universität wien

DIPLOMARBEIT

Titel der Diplomarbeit

Intertranslatability Results for Abstract Argumentation Semantics

Verfasser

Christof Spanring

angestrebter akademischer Grad

Magister der Naturwissenschaften (Mag.rer.nat)

Wien, im Dezember 2012

Studienkennzahl lt. Studienblatt:	A 405
Studienrichtung lt. Studienblatt:	Mathematik
Betreuer:	Privatdoz. Dipl.-Ing. Dr.techn. Stefan Woltran

Zusammenfassung

Diese Arbeit handelt von Beziehungen bestimmter endlicher Strukturen, von diskreten Zuständen und Transformierbarkeit.

Die (abstrakten) Strukturen setzen sich zusammen aus Argumenten und gerichteten Attacken zwischen diesen Argumenten. Als Semantik bezeichnen wir in diesem Zusammenhang Akzeptanzbedingungen für Mengen von Argumenten. Motiviert durch komplexitätstheoretische Reduktionen und inspiriert von den theoretischen Grundlagen von Graphersetzungssystemen führen wir den Begriff der lokalen Transformation ein.

Unter dem Terminus Intertranslatability werden sodann Verfahren zusammengefasst, die vorhandene Strukturen so umformen, dass unterschiedliche Semantiken vergleichbare Resultate hervorbringen. Auf diese Weise erforschen wir Zusammenhänge, Grenzen und Ausdrucksstärke der gängigsten Argumentationssemantiken.

Abstract

In this thesis we deal with finite structures which may be interpreted as (natural) language dialogues, respectively can be derived from such.

When reflecting an arbitrary dialogue one might work out arguments and conflicts. We call the resulting formal representation an argumentation framework. Depending on personal principles one might want sets of arguments to withhold certain acceptability conditions. We describe such principles with the term argumentation semantics. It is evident that neither representations nor principles remain indubitable.

Intertranslatability refers to the act of transforming argumentation frameworks with the purpose of equalizing acceptable sets of arguments for different semantics. We point out possibilities as well as impossibilities, introduce a concept of locality and flesh out the expressiveness of the most common abstract argumentation semantics.

Contents

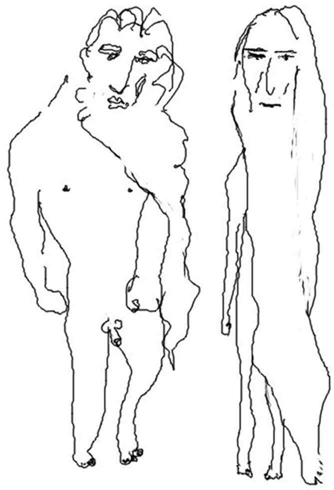


Figure 1.1: Not an Illustration of Zeus and Prometheus. [44]

1 Introduction

1.1 The Why of Abstract Argumentation

From back to Socrates much effort has been spent into categorization of natural language, into classification of speech and dispute. Long lasted the hope that a logical system, provably free of contradictions, capable of explaining world and natural language, was possible. It was Kurt Gödel who in 1931 [46] showed that any system implementing basic arithmetics is either inconsistent or can not show its own consistency. Thus shuttering the hope to pieces. Further on less ambitious approaches have gained importance and Non-Classical Logic Systems were once more put into focus. In his 1995 seminal paper Phan Minh Dung [29] came up with a formalization of argumentation, which we nowadays also use to call abstract argumentation. Of course wide agreement with this formalization in philosophical terms can not be expected without objection. In particular from a more philosophical point of view the meaning of an argument might be even more important than the syntactical representation. However abstract argumentation has almost immediately gained wide-spread acceptance in the field of artificial intelligence, computer science and formal philosophy.

Dung's work was successful in presenting not less than a generalization for human argumentation as well as other economical and social problems, such as n-person-games and the stable marriage problem, on the way also catching non-monotonic reasoning in artificial intelligence and logic programming. As it is the case with so many successful theories, abstract argumentation is built upon a simple definition: argumentation frameworks, repre-

Figure 1.2: Not an Illustration of Prometheus. [44]

sented by arguments and their conflicts, nowadays also known as Dung-style-frameworks. More applications built upon these frameworks are to be found in negotiation [3, 2, 31], law and legal reasoning [12, 14], Multi-Agent Systems and Game Theory [56, 53, 57], Decision Making and Recommendation [22, 23], Machine Learning [59] and more, part of which, we are convinced, future is still about to show.

Of course Dung's work did not come out of the blue, but to the best of our knowledge it can be seen as the first proper formalization of argumentation in simple mathematical terms. In [67] Toulmin gives an analysis of arguments in philosophical terms. The credit of refactoring arguments into moves of a person in a game can probably be given to Hamblin [48]. The interested reader might acknowledge that a comprehensive handbook of historical background for argumentation theory [69] is available. In [71] Walton gives a nice introduction into argumentation theory from a more philosophical point of view. An overview of abstract argumentation, respectively argumentation in artificial intelligence can be found in [13].

Abstract Argumentation as a term consists of two words. Just the same way abstract argumentation as a task might consist of two parts of different concern. One of these parts would be argumentation, be it argumentation in a courtyard, or argumentation between computational programs in some multi-agent-system. The other part would be abstraction, a theory built to provide tools of use for any kind of argumentation. As it is common nature in science the link between theory and application might be weak (respectively not yet practically established). One can focus on theory or on application or somewhere in between. In this thesis focus is on theory mainly, hence concrete examples of possible real world argumentation are only to be found in this introductory chapter.

Figure 1.3: Not an Illustration of Zeus. [44]

1.2 The How of Abstract Argumentation

To illustrate abstract argumentation, in the following sections we will give a short example of application (argumentation), theory (abstraction) and interaction in between the two.

Example 1.1. Eternal Argument between Prometheus and Zeus

PROMETHEUS: Humankind shall live forever.
ZEUS: Humans are ugly and shall therefor not be allowed to live on earth.
PROMETHEUS: Humans are children of eternal nature, nature itself is pure beauty by definition.
ZEUS: Speaking about eternity, humankind is likely to be a self-destructive species anyway.

If we try to make sense of this dialogue, it is inevitable to split the sayings into parts, to identify arguments and conflicts. Usually, if we happen to investigate some dialogue of natural language, it is most likely that we start by understanding one sentence after another. Also, in natural language dialogues, argumentation frameworks are often not limited to the boundaries of some determined opening and closing. Additional arguments, both older and newer, might arise at any time and extend, respectively modify the knowledge base. Additional arguments and attacks might falsify statements previously supposed to be valid. In logical reasoning one would expect valid statements inside some theory to withhold their validity, regardless of how the theory might be extended. To express this non-conformity, we classify abstract argumentation to belong to the family of non-monotonic logics. Having said this, we present an abstraction[1] of the former argument:

[1] In standard abstract argumentation the only kind of relation between arguments is a relation of attack. In [17] the authors present a concept of acceptance condition, allowing arbitrary propositional formulas to determine the relation between any arguments. An alternative definition for argumentation frameworks with the bipolarity of defending and defeating relations is presented in [1].

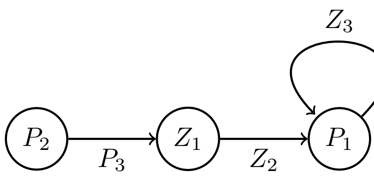
Figure 1.4: Not another Illustration of Zeus and Prometheus. [44]

Example 1.2. Dialogue Abstraction

P:	Humankind shall live forever.	$\leadsto$	P_1	humankind lives
Z:	Humans are ugly	$\leadsto$	Z_1	humans are ugly
Z:	and shall not be allowed to live.	$\leadsto$	Z_2	ugliness defeats live
P:	Humans are children of nature	$\leadsto$	P_2	humans are nature
P:	nature itself is pure beauty.	$\leadsto$	P_3	nature defeats ugly
Z:	Humankind is most likely self-destructing.	$\leadsto$	Z_3	live defeats itself

Remark. Observe that abstraction is by no means a unique transformation. Especially for natural languages we have to make use of a good part of subjective interpretation. For instance "not living on earth" and "living forever" are not necessarily contradicting. However for the greater part of this thesis we remain in the abstract layer, where no interference of this kind will appear.

For a graphical representation of this abstraction, to be able to let loose of subjective issues, we prefer the following directed graph.



In abstract argumentation we try to pick useful sets of arguments[2] and call them extensions. If for instance ancient Greek gods believed that any maximal set of conflict-free fractions of some dispute does represent a relevant interpretation, we might as well pick these sets as extensions, receiving the sets $\{P_2\}$ and $\{Z_1\}$. The collection of all extensions of one kind for one specific argumentation framework is called semantics. Thus we might name the collection of these extensions the maximal-conflict-free semantics.

[2]Alternative definitions of somewhat extended argumentation frameworks, which allow to pick sets of arguments and attacks, and also allow attacking of attacks can be found in [6, 42].

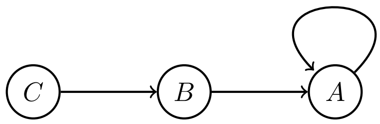
1.3 The About of this Thesis: Intertranslatability

Now for an at first sight completely different problem. We will use the famous Liar Paradox to suggest another abstract argumentation framework. Bertrand Russell is said to be the one who reintroduced the formulation, which dates back to ancient Greek philosopher Eubulides (as stated in [72]). A modified version is used for Gödel's first incompleteness result [46]. To better match Example 1.2 we extend the paradoxic sentence A with additional sentences:

Example 1.3. Liar Paradox

A:	This sentence is wrong.	$A \rightarrowtail A$
B:	Sentence A is wrong.	$B \rightarrowtail A$
C:	Sentence B is wrong.	$C \rightarrowtail B$

Now again we will put this example into a graphical representation. As it turns out Examples 1.2 and 1.3 seem to generate very similar visual output.



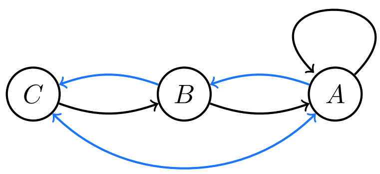
In [29] Dung states that it is an often held opinion in the artificial intelligence and logic programming community, that only systems with stable extensions are to be reasonably considered. A stable extension is a conflict-free extension, where each argument of the framework is either part of the extension or attacked by the extension, thus partitioning all the arguments into "in" and "out".

If we look for a stable extension in the Liar Paradox example however it appears that it does not provide one. The only non-empty conflict-free sets are $\{C\}$ and $\{B\}$, as C is in conflict with B and A is in conflict with itself. When looking at the possible extension $\{C\}$ obviously A is neither attacked by nor part of the extension. On the other hand for $\{B\}$, we have C not being part of the extension and not being attacked.

So when looking at abstract argumentation frameworks not only the abstract representation itself appears to be of importance, but also the semantics of interest. Thus, although Examples 1.2 and 1.3 happen to be similar in graphical representation, a severe difference pops up when taking into account the respectively related semantics. The conflicts and the arguments stay the same, but extensions differ for stable and maximal-conflict-free semantics.

Transformation 1.1. A Minor Transformation

α:	Any attacked sentence attacks back.	$(X \rightarrowtail Y) \implies (Y \rightarrowtail X)$
β:	Any self-attacking sentence is attacked by all sentences.	$(X \rightarrowtail X) \implies (Y \rightarrowtail X)$



If we apply Transformation 1.1 to the Liar Paradox we receive stable extensions $\{C\}$ and $\{B\}$. So for these particular frameworks we found a transformation such that maximal-conflict-free semantics for the Eternal Dialogue (Example 1.2) and stable semantics for the Liar Paradox (Example 1.3) modified by Transformation 1.1 coincide. More generally spoken, translations as introduced into argumentation theory in [39] focus on adding arguments and attacks to frameworks such that different extensions, respectively semantics coincide on the arguments of the original framework, sometimes with side-conditions.

In general, semantics in abstract argumentation yield hard problems concerning computational complexity. Thus even if one can think of a simple translation, this particular translation might only be of theoretical use. For instance in Section 3.4 we introduce translations, which are easily understood and reproduced, but most sure can not yield reasonable computation time. Since two semantics are likely to differ in complexity somehow, efficient translations might not be possible in both directions.

Since Turing first came up [68] with a concept of computability, various authors investigated ways of reaching efficiency, e.g. [65] and comparing computable systems, e.g. [45]. As argumentation theory can be seen as a field of non-monotonic logic, intertranslatability in abstract argumentation refers to translations between various logics (e.g. [50, 27, 51]). In [47] Gottlob shows that no modular translation from default to autoepistemic logic possibly exists. To follow translations and related results back in time, a good overview can be found in [61].

In abstract argumentation, prior to [39], effort has been put into showing strong relationship between generalized and original Dung frameworks, see e.g. [21, 11, 6, 16]. In [10] an approach of enforcing some extension by adding arguments is discussed, remarkably incorporating also negative results.

1.4 The What of this Thesis

This thesis focuses mainly on intertranslatability of argumentation frameworks as introduced by Dvořák and Woltran in [39]. The authors originally published efficiency related results with respect to grounded, stable, admissible, complete, preferred, semi-stable and stage semantics. We will extend the scope to also cover conflict-free and naive semantics.

We will introduce a new concept of locality which applies to most of the translations presented in [39]. Locality will shed light upon computational aspects (e.g. Theorem 3.1.4) as well as provide us with an implicit tool for classification of translations. We will use locality to deepen results, to gain new results and to classify given results in Section 3.1.

We will point out relations between semi-stable, stage and stable semantics (for instance Corollary 3.2.32 and Translation 3.3.13). We will present counter-examples for translations to conflict-free, naive (Theorem 3.2.4) and stage semantics (Theorem 3.2.6). We will present various partially surprising inefficient translations in Section 3.4. Most complicated and therefor maybe most notably however will be the contributions concerning finite-diameter local translations. We will use this property to show various impossibility results in Sections 3.2 and 3.3.

1.5 The Structure of this Thesis

We will give proper definitions for argumentation frameworks, semantics and translations in Chapter 2. There we will also give background information we think is necessary to understand and follow the subsequent parts, given that one might not be familiar with the discussed issues in detail. We will give definitions concerning intertranslatability and introduce concepts of locality.

Chapter 3 consists of the parts we think this thesis is able to contribute to the field of abstract argumentation in general and intertranslatability in particular. We will give an analysis of results from [39], a categorization concerning locality, new translations and new impossibility results. Finally a detailed overview of gained insights will be given in Section 3.5.

In Chapter 4 we will speak about related work, give hints and relations to show what other works seem to be interesting as far as the previously presented matter is concerned. Furthermore we will give a summary of achieved results and give account to implications as well as open questions. We will close this thesis by reflecting upon open questions and therefor possible extensions of the scope of this thesis with the ulterior motive of collecting material for future work.

Publication Statement

Due to the writer's remarkably inefficient time management habits parts of this thesis have by now already been published [37]. Ironically this concerns most of those parts with significance for inefficient intertranslatability.

Acknowledgments

I do thank my parents for being there, Marie-Theres Gallnbrunner for contributing the illustrations and Wolfgang Dvořák as well as Stefan Woltran for endless corrections and inspirational talks.

2 Behind the Scenes

One should acknowledge that without further distinction argumentation is a very wide field, from classical argumentation theory to abstract argumentation as introduced by Dung. For further distinction we refer to [69, 71, 13]. To release confusion we furthermore will only deal with argumentation frameworks and argumentation in the sense of [29] in the meantime also known as Dung frameworks and abstract argumentation. We sometimes also speak of argumentation in artificial intelligence.

Based on Dung's seminal paper we will give an introduction into argumentation frameworks in Section 2.1. We will give a short motivation and basic definitions for abstract argumentation, including attack and defense as well as basic operations on argumentation frameworks.

In Section 2.2 we will talk about extension based argumentation semantics and give the definitions for conflict-freeness and admissibility as well as the semantics needed later on, which are naive, grounded, stable, preferred, complete [29], semi-stable [18] and stage [70] semantics.

Having given the basic definitions in Section 2.3 we will take care of relations (set inclusion, etc.) described in the literature so far between given semantics partly affecting only special kind of (i.e. symmetric, acyclic and self-attack-free) argumentation frameworks.

A naive approach for computational issues for argumentation semantics would be to compute all the extensions for a given argumentation framework. Section 2.4 will give a brief overview of alternative reasoning modes and decision problems of interest. We will present verification, plain and non-empty-existence, credulous and skeptical acceptance.

Section 2.5 gives a brief introduction into complexity theory and definitions of what is needed later. However since the main results will not require complexity theory, the more important purpose of this section will be to illustrate one aspect of difference concerning semantics.

Section 2.6 can be seen as an introduction to [39]. We will give definitions for translations in abstract argumentation, including some properties of interest, namely (weak) exactness and faithfulness, modularity and monotonicity, embedding and covering. We will also introduce concepts of locality, which are supposed to add some fine-graining to the open space between modularity and monotonicity.

For readers already familiar with abstract argumentation and standard comparison procedures we still recommend to read Section 2.6. We present a bunch of definitions necessary to understand Chapter 3. In particular for the probably most complicated proofs we will make use of Definition 2.6.23 (diameter local) which itself depends on Definition 2.1.28 (diameter of argumentation frameworks).

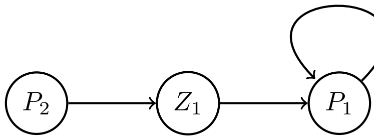


Figure 2.1.3: Abstract argumentation framework from the introductory chapter as reintroduced in Example 2.1.2.

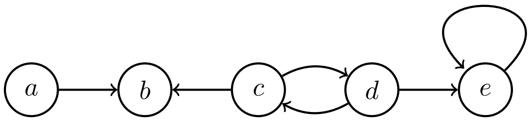


Figure 2.1.5: Graphical representation of an exemplary argumentation framework used in Example 2.1.6 and throughout the whole thesis.

2.1 Abstract Argumentation Frameworks

Definition 2.1.1. An argumentation framework (AF) is a pair

$$F = (A, R)$$

where A is a set of arguments and R is a binary relation on A, i.e. $R \subseteq A \times A$. For $(a, b) \in R$ we also write $a \rightarrowtail^R b$ and say a attacks b under R, or $a \rightarrowtail^F b$ and say a attacks b in F. In case no ambiguity arises we might just write $a \rightarrowtail b$ and say a attacks b. Sometimes for a given argumentation framework $F^* = (A^*, R^*)$ we will use A_{F^*} to denote $A_{F^*} = A^*$ and R_{F^*} to denote $R_{F^*} = R^*$.

Example 2.1.2 (Formalization of Eternal Dialogue)*.* Example 1.2 can be reformulated as a proper abstract argumentation framework with $F = (A, R)$ and

$$A = \{P_1, P_2, Z_1\} \qquad R = \{(P_2, Z_1), (Z_1, P_1), (P_1, P_1)\}$$

Remark 2.1.4*.* For most people however Figure 2.1.3 will catch the internal conception better than Example 2.1.2. For arbitrary argumentation frameworks to specify a proper definition in general it suffices to give a graphical representation. Any proper graphical representation induces a unique argumentation framework.

Example 2.1.6 (Abstract Example)*.* The graph representation from Figure 2.1.5 induces the argumentation framework[1] $F = (A, R)$ with

$$A = \{a, b, c, d, e\} \qquad R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$$

[1]This example is taken from [15].

Definition 2.1.7 (Attack, Defense and Range)**.** For an arbitrary argumentation framework $F = (A, R)$, $E \subseteq A$ and $a \in A$ we say that

- $E \rightarrowtail^R a$ (resp. $a \rightarrowtail^R E$) if for some $b \in E$, $b \rightarrowtail^R a$ (resp. $a \rightarrowtail^R b$).
- a is defended by E (respectively E defends a) with R if for each $b \in A$, such that $b \rightarrowtail^R a$, also $E \rightarrowtail^R b$ holds.
- The range[2] of E under R, denoted as E^+_R is the set $E \cup \{b \mid E \rightarrowtail^R b\}$.

Alternatively we may write $\rightarrowtail^F$, defended by E in F, range of E in F, E^+_F. If no ambiguity arises we might drop the concerning object and just write $\rightarrowtail$, defended by E and E^+.

Remark 2.1.8. Intuitively these notions can be extended a bit:

- For $E, E' \subseteq A$ sets of arguments we might also write $E \rightarrowtail E'$ to denote that for some $a \in E'$ we have $E \rightarrowtail a$ and
- E' is defended by E if for all $a \in E'$ we have that a is defended by E.
- For $a \in A$ an argument, the range of a is given by $a^+ = \{a\}^+$.

Example 2.1.9. Take into account Example 2.1.2 and the set $E = \{Z_1, P_2\}$. Now according to Definition 2.1.7 we have:

- Z_1 and P_1 are attacked by E, E is attacked by itself,
- P_1 and P_2 are defended by E,
- the range of P_2 is given by $P_2^+ = E$, the range of E is the whole framework, in other terms $E^+ = \{P_1, P_2, Z_1\} = A$.

Remark 2.1.10. Intuitively one happens to prefer sets of arguments which do not attack themselves. However the definitions of attack, defense and range do not yet handle this property. See Definition 2.2.4 for a definition of conflict-freeness.

Example 2.1.11. Take into account the argumentation framework from Figure 2.1.5. Then

- b, c and e are attacked by $\{a, d\}$,
- $\{a, d\}$ defends itself,
- the range of $\{a, d\}$ is given by $\{a, d\}^+ = A$.
- $\{c, d\}$ attacks and defends itself.

Remark 2.1.12. According to definition and graphical representation there is some similarity to graph theory. However as soon as dealing with intended meaning this similarity comes to a stop, for in graph theory an edge is intended to represent reachability, while in argumentation theory it is quite the opposite.

