122990954565	+	0	+	0.553761533836	−
			14	0.247985606634	−	0	+	0.165911827318	+	0	−
			15	0	−	0.253770360849	+	0	+	−0.572079382914	−
F_u	F	15	0	0	−	0	+	−0.080996363451	+	0	−
			1	0	−	−0.063194258147	+	0	+	−0.489898036054	−
			2	0	−	0	+	0.501408538968	+	0	−
			3	0	−	0.540077021809	+	0	+	0.350628352895	−
			4	0.026285281888	−	0	+	0.369899173025	+	0	−
			5	0	−	−0.291372437549	+	0	+	0.407158506795	−
			6	0.646150326007	−	0	+	0.053726041856	+	0	−
			7	0	−	0.265681792712	+	0	+	0.512900140129	−
			8	0.071040066888	−	0	+	0.074761360074	+	0	−
			9	0	−	0.503069154800	+	0	+	0.185909142282	−
			10	0.709359293218	−	0	+	0.386317380319	+	0	−
			11	0	−	−0.100754397467	+	0	+	−0.269323047411	−
			12	0.044188107221	−	0	+	0.384031219455	+	0	−
			13	0	−	0.219282904832	+	0	+	0.225008642064	−
			14	0.267598492819	−	0	+	−0.547728786506	+	0	−
			15	0	−	−0.487363897347	+	0	+	−0.224697723536	−

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T **75**.6*b* Symmetrized harmonics *(cont.)*

				Five-dimensional representations									
				Column of the basis									
$\mathbf{I}_h$	**I**	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±	4 Coefficient	±	5 Coefficient	±
H_g	H	2	0	0.707106781187	+	0.707106781187	+	0	−	0	−	0	+
			1	0	+	0	+	1	−	0	−	−1	+
			2	0		0		0		−1		0	
				−0.707106781187 i	+	0.707106781187 i	+	0 i	−	0 i	−	0 i	+
H_g	H	4	0	0.456435464588		−0.242481340562		0		0		0	
				0 i	+	−0.386699020961 i	+	0 i	−	0 i	−	0 i	+
			1	0		0		−0.456359553641		0		−0.235388684521	
				0 i	+	0 i	+	0.824820710530 i	−	0 i	−	0.425439679016 i	+
			2	0		−0.599071547271		0		−0.3125		0	
				0.707106781187 i	+	0.375650477505 i	+	0 i	−	0.564810071322 i	−	0 i	+
			3	0		0		−0.161588854196		0		−0.423045112488	
				0 i	+	0 i	+	0.292054439242 i	−	0 i	−	0.764608448501 i	+
			4	−0.540061724867		0.286907791336		0		0.369754986444		0	
				0 i	+	0.457548452011 i	+	0 i	−	−0.668292288848 i	−	0 i	+
H_u	H	5	0	0		0		0		−0.554632479666		0	
				0 i	−	0 i	−	0 i	+	0.716027452337 i	+	0 i	−
			1	0		0		−0.054699629983		0		−0.374916841419	
				0 i	−	0 i	−	0.070616918656 i	+	0 i	+	0.484015561010 i	−
			2	0.707106781187		−0.176776695297		0		−0.242061459138		0	
				0 i	−	−0.684653196881 i	−	0 i	+	0.3125 i	+	0 i	−
			3	0		0		−0.290486051389		0		0.438717816709	
				0 i	−	0 i	−	0.375015879779 i	+	0 i	+	−0.566382265933 i	−
			4	0		0.684653196881		0		0.09375		0	
				−0.707106781187 i	−	−0.176776695297 i	−	0 i	+	−0.121030729569 i	+	0 i	−
			5	0		0		−0.536307565142		0		−0.204851261460	
				0 i	−	0 i	−	0.692370089413 i	+	0 i	+	0.264461841362 i	−
H_g	H	6	0	0.661437827766		−0.578758099295		0		0		0	
				0 i	+	0.320217211436 i	+	0 i	−	0 i	−	0 i	+
			1	0		0		−0.063067339204		0		−0.085164426116	
				0 i	+	0 i	+	−0.244258754427 i	−	0 i	−	−0.329840404037 i	+
			2	0		−0.191366386155		0		0.174692810742		0	
				−0.395284707521 i	+	−0.345874119081 i	+	0 i	−	0.676582346707 i	−	0 i	+

Complex coefficients are given in two lines. ⇛

$\mathbf{C}_n$ 107 $\mathbf{C}_i$ 137 $\mathbf{S}_n$ 143 $\mathbf{D}_n$ 193 $\mathbf{D}_{nh}$ 245 $\mathbf{D}_{nd}$ 365 $\mathbf{C}_{nv}$ 481 $\mathbf{C}_{nh}$ 531 **O** 579 **I**

$\mathbf{C}_n$ 107 $\mathbf{C}_i$ 137 $\mathbf{S}_n$ 143 $\mathbf{D}_n$ 193 $\mathbf{D}_{nh}$ 245 $\mathbf{D}_{nd}$ 365 $\mathbf{C}_{nv}$ 481 $\mathbf{C}_{nh}$ 531 **O** 579 **I**

T **75**.6*b* Symmetrized harmonics *(cont.)*

				Five-dimensional representations									
				Column of the basis									
$\mathbf{I}_h$	**I**	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±	4 Coefficient	±	5 Coefficient	±
H_g	H	6	3	0		0		−0.239472467520		0		−0.005097467520	
				0 i	+	0 i	+	−0.927472878578 i	−	0 i	−	−0.019742406811 i	+
			4	0.250000000000		−0.21875		0		−0.171163299220		0	
				0 i	+	0.121030729569 i	+	0 i	−	−0.662912607362 i	−	0 i	+
			5	0		0		−0.034284807507		0		0.234991566549	
				0 i	+	0 i	+	−0.132784488504 i	−	0 i	−	0.910118423744 i	+
			6	0		−0.283842220697		0		−0.051822262349		0	
				−0.586301969978 i	+	−0.513014223731 i	+	0 i	−	−0.200706759042 i	−	0 i	+
H_u	H	7	0	0		0		0		0.445273949812		0	
				0 i	−	0 i	−	0 i	+	0.421553878074 i	+	0 i	−
			1	0		0		−0.270214265182		0		−0.571274403744	
				0 i	−	0 i	−	−0.255819751967 i	+	0 i	+	−0.540842194889 i	−
			2	0.478713553878		0.026179647478		0		−0.030726821724		0	
				0 i	−	0.477997168114 i	−	0 i	+	−0.029089981267 i	+	0 i	−
			3	0		0		−0.449930128930		0		−0.158429903215	
				0 i	−	0 i	−	−0.425962019096 i	+	0 i	+	−0.149990225414 i	−
			4	0		0.706048609284		0		0.439453125000		0	
				0.707106781187 i	−	−0.038669902096 i	−	0 i	+	0.416043132893 i	+	0 i	−
			5	0		0		0.275957506362		0		−0.012285631362	
				0 i	−	0 i	−	0.261257046454 i	+	0 i	+	−0.011631166718 i	−
			6	−0.520416499867		−0.028460277336		0		0.367439345979		0	
				0 i	−	−0.519637706433 i	−	0 i	+	0.347865580998 i	+	0 i	−
			7	0		0		−0.419211563123		0		0.419211563123	
				0 i	−	0 i	−	−0.396879853947 i	+	0 i	+	0.396879853947 i	−
H_g	H	8	0	0.492125492126		0.185167177355		0		0		0	
				0 i	+	0.163796345217 i	+	0 i	−	0 i	−	0 i	+
			1	0		0		0.231069506480		0		0.635723702978	
				0 i	+	0 i	+	−0.230928108103 i	−	0 i	−	0.269515184710 i	+
			2	0		0.153137712786		0		−0.467677704981		0	
				0.460101671793 i	+	0.291158089182 i	+	0 i	−	−0.144379620222 i	−	0 i	+
			3	0		0		−0.739057150537		0		0.092052487841	
				0 i	+	0 i	+	−0.162097092395 i	−	0 i	−	−0.424375697369 i	+

All coefficients are complex and are given in two lines.

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$\mathbf{C}_n$ 107 $\mathbf{C}_i$ 137 $\mathbf{S}_n$ 143 $\mathbf{D}_n$ 193 $\mathbf{D}_{nh}$ 245 $\mathbf{D}_{nd}$ 365 $\mathbf{C}_{nv}$ 481 $\mathbf{C}_{nh}$ 531 **O** 579 **I**

T **75**.6b Symmetrized harmonics *(cont.)*

				Five-dimensional representations									
				Column of the basis									
				1		2		3		4		5	
$\mathbf{I}_h$	**I**	j	m	Coefficient	±	Coefficient	±	Coefficient	±	Coefficient	±	Coefficient	±
H_g	H	8	4	−0.278605397905		−0.615979121931		0		0.190748076344		0	
				0 i	+	−0.092729490068 i	+	0 i	−	−0.184691030751 i	−	0 i	+
			5	0		0		−0.014685257746		0		0.100654253990	
				0 i	+	0 i	+	0.328480975638 i	−	0 i	−	−0.217495478723 i	+
			6	0		0.178712744984		0		−0.308486066093		0	
				0.536941758120 i	+	−0.599864620400 i	+	0 i	−	−0.505475975449 i	−	0 i	+
			7	0		0		−0.297179735082		0		−0.515150923934	
				0 i	+	0 i	+	−0.352656986102 i	−	0 i	−	−0.108777337282 i	+
			8	−0.424489731629		0.175765279502		0		−0.581256503606		0	
				0 i	+	−0.141284830262 i	+	0 i	−	0.037519945972 i	−	0 i	+
H_g	H	8	0	0		0		0		0		0	
				0 i	+	0.425524821514 i	+	0 i	−	0 i	−	0 i	+
			1	0		0		−0.456505109987		0		0.184269955213	
				0 i	+	0 i	+	0.255575893392 i	−	0 i	−	0.354844339640 i	+
			2	0.536941758120		0.599864620400		0		−0.049781724855		0	
				0 i	+	−0.178712744984 i	+	0 i	−	−0.281790156528 i	−	0 i	+
			3	0		0		0.026479507589		0		−0.710897213784	
				0 i	+	0 i	+	−0.470733588304 i	−	0 i	−	0.229757372127 i	+
			4	0		0.196756256593		0		−0.367390363863		0	
				−0.591153419673 i	+	−0.018473544365 i	+	0 i	−	0.208691524917 i	−	0 i	+
			5	0		0		0.528484208302		0		−0.384924413423	
				0 i	+	0 i	+	−0.137034885638 i	−	0 i	−	0.156328423942 i	+
			6	−0.460101671793		0.291158089182		0		−0.685803196053		0	
				0 i	+	0.153137712786 i	+	0 i	−	−0.027958993233 i	−	0 i	+
			7	0		0		−0.446918786953		0		0.025158921142	
				0 i	+	0 i	+	−0.078627293955 i	−	0 i	−	−0.329739940141 i	+
			8	0		−0.129137058153		0		0.283460962069		0	
				0.387991796832 i	+	−0.513028215732 i	+	0 i	−	−0.433740025230 i	−	0 i	+
H_u	H	9	0	0		0		0		−0.127925960108		0	
				0 i	−	0 i	−	0 i	+	−0.495455113046 i	+	0 i	−
			1	0		0		−0.173350233431		0		0.039859700083	
				0 i	−	0 i	−	−0.671382567138 i	+	0 i	+	0.154375954605 i	−

All coefficients are complex and are given in two lines.

