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Optomechanical probing of heat-bath environment of a micromechanical harmonic oscillator

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Zusammenfassung der Diplomarbeit in deutscher Sprache

Mikromechanische Resonatoren spielen eine zunehmende Rolle in der experimentellen und theoretischen Quantenphysik. Jüngste Experimente bringen mikromechanische Systeme mehr und mehr in Richtung des Quantenregimes. Theoretische Prognosen sagen voraus, dass das Verhalten der Resonatoren im Quantenregime entscheidend von ihrer Kopplung an die Umgebung abhängt. Das Verständnis von der Kopplung an die Umgebung ist daher von großer Bedeutung für das Verständnis der Dekohärenz und des Verhaltens der mechanischen Quantensysteme.

Theoretischer Teil befasst sich mit der Herleitung der Dynamik des mechanischen Oszillators linear gekoppelt an ein allgemeines thermisches Bad. Dabei wird ausgehend vom Caldeira-Leggett Modell Bewegungsgleichung aufgestellt und Spektraldichte des Bades eingeführt. Spektraldichte ist eine zentrale Größe bei der Beschreibung der Umgebung des mechanischen Systems. Es wird Abhängigkeit der messbaren Größen des mechanischen Oszillators von der Spektraldichte untersucht. Weiters wird deren Messprozess behandelt. Dies wird ermöglicht durch Kopplung des Oszillators an ein optisches Laserfeld. Durch Auslesen der optischen Quadraturen kann Dynamik des Oszillators und die Spekraldichte des Bades ausgelesen werden.

Im experimentellen Teil der Diplomarbeit wird die Messung der optischen Quadraturen untersucht. Mit Methoden aus der mathematischen Statistik wird daraus Information über die Spektraldichte der Umgebung des mechanischen Oszillators gewonnen. Das Ergebnis ist eine negative Steigung der Spektraldichte für das konkrete mechanische System.

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1 Introduction

Micromechanical resonators are playing an increasing role in experimental and theoretical quantum physics [8]-[12]. Recent experiments are pushing micromechanical systems more and more towards the quantum regime. Theoretical predictions claim that the behaviour of resonators in the quantum regime crucially depends on their coupling to environment [14]. Therefore the understanding of the environmental coupling is of great importance for understanding of the decoherence and the quantum behaviour of micromechanical systems.

Theoretical part deals with the derivation of the dynamics of the mechanical oscillator linearly coupled to a general thermal bath and discusses the measurement of the mechanical system. Furthermore it provides corrections for wrong theoretical results found in the literature [2], [3], [4]. The experimental part shows how information about the resonator's environment can be obtained.

2 Equation of motion for a harmonic oscillator in a general thermal bath

In this section we find equation of motion for a harmonic oscillator linearly coupled to a general thermal bath. We will see that the dynamics of the mechanical oscillator depends on the coupling to the environment only through its spectral density.

The mathematical describtion of the system is based on the Caldeira-Leggett model [13]. This consists of a particle in a potential and a bath described by a set of harmonic oscillators, where the position of the particle is linearly coupled to the position of each bath oscillator. In our case the potential of the particle is one of a harmonic oscillator. So the total Hamiltonian of the system is [4]:

$$H = \frac{p^2}{2M} + \frac{1}{2}M\Omega^2 q^2 + \sum_n \left(\frac{p_n^2}{2m_n} + \frac{1}{2}m_n\omega_n^2 q_n^2\right) + q\sum_n C_n q_n \tag{2.1}$$

q and p are position and momentum of the particle in a harmonic potential with mass M and bare frequency Ω. This bare frequency is not the resonance frequency of the oscillator in the presence of the bath, as we will see later. q_n and p_n are position and momentum of the nth bath oscillator with mass m_n and frequency ω_n. C_n is coupling strength of each bath oscillator to the particle.

Additionally we make 2 more assumptions: (1) The particle and the thermal bath are initially uncoupled. This means that the whole system is a product state for $t = 0$.

(2) The enviroment is initially in thermal equilibrium at temperature T. This means, that the state of the bath itself is in a product of states of each bath oscillator, where each bath oscillator is in a Gaussian state with:

$$< q_n(0) > = 0 \tag{2.2}$$
$$< p_n(0) > = 0 \tag{2.3}$$
$$< q_n p_n(0) + p_n q_n(0) > = 0 \tag{2.4}$$
$$< q_n^2(0) > = \frac{\hbar}{2m_n\omega_n} coth(\frac{1}{2}\hbar\omega_n\beta) \tag{2.5}$$
$$< p_n^2(0) > = < q_n^2(0) > (m_n\omega_n)^2 \tag{2.6}$$

We want to find the equation of motion for the position operator of the particle. Therefore we first calculate Heisenberg equations of motion for q, p, q_n, p_n by :

$$\dot{q}(t) = \frac{i}{\hbar}[H, q](t) = \frac{p(t)}{M} \tag{2.7}$$

$$\dot{p}(t) = -M\Omega^2 q(t) - \sum_n C_n q_n(t) \tag{2.8}$$

$$\dot{q}_n(t) = \frac{p_n(t)}{m_n} \tag{2.9}$$

$$\dot{p}_n(t) = -m_n \omega_n^2 q_n(t) - C_n q(t) \tag{2.10}$$

Combining both pairs of equations one can eliminate p and p_n:

$$\ddot{q}(t) + \Omega^2 q(t) = -\frac{1}{M}\sum_n C_n q_n(t) \tag{2.11}$$

$$\ddot{q}_n(t) + \omega_n^2 q_n(t) = -\frac{C_n}{m_n} q(t) \tag{2.12}$$

(2.12) as well as (2.11) are second order differential equation describing a driven undamped harmonic oscillator. Solution of (2.12) is:

$$\begin{aligned} q_n(t) &= q_n(0)\cos(\omega_n t) + \frac{p_n(0)}{m_n}\frac{\sin(\omega_n t)}{\omega_n} \\ &\quad - C_n \int_0^t ds \frac{\sin[\omega_n(t-s)]}{\omega_n}\frac{q(s)}{m_n} \end{aligned} \tag{2.13}$$

Inserting (2.13) into (2.11) gives:

$$\ddot{q}(t) + \Omega^2 q(t) + \frac{2}{M}\int_0^t ds \eta(t-s) q(s) = \frac{f(t)}{M} \tag{2.14}$$

with

$$f(t) = -\sum_n C_n \left(q_n(0)\cos(\omega_n t) + \frac{p_n(0)}{m_n}\frac{\sin(\omega_n t)}{\omega_n} \right) \tag{2.15}$$

$$\eta(s) = \frac{d}{ds}\nu(s) \tag{2.16}$$

$$\nu(s) := \int_0^\infty d\omega \frac{I(\omega)}{\omega}\cos(\omega s) \tag{2.17}$$

$$I(\omega) := \sum_n \delta(\omega - \omega_n)\frac{C_n^2}{2m_n\omega_n} \tag{2.18}$$

$I(\omega)$ is the spectral density of the environment. This is the relevant quantity, where the complete information about the coupling of the mechanical system to its environment is contained. As coupling to a thermal bath enters in the equation of motion only though $I(\omega)$.

3 Force correlation function $\langle f(t)f(t')\rangle$

Correlation function of the thermal force acting on the mechanical oscillator is an important quantity for studying the oscillator's environment. The Fourier transform of the force correlation function turns out to be proportional to the spectral density of the thermal bath.

We recall (2.15)

$$f(t) = -\sum_n C_n(q_n(0)\,cos(\omega_n t) + \frac{p_n(0)}{m_n}\frac{sin(\omega_n t)}{\omega_n}) \tag{3.1}$$

Using

$$\langle q_n(0)q_m(0)\rangle = \langle p_n(0)p_m(0)\rangle = \langle q_n(0)p_m(0)\rangle = \langle p_n(0)q_m(0)\rangle = 0$$

for $n \neq m$ one gets

$$\langle f(t)f(t')\rangle =$$

$$\langle\sum_n C_n(q_n(0)\,cos(\omega_n t)+\frac{p_n(0)}{m_n}\frac{sin(\omega_n t)}{\omega_n})\sum_m C_m(q_m(0)\,cos(\omega_m t')+\frac{p_m(0)}{m_m}\frac{sin(\omega_m t')}{\omega_m})\rangle =$$

$$\sum_n C_n^2\langle(q_n(0)\,cos(\omega_n t) + \frac{p_n(0)}{m_n}\frac{sin(\omega_n t)}{\omega_n})(q_n(0)\,cos(\omega_n t') + \frac{p_n(0)}{m_n}\frac{sin(\omega_n t')}{\omega_n})\rangle.$$

Mixed terms containing p and q can be simplified by using trigonometric theorem $sin(x)cos(y) = \frac{1}{2}(sin(x-y)+sin(x+y))$ as well as commutator and anticommutator relations

$$\langle q_n p_n - p_n q_n\rangle = \hbar, \langle q_n p_n + p_n q_n\rangle = 0 \tag{3.2}$$

Terms containing q^2 and p^2 are calculated using $cos(x)cos(y) + sin(x)sin(y) = cos(x-y)$ and

$$\langle q_n^2(0)\rangle = \frac{\langle p_n^2(0)\rangle}{(m_n\omega_n)^2} = \frac{\hbar}{2m_n\omega_n}coth(\frac{1}{2}\hbar\omega_n\beta). \tag{3.3}$$

Thus:

$$\langle f(t)f(t')\rangle = \sum_n c_n^2\frac{\hbar}{2\omega_n m_n}(coth(\frac{1}{2}\hbar\omega_n\beta)cos(\omega_n(t-t')) + \frac{sin(\omega_n(t-t'))}{i})$$

Finally using the definition

$$I(\omega) = \sum_n \frac{c_n^2}{2m_n\omega_n}\delta(\omega - \omega_n) \tag{3.4}$$

we obtain

$$\langle f(t)f(t')\rangle = \int_0^\infty \hbar I(\omega)(coth(\frac{1}{2}\hbar\omega\beta)cos(\omega(t-t')) + \frac{sin(\omega(t-t'))}{i})\, d\omega \quad (3.5)$$

$$= \int_{-\infty}^{\infty} e^{i\omega(t-t')}\frac{\hbar}{2}I(\omega)(\coth(\frac{1}{2}\hbar\omega\beta) - 1)d\omega \quad (3.6)$$

Here we expand the domain of definition of $I(\omega)$ to the whole real axis by: $I(-\omega) := -I(\omega)$.

4 Time-local form of equation of motion

We will see that equation of motion (2.14) of the mechanical oscillator can be written down in a more comfortable form in a steady state. The new form is local in time, as the convolution term in (2.14) will be replaced by a velocity-proportional damping term.

4.1 Derivation of the time-local form of equation of motion

We start with the equation of motion (2.14) for the position operator of the oscillator:

$$L_t q(t) := \ddot{q}(t) + \frac{2}{M}\int_0^t \eta(t-s)q(s)ds + \Omega^2 q(t) = \frac{f(t)}{M}$$

The Green's function of this equation is defined by:

$$L_t G(t,\lambda) := \delta(t-\lambda) \tag{4.1}$$

with

$$G(t,t) := 0 \quad \partial_t G(t,\lambda)|_{\lambda=t} := 1 \tag{4.2}$$

$G(t,\lambda)$ has the form $G(t,\lambda) = G(t-\lambda)\Theta(t-\lambda)$, with $G(t) = G(t,0)$, because:

for $t < \lambda$:

$$L_t[G(t-\lambda)\Theta(t-\lambda)] = 0$$

for $t \geq \lambda$:

$$\begin{aligned}
&L_t[G(t-\lambda)\Theta(t-\lambda)] = \\
&= \ddot{G}(t-\lambda) + \frac{2}{M}\int_0^t \eta(t-s)G(s-\lambda)\Theta(s-\lambda)ds + \Omega^2 G(t-\lambda) = \\
&= \ddot{G}(t-\lambda) + \frac{2}{M}\int_\lambda^t \eta(t-s)G(s-\lambda)ds + \Omega^2 G(t-\lambda) = \\
&= \ddot{G}(t-\lambda)\Theta(t-\lambda) + \frac{2}{M}\int_0^{t-\lambda} \eta(t-\lambda-s')G(s')ds' + \Omega^2 G(t-\lambda) = \\
&= L_t G(t)|_{t\to t-\lambda} = \delta(t-\lambda)
\end{aligned}$$

Notice, that Green function found in [4] is wrong. (See equation (60) in [4], in this reference it is denoted by $G_1(s_1, s_2)$. In particular, apart from some special cases as Ohmic spectral density, it doesn't fulfill $G_1(s_1, s_2) = G_1(s_1 - s_2, 0)$)

Solution of the equation (2.14) is:

$$\begin{aligned} q(t) &= \dot{G}(t)q(0) + G(t)\dot{q}(0) + \int_0^t G(t-s)\frac{f(s)}{M}ds \\ &:= \dot{G}q_0 + G\dot{q}_0 + q_{inh} \end{aligned} \tag{4.3}$$

By deriving this equation with respect to t one gets:

$$\begin{aligned} (I)\; q &= \dot{G}q_0 + G\dot{q}_0 + q_{inh} \\ (II)\; \dot{q} &= \ddot{G}q_0 + \dot{G}\dot{q}_0 + \dot{q}_{inh} \\ (III)\; \ddot{q} &= \dddot{G}q_0 + \ddot{G}\dot{q}_0 + \ddot{q}_{inh} \end{aligned} \tag{4.4}$$

Equations (I) and (II) can be regarded as system of equations in $q_0, \dot{q}_0$. Its solution can be inserted in (III), this eliminates q_0 and $\dot{q}_0$. The result is a differential equation for $q(t)$. In contrast to integrodiffertial equation (2.14), it is just a second order differential equation with time dependent coefficients $\gamma(t)$ and $\Omega'^2(t)$ and a renormalized force $\bar{f}'(t)$:

$$\ddot{q}(t) + \gamma(t)\dot{q}(t) + \Omega'^2(t)q(t) = \frac{\bar{f}'(t)}{M} \tag{4.5}$$

with

$$\gamma(t) = \frac{G\dddot{G} - \dot{G}\ddot{G}}{\dot{G}^2 - G\ddot{G}} \qquad \Omega'^2(t) = \frac{\ddot{G}^2 - \dot{G}\dddot{G}}{\dot{G}^2 - G\ddot{G}} \tag{4.6}$$

$$\bar{f}'(t) = (\partial_t^2 + \gamma(t)\partial_t + \Omega'^2(t))\int_0^t G(t-s)f(s)ds \tag{4.7}$$

This is a very important correction to procedure in [3], where the right hand side of (4.5) remains unnormalized $f(t)/M$, which is true only for the ohmic case. (See equation (2.18) in [3])

4.2 Stationary time-local equation of motion

Now we study the time-local form of equation of motion in a steady state with the assumpion of converging time-local coeffients.

