DIPLOMARBEIT

Titel der Diplomarbeit

„Ecological Investment: Investment under Uncertainty and Irreversibility“

Verfasser

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angestrebter akademischer Grad

Magister der Wirtschaftswissenschaften (Mag. rer. soc. oec.)

Wien, im Juni 2008

Studienkennzahl lt. Studienblatt:	A 140
Studienrichtung lt. Studienblatt:	Volkswirtschaftslehre
Betreuerin:	Dr. Julien Prat

DANKSAGUNG

An dieser Stelle möchte ich die Möglichkeit nützen, verschiedenen Personen meinen Dank auszusprechen, ohne die die vorliegende Diplomarbeit nicht entstehen hätte können.

An erster Stelle bedanke ich mich von ganzem Herzen bei meinen Eltern Lisa und Georg, die mich nicht nur finanziell während meines Studiums großzügig unterstützt haben, sondern mir vor allem immer vollstes Vertrauen und bestmögliche Unterstützung während meines Studiums und in all den Jahren davor entgegengebracht haben.

Auch bedanke ich mich bei meinem Diplomarbeitsbetreuer, Dr.Julien Prat, der mich im vergangenen Jahr bei der Themenwahl, der Literatursuche sowie insbesondere bei der Ausarbeitung der Diplomarbeit außerordentlich unterstützt hat, und dessen fachlicher Rat mir immer wieder neue Motivation gab.

Auch meinem Freundeskreis und meiner Schwester Ida möchte ich für ihre moralische Unterstützung danken, insbesondere Pamlela Nolz fürs Korrekturlesen.

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1. Introduction and Motivation

"Economics is a science which studies human behaviours as a relationship between ends and scarce means which have alternative uses." is a well known definition of economics by Lionel Robbins, a British economist in the 19^{th} century. Economics is part of everybody's daily life, ranging from planning the amount of individual daily food to calculating the future production of food by a single enterprise. The worlds largest food company Nestlé, for example, hires more than 260.000 employees and uses highly complicated methods in order to maximize the future profit flows.

The environment we live in and which our descendants will depend on as well, is the fundamental basis of our existence and thus of our daily economic decisions. But since the industrial revolution brought more welfare and broader production possibilities, the environmental system as a hole suffered more and more.

The Intergovernmental Panel on Climate Change (IPCC) Fourth Assessment Report "Climate Change 2007" states: "**Warming of the climate system is unequivocal, as is now evident from observations of increases in global average air and ocean temperatures, widespread melting of snow and ice, and rising global average sea level.**" And "Eleven of the last twelve years (1995-2006) rank among the twelve warmest years in the instrumental record of global surface temperature (since 1850)." Global warming has a lot of different effects, several of them are unpredictable, but some are straight forward like the rise in sea levels: since 1993 it has risen at an average rate of 3.1 mm/yr. Compared to the average rate since 1991, 1.8 mm/yr, a significant increase can be noted. Further, the annual average Arctic sea ice extend has shrunk by 2.7 percent per year, with larger decreases in summer of 7.4 percent per decade since 1978. Climate Change 2007: Synthesis Report (2007)
Global warming affects the whole planet: "Observational evidence from all continents and most oceans shows that many natural systems are being affected by regional climate changes, particularly temperature increases." Climate Change 2007: Synthesis Report (2007)
One of the main aspects in climate change discussions is the fact, that there is little or no doubt any more that it is caused by human activity. "Global greenhouse gas (GHG) emissions due to human activities have grown since pre-industrial times, with an increase of 70 percent

between 1970 and 2004." "Most of the observed increase in globally-averaged temperatures since the mid-20th century is very likely due to the observed increase in anthropogenic GHG concentrations. It is likely there has been significant anthropogenic warming over the past 50 years averaged over each continent (except Antarctica)." Climate Change 2007: Synthesis Report (2007)

This is the point where this thesis ties up to: the economic activity of human being and its coherence to environmental pollution. As the consequences of past actions become more and more clear and are partly irreversible, the future is always uncertain. The increase in GHG concentrations, for example, is irreversible, while at the time they were produced, their impacts on the global climate system were unknown and uncertain.

Furthermore, future economic activities have to be reconsidered very properly, not at last in order to protect our ecosystem from more damage. The earth itself, and live itself, I state, can not be destroyed by human beings, but ecological damage causes unpredictable high future costs. Who will be most affected and therefore will be forced to cope with the consequences of the environmental pollution above-average is not topic of this thesis, although it is a very important aspect.

I will mainly concentrate on a specific evaluation method of economic activities which have an impact on the ecosystem, and on determining the optimal amount and optimal timing of antipollution policies and ecological investments. The traditional cost-benefit analysis, which I will explain in more detail later, can not serve to do so because it neglects three important aspects. Two of them were already mentioned, uncertainty and irreversibility. The third is flexibility, meaning that the decision whether to make an investment or not is often not a now or never decision, but can be delayed. This possibility has a value which has to be taken into account.

The optimal timing of the environmental policy has a great impact on the result of the policy. Thus, optimal timing is an issue of this thesis, and this stochastic control problem is of the following form: At what point should society stop emission and adopt a costly policy in order to reduce pollution. This is called an *optimal stopping* problem.

The traditional cost-benefit analysis which is often used to evaluate environmental policies, recommends such a policy if the present value of the expected flow of benefits exceeds the present value of the expected flow of costs. But, as mentioned, this framework ignores three important characteristics, which I will describe now in more detail.

There are two kinds of *irreversibilities* which have to be accounted for and which work in opposite directions. In general, decision making is called irreversible when a decision considerably reduces the scope of choices available for future decision-making. As we have seen, one main reason of global warming is the concentration of GHG. Thus, an environmental policy targeting this issue certainly would make sense. An emission reducing policy, for example, raises a certain *sunk cost* on society. These sunk costs create an opportunity cost of adopting the policy now, rather than wait for more information, and biases the traditional cost-benefit analysis in favour of policy adoption. Pindyck (2000).

On the other hand, the environmental damage can be irreversible, totally or partially. For example, the concentration of GHG in the atmosphere declines very slowly, and this process can not be speeded up. Thus, adopting a policy now rather than waiting has a *sunk benefit*, a negative opportunity cost, which biases traditional cost-benefit analysis against policy adoption. Pindyck (2000). Therefore, taking these irreversibilities into account may lead to different outcomes.

Also, two kinds of *uncertainties* have to be distinguished. *Economic uncertainty* refers to the future costs and benefits of environmental damage and its reduction. For example, although the increase in GHG concentration can be measured, the resulting costs to society, like decreases in agricultural output, are unknown. The uncertainty over the evolution of the relevant ecosystem is called *ecological uncertainty*. An emission reducing policy, for example, will result in a decrease of GHG concentration, but we do not know exactly by how much, nor do we know its impacts on global or local temperatures.

The third aspect is flexibility. The adoption of a policy can be delayed, and is rarely a now or never proposition. Waiting for more information can make sense and may lead to a more efficient outcome.

Waiting has a value, and this leads to the next topic. We have seen that whenever sunk costs, uncertainties and irreversibilities are concerned, an evaluation based on the expected net present value is not efficient. A numerical example will be presented later. Thus, another evaluation method is needed, the *real options approach*, which makes it possible to deal with these problems. It argues that policies which can be delayed to gather more information should be given more weight. "This option has a value, called option value, similar to the financial call or put options." Zhao (2002). It is straight forward that an investment should not only be valued in terms of the scale, but also by the option to postpone, i.e. the timing of the investment plays an important role.

To give an example, assume an investment that costs \$84 million and will last forever. The payoff in the current period is given by p=\$10 million, and the projects future annual payoff is, due to uncertainties, either 0.5p=\$5 million or 1.5p=\$15 million. The discount rate is 10 percent. The traditional net present value (NPV) rule evaluates the expected present value of the project by (10/0.1)-84=$16 million, and suggests investing now. Assume further that the uncertainty holds for one year, and thus the investor faces the possibility to delay the investment for one year and wait for more information. Then, if the payoff one year later turns out to be 0.5p=$5 million, no investment is made, since 5/0.1-84=-$34 million. But if the payoff turns out to be 1.5p=$15 million, the total payoff is given by 15/0.1-84=$66 million and the project will be executed. Thus, the expected present value of the payoff of waiting in period one is (0.5)*(66)/1.1=$30 million, given the possibility of investing in period 2. In this example, waiting has a higher value than investing immediately. As we can see, the investment decision has to be considered in a dynamic framework, and the possibility of investing now has to compete with itself at a later date. For the sake of completeness it is mentioned that the cost of delaying arises from discounting, because the earlier an investment is undertaken, the earlier the net benefits start to account.

Zhao (2002) summarizes the conditions which make the real option approach more efficient than the expected NPV rule: (1) the outcome of the project is uncertain; (2) future information can be gathered helping to better evaluate the project; (3) the project or some of its components can be delayed; and (4) there are adjustment costs in reversing the project or its components.

The underlying thesis is structured in the following way: Chapter 2 explains the necessary mathematical tools and basic principles which will be used throughout the work. Dynamic programming, a framework for dynamic optimisation which is especially useful when uncertainty is considered, as well as Brownian Motions and Ito Processes which serve to describe stochastic processes are explained and motivated. Chapter 3 is the main part of this thesis. A model developed by Pindyck (2000, 2002), which serves as a building plot for a lot of scientific papers about ecological investment under uncertainty and irreversibility, is presented and discussed in detail. A rather simple model of determining the optimal amount and timing of an emission reducing policy is presented, which is then extended in certain directions. Different cost and benefit functions are observed as well as the impact of economic and ecological uncertainties, different levels of irreversibility, and the role of flexibility.

In Chapter 4 two different extensions of Pindyck's model are discussed. First, the role of the discount rate and its implementation on policy adoption is analysed. Special attention is given to the income-pollution pattern and the Environmental Kuznets Curve.

The second extension of the model presented in Chapter 3 concentrates on the impacts of environmental policy adoption when the possibility of extreme events, i.e. extreme changes in the evolution of the ecosystem or in the future costs and benefits associated with the environmental damage, are concerned.

In Chapter 5 some concluding remarks are given.

2. Mathematical Background

Before I will work out a detailed model of how to evaluate uncertainties and irreversibilities in investment decisions, specifically in ecological investment, I explain the relevant mathematical tools.
As the future of an ecological investment is uncertain, stochastic calculus is needed to model the dynamics of the value of a project. Specifically, Brownian Motions, also known as Wiener Processes, and a generalized form of it, the Ito Process, as well as dynamic programming, are discussed in the following chapter.

The reader must be reminded that the purpose of this chapter is only to explain the mathematical background, so it might be more convenient for experts to jump directly to Chapter 3.

2.1 Dynamic programming

Throughout this thesis the main aspect which is investigated is the optimal policy and the optimal timing of a policymaker who faces environmental and ecological costs and benefits, and who is asked to protect the environment in order to minimize the economic costs which resolve from pollution. As in any other investment decision, the decision maker has to account for the current as well as future costs and benefits, thus taking into account the option of future decisions, which are affected by uncertainties.
Dynamic programming and contingent claims analysis, which are closely related to each other, are the most useful mathematical tools for dynamic optimization, especially if uncertainty is regarded for. Here, only the basics of dynamic programming are presented and then used from chapter 3 on.
"Dynamic programming breaks a hole sequence of decisions into just two components: the immediate decision, and a valuation function that encapsulates the consequences of all subsequent decisions, starting with the position that results from the immediate decision." Dixit and Pindyck (1994)
To give an intuition, assume a firm whose state is described by the variable x_t which is known and controlled by the control variable u_t. All future values are unknown. The immediate profit

flow of this firm is given by $\pi(x_t, u_t)$. Following dynamic programming, the whole continuation period after period t is summarized into one sequence. The outcome of this period t+1 is given by $F_{t+1}(x_{t+1})$, which is random and thus we must take its expected value $\varepsilon_t[F_{t+1}(x_{t+1})]$, which is often called the continuation value.
Therefore, the firm has to choose u_t to maximize the outcomes of both periods:

$$F_t(x_t) = \max_{u_t} \left\{ \pi_t(x_t, u_t) + \frac{1}{1+\rho} \varepsilon_t[F_{t+1}(x_{t+1})] \right\}, \tag{1}$$

where the outcome of period t+1 is discounted by the factor ρ.
Equation (1) is called Bellman-equation, following the Bellman's Principle of Optimality: *An optimal policy has the property that, whatever the initial action, the remaining choices constitute an optimal policy with respect to the subproblem starting at the state that results from the initial situation.* Dixit and Pindyck (1994)

Whether considering two periods, more periods, or infinite horizon the approach of dynamic programming is in any case more or less the same as described above, where the choice in any period is binary. One particular class of this mathematical tool is called "Optimal Stopping". Here, the decision is to stop the process to take the termination payoff, or to continue for one period, where then a similar decision has to be made. In other words, stopping corresponds to making the investment and continuation corresponds to waiting. We will make use of this later.

2.2 The Wiener Process / Brownian Motion

A Wiener Process has to fulfil a set of requirements. The definitions are taken from Dixit and Pindyck (1994):

- It is a *stochastic process*, which means that the variable evolves over time in a way that is at least partially random. Examples for stochastic processes are the price of IBM stock as well as the temperature in Vienna, whereas the former is nonstationary – the expected value of its price can grow without bound, and the latter is a stationary process – the statistical properties of this variable are roughly constant over long periods of time.

- Both examples are *continuous-time* stochastic processes. Suppose a stochastic process which is defined by the probability law for the evolution of x_t of a variable x over time t. If t is a continuous variable, so is the process.
- It is a *Markov process*. The Markov property states, that the probability distribution for x_{t+1} depends only on x_t, and not additionally on what happened before time t. In words, the probability distribution for all future values of the process depends only on the current value and is unaffected by past values of the process, or any other information.
- The process has *independent increments*: The probability distribution for the change in the process over any time interval is independent of any other nonoverlapping time interval.
- Changes in the process over any finite time interval are *normally distributed*, with a variance that increases linearly with the time interval.

These properties can be explained more formally.
If z(t) is a Wiener process, than any change in z, Δz, corresponding to a time interval Δt, satisfies the following conditions:

1. The relationship between Δz and Δt is given by

$$\Delta z = \epsilon_t \sqrt{\Delta t}, \tag{2}$$

where ϵ_t is a normally distributed random variable with a mean of zero and a standard deviation of 1.

2. The random variable ϵ_t is serially uncorrelated, that is $\varepsilon[\epsilon_t, \epsilon_s] = 0$ for $t \neq s$. Thus the values of Δz for any two different intervals of time are independent. [Thus z(t) follows a Markov process with independent increments.]

Assume now a time interval T, divided into n units of length Δt, with n=T/Δt. Then the change over this time interval is given by:

$$z(s+T)-z(s)=\sum_{i=1}^{n}\in_i \sqrt{\Delta t}, \qquad (3)$$

where the ϵ_i variables are independent of each other. Thus, the change z(s+T)-z(s) is normally distributed with mean zero and variance nΔt=T which grows linearly with the time horizon. This fact makes the Wiener process a nonstationary process.
Letting Δt become infinitesimal small, the increment of a Wiener process, dz, in continuous time is

$$dz=\in_t \sqrt{dt}, \qquad (4)$$

where ϵ_t has zero mean and unit standard deviation, $\varepsilon(dz)=0,$ and $Var\left[dz\right]=\varepsilon\left[\left(dz\right)^2\right]=dt.$
A very important aspect of the Wiener process is, that it has no time derivation in a conventional sense, since $\Delta z/\Delta t=\in_t (\Delta t)^{-\frac{1}{2}}$, which becomes infinite as Δt approaches zero.

2.3 Brownian Motions and Ito Process

As mentioned, the basic Wiener Process can be generalized in certain directions. Besides the Ito Process, the *Brownian motion with drift* is especially important for us.
Thus, a generalization of equation (4) is

$$dx=\alpha dt+\sigma dz, \qquad (5)$$

where dz is the increment of a Wiener process, α is the drift parameter, and σ the variance parameter.
Note that the expected value of a change in x, Δx, over any time interval Δt, is $\varepsilon(\Delta x)=\alpha\Delta t$ and its variance $Var(\Delta x)=\sigma^2\Delta t.$

Another possibility to write this Brownian motion with drift is:

$$dx = a(x,t)dt + b(x,t)dz, \qquad (6)$$

where dz again is the increment of a Wiener process, and a(x,t) and b(x,t) are known (non-random) functions. Here, the drift and variance coefficients are functions of the current state and time. The process of equation (6) is also called an *Ito process*. The expectation is $\varepsilon(dz) = 0$, and therefore $\varepsilon(dx) = a(x,t)dt$. The variance of dx is $\varepsilon\left[dx^2\right] - \left(\varepsilon\left[dx\right]^2\right)$, which contains terms in dt, in $(dt)^2$, and in (dt)(dz), which is of order $(dt)^{3/2}$. Assume dt infinitesimal small, then terms in $(dt)^2$ and in $(dt)^{3/2}$ can be ignored. The variance becomes

$$Var\left[dx\right] = b^2(x,t)dt, \qquad (7)$$

where $b^2(x,t)$ is called the instantaneous variance rate, as a(x,t) is called the expected instantaneous drift rate of the Ito process.
This will turn out to be very useful.

The Geometric Brownian motion with drift is another type of stochastic processes, and a generalisation of equation (6), the Ito process

$$dx = \alpha x dt + \sigma x dz, \qquad (8)$$

where a(x,t) = αx, and b(x,t) = σx, where α and σ are constant.

2.4 Ito's Lemma

As mentioned in the introductory part, the aim of this thesis is to find the optimal timing and the optimal amount by which a certain policy should be adopted in order to reduce pollution. Brownian motion and Ito process respectively, will be used to model the unknown development of future economic outcomes - to get uncertainty into the model. Anyway, the Ito process is not differentiable, which would make optimization impossible. But we will make use of Ito´s Lemma, which makes it possible to differentiate Ito processes. Specifically, we will describe the value of the option to adopt a certain policy in the future to reduce pollution. This option value will partly be determined by economic uncertainty, which itself is represented by a Brownian motion. In this case we would want to determine the stochastic process that the value of the option follows. To do so we will make use of Ito´s Lemma:

Consider a function x(t) that follows the process of equation (6) and a function F(x,t) that is at least twice differentiable in x and once in t. The total differential would be

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial t}dt. \tag{9}$$

Now suppose we also include higher-order terms for changes in t

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}(dx)^2 + \frac{1}{6}\frac{\partial^3 F}{\partial x^3}(dx)^3 + \dots\ . \tag{10}$$

To see if the third and fourth term vanish in the limit, expand the right hand side and substitute equation (6) for dx to determine $(dx)^2$:

$$(dx)^2 = a^2(x,t)(dt)^2 + 2a(x,t)b(x,t)(dt)^{3/2} + b^2(x,t)dt. \tag{11}$$

And as we know, terms in $(dt)^{3/2}$ and $(dt)^2$ as well as any other term which includes dt to the power greater than one go to zero faster than dt as it becomes infinitesimally small, so the third an fourth term can be ignored and equation (11) becomes

$$(dx)^2 = b^2(x,t)dt, \tag{12}$$

and Ito´s Lemma gives the differential dF as

$$dF = \left[\frac{\partial F}{\partial t} + a(x,t)\frac{\partial F}{\partial x} + \frac{1}{2}b^2(x,t)\frac{\partial^2 F}{\partial x^2}\right]dt + b(x,t)\frac{\partial F}{\partial x}dz. \tag{13}$$

3. Ecological Investment under Uncertainty and Irreversibility

In the following chapter the basic model from Pindyck (2000) will be discussed. Later, this rather simple model will be expanded in certain directions, and it will be shown which specific problem in ecological discussion it can be useful for.