⇛

T **75**.6*b* Symmetrized harmonics *(cont.)*

				Five-dimensional representations									
				Column of the basis									
$\mathbf{I}_h$	**I**	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±	4 Coefficient	±	5 Coefficient	±
H_u	H	9	2	0.637377439199		−0.557705259299		0		0.036365208447		0	
				0 i	−	0.308569025908 i	−	0 i	+	0.140841846698 i	+	0 i	−
			3	0		0		−0.051496545045		0		0.134819705232	
				0 i	−	0 i	−	−0.199445261347 i	+	0 i	+	0.522154473102 i	−
			4	0		−0.184877493222		0		−0.075946180059		0	
				−0.381881307913 i	−	−0.334146144424 i	−	0 i	+	−0.294138290578 i	+	0 i	−
			5	0		0		0.109647011761		0		−0.104604064696	
				0 i	−	0 i	−	0.424661050513 i	+	0 i	+	−0.405129800513 i	−
			6	0.306186217848		−0.267912940617		0		0.192162091816		0	
				0 i	−	0.148231765320 i	−	0 i	+	0.744240581377 i	+	0 i	−
			7	0		0		−0.120130453451		0		−0.094194796827	
				0 i	−	0 i	−	−0.465263245588 i	+	0 i	+	−0.364814879412 i	−
			8	0		−0.288110764290		0		0.046026389693		0	
				−0.595119035712 i	−	−0.520729156248 i	−	0 i	+	0.178259440767 i	+	0 i	−
			9	0		0		−0.057827410313		0		−0.151394125680	
				0 i	−	0 i	−	−0.223964597095 i	+	0 i	+	−0.586346927472 i	−
H_g	H	10	0	0.644487845761		0.147221285753		0		0		0	
				0 i	+	−0.447465611754 i	+	0 i	−	0 i	−	0 i	+
			1	0		0		−0.332082271941		0		0.228938119664	
				0 i	+	0 i	+	−0.019049641562 i	−	0 i	−	−0.078599945645 i	+
			2	0		0.037940377475		0		0.387059870460		0	
				−0.314647791600 i	+	0.005640658652 i	+	0 i	−	−0.563646327089 i	−	0 i	+
			3	0		0		0.186428107772		0		−0.110142009037	
				0 i	+	0 i	+	0.029944617195 i	−	0 i	−	0.279404312250 i	+
			4	0.187140459880		−0.084523889350		0		0.518511177466		0	
				0 i	+	0.216942102153 i	+	0 i	−	−0.258970595297 i	−	0 i	+
			5	0		0		0.244707131721		0		0.731818240046	
				0 i	+	0 i	+	0.116062956368 i	−	0 i	−	−0.377191048726 i	+
			6	0		0.628126816550		0		0.026114535874		0	
				−0.343798977702 i	+	0.221421314252 i	+	0 i	−	0.024723395286 i	−	0 i	+
			7	0		0		−0.276200305331		0		−0.093803241339	
				0 i	+	0 i	+	0.320418015787 i	−	0 i	−	0.364498673160 i	+

All coefficients are complex and are given in two lines.

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$\mathbf{C}_n$ 107 $\mathbf{C}_i$ 137 $\mathbf{S}_n$ 143 $\mathbf{D}_n$ 193 $\mathbf{D}_{nh}$ 245 $\mathbf{D}_{nd}$ 365 $\mathbf{C}_{nv}$ 481 $\mathbf{C}_{nh}$ 531 **O** 579 **I**

$\mathbf{C}_n$ 107 $\mathbf{C}_i$ 137 $\mathbf{S}_n$ 143 $\mathbf{D}_n$ 193 $\mathbf{D}_{nh}$ 245 $\mathbf{D}_{nd}$ 365 $\mathbf{C}_{nv}$ 481 $\mathbf{C}_{nh}$ 531 **O** 579 **I**

T **75**.6*b* Symmetrized harmonics *(cont.)*

				Five-dimensional representations									
				Column of the basis									
$\mathbf{I}_h$	**I**	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±	4 Coefficient	±	5 Coefficient	±
H_g	H	10	8	0.222741700053		0.157811624403		0		−0.328829886700		0	
				0 i	+	−0.446080426565 i	+	0 i	−	0.003691457029 i	−	0 i	+
			9	0		0		−0.577073180458		0		−0.072902743085	
				0 i	+	0 i	+	0.518060225347 i	−	0 i	−	−0.162502890981 i	+
			10	0		0.224263965990		0		0.294508864792		0	
				−0.531788520159 i	+	0.068291201563 i	+	0 i	−	0.019911210725 i	−	0 i	+
H_g	H	10	0	0		−0.412928476438		0		0		0	
				0 i	+	−0.151509326440 i	+	0 i	−	0 i	−	0 i	+
			1	0		0		0.498833022415		0		−0.199410002165	
				0 i	+	0 i	+	−0.213352485814 i	−	0 i	−	0.024220092640 i	+
			2	0.281748440826		0.151398515200		0		0.341342526327		0	
				0 i	+	−0.392418930467 i	+	0 i	−	−0.536004172053 i	−	0 i	+
			3	0		0		−0.310361292595		0		−0.284587096564	
				0 i	+	0 i	+	0.145557807710 i	−	0 i	−	0.311929527690 i	+
			4	0		−0.486573291818		0		−0.324126550222		0	
				0.541390292004 i	+	−0.192777768771 i	+	0 i	−	−0.053572617128 i	−	0 i	+
			5	0		0		−0.528281832660		0		−0.439063461785	
				0 i	+	0 i	+	0.293868093557 i	−	0 i	−	−0.091261108488 i	+
			6	−0.607809568257		−0.086215175575		0		−0.075809370005		0	
				0 i	+	0.191380395442 i	+	0 i	−	0.047885455040 i	−	0 i	+
			7	0		0		−0.114748513363		0		−0.441684220277	
				0 i	+	0 i	+	0.272934096022 i	−	0 i	−	0.435145393778 i	+
			8	0		0.165352812667		0		0.458422337284		0	
				−0.454858826147 i	+	0.072640266029 i	+	0 i	−	−0.181053743919 i	−	0 i	+
			9	0		0		−0.001283134077		0		0.358877751125	
				0 i	+	0 i	+	0.367468482411 i	−	0 i	−	−0.258890275126 i	+
			10	0.226241784000		0.187190772943		0		−0.447144538068		0	
				0 i	+	−0.493949364684 i	+	0 i	−	0.193253586203 i	−	0 i	+
H_u	H	11	0	0		0		0		0.084442347761		0	
				0 i	−	0 i	−	0 i	+	−0.324927018201 i	+	0 i	−
			1	0		0		−0.148926341678		0		−0.056635327753	
				0 i	−	0 i	−	0.575046339015 i	+	0 i	+	0.124423328139 i	−

All coefficients are complex and are given in two lines.

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T **75**.6b Symmetrized harmonics *(cont.)*

Five-dimensional representations													
				Column of the basis									
$\mathbf{I}_h$	$\mathbf{I}$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±	4 Coefficient	±	5 Coefficient	±
H_u	H	11	2	0.527869144138		−0.444011719317		0		0.109760050537		0	
				0 i	−	−0.283355448183 i	−	0 i	+	−0.302697905375 i	+	0 i	−
			3	0		0		0.055060752623		0		0.197229565566	
				0 i	−	0 i	−	−0.199863385732 i	+	0 i	+	−0.665402119894 i	−
			4	0		−0.203997178600		0		0.169414645885		0	
				0.350233566926 i	−	0.331764218670 i	−	0 i	+	−0.526566872338 i	+	0 i	−
			5	0		0		0.040390600209		0		−0.059483867642	
				0 i	−	0 i	−	−0.106215594135 i	+	0 i	+	0.190046823799 i	−
			6	−0.289453945298		0.209741042394		0		−0.143728895745		0	
				0 i	−	0.140861139166 i	−	0 i	+	0.536538589652 i	+	0 i	−
			7	0		0		0.144361464535		0		0.030833225171	
				0 i	−	0 i	−	−0.452213920354 i	+	0 i	+	−0.176940580124 i	−
			8	0		−0.320618913539		0		−0.006680823256		0	
				0.614277175710 i	−	0.495500924938 i	−	0 i	+	−0.002874968579 i	+	0 i	−
			9	0		0		−0.055285362486		0		0.169257331152	
				0 i	−	0 i	−	0.247191852063 i	+	0 i	+	−0.562940284533 i	−
			10	−0.370905082492		0.338306002976		0		−0.104155632028		0	
				0 i	−	0.210426153444 i	−	0 i	+	0.397758331989 i	+	0 i	−
			11	0		0		−0.169385003840		0		0.064699314283	
				0 i	−	0 i	−	0.522554509615 i	+	0 i	+	−0.279286950269 i	−
H_u	H	11	0	0		0		0		0.025988552856		0	
				0 i	−	0 i	−	0 i	+	−0.301959697207 i	+	0 i	−
			1	0		0		−0.047836725893		0		0.076627453367	
				0 i	−	0 i	−	0.549345419131 i	+	0 i	+	−0.586514088476 i	−
			2	0		−0.013744978786		0		−0.086577001736		0	
				0 i	−	0.031940444359 i	−	0 i	+	0.617167149385 i	+	0 i	−
			3	0		0		0.004869093072		0		−0.033373260397	
				0 i	−	0 i	−	−0.095582948780 i	+	0 i	+	0.083893943038 i	−
			4	0.614277175710		−0.573485332010		0		−0.073927763459		0	
				0 i	−	−0.139399527626 i	−	0 i	+	0.451750905902 i	+	0 i	−
			5	0		0		−0.037064293807		0		0.020764754049	
				0 i	−	0 i	−	0.270773821946 i	+	0 i	+	−0.115058468397 i	−

