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„Computable General Equilibrium Models: Hybrid Top-Down Bottom-Up Energy Policy Modelling

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Abstract

This thesis presents a Computable General Equilibrium model for Austria with focus on energy and environmental policy assessment. A novel hybrid modelling structure featuring a general equilibrium top-down and a technologically oriented bottom-up model in an integrated framework is described in detail. The theoretical framework is translated into a computable model that can be used for applications. Scenarios relating to a policy issue recently discussed in Austrian politics demonstrate the applicability of the model.

JEL Classification Numbers: E2; Q4; Q52.

Keywords: Computable General Equilibrium; Hybrid Integrated Model; Top-Down Bottom-Up.

Erklärung der Selbstständigkeit

Hiermit versichere ich, die vorliegende Arbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt sowie die Zitate deutlich kenntlich gemacht zu haben.

Wien, den 20. April 2012 Michael Gregor Miess

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CHAPTER 1

Introduction

The main scope of this thesis is to present a Computable General Equilibrium (CGE) model for Austria focusing on energy and environmental policy assessment. This model follows a hybrid modelling structure recently developed by Böhringer and Rutherford (2008, [8]) that allows for the direct integration of a technologically oriented bottom-up sector into an economy-wide top-down general equilibrium model.

CGE models are often referred to as an attempt to translate the Walrasian general equilibrium framework into a numerically solvable model. The Walrasian general equilibrium structure is essentially a system of equations depicting demand by consumers and supply by producers under the equilibrium condition of market clearance on every market.

Thus, an abstract depiction of the economy is replaced by a computable one that can be used for applied economic analysis.

Since the late 1970's and early 1980's, CGE models have been used extensively by the World Bank and the International Monetary Fund to assess the implications of policy-induced changes in a general equilibrium framework, very often for developing economies (see Mitra-Kahn, 2008, [47]). The recent improvements in CGE modelling practice put forth by Böhringer and Rutherford (2008, [8]) allow for its further application to a comprehensive cost-benefit assessment of energy and environmental policies.

A CGE modelling structure suitable for various issues of energy and environmental economics will be described in detail, including its theoretical background, in the following chapters.

The main innovation by Böhringer and Rutherford (2008, [8]) that allows for a thorough treatment of the energy system on a technological level in interaction with an economy in general equilibrium is a hybrid top-down bottom-up formulation of a CGE model as a Mixed Complementarity Problem (MCP). An MCP is a problem formulation in mathematical optimisation that offers several advantages over comparable problem formulations such as Non-Linear Programs (NLPs), see section 3.2.1.

The top-down paradigm usually relates to economy-wide models focusing on policy implications as regarding public finances, economic competitiveness and employment, among others.

Bottom-up models concentrate on the direct depiction of technologies, giving a detailed picture of the supply side of the economy that can be related to pollutants and other environmental factors. Therefore, while top-down models usually miss out on technological detail but treat household demand for the different goods as an endogenous variable, bottom-up models have to take economy-wide demand for goods as exogenous while being able to provide a production function incorporating high technological detail.

It seems obvious that the conjunction of top-down and bottom-up model features in one common framework is highly desirable. This can be achieved by formulating an economic equilibrium as an MCP.

Casting an economic equilibrium problem as an MCP (see Mathiesen, 1985a, [43]) makes the problem of finding an equilibrium both more efficient and transparent to solve.

The problem becomes more efficient in the sense that powerful solution algorithms existing for MCP problems, such as the PATH solver (Dirkse and Ferris, 1995, [20]), can be employed to solve the model numerically. Also, the MCP format allows for the formulation of a wider, more complex range of economic models with an environmental focus explicitly considering income effects, taxes and tax distortions, as well as market imperfections and failures, more than models relying on a linear or non-linear mathematical programming approach. This is due to the fact that the integrability conditions inherent to economic models cast as optimisation problems can be relaxed, as is further explained in section 3.1.

The MCP format reaches a higher transparency by establishing a logical connection between price formation and market clearing conditions in the form of weak inequalities. An equational system of interlocking zero profit (revenue from output has to equal factor costs) and market clearance (supply has to equal demand for positive prices) allows for the direct integration of a technology-oriented bottom-up representation of the electricity sector into the top-down general equilibrium model in the MCP format.

Hence, the consideration of general equilibrium effects is combined with a technological representation of the electricity sector. Environmental externalities due to the production of electricity, capacity and resource constraints can be directly linked to the rest of the economy.

This application of CGE models to environmental and energy issues sounds highly promising, especially facing the imminent threat that global warming might substantially change the present structure of our society. It seems strictly reasonable to assess the costs we have to face to transform our economic system in order to reduce the emission of greenhouse gases (GHGs) and other pollutants in an economy-wide framework. Trying to quantify the costs of global warming, which could be pinned against the economic costs of taking action against climate change, seems hard or impossible due to the high complexity and uncertainty as regarding changes and feedback cycles within the climatic system. However, more or less recent undertakings such as the widely cited Stern Review (Stern, 2007, [64]) have tried to quantify the economic impacts of climate change and come to the conclusion that the costs of not acting on climate change might outweigh the costs of doing so.

In this process of weighing costs against benefits, i.e. determining which policies truly make sense to combat climate change, and which do not, CGE models seem to present themselves as a highly useful tool. However, to verify this assumption, it is necessary to re-consider the theoretical roots of the CGE modelling concept and its direct applicability to the quantitative dimension of policy assessment.

The structure of CGE models is commonly perceived to be derived from the theoretical foundations laid by general equilibrium theory, which originated from the work of Léon Walras in the 1870's and was formalised by Arrow, Debreu and others in the 1950's. However, concepts such as calibrating the model to a benchmark data set, assuming the economy to be in state of equilibrium in the benchmark year, are one of several departures of CGE modelling practice from rigorous general equilibrium theory.

One could even say that CGE models are a separate class of models only loosely connected to general equilibrium theory as such. They are rather in line with a tradition of balanced macro models such as the Johansen (1960, [33]) model shortly sketched in section 2.2.1 that rely heavily on exogenous parameter values to explain price-endogenous adjustment processes in an economy-wide setting. Johansen's model relied on a linear solving procedure, recently developed CGE models are mostly solved with an algorithm of Newton type.

This stands in contrast to a general equilibrium model that is merely rendered computable. In this type of model, first the existence of an economic equilibrium is established through the standard Arrow-Debreu exposition, then economic data are assigned to the various sectors, and lastly a fixed-point algorithm of a type similar to the one developed by Scarf (1967, [58], 1973, [59]) is applied to calculate the set of equilibrium prices clearing all markets (see Mitra-Kahn, 2008, [47, p. 53]). However, due to the tediousness of this approach and the fact that it did not generally yield a solution (see Mitra-Kahn, 2008, [47, p. 53]), models of this type, which were usually termed Applied General Equilibrium (AGE) models, have hardly been used since the mid-1980s (Mitra-Kahn, 2008, [47, pp. 72-73]). According to Mitra-Kahn (2008, [47]), AGE models have become synonymous with CGE models after this point, as the CGE approach proved to be more practical, with the AGE approach being merged in the CGE approach.

The results of a CGE model, now, depend largely on the choices of the modeller as regarding exogenous parameters and the macro balancing equations, a fact that should be set out clearly in every paper dealing with CGE modelling.

In regard of these discussions, chapter 2 delivers a short description of the origins of general equilibrium theory, the structure of a general equilibrium model and its relation to the CGE modelling concept. The model developed by Johansen (1960, [33]) is illustrated shortly. Furthermore, the process of forming a numerical CGE model from a stylised economic model and the calibration procedure are displayed briefly.

In chapter 3, after showing how to cast an economic equilibrium as an MCP in section 3.2.1, sections 3.2.2 and 3.2.3 deliver a theoretical model as described in Böhringer and Rutherford

(2008, [8]) featuring the (static) hybrid top-down bottom-up structure delineated above. A dynamic version of this model is proposed in section 3.2.4.

Consequently, in chapter 4 data and the equational structure of a hybrid CGE model for Austria constructed according to the theoretical model as set forth in the previous chapter are described in comprehensive detail in sections 4.1 to 4.2. Section 4.3 provides a brief picture of the calibration and solving procedures and elaborates on the concept of a numéraire in a CGE model.

In chapter 5, the model is applied to an environmental policy issue that has been discussed in Austrian politics recently.

Chapter 6 concludes the thesis, gives a critical viewpoint on the model presented and proposes improvements to the model as well as future research efforts.

CHAPTER 2

A Brief Guide to Computable General Equilibrium Models

2.1 From the Walrasian General Equilibrium Framework to Computable General Equilibrium Modelling

In principle, the literature often attributes CGE modelling to be the attempt to transform the Walrasian general equilibrium structure, which was formalised by Kenneth Arrow, Gerard Debreu and others (see e.g. Arrow and Debreu, 1954, [3], Debreu, 1959, [18], Arrow and Hahn, 1971, [4]) in the 1950's, from a merely theoretical, abstract representation of the economy into a (numerically) solvable model which can be used for applied economic analysis. Therefore, firstly the Walrasian general equilibrium system and the intentions behind its formulation are described in brief. Then, some essential features of the Walrasian system are presented in order to enhance the understanding of the theoretical foundations of the general equilibrium model structure that will be described in section 2.1.1.

Lèon Walras was first to formulate the economic system for a given point in time as a system of simultaneous equations depicting the demand for goods by consumers, the supply of goods by producers, and using the equilibrium condition that supply was to equal demand on every market of the economy. Utility maximisation for consumers and profit maximisation for producers were assumed, as well as the prevailing of perfect competition (consumers and producers regard the prices as independent of their own choices) (see Arrow, Debreu, 1954, [3, p. 265]). The assumption of perfect competition, as well as the assumption that the economy is in equilibrium at the point of time at which it is examined, have drawn a substantial amount of criticism against using CGE models to evaluate policy implications. Since the notion of perfect competitions it is crucial from the very beginning of the analysis, a few words on Walras' idea of an equilibrium and its relationship to the assumption of perfect (free) competition will be said here.

The notion of an equilibrium within L. Walras' models involves the equality of effective demand and supply for commodities and services, as well as a stable equilibrium price system.

Furthermore, the zero profit condition has to hold for entrepreneurs[1], meaning that the *factor costs* have to equal the *selling price for commodities* (see Van Daal, Jolink, 1993, [66, p. 11f]). Walras, however, was well aware that

> equilibrium is not a real state but rather an ideal. (Van Daal, Jolink, 1993, [66, p. 12])

It is further argued that

> Only in a situation of free competition, in which entrepreneurs are free to flow in and out of production sectors, will the market transactions tend to a state of equilibrium, which as such is denominated by Walras 'the normal state'. [66, p. 12]

Actually, Walras considered the fact that in a market system subjected to continuous change an equilibrium would never be completely attained:

> Such is the continuous market, which is perpetually tending towards equilibrium without ever actually attaining it, because the market has no other way of approaching equilibrium except by groping, and, before the goal is reached, it has to renew its efforts and start over again, all the basic data of the problem, e.g. the initial quantities possessed, the utilities of goods and services, the technical coefficients, the excess of income over consumption, the working capital requirements, etc., having changed in the meantime. (see Walras, 1954, [68, p. 380, Section 322])

Walras' idea behind a market exhibiting free competition, which had to be highly organised, and where prices are known to all agents because of a centralised pricing system, seemed to be that of the (Paris) Stock Exchange [66, p. 9]. Later on he also provided an idea behind the dynamics of the pricing mechanism on such a market, the so-called *tâtonnement* process, which is essentially a price groping mechanism of successive bidding by sellers and consumers in a disequilibrium situation until an equilibrium price has been reached (see [66, pp. 9]). However, one has to keep in mind that Walras' models always have to be seen as static equilibrium models, where a situation of disequilibrium is never taken into account. The tâtonnement process has received much attention in economic literature (see e.g. the recent publication by Gintis, 2007, [24]) as according to its convergence to a stable price system, but since at this point only static equilibrium models are considered, there is no need for further elaboration on this matter.

Most importantly, maybe because of the nature of his mathematical education, Walras was not aware of the fact that the system of simultaneous equations determining an equilibrium situation in his models *did not necessarily have a solution* (see [66, p. 172], [3, p. 265]). In fact his sole argument was that a system of n equations in n variables has a solution, which is an inadequate argument for non-linear systems of equations.

1 Entrepreneurs are the agents responsible for founding and managing firms in Walras' models; for more information see e.g. Van Daal, Jolink, 1993, [66], chapter 7.

Thus, the argument of the equality of the number of prices that have to be determined on the markets with the number of equations denoting the equality of supply and demand, both equal to the number of commodities, let's say n, does not suffice to guarantee the existence of an equilibrium solution. However, the following points were realised by Walras from this system of equations and unknowns (see Arrow, Hahn 1971, [4, p. 4]):

- The demand and supply by households and firms is only determined by changes in the system of *relative prices*. Thus, the system of equations has only $n-1$ unknowns, and one commodity can subsequently be chosen as a *numéraire*. The prices of all other goods can then be measured relative to this numéraire good.
- As *all income is spent*, the budgetary balance of each household between its income and the value of its consumption has to hold, and because of perfect competition, the zero profit condition holds for all firms. These conditions together imply to what has been commonly come to known as *Walras' Law*: The market value of the economy's supply equals that of demand for any set of feasible prices, not only for equilibrium prices. This means that the relations of supply and demand are not independent from each other. Thus, if $n-1$ markets clear, the n^{th} market also has to clear.

As mentioned above, only in the 1950's did economic theory develop concise arguments for the existence of an equilibrium under conditions similar to those supposed by Walras, mostly due to the work of Arrow, Debreu, Scarf, and others.

2.1.1 Structure of the General Equilibrium Framework (The Arrow-Debreu Model)

The simple general equilibrium *trade model* as set out in Shoven and Whalley (1992, [63])[2], in the spirit of the general equilibrium concept elaborated e.g. in Arrow and Hahn (1971, [4]), can be described as an economy with a (finite) set of n commodities, $i \in \{1, \ldots, n\}$, where each commodity has a nonnegative price $p_i \geq 0$. Market prices are represented by the price vector $p = (p_1, \ldots, p_n)$. w_i symbolises the (nonnegative) economy-wide endowment of commodity i that is owned by the consumers, which is assumed to be strictly positive for at least one i. d_i represent the market demand functions, which are assumed to be nonnegative, continuous, and homogeneous of degree zero in p.

Homogeneity of degree zero means that a proportional increase of all prices would also increase the income of the households by the same proportion, leaving the system of relative prices unchanged. Thus, the physical quantities of goods demanded by the household would also remain the same. Because of the homogeneity assumption the prices can be normalised to sum up to one, so that they lie in a unit simplex (Shoven and Whalley, 1992, [63, p. 10]):

$$\sum_{i=1}^{n} p_i = 1 \tag{2.1}$$

2 The notation of all models presented henceforth has been changed to keep one common standard for this thesis.

In the trade model, Walras' Law can be stated as the market value of demand equaling the market value of the economy's endowments (Shoven and Whalley, 1992, [63, p. 10]):

$$\sum_{i=1}^{n} p_i(d_i(p) - w_i) = 0 \tag{2.2}$$

As set out before, Walras' Law has to hold for any price system of the economy, not only in equilibrium. Therefore, Walras' Law can be used as a basic checking system for any CGE model. If Walras' law does not hold any more, the model usually suffers from misspecification or another serious impairment, since the *budget balance* for the sum of all households *is violated* in the model. A price system setting this economy into a general equilibrium now is a price system p^* such that [63, p. 10f]

$$d_i(\mathbf{p^*}) - w_i \leq 0 \qquad \forall i \in \{1, \ldots, n\} \tag{2.3}$$

If $p_i^* > 0$, then the above constraint has to be binding, i.e. equality has to hold and supply has to equal demand. When supply permanently exceeds demand in this system, the price p_i^* has to equal to zero. This situation can be seen as a commodity freely provided by nature, such as e.g. air. As soon as demand exceeds supply at a zero price, the goods will have a positive price p_i^* equating supply and demand. The equilibrium price system given above will thus clear all markets.

For a general equilibrium model with production, now, a production technology has to be specified. An example of such a technology would be a finite number m of constant-returns-to-scale (CRTS) production technologies (production methods or activities). Each technology can then be described by fixed coefficients a_{ij}, symbolising the amount of good/factor i used for the activity j when the activity is operated at unit intensity. Negative signs on the coefficients then denote inputs, a positive sign indicates an output (see [63, p. 11]). *Non-reversibility* is assumed for all these production activities, meaning that the inputs can not be produced from the respective outputs. Now let the vector $Y = (Y_1, \ldots, Y_m)$ represent the (nonnegative) levels of intensity at which each activity operates. Infinite amounts of outputs stemming from finite amounts of inputs are ruled out, i.e. the production is assumed to be bounded (the "no free lunch" assumption). Technically, this implies that the set of Y is such that [63, p. 12]

$$\sum_{j=1}^{m} a_{ij} Y_j + w_i \geq 0 \qquad \forall i \in \{1, \ldots, n\} \tag{2.4}$$

is contained within a bounded set.

A general equilibrium for this model now is represented by a set of equilibrium prices p_i^* and activity levels Y_j^* such that [63, p. 12]

- demand is equal to supply for all commodities:

$$d_i(p*) = \sum_{j=1}^{m} a_{ij} Y_j + w_i \qquad \forall i \in \{1, \dots, n\} \tag{2.5}$$

- the zero profit condition holds for all activities, i.e. no production activity makes positive profits, while those that are active break even:

$$\sum_{i=1}^{n} p_i^* a_{ij} \leq 0 \qquad (= 0 \quad if \quad Y_j^* > 0) \qquad \forall j \in \{1, \dots, m\} \tag{2.6}$$

For those activities where supply permanently exceeds demand, we assume disposal via a disposal activity (this can be done because the possibility of "free disposal" is assumed). These disposal technologies are incorporated in the technology matrix A. Therefore, we have no complementary slackness condition in equation (2.5) any more (we have an equality instead of an inequality).

As mentioned above, it was the main contribution by Arrow and Debreu (1954, [3]), later on set out more thoroughly in Debreu (1959, [18]) or Arrow and Hahn (1971, [4]), to prove the existence of an equilibrium for an economic model as proposed above.

The mathematical proof basically involves the application of fixed point theorems to economic modelling (see [63, p. 12ff]). The intuition behind the use of fixed point theorems is to find a stable set of equilibrium prices by constructing a map of the unit simplex (the normalised price system) into itself where one can apply a fixed point theorem to prove the existence of an equilibrium. The fixed point of the system then corresponds to equilibrium prices, where prices and demands remain stable.

The two theorems used for the proof of the existence of a general equilibrium are Brouwer's fixed point theorem for point-to-point mappings and Kakutani's fixed point theorem for point-to-set mappings. These theorems are e.g. discussed in Scarf (1973, [59, p. 28]). The complete proof for the existence can e.g. be found in Debreu (1959, [18]), a sketch of the proof is provided in Shoven and Whalley (1992, [63, p. 12-21]).

The above mentioned proofs apply to a standard Arrow-Debreu model (as presented above in its basic structure) without the participation of a government agent in the economy. The incorporation of taxes into the model complicates matters considerably, since the incomes of the agents will partially depend on the amount and form of the reimbursement of government revenues to the agents. Therefore, consumer income will not only depend on prices, but also on the demands of all other consumers and the production of all producers, which in turn determine government income (see [63, p. 21f] for a discussion of this point). An existence proof, as well as a computational procedure, for a general equilibrium with taxes can be obtained from Shoven and Whalley (1973, [61]). Nothing is said here about the efficiency of such an equilibrium with taxes, but since taxes of various forms certainly are economic reality, they are important features of an applied general equilibrium analysis.

The computation of such an economic equilibrium is another issue, since the proofs by Arrow, Debreu and others are rather non-constructive, i.e. they show the existence of equilibria, but not how the latter are reached [63, p. 20f]. It is due to the work of Scarf (1967, [58], 1973, [59]), who developed fixed point algorithms to actually calculate general equilibrium prices, that equilibria were rendered computable based on theoretical work. However, because of speed and efficiency matters, algorithms used in modern CGE models are mostly of Newton type. An algorithm of Newton type will be very briefly touched later on in section 4.3.

2.2 The Computable General Equilibrium Model

2.2.1 The Johansen (1960) Model

As mentioned before, CGE models in the literature often refer to the Arrow-Debreu model set out in the previous chapter as their theoretical background. However, the person usually credited to formulate

> the first empirically based, multi-sector, price-endogenous model analyzing resource allocation issues (Shoven and Whalley, 1984, [62, p. 1008])

was Leif Johansen (1960, [33]). He built a multi-sectoral model for Norway in order to analyse sectoral "*deviations* from the uniformity of the growth process" (Johansen, 1960, [33, p. 5], his emphasis), standing in contrast to the usual Arrow-Debreu general equilibrium or growth theory at this time, which spoke of a balanced growth path across sectors (Mitra-Kahn, 2008, [47, p. 9]).

To do so, Johansen used the national accounting framework to construct a data set for a specific point in time. He used the year 1950 as his base year. This data framework is of the input-output type, representing the circular flows of the economy between sectors in matrix form (see section 4.1 for a description of a similar kind of data set). He explicitly edited this data set in order to balance the rows and columns of this matrix (see Johansen, 1960, [33, 65-66] for the changes implemented)[3] to achieve an equilibrium in the benchmark year for this economy. He then applied production functions for firms and demand functions for households, as well as elasticities of substitution for capital and labor across sectors, allowing for capital to move freely, but be rewarded differently within the sectors (see [47, p. 10]). He assumed the economy to be in full employment, justifying this with the "institutional" conditions in Norway in the 1950's [33, p. 19], and fixed investment and exports exogenously. Equilibrium was then defined by what Johansen referred to as a "book keeping relationship" [33, p. 48]: $Y_i = \sum_j y_{ij} + C_i + Z_i$, the equality of gross production Y_i by a sector with the sum of intermediate inputs of good i demanded by sectors j, $\sum_j y_{ij}$, household demand for good i, C_i, and an exogenous demand aggregate Z_i for good i consisting of government demand,

3 As with all data, certain corrections have to be made to satisfy the restrictions imposed by economic theory. However, the concern here is that an equilibrium is *facilitated* in order to be able to apply the CGE modelling approach based on economic theory, rather than an equilibrium situation being represented by the data set chosen.

investment, exports, and competitive imports from the sectoral goods, where investment plus exports equaled savings plus imports as laid down by the national accounting framework. Given the equilibrium data set, the model devised by Johansen found an equilibrium in the macro balancing equations [47, p. 10].

However, there is no reference to be found in Johansen's work to the Arrow-Debreu general equilibrium literature or theory. Furthermore, the optimality of the model solution (which underlies a competitive equilibrium in the Arrow-Debreu framework) is not guaranteed in Johansen's framework, as

> a development path satisfying the equations of our model is not necessarily an optimal one, even though the model is characterised by marginalistic adaptations (see [33, p. 172]).

However, he talks of optimality in reference to "optimal for given values of exogenous variables" [33, p. 172], and states that an optimal expansion pattern could be described in the formal framework of the model (ibid.). Basically, Johansen never claimed to be connected with the Arrow-Debreu general equilibrium theory (see [47, p. 11]), and what he has achieved was a "balanced macro model (not a *balanced growth model*), successfully solved with linear equations" ([47, p.12], his emphasis). Johansen furthermore undertook an analysis of the parameter values he chose for his elasticities of substitution relating to empirically observed data to verify his choices.

What drove Johansen's model, basically, was the choice of exogenous variables, which would determine the level of economic growth, and the model would spread the growth throughout the sectors, according to the parameter values chosen [47, p. 11]. Therefore, growth is not an endogenous variable in this model (nor in the model described in detail in chapter 4). Johansen wanted to explain growth differentials between sectors, and to achieve this he "invented a method for economic analysis through a combination of the national accounts, macro economic balancing equations and input output analysis" [47, p. 12].

All of the CGE models described in later chapters follow similar patterns: first, an equilibrium data set is constructed, macro balancing equations are set up, and via a choice of exogenous variables and a numerical procedure the model is solved. This approach does not share the theoretical rigour of the Arrow-Debreu model shown in Section 2.1.1, as the equilibrium condition for the base year is assumed, not guaranteed, with all results then depending on this presupposed initial benchmark equilibrium, disregarding to a certain extent whether a solution obtained with parameters based on a presupposed benchmark equilibrium guarantees optimality (the optimal competitive equilibrium). Furthermore, the empirical data for the parameter values chosen in the model, i.e. exogenously assumed elasticities of substitution and other parameters, are hard or in some cases even impossible to estimate empirically, and therefore sometimes vary greatly across the literature.

The model building approach depicted above proved to be more efficient than the solving procedure developed by Scarf (1973, [59]) and his scholars depending on fixed point theorems. Departing from the linear solving procedure used by Johansen, CGE modellers nowadays mostly use Newton type algorithms, where convergence to an equilibrium solution is not always guaranteed, but "applied modellers who have used them seem not to have encountered nonconvergence difficulties" (Shoven and Whalley, 1992, [63, p, 67]). This further demonstrates the applied nature of CGE modelling practice.

Nonetheless, CGE models provide a useful tool for policy analysis, as they can simulate the impact of changes of exogenous variables throughout the economy considering endogenously the change of equilibrium prices and quantities for all sectors of the economy. With this said, the process transforming a stylised model into an applied economic model is set out shortly.

2.2.2 What is a Computable General Equilibrium Model?

As can be inferred from the statements above, CGE models are to some point rooted in economic theory, most of all in the competitive general equilibrium model set forth by Arrow and Debreu at the end of the 1950's and beginning of the 1960's (see e.g. Arrow and Debreu 1954 [3], Debreu 1959 [18], or Arrow and Hahn 1971 [4]). On the other hand, as can be seen from section 2.2.1, their main idea is a rather pragmatic one, namely the analysis of sectoral growth differentials or the effects of the implementation of exogenous policy instruments on the economic system, amongst others.

The simple procedure of equipping a model with real-world data and choosing exogenous elasticities moves a CGE model away from rigorous theory to a certain extent. Based on this rigorous economic theory, however, CGE models could be said to be tending towards a more pragmatic approach of trying to depict the economic system, considering equilibrium price and quantity formation for goods and services endogenously and based on the optimisation of a representative household agent, but depending on exogenously chosen parameters and, to some point arbitrarily selected, data. Regarding this benchmark data set, one never knows whether the highly theoretical concept of equilibrium is or was actually satisfied by the dataset chosen.

To explain the nature of a CGE model to a before unexperienced reader, one of the first attempts for this thesis to grasp the idea of a CGE model and explain it was to present something like a *"generic"* CGE model. However, it seems safe to say that the generic CGE model *is* the competitive general equilibrium model as explained in short in section 2.1.1, but bearing in mind the concerns about the relation between CGE models and general equilibrium theory voiced in the introduction.

A CGE model would then go ahead and distort or amend this model by assuming concrete functional forms for the utility and production functions, in many cases choosing CES functions, and adding more conditions and constraints that do not allow the model to be analytically solvable anymore. The next step is to find a computerised algorithm that gives the modeller an approximate, numerical solution for the static equilibrium conditional on a benchmark dataset that will serve to determine the parameters of the model, the number and qualitative influence of which of course are defined by the concrete functional forms chosen for the model. The conclusion from this train of thought is the following: there is no so called *"generic"* CGE model, only specific ones, as a CGE model is always a "variation" of a Walrasian general equilibrium model tailored to a specific economic or policy question in the static case, and often a "variation" of a classic Ramsey model in the dynamic one.

Thus, this section shall rather serve to describe how to construct a CGE model from a stylised model, the goals one wants to achieve with the thereby constructed model, and what is necessary to do that. The prime motivation to formulate and solve such a model is that CGE models often like to distinguish themselves from stylised models stating that they incorporate "real-world complexities" (Böhringer et al., 2003, [10]). The sensibility of constructing a CGE model thus being assumed, the following section wants to give some hints as of how to build an applied CGE model from a stylised economic model.