But anyway, there are some set-theoretic tools also used in graph theory, we want to ensure to be able to use with argumentation frameworks too, most important of which appear to be union and isomorphism.

[2] The concept of range was first introduced in [70], although general agreement on the corresponding formalization has changed slightly in between.

Definition 2.1.13 (Set Operations)**.** For argumentation frameworks $F = (A, R)$ and $F' = (A', R')$ we give definitions for basic set operations.

- The union of two argumentation frameworks is given as $F \cup F' = (A \cup A', R \cup R')$.
- The intersection of two argumentation frameworks is given as $F \cap F' = (A \cap A', R \cap R')$.

Remark 2.1.14. Intuitively for any set $\mathscr{F} = \{F_1, F_2 \dots\}$ of argumentation frameworks also union and intersection of this set (respectively of these argumentation frameworks) are defined:

$$\bigcup_{F \in \mathscr{F}} F = \bigcup \mathscr{F} = F_1 \cup \bigcup (\mathscr{F} \setminus F_1) \qquad \bigcap_{F \in \mathscr{F}} F = \bigcap \mathscr{F} = F_1 \cap \bigcap (\mathscr{F} \setminus F_1)$$

Remark 2.1.15 (Finite Argumentation Frameworks Only). Concerning classical set theory[3] $\bigcup \mathscr{F}$ and $\bigcap \mathscr{F}$ are well-defined also for infinite sets $\mathscr{F}$, possibly resulting in infinite argumentation frameworks. However, since it is not part of the investigation, we restrict ourselves to finite[4] argumentation frameworks ($|A| < \infty$) for the rest of this work. Thus the union of an infinite amount of argumentation frameworks has to be handled with care. As a side note intertranslatability for infinite frameworks also serves a quiet different purpose.

Definition 2.1.16 (Subframework)**.** For argumentation frameworks $F = (A, R)$, $F' = (A', R')$ we call F a subframework of F' if the corresponding subset relation holds with respect to arguments and attacks:

$$F \subseteq F' \iff A \subseteq A', R \subseteq R' \qquad F \subsetneq F' \iff A \subsetneq A', R \subsetneq R'$$

Definition 2.1.17 (Restriction)**.** The restriction of an argumentation framework $F = (A, R)$ to a set $E \subseteq A$ of arguments is given by:

$$F|_E = (E, R \cap (E \times E)) .$$

$F|_E$ is then called the by E induced subframework of F.

Definition 2.1.18 (Isomorphism)**.** Two argumentation frameworks $F = (A, R)$ and $F' = (A', R')$ are called isomorphic ($F \cong F'$) if there exists a bijective function $\varphi : A \to A'$ such that

$$(a, b) \in R \iff (\varphi(a), \varphi(b)) \in R'.$$

If no ambiguity arises for any isomorphic frameworks we take an isomorphism $\varphi : A \to A'$ as given. Otherwise we write $\varphi_{F,F'}$, $\varphi_{A,A'}$ or similar. To denote relations we might also make use of an asymmetric equivalence symbol:

$$F \cong^{\varphi} F' \iff F' \cong^{\varphi^{-1}} F.$$

[3]For an introduction into classical set theory we refer to [40, 24]. A comprehensive collection of set theoretic topics can be found in [54]. For the purpose of this work, to be familiar with problems of contemporary logic and set theory probably [41] suffices.

[4]Actually argumentation frameworks in the sense of [29] include infinity, Dung also introduces categories for infinite frameworks. Further discourses on infinite argumentation frameworks are to be found in e.g. [20, 62, 5].

Remark 2.1.19. For any argumentation framework $F = (A, R)$ and any mapping $\varphi : A \to A'$ transforming the argumentation framework F into another argumentation framework $F' = (A', R')$:

$$A' = \{\varphi(a) \mid a \in A\} \qquad R' = \{(\varphi(a), \varphi(b)) \mid (a, b) \in R\}$$

we can extend the definition of φ in a natural way:

$$\begin{aligned} \varphi(A^{\subseteq}) &= \left\{\varphi(a) \mid a \in A^{\subseteq} \cap A\right\} \\ \varphi(r) = \varphi(a, b) &= (\varphi(a), \varphi(b)) && \textit{for } (a, b) = r \in R \\ \varphi(R^{\subseteq}) &= \left\{\varphi(r) \mid r \in R^{\subseteq} \cap R\right\} \\ \varphi(F^{\subseteq}) &= (\varphi(A^{\subseteq}), \varphi(R^{\subseteq})) && \textit{for } F^{\subseteq} = (A^{\subseteq}, R^{\subseteq}) \end{aligned}$$

Remark 2.1.20 (Universe, $\mathscr{F}^{\infty}$). Technically there is a proper definition for argumentation frameworks to be elements of the following class:

$$\mathscr{C}_{\mathscr{F}} = \{F = (A, R) \mid A \subsetneq \{a_i \mid i \textit{ an ordinal}\}, R \subseteq A \times A\}$$

Thus we have that the universe of all possible argumentation frameworks forms a proper class. If no ambiguity arises we can use other letters and signs to denote argumentation frameworks, arguments and attacks, sets of arguments and sets of attacks. By definition any x in the domain of abstract argumentation, x being an argument, an attack, a set of arguments, a set of attacks, an argumentation framework or a set of argumentation frameworks can be identified as such. Nonetheless we will mostly use easily distinguishable names.

Observe that there are only countably many isomorphically different finite argumentation frameworks. Thus there is a proper set $\mathscr{F}^{\infty}$ of standardized finite argumentation frameworks, such that for any finite argumentation framework F there is exactly one $L \in \mathscr{F}^{\infty}$ such that $F \cong L$. However already all isomorphically equivalent frameworks with one argument only $\mathscr{F} = \{F \mid F \cong (\{a_1\}, \emptyset)\}$ do not form a proper set anymore. By allowing any ordinal to be part of the name of an argument we blow up the number of isomorphically equivalent frameworks.

Example 2.1.21 (Isomorphism). Take into account the argumentation framework $F = (A, R)$ with:

$$A = \{a_1, a_2, a_3\} \qquad R = \{(a_1, a_2), (a_2, a_3), (a_3, a_3)\}$$

This argumentation framework is isomorphically equivalent to the framework from Example 2.1.2 as well as Example 1.2 and 1.3. Furthermore without loss of generality we have

$$F \in \mathscr{F}^{\infty}.$$

Definition 2.1.22 (Cardinality)**.** The cardinality of an argumentation framework $F = (A, R)$ is given by the number of its arguments:

$$|F| = |A|$$

Cardinality is mostly of theoretical use as a first comparison of structure of different argumentation frameworks. In terms of computational complexity however, where we will be referring to an input size, we need a different measure. Thus we introduce a measure of size which does not reflect any structural issues, but reflects how much space some argumentation framework needs for representation.

Definition 2.1.23 (Size)**.** The size of an argumentation framework $F = (A, R)$ involves numbers of arguments and attacks:

$$\|F\| = |A| + |R|$$

Remark 2.1.24 (Ordering)*.* Observe that neither cardinality nor size are capable of identifying isomorphic argumentation frameworks. Although for any isomorphic frameworks $F \cong F'$ we have $|F| = |F'|$ and $\|F\| = \|F'\|$, the reverse does not necessarily hold. It is possible to give a total ordering for argumentation frameworks, such that cardinality is respected, size is partially reflected and isomorphism enacts as the relation of equality.

Example 2.1.25 (Size and Cardinality)*.* Consider the argumentation frameworks $F = (A, R)$ and $F' = (A', R')$ with:

$$A = \{a_1, a_2, a_3\} \qquad A' = \{a_1, a_2\}$$
$$R = \{(a_1, a_2), (a_2, a_3)\} \qquad R' = \{(a_1, a_1), (a_1, a_2), (a_2, a_1), (a_2, a_2)\}$$

Without loss of generality $F, F' \in \mathscr{F}^\infty$ are the minimal examples for different behaviour of cardinality and size.

$$|F| = 3 \qquad |F'| = 2 \qquad \textit{but} \qquad \|F\| = 5 \qquad \|F'\| = 6$$

Definition 2.1.26 (Connection and Path)**.** Two arguments a and b are called connected in some argumentation framework $F = (A, R)$ with $a, b \in A$ if $a = b$ or there exists a sequence $P = \{(a, c_1), (c_1, c_2) \dots (c_n, b)\}$ such that for each $(s_1, s_2) \in P$ we have[5] $\{(s_1, s_2), (s_2, s_1)\} \cap R \neq \emptyset$. If P satisfies this condition P is called an (undirected) path from a to b in F. If $a = b$ then the empty set $\emptyset$ is a path from a to b.

Take into account the argumentation frameworks F, $F_1 = (A_1, R_1)$ and $F_2 = (A_2, R_2)$, with $F_1 \subseteq F$ and $F_2 \subseteq F$. F_1 and F_2 are called connected in F if there exist $a_1 \in A_1$ and $a_2 \in A_2$ such that a_1 and a_2 are connected in F.

Sometimes for argumentation frameworks F_1 and F_2 we might also speak of connectedness without explicitly referring to some comprising F. In this case the respective argumentation framework F is implicitly defined as the union of F_1 and F_2, $F = F_1 \cup F_2$.

[5] Observe that for the purpose of this thesis we make use of undirected connectedness only. All definitions are easily to be converted into corresponding definitions with directed quality by requiring $(s_1, s_2) \in R$ at this point.

Definition 2.1.27 (Distance)**.** For some argumentation framework $F = (A, R)$, the distance of two arguments $a, b \in A$ in F, is given as size of the shortest path[6] from a to b:

$$dist(a,b)^F = \min_{P \text{ is a path from } a \text{ to } b \text{ in } F} (|P|)$$

The distance of two argumentation frameworks $F_1 = (A_1, R_1)$ and $F_2 = (A_2, R_2)$ (or sets A_1 and A_2) in F is given by

$$dist(F_1, F_2)^F = dist(A_1, A_2)^F = \min_{a_1 \in A_1, a_2 \in A_2} dist(a_1, a_2)^F$$

In case no ambiguity arises we might just write $dist(a, b)$ and $dist(F_1, F_2)$ for $F_1, F_2 \subseteq F$. If no argumentation framework F with $F_1, F_2 \subseteq F$ is specified, we will implicitly use $F = F_1 \cup F_2$ and therefor $dist(F_1, F_2) = dist(F_1, F_2)^{F_1 \cup F_2}$. Observe that the implicit distance of two argumentation frameworks defaults to either 0 or ∞.

Definition 2.1.28 (Diameter)**.** For argumentation frameworks $F = (A, R)$, $F \subseteq F'$ the diameter of F (or A) in F' is given by[7]

$$dia(F)^{F'} = dia(A)^{F'} = \max_{a,b \in A} (dist(a, b)^{F'})$$

Often enough we will simply use the term diameter of F, to denote $dia(F) = dia(F)^F$.

Definition 2.1.29 (Connected Components)**.** For any argumentation framework F, a set of argumentation frameworks $\mathscr{F}$ is called a fragmentation (into connected components) of F if $\bigcup \mathscr{F} = F$ and for any $F' \in \mathscr{F}$ we have $dia(F') < \infty$.

If furthermore for any two argumentation frameworks $F_1, F_2 \in \mathscr{F}$, we have F_1 and F_2 not being connected ($dist(F_1, F_2)^F = dia(F_1 \cup F_2) = \infty$), we call $\mathscr{F}$ a minimal fragmentation into connected components.

Lemma 2.1.30. *If an argumentation framework F has diameter $d < \infty$, then F consists of at most 1 connected component.*

Example 2.1.31. Take into account the argumentation frameworks F_1, F_2 and F_3 with:

$$F_1 = (\{b\}, \emptyset) \qquad F_2 = (\{a, c\}, \{(a, c)\}) \qquad F_3 = (\{a, b, c\}, \{(a, b), (b, c)\})$$

We now have the following:

	F_1	F_2	F_3	$F_1 \cup F_2$	$F_1 \cup F_2 \cup F_3$
diameter	0	1	2	∞	1
distance of a and c	–	1	2	1	1
distance of a and b	–	–	1	∞	1
connected components	1	1	1	2	1
minimal fragmentation	$\{F_1\}$	$\{F_2\}$	$\{F_3\}$	$\{F_1, F_2\}$	$\{F_2 \cup F_3\}$

[6]We declare that $\min_{x \in \emptyset}(x) = \infty$, such that the distance between arguments not being connected is infinite.
[7]We declare that $\max_{x \in \emptyset}(x) = 0$, such that the empty framework has a diameter of 0.

2.2 Argumentation Semantics: What Good is an Argument?

When dealing with argumentation frameworks the main task is to find good sets of arguments. Semantically we might be interested in true or right sets, for the purpose of being general we only think about good sets. This also implies that inversion of a good set will in general not yield any good set. This chapter still is mostly based on [29], semi-stable semantics is due to [18], stage semantics to [70]. For a comprehensive overview of semantics for abstract argument systems see [8, 4].

Definition 2.2.1 (Extensions and Semantics)**.** Given an argumentation framework $F = (A, R)$ any partition $\sigma(F) \subseteq \wp(A)$ is called a semantics of F. The sets $E \in \sigma(F)$ are called extensions of F.

Remark 2.2.2. Further on we will prefer to give rules an extension has to fulfill to belong to a specific semantics and implicitly build semantics upon these rules.

Note that although semantics and extensions are defined with respect to some argumentation framework, we will often omit this reference when the context clearly indicates the respective argumentation framework.

We only consider[8] semantics σ of interest such that for any $F \cong^{\varphi} F'$ we have $\sigma(F') = \varphi(\sigma(F))$. Thus the semantics we have in mind are independent of names of arguments.

Remark 2.2.3 (Properly Defined Semantics). Semantics for argumentation frameworks pick a set of sets of arguments. Formally (compare Remark 2.1.20) we can define a semantics σ as a mapping from argumentation frameworks to sets of sets of arguments.

$$\sigma : \{F_1, F_2 \dots\} \to \wp(\wp(\{a_1, a_2 \dots\})) \qquad\qquad \sigma(F) \subseteq \wp(A_F)$$

The first thing that comes to mind probably is that good sets should not[9] attack themselves. By introducing conflict-free sets we take care of this issue. To our knowledge there is no serious work on semantics which do not fulfill conflict-freeness.

Definition 2.2.4 (Conflict-Freeness)**.** Consider an argumentation framework $F = (A, R)$, a set $E \subseteq A$ is called conflict-free (cf) in F if there are no arguments $a, b \in E$ such that $a \rightarrowtail b$.

Now that we have introduced conflict-freeness, we will augment this definition to also consider defense. We recall that by definition of defense (Definition 2.1.7), defense is attacking of attackers. Now intuitively if a set E of arguments is defending itself against all attacking arguments we consider this E to be more interesting than sets which do not possess this feature.

Definition 2.2.5 (Admissibility)**.** Consider any argumentation framework $F = (A, R)$, a conflict-free set $E \subseteq A$ is called admissible (adm) in F if for any $a \in A$ with $a \rightarrowtail E$ also $E \rightarrowtail a$.

[8] Observe that in abstract argumentation, since we skip deeper meaning of frameworks, we do not consider semantics based upon subjective values, like truth or morality.

[9] For non-standard argumentation frameworks with recursive attacks or similar concepts, extensions usually consist of arguments and attacks. Thus e.g. self-attacking arguments become conflict-free if the self-attack is not included in the respective extension.

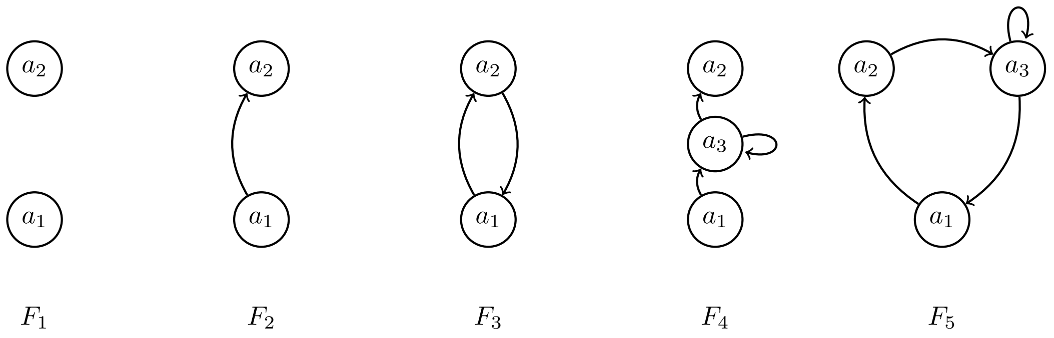


Figure 2.2.6: Minimalistic argumentation frameworks for the purpose of illustration.

Example 2.2.7 (Conflict-freeness vs. Admissibility). Having a look at the frameworks in Figure 2.2.6, we compare conflict-free and admissible sets:

	F_1		F_2		F_3		F_4		F_5	
	cf	adm	cf	adm	cf	adm	cf	adm	cf	adm
$\emptyset$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$\{a_1\}$	✓	✓	✓	✓	✓	✓	✓	✓	✓	-
$\{a_2\}$	✓	✓	✓	-	✓	✓	✓	-	✓	-
$\{a_1, a_2\}$	✓	✓	-	-	-	-	✓	✓	-	-

Admissibility seems to give an additional impression, compare F_2,F_3 respectively F_1,F_4. On the other hand for some frameworks admissibility does not decide anything, for instance in F_5 and other odd length cycles only the empty set is admissible, while a lot of sets are still conflict-free. It is a much discussed[10] and still open question in the abstract argumentation community whether semantics without admissibility are of practical interest.

Example 2.2.8. Consider the framework F from Figure 2.1.5. We have

$$cf(F) = \{\emptyset, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b\}, \{b, d\}\}$$
$$adm(F) = \{\emptyset, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}\}$$

Definition 2.2.9 (Naive Semantics)**.** For an arbitrary argumentation framework $F = (A, R)$ a conflict-free set $E \in cf(F)$ is called a naive extension of F if it is maximal with respect to set inclusion:

$$naive(F) = \{E \in cf(F) \mid \textit{for } E' \in cf(F), E \subseteq E' \textit{ already } E = E'\}$$

Definition 2.2.10 (Preferred Semantics)**.** For an arbitrary argumentation framework F an admissible set $E \in adm(F)$ is called a preferred extension of F if it is maximal with respect to set inclusion:

$$prf(F) = \{E \in adm(F) \mid \textit{for } E' \in adm(F), E \subseteq E' \textit{ already } E = E'\}$$

[10]See for instance [9] for a discussion, including semantics based on conflict-freeness of strongly connected components.

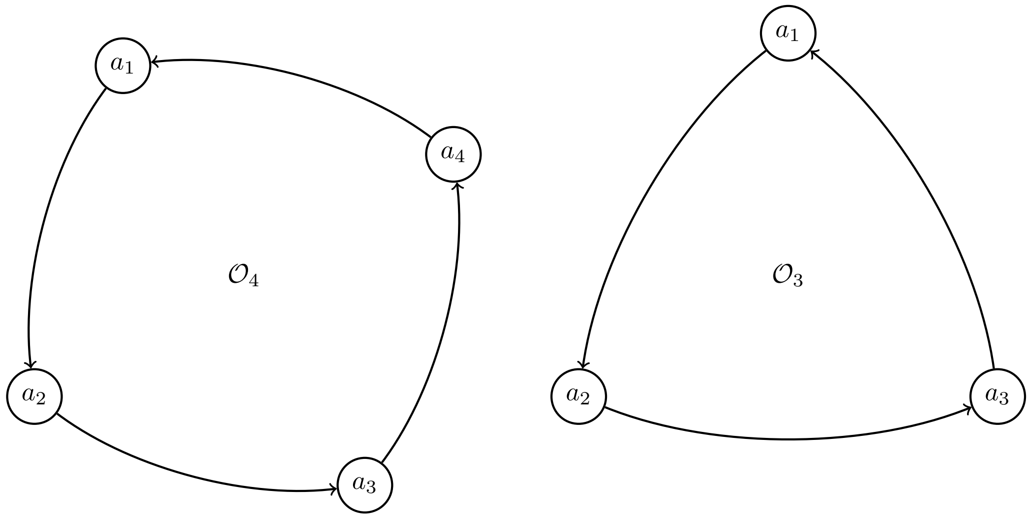


Figure 2.2.15: Circular argumentation frameworks $\mathcal{O}_4$ of 4 and $\mathcal{O}_3$ of 3 arguments.

Example 2.2.11. Having a look at the argumentation frameworks from Figure 2.2.6. We compare naive with preferred semantics:

	F_1		F_2		F_3		F_4		F_5	
	naive	*prf*	*naive*	*prf*	*naive*	*prf*	*naive*	*prf*	*naive*	*prf*
$\emptyset$	-	-	-	-	-	-	-	-	-	✓
$\{a_1\}$	-	-	✓	✓	✓	✓	-	-	✓	-
$\{a_2\}$	-	-	✓	-	✓	✓	-	-	✓	-
$\{a_1, a_2\}$	✓	✓	-	-	-	-	✓	✓	-	-

Example 2.2.12. Consider the argumentation framework F from Figure 2.1.5. We have:

$$naive(F) = \{\{a, c\}, \{a, d\}, \{b, d\}\}$$
$$prf(F) = \{\{a, c\}, \{a, d\}\}$$

Definition 2.2.13 (Characteristic Function)**.** For an argumentation framework $F = (A, R)$ and a set of arguments A' the characteristic function $\mathcal{F}_F : \wp(A) \to \wp(A)$ is given as

$$\mathcal{F}_F(A') = \{a \mid a \textit{ is defended by } A'\}$$

Remark 2.2.14. Although the characteristic function also works on self-conflicting sets of arguments, we intend to use it on conflict-free sets and as far as we know there is no utilization of the characteristic function on self-conflicting sets.