We assume that time dependent coefficients converge to constant values for infinite times, as this assumption is consistent with the experiment:

$$\gamma(t) \to \gamma \qquad \Omega'^2(t) \to \Omega'^2 \tag{4.8}$$

In this case the renormalized force converges to:

$$\bar{f}'(t) \to (\partial_t^2 + \gamma\partial_t + \Omega'^2)\int_{-\infty}^{t} G(t-s)f(s)ds =: f'(t) \tag{4.9}$$

So the time-local equation of motion converges to:

$$\ddot{q}(t) + \gamma\dot{q}(t) + \Omega'^2 q(t) = \frac{f'(t)}{M} \tag{4.10}$$

We are interested in autocorrelation funcion of $f'(t)$. For doing that we regard the fourier transform of (4.9):

$$\begin{aligned} \tilde{f}'(\omega) &= (-\omega^2 + i\gamma\omega + \Omega'^2)\tilde{G}(\omega)\tilde{f}(\omega) \\ &:= A(\omega)\tilde{f}(\omega) \end{aligned} \tag{4.11}$$

We recall (3.6), the correlation function for the original thermal force $f(t)$:

$$\begin{aligned} \langle f(t)f(t')\rangle &= \int_{-\infty}^{\infty} e^{i\omega(t-t')}\frac{\hbar}{2}I(\omega)(\coth(\frac{1}{2}\hbar\omega\beta)-1)d\omega \\ \Rightarrow \langle \tilde{f}(\omega_1)\tilde{f}^*(\omega_2)\rangle &= (2\pi)^2\frac{\hbar}{2}I(\omega)(\coth(\frac{1}{2}\hbar\omega\beta)-1)\delta(\omega_1-\omega_2) \end{aligned} \tag{4.12}$$

In the last step we used the equivalence, which is valid for any time dependent complex operator $F(t)$:

$$\begin{aligned} F(t)F^*(t') &= \int_{-\infty}^{\infty} e^{i\omega(t-t')}c(\omega)d\omega \\ \Leftrightarrow \tilde{F}(\omega_1)\tilde{F}^*(\omega_2) &= (2\pi)^2 c(\omega_1)\delta(\omega_1-\omega_2)\,, \end{aligned} \tag{4.13}$$

where we use the following convention of the fourier transform:

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt \tag{4.14}$$

$$\Rightarrow f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{f}(\omega)e^{i\omega t}d\omega\ . \tag{4.15}$$

Inserting (4.11) into (4.12) we find:

$$\langle \frac{\tilde{f}'(\omega_1)}{A(\omega_1)}\frac{\tilde{f}'^*(\omega_2)}{A^*(\omega_2)}\rangle = (2\pi)^2\frac{\hbar}{2}I(\omega_1)(\coth(\frac{1}{2}\hbar\omega_1\beta) - 1)\delta(\omega_1 - \omega_2) \tag{4.16}$$

$$\Rightarrow \langle \tilde{f}'(\omega_1)\tilde{f}'^*(\omega_2)\rangle = (2\pi)^2|A(\omega_1)|^2\frac{\hbar}{2}I(\omega_1)(\coth(\frac{1}{2}\hbar\omega_1\beta) - 1)\delta(\omega_1 - \omega_2)$$

Using (4.13) again we obtain:

$$\begin{aligned}\langle f'(t)f'(t')\rangle &= \int_{-\infty}^{\infty} |A(\omega)|^2\frac{\hbar}{2}I(\omega)(\coth(\frac{1}{2}\hbar\omega\beta) - 1)e^{i\omega(t-t')} \\ &= \int_{-\infty}^{\infty} ((\Omega'^2 - \omega^2)^2 + \gamma^2\omega^2)|\tilde{G}(\omega)|^2\frac{\hbar}{2}I(\omega)(\coth(\frac{1}{2}\hbar\omega\beta) - 1)e^{i\omega(t-t')}\end{aligned}$$

$\tilde{G}(\omega)$ can be found by performing a Fourier transform on (4.1) for $\lambda = 0$:

$$\tilde{G}(\omega) = \frac{1}{-\omega^2 + \frac{2}{M}\tilde{\eta}(\omega) + \Omega^2} \tag{4.17}$$

,where $\tilde{\eta}(\omega) = \int_{\mathbf{0}}^{\infty} \eta(t)e^{-i\omega t}dt$, as the upper limit of $\int_0^{\mathbf{t}} \eta(t-s)G(s)ds$ is t.

Finally we get:

$$\langle f'(t)f'(t')\rangle = \int_{-\infty}^{\infty} \frac{(\Omega'^2 - \omega^2)^2 + \gamma^2\omega^2}{(\Omega^2 - \omega^2 + \frac{2}{M}Re[\tilde{\eta}(\omega)])^2 + Im[\frac{2}{M}\tilde{\eta}(\omega)]^2}\frac{\hbar}{2}I(\omega)(\coth(\frac{1}{2}\hbar\omega\beta) - 1)e^{i\omega(t-t')}d\omega \tag{4.18}$$

$\langle f'(t)f'(t')\rangle$ can be written in the same form as $\langle f(t)f(t')\rangle$ but with a modified spectral density:

$$\langle f'(t)f'(t')\rangle = \int_{-\infty}^{\infty} \frac{\hbar}{2}I'(\omega)(\coth(\frac{1}{2}\hbar\omega\beta) - 1)e^{i\omega(t-t')}d\omega \tag{4.19}$$

with

$$I'(\omega) = \frac{(\Omega'^2 - \omega^2)^2 + \gamma^2\omega^2}{(\Omega^2 - \omega^2 + \frac{2}{M}Re[\tilde{\eta}(\omega)])^2 + Im[\frac{2}{M}\tilde{\eta}(\omega)]^2} I(\omega) \tag{4.20}$$

The denominator is squared absolute value of a Fourier transform of a real function $G(t)$ and thus symmetric, whereas $I(\omega)$ is asymmetric by definition. Thus $I'(\omega)$ is asymmetric too.

Notice, that γ and Ω'^2 actually depend on $G(t)$ and therefore can be expressed by Ω and $\tilde{\eta}(\omega)$.

4.3 Time-local coefficients

In this part we explicitly calculate the steady-state values of time-dependent time-local coefficients under assumption of a rational Green function in the Laplace space.

According to definition (4.1) $G(t)$ fulfills:

$$\ddot{G}(t) + \frac{2}{M}\int_0^t \eta(t-s)G(s)ds + \Omega^2 G(t) \quad = \quad \delta(t) \tag{4.21}$$

The conditions $G(0) = 0$, $\dot{G}(0) = 1$ implicit $G(0^-) = 0$ and $\dot{G}(0^-) = 0$ (as this gives rise to $\ddot{G}(0) + ... = \delta(0)$).

We use the defintion of Laplace transform:

$$\mathcal{L}\{G\}(s) \quad := \quad \int_{0^-}^{\infty} G(t)e^{-ts}dt \tag{4.22}$$

and the rules:

$$\mathcal{L}\{\dot{G}\}(s) \quad = \quad s\mathcal{L}\{G\} - G(0^-) \tag{4.23}$$

$$\lim_{s->\infty} s\mathcal{L}\{G\}(s) \quad = \quad G(0^+) \tag{4.24}$$

Thus we find:

$$\mathcal{L}\{G\}(s) \quad = \quad \frac{1}{s^2 + \frac{2}{M}\mathcal{L}\{\eta\}(s) + \Omega^2} \tag{4.25}$$

$\mathcal{L}\{G\}(s)$ is real as $G(t)$ is real.

We assume that $G(t)$ is smooth for $t \in [0, \epsilon)$. Then $G(0) = G(0^+) = 0$ and $\dot{G}(0) = \dot{G}(0^+) = 1$ implies:

$$\lim_{s->\infty} s\mathcal{L}\{G\}(s) \quad = \quad 0 \tag{4.26}$$

$$\lim_{s->\infty} s^2\mathcal{L}\{G\}(s) \quad = \quad 1 \tag{4.27}$$

So if $\mathcal{L}\{G\}(s)$ is or can be approximated by a rational function, it has to be of the form:

$$\mathcal{L}\{G\}(s) \quad = \quad \frac{s^n + A_{n-1}s^{n-1} + A_{n-2}s^{n-2} + ...}{s^{n+2} + B_{n+1}s^{n+1} + B_n s^n + ...} \tag{4.28}$$

with real coefficients A_n and B_n. The last fact implies that roots of both polynomials are either real or for each complex root there exists a complex conjugate root.

According to the partial fraction decomposition theorem, any rational function $f : \mathbb{C} \to \mathbb{C}$ with poles x_i ($i = 1, 2, ..., n$) of the order r_i can be written in the form:

$$f(s) \quad = \quad \sum_{i=1}^{n} \sum_{j=1}^{r_i} \frac{a_{ij}}{(s - x_i)^j} + polynomial(s) \tag{4.29}$$

So $\mathcal{L}\{G\}(s)$ can be written in the form:

$$\mathcal{L}\{G\}(s) \quad = \quad \sum_{i=1}^{n} \sum_{j=1}^{r_i} \frac{a_{ij}}{(s - x_i)^j} \tag{4.30}$$

Note that $polynomial(s) \equiv 0$ due to $\lim_{s->\infty} s\mathcal{L}\{G\}(s) = 0$. Now we know the shape of $G(t)$

$$\mathcal{L}\{\frac{t^n}{n!}e^{-at}\}(s) \quad = \quad \frac{1}{(s+a)^{n+1}} \tag{4.31}$$

$$\Rightarrow G(t) \quad = \quad \sum_{i=1}^{n} \sum_{j=1}^{r_i} a_{ij} \frac{t^{j-1}}{(j-1)!} e^{x_i t} \tag{4.32}$$

G(t) equals the expectation value of the position $< q(t) >$ of the particle if an external force $f(t) = \delta(t)$ is acting on it. In our case, where the particle cannot "diffuse away" under the influence of a singe deltalike kick, it is clear that

$$Re[x_i] \quad < \quad 0 \tag{4.33}$$

Let the pair $\{-\beta, -\delta\}$ be the two real parts of the poles, which are closest to 0. Then:

$$G(t) \quad = \quad f_0(t)e^{-\beta t} + g_0(t)e^{-\delta t} + O(e^{-\kappa t}) \qquad \kappa > \delta > \beta \tag{4.34}$$

$$\dot{G}(t) \quad = \quad f_1(t)e^{-\beta t} + g_1(t)e^{-\delta t} + O(e^{-\kappa t}) \tag{4.35}$$

$$\ldots \tag{4.36}$$

$$\dddot{G}(t) \quad = \quad f_3(t)e^{-\beta t} + g_3(t)e^{-\delta t} + O(e^{-\kappa t}) \tag{4.37}$$

with some real functions $f_{0,1,2,3}$ and $g_{0,1,2,3}$, where $f_{1,2,3}$ and $g_{1,2,3}$ can be expressed by f_0 and g_0 respectively.

We recall (4.6):

$$\gamma(t) = \frac{G\dddot{G} - \dot{G}\ddot{G}}{\dot{G}^2 - G\ddot{G}} \tag{4.38}$$

$$= \frac{(f_0f_3 - f_1f_2)e^{-2\beta t} + (f_0g_3 + f_3g_0 - f_1g_2 - f_2g_1)e^{-(\beta+\delta)t} + ...}{(f_1^2 - f_0f_2)e^{-2\beta t} + (2f_1g_1 - f_0g_2 - f_2g_0)e^{-(\beta+\delta)t} + ...} \tag{4.39}$$

Now we distinguish two general cases:

I). There exists more than one pole x_i with $Re[x_i] = -\beta$ or at least one pole x_i with $r_i > 1$. So the poles of $\mathcal{L}\{G\}(s)$ can be written down as:

$$\{x_i\} = \{-\beta + i\omega_1, -\beta + i\omega_2, -\beta + i\omega_3, ... , -\delta + i\alpha_1, ...\} \tag{4.40}$$

Furthermore we regard 3 more distinctions of cases:

a).There exist ω_i and ω_j with $|\omega_i| \neq |\omega_j|$.
In this case $\gamma(t)$ and $\Omega'^2(t)$ do not converge. It can be checked easily with exatly the same technique, that I will use to calculate the values of $\gamma(\infty)$ and $\Omega'^2(\infty)$ for the next case.

b).Imaginary parts of all poles $\{x_i\}$ with $Re[x_i] = -\beta$ have the same absolut value $\omega \neq 0$.

$$\Rightarrow f_0(t) = \sum_n c_n t^n e^{i\omega t} + \sum_n c_n^* t^n e^{-i\omega t} \quad c_i \in \mathbb{C} \tag{4.41}$$

$$= \sum_n d_n t^n \sin(\omega + \phi_n) \quad d_n \in \mathbb{R} \tag{4.42}$$

$$= d_n t^n \sin(\omega t + \phi_n) + o(t^n) \tag{4.43}$$

Using (4.34)-(4.37) one can calculate f_1,f_2 and f_3. The results lead to:

$$(f_0f_3 - f_1f_2)(t) = A\ t^{2n}2\beta\omega^2 + o(t^{2n}) \tag{4.44}$$

$$(f_1^2 - f f_2)(t) = A\ t^{2n}\omega^2 + o(t^{2n}) \tag{4.45}$$

with some constant factor A.