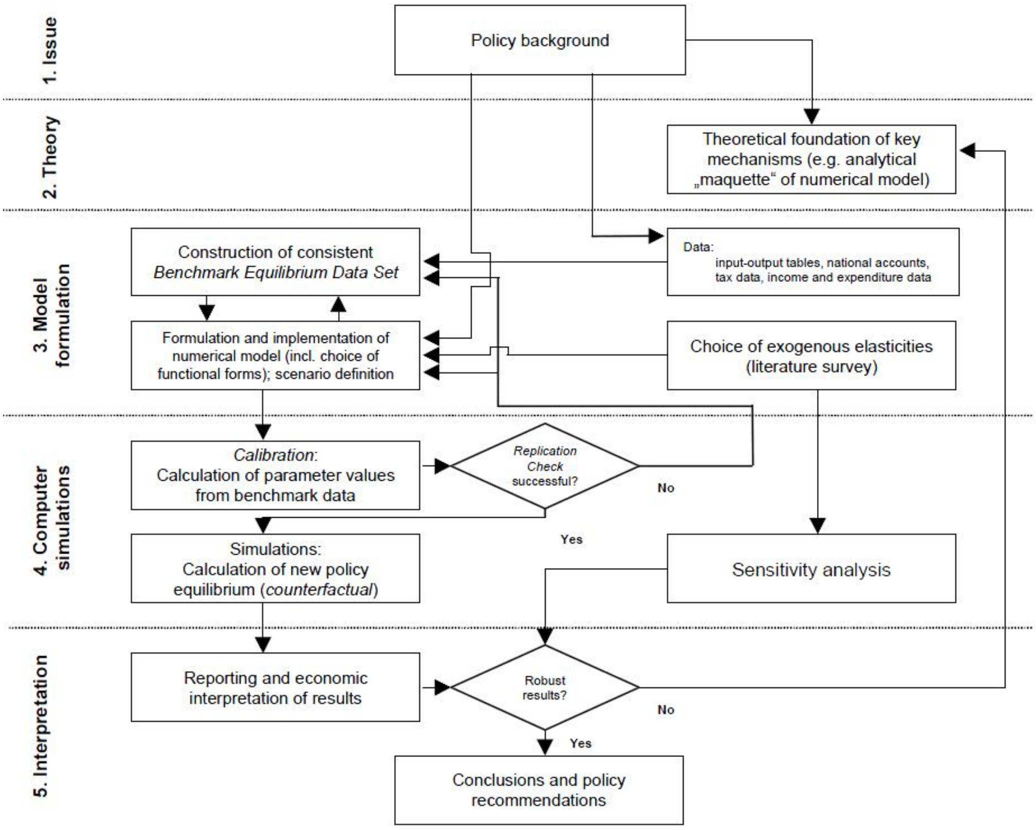
The usual approach to solving analytical models in the theoretic literature is the following (Böhringer et al., 2003, [10, p. 2]): deriving the total differential of the market equilibrium conditions of the model, and then solving this equational system for exogenous changes of variables that are of interest to the modeller. However, for many policy questions this approach will not be feasible because the model needed to depict these relationships, as mentioned above, is not analytically solvable any more. These complexities might include a more detailed production structure, heterogeneous household agents, an elaborated tax system, etc. Thus, the analytical approach might provide qualitative insights, but its application to actual political policy questions will be limited (Böhringer et al., 2003, [10, p. 2]).

Departing from the policy question one wants to quantify, any modeller can safely follow a step-wise approach to produce a numerically solvable CGE model. The five main steps involved in constructing and using CGE models are the following (Böhringer et al., 2003, [10, p. 2]):

1. The policy measure to be analyzed is examined carefully → the suitable model designs, also according to scope and availability of the required data, are chosen.
2. Rigorous economic theory (maybe by drafting a simple stylised analytical model) should make the key economic mechanisms that act as a driving force of the economic results of the model clear.
3. Construction of a data base for the model, the mathematical model formulation and software implementation, scenario definition (choice of exogenous parameter values, e.g. elasticities).
4. Calibration.
5. Simulations, counterfactuals (scenarios), sensitivity analysis and economic validation.

The process described above is visualised in figure 2.1.

Figure 2.1: General Equilibrium Analysis



Source: Böhringer et al. (2003, [10, p. 3])

One of the most important terms to be elaborated on maybe is the *scenario definition.* This involves the setting up of alternative policy instruments and strategies that can stimulate changes in comparison to the benchmark or reference situation. The choice of the concrete functional forms, the exogenous parameters (elasticities of substitution, etc.) will of course essentially determine the outcome of any policy or scenario simulation (Böhringer et al., 2003, [10, p. 2]). The most common and widely applied procedure to choose these exogenous parameter values is known as **calibration** (see Mansur and Whalley, 1984, [42] for more detailed information on the calibration procedure). The calibration procedure is outlined in section 2.2.3 below.

The most important check of consistency that the model has to fulfill is the replication of the benchmark equilibrium as a solution of the model without computational work (Böhringer et al., 2003, [10, p. 3], see also section 2.2.3 below). Only after this step has been achieved, the modeller can proceed to policy analysis.

As soon as the model has been constructed, the way to use it is via **business as usual** and **policy scenarios or simulations**. The business as usual scenario is simply the continuation of the status quo, the policy scenario changes aspects of the initial equilibrium to obtain a different economic development which is then seen as the *economic effects of a policy measure.* This involves, amongst others, changes in exogenous variables or single parameters (Böhringer et al., 2003, [10, p. 3]).

In the static case this means that the initial equilibrium, the benchmark obtained from the benchmark dataset used for calibration (see the next section 2.2.3), is the same as the model solution, for the economy is already assumed to be in the state of equilibrium. A policy scenario would exogenously distort a parameter, such as a tax or an investment parameter, or maybe an elasticity, and then calculate a different state of equilibrium. The differences between these equilibria would be the economic effect of the policy measure.

The comparison can provide information on the changes in economic variables such as the sectoral composition of the economy, employment, production, household consumption, the relative price system, investment, tax revenues, etc. (cf. Böhringer et al., 2003, [10, p. 3]).

In a dynamic model, the business as usual scenario would be defined by exogenously set quantity and price reference paths, i.e. exogenous growth path and inflation rate. Departing from the benchmark data set, these reference paths would lead to an adaptation process until a final steady state in infinite time or at a defined end of the modelling period. The policy scenario would distort an initial parameter, or another parameter maybe even changing during every time period of the model, and calculate a new reference path leading to a new final steady state. The effect of the policy measure would then be determined by the differences between the business as usual scenario and the policy scenario in every period.

At the end of the modelling process, the results have to be interpreted according to "sound economic theory" (Böhringer et al., 2003, [10, p. 3]). As the modeller had to choose exogenous elasticities and a single data base-year observation[4], which crucially determine the mechanics of the model, the outcome of the policy simulation will strongly depend on these.

Thus, comprehensive *sensitivity analysis*, which essentially means a couple of subsequent model runs with varying exogenous (key) parameters and elasticities, or maybe also different assumptions on economic incentives for the agents within the model, validates the model results and enables the modeller to provide credible policy advice (see Böhringer et al., 2003, [10, p. 3]). Furthermore, if one has constructed a dynamic model, a backcast of historic data might be a good idea. If the model does not depict past economic relationships, one might respecify the parameters, elasticities and/or economic incentives.

However, since past economic development strongly depends on exogenous shocks, which often are subject to chance (probability distributions) or maybe real-world complexities involving irrationalities in human behavior and other factors that are mostly not depicted in (deterministic) CGE models, this procedure of backcasting and specifying parameters accordingly is certainly limited up to some point.

4 Or an average of years, which brings about its own problems such as whether a couple of years can be interpreted as a single year, and how one should consider the economic cycles over certain time periods.

Finally, what is the relation between a CGE model and the text-book example of a representative consumer maximising its welfare (thus being equal to social welfare)? As shown in the publication on the 1-2-3 model (Devarajan et al., 1990, [19, p. 629, Table 2]), itself one of the classic textbook examples of a CGE model, a CGE model in an uncomplicated case as the 1-2-3 model corresponds to a simple programming problem (see Devarajan et al., 1990, [19, pp. 632f] in the text, Figure 1 in [19, p. 632] for a graphical representation). Here, it is emphasised that a single-consumer general equilibrium model can be represented by a programming model where a representative consumer maximises utility, subject to the constraints that correspond to some of the equations of the CGE model. Because the consumer is representative, the utility of this consumer is equivalent to social welfare (Devarajan et al., 1990, [19, p. 633]). For such a model, the shadow prices of the constraint equations would then correspond to market prices in the CGE model (Devarajan et al., 1990, [19, p. 633]).

2.2.3 Calibration

As already mentioned in section 2.2.1, many parameter values for a CGE model have to be estimated from the benchmark data set and other data sources. To determine values for the free parameters, the main requirement is a consistent data set of one year, or a single observation that depicts an average over a range of years of economic data, and the exogenous elasticity parameters, where one mostly has to rely on econometric estimates from the literature (Böhringer et al., 2003, [10, p. 3]).

In any CGE model, the quantity and function of parameters range, depending on the functional form of the equation, from single parameter equations, over two-parameter Cobb-Douglas production functions to the three-parameter CES (CET)[5] equations or even more complex functional forms. Econometrically, it is nearly impossible to measure all of these parameters, even if the respective data series were available (Robinson et al., 1999, [51, p. 24]). Thus, the standard CGE procedure is to parameterise the model to the benchmark data set, in this case the *SAM (Social Accounting Matrix)*[6] for the base year, supplemented by additional sources, mainly from the literature, and by econometric estimates, if this is possible.

This parameterisation of the model, in principle already set out in the previous section, is commonly called **calibration**. Specifically, for the class of multi-sectoral CGE models that will be focused on in this thesis, using mostly CES or Cobb-Douglas production functions, it "involves determining a set of parameters and exogenous variables so that the CGE model solution exactly replicates the economy represented in the SAM" (Robinson et al., 1999, [51, p. 25]).

5 CES - Constant Elasticity of Substitution, CET - Constant Elasticity of Transformation
6 The so-called SAM or social accounting matrix is a benchmark dataset for an economy. The SAM concept will be further described in section 4.1.

The main procedure for calibration is to *assume that the base year of the model is also the base year for all prices.* Thus, **all prices are set to unity**[7], and given that the benchmark values of the *variables* are known from the benchmark data set, we can solve the model equations for the share parameters of the production and trade functions. This is possible because, if the data from the SAM actually add up, total factor payments equal total value added in each sector. Together with a competitive set up this implies that *constant returns to scale* prevail in this economy, meaning that all share parameters in the production and trade aggregation functions *add up to one.* We then have only one unknown in the functions, and the share parameter, with all other variables being set exogenously, *can be directly solved for* (or taken directly from the benchmark data set). Similarly, we can solve for the shift parameters in the respective equations (see [51, p. 27f]).

Calibration thus verifies whether the model can re-create the benchmark equilibrium, which should be replicable without computing power (Böhringer and Rutherford 2003, [10]). If this is not the case, the modeller has to go back to the model equations and respecify the model. The first basic consistency test is whether Walras' Law is satisfied for the model. The second is whether the benchmark data set can be verified as a solution by the modelling system (e.g. GAMS[8]). The third is whether the model as a whole is homogeneous of degree zero, i.e. whether a multiplication of all prices with the same scalar leaves the model solution unchanged (remember that a CGE model only determines relative prices). This can be obtained by multiplying the price of the numéraire good with a certain scalar. The result should be that all prices rise by the same proportion, but that quantities remain unchanged. If this is not the case, the model is not homogeneous of degree zero, and the modeller should look at whether all price relations are depicted correctly in the equational specifications (see [51, p. 28f]).

Furthermore, it is very important to mention that calibration is a **deterministic** procedure, thus not allowing for statistical tests of the model specifications (Böhringer et al., 2003, [10, p. 3]). This is another point why the backcasting procedure described above has clear limits when it comes to deterministically specified CGE models.

2.2.4 Critique

Any multi-sectoral CGE model might offer a good insight into selected policy questions on a quantitative scale, modelling the behaviour of all economic agents in a complex and logically comprehensive way. The question is whether these insights can be trusted.

What can always be criticised about a CGE model, as already mentioned in section 2.2.1, is the fact that the assumption of the economy being in equilibrium in the benchmark year seems rather constructed to be able to calibrate the model than rigorously justified. As could

7 Setting all prices to unity in the benchmark year raises some questions about the validity of the concept of a numéraire in this context. This issue is discussed in section 4.3.2.

8 The General Algebraic modelling System, a modelling software. Please see section 3.1 for further information on GAMS.

be seen in the previous section, the solving process of the model critically hinges on this assumption.

What should be also considered is the applied nature of CGE modelling, and that the quantitative magnitude of a reaction to an external policy shock is analysed only subject to very narrow constraints under certain assumptions.

This maybe casts some doubts on some of the arguments e.g. given by Robinson et al. (1999, [51]) as of why to construct numerical models from stylised ones, such as:

> Although stylised models may tell us the direction of change in response to a tariff increase, often we are concerned more with the magnitude of the change. Policymakers wish to know, 'By how much will exports and imports decline if we raise import tariffs?' (Robinson et al, 1999, [51, p. 2])

One has to be very careful with such statements, especially if the model is still quite abstracted from real world complexities (even if maybe claimed differently). In the opinion of the author, a CGE model mainly serves to learn about the mechanics of the economy under certain conditions. One, then, should be very careful when directly interpreting the quantitative dimension of projected changes due to exogenous policy shocks. Still, the insights of such models might be of great use to a policy maker, especially regarding the qualitative dimension (what direction of change) of a policy analysis, taking into account complexities too prohibitive for analytically solvable models.

CHAPTER 3

Top-Down Bottom-Up Energy Policy Modelling

3.1 Top-Down Bottom-Up CGE Models

The structure of a top-down general equilibrium model as described in chapter 2 can now be applied to assess the economic implications of energy policies. As energy policies will often coincide with environmental regulations, the increased importance of the quantitative assessment of the costs of stricter environmental regulations will require modelling tools that can fulfill this purpose.

As global climate change is a multidisciplinary subject, which was taken on by economists only after physical scientists and ecologists, models concerned with the assessment of the implications of environmental policies have to incorporate not only economic relationships, but also a certain amount of technical detail in the production and consumption process of energy. It is not hard for the scientific community to agree on "win-win" strategies, e.g. where greenhouse gas emissions and economic costs are reduced at the same time. However, to reach an understanding of "cost-benefit" strategies - international protocols in which one set of nations or industries incur short or medium term costs that benefit other nations (see Manne, 2005, [38, p. 255]) or industries is a cumbersome process for the scientific community, not to speak of the political dimension of the problem.

Thus, the modelling of environmental and energy policies is a delicate subject both in regard to the political and the economic sphere. Generally, a substantial reduction in emissions will pose considerable costs in one or several sectors of an economy or a nation. The subsequent change in the relative price system will cause general equilibrium effects throughout the whole economy. Therefore, the analysis of energy and environmental policy within the framework of a CGE model is a useful way to quantify costs and benefits of a given regulation (Conrad, 1999, [15, p. 1060]). Single- and multi-market partial equilibrium models will allow to estimate the short run costs of energy and environmental policy measures, considering substitution processes for production and consumption as well as market clearing conditions on the goods market. CGE models, additionally, will take account of the adjustments in all sectors of the economy, and depict the ties between the factor incomes and consumer expenditures (see Conrad, 1999, [15, p. 1060]).

The two modelling approaches representing the technically oriented, partial equilibrium models and the economically focused general equilibrium models are the so called **bottom-up** and **top-down** modelling paradigms (see International Panel on Climate Change (IPCC), 1996, [32]).

Bottom-up models focus on current and prospective competition of energy technologies in detail, on the supply-side of the economy (possibilities of substitution of primary forms of energy in the production process) and on the demand-side (potential for energy efficiency in final uses and fuel substitution). These models assist in depicting how different technologies create substantially different environmental results. However, their weaknesses lie firstly in an unrealistic illustration of decision making on a micro level by firms and consumers as regarding the selection of technologies used to produce and consume goods such as energy. Secondly, they usually neglect macro-economic feedback cycles for different structures of energy use and energy policies when it comes to questions of economic structure, productivity and trade issues affecting the rate, direction and distribution of economic growth (Hourcade et al., 2006, [30, p. 2]).

Top-down models incorporate policy implications in regard to public finances, economic competitiveness and employment. Since the end of the 1980's this class of models has been dominated by CGE models, showing the decline of the influence of other macroeconomic paradigms, such as disequilibrium models (Hourcade et al., 2006, [30, p. 2]). As shown in chapter 2, CGE models feature microeconomic optimisation behaviour of economic agents, inducing corresponding behavioural responses to energy policies involving substitution of energy for other intermediate inputs or consumption goods. They account for initial market distortions, pecuniary spillovers, as well as income effects for economic agents such as households and the government (Böhringer, Rutherford, 2008, [8, p. 575]).

CGE models, however, are usually quite aggregated on a technological scale, so that they do not generally allow for technological options beyond the current technological practice. As the substitution elasticities are mostly measured from historical data series, there is no guarantee that these will remain the same in the face of technological changes. Thus, the incentive to using environmentally friendly technologies, e.g. exhibiting low greenhouse gas emissions, could be underestimated. Also, because of a lack of detail on the technical side, the projections of energy use and supply made by top-down models are possibly not underpinned by a technically feasible system (see [30, p. 2f]). This may lead a top-down model to violate some basic physical restrictions such as the conservation of matter and energy (Böhringer, Rutherford, 2008, [8, 575]).

The debate regarding the dichotomy between top-down and bottom-up models first began in the course of discussions about the efficiency-gap in the 1980's and early 1990's (see Grubb et al., 1993, [25]), which was concerned with a possible efficiency gap between current and future energy use. Competitive markets, mostly assumed for the setup of top-down models, would deny the possibility of the existence of such an efficiency gap - that society could profitably increase the efficiency in energy use. Bottom-up models, however, suggested a series of "no-regret" ("win-win") possibilities to increase energy efficiency (see [30, p. 3]). This debate still is not completely resolved according to Hourcade et al. (2006, [30]), thus bearing implications for the construction of models to analyse energy and environmental policies.

As can be seen, the integration of the top-down and bottom-up approaches to energy policy modelling is highly desirable, explaining the recent efforts to construct hybrid models described in Hourcade et al. (2006, [30]). These modelling efforts can be divided into three overarching categories (Böhringer, Rutherford, 2008, [8, p. 575f]):

Firstly in the so-called **"soft link"** approach, bottom-up and top-down models that have been developed separately can be linked to form a hybrid model. This approach is being followed since the 1970's, however, the coherence of the hybrid model is threatened because of inconsistencies regarding behavioral assumptions and accounting concepts within the "soft-linked" models, most probably occurring because the two formally independent models cannot be reconciled without grave difficulties. Examples for models of this type can be found in Hoffman and Jorgenson (1977, [27]), Hogan and Weyant (1982, [28]), Drouet et al. (2005, [22]), or Schäfer and Jacoby (2006, [60]), amongst others.

Secondly, it is possible to concentrate on one type of model - either the top-down or bottom-up part - and employ a "reduced" form of the other. A well-known example of this type is the ETA-Macro Model (Manne, 1977, [41]) and its follow-up MERGE (Manne, Mendelssohn and Richels, 2006, [39]). Here, a detailed bottom-up system for energy provision is coupled with a highly aggregated one-sector macroeconomic model of production and consumption within one single framework of optimisation. Other examples of modelling efforts using the same approach can e.g. be obtained from Bahn et al. (1999, [5]), Messner and Schrattenholzer (2000, [45]), and also Bosetti et al. (2006, [11]).[1]

The third approach, which is also followed by Böhringer and Rutherford (2008), is to completely integrate top-down and bottom up models in a single modelling framework formulated as an **MCP (Mixed Complementarity Problem)**. This modelling innovation relies on the development of powerful solving algorithms in the 1990's (Dirkse and Ferris, 1995, [20]) and their implementation in GAMS (General Algebraic modelling System)[2] software (Rutherford, 1995, [55]). In an earlier paper, Böhringer (1998, [7]) already showed how the complementarity format can be employed to formulate a hybrid description of the economy in a CGE model, where the energy sectors are represented by a bottom-up activity analysis, and the other producing sectors of the economy are characterised by regular (mostly CES) production functions typical for a top-down CGE model.

1 For further information on energy and environmental models, one can also consult the documentations for the WITCH [12], PRIMES [14], MARKAL [37], MERGE [40], and MESSAGE [46] models.

2 For more information on the GAMS software package, please visit `www.gams.com` and see Brooke et al. (1996, [13]).

This paper by Böhringer (1998, [7]) demonstrated that the difference between bottom-up activity analysis, which essentially is a partial equilibrium activity analysis (Koopmans, 1951, [36]) providing a "precise technological description of the energy system from primary energy processing via several conversion, transport and distribution processes to final energy use systems while neglecting the interactions with the rest of the economy" (Böhringer, 1998, [7, 234]) and elasticity-based CGE models is not a real dichotomy, but simply refers to the level of aggregation and the extent of the ceterus paribus assumptions. Thus, economic theory provides a unifying concept for both approaches (Böhringer, 1998, [7, p. 235]). This has been shown in previous methodological papers (Weyant, 1985, [70] and Mathiesen, 1985a, [43]).

Mathiesen (1985a, [43]) in particular demonstrates how to formulate a general economic equilibrium for an Arrow-Debreu economy in a complementarity format. Böhringer and Rutherford (2008, [8]) then proceed to show that "complementarity is a feature of economic equilibrium rather than an equilibrium condition per se" (Böhringer, Rutherford, 2008, [8, p. 576]). The complementarity format allows to cast an equilibrium in the form of weak inequalities, establishing a logical connection between prices and market clearing conditions. The properties of this format then make it possible to directly integrate bottom-up activity analysis into a general equilibrium top-down representation of the whole economy (see Böhringer, Rutherford, 2008, [8, p. 576]). Other advantages of the mixed complementarity format are that the so-called *integrability conditions* (see Pressman, 1970, [49, p. 308ff] or Takayama and Judge, 1971, [65]) inherent to economic models cast as optimisation problems (as the integrability conditions follow from the first order conditions of the optimisation procedure) can be relaxed (see [8, p. 576]).

The integrability conditions can be written in the following form (Pressman, 1970, [49, p. 308]):

$$\frac{\partial d_i(p)}{\partial p_j} = \frac{\partial d_j(p)}{\partial p_i} \qquad \forall i,j \tag{3.1}$$

where $p_{i,j}$ are the prices of goods i,j and $d_{i,j}(p)$ are the associated demand functions (quantities) for this good. An economic intuition for this condition is that the components of the cross-price effects, i.e. the substitution and income effects that determine the change in demand for one commodity when the price of another commodity changes, are equal (see Pressman, 1970, [49, p. 310]). Generally, non-integrability of an economic model also implies that there may be no optimisation model that leads to the complementarity problem. This will typically be the case when distributional and equity issues are in the focus of the modeller (Mathiesen 1985a, [43, p. 144]).

The first order conditions resulting from primal or dual mathematical programs (from the optimising approach to find an equilibrium) require efficient allocation, ruling out common second-best solutions such as equilibrium with taxes, distortions resulting from these taxes, as well as market imperfections and failures. Thus, the optimisation approach is restricted when it comes to assess the implications of economic policies [8, p. 576]. The complementarity format is more flexible in this regard. Furthermore, non-integrabilities in real world applications relate to empirical evidence that the individual demand functions do not only depend on the prices of commodities, but also on the factor incomes earned by this individual [8, p. 576]. In cases as these, the demand functions cannot be integrated into an economy-wide utility function (see Rutherford 1999b, [57]), and thus income effects matter when solving the model (see Hurwicz, 1995, [31] or Russell, 1995, [52]). As follows from equation (3.1) above, an optimisation problem which is associated to an MCP and that can be applied to solve for equilibrium prices and quantities will exist only if the matrix of cross-price elasticities is symmetric (see [8, p. 576]).

The following section 3.2.1 shows how to cast an Arrow-Debreu economy in the MCP format. Consequently, section 3.2.2 spells out an Arrow-Debreu economy in a complementarity format. Section 3.2.3 provides the model structure to integrate a bottom-up energy sector into the top-down general equilibrium model. A dynamic formulation of the model is set forth in section 3.2.4. After this, in chapter 4 the model developed at the *IHS (Institute for Advanced Studies in Vienna)* on the basis of the model taken from the literature (Böhringer, Rutherford, 2008, [8]) is described in sufficient detail regarding data and model equations.

3.2 Methodology and Structure of an Integrated Hybrid Dynamic CGE Model in Complementarity Format

3.2.1 The Formulation of Economic Equilibrium as a Mixed Complementarity Problem (MCP)

As was shown by Mathiesen (1985a, 1985b, [43] [44]), a general (Walrasian) equilibrium problem can be reformulated as an MCP, which offers a more flexible format both regarding the complexity of the model and computing efficiency. So far, we are only concerned with a competitive economy without any price distortions. However, according to Mathiesen (1985b, [44, p. 1228]), extensions to distortive ad valorem taxes, a public sector, a foreign sector with imports and exports, institutional constraints on prices, or non-competitive behaviour (imperfect competition etc.) could be easily incorporated into the model.

A complementarity problem can be written down in the following form (Mathiesen, 1985a, [43, p. 144]):

$$
\begin{aligned}
&(CP) \qquad Find\ z \in \mathbf{R}^l\ that\ solves \\
&F(z) \geq 0,\ z \geq 0 \quad and \\
&z^T F(z) = 0.
\end{aligned}
$$

In order to cast a Walrasian general equilibrium model as an MCP, we have to start from the following definition by Scarf (1973, [59, p. 100, Definition 5.1.3]

5.1.3 [Definition] A price vector p^* of commodities $i \in \{1, \dots, n\}$ and a vector of activity levels y^* of sectors $j \in \{1, \dots, m\}$ constitute a competitive equilibrium if:
a. Supply equals demand in all markets, or

$$d(p^*) = Ay^* + w \text{ ; and} \tag{3.2}$$

b. production is consistent with profit maximisation in the sense that

$$\sum_i p_i^* a_{ij} \leq 0 \quad \forall j, \text{ with equality if } y_j^* > 0. \tag{3.3}$$

where the matrix A with components a_{ij} is an activity analysis matrix describing production technology where inputs are denoted with a negative sign, outputs with a positive sign (which according to Scarf (1973, [59, p. 99]) can easily be extended to production functions or more general sets if the usual convexity assumptions are fulfilled), $d(p^*)$ is the vector containing the market demand functions for the commodities $i \in \{1, \dots, n\}$, and w is the vector of initial resource endowments. If one wants to assure the existence of a competitive equilibrium for this economy, one has to assume that the set of activity levels that induce non-negative net supply of all commodities (Scarf 1973, [59, p. 100])

$$\{y|\ y \geq 0,\ Ay + w \geq 0\}, \tag{3.4}$$

is a bounded set, see equation (2.4). The proof can be found in Scarf (1973, [59, chapter 5]). What is being said above in short rephrases the market clearance condition we have encountered in chapter 2, equation (2.5), and the zero profit condition, equation (2.6).

To describe a general equilibrium model, it is often stated according to three basic components: the total endowments of the economy, the production technology and the preferences of the agents in the economy. All individuals in the economy act as price takers, i.e. they regard the prices in the economy as given, and they behave competitively. This is a standard Arrow-Debreu (1954, [3]) problem, which could be presented (exemplarily here the two-goods case is shown) as a constrained optimisation problem subject to income, technology, and feasibility constraints (Paltsev, 2004, [48, p. 3]):

$$\begin{aligned}
&max\ U(X,Y)\ s.t. \\
&p_x X + p_y Y = wL + rK \\
&X = F(K_x, L_x) \\
&Y = G(K_y, L_y) \\
&L = L_x + L_y \\
&K = K_x + K_y
\end{aligned} \tag{3.5}$$

where $U(X,Y)$ is a utility function (of the representative household); p_x and p_y are prices of the respective goods X and Y; r is the rental price of capital; w is the price of labour (wage rate); K_x, K_y and L_x, L_y are capital and labour used in the production of the goods X and Y, respectively. This is a pretty standard textbook optimisation problem, here it is possible to solve the optimisation problem "directly". This is usually done via Lagrange-Multipliers, in GAMS it can be solved as an NLP (non-linear programming) problem.

However, non-integrable models cannot be represented as a single NLP problem (since the resulting allocation would be inefficient, see Rutherford 1995, [55, p. 1308]), which is mainly the case with the presence of several consumers, taxes, or other market distortions. Thus, turning an NLP into an MCP enlarges the possibilities to construct economic models, as NLP problems are a subset of MCP (see Paltsev, 2004, [48, p. 3], or Cretegny and Rutherford, 2004, [17, p. 1]).

Therefore, formulating an economic equilibrium as an MCP makes the problem both more efficient and more transparent to solve. To do this, one has to rewrite Definition 5.1.3 from Scarf (1973, [59]) for an economic equilibrium in order to obtain an MCP that allows to find the optimal solution, i.e. the equilibrium of the system (see Mathiesen, 1985a, [43, p. 147ff]), which will be done in the following.

As before, consider a *closed economy with production.* There are n commodities and m activities exhibiting *constant returns to scale* production. If there are production sectors with *decreasing* returns to scale, the profit maximising vector of supplies and demands is derived and added to the sum of household demands in order to obtain net market demands (Mathiesen 1985a, [43, p. 147]). Thus, the household sector "swallows" the net demand of these decreasing returns to scale production sectors, and we can proceed with finding an equilibrium as though they did not exist.

Now, following [43, p. 147], let for
$i \in \{1, \dots, n\}$ commodities and
$j \in \{1, \dots, m\}$ production activities
$w = (w_i)$ denote the vector of endowments,
$p = (p_i)$ denote the vector of prices,
$d(p) = (d_i(p))$ denote the vector of net market demand functions,
$\Pi(p) = (\Pi_j(p))$ denote the vector of unit profit functions (for the constant return to scale activities), i.e. the difference between unit revenue and unit cost, from which one can derive

$$a_j(p) = (a_{ij}(p)) = (\partial \Pi_j(p)/\partial p_i), \tag{3.6}$$

which, by Hotelling's Lemma, see equation (3.19), is the vector of profit maximising input-output coefficients (the "production technology") scaled to *unit production* in activity j. Thus, the matrix A tells us how the goods are produced from intermediate inputs, capital and labour, with negative entries denoting inputs, positive entries denoting outputs. Finally, we have
$y = (y_j)$ denote the vector of activity levels.

It is assumed that the demand functions $d_i(p)$ and the profit maximising technological coefficient-functions $a_{ij}(p)$ are point-to-point and continuously differentiable. Furthermore, for notation, we have

$A = A(p) = [a_1(p), \dots, a_n(p)]$ denote the technology matrix of input-output coefficients consistent with unit production.