All coefficients are complex and are given in two lines. ⇛

T **75**.6b Symmetrized harmonics *(cont.)*

				Five-dimensional representations									
				Column of the basis									
				1		2		3		4		5	
$\mathbf{I}_h$	$\mathbf{I}$	j	m	Coefficient	±	Coefficient	±	Coefficient	±	Coefficient	±	Coefficient	±
H_u	H	11	6	0		0.142315886268		0		−0.027619186771		0	
				−0.557447453624 i	−	−0.557175262484 i	−	0 i	+	0.374576632933 i	+	0 i	−
			7	0		0		−0.059458253341		0		0.068131343283	
				0 i	−	0 i	−	0.355274152313 i	+	0 i	+	−0.602195504744 i	−
			8	−0.350233566926		0.376227464472		0		0.026695207581		0	
				0 i	−	0.100673932296 i	−	0 i	+	−0.217300584336 i	+	0 i	−
			9	0		0		−0.051677352706		0		−0.036778616933	
				0 i	−	0 i	−	0.488473732772 i	+	0 i	+	0.140268656299 i	−
			10	0		−0.095523468426		0		−0.029014016947		0	
				0.435031420071 i	−	0.398707995836 i	−	0 i	+	0.346937183109 i	+	0 i	−
			11	0		0		0.077858256622		0		0.050421229192	
				0 i	−	0 i	−	−0.484752693378 i	+	0 i	+	−0.487294874007 i	−
H_g	H	12	0	0.508072849927		0.342370043531		0		0		0	
				0 i	+	0.372860847473 i	+	0 i	−	0 i	−	0 i	+
			1	0		0		0.088686526027		0		−0.481877333248	
				0 i	+	0 i	+	0.034805959528 i	−	0 i	−	−0.192202858864 i	+
			2	0		0.316486508278		0		0.001511745346		0	
				0.364873518395 i	+	−0.258452075530 i	+	0 i	−	0.022335605508 i	−	0 i	+
			3	0		0		0.446644971206		0		0.054563575050	
				0 i	+	0 i	+	0.177732971135 i	−	0 i	−	0.034293652869 i	+
			4	−0.215003782611		−0.134209392426		0		−0.595712250320		0	
				0 i	+	−0.116448739396 i	+	0 i	−	−0.276149391364 i	−	0 i	+
			5	0		0		0.099969717289		0		0.515616533261	
				0 i	+	0 i	+	0.083236597923 i	−	0 i	−	0.221838320922 i	+
			6	0		0.276273081947		0		0.324549346049		0	
				0.386893033360 i	+	−0.258734716059 i	+	0 i	−	0.135065059139 i	−	0 i	+
			7	0		0		0.554858203993		0		−0.278163966137	
				0 i	+	0 i	+	0.258807201945 i	−	0 i	−	−0.138754937643 i	+
			8	−0.238496883249		−0.155581951182		0		0.352089002995		0	
				0 i	+	−0.155151803315 i	+	0 i	−	0.177865414563 i	−	0 i	+
			9	0		0		−0.524728354779		0		0.035619536658	
				0 i	+	0 i	+	−0.222821282256 i	−	0 i	−	−0.019187551130 i	+

All coefficients are complex and are given in two lines.

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$\mathbf{C}_n$ 107 $\mathbf{C}_i$ 137 $\mathbf{S}_n$ 143 $\mathbf{D}_n$ 193 $\mathbf{D}_{nh}$ 245 $\mathbf{D}_{nd}$ 365 $\mathbf{C}_{nv}$ 481 $\mathbf{C}_{nh}$ 531 $\mathbf{O}$ 579 $\mathbf{I}$

T **75**.6*b* Symmetrized harmonics *(cont.)*

$\mathbf{I}_h$	$\mathbf{I}$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±	4 Coefficient	±	5 Coefficient	±
				Five-dimensional representations — Column of the basis									
H_g	H	12	10	0		0.310219684085		0		0.452461235171		0	
				0.466026926595 i	+	−0.305830170578 i	+	0 i	−	0.173658766800 i	−	0 i	+
			11	0		0		−0.190713461237		0		0.507191059542	
				0 i	+	0 i	+	−0.081918505801 i	−	0 i	−	0.236893645394 i	+
			12	−0.372497770879		−0.260457425732		0		0.219167944242		0	
				0 i	+	−0.309950279185 i	+	0 i	−	0.079224487820 i	−	0 i	+
H_g	H	12	0	0		0.042155543977		0		0		0	
				0 i	+	−0.010884514651 i	+	0 i	−	0 i	−	0 i	+
			1	0		0		0.058112182721		0		−0.275105378628	
				0 i	+	0 i	+	0.067244522352 i	−	0 i	−	−0.313630086532 i	+
			2	0.587138739062		−0.145408578346		0		−0.285473164823		0	
				0 i	+	0.538293361847 i	+	0 i	−	−0.363516824447 i	−	0 i	+
			3	0		0		0.260485451711		0		−0.133941050828	
				0 i	+	0 i	+	0.297693288918 i	−	0 i	−	−0.174645812010 i	+
			4	0		−0.500284655184		0		0.167712969898		0	
				−0.498203740666 i	+	−0.112144494359 i	+	0 i	−	0.258709474650 i	−	0 i	+
			5	0		0		−0.514244450457		0		0.081213392692	
				0 i	+	0 i	+	−0.662211319813 i	−	0 i	−	0.064248386339 i	+
			6	−0.092287083266		0.013338367503		0		0.111313576608		0	
				0 i	+	−0.121469106565 i	+	0 i	−	0.117067170641 i	−	0 i	+
			7	0		0		−0.177241539153		0		0.207547067604	
				0 i	+	0 i	+	−0.267738387581 i	−	0 i	−	0.285315957710 i	+
			8	0		−0.251746970655		0		−0.292150292227		0	
				−0.239535068790 i	+	−0.051024010625 i	+	0 i	−	−0.398625140209 i	−	0 i	+
			9	0		0		−0.121348687986		0		0.460326127546	
				0 i	+	0 i	+	−0.114648300597 i	−	0 i	−	0.583283201378 i	+
			10	−0.383081186376		0.079788628350		0		0.348050782263		0	
				0 i	+	−0.409631200988 i	+	0 i	−	0.408720894531 i	−	0 i	+
			11	0		0		−0.031802370729		0		−0.166234144053	
				0 i	+	0 i	+	−0.026008875848 i	−	0 i	−	−0.250108288680 i	+
			12	0		0.396073029563		0		0.233075285039		0	
				0.440926279106 i	+	0.111308049950 i	+	0 i	−	0.280065925049 i	−	0 i	+

All coefficients are complex and are given in two lines.

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T **75**.6b Symmetrized harmonics *(cont.)*

				Five-dimensional representations									
				Column of the basis									
$\mathbf{I}_h$	**I**	j	m	1		2		3		4		5	
				Coefficient	±	Coefficient	±	Coefficient	±	Coefficient	±	Coefficient	±
H_u	H	13	0	0		0		0		0.355696653532		0	
				0 i	−	0 i	−	0 i	+	0.099684507878 i	+	0 i	−
			1	0		0		0.489958853946		0		0.421121249210	
				0 i	−	0 i	−	0.161730136167 i	+	0 i	+	−0.179203531778 i	−
			2	0.613434661014		0.416582674853		0		0.021496501123		0	
				0 i	−	0.139242453250 i	−	0 i	+	−0.241980054368 i	+	0 i	−
			3	0		0		−0.216712571494		0		0.490453345236	
				0 i	−	0 i	−	0.028545076907 i	+	0 i	+	0.171551846791 i	−
			4	0		0.200912308556		0		0.032269478049		0	
				−0.238475165189 i	−	0.147979584519 i	−	0 i	+	−0.047382059614 i	+	0 i	−
			5	0		0		0.232764514601		0		0.348761290029	
				0 i	−	0 i	−	0.262218961927 i	+	0 i	+	0.209971314067 i	−
			6	0.200049779344		0.113651327279		0		−0.336927863386		0	
				0 i	−	−0.359966882235 i	−	0 i	+	−0.143632278938 i	+	0 i	−
			7	0		0		−0.436439324630		0		−0.245980335988	
				0 i	−	0 i	−	−0.015177759835 i	+	0 i	+	0.003736201477 i	−
			8	0		−0.469541828429		0		0.379771681274		0	
				−0.438287107083 i	−	0.317907957799 i	−	0 i	+	−0.188303692447 i	+	0 i	−
			9	0		0		0.499313293822		0		0.328965572965	
				0 i	−	0 i	−	−0.046901228251 i	+	0 i	+	−0.032240030786 i	−
			10	0.289271503005		0.211798153690		0		0.265937164586		0	
				0 i	−	0.346004589305 i	−	0 i	+	−0.018826606982 i	+	0 i	−
			11	0		0		0.152725351956		0		0.223366592941	
				0 i	−	0 i	−	0.183287706193 i	+	0 i	+	−0.083356266556 i	−
			12	0		0.088591808536		0		0.618915443732		0	
				−0.501032940387 i	−	0.329169834615 i	−	0 i	+	0.182404205551 i	+	0 i	−
			13	0		0		0.187691038854		0		0.285324391850	
				0 i	−	0 i	−	−0.169398759352 i	+	0 i	+	0.216372628214 i	−
H_u	H	13	0	0		0		0		0.124115356274		0	
				0 i	−	0 i	−	0 i	+	−0.131099102143 i	+	0 i	−
			1	0		0		0.159824050240		0		0.282543986114	
				0 i	−	0 i	−	−0.238796804448 i	+	0 i	+	0.553356975362 i	−

All coefficients are complex and are given in two lines.