Example 2.2.16. Consider the circular frameworks shown in Figure 2.2.15. If we take the set $E_1 = \{a_1\}$, then the characteristic function yields $\{a_3\} = \mathcal{F}_F(E_1)$ for both frameworks. If we take the set $E_2 = \{a_1, a_3\}$, then we have $\mathcal{F}_{\mathcal{O}_4}(E_2) = E_2$, but $\mathcal{F}_{\mathcal{O}_3}(E_2) = \{a_2, a_3\}$. If we take the set $E_3 = \{a_1, a_2, a_3\}$, then we have $\mathcal{F}_{\mathcal{O}_3}(E_3) = E_3$, but $\mathcal{F}_{\mathcal{O}_4}(E_3) = \{a_1, a_3, a_4\}$.

The following deductions are important for well-definedness of admissible semantics and semantics based on the characteristic function. The same results, although different in presentation, can be found in [29] as Fundamental Lemma 10 and as Lemma 18.

Lemma 2.2.17. *The characteristic function preserves conflict-freeness and admissibility. For an arbitrary argumentation framework $F = (A, R)$, and a set of arguments $E \subseteq A$, we have:*

$$E \in cf(F) \implies \mathcal{F}_F(E) \in cf(F) \quad (1)$$

$$E \in adm(F) \implies \mathcal{F}_F(E) \in adm(F) \quad (2)$$

Proof. (1) For a contradiction assume that there are $a, b \in \mathcal{F}_F(E)$ with $a \rightarrowtail b$. Since E defends b we have $E \rightarrowtail a$. But then, since E defends a against any attack, we also have $E \rightarrowtail E$, a contradiction.

(2) Now observe that due to admissibility E defends itself and therefor $E \subseteq \mathcal{F}_F(E)$. Now since $\mathcal{F}_F(E)$ adds only arguments to E which are defended already by E alone, also $\mathcal{F}_F(E)$ is admissible. □

Corollary 2.2.18 (Characteristic Function)**.** *For any argumentation framework F a conflict-free set E of arguments is admissible if and only if $E \subseteq \mathcal{F}_F(E)$.*

Definition 2.2.19 (Fixed Point)**.** Consider some argumentation framework $F = (A, R)$, a fixed point of the characteristic function is a set $E \subseteq A$ such that $\mathcal{F}_F(E) = E$.

Example 2.2.20 (Fixed Points). When looking at Figure 2.2.6 we can state that

- the only fixed point of F_1 is $\{a_1, a_2\}$,
- the only fixed point of F_2 is $\{a_1\}$,
- the fixed points of F_3 are $\emptyset, \{a_1\}, \{a_2\}, \{a_1, a_2\}$,
- the only fixed point of F_4 is $\{a_1, a_2\}$,
- the fixed points of F_5 are $\emptyset, \{a_1, a_2, a_3\}$.

Definition 2.2.21 (Complete Semantics)**.** For an arbitrary argumentation framework $F = (A, R)$ a conflict-free set $E \subseteq A$ is called a complete extension of F if it is a fixed point of the characteristic function.

$$com(F) = \{E \in cf(F) \mid \mathcal{F}_F(E) = E\}$$

Remark 2.2.22. Due to Corollary 2.2.18 any set which is a conflict-free fixed point is also admissible. For practical purposes complete semantics can be obtained by creating fixed points out of conflict-free (respectively admissible) sets.

Example 2.2.23. Considering the framework F from Figure 2.1.5 the complete extensions are given as:

$$com(F) = \{\{a\}, \{a, c\}, \{a, d\}\}.$$

Example 2.2.24. The complete semantics for the argumentation frameworks from Figure 2.2.15 are given as:

$$com(\mathcal{O}_4) = \{\emptyset, \{a_1, a_3\}, \{a_2, a_4\}\}$$
$$com(\mathcal{O}_3) = \{\emptyset\}$$

Remark 2.2.25. Observe that if for some argumentation framework $F = (A, R)$ we have $E_1 \subseteq E_2 \subseteq A$, immediately also $\mathcal{F}_F(E_1) \subseteq \mathcal{F}_F(E_2)$. The empty set is admissible in any argumentation framework and thus there has to be a unique least fixed point of the characteristic function.

Definition 2.2.26 (Grounded Semantics)**.** For any argumentation framework $F = (A, R)$ the unique grounded extension of F is given as the least fixed point of the characteristic function.

$$grd(F) = \{E \subseteq A \mid \mathcal{F}_F(E) = E, \textit{for } E' \subseteq E \textit{ with } E' = \mathcal{F}_F(E') \textit{ already } E' = E'\}$$

Remark 2.2.27. Since for any argumentation framework F there is always exactly one grounded extension E, we sometimes also speak of the grounded extension E and if no ambiguity arises write $grd(F) = E$. The grounded semantics can be obtained by iteratively applying the characteristic function to the empty set.

$$grd(F) = \mathcal{F}_F^{\lceil \frac{|A|}{2} \rceil}(\emptyset)$$

Example 2.2.28. The grounded extension of framework F from Figure 2.1.5 is given as $grd(F) = \{a\}$. The grounded extensions for the argumentation frameworks from Figure 2.2.6 are given as $grd(F_1) = \{a_1, a_2\}$, $grd(F_2) = \{a_1\}$, $grd(F_3) = \emptyset$, $grd(F_4) = \{a_1, a_2\}$, $grd(F_5) = \emptyset$.

Remark 2.2.29. Abstract argumentation frameworks are based on an attack relation. For a specific extension this relation tells us something about the remaining arguments too. We receive an implicit partitioning of: arguments in the extension, arguments attacked by the extension and arguments neither being part of nor being attacked by the extension. Observe that in this way e.g. for F from Figure 2.1.5 the extension $\{a, c\}$ implies three partitions, while $\{a, d\}$ implies only two partitions, with all arguments either being part of or being attacked by the extension. We might consider extensions where any argument is either part of or attacked by the extension of greater importance, compare Section 1.3.

Definition 2.2.30 (Stable Semantics)**.** For any argumentation framework $F = (A, R)$ a conflict-free set $E \subseteq A$ is called a stable extension of F if it attacks any $a \in A \setminus E$, in other words the range spans all arguments in the framework.

$$stb(F) = \left\{E \in cf(F) \mid E^+ = A\right\}$$

Remark 2.2.31 (Admissibility of Stable Semantics)*.* Since a stable extension is conflict-free and the range encompasses all arguments it becomes immediately clear, that we can replace conflict-free in the preceding definition by admissible. In other words, each stable extension is also admissible. However not every argumentation framework provides a stable extension. For instance in Figure 2.2.6 we have $stb(F_5) = \emptyset$.

Example 2.2.32. Consider the argumentation frameworks from Figure 2.2.6 we investigate stable, grounded and complete semantics:

	F_1			F_2			F_3			F_4			F_5		
	stb	*com*	*grd*	*stb*	*com*	*grd*	*stb*	*com*	*grd*	*stb*	*com*	*grd*	*stb*	*com*	*grd*
$\emptyset$	-	-	-	-	-	-	-	✓	✓	-	-	-	-	✓	✓
$\{a_1\}$	-	-	-	✓	✓	✓	✓	✓	-	-	-	-	-	-	-
$\{a_2\}$	-	-	-	-	-	-	✓	✓	-	-	-	-	-	-	-
$\{a_1, a_2\}$	✓	✓	✓	-	-	-	-	-	-	✓	✓	✓	-	-	-

Observe that for F_5 complete and grounded semantics contain the empty set while stable semantics is empty itself.

Example 2.2.33. Consider the framework from Figure 2.1.5. The only stable extension consists of a and d.

$$stb(F) = \{\{a, d\}\}$$

Remark 2.2.34. As Dung already pointed out in [29] not every useful argumentation framework possesses a stable extension. In the following we present two argumentation semantics, which prove to be equivalent to stable semantics if some stable extension exists, yet stage and semi-stable semantics lack the annoyance of collapsing where no stable extension exists.

As far as motivation for stage and semi-stable semantics is concerned a comparison with naive and preferred semantics (Definitions 2.2.9 and 2.2.10) seems natural. Previously we had semantics maximizing extensions themselves, in the following we will present semantics maximizing the range of extensions.

Definition 2.2.35 (Stage Semantics)**.** For a given argumentation framework $F = (A, R)$ a conflict-free set of arguments $E \subseteq A$ is called a stage extension of F if it is maximal in range:

$$stg(F) = \left\{E \in cf(F) \mid \textit{for } E' \in cf(F), E'^{+} \subseteq E^{+} \textit{ already } E'^{+} = E^{+}\right\}.$$

Definition 2.2.36 (Semi-Stable Semantics)**.** For a given argumentation framework $F = (A, R)$ an admissible set of arguments $E \subseteq A$ is called a semi-stable extension of F if it is maximal in range:

$$sem(F) = \left\{E \in adm(F) \mid \textit{for } E' \in adm(F), E'^{+} \subseteq E^{+} \textit{ already } E'^{+} = E^{+}\right\}.$$

Example 2.2.37. Consider the argumentation frameworks from Figure 2.2.15. We have:

$$stg(\mathcal{O}_4) = \{\{a_1, a_3\}, \{a_2, a_4\}\} \qquad stg(\mathcal{O}_3) = \{\{a_1\}, \{a_2\}, \{a_3\}\}$$
$$sem(\mathcal{O}_4) = \{\{a_1, a_3\}, \{a_2, a_4\}\} \qquad sem(\mathcal{O}_3) = \{\emptyset\}$$

	$\emptyset$	$\{a_1, a_3\}$	$\{a_2, a_4\}$	$\{a_1\}, \{a_2\}, \{a_3\}, \{a_4\}$		$\emptyset$	$\{a_1\}$,$\{a_2\}$,$\{a_3\}$
$cf(\mathcal{O}_4)$	✓	✓	✓	✓	$cf(\mathcal{O}_3)$	✓	✓
$adm(\mathcal{O}_4)$	✓	✓	✓	-	$adm(\mathcal{O}_3)$	✓	-
$grd(\mathcal{O}_4)$	✓	-	-	-	$grd(\mathcal{O}_3)$	✓	-
$stb(\mathcal{O}_4)$	-	✓	✓	-	$stb(\mathcal{O}_3)$	-	-
$com(\mathcal{O}_4)$	✓	✓	✓	-	$com(\mathcal{O}_3)$	✓	-
$naive(\mathcal{O}_4)$	-	✓	✓	-	$naive(\mathcal{O}_3)$	-	✓
$prf(\mathcal{O}_4)$	-	✓	✓	-	$prf(\mathcal{O}_3)$	✓	-
$stg(\mathcal{O}_4)$	-	✓	✓	-	$stg(\mathcal{O}_3)$	-	✓
$sem(\mathcal{O}_4)$	-	✓	✓	-	$sem(\mathcal{O}_3)$	✓	-

Table 2.3.1: Semantical differences for the argumentation frameworks from Figure 2.2.15

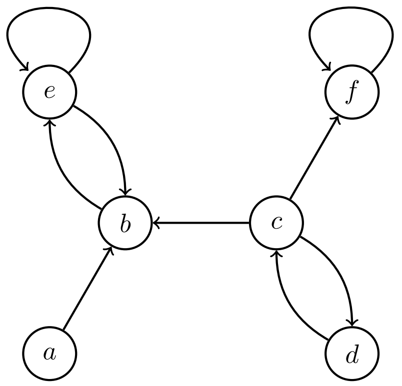


Figure 2.3.2: Framework illustrating aspects of argumentation semantics.

2.3 A First Comparison of Argumentation Semantics

After introducing various argumentation semantics in the previous section, we will focus on showing differences and relations between them in this section. So far we presented semantics relying on different principles. Some of the introduced semantics are based upon conflict-freeness (naive and stage), some are based on admissibility (preferred and semi-stable), others do not depend on whether they are built upon conflict-free or admissible sets (complete, grounded and stable).

The observation of concrete argumentation frameworks and semantics can solely give hints on relational properties, or with a bit of luck disprove assumptions. The first relational

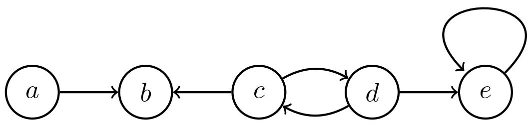


Figure 2.3.3: Example illustrating aspects of argumentation semantics.

	$\emptyset$	$\{a\}$	$\{b\}$	$\{c\}$	$\{d\}$	$\{a,c\}$	$\{a,d\}$	$\{b,d\}$
cf	✓	✓	✓	✓	✓	✓	✓	✓
adm	✓	✓	-	✓	✓	✓	✓	-
grd	-	✓	-	-	-	-	-	-
stb	-	-	-	-	-	-	-	-
com	-	✓	-	-	-	✓	✓	-
naive	-	-	-	-	-	✓	✓	✓
prf	-	-	-	-	-	✓	✓	-
stg	-	-	-	-	-	✓	-	✓
sem	-	-	-	-	-	✓	-	-

Table 2.3.4: Semantical differences for the argumentation framework from Figure 2.3.2

	$\emptyset$	$\{a\}$	$\{b\}$	$\{c\}$	$\{d\}$	$\{a,c\}$	$\{a,d\}$	$\{b,d\}$
cf	✓	✓	✓	✓	✓	✓	✓	✓
adm	✓	✓	-	✓	✓	✓	✓	-
grd	-	✓	-	-	-	-	-	-
stb	-	-	-	-	-	-	✓	-
com	-	✓	-	-	-	✓	✓	-
naive	-	-	-	-	-	✓	✓	✓
prf	-	-	-	-	-	✓	✓	-
stg	-	-	-	-	-	-	✓	-
sem	-	-	-	-	-	-	✓	-

Table 2.3.5: Semantical differences for the argumentation framework from Figure 2.3.3

property we are going to visit is set inclusion, namely the question whether extensions of one kind are always also extensions of another kind.

Lemma 2.3.6 ($\sigma \subseteq cf$). *Any σ-extension is always also a conflict-free extension, for $\sigma \in \{adm, grd, com, stb, naive, prf, stg, sem\}$.*

Proof. Follows immediately from the definitions. □

Lemma 2.3.7 ($\sigma \subseteq adm$). *Any σ-extension is always also an admissible extension, for $\sigma \in \{grd, com, stb, prf, sem\}$.*

Proof. For prf and sem extensions this follows immediately from the definition. Admissibility of stable extensions is already covered with Remark 2.2.31. Since com and grd extensions are based on fixed points of the characteristic function, we only have to reconsider Corollary 2.2.18 (cf of E and $E \subseteq \mathcal{F}_F(E) \implies adm$ of E) to finish this proof. □

Lemma 2.3.8 ($grd \in com$). *For any argumentation framework the grounded extension is also a complete extension.*

Proof. Grounded extensions are the least fixed point, built upon the empty set. Conflict-freeness follows by Lemma 2.2.17. Any conflict-free fixed point is a complete extension. □

Corollary 2.3.9 ($grd(F) = \bigcap com(F)$). *For any argumentation framework F we have*

$$grd(F) = \bigcap com(F) \in com(F).$$

Lemma 2.3.10 ($prf \subseteq com$)**.** *For any argumentation framework F a preferred extension E is also a complete extension.*

Proof. Since preferred extensions are admissible, with Corollary 2.2.18 we have $E \subseteq \mathcal{F}_F(E)$. According to the definition of preferred semantics (Definition 2.2.10) E is a maximal admissible set, therefor either $E = \mathcal{F}_F(E)$ or $\mathcal{F}_F(E)$ is not admissible. The later anyhow contradicts Lemma 2.2.17. □

Lemma 2.3.11 ($sem \subseteq prf$)**.** *For an arbitrary argumentation framework $F = (A, R)$ any semi-stable extension E is also a preferred extension.*

Proof. Since semi-stable extensions are maximal admissible with respect to range, no set of arguments $E' \supsetneq E$ can be admissible in F. Otherwise with $a \in E' \setminus E$ and $E \not\rightarrowtail a$ also $E'^+ \supsetneq E^+$. □

Lemma 2.3.12 ($stg \subseteq naive$)**.** *Any stage extension is also a naive extension.*

Proof. Follow the previous proof, but replace admissible with conflict-free and semi-stable with stage. □

Lemma 2.3.13 ($stb \subseteq \sigma$)**.** *For an arbitrary argumentation framework F any stable extension E is also a σ-extension, for $\sigma \in \{sem, stg, prf, naive, com, adm, cf\}$.*

Proof. With the previous lemmata it suffices to show this lemma for $\sigma \in \{sem, stg\}$. Now with Lemma 2.3.7 we know that E is admissible (and conflict-free by definition), since it clearly is maximal in range we have $E \in sem(F)$ and $E \in stg(F)$. □

Lemma 2.3.14 (Non-Empty Stable Semantics)**.** *For any argumentation framework $F = (A, R)$, if there is at least one stable extension, instantly stable, semi-stable and stage extensions coincide.*

Proof. For any stable extension E by definition $E^+ = A$, therefor any range-maximal conflict-free or admissible set E' must also fulfill $E'^+ = A$, considering Lemma 2.3.13 the desired property can be derived immediately. □

Lemma 2.3.15 (Range-Maximizing and Selfattack)**.** *If for some argumentation framework $F = (A, R)$ without self-attacks (for all $a \in A$ we have $a \not\rightarrowtail a$) semi-stable and stage extensions coincide ($sem(F) = stg(F)$) then already $sem(F) = stg(F) = stb(F)$.*

Proof. If any semi-stable extension E is also a stage extension, then for any argument $a \in A \setminus E^+$ we need to have $(a, a) \in R$. Otherwise either $a \rightarrowtail E$ and E is therefor no semi-stable extension, or $E \cup \{a\}$ is conflict-free and covers greater range than E, yielding E not being a stage extension. Since the framework of interest does not provide self-attacks, however we need $E^+ = A$, yielding non-empty stable semantics. □

Lemma 2.3.16 (Conflict-Free Extensions are Covered by Naive Extensions)**.** *For an arbitrary argumentation framework $F = (A, R)$ any conflict-free set is covered by some naive extension.*

$$A \in cf(F) \implies \text{there is some } E \in naive(F) \text{ such that } A \subseteq E$$

Corollary 2.3.17. *Any σ-extension is covered by some naive extension, for all introduced semantics* $\sigma \in \{cf, adm, com, stb, grd, naive, prf, stg, sem\}$.

Lemma 2.3.18 (Admissible Extensions are Covered by Preferred Extensions)**.** *For an arbitrary argumentation framework $F = (A, R)$ any admissible set is covered by some preferred extension.*

$$A \in adm(F) \implies \text{there is some } E \in prf(F) \text{ such that } A \subseteq E$$

Corollary 2.3.19. *Any admissible set is covered by some complete extension.*

Corollary 2.3.20. *Any σ-extension is covered by some σ'-extension for* $\sigma \in \{adm, grd, com, stb, sem, prf\}$ *and* $\sigma' \in \{adm, com, prf\}$.

Remark 2.3.21 (Acyclic Argumentation Frameworks). Acyclic argumentation frameworks $F = (A, R)$ do not contain any circular sequence of attacks $(a_0, a_1), (a_1, a_2) \dots (a_n, a_0)$. Acyclic in this context matches the definition of well-founded for finite argumentation frameworks from [29] (Definition 29 in there). Dung shows that for any well-founded argumentation framework automatically grounded, preferred and stable semantics coincide. We conclude that for acyclic argumentation frameworks grounded, complete, preferred, semi-stable, stable and stage semantics coincide.

Remark 2.3.22 (Symmetric Argumentation Frameworks). For symmetric argumentation frameworks $F = (A, R)$ we have $(a, b) \in R$ if and only if $(b, a) \in R$. In such frameworks any argument defends itself, thus if F is symmetric we have $cf(F) = adm(F)$ implying $naive(F) = prf(F)$ and $stg(F) = sem(F)$.

We observe that so far only stable semantics is capable of producing no extension at all. This observation appears to be related with the following semantical property which is fulfilled by any but stable semantics. The intuition is that we would prefer semantics to treat distinct parts of any framework independently.

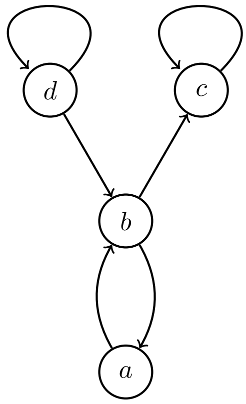
Definition 2.3.23 (Property of Non-Interference)**.** We say that a semantics σ satisfies the non-interference property[11] if and only if for any argumentation framework $F = F_1 \cup F_2$ where $F_1 \cap F_2 = (\emptyset, \emptyset)$, we have $\sigma(F) = \{E_1 \cup E_2 \mid E_1 \in \sigma(F_1), E_2 \in \sigma(F_2)\}$.

If we take into account the definitions of introduced semantics it appears that to check $E \in \sigma(F)$ intuitively for some semantics it suffices to look at the direct impact of E in F, for others we have to compare E with other sets of arguments. We express this intuitive difference by labelling the respective semantics in the following definition.

Definition 2.3.24 (Conditional Semantics)**.** We sometimes speak of conditional semantics, referring to naive, stage, semi-stable and preferred semantics; and opposed to these of unconditional semantics, referring to conflict-free, admissible, complete, grounded and stable semantics.

We close this section with a final example (Figure 2.3.25) and a graphical illustration of discussed set inclusions (Figure 2.3.26). For the final example observe how much stage and

[11] Opposed to [4] and prior occurrences we use a stronger version of non-interference which we believe to match the intent better.



$stg(F) = \{\{b\}\}$

$sem(F) = \{\{a\}\}$

Figure 2.3.25: Illustrating aspects of semi-stable and stage semantics.

semi-stable extensions differ. When looking at the introductory examples of this section, one might assume that each semi-stable extension is covered by some stage extension. However this example proves that assumption wrong.