By inserting these equations into (4.39) we obtain:

$$\gamma(t) = \frac{(t^{2n}2\beta\omega^2 + o(t^{2n}))e^{-2\beta t} + O(e^{-(\beta+\delta)t})}{(t^{2n}\omega^2 + o(t^{2n}))e^{-2\beta t} + O(e^{-(\beta+\delta)t})} \tag{4.46}$$

$$= \frac{(2\beta\omega^2 + o(1)) + O(e^{-(\delta-\beta)t})}{(\omega^2 + o(1)) + O(e^{-(\delta-\beta)t})} \tag{4.47}$$

$$\Rightarrow \gamma =: \gamma(\infty) \tag{4.48}$$

$$= 2\beta \tag{4.49}$$

In the same way one can calculate:

$$\Omega'^2 =: \Omega'^2(\infty) \tag{4.50}$$

$$= \beta^2 + \omega^2 \tag{4.51}$$

c).In the third case we consider $\omega_i = 0$.

$$\Rightarrow f_0(t) = c_n t^n + o(t^n) \tag{4.52}$$

with $n \geq 1$ as $r_i > 1$.

$$\Rightarrow (f_0 f_3 - f_1 f_2)(t) = B\ 2nt^{2n-2}\beta + o(t^{2n-2}) \tag{4.53}$$

$$(f_1^2 - f_0 f_2)(t) = B\ nt^{2n-2} \tag{4.54}$$

for some constant B.
The time-local coefficients converge to:

$$\Rightarrow \gamma = 2\beta \tag{4.55}$$

$$\Omega'^2 = \beta^2 \tag{4.56}$$

II). There exist only one pole x_i with $Re[x_i] = -\beta$ with $r_i = 1$. So the poles of $\mathcal{L}\{G\}(s)$ can be written down as:

$$\{x_i\} = \{-\beta, -\delta + i\alpha_1, -\delta + i\alpha_2, ...\} \tag{4.57}$$

It can be shown, that the time-local coefficients converge only in the case if $|\alpha_j| = 0 \; \forall j$.

In this case $f_0(t)$ is a constant and it turns out, that:

$$(f_0 f_3 - f_1 f_2)(t) = (f_1^2 - f_0 f_2)(t) \equiv 0 \tag{4.58}$$

Furthermore:

$$\begin{aligned} g_0(t) &= c_n t^n + o(t^n) && (4.59) \\ (f_0 g_3 + f_3 g - f_1 g_2 - f_2 g_1)(t) &= C\, t^n (\beta - \delta)^2 (\beta + \delta) + o(t^n) && (4.60) \\ (2 f_1 g_1 - f_0 g_2 - f_2 g_0)(t) &= C\, t^n (\beta - \delta)^2 + o(t^n) && (4.61) \end{aligned}$$

By inserting (4.58), (4.60) and (4.61) into (4.39) we obtain:

$$\begin{aligned} \gamma &= \beta + \delta && (4.62) \\ \Omega'^2 &= \beta\delta && (4.63) \end{aligned}$$

Summing up all cases we can find a simple rule for calculating time-local coefficients:

If $\{x_i\}$ is list of the poles of $\mathcal{L}\{G\}(s)$, where each x_i appears r_i times, and $\{a, b\}$ are elements of the list with the biggest real part, then:

$$\begin{aligned} \gamma &= -Re[a + b] && (4.64) \\ \Omega'^2 &= |ab| && (4.65) \end{aligned}$$

This result for Ω'^2 is different from that in [2], where Ω'^2 is supposed to be $-\frac{2}{M}\int_0^\infty \frac{I(\omega)}{\omega} d\omega + \Omega^2$. This is only true in the weak coupling limit, but not generally, as can be checked numerically. (See (2.44) and (2.45) and the text below in [2])

4.4 Time-local coefficients in the weak-coupling regime

In the previous subsection we have found, that the time-local coefficients for infinite times γ and Ω'^2 are given by the 2 poles of $\mathcal{L}\{G\}(s)$ with the biggest real part.
Now we try to find a simple approximated formula for the relevant poles and therefore for γ and Ω'^2 in the case, where the coupling of the harmonic oscillator to the thermal bath is small, and give the conditions where this approximation applies.

Using the definition of $\mathcal{L}\{G\}(s)$ we obtain:

$$\begin{aligned}\mathcal{L}\{G\}(s)^{-1} &= s^2 + \frac{2}{M}\mathcal{L}\{\eta\}(s) + \Omega^2 &(4.66)\\ &= s^2 + \frac{2}{M}s\mathcal{L}\{\nu\}(s) - \frac{2}{M}\nu(0) + \Omega^2 &(4.67)\end{aligned}$$

In the last step we used (2.16).

We define

$$\sqrt{-\frac{2}{M}\nu(0) + \Omega^2} =: K \tag{4.68}$$

and x a pole of $\mathcal{L}\{\nu\}(s)$.

Now we expand $\mathcal{L}\{\nu\}(s)$ around iK', with K' being a guess for the imaginary part of x, with $K' \in \mathbb{R}$, $\delta \in \mathbb{C}$:

$$\mathcal{L}\{\nu\}(iK' + \delta) = \mathcal{L}\{\nu\}(iK') + \partial_s\mathcal{L}\{\nu\}(s)|_{s=iK'}\,\delta + \partial_s^2\mathcal{L}\{\nu\}(s)|_{s=iK'}\frac{\delta^2}{2} + ... \tag{4.69}$$

If

$$\mathcal{L}\{\nu\}(iK') + \partial_s\mathcal{L}\{\nu\}(s)|_{s=iK'}\,\delta \gg \partial_s^2\mathcal{L}\{\nu\}(s)|_{s=iK'}\frac{\delta^2}{2} + ... \tag{4.70}$$

in the intervall for $\delta = s - iK$, where $|s^2 + K^2| \ngtr |\frac{2}{M}s\mathcal{L}\{\nu\}(s)|$ (i.e. the terms are comparable and therefore roots can occur), its only the first two terms in the taylor expansion that matter.

To find a pole of $\mathcal{L}\{G\}(s)$ we insert the linearized $\mathcal{L}\{\nu\}(iK' + \delta)$ into (4.67) and set the resulting expression to 0:

$$
\begin{aligned}
(iK'+\delta)^2 + \frac{2}{M}(iK'+\delta)\Big(\mathcal{L}\{\nu\}(iK') + \partial_s\mathcal{L}\{\nu\}(s)|_{s=iK'}\,\delta\Big) + K^2 &= 0 \quad (4.71)\\
\Leftrightarrow \frac{2}{M}iK'\mathcal{L}\{\nu\}(iK') + K^2 - K'^2 & \\
+\delta\Big(2iK' + \frac{2}{M}iK'\partial_s\mathcal{L}\{\nu\}(s)|_{s=iK'} + \frac{2}{M}\mathcal{L}\{\nu\}(iK')\Big) + & \\
+\delta^2\Big(1 + \frac{2}{M}\partial_s\mathcal{L}\{\nu\}(s)|_{s=iK'}\Big) &= 0 \quad (4.72)
\end{aligned}
$$

Under conditions:

$$
\begin{aligned}
|\frac{2}{M}\partial_s\mathcal{L}\{\nu\}(s)|_{s=iK'}| &\ll 1 \quad (4.73)\\
|\frac{1}{M}\mathcal{L}\{\nu\}(iK')| &\ll K' \quad (4.74)
\end{aligned}
$$

(4.72) simplifies to:

$$
K^2 - K'^2 + \frac{2}{M}iK'Re[\mathcal{L}\{\nu\}(iK')] + \delta\Big(2iK' + \frac{2}{M}Re[\mathcal{L}\{\nu\}(iK')]\Big) + \delta^2 = 0 \quad (4.75)
$$

$$
\begin{aligned}
\Rightarrow \delta_{1,2} &= -iK' - \frac{1}{M}\mathcal{L}\{\nu\}(iK') \pm \sqrt{-K^2 + (\frac{1}{M}\mathcal{L}\{\nu\}(iK'))^2} \quad (4.76)\\
&\approx -i(K' \mp K) - \frac{1}{M}\mathcal{L}\{\nu\}(iK') \quad (4.77)
\end{aligned}
$$

To be precise we also assume that:

$$
|\frac{1}{M}\mathcal{L}\{\nu\}(iK')| \ll K \quad (4.78)
$$

So the first guess for two poles $x_{1,2}$ of $\mathcal{L}\{\nu\}(s)$ is:

$$
\begin{aligned}
x_{1,2} &\approx \pm iK - \frac{1}{M}\mathcal{L}\{\nu\}(iK') \quad (4.79)\\
&\approx \pm iK - \frac{1}{M}Re[\mathcal{L}\{\nu\}(iK')] \quad (4.80)
\end{aligned}
$$

Now we can find a better approximation for $x_{1,2}$, if we repeat the procedure from step (4.69) but expanding $\mathcal{L}\{\nu\}(s)$ around iK. The result is:

$$x_{1,2} \approx \pm iK - \frac{1}{M} Re[\mathcal{L}\{\nu\}(iK)]. \tag{4.81}$$

Untill now we do not know if the poles $x_{1,2}$, the approximation to which we have found, are poles with the biggest real part, which we denote by $x_{max1,2}$:

If $Im[x_{max1,2}] \approx \pm K$, it could still be, that there is another pole with the same imaginary part but with smaller real part and therefore not the pole of interest. However the approximation of (4.69), which we insert in (4.67) to calculate δ, works the better the smaller $|\delta|$. So the resulting δ is an approxamtion for the real part of the pole with the real part closest to 0. Because real part of all poles is negative (otherwise there would be no damping), this pole is one with the biggest real part.

If $Im[x_{max1,2}] \approx \pm K' \not\approx \pm K$ and we would try to find an approximation for the real part, we would get $\delta_{1,2} \approx -i(K' \mp K) - \frac{1}{M} Re[\mathcal{L}\{\nu\}(iK')]$, which contains a correction for the imaginary part. This means that $\pm K' \not\approx Im[x_{max1,2}]$.

Of course these arguments work only if the linearisation of (4.69) around $iIm[x_{max}]$ for $\delta = Re[x_{max}]$ is a good approximation.

Summing up we can state, that if conditions characterizing the smallness of $I(\omega)$, namely (4.70), (4.73), (4.74) and (4.78) hold, poles with the biggest real part are given by:

$$x_{max1,2} \approx \pm iK - \frac{1}{M} Re[\mathcal{L}\{\nu\}(iK)] \tag{4.82}$$

and the time-local coefficients at infinity are therefore:

$$\gamma \approx \frac{2}{M} Re[\mathcal{L}\{\nu\}(iK)] \tag{4.83}$$

$$\Omega'^2 \approx K^2 \tag{4.84}$$

We can also express there coefficients through $I(\omega)$ and the bare oscillator frequency Ω. Using (2.17) we find:

$$Re[\mathcal{L}\{\nu\}(iK)] = Re[\int_0^\infty \nu(s)e^{-iKs}ds] \tag{4.85}$$

$$= \int_0^\infty \nu(s)\cos(Ks)ds \tag{4.86}$$

$$= \int_0^\infty \int_0^\infty \frac{I(\omega)}{\omega}\cos(\omega s)\cos(Ks)d\omega ds \tag{4.87}$$

$$= \int_0^\infty \int_0^\infty \frac{I(\omega)}{2\omega}(\cos(s(\omega+K)) + \cos(s(\omega-K)))d\omega ds$$

$$= \int_0^\infty \frac{\pi I(\omega)}{2\omega}(\delta(\omega+K) + \delta(\omega-K))d\omega \tag{4.88}$$

$$= \frac{\pi I(K)}{2K} \tag{4.89}$$

$$\Rightarrow \gamma \approx \frac{\pi I(K)}{MK} \tag{4.90}$$

Now we use (4.68):

$$K^2 = -\frac{2}{M}\nu(0) + \Omega^2 \tag{4.91}$$

$$= -\frac{2}{M}\int_0^\infty \frac{I(\omega)}{\omega}d\omega + \Omega^2 \tag{4.92}$$

$$\Rightarrow \Omega'^2 \approx -\frac{2}{M}\int_0^\infty \frac{I(\omega)}{\omega}d\omega + \Omega^2 \tag{4.93}$$