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T **75**.6*b* Symmetrized harmonics *(cont.)*

				Five-dimensional representations — Column of the basis									
$\mathbf{I}_h$	$\mathbf{I}$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±	4 Coefficient	±	5 Coefficient	±
H_u	H	13	2	0		0.427579327465		0		0.120645828096		0	
				0 i	−	−0.023418289497 i	−	0 i	+	0.583310608199 i	+	0 i	−
			3	0		0		−0.116349834308		0		0.155578946496	
				0 i	−	0 i	−	−0.132964332975 i	+	0 i	+	−0.262063282321 i	−
			4	0.365902724671		−0.228823200499		0		0.037002554802		0	
				0 i	−	0.275919260080 i	−	0 i	+	0.122623064756 i	+	0 i	−
			5	0		0		−0.008649464047		0		0.070493436134	
				0 i	−	0 i	−	−0.555398270282 i	+	0 i	+	−0.396096211200 i	−
			6	0		0.313794941347		0		−0.095116625878		0	
				0.581579998038 i	−	0.370533618745 i	−	0 i	+	0.241490963380 i	+	0 i	−
			7	0		0		−0.201166646984		0		−0.118986152115	
				0 i	−	0 i	−	−0.094547479576 i	+	0 i	+	−0.082587728875 i	−
			8	−0.530907473199		0.361952909889		0		0.266980502944		0	
				0 i	−	0.146333270582 i	−	0 i	+	0.562665606219 i	+	0 i	−
			9	0		0		0.259466250137		0		0.171556957542	
				0 i	−	0 i	−	0.261374727928 i	+	0 i	+	0.175397145555 i	−
			10	0		0.081051669362		0		0.135385985777		0	
				−0.402199833270 i	−	−0.272572378461 i	−	0 i	+	0.124540554572 i	+	0 i	−
			11	0		0		−0.010801429028		0		0.144528454891	
				0 i	−	0 i	−	−0.391203925012 i	+	0 i	+	0.265625336607 i	−
			12	0.290262727508		−0.169615194895		0		0.211877654293		0	
				0 i	−	0.436252054934 i	−	0 i	+	−0.249455468280 i	+	0 i	−
			13	0		0		0.166773030408		0		0.037326773194	
				0 i	−	0 i	−	0.460061251254 i	+	0 i	+	−0.430358549012 i	−
H_g	H	14	0	0.634946889183		−0.132230619525		0		0		0	
				0 i	+	−0.048378353568 i	+	0 i	−	0 i	−	0 i	+
			1	0		0		0.349900763842		0		−0.179246215412	
				0 i	+	0 i	+	0.260517290130 i	−	0 i	−	0.359044739604 i	+
			2	0		−0.148054874254		0		−0.073272963858		0	
				−0.270658886297 i	+	0.090039413776 i	+	0 i	−	0.139587474097 i	−	0 i	+
			3	0		0		−0.096588855287		0		0.109585590271	
				0 i	+	0 i	+	0.514301554305 i	−	0 i	−	−0.278059367901 i	+

All coefficients are complex and are given in two lines.

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T **75**.6*b* Symmetrized harmonics *(cont.)*

				Five-dimensional representations									
				Column of the basis									
$\mathbf{I}_h$	$\mathbf{I}$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±	4 Coefficient	±	5 Coefficient	±
H_g	H	14	4	0.158614962064		0.286560234198		0		0.076780935914		0	
				0 i	+	−0.040344293374 i	+	0 i	−	0.437677959211 i	−	0 i	+
			5	0		0		−0.116040573537		0		−0.313732266487	
				0 i	+	0 i	+	0.093143656994 i	−	0 i	−	0.011784199803 i	+
			6	0		0.513761720224		0		−0.158363007945		0	
				−0.282257393651 i	+	−0.137958813987 i	+	0 i	−	−0.249360925865 i	−	0 i	+
			7	0		0		0.357424641827		0		−0.475799866708	
				0 i	+	0 i	+	0.105447225548 i	−	0 i	−	−0.147612985048 i	+
			8	0.170207612553		−0.150760063072		0		−0.374180777141		0	
				0 i	+	0.445321661493 i	+	0 i	−	0.482360101286 i	−	0 i	+
			9	0		0		0.092500092821		0		−0.054176483564	
				0 i	+	0 i	+	−0.346515169979 i	−	0 i	−	0.064619498640 i	+
			10	0		−0.232868993893		0		−0.160566984101		0	
				−0.316146608760 i	+	−0.360759094078 i	+	0 i	−	0.047066772557 i	−	0 i	+
			11	0		0		−0.300634530943		0		0.121620354243	
				0 i	+	0 i	+	0.063387234443 i	−	0 i	−	−0.408637512183 i	+
			12	0.206671503490		−0.193350879242		0		0.370880324362		0	
				0 i	+	−0.371491086055 i	+	0 i	−	0.295131917051 i	−	0 i	+
			13	0		0		0.154782310599		0		−0.095887618439	
				0 i	+	0 i	+	−0.357584112087 i	−	0 i	−	−0.452557635402 i	+
			14	0		0.013631472443		0		−0.237032457345		0	
				−0.497117544216 i	+	0.049274834705 i	+	0 i	−	0.094394809480 i	−	0 i	+
H_g	H	14	0	0		−0.303587540618		0		0		0	
				0 i	+	0.263502529246 i	+	0 i	−	0 i	−	0 i	+
			1	0		0		−0.218655661253		0		0.331698617829	
				0 i	+	0 i	+	0.194778660624 i	−	0 i	−	0.193488250752 i	+
			2	0.352784613347		−0.152996848234		0		0.242260635466		0	
				0 i	+	0.039574539257 i	+	0 i	−	0.204689111172 i	−	0 i	+
			3	0		0		0.448171716096		0		0.197404605846	
				0 i	+	0 i	+	0.444623498195 i	−	0 i	−	0.350755843920 i	+
			4	0		0.220072551564		0		−0.096527213709		0	
				0.473870433650 i	+	0.337549589950 i	+	0 i	−	0.011312820404 i	−	0 i	+

All coefficients are complex and are given in two lines.

⇛

T 75.6b Symmetrized harmonics *(cont.)*

				Five-dimensional representations									
				Column of the basis									
$\mathbf{I}_h$	**I**	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±	4 Coefficient	±	5 Coefficient	±
H_g	H	14	5	0		0		−0.227409442956		0		0.252592666450	
				0 i	+	0 i	+	−0.406868927175 i	−	0 i	−	−0.091077376582 i	+
			6	−0.490432996567		−0.280182980924		0		−0.097808797297		0	
				0 i	+	−0.300155251897 i	+	0 i	−	−0.329711178838 i	−	0 i	+
			7	0		0		−0.324082666270		0		0.135906883912	
				0 i	+	0 i	+	0.068947369861 i	−	0 i	−	−0.441575874647 i	+
			8	0		−0.105678162699		0		0.354539901602		0	
				0.163827091707 i	+	−0.234163445279 i	+	0 i	−	−0.013705816657 i	−	0 i	+
			9	0		0		−0.174936915084		0		−0.048179836761	
				0 i	+	0 i	+	−0.115295875987 i	−	0 i	−	−0.119967302289 i	+
			10	−0.264779428592		0.226078237110		0		−0.176487230817		0	
				0 i	+	−0.381732640418 i	+	0 i	−	−0.405235249711 i	−	0 i	+
			11	0		0		0.004450845368		0		−0.454281762059	
				0 i	+	0 i	+	−0.364567900267 i	−	0 i	−	−0.413485714161 i	+
			12	0		−0.305910146505		0		−0.047364750459		0	
				−0.498605551649 i	+	0.128249591517 i	+	0 i	−	0.425720617584 i	−	0 i	+
			13	0		0		−0.079485565170		0		0.048928521908	
				0 i	+	0 i	+	0.074288340943 i	−	0 i	−	−0.092369907247 i	+
			14	0.254775090343		−0.318797549803		0		0.453530065835		0	
				0 i	+	−0.089258536941 i	+	0 i	−	0.247527616991 i	−	0 i	+
H_g	H	14	0	0		0.204602021716		0		0		0	
				0 i	+	−0.424112231870 i	+	0 i	−	0 i	−	0 i	+
			1	0		0		−0.129264615554		0		−0.114109640406	
				0 i	+	0 i	+	−0.447479174449 i	−	0 i	−	0.111012323633 i	+
			2	0		0.244522181758		0		0.085975237720		0	
				0 i	+	0.287959367313 i	+	0 i	−	0.111856837828 i	−	0 i	+
			3	0		0		−0.029504426010		0		0.595575446401	
				0 i	+	0 i	+	0.057256393874 i	−	0 i	−	0.197428561662 i	+
			4	0.381512487078		0.375875025340		0		−0.425935558215		0	
				0 i	+	−0.088687957140 i	+	0 i	−	−0.318171927568 i	−	0 i	+
			5	0		0		−0.472464095499		0		−0.036541063902	
				0 i	+	0 i	+	−0.118652191291 i	−	0 i	−	0.311141126057 i	+

All coefficients are complex and are given in two lines.