2.4 The Better Good of Reasoning

Example 2.4.2 (Exponentially Many Extensions). For some finite set I with $|I| = n$ consider the argumentation framework $F = (A, R)$ with:

$$A = \{a_i, b_i \mid i \in I\} \qquad R = \{(a_i, b_i), (b_i, a_i) \mid i \in I\}$$

We have $|A| = 2n$ and $|R| = 2n$, thus $\|F\| = 4n$. We present stable semantics for this framework:

$$stb(F) = \Big\{ \{a_i \mid i \in I \setminus J\} \cup \{b_j \mid j \in J\} \,\Big|\, J \subseteq I \Big\}$$

It follows that $|stb(F)| = 2^n = 2^{\frac{\|F\|}{4}}$. Thus stable semantics and with earlier subset relations most other introduced semantics (except grounded) appear to possibly invoke exponentially many extensions with respect to the size of the argumentation framework of concern.

In the previous examples, for the semantics of interest we used to present all valid extensions. In general, for bigger frameworks especially, it might be more comfortable to compute smaller amounts of information. For instance, when looking at the argumentation framework F from Figure 2.4.1, one might occasionally wonder if it provides a stable extension, $stb(F) \neq \emptyset$. Or if any argument in the innermost circle $a \in \{a_1, a_2, a_3\}$ is part of some or maybe even all preferred extensions. An oracle might conclude that $F = (A, R)$ does not provide a stable extension, that each $a \in \{a_1, a_2, a_3\}$ is contained in some preferred extension and therefor no a_i in all preferred extensions. Naively one might consider to generally compute all extensions for one specific semantics and deduce questions like these from this computation. Since semantics in general are defined by subsets of the power set $\sigma(F) \subseteq \wp(A)$ of the arguments however, it might be the case that there are exponential

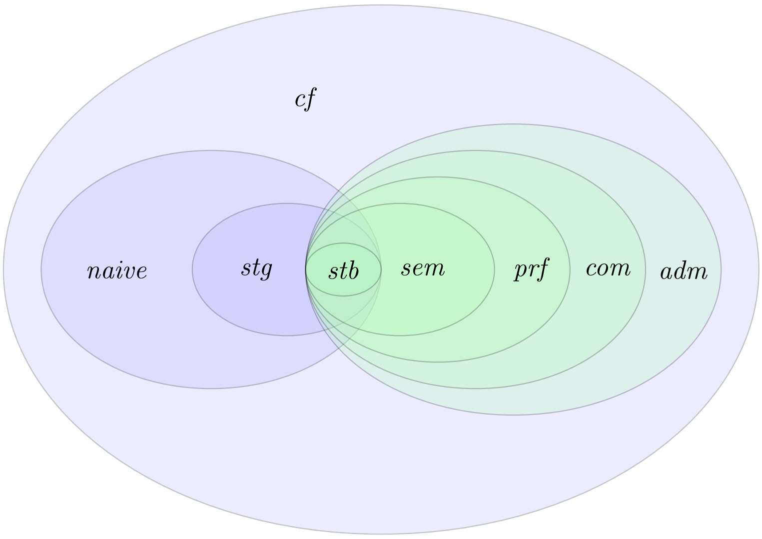


Figure 2.3.26: A Venn diagram illustrating set inclusion concerning common argumentation semantics. Observe that we refer to grounded semantics as just one special complete extension.

many different extensions (see also Example 2.4.2). A comparison of that many extensions will certainly prove to be kind of hard. In this section we will present standard reasoning modes and decision problems established in the community, narrowing down semantics without actually requiring access to all the extensions. For the remainder of this section we occasionally refer to some semantics σ.

When thinking about the origins of abstract argumentation, the probably first question that comes to mind is whether one specific argument is part of at least one extension, in other words, if the semantics of choice supports an argument of choice.

Example 2.4.3 (Credoulous Acceptance). Consider preferred semantics, the argumentation framework shown in Figure 2.4.1, and argument a_1. Obviously a_1 defends itself, thus $\{a_1\}$ is an admissible set, as a consequence some preferred extension E includes a_1.

Definition 2.4.4 (Credoulous Acceptance)**.**
Cred_σ: Given some argumentation framework $F = (A, R)$ and some argument $a \in A$. Is a contained in some σ-extension: $\exists E \in \sigma(F)(a \in E)$?

If we happen to investigate argumentation frameworks for the purpose of establishing some truth, we would much likely prefer a stronger support for at least some arguments, we might wonder if for some semantics and some argument effectively every extension contains this argument.

Example 2.4.5 (Skeptical Acceptance). Consider preferred semantics, the argumentation framework shown in Figure 2.4.1, and argument a_1. Since a_2 is contained in some preferred extension E, clearly a_1 is not member of all preferred extensions.

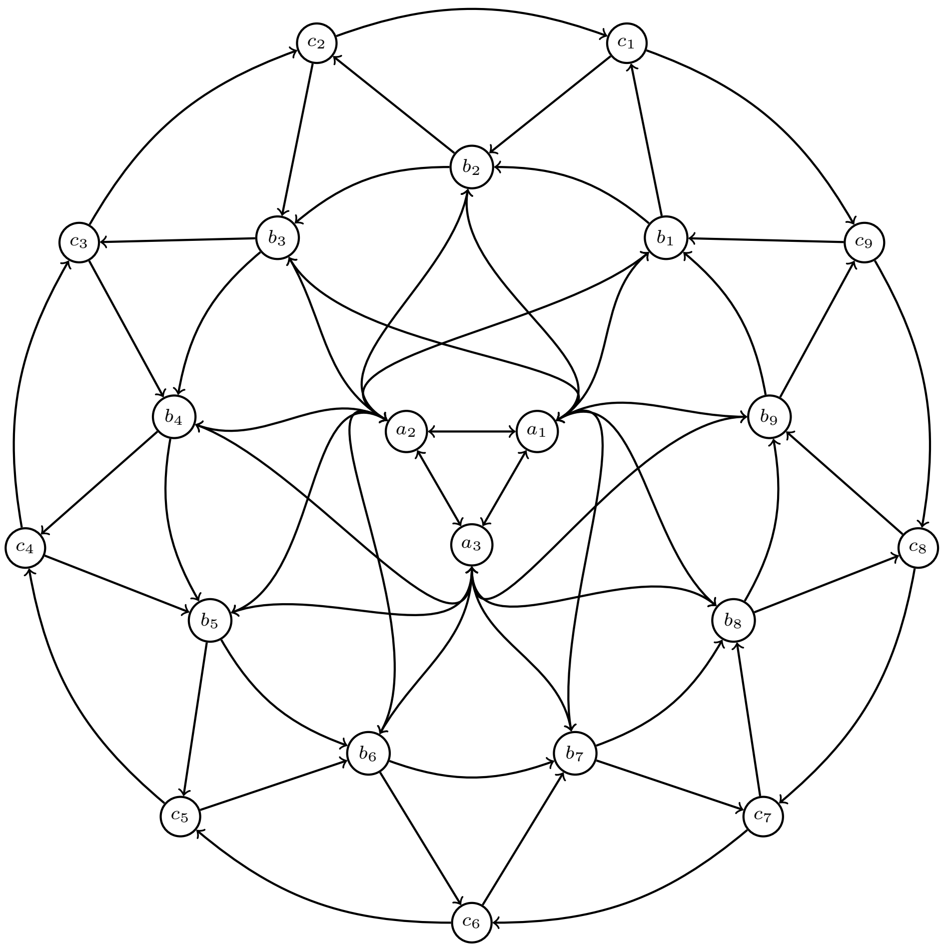


Figure 2.4.1: A bigger abstract argumentation framework.

Definition 2.4.6 (Skeptical Acceptance)**.**
Skept_σ: Given some argumentation framework $F = (A, R)$ and some argument $a \in A$. Is a contained in all σ-extensions: $\forall E \in \sigma(F)(a \in E)$?

Furthermore we might be interested in applying acceptance to more complicated cases, to sets of arguments. We might feel the need to ask, whether some specific set of arguments represents an extension.

Example 2.4.7 (Verification)*.* Consider complete semantics, the empty set and the argumentation framework shown in Figure 2.4.1. Any argument is attacked by some other argument, thus the grounded extension remains empty and therefor the empty set proves to be a complete extension.

Definition 2.4.8 (Verification)**.**
Ver_σ: Given some argumentation framework $F = (A, R)$ and a set of arguments $E \subseteq A$. Is E a σ-extension: $E \in \sigma(F)$?

The questions introduced so far might also be thought of as reasoning modes. By asking these questions we try to narrow down semantics. We believe that questions of practical use can be stated simply by using these three reasoning modes. On the theory level however some more questions might bother us. For example if for some specific semantics some argumentation framework provides an extension.

Example 2.4.9 (Existence). Consider stable semantics and the argumentation framework $F = (A, R)$ shown in Figure 2.4.1. Now assume that there is some stable extension E. Without loss of generality assume $b_3 \in E$. Now $c_2 \rightarrowtail b_3$ but only b_2 and c_3 defend b_3 against c_2, both of which are in conflict with b_3 themselves. The conclusion is that $b \notin E$ for all $b \in \{b_1, b_2 \dots b_9\}$. Furthermore $\{c_1, c_2 \dots c_9\}$ represents an odd circle and thus there is no conflict-free $C \subseteq \{c_1, c_2 \dots c_9\}$ such that $C^+ = \{c_1, c_2 \dots c_9\}$. It follows that there is no admissible set $E \subseteq A$ with $E^+ = A$ and therefor F does not have any stable extension, in other terms $stb(F) = \emptyset$.

Definition 2.4.10 (Existence).
Exists_σ: Given some argumentation framework $F = (A, R)$. Does F provide some σ-extension: $\exists E \in \sigma(F)$?

So far we have learned, that there are frameworks, which do not provide a stable extension. However it does not take long to recognize, that all the other semantics we introduced provide some extension for any framework. Consequently the question arises, whether an extension other than the empty set exists for arbitrary semantics.

Example 2.4.11 (Non-Empty Existence). Consider semi-stable semantics and the argumentation framework shown in Figure 2.4.1. We notice that a_1 defends itself against all attacks, thus a_1 is admissible and therefor there has to be some non-empty $E \in sem(F)$ with $a_1 \in E^+$.

Definition 2.4.12 (Non-Empty Existence).
$\mathsf{Exists}_\sigma^{\neg\emptyset}$: Given some argumentation framework $F = (A, R)$. Does F provide some non-empty σ-extension: $\exists E \in \sigma(F)(E \neq \emptyset)$?

2.5 Complexity of Abstract Argumentation

The introduction of reasoning, reasoning modes and decision problems comes along with the hope of reduced complexity. We have to keep in mind that without further information for some semantics σ and some argumentation framework $F = (A, R)$ there might be exponential many different extensions, $|\sigma(F)| \approx 2^{|A|}$. Thus computational issues might also require exponential time. By downsizing questions we hope that also time and space requirements decrease. In this section we will present a short summary of results concerning complexity of abstract argumentation semantics.

In the following we briefly recall terms from computational complexity. We skip meaning and motivation and just present very reduced definitions. For a more comprehensive discourse on computational complexity we refer to [60]. For an overview on complexity of abstract argumentation we refer to [35] and the references in there.

Definition 2.5.1 (Big-Oh). For functions $f, g : \mathbb{N} \to \mathbb{R}$ we say that f is in big-oh of g ($f = O(g)$) if and only if there exist $n_0, m_0 \in \mathbb{N}$ such that for all $n > n_0$ we have $f(n) < m_0 g(n)$.

Definition 2.5.2 (Algorithm)**.** An algorithm α is a finite set of rules that transforms any valid finite input into some finite output. Each application of some rule is called a step, the number of steps needed for an application of the algorithm is called time. While processing the algorithm may use some buffer memory, which we can think of as to be similar to a blackboard that can be deleted and reused. The amount of buffer memory an algorithm needs while processing is called space. Any input s can be represented by a unique finite string of characters, we call the length of this string, $|s| = n$ the size of the input. If there is a function f such that $\alpha(s)$ needs $O(f(n))$ time (respectively space) we say that α is an f-time (respectively f-space) algorithm.

Definition 2.5.3 (First Complexity Classes)**.** For given input size n we call

- *trivial*: the class of k-time algorithms, for k being independent of n,
- L: the class of $\log(n)$-space algorithms,
- P: the class of n^k-time algorithms, for $k < \infty$,
- PSPACE: the class of n^k-space algorithms, for $k < \infty$,
- EXPTIME: the class of b^n-time algorithms, for $b < \infty$.

Definition 2.5.4 (Reduction)**.** Given two computational tasks T_1 and T_2 a reduction from T_1 to T_2 is an algorithm R, that can be applied to any instance I of T_1 such that

- $R(I)$ is an instance of T_2,
- $R(I)$ needs at most $O(\log(|I|))$ space,
- the answers to I in T_1 and $R(I)$ in T_2 match exactly.

Definition 2.5.5 (Hardness and Completeness)**.** Given some computational task T, a Complexity Class $\mathrm{C_T}$ is implicitly given by all computational tasks T', that can be reduced to T, i.e. there is a reduction from T' to T. For each such T, T' we say that T' is in $\mathrm{C_T}$ and T is $\mathrm{C_{T'}}$-hard. If furthermore for some $T^* \in \mathrm{C_T}$ we have $\mathrm{C_{T^*}} = \mathrm{C_T}$, i.e. each $\mathrm{C_T}$-problem can be reduced to T^*, T^* is in $\mathrm{C_T}$ and $\mathrm{C_T}$-hard, we say that T^* is $\mathrm{C_T}$-complete.

Definition 2.5.6 (Polynomial Hierarchy)**.** Given a logical formula α, without loss of generality $\alpha = \bigwedge_{i\in\{1,2\ldots k\}}(a_{i,1} \vee a_{i,2} \vee a_{i,3})$ for $a_{i,j} \in \{a_1, a_2 \ldots a_m\}$, a sequence $\mathcal{Q}_n = Q_n Q_{n-1} \ldots Q_1$ with $Q_i = \exists E_i$ for odd $i = 2k+1$ and $Q_i = \forall E_i$ for even $i = 2k$ and for $i \neq j : E_i \cap E_j = \emptyset$ and $\bigcup E_i = \{a_1, a_2 \ldots a_m\}$. Any $\mathcal{Q}_n\alpha$ forms a decision problem. We define the polynomial hierarchy:

- Σ_n^P: class of tasks reducible to some $\mathcal{Q}_n\alpha$,
- Π_n^P: class of tasks reducible to some $\neg\mathcal{Q}_n\alpha$,
- $\mathrm{NP} = \Sigma_1^P$,
- $\mathrm{coNP} = \Pi_1^P$.

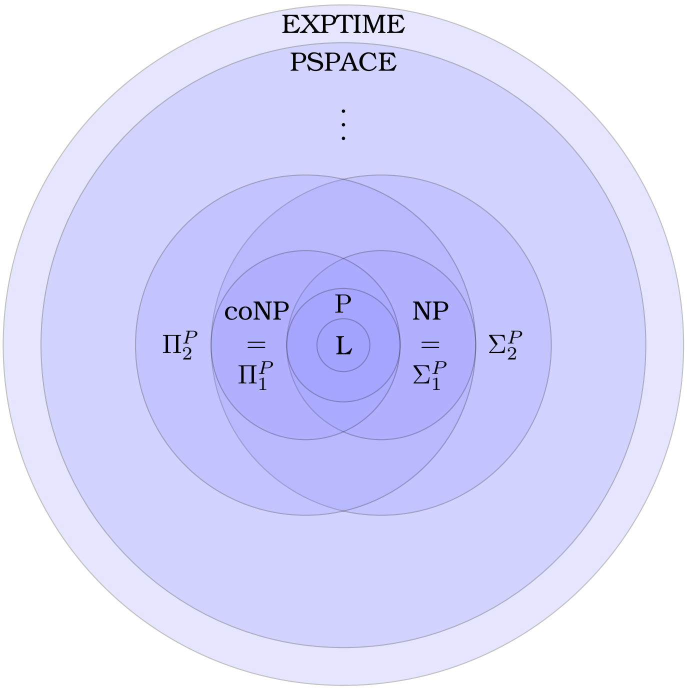


Figure 2.5.7: A Venn diagram illustrating Complexity classes of our concern.

Remark 2.5.8 (Polynomial Hierarchy - Without Proof). We have (see also Figure 2.5.7)

$$
\begin{gathered}
\mathrm{L} \subseteq \mathrm{P} = \Sigma_0^P = \Pi_0^P \\
\Sigma_n^P, \Pi_n^P \subseteq \Sigma_{n+1}^P, \Pi_{n+1}^P \\
\bigcup \Sigma_n^P = \bigcup \Pi_n^P = \text{PSPACE} \subseteq \text{EXPTIME} \\
\mathrm{P} \subsetneq \text{EXPTIME}
\end{gathered}
$$

To the best of our knowledge these are all the set-theoretical relations known to hold for the presented complexity classes. It is commonly believed, but not proven, that above subset relations are real subset relations (e.g. $\mathrm{L} \subsetneq \mathrm{P}$).

We now turn to concrete results as far as complexity of argumentation semantics is concerned. Luckily as far as the introduced complexity classes are concerned there are no open questions left. We hereby admit that for this work we do not consider non-trivial subclasses of L to be of importance. Thus some results are well known in the literature, other results are trivial in comparison. We begin with a selection of the later ones.

Lemma 2.5.9 (Some Results for Trivial Complexity). *We have that* Cred_{cf}, Cred_{naive}, Skept_{cf}, Exists_{cf}, Exists_{naive},Exists_{stg},Exists_{sem} *are trivial*

Proof. Take for instance conflict-freeness and sceptical acceptance of some argument a for any argumentation framework F. The empty set is always a conflict-free extension and $a \notin \emptyset$. □

σ	Cred_σ	Skept_σ	Ver_σ	Exists_σ	$\mathsf{Exists}_\sigma^{\neg\emptyset}$
cf	in L	*trivial*	in L	*trivial*	in L
naive	in L	in L	in L	*trivial*	in L
grd	P-c	P-c	P-c	*trivial*	in L
stb	NP-c	coNP-c	in L	NP-c	NP-c
adm	NP-c	*trivial*	in L	*trivial*	NP-c
com	NP-c	P-c	in L	*trivial*	NP-c
prf	NP-c	Π_2^P-c	coNP-c	*trivial*	NP-c
sem	Σ_2^P-c	Π_2^P-c	coNP-c	*trivial*	NP-c
stg	Σ_2^P-c	Π_2^P-c	coNP-c	*trivial*	in L

Table 2.5.11: Complexity of abstract argumentation, where $\mathcal{C}$-c denotes $\mathcal{C}$-completeness.

Lemma 2.5.10 (Some Results for Logspace Complexity)**.** *We have that* Ver_{cf}, Ver_{naive}, Skept_{naive}, $\mathsf{Exists}_{cf}^{\neg\emptyset}$, $\mathsf{Exists}_{naive}^{\neg\emptyset}$, $\mathsf{Exists}_{stg}^{\neg\emptyset}$ *are in* L*.*

Proof. Take for instance verification of naive semantics in some argumentation framework $F = (A, R)$ for some set $E \subseteq A$. To check pairwise conflict-freeness of E we need space for two pointers. To check if there is some argument outside of E that is not attacked by E and not attacked by itself the two pointers are still sufficient. □

In his 1995 paper Dung [29] already explored to a good part admissible, grounded, stable and preferred semantics. He showed that Skept_{adm}, Exists_{adm}, Exists_{grd} and Exists_{prf} are trivial, that Ver_{adm}, Ver_{stb} and $\mathsf{Exists}_{grd}^{\neg\emptyset}$ are in L and that Cred_{grd}, Skept_{grd} and Ver_{grd} are in P. Dvořák and Woltran in [39] showed that Cred_{grd}, Skept_{grd} and Ver_{grd} are even P-hard, thus concluding P-completeness. In the very same paper they showed coNP-completeness of Ver_{stg}. The same authors showed in [38] Σ_2^P-completeness of Cred_{stg}, Π_2^P-completeness of Skept_{stg}, coNP-completeness of Ver_{sem} and NP-completeness of $\mathsf{Exists}_{sem}^{\neg\emptyset}$.

By connecting graph theoretical structures and various logics and subsequent results in their 1996 paper [28] Dimopoulos and Torres implicitly also affected complexity of abstract argumentation. We derive NP-completeness of Cred_{adm}, $\mathsf{Exists}_{adm}^{\neg\emptyset}$, $\mathsf{Exists}_{com}^{\neg\emptyset}$, Cred_{stb}, Exists_{stb}, $\mathsf{Exists}_{stb}^{\neg\emptyset}$, Cred_{prf} and $\mathsf{Exists}_{prf}^{\neg\emptyset}$ as well as coNP-completeness of Skept_{stb} and Ver_{prf}.

Coste-Marquis, Devred and Marquis contributed to the realm of abstract argumentation complexity in their 2005 work [26] which focussed on symmetric frameworks by declaring NP-completeness of Cred_{com}, P-completeness of Skept_{com}, efficiency of Ver_{com} and triviality of Exists_{com}.

Credits for showing Π_2^P-completeness of Skept_{prf} can be given to Dunne and Bench-Capon [33]. Credits for showing showed Σ_2^P-membership of Cred_{sem} and Π_2^P-membership of Skept_{sem} in [34] can be given to Dunne and Caminada, credits for showing hardness of the latest memberships is again due to [38].

It appears thus that for the semantics introduced in this work there are no open problems as far as mainstream complexity issues are concerned. We present all of these complexity related results in Table 2.5.11.