4.5 Renormalized force correlation function $\langle f'(t)f'(t')\rangle$ in the weak coupling regime

We recall (4.19):

$$\langle f'(t)f'(t')\rangle = \int_{-\infty}^{\infty} \frac{\hbar}{2} I'(\omega)(\coth(\frac{1}{2}\hbar\omega\beta) - 1)e^{i\omega(t-t')}d\omega \qquad (4.94)$$

with

$$I'(\omega) = \frac{(\Omega'^2 - \omega^2)^2 + \gamma^2\omega^2}{(\Omega^2 - \omega^2 + \frac{2}{M}Re[\tilde{\eta}(\omega)])^2 + Im[\frac{2}{M}\tilde{\eta}(\omega)]^2} I(\omega) \qquad (4.95)$$

$$= \frac{(\Omega'^2 - \omega^2)^2 + \gamma^2\omega^2}{(K^2 - \omega^2 - \omega\frac{2}{M}Im[\tilde{\nu}(\omega)])^2 + \omega^2 Re[\frac{2}{M}\tilde{\nu}(\omega)]^2} I(\omega) \qquad (4.96)$$

We want to find out under which conditions

$$\frac{(\Omega'^2 - \omega^2)^2 + \gamma^2\omega^2}{(K^2 - \omega^2 - \omega\frac{2}{M}Im[\tilde{\nu}(\omega)])^2 + \omega^2 Re[\frac{2}{M}\tilde{\nu}(\omega)]^2} \approx 1 \qquad (4.97)$$

The argumentation and conditions are the same as in the previous subsection:

If $\frac{2}{M}\tilde{\nu}(\omega)$ is much smaller and varies slower then K for ω in the region around K, its only the value of $\tilde{\nu}(\omega)$ for $\omega = K$ which matters. More precisely:

$$|\frac{2}{M}\partial_\omega \tilde{\nu}(\omega)|_{\omega=K}| \ll 1 \qquad (4.98)$$

$$|\frac{1}{M}\tilde{\nu}(K)| \ll K \qquad (4.99)$$

and the linearization of $\tilde{\nu}(\omega)$ around $\omega = K$

$$\tilde{\nu}(\omega) \approx \tilde{\nu}(K) + \partial_\omega \nu(\omega)|_{\omega=K}(\omega - K) \qquad (4.100)$$

should be valid in the region, where $|K^2 - \omega^2| \not\gg |\omega\frac{2}{M}\tilde{\nu}(\omega)|$.

Then with exactly the same argumentation as in the previous subsection one can apply the following approximations:

$$\begin{aligned}
(K^2-\omega^2-\omega\frac{2}{M}Im[\tilde{\nu}(\omega)])^2+\omega^2 Re[\frac{2}{M}\tilde{\nu}(\omega)]^2 &\approx (K^2-\omega^2-\omega\frac{2}{M}Im[\tilde{\nu}(K)])^2+\omega^2 Re[\frac{2}{M}\tilde{\nu}(K)]^2 \\
&\approx (K^2-\omega^2-K\frac{2}{M}Im[\tilde{\nu}(K)])^2+\omega^2 Re[\frac{2}{M}\tilde{\nu}(K)]^2 \\
&\approx (K^2-\omega^2)^2+\omega^2 Re[\frac{2}{M}\tilde{\nu}(K)]^2
\end{aligned}$$

Finally using (4.83) and (4.84) we find, that the renormalized spectral density is well approximated by the original one:

$$\begin{aligned}
(K^2-\omega^2-\omega\frac{2}{M}Im[\tilde{\nu}(\omega)])^2+\omega^2 Re[\frac{2}{M}\tilde{\nu}(\omega)]^2 &\approx (\Omega'^2-\omega^2)^2+\omega^2\gamma^2 \\
\Rightarrow I'(\omega) &\approx I(\omega) \qquad (4.101)
\end{aligned}$$

For the correlation function of the renormalized thermal force this means by comparing (4.19) with (3.6):

$$\langle f'(t)f'(t)\rangle \approx \langle f(t)f(t)\rangle \qquad (4.102)$$

5 Mean square steady-state displacement of a harmonic oscillator coupled to a thermal bath

We assume that $\gamma(t)$ and $\Omega'^2(t)$ converge to constant values γ and Ω'^2 for $t \to \infty$:

$$\ddot{q}(t) + \Omega'^2(t)q(t) + \gamma(t)\dot{q}(t) = \frac{f'(t)}{M} \tag{5.1}$$

$$\xrightarrow{t\to\infty} \ddot{q}(t) + \Omega'^2 q(t) + \gamma\dot{q}(t) = \frac{f'(t)}{M} \tag{5.2}$$

First we solve (5.2) for $q(t)$ using the Fourier transform of the equation:

$$\tilde{q}(\omega) = \frac{\tilde{f}'(\omega)}{M}\frac{1}{\Omega'^2 - \omega^2 + i\gamma\omega} \tag{5.3}$$

$$\Rightarrow q(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\tilde{f}'(\omega)}{M}\frac{e^{i\omega t}}{\Omega'^2 - \omega^2 + i\gamma\omega}\,d\omega \tag{5.4}$$

$$\Rightarrow \langle q(t)q(t')\rangle = \frac{1}{(2\pi)^2}\iint\frac{\langle\tilde{f}'(\omega)\tilde{f}'(\omega')\rangle e^{i(\omega t+\omega' t')}}{M^2(\Omega'^2 - \omega^2 + i\gamma\omega)(\Omega'^2 - \omega'^2 + i\gamma\omega')}\,d\omega\,d\omega' \tag{5.5}$$

Now we have to find the formula for $\langle\tilde{f}'(\omega)\tilde{f}'(\omega')\rangle$:

$$\begin{aligned}\langle\tilde{f}'(\omega)\tilde{f}'(\omega')\rangle &= \langle\int_{-\infty}^{\infty} f'(t)e^{-i\omega t}\,dt\int_{-\infty}^{\infty} f'(t')e^{-i\omega' t'}\,dt'\rangle \\ &= \iint\langle f'(t)f'(t')\rangle e^{-i(\omega t+\omega' t')}\,dt\,dt' \end{aligned} \tag{5.6}$$

Force correlation function can be written in the following way:

$$\begin{aligned}\langle f'(t)f'(t')\rangle &= \int_0^{\infty} I'(\bar{\omega})\hbar(\coth(\frac{1}{2}\hbar\bar{\omega}\beta)\cos(\bar{\omega}(t-t')) + \frac{\sin(\bar{\omega}(t-t'))}{i})\,d\bar{\omega} \\ &= \int_0^{\infty} a(\bar{\omega})\cos(\bar{\omega}(t-t')) + ib(\bar{\omega})\sin(\bar{\omega}(t-t'))\,d\bar{\omega} \\ &= \int_{-\infty}^{\infty}\frac{a(\bar{\omega}) + b(\bar{\omega})}{2}e^{i\bar{\omega}(t-t')}d\bar{\omega}. \end{aligned} \tag{5.7}$$

In the last step we extend the definition area of $a(\bar{\omega})$ and $b(\bar{\omega})$ to the whole real axis with the properties: $a(-\bar{\omega}) = a(\bar{\omega})$ and $b(-\bar{\omega}) = -b(\bar{\omega})$. Now we insert (5.7) in (5.6):

$$
\begin{aligned}
\langle \tilde{f}'(\omega)\tilde{f}'(\omega')\rangle &= \iiint \frac{a(\bar{\omega})+b(\bar{\omega})}{2} e^{i\bar{\omega}(t-t')} e^{-i(\omega t+\omega' t')}\, dt\, dt'\, d\bar{\omega} \\
&= (2\pi)^2 \iiint \frac{a(\bar{\omega})+b(\bar{\omega})}{2} \frac{1}{2\pi} e^{it(\bar{\omega}-\omega)} \frac{1}{2\pi} e^{it'(-\bar{\omega}-\omega')}\, dt\, dt'\, d\bar{\omega} \\
&= (2\pi)^2 \int \frac{a(\bar{\omega})+b(\bar{\omega})}{2} \delta(\bar{\omega}-\omega)\, \delta(\bar{\omega}+\omega')\, d\bar{\omega} \\
&= (2\pi)^2 \frac{a(\omega)+b(\omega)}{2} \delta(\omega+\omega') \qquad (5.8)
\end{aligned}
$$

Substituting (5.8) in (5.5) we obtain:

$$
\begin{aligned}
\langle q(t)q(t')\rangle &= \iint \frac{a(\omega)+b(\omega)}{2} \delta(\omega+\omega') \frac{e^{i(\omega t+\omega' t')}}{M^2(\Omega'^2-\omega^2+i\gamma\omega)(\Omega'^2-\omega'^2+i\gamma\omega')}\, d\omega\, d\omega' \\
&= \int \frac{a(\omega)+b(\omega)}{2} \frac{e^{i\omega(t-t')}}{M^2(\Omega'^2-\omega^2+i\gamma\omega)(\Omega'^2-\omega^2-i\gamma\omega)}\, d\omega \\
&= \int_{-\infty}^{\infty} \frac{a(\omega)+b(\omega)}{2M^2} \frac{e^{i\omega(t-t')}}{(\Omega'^2-\omega^2)^2+(\gamma\omega)^2}\, d\omega \\
&= \frac{1}{M^2} \int_{-\infty}^{\infty} \frac{I'(\omega)\hbar\left(\coth(\frac{\hbar\omega\beta}{2})-1\right)}{(\Omega'^2-\omega^2)^2+(\gamma\omega)^2} e^{i\omega(t-t')}\, d\omega, \qquad (5.9)
\end{aligned}
$$

By inserting (4.20) into (5.9), one gets a general expression for $\langle q(t)q(t')\rangle$ in a steady state, which holds even if $\gamma(t)$ and $\Omega'^2(t)$ do not converge. It can be derived directly from (2.14):

$$
\langle q(t)q(t')\rangle = \frac{1}{M^2} \int_{-\infty}^{\infty} \frac{I(\omega)\hbar\left(\coth(\frac{\hbar\omega\beta}{2})-1\right)}{(\Omega^2-\omega^2+\frac{2}{M}Re[\tilde{\eta}(\omega)])^2+Im[\frac{2}{M}\tilde{\eta}(\omega)]^2} e^{i\omega(t-t')}\, d\omega \qquad (5.10)
$$

6 Mean square steady-state displacement of a harmonic oscillator coupled to laser and a thermal bath

Now we consider a system where the harmonic oscillator described above is coupled to a laser field within an optical cavity. This can be realized as follows:

A vibrational mode of a high reflective micromirror is modeled as harmonic oscillator. The micromirror together with another solid mirror forms an optical cavity, which is driven by a laser beam.

The total Hamiltonian of a harmonic oscillator coupled to thermal bath and laser field within a driven optical cavity is [1]:

$$\begin{aligned} H &= \frac{p^2}{2M} + \frac{1}{2}M\Omega^2 q^2 + \sum_n \left(\frac{p_n^2}{2m_n} + \frac{1}{2}m_n\omega_n^2 q_n^2\right) + q\sum_n C_n q_n \\ &\quad + \hbar\omega_c a^\dagger a - \hbar g_0 a^\dagger a q + i\hbar E(a^\dagger e^{-i\omega_0 t} - a e^{i\omega_0 t}) \end{aligned} \tag{6.1}$$

a and $a^\dagger$ are annihilation and creation operators of the laser field,
$g_0 = \omega_c/L$ is coupling constant of the mechanics to the laser field, where ω_c is the resonance frequency of the cavity with length L and decay rate κ,
$|E| = \sqrt{2P\kappa/\hbar\omega_0}$ where P is the input power of laser with frequency ω_0.

Heisenberg equations of motion written in the interaction picture with respect to $\hbar\omega_0 a^\dagger a$ are:

$$\dot{q} = \frac{p}{M} \tag{6.2}$$

$$\dot{p} = -M\Omega^2 q - \sum_n C_n q_n + \hbar g_0 a^\dagger a \tag{6.3}$$

$$\dot{a} = -(\kappa + i\Delta_0)a + i g_0 a q + E + \sqrt{2\kappa}a^{in} \tag{6.4}$$

$$\dot{q}_n = \frac{p_n}{m_n} \tag{6.5}$$

$$\dot{p}_n = -m_n\omega_n^2 q_n - C_n q \tag{6.6}$$

with $\Delta_0 = \omega_c - \omega_0$.

Exactly as in section (2) by solving the last two equations we can find $q_n(q(t))$ and insert it in equation for $\dot{p}$. The result is:

$$\begin{aligned}\ddot{q}(t) + \Omega^2 q(t) + \frac{2}{M}\int_0^t ds\eta(t-s)q(s) &= \frac{f(t)}{M} + \frac{\hbar g_0}{M}a^\dagger a \\ \dot{a} &= -(\kappa + i\Delta_0)a + ig_0 aq + E + \sqrt{2\kappa}a^{in}\end{aligned} \tag{6.7}$$

If we are interested in a time-local form we can proceed exactly as in subsection (4.1) but substituting $\frac{f(t)}{M} + \frac{\hbar g_0}{M}a^\dagger a$ for $\frac{f(t)}{M}$:

$$\begin{aligned}(6.7) \Leftrightarrow \ddot{q}(t) + \gamma(t)\dot{q}(t) + \Omega'^2(t)q(t) = \\ = \frac{\bar{f}'(t)}{M} + (\partial_t^2 + \gamma(t)\partial_t + \Omega'^2(t))\int_0^t G(t-s)\frac{\hbar g_0}{M}a^\dagger(s)a(s)ds\end{aligned} \tag{6.8}$$

Note, that the Green function $G(t)$ and therefore $\gamma(t)$ and $\Omega'^2(t)$ are not affected by the presence of the coupling to the light field.