⇛

$\mathbf{C}_n$ 107 $\mathbf{C}_i$ 137 $\mathbf{S}_n$ 143 $\mathbf{D}_n$ 193 $\mathbf{D}_{nh}$ 245 $\mathbf{D}_{nd}$ 365 $\mathbf{C}_{nv}$ 481 $\mathbf{C}_{nh}$ 531 **O** 579 **I**

T **75**.6b Symmetrized harmonics *(cont.)*

				Five-dimensional representations									
				Column of the basis									
				1		2		3		4		5	
$\mathbf{I}_h$	$\mathbf{I}$	j	m	Coefficient	±	Coefficient	±	Coefficient	±	Coefficient	±	Coefficient	±
H_g	H	14	6	0		−0.004125320330		0		−0.056793617066		0	
				−0.385904716926 i	+	−0.132362287323 i	+	0 i	−	0.152671670102 i	−	0 i	+
			7	0		0		−0.107590374186		0		−0.199921750755	
				0 i	+	0 i	+	−0.435656973947 i	−	0 i	−	0.409428980503 i	+
			8	−0.568844955943		0.065884668550		0		−0.399160022205		0	
				0 i	+	−0.296670473271 i	+	0 i	−	0.166967424897 i	−	0 i	+
			9	0		0		0.198555198067		0		−0.170861458421	
				0 i	+	0 i	+	0.021580491631 i	−	0 i	−	−0.031846981774 i	+
			10	0		0.318168061140		0		−0.417134490344		0	
				0.574229680044 i	+	−0.131754939252 i	+	0 i	−	−0.041596025135 i	−	0 i	+
			11	0		0		−0.362685616653		0		−0.046257656472	
				0 i	+	0 i	+	0.131926129030 i	−	0 i	−	−0.127418746505 i	+
			12	0.175680500630		−0.191741298351		0		0.064172940969		0	
				0 i	+	−0.000595245953 i	+	0 i	−	−0.375186817301 i	−	0 i	+
			13	0		0		0.385868840068		0		0.363137935449	
				0 i	+	0 i	+	0.050008840456 i	−	0 i	−	0.307847238657 i	+
			14	0		0.338690761939		0		0.206763677124		0	
				−0.146074720643 i	+	0.326266456537 i	+	0 i	−	0.346893194287 i	−	0 i	+
H_u	H	15	0	0		0		0		−0.617453670117		0	
				0 i	−	0 i	−	0 i	+	0.212161575233 i	+	0 i	−
			1	0		0		0.072322801321		0		0.085022866910	
				0 i	−	0 i	−	0.104578693417 i	+	0 i	+	−0.034238353250 i	−
			2	0.546279437613		0.128288542637		0		−0.133521090018		0	
				0 i	−	−0.172528352914 i	−	0 i	+	0.421538613470 i	+	0 i	−
			3	0		0		0.157996611168		0		−0.105917049487	
				0 i	−	0 i	−	0.227218870621 i	+	0 i	+	0.153427387776 i	−
			4	0		0.162900950846		0		0.252477009827		0	
				0.218530144883 i	−	−0.590248679682 i	−	0 i	+	−0.191489148394 i	+	0 i	−
			5	0		0		0.163199341848		0		0.111074765410	
				0 i	−	0 i	−	0.224349013003 i	+	0 i	+	−0.405586979182 i	−
			6	−0.184901439533		0.338745215394		0		0.303049343042		0	
				0 i	−	0.222855113078 i	−	0 i	+	−0.028621762377 i	+	0 i	−

All coefficients are complex and are given in two lines. ⇛

$\mathbf{C}_n$ 107 $\mathbf{C}_i$ 137 $\mathbf{S}_n$ 143 $\mathbf{D}_n$ 193 $\mathbf{D}_{nh}$ 245 $\mathbf{D}_{nd}$ 365 $\mathbf{C}_{nv}$ 481 $\mathbf{C}_{nh}$ 531 $\mathbf{O}$ 579 $\mathbf{I}$

T **75**.6*b* Symmetrized harmonics *(cont.)*

				Five-dimensional representations									
				Column of the basis									
$\mathbf{I}_h$	$\mathbf{I}$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±	4 Coefficient	±	5 Coefficient	±
H_u	H	15	7	0		0		−0.063728295305		0		−0.394975226073	
				0 i	−	0 i	−	−0.158264811950 i	+	0 i	+	0.230092085954 i	−
			8	0		−0.112097039250		0		−0.060438732073		0	
				0.411638486446 i	−	−0.138284804040 i	−	0 i	+	−0.137749724273 i	+	0 i	−
			9	0		0		0.099092453724		0		0.576788494285	
				0 i	−	0 i	−	−0.116745613828 i	+	0 i	+	0.013707580780 i	−
			10	−0.280709006413		0.124783925507		0		−0.230851203047		0	
				0 i	−	0.170721439610 i	−	0 i	+	−0.299617102266 i	+	0 i	−
			11	0		0		0.504581992923		0		−0.202773448018	
				0 i	−	0 i	−	0.038010396133 i	+	0 i	+	0.274231654351 i	−
			12	0		−0.277117064955		0		0.063253228632		0	
				0.531787863959 i	−	0.128792373303 i	−	0 i	+	0.099121361236 i	+	0 i	−
			13	0		0		0.480450187420		0		0.132019758338	
				0 i	−	0 i	−	−0.385793694906 i	+	0 i	+	−0.038957983704 i	−
			14	−0.297645237521		−0.487161541099		0		0.042587622430		0	
				0 i	−	−0.085557523094 i	−	0 i	+	0.061763010971 i	+	0 i	−
			15	0		0		0.247541377828		0		−0.198490954164	
				0 i	−	0 i	−	−0.284118447059 i	+	0 i	+	−0.252023268379 i	−
H_u	H	15	0	0		0		0		−0.241260802749		0	
				0 i	−	0 i	−	0 i	+	0.376319625911 i	+	0 i	−
			1	0		0		−0.113860834415		0		0.038737884660	
				0 i	−	0 i	−	−0.153645144525 i	+	0 i	+	−0.047566014181 i	−
			2	0		0.198508989574		0		−0.464664635451		0	
				0 i	−	−0.461293206416 i	−	0 i	+	−0.236632294846 i	+	0 i	−
			3	0		0		−0.247374683813		0		−0.169894168653	
				0 i	−	0 i	−	−0.334600507417 i	+	0 i	+	−0.034519762548 i	−
			4	0.638218380162		−0.227863906042		0		0.213657224533		0	
				0 i	−	0.168005778681 i	−	0 i	+	−0.065214279659 i	+	0 i	−
			5	0		0		−0.244153526985		0		0.446847329423	
				0 i	−	0 i	−	−0.336855328522 i	+	0 i	+	0.243338126759 i	−
			6	0		−0.242638861509		0		0.035500224663		0	
				−0.452576103582 i	−	0.125408661515 i	−	0 i	+	−0.248619934733 i	+	0 i	−

All coefficients are complex and are given in two lines.

⇛

$\mathbf{C}_n$ 107 $\mathbf{C}_i$ 137 $\mathbf{S}_n$ 143 $\mathbf{D}_n$ 193 $\mathbf{D}_{nh}$ 245 $\mathbf{D}_{nd}$ 365 $\mathbf{C}_{nv}$ 481 $\mathbf{C}_{nh}$ 531 $\mathbf{O}$ 579 $\mathbf{I}$

T **75**.6*b* Symmetrized harmonics *(cont.)*

$\mathbf{I}_h$	$\mathbf{I}$	j	m	1 Coefficient	±	2 Coefficient	±	3 Coefficient	±	4 Coefficient	±	5 Coefficient	±
				Five-dimensional representations — Column of the basis									
H_u	H	15	7	0		0		0.172926365467		0		−0.257959941831	
				0 i	−	0 i	−	0.191354330330 i	+	0 i	+	0.160833221032 i	−
			8	0.049282415492		−0.350944700942		0		0.150443974181		0	
				0 i	−	−0.130477521603 i	−	0 i	+	0.171025831203 i	+	0 i	−
			9	0		0		0.129524041723		0		−0.007301260065	
				0 i	−	0 i	−	0.009611829597 i	+	0 i	+	−0.530913197125 i	−
			10	0		−0.189555865063		0		0.325892614325		0	
				−0.225840399420 i	−	0.221705138201 i	−	0 i	+	0.461482538101 i	+	0 i	−
			11	0		0		−0.034957212217		0		−0.303844768257	
				0 i	−	0 i	−	−0.486475559585 i	+	0 i	+	−0.049580609998 i	−
			12	−0.300413952316		−0.428659863879		0		−0.107990278974		0	
				0 i	−	−0.309703492051 i	−	0 i	+	−0.140859564152 i	+	0 i	−
			13	0		0		0.430076637339		0		0.044551775922	
				0 i	−	0 i	−	−0.105983295844 i	+	0 i	+	−0.085884143985 i	−
			14	0		0.083400936846		0		−0.067246625263		0	
				0.494136605056 i	−	0.284888572470 i	−	0 i	+	−0.090628190531 i	+	0 i	−
			15	0		0		0.315302511654		0		0.274067028924	
				0 i	−	0 i	−	0.017643504234 i	+	0 i	+	0.392057236996 i	−

All coefficients are complex and are given in two lines.

T **75**.6*c* Spin harmonics

§ **16**–6, pp. 74, 75

They must be obtained from T **74**.6*c*.

For gerade spin harmonics use the symmetrized harmonics for the gerade representations (subscript *g*) listed in T **75**.6*b*; for ungerade spin harmonics use the symmetrized harmonics for the ungerade representations (subscript *u*) listed in T **75**.6*b*.