Figure 2.6.1: An Illustration of Intertranslatability in real life. [44]

2.6 Translations and Properties

So far we have compared argumentation semantics with respect to set inclusion and complexity. One might wonder, if some fine-graining to these comparisons can be added. In [39] Dvořák and Woltran present a new concept of comparison for argumentation semantics. In the previous section we pointed out, that in comparing problems of computational complexity, reductions play an important role. Translations as introduced into abstract argumentation in [39] apply the concept of computational task reduction to argumentation semantics.

A translation informally is a mapping from argumentation frameworks to argumentation frameworks, such that some property is one to one kind of mapped onto some possibly different property. In this context, when speaking about “property” we think about semantics, with “one to one kind of” we mean that there is some automated way to derive the semantics of the original framework from the other. Thus we speak of translating semantics and write e.g. $Tr : \sigma \Rightarrow \sigma'$ for a translation from semantics σ to σ'. In the following we will sometimes use “transformations” and sometimes “translations” with the intention of using the later only if the “one to one kind of” part is fulfilled.

For translations we try to keep the idea of efficiency as used for reductions, i.e. reductions are bound to $\log$-space transformations. Since we also try to interlock semantics in the original and in the translated framework it becomes immediately clear, that some translations will not be possible due to reasons of computational complexity. Think about existence and the possibility of some translation $stb \Rightarrow adm$ for instance. In this section we present a brief introduction into intertranslatability for argumentation semantics along with known results, all of which are strongly related and to a great part taken from [39].

Definition 2.6.2 (Predictability)**.** We call a transformation Tr predictable if for any isomorphic argumentation frameworks $F = (A, R)$, $F' = (A', R')$, $F \cong^{\varphi} F'$, there is a mapping ψ such that $Tr(F) \cong^{\psi} Tr(F')$ and even $\psi(a) = \varphi(a)$ for all $a \in A$.

Sometimes we will give characterizations (e.g. remainder sets in Definition 2.6.12) for specific arguments or sets of arguments in the translated framework. For predictable frameworks we require also these characterizations to be isomorphic.

Remark 2.6.3 (Predictability). For applications a translation implementing random algorithms, considering names of arguments, referring to some order of arguments or similar might be of interest and also in conflict with predictability. Until proven otherwise however we actually believe that only predictable translations are useful. For the purpose of this work we therefor restrict ourselves to predictable translations.

Remark 2.6.4 (Proper Definition of Translations). Technically for some argumentation framework $F = (A, R)$ and some translational transformation Tr there is a proper definition (compare Remark 2.1.20) concerning the names of the arguments in the translated framework $Tr(F) = (A', R')$ with:

$$A' \subsetneq A \cup \left\{ a_i^{F^{\subseteq}} \mid i \in \mathbb{N}, F^{\subseteq} \subseteq F \right\}, \qquad\qquad R' \subseteq A' \times A'$$

The intuition is that translations operate on argumentation frameworks, and therefor build their structure, in names their arguments, from the original framework (A), from properties of subframeworks ($a_i^{F^{\subseteq}}$ with $F^{\subseteq} \neq \emptyset$) and from properties not depending on any concrete framework ($a_i^{\emptyset}$). Observe that the a in $a_i^{F^{\subseteq}}$ does not refer to any $a \in A$ but is fixed. Reference to some argument $b \in A$ can formally be managed by using $a_i^{\{b\}}$.

If no ambiguity arises we can use other letters and signs to denote arguments of translated frameworks. We just need to remind ourselves from time to time, that there is a proper definition underneath, distinguishing three kinds of arguments after translating.

We call a translation Tr a finite translation if for any finite argumentation framework F we have $|Trans(F)| < \infty$. If not mentioned explicitly we will further on only deal with finite translations.

In [55] Liberatore investigates relationships between variants of default logic. He introduces a concept of faithfulness, which interlocks models of the original logic and its translation in a strong way. In relation with this we introduce exactness.

Definition 2.6.5 (Exactness)**.** For semantics σ, σ' we call a transformation $Tr : \sigma \Rightarrow \sigma'$ an exact translation if for every argumentation framework F we have

$$\sigma(F) = \sigma'(Tr(F))$$

So if a translation is exact, for solving most semantical reasoning problems one can think of, it will not make a difference to take F with σ or to take $Tr(F)$ with σ'. If the problem on one side is solved, the solution can immediately be turned into a solution on the other side. It is easily to be seen, that for any semantics σ the identical translation Tr_{id}, which simply copies the original framework, is an exact translation for $\sigma \Rightarrow \sigma$.

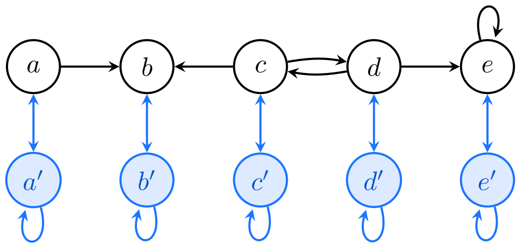


Figure 2.6.7: A first translation applied to Example 2.1.6.

Example 2.6.6 (Exact Translation). For an arbitrary argumentation framework $F = (A, R)$, we define a transformation $Tr(F) = (A', R')$ with:

$$A' = \{a, a' \mid a \in A\}, \qquad R' = R \cup \{(a, a'), (a', a'), (a', a) \mid a \in A\}.$$

An illustration of this transformation as applied to the argumentation framework from Example 2.1.6 is shown in Figure 2.6.7. For frameworks received from applying this transformation it appears that any admissible set is a fixed point of the characteristic function. Obviously any set E of arguments can defend some argument a only if $a \in E$. Moreover admissible sets of arguments are the same for F and $Tr(F)$. Thus we get an exact translation from admissible to complete semantics, $Tr : adm \Rightarrow com$

Example 2.6.8 (No Exact Translation). Observe that the argumentation framework from Figure 2.1.5 has as admissible sets $\{\emptyset, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}\}$. Now clearly $\{a\} \subsetneq \{a, c\}$, immediately we have that there can not be an exact translation $Tr : adm \Rightarrow prf$.

So there are semantics which can not be translated exactly. Observe that for the reasoning problems introduced however, due to Class-Completeness, there has to be a reduction from some admissible problem to the corresponding problem in preferred semantics. Following [51] we introduce a concept of faithfulness.

Definition 2.6.9 (Faithfulness). For semantics σ, σ' we call a transformation $Tr : \sigma \Rightarrow \sigma'$ a faithful translation if for every argumentation framework $F = (A, R)$ we have $|\sigma(F)| = |\sigma'(Tr(F))|$ and

$$\sigma(F) = \{E \cap A \mid E \in \sigma'(Tr(F))\}$$

And indeed there is a faithful translation $Tr : adm \Rightarrow prf$, for more details we refer to [39] or Translation 3.1.85.

For most reasoning problems given some faithful translation it is still an easy task to retrieve a solution for the original framework, assuming prior knowledge about the corresponding problem in the translated framework. However, some reasoning problems might change their meaning, as it is for non-empty-existence. Think about semantics σ_1 and σ_2 with a faithful translation $Tr : \sigma_1 \Rightarrow \sigma_2$. If for some argumentation framework F we know that there is some non-empty σ_2-extension E of $Tr(F)$, then without further knowledge we do not know anything about non-empty extensions for $\sigma_1(F)$, since E might consist of additional arguments only.

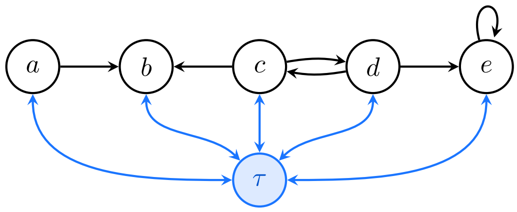


Figure 2.6.13: Another simple translation applied to Example 2.1.6.

Example 2.6.10 (No Faithful Translation). Take any argumentation framework F with non-empty grounded extension $grd(F) \neq \emptyset$. Then obviously there can not be an exact or faithful translation $Tr : grd \Rightarrow adm$, since the empty set is always admissible.

Example 2.6.11 (No Translation). Take any framework F and any semantics σ such that $|\sigma| \geq 2$, then clearly there can not be any exact or faithful translation $Tr : \sigma \Rightarrow grd$, since grounded semantics consists of exactly one extension only.

With the former two examples we run into trouble. To our knowledge there is no useful concept of a translation for the later example. For the first of these examples however [39] provides a workaround.

Definition 2.6.12 (Weakly Exact/Faithful). For semantics σ, σ', we call a transformation Tr a

- weakly exact translation with respect to $\mathcal{S}$ for $\sigma \Rightarrow \sigma'$ if there exists a proper set $\mathcal{S}$ of sets of arguments, such that for any argumentation framework F, $\sigma(F) = \sigma'(Tr(F)) \setminus \mathcal{S}$,
- weakly faithful translation with respect to $\mathcal{S}$ for $\sigma \Rightarrow \sigma'$ if there exists a proper set $\mathcal{S}$ of sets of arguments, such that for any argumentation framework F, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(Tr(F)) \setminus \mathcal{S}\}$ and $|\sigma(F)| = |\sigma'(Tr(F)) \setminus \mathcal{S}|$.

For a given weakly exact or faithful translation with respect to some set $\mathcal{S}$, the elements of $\mathcal{S}$ are also called remainder sets.

Example 2.6.14 (Weakly Exact Translation). For any argumentation framework $F = (A, R)$, with respect to $\{\{\tau\}\}$ we define a transformation $Tr(F) = (A', R')$ with:

$$A' = A \cup \{\tau\}, \qquad R' = R \cup \{(a, \tau), (\tau, a) \mid a \in A\}$$

See Figure 2.6.13 for an application of this transformation to the argumentation framework from Example 2.1.6. Now $\{\tau\}$ clearly is a stable extension in $Tr(F)$. If F has a stable extension E, then clearly E is also a stable extension of $Tr(F)$. Thus with Lemma 2.3.14 it follows that this is a weakly exact translation $Tr : stb \Rightarrow \sigma$ for $\sigma \in \{sem, stg\}$.

Remark 2.6.15. For a given weakly exact or faithful translation with respect to $\mathcal{S}$ by definition and with reference to Remark 2.6.4, each $S \in \mathcal{S}$ consists only of arguments introduced for the translation, independent of any specific argumentation framework.

$$S \subsetneq \left\{a_1^\emptyset, a_2^\emptyset \dots\right\}$$

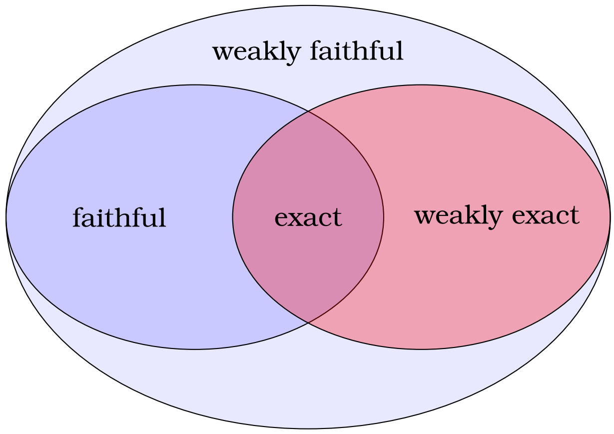


Figure 2.6.16: A Venn diagram illustrating subset relations of (weakly) exact/faithful translations

To get a more clear differentiation, we might refer to the arguments of which any S is built of as

$$S \subsetneq \{\tau_1, \tau_2 \ldots\}.$$

Observe that $\mathcal{S}$ is supposed to be a set, although it might be useful to use proper classes. However we believe that $\mathcal{S}$ being a proper class is highly pathological in nature and only of theoretical use. If not mentioned explicitly we will further on thus only deal with a finite set of remainder sets $\mathcal{S} : |\mathcal{S}| < \infty$.

Lemma 2.6.17. *If a translation is faithful or weakly exact, then it is also weakly faithful. If it is exact it is also faithful and weakly exact. See Figure 2.6.16 for an illustration of these subset relations.*

Since part of the motivation for introducing translations is to find other ways of computing solutions for reasoning problems, respectively computing semantics for arbitrary argumentation frameworks, we would like translations to fulfill further conditions.

Definition 2.6.18 (Efficiency)**.** A transformation Tr is called efficient if for every argumentation framework F, $Tr(F)$ can be computed using logarithmic space with respect to the size of F (Definition 2.1.23).

Any efficient transformation will also meet the requirements to transform statements of computational complexity. Dvořák and Woltran [39] used this to show some impossibility results.

With the following translational property we introduce a concept intended to ease computational efforts by a great part. The idea is to be able to split computation into several independent parts, thus enabling parallel computing.

Definition 2.6.19 (Modularity)**.** A transformation Tr is called modular if for any argumentation frameworks F, F', we retrieve

$$Tr(F) \cup Tr(F') = Tr(F \cup F').$$

It appears that for modular transformations we can retrieve the transformation of any framework by dividing it into smaller and smaller fractions.

$$Tr(F) = Tr(F_1 \cup F_2 \dots F_n) = Tr(F_1) \cup Tr(F_2) \dots Tr(F_n)$$

At some point anyway we will arrive at smallest fractions. These smallest fractions can then be divided into classes of isomorphic frameworks. Since we restrict ourselves to predictable frameworks (compare Remark 2.6.2) for each of these classes it suffices to translate just one representative. Generalizing this principle we present a concept of locality.

Definition 2.6.20 (Locality With Respect to $\mathscr{L}$)**.** A transformation Tr is called local with respect to a set $\mathscr{L}$ of argumentation frameworks ($\mathscr{L} = \{F_1, F_2, \dots\}$) if for any argumentation framework F, we retrieve

$$Tr(F) = \bigcup_{L \in \mathscr{L}, L \cong F' \subseteq F} Tr(F')$$

Remark 2.6.21. Without loss of generality the argumentation frameworks in $\mathscr{L}$ can be chosen isomorphically different, i.e. for $L_1, L_2 \in \mathscr{L}$ with $L_1 \cong L_2$ already $L_1 = L_2$.

Definition 2.6.22 (k-Component Locality)**.** A transformation Tr is called k-component local if it is local with respect to $\mathscr{L}$ and each $F \in \mathscr{L}$ consists of at most k connected components (compare Definition 2.1.29), in other words for each $L \in \mathscr{L}$ there is a fragmentation $\mathscr{F}_L$ into connected components[12] such that $|\mathscr{F}_L| \leq k$.

Definition 2.6.23 (Local with Diameter)**.** A transformation Tr is called local with diameter d if for any argumentation framework F we have:

$$Tr(F) = \bigcup_{F' \subseteq F, dia(F') \leq d} Tr(F')$$

Lemma 2.6.24 (Diameter vs. 1-Component)**.** *Any transformation which is local with finite diameter is also* 1*-component local with respect to some possibly infinite set* $\mathscr{L}$*.*

Proof. For an arbitrary argumentation framework $F = (A, R)$ and transformation Tr with diameter d, consider the (for $d > 0$ infinite) set of universal (compare Remark 2.1.20) argumentation frameworks $\mathscr{L} \subsetneq \mathscr{F}^\infty$ with diameter of at most d.

$$\mathscr{L} = \{F \in \mathscr{F}^\infty \mid dia(F) \leq d\}$$

Then for $F' \subseteq F$ with $dia(F') \leq d$ consequently there is some $L \in \mathscr{L}$ such that $L \cong F'$. Therefor

$$Tr(F) = \bigcup_{L \in \mathscr{L}, L \cong F' \subseteq F} Tr(F')$$

□

Definition 2.6.25 (Finite Locality)**.** A transformation Tr is called finitely local if it is local with respect to $\mathscr{L}$ and $|\mathscr{L}| < \infty$.

[12] We have $\bigcup \mathscr{F}_L = \mathscr{L}$ and for any $F \in \mathscr{F}_L$: $dia(F) < \infty$.

Lemma 2.6.26. *A transformation Tr is finitely local with respect to $\mathscr{L}$ if and only if there is some $n < \infty$ such that for each $L \in \mathscr{L}$, $|L| \leq n$.*

Proof.

$\Longrightarrow$: If $\mathscr{L} < \infty$, then there is some $L_m \in \mathscr{L}$ such that $|L'| \leq |L_m|$ for all $L' \in \mathscr{L}$. With Remark 2.1.15 any argumentation framework is finite, now simply take $n = |L_m|$.

$\Longleftarrow$: There are only a finite number of isomorphically different argumentation frameworks F with $|F| \leq n$ for finite n.

□

Corollary 2.6.27. *Any finitely local transformation is k-component local for some $k < \infty$.*

Definition 2.6.28 (Strict Locality)**.** A transformation is called strictly local if it is 1-component finitely local.[13] In case ambiguity arises we also speak of strictly local with respect to a set $\mathscr{L}$ of 1-component argumentation frameworks, or similar.

Remark 2.6.29 (Locality). We will present further details on relations between modular, local and faithful translations in Section 3.1. For now we keep in mind that strictly local is the most restricted form of locality and the following relations: Each strictly local transformation is local with finite diameter and finitely local. Each local with finite diameter transformation is 1-component local. Each finite transformation is k-component local for some finite k.

We will show that strict locality is in some way an immediate extension of modular in Lemma 3.1.1 and we will show that monotonicity as introduced below is equivalent to locality in Lemma 3.1.3. We will use finite locality to derive efficiency in Theorem 3.1.4.

Besides relating efficiency and modularity, Chapter 3 will work with locality in various ways. We will use strict locality to broaden proofs based upon modularity. We will use k-component locality to classify concrete transformations. We will use finite-diameter and 1-component locality to acquire impossibility results.

One might wish that subframeworks translate into subframeworks. With the following definition we introduce monotonicity as presented in [39]. We will show in Section 3.1 that monotonicity serves as the most general form of locality.

Definition 2.6.30 (Monotonicity)**.** A transformation Tr is called monotone if for any argumentation frameworks F, F', we retrieve

$$F \subseteq F' \implies Tr(F) \subseteq Tr(F').$$

When thinking about practical applications of translations it appears that we might want arguments and conflicts (maybe even non-conflicts) to remain after translating. Thus, especially on the meta-level, the following conditions might be of interest.

[13]Recall that for predictable transformations Tr (compare Definition 2.6.2) for computational issues it suffices to compute $Tr(L)$ for all $L \in \mathscr{L}$ and apply the results to isomorphic subsets of F.

Definition 2.6.31 (Covering)**.** A transformation Tr is called covering if for every argumentation framework F we have $F \subseteq Tr(F)$.

Definition 2.6.32 (Embedding)**.** A transformation Tr is called embedding if for every argumentation framework $F = (A, R)$ we have $F \subseteq Tr(F)$ and $Tr(F)|_A = F$, in other words if the transformation is covering and no attacks between original arguments are added.

A transformation fulfilling embedding will preserve the inner structure of the original framework. A covering transformation will still preserve much of the inner structure, although there might be additional attacks between original arguments. Justification for both is that we want to preserve original arguments and conflicts (and conflict-freeness), to ensure to still be talking about the same argumentation process on a meta-level. Clearly an embedding transformation is always also a covering transformation but not vice versa.

3 Contributions

This thesis started out with repeated attempts of translating semi-stable to stage semantics. As a first substantial result impossibility of weakly exact translations was detected and already included in [39]. This first impossibility was soon followed up by impossibility of modular weakly faithful translations. Up to now existence of efficient weakly faithful translations is still unknown, but by enhancing modularity to levels of locality by now we know that possibly existing efficient translations are probably not all that intuitive. See for instance Example 3.2.27 for a partial translation. Consequently the results of this thesis can be seen as spin-off products of the initial attempts. We tried to complete the picture, yet for the time being results of non-restricted local translations are rare.

However by investigating locality and thus applicability of parallel computing we developed techniques suitable for categorization of translations as well as detection of impossibilities. We would like to emphasize the concept of strict locality, for we actually believe that translations possessing this property might build a well distinguishable class of transformations (at least compared to the property of being efficient).

In the previous chapter we have given definitions, basic results and background information for argumentation frameworks and intertranslatability. In this chapter we will focus on intertranslatability results for the introduced semantics. We will deal with open questions from [39], we will present modified as well as new translations and we will present new impossibility results for several demands. We will give a motivation and make use of the different levels of locality. We will present results affecting interplay between various semantics.

Dvořák and Woltran investigated intertranslatability with respect to stage, stable, semi-stable, preferred, complete, admissible and grounded semantics. As a first step we will expand the semantics of interest to also include conflict-free and naive semantics. A summarizing presentation of results from [39] is given in Table 3.0.1. Colored cells indicate enhancement by this thesis. As far as the encoding of the results is concerned for now we only declare that

- each cell consists of an upper (translations) and a lower (impossibilities) area;
- for each result the part following the colon is a reference, for Table 3.0.1 referring to some result from [39];
- $\mathbb{E}$ indicates exactness, E indicates weak exactness, $\mathbb{F}$ indicates faithfulness and F indicates weak faithfulness.

We will give a detailed definition, especially for the remaining parts of the encoding in Definition 3.1.14. Some enhancements are also due to broadening translations of interest also to inefficient translations, thus enabling additional results (Section 3.4).