As already mentioned, the analytical framework used by Pindyck (2000) is not a typical net present value rule calculation. Rather it takes into account certain types of irreversibility, uncertainty and flexibility using the real option approach described in the introductory part. The aim is to determine the optimal timing and the optimal amount of an emission reducing policy, adopted by a decision maker. The stock of pollution, as we will see within a short time, is given and controlled by an emission rate. Thus, a costly policy concentrates on this control variable.

3.1 Analytical Framework

Unsurprisingly the first variable to explain is the one of environmental pollution, M_t, which summarizes one or more stocks of environmental pollutants. It might stand for the concentration of CO_2 in the atmosphere, for the acid level of a lake or forest, or for something comparable.

And of course M_t is very unlikely to be constant over a longer period of time, so let E_t be the flow variable that controls $M_{t.}$ E_t might stand for the rate of CO_2 emission for example; actually whenever emissions are mentioned in this thesis, which will be the case quite often, it can be regarded as an example for any other type of pollution.

It will be assumed that E_t follows an exogenous trajectory as long as no policy controls it. So, the endogenous evolution of M_t is given by

$$dM = [\beta E(t) - \delta M(t)] + d\varnothing(t), \qquad (1)$$

where dø(t) is the increment of a stochastic process. As dM_t characterises the ecological evolution, dø(t) will only be considered when taking into account ecological uncertainty. The parameter δ is the natural rate at which the stock of pollution dissipates over time, and β measures the quantity of emissions absorbed by the ecosystem.

M_t, as the stock of pollution, is also assumed to entail a certain cost, which is specified by the function $B(M_t, \Theta_t)$, where B is assumed to be linear in M_t, and Θ_t shifts over time and reflects certain changes in tastes and technology. For example, the development of new technologies decreases the social cost of higher pollution, or on the other hand demographic changes raise the cost. Uncertainty over the future costs and benefits of policy adoption can be reached by letting Θ_t follow a stochastic process. This will be shown in chapter 3.2. And as we will see, Θ_t determines the timing of policy adoption.

Due to the fact that pollution results in costs for society, a policy has to be induced which has an impact on the evolution of E_t and therefore reduces pollution. Consider a policy at time T which changes the evolution of E_t to some new trajectory E_t^*.

The pollution reduction can be done gradually or at once. For now consider the latter, a reduction to a new and permanent level E_1, with $0 \leq E_1 \leq E_0$.

Not only pollution leads to a certain cost, but also the induction of the policy leads to a certain social cost, which is assumed to be completely sunk and denoted by $K(E_1)$.

It is quite trivial that the policy objective which has to be maximized involves these two different costs, the one of pollution and the one of policy adoption which reduces pollution. It is denoted by

$$W = \varepsilon_0 \int_0^\infty B(M_t, \theta_t) e^{-rt} - \varepsilon_0 K(E_1) e^{-rT}, \qquad (2)$$

where ε_0 is the expectation at time t = 0, T is the point of time, where the policy is adopted, $E_0 - E_1$ is the amount that emissions are reduced, and r is the discount rate.

So what has to be determined, on the one hand, is the optimal time at which the policy should be induced - thus we deal with an optimal stopping problem, and on the other hand, by how much emissions should be reduced, not forgetting about the dependence of M_t on E_t, and the possibly stochastic evolution of M_t and Θ_t.

In the next part of the thesis a model will be presented which only contains economic uncertainty but doesn't regard ecological uncertainty, thus Θ_t evolves stochastically but M_t does not. Chapter 3.4 presents a model with ecological uncertainty.

3.2 Economic Uncertainty

Now we deal with a model where the evolution of Θ_t includes a stochastic term, and where the policy adoption implies a once and for all reduction from its initial level E_0 to zero.
The cost of emission reduction is assumed to be a linear function of the size of the reduction, denoted by K. The cost function depending on the pollution, $B(M_t, \Theta_t)$ is assumed to be linear in M_t, i.e.

$$B(M_t, \theta_t) = -\theta_t M_t \tag{3}$$

Equation (1) loses the stochastic part since there is no ecological uncertainty considered now, and turns to

$$\frac{dM}{dt} = [\beta E(t) - \delta M(t)]dt$$

There is economic uncertainty, described by a geometric Brownian motion, which has been discussed in chapter 2

$$d\theta = \alpha\theta dt + \sigma\theta dz \tag{4}$$

The function $B(M_t, \Theta_t)$ is known for some parts but for others not. The flow of social cost from M_t is known, but the flow of future cost is uncertain, and this uncertainty grows along the time horizon.
The policy which may be adopted to reduce E from E_0 to zero raises a certain cost K as well. Here, we suppose that $K = kE_0$.

Thus, we have to find a policy adoption rule that maximizes the net present value function of equation (2), subject to equations (4) and (1), for the evolution of Θ_t and M_t.

This problem will be solved by dynamic programming, which was introduced in chapter 1. Two regions will be defined, the "no-adopt" region, indicating the time before policy adoption in which $E_t = E_0$, and the "adopt" region where $E_t = 0$, denoting the time after policy adoption. For both regions a net present value function has to be defined: $W^N(\Theta, M)$ for the "no-adopt" region and $W^A(\Theta, M)$ for the "adopt" region.

Since $B(M_t, \theta_t) = -\theta_t M_t$, W(Θ, M) must satisfy the following general Bellman equation:

$$rW = -\theta M + \frac{1}{dt}\varepsilon_t(dW), \tag{5}$$

where -ΘM is the social cost from the stock of pollutant, and the second term on the right hand side is the expected rate of increase in W – the capital gain.

In order to find a solution for equation (5), let's have a look at dW first, remembering equation (10) introduced in chapter 2.4. We expand dW by Ito´s Lemma:

$$dW = \frac{dW}{dM}dM + \frac{dW}{d\theta}d\theta + \frac{1}{2}\frac{d^2W}{d\theta^2}(d\theta)^2 \tag{6}$$

Taking into account equation (12) from chapter 2.4, dΘ is given by

$$(d\theta)^2 = (\alpha\theta dt + \sigma\theta dz)^2 = \sigma^2\theta^2. \tag{7}$$

Inserting equation (7) into (6) yields

$$dW = \frac{dW}{dM}dM + \alpha\theta\frac{dW}{d\theta}dt + \sigma\theta\frac{dW}{d\theta}dz + \frac{1}{2}\frac{d^2W}{d\theta^2}(\sigma^2\theta^2 dt)$$

Thus, taking into account that $\sigma\theta\frac{dW}{d\theta}dz = 0$, the expectation of dW is given by

$$E[dW] = E[\frac{dW}{dM}dM + \alpha\theta\frac{dW}{d\theta}dt + \frac{1}{2}\sigma^2\theta^2\frac{d^2W}{d\theta^2}dt],$$

which is now inserted into equation (5):

$$rW = -\theta M + \frac{1}{dt}\varepsilon_t(dW) = -\theta M + [\beta E(t) - \delta M(t)]dW_M + \alpha\theta W_\theta + \frac{1}{2}\sigma^2\theta^2 W_{\theta\theta}.$$

To get the Bellman equations for the „no adopt“ region denoted by $W^N(\Theta, M)$, and the „adopt“ region denoted by $W^A(\Theta, M)$, respectively, we distinguish in E:

$W^N(\Theta, M)$ must satisfy:

$E_t = E_0$:

$$rW^N = -\theta M + (\beta E_0 - \delta M)W_M^N + \alpha\theta W_\theta^N + \frac{1}{2}\sigma^2\theta^2 W_{\theta\theta}^N, \tag{8}$$

and likewise $W^A(\Theta, M)$ must satisfy the Bellman equation:

$E_t = 0$:

$$rW^A = -\theta M - \delta M W_M^A + \alpha\theta W_\theta^A + \frac{1}{2}\sigma^2\theta^2 W_{\theta\theta}^A. \tag{9}$$

These two differential equations have to be solved subject to the following set of boundary conditions:

$$W^N(0, M) = 0, \tag{10}$$

$$W^N(\theta^*, M) = W^A(\theta, M) - K, \tag{11}$$

$$W_\theta^N(\theta^*, M) = W_\theta^A(\theta^*, M). \tag{12}$$

Equation (10) simply states, that if Θ is zero, it will stay at zero thereafter. Equation (11) is the so called value matching condition. It states that, if Θ reaches the critical level Θ*, at or above which the policy should be adopted, and society adopts the policy, a certain cost K = kE_0 incurs, and the net present value is $W^A(\Theta, M) - K$. Thus, the parameter Θ determines the timing of policy adoption.

Equation (12) is the smooth pasting condition. It states that if adoption is indeed optimal at the critical value Θ*, then the derivates of the value functions must be continuous at Θ*.

The solutions of these two differential equations are:

$$W^N(\theta, M) = A\theta^\gamma - \frac{\theta M}{r+\delta-\alpha} - \frac{\beta E_0 \theta}{(r-\alpha)(r+\delta-\alpha)}, \qquad (13)$$

and

$$W^A(\theta, M) = \frac{-\theta M}{r+\delta-\alpha}. \qquad (14)$$

First consider equation (13): $A\Theta^\gamma$ is the general (homogenous) solution of this type of differential equation, and the other parts are the inhomogeneous solutions. Consider the second term on the right hand side of equation (13), which is equal to the right hand side of equation (14). The current stock of pollutant, M, decays at a rate δ, and α is the growth rate of θ. Thus, the present value of the flow of social cost resulting from M is given by $- \theta M / (r+\delta-\alpha)$. As the emission reducing policy is not yet adopted in the "no-adopt" region, also the present value of the flow of social cost that results from emission continuation has to be considered, which is the third term on the right hand side of equation (13). The present value of the flow of cost from emissions E_0 now is $\beta E_0 \theta / (r+\delta-\alpha)$, and taking into account the cost from emissions E_0 in all future periods leads to $\beta E_0 \theta / (r-\alpha)(r+\delta-\alpha)$.
If the emission reducing policy is adopted, E = 0, and the value function W^A applies.

A, Θ^* and γ are unknowns which have to be determined. This problem can be solved using the boundary conditions (10) – (12). Detailed calculations can be found in the technical appendix 6.a.

Inserting equation (8) into boundary conditions (10) and (11) yields

$$A = (\frac{\gamma-1}{k})^{\gamma-1}[\frac{\beta}{(r-\alpha)(r+\delta-\alpha)\gamma}]^\gamma E_0, \qquad (15)$$

$$\theta^* = (\frac{\gamma}{\gamma-1})\frac{k(r-\alpha)(r+\delta-\alpha)}{\beta}, \qquad (16)$$

where A and therefore the value of the option is linear in E_0.

Finally by using boundary equation (10) and inserting it in equation (8), we obtain the following quadratic equation which determines γ:

$$\frac{1}{2}\sigma^2\gamma(\gamma-1)+\alpha\gamma-r=0. \qquad (17)$$

So γ can now be calculated by solving the quadratic equation

$$\gamma=\frac{1}{2}-\frac{\alpha}{\sigma^2}+\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}. \qquad (18)$$

From equation (16) we can determine when it is optimal to adopt the policy in the absence of any uncertainty, namely if γ is zero:

$$\frac{\beta\theta}{(r-\alpha)(r+\delta-\alpha)}=k. \qquad (19)$$

The left-hand side of equation (19) is the present value of the flow of social cost from one unit of emissions now and throughout the future, and the right hand side is the cost of permanently reducing emissions by one unit. This is called the standard net present value or standard cost benefit calculation.

It can be rewritten as

$$\theta^*=\frac{k(r-\alpha)(r+\delta-\alpha)}{\beta}, \qquad (20)$$

where θ is increased by the factor $\gamma/(\gamma-1)$ if there is uncertainty.

Let's now compare the opportunity costs of current adoption with the opportunity "benefits" of current adoption. To do so, we define W* as the value when the adoption is made optimally, and W_0 as the value when adoption is made immediately, and calculate $W^* - W_0$.
Suppose $\theta < \theta^*$, and $W^* = W^N$. We know that $W_0 = W^A - K$, and so
$W^* - W_0 = W^N - W^A + K$, or

$$W^*-W_0=K+A\theta^\gamma-\frac{\beta E_0\theta}{(r-\alpha)(r+\delta-\alpha)}. \qquad (21)$$

So the difference between the opportunity costs and benefits of current adoption is equal to K, the direct cost from policy adoption, to the value of the option to adopt, which counts as an opportunity cost of current adoption – the second term on the right hand side, minus the present value of the additional flow of social cost from continued emissions, an opportunity "benefit" from current adoption – the last term on the right hand side of equation (21).

Numerical example

For a better understanding of the model presented above, a numerical example is now presented. The relevant parameter values, also used by Pindyck (2000), are summarized in Table 1.

Parameter	Interpretation	Value
α	expected percentage rate of growth of θ	0.0
r	interest rate	0.04
δ	pollutant decay rate	0.02
σ	volatility rate of θ	0.2
β	emissions absorbed by the ecosystem	1.0
E_0	emission rate	300,000 tons per year
θ_0	current social cost	$20 per ton

Table 1: Parameter values

Thus, $K = k^*E_0$ = $2 billion. Equation (18) yields γ = 2,0, equation (15) is equal to A = 1,953,125, and equation (16) gives θ^* = $32 per ton.

At the current value of θ_0 = $20 per ton, the value of the option to adopt it in the future is $A\theta_0^\gamma = \$0.78$ billion and the policy should not be adopted immediately. But at the critical value θ^* = $32 per ton, $A\theta^\gamma = \$2.0$ billion, and the policy should be adopted.

More precisely, equation (21) compares the opportunity costs of current adoption with the opportunity "benefits" of current adoption:

$$W^* - W_0 = K + A\theta^\gamma - \frac{\beta E_0 \theta}{(r-\alpha)(r+\delta-\alpha)} = 281{,}350{,}000,$$

which is positive, and thus it is optimal to delay the emission reducing policy.

The solution can also been shown graphically, as illustrated in Figure 1, where M = 0 is assumed. Note that the critical value $\theta^* = \$32$ per ton is found at the tangency of W^N and $W^A - K$. Assuming M > 0 rotates both curves downwards.

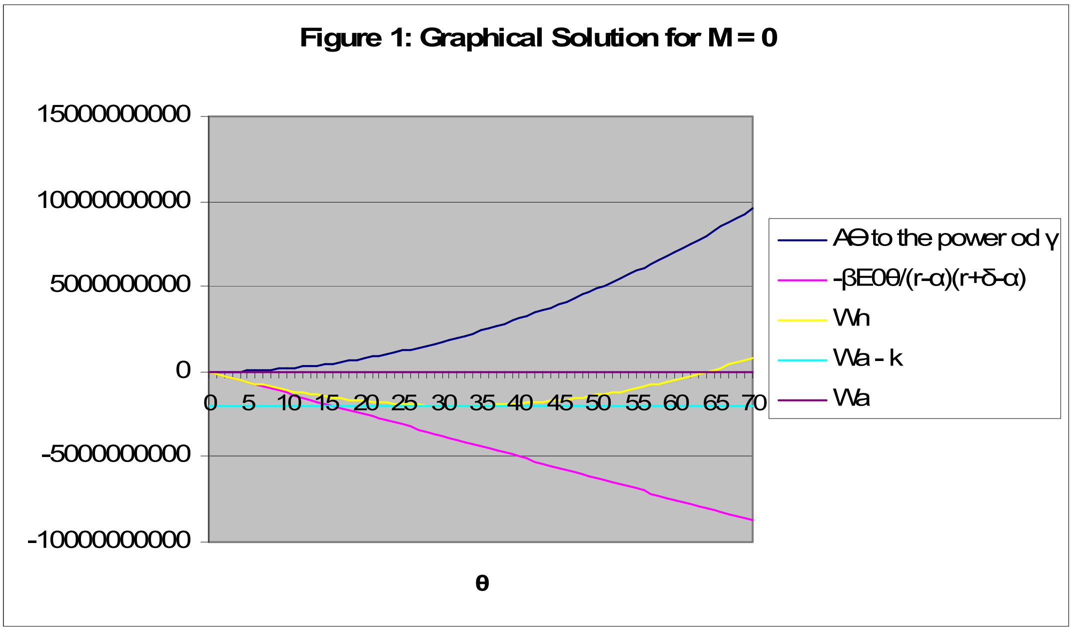

Figure 1

Let’s have a look at some dynamic effects of the outcomes:

As mentioned, we consider economic uncertainty in this model, which is described by equation (4). The more uncertainty there is, the higher is σ. Considering equation (18), as σ increases, γ decreases, and hence θ* increases. In words, the greater the uncertainty of the future social cost of the pollutant, the greater is the incentive to wait rather than to adopt the policy now, in order to gather more information. Remember that θ determines the timing of policy adoption. Of course, the greater the current cost of pollutant, the greater the incentive to trigger policy adoption.

Let’s have a look at the graphical solution for the case of increasing uncertainty. In Figure 1 it is assumed that σ = 0.2. Let’s now assume that σ = 0.25, and all other variables given by table 1. Then, γ decreases to 1.73 and θ* increases to 37.7. This is plotted in Figure 2:

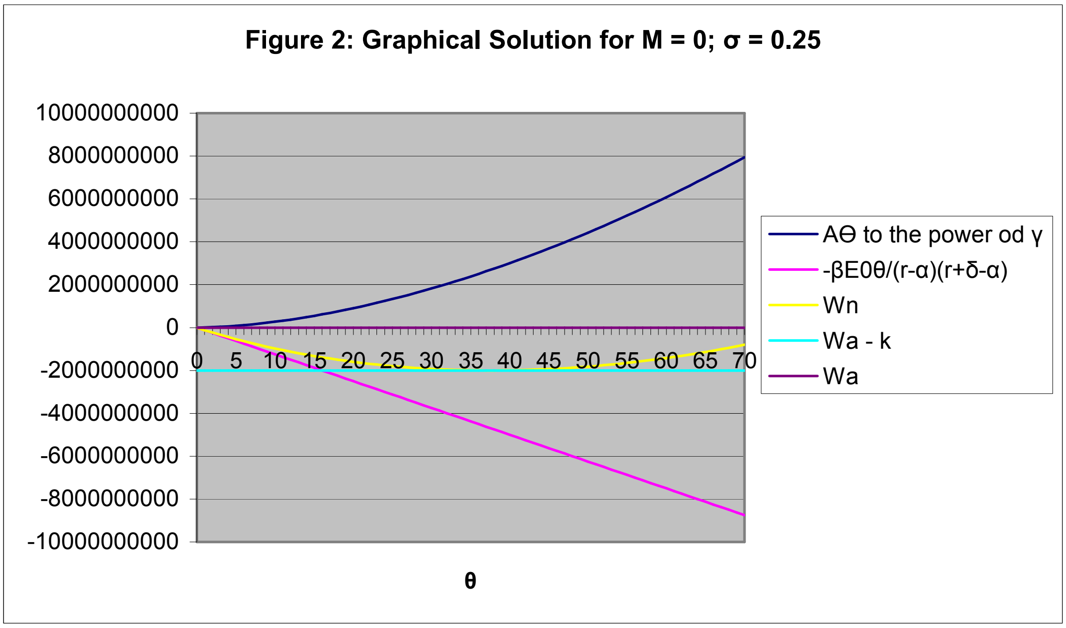

Figure 2

The same increase in θ* is true if the discount rate r increases, which also implies a greater reduction in the present value of K.