Furthermore, since the unit profit function is homogeneous of degree one in all prices (as the system only determines relative prices, unit profit will only rise nominally if all prices increase by the same proportion, not in real terms), we have (following from Euler's homogeneous function theorem)

$$\Pi_j(p) = (\nabla \Pi_j(p))^T p = a_j^T p. \tag{3.7}$$

This relation is used in (3.8) below. Now, Mathiesen (1985a, [43, p. 148]) proposes the following equivalent definition to the one by Scarf (1973, [59]) given above[3]:

> A price vector p^* and a vector of activity levels y^* constitute a *competitive equilibrium* if:
>
> No activity earns a positive profit: $\quad - A^T p^* \geq 0.$ (3.8)
>
> No commodity is in excess demand: $\quad w + Ay^* - d(p^*) \geq 0.$ (3.9)
>
> No prices or activity levels are negative: $\quad p^* \geq 0, \quad y^* \geq 0.$ (3.10)
>
> An activity earning a deficit is not run, and an operated activity runs at zero profits: $\quad (A^T p^*)^T y^* = 0$ (3.11)
>
> A commodity in excess supply has a zero price, and a positive price implies that supplies equal demands: $\quad {p^*}^T (w + Ay^* - d(p^*)) = 0$ (3.12)

The demand functions will normally be defined in such a way that they are consistent with individual household utility maximisation, i.e. $d_i(p) = \sum_h x_i^h$, where x_i^h is the h^{th} household's utility maximising demand of commodity i (Mathiesen 1985b, [44, p. 1230]). Households' excess demands are now given by $d(p) - w$. If these demands for products satisfy each household's budget constraint and if we assume non-satiation, then we have $p^T d(p) = p^T w$. This means nothing more than that all income is spent, which is yet another denotation of *Walras Law* that we have already encountered in section 2.1, equation (2.2).

Thus, the demand functions are homogeneous of degree 0, and the system determines relative prices only, as always in a Walrasian equilibrium system. This means that if p^* is an equilibrium price system, λp^* is also an equilibrium system for any scalar $\lambda > 0$. Therefore, we are free to normalise prices in terms of a numéraire good (see Mathiesen 1985a, 1985b, [43, p. 148] [44, p. 1229]).

3 However, this definition does not include disposal activities in the matrix A, which are contained in the matrix also named A in Scarf's definition. This leads to the inequality in condition (3.9).

In order to characterise market distortions that often are part of economic reality, one can now introduce a 'cost-coefficients' matrix B, which is different from the technology coefficients matrix A [43, p. 148]. Without any distortions in the market system, $B = A$. For the case of ad-valorem taxes (t_{ij}) which are put on the input factor of commodity i to activity j, the cost coefficients b_{ij} can be defined as $b_{ij} \equiv (1 + t_{ij})a_{ij}$. Another problem that can be fit in this framework is the invariant capital stock problem, see Hansen and Koopmans (1972, [26]) and Mathiesen (1985a, [43, p. 148], 1985b, [44, p. 1231]).

According to Mathiesen (1985a, [43, p. 148f]), this format allows for so called "mixed models", where one particular sector (Mathiesen here explicitly names the energy sector as an example) can be depicted by a linear activity analysis matrix or nonlinear programming constraints, while the other sectors of the economy are described by neoclassical production and demand functions. This feature of the MCP format is then exploited for a hybrid bottom-up top-down energy model, as will be described in the next chapter 3.2.2.

Before proceeding, one should stress the similarity between a general equilibrium (GE) problem and a partial equilibrium (PE) problem in the MCP format. The only difference between them is a vector of exogenously given unit costs c [43, p. 149]. In a PE, this vector denotes purchases from outside of the sector depicted in the PE problem. Therefore, in a PE problem one would write instead of equation (3.8) [43, p. 149]

$$c - A^T p \geq 0 \tag{3.13}$$

and equation (3.11) would be changed accordingly.
Now, to repeat, the general depiction of the complementarity format is [43, p. 149]

$$\begin{aligned} &(CP) \qquad Find\ z \in \mathbf{R}^l\ that\ solves \\ &F(z) \geq 0,\ z \geq 0 \quad and \\ &z^T F(z) = 0. \end{aligned} \tag{3.14}$$

If the mapping of the function F of $\mathbf{R}^l$ into itself is an affine transformation, e.g. $F(z) = q + Mz$, the corresponding complementarity problem is called linear (see Cottle et al., 1992, [16] for a thorough treatment of linear complementarity problems), otherwise it is nonlinear [43, p. 149]. In the iterative solving process described by Mathiesen (1985a, [43]), the MCP is approximated by a series of linear complementarity problems.

Most importantly, the relation between the equilibrium problem posed in equations (3.8) to (3.12) and the complementarity format presented in (3.14) is given by (Mathiesen, 1985a, [43,

p. 149]):

$$z = \begin{pmatrix} y \\ p \end{pmatrix} \quad \text{and} \quad F\begin{pmatrix} y \\ p \end{pmatrix} = \begin{pmatrix} -A^T p \quad (= -\Pi(p)) \\ w + Ay - d(p) \end{pmatrix} \tag{3.15}$$

One can easily check the equivalence between the equilibrium definition from equations (3.8) to (3.12) and the one given above as an MCP. From equation (3.15) it is now clear that if the market demand functions are of linear structure in prices, i.e. $d(p) = d + Dp$, and if the input output coefficients of the matrix A are fixed, then the definition of an economic equilibrium given above will be a *linear* complementarity problem [43, p. 149]. In general, however, this will not be the case and a mixed complementarity problem has to be solved.

3.2.2 An Arrow-Debreu Economy in a Complementarity Format

The section above illustrated how in principle an economic equilibrium can be denoted as an MCP. Next, according to Böhringer and Rutherford (2008, [8]), it is further shown that complementarity is actually a characteristic of economic equilibrium, which would make conditions (3.11) - (3.12) obsolete, or rather demonstrate that they follow from conditions (3.8) - (3.10). Thus, the fact is established that a competitive economic equilibrium as in definitions (3.2) - (3.3) can always be cast as an MCP.

After adding in a bottom-up part in the form of an electricity sector into the model formulated as an MCP, the described general equilibrium hybrid model is a direct combination of bottom-up and top-down in a complementarity format.

The general formulation of an Arrow-Debreu Equilibrium as a complementarity problem for the static case is shown next. The interested reader can download a simple one-good version of the basic model as set out in Böhringer, Rutherford (2008, [8]) model in GAMS - code from `http://www.mpsge.org/td_bu.zip`[4].

Now consider a competitive economy with n commodities (including the primary factors capital and labour), m sectors of production and k households. The decision variables can then be classified into the following categories (see Mathiesen, 1985a, [43], and Böhringer, Rutherford, 2008, [8]):

y a nonnegative m-vector (with running index j) of activity levels for the constant-returns-to-scale (CRTS) producing sectors,

p a nonnegative n-vector (with running index i) of prices for all goods and factors,

M a nonnegative k-vector (with running index h) of household income (including any government entities)

4 Last accessed on March 19^{th}, 2012.

As described before, the complementarity format facilitates weak inequalities and is a logical connection between prices and market conditions, exemplified by **zero profit**, **market clearance** and **income balance** equations. A competitive equilibrium for all markets now is described by a vector of activity levels ($y_j \geq 0$), a vector of prices ($p_i \geq 0$), and a vector of incomes (M_h) fulfilling the following conditions:

- **The Zero Profit Condition** requires that any activity operated at a positive intensity must earn zero profit (i.e. the value of inputs must be equal or greater than the value of outputs). Activity levels y_j for constant return to scale production sectors are the complementary (associated) variables with this conditions. It means that either $y_j > 0$ (a positive amount of good j is produced) and profit is zero, or profit is negative and $y_j = 0$ (no production activity takes place). Specifically, the following condition should be satisfied for every sector of the economy [8, p. 577]:

$$-\Pi_j(p) \geq 0 \tag{3.16}$$

where: $\Pi_j(p)$ denotes the *unit profit* function for the CRTS production activity j, which is determined as the difference between unit revenue and unit cost. This can be written as $\Pi_j(p) = r_j(p) - c_j(p)$ for $j \in \{1, \ldots, m\}$. Since we assume the technologies to exhibit constant returns to scale, it holds that (equivalently to equation (3.7) in the previous chapter) the unit-profit function is homogeneous of degree one, and thus by Euler's homogeneous function theorem we have

$$\Pi_j(p) = (\nabla \Pi_j(p))^T p = \sum_i^n p_i \frac{\partial \Pi_j(p)}{\partial p_i} \tag{3.17}$$

- **The Market Clearance Condition** requires that any good with a positive price must have equality in supply and demand and any good in excess supply must have a zero price. The price vector p (which includes prices of all goods and factors of production) is the complementary variable. Using the MCP approach, the following condition should be satisfied for every good and every factor of production [8, p. 577]:

$$\sum_j^m y_j \frac{\partial \Pi_j(p)}{\partial p_i} + \sum_h^k w_{ih} \geq \sum_h^k d_{ih}(p,M_h) \quad \forall i \tag{3.18}$$

where:

w_{ih}	signifies the initial endowment by commodity and household,
$\frac{\partial \Pi_j(p)}{\partial p_i}$	indicates (by Hotelling's Lemma) the compensated supply of good i per unit of operation of activity j, and
d_{ih}	is the utility maximising demand for good i by household h.

Hotelling's Lemma is a basic result from microeconomics relating the supply of a good to the profit the producing entity of this good receives. The Lemma can be stated in the following way (see Hotelling, 1932, [29], or Varian (1992, [67, p. 43]):

If $\tilde{y_j}(p_i)$ is the net supply function for the good i of a firm j in terms of the good's price p_i, then

$$\tilde{y_j}(p_i) = \frac{\partial \tilde{\Pi}_j(p)}{\partial p_i} \tag{3.19}$$

for $\tilde{\Pi}_j$ the profit function of firm j in terms of the good's price, assuming that $p_i > 0$.

It is a corollary of the envelope theorem from microeconomics. The compensated supply function of good i captures only substitution effects, and no income effects.

Please note that in (3.19) above $\tilde{\Pi}_j$ signifies the profit function of sector (firm) j, and $\tilde{y_j}(p_i)$ the net supply function of a firm j for a good i in relation to its price p_i.

This stands in difference to the formulation in equation (3.18) above, where the term $\frac{\partial \Pi_j(p)}{\partial p_i}$ denotes the partial derivative of the unit profit function of sector j with respect to the price, and y_j is the activity level of that firm (sector). Thus, the term $y_j \frac{\partial \Pi_j(p)}{\partial p_i}$ denotes an expression of the type "quantities times quantities", i.e. the activity level of a certain sector y_j times the input quantities of good i needed for the production of one *unit* of the sectoral good j. Thus, the net supply function $\tilde{y_i}$ of a good i is depicted by the sum $\sum_j^m y_j \frac{\partial \Pi_j(p)}{\partial p_i}$, considering total supply of that good minus the fraction of total production that is used as intermediate inputs into production by other sectors.

This stems from the fact that if $i \neq j$, then $\frac{\partial \Pi_j(p)}{\partial p_i}$ has a negative sign, denoting an input of good/factor i in sector $j's$ unit production, if $i = j$, $\frac{\partial \Pi_i(p)}{\partial p_i}$ is equal to unity[5]. Especially, $y_i \frac{\partial \Pi_i(p)}{\partial p_i}$ will denote the total supply of good i by a sector, excluding the endowments of the households with this good.

- **The Income Balance Condition** requires that for each household h expenditure must equal factor income [8, p. 577]:

$$M_h = \sum_i p_i w_{ih} \tag{3.20}$$

This condition is introduced as a vector of intermediate variables to simplify the implementation and to increase the transparency of the model. They can be substituted out of the model without changing the underlying model structure, as in the form presented by Mathiesen (1985a, [43]) and described in the previous Chapter 3.2.1 [8, p. 577].

5 Referring back to the mere fact that one unit of supply of good i is provided per unit of operation of sector i.

An economic equilibrium in an MCP format now is described by the conditions (inequalities) (3.16) and (3.18), as well as the equality (3.20), and by adding two additional requirements [8, p. 577]:

- *Irreversibility:* all activities produce at non-negative levels:

$$y_j \geq 0 \quad \forall j \tag{3.21}$$

- *Free disposal:* prices stay non-negative:

$$p_i \geq 0 \quad \forall i \tag{3.22}$$

Now, if the utility function underlying the optimisation process of the households has the property of non-satiation, the expenditure by the households will completely exhaust their income (i.e. Walras Law has to hold), such that (see [8, p. 577], where the term has been corrected in this thesis, and section 3.2.1 above):

$$\sum_i p_i d_{ih}(p,M_h) = M_h = \sum_i p_i w_{ih} \qquad \forall h \tag{3.23}$$

In a first step, we just rearrange equation (3.18) slightly (as $p_i \geq 0 \quad \forall i$, the unequal sign does not flip):

$$\begin{aligned}
&\sum_j^m y_j \frac{\partial \mathit{\Pi}_j(p)}{\partial p_i} + \sum_h^k w_{ih} \geq \sum_h^k d_{ih}(p,M_h) \quad \forall i \Leftrightarrow \\
&\sum_j^m p_i y_j \frac{\partial \mathit{\Pi}_j(p)}{\partial p_i} \geq \sum_h^k p_i (d_{ih}(p,M_h) - w_{ih}) \quad \forall i
\end{aligned} \tag{3.24}$$

Now, if one substitutes the expression $p^T(d_h(p,M_h) - w_h) = \sum_i p_i(d_{ih}(p,M_h) - w_{ih}) = 0$ into condition (3.24), after having taken the sum over all i, one gets the following inequality (see [8, p. 578], again the term has been corrected):

$$\begin{aligned}
&\sum_i \sum_j p_i y_j \frac{\partial \mathit{\Pi}_j(p)}{\partial p_i} \geq \sum_i \sum_h p_i (d_{ih}(p,M_h) - w_{ih}) \Leftrightarrow \\
&\sum_i \sum_j p_i y_j \frac{\partial \mathit{\Pi}_j(p)}{\partial p_i} \geq \sum_h \underbrace{\sum_i p_i (d_{ih}(p,M_h) - w_{ih})}_{=0} \Leftrightarrow \\
&\sum_j \sum_i y_j p_i \frac{\partial \mathit{\Pi}_j(p)}{\partial p_i} = \sum_j y_j \mathit{\Pi}_j(p) \geq 0
\end{aligned} \tag{3.25}$$

where we have used the fact that $\Pi_j(p) = \sum_i p_i \frac{\partial \Pi_j(p)}{\partial p_i}$. On the contrary, however, conditions (3.21) and (3.16) imply that $y_j \Pi_j(p) \leq 0 \quad \forall j$. Now, in order for the sum $\sum_j y_j \Pi_j(p)$ to be greater or equal to zero, each of its elements has to be equal to zero. Thus, we get the result that in an equilibrium situation, every activity which exhibits a negative unit profit remains idle [8, p. 578]:

$$y_j \Pi_j(p) = 0 \quad \forall j, \tag{3.26}$$

and that every commodity that is in excess supply must have a price of zero [8, p. 578]:

$$\begin{aligned} &p_i \left[\sum_j^m y_j \frac{\partial \Pi_j(p)}{\partial p_i} + \sum_h^k w_{ih} - \sum_h^k d_{ih}(p, M_h) \right] = 0 \quad \forall i \Leftrightarrow \\ &p_i \left[\sum_j^m a_{ij}(p) y_j + \sum_h^k w_{ih} - \sum_h^k d_{ih}(p, M_h) \right] = 0 \quad \forall i \end{aligned} \tag{3.27}$$

where we have used the fact, as in equation (3.6) above, that $a_j(p) = (a_{ij}(p)) = (\partial \Pi_j(p)/\partial p_i)$, where a_{ij} is a coefficient in the technology matrix of activity (sector) j, with positive entries denoting outputs, and negative entries denoting inputs.

Equation (3.27) can be derived as following: Firstly, we know from equations (3.23), (3.25) and (3.26) that

$$\sum_i p_i \left[\sum_j^m y_j \frac{\partial \Pi_j(p)}{\partial p_i} + \sum_h^k w_{ih} - \sum_h^k d_{ih}(p, M_h) \right] = 0. \tag{3.28}$$

Furthermore, as we now that $p_i \geq 0 \quad \forall i$ from (3.22) and by (3.18) that

$$\sum_j^m y_j \frac{\partial \Pi_j(p)}{\partial p_i} + \sum_h^k w_{ih} - \sum_h^k d_{ih}(p, M_h) \geq 0 \quad \forall i,$$

we have to conclude that

$$p_i \left[\sum_j^m y_j \frac{\partial \Pi_j(p)}{\partial p_i} + \sum_h^k w_{ih} - \sum_h^k d_{ih}(p, M_h) \right] = 0 \quad \forall i,$$

since the sum displayed in (3.28) can only be equal to zero if each of it elements is equal to zero, given all other conditions described above.

3.2.3 Integrating Bottom-Up in Top-Down

Because of the insights gained above, Böhringer and Rutherford (2008, [8, p. 578]) conclude that *"complementarity is a characteristic rather than a condition for equilibrium in the Arrow-Debreu model"*. It is this characteristic of an equilibrium allocation that motivates to formulate an economic equilibrium in the mixed complementarity format. Their approach now, because of the properties of an MCP described above, allows one to include a bottom-up activity analysis in the model, where alternative production technologies may produce a good (e.g. some form of energy good) subject to process-oriented (technical feasibility, etc.) capacity constraints [8, p. 578].

As an example, Böhringer and Rutherford (2008, [8]) name an "energy sector linear programming problem which seeks to find the least-cost schedule for meeting an exogenous set of energy demands using a given set of energy technologies" [8, p. 578], where the energy technologies are indexed by *tec*:

$$
\begin{aligned}
&\min \quad \sum_{tec} \bar{c}_{tec}\, y_{tec} && (3.29)\\
&\text{subject to}\\
&\sum_{tec} a_{j,tec}\, y_{tec} = \bar{d}_j \quad \forall j \in \{\text{energy goods}\}\\
&\sum_{tec} b_{k,tec}\, y_{tec} \le \kappa_k \quad \forall k \in \{\text{energy resources}\}\\
&y_{tec} \ge 0
\end{aligned}
$$

where:

y_{tec}	denotes the activity level of the energy technology *tec*,
$a_{j,tec}$	stands for the "netput" (energy goods may be inputs as well as outputs for a technology) of energy good j by technology *tec*
$\bar{c}_{tec}$	is the exogenous, constant marginal unit cost of producing the energy good by the means of technology *tec*
$\bar{d}_j$	denotes the market demand for energy good j (which is derived from the top-down general equilibrium part of the model)
$b_{k,tec}$	represents the unit demand for the energy resource k by technology *tec*, and
κ_k	stands for the aggregate supply of the energy resource k.

These resources may be capacities of the economy in regard to the generation or transmission of the energy good. Some of them may be specific to an individual technology (such as the amount of wind available to an economy to produce electricity), others can be traded in markets, thus being allocated to the most efficient use [8, p. 578].

The bars over c_{tec} and d_j here shall indicate that these coefficients are taken as given in the maximisation process of the firms in the energy sector. The values of these coefficients are determined in the price framework of the outer, top-down general equilibrium model [8, p. 579].

When one derives the Karush-Kuhn-Tucker conditions characterising optimality for this linear programming problem, one has [8, p. 579]:

$$\sum_{tec} a_{j,tec}\, y_{tec} = \bar{d}_j, \quad \pi_j \geq 0, \quad \pi_j \left(\sum_{tec} a_{j,tec}\, y_{tec} - \bar{d}_j \right) = 0 \tag{3.30}$$

and

$$\sum_{tec} b_{k,tec}\, y_{tec} \leq \kappa_k, \quad \mu_k \geq 0, \quad \mu_k \left(\sum_{tec} b_{k,tec}\, y_{tec} - \kappa_k \right) = 0 \tag{3.31}$$

where:

π_j is the Lagrange multiplier on the balance between price and demand for good j, and

μ_k is the shadow price placed on the energy sector resource k.

When one compares now the Kuhn-Tucker conditions given above with the top-down general equilibrium model, see equation (3.27), one can see the equivalence between the shadow prices on the mathematical programming constraints and the market prices of the top-down model [8, p. 579]. Thus, the mathematical linear program can be viewed as a particular case of the general equilibrium problem where [8, p. 579]

1. all income constraints are dropped
2. the energy demands are given exogenously from the top-down model
3. the cost coefficients of the energy supply technologies are held fixed, contrary to the price-responsive coefficients obtained from the general equilibrium problem.

Thus, one can replace the aggregate top-down description of the energy good producing sector (e.g. a neoclassical production function) by the Kuhn-Tucker conditions obtained from the linear program characterising minimum costs while fulfilling the supply schedule of the energy sector that is derived from the energy demand from the general equilibrium top-down model. Therefore, technological details can be incorporated, while all prices remain endogenous [8, p. 579].

Now the *weak duality theorem* relates the optimising value of the linear programming problem to the shadow prices and constants that come from the constraint equations [8, p. 579]:

$$\sum_j \pi_j \bar{d}_j = \sum_{tec} \bar{c}_{tec}\, y_{tec} + \sum_k \mu_k \kappa_k \tag{3.32}$$

Further insight into the connection between the bottom-up linear programming model and the top-down outer economic environment can be obtained from equation (3.32). It represents no more than a *zero profit condition*, see equation (2.6), which is applied to the aggregate energy subsector of the economy: in an equilibrium situation, the value of the energy goods and services produced must equal the variable costs for the production of energy plus the market value of the rents paid for the natural resources [8, p. 579].

As has been mentioned before, the MCP formulation of an economic equilibrium provides some flexibility regarding the depiction of features known from economic reality such as income effects, or second-best characteristics such as tax distortions or market failures (e.g. environmental and other externalities) [8, p. 580]. The latter can be included in the model e.g. via explicit bounds on the decision variables (another useful possibility for an MCP) such as prices and activity levels. Such examples may include politically or otherwise motivated upper bounds on variables (e.g. price caps on certain energy goods), or lower bounds such as minimum real wages [8, p. 579]. Examples for quantity constraints can represent certain bounds on the share of a certain production technology in total energy production [8, p. 579]. Thus, quotas for renewable energy production or other desired policy goals can be incorporated in the model.

With these constraints, there exist associated *complementary variables*. These enable the model to keep the equilibrium situation while applying the constraints. For price constraints, a rationing variable will be activated as soon as the price constraint becomes binding; for quantity constraints, a complementary endogenous subsidy or tax will apply [8, p. 579].

An example for a one-sector economy with separate energy goods for the static model set out above can be found in Böhringer, Rutherford (2008, [8]). Here, in the next step the dynamisation of the framework above is described, before proceeding to an example for a dynamic model presented in section 4.

3.2.4 The Dynamics of the Ramsey Model in an MCP Formulation

When assessing the long term effects of technological and structural change for the energy sector, in hindsight to environmental issues, a potential policy maker will be interested in a model that can give an evaluation of long term costs and benefits for energy policies. Thus, an endogenous formulation of investment decisions, which can only be described in an intertemporal framework, will allow an explicit description of the sector- and technology-specific capital stock evolvement, as well as a certain technology mix (see Frei et al., 2003, [23, p. 1017]).

The underlying model paradigm will determine the way the behavior and formation of expectations by the agents of the economy is modelled. Different optimisation concepts such as short to medium term thinking by the individuals of the economy (myopic profit and utility maximisation) or perfect foresight, where the agents are supposed to know as much as the modeller and perfectly anticipate all future and current changes, will decisively shape model output and policy evaluations (Frei et al., 2003, [23, p. 1017]).

Modelling the behavior of the agents with perfect foresight might not seem to be a very realistic depiction of human behavior, however, it guarantees logical consistency of the model and is thus the first modelling approach when choosing a deterministic setting for a CGE model. Assuming perfect foresight, the static model described in the previous section can be extended to a dynamic one by taking only a couple of steps. In this framework, the realised prices of the model are equal to the prices expected by the agents of the economy (Böhringer, Rutherford 2008, [8, p. 586]). If one adheres to the standard *Ramsey Model* of investment and savings, the notion of perfect foresight is connected to the assumption of an infinitely-lived representative household, making choices trading off the consumption levels of future and current generations [8, p. 586]. This representative agent maximises her utility subject to an intertemporal budget constraint. The marginal cost of capital formation and the marginal return to investment are equalised via a savings rate. Optimisation requires that the rates of return to capital and investment are formed in such a way so that the marginal utility of a unit of investment, and a marginal utility of a unit of consumption foregone by the household are equalised [8, p. 586].

Formulated as a primal non-linear program, the basic Ramsey model takes the following form (see Rutherford et al., 2002, [54, pp. 579]):

A social planner maximises the present value of lifetime utility for the representative household:

$$U = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t u(C_t) \tag{3.33}$$

where ρ is the time preference rate, C_t is the aggregate consumption in year t, and $u(.)$ is the instantaneous utility of consumption. The representative agent can then choose whether the output good is consumed or invested, which is the maximisation constraint for the agent:

$$C_t + I_t = f(K_t) \tag{3.34}$$

where I_t is investment in year t, K_t is the capital stock in year t, and $f(K_t)$ the economy-wide production function. Usually, the neoclassical assumptions are placed on the production function, i.e. strict monotonicity ($f'(K_t) > 0$) and concavity ($f''(K_t) < 0$). Furthermore, it makes life easy for the modeller to assume the production function to exhibit constant returns to scale in capital and a second factor, usually labour, where the supply is specified exogenously, e.g. by population growth, i.e.

$$f(K_t) = F(K_t, \bar{L}_t) \tag{3.35}$$

The capital stock in period t is now equal to the capital stock remaining from the last period after depreciation, plus the investment in capital good from the last period, which can be written as:

$$K_t = (1-\delta)K_{t-1} + I_{t-1}, \quad K_0 = \bar{K}_0, \quad I_t \geq 0 \tag{3.36}$$

where δ is the annual rate of capital depreciation, and the initial capital stock K_0 is specified exogenously.

Casting the Ramsey model as an MCP, however, only requires a few modifications to the static framework set out in section 3.2.2, because most relations described in this static model are intra-period, thus being still valid on a period-by-period basis in the dynamic extension of the model [8, p. 586]. When it comes to capital stock formation and investment, capital has to be allocated efficiently across periods (which is done by investment per period) as is shown in equation (3.36). This implies two central intertemporal zero profit conditions connecting the purchase price of a unit of capital stock in period t to the cost of a unit of investment and the return to capital [8, p. 586].

In the equations below, the following variables are used amongst others:

p_t^K	denotes the market value (the purchase price) of a unit of capital stock at the beginning of period t
K_t	is the associated dual variable depicting the activity level of the capital stock formation in period t, and
I_t	is the associated dual variable indicating the activity level of aggregate investment in period t
r_t^K	is the rental rate of capital, i.e. the value of rental services of capital (the households own the capital stock and rent it to the sectors)
p_t^Y	is the price of the output good (or a weighted index of sectoral prices)

First of all, the market value of a unit of already depreciated capital purchased at the beginning of period t (p_t^K) has to be greater or equal to the value of capital rental services through that period (r_t^K) plus the (depreciated) value of a unit of capital if sold at the beginning of the next time period (p^K_{t+1})[8, p. 586], which is the *zero profit condition on capital formation*:

$$-\Pi^K_t = p^K_t - r^K_t - (1-\delta)p^K_{t+1} \geq 0 \tag{3.37}$$

The idea behind this formulation is that of a *no arbitrage* condition: the price of capital can be no less than the profit made from the renting out of capital as a producing factor to the sectors by the household this period plus the value at which the household can sell the capital in the next period, subject to depreciation. If this condition were not met, capital would be undervalued in relation to its productivity in the economy. As it would be profitable to purchase capital this period, rent it out and sell it of next period if this condition were not met, and since we have a representative agent, the price of capital this period instantaneously has to rise to meet this condition. The price of the capital stock in the next period, then, is limited in the next equation (3.38).

Secondly, the opportunity to make investments in the year t puts a restraint on the price of capital in period $t+1$ [8, p. 586], which is the *zero profit condition of investment*:

$$-\Pi^I_t = -p^K_{t+1} + p^Y_t \geq 0 \tag{3.38}$$

where p^Y_t is the price of an output good that can be used either for consumption or investment in period t, calculated as a weighted index of all sectoral prices. Here, we have another no arbitrage condition reflected: the price of capital in the next period mus be less than or equal to that of the investment good in this period, which becomes part of the capital stock in the next period. Otherwise, it would be profitable to undertake further investment this period and sell off the resulting capital stock in the next period. The prices of capital and the investment good have to adapt to this condition.

Every year, the sectoral capital stock changes by the depreciation of the capital stock from the previous year and by the investment of the past period, thus [8, p. 586f]:

$$K_{i,t+1} = (1-\delta)K_{i,t} + I_{i,t} \tag{3.39}$$

Now, as investment has been added to the equational system as a demand category, the whole output $Y_{t,i}$ for a good i at time t must equal total demand for this good, consisting of final household demand, intermediate demand by sectors and investment demand (cf. [8, p.

586][6], see also equation (4.31)):

$$Y_{t,i} = \sum_j \frac{\partial \Pi_{t,i}(p)}{\partial p_{t,j}} \geq \frac{\partial \Pi_t^{C_i}}{\partial p_{C_{t,i}}} C_{t,i} + \sum_{tec} a_{tec}^{Y_i} ELE_{t,tec} + I_{t,i} \tag{3.40}$$

where

$\sum_j \frac{\partial \Pi_{t,i}(p)}{\partial p_{t,j}}$ by Hotelling's lemma captures total supply minus intermediate inputs (as the expression will be negative for input good/factor $i \neq j$ and positive for the output good i),

$\frac{\partial \Pi_t^{C_i}}{\partial p_{C_{t,i}}} C_{t,i}$ is total final consumption demand by households for good i at time t, where $p_{C_{t,i}}$ is price of consumption for good i,

$\sum_{tec} a_{t,tec}^{Y_i} ELE_{t,tec}$ are the inputs demanded from the macro production good i by an electricity producing technology tec to produce electricity (the bottom-up part) and

$I_{t,i}$ is the amount of good i devoted to investment.