T **75**.7 Matrix representations

Use T **74**.7 ▪. § **16**–7, p. 77

T **75**.8 Direct products of representations

§ **16**–8, p. 81

$\mathbf{I}_h$	A_g	T_{1g}	T_{2g}	F_g	H_g
A_g	A_g	T_{1g}	T_{2g}	F_g	H_g
T_{1g}		$A_g \oplus \{T_{1g}\} \oplus H_g$	$F_g \oplus H_g$	$T_{2g} \oplus F_g \oplus H_g$	$T_{1g} \oplus T_{2g} \oplus F_g \oplus H_g$
T_{2g}			$A_g \oplus \{T_{2g}\} \oplus H_g$	$T_{1g} \oplus F_g \oplus H_g$	$T_{1g} \oplus T_{2g} \oplus F_g \oplus H_g$
F_g				$A_g \oplus \{T_{1g}\} \oplus \{T_{2g}\} \oplus F_g \oplus H_g$	$T_{1g} \oplus T_{2g} \oplus F_g \oplus 2H_g$
H_g					$A_g \oplus \{T_{1g}\} \oplus \{T_{2g}\} \oplus F_g \oplus \{F_g\} \oplus 2H_g$

⇛

T **75**.8 Direct products of representations *(cont.)*

$\mathbf{I}_h$	A_u	T_{1u}	T_{2u}	F_u	H_u
A_g	A_u	T_{1u}	T_{2u}	F_u	H_u
T_{1g}	T_{1u}	$A_u \oplus T_{1u} \oplus H_u$	$F_u \oplus H_u$	$T_{2u} \oplus F_u \oplus H_u$	$T_{1u} \oplus T_{2u} \oplus F_u \oplus H_u$
T_{2g}	T_{2u}	$F_u \oplus H_u$	$A_u \oplus T_{2u} \oplus H_u$	$T_{1u} \oplus F_u \oplus H_u$	$T_{1u} \oplus T_{2u} \oplus F_u \oplus H_u$
F_g	F_u	$T_{2u} \oplus F_u \oplus H_u$	$T_{1u} \oplus F_u \oplus H_u$	$A_u \oplus T_{1u} \oplus T_{2u} \oplus F_u \oplus H_u$	$T_{1u} \oplus T_{2u} \oplus F_u \oplus 2H_u$
H_g	H_u	$T_{1u} \oplus T_{2u} \oplus F_u \oplus H_u$	$T_{1u} \oplus T_{2u} \oplus F_u \oplus H_u$	$T_{1u} \oplus T_{2u} \oplus F_u \oplus 2H_u$	$A_u \oplus T_{1u} \oplus T_{2u} \oplus F_u \oplus F_u \oplus 2H_u$
A_u	A_g	T_{1g}	T_{2g}	F_g	H_g
T_{1u}		$A_g \oplus \{T_{1g}\} \oplus H_g$	$F_g \oplus H_g$	$T_{2g} \oplus F_g \oplus H_g$	$T_{1g} \oplus T_{2g} \oplus F_g \oplus H_g$
T_{2u}			$A_g \oplus \{T_{2g}\} \oplus H_g$	$T_{1g} \oplus F_g \oplus H_g$	$T_{1g} \oplus T_{2g} \oplus F_g \oplus H_g$
F_u				$A_g \oplus \{T_{1g}\} \oplus \{T_{2g}\} \oplus F_g \oplus H_g$	$T_{1g} \oplus T_{2g} \oplus F_g \oplus 2H_g$
H_u					$A_g \oplus \{T_{1g}\} \oplus \{T_{2g}\} \oplus F_g \oplus \{F_g\} \oplus 2H_g$

⇛

T **75**.8 Direct products of representations *(cont.)*

$\mathbf{I}_h$	$E_{1/2,g}$	$E_{7/2,g}$	$F_{3/2,g}$	$I_{5/2,g}$
A_g	$E_{1/2,g}$	$E_{7/2,g}$	$F_{3/2,g}$	$I_{5/2,g}$
T_{1g}	$E_{1/2,g} \oplus F_{3/2,g}$	$I_{5/2,g}$	$E_{1/2,g} \oplus F_{3/2,g} \oplus I_{5/2,g}$	$E_{7/2,g} \oplus F_{3/2,g} \oplus 2I_{5/2,g}$
T_{2g}	$I_{5/2,g}$	$E_{7/2,g} \oplus F_{3/2,g}$	$E_{7/2,g} \oplus F_{3/2,g} \oplus I_{5/2,g}$	$E_{1/2,g} \oplus F_{3/2,g} \oplus 2I_{5/2,g}$
F_g	$E_{7/2,g} \oplus I_{5/2,g}$	$E_{1/2,g} \oplus I_{5/2,g}$	$F_{3/2,g} \oplus 2I_{5/2,g}$	$E_{1/2,g} \oplus E_{7/2,g} \oplus 2F_{3/2,g} \oplus 2I_{5/2,g}$
H_g	$F_{3/2,g} \oplus I_{5/2,g}$	$F_{3/2,g} \oplus I_{5/2,g}$	$E_{1/2,g} \oplus E_{7/2,g} \oplus F_{3/2,g} \oplus 2I_{5/2,g}$	$E_{1/2,g} \oplus E_{7/2,g} \oplus 2F_{3/2,g} \oplus 3I_{5/2,g}$
A_u	$E_{1/2,u}$	$E_{7/2,u}$	$F_{3/2,u}$	$I_{5/2,u}$
T_{1u}	$E_{1/2,u} \oplus F_{3/2,u}$	$I_{5/2,u}$	$E_{1/2,u} \oplus F_{3/2,u} \oplus I_{5/2,u}$	$E_{7/2,u} \oplus F_{3/2,u} \oplus 2I_{5/2,u}$
T_{2u}	$I_{5/2,u}$	$E_{7/2,u} \oplus F_{3/2,u}$	$E_{7/2,u} \oplus F_{3/2,u} \oplus I_{5/2,u}$	$E_{1/2,u} \oplus F_{3/2,u} \oplus 2I_{5/2,u}$
F_u	$E_{7/2,u} \oplus I_{5/2,u}$	$E_{1/2,u} \oplus I_{5/2,u}$	$F_{3/2,u} \oplus 2I_{5/2,u}$	$E_{1/2,u} \oplus E_{7/2,u} \oplus 2F_{3/2,u} \oplus 2I_{5/2,u}$
H_u	$F_{3/2,u} \oplus I_{5/2,u}$	$F_{3/2,u} \oplus I_{5/2,u}$	$E_{1/2,u} \oplus E_{7/2,u} \oplus F_{3/2,u} \oplus 2I_{5/2,u}$	$E_{1/2,u} \oplus E_{7/2,u} \oplus 2F_{3/2,u} \oplus 3I_{5/2,u}$
$E_{1/2,g}$	$\{A_g\} \oplus T_{1g}$	F_g	$T_{1g} \oplus H_g$	$T_{2g} \oplus F_g \oplus H_g$
$E_{7/2,g}$		$\{A_g\} \oplus T_{2g}$	$T_{2g} \oplus H_g$	$T_{1g} \oplus F_g \oplus H_g$
$F_{3/2,g}$			$\{A_g\} \oplus T_{1g} \oplus T_{2g} \oplus F_g \oplus \{H_g\}$	$T_{1g} \oplus T_{2g} \oplus 2F_g \oplus 2H_g$
$I_{5/2,g}$				$\{A_g\} \oplus 2T_{1g} \oplus 2T_{2g} \oplus F_g \oplus \{F_g\} \oplus H_g \oplus 2\{H_g\}$

$\twoheadrightarrow$

T **75**.8 Direct products of representations *(cont.)*

$\mathbf{I}_h$	$E_{1/2,u}$	$E_{7/2,u}$	$F_{3/2,u}$	$I_{5/2,u}$
A_g	$E_{1/2,u}$	$E_{7/2,u}$	$F_{3/2,u}$	$I_{5/2,u}$
T_{1g}	$E_{1/2,u} \oplus F_{3/2,u}$	$I_{5/2,u}$	$E_{1/2,u} \oplus F_{3/2,u} \oplus I_{5/2,u}$	$E_{7/2,u} \oplus F_{3/2,u} \oplus 2I_{5/2,u}$
T_{2g}	$I_{5/2,u}$	$E_{7/2,u} \oplus F_{3/2,u}$	$E_{7/2,u} \oplus F_{3/2,u} \oplus I_{5/2,u}$	$E_{1/2,u} \oplus F_{3/2,u} \oplus 2I_{5/2,u}$
F_g	$E_{7/2,u} \oplus I_{5/2,u}$	$E_{1/2,u} \oplus I_{5/2,u}$	$F_{3/2,u} \oplus 2I_{5/2,u}$	$E_{1/2,u} \oplus E_{7/2,u} \oplus 2F_{3/2,u} \oplus 2I_{5/2,u}$
H_g	$F_{3/2,u} \oplus I_{5/2,u}$	$F_{3/2,u} \oplus I_{5/2,u}$	$E_{1/2,u} \oplus E_{7/2,u} \oplus F_{3/2,u} \oplus 2I_{5/2,u}$	$E_{1/2,u} \oplus E_{7/2,u} \oplus 2F_{3/2,u} \oplus 3I_{5/2,u}$
A_u	$E_{1/2,g}$	$E_{7/2,g}$	$F_{3/2,g}$	$I_{5/2,g}$
T_{1u}	$E_{1/2,g} \oplus F_{3/2,g}$	$I_{5/2,g}$	$E_{1/2,g} \oplus F_{3/2,g} \oplus I_{5/2,g}$	$E_{7/2,g} \oplus F_{3/2,g} \oplus 2I_{5/2,g}$
T_{2u}	$I_{5/2,g}$	$E_{7/2,g} \oplus F_{3/2,g}$	$E_{7/2,g} \oplus F_{3/2,g} \oplus I_{5/2,g}$	$E_{1/2,g} \oplus F_{3/2,g} \oplus 2I_{5/2,g}$
F_u	$E_{7/2,g} \oplus I_{5/2,g}$	$E_{1/2,g} \oplus I_{5/2,g}$	$F_{3/2,g} \oplus 2I_{5/2,g}$	$E_{1/2,g} \oplus E_{7/2,g} \oplus 2F_{3/2,g} \oplus 2I_{5/2,g}$
H_u	$F_{3/2,g} \oplus I_{5/2,g}$	$F_{3/2,g} \oplus I_{5/2,g}$	$E_{1/2,g} \oplus E_{7/2,g} \oplus F_{3/2,g} \oplus 2I_{5/2,g}$	$E_{1/2,g} \oplus E_{7/2,g} \oplus 2F_{3/2,g} \oplus 3I_{5/2,g}$
$E_{1/2,g}$	$A_u \oplus T_{1u}$	F_u	$T_{1u} \oplus H_u$	$T_{2u} \oplus F_u \oplus H_u$
$E_{7/2,g}$	F_u	$A_u \oplus T_{2u}$	$T_{2u} \oplus H_u$	$T_{1u} \oplus F_u \oplus H_u$
$F_{3/2,g}$	$T_{1u} \oplus H_u$	$T_{2u} \oplus H_u$	$A_u \oplus T_{1u} \oplus T_{2u} \oplus F_u \oplus H_u$	$T_{1u} \oplus T_{2u} \oplus 2F_u \oplus 2H_u$
$I_{5/2,g}$	$T_{2u} \oplus F_u \oplus H_u$	$T_{1u} \oplus F_u \oplus H_u$	$T_{1u} \oplus T_{2u} \oplus 2F_u \oplus 2H_u$	$A_u \oplus 2T_{1u} \oplus 2T_{2u} \oplus F_u \oplus F_u \oplus H_u \oplus 2\{H_u\}$
$E_{1/2,u}$	$\{A_g\} \oplus T_{1g}$	F_g	$T_{1g} \oplus H_g$	$T_{2g} \oplus F_g \oplus H_g$
$E_{7/2,u}$		$\{A_g\} \oplus T_{2g}$	$T_{2g} \oplus H_g$	$T_{1g} \oplus F_g \oplus H_g$
$F_{3/2,u}$			$\{A_g\} \oplus T_{1g} \oplus T_{2g} \oplus F_g \oplus \{H_g\}$	$T_{1g} \oplus T_{2g} \oplus 2F_g \oplus 2H_g$
$I_{5/2,u}$				$\{A_g\} \oplus 2T_{1g} \oplus 2T_{2g} \oplus F_g \oplus \{F_g\} \oplus H_g \oplus 2\{H_g\}$