Figure 3 shows the solution for r = 0.06 and all other variables given by Table 1. Now θ* is 56.6:

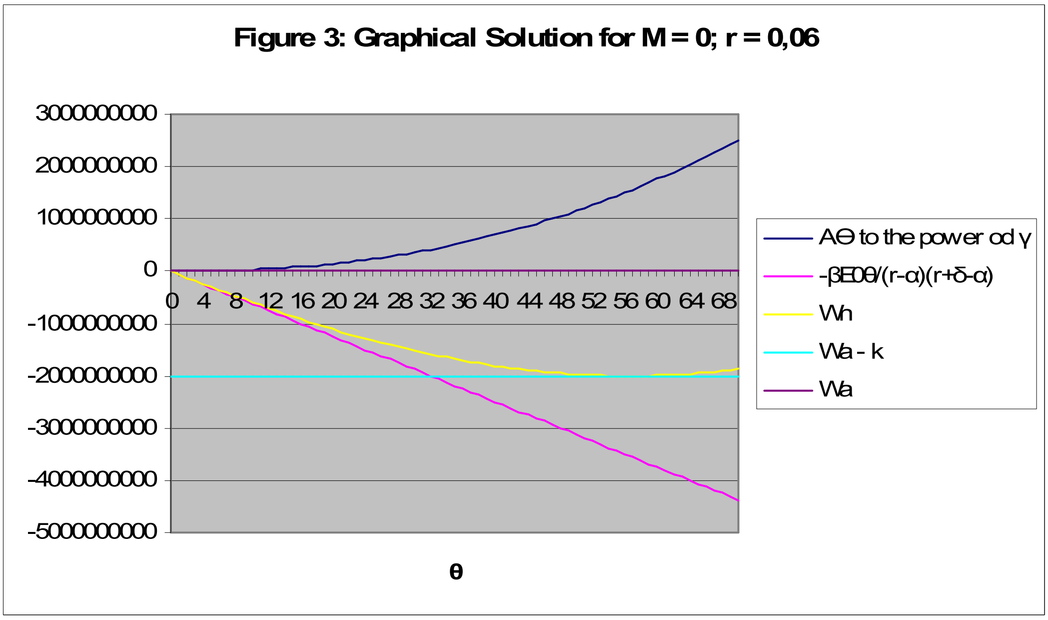

Figure 3

Besides uncertainty, irreversibility plays an important role in calculating the optimal timing of policy adoption. As the rate of "depreciation" of the stock of pollutant, δ, increases, the reversibility of the damage form environmental pollution rises. This makes the sunk benefit of adopting the policy now rather than waiting smaller.

Have a look at Figure 4 below: now δ = 0.04, which is twice as large as in Figure 1. θ* is increased to 42.6 in this case.

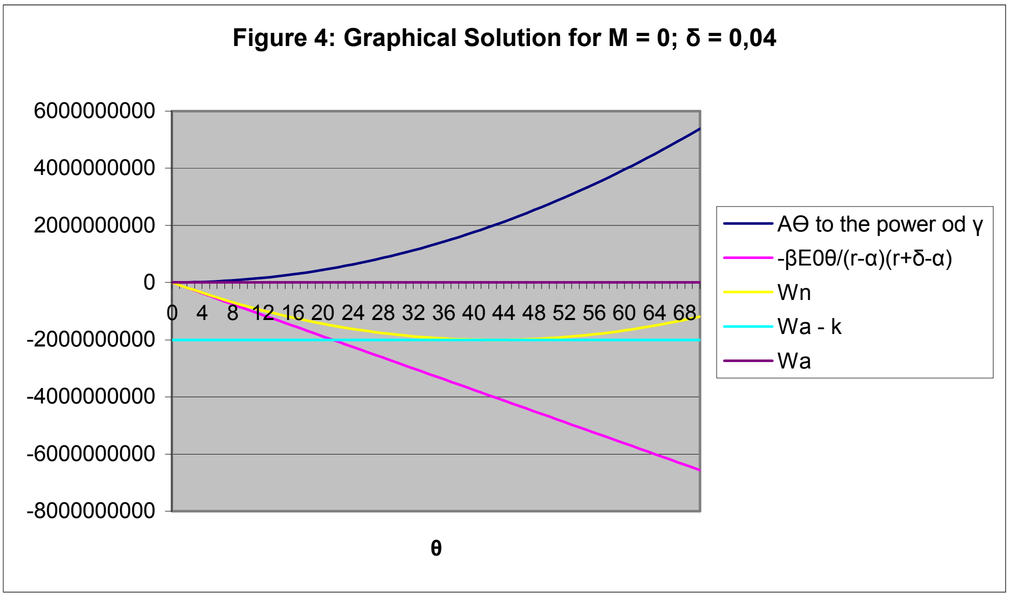

Figure 4

Further, as α increases, θ* monotonically decreases. Having a look at equation (4) shows that an increase in α leads to a higher future flow of social costs and thus makes an earlier policy adoption favourable. Figure 5 shows the solution for θ* when its growth rate α is slightly increased to 0.01. Here, θ* decreases to 24.6, thus the pollution reducing policy is adopted earlier.

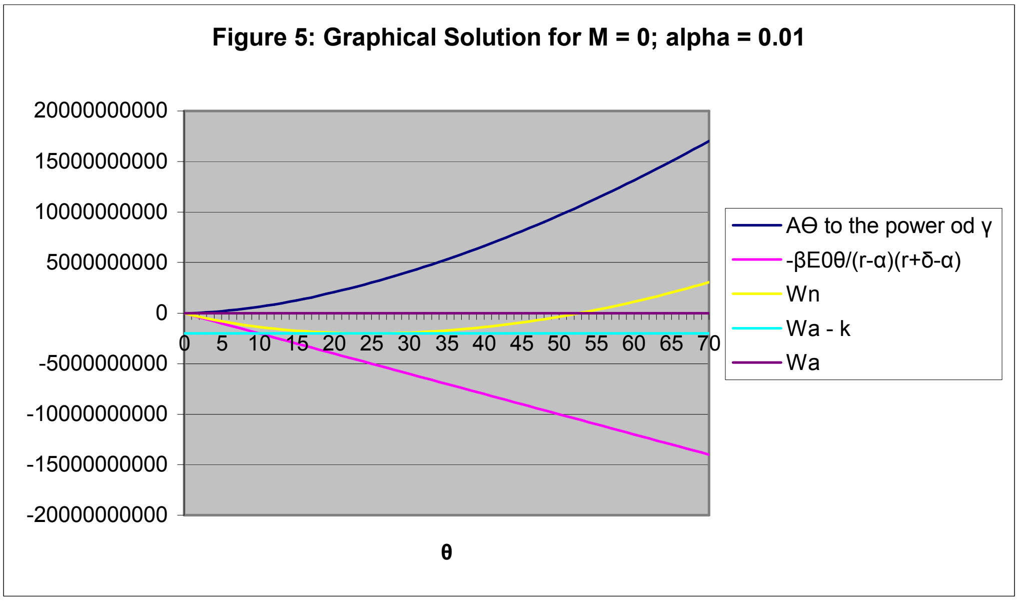

Figure 5

Finally, θ* is independent of M_t and E_0. A increases linearly with E_0 and augments the value of society's option to adopt the policy.

As an example, assume that E_0 = 600,000, all other parameters stay unchanged. See figure 6 below, where θ* is still 32. Although A^* increases to 3906054, W^N and W^A – K don't change, and thus its tangency is still at 32.

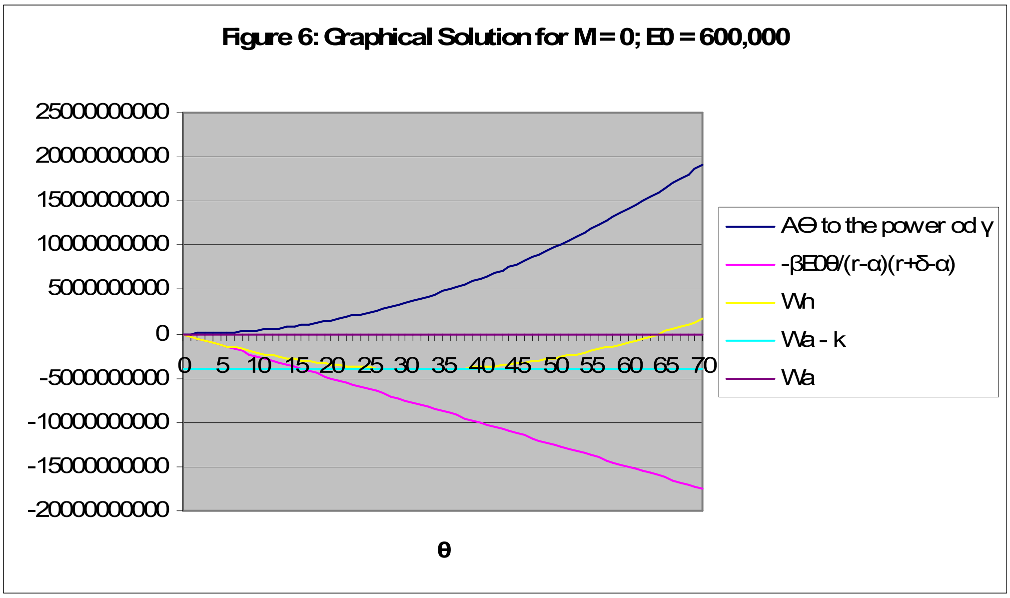

Figure 6

To see the dependency of θ^* on σ and δ more precisely, have a look at Figure 2, for δ = 0.01 to 0.04. As σ increases, θ^* increases as well, as we have already seen in figure 2. This is partly due to the fact that a model assuming an all-or-nothing reduction is regarded, as Pindyck (2000) states. But it shows clearly that taking uncertainty into account plays an important role for emission reducing policy optimization.

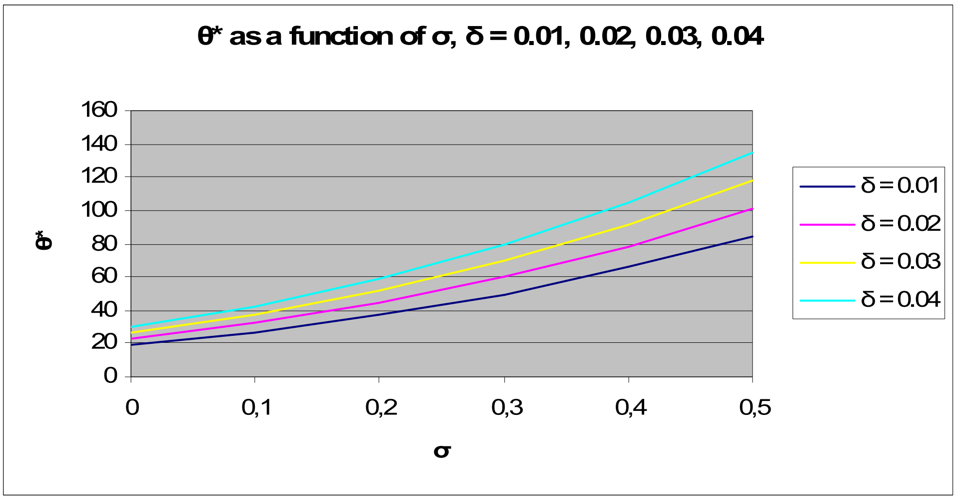

Figure 7

3.2.1 Convex Costs and Partial Reduction in Emissions

Now a policy is considered which partially reduces emissions. Suppose that the sunk cost from policy adoption is a quadratic function of the amount of reduced emissions:

$$K = k_1(E_0 - E_1) + k_2(E_0 - E_1)^2, \tag{22}$$

where $k_1, k_2 > 0$.

So a permanent 1-unit reduction in E_l leads to a cost of

$$k(E) = -\frac{dK}{dE_1} = k_1 + 2k_2(E_0 - E_1).$$

In this model, not only the optimal time of adoption, in the form of the variable θ* as in chapter 3.2, but also the optimal amount by which the emissions should be reduced, the value of E_1, has to be found.

Again, there is an "adopt" and a "no-adopt" region, denoted by $W^A(\theta, M)$ and $W^N(\theta, M)$.

For the "no-adopt" region equation (8) still holds:

$$rW^N = -\theta M + (\beta E_0 - \delta M)W_M^N + \alpha W_\theta^N + \frac{1}{2}\sigma^2\theta^2 W_{\theta\theta}^N.$$

but, since after policy adoption $\frac{dM}{dt} = \beta E_1 - \delta M$ because emissions are not reduced to zero, an additional term, $\beta E_1 W_M^A$ has to be added to equation (9):

$$rW^A = -\theta M + (\beta E_1 - \delta M)W_M^A + \alpha\theta W_\theta^A + \frac{1}{2}\sigma^2\theta^2 W_{\theta\theta}^A. \tag{23}$$

Thus, the solution for $W^A(\theta, M)$ is

$$W^A(\theta, M) = -\frac{\theta M}{r+\delta-\alpha} - \frac{\beta E_1 \theta}{(r-\alpha)\left(r+\delta-\alpha\right)}, \tag{24}$$

while the solution for $W^N(\theta, M)$ is still

$$W^N(\theta, M) = A\theta^\gamma - \frac{\theta M}{r+\delta-\alpha} - \frac{\beta E_0 \theta}{(r-\alpha)(r+\delta-\alpha)}. \tag{13}$$

To determine the optimal amount E*, by which emissions should be reduced, equations (22) and (24) are used to maximize the net payoff of policy adoption:

$$\max_E [W^A(\theta, M; E) - K(E)] = -\frac{\theta M}{r+\delta-\alpha} - \frac{\beta E_1 \theta}{(r-\alpha)\left(r+\delta-\alpha\right)} - k_1(E_0 - E) - k_2(E_0 - E)^2$$

$$= -\frac{\theta M}{r+\delta-\alpha} - \frac{\beta E_1 \theta}{(r-\alpha)\left(r+\delta-\alpha\right)} - k_1(E_0 - E) - k_2(E_0^2 - 2E_0E + E^2) \tag{25}$$

$$FOC: -\frac{\beta\theta}{(r-\alpha)(r+\delta-\alpha)} + k_1 + k_2 2E_0 - sk_2 E = 0$$

$$E^* = E_0 + \frac{k_1}{2k_2} - \frac{\beta\theta}{2k_2(r-\alpha)(r+\delta-\alpha)}. \tag{26}$$

The optimal amount of emission reduction is now found, and in the same way as in chapter 3.2, we use boundary conditions (10) and (11) and assume that $\rho = (r-\alpha)(r+\delta-\alpha)$ to find A and the constant θ*. Detailed calculations can be found in appendix 6.b.

Using equation (11) yields

$$A = \frac{\beta^2}{4k_2\rho^2\theta^{\gamma-2}} + \frac{k_1^2}{4k_2\theta^\gamma} - \frac{\beta k_1}{2k_2\rho\theta^{\gamma-1}}, \quad (27)$$

and using equation (12) yields

$$(\gamma-2)\beta^2\theta^2 + 2\rho(\gamma-1)\beta k_1\theta + \gamma\rho^2 k_1^2 = 0. \quad (28)$$

Assuming that $\gamma > 2$, which is given by equation (12), and taking into account that $W^A(\theta) - W^N(\theta) - K(E(\theta^*))$ is convex in θ, θ^* is the largest root of this quadratic equation:

$$\theta^* = \frac{\rho k_1(\gamma-1)}{\beta(\gamma-2)}\left[1 + \sqrt{1 - \frac{\gamma(\gamma-2)}{(\gamma-1)^2}}\right]. \quad (29)$$

Now $E^*(\theta^*)$ can easily be calculated with equation (26), and lies between 0 and E_0. $E^*(\theta^*)$ decreases as σ increases (and γ decreases), while θ^* increases in this case, as in chapter 3.2.

Numerical Example

Again, a numerical example will help to better understand the outcomes. Suppose k1 = 5000, k2 = 0.0055, and $\sigma = 0.045$, while all other values stay the same as given in Table 1 in chapter 3.2. Furthermore, assume that θ_{max} is the value of θ for which $E^* = 0$, which, by equation (26) is given as $\theta_{\max} = \rho k_1 / \beta + 2\rho k_2 E_0 / \beta$, and $\theta_{min} = 12$, at which the policy is never adopted. Then $\theta_{max} = 20$, and equations (28) and (29) yield $\gamma = 6.8$, and $\theta^* = 17$, i.e., $\theta^* < \theta_{max}$. Equation (26) gives that $E^* = 110{,}606$ tons per year, which is positive.
In Figure 8 we can see the dependency of E^* and θ^* on σ. As in chapter 3.2, θ^* increases with σ, but now also the amount by which emissions are reduced depends on the degree of uncertainty over the future benefits of a reduction. Note that, if $\sigma = 0$, the NPV rule applies, and as we are assuming $\alpha = 0$, θ^* cannot rise if $\sigma = 0$ as well. The parameter E^* falls as σ and θ^* increase together, and for any value of $\sigma > 0.063$, $E^* = 0$. One can also see, if θ^* is smaller than 12, emissions are not reduced at all, and as θ^* increases, emissions get reduced more and more.

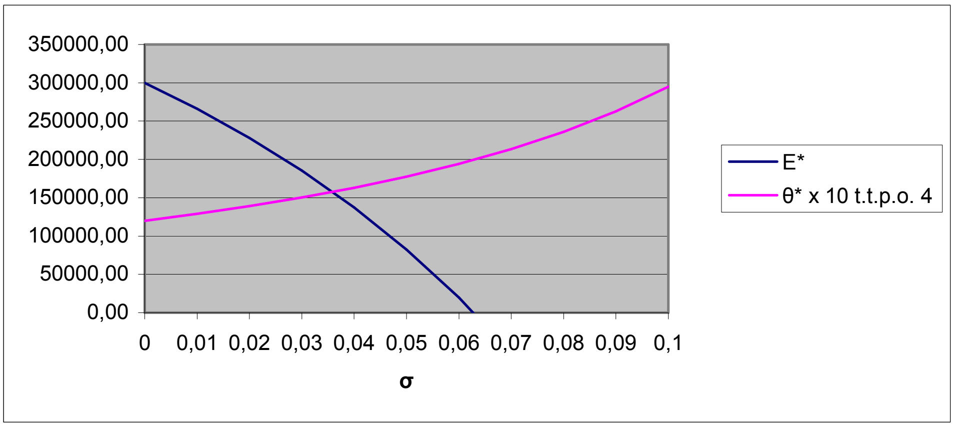

Figure 8: Partial Reduction – Dependency of E^* and θ^* on σ

A higher initial value of E_0 does not affect θ^*, thus ending in a higher ending level E^*. For example, assume that $E_0 = 600{,}000$. The solution is plotted in Figure 9.

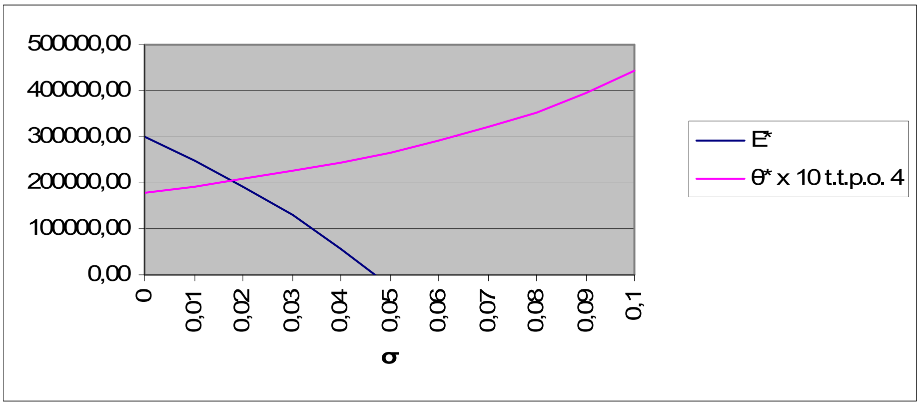

Figure 9

The parameter k_1 is positively correlated with θ^*, and k_2 does not affect θ^*, but increases E^*. Figure JHG shows the solution for $k_1 = 7500$, all other parameters unchanged.