In this equation, intermediate demand enters the left hand side negatively, and can be brought to the right hand side for a better understanding. Other demand categories, which will be present in the model described in section 4, such as government or export demand, can be subsumed under the household demand here. The demand of the electricity technologies is determined by what could be called 'input-output coefficients' $a_{tec}^{Y_i}$ which can be assumed fixed for a given technology (mix). Only the level at which a technology produces at time t, $ELE_{t,tec}$, changes according to the demand schedule, subject to capacity constraints and resource availabilities.

As in the standard Ramsey model, the intertemporal demand responses within the model arise from the optimisation of an infinitely lived representative household. This household allocates her lifetime income, which is the intertemporal budget constraint, according to intertemporal utility maximisation by solving [8, p. 587]:

$$\max \sum_t \left(\frac{1}{1+\rho}\right)^t u(C_t) \tag{3.41}$$

subject to

$$\sum_t p_t^C C_t = M \tag{3.42}$$

where

$u(.)$	indicates the instantaneous utility function of the representative household
ρ	denotes the time preference rate, and
M	is lifetime household income
p_t^C	is the price for the aggregate final consumption good at time t

6 The expression has been changed here for a better understanding.

C_t is aggregate final consumption

An instantaneous utility function featuring isoelastic lifetime utility is given by:

$$u(C) = \frac{c^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}} \tag{3.43}$$

where η represents a constant intertemporal elasticity of substitution indicating how the household values consumption at certain time periods when optimising from the present point in time.

A considerable issue for the dynamic formulation of the model is the *terminal capital stock constraint problem.* A finite model horizon causes a problem when it comes to capital accumulation [8, p. 587]. This is the case because in the last period of the model the capital stock would lose all its value, since the "model world" ends after this last period. This would have significant effects on the behavior of economic agents before this period, affecting investment rates in the periods leading up to the end of the model horizon [8, p. 587]. To correct for this effect, Böhringer and Rutherford (2008, [8, p. 587f]) propose to define a terminal constraint forcing investment to increase in proportion to the change in consumption demand. Here, the mixed complementarity format allows one to include the *post-terminal capital stock* as an endogenous variable. Lau, Pahlke and Rutherford (2002, [54]) show that, using state variable targeting for the post-terminal capital stock, the growth of investment in the terminal period can be related to the growth rate of capital or any other "stable" quantity variable of the model [8, p. 588].

As already mentioned before, the following chapter shall introduce a dynamic model at the IHS Vienna that has been developed from the basic model as proposed in Böhringer, Rutherford (2008, [8]).

CHAPTER 4

A Hybrid Top-Down Bottom-Up CGE Model for Austria

The theoretical model delivered in chapter 3 can be transformed into a computable general equilibrium model that can be used for policy evaluation and other economic research efforts. This step has been taken at the IHS Vienna by creating a CGE model for the Austrian economy with a detailed depiction of the electricity sector, which will be described in comprehensive detail in the following.

The model presented in this chapter is in principle an extended version of the one proposed in Böhringer, Rutherford (2008, [8]), where the basic model has been adapted to represent a model version of the Austrian economy in an MCP format. This includes a data set, a so-called **Micro Consistent Social Accounting Matrix** or **MCM**, see e.g. Böhringer, Rutherford (2008, [8]) in regard to this concept, which has been tailored to fit data of the Austrian economy as provided by Statistics Austria[1].

Furthermore, expansions of the model have been added to feature *multiple sectors*, an *extended government agent* levying various taxes and redistributing income, the explicit consideration of *intermediate flows* between the various sectors, and the extension from a closed economy to a *small open economy* featuring a Rest of the World (RoW) agent demanding exports and providing import goods. Furthermore, scenarios to investigate Austria's fulfillment of obligations posed e.g. in the EU 20-20-20 targets (20% reduction in greenhouse gas emissions, 20% of total EU energy production from renewable energy sources, as well as a 20% increase in energy efficiency until 2020[2]) have been implemented and will be described in chapter 5.

4.1 Data

The **SAM (Social Accounting Matrix)** is a useful way to represent the circular flows of an economy for modelling purposes. King (1985, [34]) states the two main objectives of a SAM to be as following :

1 The national accounting tables published by statistics Austria, see e.g. http://www.statistik.at/web_de/statistiken/volkswirtschaftliche_gesamtrechnungen/index.html for further information on this data set. Last accessed on March 19^{th}, 2012.

2 see e.g. http://ec.europa.eu/climateaction/eu_action/index_en.htm for more information on the EU 20-20-20 targets. Last accessed on March 19^{th}, 2012.

- to organise information about the economic and social structure of a country for a certain time period, and
- to provide the statistical base for a plausible model that represents a static image of the economy, while being able to simulate policy interventions in this economy.

Therefore, SAMs have a tradition of being used to provide a snapshot of developing economies for further analysis. At the same time, following the tradition of Johansen (1960), this type of data structure has been used as a benchmark equilibrium data set for CGE models. A treatment of SAMs, their development and applications can be obtained e.g. from King (1985, [34]) or Pyatt and Round (1977, [50]).

Basically, a SAM forges two basic ideas of economics (see Robinson et al., 1999, [51, 6ff]) into one concept:

- Firstly, corresponding to the well known input-output figures, a SAM provides the *linkages between the different sectors* of an economy. This means that each purchase of an intermediate input used in the production process by one sector corresponds to a sale by another sector. Thus, if we let a_{ij} be the element of the matrix in the $i-th$ row and the $j-th$ column, a_{ij} represents the expenditures of account j on intermediate inputs which are received by account i. Entering the SAM matrix as a single cell entry (intersection of row i and column j), it appears as two separate entries in the accounts of the respective sectors using traditional double-entry book-keeping.

  However, the SAM generalises this idea from the intermediate flows to include all transactions within the economy: a purchase of a consumption good by the household agent(s) would enter as an expenditure in the respective columns of the household agent, similar to a tax payment (an expenditure of the household, a receipt by the government). Thus, a SAM matches every *expenditure (input)* within the economy to a corresponding *receipt (output)*. Expenditures are denoted column-wise, receipts row-wise (see Table 4.2).

- Secondly, as can be inferred from above, a SAM embodies the fact stemming from the national accounting framework that *income always equals expenditure*. As this has to be true for every industry (sector) of the economy, the sum of the columns always has to equal the sum of the rows in order to facilitate a benchmark equilibrium (all markets have to clear). Thus, for every sector, the revenue from sales (exports, domestic final consumption, intermediate consumption) has to equal expenditures (intermediate inputs, factors, taxes, etc.). This corresponds to the two main conditions set forth in section 2.1.1:

  The *zero profit condition*, see (2.6), requiring every activity of production to make non-positive profits, can be read as the equality of the value of inputs and outputs for the sectors, thus the row sum being equal to the column sum for every sector.

  As can already be inferred from above, the *market clearance condition*, see (2.5), requires all markets to clear in equilibrium, which is also described by the equality of output (generating corresponding receipts, sum of each row) and consumption (sum of each column) for every sector.

The accounts of a SAM form the framework for the model of a small, open economy. The model equations will specify the market, behavioral and systemic relationships that underlie each account within the SAM (as also in [51, p. 8]). The specification of behaviour on the market (demand, supply and market clearance conditions) is required for the activity and factor accounts. The household and government accounts stand for the budget restrictions of the government and households agents (income has to equal expenditure). The capital and RoW accounts reflect the two macroeconomic balance conditions: internal balance (investment equals savings) and external balance (exports plus capital inflows have to equal imports) (see also Robinson et al., 1999, [51, p. 8]).

The data set for the hybrid top-down bottom-up CGE model developed at IHS Vienna consists of a very similar type of SAM to the one described above. It is constructed mainly from national accounting data and input-output tables. These are compiled every year for the Austrian economy by Statistik Austria, with a lag of about five years, so that the newest version of these tables at the point of composition of this thesis is for the year 2007. One might remark on these data sets that they generally are measured in *monetary units* at current prices, i.e. in values produced or consumed.

Therefore, physical units such as product quantities are not explicitly measured with this type of data. However, the values provided for certain goods or sectors can be related back to physical quantities via average prices for a quantity measure, such as prices per ton/item produced/consumed, which have to be taken from outside the data set. This is not usually done within CGE models, where one is only interested in a system of relative prices. Only for certain interpretations and applications, it might be useful to extract physical quantities from the model results. Further elaboration on this topic is conducted in section 4.3.2 of this thesis.

This data set has been constructed for the benchmark year 2005. The logic of the matrix strictly follows the accounting ideas delineated above, however, some changes as regarding sign conventions and the treatment of sectoral production, among others, have to be mentioned.

Firstly, the sign conventions of how a benchmark equilibrium data set is facilitated vary among certain types of SAMs. For input into the CGE model developed at IHS Vienna, the data are arranged in such a way that inputs into production/expenditures by the producing sector enter the matrix with a *negative sign*, while output/revenues of producing sectors enter the matrix with a *positive sign*. Thus, the **zero profit conditions** (total costs for production equal total revenues from production) for the production sectors are depicted in the matrix by the **column entries**, where inputs and outputs have to be equal, thus sum up to zero.

Similarly, for the **row entries**, consumers', or households', expenditures on consumption of goods are denoted with a *negative sign*, whereas income/revenue is depicted with a *positive sign*. Thus, market clearance is represented in the benchmark data set by expenditures/consumption equaling revenues/income. The **market clearance condition** is thus ensured by the **row entries** summing up to zero.

This type of SAM is called a **Micro Consistent Matrix (MCM)**, another concept that facilitates an equilibrium for the benchmark data set. The benchmark equilibrium makes sure that the model calibrates if a computable functional form has been chosen for the model equations. An example for a SAM for the Austrian economy following the conventions above can be found in table 4.2. The sign conventions on inputs/outputs allow for an easy checking system whether the benchmark data set actually represents an equilibrium, i.e. whether the row/columns sums are equal to zero.

Secondly, often a SAM distinguishes between a so called *activity account* and a *commodity account*, where the activity account produces one or more commodities. A commodity is then bought by the commodity account, which in turn sells the product to the households. This allows for some detailed distinctions of production processes, e.g. to distinguish between small and large-scale farming, which produce the same product, food, by different processes. This distinction is not made for the model developed at IHS Vienna, since it is not the focus of research and would complicate matters too much at the current state of model development. In the data set presented here, a different approach is followed, which is standard when it comes to national accounting tables: the matrix describes what goods and factors are used to produce a good. This means that the sectoral production of different goods has been aggregated, and therefore we see here some sort of "aggregate" production technology for the Austrian economy.

Specifically, if two firms from different sectors produce at least some amount of a certain good, these two productions are aggregated in the matrix account of this one good.[3] Usually, national accounting tables (input-output tables) are given in $product \times product$ form, to which the data set presented here has been adapted.

Here, one might argue that by merging the sector and commodity account one loses technological detail, but on the other hand this interpretation is quite useful for environmental applications for the following reason:

One is not interested in what sector provides the intermediate input to produce some good, which might be an in the long run ecologically harmful energy input such as coal or another fossil fuel, but what type of intermediate input is used to produce this good. In this way, a focus on the technology mix used to manufacture certain goods or provide certain services can be implemented more directly.

3 The difference in these classification corresponds to the national accounting classification standards for sectors, NACE (which is the acronym for "Nomenclature statistique des Activités économiques dans la Communauté Européenne", see `http://www.statistik.at/web_en/classifications/implementation_of_the_onace2008/index.html` for further reference for its Austrian implementation), and the classification system for products, CPA (Classification of Product by Activity, see `http://ec.europa.eu/eurostat/ramon/index.cfm?TargetUrl=DSP_PUB_WELC` for further reference). Both links last accessed on March 19^{th}, 2012.

In total, the aggregation of the data features 13 sectors of production for the economy, a list of which is given in table 4.1. As can be inferred from the level of disaggregation for certain sectors, the focus of this model lies on energy production and consumption: the energy sector is given in relatively high detail, while the service sector is highly aggregated. Also, the detail of the government sector has been increased to include various taxes and transfers between the government and the households.

As can be inferred from table 4.2, the sectors use intermediate inputs from other sectors, as well as the primary factors capital and labour to produce their output, which is denoted as a positive value in the so-called *diagonal*, i.e. the intersections of the row and column attributed to a certain sector. Firms rent capital from the household to use it for production, and the households provide labour input.

The factor incomes from capital and labour accrue to households (HH), who can use them for consumption and other purposes. The latter are mainly tax payments and transfers from/to the government. Labour taxes (including social security contributions), taxes on refined oil products, consumption taxes, taxes on production and a residual of other taxes have been included within the model. The taxes on production and the residual taxes are held fixed as a transfer from the households to the government, the others are calculated endogenously in the model, either as a fixed rate (flexible absolute flow) or as a flexible rate adapting to a fixed absolute monetary flow.

Furthermore, to depict the macroeconomic flows of the Austrian economy correctly, pension payments, unemployment benefits and a residual of other transfers, mostly social security payments, have been included in the model as fixed transfers. These are given exogenously, and do not adapt endogenously, as this would require too much complexity of the model, which has a focus on the production and also consumption of energy goods.

Next, the equations of the model that, as the core of the model, actually depict these relations outlined in the data, are described in greater detail.

Table 4.1: Sectors of the MCM - SAM

Abbreviation	Sector Name	CPA Sectors[1]
AGR	Agriculture	1,2,5
FERR	Ferrous, Non-Ferrous Ore and Metals	27
CHEM	Chemical Products	24
ENG	Engineering	28-32, 34, 35
OTH	Other Production	17-19, 21, 22, 25, 33, 36, 37, 15, 16, 26
BUI	Building and Construction	45
TRA	Transport	60-62
SERV	Services	41,50-52,55,63-67,70-75,80,85,90,91-93,95
ELE	Electricity	40A
FW	Steam and Hot Water Supply	40C
EN	Fossil Fuel Energy	10,11,23,40B
Foss	Imports of Fossil Fuels	-
OINT	Intermediate Input within aggregated sectors	-
G	Government Consumption	-
GOVT	Government Agent	-
L	Labour	-
K	Capital	-
HH	Household Agent	-
INV	Benchmark Investment	-
IMP	Imports	-
LTAX	Wage Tax including employers' and employees' social security benefits	-
PENSION	Pensions	-
MSt	Tax on Refined Oil Products	-
CTAX	Consumption Tax	-
ITAX	Taxes on Production (are attributed to the household for technical reasons)	-
UEBEN	Unemployment Benefits	-
OTAX	Other taxes on Production	-
OTRANS	Other Social Transfers	-
ROW	Rest of the World	-

[1] These Sector classifications refer to the CPA classification of Statistik Austria in the input-output tables of 2005. The input-output tables can be obtained from `http://www.statistik.at/web_en/dynamic/statistics/national_accounts/input_output_statistics/publikationen?id=&webcat=358&nodeId=1096&frag=3&listid=358`. Last accessed on March 19^{th}, 2012.

Table 4.2: The Microconsistent SAM of the Hybrid Top-Down Bottom-Up Model Developed at IHS Vienna for the year 2005 (in Million Euro)

	AGR	FERR	CHEM	ENG	OTH	BUI1	BUI2	TRA	FuE	SERV	ELE	FW	EN	FOSS	OINT	G	HH	INV	GOVT	ROW	TOT
AGR	9037	-5	-5	-6	-3849	-16	-1	-3	0	-372	-1	0	0	0	-1880	-198	-1850	-320	0	-531	0
FERR	0	16939	-32	-3640	-479	-321	-436	-12	0	-47	-2	-3	0	0	-3766	0	-7	-286	0	-7908	0
CHEM	-159	-102	17443	-625	-2177	-56	-220	-19	-37	-1776	-2	-1	-25	0	-1650	-1078	-1329	-125	0	-8062	0
ENG	-228	-231	-137	94739	-1637	-788	-1732	-596	-24	-4558	-376	-36	-96	0	-18698	0	-4507	-15629	0	-45466	0
OTH	-468	-307	-538	-2483	79024	-3788	-1143	-225	-36	-9210	-43	-77	-14	0	-12559	-201	-16647	-3974	0	-27311	0
BUI1	-73	-19	-22	-69	-162	21558	-181	-96	-2	-3061	-37	-3	-9	0	-1684	0	0	-15568	0	-572	0
BUI2	-35	-18	-19	-66	-137	-456	12078	-83	-4	-3650	-15	-2	-5	0	-368	0	-1206	-5818	0	-196	0
TRA	-23	-341	-257	-750	-2203	-398	-65	19696	-5	-3271	-59	-3	-64	0	-1506	-394	-4566	-137	0	-5654	0
FuE	0	-20	-46	-267	-81	-2	-2	-18	1730	-177	-6	-3	-2	0	-181	-64	-10	0	0	-851	0
SERV	-616	-1244	-1010	-8031	-9264	-2698	-1997	-4639	-359	260697	-423	-84	-468	0	-66164	-45848	-86267	-10039	0	-21546	0
ELE	-85	-289	-126	-212	-539	-39	-26	-273	-6	-1343	6022	-21	-10	0	0	0	-3053	0	0	0	0
FW	-3	-4	-11	-25	-31	-1	-3	-16	-1	-200	-2	499	-1	0	-12	0	-186	0	0	-3	0
EN	-203	-1442	-828	-173	-707	-445	-79	-1053	-8	-1572	-593	-95	15471	0	-3293	0	-5004	23	0	1	0
Foss	0	0	0	0	0	0	0	0	0	0	0	0	-8933	8933	0	0	27	-27	0	0	0
OINT	-1880	-3766	-1650	-18698	-12559	-1684	-368	-1506	-181	-66164	0	-12	-3293	0	111761	0	14	-14	0	0	0
G	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	47783	3011	-3010	-47784	0	0
IMP	-2211	-5928	-10098	-43286	-26292	-414	-315	-4383	-240	-12980	0	-25	-2233	-8933	0	0	-13	13	0	117338	0
L	-382	-1421	-921	-10296	-10542	-5422	-2859	-4511	-822	-80592	-1387	-47	-318	0	0	0	119520	0	0	0	0
K	-2671	-1802	-1743	-6112	-8365	-5030	-2651	-2263	-5	-71724	-3076	-87	0	0	0	0	52872	51896	0	761	0
LTAX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-53967	0	53967	0	0
PENS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	34240	0	-34240	0	0
MSt	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-3565	0	3565	0	0
CTAX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-19466	0	19466	0	0
ITAX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-10521	3015	7506	0	0
UEBEN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1850	0	-1850	0	0
OTAX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-10792	0	10792	0	0
TRANS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11422	0	-11422	0	0
TOT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

4.2 Model Equations - a Detailed Depiction of the Model

Following the logic of the MCP format, the equations of the model developed at IHS Vienna is grouped into zero profit, market clearance, and income balance conditions.

The notation is as follows: goods (= sectors) including the energy sectors (fossil fuels and electricity) will be indexed by s, macro goods and sectors (excluding the energy sectors) by i, the fossil fuel energy sector is given by en, the electricity sector by ele. All activities that have a capital stock (macro sectors, energy sectors, technologies) will be denoted by act, all goods and factors are denoted by the set sf. Endogenous variables are written as upper case Latin letters, parameters and exogenous variables as lower case or Greek letters. Primary factors are subscribed with the letter f, energy technologies with tec, natural resources are denoted by res. There is no need to index households as there is only one representative household in this model. Time (in years) is given by the index t. The time horizon of this dynamic model runs until the year 2040 (which can easily be changed).

The structure is similar to that of an Arrow-Debreu type of economy, with the assumption of a small, open economy. The following variables determine the model:

- Activity Levels

$Y_{t,i}$	sectoral output in monetary units
$ELE_{t,tec}$	production level of electricity in monetary units
EN_t	Supply of energy good (fossil fuels)
EXP_t	exports of sectoral products
$IMP_{t,sf}$	imports of sectoral products including raw energy
$FFIMP_t$	fossil fuel imports
C_t	final consumption of household (aggregate of sectoral goods)
LSP_t	labor supply
W	intertemporal welfare (utility) of household
CLS_t	full consumption including leisure
G	intertemporal government consumption good
$I_{t,act}$	investment
$K_{t,act}$	capital stock
$KD_{t,act}$	capital demand
$LD_{t,sf}$	labor demand
$IDM_{t,sf}$	intermediate demand of good i in sector j
$CD_{t,sf}$	Final consumption demand of energy and sectoral goods

- Market prices

$PY_{t,i}$	domestic price index
$PELE_t$	price of electricity
PE_t	price of energy
PF_t	price for fossil fuel resources

PL_t	wage rate index
PLS_t	price of leisure
$RK_{t,act}$	rental price of capital
$PCLS(t)$	price for full consumption including leisure
$PK_{t,act}$	price of one unit of capital stock
PKT	price of one unit of post-terminal capital stock
PC_t	price of final consumption good for household
PW	price aggregate of intertemporal utility good (consumption and leisure) - price for welfare of household
PG	price of aggregate government consumption
PIM_t	price of import good composite
$PIME_t$	price of refined energy import good
$PN_{t,res}$	Shadow price of natural resources (complementary variable for available natural resource supply)
$PCAP_{t,tec}$	shadow price on capacity (complementary variable for technology specific capacity constraints)
$pcarb_t$	price for carbon = carbon tax

- Income levels of agents

HH	income level of representative household agent
$GOVT$	income level of government agent

- Other variables

CA	current account
KT	post-terminal capital stock
ψ, ϕ, κ	refinancing instruments for endogenous taxes or subsidies
$\eta_{imp(t,i,f,tec)}$	endogenous share of imports by sector, factor, technology

Exogenous parameters will be described in the equations itself, a complete list will be provided in the appendix, chapter A (tables A.1 and A.2).

4.2.1 Nesting Structure

The nesting structure is crucial for understanding the model. The production of sectoral goods, as well as consumption, is determined via so-called *nested CES* functions. This means that the sectors can substitute between different inputs into production with a certain fixed, exogenously given elasticity of substitution, while consumers can substitute between different consumption goods with a certain exogenous elasticity. The CES functions are mostly given in the so-called *calibrated share form*. Basically, the calibrated share form is a normalisation of a CES function with respect to the relation of variables to their benchmark values (see Klump and Saam, 2007, [35]). Further information on the calibrated share form and its equivalence to the so-called coefficient form of CES functions can be obtained from Böhringer et al. (2003, [10, pp. 7-11]).

In short, the coefficient form of a CES production function takes the following shape (see Böhringer et al. 2003, [10, pp. 7-9]):

$$Y = \gamma \cdot \left(\sum_i \alpha_i x_i^{\rho} \right)^{\frac{1}{\rho}} \tag{4.1}$$

where

Y	denotes the level (output) of production
γ	is a shift (scaling) parameter
α_i	is a distribution parameter for input i
x_i	signifies the demand for input i
ρ	denotes a substitution parameter, derived from an elasticity of substitution σ ($\rho := \frac{\sigma-1}{\sigma}$)

The calibrated share form takes a slightly different appearance:

$$Y = Y_0 \cdot \left[\sum_i \left(\theta_i \left(\frac{x_i}{x_{i_0}} \right)^{\rho} \right) \right]^{\frac{1}{\rho}} \tag{4.2}$$

where

Y_0	denotes the benchmark output level of production,
θ_i	is the benchmark value share of input i into production, with
$\theta_i = \frac{x_{i_0} w_{i_0}}{Y_0 p_0}$	Here, x_{i_0} is the benchmark demand for input i, w_{i_0} is the benchmark price for input i, Y_0 is benchmark output, and p_0 is the benchmark output price,
ρ	is a substitution parameter defined as above.

The equivalence between these depictions of CES functions can be obtained from Böhringer et al. (2003, [10, p. 8]). Generally speaking, $\frac{Y}{Y_0}$ represents a normalised output (normalised to the benchmark output level) represented by a weighted general mean of order ρ taken over normalised inputs $\frac{x_i}{x_{i_0}}$ (cf. Klump and Saam, 2008, [35, p. 257]). For the model described in the following, this calibrated share form substantially facilitates calibration.

There, the benchmark values of the variables are taken from the data, as well as the benchmark value shares of input i into production (which can easily be calculated from SAM data). As we have an equilibrium in the benchmark and a CRTS economy, all shares sum up to one. Further, in the benchmark run the level of the variables corresponds to their benchmark value. Thus, all level variables as described above, which are subsequently determined endogenously within the model, are set to *unity*. As a consequence, the benchmark equilibrium can be replicated without any computing power, as described in section 2.2.3 above.

All other CES functions (cost and demand functions for production, utility, expenditure and demand functions for consumption) can be cast in calibrated share form in a very similar manner, see Böhringer et al. (2003, [10, pp. 7-11]) for further elaboration on this. Specifically, the *unit cost functions*, i.e. the costs for one unit of output, corresponding to the production function in calibrated share form take the following form (see Rutherford, 2002, [53, p. 6])[4]:

$$C(w) = C_0 \cdot \left[\sum_i \theta_i \left(\frac{w_i}{w_{i_0}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{4.3}$$

where

C	is the unit cost level, i.e. the cost level for one unit of production,
C_0	denotes benchmark unit costs,
w	signifies the vector of input prices w_i,
θ_i	represents the benchmark value share of input i as above , and
σ	is an elasticity of substitution

A large part of the basic structure of the model is determined through the nesting chosen. The nesting structure for sectoral production is shown in the following figure 4.1, that of consumption in figure 4.2.

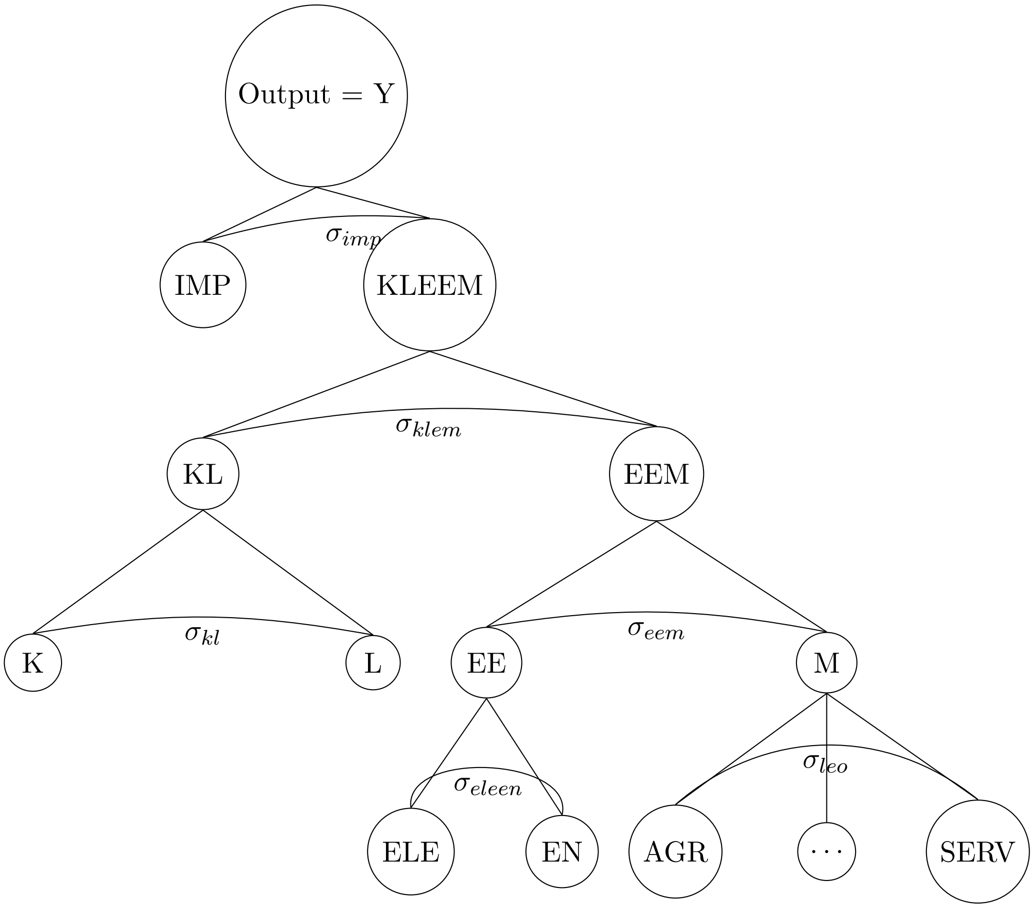
In this nesting structure, output Y is a composite of imported goods (IMP) and a nest of capital (K), labour (L), energy (EE) and material (M), $KLEEM$, where the sectors can substitute with the elasticity σ_{IMP}. This means that a good can either be produced domestically or imported, which essentially is a reduced form of the Armington assumption (see Armington, 1969, [2]).

In the next step, there exists a possibility of substitution between a capital and labour composite (nest KL) and an energy and material (EEM) nest for domestic production with the elasticity σ_{klem}.

Then again, in the different nests, the sectors can substitute between capital and labour (nest KL, elasticity σ_{kl}), and between the energy composite (nest EE) and the material composite M with the elasticity σ_{eem}. On the bottom level, the sectors can choose between different material inputs, either between electricity ELE and fossil energy EN in the energy domain (nest EE, elasticity σ_{EE}), or between sectoral goods in the material nest M, with the elasticity σ_{leo}. The material nest is usually chosen as a Leontief-Nest (zero possibility of substitution), or with a low elasticity of substitution. Some values chosen for these elasticities will be presented in chapter A.