⇒	*stg*	*stb*	*sem*	*prf*	*com*	*adm*	*grd*
stg	$\mathbb{E}_0^e$: id	F_e: Th. 10	$\mathbb{E}_2^c$: Tr_2 $\mathbb{F}_0^e$: Tr_5 E^e: Th. 14	F_e: Th. 9	F_e: Th. 10	F_e: Th. 10	F: Prop. 3
stb	E_0^e: Tr_3	$\mathbb{E}_0^e$: id	E_0^e: Tr_3	(E_2^e: Tr_4)	(E_2^e: Tr_4)	(E_2^e: Tr_4)	F: Prop. 3
sem	E: Th. 15	F_e: Th. 10	$\mathbb{E}_0^e$: id	F_e: Th. 9	F_e: Th. 10	F_e: Th. 10	F: Prop. 3
prf	E: Th. 15	F_e: Th. 10	$\mathbb{E}_0^e$: Tr_1	$\mathbb{E}_0^e$: id	F_e: Th. 10	F_e: Th. 10	F: Prop. 3
com	$\mathbb{F}_0^e$: Tr_7 E: Th. 11	$\mathbb{F}_0^e$: Tr_7 E: Th. 11	$\mathbb{F}_0^e$: Tr_7 E: Th. 11	$\mathbb{F}_2^e$: $Tr_4 \circ Tr_7$ E: Th. 11	$\mathbb{E}_0^e$: id	F_2^e: $Tr_4 \circ Tr_7$ E: Th. 12	F: Prop. 3
adm	$\mathbb{F}_1^e$: Tr_6 E: Th. 11	$\mathbb{F}_1^e$: Tr_6 E: Th. 11	$\mathbb{F}_1^e$: Tr_6 E: Th. 11	$\mathbb{F}_2^e$: $Tr_4 \circ Tr_6$ E: Th. 11	$\mathbb{E}_0^e$: Tr_1	$\mathbb{E}_0^e$: id	F: Prop. 3
grd	$\mathbb{F}_e^e$: Tr_8	$\mathbb{F}_e^e$: Tr_8 E_e: Th. 13	$\mathbb{F}_e^e$: Tr_8	$\mathbb{F}_e^e$: Tr_8	$\mathbb{F}_e^e$: Tr_8 E_e: Th. 13	F_e^e: $Tr_4 \circ Tr_8$ E_e: Th. 13	$\mathbb{E}_0^e$: id

Table 3.0.1: A summary of intertranslatability results with references to results from [39], decryption by Definition 3.1.14, colored cells indicate enhancement by this thesis.

In the following among other results we will present new facts for all question marks in Table 2 from [39]. As far as intertranslatability for abstract argumentation semantics is concerned a detailed summary of all results from this thesis can be found in Section 3.5 (Table 3.5.1). Highlighting of singular results turns out to be a difficult task, however we would like to hint to

- relation of stage, stable and semi-stable semantics in self-attack-free argumentation frameworks (Corollary 3.2.32),
- efficiency of finitely local and thus modular translations (Theorem 3.1.4),
- impossibility of weakly faithful translations $\sigma \Rightarrow (cf|naive)$ (Theorem 3.2.4),
- impossibility of weakly exact translations $(sem|prf|com|adm) \Rightarrow (cf|naive|stg)$ (Corollary 3.2.8),
- impossibility of efficient exact translations $grd \Rightarrow stg$ (Corollary 3.3.15),
- impossibility of finite-diameter local weakly faithful translations $(sem|prf) \Rightarrow stg$ (Theorem 3.2.24),
- impossibility of finite-diameter local weakly exact translations $grd \Rightarrow sem$ (Theorem 3.3.18),
- impossibility of finite-diameter local weakly faithful translations $grd \Rightarrow (stg|prf)$ (Theorem 3.3.20) and $grd \Rightarrow (stb|com|adm)$ (Corollary 3.3.21),
- the surprisingly simple modular embedding faithful Translation 3.1.72 for $com \Rightarrow (stg|stb|sem|prf)$,
- the modular embedding faithful Translation 3.3.11 for $grd \Rightarrow sem$,
- the oracle embedding exact Translation 3.4.17 for $sem \Rightarrow prf$.

In Section 3.1 we begin the contributions chapter with a deeper look into known translations, as well as modifications of these translations and genuinely new translations with similar effect. We will also analyze relations between efficient, modular, local and monotone transformations. The aim of new and modified translations will be to meet a stronger level of locality, generality or simplicity.

In translating argumentation frameworks a lot of effort has been put into investigation of similar semantics. In Section 3.2 we will focus on intertranslatability of conditional semantics (naive, stage, semi-stable and preferred semantics, see Definition 2.3.24).

For the semantics presented in this work, grounded semantics stands out of the crowd for possessing a unique extension for every argumentation framework and being comparably simply computable. In Section 3.3 we will present new results concerning impossibilities of finite-diameter translations and efficient translations $grd \Rightarrow \sigma$.

When looking for efficient translations or at efficiency of translations it is also good to know upper boundaries. By focusing on transformations without the limits of computational issues, we finalize this chapter by giving results for inefficient translations in Section 3.4.

Section 3.5 is dedicated to outlining all results of this thesis. We will present a detailed table as well as illustrations for selected classes of translations.

Remark 3.0.2 (Empty Argumentation Frameworks). In contrast to [39] we allow arbitrary argumentation frameworks F to be empty ($F = (\emptyset, \emptyset)$). As far as intertranslatability is concerned this mainly affects behaviour of stable semantics. The empty set is a stable extension if and only if the argumentation framework of interest is empty. Thus we will mark some translations as non-general (e.g. $stb \Rightarrow adm$) but on the other hand allow some exact translations (e.g. $naive \Rightarrow stb$).

3.1 Translational Basics

So far we have introduced various translational properties. While embedding and covering serve structure-preserving purpose, efficiency, modularity, locality and monotonicity are intended to give measures for computational issues, exactness and faithfulness are used to give basic conditions for the use of translations. With the hitherto introduced characteristics the strongest attributes a translation can have are modular, embedding and exact. We observe the following correlations:

- If a translation is embedding it is also covering. In general monotone translations will at least be argument-covering. Since the semantics of interest in this thesis operate on the same concept of conflict-freeness furthermore monotonicity and the covering property will appear to be in a strong correlation.

- If a translation is exact it is also faithful and weakly exact. If a translation is faithful or weakly exact it is also weakly faithful. We hint to Figure 2.6.16 for a Venn diagram illustrating these relations.

- If a translation is modular it is also efficient and strictly local, as will be shown shortly. As far as the levels of locality are concerned we will present a Venn diagram in Figure 3.1.9, illustrating significance and correlation also with the concept of efficiency. Subsection 3.1.1 will deal in great parts with resolving these issues.

For a starter in Subsection 3.1.1 we will clarify relations of computational properties. We will follow up by analyzing and giving first results in the subsequent subsections, namely translations and impossibilities as far as intertranslatability between argumentation semantics is concerned. For this section we will overall stay close to [39].

A few words about locality

In Section 2.6 we have introduced various concepts of locality. Local Translations (Definition 2.6.20) can be declared by translating a possibly infinite set $\mathscr{L}$ of argumentation frameworks.

In Lemma 3.1.3 we will point out the relation between local and monotone translations. For finitely local translations (Definition 2.6.25) we deal with a finite set $\mathscr{L}$ of representative argumentation frameworks determining the translation; we will focus on implications of this definition for efficiency in Theorem 3.1.4. In Lemma 3.1.1 we will point out the relationship between modularity (Definition 2.6.19) and strict locality (Definition 2.6.28). We will use finite-diameter locality (Definition 2.6.23) to give impossibility results. We will use k-component locality (Definition 2.6.22) to classify actual translations.

For now we point out that while diameter local translations are also 1-component local (Lemma 2.6.24) the reverse does not hold. While finite-diameter translations are not necessarily efficient we will give 1-component local translations in Section 3.4 where finite-diameter local translations happen to fail and an efficient 1-component local Translation 3.3.9 which appears not to be finite-diameter local.

The main purpose of introducing finite-diameter locality is to toughen impossibility results, originally revealed with respect to modularity. By thinking of finite-diameter local translations we have transformations in mind which not necessarily possess efficiency but do so for sparsely populated argumentation frameworks.

3.1.1 Essential Properties of Local Translations

The original intent of modular translations is to ensure usability for parallel computing. In order to achieve this goal, modular translations can be built by splitting the transformation into small, smaller and smallest parts. When thinking about modularity we think about breaking the framework into bits.

What follows is a classification of modularity as the most restricted and still useful form of locality and of monotonicity as the most general form of locality, thus presenting concepts of locality as fine-graining between monotonicity and modularity. To decorate this categorization with Theorem 3.1.4 we will show that finitely local and therefor modular translations happen to already be efficient.

Lemma 3.1.1 (Modular vs. Local)**.** *Any modular translation Tr is also strictly local.*

Proof. Take into account the set $\mathscr{L}_m = \{F_0, F_1, F_2, F_3\}$ with

$$F_0 = (\emptyset, \emptyset) \qquad F_2 = (\{a_0\}, \{(a_0, a_0)\})$$
$$F_1 = (\{a_0\}, \emptyset) \qquad F_3 = (\{a_0, a_1\}, \{(a_0, a_1)\})$$

Now obviously if Tr is local with respect to $\mathscr{L}_m$ it is already strictly local (finitely 1-component local, see Definition 2.6.28). We observe that for any argumentation framework F we have that F is the union of sets which are isomorphic to some F_i with $i \in \{0, 1, 2, 3\}$:

$$F = \bigcup_{F' \cong F_i, F' \subseteq F} F'$$

But then for any modular translation Tr (as of Definition 2.6.19) we retrieve

$$Tr(F) = Tr(\bigcup_{F' \cong F_i, F' \subseteq F} F') = \bigcup_{F' \cong F_i, F' \subseteq F} Tr(F')$$

and therefor any modular translation is also local with respect to $\mathscr{L}_m$, respectively strictly local. □

Remark 3.1.2*.* Observe that in the previous proof $\mathscr{L}_m$ is minimal. For any proper subset $\mathscr{L} \subsetneq \mathscr{L}_m$ we can think of some modular translation not being local with respect to $\mathscr{L}$. Furthermore F_1, F_2 and F_3 are necessary to build the identical transformation (Transformation 3.1.11). As a side note we can think[1] of a weakly faithful translation with remainder set $\emptyset$, making use also of F_0. Thus we claim modular translations to be the most restricted and still reasonable local translations.

Lemma 3.1.3 (Monotone vs. Local)**.** *Any translation Tr is monotone if and only if it is local with respect to some possibly infinite set $\mathscr{L}$.*

Proof.

$\Longleftarrow$: Assume that Tr is local with respect to some $\mathscr{L}$. For arbitrary argumentation frameworks $F \subseteq F'$ we have that for any subframework $F^< \subseteq F$ with $F^< \cong L$ for some $L \in \mathscr{L}$, also $F^< \subseteq F'$ and therefor $Tr(F^<) \subseteq Tr(F)$ and $Tr(F^<) \subseteq Tr(F')$. Thus monotonicity follows:

$$Tr(F) = \bigcup_{F^< \subseteq F} Tr(F^<) \subseteq \bigcup_{F^< \subseteq F'} Tr(F^<) = Tr(F')$$

$\Longrightarrow$: Take into account the universe of all possible isomorphically different abstract argumentation frameworks $\mathscr{F}^\infty$ (compare Remark 2.1.20). Now observe that for any argumentation framework F and for all $F' \subseteq F$ there is some $L' \in \mathscr{F}^\infty$ such that $L' \cong F'$. Thus any monotone translation Tr is also local with respect to $\mathscr{F}^\infty$.

$$Tr(F) = \bigcup_{F' \subseteq F} Tr(F') = \bigcup_{L' \in \mathscr{F}^\infty, L' \cong F' \subseteq F} Tr(F')$$ □

[1] Take into account Translation 3.1.66 for $stb \Rightarrow adm$. The empty argumentation framework translates into a single-argument framework and the empty set is used as remainder set.

Theorem 3.1.4 (Efficient vs. Local)**.** *Any finitely local translation is already efficient.*

Proof. We look at any argumentation framework $F = (A, R)$ and investigate some translation Tr which is local with respect to some finite (Definition 2.6.25) set $\mathscr{L}$ of argumentation frameworks. For given $L \in \mathscr{L}$, since $\|L\|$ is finite, observe that we need some fixed amount of finite space to compute $Tr(L)$. Furthermore, since $\mathscr{L}$ is finite, we can fix some $L_1 \in \mathscr{L}$ for which space requirement of Tr is maximal but constant and $L_2 \in \mathscr{L}$ for which size $\|L_2\|$ is maximal. Note that subgraph isomorphism in general is NP-complete [45], however by restricting the size of the graph of interest efficiency can be achieved. We need $O(\|L_2\| \log(\|F\|))$ space to check for subframeworks in F which are isomorphic to L_2 and thus to translate any subgraph isomorphic to some $L \in \mathscr{L}$. It follows that any finitely local translation can be computed efficiently (compare Definition 2.6.18). □

Corollary 3.1.5.

1. *Any strictly local translation is already efficient.*
2. *Any modular translation is already efficient.*

We refer to [63] for efficiency of graph connectivity. Combined with the semantical property of non-interference (Definition 2.3.23) we conclude an interesting observation.

Corollary 3.1.6. *Semantics fulfilling the non-interference property which can be translated finitely local can also be translated efficiently* 1*-component local.*

Remark 3.1.7. The reverse of Theorem 3.1.4 (efficient $\implies$ finitely local) is not true in general, even for useful translations. Tr_8 from [39] is efficient but not finitely local.[2]

It is an open and to speak of interesting question whether the reverse of Corollary 3.1.6 is true or false. Does existence of efficient 1-component local translations tell us something about existence of finitely local translations?

This leads us to some more questions about which properties translations from [39] fulfill and which properties they might fulfill with minor modifications, most of which we will deal with in this section.

Corollary 3.1.8 (Subset Relations for Computational Properties)**.** *With reference to Remark 2.6.29 and the previous results for local translations we present the following summary. Consider some translation* Tr *then the following statements hold:*

- *If* Tr *is modular it is also strictly local.*
- Tr *is strictly local if and only if it is* 1*-component finitely local.*
- *If* Tr *is local with diameter* d *it is* 1*-component local.*
- *If* Tr *is* 1*-component local it is local with possibly infinite diameter* d*.*
- *If* Tr *is* k*-component local it is also* $k+1$*-component local.*

[2]In fact Tr_8 is not monotone. However we will give a simple modification (Translation 3.3.9) such that Tr_8 becomes an efficient 1-component local faithful translation.

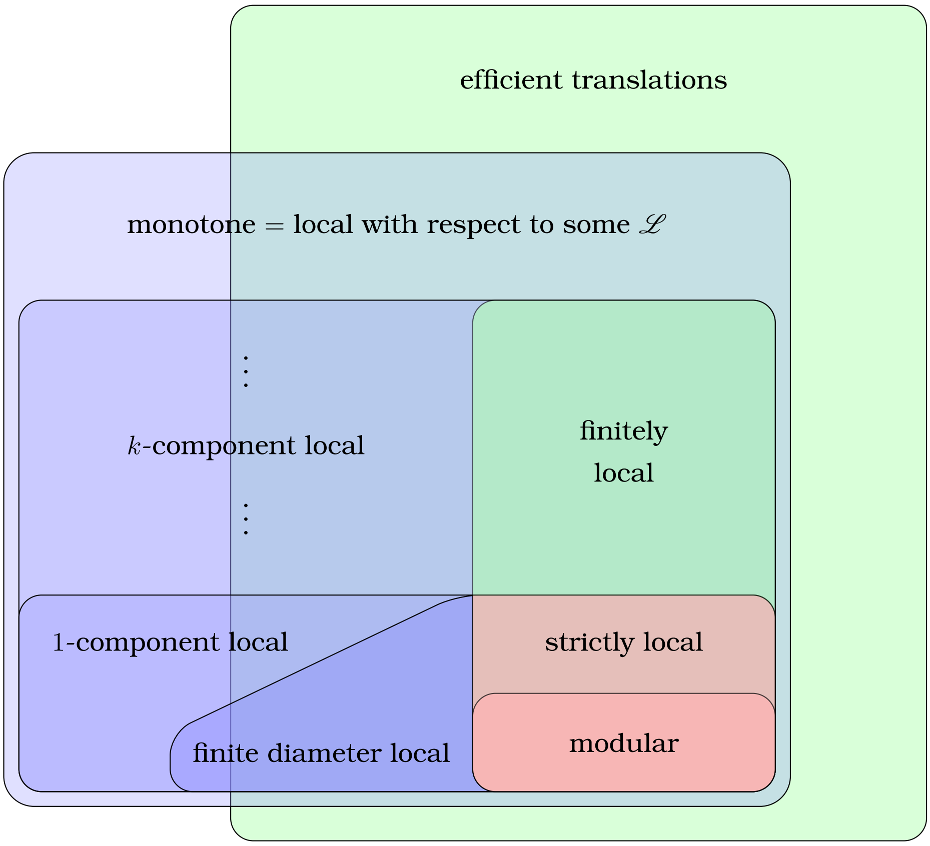


Figure 3.1.9: Illustration of relations between computational properties of translations as presented in Corollary 3.1.8.

- *If Tr is finitely local it is k-component local for some finite k.*
- *If Tr is finitely local it is also efficient.*
- *Tr is monotone if and only if it is local with respect to some $\mathscr{L}$.*

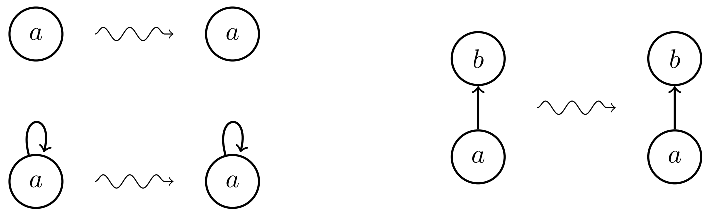
A graphical illustration of these concluding relations can be found in Figure 3.1.9

Remark 3.1.10. When presenting translations which are local with respect to some finite set $\mathscr{L}$ of argumentation frameworks, a visually very comforting way is to present the transformations for all $L \in \mathscr{L}$ separately. We will follow this idea in the following and often even skip text representations later on.

Transformation 3.1.11 (Identity)**.** Identity for argumentation frameworks can be defined as a modular transformation, $F = (A, R), Tr_{id}(F) = (A', R')$, with $A = A'$ and $R = R'$:

$A = \{a\}$	$R = \emptyset$	$\Longrightarrow$	$A' = \{a\}$	$R' = \emptyset$
$A = \{a\}$	$R = \{(a, a)\}$	$\Longrightarrow$	$A' = \{a\}$	$R' = \{(a, a)\}$
$A = \{a, b\}$	$R = \{(a, b)\}$	$\Longrightarrow$	$A' = \{a, b\}$	$R' = \{(a, b)\}$

As mentioned above we can also visually represent the identical transformation with the following scheme:



Translation 3.1.12 ($\mathbb{E}_0^e$: $\sigma \Rightarrow \sigma$)**.** Obviously for any argumentation framework F and any semantics σ we have that Tr_{id} is an efficient modular embedding translation for $\sigma \Rightarrow \sigma$.

Remark 3.1.13. In general there will not be any local with diameter 0 translation for semantics making use of conflict-freeness. Any such translation does not have access to attacks between original arguments.

As already used for Translation 3.1.12 by the end of this subsection we are able to categorize translations according not only to (weak) exactness/faithfulness, efficiency and embedding/covering property but also to their respective level of locality (compare also Figure 3.1.9). The following Definition 3.1.14 presents an encoding we will subsequently use throughout this thesis.

Definition 3.1.14 (Short Form of Translational Properties)**.** We use $\mathbb{E}$ to denote exactness, E to denote weak exactness, $\mathbb{F}$ to denote faithfulness and F to denote weak faithfulness. We call $X \in \{\mathbb{E}, E, \mathbb{F}, F\}$ the type of translation, for X being the type of translation we use X^e to denote the embedding property and X^c to denote the covering property. Furthermore we use X_i to denote the level of locality respectively efficiency where i is decoded by value as follows:

0	modular	5	efficient monotone
1	strictly local	6	1-component local
2	finitely local	7	monotone
3	efficient 1-component local		
4	finite-diameter local	e	efficient

We refer to impossibility results by coloring, e.g. $\mathbb{E}_0$ means that some modular exact translation is not possible. Furthermore we use parentheses to denote applicability of some translation only for specific argumentation frameworks. If some covering efficient monotone weakly faithful translation applies only to non-empty argumentation frameworks we might use (F_5^e).

3.1.2 Foundational Transformations and Translations

We point out that any mapping and therefor any translation Tr can be represented as a class of ordered pairs $Tr = \{(F, F') \mid F$ *some argumentation framework and* $Tr(F) = F'\}$.

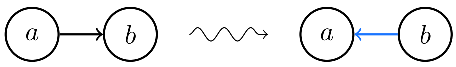
Intuitively for arbitrary transformations Tr_1 and Tr_2 with $(F, F_1) \in Tr_1$ and $(F, F_2) \in Tr_2$ we can build a new transformation Tr_{1+2} with $(F, F_1 \cup F_2) \in Tr_{1+2}$. The motivation for this observation is that for local transformations with respect to $\mathscr{L}$ for any $L \in \mathscr{L}$ we have a separate transformation which is local with respect to $\{L\}$. Thus we can build a transformation which is local with respect to $\mathscr{L}$ by breaking the definition into partial transformations $Tr_1, Tr_2 \ldots$ being local with respect to $\mathscr{L}_1, \mathscr{L}_2 \ldots$ such that $\mathscr{L} = \mathscr{L}_1 \cup \mathscr{L}_2 \ldots$ In the following we will introduce partial transformations on possibly similar sets $\mathscr{L}$ (e.g. $\mathscr{L}_1 \cap \mathscr{L}_2 \neq \emptyset$). As touched in Remark 2.6.21 this does not affect well-definedness of the presented local transformations.

Definition 3.1.15 (Union of Transformations)**.** For translations Tr and Tr' we define the union of transformations $Tr \cup Tr'$ as

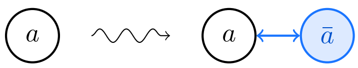
$$(Tr \cup Tr')(F) = Tr(F) \cup Tr'(F)$$

Remark 3.1.16. The intent of the union of transformations is that for transformations Tr and Tr' which are local with respect to $\mathscr{L}$ and $\mathscr{L}'$ we receive a transformation $Tr \cup Tr'$ which is local with respect to $\mathscr{L} \cup \mathscr{L}'$. In general however we need to take care, as names of new arguments might be used in both transformations. Observe that for any argumentation framework F and any monotone or predictable transformation Tr we get $Tr(F) = Tr(F)$ and thus $(Tr \cup Tr)(F) = Tr(F)$.