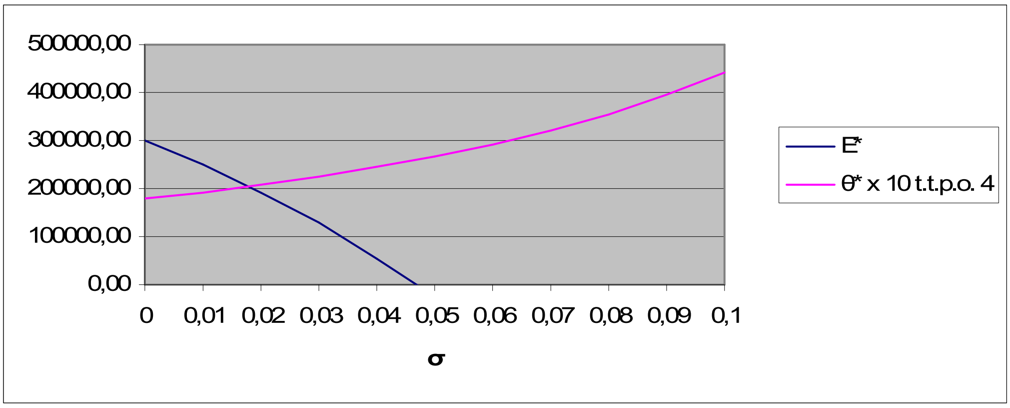

Figure 10

And Figure 11 shows the solution for $k_2 = 0{,}007$.

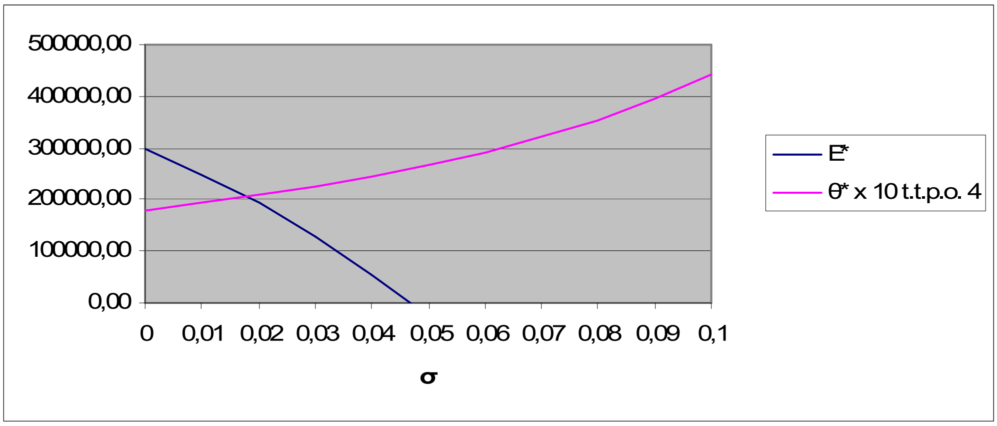

Figure 11

3.2.2 Convex Benefit Function.

Until now, we assumed that the benefit function $B(M_t,\theta_t)$ is linear in M_t. And we saw that the policy adoption rule, in this case, is independent of the stock of pollutant, M_t. Now, we assume the benefit function to be convex in M_t, which means that the damage from a certain stock of pollutant rises more than proportionally. The policy adoption will not be partial but a once and for all reduction, and the costs will not be convex, but given by $K=kE_0$, as in chapter 3.2.

The benefit function $B(M_t,\theta_t)$ is quadratic in M_t now, i.e.

$$B(M,\theta) = -\theta_t M_t^2. \qquad (30)$$

Again, benefit functions for the "adopt" and "no-adopt" regions have to be defined. They are indeed only a little different from equations (8) and (9); only the term -θM is replaced by -θM^2:

$$rW^N = -\theta M^2 + (\beta E_0 - \delta M)W_M^N + \alpha W_\theta^N + \frac{1}{2}\sigma^2\theta^2 W_{\theta\theta}^N \tag{8′}$$

$$rW^A = -\theta M^2 - \delta M W_M^A + \alpha\theta W_\theta^A + \frac{1}{2}\sigma^2\theta^2 W_{\theta\theta}^A \tag{9′}$$

s.t. boundary equations (10) – (12).

These Bellman equations have the following solutions

$$W^N(\theta, M) = A\theta^\gamma - \frac{\theta M^2}{r + 2\delta - \alpha} - \frac{2\beta^2 E_0^2 \theta}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)} - \frac{2\beta E_0 \theta M}{(r+2\delta-\alpha)(r+\delta-\alpha)},$$

and

$$W^A(\theta, M) = -\frac{\theta M^2}{r - 2\delta - \alpha},$$

where A and θ have to be determined, and γ is given by equation (18).

Now boundary equations (11) and (12) are used to determine A and the critical value θ*. Detailed calculations can be found in appendix 6.c.

$$\theta^* = \frac{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)k\gamma}{2\beta(\gamma-1)[\beta E_0 + (r-\alpha)M]} \tag{31}$$

$$A = E_0\left(\frac{\gamma-1}{k}\right)^{\gamma-1}\left[\frac{2\beta^2 E_0 + 2\beta(r-\alpha)M}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)\gamma}\right]^\gamma \tag{32}$$

Here, θ^* depends on M_t, where the higher M_t, the lower θ^*. A higher M implies a higher marginal cost from additional emissions, and therefore an emission reducing policy will be adopted earlier. A higher current value of E_0 (together with rising marginal costs of emission) implies a higher value of θ^*, as does a higher cost of emission reduction, k, and a higher decay rate, δ.

The convex cost function, however, does not change for the fact that uncertainty affects the timing of the policy in the same way as without convexity, where $B(M_t, \theta_t)$ in linear in M, although it does affect the policy adoption rule. But the timing is determined by θ^*, which gets affected by the parameter σ through the multiplier $(\gamma-1)/\gamma$, and γ is given by equation (18), as in chapter 3.2.

3.3 Gradual Emission Reductions

In this modification of the basic model, I consider that there is only one possibility for a policy implication. But, and this is the important aspect in this chapter, emissions can be reduced gradually and continuously.

The policy maker must now sight both variables θ_t and the stock variable M_t to decide when and by how much emissions should be reduced.

We assume that the cost of policy adoption is completely sunk and a quadratic function of the amount of emission reductions, as in equation (22). The cost of a one unit reduction in E is therefore: $\Delta K = k_1 + 2k_2(E_0 - E_1)$. The benefit function $B(\theta_t, M_t)$ is assumed to be linear in θ_t and M_t, i.e. $B_t = -\theta_t M_t$.

Defining $m_1 = k_1 + 2k_2 E_0$ and $m_2 = 2k_2$, gives

$$\Delta K = m_1 - m_2 E. \tag{33}$$

A small reduction in the stock of the pollutant, ΔM_t, leads to a benefit of $\Delta B_t = -\theta_t \Delta M_t$. The stock variable M_t is controlled by the flow variable E_t. If emissions are reduced by ΔE at time t=0, M_t changes by

$$\Delta M_t = -\frac{\beta \Delta E}{\delta}\left[1 - e^{-\delta t}\right], \tag{34}$$

which now allows to define the social benefit of an incremental reduction in emissions at time t, which is

$$\Delta W_t = \varepsilon_t \int_t^{\infty} \Delta B_\tau e^{-r(\tau - r)} d\tau = \beta \theta_t \Delta E / \rho, \qquad (35)$$

where $\rho \equiv (r-\alpha)(r+\delta-\alpha)$.

It has to be determined how much emissions should be reduced initially, and how much in future periods. This last decision depends on the change in θ.

Assume that E_t = E currently, and given E_t together with θ_t and M_t, the value function is W(E;θ,M). The value of society's option to permanently reduce E by one unit is ΔF, and exercising that option results in the cost ΔF(E;θ,M) + K(E), and the payoff ΔW(θ). Then ΔF must satisfy the Bellman equation

$$r\Delta F = (\beta E - \delta M)\Delta F_M + \alpha \theta F_\theta + \frac{1}{2}\sigma^2 \theta^2 \Delta F_{\theta\theta}, \qquad (36)$$

subject to:

$$\Delta F(E;0,M) = 0, \qquad (37)$$

$$\Delta F(E;\theta^*,M) = \Delta W(\theta^*) - \Delta K(E), \qquad (38)$$

$$\Delta F_\theta(E;\theta^*,M) = \Delta W_\theta(\theta^*), \qquad (39)$$

which has the solution:

$$\Delta F = a\theta^\gamma, \qquad (40)$$

where γ >1 is given by equation (18). The policy of emission reductions should be exercised whenever θ exceeds the critical value θ*(E), with dθ*/dE < 0.
The constant α and the critical value θ*(E) are determined by solving boundary equations (38) and (39):

$$\theta^* = \frac{\rho\gamma(m_1 - m_2 E)}{\beta\Delta E(\gamma - 1)} \tag{41}$$

$$a = \left(\frac{\beta\Delta E}{\gamma\rho}\right)^{\gamma} \left(\frac{\gamma - 1}{m_1 - m_2 E}\right)^{\gamma - 1} \tag{42}$$

Detailed calculations can be found in appendix 6.d.

Consider equation (41), $\rho(m_1\text{-}m_2E)$ is the amortized sunk cost of an incremental reduction in emissions, β is the absorption rate. Because of uncertainty, this amortized sunk cost exceeds the threshold value without uncertainty by the multiple $(\gamma\text{-}1)/\gamma$. Furthermore, as E decreases, θ^* increases and a falls. If σ increases, θ^* increases as well, and if δ increases, θ^* decreases, for any value of E. Note that depending on the initial level of θ, it might be the best choice to reduce emissions in period 1, and then as θ reaches its threshold value θ^*, to gradually continue reducing emissions.

Numerical example

Uncertainty does not only affect the level of emission reduction over time, but also the level of initial reduction. To better understand the dependency on σ, a numerical example by Pindyck (2000) is presented. Pindyck ran a Monte Carlo Simulation to examine these impacts. The parameter values used are the following: E_0 = 300.000 tons per year, k_1 = 5000, and k_2 = 0.055, r= 0.4, δ = 0.02, β = 1, and α = 0.01, σ varies from 0 to 0.15, in increments of 0.005. The simulation is plotted in Figure 12.

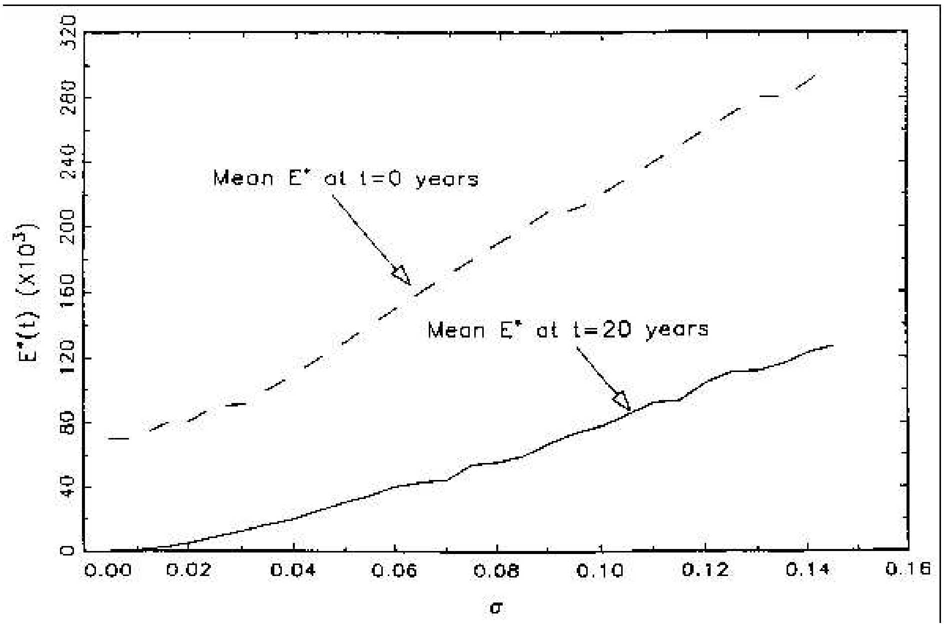

Figure 12: Mean Optimal Emissions Level at t = 0 and 20 Years

Figure 12 shows the mean optimal emissions level initially and after 20 years. Without uncertainty, emissions are reduced from 300.000 to about 70.000 tons per year, and then reduced gradually to zero. If σ is increased, less emissions are reduced immediately, which reflects the value of waiting. E^* also increases with σ over time.

The value of waiting is especially important. As Pindyck (2000) states, "because of the possibility that θ will not increase as much as expected, there are indeed realizations in which it takes a very long time for θ to grow to the point where elimination is justified."

3.4 Ecological Uncertainty

Until now we only considered economic uncertainty, but no ecological uncertainty. To outline this problem as well, we now keep θ fixed, and use equation (1) for the evolution of M_t:

$$dM = [\beta(E(t) - \delta M(t)] + d\varnothing(t). \qquad (1)$$

Because dø(t) is the increment of a stochastic process, even if we know the trajectory of E_t, future values of M_t are still uncertain.

In addition, we assume the benefit function $B(\theta_t, M_t)$ to be convex in M_t, because otherwise – if it would be linear in M_t – the stochastic evolution of M_t would have no effect on the expected marginal social return from reductions in E_t, even if K were a nonlinear function.

So, $B(\theta, M) = -\theta M^2$, as already defined in chapter 3.2.2, and $K = kE_0$.

The Bellman equations for the "adopt" and the "no-adopt" regions are

$$rW^N = -\theta M^2 + (\beta E_0 - \delta M)W_M^N + \frac{1}{2}\sigma^2 W_{MM}^N, \tag{43}$$

and

$$rW^A = -\theta M^2 - \delta M W_M^A + \frac{1}{2}\sigma^2 W_{MM}^A. \tag{44}$$

Subject to the following set of boundary conditions.

$$W_M^A(0) = 0, \tag{45}$$

$$W^N(M^*) = W^A(M^*) - K, \tag{46}$$

$$W_M^N(M^*) = W_M^A(M^*), \tag{47}$$

where M* is the critical value of M_t, at which the policy should be adopted, the solution of (44) and boundary condition (45) for W^A is

$$W^A(M) = -\frac{\theta M^2}{r + 2\delta} - \frac{\sigma^2\theta}{r(r + 2\delta)}. \tag{48}$$

W^N, the solution from equation (43) can not be solved analytically without making certain restrictions on the parameter values, but only numerically, which will be shown soon. First, we consider the special case, where $\delta=0$.

3.4.1 Complete Irreversibility

In this chapter we assume complete irreversibility, i.e. we set δ=0. Then, equation (48) reduces to:

$$W^A(M) = \frac{-\theta(rM^2 + \sigma^2)}{r^2}. \tag{49}$$

The homogenous solution of equation (43) is $W^N = B_1 e^{\phi_1 M} + B_2 e^{\phi_2 M}$, which together with the inhomogeneous solution gives

$$W^N(M) = Be^{\phi M} - \frac{\theta(rM^2 + \sigma^2)}{r^2} - \frac{2\beta E_0 \theta(\beta E_0 + rM)}{r^3}, \tag{50}$$

where

$$\phi = -\frac{\beta E_0}{\sigma^2}\left[1 - \sqrt{1 + 2r\sigma^2 / \beta^2 E_0^2}\right] > 0, \tag{51}$$

and B and M^* are constants which have to be determined.

Consider equation (50). The first term on the right hand side is the value of the option to adopt the policy. The second term is the present value of the flow of social cost from the current stock of pollutant, and the third term is the present value of the flow of social cost that would result if emissions continued at the rate E_0 forever.

The constants B and M^* are determined by equation (49) and the boundary conditions (46) and (47):

$$B = \frac{2\beta E_0 \theta}{r^2 \phi} e^{-\phi M^*}, \tag{52}$$

and

$$M^* = -\frac{\beta E_0}{r} - \frac{\sigma^2}{\beta E_0 \left(1 - \sqrt{1 + 2r\sigma^2 / \beta E_0^2}\right)} + \frac{r^2 K}{2\beta E_0 \theta}, \tag{53}$$

where $\partial M / \partial K > 0, \partial M^* / \partial r > 0, \partial M^* / \partial \theta < 0,$ and $\partial M^* / \partial \sigma > 0$. So, stochastic fluctuations in M create an incentive to delay policy adoption.

Numerical Example

The parameters are r = 0.04, K = 4, E_0 = 0.3, β = 1, and θ = 0.002. If σ = 0 the policy should be adopted immediately. And if σ = 1, the policy will be adopted when $M \geq M^* = 6.74$, and if σ = 4, the policy should be adopted when $M \geq M^* = 16.21$.

3.4.2 General Case

Now a more general case, where δ > 0, is considered. As already mentioned in chapter 3.4, $W^N(M)$ and the critical value M^* can then only be found numerically.

The stock of pollutant, M_t, is measured in millions of tons, the emission rate in millions of tons per year, the value and cost functions in billions of dollars, and θ in billion dollars/(million tons)^2. The parameter values are: K = 4, E_0= 0.3, θ = 0.002, σ = 1, α = 0, r = 0.04, δ = 0.02 and β = 1.

The solution for M^* is 13.05, which is not trivial to find. Pindyck (2000) derives this solution by first using any candidate number for M^*, denoted by M_0^*, equation (48), and the boundary conditions (46) and (47) to get $W^N(M_0^*)$ and $W_M^N(M_0^*)$. These are used to solve equation (43) backwards to determine a corresponding candidate solution for $W^N(M)$ for all M between 0 and M_0^*. The so found candidate numbers are then adjusted up and down until the conditions $W_M^N < 0$ for all values of M, and $W_{MM}^N < 0$ at M = 0, are fulfilled.

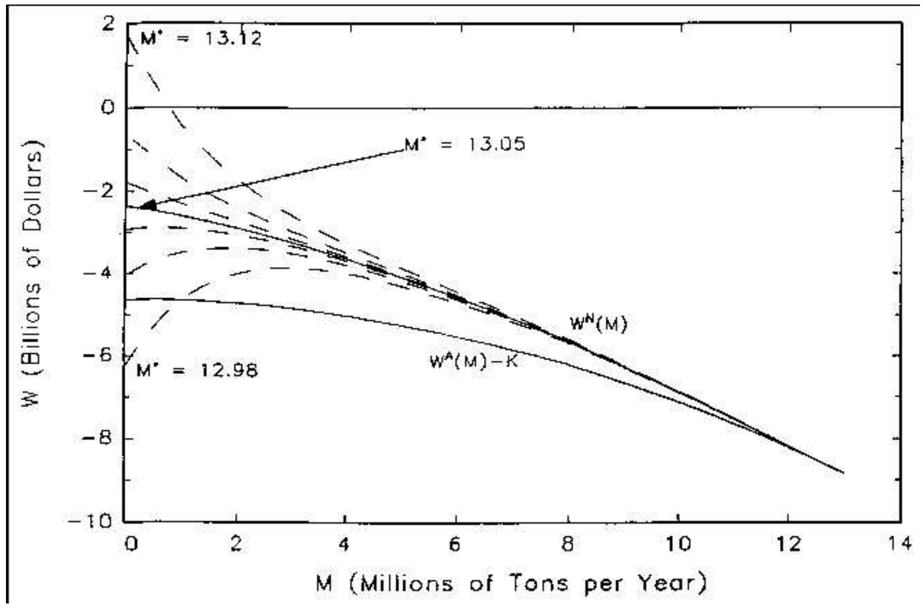

Figure 13: Solution method for stochastic M

Figure 13 shows the solution method for stochastic M. Different candidate solutions for $W^N(M)$ corresponding to different values of M^* are plotted, as well as $W^A(M)$-K. For candidate values of M^* below 13.05, $W_M^N(M) > 0$ for small values of M, and above 13.05, $W_{MM}^N(M) > 0$ for small values of M.

σ	M*	
	δ = 0	δ = 0.02
0,3	5,48	11,08
0,5	5,73	11,59
0,8	6,28	12,45
1	6,47	13,05
2	9,59	16,47
4	16,21	25,75

Table 2: Solutions for M^*

Table 2 shows different solutions of M^* for values of σ ranging from 0.3 to 4.0, and for δ equal to 0 and 0.02. M^* increases with σ and also with δ. A higher δ implies more reversible emissions, and therefore the drift rate of M declines. This leads to a lower present value of the flow of social cost for any current value of M, and so a higher M is needed to justify the sunk cost of policy adoption. Pindyck (2000)

4. Extensions of the model

The model by Pindyck 2000 and its extension, the model of Pindyck 2002, can both be used in several ways for further investigations of environmental problems, especially if uncertainties and irreversibilities are concerned and if the real option approach is asked for.