T **75**.9 Subduction (descent of symmetry) § **16**–9, p. 82

$\mathbf{I}_h$	$\mathbf{I}$	$\mathbf{T}_h$	$\mathbf{T}$	$(\mathbf{C}_{5v})$	$(\mathbf{C}_{3v})$
A_g	A	A_g	A	A_1	A_1
T_{1g}	T_1	T_g	T	$A_2 \oplus E_1$	$A_2 \oplus E$
T_{2g}	T_2	T_g	T	$A_2 \oplus E_2$	$A_2 \oplus E$
F_g	F	$A_g \oplus T_g$	$A \oplus T$	$E_1 \oplus E_2$	$A_1 \oplus A_2 \oplus E$
H_g	H	${}^1E_g \oplus {}^2E_g \oplus T_g$	${}^1E \oplus {}^2E \oplus T$	$A_1 \oplus E_1 \oplus E_2$	$A_1 \oplus 2E$
A_u	A	A_u	A	A_2	A_2
T_{1u}	T_1	T_u	T	$A_1 \oplus E_1$	$A_1 \oplus E$
T_{2u}	T_2	T_u	T	$A_1 \oplus E_2$	$A_1 \oplus E$
F_u	F	$A_u \oplus T_u$	$A \oplus T$	$E_1 \oplus E_2$	$A_1 \oplus A_2 \oplus E$
H_u	H	${}^1E_u \oplus {}^2E_u \oplus T_u$	${}^1E \oplus {}^2E \oplus T$	$A_2 \oplus E_1 \oplus E_2$	$A_2 \oplus 2E$
$E_{1/2,g}$	$E_{1/2}$	$E_{1/2,g}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{7/2,g}$	$E_{7/2}$	$E_{1/2,g}$	$E_{1/2}$	$E_{3/2}$	$E_{1/2}$
$F_{3/2,g}$	$F_{3/2}$	${}^1F_{3/2,g} \oplus {}^2F_{3/2,g}$	${}^1F_{3/2} \oplus {}^2F_{3/2}$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}$
$I_{5/2,g}$	$I_{5/2}$	$E_{1/2,g} \oplus {}^1F_{3/2,g} \oplus {}^2F_{3/2,g}$	$E_{1/2} \oplus {}^1F_{3/2} \oplus {}^2F_{3/2}$	$E_{1/2} \oplus E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}$	$2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}$
$E_{1/2,u}$	$E_{1/2}$	$E_{1/2,u}$	$E_{1/2}$	$E_{1/2}$	$E_{1/2}$
$E_{7/2,u}$	$E_{7/2}$	$E_{1/2,u}$	$E_{1/2}$	$E_{3/2}$	$E_{1/2}$
$F_{3/2,u}$	$F_{3/2}$	${}^1F_{3/2,u} \oplus {}^2F_{3/2,u}$	${}^1F_{3/2} \oplus {}^2F_{3/2}$	$E_{1/2} \oplus E_{3/2}$	$E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}$
$I_{5/2,u}$	$I_{5/2}$	$E_{1/2,u} \oplus {}^1F_{3/2,u} \oplus {}^2F_{3/2,u}$	$E_{1/2} \oplus {}^1F_{3/2} \oplus {}^2F_{3/2}$	$E_{1/2} \oplus E_{3/2} \oplus {}^1E_{5/2} \oplus {}^2E_{5/2}$	$2E_{1/2} \oplus {}^1E_{3/2} \oplus {}^2E_{3/2}$

⇒

T **75**.9 Subduction (descent of symmetry) *(cont.)*

$\mathbf{I}_h$	$(\mathbf{D}_{5d})$	$(\mathbf{D}_{3d})$	$\mathbf{D}_{2h}$
A_g	A_{1g}	A_{1g}	A_g
T_{1g}	$A_{2g} \oplus E_{1g}$	$A_{2g} \oplus E_g$	$B_{1g} \oplus B_{2g} \oplus B_{3g}$
T_{2g}	$A_{2g} \oplus E_{2g}$	$A_{2g} \oplus E_g$	$B_{1g} \oplus B_{2g} \oplus B_{3g}$
F_g	$E_{1g} \oplus E_{2g}$	$A_{1g} \oplus A_{2g} \oplus E_g$	$A_g \oplus B_{1g} \oplus B_{2g} \oplus B_{3g}$
H_g	$A_{1g} \oplus E_{1g} \oplus E_{2g}$	$A_{1g} \oplus 2E_g$	$2A_g \oplus B_{1g} \oplus B_{2g} \oplus B_{3g}$
A_u	A_{1u}	A_{1u}	A_u
T_{1u}	$A_{2u} \oplus E_{1u}$	$A_{2u} \oplus E_u$	$B_{1u} \oplus B_{2u} \oplus B_{3u}$
T_{2u}	$A_{2u} \oplus E_{2u}$	$A_{2u} \oplus E_u$	$B_{1u} \oplus B_{2u} \oplus B_{3u}$
F_u	$E_{1u} \oplus E_{2u}$	$A_{1u} \oplus A_{2u} \oplus E_u$	$A_u \oplus B_{1u} \oplus B_{2u} \oplus B_{3u}$
H_u	$A_{1u} \oplus E_{1u} \oplus E_{2u}$	$A_{1u} \oplus 2E_u$	$2A_u \oplus B_{1u} \oplus B_{2u} \oplus B_{3u}$
$E_{1/2,g}$	$E_{1/2,g}$	$E_{1/2,g}$	$E_{1/2,g}$
$E_{7/2,g}$	$E_{3/2,g}$	$E_{1/2,g}$	$E_{1/2,g}$
$F_{3/2,g}$	$E_{1/2,g} \oplus E_{3/2,g}$	$E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$	$2E_{1/2,g}$
$I_{5/2,g}$	$E_{1/2,g} \oplus E_{3/2} \oplus {}^1E_{5/2,g} \oplus {}^2E_{5/2,g}$	$2E_{1/2,g} \oplus {}^1E_{3/2,g} \oplus {}^2E_{3/2,g}$	$3E_{1/2,g}$
$E_{1/2,u}$	$E_{1/2,u}$	$E_{1/2,u}$	$E_{1/2,u}$
$E_{7/2,u}$	$E_{3/2,u}$	$E_{1/2,u}$	$E_{1/2,u}$
$F_{3/2,u}$	$E_{1/2,u} \oplus E_{3/2,u}$	$E_{1/2,u} \oplus {}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$	$2E_{1/2,u}$
$I_{5/2,u}$	$E_{1/2,u} \oplus E_{3/2} \oplus {}^1E_{5/2,u} \oplus {}^2E_{5/2,u}$	$2E_{1/2,u} \oplus {}^1E_{3/2,u} \oplus {}^2E_{3/2,u}$	$3E_{1/2,u}$

Other subgroups: $\mathbf{D}_5$, $\mathbf{S}_{10}$, $\mathbf{C}_5$ (see $\mathbf{D}_{5d}$); $\mathbf{D}_3$, $\mathbf{S}_6$, $\mathbf{C}_3$ (see $\mathbf{D}_{3d}$); $\mathbf{C}_{2h}$, $\mathbf{C}_{2v}$, $\mathbf{D}_2$, $\mathbf{C}_s$, $\mathbf{C}_i$, $\mathbf{C}_2$ (see $\mathbf{D}_{2h}$).