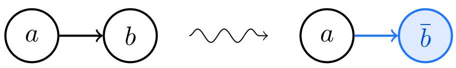
Transformation 3.1.17 (Symmetry)**.** With respect to $\mathscr{L} = \{(\{a, b\}, \{(a, b)\})\}$ we define a symmetrizing modular transformation Tr_{sym}:



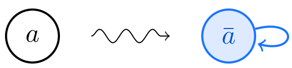
Transformation 3.1.18 (Emphasizing Arguments)**.** With respect to $\mathscr{L} = \{(\{a\}, \emptyset)\}$ we define a modular transformation Tr_{emph0} emphasizing arguments:



Transformation 3.1.19 (Emphasizing Attacks)**.** With respect to $\mathscr{L} = \{(\{a, b\}, \{(a, b)\})\}$ we define a modular transformation Tr_{emph1} emphasizing attacks:



Transformation 3.1.20 (Emphasizing with Conflict)**.** With respect to $\mathscr{L} = \{(\{a\}, \emptyset)\}$ we define a modular transformation $Tr_{\overline{emph}}$ self-conflicting additional arguments of the form $\bar{A}$:



Remark 3.1.21. Observe, that the previous four transformations in general do not give any useful translation on their own. In Tr_{sym} we have that arguments not being member of a classical attack relation do not occur in the transformed framework, in Tr_{emph0} attacks are not translated at all, in Tr_{emph1} non-attacking arguments do not reappear, in $Tr_{\overline{emph}}$ original arguments disappear completely. To be of practical use Transformations 3.1.17 to 3.1.20 will be used in union with other transformations.

The identical Translation 3.1.12 will not constitute a disturbing factor for most cases as far as union of transformations (Definition 3.1.15) is concerned. As far as efficient translations are concerned, to our knowledge there are very few results negating the

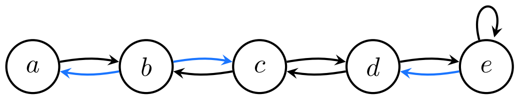


Figure 3.1.23: Translation 3.1.22 ($cf \Rightarrow adm$,$naive \Rightarrow prf$) as applied to Example 2.1.6.

covering property[3] (Definition 2.6.31). Viewed in this light it comes clear that the intentional use of Tr_{sym}, Tr_{emph0} and Tr_{emph1} is in union with Tr_{id}, the intentional use of $Tr_{\overline{emph}}$ is in union with Tr_{emph0} and/or Tr_{emph1}.

Thus also the definitions and names of the preceding four transformations can be declared: The transformation given by $Tr = Tr_{id} \cup Tr_{sym}$ transforms any framework into a similar framework, with the same set of arguments and similar but symmetric conflicts. With $Tr_{id} \cup Tr_{emph0}$ we accentuate importance of arguments, with $Tr_{id} \cup Tr_{emph1}$ we accentuate importance of attacks.

Observe that Tr_{emph0}, Tr_{emph1} and $Tr_{\overline{emph}}$ use the same additional arguments, thus e.g. in $Tr_{emph0} \cup Tr_{emph1}$ accentuation of some argument a and accentuation of some attack (b, a) operate on the same additional argument $\bar{a}$.

Translation 3.1.22 ($\mathbb{E}_0^c$: $cf \Rightarrow adm$, $naive \Rightarrow prf$)**.** For an arbitrary argumentation framework F we have that $Tr = Tr_{id} \cup Tr_{sym}$ is a covering modular exact translation for $cf \Rightarrow (cf|adm)$ and $naive \Rightarrow (naive|prf)$.

Proof. We have that $Tr(F)$ is a symmetric framework (compare Remark 2.3.22), thus any conflict-free set is also admissible. For any argumentation framework any admissible set is also conflict-free. Furthermore any conflict-free set of arguments in $Tr(F)$ is also conflict-free in F and vice versa, since neither Tr_{id} nor Tr_{sym} add new conflicts or arguments. Thus the first part ($cf \Rightarrow (cf|adm)$) turns out to be true. If for some argumentation framework conflict-free and admissible sets coincide as a matter of course also maximized conflict-free and maximized admissible sets coincide, hence Tr also proves to be a translation for $naive \Rightarrow (naive|prf)$. □

Lemma 3.1.24 (E^e: $cf \Rightarrow adm$)**.** *There is no embedding weakly exact translation for* $cf \Rightarrow adm$.

Proof. Take into account the argumentation framework $F = (\{a, b\}, \{(a, b)\})$. We have $cf(F) = \{\{a\}, \{b\}, \emptyset\}$. Since b is attacked by a in F and with Definition 2.6.32 (Embedding) in mind for any admissible set E with $b \in E$ we need E to attack a. So either b attacks a and the translation is not embedding or some argument different from b attacks a and the translation is not exact. □

We observe that the problematic extensions for conflict-free and for naive semantics coincide for the argumentation framework from the proof above. Furthermore we observe that for any semantics σ based on admissibility we have $E \in \sigma(F)$ only if $E \in adm(F)$.

[3] Lemma 3.1.45 represents a pathological and at the same time the only known impossibility result for covering translations.

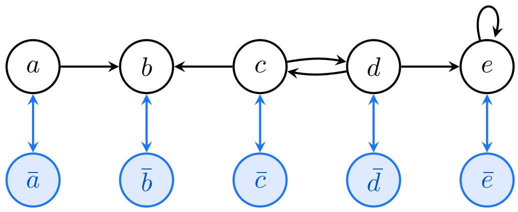


Figure 3.1.28: Translation 3.1.27 ($cf \Rightarrow naive$) as applied to Example 2.1.6.

Corollary 3.1.25 (E^e: $(cf|naive) \Rightarrow (stb|sem|prf|com|adm)$)**.** *There is no embedding weakly exact translation for* $(cf|naive) \Rightarrow (stb|sem|prf|com|adm)$*.*

Lemma 3.1.26 (E: $(cf|com|adm) \Rightarrow (naive|stg|stb|sem|prf)$)**.** *There is no weakly exact translation for* $(cf|com|adm) \Rightarrow (naive|stg|stb|sem|prf)$*.*

This lemma is already part of [39], the idea is that for admissible, complete and conflict-free semantics the selected extensions can be proper subsets of each other for one particular framework whereas this is not possible for stable, preferred, naive, semi-stable and stage semantics.

Translation 3.1.27 ($\mathbb{F}_0^e$: $cf \Rightarrow (naive|stg|stb|sem|prf)$)**.** For an arbitrary argumentation framework F we have that $Tr = Tr_{id} \cup Tr_{empho}$ is an embedding modular faithful translation for $cf \Rightarrow (naive|stg|stb|sem|prf)$.

Proof. For an arbitrary $E \in cf(F)$ we take into account the set $E' = E \cup \{\bar{a} \mid a \in A_F \setminus E\}$. Now E' is conflict-free and for any argument $a \in A_F$ either a or $\bar{a}$ is a member of E'. By definition of Tr_{empho} we have $E'^+ = A_{Tr(F)}$. Thus E' is a stable and therefor also naive, stage, semi-stable and preferred extension.

For $\sigma \in \{naive, stg, stb, sem, prf\}$ we observe that by the embedding property for any $E' \in \sigma(Tr(F))$ and $E = E' \cap A_F$, immediately $E \in cf(F)$. Furthermore by maximality of σ for any $a \in A_F$ and any $E' \in \sigma(Tr(F))$ we have either $a \in E'$ or $\bar{a} \in E'$. Thus from $E_1', E_2' \in \sigma(Tr(F))$ with $E_1' \cap A_F = E_2' \cap A_F$ it follows that already $E_1' = E_2'$, implying $|\sigma(Tr(F))| = |cf(F)|$. □

Remark 3.1.29*.* Observe that despites superficial similarity the task of translating $adm \Rightarrow prf$ is not quite as simple as translating $cf \Rightarrow naive$. By combining other translations anyhow we will present the embedding modular faithful Translation 3.1.85 for $adm \Rightarrow prf$ in Subsection 3.1.6.

Translation 3.1.30 ($\mathbb{E}_0^e$: $adm \Rightarrow com$,$naive \Rightarrow stg$,$prf \Rightarrow sem$)**.** For an arbitrary argumentation framework F we have that $Tr = Tr_{id} \cup Tr_{empho} \cup Tr_{\overline{emph}}$ is an embedding modular exact translation for $adm \Rightarrow (com|adm)$, $naive \Rightarrow (naive|stg)$ and $prf \Rightarrow (sem|prf)$.

Proof. Observe that by definition Tr equals Tr_1 from [39]. A detailed proof of $adm \Rightarrow (com|adm)$ and $prf \Rightarrow (sem|prf)$ is to be found there. We are left with the apparently similar task of showing that Tr is an exact translation for $naive \Rightarrow (naive|stg)$. In other words for any argumentation framework F we have 1. $naive(F) = naive(Tr(F))$ and 2. $naive(Tr(F)) = stg(Tr(F))$. We hereby specify the set of additional arguments $\bar{A} = \{\bar{a} \mid a \in A_F\}$.

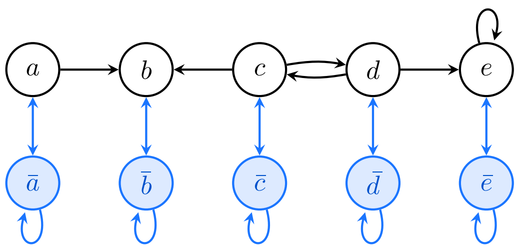


Figure 3.1.31: Translation 3.1.30 ($adm \Rightarrow com$, $prf \Rightarrow sem$, $naive \Rightarrow stg$) as applied to Example 2.1.6.

1. Since neither Tr_{id} nor $Tr_{emph0} \cup Tr_{\overline{emph}}$ add additional attacks between the original arguments and any new argument $\bar{a} \in \bar{A}$ is self-conflicting by definition ($Tr_{\overline{emph}}$) we have that any $E \subseteq A_{Tr(F)}$ is conflict-free in F if and only if it is conflict-free in $Tr(F)$. Thus immediately $naive(F) = naive(Tr(F))$.
2. As pointed out in Lemma 2.3.12 any stage extension is also a naive extension. If $E \in naive(Tr(F))$ then $E^+_{Tr(F)} = E^+_F \cup \{\bar{a} \mid a \in E\}$. Assuming there exists some conflict-free E' such that $E^+_{Tr(F)} \subsetneq E'^+_{Tr(F)}$ we receive $E \subsetneq E'$ since any $\bar{a}$ with $a \in E$ is attacked only by a and $\bar{a}$. Thus with maximality of naive extensions immediately $E' = E$ and therefor also $naive(Tr(F)) \subseteq stg(Tr(F))$. □

Remark 3.1.32. Apparently as far as Tr_{emph0} is concerned for any argumentation framework $F = (A, R)$ with $a \in A$ the attack $(\bar{a}, a)$ is not necessary for Translation 3.1.27 and parts of Translation 3.1.30. But omitting this attack does neither simplify any proof nor would such a translation provide stronger translational properties. We therefor abstain from splitting Tr_{emph0} into two variations with the benefit of keeping the number of introduced transformations a bit lower.

3.1.3 Stage Semantics

As we have seen in Translation 3.1.22 translating conflict-freeness into admissibility is fairly easy, as is naive semantics into preferred semantics. Surprisingly for $stg \Rightarrow sem$ the task turns out to be much more difficult. In the following we present various results in respect of translations from stage semantics.

Lemma 3.1.33 (F_e: $(stg|sem) \Rightarrow prf$, $(stg|sem|prf) \Rightarrow (stb|com|adm)$).

- *There is no efficient weakly faithful translation for* $(stg|sem) \Rightarrow prf$ *unless* $\Sigma_2^P = \text{NP}$.
- *There is no efficient weakly faithful translation for* $(stg|sem|prf) \Rightarrow (stb|com|adm)$ *unless* $\Sigma_2^P = \text{NP}$.

These results are taken from [39] and proofs are to be found there. Both proofs are based on complexities of the various semantics, the first proof makes use of credulous, the second of skeptical acceptance.

We will present the general impossibility of modular exact translations for $stg \Rightarrow sem$ with Lemma 3.1.38. The reason for this impossibility is already utilized in the following (partial) modular exact Translation 3.1.34 for $stg \Rightarrow sem$. We restrict the argumentation frameworks of interest such that all self-attacking arguments are attacked by all arguments in their respective range. Observe that the chosen subclass of argumentation frameworks is maximal with regard to modular exact intertranslatability.

Translation 3.1.34 (($\mathbb{E}_0^c$): $stg \Rightarrow sem$)**.** We investigate any argumentation framework $F = (A, R)$, where from $a \in A$ with $(a, a) \in R$ and $(a, b) \in R$ for some b it follows that also $(b, a) \in R$.[4] For frameworks of this kind the transformation

$$Tr = Tr_{\mathit{id}} \cup Tr_{\mathit{sym}} \cup Tr_{\mathit{emph0}} \cup Tr_{\mathit{emph1}} \cup Tr_{\overline{\mathit{emph}}}$$

is a covering modular exact translation for $stg \Rightarrow (stg|sem)$.

Proof. Observe that this transformation is similar to Tr_2 from [39]. We already know that $Tr_{\mathit{id}} \cup Tr_{\mathit{sym}}$ is an exact translation for $naive \Rightarrow (naive|prf)$. Conflict-freeness is not altered by $Tr_{\mathit{emph0}} \cup Tr_{\mathit{emph1}} \cup Tr_{\overline{\mathit{emph}}}$, as a consequence and in connection with Lemmata 2.3.11 and 2.3.12 ($sem \subseteq prf, stg \subseteq naive$) focus has to be put on range of possible extensions.

This said, apparently as far as $Tr(F)$ is concerned, due to Tr_{sym} stage and semi-stable extensions coincide, thus what remains to show is that for the frameworks of interest we have $E \in stg(F)$ if and only if $E \in stg(Tr(F))$.

$\Longrightarrow$: Postulating $E \in stg(F)$ we take a look at $E^+_{Tr(F)}$. We get

$$E^+_{Tr(F)} = E^+_F \cup \{\bar{a} \mid a \in E^+_F\} \cup \{a \mid a \rightarrowtail^F E\}$$

Assuming there is some $E' \in cf(Tr(F))$ with $E^+_{Tr(F)} \subseteq E'^+_{Tr(F)}$ clearly we also need

$$\{\bar{a} \mid a \in E^+_F\} = \left\{\bar{a} \mid \bar{a} \in E^+_{Tr(F)}\right\} \subseteq \left\{\bar{a} \mid \bar{a} \in E'^+_{Tr(F)}\right\} = \{\bar{a} \mid a \in E'^+_F\}$$

But then since E is a stage extension in F above subset relations become equality relations and E' is another (or the same) stage extension in $Tr(F)$.

$\Longleftarrow$: For $E \in stg(Tr(F))$ we observe that also $E \subseteq A_F$, furthermore $E \in cf(F)$ and even $E \in naive(F)$. For a contradiction we assume $E \notin stg(F)$. Then there is some $E' \in stg(F)$ such that $E^+_F \subsetneq E'^+_F$. We have a look at $A_0 = E^+_{Tr(F)} \setminus E'^+_{Tr(F)}$ Since elements of type $\bar{a} \in E^+_{Tr(F)}$ are predetermined as $\{\bar{a} \mid a \in E^+_F\}$ we have that A_0 consists of elements from A_F only. For the very same reason we need $\emptyset \neq A_0$, otherwise also $E^+_{Tr(F)} \subsetneq E'^+_{Tr(F)}$.

Now for any $a_0 \in A_0$ with $a_0 \not\rightarrowtail^{Tr(F)} a_0$ on the one hand we have that a_0 must be in conflict with E' (in F and $Tr(F)$), otherwise $E' \notin naive(F)$. On the other hand if a_0 is in conflict with E' due to Tr_{sym} also $E' \rightarrowtail^{Tr(F)} a_0$. Thus it follows that a_0 is self-attacking, in symbols $a_0 \rightarrowtail^{Tr(F)} a_0$. But then with $E \rightarrowtail^{Tr(F)} a_0$, due to the very nature of the argumentation frameworks of interest, already $E \rightarrowtail^F a_0$, contradicting $E^+_F \subsetneq E'^+_F$. □

[4] Compare Translation 3.1.42 for a prototype of frameworks that do not satisfy this condition.

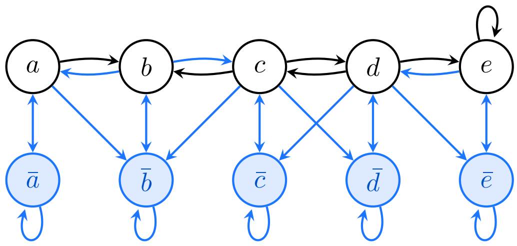


Figure 3.1.35: Translation 3.1.34 ($stg \Rightarrow sem$) as applied to Example 2.1.6

When looking at the previous proof it occurs that the used translation might not work for general argumentation frameworks just for the possible existence of non-attacked by others but attacking and self-attacking arguments. As it turns out this is a first limitation of modular translations.

Lemma 3.1.36 (E^e: $stg \Rightarrow sem$). *There is no embedding weakly exact translation for* $stg \Rightarrow sem$.

Proof. This lemma is already to be found in [39], but the proof touches characteristics of exact translations $stg \Rightarrow sem$ we will soon enough be referring to. Consider the argumentation framework $F = (\{a, b, c\}, \{(a, b), (b, c), (c, c)\})$. Now $stg(F) = \{\{a\}, \{b\}\}$. For any embedding weakly exact translation Tr we observe that if $stg(F) \subseteq sem(Tr(F))$ firstly $(a, b) \in R_{Tr(F)}$ and therefor secondly b has to defend itself against a, in other words also $(b, a) \in R_{Tr(F)}$. □

We observe that in the previous proof as far as semi-stable semantics was concerned we did make use only of the admissibility property. Thus the very same proof immediately also applies to any semantics (on the right side) relying on admissibility.

Corollary 3.1.37 (E^e: $stg \Rightarrow \sigma$). *There is no embedding weakly exact translation for* $stg \Rightarrow (stb|sem|prf|com|adm)$.

Lemma 3.1.38 (E_0: $stg \Rightarrow sem$). *There is no modular weakly exact translation for* $stg \Rightarrow sem$.

Recall that modularity resolves to locality with respect to the argumentation frameworks

$$(\emptyset, \emptyset) \qquad (\{a_1\}, \emptyset) \qquad (\{a_1\}, \{(a_1, a_1)\}) \qquad (\{a_1, a_2\}, \{(a_1, a_2)\})$$

Proof. For a contradiction we assume that there exists some modular weakly exact with remainder set $\mathcal{S}$ translation $Tr : stg \Rightarrow sem$. Then by definition of modularity (Definition 2.6.19) and considering the singular framework $(\{a\}, \emptyset)$ we obtain that for any argumentation framework F and any $a \in A_F$ also $a \in A_{Tr(F)}$. Due to weak exactness furthermore $stg(F) \subseteq sem(Tr(F)) \subseteq stg(F) \cup \mathcal{S}$.

We now consider the frameworks $F_1 = (\{a, b\}, \{(a, b)\})$, $F_2 = (\{b, c\}, \{(c, b), (c, c)\})$ and $F = F_1 \cup F_2$. Due to modularity $Tr(F) = Tr(F_1) \cup Tr(F_2)$. Furthermore $stg(F_1) = \{\{a\}\}$, $stg(F_2) = \{\{b\}\}$ and $stg(F) = \{\{a\}\}$. Due to monotonicity b can not be self-attacking in $Tr(F)$.

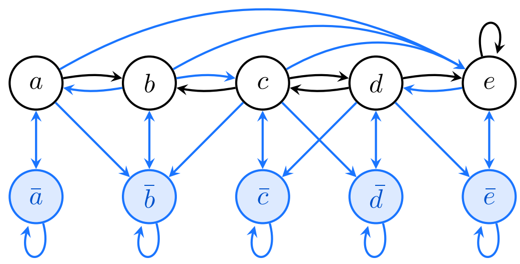


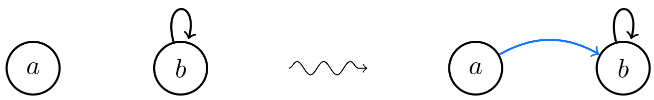
Figure 3.1.41: Translation 3.1.40 ($stg \Rightarrow sem$) as applied to Example 2.1.6.

Since $\{b\} \in sem(Tr(F_2))$ we have that $\{b\}$ is defending itself against all attacks in $Tr(F_2)$. Due to subset isomorphism and modularity the same holds for $\{b\}$ in $Tr(F_1)$. We conclude admissibility of $\{b\}$ in $Tr(F)$. This implies some bidirectional conflict between a and b in $Tr(F_1)$ and thus the same for a and b as well as b and c in $Tr(F)$.

With the previous results and due to modularity and exactness we have $\{a\}, \{b\} \in adm(Tr(F))$, $\{a, b, c\} \subseteq \{b\}^+_{Tr(F)}$ and $c \notin \{a\}^+_{Tr(F)}$. Thus there has to be some $S \in \mathcal{S}$ such that $\{b\}^+_{Tr(F)} \subsetneq S^+_{Tr(F)}$. But then due to modularity already $\{b\}^+_{Tr(F_2)} \subsetneq S^+_{Tr(F_2)}$. □

Remark 3.1.39 ($\mathbb{F}^e_0$: $stg \Rightarrow sem$). For the previous proof we put some more light into problems regarding translations for $stg \Rightarrow sem$. We refer to [39] for a modular embedding faithful translation $Tr_5 : stg \Rightarrow sem$. In the following we present two workarounds for exact translations.