4.1 Discount rate and Environmental Kuznets Curve

A paper which particularly arouse my interest and which builds up on Pindyck (2002) is "Is the Discount Rate Relevant in Explaining the Environmental Kuznets Curve?" by Giuseppe Di Vita (2003). The question he deals with in this paper is, in short, the role of the interest rate in explaining the income-pollution pattern. Although Di Vita does not take into account irreversibilities and uncertainties in particular, this model can be seen as an extrapolation of Pindyck (2002), especially concentrating on the role of the interest rate. Di Vita writes: "*The choice of a correct rate to make up-to-date future benefits is particularly relevant in cases where problems of irreversibilities (for example lost of biodiversity) and uncertainties (like future costs of pollution accumulation or climatic changes) emerge about some environmental damage, and we want to know if it is worth implementing some environmental policy now rather than delaying its adoption.*"

4.1.1 Introduction

Before discussing this model and its implications in detail, some basic explanations about income-pollution patterns and the theory of "Environmental Kuznets Curves" (EKC) are presented.

Grossman and Krueger (1991) as well as Lucas, Wheeler et al. (1992) and others were the first to analyse the correlation between income levels and adoption of a pollution abatement policy for different countries. The hypothesis of the EKC states that pollution follows an inverted U-shaped path with respect to economic growth, see Figure 14. An industrialising country faces low income and high interest rates, whereas an industrialized country, as a consequence of growth, has a high income level and faces low interest rates. These interest rates play an important role when deciding to adopt an emission reducing policy or not, as will be shown.

Although there are many different theories to explain the income-pollution pattern, Di Vita investigates the role of interest rates, and is so “making use of an argument that has never yet been introduced in the economic debate on this issue.” Di Vita (2003).

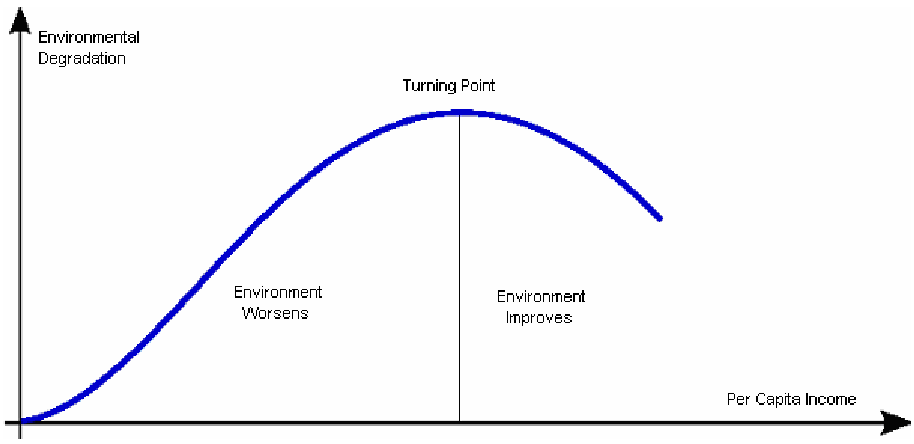

Figure 14: The Environmental Kuznets Curve

Some of the other theories which work on an explanation of the EKC are worth to be mentioned as well. Arrow, B. Bolin et al. (1995) suggest that an economy which runs trough a transition from a rural economy to a more polluted world as a consequence of growth and industrialization, then, as a more developed world, may pay more for “green” services.
The change in preferences for environmental quality has also been consulted to explain the income-pollution pattern by Di Vita (2002).
Jaeger (1998) presents a quite similar model, where as a consequence of economic growth more emissions are absorbed and people get willing to devote scare resources to invest in less economic but more ecological technologies. Suri and Duane (1998) and Rothman (1998) investigate the role of transferring polluting productions from rich to poor countries. Also differences in corruption levels have been analyzed to justify the EKC by López and Mitra (2000).
Although these are by far not all papers which address the hypothesis of EKC, as already mentioned, Di Vita (2003) is the first paper to explain the inverse U-shaped relationship between income and pollution through the differences in discount rates among countries.

Now the debate is coming along about why the discount rate is high in poor and developing countries while it is low in rich and developed ones.

Hibbs (2001) affirms that the neoclassical school of thought is not able to explain this difference due to the following facts. In the Solow model, a fall in the discount rate is similar to an increase in capital stock and income. Romer (1996). Joshi (1993) showed that the fact that the discount rate is assumed to be exogenously given in the neoclassical growth model may bias the result because it does not consider that this variable declines as income and consumption rise.

But there are other theoretical concepts which try to clarify the question why interest rates differ in poor and rich countries, like the scarcity of capital, the high risk premium in the financial market, the high leverage of dept, political instability, and the necessity to devote scare resources to satisfy present needs. All of those support the assumptions of Di Vita (2003).

4.1.2 Analytical Framework

As in the models of Pindyck (2000) and Pindyck (2002), a social planner has to decide at some point in time whether to adopt a costly policy to reduce pollution or to delay this decision to some future date. Under the net present value rule, the policy will be undertaken if the present value of the net social benefit is positive. Taking into account irreversibilities and uncertainties may change the outcome of these calculations but does not change for the fact that a low discount rate encourages the policy adoption.

Thus, differences in discount rates may explain why in poor countries we find a positive relationship between income and pollution, while in developed countries policies to protect and preserve the ecological system are adopted.

In Chapter 3 we discussed Pindycks model from 2000 in detail. Although Di Vita uses the analytical framework from Pindyck (2002), it is very similar to Pindyck (2000), and thus will not be described here too precisely.

Similar to chapter 3, but ignoring the increment of a stochastic process, the evolution of M_t is given by

$$\dot{M} = \beta E(t) - \delta M(t), \qquad (1')$$

where the parameter $0 < \beta < 1$ measures the quantity of emissions absorbed by the ecosystem, and δ represents the rate at which M_t dissipates over time.

The social cost function is again given by equation (3) from chapter 3:

$$B(M_t, \theta_t) = -\theta_t M_t, \qquad (3)$$

where B is assumed to be linear, and the costs are assumed to be completely sunk.
Thus, the present value of social welfare is given by equation (2) from chapter 3:

$$W = \varepsilon_0 \int_0^\infty B(M_t, \theta_t) e^{-rt} - \varepsilon_0 K(E_1) e^{-rT}. \qquad (2)$$

For the moment we do not account for uncertainty and irreversibility brought about by pollution emission, thus we are assuming that there is no ecological uncertainty. Economic uncertainty, however, is introduced into this model by the evolution of θ. In Pindyck (2002) it is assumed that θ_T will equal $\underline{\theta}$ or $\overline{\theta}$ with equal probability, with $\underline{\theta} < \overline{\theta}$ and $½(\underline{\theta} + \overline{\theta}) = \theta_0$, the current value of θ. Further, θ is assumed not to change after time T, at which the policy will only be adopted if $\theta_T = \overline{\theta}$.
For the moment the policy maker can choose between two different points in time to adopt an emission reducing policy, either now, at $t = 0$, or at $t = T$.

Solving equation (1′) gives M_t as a function of time, supposing that $E_t = E_0$ for $t < T$, and $E_t = 0$ for $t \geq T$.

$$M_t = \{ \begin{matrix} (\beta E_0 / \delta)(1 - e^{-\delta t}) + M_0 e^{-\delta t} \\ (\beta E_0 / \delta)(e^{\delta T} - 1) e^{-\delta t} + M_0 e^{-\delta t} \end{matrix} \quad \text{for } 0 \leq t \leq T, \text{ for } t > T \qquad (54)$$

Thus, if the policy is never adopted, the value of the social welfare function W_N, is given by

$$W_N = -\int_0^{\infty} \theta_0 M_t e^{-rt} dt = -\frac{\theta_0 M_0}{(r+\delta)} - \frac{\beta E_0 \theta_0}{r(r+\delta)}, \quad (55)$$

and if the policy is adopted at time t = 0, where $E_t = 0$, the social welfare function W_0 is given by

$$W_0 = -\frac{\theta_0 M_0}{r+\delta} - K. \quad (56)$$

For now, the policy should be adopted if $W_0 > W_N$, following the simple net present value rule.
The simple framework presented just now, is extended to analyse the role of the discount rate in determining the optimal emission reducing policy in the following chapters.

4.1.3 Role of the discount rate

The concentration will now focus on the consequences of the discount rate. Its level determines which of the social welfare functions W_N and W_0 will be greater than the other, and so determines if a policy is implemented or not, assuming ceteris paribus conditions concerning all other parameters.
Setting $W_N = W_0$ gives the watershed discount rate, denoted by $\hat{r}$ which lies exactly between the two areas where pollution will be stopped and where pollution will keep on rising on its path given by the motion equation (1´). If $r > \hat{r}$, it will always be true that $W_0 < W_N$ and an emission reducing policy will be adopted, and vice versa.

$$W_N = W_0$$
$$-\frac{\theta_0 M_0}{r+\delta} - \frac{\beta E_0 \theta_0}{r\left(r+\delta\right)} = -\frac{\theta_0 M_0}{r+\delta} - K$$
$$Kr^2 + rK\delta - \beta E_0 \theta_0 = 0, \quad (57)$$

Equation (57) is the quadratic equation from which we can easily obtain the values of the watershed discount rate $\hat{r}$.

Di Vita (2003) uses the same parameters as in Pindyck (2002) to give a numerical example. They are presented in Table 3.

Parameter	Interpretation	Value
β	emissions absorbed by the ecosystem	1
K	PV of costs of policy adoption	$ 2000.000.000
E0	emission rate	300.000 tons/year
θ_0	current social cost	$ 20 tons/year
δ	pollutant decay rate	0.02

Table 3: Parameter Values

Using equation (57), $r_{1,2} = \dfrac{-K\delta \pm \sqrt{K\delta^2 - 4K\beta E_0}}{2K}$, we find two possible values for $\hat{r}$: $\hat{r}_1$ = 0.04863 and $\hat{r}_2$ = -0.05586. Pindyck (2002) uses r = 0.04 for his calculations, and for all values of r < $\hat{r}_1$, the positive threshold value, it is always true that $W_0 > W_N$.

To describe the discount rate-pollution emission pattern more precisely, we take the partial derivative of E with respect to r, using equation (56):

$$\beta E_0 \theta_0 = Kr^2 + Kr\delta$$

$$E_0 = \frac{Kr^2 + Kr\delta}{\beta\theta_0}$$

$$\frac{\partial E}{\partial r} = \frac{2Kr + K\delta}{\beta\theta_0}$$

$$\frac{\partial E}{\partial r} = K\frac{2r + \delta}{\beta\theta_0} > 0, \qquad (58)$$

such that $\partial^2 K / \theta r^2 = 2K / \beta\theta_0 > 0.$ Equation (58) is presented in Figure 15.

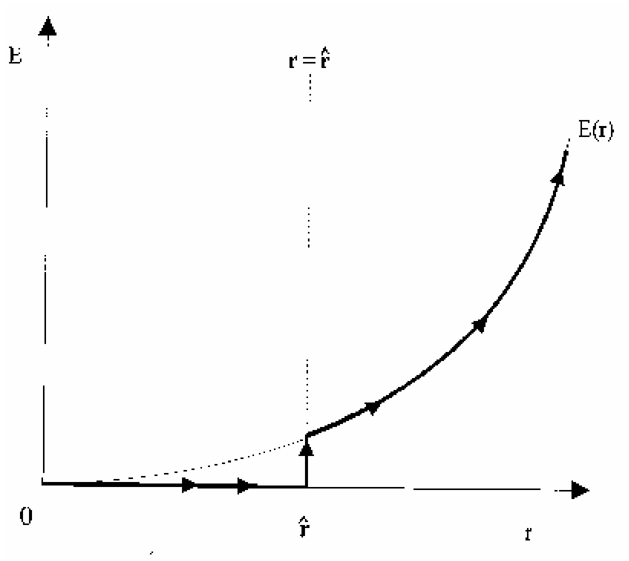

Figure 15: Discount rate-Pollution emission pattern

As we can see, the horizontal axis shows the discount rate and the vertical axis the evolution of pollution emissions. The curve E(r) describes the increasing and convex discount rate-emission pollution pattern which is divided into two parts by the threshold value $\hat{r}$. For any r $< \hat{r}$, pollution will be zero because the emission reducing policy will be adopted, and if r $> \hat{r}$ emissions will continue to rise - M_t will follow equation (1′). Considering different countries implies that those with a discount rate lower than $\hat{r}$ will show declining emissions, whereas in those with r $> \hat{r}$ the opposite will be true, assuming ceteris paribus conditions.

So far we characterized countries with the help of two parameters, the discount rate and the income level. The first is part of Pindycks model, whereas the latter is not. Thus, income I will be introduced now, assuming that:

$$K = f(I, E) = I^{\alpha} + E^{1-\gamma}, \tag{59}$$

where the following conditions are respected: $\partial K / \theta I > 0$, $\partial K / \partial E < 0$, $\partial^2 K / \partial E^2 > 0$, and $\partial^2 K / \partial I^2 < 0$. Thus, k is convex-decreasing in E, while the social cost of pollution abatement policy is concave-increasing in I (output) – the cost of policy adoption rises together with income, but K increases less than proportionally.

K is a direct function of income with 0 < α < 1, and γ > 1 considers pollution abatement costs change as the emission changes.

Given α, γ, the income level I and E, equation (59) displays the social costs of environmental policies at any time.

Substituting (59) in (57) and deriving E with respect to I gives

$$\frac{\partial E}{\partial I} = I^{\alpha-1}\alpha \frac{E^{\gamma}}{-1+\gamma} > 0. \tag{60}$$

In order to analyze the link between the discount rate and I, we again substitute (59) in (57) and take the derivative of r with respect to I, which gives

$$\frac{\partial r}{\partial I} = -\frac{I^{\alpha-1}\alpha r(r+\delta)}{(I^{\alpha}+E^{1-\gamma})(2r+\delta)} < 0. \tag{61}$$

Equation (60) shows that pollution emissions are increasing with income. In the absence of an emission reducing policy, as shown by Di Vita (2008) in a numerical example, income increases more than proportionally.

Furthermore, equation (61) shows that there is an inverse relationship between r and I. So, countries with a low income level will face a high discount rate, and vice versa. Therefore, growth in developing countries will lead to higher ecological damage. Only if $r < \hat{r}$, growth implies a reduction in emissions.

Next, we analyze the changes of M_t over time with changes in I. To derive the income pollution pattern, we substitute K for (59), (54) for E_0, and (1′) for M_t, and get

$$\dot{M} = \left\{ \begin{array}{l} \frac{(I^{\alpha}+E^{1-\gamma})r(r+\delta)}{\theta_0} e^{-\delta t} - \delta M_0 e^{-\delta t}, \\ -\frac{I^{\alpha} r(r+\delta)}{\theta_0}(e^{\delta T}-1)e^{-\delta t} - \delta M_0 e^{-\delta t}, \end{array} \right. \tag{62}$$

where the first row describes $\dot{M}$ when $0 \leq t \leq T$ and $r > \hat{r}$, while the second row describes $\dot{M}$ for $t > T$ and $r > \hat{r}$.

Taking the first partial derivative of $\dot{M}$ with respect to I, gives

$$\frac{\partial \dot{M}}{\partial I} = \begin{cases} I^{\alpha-1}\alpha \frac{r(r+\delta)}{\theta_0} e^{-\delta t} > 0, \\ -I^{\alpha-1}\alpha \frac{r(r+\delta)}{\theta_0}(e^{\delta T}-1)e^{-\delta t} < 0, \end{cases} \qquad (63)$$

where the first row counts for $r > \hat{r}$ where $\dot{M}$ is a concave-increasing function of income , and the second for $r < \hat{r}$, where $\dot{M}$ is a convex-decreasing function of I.

Equation (63) states that the pollution stock increases as income does, which is described by equation (1′), while the discount rate decreases. At the point where r is equal to the watershed value $\hat{r}$, M_t reaches its maximum level and from that point on $E_t = 0$ and M_t will continually decrease by δM_t.

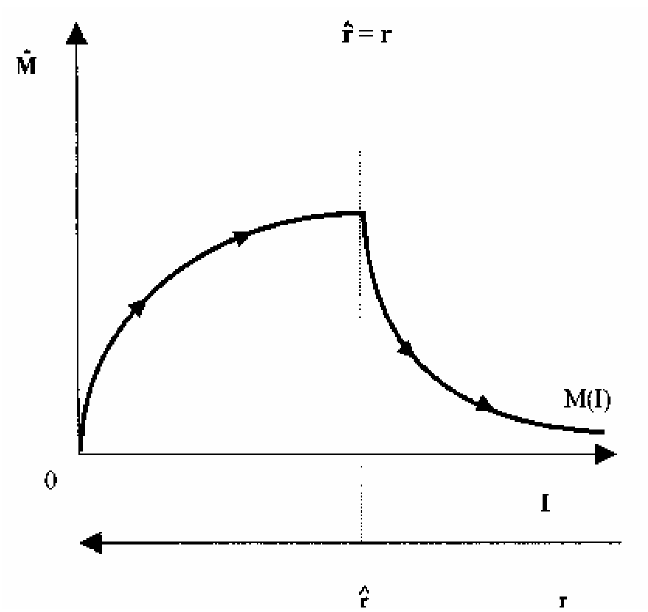

Figure 16: Income-pollution pattern

Figure 16 describes what we have just analyzed. The vertical axis reports the dynamics of the pollution stock, and the horizontal axis reports income as well as the discount rate. As explained before, the curve is concave-increasing until the threshold value is reached, and then convex-decreasing. Remember that this rather unrealistic dynamic is due to the fact that in the simple model by Pindyck it is assumed that emissions fall immediately to zero as soon as an environmental policy is adopted.

Furthermore, as Di Vita (2003) describes, empirical results show a different finding, where pollution emissions first increase quickly and, after policy implementation, decline slowly.

4.1.4 Partial reduction of pollution emissions

To allow for these effects, Di Vita (2003) also develops a model where pollution emissions are partially reduced, such that $E_t \neq 0$ at the moment when $r = \hat{r}$ and the policy is adopted.
To show his outcome graphically, have a look at Figure 9:

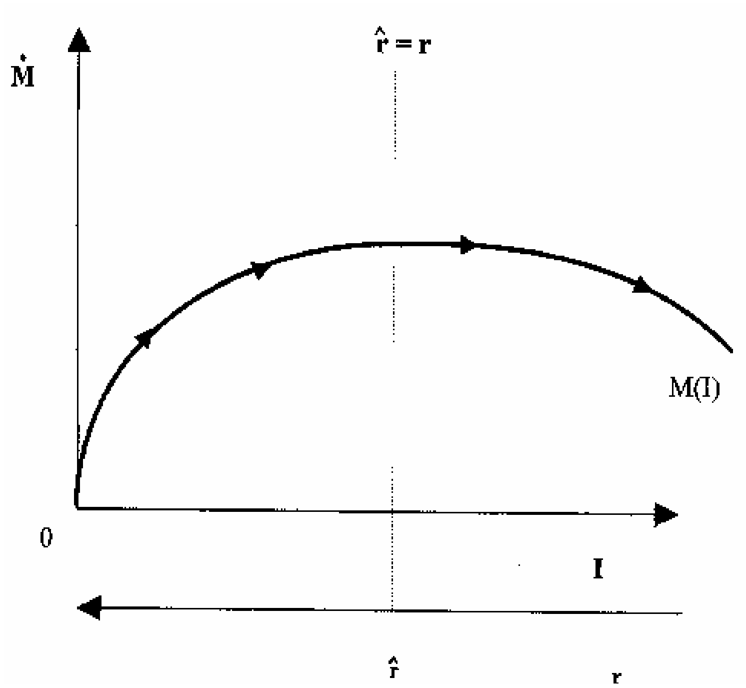

Figure 17: Income-pollution pattern in case of partial reduction of pollution emissions

Here, $\dot{M}$ is concave-increasing in income for $r > \hat{r}$, and convex-decreasing in income for $r < \hat{r}$. The figure shows an inverse U-shaped EKC, in which the decreasing part of the curve falls more slowly than the rising one. Thus, a more realistic income-pollution pattern is reached, although the basic results are the same in chapter 4.1.1.