4 Remark: During the calibration procedure, the benchmark unit costs C_0 will be set to unity, therefore they are not considered in the equations given in section 4.2.2.

Figure 4.1: The Nesting Structure of Producing Sectors



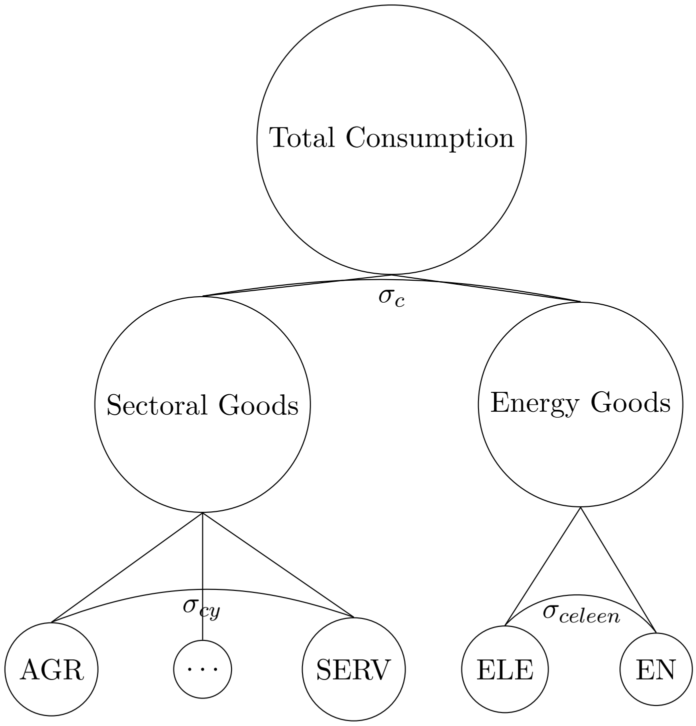
Consumption is organised similarly. The households decide between bundles of consumption goods, with a certain, exogenously specified elasticity.

On the top levels, households decide whether to consume energy goods or the sectoral goods with the elasticity σ_c. Then, on the levels below, they can decide between the sectoral goods themselves, with a uniform elasticity σ_{cy}, and how they form their energy goods composite, where they choose between electricity and fossil fuels, with an elasticity σ_{celeen}.

As the focus of the model is put on fossil energy production, and here, at least for the Austrian economy, imports of fossil fuels play a major role, the production structure of energy sectors is organised differently, as shown in figure 4.3. In the following, when the term energy is used, it is meant as a synonym to fossil energy products, unless explicitly defined otherwise.

The sectors decide on the top level whether to produce fossil energy domestically (FLYE), or whether to import refined energy products (Imports). On the next level, a composite of labour and fossil fuel imports (FL) can be substituted for inputs from other sectors (YE), including energy and electricity with the elasticity σ_{FL}. Fossil fuel imports and labour can be substituted for each other with a (low) elasticity σ_{FL}.

Figure 4.2: The Nesting Structure of Household Consumption

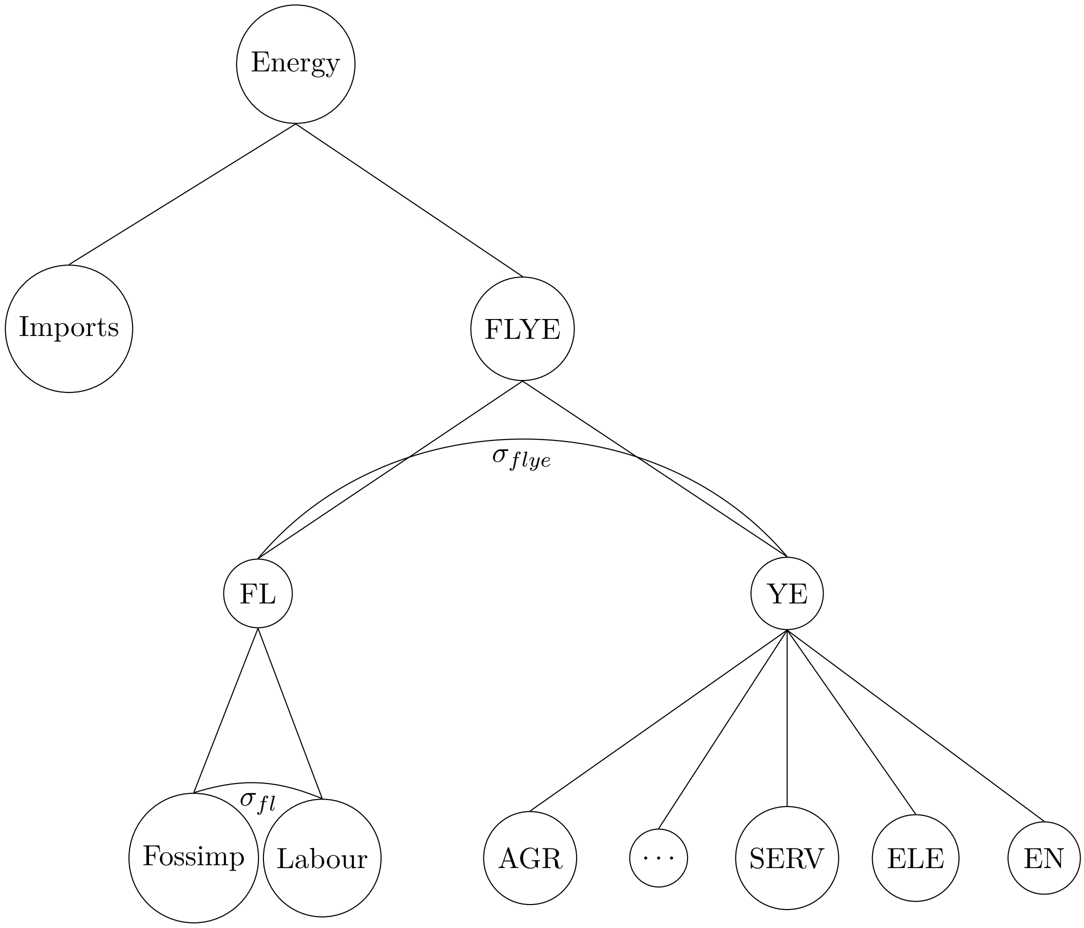


In this nesting, the imports of fossil fuels substitute for capital in the FL nest. This means that labour and raw energy can be combined, with a certain (low) elasticity, into a composite, which can then be refined using the products and inputs from other sectors (YE). Thus, the technical process of refining fossil energy is depicted by using a raw-energy - labour composite and inputs from other sectors. This technical process is held fixed in a Leontief-nest (zero possibility of substitution). Also, on the top level, domestically produced fossil energy and imports are held in fixed proportions, assuming that products which are imported in a refined form cannot be substituted for domestic products in the medium term for technical reasons (no adequate refinery plants, etc.).

The electricity sector, as mentioned before, is modeled in a separate bottom-up part, where the nesting structure of the CES functions given above is replaced by a step-wise supply function based on available technological options to produce electricity, such as hydropower, wind power, thermal power plants, etc., the structure of which will be described within the equations concerning the electricity sector in the remaining sections of this chapter.

In the following, the terminology introduced for the nesting structure above will be used for the model equations given in the next section.

Figure 4.3: The Nesting Structure of Energy Production



4.2.2 Zero Profit Conditions

As elaborated on in sections 3.2.1 and 3.2.2, the zero profit conditions prescribe that the **unit profit function** for the representative sectoral firms has to be smaller or equal to zero for all goods in all years, that is $-\Pi^Y_{t,i} \geq 0 \quad \forall t,i$. The firms minimise their costs subject to CES functions, which tell us the price-dependent use of factors and intermediate inputs for each sector (see Böhringer, Rutherford, 2008, [8, p. 581]). This intuitively means that the market value of the inputs has to equal the market value of the outputs (with simultaneous market clearance, which is ensured by the market clearance conditions). This can be written as

$$\begin{aligned} \Pi^Y_{t,i} \text{ (unit profit of macro sector i at time t) } &= p^Y_{t,i} \text{ (output price of good)} \\ &- \textit{unit costs} \text{ (market value of inputs for unit production)} \leq 0, \end{aligned}$$

The structure of the zero profit conditions described in the following will follow this pattern. These CES functions are similar to the ones described in the previous section, given in calibrated share form. However, as we use unit profit functions, benchmark levels do not have to be considered for normalisation, and we can solely rely on prices and benchmark value shares for the representation of the zero profit conditions. The functional form for the unit costs follows that presented in equation (4.3).

The zero profit condition for the macro sectors (excluding energy and electricity), now, reads as follows:

$$\Pi^{Y}_{t,i} = PY_{t,i} - total\ unit\ cost \leq 0 \Leftrightarrow$$
$$\eta_{imp_{t,i}} \cdot PIM_t + (1 - \eta_{imp_{t,i}}) \cdot \left[\left(\theta_{klem_i} \cdot KLcomp_{t,i}^{1-\sigma_{klem_i}} + (1 - \theta_{klem_i}) \cdot EEMcomp_{t,i}^{1-\sigma_{klem_i}} \right)^{\frac{1}{1-\sigma_{klem_i}}} \right] \geq PY_{t,i} \quad (4.4)$$

where

$PY_{t,i}$	is the output price of the sectoral good
PIM_t	is the fixed world market price of the good
$\eta_{imp_{t,i}}$	is the endogenous share of imports
θ_{klem_i}	is the share of the capital and labour composite in total sectoral production
$1 - \theta_{klem_i}$	is then the share of energy, electricity and material in total production (as all shares add up to one)
$KLcomp_{t,i}$	is the composite of capital and labour as shown in figure 4.1
$EEM_{t,i}$	is the composite of energy, electricity and material (intermediate inputs) as shown in figure 4.1
σ_{klem_i}	is the elasticity of substitution between the composites described above
Y_i	is the associated complementary variable

The composites themselves, now, are of CES form:

- The capital-labour nest:

$$\left[\theta_{k_i} \cdot RK_{t,i}^{1-\sigma_{kl_i}} + (1 - \theta_{kl_i}) \cdot PL_t^{1-\sigma_{kl_i}} \right]^{\frac{1}{1-\sigma_{kl_i}}} \quad (4.5)$$

where

θ_{k_i}	denotes the share of capital
$RK_{t,i}$	is the rental rate of capital of sector i
PL_t	is the price of labour
σ_{kl_i}	is the elasticity of substitution between labour and capital

- The electricity, energy, material nest:

$$\left[\theta_{ee_i} \cdot EEcomp_{t,i}^{1-\sigma_{eem_i}} + (1-\theta_{ee_i}) \cdot Mcomp_{t,i}^{1-\sigma_{eem_i}}\right]^{\frac{1}{1-\sigma_{eem_i}}} \tag{4.6}$$

where

σ_{eem_i}	is the elasticity of substitution between the energy-electricity and the material composites
θ_{ee_i}	is the share of the electricity-energy composite
$EEcomp_{t,i}$	is the electricity-energy composite
$Mcomp_{t,i}$	is the material composite

- The electricity-energy subnest:

$$\left[\theta_{ele_i} \cdot \left(PELE_t \cdot \left(1 + t_{yele} \cdot \frac{\psi_{t,t_{yele}}}{1+t_{yele}}\right)\right)^{1-\sigma_{elee_i}} + (1-\theta_{ele_i}) \cdot \left(PE_t \cdot \left(1 + t_{ye} \cdot \frac{\psi_{t,t_{ye}}}{(1+t_{ye})}\right)\right)^{1-\sigma_{elee_i}}\right]^{\frac{1}{1-\sigma_{elee_i}}} \tag{4.7}$$

where

θ_{ele_i}	denotes the share of electricity
$PELE_t$	is the price of electricity
t_{yele}	represents the (exogenous) tax rate on electricity consumption by sectors
$\psi_{t,t_{yele}}$	is the endogenous adjustment instrument for the electricity tax
σ_{elee_i}	is the elasticity of substitution between electricity and energy
PE_t	is the price of fossil energy
t_{ye}	is the (exogenous) tax rate on fossil energy use by sectors
$\psi_{t,t_{ye}}$	is the endogenous adjustment instrument for the energy tax

- The material subnest:

$$\left[\sum_j \theta_{leo_{j,i}} \cdot PY_{t,j}^{1-\sigma_{leo}}\right]^{\frac{1}{1-\sigma_{leo}}} \tag{4.8}$$

where

$\theta_{leo_{j,i}}$	is the share of input good j in sector i's production
σ_{leo}	is the elasticity of substitution between material inputs
$PY_{t,j}$	is the price of the output good of sector j

Thus, the nesting structure given in section 4.2.1 is exemplified in the form of CES functions. The endogenous share of imports in the economy is determined by the relation between the domestic price level and the world price level (fixed to unity): if the domestic price level rises, imports increase, since it is relatively cheaper to import goods compared to producing domestically now. The converse applies for a fall in the domestic price level: imports will diminish, since it becomes relatively cheaper to produce domestically.

The zero profit function for the production of *fossil fuel energy*, now, works similarly:

- The total energy nest at the top level:

$$\Pi_t^{en} \leq 0 \Leftrightarrow$$
$$\eta_{impt_{t,en}} \cdot PIME_t + (1 - \eta_{impt_{t,en}}) \cdot \left[FLYEcomp_t \cdot \frac{carbon}{Y_{en}} \cdot pcarb_t \right] \geq PE_t \quad (4.9)$$

where

$\eta_{impt_{t,en}}$	is the endogenous import share of refined fossil fuel energy
$PIME_t$	is the price for the import of refined fossil fuel energy
$FLYEcomp_t$	is the composite for domestic production of energy
$carbon$	is the baseline amount of carbon emission resulting from the total amount of fossil fuel energy available in the economy
Y_{en}	is the total output of fossil fuel energy in the baseyear
$pcarb_t$	is the price to emit carbon, or carbon tax
EN	is the associated complementary variable

The total output of carbon emissions in this model is assumed to adhere to the quantity of fossil fuels only, thus abstracting from the fact that different technical processes will influence the amount of carbon emitted by one unit of fossil fuel energy, which is incorporated exogenously in the model. The carbon emissions are then brought to the average emissions by one unit of fossil fuel energy by dividing it by the total output Y_{en}, and then a price of carbon (a carbon tax) is placed on this one unit of fossil fuel energy, thus raising the price of fossil energy PE.

- The nest of the fossil fuel imports/labour composite and the inputs from other sectors reads as follows:

$$\left[\theta_{fl} \cdot FLcomp_t^{1-\sigma_{flye}} + (1 - \theta_{fl}) \cdot YEcomp_t^{1-\sigma_{flye}} \right]^{\frac{1}{1-\sigma_{flye}}} \quad (4.10)$$

where

θ_{flye}	is the share of the fossil fuel imports/labour composite
$FLcomp_t$	is the fossil fuel/labour composite in energy production
σ_{flye}	is the elasticity of substitution between the composites

$YEcomp_t$ is the composite of sectoral inputs into energy production

- The fossil fuel imports/labour nest:

$$\left[\theta e_f \cdot PF_t^{1-\sigma_f} + (1-\theta e_{fl}) \cdot PL_t^{1-\sigma_{fl}}\right]^{\frac{1}{1-\sigma_{fl}}} \tag{4.11}$$

where

θe_f	is the share of fossil fuel imports in the composite
PF_t	is the price of fossil fuel imports
PL_t	is the price of labour (wage)

- In the sectoral inputs nest, the absence of an elasticity of substitution indicates a Leontief nest (we assume the technical process of producing energy to be fixed, determining energy efficiency exogenously):

$$\sum_j \left(\theta_{leoe_{j,en}} \cdot PY_{t,j}\right) + \theta_{leo_{ele,en}} \cdot PELE_t \cdot \left(1 + t_{yele} \cdot \frac{\psi_{t,t_{yele}}}{1+t_{yele}}\right) + \theta_{leo_{en,en}} \cdot PE_t \tag{4.12}$$

$\theta_{leoe_{j,en}}$	is the share of the input good of sector j in the production process of the energy sector
$PY_{t,j}$	is the price of the sectoral macro goods
$\theta_{leo_{ele,en}}$	is the share input of electricity in energy production
$PELE_t$	is the price of electricity
$\psi_{t,t_{yele}}$	is the endogenous adjustment factor for the electricity tax
t_{yele}	is the electricity tax
PE_t	is the price of energy

Here, it is assumed that the energy sector pays no tax on the energy it produces and uses in the production process of energy.

The production of the electricity sector, now, is not captured by a CES production function, but in a separate bottom-up representation for the technical production possibilities in this sector. Thus, electricity production is depicted by Leontief technologies which are active or inactive in equilibrium. The profitability determines the activity of such a technology. This focus on technologies in electricity production is the key to simulate policy issues such as green quotas or carbon limits, as these often require certain electricity production technologies (e.g. fossil fuels, nuclear power, etc.) to become inactive (see Böhringer, Rutherford, 2008, [8, p. 582]):

$$\Pi_t^{ele} \leq 0 \Leftrightarrow$$
$$KLEM_{ELE_{t,tec}} + \sum_{res} a_{res,tec} \cdot PN_{t,res} + PCAP_{t,tec} \; if \; capacity_{t,tec}$$
$$\geq PELE_t \cdot (1 + \phi_t \; if \; quota_{tec}) \quad (4.13)$$

where

$KLEM_{ELE_{t,tec}}$	denotes the capital-material-labour-energy composite in electricity production
$a_{res,tec}$	is the input-coefficient of resources for technologies (e.g. wind power requires wind as input, etc.)
$PN_{t,res}$	is the price of the natural resources
$PCAP_{t,tec}$	is the shadow price on capacity for a technology (capacity constraints as regarding plants to produce energy, etc.)
$capacity_{t,tec}$	is the capacity constraint for technologies
$PELE_t$	is the price of domestically produced electricity
ϕ_t	is an endogenous subsidy adaption parameter for desired technologies
$quota_{tec}$	sets the quota obligations for certain technologies
ELE_t	is the associated complementary variable

The capital-labour-energy-material (KLEM) nest in electricity production takes the following form:

$$a_{tec}^K \cdot RK_{t,tec} + a_{tec}^L \cdot PL_t + a_{tec}^{en} \cdot PE_t \cdot \left(1 + \frac{t_{ye} \cdot \psi_{t,t_{ye}}}{1 + t_{ye}}\right) + \sum_i a_{tec}^i \cdot PY_{t,i} \quad (4.14)$$

where

a_{tec}^K	is the input coefficient for capital by technology
$RK_{t,tec}$	is the return to capital by technology
a_{tec}^L	is the input coefficient of labour by technology
PL_t	is the price of labour
a_{tec}^{en}	is the input coefficient for fossil energy by technology
PE_t	is the price of fossil energy
t_{ye}	is the tax on fossil energy for sectors
$\psi_{t,t_{ye}}$	is the endogenous adjustment factor for the energy tax
a_{tec}^i	is the input of sectoral good i in technology tec
$PY_{t,i}$	is the price of this sectoral good

On the household side, utility is determined by consumption and leisure. The *final consumption zero profit condition* is again characterised by a nested CES function (see also figure 4.2):

$$\Pi_t^C \leq 0 \Leftrightarrow \left[\theta c_y \cdot YcompC_t^{1-\sigma_c} + (1-\theta c_y) \cdot EEcompC_t^{1-\sigma_c}\right]^{\frac{1}{1-\sigma_c}} \geq PC_t \quad (4.15)$$

where

θc_{yee}	is the share of sectoral goods in total consumption
$YcompC_t$	is the sectoral goods composite in total consumption
σ_c	is the elasticity of substitution between the sectoral goods composite and the energy-electricity composite in total consumption
$EEcompC_t$	is the energy-electricity composite in total consumption
C_t	is the associated complementary variable

The CES-form of the composites is depicted as follows:

- The nest of the sectoral goods in consumption is given by:

$$\left[\sum_i \theta c_{y_i} \cdot \left(PY_{t,i} \cdot \left(1 + t_c \cdot \frac{\psi_{t,t_c}}{1+t_c}\right)\right)^{1-\sigma_{cy}}\right]^{\frac{1}{1-\sigma_{cy}}} \quad (4.16)$$

where

θc_{y_i}	is the share of good i in the sectoral consumption composite
$PY_{t,i}$	is the price of good i
t_c	is the tax on final consumption
ψ_{t,t_c}	is the endogenous adjustment factor for the consumption tax
σ_{cy}	is the elasticity of substitution for consumption goods

- The energy-electricity nest in consumption is written as:

$$\left[\theta c_{ele} \cdot \left(PELE_{t,i} \cdot \left(1 + \frac{t_{hele} \cdot \psi_{t,t_{hele}} + t_c \cdot \psi_{t,t_c}}{1 + t_{hele} + t_c}\right)\right)^{1-\sigma_{celeen}} + (1-\theta c_{ele}) \cdot \left(PE_t \cdot \left(1 + \frac{t_{he} \cdot \psi_{t,t_{he}} + t_c \cdot \psi_{t,t_c}}{1 + t_{he} + t_c}\right)\right)^{1-\sigma_{celeen}}\right]^{\frac{1}{1-\sigma_{celeen}}} \quad (4.17)$$

where

θc_{ele}	stands for the share of electricity in household consumption
t_{hele}	denotes the tax on household consumption of electricity
$\psi_{t,t_{hele}}$	is the corresponding endogenous adjustment parameter
t_c	is the tax on household final consumption
ψ_{t,t_c}	is the corresponding endogenous adjustment parameter

σ_{celeen} is the elasticity of substitution between energy and electricity in final household consumption

Total household consumption is then combined with leisure and linked over the periods to obtain full household utility, which we denote by *welfare* in this model. First of all, the price of leisure is determined via the wage level and the wage tax and a corresponding *zero profit function for labour supply.* As the elastic labour supply will be a variable determining household demand for leisure and the according factor prices, the zero profit function for labour supply relates the price of labour and the price of leisure to each other:

$$\Pi_t^{LS} \leq 0 \Leftrightarrow PLS_t \geq PL_t \cdot (1 - t_w \cdot \psi_{t,t_w}) \tag{4.18}$$

where

PLS_t is the price of leisure at time t
PL_t is the wage at time t
t_w is the wage tax (incl. social security benefits for employer and employee)
$\psi t,t_w$ is the corresponding endogenous adjustment parameter
LSP_t (labour supply at time t) is the corresponding complementary variable

Here, the representative household will supply additional labour as long as the marginal benefit from working (wage adjusted by taxes) is larger than the marginal disutility from working (the price for leisure). Therefore, the "costs" for not working (the price of leisure) will adapt so that the price for leisure has to be at least as large as the wage level (corrected by labour taxes). The mechanics behind this are the following: if the price of leisure were lower than the wage level after taxes, the household would supply additional labour, thus lowering the price for labour (until it is equal to the price of leisure). This elastic labour supply is one of the key driving forces in the model, together with the equations concerning investment and capital formation. Thus, if e.g. the wage level rises, the corresponding price for leisure has to increase, which then in turn has implication on the consumption behaviour (which itself is a mix of leisure and material goods consumption) of the household according to the equational framework of the consumption-leisure-welfare block of the model equations, see also equations (4.19) and (4.20).

On the next level leisure is combined with aggregate consumption in a CES function:

$$\Pi_t^{CLS} \leq 0 \Leftrightarrow$$
$$\left[\theta cls_c \cdot PC_t^{1-\sigma_{cls}} + (1 - \theta cls_c) \cdot PLS_t^{1-\sigma_{cls}}\right]^{\frac{1}{1-\sigma_{cls}}} \geq PCLS_t \tag{4.19}$$

where

θcls_c is the share in consumption in the consumption and leisure composite
PC_t is the price of consumption
σ_{cls} is the elasticity of substitution between consumption and leisure
PLS_t is the price of leisure
$PCLS_t$ is the price of full consumption including leisure
CLS_t full household consumption including leisure is the complementary variable

Here, the price for consumption and the price for leisure determine, according to the substitution elasticity between consumption and leisure, how the household composes its full consumption aggregate CLS_t. The relation between the prices of consumption and leisure in this equation then, of course, will determine the labour supply behaviour of the household decisively. For example, if the wage level rises, this would at first increase the labour supply by the household, as the price for leisure PLS_t has to increase in turn with the wage level subject to taxes, equation (4.18), which means that less leisure is demanded by the household (thus, the labour supply is one of the quantitative variables that will adapt in equation (4.37), the market clearance condition for labour supply and leisure, respectively). However, an increase in the wage level and the level of household consumption might in turn have influence on the sectoral prices (higher production costs and higher demand level), and thus also the aggregate price of consumption. Higher prices will further stimulate sectoral production and increase factor demand, which would meet the increased labour supply. It will depend on the specification of the exogenous parameters (elasticities etc.), whether a wage increase in the end will actually lead to an increased employment of the factor labour (as the sectors can e.g. substitute between capital and labour in the production process).

The full intertemporal household utility is then obtained by linking up the different time periods, which is done in the following equation:

$$\Pi^W \leq 0 \Leftrightarrow \left[\sum_t \left(\theta wcls_t \cdot \frac{PCLS_t}{pref_t} \right)^{1-\sigma_t} \right]^{\frac{1}{1-\sigma_t}} \geq PW \tag{4.20}$$

where

Π^W is the intertemporal profit function of household welfare
$\theta wcls_t$ is the share of household welfare obtained in time t
$PCLS_t$ is the price of full consumption including leisure at time t
$pref_t$ is the price reference path, a discount factor that is applied to all prices in the economy (exponentially)
σ_t denotes the intertemporal elasticity of substitution of the household
PW is the intertemporal price of welfare
W welfare is the associated complementary variable

The condition above, as described in section 3.2.4, states that the optimising infinitely lived agent allocates lifetime income to maximise full consumption, i.e. consumption of goods and leisure (consumption of time), intertemporally (subject to the intertemporal elasticity of substitution). Here, the share parameter $\theta wcls_t$ measures the share of the time endowment in period t in the total time endowment, which is the sum of all time endowments over all periods. The evolvement of the time endowment in period t is determined by the quantity, see equation (4.43), and price, see equation (4.44), reference paths. As the price reference path is reflected both in the price for full consumption including leisure $PCLS_t$ and the share parameter $\theta wcls_t$, $PCLS_t$ is divided by $pref_t$ to avoid double-accounting of the price development path.

Furthermore, we need a zero profit condition for investment and capital. The inter-period relation of the yearly prices for capital is described by the *zero profit condition for investment*, see also (3.38):

$$\begin{aligned} \Pi_t^I \leq 0 \Leftrightarrow \\ \sum_i PY_{t,i} \cdot a_i^I \geq PK_{t+1,act} \; if \; (t < T), \\ \sum_i PY_{t,i} \cdot a_i^I \geq PK_{t+1,act} + PKT \; if \; (t = T) \end{aligned} \tag{4.21}$$

where

PY_t	is the price of the output good of sector i at time t
a_i^I	is the share of output of sector i that is put into investment
$PK_{t+1,act}$	is the price of capital in period $t+1$ for sector/technology act
PKT	is the price of the post-terminal capital stock in the terminal period T
$I_{t,act}$	is the associated complementary variable, investment of activity act at time t

Thus, the market price of capital in year $t+1$ is limited by undertaking investment, composed of sectoral goods, in period t (if the price of capital in period $t+1$ rises too much, investments in this periods would go up, thus lowering the price of capital in $t+1$). Basically, this zero profit condition relates the price of the investment (output) goods to the price of capital for each activity (production sectors and technologies). As long as it is more expensive to buy a unit of an investment good from the other sectors than to buy a unit of capital, no investment occurs. Only if the price of capital is equal to the price of the investment goods for an activity, investment is undertaken (complementary slackness). Investment is then undertaken for each sector so to satisfy the capital market clearance given in equation (4.28).

As described in section 3.2.4, state variable targeting can be used to approximate terminal investment and post-terminal capital stock. Investment in the terminal period can be related to other stable variables such as e.g. consumption or previous investment variables. Here, the terminal constraint forces investment to increase in proportion to investment of the pre-terminal (the "second last") period of the modelling horizon, see equation (4.30). In the terminal period $t = T$, the price of capital for the next period, PK_{t+1} will equal zero, as the model world comes to an "end" and no value will be placed on capital in the post-terminal period any more. This would lead to a price of zero for capital also in the terminal period, and, given perfect foresight, would thus have repercussions on all previous periods of the model.

Therefore, the equation is amended by the price for the post-terminal capital stock PKT, the complementary variable to equation (4.29), which is unequal to zero due to the constraint placed on investment for the terminal period in equation (4.30). This price for post-terminal capital approximates the infinite horizon price for capital (see Lau, Pahlke and Rutherford, 2002, [54]).

The *zero-profit condition for capital*, which of course interlocks with the zero profit condition for investment, takes the following form, see also (3.37):

$$\begin{aligned} \Pi_t^K \leq 0 \Leftrightarrow \\ PK_{t,act} \geq RK_{t,act} \cdot (r+\delta) + PK_{t+1,act} \cdot (1-\delta) + \ if\ (t<T), \\ PK_{t,act} \geq RK_{t,act} \cdot (r+\delta) + PK_{t+1,act} \cdot (1-\delta) + [PKT \cdot (1-\delta)]\ if\ (t=T) \end{aligned} \tag{4.22}$$

where

$PK_{t,act}$ is the price of one unit of capital stock for activity act at time t
$RK_{t,act}$ is the rental price of one unit of capital at time t for activity *act*
r is the exogenously determined baseline interest rate
δ is the exogenously determined depreciation rate of capital
PKT is the price of the post-terminal capital stock
$K_{t,act}$ is the associated complementary variable, capital stock of activity act at time t

Again, as described in section 3.2.4, this condition implies that the market value of a unit of capital purchased at time t must be greater or equal to the value of capital rental services through that period plus the price of capital in the next period, which is subject to depreciation. The intuition is the following: the households will adjust their capital in such a way, so that they are indifferent between buying marginally more capital stock in this period, and keeping it (not selling it off) at the beginning of the next period. This, then, is their optimal period-by-period capital stock, and prices will form correspondingly. The zero profit condition for investment, (4.21), interacting with this condition, will take care that the capital stock of the households, who rent it out to the activities, is adjusted accordingly.