T **75**.10 ♣ Subduction from O(3)

§ **16**–10, p. 82

j	$\mathbf{I}_h$
$30n$	$(n+1)\,A_g \oplus n\,(3T_{1g} \oplus 3T_{2g} \oplus 4F_g \oplus 5H_g)$
$30n+1$	$n\,(A_u \oplus 2T_{1u} \oplus 3T_{2u} \oplus 4F_u \oplus 5H_u) \oplus (n+1)\,T_{1u}$
$30n+2$	$n\,(A_g \oplus 3T_{1g} \oplus 3T_{2g} \oplus 4F_g \oplus 4H_g) \oplus (n+1)\,H_g$
$30n+3$	$n\,(A_u \oplus 3T_{1u} \oplus 2T_{2u} \oplus 3F_u \oplus 5H_u) \oplus (n+1)(T_{2u} \oplus F_u)$
$30n+4$	$n\,(A_g \oplus 3T_{1g} \oplus 3T_{2g} \oplus 3F_g \oplus 4H_g) \oplus (n+1)(F_g \oplus H_g)$
$30n+5$	$n\,(A_u \oplus 2T_{1u} \oplus 2T_{2u} \oplus 4F_u \oplus 4H_u) \oplus (n+1)(T_{1u} \oplus T_{2u} \oplus H_u)$
$30n+6$	$(n+1)(A_g \oplus T_{1g} \oplus F_g \oplus H_g) \oplus n\,(2T_{1g} \oplus 3T_{2g} \oplus 3F_g \oplus 4H_g)$
$30n+7$	$n\,(A_u \oplus 2T_{1u} \oplus 2T_{2u} \oplus 3F_u \oplus 4H_u) \oplus (n+1)(T_{1u} \oplus T_{2u} \oplus F_u \oplus H_u)$
$30n+8$	$n\,(A_g \oplus 3T_{1g} \oplus 2T_{2g} \oplus 3F_g \oplus 3H_g) \oplus (n+1)(T_{2g} \oplus F_g \oplus 2H_g)$
$30n+9$	$n\,(A_u \oplus 2T_{1u} \oplus 2T_{2u} \oplus 2F_u \oplus 4H_u) \oplus (n+1)(T_{1u} \oplus T_{2u} \oplus 2F_u \oplus H_u)$
$30n+10$	$(n+1)(A_g \oplus T_{1g} \oplus T_{2g} \oplus F_g \oplus 2H_g) \oplus n\,(2T_{1g} \oplus 2T_{2g} \oplus 3F_g \oplus 3H_g)$
$30n+11$	$n\,(A_u \oplus T_{1u} \oplus 2T_{2u} \oplus 3F_u \oplus 3H_u) \oplus (n+1)(2T_{1u} \oplus T_{2u} \oplus F_u \oplus 2H_u)$
$30n+12$	$(n+1)(A_g \oplus T_{1g} \oplus T_{2g} \oplus 2F_g \oplus 2H_g) \oplus n\,(2T_{1g} \oplus 2T_{2g} \oplus 2F_g \oplus 3H_g)$
$30n+13$	$n\,(A_u \oplus 2T_{1u} \oplus T_{2u} \oplus 2F_u \oplus 3H_u) \oplus (n+1)(T_{1u} \oplus 2T_{2u} \oplus 2F_u \oplus 2H_u)$
$30n+14$	$n\,(A_g \oplus 2T_{1g} \oplus 2T_{2g} \oplus 2F_g \oplus 2H_g) \oplus (n+1)(T_{1g} \oplus T_{2g} \oplus 2F_g \oplus 3H_g)$
$30n+15$	$(n+1)(A_u \oplus 2T_{1u} \oplus 2T_{2u} \oplus 2F_u \oplus 2H_u) \oplus n\,(T_{1u} \oplus T_{2u} \oplus 2F_u \oplus 3H_u)$
$30n+16$	$(n+1)(A_g \oplus 2T_{1g} \oplus T_{2g} \oplus 2F_g \oplus 3H_g) \oplus n\,(T_{1g} \oplus 2T_{2g} \oplus 2F_g \oplus 2H_g)$
$30n+17$	$n\,(A_u \oplus T_{1u} \oplus T_{2u} \oplus 2F_u \oplus 2H_u) \oplus (n+1)(2T_{1u} \oplus 2T_{2u} \oplus 2F_u \oplus 3H_u)$
$30n+18$	$(n+1)(A_g \oplus T_{1g} \oplus 2T_{2g} \oplus 3F_g \oplus 3H_g) \oplus n\,(2T_{1g} \oplus T_{2g} \oplus F_g \oplus 2H_g)$
$30n+19$	$n\,(A_u \oplus T_{1u} \oplus T_{2u} \oplus F_u \oplus 2H_u) \oplus (n+1)(2T_{1u} \oplus 2T_{2u} \oplus 3F_u \oplus 3H_u)$
$30n+20$	$(n+1)(A_g \oplus 2T_{1g} \oplus 2T_{2g} \oplus 2F_g \oplus 4H_g) \oplus n\,(T_{1g} \oplus T_{2g} \oplus 2F_g \oplus H_g)$
$30n+21$	$(n+1)(A_u \oplus 3T_{1u} \oplus 2T_{2u} \oplus 3F_u \oplus 3H_u) \oplus n\,(T_{2u} \oplus F_u \oplus 2H_u)$
$30n+22$	$(n+1)(A_g \oplus 2T_{1g} \oplus 2T_{2g} \oplus 3F_g \oplus 4H_g) \oplus n\,(T_{1g} \oplus T_{2g} \oplus F_g \oplus H_g)$
$30n+23$	$n\,(A_u \oplus T_{1u} \oplus F_u \oplus H_u) \oplus (n+1)(2T_{1u} \oplus 3T_{2u} \oplus 3F_u \oplus 4H_u)$
$30n+24$	$(n+1)(A_g \oplus 2T_{1g} \oplus 2T_{2g} \oplus 4F_g \oplus 4H_g) \oplus n\,(T_{1g} \oplus T_{2g} \oplus H_g)$
$30n+25$	$(n+1)(A_u \oplus 3T_{1u} \oplus 3T_{2u} \oplus 3F_u \oplus 4H_u) \oplus n\,(F_u \oplus H_u)$
$30n+26$	$(n+1)(A_g \oplus 3T_{1g} \oplus 2T_{2g} \oplus 3F_g \oplus 5H_g) \oplus n\,(T_{2g} \oplus F_g)$
$30n+27$	$(n+1)(A_u \oplus 3T_{1u} \oplus 3T_{2u} \oplus 4F_u \oplus 4H_u) \oplus n\,H_u$
$30n+28$	$(n+1)(A_g \oplus 2T_{1g} \oplus 3T_{2g} \oplus 4F_g \oplus 5H_g) \oplus n\,T_{1g}$
$30n+29$	$n\,A_u \oplus (n+1)(3T_{1u} \oplus 3T_{2u} \oplus 4F_u \oplus 5H_u)$

$n = 0, 1, 2, \ldots$ ↠

T **75**.10 ♣ Subduction from O(3) *(cont.)*

j	$\mathbf{I}_h$
$15n+\frac{1}{2}$	$(n+1)\,E_{1/2,g} \oplus n\,(E_{7/2,g} \oplus 2F_{3/2,g} \oplus 3I_{5/2,g})$
$15n+\frac{3}{2}$	$n\,(E_{1/2,g} \oplus E_{7/2,g} \oplus F_{3/2,g} \oplus 3I_{5/2,g}) \oplus (n+1)\,F_{3/2,g}$
$15n+\frac{5}{2}$	$n\,(E_{1/2,g} \oplus E_{7/2,g} \oplus 2F_{3/2,g} \oplus 2I_{5/2,g}) \oplus (n+1)\,I_{5/2,g}$
$15n+\frac{7}{2}$	$n\,(E_{1/2,g} \oplus 2F_{3/2,g} \oplus 2I_{5/2,g}) \oplus (n+1)(E_{7/2,g} \oplus I_{5/2,g})$
$15n+\frac{9}{2}$	$n\,(E_{1/2,g} \oplus E_{7/2,g} \oplus F_{3/2,g} \oplus 2I_{5/2,g}) \oplus (n+1)(F_{3/2,g} \oplus I_{5/2,g})$
$15n+\frac{11}{2}$	$(n+1)(E_{1/2,g} \oplus F_{3/2,g} \oplus I_{5/2,g}) \oplus n\,(E_{7/2,g} \oplus F_{3/2,g} \oplus 2I_{5/2,g})$
$15n+\frac{13}{2}$	$(n+1)(E_{1/2,g} \oplus E_{7/2,g} \oplus F_{3/2,g} \oplus I_{5/2,g}) \oplus n\,(F_{3/2,g} \oplus 2I_{5/2,g})$
$15n+\frac{15}{2}$	$n\,(E_{1/2,g} \oplus E_{7/2,g} \oplus F_{3/2,g} \oplus I_{5/2,g}) \oplus (n+1)(F_{3/2,g} \oplus 2I_{5/2,g})$
$15n+\frac{17}{2}$	$n\,(E_{1/2,g} \oplus F_{3/2,g} \oplus I_{5/2,g}) \oplus (n+1)(E_{7/2,g} \oplus F_{3/2,g} \oplus 2I_{5/2,g})$
$15n+\frac{19}{2}$	$(n+1)(E_{1/2,g} \oplus E_{7/2,g} \oplus F_{3/2,g} \oplus 2I_{5/2,g}) \oplus n\,(F_{3/2,g} \oplus I_{5/2,g})$
$15n+\frac{21}{2}$	$(n+1)(E_{1/2,g} \oplus 2F_{3/2,g} \oplus 2I_{5/2,g}) \oplus n\,(E_{7/2,g} \oplus I_{5/2,g})$
$15n+\frac{23}{2}$	$(n+1)(E_{1/2,g} \oplus E_{7/2,g} \oplus 2F_{3/2,g} \oplus 2I_{5/2,g}) \oplus n\,I_{5/2,g}$
$15n+\frac{25}{2}$	$(n+1)(E_{1/2,g} \oplus E_{7/2,g} \oplus F_{3/2,g} \oplus 3I_{5/2,g}) \oplus n\,F_{3/2,g}$
$15n+\frac{27}{2}$	$n\,E_{1/2,g} \oplus (n+1)(E_{7/2,g} \oplus 2F_{3/2,g} \oplus 3I_{5/2,g})$
$15n+\frac{29}{2}$	$(n+1)(E_{1/2,g} \oplus E_{7/2,g} \oplus 2F_{3/2,g} \oplus 3I_{5/2,g})$

$n = 0, 1, 2, \ldots$

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