4.1.5 Uncertainty and Irreversibility

In the model described above, we used the framework of Pindyck (2002) but did not take uncertainties and irreversibilities into account. To remember, Pindyck assumed that a policy would be adopted, after waiting until time T, if and only if $\theta_T = \overline{\theta}$ (where θ is the future social cost, that would be either high $\overline{\theta}$ or low $\underline{\theta}$). This more complicated version of Pindyck, in contrast to the simpler one with which is dealt here, does not change the conclusions described above. If uncertainty and irreversibility are taken into account, the role of the discount rate in the choice whether to adopt a costly policy or not is emphasized, and the threshold value of $\hat{r}$ is affirmed to be even higher. Pindyck himself attests that a higher discount rate reduces future benefits and raises future costs, thus motivating the decision to delay adoption of the policy. Di Vita (2003).

4.1.6 Summary and Implications

On the one hand, a direct relationship between the discount rate and pollution emissions was shown. More precisely, a threshold value of the discount rate $\hat{r}$ was found, such that for all countries with a value of r higher than $\hat{r}$, environmentally harmful production processes are adopted. Contrary, countries with a lower discount rate than the threshold value, adopt policies to reduce pollution emissions. And on the other hand, it was shown that the discount rate and income move in opposite directions. These two dynamics of r allow a better understanding of the relationship between pollution and the economic deployment of a country. Countries with low levels of income and high discount rates show a positive relationship between economic growth and pollution, whereas those with high income and low discount rates show the opposite, a negative relationship between growth and pollution.
This was pointed out by the means of two models whose groundwork was given by Pindyck (2002). First, a rather simple model was developed where emissions were assumed to be zero after policy adoption, and secondly the outcomes of a model with gradual emission reduction were shortly presented. Although the main findings were the same, the more precise model showed a income-pollution pattern that was first concave-increasing until the threshold value of the discount rate was reached, and from this point on convex-decreasing, thus finishing in a more realistic behaviour of the Environmental Kuznets Curve.

To summarize the outcome in a single sentence, the problem for the ecosystem in and for developing countries is the fact that they face high interest rates, and as a consequence can't afford costly emission reducing policies. Only economic growth can lead to environmental policies, but not directly, only through the discount rate reduction channel.

Enforcing environmentally harmful economic growth in order to reach a lower discount rate to afford environmental policies is a dilemma, in my opinion.

4.2 The Timing of Environmental Policies in the Presence of Extreme Events

4.2.1 Introduction

The models of Pindyck 2000 and 2002 serve as an analytical framework for a lot of different scientific papers dealing with environmental policy issues. In particular this framework is widely used if uncertainties and irreversibilities are concerned, which is the case if the optimal policy is determined by real option theory instead of the traditional cost benefit analysis.
In the last chapter, I concentrated on the role of the discount rate within this framework, and I will now discuss in more detail the dynamics of uncertainty and irreversibility. The paper by Dotsis, Makropoulou, and Psychogios "Environmental Policy Implications of Extreme Variations in Pollutant Stock Levels and Socioeconomic Costs" will serve as the building block in the following chapter. They investigate "the impact of jumps in carbon dioxide emission levels and abrupt increases in pollutant-related socio-economic costs, with respect to the optimal timing of environmental policies and the optimal emission abatement level." Dotsis, Makropoulou et al.) Jump diffusion processes are used to describe the evolution of these variables.

Whenever ecological investment is regarded, uncertainties about the future have to be accounted for. The evolution of an ecosystem also faces sudden changes. Its implementations for environmental policies are the issue of this chapter. To give an example, the average rise of CO_2 levels has been about 1.6 parts per million by volume in the recent decades. However, the newspaper "Independent" reported on October 11, 2004 that the levels of CO_2 made a sudden jump which could not be explained by terrestrial emissions from power stations and motor vehicles, and in the last two years the level had risen by 2.08ppm and 2.54ppm. Some scientists argue that these jumps can be explained by the theory of "feedback mechanism". This theory states that climate change and global warming alters the earth's environmental system, changing warming to increase even faster than before. Thus, we can see that the future evolution of the world wide ecosystem is unpredictable, and that sudden changes have to be accounted for when exploring the optimal timing and amount of emission reducing policies.

Dotsis et al. use the framework of Pindyck (2000, 2002) which has been discussed in detail above. They modified the given model to investigate their questioning not very much, so I will present their equations and assumptions as well as their findings, but I will not present the detailed calculations since they are very similar to those discussed in chapter 3 of this thesis.

Here, two kinds of uncertainty will be regarded: first environmental uncertainty, that is, uncertainty over the concentration of CO_2, and second economic uncertainty, that is, uncertainty over the future costs and benefits associated with the environmental change. Both uncertainties will not be modelled simultaneously.
However, uncertainty is assumed to be endogenous, and the benefits from a reduction in emissions depend on the current level of the stock of pollutant.

4.2.2 Ecological uncertainty

In the following section the stock of pollutant is assumed to exhibit discontinuities, and the optimal policy has to be found. More specifically, the evolution of the stock of pollutant, M_t, is now a little different from the one Pindyck (2000) uses, and follows a Gaussian mean reverting process augmented by jumps:

$$dM_t = \left[\beta E(t) - \delta M(t)\right] dt + \sigma dZ + \phi dq, \tag{64}$$

where dZ_t is a standard Wiener process, and the jump part is controlled by compound Poisson process (dq_t) with constant parameter λ, and positive jump size φ, which is assumed to be constant in this model. After a policy adoption, it is assumed that the probability of a jump drops to zero. The parameters β, δ, and σ are the same as in Pindyck (2000, 2002). Also the flow variable E(t), as well as the convex social cost function $B(M_t, \theta_t)=-\theta M^2$ possess the same characteristics as in Pindyck (2000, 2002). Differences only lie in the following facts: The parameter θ is assumed to be constant, and whenever emissions are reduced, E(t) drops to a new value E_1, as applied before, and λ becomes zero.

The cost function K, which occurs when a policy is adopted, is $K = kE_0$, with $E_1 = 0$.

Standard dynamic programming is again used to find the optimal level of M_t where an emission reducing policy should be adopted.

The policy objective is given by

$$W = E_0\left\{\int_0^\infty B(M_t,\theta_t)e^{-rt}dt + K(E_1)e^{-r\tilde{T}}\right\}, \tag{65}$$

where T is the unknown time at which the policy should be adopted.

The Bellman equations for the “no-adopt” and “adopt” regions, respectively, are

$$\left[\beta E - \delta M\right]W_M^N + \frac{1}{2}\sigma^2 W_{MM}^N - \left(r+\lambda\right)W^N + \lambda W^N(M+\phi) = \theta M^2, \tag{66}$$

$$\left[\beta E_1 - \delta M\right]W_M^A + \frac{1}{2}\sigma^2 W_{MM}^A - (r+\lambda)W^A + \lambda W^A(M+\phi) = \theta M^2, \tag{67}$$

which have to satisfy the following set of boundary equations:

$$W_M^A(0) = 0 \tag{68}$$

$$W^N(M^*) = W^A(M^*) - K \tag{69}$$

$$W_N^M(M^*) = W_N^A(M^*), \tag{70}$$

where M^* is the critical value of M where the policy should be adopted. Also the set of the boundary conditions has the same characteristics as in Pindyck (2000, 2002).

To obtain a closed form solution, δ is assumed to be zero, i.e. the environmental damage is completely irreversible, because otherwise only a numerical solution for the "no adopt" region could be found, as explained in chapter 3.4.

Thus, the value function by the "no-adopt" region is given by

$$\beta E W_M^N + \frac{1}{2}\sigma^2 W_{MM}^N - (r+\lambda)W^N + \lambda W^N(M+\phi) = \theta M^2, \tag{71}$$

where the general solution is

$$W = C_1 e^{m_1 M} + C_2 e^{m_2 M}, \tag{72}$$

and m_1 and m_2 satisfy

$$\frac{1}{2}\sigma^2 m^2 + \beta E_0 m - (\lambda + r) + \lambda e^{m\phi} = 0, \tag{73}$$

which can only be solved numerically.

So, the solution for the no adopt area is given by

$$W^N = C_1 e^{m_1 M} - \frac{(\sigma^2 + \lambda\phi^2 + rM^2)\theta}{r^2} - \frac{2\theta(\beta E_0 + \lambda\phi)(\beta E + \lambda\phi + rM)}{r^3}, \tag{74}$$

taking into account that C_2 is zero. The right hand side of equation (69) is the value of the option to adopt the policy. As k_1 is positive, k_2 is negative, and thus as M_t tends to infinity, $e^{k_2 M}$ tends to zero. But the value of the option can not tend to zero as M_t tends to infinity, thus C_2 is zero.

The value function for the "adopt" region is given by

$$\frac{1}{2}\sigma^2 W_{MM}^A - (r+\lambda)W^A + \lambda W^A(M+\phi) = \theta M^2, \tag{75}$$

and has the solution

$$W^A(M) = -\frac{(\sigma^2 + \lambda\phi^2 + rM^2)\theta}{r^2} - \frac{2\theta\lambda\phi(\lambda\phi + rM)}{r^3}. \tag{76}$$

The parameters C1 and M^* can be determined from the boundary equations:
Similar calculations have been done in chapter 3 of this thesis.

$$C_1 = \frac{\frac{2}{r^2}\beta E_0 \theta}{m_1 e^{m_1 M^*}}, \tag{77}$$

and

$$M^* = \frac{1}{m_1} - \frac{1}{r}(\beta E_0 + 2\lambda\phi)\frac{Kr^2}{2\theta\beta E_0}, \tag{78}$$

which is equal to the one found in Pindyck (2000), if $\lambda = 0$.
Equation (76) is decreasing in λ and φ, and therefore an emission reducing policy is adopted earlier if the stock of pollutant is assumed to face large unexpected changes.
This is one of the main outcomes of this chapter and of the paper by Dotsis et al. They also pay attention to the case where λ is not assumed to be zero after policy adoption, but drops to a new value λ^*. However this fact, even if more realistic, does not change the main findings.

4.2.3 Economic uncertainty

In the last chapter I dealt with the problem of unexpected and large changes in the pollution rate, and found that a policy should be adopted earlier if such discontinuities are regarded. Now, economic instead of ecological uncertainty is introduced, and again the calculations will only be presented shortly because they are very similar to the ones in chapter 3. The policy implications are of interest.

Economic uncertainty, as in chapter 3, is introduced by θ, which now follows a geometric Brownian motion augmented by jumps:

$$d\theta = \alpha\theta dt + \sigma\theta dz + \phi\theta dq, \tag{79}$$

where the jumps are again controlled by a Poisson process with positive constant jump sizes.

The stock of pollutant is now given by

$$dM(t)/t = \beta E(t) - \delta M(t). \tag{80}$$

The discontinuous part of the economic costs remains, emissions are assumed to be zero after policy implementation, and the cost function is given by: $K=kE_0$.

The Bellman equations for the "adopt" and "no-adopt" areas, respectively are given by

$$rW^N = -\theta M - (\beta E_0 - \delta M)W_M^N + \alpha\theta W_\theta^N + \frac{1}{2}\sigma^2\theta W_{\theta\theta}^N - \lambda W^N(\theta, M) + \lambda M W^N(\theta(\phi+1), M) \tag{81}$$

$$rW^A = -\theta M - (\beta E_1 - \delta M)W_M^A + \alpha\theta W_\theta^A + \frac{1}{2}\sigma^2\theta W_{\theta\theta}^A - \lambda W^A(\theta, M) + \lambda M W^A(\theta(\phi+1), M), \tag{82}$$

subject to the following set of boundary conditions:

$$W^N(0, M) = 0 \tag{83}$$

$$W^N(\theta^*, M) = W^A(\theta^*, M) - K \tag{84}$$

$$W_\theta^N(\theta^*, M) = W_\theta^N(\theta^*, M) \tag{85}$$

The solutions are:

For the "no-adopt" area

$$W^N(\theta, M) = A\theta^\gamma - \frac{\theta M}{r + \delta - (\alpha + \lambda\phi)} - \frac{\beta E_0 \theta}{(r - (\alpha + \lambda\phi))(r + \delta - (\alpha + \lambda\phi))}, \tag{86}$$

where γ is characterised by

$$\frac{1}{2}\sigma^2\gamma(\gamma-1)+\alpha\gamma-(r+\lambda)+\lambda(1+\phi)^{\gamma}=0, \tag{87}$$

and for the "adopt" area

$$W^A(\theta,M)=-\frac{\theta M}{r+\delta-(\alpha+\lambda\phi)}. \tag{88}$$

Finally, from the boundary conditions we get

$$A=E_0\left(\frac{\gamma-1}{k_1}\right)^{\gamma-1}\left[\frac{\beta}{(r-(a+\lambda\phi))(r+\delta-(\alpha+\lambda\phi))\gamma}\right]^{\gamma}, \tag{89}$$

and

$$\theta^* = \left(\frac{\gamma}{\gamma-1}\right)k_1(r-(a+\lambda\phi))(r+\delta-(\alpha+\lambda\phi))/\beta. \tag{90}$$

Equation (90) is strictly decreasing in γ and φ. So the same result for environmental uncertainty is reached as for ecological uncertainty: An increase in the probability of large future jumps will cause an earlier adoption of environmental policies. Here, in equation (90) these jumps concern the future flow of social costs, whereas in equation (78) the jumps concern sudden changes in the stock of pollutant M_t. Furthermore, an increase in the jumps size will have a similar effect.

4.2.4 Summary

Pindyck (2000, 2002) investigates irreversibilities and uncertainties to determine the optimal timing of environmental policies. He uses the real option theory instead of the traditional cost benefit analysis. Dotsis et al. use this analytical framework to determine in more detail the implications of uncertainties, such as taking into account sudden unpredictable changes and jumps in the stock of pollutant and in the social cost per unit of stock of pollutant caused by climate change. For example, so-called "feedback mechanisms" can be made responsible for these jumps. In both cases they find that policies are about to be adopted earlier if such changes are accounted for. Thus, neglecting these outcomes could lead to semi-optimal timing in environmental policies.

5. Summary and Conclusion

5.1 English Abstract

Investing in the earths ecosystem, as it is the fundament of our existence, is an absolutely necessary undertaking, not at least because of the substantiated damage from which it already suffered as a result of human activities. As the future is uncertain, and environmental harmful economic actions are partly irreversible, a precise valuation of ecological investment becomes more and more important, and part of scientific economic discussion. The traditional Net-Present Value rule, as pointed out clearly in this thesis, is not suitable enough to account for uncertainty, irreversibility and flexibility. Thus, the real option approach is used to evaluate and determine the optimal timing as well as the optimal amount of an emission reducing policy carried out by a policy maker. This approach, which refers to an option value comparable to the financial call or put option, allows giving future possibilities of investment a value which is not considered under the NPV-rule.

The mathematical background used in the main part of this thesis, chapter 3, is presented in chapter 2, as dynamic programming and certain types of Brownian motion, especially the Ito-process. The latter allows to model stochastic trends which face uncertainties and irreversibilities. Dynamic programming is used to break the whole future which is regarded in two sequences, the present decision and a second one which encapsulates all future consequences and possibilities.

Chapter 3 describes the model by Pindyck (2000): “Irreversibilities And The Timing of Environmental Policy” in detail. First of all, the analytical framework is presented, which is then extended to analyse the implications of ecological and economic uncertainty, irreversibility and the possibility of delaying the investment decision, as well as the possibility of reducing emissions at once or gradually.

Economic uncertainty together with a once and for all reduction in emissions is modelled in chapter 3.1. Here, the greater the uncertainty over future social cost of pollution, the greater the incentive to wait rather than adopt the policy immediately. The same incentive to wait is given if the discount rate r increases. Under ceteris paribus conditions, a greater current cost of pollution makes immediate policy adoption favourable, which the traditional NPV-rule takes into account as well. The timing of an emission reducing policy is also affected by irreversibility. In this chapter, the higher the natural rate at which pollution depreciates, which means that pollution gets more reversible, the smaller the sunken benefit of adopting the policy now rather than waiting, and the policymaker tends to delay the adoption. Furthermore, the timing of policy adoption does not depend on the initial level of pollution, but the value of society’s option to adopt the policy increases linearly with the initial level of pollution. Numerical as well as graphical solution can be found at the end of chapter 3.2.

In chapter 3.2.1 economic uncertainty, a convex cost function, and the possibility of partial emission-reductions are assumed. As in chapter 3.2, the more uncertainty, the later an emission reducing policy is adopted. But now, uncertainty also determines the amount by which emissions are reduced. The higher uncertainty, the lower the emission reduction, whereas an increase in the convex cost function increases the amount by which emissions are reduced.

Next, a convex benefit function together with economic uncertainty is modelled in chapter 3.2.2, in contrast to before, where the benefit function was linearly correlated with the stock of pollutant. The higher this stock, the earlier a policy gets adopted, because a higher stock of pollution implies a higher marginal cost of additional emissions. Besides, a higher emission level, a higher cost of emission reduction and a higher decay rate – more reversibility, lead to later adoption.

Gradual emission reduction is then assumed in chapter 3.3. Here, a policy maker faces the possibility to reduce emissions gradually and continuously. Thus, the optimal timing and the optimal amount of emission reduction has to be determined. The cost function is assumed to be convex, and the benefit function is assumed to be linear. Uncertainty then affects not only the emission reduction over time, but also the initial reduction.

From chapter 3.4 onwards, ecological uncertainty is assumed but no economic uncertainty. After a short overview of the new analytical framework, in chapter 3.2.1 complete reversibility is assumed in order to find an analytical solution. As ecological uncertainty is assumed to control the stock of pollutant stochastically, the future emission rates are known but still the evolution of the ecosystem is uncertain. This uncertainty, analogous to the economic uncertainty, delays policy adoption. A numerical example shows that the greater uncertainty, the later an emission reducing policy is adopted.

A more general case with ecological uncertainty is modelled in chapter 3.4.2, where environmental damage is partly irreversible. A solution is found only numerically in this case. The higher uncertainty over the evolution of the stock of pollutant, the later a policy is adopted. The same delay is true for lower irreversibility. Thus, a higher pollution decay rate implies that the stock of pollution faces a lower drift rate, and as a consequence the present value of the flow of social cost for any current value of the pollution stock is smaller. Therefore, to compensate for the sunk cost of policy adoption, a higher stock of pollutant is needed to trigger policy implementation.

Summarizing chapter 3, there is a possibility to delay policy adoption, called flexibility, and if the costs for a policy adoption are assumed to be sunk, then immediate emission reduction imposes an opportunity cost on society. However, there is also an opportunity "benefit" of early adoption because the stock of pollutant gets reduced, which otherwise would impose a nearly irreversible cost on society. And the higher uncertainty of the future costs and benefits of reduced emissions, i.e. economic uncertainty, or of the evolution of the stock of pollutant, i.e. ecological uncertainty, is assumed, the more the policy adoption gets delayed in order to gain more information. This is true for once and for all reduction as well as for gradual emission reductions, and in both cases the delay gets reduced the greater irreversibility.

This basic framework is then extended to two directions in order to analyse more precisely the impacts of the discount rate on the one hand and of the possibility of extreme events on the other hand. This is done in chapter 4.