So in case time-local coefficients converge, the entire optomechanical system for $t \to \infty$ is described by:

$$\begin{aligned}\ddot{q}(t) + \gamma\dot{q}(t) + \Omega'^2 q(t) = \\ = \frac{f'(t)}{M} + (\partial_t^2 + \gamma\partial_t + \Omega'^2)\int_0^t G(t-s)\frac{\hbar g_0}{M}a^\dagger(s)a(s)ds\end{aligned} \tag{6.9}$$

$$\dot{a}(t) = -(\kappa + i\Delta_0)a(t) + ig_0 a(t)q(t) + E + \sqrt{2\kappa}a^{in}(t) \tag{6.10}$$

The term $(\partial_t^2 + \gamma\partial_t + \Omega'^2)\int_0^t G(t-s)\frac{\hbar g_0}{M}a^\dagger(s)a(s)ds$ in (6.9) describes modified radiation pressure force. I want to consider two special cases:

1). If the coupling of the mechanics to the laser field is small and therefore radiation pressure effects can be neglected, this term can be ignored completely. Then the dynamics of the mechanics is independent of that of the optics and can be solved as described in previous sections. Equation (6.10) then describes a cavity mode which acts as a readout apparatus.

2). Untill the end of the section we will deal with the second case: arbitrary optomechanical coupling but weak coupling of the mechanical oscillator to the thermal bath, described by conditions in subsection (4.5). Under those the approximation is true:

$$\tilde{G}(\omega) \quad \approx \quad \frac{1}{-\omega^2 + i\omega\gamma + \Omega'^2} \tag{6.11}$$

and therefore for any function or operator $F(t)$ (given for $t \in [0, \infty]$):

$$(-\omega^2 + i\omega\gamma + \Omega'^2)\tilde{G}(\omega)\tilde{F}(\omega) \quad \approx \quad \tilde{F}(\omega) \tag{6.12}$$

$$\Leftrightarrow (\partial_t^2 + \gamma\partial_t + \Omega'^2)\int_0^t G(t-s)F(s)ds \quad \approx \quad F(t) \tag{6.13}$$

So in the bath-weak-coupling regime equations (6.9) and (6.10) turn into:

$$\ddot{q} \quad = \quad -\gamma\dot{q} - \Omega'^2 q + \frac{f(t)}{M} + \frac{\hbar g_0}{M}a^\dagger a \tag{6.14}$$

$$\dot{a} \quad = \quad -(\kappa + i\Delta_0)a + ig_0 aq + E + \sqrt{2\kappa}a^{in}(t) \tag{6.15}$$

From now on I will write all formulas in dimensionless units in order to make it easier to compare the results with those of [1]. Dimensionless units are defined by:

$$\begin{aligned} q_{dimensionless} &:= \frac{q}{l} \\ p_{dimensionless} &:= \frac{p\, l}{\hbar} \\ l &:= \sqrt{\frac{\hbar}{M\,\omega_m}} \end{aligned} \tag{6.16}$$

$$\Rightarrow [q_{dimensionless}, p_{dimensionless}] = i \tag{6.17}$$

and define:

$$\omega_m \quad := \quad \Omega' \tag{6.18}$$

$$G_0 \quad = \quad g_0 l \tag{6.19}$$

$$\xi(t) \quad = \quad f(t)\frac{l}{\hbar} \tag{6.20}$$

The equations of motion in dimensionless units are (omitting the subindex "dimensionless"):

$$\begin{aligned}
\dot{q} &= \omega_m p && (6.21)\\
\dot{p} &= -\gamma p - \omega_m q + \xi(t) + G_0 a^\dagger a && (6.22)\\
\dot{a} &= -(\kappa + i\Delta_0)a + iG_0 aq + E + \sqrt{2\kappa}a^{in}(t) && (6.23)
\end{aligned}$$

Comparing this set of equations with (2a)-(2c) in [1], the only difference lies in $\xi(t)$ which here describes thermal force of a general thermal bath as opposed to an Ohmic one in the mentioned paper.

So we can proceed exactly as in [1] by calculating mean values in a steady state:

$$\begin{aligned}
q_s &:= \langle q(t)\rangle_{t\to\infty} = \frac{G_0|\alpha_s|^2}{\omega_m} && (6.24)\\
\alpha_s &:= \langle a(t)\rangle_{t\to\infty} = \frac{E}{\kappa + i\Delta} && (6.25)
\end{aligned}$$

$$\langle p(t)\rangle_{t\to\infty} = 0 \qquad (6.26)$$

with

$$\Delta = \Delta_0 - \frac{G_0^2|\alpha_s|^2}{\omega_m} \qquad (6.27)$$

Then equations of motion for $\delta q := q - q_s$, $\delta p := p$, $\delta a := a - \alpha_s$ can be linearized under condition $|\alpha_s| \gg 1$ [1]:

$$\begin{aligned}
\delta\dot{q} &= \omega_m \delta p && (6.28)\\
\delta\dot{p} &= -\omega_m \delta q - \gamma\delta p + G\delta X + \xi && (6.29)\\
\delta\dot{X} &= -\kappa\delta X + \Delta\delta Y + \sqrt{2\kappa}X^{in} && (6.30)\\
\delta\dot{Y} &= -\kappa\delta Y - \Delta\delta X + G\delta q + \sqrt{2\kappa}Y^{in} && (6.31)
\end{aligned}$$

with $\delta X \equiv (\delta a + \delta a^\dagger)/\sqrt{2}$, $\delta Y \equiv (\delta a - \delta a^\dagger)/i\sqrt{2}$ and $X^{in} = (a^{in} + a^{in,\dagger})/\sqrt{2}$, $Y^{in} = (a^{in} - a^{in,\dagger})/i\sqrt{2}$.

$$G \equiv G_0\alpha_s\sqrt{2} = \frac{2\omega_c}{L}\sqrt{\frac{P\kappa}{M\omega_m\omega_0(\kappa^2 + \Delta^2)}} \qquad (6.32)$$

is the effective optomechanical coupling.

This set of equations for Heisenberg operators can be written in the following form:

$$\dot{u}(t) = Au(t) + n(t) \tag{6.33}$$

with

$$u(t) = \begin{pmatrix} \delta q(t) \\ \delta p(t) \\ \delta x(t) \\ \delta y(t) \end{pmatrix} \; n(t) = \begin{pmatrix} 0 \\ \xi(t) \\ \sqrt{2\kappa} X^{in}(t) \\ \sqrt{2\kappa} Y^{in}(t) \end{pmatrix} \; A = \begin{pmatrix} 0 & \omega_m & 0 & 0 \\ -\omega_m & -\gamma & G & 0 \\ 0 & 0 & -\kappa & \Delta \\ G & 0 & -\Delta & -\kappa \end{pmatrix} \tag{6.34}$$

The Fourier transform of (6.33) produces:

$$\begin{aligned} i\omega\tilde{u}(\omega) &= A\tilde{u}(\omega) + \tilde{n}(\omega) \\ \Leftrightarrow (i\omega - A)\tilde{u}(\omega) &= \tilde{n}(\omega) \\ \Leftrightarrow B\tilde{u}(\omega) &= \tilde{n}(\omega) \\ \Leftrightarrow \tilde{u}(\omega) &= B^{-1}\tilde{n}(\omega) \end{aligned} \tag{6.35}$$

with

$$B := i\omega - A \tag{6.36}$$

Now we are interested only in the first component of $\tilde{u}(\omega)$:

$$\begin{aligned} \delta\tilde{q}(\omega) = \tilde{u}(\omega)_1 &= \sum_{k=1}^{4} B^{-1}_{1k}(\omega)\tilde{n}_k(\omega) \\ &=: \sum_k b_k(\omega)\tilde{n}_k(\omega) \\ \Rightarrow \delta\tilde{q}(\omega)\delta\tilde{q}(\omega')^* &= \sum\sum b_i(\omega)b_k(\omega')^*\tilde{n}_i(\omega)\tilde{n}_k(\omega')^* \\ &= \sum_k b_k(\omega)b_k(\omega')^*\tilde{n}_k(\omega)\tilde{n}_k(\omega')^* + \sum_i\sum_{k\neq i} b_i(\omega)b_k(\omega')^*\tilde{n}_i(\omega)\tilde{n}_k(\omega')^* \\ \Rightarrow \langle\delta\tilde{q}(\omega)\delta\tilde{q}(\omega')^*\rangle &= \sum_k b_k(\omega)b_k(\omega')^*\langle\tilde{n}_k(\omega)\tilde{n}_k(\omega')^*\rangle + \\ &\quad + b_3(\omega)b_4(\omega')^*\langle\tilde{n}_3(\omega)\tilde{n}_4(\omega')^*\rangle + b_4(\omega)b_3(\omega')^*\langle\tilde{n}_4(\omega)\tilde{n}_3(\omega')^*\rangle \end{aligned}$$

Other mixed summands vanish, because stochastic force of the mechanics is uncorrelated with vacuum radiation input noise of the light field.

The spectrum of the Brownian stochastic force in Fourier space can be obtained using (4.12) by multiplying with $(\frac{l}{\hbar})^2 = \frac{1}{M\omega_m\hbar}$:

$$\left\langle\tilde{\xi}(\omega)\tilde{\xi}(\omega')^*\right\rangle = (2\pi)^2\frac{I(\omega)\left(\coth(\frac{\hbar\omega\beta}{2})-1\right)}{2M\omega_m}\delta(\omega-\omega') \tag{6.37}$$

Correlation functions for the vacuum radiation input noise can be easily calculated using the fact, that in a good approximation for $\frac{\hbar\omega_c}{k_B T} \gg 1$ the only nonzero correlation function of a^{in} and $a^{in,\dagger}$ [1] is:

$$\left\langle a^{in}(t)a^{in,\dagger}(t')\right\rangle = \delta(t-t') \tag{6.38}$$

After applying the definition $X^{in} = \frac{a^{in}+a^{in,\dagger}}{\sqrt{2}}$ and $Y^{in} = \frac{a^{in}-a^{in,\dagger}}{i\sqrt{2}}$ and performing the Fourier transform we obtain:

$$\begin{aligned} \left\langle X^{in}(t)X^{in}(t')\right\rangle = \left\langle Y^{in}(t)Y^{in}(t')\right\rangle &= \frac{\delta(t-t')}{2} \\ \Rightarrow \left\langle \tilde{X}^{in}(\omega)\tilde{X}^{in}(\omega')^*\right\rangle = \left\langle \tilde{Y}^{in}(\omega)\tilde{Y}^{in}(\omega')^*\right\rangle &= \pi\delta(\omega-\omega') \\ \Rightarrow \langle \tilde{n}_3(\omega)\tilde{n}_3(\omega')^* \rangle = \langle \tilde{n}_4(\omega)\tilde{n}_4(\omega')^* \rangle &= 2\pi\kappa\delta(\omega-\omega') \end{aligned} \tag{6.39}$$

and:

$$\begin{aligned} \left\langle X^{in}(t)Y^{in}(t')\right\rangle = \left\langle Y^{in}(t)X^{in}(t')\right\rangle^* &= -\frac{\delta(t-t')}{2i} \\ \Rightarrow \langle \tilde{n}_3(\omega)\tilde{n}_4(\omega')^* \rangle = -\langle \tilde{n}_4(\omega)\tilde{n}_3(\omega')^* \rangle &= i2\pi\kappa\delta(\omega-\omega') \end{aligned} \tag{6.40}$$

[1] the exact formula is

$$\begin{aligned} \left\langle a^{in}(t)a^{in,\dagger}(t')\right\rangle &= [N(\omega_c)+1]\delta(t-t') \\ \left\langle a^{in,\dagger}(t)a^{in}(t')\right\rangle &= N(\omega_c)\delta(t-t') \end{aligned}$$

So for the mechanical spectrum $S_q^\Delta(\omega)$ we get:

$$\begin{aligned}
\langle\delta\tilde{q}(\omega)\delta\tilde{q}(\omega')^*\rangle &= (2\pi)^2\delta(\omega-\omega')\Big(b_2(\omega)b_2(\omega')^*\frac{I(\omega)\left(\coth(\frac{\hbar\omega\beta}{2})-1\right)}{2M\omega_m}+ \\
&\quad +b_3(\omega)b_3(\omega')^*\frac{\kappa}{2\pi}+b_4(\omega)b_4(\omega')^*\frac{\kappa}{2\pi}+ \\
&\quad +b_3(\omega)b_4(\omega')^*\frac{i\kappa}{2\pi}-b_4(\omega)b_3(\omega')^*\frac{i\kappa}{2\pi}\Big) \qquad (6.41)\\
&= (2\pi)^2\delta(\omega-\omega')\Big(|b_2(\omega)|^2\frac{I(\omega)\left(\coth(\frac{\hbar\omega\beta}{2})-1\right)}{2M\omega_m}+\frac{\kappa}{2\pi}|b_4(\omega)+ib_3(\omega)|^2\Big) \\
&=: 2\pi\delta(\omega-\omega')S_q^\Delta(\omega) \qquad (6.42)
\end{aligned}$$

After calculating the matrix B with Mathematica and inserting its elements in (6.42) we arrive at:

$$\begin{aligned}
\langle\delta q(t)\delta q(t')\rangle &= \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}S_q^\Delta(\omega)e^{i\omega(t-t')} \qquad (6.43)\\
&= \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}|\chi_{eff}^\Delta(\omega)|^2[S_{th}(\omega)+S_{rp}(\omega,\Delta)]e^{i\omega(t-t')} \qquad (6.44)\\
&= \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}|b_2(\omega)|^2\left(\frac{\pi I(\omega)\left(\coth(\frac{\hbar\omega\beta}{2})-1\right)}{M\omega_m}+\kappa\frac{|b_4(\omega)+ib_3(\omega)|^2}{|b_2(\omega)|^2}\right)e^{i\omega(t-t')}
\end{aligned}$$

with

$$\begin{aligned}
S_{th}(\omega) &= \frac{\pi I(\omega)}{M\omega_m}\left(\coth(\frac{\hbar\omega\beta}{2})-1\right)\\
S_{rp}(\omega,\Delta) &= \frac{\kappa\, G^2}{\Delta^2+\kappa^2+\omega^2+2\Delta\omega}\\
\chi_{eff}^\Delta(\omega) &= \omega_m\Big(\omega_m^2+i\gamma\omega-\omega^2-\frac{G^2\Delta\omega_m}{\Delta^2+(\kappa+i\omega)^2}\Big)^{-1} \qquad (6.45)
\end{aligned}$$

Apart from the obviously different thermal noise spectrum $S_{th}(\omega)$ compared to [1], the radiation pressure noise spectrum $S_{rp}(\omega,\Delta)$ differs as well. The reason is, that we have calculated $\langle\delta q(t)\delta q(t')\rangle$ as opposed to $\langle\delta q(t)^2\rangle$ in [1], where only the symmetric part of the whole $S_q^\Delta(\omega)$ survives.