Using state variable targeting as described in section 3.2.4 and similar to the condition for the terminal period for equation (4.21), the price for the post-terminal capital stock is added to the equation only in the terminal period of the model.

4.2.3 Market Clearance Conditions

As already described in section 3.2.2, Equation (2.5), using *Hotelling's Lemma*, Equation (3.19), we can derive compensated supply and demand functions for goods and factors. The balance of supply and demand is assured by the flexible prices on competitive markets [8, p. 583]. These flexible prices are determined in concordance with the zero profit conditions. The interdependence between zero profit and market clearance conditions is thus mutual: prices and quantities are determined simultaneously by the zero profit and market clearance conditions. Each condition puts a restraining factor on the other. If a sector makes negative profit, nothing is produced, thus quantities are zero, and supply will equal demand.

If supply permanently exceeds demand, on the other hand, prices will be zero. Only in the case of zero but nonnegative profit and equality of supply and demand we have positive quantities and positive prices for a sector. As then all goods and factor markets have to clear, we get a system of interdependent endogenous prices and quantities constituting our model economy.

We will start with the market clearance conditions of the **factor markets**. To give an example, the market clearance condition for *labour* in this model economy then looks like (see also [8, p. 583]):

$$\bar{Z}_t - \frac{\partial \Pi^W}{\partial PLS_t} \cdot W = LSP_t \geq \sum_i \frac{\partial \Pi_t^{Y_i}}{\partial PL_t} \cdot Y_{t,i} + \frac{\partial \Pi_t^{en}}{\partial PL_t} \cdot EN_t + \sum_{tec} a_{t,tec}^L \cdot ELE_{t,tec} \quad (4.23)$$

where

$\bar{Z}_t$	is the total endowment of time per year by the household
$\frac{\partial \Pi^W}{\partial PLS_t}$	is unit demand for leisure at time t
LSP_t	is the level of labour supply by the household at time t, which is equal to total household time endowment minus the demand for leisure
$\frac{\partial \Pi_t^{Y_i}}{\partial PL_t}$	is unit labour input required by sector i at time t
$\frac{\partial \Pi_t^{en}}{\partial PL_t}$	is unit labour input required by fossil fuel production sector *en* at time t
$Y_{t,i}$	is the level of production of sector i at time t
$a_{t,tec}^L \cdot ELE_{t,tec}$	is unit labour input required by technology tec for electricity production at time t

Here, the price for labour PL_t is the associated complementary variable. In the model equations, this market clearance condition takes the following form:

$$LSP_0 \cdot LSP_t \geq \sum_i \left[(-Y^L_{0,i}) \cdot (Y_{t,i}) \cdot \left(\frac{PY_{t,i}}{KLcomp_{t,i}} \right)^{\sigma_{klem_i}} \cdot \left(\frac{KLcomp_{t,i}}{PL_t} \right)^{\sigma_{kl_i}} \right]$$
$$+ (-EN^L_0) \cdot (EN_t) \cdot \left(\frac{FLYEcomp_t}{FLcomp_t} \right)^{\sigma_{flye}} \cdot \left(\frac{FLcomp_t}{PL_t} \right)^{\sigma_{fl}}$$
$$+ \sum_{tec} a^L_{t,tec} \cdot (-ELE^L_{0,tec}) \cdot ELE_{t,tec} \quad (4.24)$$

where the composites are defined as in figure 4.1, $(-Y^L_{0,i})$ is the benchmark demand for labour by sector i, LSP_0 is the benchmark supply of labour by the households, $(-EN^L_0)$ is the benchmark labour demand of the energy sector, and $(-ELE^L_{0,tec})$ is the benchmark labour demand of an electricity producing technology tec (all coming from SAM data). The negative sign in front of a benchmark demand variable such as $Y^L_{0,i}$ is due to the signing conventions of the SAM, as inputs into productions are denoted with a negative sign (see section 4.1).

Henceforth, if a level variable is given such as $Y_{t,i}$ and is connected to the unit demand derived from the profit function, such as
$\frac{\partial \Pi^{Y_i}_t}{\partial PL_t}$,
what is actually denoted here is the benchmark demand of a certain sector for a certain input factor, e.g. $(-Y^L_{0,i})$, times the activity level of this sector $Y_{t,i}$, multiplied with the price-responsive (responsive to the good's price, in this case the wage level) compensated unit demand function for this good. Similarly, any variable which we write as LSP_t actually denotes the benchmark level LSP_0 times a level variable also signified by LSP_t (which starts from unity for all goods and factors in the economy, see section 4.3) that is determined endogenously within the model. This satisfies the calibrated share form, which in turn facilitates the calibration procedure. For sake of simplicity, however, these benchmark level variables will be omitted in most of the equations presented henceforth. When referring back to the calibrated share form, however, it will be important for the reader to bear the facts sketched above in mind. What is called a *level variable* henceforth is nothing more than a variable indicating the relative change of a certain variable in relation to its benchmark value.

The derivation of this is exemplarily set out to the reader in this case according to the chain rule (the share parameters resulting from the "innermost" derivatives are not considered), which will not be done for the other market clearance conditions:

- The partial derivative of the macro-sectoral profit function w.r.t. the price of labour:

$$\sum_i \frac{\partial \Pi_t^{Y_i}}{\partial PL_t} \cdot Y_{i,t} = \sum_i Y_{i,t} \cdot \frac{1}{1-\sigma_{klem_i}} \cdot \underbrace{\left(\left(\theta_{klem_i} \cdot KLcomp_{t,i}^{1-\sigma_{klem_i}} + (1-\theta_{klem_i}) \cdot EEMcomp_{t,i}^{1-\sigma_{klem_i}}\right)^{\frac{\sigma_{klem_i}}{1-\sigma_{klem_i}}}\right)}_{=PY_{t,i}^{\sigma_{klem_i}}} \cdot (1-\sigma_{klem_i}) \cdot KLcomp^{-\sigma_{klem_i}} \cdot \frac{1}{1-\sigma_{kl_i}} \cdot \underbrace{\left(\theta_{k_i} \cdot RK_{t,i}^{1-\sigma_{kl_i}} + (1-\theta_{kl_i}) \cdot PL_t^{1-\sigma_{kl_i}}\right)^{\frac{\sigma_{kl_i}}{1-\sigma_{kl_i}}}}_{=KLcomp^{\sigma_{kl_i}}} \cdot (1-\sigma_{kl_i}) \cdot PL_t^{-\sigma_{kl_i}} = \sum_i Y_{i,t} \cdot \left(\frac{PY_{t,i}}{KLcomp_{t,i}}\right)^{\sigma_{klem_i}} \cdot \left(\frac{KLcomp_{t,i}}{PL_t}\right)^{\sigma_{kl_i}} \quad (4.25)$$

- The partial derivative of the energy sectors' profit function w.r.t. the price of labour:

$$\frac{\partial \Pi_t^{en}}{\partial PL_t} \cdot EN_t = EN_t \cdot \frac{1}{1-\sigma_{flye}} \cdot \underbrace{\left(\theta_{fl} \cdot FLcomp_t^{1-\sigma_{flye}} + (1-\theta_{fl}) \cdot YEcomp_t^{1-\sigma_{flye}}\right)^{\frac{\sigma_{flye}}{1-\sigma_{flye}}}}_{=FLYEcomp_t^{\sigma_{flye}}} \cdot (1-\sigma_{flye}) \cdot FLcomp^{-\sigma_{flye}} \cdot \frac{1}{1-\sigma_{fl}} \cdot FLcomp^{\sigma_{fl}} \cdot (1-\sigma_{fl}) \cdot PL_t^{-\sigma_{fl}} = EN_t \cdot \left(\frac{FLYEcomp_t}{FLcomp_t}\right)^{\sigma_{flye}} \cdot \left(\frac{FLcomp_t}{PL_t}\right)^{\sigma_{fl}} \quad (4.26)$$

As the factor capital has a decisive role in the intertemporal framework via the association of investment in the current period and the (desired) capital stock in the future, we have several market clearing conditions for it.

First of all, the firms rent capital from the households as a producing factor, paying the *rental price of capital* for this purpose. The market clearance condition for the rental price of capital, now, takes the following form:

$$K_{t,act} \cdot (r+\delta) \geq \sum_i \frac{\partial \Pi_t^{Y_i}}{\partial RK_t} \cdot Y_{i,t} + \sum_{tec} a_{t,tec}^K \cdot ELE_{t,tec} \Leftrightarrow$$

$$K_{t,act} \cdot (r+\delta) \geq \sum_i Y_{i,t} \cdot \left(\frac{PY_{t,i}}{KLcomp_{t,i}}\right)^{\sigma_{klem_i}} \cdot \left(\frac{KLcomp_{t,i}}{RK_t}\right)^{\sigma_{kl_i}} + \sum_{tec} a_{t,tec}^K \cdot ELE_{t,tec} \quad (4.27)$$

where

$K_{t,act}$	is the capital stock of activity act at time t
r	is the baseline interest rate, chosen exogenously
δ	is the depreciation rate (exogenous)
$\frac{\partial \Pi_t^{Y_i}}{\partial RK_t}$	is the unit demand for capital rental services of activity act at time t
$a_{t,tec}^{K}$	is the capital input coefficient of technology tec at time t
$KLcomp_{t,i}$	is the capital-labour composite in Y production
RK_t	the rental price of capital, is the associated complementary variable

The partial derivation of the profit function for capital follows analogously to the one shown above for the factor labour.

On the left hand side, we have the capital stock of an activity at time t times the baseline interest and depreciation rate. This is exactly the sum to pay for capital rental services in period t, RK_t, since it compensates for the market interest rate (opportunity costs for not selling off the capital stock by the household) and the depreciation of capital. As the capital stock is fixed within one period and can only be changed by investment on an inter-period basis, this corresponds to the supply of capital rental services for one period, which is owned by the representative household in this model. This supply has to be greater or equal to the demand for capital rental services by the producing sectors and the electricity sector (considering also the substitutability with labour in the capital-labour nest), again only allowing for a positive price if supply is equal to demand. The fossil fuel sector will not demand capital services, as capital inputs are substituted by fossil fuel imports for this sector (see Figure 4.3).

The fact that investment today enters the capital stock tomorrow is depicted in a "market clearance condition", associated with the price of capital in period t, PK_t. Thus, the capital stock movement law is incorporated into the MCP framework.

$$K_{t-1,act} \cdot (1-\delta) + I_{t-1,act} \geq K_{t,act} \tag{4.28}$$

Here, it is described that the capital stock of an activity (i.e. macro sectors and technologies) this period, $K_{t,act}$, is no less than last period's capital stock $K_{t-1,act}$, depreciated with the depreciation rate δ, plus last period's investment $I_{t-1,act}$. As this year's capital stock can only be changed via investment at the end of the period (negative investment meaning the selling off of capital stock), investment is the decision variable is this equation. Investment will adapt in such a way as to satisfy the demand for capital foreseen by the sectors (perfect foresight) as long as the price of capital is equal to the price for the investment goods for sectors, see the zero profit condition for investment, (4.21). Thus, the sectors will invest as long as the further production resulting from an additional unit of capital stock in the next period will generate enough revenue to cover the investment costs, the prices of all other inputs required for production (labour, intermediate inputs, fossil energy, electricity) taken into consideration, as they are substitutes for capital in the production process.

The terminal capital constraint is written into the model in the following form, only applying to the terminal period T of the modelling horizon:

$$\sum_{act} (1-\delta) \cdot K_{t,act} + I_{t,act} \geq KT \quad if \ (t=T) \tag{4.29}$$

Here, we determine the post-terminal capital stock, which is endogenous within the model. The post-terminal capital stock is simply the depreciated capital stock of the period of terminal period plus investment within this period, which is determined by the model to evolve according to the investment level of the second-last period of the modelling horizon times the exogenously specified growth factor (see equation (4.30) below), thus avoiding that in the last period the household sells off all the capital stock as the model world comes to an "end". Furthermore, the price for the post-terminal capital stock is incorporated in equations (4.21) and (4.22) in order to avoid a price of zero for the capital stock in the terminal period, which of course due to perfect foresight would have negative repercussions on all modelling periods before.

For equation (4.29), the price for the post-terminal capital stock PKT is the associated complementary variable.

$$I_{t,act} = (1+g) \cdot I_{t-1,act} \quad if \ (t=T) \tag{4.30}$$

The market clearance for the sectoral goods markets is described, again using Hotelling's Lemma, in the following equation:

$$Y_{t,i} \geq \sum_{(j\neq i),en} \frac{\partial \Pi_t^{Y_{j,en}}}{\partial PY_{i,t}} \cdot IDM_{t,i,j} + \sum_{tec} a_{t,tec}^{Y_i} ELE_{t,tec} + \frac{\partial \Pi_t^{Y_i}}{\partial PY_{exp_{i,t}}} \cdot EXP_{t,i} + GD_{t,i} + \sum_{act} I_{t,act} \cdot a_i^I + \frac{\partial \Pi_t^{C_i}}{\partial PC_{t,i}} \cdot CD_{t,i} \tag{4.31}$$

$\frac{\partial \Pi_t^{Y_i}}{\partial PY_{j,t}}$	is the unit demand for intermediate inputs of sectoral good i at time t (by macro and fossil energy sectors)
$IDM_{t,i,j}$	is the level of intermediate demand by sector j (macro sectors and fossil fuel energy sectors) for intermediate good i at time t
$a_{t,tec}^{Y_i}$	is the input of sectoral good i into production of electricity generating technology tec
ELE_{tec}	is the level of electricity generation by technology tec
$\frac{\partial \Pi_t^{Y_i}}{\partial PY_{exp_{i,t}}}$	is the sectoral export demand by the rest of the world (RoW), determined by the relation of the domestic price to

	the exogenously given world price $PY_{exp_{i,t}}$
$EXP_{t,i}$	is the level of export of good i at time t
$GD_{t,i}$	is government demand for sectoral good i
$I_{t,act}$	is investment of activity act at time t
a_i^I	is the fraction of sectoral good i devoted to investment in relation to the total output
$\frac{\partial \Pi_t^C}{\partial PC_{i,t}}$	is the unit final consumption demand for sectoral good i
$CD_{t,i}$	is the level of consumption demand for sectoral good i at time t, and
$PC_{t,i}$	is the price for final consumption of good i, which is different from the output price because of the consumption tax
$PY_{t,i}$	is the associated complementary variable, the price of output good i at time t

Here, we see how sectoral production has to be equal in monetary values to the sum of intermediate inputs used for production (including fossil fuel energy and electricity demand), exported goods, investment by sector, and final consumption demand of the sectoral good.

Intermediate demand here is again determined according to Hotelling's Lemma, analogously to the previously described market clearance conditions and according to the nesting structure laid down in section 4.2.1, figure 4.1 for the macro sectors and figure 4.3 for the fossil fuel energy sectors. Therefore, they do not need to be elaborated on in detail here. For the electricity sector, the input structure for the technologies is given by a Leontief structure (i.e. fixed input coefficients).

Export demand is set into relation to the exogenously determined world price. While the world prices are assumed to be constant throughout the periods, the domestic price level changes. If the domestic price level rises above the world prices, domestic products become more expensive in relation to foreign goods and exports decrease. If the domestic price level falls below world prices, the converse applies and exports increase.

Government demand is held fixed in this model, only growing according to the exogenously fixed growth path. As the government agent is the main source of change in the scenarios, subsidising certain technologies, changing present or introducing new taxes, maybe setting quotas for certain technologies, to name some examples, government demand substantially changes only in the course of counterfactual policy scenarios.

Consumption demand is determined by Hotelling's Lemma, according to the nesting structure of consumption shown in section 4.2.1, figure 4.2 and analogously to the market clearance conditions already described in detail.

The share of the sectoral good i in the *investment* good (thus a weighted average of sectoral goods) is determined from benchmark data. Therefore, the composition of the investment good is given exogenously within the model, i.e. there is no investment function by the sectors (firms) that changes the composition of goods that will make the capital good via investment. This is also one of the aspects of the model that are open for further development and research.

The market clearance conditions for the *fossil energy* goods and the electricity good are only slightly different to the previously described ones:

$$EN_{t,en} \geq \sum_i \frac{\partial \Pi_t^{Y_i}}{\partial PE_{en,t}} \cdot IDM_{t,en,i} + \sum_{tec} a_t^{Y_{en}} ELE_{t,tec} + \frac{\partial \Pi_t^C}{\partial PC_{en,t}} \cdot CD_{t,en} \tag{4.32}$$

where

$EN_{t,en}$	corresponds to the supply of energy good *en*
$\frac{\partial \Pi_t^{Y_i}}{\partial PE_{en,t}}$	is the compensated unit demand for energy good *en* by the sectors
$IDM_{t,en,i}$	is the level of intermediate demand for the energy good by sectors
$a_t^{Y_e n}$	is the input of the energy good *en* into the production of electricity by technology *tec*
ELE_{tec}	signifies the level of electricity generation by technology *tec*
$\frac{\partial \Pi_t^C}{\partial PC_{en,t}}$	is the unit demand for final consumption of the energy good *en*
$PC_{en,t}$	is the consumption price of the energy good *en*, incl. energy taxes
$CD_{t,en}$	denotes the level of final consumption demand for the energy good
$PE_{en,t}$	is the associated complementary variable, the price for energy good *en*

$$\sum_{tec} ELE_{t,tec} \geq \sum_i \frac{\partial \Pi_t^{Y_i}}{\partial PELE_{ele,t}} \cdot IDM_{t,ele,i} + \frac{\partial \Pi_t^{EN}}{\partial PELE_t} \cdot IDM_{t,ele,en} + \frac{\partial \Pi_t^C}{\partial PC_{ele,t}} \cdot CD_{t,ele} \tag{4.33}$$

where

$ELE_{t,tec}$	denotes the level of electricity generation by technology *tec* at time t
$\frac{\partial \Pi_t^{Y_i}}{\partial PELE_{ele,t}}$	is the compensated unit demand for electricity by macro sector i at t
$IDM_{t,ele,i}$	signifies the level of intermediate demand for electricity by sector i
$\frac{\partial \Pi_t^{EN}}{\partial PELE_{ele,t}}$	is the compensated unit demand for electricity by energy sector *en* at t
$IDM_{t,ele,en}$	is the level of intermediate demand for electricity by energy sector *en*
$\frac{\partial \Pi_t^C}{\partial PC_{ele,t}}$	is the compensated unit demand for electricity by the household
$CD_{t,ele}$	corresponds to the level of consumption demand by the household at t
$PC_{ele,t}$	is the consumption price of electricity at t (incl. electricity taxes)
$PELE_{ele,t}$	is the associated complementary variable, the price of electricity at t

The structure of the latter two market clearance conditions corresponds to the one of sectoral macro goods described in equation (4.31), and needs no further comments. The special role the electricity good occupies in this model is rather determined its *supply side*. Here, the availability of natural resources, such as water, wind, biofuels, etc., is incorporated into the supply curve of electricity, as well as capacity constraints, such as power plants, the construction of which takes time and capital. The market clearance condition for natural resources given in equation (4.34), that for capacity constraints in equation (4.35)

$$NRSUP_{t,res} \geq \sum_{tec \in ntec} RESSUP_{tec} \cdot ELE_{t,tec} + \sum_{tec \in xtec} RESSUP_{tec} \cdot ELE_{t,tec} \tag{4.34}$$

where

$NRSUP_{t,res}$ corresponds to the total amount of natural resources available for electricity generation
$ntec$ is the set of electricity generating technologies using natural resources, namely water, wind and solar power, as well as biofuels
$xtec$ is the set of electricity generating technologies using fossil fuel resources, particularly coil, oil and gas
$RESSUP_{tec}$ is the exogenously given amount of resource supplies by technology
$ELE_{t,tec}$ is the level of electricity generation by technology *tec* at time t
$PN_{t,res}$ signifies the associated complementary variable, the shadow price for a natural resource

$$\overline{E_{t,tec}} \geq ELE_{t,tec} \tag{4.35}$$

where

$\overline{E_{t,tec}}$ is the exogenously set capacity constraint for technology *tec*
$ELE_{t,tec}$ is the level of electricity generation by technology *tec*
$PCAP_{t,tec}$ is the associated complementary variable, the shadow price on capacity for technology *tec*

The shadow prices for natural resources and capacity constraints enter the zero profit condition for electricity, equation (4.13). Thus, the availability of natural resources and power plant capacities directly influence the price for electricity, exercising control over supply and demand of electricity endogenously, based on the exogenous data about these resources and capacities.

Analogously to the resource constraints modeled above, a carbon limit on CO_2 emissions can be applied exogenously in the following form:

$$CARBLIM_t \geq \sum_{en} carbon * EN_{t,en} \tag{4.36}$$

where

$CARBLIM_t$	is the exogenously set limit on carbon emissions, e.g. determined by an emission certificate system of a treaty agreement
carbon	is the baseline (2005) amount of carbon emissions
$EN_{t,en}$	is the level of production a fossil fuel energy source *en*
$pcarb_t$	is the associated complementary variable, the price of carbon emissions (or carbon tax)

Additionally, the model features *market clearance for the consumption, leisure and welfare* "markets". Leisure and welfare (which is a composite of consumption and leisure) are viewed as goods that the household prices like any other good in the economy and that can be traded off like all other goods with an exogenously defined elasticity of substitution.

The demand for leisure is determined by the prices for consumption and leisure, basically, considering labour taxes that change the wage, which in turn, due to the equilibrium situation and the underlying optimisation procedure, will equal the price of leisure, again after taxes, see equation (4.18). Since the benchmark variables are important for the understanding of several equations below, and since the structure is slightly different than in the market clearance conditions described above, they are not omitted in some of the equations below.

$$qref_t \cdot [LSP_0 \cdot (1 - t_w \cdot \psi_{t,t_w}) + LSD_0 \cdot (1 - t_{w0})] \geq \\ LSP_0 \cdot LSP_t \cdot (1 - t_w \cdot \psi_{t,t_w}) + LSD_0 \cdot (1 - t_{w0}) \cdot CLS_t \cdot \frac{\partial \Pi_t^{CLS}}{\partial PLS_t} \quad (4.37)$$

where

$qref_t$	denotes the exogenously set quantity reference path (i.e. growth path)
LSP_0	indicates the benchmark labour supply
LSP_t	is the (endogenous) level of labour supply in period t
LSD_0	is benchmark leisure demand
t_w	is the wage tax
ψ_{t,t_w}	depicts the endogenous adjustment parameter for the wage tax
$\frac{\partial \Pi_t^{CLS}}{\partial PLS_t}$	is the compensated demand for leisure by households determined by Hotelling's Lemma, which is determined by the price for consumption, the price for leisure (wage) and a substitution elasticity between consumption and leisure
PLS_t	is the associated complementary variable, the price for leisure at time t

The intuition behind the equation given above is the following: the quantity reference path is something like an economic growth variable (time endowment is assumed to increase in line with economic growth, which would in this case also correspond to a population growth variable or a labour productivity growth variable). Total labour supply LSP_t and the leisure demand derived from the full consumption including leisure (CLS_t) zero profit function by Hotelling's Lemma always have to "clear" the total time endowment of the household.

Thus, the "leisure market" always has to grow according to the growth of the total time endowment (denoted at the left hand side of the equation), and in turn to a large extent determines the labour supply of the household, which is the residual of the leisure demand by the household. As a result, the prices for labour (wage after taxes), the labour demand by the sectors, and the price for leisure simultaneously determine the labour supply of the household, and thus also its full consumption composite including leisure.

Therefore, the consumption path described below moves according to the development of full consumption including leisure:

$$C_t \geq CLS_t \frac{\partial \Pi_t^{CLS}}{\partial PC_t} \tag{4.38}$$

where

C_t	is the level of consumption
CLS_t	is the level of full consumption incl. leisure
$\frac{\partial \Pi_t^{CLS}}{\partial PC_t}$	is the compensated demand function of the household for aggregate final consumption, again derived by Hotelling's Lemma
PC_t	is the associated complementary variable, the price for aggregate consumption

Aggregate final consumption is determined by the consumption of material goods and leisure in this economy through the equational framework given above. The *price for consumption* is in turn determined by the interaction of the household demand for consumption derived above and the structure of the aggregate material consumption good, which is given as a CES function of the macro and energy output goods, see equation (4.15). Thus, the supply side of the economy can be related to the demand side, considering the household optimisation in relation to technical factors of production in a common framework.

Furthermore, there exists an additional market for full consumption including leisure, which has to clear simultaneously (this corresponds to the clearance of the total consumption level of the household for each period). Here, the demand for full consumption incl. leisure is derived from the zero profit function for welfare by Hotelling's Lemma. Thus, the intertemporal substitution behaviour of the household is considered accordingly.

$$\begin{gathered} CLS_t \cdot (C_0 + LSD_0 \cdot (1 - t_{w_0})) \geq (C_0 + LSD_0 \cdot (1 - t_{w_0})) \cdot qref_t \cdot W \frac{\partial \Pi_t^W}{\partial PCLS_t} \Leftrightarrow \\ CLS_t \cdot (C_0 + LSD_0 \cdot (1 - t_{w_0})) \geq (C_0 + LSD_0 \cdot (1 - t_{w_0})) \cdot qref_t \cdot W \left(\frac{PW}{PCLS_t} \cdot pref_t \right)^{\sigma_t} \end{gathered} \tag{4.39}$$

where

CLS_t	indicates the level of full consumption incl. leisure
C_0	is benchmark aggregate final consumption
LSD_0	denotes benchmark leisure demand
t_{w_0}	is the benchmark wage tax
W	is the aggregate welfare of the household
$\frac{\partial \Pi_t^W}{\partial PCLS_t}$	is the compensated demand for full consumption incl. leisure
$qref_t$	is the exogenously set quantity reference path (growth parameter)
$pref_t$	is the exogenously set price reference path (present value price path)
PW	is the price for aggregate welfare
σ_t	is the intertemporal elasticity of substitution
$PCLS_t$	is the associated complementary variable, the price of full consumption including leisure

Here, the household can substitute full consumption including leisure intertemporally. Looking at the zero profit function for welfare (equation (4.20)), one notices the intertemporal elasticity of substitution entering the equation via the partial derivative. As in a Ramsey model, the household will try to "smoothen" consumption, in relation to the personal *time preference rate* (depicted by the intertemporal elasticity of substitution), as well as by the quantity reference path (growth path) $qref_t$ and the present value price path $pref_t$, all of which are defined exogenously. The price of aggregate welfare PW is chosen as numéraire for the model (i.e. set to unity, see section 4.3).

The final equation for this "consumption-leisure-welfare" block of market clearance conditions is the requirement that all income is spent. Household expenditure on aggregate consumption (on "welfare") has to be at least as large as household income, intertemporally:

$$\left[\sum_t qref_t \cdot (C_0 + LSD_0 \cdot (1 - t_{w_0})) \cdot pref_t\right] \cdot (W \cdot PW) \geq M \tag{4.40}$$

where

$[\sum_t qref_t \cdot (C_0 + LSD_0 \cdot (1 - t_{w_0})) \cdot pref_t]$	is the sum of full consumption incl. leisure over all time periods at present value
W	is aggregate welfare
PW	is the price for aggregate welfare
M	denotes the total intertemporal household income (at present value)

Total household income is of course nothing else than the intertemporal sum of total leisure and aggregate material consumption, i.e. full consumption incl. leisure over all time periods, which is defined as welfare in this model. Therefore, the inequality stated above has to hold as an equality by definition, and all household income is spent.

This concludes the description of the most important market clearance and zero profit conditions, that together form the core to modelling the interlocking network of an economy.

4.2.4 Income Balance Equations

The income balance equations for the household and government agent that are described in the following, as has already been mentioned, could be substituted out of the model. However, for logical clarity and ease of use and better understanding of the model, they are incorporated into the equational system, showing the exact composition of household and government income.

$$
\begin{aligned}
&M \leq \sum_{act} PK0_{act} \cdot K0_{act} + \\
&\sum_{t}\Big[PLS_t \cdot qref_t \cdot \bar{Z}_t - PIM_t \cdot CA_t + \sum_{res} PN_{t,res} \cdot NRSUP_{t,res} + \\
&\sum_{tec} PCAP_{t,tec} \cdot \overline{E_{t,tec}} - \sum_{tec} ELE_{t,tec} \cdot \phi_t \cdot PELE_t \ if \ quota_{tec} + \\
&\qquad qref_t \cdot pref_t \cdot (UEBEN \ + \ SOCIALTRANS)\Big] \quad - \quad PKT \cdot KT
\end{aligned} \tag{4.41}
$$

where

M	is total household income
$PK0_{act}$	denotes the benchmark price for capital
$K0_{act}$	is the benchmark capital stock of activity act
$PLS_t \cdot qref_t \cdot \bar{Z}_t$	is the value of the time endowment of the household at time t, subject to the growth path $qref_t$
$PIM_t \cdot CA_t$	is the value of the trade balance (current account), i.e. the difference between imports and exports valued at the exogenous import (world market) price PIM
$PN_{t,res} \cdot NRSUP_{t,res}$	are the rents on national resources that accrue to the households
$PCAP_{t,tec} \cdot \overline{E_{t,tec}}$	are the rents resulting from the scarcity of capacities for producing electricity that are paid to the households
$ELE_{t,tec} \cdot \phi_t \cdot PELE_t \ if \ quota_{tec}$	are subsidies for certain electricity producing technologies in order to fulfill quota obligations
$qref_t \cdot pref_t \cdot (UEBEN \ + \ SOCIALTRANS)$	is the value of social transfers by the government at time t (subject to growth and inflation)
$PKT \cdot KT$	is the value of the post-terminal capital stock (all activities)

This is the **full intertemporal household income** (including leisure). Firstly, the (representative) household receives income from its endowment with the primary production factors capital and labour. Capital income is simply the difference between the sum of the value of the capital stock of all activities at the benchmark minus the total value of capital stock in the terminal period. Any difference between the value of capital stock at the first period and the present value of capital in the post-terminal period would then correspond to the capital income accrued to the household, as the post-terminal capital stock approximates the infinite horizon capital stock. This capital income is considered after depreciation and inflation rates, in effect summing up the total capital rents paid to the households by the sectors. Further, the households receive income from their total time endowment (labour income and leisure consumed). The time endowment is valued at the price of leisure, which denotes the value the household places on one unit of its time endowment.