First, a model by Di Vita (2003) "Is the Discount Rate Relevant in Explaining the Environmental Kuznets Curve?" is presented and discussed. The matter of investigation is the question how the interest rate affects environmental policies. If the discount rate is high due to economic growth as found in developed countries, the income-pollution pattern states that environmental policies are more likely to be adopted than in developing countries which face high interest rates. Thus, the discount rate and income move in opposite directions, such as the discount rate and the willingness to adopt emission reducing policies. Furthermore, the coherence between economic growth and pollution depends on the discount rate in the following way: countries with a low level of income and high discount rates show a positive relationship between economic growth and pollution, whereas developed countries illustrate the opposite. Both, a once-and-for all reduction and a gradual emission reduction were modelled, although the main findings were the same.

In chapter 4.2 extreme variations in pollution stock levels and in the socioeconomic costs were assumed. Again, the optimal timing and the optimal amount of an emission reducing policy were determined, but now accounting for the possibility of sudden jumps in the emission level and in pollutant-related socio-economic costs. Economic and ecological uncertainties were not modelled together. First, in chapter 4.2.1 sudden jumps were assumed in the stock of pollution. Then, in chapter 4.2.2 large and unexpected changes in the future flow of social cost were regarded. In both cases, the possibility of sudden unpredictable changes lead to earlier policy adoption.

This thesis gives on overview of ecological investment theory, especially if uncertainty, irreversibility and flexibility are concerned. The real option approach is explained and two different extensions of Pindyck's basic evaluation method were discussed. Needless to say, more research on this fields has to be done, and actually is done, in order to minimize the extremely high costs which may result from environmentally harmful production techniques.

5.2 German Abstract / Deutsche Zusammenfassung

Diese Diplomarbeit befasst sich mit dem Thema des ökologischen Investments. Die Debatte über den globalen Klimawandel und dessen Folgen stellt einen wichtigen Teil in der derzeitigen öffentlichen und politischen Diskussion dar. Der Einfluss der wirtschaftlichen Tätigkeiten auf die Umwelt ist unübersehbar, und durch zahlreiche wissenschaftliche Arbeiten belegt. Dies wurde in der Einführung dieser Arbeit ausreichend aufgezeigt und dokumentiert. Welche Folgen und welche daraus resultierenden Kosten aus dem Klimawandel resultieren kann jedoch nur geschätzt werden. Auch die optimalen Strategien wie man dem Wandel entgegenwirken kann, und sollte, liegen nicht auf der Hand. Wie jede Investition, so müssen auch die Investitionen in unser Ökosystem bestmöglich bewertet und verglichen werden. Jede Reduktion von Treibhausgasen, die als einer der Hauptverursacher des Klimawandels gelten, bedarf umfangreicher Forschungsarbeit und hohem Investitionsvolumen, sprich, kostet Geld. Der Nutzen daraus, etwa eine langsamer voranschreitende Temperaturerhöhung, entspricht jenen Kosten, die ansonsten durch eine höhere Umweltbelastung enstanden wären. Diese sind jedoch unvorhersehbar, also unsicher auf der einen Seite. Auf der anderen Seite sind die Folgen des Klimawandels teilweise irreversibel, so ist der bisherige Ausstoß von Treibhausgasen nicht mehr rückgängig zu machen. Selbiges gilt für umweltschohnende Maßnahmen. Deren genauer positiver Einfluß ist ebenso unsicher, und sobald diese Investition getätigt worden ist, sind die dafür anfallenden Kosten, zumindest teilweise, gesunkene Kosten, also ebenfalls irreversibel.

Die Bewertung einer Investition erfolgt traditionellerweise auf Basis der Kapitalwertregel, der net-present-value (NPV) rule. Sie besagt, dass sobald der erwartete abdiskundierte, also gegenwärtige Profit grösser ist als die erwarteten abdiskundierten Kosten, die Investition getätigt werden soll. Unberücksichtigt bleiben hier jedoch die angesprochene Unsicherheit und die Irrversibiliät der Investitionen sowie die Möglichkeit diese auf einen andern Zeitpunkt zu verschieben. Diese Möglichkeit der Flexibilität hat einen Wert, ähnlich dem der call- und put-options am Aktienmarkt, etwa wenn durch die Verzögerung der Investition neue wertvolle Informationen gesammelt werden können.

Um bei der Bewertung, in diesem Falle umweltschonender Investitionen, diese drei Kriterien nicht ausser Acht zu lassen verwendet man den sogenannten real – option – approach.

Der Hauptaspekt dieser Diplomarbeit widmet sich genau dieser Bewertungsmethode. Nach einer Einleitung, die die Unterschiede der NPV – rule und der Realoptionstheorie genau erklärt, und die Herangehensweise motiviert, erkläre ich die fundamentalen mathematischen

Methoden die dafür nötig sind. Um die Unsicherheiten der getätigten Investitionen sowie der Folgen eines erhöhten Treibhausgasausstoßen zu modellieren, wird der sogenannte „Wiener Process“, auch „Brownian Motion“ genannt, verwendet.

In Kapitel 3 wird das Modell von „Irreversibilities and the Timing of Environmental Policies“, Pindyck (2000), detailiert besprochen und verschiedene Dynamiken unterschiedlicher Parameter dargestellt. Kurz zusammengefasst handelt es sich bei diesem Modell um die Bewertung von Emissionsreduktionen anhand der Realoptionenmethode. Ökonomische und ökologische Unsicherheit, die Folgen von Irreversibilität, die Möglichkeit der Flexibilität, sowie verschieden Kostenfunktionen werden berücksichtigt, und deren unterschiedlichen Implikationen untersucht um den optimalen Zeitpunkt, sowie den optimalen Umfang der Emissionsreduktion zu bestimmen. Auch die Möglichkeit von einmaliger sowie gradueller Reduktionen der Treibhausgase wird modelliert, und deren Ergebnisse verglichen. Eines der wichtigsten Ergebnisse ist, dass eine umweltschohnende Politik desto später durchgeführt werden soll, je grösser die Unsicherheit ist. Eine Verzögerund der Politik ist auch dann optimal, wenn die Irreversibilität abnimmt.

In Kapitel 4 werden zwei Erweiterungen des in Kapitel 3 besprochenen Modelles dargestellt und analysiert.

Anhand von Di Vita’s Modell „Is the Discount Rate Relevant in Explaining the Environmental Kuznets Curve?“ wird die Rolle der Zinsrate genauer unter die Lupe genommen. Di Vita kommt zu dem Schluß, dass die Unterschiede in der Zinsrate in entwickelten Ländern und Entwicklungsländern dazu führt, dass in ersteren umweltschohnende Maßnahmen getroffen werden, in letzteren jedoch nicht. Er führt dies darauf zurück, dass unter einer niedrigen Zinsrate eine umweltschohnende Politik eher möglich ist, was in Kapitel 4.1 auch modelliert wird.

Die zweite Erweiterung des zugrundeliegenden Modells beschäftigt sich mit den Folgen von extremen Ereignissen, einerseits im Ökosystem, etwa unvorhersehbaren extremen Steigerungen von Treibhausgasen in der Atmosphäre, und andererseits von extremen Sprüngen der sozial-ökonomischen Kosten die durch den Klimawandel hervorgerufen werden. Unter Anbetracht dieser Möglichkeiten wird, ähnlich wie in Kapitel 3, der optimale Zeitpunkt und der optimale Umfang von treibhausgasreduzierenden Investitionen errechnet. Je grösser die Wahrscheinlichkeit von großen unvorhersehbaren Änderungen der Emissionen beziehungsweise der damit verbundenen Kosten, desto früher wird eine emsionsreduzierende Politik optimal.

Kapitel 5.1 fasst die wichtigsten Erkenntnisse der Arbeit zusammen und vergleicht deren Implikationen. In Kapitel 6 sind die detailierten mathematischen Kalkulation zu finden.

6. Technical appendix

6.a)

Determining A, Θ* and γ from equation (13) in chapter 3.2:

First using equation (8) from chapter 3.2:

$$W^N(\theta, M) = W^A(\theta, M) - K$$

$$A\theta^\gamma - \frac{\theta M}{r+\delta-\alpha} - \frac{\beta E_0 \theta}{(r-\alpha)(r+\delta-\alpha)} = -\frac{\theta M}{r+\delta-\alpha} - K$$

$$A\theta^\gamma - \frac{\beta E \theta}{(r-\alpha)(r+\delta-\alpha)} = -K$$

$$A\theta^\gamma = \frac{\beta E \theta}{(r-\alpha)(r+\delta-\alpha)} - K$$

$$A = \frac{\beta E_0 \theta}{(r-\alpha)(r+\delta-\alpha)} \frac{1}{\theta^\gamma} - \frac{K}{\theta^\gamma}$$

$$A = \frac{\beta E_0}{(r-\alpha)(r+\delta-\alpha)} \theta^{1-\gamma} - \frac{K}{\theta^\gamma} \qquad (1^*)$$

Taking into account that

$$W_\theta^N(\theta^*, M) = A\gamma\theta^{\gamma-1} - \frac{M}{r+\delta-\alpha} - \frac{\beta E_0}{(r-\alpha)(r+\delta-\alpha)}$$

$$W_\theta^A(\theta^*, M) = -\frac{M}{r+\delta-\alpha},$$

and then using equation (9) from chapter 3.2:

$$W_\theta^N(\theta^*, M) = W_\theta^A(\theta^*, M),$$

$$A\gamma\theta^{\gamma-1} - \frac{M}{r+\delta-\alpha} - \frac{\beta E_0}{(r-\alpha)(r+\delta-\alpha)} = \frac{M}{r+\delta-\alpha}$$

$$A\gamma\theta^{\gamma-1} - \frac{\beta E_0}{(r-\alpha)(r+\delta-\alpha)} = 0$$

$$A\gamma\theta^{\gamma-1} = \frac{\beta E_0}{(r-\alpha)(r+\delta-\alpha)}$$

Inserting (1*) for A

$$\left[\frac{\beta E_0}{(r-\alpha)(r+\delta-\alpha)}\theta^{1-\gamma}-\frac{K}{\theta^{\gamma}}\right]\gamma\theta^{\gamma-1}=\frac{\beta E_0}{(r-\alpha)(r+\delta-\alpha)}$$

$$\frac{\beta E_0}{(r-\alpha)(r+\delta-\alpha)}\gamma-\gamma K\theta^{-1}=\frac{\beta E_0}{(r-\alpha)(r+\delta-\alpha)}$$

$$\gamma K\theta^{-1}=\frac{\beta E_0}{(r-\alpha)(r+\delta-\alpha)}(\gamma-1)$$

$$\theta^{-1}=\frac{\beta E_0}{(r-\alpha)(r+\delta-\alpha)}(\gamma-1)\frac{1}{K\gamma}$$

$$\theta^{*}=\frac{K\gamma}{(\gamma-1)}\left[\frac{(r-\alpha)(r+\delta-\alpha)}{\beta E_0}\right] \qquad (2^*)$$

Then inserting equation (2*) in equation (1*) to get A*:

$$A\theta^{\gamma}-\frac{\beta E_0\theta}{(r-\alpha)(r+\delta-\alpha)}=-K$$

$$A\left[\frac{\gamma K}{(\gamma-1)}\frac{(r-\alpha)(r+\delta-\alpha)}{\beta E_0\theta}\right]^{\gamma}-\frac{\beta E_0\theta}{(r-\alpha)(r+\delta-\alpha)}\left[\frac{\gamma K}{(\gamma-1)}\frac{(r-\alpha)(r+\delta-\alpha)}{\beta E_0\theta}\right]=-K$$

$$A\left(\frac{\gamma K}{(\gamma-1)}\right)^{\gamma}\left(\frac{(r-\alpha)(r+\delta-\alpha)}{\beta E_0\theta}\right)^{\gamma}-\frac{\gamma K}{(\gamma-1)}=-K$$

$$A=\left(\frac{\gamma K}{(\gamma-1)}-K\right)\left(\frac{\gamma K}{(\gamma-1)}\right)^{-\gamma}\left(\frac{(r-\alpha)(r+\delta-\alpha)}{\beta E_0\theta}\right)^{-\gamma}$$

$$A=\left[\frac{\gamma K}{(\gamma-1)}\left(\frac{\gamma K}{(\gamma-1)}\right)^{-\gamma}-K\left(\frac{\gamma K}{(\gamma-1)}\right)^{-\gamma}\right]\left(\frac{\beta E_0\theta}{(r-\alpha)(r+\delta-\alpha)}\right)^{\gamma}$$

$$A=\left[\left(\frac{\gamma-1}{\gamma K}\right)^{\gamma-1}-K\left(\frac{\gamma-1}{\gamma K}\right)^{\gamma}\right]\left(\frac{\beta E_0\theta}{(r-\alpha)(r+\delta-\alpha)}\right)^{\gamma}$$

$$A=\left(\frac{\gamma-1}{\gamma K}\right)^{\gamma}\left[\left(\frac{\gamma-1}{\gamma K}\right)^{-1}-K\right]\left(\frac{\beta E_0\theta}{(r-\alpha)(r+\delta-\alpha)}\right)^{\gamma}$$

$$A=\left(\frac{\gamma-1}{\gamma K}\right)^{\gamma}\left[\frac{\gamma K}{\gamma-1}-\frac{(\gamma-1)K}{\gamma-1}\right]\left(\frac{\beta E_0\theta}{(r-\alpha)(r+\delta-\alpha)}\right)^{\gamma}$$

$$A=\left(\frac{\gamma-1}{\gamma K}\right)^{\gamma}\left(\frac{K}{\gamma-1}\right)\left(\frac{\beta E_0\theta}{(r-\alpha)(r+\delta-\alpha)}\right)^{\gamma}$$

$$A = \left(\frac{\gamma-1}{\gamma K}\right)^{\gamma}\left(\frac{\gamma-1}{K}\right)^{-1}\left(\frac{\beta E_0\theta}{(r-\alpha)(r+\delta-\alpha)}\right)^{\gamma}$$

$$A = \left(\frac{1}{\gamma}\right)^{\gamma}\left(\frac{\gamma-1}{K}\right)^{\gamma}\left(\frac{\gamma-1}{K}\right)^{-1}\left(\frac{\beta E_0\theta}{(r-\alpha)(r+\delta-\alpha)}\right)^{\gamma}$$

$$A = \left(\frac{1}{\gamma}\right)^{\gamma}\left(\frac{\gamma-1}{K}\right)^{\gamma-1}\left(\frac{\beta E_0\theta}{(r-\alpha)(r+\delta-\alpha)}\right)^{\gamma}$$

$$A^* = \left(\frac{\gamma-1}{K}\right)^{\gamma-1}\left(\frac{\beta E_0\theta}{(r-\alpha)(r+\delta-\alpha)}\right)^{\gamma} \tag{3*}$$

and making use of our assumption, that $K = kE_0$, equation (2*) and (3*) become equations (15) and (16) from chapter 3.2:

$$A = (\frac{\gamma-1}{k})^{\gamma-1}[\frac{\beta}{(r-\alpha)(r+\delta-\alpha)\gamma}]^{\gamma} E_0, \tag{15}$$

$$\theta^* = (\frac{\gamma}{\gamma-1})\frac{k(r-\alpha)(r+\delta-\alpha)}{\beta}. \tag{16}$$

Inserting (10) in (8) in chapter 3.2:

$$W^N(0,M) = 0 \tag{10}$$

$$rW^N = -\theta M + (\beta E_0 - \delta M)W_M^N + \alpha W_\theta^N + \frac{1}{2}\sigma^2\theta^2 W_{\theta\theta}^N \tag{8}$$

where

$W^N = A\theta^\gamma$;

$W_\theta^N = A\gamma\theta^{\gamma-1}$;

$W_{\theta\theta}^N = A\gamma(\gamma-1)\theta^{\gamma-2}$

$$\frac{1}{2}\sigma^2\theta^2 A\gamma(\gamma-1)\theta^{\gamma-2} + \alpha\theta A\gamma\theta^{\gamma-1} - rA\theta^\gamma = 0$$

$$\frac{1}{2}\sigma^2 A\gamma(\gamma-1)\theta^\gamma + \alpha A\gamma\theta^\gamma - rA\theta^\gamma = 0$$

$$A\theta^\gamma[\frac{1}{2}\sigma^2\gamma(\gamma-1) + \alpha\gamma - r] = 0,$$

receiving the following quadratic equation:

$$\frac{1}{2}\sigma^2\gamma(\gamma-1)+\alpha\gamma-r=0\,,$$

and solving it:

$$\frac{1}{2}\sigma^2\gamma^2+\left(\alpha-\frac{1}{2}\sigma^2\right)\gamma-r=0$$

$$y_{1,2}=\frac{-\left(\alpha-\frac{1}{2}\sigma^2\right)\pm\sqrt{\left(\alpha-\frac{1}{2}\sigma^2\right)^2-4\frac{1}{2}\sigma^2(-r)}}{2\frac{1}{2}\sigma^2}=$$

$$=\frac{-\alpha+\frac{1}{2}\sigma^2\pm\sqrt{\left(\alpha-\frac{1}{2}\sigma^2\right)^2+2\sigma^2 r}}{\sigma^2}=$$

$$=\frac{1}{2}-\frac{\alpha}{\sigma^2}\pm\sqrt{\frac{\left(\alpha-\frac{1}{2}\sigma^2\right)^2+2\sigma^2 r}{\sigma^4}}=$$

$$=\frac{1}{2}-\frac{\alpha}{\sigma^2}\pm\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}=$$

yields:

$\gamma=\frac{1}{2}-\frac{\alpha}{\sigma^2}+\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}$ which is equation (18) in chapter 3.2.