We also get immediately $\langle \delta p(t) \delta p(t') \rangle$:

$$
\begin{aligned}
\delta \dot{q} &= \omega_m \delta p \\
\Rightarrow i\omega \delta \tilde{q}(\omega) &= \omega_m \delta \tilde{p}(\omega) \\
\Rightarrow \delta \tilde{p}(\omega) &= i \frac{\omega}{\omega_m} \delta \tilde{q}(\omega) \\
\Rightarrow \langle \delta \tilde{p}(\omega) \delta \tilde{p}(\omega')^* \rangle &= \frac{\omega \omega'}{\omega_m^2} \langle \delta \tilde{q}(\omega) \delta \tilde{q}(\omega')^* \rangle \\
&= \frac{\omega^2}{\omega_m^2} \langle \delta \tilde{q}(\omega) \delta \tilde{q}(\omega')^* \rangle
\end{aligned}
$$

In the last step we again use the fact that $\langle \tilde{q}(\omega) \tilde{q}(\omega')^* \rangle$ has $\delta(\omega - \omega')$ as a multiplicative factor.

$$
\Rightarrow \langle p(t) p(t') \rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\omega^2}{\omega_m^2} S_q^{\Delta}(\omega) e^{i\omega(t-t')} \tag{6.46}
$$

We want to test the formula for $S_{th}(\omega)$ in the Ohmic case:

$$
\begin{aligned}
I(\omega) &= c\omega \\
\Rightarrow \nu(t) &= c\pi \delta(t) \\
\Rightarrow \eta(t) &= c\pi \dot{\delta}(t) \\
\Rightarrow \gamma &= \gamma(t) = \frac{c\pi}{M} \\
\Rightarrow c &= \frac{M\gamma}{\pi} \\
\Rightarrow I(\omega) &= \frac{M\gamma\omega}{\pi} \\
\Rightarrow S_{th}(\omega) &= \frac{\gamma\omega}{\omega_m} \left(\coth(\frac{\hbar\omega\beta}{2}) - 1 \right)
\end{aligned} \tag{6.47}
$$

, the symmetric part of which is exactly the formula for $S_{th}(\omega)$ in [1].

Another test is setting the optomechanical coupling G to 0. Then $\langle \delta q(t) \delta q(t') \rangle$ becomes (5.9), which is correlation function of the oscillator's position in a thermal bath in the absence of laser field.

7 Measuring the optical spectrum of the optomechanical system

In this section I want to discuss the measurement of the optomechanical system and how the information about the motion of the mechanics can be obtained. As we will see homodyne measurement of the optical quadratures provides direct access to the spectral density $I(\omega)$. As in the previous section, coupling of the mechanical system to thermal bath should be weak in the sence that $I'(\omega) \approx I(\omega)$.

In case $\Delta = 0$, which means that the effective (i.e. measurable) detuning between the cavity and the laser driving the cavity is 0, equations for the phase quadrature of light (6.31) is:

$$\delta\dot{Y} = -\kappa\delta Y + G\delta q + \sqrt{2\kappa}Y^{in} \tag{7.1}$$

So in this case measurement of the phase quadrature can be used to get information about the mechanics. In real experiments $\Delta = 0$ would never be exactly achieved. Additionally stability conditions are more likely violated if $\Delta > 0$. Thats why one usually makes Δ slightly smaller then 0 in order to avoid instability. However if $|\Delta| \ll \kappa$ we will see that (7.1) is still a good approximation. Combining (6.30) and (6.31) one gets:

$$\begin{aligned}
\delta\ddot{X} + 2\kappa\delta\dot{X} + (\kappa^2 + \Delta^2)\delta X &= \kappa\sqrt{2\kappa}X^{in} + \sqrt{2\kappa}\dot{X}^{in} + \Delta\sqrt{2\kappa}Y^{in} + \Delta G\delta q \qquad (7.2)\\
\delta\ddot{Y} + 2\kappa\delta\dot{Y} + (\kappa^2 + \Delta^2)\delta Y &= \kappa\sqrt{2\kappa}Y^{in} + \sqrt{2\kappa}\dot{Y}^{in} - \Delta\sqrt{2\kappa}X^{in} + \kappa G\delta q + G\dot{q}
\end{aligned}$$

So for $|\Delta| \ll \kappa$ these equations are well approximated by:

$$\begin{aligned}
\delta\ddot{X} + 2\kappa\delta\dot{X} + \kappa^2\delta X &= \kappa\sqrt{2\kappa}X^{in} + \sqrt{2\kappa}\dot{X}^{in} + \Delta G\delta q \qquad (7.3)\\
\delta\ddot{Y} + 2\kappa\delta\dot{Y} + \kappa^2\delta Y &= \kappa\sqrt{2\kappa}Y^{in} + \sqrt{2\kappa}\dot{Y}^{in} + \kappa G\delta q + G\dot{q} \qquad (7.4)
\end{aligned}$$

The last equation is a sum of κ(7.1) and ∂_t(7.1). Therefore equations for quadratures of light for small detunings read:

$$\begin{aligned}
\delta\ddot{X} + 2\kappa\delta\dot{X} + \kappa^2\delta X &= \kappa\sqrt{2\kappa}X^{in} + \sqrt{2\kappa}\dot{X}^{in} + \Delta G\delta q \qquad (7.5)\\
\delta\dot{Y} &= -\kappa\delta Y + G\delta q + \sqrt{2\kappa}Y^{in} \qquad (7.6)
\end{aligned}$$

To examine the dependence of δY on δq we regard the Fourier transform of (7.1):

$$\Leftrightarrow \delta Y = \delta q\frac{G}{i\omega + \kappa} + \frac{\sqrt{2\kappa}}{i\omega + \kappa}Y^{in} \tag{7.7}$$

If $\kappa \gg \omega_m \gg \gamma$ then $i\omega + \kappa \approx \kappa$ for ω where the spectrum of the mechanics is situated and therefore:

$$\delta Y \;=\; \delta q \frac{G}{\kappa} + \sqrt{\frac{2}{\kappa}} Y^{in} \tag{7.8}$$

The output quadrature δY^{out} is related to δY through the usual input-output relation. If the detector has quantum efficiency $\eta < 1$, the relation for the **measured** output quadrature can be generalized [1] to:

$$\begin{aligned} \delta Y^{out} &= \sqrt{\eta}(\sqrt{2\kappa}\delta Y - Y^{in}) - \sqrt{1-\eta} Y^{\upsilon} && (7.9) \\ &= \sqrt{\frac{2\eta}{\kappa}} G\delta q + \sqrt{\eta} Y^{in} - \sqrt{1-\eta} Y^{\upsilon} && (7.10) \end{aligned}$$

Here Y^{υ} is delta-correlated noise, which is uncorrelated with other noise operators introduced before. It arises due to the fact that some part of the output light gets lost because of $\eta < 1$ in the experiment.

Going into the Fourier space we obtain:

$$\delta \tilde{Y}^{out}(\omega) \;=\; \sqrt{\frac{2\eta}{\kappa}} G\delta\tilde{q}(\omega) + \sqrt{\eta}\tilde{Y}^{in}(\omega) - \sqrt{1-\eta}\tilde{Y}^{\upsilon}(\omega) \tag{7.11}$$

The dynamics of the mechanics for $\Delta \ll \kappa$ is described by (6.28),(6.29) and (7.3):

$$\begin{aligned} \delta\dot{q} &= \omega_m \delta p && (7.12) \\ \delta\dot{p} &= -\omega_m \delta q - \gamma\delta p + G\delta X + \xi && (7.13) \\ \delta\ddot{X} + 2\kappa\delta\dot{X} + \kappa^2\delta X &= \kappa\sqrt{2\kappa}X^{in} + \sqrt{2\kappa}\dot{X}^{in} + \Delta G\delta q && (7.14) \end{aligned}$$

and therefore independent of Y^{in}. This means that δq and Y^{in} are uncorrelated. So:

$$\begin{aligned} \langle \delta\tilde{Y}^{out}(\omega)\delta\tilde{Y}^{out}(\omega')^*\rangle &= \frac{2\eta}{\kappa} G^2 \langle \delta\tilde{q}(\omega)\delta\tilde{q}(\omega')^*\rangle + \eta\langle \tilde{Y}^{in}(\omega)\tilde{Y}^{in}(\omega')^*\rangle + (1-\eta)\langle \tilde{Y}^{\upsilon}(\omega)\tilde{Y}^{\upsilon}(\omega)^*\rangle \\ &= 2\pi\left(\frac{2\eta}{\kappa} G^2 \, S_q^{\Delta}(\omega) + \frac{1}{2}\right)\delta(\omega-\omega') && (7.15) \end{aligned}$$

In the last step we used (6.42) and (6.39).

Measured spectrum $S_f(\omega)$ of an observable f [7] is given by:

$$\langle \tilde{f}(\omega)\tilde{f}(\omega')\rangle = 2\pi S_f(\omega)\delta(\omega-\omega') \tag{7.16}$$

$$\Rightarrow S_{\delta Y^{out}}(\omega) = \frac{2\eta}{\kappa}G^2\, S_q^{\Delta}(\omega) + \frac{1}{2} \tag{7.17}$$

$$= \frac{2\eta}{\kappa}G^2\, |\chi_{eff}^{\Delta}(\omega)|^2\{S_{th}(\omega) + S_{rp}(\omega,\Delta)\} + \frac{1}{2} \tag{7.18}$$

In our case of $\kappa \gg \omega_m \gg \gamma$, $\kappa \gg \Delta$

$$S_{rp}(\omega,\Delta) = \frac{\kappa\, G^2}{\Delta^2+\kappa^2+\omega^2+2\Delta\omega} \tag{7.19}$$

$$\approx \frac{G^2}{\kappa} \tag{7.20}$$

$$\chi_{eff}^{\Delta}(\omega) \approx \omega_m[\omega_{m_{eff}}^2 - \omega^2 + i\omega\gamma_{eff}] \tag{7.21}$$

$$\omega_{m_{eff}}^2 := \omega_m^2 - \frac{G^2\Delta\omega_m}{\kappa^2} \approx \omega_m^2 \tag{7.22}$$

$$\gamma_{eff} := \gamma + \frac{2G^2\Delta\omega_m}{\kappa^3} \approx \gamma \tag{7.23}$$

The last two approximations set two more conditions, which will be mentioned in the end of the section.

If light radiation pressure noise spectrum is negligible, i.e.

$$S_{rp}(\omega,\Delta) \ll S_{th}(\omega) \tag{7.24}$$

$$\Leftrightarrow \frac{G^2}{\kappa} \ll \frac{\pi I(\omega)}{M\omega_m}\left(\coth(\frac{\hbar\omega\beta}{2}) - 1\right) \tag{7.25}$$

$$\ll \approx \frac{\pi I(\omega_m)}{M\omega_m}\left(\coth(\frac{\hbar\omega_m\beta}{2}) - 1\right) \tag{7.26}$$

$$\ll \approx \gamma\left(\coth(\frac{\hbar\omega_m\beta}{2}) - 1\right) \tag{7.27}$$

the mechanical spectrum $S_q^{\Delta}(\omega)$ is the same as in case of a harmonic oscillator coupled to a thermal bath only:

$$S_q^{\Delta}(\omega) = |\chi_{eff}^{\Delta}(\omega)|^2\{S_{th}(\omega) + S_{rp}(\omega,\Delta)\} \tag{7.28}$$

$$\approx |\chi_{eff}^{0}(\omega)|^2 S_{th}(\omega) \tag{7.29}$$

$$\approx S_q(\omega) := \frac{\pi\omega_m}{M}\frac{I(\omega)\left(\coth(\frac{\hbar\omega\beta}{2}) - 1\right)}{(\omega_m^2-\omega^2)^2+(\gamma\omega)^2} \tag{7.30}$$

$$\approx \frac{2\pi\omega_m}{M\hbar\beta}\frac{I(\omega)/\omega}{(\omega_m^2-\omega^2)^2+(\gamma\omega)^2} \tag{7.31}$$

In the last step we applied the high temperature limit $\frac{\hbar\omega\beta}{2} \ll 1$. So summing up we can state that under conditions of weak mechanics-to-bath coupling (i.e. $I(\omega) \approx I'(\omega)$) and additionally large cavity bandwidth κ, small optomechanical coupling G, small cavity detuning Δ and high temperature T, expressed by:

$$\kappa \gg \Delta \tag{7.32}$$

$$\kappa \gg \omega_m \gg \gamma \tag{7.33}$$

$$\frac{G^2}{\kappa} \ll \frac{2\gamma}{\hbar\omega_m\beta} \tag{7.34}$$

$$\frac{G^2\Delta}{\kappa^2} \ll \omega_m \tag{7.35}$$

$$\frac{2G^2\Delta\omega_m}{\kappa^3} \ll \gamma \tag{7.36}$$

$$\frac{\hbar\omega_m\beta}{2} \ll 1 \tag{7.37}$$

the spectrum of the measured phase quadrature reads:

$$S_{\delta Y^{out}}(\omega)_{\text{dimensionless units}} \approx \frac{4\pi\eta G^2\omega_m}{M\hbar\beta\kappa}\frac{I(\omega)/\omega}{(\omega_m^2-\omega^2)^2+(\gamma\omega)^2}+\frac{1}{2} \tag{7.38}$$

$$\approx \frac{4\pi\eta G^2\omega_m}{M\hbar\beta\kappa}\frac{I(\omega)/\omega}{(\omega_m^2-\omega^2)^2+(\gamma\omega)^2} \tag{7.39}$$

We will see, that for all reasonable parameters present in the experiment the additive constant "$\frac{1}{2}$" can be ignored completely in the frequency interval around ω_m, which we will analyze. We will use this formula for fitting the experimental data, as the voltage at the output of the homodyne detection is proportional to δY^{out}.