Additionally, the households in this stylised economy are assumed to be endowed with all natural resources and capacities for electricity production. The rents paid to the households by the electricity producing technologies for capacities and natural resources, thus, enter the household budget. However, any subsidies doled out to technologies to reach certain quota targets have to be subtracted from household income. These subsidies can be seen as higher electricity prices paid by the households to finance a different "technology mix" to produce this electricity, where "greener" but more expensive technologies take a higher share in electricity production.

Finally, unemployment benefits and other social transfers from the government are part of household income, which are assumed to grow according to the growth and inflation parameters unless there are any policy changes implemented.

Government income, put simply, consists of tax revenues minus social transfers:

$$\begin{aligned} &GOVT_t \leq LSP_t \cdot t_w \cdot \psi_{t,t_w} + C_0^{en} \cdot PE_t \cdot t_{he} \cdot \psi_{t,t_{he}} \cdot CD_t^{en} + \\ &C_0^{ele} \cdot PELE_t \cdot t_{hele} \cdot \psi_{t,t_{hele}} \cdot CD_t^{ele} + \sum_{s \backslash en} IDM_{t,s,en,s,en} \cdot PE_t \cdot t_{ye} \cdot \psi_{t,t_{ye}} + \\ &\sum_{s \backslash ele} IDM_{t,s,ele,s,ele} \cdot PELE_t \cdot t_{yele} \cdot \psi_{t,t_{yele}} + \\ &\sum_s C_0^s \cdot t_c \cdot \psi_{t,t_c} \cdot \sum_s CD_t^s \cdot PY_t^s \cdot (1 + t_{he} \cdot \psi_{t,t_{he}}) if_{s=en} \cdot (1 + t_{hele} \cdot \psi_{t,t_{hele}}) if_{s=ele} \\ &\qquad - qref_t \cdot pref_t \cdot (UEBEN \; + \; SOCIALTRANS) \end{aligned} \tag{4.42}$$

where

$LSP_t \cdot t_w \cdot \psi_{t,t_w}$ is the total labour tax income in period t

$C_0^{en} \cdot PE_t \cdot t_{he} \cdot \psi_{t,t_{he}} \cdot CD_t^{en}$ is the total income from the household energy tax

$C_0^{ele} \cdot PELE_t \cdot t_{hele} \cdot \psi_{t,t_{hele}} \cdot CD_t^{ele}$ is the total income from the household electricity tax

$\sum_{s \backslash en} IDM_{t,s,en,s,en} \cdot PE_t \cdot t_{ye} \cdot \psi_{t,t_{ye}}$ is the total tax income from the tax on industry energy consumption

$\sum_{s \backslash ele} IDM_{t,s,ele,s,ele} \cdot PELE_t \cdot t_{yele} \cdot \psi_{t,t_{yele}}$ is the total tax income on industry electricity consumption

$\sum_s C_0^s \cdot t_c \cdot \psi_{t,t_c} \cdot \sum_s CD_t^i \cdot PY_t^i \cdot (1 + t_{he} \cdot \psi_{t,t_{he}}) if_{s=en} \cdot (1 + t_{hele} \cdot \psi_{t,t_{hele}}) if_{s=ele}$

is the total consumption (value added) tax income

$qref_t \cdot pref_t \cdot (UEBEN \; + \; SOCIALTRANS)$ are social transfers to the households and government consumption

Contrary to the household income, which is allocated across all periods, the government income constraint has to hold on a period-by-period basis. This, of course, is equivalent to a government agent that cannot incur any debts or surpluses at any period. Thus, as mentioned before, the government agent merely serves as some sort of "redistribution agent" levying taxes to provide some sort of public good, as well as unemployment benefits and social transfers (e.g. pension payments). The provision of these transfers is held fixed, and is only changed according to the quantity and price reference paths, while taxes are allowed to adapt endogenously. As goes for the tax on consumption (value added tax), a distinction has to be made between energy/electricity goods and all other macro-sectoral goods, since the consumption tax is levied on the total of the price for these goods plus the primary energy taxes. The total government income described above is used to provide the public good(s) (which can be thought of as health and education services, national defense, etc.). The public good is a Leontief-composite of sectoral goods (thus subject to price developments in the economy), and is assumed to grow according to the quantity reference path.

The income balance equations conclude the detailed description of model equations. In the following, a short overview of the calibration procedure is given in section 4.3, prior to an exemplary policy simulation (chapter 5).

4.3 Computational Procedure

4.3.1 Calibration

As elaborated on in section 2.2.3, calibration involves determining a set of exogenous elasticities and share parameters either from econometric estimates or economic literature (elasticities), or from the benchmark data set (share parameters). As the functional forms of the equations are given in calibrated share form (see section 4.2.1), the benchmark variables (quantities) and value shares of input factors can be directly taken from the benchmark data set, while the *level variables are set to unity* for the benchmark year. All *prices* for the benchmark year are also *set to unity.*

The dynamic evolvement of quantity level variables is then determined according to the *quantity reference path* (which can be seen as the baseline growth rate of the economy) from their benchmark level, that of price variables according to the *price reference path* (baseline present value price path, inflation parameter). This then means that the quantity reference path is an exponential growth path, determined by the equation

$$qref(t) = (1 + g)^t \tag{4.43}$$

where g is an exogenously specified annual growth rate. The price reference path, which determines the present value of future prices, is set by

$$pref(t) = \left(\frac{1}{1+r}\right)^t \tag{4.44}$$

where r is the exogenously specified baseline interest rate of the economy. Thereby, while the quantities of production grow within the model, prices are devalued in relation to the numéraire good.

As a numéraire within the model, the price of welfare PW is set to unity for all periods. Thus, all future prices are measured in present value terms of household welfare (utility), which itself is a composite of consumption and leisure as determined in the framework described in section 4.2.

First of all in the calibration procedure, during the *benchmark replication check (benchmark run)*, the equations of the model are verified by checking whether the benchmark equilibrium can be reproduced without computing power as described in section 2.2.3.

Quantity and price level variables for each year are now solely determined by the values of the $qref$ and $pref$ parameters, respectively, and because of the functional form chosen, all equations have to hold if they are specified correctly. As the figures of the SAM described in section 4.1 add up (i.e. the value of inputs/expenditures equals the value of outputs/revenues), and because we further impose constant returns to scale in the economy, the share parameters all add up to one.

Since the conventions of the SAM relate inputs to outputs column-wise (i.e. input costs equal revenues from outputs, meaning that the sum of column entries has to be zero), we can simply take these share parameters from the data. Starting with prices from unity levels, thus, the zero profit conditions have to hold for the benchmark year, and also for all other years, since all prices are subject to the same price development path.

Similarly, as supply has to equal demand for all sectors (i.e. all goods produced are consumed, which is depicted by the sum of row entries being zero), the market clearance conditions have to be satisfied for the benchmark year according to SAM data, and for all years thereafter, because quantities produced and consumed are subject to the same quantity reference path.

After the benchmark replication check has been completed and the equational system of the model has been verified, resource constraints for natural resources and capacity limits power plants are set exogenously, and a new equilibrium path is computed. This is called the *business as usual* run, simulating a development of the economy without policy interference, but subject to exogenous resource and capacity constraints for the energy system. These resource constraints will change the dynamic evolvement of quantity and reference path. Not only will these resource scarcities limit production possibilities on the one hand, on the other they imply rents from these capacities that accrue to the (representative) household, which in turn will have implications on her consumption and labour supply patterns. This *business as usual (BAU) scenario* is used as the reference scenario for all policy simulations.

Finally, one or more *policy scenarios (policy simulations)* will vary or distort the business as usual scenario regarding exogenous parameters and/or resource and capacity constraints. This might include changes in investment parameters (e.g. quota obligations for renewable energy financed by quota subsidies), exogenous elasticities, technological availabilities (e.g. nuclear phase out, meaning that an electricity producing technology is not available any more), variations in taxation parameters, or limits on CO_2 emissions (e.g. according to the Kyoto or any other possible international protocol), amongst others. The model will compute a new equilibrium path for the changed exogenous parameter(s), if this is possible (for some extreme assumption, there will be no equilibrium solution any more).

The difference in the various economic variables depicted by the model between the BAU and the policy scenario is then interpreted as the economic effect of a certain policy.

An important part of the calibration procedure is the so-called *sensitivity analysis*, where the results of the policy scenario(s) are subjected to changes in exogenous parameters. Possible questions in this context could be: does the result, at least qualitatively, still hold if I change a certain, central exogenous elasticity of substitution, for which I maybe even found differing estimates in the literature? Does my model, even though deterministic, give a reasonable explanation for past events (backcast), especially if I even simulate past exogenous policy shocks? What happens if the price for crude oil imports (fossil fuel resources) changes substantially?

Since the exogenously given elasticities are important determinants of model results, some values are exemplarily given in tables A.1 and A.2 in the appendix, chapter A.

4.3.2 Numéraire Good

In the benchmark period of the model, all prices are set to unity as described above, and are then assumed to follow the growth and price reference paths for the benchmark replication check. This feature of the calibration procedure might raise the question whether the concept of a numéraire in the model can still be viewed in line with its common notion (the numéraire being the good in relation to which the value of all other goods is measured in the economy).

One could argue that if all goods in the benchmark period have a price of one, the concept of a numéraire becomes obsolete (e.g. every good is a numéraire in the benchmark period, and all goods have the same price). If these equal benchmark prices are then projected into the future with the same price and quantity reference paths for all goods, the numéraire concept is invalid or at least redundant for all periods of the benchmark replication check. Furthermore, the idea that all goods have the same price is of course highly counter-intuitive or seems simply wrong.

To show why the notion of a numéraire still makes sense within the CGE model presented in this thesis, one has to go back to the construction of the data set as explained in section

4.1, the functional forms chosen for the equational framework of the model (CES functions in calibrated share form) and the fact that we are solving a system of *relative prices* only.

Firstly, as explained in section 4.1, the data in the SAM are all measured in terms of value. This means that all variables depicted by the SAM, e.g. total production $Y_{t,i}$ of a certain good i at time t, are measured in monetary terms such as in Euro. This value of course corresponds to a physical quantity of a good, such as tons of steel or a number of items, *times* the price of this good per physical quantity. Therefore, neither the price for one unit of a physical quantity of the good nor the number of physical quantities are depicted within the data set. If either the price for a good or the physical quantities are known to the modeller, the other can of course be calculated from the aggregate value terms in the SAM. This information, however, has to be gathered from outside the data set.

Thus, because the zero profit and market clearance conditions hold and due to the assumption of CRTS, both production and consumption functions can use benchmark value shares as input for their parameter specification since the benchmark value shares will add up to one. The zero profit functions of the model described in section 4.2.2 are scaled to unit production. Therefore, the zero profit functions, taking account the nesting structure as described in section 4.2.1, describe nothing more than the *composition* of one unit (one unit of value, one *value unit*) of output of a respective good by input factors, all in terms of value. Therefore, we e.g. account for the fact that one “value” unit of production of a good i carries a labour share of e.g. one third, meaning that one third of a “value” unit of production of good i is paid to the factor labour as input cost. The benchmark value shares are directly taken from SAM data and are assumed to be invariant across time, the elasticities of substitution are set exogenously. If we set a price of unity for all goods, and because all share parameters add up to one, the functional forms are satisfied and equilibrium is verified in the benchmark.

However, the concept *price* as used in a zero profit function or market clearance condition fundamentally differs from the notion of a price with respect to a physical quantity of a good, e.g. the price of a ton of wheat. The price in a zero profit function rather describes a *price level*, namely that of the benchmark period. This price level simply serves as a reference point for other periods of the model run.

Therefore, if we have a “price” vector $p = (1, \ldots, 1)$ carrying only ones denoting the relative prices for all goods in the economy, we simply state that the *benchmark price level* is unity for all goods. If one were to measure all of these goods in physical quantities, and denote prices per physical quantity in terms of a numéraire good, then the vector above would in general carry coefficients *different from one*, only the numéraire good will always have a price of one.

It is not necessary to talk about physical quantities of goods within a CGE model, since we are only interested in a system of relative prices that we determine starting from a benchmark data set in value terms. For CGE modelling purposes, the mere fact that one can refer back to physical quantities from the system of relative prices, if the corresponding data is available, suffices for almost all applications. Thus, if we talk of a price in a CGE model, we always mean a *price relative to its benchmark price level.* This substantially facilitates data collection and model solution, while no information is lost about the price-dependent relations of the endogenous variables to each other within the model.

The single "CGE model prices" of the various goods are all scaled in such a way that we can also represent all quantity level variables of the different goods with unity in the benchmark year. This is one of the great advantages of the calibrated share form for CES functions: all CES functions are scaled to their benchmark output levels in the market clearance conditions (see section 4.2.3). It suffices to multiply the benchmark level of a certain variable with the quantity level of this variable *relative to the benchmark year* in a certain time period to obtain the total value of this variable in the period looked at. This means that e.g. the total level of output $Y_{t,i}$ for a good i at time t is obtained by multiplying the benchmark output level $Y_{0,i}$ with the *activity level* of the sector producing good i in period t. Benchmark levels are held fixed, and only the level variables are allowed to adapt endogenously, all starting from a level of unity. The activity level in relation to a benchmark value of unity is what is denoted as a (quantity) *level variable* in this context. Therefore, we scale both the benchmark price *and* quantity level variables to unity in the benchmark year. The only scaling factor we need are the benchmark levels such as $Y_{0,i}$, which only lead us back to variables denoted in terms of value, of course, not physical quantities, as described above.

The procedure sketched above not only facilitates the calibration procedure, but also the numerical model solution process. As the zero profit functions and the base year quantity level variables are both scaled to unity, we always measure a system in relation to a benchmark level. In short, the arguments above indicate that the numéraire concept is neither violated nor obviously applied in the benchmark run, as we consider price levels in relation to a benchmark level rather than actual prices, and due to the fact that the data set is given in value terms. Only if one was to refer back to actual physical quantities and prices per unit of physical quantity such as a ton of product or a single discrete item, only then one would need to pick a numéraire to find a common unit of measurement for the goods denoted in physical quantities.

However, the real importance of a numéraire good in the context of this model relates to the numerical solving process for the business as usual and policy runs, and the **ambiguity of any solution** found since *we only determine a system of relative prices.* This means that if $p^* = (p_1, \ldots, p_n)$ is an equilibrium price system and thus also a solution for the model, also λp^* is a solution (see section 3.2.1). Therefore, if the numerical solver finds a solution for the model, it is ambiguous in relation to an unknown scalar. Therefore, we always have to relate the model solution, i.e. the equilibrium price system in the business as usual and policy runs, to the price level we have chosen in the benchmark year. Here, now, the concept of a numéraire is satisfied exactly as one might expect: future prices of all goods are valued in relation to the benchmark price of the numéraire good.

Choosing the price of welfare PW as numéraire then carries the additional advantage of measuring all future prices (or price levels) in terms of present value of household utility (welfare). This seems as the natural unit of measurement in a CGE model, where we always only refer to a system of relative prices.

4.3.3 Solver

The model is solved by using the numerical solving algorithm PATH. It would certainly exceed the scope of this thesis to display any mathematical detail or elaborations for this solver. Rather, a very brief impression of the way such a solver works should be given, along with some references to conduct further research.

As described briefly in section 2.2.1, CGE modellers nowadays mostly use Newton type algorithms to solve their models. The PATH solver developed by Dirkse and Ferris (1995, [20]) is a stabilised Newton method for solving Mixed Complementarity Problems (MCPs). Important features of the PATH solver are a stabilisation scheme for Newton methods for nonsmooth equations, as well as a global convergence result for the algorithm (Dirkse and Ferris, 1995, [20, p. 2]).

In order to solve the MCP in the PATH solver, it is expressed in the form of the so-called normal equation (see [20, p. 3] for the formulation of the normal equation). Thus, the MCP can be viewed as the problem of finding a zero of an equation, however of a potentially non-smooth one. Thereby, Newton type techniques from equation solving can be applied to solve an MCP [20, pp. 3]. The classical Newton's method (see [20, pp. 4] for a short description) has excellent local convergence properties, but global convergence is not guaranteed. In order to improve these convergence properties, the PATH solver employs methods reaching a global convergence result under assumptions generalising those necessary to attain similar results as for smooth functions [20, p. 1]. Briefly speaking, the PATH solver guarantees convergence for the model if it is correctly specified. For more detailed information on the PATH solver, the reader is advised to consult the paper by Dirkse and Ferris (1995, [20]) and the references therein.

CHAPTER 5

Application of the Model: Scenario Simulation

The scenario described below was conducted for the IHS study *'Green Jobs for a sustainable, low-carbon Austrian economy'* (Balabanov et al. 2010, [6][1]) commissioned by the Austrian Ministry for Labour, Social Affairs and Consumer Protection. It shall serve as an example for the application of a CGE-model to a policy issue.

The overarching objective of the study was to analyse the economic effects and possibilities of Austria meeting the EU 20-20-20 targets[2]. The EU 20-20-20 targets involve cutting greenhouse gases by 20% compared to levels in 1990, achieving a total share of 20% of renewable energy in the total energy mix, and saving 20% in total energy consumption compared to projections. In order to achieve these targets, the Austrian government has set up the Austrian Energy Strategy[3]. Therein, a set of measures such as the thermal renovation of existing buildings, improving the share of renewable energy in electricity and heat production, expanding and upgrading the existing Austrian electricity grid and promoting Research and Development (R&D) in this area, is defined that shall make sure that Austria can meet its commitments.

For the first analytical part of the study involving CGE modelling (see Balabanov et al. 2010, [6, pp. 23 - 82]), a static model, which is structurally very similar to the one presented in detail in this thesis, was used to assess the yearly effects of selected measures on the labour market, GDP, consumption and the sectoral composition of the economy, amongst others. The focus of the static model was mainly on labour market effects in order to assess the potential of so called *green jobs*[4]. Generally speaking, green jobs in the context of this study can be thought of as jobs that help to reduce CO_2 emissions in the economy.

The second analytical part of the study conducted with a CGE model involved a dynamic analysis with the model presented in this thesis (see Balabanov et al., 2010, [6, pp. 83-95]), where focus was placed on long-term economic development in relation to achieving the CO_2

1 German with English executive summary
2 See e.g. `http://ec.europa.eu/climateaction/eu_action/index_en.htm`. Last accessed on March 19^{th}, 2012.
3 See `http://www.energiestrategie.at` (in German). Last accessed on March 19^{th}, 2012.
4 For the exact definition of green jobs for this study see Balabanov et al. (2010, [6, pp. 12]).

targets and the proposed energy mix by investment and tax measures. The total sum of investment by sector is listed in table 5.1. These political measures are simulated within different scenarios in the model. Thus, the term (political) measure(s) always can be identified with a certain scenario in the context of this chapter.

Table 5.1: Total Sums of Investment by Sector (in Mio. Euro) Simulated in Balabanov et al. (2010, [6, p. 83])

Year	**Sector**		
	ENG	**BUI1**	**BUI2**
2011	224	172	3.103
2012	261	220	3.375
2013	275	241	3.65
2014	348	322	3.925
2015	374	373	4.201
2016	403	435	4.478
2017	448	514	4.758
2018	497	616	5.04
2019	554	736	5.323
2020	628	885	5.611

The measures depicted in these aggregate sums are listed below (in brief)[5]. As before, the term energy tax always refers to a tax on the consumption of fossil fuel energy.

- *Electricity Scenario* Investments into the promotion of technologies employing renewable energy sources (RES) for electricity production such as hydropower, wind energy, photovoltaics, concentrated sun energy, biomass and combined heat and power generation. These are distributed among the sectors AGR (agriculture)[6], ENG (engineering), BUI1 (building of complete constructions or parts thereof) and BUI2 (building installation/completion)[7] and range from 324 million Euros in the year 2012 until 797 million Euros in the year 2020. The investment figures are chosen (with some variations) according to the National Renewable Energy Action Plan for Austria (NREAP-AT[8]). The refunding of the investment by the government is conducted via a tax on electricity consumption by the households, which shall model the Austrian Eco-Electricity Act ("Ökostromverordnung"[9]).

- *Pump Storage Hydro Power Plants and Electricity Grid* Construction of additional pump storage power plants and modernisation of electricity grid until 2020 (based on the

5 For more detailed information on these measures, please see Balabanov et al. (2010, [6, pp. XV - XXIII (English), pp. 23 - 82 (German)]).

6 In difference to the other sectors, investments into the agriculture sector are depicted not directly in the top-down part of the model, but via additional inputs into relevant technologies such as biomass within the bottom-up part of the model.

7 For a complete list of sectors, please see table 4.1 in section 4.1. Please note that the building sector has been further disaggregated for this study, since it is the focus of a large amount of investment activities.

8 See `http://ec.europa.eu/energy/renewables/transparency_platform/doc/national_renewable_energy_action_plan_austria_en.pdf`. Last accessed on March 19^{th}, 2012.

9 For more information please see `http://www.e-control.at/de/marktteilnehmer/news/themen-archiv/oeko-energie-news/oekostromverordnung-2011`. Last accessed on March 19^{th}, 2012.

Masterplan by the Verbund AG[10]). The investments make up about 500 million Euros yearly until 2020, which are divided up between the sectors ENG and BUI1. The total sum is about 5 billion Euros from 2011 until 2020. The refunding for these investments again takes place via a tax on electricity. This corresponds to the assumption that these investments into power plants and grids will eventually be brought in via the price of electricity.

- *Heating Scenario* Currently, more than one million coal and oil heating systems in Austria, as well as more than 260,000 installations of electricity heatings, offer a large potential for increased penetration of renewables in the Austrian heating sector. Possible measures include the replacement of oil heatings by wood-pellets heating installations, communal heating installations based on wood chips, solar thermal energy and other biogenic fuels such as biogas, and the like. The figures implemented for this scenario are based on the NREAP-AT, and include investment and operational costs (such as increased use of wood pellets input for biomass heating systems).

  The costs for these measures are split up between the sectors AGR, ENG, BUI1 and BUI2, and range from 218 million Euros in the year 2012 to 752 million euros in the year 2020. An important feature of this scenario is that only 20% of total investments are undertaken by the state (government), while 80% of total investments are carried out by the households. This shall simulate the incentive structure posed by the government, which grants a subsidy of about 20% for the purchase of a new heating system. Thus, the additional tax burden incurred by investments into renewable heating systems is lowered. The refunding of the public part of the subsidies is conducted via a tax on household energy consumption, which has further beneficial environmental effects. In a second model run, several methods of tax refunding are discussed (see Balabanov et al., 2010, [6, pp. 47 - 53]).

- *Thermal Renovation of Buildings* In total investment sums, this scenario depicts the quantitatively largest policy measure. Thermal renovation reduces energy demand for household heating, and thus has large direct effects as regarding the decrease of CO_2 emissions. The target simulated for this policy measure is an increase of the annual rate of thermal building renovation of 1.2% (in 2008) up to 3% in 2020. This measure requires relatively large sums of investment. The study simulates a (linear) increase from 450 million Euros public investment in the year 2008 up to 1.1 billion Euros in the year 2020.

  Again, this scenario models the incentive structure of government interventions: as the government doles out a subsidy of 20% for the renovation of thermal buildings, it induces a further 80% of private investment measured in terms of total investment. This corresponds to a "leverage" between public and private investment of 1 : 4. Thus, overall investments into the thermal renovation of buildings are supposed to increase from 2.25

10 See `http://www.verbund.com/~/media/81595B8816F64ABBAACE152B33BACC45.ashx`. Last accessed on March 19^{th}, 2012.

billion Euros in the year 2008 up to 5.5 billion euros in the year 2020. The sector that is solely promoted is the sector BUI2, which is relatively small in comparison to the total economy. The tax refunding of the public subsidies is carried out via a tax on energy consumption of both households and firms. Again, a simulation of different tax refunding options is conducted and the results discussed (see Balabanov et al., 2010, [6, pp. 67 - 74]).

- *Promotion of R&D for Renewable Energies* This scenario simulates an increase of public expenditures for the research of alternative energies, the improvement of energy efficiency, and other climate-related research topics for the energy system. This scenario relates to a rise in public investments into research within the field of energy from 70 million Euros in the year 2008 up to 120 million Euros in the year 2020. Furthermore, an additional scenario with public investment of 120 million Euros and simultaneous 120 million of private investment in the year 2020 is set forth. This shall simulate a changed incentive structure within the economy that induces further private investment into energy research.

All of these scenarios are combined into one "collective scenario" for the analysis with the dynamic model that is the focus of this thesis. The R&D scenario is considered by increased intermediate inputs of research services for the bottom-up energy technologies. The advantages of the dynamic model in relation to the static model used for previous analysis within the study are a dynamic adjustment of taxes, the bottom-up depiction of electricity production, and dynamic CO_2 accounting. The static model, however, has an explicit focus on labour market effects, featuring household disaggregation into three skill groups[11], and was therefore well suited to assess the potential for green jobs of these measures

The main objective of the dynamic analysis conducted with the model presented in this thesis is to assess the overall economic cost of meeting the CO_2 targets, and whether it is realistic to meet them with the measures described above. For this assessment, *two scenarios were constructed: a medium- and a long-run scenario.* In the medium-run scenario, public subsidies are doled out until the year 2020, when the targets have to be reached, and are faded out afterwards. In the case of the long-run scenario, subsidies are continued until the year 2035 on the level of the year 2020.

The underlying assumption for the medium-run scenario is that the public is informed about the fact that the public subsidies run out in the year 2020, whereas the public knows that the subsidies will continue on their 2020 level until 2035 in the long-run scenario. This implies that a higher burden of taxation is anticipated by the household in the model, and behaviour is adjusted accordingly. This notion is of course coupled with a household agent optimising according to perfect foresight, i.e. the household agent knows as much as the modeller, anticipates all future and current changes and acts accordingly (see chapter 3.2.4).

11 High, Medium and Low-Skilled households according to highest level of education attained.

Both scenarios assume additional behavioural changes within the population (of the representative household), such as decreased consumption of energy due to increased awareness of negative environmental effects. Thus, energy consumption is assumed to decrease exogenously respective to the BAU scenario. Further, technical innovations will increase energy efficiency, and thus also reduce energy consumption. These two effects are combined in an exogenous efficiency parameter (see table 5.2). Also, the base level and amount of increase of certain taxes is assumed differently for the two scenarios (see table 5.2). In the table below, the refinancing instruments are sorted according to their absolute contribution to refinancing the measures described above.

Table 5.2: Scenario Assumptions (see Balabanov et al., 2010, [6, p. 84])

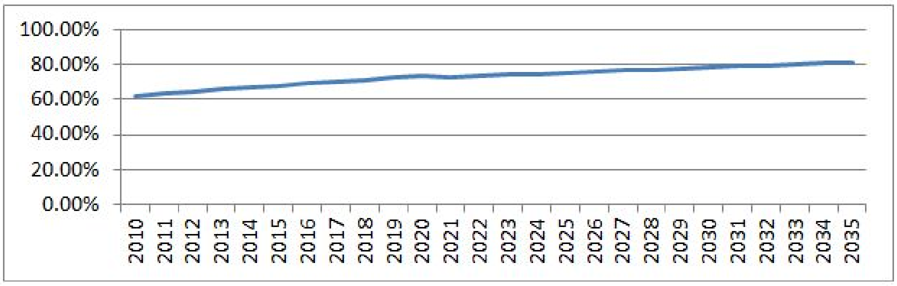
Parameter	**Measures until 2020**	**Measures until 2035**
Base Level of Firm Energy Tax	10%	15%
Elasticity of Substitution between Energy and Consumption Goods for Households	0,5	0,6
Efficiency Parameter	1,8 % yearly until 2020 0,5 % yearly from 2021	1,8 % yearly until 2020 1,5 % yearly from 2021 until 2030 1 % yearly from 2031
Refinancing Instrument	Household Energy Tax Firm Energy Tax Consumption Tax	Household Energy Tax Firm Energy Tax Consumption Tax Wage Tax (small)

Refinancing in both scenarios is mainly conducted via introducing a tax on (fossil) energy consumption by firms (sectors) and by increasing the tax on household (fossil) energy consumption, but also by a slight increase of the consumption tax. In the long-run scenario, the wage tax is raised by a small amount in order to avoid large distortive effects by the other taxes. Taxing the energy consumption of households and firms of course will actually function as a price increase for these goods, further reducing their use and thus also decrease associated CO_2 emissions. Thus, the model considers the fact that choosing the right refinancing instrument for investments into the increased penetration of RES in electricity and heat production, thermal insulation and the like will increase the associated effects. Furthermore, the efficiency parameter is assumed to develop differently for the two scenarios, modelling an increased awareness for environmental concerns in the population and faster technological progress in the long-run scenario.