6. b)

Calcualting A and θ* from chapter 3.2.1:

$$\mathrm{W}^{\mathrm{N}}\left(\theta^*,\mathrm{M}\right)\;=\;\mathrm{W}^{\mathrm{A}}\left(\theta,\,\mathrm{M}\right)\;-\;\mathrm{K}, \tag{11}$$

$$A\theta^\gamma-\frac{\theta M}{r+\delta-\alpha}-\frac{\beta E_0\theta}{\rho}=-\frac{\theta M}{r+\delta-\alpha}-\frac{\beta E_1\theta}{\rho}-K$$

$$A\theta^\gamma-\frac{\beta E_0\theta}{\rho}=-\frac{\beta E_1\theta}{\rho}-K$$

$$A\theta^\gamma-\frac{\beta E_0\theta}{\rho}+\frac{\beta E_1\theta}{\rho}+K=0$$

$$A\theta^{\gamma} - (E_0 - E_1)\frac{\beta\theta}{\rho} + k_1(E_0 - E_1) + k_2(E_0 - E_1)^2 = 0$$

$$A\theta^{\gamma} + (E_0 - E_1)\left[-\frac{\beta\theta}{\rho} + k_1 + k_2(E_0 - E_1)\right] = 0$$

$$A\theta^{\gamma} + \left(-\frac{k_1}{2k_2} + \frac{\beta\theta}{2k_2\rho}\right)\left[-\frac{\beta\theta}{\rho} + k_1 + k_2\left(\frac{k_1}{2k_2} - \frac{\beta\theta}{2k_2\rho}\right)\right] = 0$$

$$A\theta^{\gamma} + \left(-\frac{k_1}{2k_2} + \frac{\beta\theta}{2k_2\rho}\right)\left[-\frac{\beta\theta}{\rho} + k_1 + \frac{\beta\theta k_2}{2k_2\rho} - \frac{k_1 k_2}{2k_2}\right] = 0$$

$$A\theta^{\gamma} + \left(\frac{\beta\theta}{2k_2\rho} - \frac{k_1}{2k_2}\right)\left(-\frac{\beta\theta}{\rho} + k_1 + \frac{\beta\theta}{2\rho} - \frac{k_1}{2}\right) = 0$$

$$A\theta^{\gamma} + \left(\frac{\beta\theta}{2k_2\rho} - \frac{k_1}{2k_2}\right)\left(\frac{k_1}{2} - \frac{\beta\theta}{2\rho}\right) = 0$$

$$A\theta^{\gamma} + \frac{\beta\theta k_1}{4k_2\rho} - \frac{\beta^2\theta^2}{4k_2\rho^2} - \frac{k_1^2}{4k_2} + \frac{k_1\beta\theta}{4k_2\rho} = 0$$

$$A\theta^{\gamma} + \frac{\beta\theta k_1}{2k_2\rho} - \frac{\beta^2\theta^2}{4k_2\rho^2} - \frac{k_1^2}{4k_2} = 0$$

$$A = \left(\frac{\beta^2\theta^2 k}{4k_2\rho} + \frac{k_1^2}{4k_2} - \frac{\beta\theta k_1}{2k_2\rho}\right)\theta^{-\gamma}$$

$$A = \frac{\beta^2\theta^2}{4k_2\rho^2\theta^{\gamma}} + \frac{k_1^2}{4k_2\theta^{\gamma}} - \frac{\beta\theta k_1}{2k_2\rho\theta^{\gamma}}$$

$$A = \frac{\beta^2}{4k_2\rho^2\theta^{\gamma-2}} + \frac{k_1^2}{4k_2\theta^{\gamma}} - \frac{\beta k_1}{2k_2\rho\theta^{\gamma-1}}. \tag{27}$$

$$\mathrm{W}_{\theta}^{N}\left(\theta^*,\ \mathrm{M}\right) \ = \ \mathrm{W}_{\theta}^{A}\ \left(\theta^*,\ \mathrm{M}\right). \tag{12}$$

where

$$W_{\theta}^{N}(\theta, M) = A\gamma\theta^{\gamma-1} - \frac{M}{r+\delta-\alpha} - \frac{\beta E_0}{\rho},$$

$$W_{\theta}^{A}(\theta, M) = -\frac{M}{r+\delta-\alpha} - \frac{\beta E_1}{\rho},$$

$$A\gamma\theta^{\gamma-1} - \frac{M}{r+\delta-\alpha} - \frac{\beta E_0}{\rho} = -\frac{M}{r+\delta-\alpha} - \frac{\beta E_1}{\rho}$$

$$A\gamma\theta^{\gamma-1} - \frac{\beta E_0}{\rho} + \frac{\beta E_1}{\rho} = 0$$

$$A\gamma\theta^{\gamma-1} - \frac{\beta}{\rho}(E_0 - E_1) = 0$$

$$\left(\frac{\beta^2\theta^2}{4k_2\rho^2} + \frac{k_1^2}{4k_2} - \frac{\beta\theta k_1}{2k_2\rho}\right)\theta^{-\gamma}\gamma\theta^{\gamma-1} - \frac{\beta}{\rho}(E_0 - E_1) = 0$$

$$\left(\frac{\beta^2\theta^2}{4k_2\rho^2} + \frac{k_1^2}{4k_2} - \frac{\beta\theta k_1}{2k_2\rho}\right)\gamma\theta^{-1} = \frac{\beta}{\rho}(E_0 - E_1)$$

$$\left(\frac{\beta^2\theta^2}{4k_2\rho^2} + \frac{k_1^2}{4k_2} - \frac{\beta\theta k_1}{2k_2\rho}\right)\gamma\theta^{-1} = \frac{\beta}{\rho}\left(\frac{\beta\theta}{2k_2\rho} - \frac{k_1}{2k_2}\right)$$

$$\left(\frac{\beta^2\theta^2}{4k_2\rho^2} + \frac{k_1^2}{4k_2} - \frac{\beta\theta k_1}{2k_2\rho}\right)\gamma\theta^{-1} = \frac{\beta^2\theta}{2k_2\rho^2} - \frac{k_1\beta}{2k_2\rho}$$

$$\left(\frac{\beta^2\theta^2}{4k_2\rho^2} + \frac{k_1^2}{4k_2} - \frac{\beta\theta k_1}{2k_2\rho}\right)\gamma = \frac{\beta^2\theta^2}{2k_2\rho^2} - \frac{k_1\beta\theta}{2k_2\rho}$$

$$\left(\frac{\beta^2\theta^2}{4k_2\rho^2} + \frac{k_1^2}{4k_2} - \frac{\beta\theta k_1}{2k_2\rho}\right)\gamma - \frac{\beta^2\theta^2}{2k_2\rho^2} + \frac{k_1\beta\theta}{2k_2\rho}$$

$$\frac{\beta^2\theta^2}{4k_2\rho^2}(\gamma-2) + \frac{k_1^2}{4k_2}\gamma + \frac{\beta\theta k_1}{2k_2\rho}(\gamma-1) = 0$$

$$\frac{\beta^2\theta^2}{4k_2\rho^2}(\gamma-2) + \frac{k_1^2\rho^2}{4k_2\rho^2}\gamma + \frac{2\rho\beta\theta k_1}{4k_2\rho^2}(\gamma-1) = 0 \qquad (28)$$

$$(\gamma-2)\beta^2\theta^2 + 2\rho(\gamma-1)\beta k_1\theta + \gamma\rho^2 k_1^2 = 0.$$

Assuming that $\gamma > 2$, and taking into account that $W^A(\theta) - W^N(\theta) - K(E(\theta^*))$ is convex in θ, θ^* is the largest root of this quadratic equation (28):

$$\theta^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\theta^* = \frac{2\rho(\gamma-1)\beta k_1 \pm \sqrt{[2\rho(\gamma-1)\beta k_1]^2 - 4(\gamma-2)\beta^2\gamma\rho^2 k_1^{\,2}}}{2(\gamma-2)\beta^2}$$

$$\theta^* = \frac{2\rho(\gamma-1)\beta k_1 \pm \sqrt{[4\rho^2(\gamma-1)^2\beta^2 k_1^2 - 4(\gamma-2)\beta^2\gamma\rho^2 k_1^{\,2}}}{2(\gamma-2)\beta^2}$$

$$\theta^* = \frac{2\rho(\gamma-1)\beta k_1 \pm \sqrt{\rho^2 k_1^2[(\gamma-1)^2 - \gamma(\gamma-2)]}}{(\gamma-2)\beta}$$

$$\theta^* = \frac{\rho(\gamma-1)k_1}{\beta(\gamma-2)} \pm \frac{\sqrt{\rho^2 k_1^2[(\gamma-1)^2 - \gamma(\gamma-2)]}}{\beta(\gamma-2)}$$

$$\theta^* = \frac{\rho(\gamma-1)k_1}{\beta(\gamma-2)} \pm \sqrt{\frac{\rho^2 k_1^2[(\gamma-1)^2 - \gamma(\gamma-2)]}{\beta^2(\gamma-2)^2}}$$

$$\theta^* = \frac{\rho(\gamma-1)k_1}{\beta(\gamma-2)} \pm \sqrt{\frac{\rho^2 k_1^2(\gamma-1)^2}{\beta^2(\gamma-2)^2} - \frac{\rho^2 k_1^2 \gamma(\gamma-2)}{\beta^2(\gamma-2)^2}}$$

$$\theta^* = \frac{\rho k_1(\gamma-1)}{\beta(\gamma-2)} \pm \sqrt{\frac{\rho^2 k_1^2(\gamma-1)^2}{\beta^2(\gamma-2)^2} - \frac{\rho^2 k_1^2 \gamma}{\beta^2(\gamma-2)}}$$

$$\theta^* = \frac{\rho k_1(\gamma-1)}{\beta(\gamma-2)} \left[1 + \sqrt{1 - \frac{\gamma(\gamma-2)}{(\gamma-1)^2}}\right]. \tag{29}$$

c) Calculating A and θ* in chapter 3.2.2:

$$W^N(\theta^*, M) = W^A(\theta, M) - K, \tag{11}$$

$$A\theta^\gamma - \frac{\theta M^2}{r+2\delta-\alpha} - \frac{2\beta^2 E_0^2 \theta}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)} - \frac{2\beta E_0 \theta M}{(r+2\delta-\alpha)(r+\delta-\alpha)} = -\frac{\theta M^2}{r-2\delta-\alpha} - K$$

$$A\theta^\gamma - \frac{2\beta^2 E_0^2 \theta}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)} - \frac{2\beta E_0 \theta M}{(r+2\delta-\alpha)(r+\delta-\alpha)} = -K$$

$$A = \left[\frac{2\beta^2 E_0^2 \theta + 2\beta E_0 \theta M(r-\alpha)}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}\right]\theta^{-\gamma} - \frac{K}{\theta^\gamma}$$

$$A = \left[\frac{2\beta E_0 \theta[\beta E_0 + M(r-\alpha)]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}\right]\theta^{-\gamma} - \frac{K}{\theta^\gamma} \tag{4*}$$

$$W_\theta^N(\theta^*, M) = W_\theta^A(\theta^*, M), \tag{9}$$

where

$$W_\theta^N(\theta, M) = A\gamma\theta^{\gamma-1} - \frac{M^2}{r+2\delta-\alpha} - \frac{2\beta^2 E_0^2}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)} - \frac{2\beta E_0 M}{(r+2\delta-\alpha)(r+\delta-\alpha)},$$

$$W_\theta^A(\theta^*, M) = -\frac{M^2}{r+2\delta-\alpha}$$

$$A\gamma\theta^{\gamma-1} - \frac{M^2}{r+2\delta-\alpha} - \frac{2\beta^2 E_0^2}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)} - \frac{2\beta E_0 M}{(r+2\delta-\alpha)(r+\delta-\alpha)} = -\frac{M^2}{r+2\delta-\alpha}$$

$$A\gamma\theta^{\gamma-1} - \frac{2\beta^2 E_0^2}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)} - \frac{2\beta E_0 M}{(r+2\delta-\alpha)(r+\delta-\alpha)} = 0$$

$$A\gamma\theta^{\gamma-1} = \frac{2\beta^2 E_0^2 + 2\beta E_0 M(r-\alpha)}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}$$

$$A\gamma\theta^{\gamma-1} = \frac{2\beta E_0[\beta E_0 + (r-\alpha)M]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}$$

inserting equation (4*) for A:

$$\left\{\left[\frac{2\beta E_0\theta[\beta E_0 + M(r-\alpha)]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}\right]\theta^{-\gamma} - \frac{K}{\theta^\gamma}\right\}\gamma\theta^{\gamma-1} =$$
$$= \frac{2\beta E_0[\beta E_0 + (r-\alpha)M]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}$$

$$\left[\frac{2\beta E_0\theta[\beta E_0 + M(r-\alpha)]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}\right]\gamma\theta^{-1} - \frac{K}{\theta^\gamma}\gamma\theta^{\gamma-1} = \frac{2\beta E_0[\beta E_0 + (r-\alpha)M]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}$$

$$\left[\frac{2\beta E_0\theta[\beta E_0 + M(r-\alpha)]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}\right]\gamma\theta^{-1} - \frac{K}{\theta}\gamma = \frac{2\beta E_0[\beta E_0 + (r-\alpha)M]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}$$

$$\left[\frac{2\beta E_0[\beta E_0 + M(r-\alpha)]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}\right]\gamma - \frac{K}{\theta}\gamma = \frac{2\beta E_0[\beta E_0 + (r-\alpha)M]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}$$

$$-K\gamma\theta^{-1} = \frac{2\beta E_0[\beta E_0 + (r-\alpha)M]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)} - \left[\frac{2\beta E_0[\beta E_0 + (r-\alpha)M]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}\right]\gamma$$

$$-K\gamma\theta^{-1} = \left[\frac{2\beta E_0[\beta E_0 + (r-\alpha)M]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}\right](1-\gamma)$$

$$\theta^{-1} = \left[\frac{2\beta E_0[\beta E_0 + (r-\alpha)M]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}\right](\gamma-1)\frac{1}{K\gamma}$$

$$\theta = \left[\frac{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}{2\beta E_0[\beta E_0 + (r-\alpha)M]}\right]\frac{K\gamma}{(\gamma-1)}$$

$$\theta = \frac{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)kE_0\gamma}{2\beta E_0[\beta E_0 + (r-\alpha)M](\gamma-1)}$$

$$\theta^* = \frac{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)k\gamma}{2\beta(\gamma-1)[\beta E_0 + (r-\alpha)M]} \tag{31}$$

now inserting equation (31) in (4*) to get A:

$$A\left[\frac{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)k\gamma}{2\beta(\gamma-1)[\beta E_0+M(r-\alpha)]}\right]^{\gamma}-\frac{2\beta E_0\theta[\beta E_0-M(r-\alpha)]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}=-K$$

$$A\left[\frac{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)k\gamma}{2\beta(\gamma-1)[\beta E_0+M(r-\alpha)]}\right]^{\gamma}-\frac{2\beta E_0[\beta E_0-M(r-\alpha)]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}\theta=-K$$

$$A\left[\frac{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)k\gamma}{2\beta(\gamma-1)[\beta E_0+M(r-\alpha)]}\right]^{\gamma}-\frac{2\beta E_0[\beta E_0-M(r-\alpha)]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)}\frac{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)k\gamma}{2\beta(\gamma-1)[\beta E_0+M(r-\alpha)]}=-k$$

$$A\left[\frac{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)k\gamma}{2\beta(\gamma-1)[\beta E_0+M(r-\alpha)]}\right]^{\gamma}-\frac{E_0k\gamma}{(\gamma-1)}=-K$$

$$A\left[\frac{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)k\gamma}{2\beta(\gamma-1)[\beta E_0+M(r-\alpha)]}\right]^{\gamma}-\frac{E_0k\gamma}{(\gamma-1)}+kE_0=0$$

$$A\left[\frac{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)k\gamma}{2\beta(\gamma-1)[\beta E_0+M(r-\alpha)]}\right]^{\gamma}-\frac{E_0k\gamma+kE_0(\gamma-1)}{(\gamma-1)}=0$$

$$A\left[\frac{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)k\gamma}{2\beta(\gamma-1)[\beta E_0+M(r-\alpha)]}\right]^{\gamma}-\frac{kE_0}{(\gamma-1)}=0$$

$$A=\frac{kE_0}{(\gamma-1)}\left[\frac{2\beta(\gamma-1)[\beta E_0+M(r-\alpha)]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)k\gamma}\right]^{\gamma}$$

$$A=E_0\frac{k}{(\gamma-1)}\left(\frac{\gamma-1}{k}\right)^{\gamma}\left[\frac{2\beta[\beta E_0+M(r-\alpha)]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)\gamma}\right]^{\gamma}$$

$$A=E_0\left(\frac{\gamma-1}{k}\right)^{-1}\left(\frac{\gamma-1}{k}\right)^{\gamma}\left[\frac{2\beta[\beta E_0+M(r-\alpha)]}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)\gamma}\right]^{\gamma}$$

$$A=E_0\left(\frac{\gamma-1}{k}\right)^{\gamma-1}\left[\frac{2\beta^2E_0+2\beta(r-\alpha)M}{(r-\alpha)(r+2\delta-\alpha)(r+\delta-\alpha)\gamma}\right]^{\gamma} \tag{32}$$

d)

Calculating θ* and α from chapter 3.3:

$$\alpha\theta^{\gamma}=\frac{\beta\theta_t\Delta E}{\rho}-K \tag{38}$$

$$\alpha\theta^{\gamma} = \frac{\beta\theta_t \Delta E}{\rho} - (m_1 - m_2 E)$$

$$\alpha = \left[\frac{\beta\theta_t \Delta E}{\rho} - (m_1 - m_2 E)\right]\theta^{-\gamma}$$

$$\alpha\gamma\theta^{\gamma-1} = \frac{\beta\Delta E}{\rho} \tag{39}$$

$$\left[\frac{\beta\theta_t \Delta E}{\rho} - (m_1 - m_2 E)\right]\theta^{-\gamma}\gamma\theta^{\gamma-1} = \frac{\beta\Delta E}{\rho}$$

$$\left[\frac{\beta\theta_t \Delta E}{\rho} - (m_1 - m_2 E)\right]\gamma\theta^{-1} = \frac{\beta\Delta E}{\rho}$$

$$\frac{\beta\Delta E}{\rho}\gamma - (m_1 - m_2 E)\gamma\theta^{-1} = \frac{\beta\Delta E}{\rho}$$

$$(m_1 - m_2 E)\gamma\theta^{-1} = \frac{\beta\Delta E}{\rho}(\gamma - 1)$$

$$\theta^{-1} = \frac{\beta\Delta E(\gamma-1)}{\rho(m_1 - m_2 E)\gamma}$$

$$\theta^{*} = \frac{\rho\gamma(m_1 - m_2 E)}{\beta\Delta E(\gamma-1)} \tag{41}$$

inserting in (38) yields α:

$$\alpha\theta^{\gamma} = \frac{\beta\theta_t \Delta E}{\rho} - (m_1 - m_2 E)$$

$$\alpha\left[\frac{\rho\gamma(m_1 - m_2 E)}{\beta\Delta E(\gamma-1)}\right]^{\gamma} = \frac{\beta\Delta E}{\rho}\frac{\rho\gamma(m_1 - m_2 E)}{\beta\Delta E(\gamma-1)} - (m_1 - m_2 E)$$

$$\alpha\left(\frac{\gamma(m_1 - m_2 E)}{\gamma-1}\right)^{\gamma}\left(\frac{\rho}{\beta\Delta E}\right)^{\gamma} = \frac{\gamma(m_1 - m_2 E)}{\gamma-1} - (m_1 - m_2 E)$$

$$\alpha = \left(\frac{\gamma(m_1 - m_2 E)}{\gamma-1} - (m_1 - m_2 E)\right)\left(\frac{\gamma(m_1 - m_2 E)}{\gamma-1}\right)^{-\gamma}\left(\frac{\rho}{\beta\Delta E}\right)^{-\gamma}$$

$$\alpha = \left[\left(\frac{\gamma(m_1 - m_2 E)}{\gamma-1}\right)\left(\frac{\gamma(m_1 - m_2 E)}{\gamma-1}\right)^{-\gamma} - (m_1 - m_2 E)\left(\frac{\gamma(m_1 - m_2 E)}{\gamma-1}\right)^{-\gamma}\right]\left(\frac{\rho}{\beta\Delta E}\right)^{-\gamma}$$

$$\alpha = \left[\left(\frac{\gamma-1}{\gamma(m_1 - m_2 E)}\right)^{\gamma-1} - (m_1 - m_2 E)\left(\frac{\gamma-1}{\gamma(m_1 - m_2 E)}\right)^{\gamma}\right]\left(\frac{\rho}{\beta\Delta E}\right)^{-\gamma}$$

$$\alpha = \left(\frac{\gamma - 1}{\gamma(m_1 - m_2 E)}\right)^{\gamma} \left[\left(\frac{\gamma - 1}{\gamma(m_1 - m_2 E)}\right)^{-1} - (m_1 - m_2 E)\right]\left(\frac{\beta \Delta E}{\rho}\right)^{\gamma}$$

$$\alpha = \left(\frac{(\gamma - 1)}{\gamma(m_1 - m_2 E)}\right)^{\gamma} \left[\frac{\gamma(m_1 - m_2 E)}{\gamma - 1} - \frac{(\gamma - 1)(m_1 - m_2 E)}{\gamma - 1}\right]\left(\frac{\beta \Delta E}{\rho}\right)^{\gamma}$$

$$\alpha = \left(\frac{\gamma - 1}{\gamma(m_1 - m_2 E)}\right)^{\gamma} \left(\frac{m_1 - m_2 E}{\gamma - 1}\right)\left(\frac{\beta \Delta E}{\rho}\right)^{\gamma}$$

$$\alpha = \left(\frac{1}{\gamma}\right)^{\gamma} \left(\frac{\gamma - 1}{m_1 - m_2 E}\right)^{\gamma} \left(\frac{\gamma - 1}{m_1 - m_2 E}\right)^{-1} \left(\frac{\beta \Delta E}{\rho}\right)^{\gamma}$$

$$\alpha = \left(\frac{\beta \Delta E}{\gamma \rho}\right)^{\gamma} \left(\frac{\gamma - 1}{m_1 - m_2 E}\right)^{\gamma - 1} \tag{42}$$

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