8 Experimental setup

The whole experimental setup is illustrated on (Fig 2). Our harmonic oscillator is a Si_3N_4 100 μm $\times$ 50 μm $\times$ 1 μm micromechanical resonator, which carries a high-reflectivity Bragg mirror (Fig 1). It serves together with a fixed macroscopic input mirror (IM) as the end mirror of a 25 mm Fabry-Perrot cavity, which is kept under pressure of 10^{-7} mbar. The beam of tunable 1064 nm Nd:YAG laser is split at a polarizing beamsplitter (PBS) into a strong field acting as local oscillator (LO) and a $200\mu W$ weak one, the signal. The signal is fed through an electro-optical modulator (EOM) to generate a Pound-Drever-Hall error signal for locking the laser to a resonance frequency of the cavity. The signal passes through a PBS and $\lambda/4$ waveplate before entering the cavity in order to separate the input from the output signal. The phase quadrature of the latter is measured in a homodyne detection scheme. The phase ($\mathbf{\Phi}$) of the local oscillator is stabilized by a proportional-integral-derivative controller (PID). The sum of the outputs of both detectors is multiplied with the electronical sinusoidal signal created by a function generator (FG) and driving the EOM. The resulting voltage is the error signal which passing through a further PID controller locks the laser to the cavity. The difference of the detector outputs is proportional to the phase quadrature of the signal δY^{out}, which is proportional to the excitation of the micromechanical resonator. Its spectrum is monitored by a spectrum analyzer (SA).

Essential experimental parameters are:

- cavity length $L = 25 * 10^{-3}$m
- free cavity frequency and the cavity driving frequency $\omega_c \approx \omega_0 \approx 3*10^{14}$Hz
- input laser power $P = 200 * 10^{-6}$W
- cavity decay rate i.e. half width at half maximum $\kappa = 2\pi * 30 * 10^6$ Hz
- effective mass of the mechanical resonator (depending on the exact position of the laser beam on the mechanical oscillator) $M = 200 * 10^{-9} kg$
- effective cavity detuning with respect to laser freqency $\Delta = -2100$ Hz
- $\omega_m \approx 2\pi * 942000$ Hz
- $\gamma \approx 3200$ Hz

Using (6.32) we find: $G = 1312$ Hz

One can easily check that conditions (7.32)-(7.37) necessary for derivation of the formula for the measured spectrum of the homodyning output (7.39) are satisfied. The only condition which could be improved to achieve better validity of approximations, is $\kappa/\omega_m \gg 1$, which in our case is ≈ 30. Additionally analyzing the experimental data one will have to make sure that the assumption of weak bath coupling leading to $I'(\omega) \approx I(\omega)$ is justified, as this assumption is put in the derivation of (7.39) too. Justification of this assumption is presented in the next section.

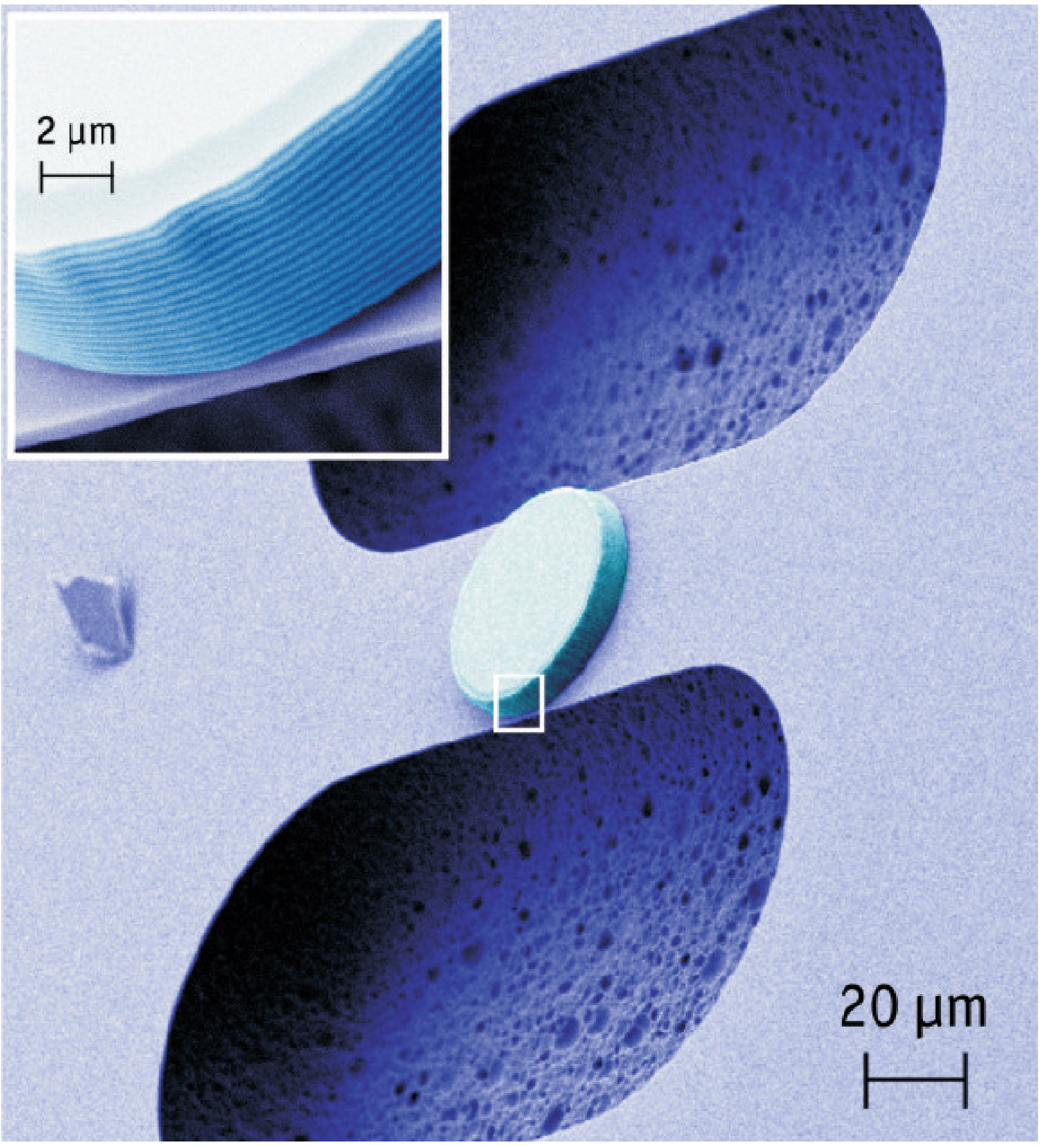

Figure 1: Our harmonic oscillator: a Si_3N_4 100 μm $\times$ 50 μm $\times$ 1 μm micromechanical resonator carrying a high-reflectivity Bragg mirror [15]

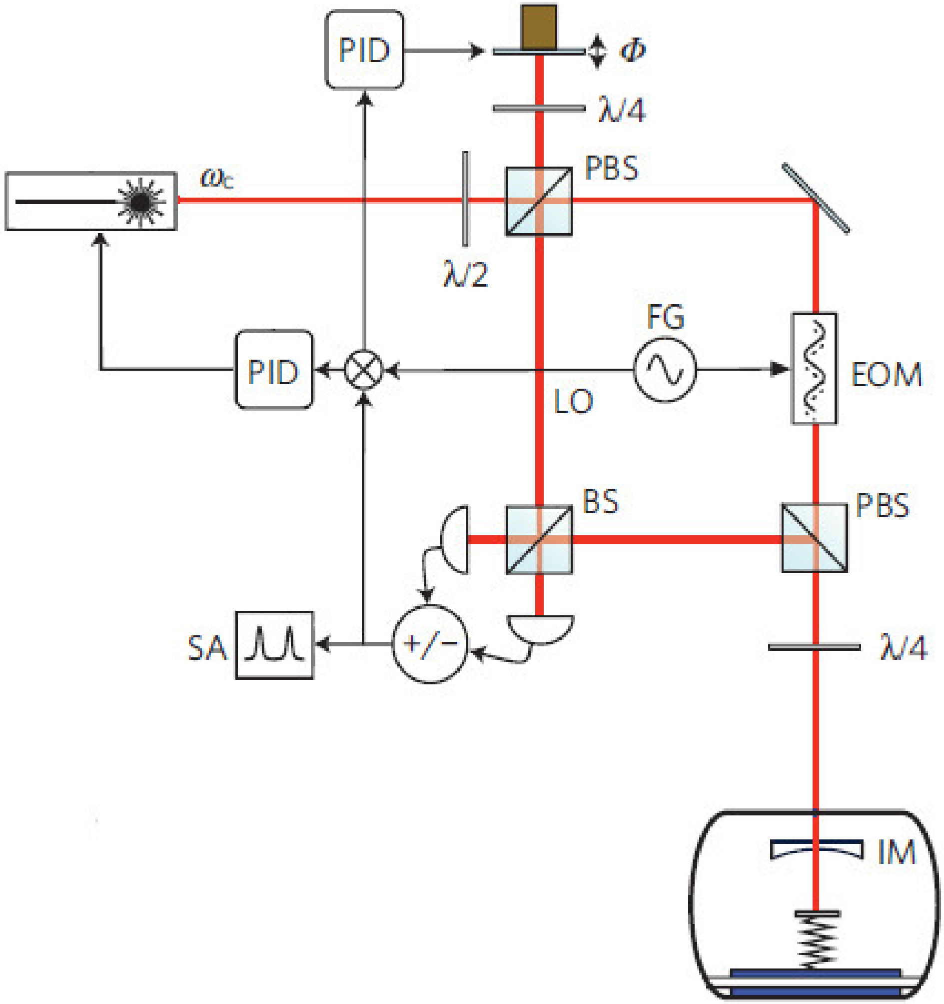

Figure 2: The micromechanical oscillator serves together with a fixed macroscopic input mirror (IM) as the end mirror of a Fabry-Perrot cavity. The laser beam is split at a polarizing beamsplitter (PBS) into a strong field acting as local oscillator (LO) and a weak one, the signal. The signal is fed through an electro-optical modulator (EOM) to generate a Pound-Drever-Hall error signal for locking the laser to a resonance frequency of the cavity. The signal passes through a PBS and $\lambda/4$ waveplate before entering the cavity in order to separate the input from the output signal. The phase quadrature of the latter is measured in a homodyne detection scheme. The phase ($\mathbf{\Phi}$) of the local oscillator is stabilized by a proportional-integral-derivative controller (PID). The sum of the outputs of both detectors is multiplied with the electronical sinusoidal signal created by a function generator (FG) and driving the EOM. The resulting voltage is the error signal which passing through a further PID controller locks the laser to the cavity. The difference of the detector outputs is proportional to the phase quadrature of the signal δY^{out}, which is proportional to the excitation of the micromechanical resonator. Its spectrum is monitored by a spectrum analyzer (SA).