5.1 Medium-run Scenario (Measures until 2020)

The public and private investment measures described above start in the year 2011 and end in the year 2020. The explicit depiction of electricity technologies in the bottom-up part of the model facilitates a detailed perspective on the share of renewables in Austrian electricity production (see figure 5.1).

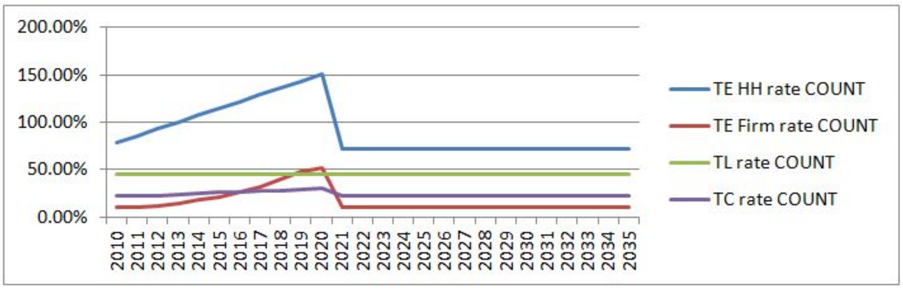
Figure 5.1: Quota of RES-based Technologies in Total Electricity Production for Austria - Model Projections



Source: Balabanov et al. (2010)

The high share of renewables in electricity production in Austria is of course mostly due to the large availability of hydropower as a natural resource. The figure above shows that the ambitious target of a share of 80% renewables in electricity production from a level of about 60% in 2010 can be met until the year 2035, with a value of more than 70% in 2020. This shall contribute to the overall Austrian target of 34% of renewables in total energy production until 2020 (in the year 2009, the share of renewables in Austria was about 29 %[12]). The rise of renewables in electricity production that is depicted in this scenario is largely due to increased taxation of fossil energy resources (see figure 5.2). Even after the end of the period of increased taxation of fossil energy in 2020, the share of renewables in electricity production continues to rise, which is due to the higher potential for capacity extension for renewable energy technologies and the increased world market price for fossil fuels (the import costs for fossil energy resources are assumed to rise by up to 80% until the year 2035).

Figure 5.2: Change of Tax Rates Levied to Finance Measures



Source: Balabanov et al. (2010)

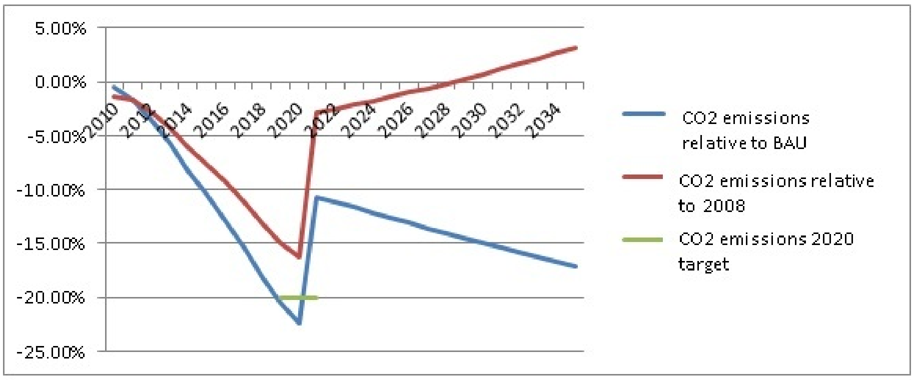
Figure 5.2 above shows the taxrates calculated endogenously within the model to finance

12 See e.g. http://www.umweltbundesamt.at/umweltsituation/energie/energie_austria/. Last accessed on March 19^{th}, 2012.

the investment measures described above and to balance the government budget. One can notice a rather steep rise until the year 2020, and a subsequent fall in the tax rates thereafter. As the government is forced to balance its budget, the investments induce seemingly rather prohibitive tax rate increases. Thus, the scenario here shall not propose "realistic" political options, but rather indicate the costs that have to be faced to foster the necessary change towards a low-carbon economy with a limited amount of tax instruments as proposed here. Thus, the rise mostly of the consumption tax from about 20% in the year 2010 to ca. 29 % in the year 2020 should not be seen as a proposal to increase this tax to such a high level, but rather quantify costs that have to be refinanced in term of tax revenues, as mentioned before.

However, the rise of the tax on household energy consumption from about 79% in 2010 (mostly the mineral oil tax) to more than 150% in 2020 can be seen as a viable economic measure. The same goes for the tax increase for energy consumption by the firms. This tax is introduced in the model in 2010 with a level of 10%, and increases up to 53% in 2020. It is a well-known economic fact that only price increases for a good that has negative externalities (such as fossil energy) can permanently reduce its use. Of course, since such steep tax increases as simulated in this scenario would have severe social impact due to the reliance of low-income households on low energy prices to finance their living and mobility standards, accompanying political measures absorbing negative distributional impacts of such taxes would have to be implemented if taxes on fossil energy were increased to levels as indicated here.

Figure 5.3: Change of CO_2 in Relation to BAU and 2008 Levels (with emission target)



Source: Balabanov et al. (2010)

However, despite of the prohibitive tax rates on fossil energy and assumed rises in oil import prices, the EU 20-20-20 target of a 20 % reduction in CO_2 emissions relative to 1990 levels is not reached by far. For this model simulation, since the target of 20% reduction with respect to 1990 levels was out of reach, it was decided to aim at a target of 20% reduction in 2020 versus the first scenario year of the model, which is 2008. As one can see in figure 5.3, even this less ambitious target was not met with a mere reduction of 16.3% instead of the planned 20%. However, the reductions versus the BAU scenario are substantial with more that 20%.

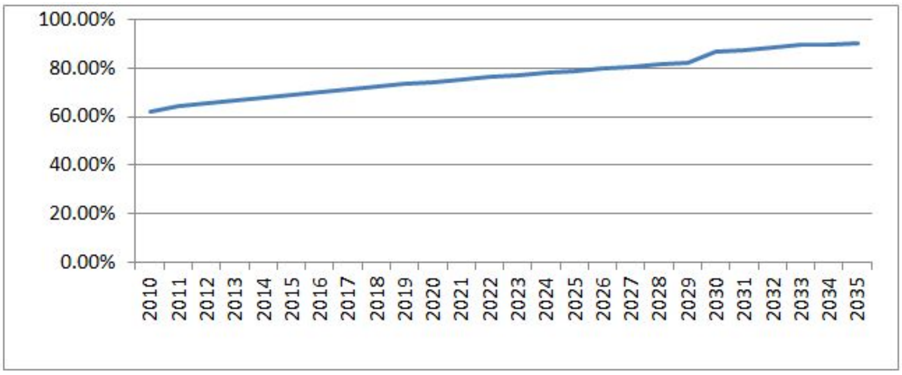
Thus, in spite of seemingly prohibitive tax rates on fossil energy use, as well as exogenous assumptions about energy efficiency and behavioural change within the population, the EU

20-20-20 or even less ambitious targets are not met regarding CO_2 emissions with medium-term measures.

5.2 Long-run Scenario (Measures until 2035)

For the long-run scenario, measures are started in the year 2011 and continued until the year 2035. From 2011 until 2020 the investments are equal to those of the medium-term scenarios, from 2020 until 2035 they remain on the level of 2020.

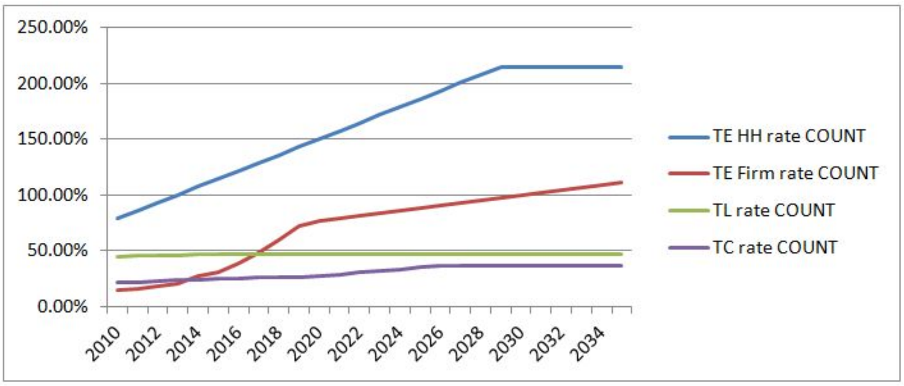
Figure 5.4: Quota of RES-based Technologies in Total Electricity Production for Austria - Model Projections



Source: Balabanov et al. (2010)

As can be seen from figure 5.4, the quota of renewables rises more than in the medium-term scenario, with figures of up to 90% in 2035. Also, the 2020 levels are with about 75% higher than those in the medium-term scenario. The jump in the year 2029 shows the use of photovoltaics to produce electricity.

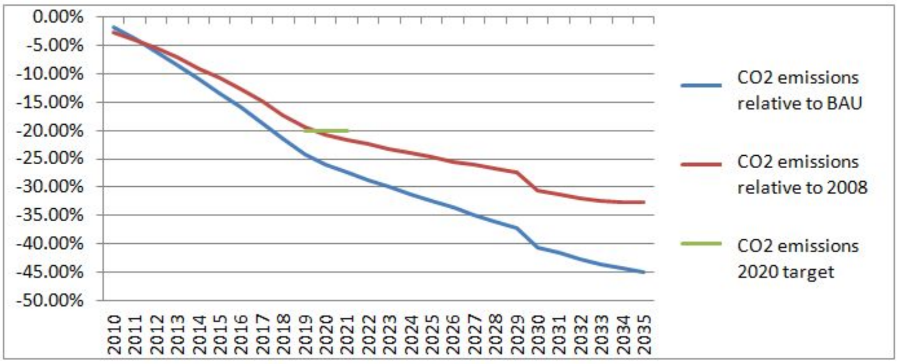
Figure 5.5: Change of Tax Rates Levied to Finance Measures



Source: Balabanov et al. (2010)

However, as shown in figure 5.5, the "economic price" that has to be paid for environmental benefits is rather high. Consumption tax increase is the most moderate, with an increase from about 20% in the year 2011 up to 29% in the year 2026, when it reaches an exogenously defined maximum which was put in place to limit its increase. The tax on household energy use rises steeply up to about 215 % in the year 2030, when it reaches its upper limit. The tax on energy use by firms keeps rising up to 110% until 2035 without reaching its upper limit. Due to adverse economic effects, but because the distortive effects of the other taxes are already so high that an additional tax has to be used, the labour tax is increased only slightly up to 49% of gross labour income (including employees' social security benefits).

Figure 5.6: Change of CO_2 in Relation to BAU and 2008 Levels (with emission target)



Source: Balabanov et al. (2010)

Figure 5.6 shows the beneficial environmental effects resulting from the high taxation of fossil energy described above, as well as from the increases in energy efficiency and behavioural changes laid down in table 5.2. The reduction of CO_2 emissions versus the BAU scenario constitutes more than 45%, and versus 2008 levels more than 32% until 2035. Furthermore, a 20% cut in CO_2 emissions versus 2008 levels until the year 2020 is met (with reductions of about 20.7%). However, the EU 20-20-20 target of emission cuts of 20% with respect to levels in 1990 is still not reached. The reduction versus 1990 levels only amounts to about 12%. This might indicate that the measures implemented in this scenario, even though ambitious and accompanied by tax increases that might stir fierce political turmoil, do not suffice to cut CO_2 emissions by the amount foreseen by the EU. One might argue that the assumptions on technological change and energy efficiency are very conservative, but since these cannot be relied upon at this point in time if one wants to achieve definite targets, this seems like a better approach in the view of the author.

5.3 Conclusion from Policy Application

As could be seen from the policy application, the model delivered quantitative results relating political measures such as investment in greener technologies for energy production or the increase in energy efficiency to various refinancing instruments (taxes), the technological composition of electricity production and CO_2 emissions.

Even though in the long-term scenario very high taxrates and continuous investments were assumed, the EU 20-20-20 targets were not fully met. This might be due to conservative assumptions relating to technological innovations, behavioural changes, and/or energy efficiency as described above. Another factor might be the assumption of exogenous economic growth for these scenarios, which was set to about 1.5% per year. If one assumes less growth, or a recession from the year 2012 on, which seemed quite likely at the point in time when this thesis was composed, figures might change to a large extent.

However, this scenario shall show that current measures taken to cut emissions and to save energy in the developed world, with Austria as an example, are not ambitious enough when looking at the global warming challenge. Even under high economic costs, the ambitious targets cannot be achieved in the current setting. Thus, a tax on CO_2 emissions, also for the non-industrial sectors[13], for Austria and the EU should definitely be taken into consideration, and could be subject to further studies with the model presented in this thesis.

13 The industry sectors are currently covered by the EU Emission trading scheme (ETS), which is economically equivalent to a CO_2 tax under certain conditions. More information about the EU-ETS can be obtained from `http://ec.europa.eu/clima/policies/ets/index_en.htm`. Last accessed on March 19^{th}, 2012.

CHAPTER 6

Critique, Conclusion and Outlook

After having reviewed the theoretical foundations, the model methodology, the formulation of a hybrid top-down bottom-up CGE model for the Austrian economy and its subsequent application to a policy issue, it seems appropriate to draw the conclusion that this framework is well suited for further research and case studies.

Casting a CGE model as an MCP and fully integrating an energy sub-sector displaying technological detail combines two modelling paradigms that were formerly perceived to be dichotomous, increasing the strengths of both approaches and alleviating their weaknesses.

One of the specious strengths of this modelling framework is that price-endogenous adjustment processes for all variables can be determined in an economy-wide framework, subject to the technological feasibility of production in the electricity or any other energy subsector and taking account of negative externalities such as GHGs or environmental pollutants.

Furthermore, the SAM data structure offers an efficient and flexible format to organise national data from the System of National Accounts and other sources. SAM data can be tailored to policy-specific problem statements, where the national Input/Output tables are amended with microdata sources such as household or consumer surveys. Providing a snapshot of the economy at a certain point of time, the SAM offers a good starting point for "What-if" analyses: what is the effect of a certain policy measure departing from the base year of SAM data?

Via the construction of scenarios and comparing a policy scenario to a business as usual scenario, a CGE analysis always makes a relative statement: what would happen/have happened if things are/had been different?

Furthermore, since the integrability conditions inherent to economic equilibrium models formulated as optimisation problems can be relaxed, see (3.1), the MCP format allows for the construction of complex, large-scale models incorporating technological detail and accounting for income effects, taxes and tax distortions, as well as market imperfections and failures that cannot be considered equally by economic models that are cast as linear or non-linear mathematical programs.

The direct integration of the bottom-up sector in the top-down CGE model, however, restricts the number of technologies depicted and the complexity of the bottom-up subsector in general. This problem can be remedied by choosing a *decomposed* model structure as described in Böhringer and Rutherford (2009, [9]). This modelling approach features a decomposition of the top-down and bottom-up parts of the integrated model that is solved iteratively. Therein, the top-down model is solved using complementarity methods, while the bottom-up model is solved separately as a quadratic program. The iterative (Jacobi) algorithm has been observed to converge rapidly when the energy subsector is small in comparison to the rest of the economy (see Böhringer and Rutherford, 2009, [9, p. 1649]), which usually is the case for developed economies.

As can be inferred from above, the framework presented offers a broad range of applications. The model implementation in GAMS is facilitated by an especially designed software package (GAMS-MPSGE, see Rutherford, 1999a, [56]) that allows for a rather intuitive programming approach where many technicalities of the MCP approach do not have to be considered by the modeller any more.

Furthermore, the PATH solver represents a powerful solution algorithm, usually converging quite fast if the model complexity is modest according to the experience of the author. It has shown to be globally convergent for quite general assumptions that suffice for most applications (see Dirkse and Ferris, 1995, [20]).

The hybrid model formulation reconciles two modelling paradigms into a common framework, combining their strengths and reducing their weaknesses. Thus, the hybrid structure probably is the biggest advantage of this modelling approach. Its application to the identification and assessment of effective policies acting on the global warming challenge in a comprehensive economy-wide cost-benefit analysis might lead to viable results. However, any modelling paradigm has its disadvantages, some of which have been touched on in this thesis.

The model structure presented in the preceding chapters exhibits several weaknesses, which provide an agenda for further research, given they do not inherently lie in the structure of a CGE model itself, such as the heavy reliance on exogenous parameters. In the latter case, there presently seems no way to amend this problem directly in the model, it is rather part of the method. The solution to this problem must lie in getting the right values for these exogenous parameters. One approach is to conduct surveys and use statistical estimations tailored to the needs of a CGE model, providing some of the elasticities required for a specifically designed CGE model. Efforts in this direction are currently being undertaken.

In general it must be clear that every method has its strengths and weaknesses. Due to the complexity of the economic system, no model can replicate reality, thus all models have to focus on certain aspects, which makes them vulnerable to the aspects they do not consider.

However, an analysis that is clear about its objectives will also duly refer to the aspects it did not consider. Thus, an open approach to CGE analysis, disclosing all relevant information to the reader in order to make the results open to scrutiny, should guarantee scientific clarity and offer an honest disclaimer about the aspects of reality not covered by a CGE model.

Especially regarding this model, several construction zones have to be noted, all subject to further research and development in order to enhance the model:

- monopolistic competition (see Dixit and Stiglitz, 1977, [21]) instead of perfect competition,
- public finances: consideration of government debt both as a stock and as a flow variable,
- disaggregation of the representative household agent, aiming at a distinction according to the ownership of productive capital and/or highest education attained,
- incorporation of a simplified depiction of financial markets, possibly in relation to government debt, and
- depiction of uncertainty, taking account of business cycles and fluctuations in financial and asset markets.

Additionally, the calibration procedure can and should be amended by some sort of automated sensitivity analysis. The model should be subjected to computerised testing for random variations of its exogenously set elasticities and of the benchmark data (SAM). The outcomes should be analysed and interpreted according to sound economic theory, and the parameters should be changed if necessary.

The parameters could also be adjusted in such a way so that they reflect past data (backcasting), e.g. replicating the development of the economy between various past SAM data sets. Setting the parameters to values that would match the developments of these SAMs would at least suggest that past economic developments can be used to forecast policy-induced economic effects. Still, the model would not be able to account for changes in these parameters, which might also occur due to policies or other structural events in the economy.

Especially if uncertainty was depicted in the model, Bayesian methods such as those used to estimate and evaluate Dynamic Stochastic General Equilibrium (DSGE) Models (see for example An and Schorfheide, 2007, [1]) might be adapted and applied to the CGE modelling framework presented in this thesis.

Of course, taking into account the complexity the model already exhibits, all these points, especially in their totality, represent a multi-year research program for several scientists.

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Nomenclature

AGE	Applied General Equilibrium
CES	Constant Elasticity of Substitution
CET	Constant Elasticity of Transformation
CGE	Computable General Equilibrium
CPA	Classification of Producty by Activity
CRTS	Constant-Returns-To-Scale
DSGE	Dynamic Stochastic General Equilibrium
GE	General Equilibrium
GHG	Greenhouse Gases
IHS	Institut für Höhere Studien (Institute for Advanced Studies)
MCM	Micro Consistent Matrix
MCP	Mixed Complementarity Problem
NACE	Nomenclature statistique des Activités économiques dans la Communauté Européenne
NLP	Non-Linear Program
NREAP-AT	National Renewable Energy Action Plan for Austria
PE	Partial Equilibrium
R&D	Research & Development
RES	Renewable Energy Sources
SAM	Social Accounting Matrix

APPENDIX A

Appendix

Table A.1: Exogenous Parameters in the Dynamic Model - Elasticities, Growth Rate and Interest Rate

Parameter Name	**Description**	**Value**
σ_t	Intertemporal elasticity of substitution	0.50
r	Baseline interest rate	3 %
g	Baseline growth rate	1.5 %
σ_{kl}	Elasticity of substitution between capital and labour in sectoral production	0.70
σ_{klem}	Elasticity of substitution between capital/labour composite and energy/material composite	0.30
σ_{eleen}	Elasticity of substitution between electricity and energy in sectoral production	0.75
σ_{eem}	Elasticity of substitution between energy/electricity composite and material composite in sectoral production	0.30
σ_{leo}	Elasticity of substitution between material goods in sectoral production	0.10
σ_{fl}	Elasticity of substitution between fossil fuel and labour in energy production	0.05
σ_{flye}	Elasticity of substitution between fossil/labour composite and sectoral goods in energy production	0.20
σ_{celeen}	Elasticity of substitution between electricity and energy in consumption	0.70
σ_{cy}	Elasticity of substitution between sectoral goods in consumption	0.50
σ_c	Elasticity of substitution between sectoral goods and energy goods in consumption	0.30
σ_{cls}	Elasticity of substitution between consumption and leisure	0.50

These parameters have either been taken from the literature, mostly Böhringer and Rutherford (2008, [8]), or have evolved from discussions with colleagues at various workshops, especially during the project *Green Jobs for a Sustainable, Low-carbon Austrian Economy* (Balabanov et al., 2010, [6]).

Table A.2: Exogenous Parameters in the Dynamic Model - Sectoral Import Elasticities

Parameter Name	Description	Value
$\sigma_{imp,agr}$	Import elasticity of sector agriculture	0.58
$\sigma_{imp,ferr}$	Import elasticity of sector ferrous, non ferrous ore and metals	0.53
$\sigma_{imp,chem}$	Import elasticity of sector chemical products	0.97
$\sigma_{imp,eng}$	Import elasticity of sector engineering	1.32
$\sigma_{imp,other}$	Import elasticity of sector other production	0.22
$\sigma_{imp,bui1}$	Import elasticity of sector building of complete constructions or parts thereof	0.10
$\sigma_{imp,bui2}$	Import elasticity of sector building installation/completion	0.10
$\sigma_{imp,tra}$	Import elasticity of sector transport	0.20
$\sigma_{imp,serv}$	Import elasticity of sector services	0.10
$\sigma_{imp,fw}$	Import elasticity of sector steam and hot water supply	0.10
$\sigma_{imp,en}$	Import elasticity of sector fossil fuel energy	0.05

Source: Welsch (2008, [69]).

A.1 Abstract (Deutsch)

Diese Diplomarbeit präsentiert ein rechenbares allgemeines Gleichgewichtsmodell mit Schwerpunkt auf Energie- und Umweltpolitikanalyse. Eine neuartige, hybride Modellstruktur, die ein allgemeines Gleichgewichtsmodell (top-down) mit einem technologisch orientierten bottom-up Modell in einem gemeinsamen Rahmenwerk verbindet, wird im Detail beschrieben. Das vorgestellte theoretische Grundmodell wird in ein rechenbares Modell übersetzt, das für Anwendungen geeignet ist. Szenarien, die einen kürzlich in der österreichischen Politik diskutierten Sachverhalt betreffen, zeigen die Anwendbarkeit des Modells.

A.2 Deutsche Zusammenfassung

Die zentrale Fragestellung dieser Diplomarbeit ist die Präsentation eines rechenbaren allgemeinen Gleichgewichtmodells für Österreich mit Schwerpunkt auf Bewertung und Analyse von Energie- und Umweltpolitikmaßnahmen. Das vorgestellte Modell folgt einer kürzlich entwickelten hybriden Modellstruktur, die die direkte Einbindung eines technologisch feingliedrigen bottom-up Teilsektors in ein gesamtwirtschaftliches allgemeines Gleichgewichtsmodell erlaubt.

Allgemeine rechenbare Gleichgewichtsmodelle werden oft als Versuch gewertet, die Walrasianische allgemeine Gleichgewichtstheorie in ein numerisch lösbares Modell zu übersetzen. Damit soll aus einer abstrakten Repräsentation des Wirtschaftssystems ein rechenbares Modell gewonnen werden, das für angewandte wirtschaftswissenschaftliche Forschung eingesetzt werden kann. Seit dem Ende der 1970er und seit Anfang der 1980er Jahre werden rechenbare allgemeine Gleichgewichtsmodelle von der Weltbank und dem Internationalen Währungsfonds für die Analyse der Auswirkungen von politikmaßnahmeninduzierten Änderungen des Wirtschaftssystems im allgemeinen Gleichgewicht herangezogen.

Kürzlich erfolgte Verbesserungen in der allgemeinen Gleichgewichtsmodellierung erlauben nunmehr die weitere Anwendung auf eine umfassende Kosten-Nutzen Analyse verschiedener Maßnahmen im Energie- und Umweltbereich. Die hauptsächliche Innovation durch Böhringer und Rutherford (2008, [8]), die eine solche systematische Einbindung eines technologisch disaggregierten Produktionssektors, in diesem Falle des Elektrizitätssektors, erlaubt, ist die Formulierung eines allgemeinen Gleichgewichtsproblems als gemischtes Komplementaritätsproblem (MCP - Mixed Complementarity Problem). Die Formulierung als gemischtes Komplementaritätsproblem hat mehrere Vorteile gegenüber vergleichbaren Problemformulierungen wie nicht-linearen mathematischen Programmen und erlaubt eine effizientere Handhabung von Modellen, die komplexer ausgestaltet werden können als unter anderen mathematischen Optimierungsansätzen.

Das top-down Modellierungsparadigma bezieht sich üblicherweise auf gesamtwirtschaftliche Modelle mit Schwerpunkt auf den öffentlichen Haushalt, Wettbewerbsfähigkeit, Arbeitsmarktfragen, Außenhandel, und andere makroökonomische Parameter. Bottom-up Modelle behandeln meistens eher die Produktionsseite der Wirtschaft, und stellen die Produktion eines oder mehrerer spezifischer Sektoren der Wirtschaft im technologischen Detail dar. Dabei werden, vom Standpunkt der Analyse aus gesehen, externe Sektoren, Kosten- oder Nachfragefaktoren als exogen betrachtet und in das Modell somit auch als exogene Parameter eingeführt.

An top-down Modellen ist im Normalfall also die technologische Feingliedrigkeit zu vermissen, jedoch werden Haushaltsnachfrage und andere Faktoren als endogen betrachtet. Bottom-up Modelle müssen die gesamtwirtschaftliche Nachfrage und externe Kostenfaktoren als exogen betrachten, können dabei jedoch auf technologisch disaggregierte Produktionsfunktionen abstellen.

Somit erscheint klar, dass eine Vereinigung dieser beiden Ansätze die Schwächen des jeweiligen Ansatzes vermindert, und die Stärken beider Methoden ergänzend zusammenführt. Dies kann durch die hybride Formulierung eines top-down rechenbaren allgemeinen Gleichgewichtsmodells und eines bottom-up Optimierungsmodells in einem integrierten Rahmenwerk erreicht werden.

Die Anwendung von solch hybriden Modellstrukturen auf Energie- und Umweltfragen scheint aus jetziger Sicht, in Anbetracht des von Klimaforschern festgestellten und prognostizierten Klimawandels, als wirtschaftswissenschaftliche Methode höchst relevant. Dadurch wird die Entgegenstellung und der Vergleich von gesamtwirtschaftlichen Kostenfaktoren mit technologischen Möglichkeiten der Energieherstellung sowie weiteren umwelt- und ressourcenbezogenen Notwendigkeiten, die sich wiederum innerhalb großangelegter Umweltmodelle teilweise quantifizieren lassen, innerhalb eines modellhaften Rahmens ermöglicht.

In diesem Zusammenhang wird in dieser Arbeit auch eine Anwendungsmöglichkeit des Modells vorgestellt, worin eine Analyse von Teilen der österreichischen Maßnahmen zur Erreichung der EU 20-20-20 Ziele, die österreichische Energiestrategie, als Kurzzusammenfassung präsentiert wird.

A.3 Curriculum Vitae

Family name:	MIESS
First names:	Michael Gregor
Date of birth:	25-09-1983
Nationality:	Austria

Education

Spring 2012 (forthcoming)	Diploma in Economics
03/2005 - current	University of Vienna, Economics
10/2006 - current	University of Vienna, Mathematics
10/2004 - current	University of Vienna, Philosophy
10/2003 - 09/2005	University of Vienna, Comparative Literature

Present position Since 01/2011: Researcher in the field of general equilibrium modelling at the Department for Economics and Finance at the Institute for Advanced Studies, research fields: environmental and energy economics, labour economics.

Years with firm 4

Professional Experience Record

Date	Since 01/2011
Location	Vienna
Company	Institute for Advanced Studies in Vienna
Position	Researcher

Date	07/2008 - 01/2011
Location	Vienna
Company	Institute for Advanced Studies in Vienna
Position	Student Assistant

Date	11/2006 - 07/2008
Location	Vienna, Neusiedl am See
Company	East - West Business Connecting GmbH
Position	Managing Director

Selected Publications

- Miess, M., Schmelzer, S. and Schnabl, A. (2011). Evaluierung der regionalen Beschäftigungs- und Wachstumsoffensive 2005/2006 (Evaluation of the Regional Employment and Growth

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- Berger, J., Keuschnigg, C., Keuschnigg, M., Miess, M., Strohner, L., Winter-Ebmer, R. (2009). Modelling of Labour Markets in the European Union. Final Report. *Project Report for the European Commission as part of the European Community Programme for Employment and Social Solidarity (2007 - 2013), DG for Employment, Social Affairs and Equal Opportunities, Ref. No. VC/2007/0344.*

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