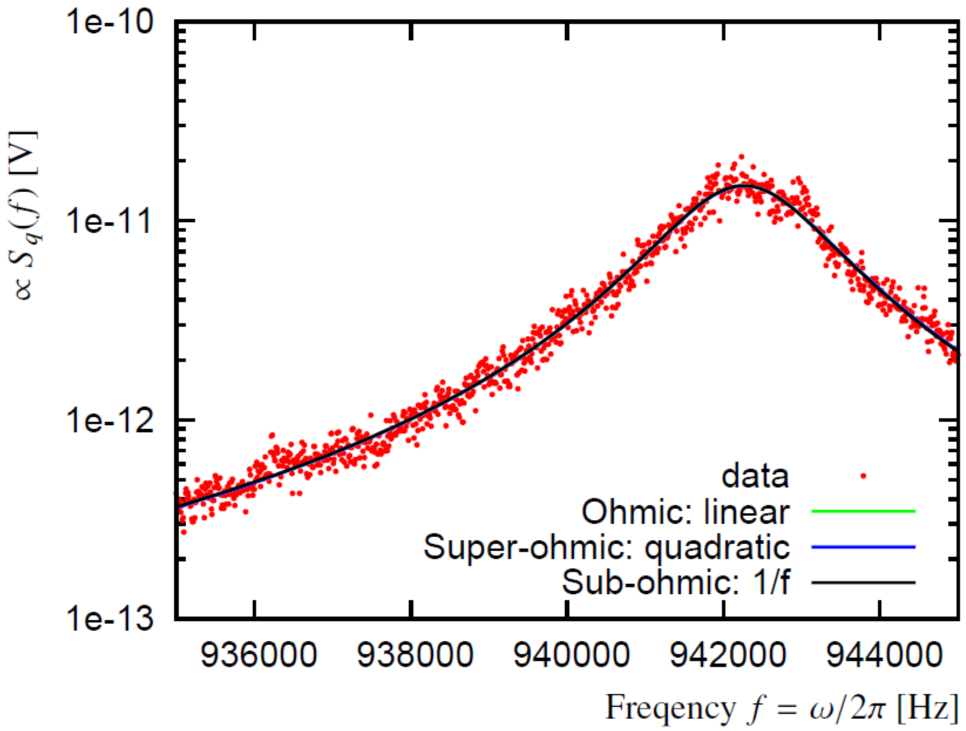

Figure 3: Experimental data and fits with different spectral densities, data were measured by Mag. Simon Groeblacher

9 Fitting the experimental data

Red points in (Fig 3) are the logarithmic plot of the power spectrum of the output voltage of homodyne detection. They were obtained after performing digital Fourier transform on the data in time domain. In the parameter regime present in the experiment we have shown that the spectrum of the measurement data is given by (7.39), if the assumption of weak mechanics-to-bath coupling is justified.

We want to find out under the assumption of weak coupling, which spectral density $I(\omega)$ from a parametric class of spectral densities fits best with the data points. For this purpose we assume a power law

$$I_k(\omega) \quad \propto \quad \omega^k \tag{9.1}$$

with parameter k.

First we try to find a fit for $k = -1, 1, 2$. It turns out that all three fit curves are nearly undistinguishable and fit well with the data. They are marked on (Fig 3) with different colours, the curves however overlap. This fact, that the data fits well with a Lorentzian multiplied by a correction function $\propto \omega^{k-1}$ i.e. fits well with a function of type (7.39), I use as justification for the weak coupling assumption. Additionally I will present a proposal for a more rigorous test for this assumption in the next section.

Although different models for $I(\omega)$ don't seem to change the power spectrum of the mechanical displacement such that it can be seen with bare eye, it is still possible to discriminate different bath models with a technique well known in mathematical statistics called "Bootstrapping" [6]. Its main idea is as follows:

Our data consist of a set of N data points. Randomly N points are chosen from the whole set with repitition, which means that some points appear more than once. The chosen points are fitted with the formula:

$$\log[a^2 \frac{\omega^{k-1}}{(\omega_m^2 - \omega^2)^2 + (\gamma\omega)^2}] \tag{9.2}$$

with free parameters $\{a, k, \omega_m, \gamma\}$. Fitting parameters are found by minimizing the sum of squares of the distance between the data points and the fitted curve. In this step it is important to find a global, not a local minimum. After the fitting is performed, the parameter k is recorded. The procedure of chosing points randomly, fitting by minimization and finding k is repeated several times. The result is a list of numbers for k. This list is plotted in form of a histogramm (4) , the mean value and the standard deviation of which can be calculated.

We are interested in the slope of $I(\omega)$ i.e. $\partial_\omega I(\omega) \propto k\omega^{k-1}$ around ω_m.
Using the relation $\partial_\omega I(\omega)|_{\omega=\omega_m} = I(\omega_m)\frac{k}{\omega_m}$ we find:

$$\partial_\omega I(\omega)|_{\omega=\omega_m} \propto -2 \pm 1,5 \tag{9.3}$$

which means a **negative slope of the spectral density** within the error bars.

The bootstrapping procedure was implemented in mathematica (5) by Dr. Konrad Kieling (Universitt Potsdam).

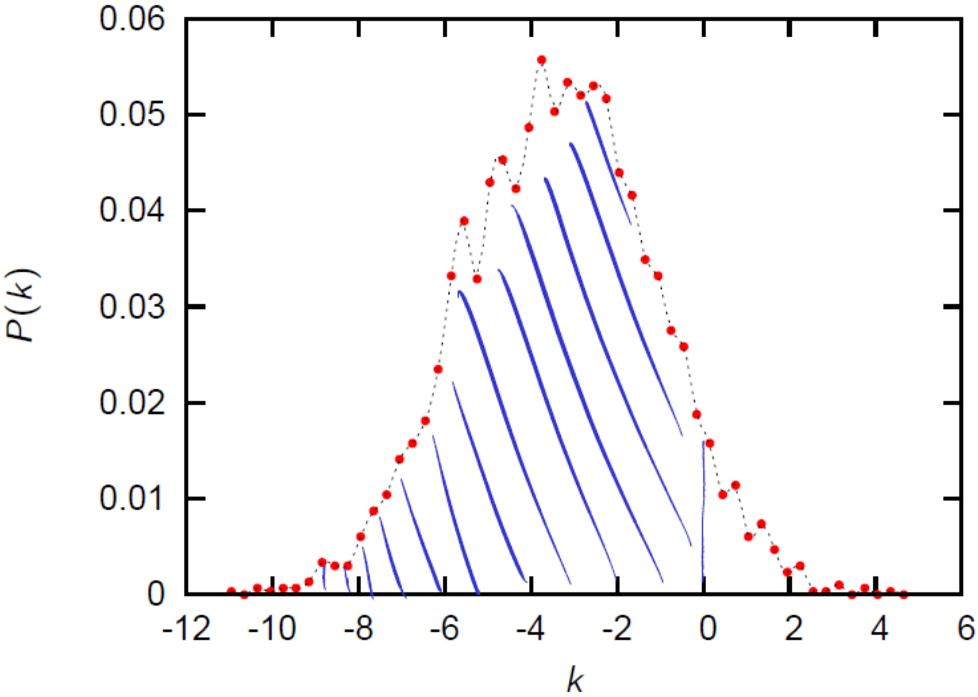

Figure 4: Histogramm for k ($I(\omega) \propto \omega^k$) [by Dr.Konrad Kieling]: The probability distribution has a peak located in the area with negative k (dashed area). This corresponds to a negative slope of spectral density.

10 Summary and outlook

In the last section we have analyzed the behaviour of the spectral density $I(\omega)$ of the thermal bath in the frequency region around the resonance frequency of the mechanics. The result was a negative slope of the spectral density in the analyzed interval. This is a contradiction to the usual assumption of an Ohmic spectral density i.e. $I(\omega) = c\omega$, $c > 0$. This contradiction wasn't observed in experiments till now, because the measurable power spectrum of the mechanical displacement is hardly influenced by the shape of $I(\omega)$ in the weak coupling regime of the mechanical device. This weak dependence however can be measured by the statistical methods, which we applied.

As the dependence is small, there exist loopholes for possible unconsidered experimental factors, which may lead to a slight distortion in the measurement outcome and therefore to a wrong estimation of the spectral density. Additionally a lot of approximations were made in the derivation of the function used for fitting the experimental results. The problem can be posed this way: We assume that force with correlation function given by (3.6) or in other words with spectrum $\propto I(\omega)/\omega$ is acting on the mechanics. After performing the experiment, one gets an estimation for the slope of $I(\omega)$. How can this estimation be tested?

A good test for the correctness of this estimation is to drive the mechanics with a *known* external mechanical force with spectrum $J(\omega)/\omega$, where $J(\omega)$ has the same slope as the estimated one of $I(\omega)$ but higher amplitude, such that the thermal force can be neglected. In addition one should vary the slope of $J(\omega)$ slightly and check, if the bootstrapping technique provides correct results for slightly varying slopes of $J(\omega)$. Then one could assure himself that the procedure of estimation of the force spectrum acting on the mechanics is correct. This would also be a test for the validity of the weak coupling assumption in the sence that $I'(\omega) \approx I(\omega)$, as non-weak coupling leads to renormalization of any force acting on the mechanics. The external "test force" could be e.g. excitation of a piezoelectric crystal, on which the micromechanical oscillator could be mounted. In this case the excitation of the piezo is proportional to the voltage applied on it.

The technique of probing the oscillator's spectral density presented in this work is an important step towards characterization of the heat-bath environment in the view of the fact that different thermal baths may lead to a dramatically different behaviour of the oscillator in the quantum regime. [14]

11 Appendix

- **helper functions**

Get data in a given window around a central point.

```
getData[w_List, c_, f_: Log] :=
 {#[[1]], f[#[[2]]]} & /@ Select[data, #[[1]] ≤ c + w[[2]] && #[[1]] ≥ c - w[[1]] &]
getData[w_, c_, f_: Log] := getData[{w, w}, c, f];
```

Access hash list *h* for the given key.

```
hash[h_, key_] := key /. h;
```

Set empty fields of the hashlist to common default values.

```
setdefaults[myrun_] := Module[{myrun0, mr, dataset, ii}, myrun0 =
    {"samplefunction" → (RandomChoice[#1, #2] &), "samples" → 1000, "constraints" → {},
     "method" → "SimulatedAnnealing", "solutions" → {}, "comment" → ""};
   mr = Join[myrun, #[[2]] → hash[myrun0, #[[2]]] & /@
      Select[(ii = #[[1]]; {Length[Select[myrun, #[[1]] == ii &]] == 1, ii}) & /@ myrun0,
       #[[1]] == False &]];
   If[Length[Select[mr, #[[1]] == "region" &]] == 0, mr = Append[mr, "region" →
       {Min[#[[1]] & /@ hash[myrun, "data"]], Max[#[[1]] & /@ hash[myrun, "data"]]}]];
   If[Length[Select[mr, #[[1]] == "samplesize" &]] == 0,
    mr = Append[mr, "samplesize" → (hash[myrun, "data"] // Length)]];
   Return[mr];];
```

The job

- **define the dataset**

A set of replacement rules to parametrise the jobs, such a dataset contains information about the function to fit, the region, the free parameters, the datasource, etc.

```
mydata = {
   "comment" → "oldest data from vienna, log, 4k window",
   "model" → (Log[a^2 #1^(d - 1) / ((b^2 - #1^2)^2 + (c #1)^2)] &),
   "samplesize" → Length[getData[4000, 942 500]],
   "solutions" → {},
   "samples" → 10,
   "method" → "SimulatedAnnealing",
   "constraints" → {940 000 ≤ b ≤ 945 000},
   "parameters" → {a, b, c, d},
   "data" → getData[4000, 942 500, Log]
  };
```

Here we want to fit the function $f(x) = \mathrm{Log}\left(\frac{a^2 x^{d-1}}{(b^2-x^2)^2+(c\,x)^2}\right)$ to data from a window of size 2x4kHz around the center of 942 500Hz. The free parameters are a, b, c, d with the constraint of $940000 \leq b \leq 945000$. For the bootstrapping, Length[getData[4000, 942 500]] points are drawn using RandomChoice. This is done 10 times (use maybe 1000 for better statistics) and for each sample the sum of squares of $f(x)$ to Log(data) is minimised using simulated annealing.

Figure 5: Mathematica file used for bootstrapping [Dr. Konrad Kieling]: page 1

run the minimisation

set some defaults

```
mydata = setdefaults[mydata];
```

First, we chose the random samples.

```
d1 = Table[hash[mydata, "samplefunction"][Table[i, {i, 1, hash[mydata, "data"] // Length}],
     hash[mydata, "samplesize"], i], {i, 1, hash[mydata, "samples"]}];
```

Easier access to the solutions element in the hash

```
sol = Position[#[[1]] & /@ mydata, "solutions"][[1]][[1]];
```

The bootstrapping + minimisation. It is all built into *Mathematica* anyways.

```
For[i = 1, i ≤ Length[d1], i++,
  F = Plus @@
    ((#[[2]] - hash[mydata, "model"][#[[1]]])^2 & /@ hash[mydata, "data"][[d1[[i]]]]);
  (* define the objective function: sum of squared differences *)
  solmin = NMinimize[Join[{F}, hash[mydata, "constraints"]], hash[mydata, "parameters"],
    Method → hash[mydata, "method"]]; (* numerical minimisation *)

  mydata[[sol]] = "solutions" → Append[mydata[[sol]][[2]], Append[solmin, d1[[i]]]];
  (* append the new result to the hash entry with key "solutions" *)
  Print[ToString[i] <> "/" <> ToString[Length@d1]]; (* some status message *)
 ];

"solutions" /. mydata
```

Having a look at the data

- **some more helper functions**

Bin the data

```
binData[binwidth_: 1] := Module[{}, (
     Select[If[Length[#] > 0, {(Floor[#[[1]]] + .5) binwidth, Length[#]}, {}] & /@
       (Abs[(d /. #[[2]])] / binwidth & /@ ("solutions" /. mydata) // BinLists),
      Length[#] > 0 &]
    )];
```

Show a plot of binned data (basically, histograms)

```
showPlot[binwidth_: 1] := Module[{}, ListPlot[binData[binwidth]]];

binData[]

{{0.5, 2}, {1.5, 2}, {2.5, 2}, {3.5, 1}, {4.5, 2}, {5.5, 1}}
```

Figure 6: Mathematica file used for bootstrapping [Dr.Konrad Kieling]: page 2

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