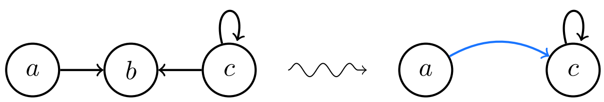
Translation 3.1.40 ($\mathbb{E}^c_2$: $stg \Rightarrow sem$). With respect to $\mathcal{L} = \{(\{a, b\}, \{(b, b)\})\}$ we present a 2-component finitely local transformation Tr':



The transformation $Tr = Tr_{id} \cup Tr_{emph0} \cup Tr_{emph1} \cup Tr_{\overline{emph}} \cup Tr'$ is a covering 2-component finitely local exact translation for $stg \Rightarrow sem$.

Proof. Upon inspection this transformation proves to be identical to Tr_2 from [39], thus a detailed proof is to be found there. Basically with this transformation we resolve the problem presented in Lemma 3.1.38 by putting self-attacking arguments into the range of all arguments. Since with Tr_{emph1} anyway attacks and therefor range are emphasized this tactic appears to work out. □

Translation 3.1.42 ($\mathbb{E}^c_1$: $stg \Rightarrow sem$). With respect to $\mathcal{L} = \{(\{a, b, c\}, \{(a, b), (c, b), (c, c)\})\}$ we present a strictly local transformation Tr':



The transformation $Tr = Tr_{id} \cup Tr_{emph0} \cup Tr_{emph1} \cup Tr_{\overline{emph}} \cup Tr'$ is a covering strictly local exact translation for $stg \Rightarrow sem$.

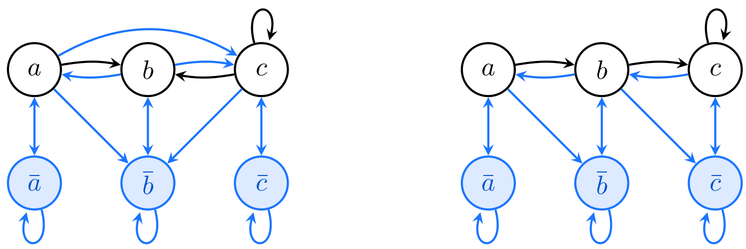


Figure 3.1.43: Translation 3.1.42 ($stg \Rightarrow sem$) as applied to two slightly different argumentation frameworks.

Proof. The proof for Translation 3.1.34 applies mostly. Only this time any self-attacking argument c from F becomes attacked by all arguments a attacking any b for which $c \rightarrowtail b$. So as far as the last few sentences of above proof are concerned we are on the safe side.

What remains to show is that Tr' does not destroy extensions in another way. Firstly $prf(Tr(F))$ is still the same for any argumentation framework F since Tr' produces attacks only on self-attacking arguments.

Secondly we think about F to be the argumentation framework from the definition of Tr', $F \in \mathscr{L}$ and F to be a subframework of some F' to be translated, $F \subseteq F'$. Apparently whether a or b is member of some extension is independent from the attack (a, c) since the range of F' is reproduced with $Tr_{emph0} \circ Tr_{emph1} \circ Tr_{\overline{emph}}$ with arguments of the form $\bar{A}$. □

3.1.4 Stable Semantics

For the semantics we have introduced, stable semantics stands out, as for general frameworks it is possible that no extension exists at all. This implies that only weakly exact respectively weakly faithful translations will be possible for $stb \Rightarrow \sigma$ in general.

Lemma 3.1.44 ($\mathbb{F}$: $stb \Rightarrow \sigma$)**.** *There is no faithful translation for $stb \Rightarrow \sigma$ where σ is non-empty for all argumentation frameworks F ($\sigma(F) \neq \emptyset$).*

For translations $\sigma \Rightarrow stb$ on the other hand we have to deal with the need of relating connected components of argumentation frameworks. Thus in general there will be no covering weakly exact translation $\sigma \Rightarrow stb$. Furthermore all other semantics permit the empty set to be an extension for non-empty argumentation frameworks where $\emptyset$ is a stable extension only for $F = (\emptyset, \emptyset)$.

Lemma 3.1.45 (E^c: $\sigma \Rightarrow stb$)**.** *There is no covering weakly exact translation for $\sigma \Rightarrow stb$ with $\sigma \in \{cf, naive, stg, sem, prf, com, adm, grd\}$.*

Proof. Take into account the argumentation framework $F = (\{a\}, \{(a, a)\})$. Assuming $Tr : \sigma \Rightarrow stb$ is a covering translation then $\emptyset \in stb(Tr(F))$. But this can only be the case for $Tr(F) = (\emptyset, \emptyset)$ and thus Tr can not fulfill the covering property. □

Corollary 3.1.46 (E_7: $\sigma \Rightarrow stb$)**.** *We take into account that any monotone translation has to cover all arguments. Thus there is no monotone weakly exact translation for $\sigma \Rightarrow stb$.*

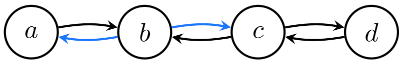


Figure 3.1.48: Translation 3.1.47 ($naive \Rightarrow stb$) as applied to Example 2.1.6.

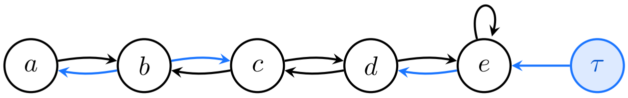


Figure 3.1.50: Translation 3.1.49 ($naive \Rightarrow stb$) as applied to Example 2.1.6.

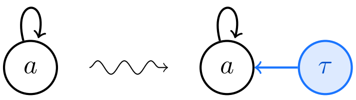
Translation 3.1.47 ($\mathbb{E}_e$: $naive \Rightarrow stb$)**.** For an arbitrary argumentation framework $F = (A, R)$ take into account the transformation $Tr(F) = (A', R')$ with

$$A' = A \setminus \{a \mid (a, a) \in R\} \qquad R' = \{(a, b), (b, a) \mid a, b \in A', (a, b) \in R\}$$

then Tr forms an efficient exact translation for $naive \Rightarrow stb$.

Proof. Firstly we observe that $naive(Tr(F)) = naive(F)$. Since any $a \in A_{Tr(F)}$ is self-defending and not self-attacking immediately also $stb(Tr(F)) = naive(Tr(F))$. □

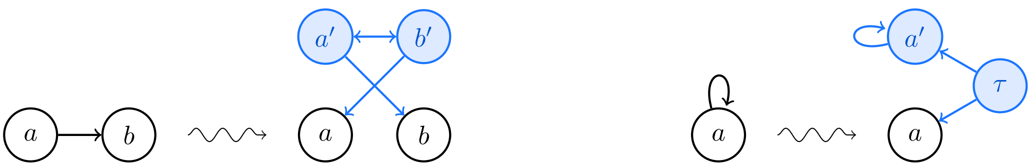
Translation 3.1.49 ($\mathbb{F}_0^c$: $naive \Rightarrow stb$)**.** With respect to $\mathscr{L} = \{(\{a\}, \{(a, a)\})\}$ take into account the modular self-attack-removing transformation Tr':



We claim that the transformation $Tr = Tr_{id} \cup Tr_{sym} \cup Tr'$ is a covering modular faithful translation for $naive \Rightarrow stb$.

Proof. We observe that if there is a stable extension $E \in stb(Tr(F))$ we have $\tau \in E$ and furthermore $E \setminus \{\tau\} \in stb(Tr(F) \setminus \tau^+_{Tr(F)})$. Now the proof for Translation 3.1.47 applies. □

Translation 3.1.51 ($\mathbb{F}_0^c$: $naive \Rightarrow stb$)**.** With respect to $\mathscr{L} = \{(\{a, b\}, \{(a, b)\}), (\{a\}, \{(a, a)\})\}$ we define a modular parallel stabilizing transformation Tr':



We claim that $Tr = Tr_{id} \cup Tr'$ is an embedding modular faithful translation for $naive \Rightarrow stb$.

Proof. We observe that for an arbitrary argumentation framework $F = (A, R)$ the restriction of the translated framework to new arguments $\bar{F} = Tr(F)|_{A_{Tr(F)} \setminus A_F}$ is isomorphic to the by Translation 3.1.49 (hereby referenced as Tr_0) translated framework.

$$\bar{F} \cong Tr_0(F)$$

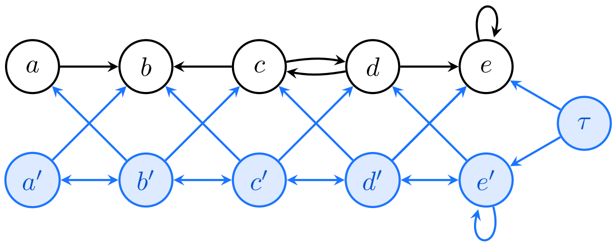


Figure 3.1.52: Translation 3.1.51 ($naive \Rightarrow stb$) as applied to Example 2.1.6.

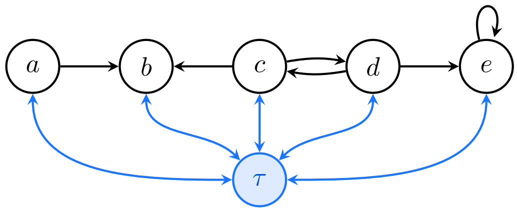
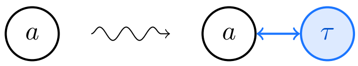


Figure 3.1.55: Translation 3.1.54 ($stb \Rightarrow (stg|sem)$) as applied to Example 2.1.6

Furthermore there are no attacks from A to $A_{\bar{F}}$ in $Tr(F)$, thus any possibly stable extension of $Tr(F)$ is extending some stable extension of $\bar{F}$. Now for $a' \in A_{\bar{F}}$ (with $(a, a) \notin R$) we observe that a' defends a against all attacks, τ attacks all self-attacking arguments and thus Tr is indeed an embedding modular faithful translation for $naive \Rightarrow stb$. □

Transformation 3.1.53 (Stabilizing by Range)**.** With respect to $\mathscr{L} = \{(\{a\}, \emptyset)\}$ we define a modular transformation Tr_{stb0} with the aim of stabilizing by range:



Translation 3.1.54 (E_0^e: $stb \Rightarrow \sigma$, Range)**.** For an arbitrary argumentation framework F we have that the transformation Tr with

$$Tr = Tr_{id} \cup Tr_{stb0}$$

is an embedding modular weakly exact translation for $stb \Rightarrow (stg|stb|sem)$ with remainder set $\{\tau\}$.

Proof. This transformation proves to be equal to Tr_3 from [39]. A detailed proof for the claimed qualities can be found there. Basically the requirements are accomplished by creating an artificial stable extension $\{\tau\}$. Since possibly existing stable extensions survive with Lemma 2.3.14 the desired property is fulfilled. In contrast with [39] we allow the empty argumentation framework and thus additionally need to consider $F_0 = (\emptyset, \emptyset)$. Now $Tr(F_0) = F_0$ and the claimed qualities hold. □

Transformation 3.1.56 (Stabilizing by Admissibility)**.** With the aim of stabilizing by admissibility and with respect to $\mathscr{L} = \{(\{a, b\}, \emptyset), (\{a\}, \emptyset)\}$ we define the 2-component finitely local transformation Tr_{stb1}:

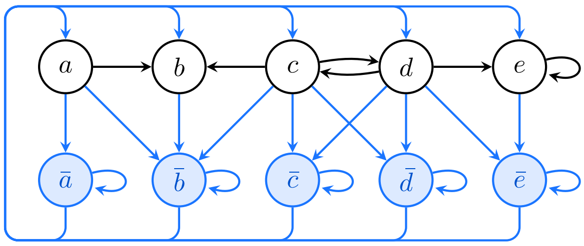
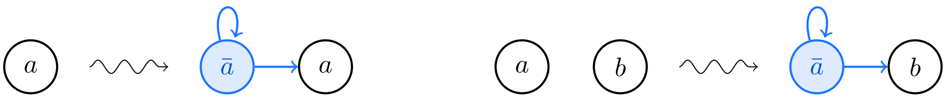


Figure 3.1.58: Translation 3.1.57 ($stb \Rightarrow (sem|prf|com|adm)$) as applied to Example 2.1.6.



Translation 3.1.57 ((E_2^e): $stb \Rightarrow \sigma$, Admissibility)**.** For any non-empty argumentation framework $F \neq (\emptyset, \emptyset)$ we have that the transformation Tr with

$$Tr = Tr_{id} \cup Tr_{emph0} \cup Tr_{emph1} \cup Tr_{\overline{emph}} \cup Tr_{stb1}$$

is an embedding 2-component finitely local weakly exact translation with remainder set $\emptyset$ for $stb \Rightarrow (stb|adm)$.

Proof. This transformation happens to be equal to Tr_4 from [39]. A detailed proof can be found there. Basically the intention of this translation is that any argument from $\bar{A}$ attacks all original arguments, with Tr_{emph0} and Tr_{emph1} any original argument is defended by some extension E if and only if $E_F^+ = A_F$. □

Corollary 3.1.59 ((E_2^e): $stb \Rightarrow (sem|prf|com)$)**.** *The previous translation applies also to translating* $stb \Rightarrow (sem|prf|com)$.

Proof. This follows immediately from set inclusion properties discussed in Chapter 2.3, namely

$$stb \subseteq sem \subseteq prf \subseteq com \subseteq adm.$$

□

In the following we will show that for general argumentation frameworks there is no weakly exact translation for $stb \Rightarrow adm$. We will conclude implications and follow up with a weakly faithful translation for $stb \Rightarrow (stg|stb|sem|prf|com|adm)$.

Lemma 3.1.60 (E: $\sigma \Rightarrow (cf|adm)$)**.** *For any argumentation framework* F *and any semantics* σ *with possibly but not necessarily* $\emptyset \in \sigma(F)$ *there is no weakly exact translation* $\sigma \Rightarrow (cf|adm)$*. This especially applies to semantics* $\sigma \in \{naive, stg, stb, sem, prf, com, grd\}$.

Proof. This result is touched in [39], we as few as broaden the consequences. Any conflict-free or admissible semantics involves the empty set as an extension. So if $\emptyset \notin \sigma(F)$ we have to include $\emptyset$ in the remainder sets. On the other hand for the cases where $\emptyset \in \sigma(F)$ for weakly exact translations we can not include $\emptyset$ in the remainder sets. □

Lemma 3.1.61 ($\mathbb{F}$: $\sigma \Rightarrow (cf|adm)$)**.** *There is no faithful translation* $\sigma \Rightarrow (cf|adm)$ *with*

$$\sigma \in \{naive, stg, stb, sem, prf, com, grd\}.$$

Proof. We observe that for any argumentation framework $\emptyset \in (cf|adm)(F)$, yet there are argumentation frameworks F' such that $\emptyset \notin \sigma(F')$. □

Lemma 3.1.62 (E: $stb \Rightarrow com$)**.** *For possibly empty argumentation frameworks there is no weakly exact translation* $stb \Rightarrow com$*.*

Proof. Observe that for the empty argumentation framework $F_0 = (\emptyset, \emptyset)$ we have the empty set as a stable extension $stb(F_0) = \{\emptyset\}$. Thus the empty set can not be a member of the remainder sets.

Now consider the argumentation framework $F_2 = (\{a, b\}, \{(a, b), (b, a)\})$. We have $stb(F_2) = \{\{a\}, \{b\}\}$ and thus $\emptyset \notin stb(F_2)$. Assuming for a contradiction that there exists some weakly exact translation Tr for $stb \Rightarrow com$ then it follows that $\emptyset \notin com(Tr(F_2))$. If the empty set is not a complete extension however it follows that the grounded extension is not empty either. The grounded extension in turn is a subset of every complete extension and therefor there is some argument a_0 such that for any $E \in com(Tr(F_2))$ we have $a_0 \in E$. Since we need $stb(F_2) \subseteq com(Tr(F_2))$ it follows that Tr can not be weakly exact. □

Considering the simple symmetric argumentation framework F_2 as introduced in the previous proof, with stable, semi-stable, preferred, stage and naive semantics $\sigma(F) = \{\{a\}, \{b\}\}$ it becomes clear that the very same proof for possibly empty argumentation frameworks and weakly exact translations $stb \Rightarrow com$ can be applied also to these semantics. In this context if $\sigma \neq stb$ we have $\sigma(F) \neq \emptyset$ for any argumentation framework, but it still remains to be a computationally hard question whether there is some $E \in \sigma(F)$ such that $E \neq \emptyset$.

Corollary 3.1.63 (E: $(naive|stg|sem|prf) \Rightarrow com$)**.** *There is no weakly exact translation* Tr *for* $(naive|stg|sem|prf) \Rightarrow com$*.*

Lemma 3.1.64 ($\mathbb{F}$: $(naive|stg|stb|sem|prf) \Rightarrow com$)**.** *There is no faithful translation* Tr *for* $(naive|stg|stb|sem|prf) \Rightarrow com$*.*

Proof. Take into account the argumentation framework $F = (\{a, b\}, \{(a, b), (b, a)\})$. Obviously we have $\sigma(F) = \{\{a\}, \{b\}\}$. Thus we need $\{E_1, E_2\} = com(Tr(F))$ with $\{a\} = A \cap E_1$ and $\{b\} = A \cap E_2$. Due to Corollary 2.3.9[5] there has to be some $E \in com(Tr(F))$ such that $E \subseteq E_1 \cap E_2$ and therefor $\emptyset = A \cap E$, a contradiction to $com(Tr(F)) = \{E_1, E_2\}$. □

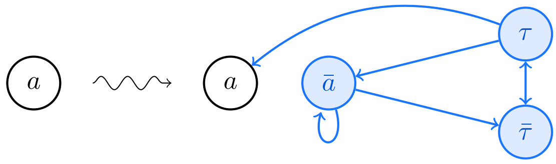
In Translation 3.1.57 we excluded empty argumentation frameworks, for if and only if $F = (\emptyset, \emptyset)$ then $stb(F) = \{\emptyset\}$. Thus we can not use the empty set as a remainder set for any translation $stb \Rightarrow \sigma$ and possibly empty argumentation frameworks. What follows is a workaround making use of faithful intertranslatability (Definition 2.6.9).

[5]The grounded extension is also a complete extension and can be defined as the intersection of all complete extensions.

Transformation 3.1.65 (Faithful Stabilizing)**.** With respect to $\mathscr{L} = \{L_0, L_1\}$ with

$$L_0 = (\emptyset, \emptyset) \qquad\qquad L_1 = (\{a\}, \emptyset)$$

we define a modular transformation Tr_{stb2}, where $Tr_{stb2}(L_0) = (\{\bar{\tau}\}, \emptyset)$, with the aim of stabilizing by range and admissibility:



Translation 3.1.66 (F_0^e: $stb \Rightarrow \sigma$)**.** For an arbitrary argumentation framework F we have that the transformation Tr with

$$Tr = Tr_{id} \cup Tr_{empho} \cup Tr_{emph1} \cup Tr_{\overline{emph}} \cup Tr_{stb2}$$

is an embedding modular weakly faithful translation for $stb \Rightarrow (stg|stb|sem|prf|com|adm)$ with remainder sets $\{\tau\}$ and $\emptyset$.

Proof. As a starter we consider the empty argumentation framework $F_0 = (\emptyset, \emptyset)$. It appears that $Tr(F_0) = (\{\bar{\tau}\}, \emptyset)$, thus the translation fulfills the desired properties for F_0. For non-empty argumentation frameworks also the transformed framework is non-empty due to Tr_{id}, thus for the rest of this proof we consider only non-empty argumentation frameworks.

Now observe that τ is attacking any from τ different argument. Thus $\{\tau\}$ is a stable extension in $Tr(F)$ and immediately

$$E \in stb(F) \iff E \cup \{\bar{\tau}\} \in stb(Tr(F)) \text{ for } E \neq \{\tau\}$$

Of course any stable extension is also admissible, what remains to show is that for any $E \in adm(Tr(F))$ with $E \neq \emptyset$ already $E \in stb(Tr(F))$.

In the following we try to think about some non-empty admissible set $E \subseteq Tr(F)$ different from $\{\tau\}$. Since τ is attacked only by $\bar{\tau}$ we observe that $\bar{\tau}$ needs to be a member of E. Since $\bar{\tau}$ is attacked by all $\bar{a}$ for $a \in A_F$, all $\bar{a}$ need to be attacked in return.

So far we have $\{\tau, \bar{\tau}\} \cup \{\bar{a} \mid a \in A_F\} \subseteq E^+_{Tr(F)}$. But then also $A_F \subseteq E^+_{Tr(F)}$ since $\bar{A}$ reflects the range of arguments from A_F due to Tr_{empho}, Tr_{emph1} and $Tr_{\overline{emph}}$. So obviously $A_{Tr(F)} = E^+_{Tr(F)}$ and thus E is a stable extension in $Tr(F)$ and $E \cap A_F$ is a stable extension in F. □

Lemma 3.1.68 (E_6: $stb \Rightarrow (prf|com|adm)$)**.** *There is no* 1*-component local weakly exact translation for* $stb \Rightarrow \sigma$ *with* $\sigma \in \{prf, com, adm\}$*.*

Proof. Take into account the argumentation framework $F = (\{a, b\}, \{(b, b)\})$. Now for any 1-component local translation Tr by definition we can split the act of translating into translating of two distinct argumentation frameworks $F_a = (\{a\}, \emptyset)$ and $F_b = (\{b\}, \{(b, b)\})$. Obviously F does not have a stable extension and therefor we expect $Tr(F)$ not to have any σ-extension.

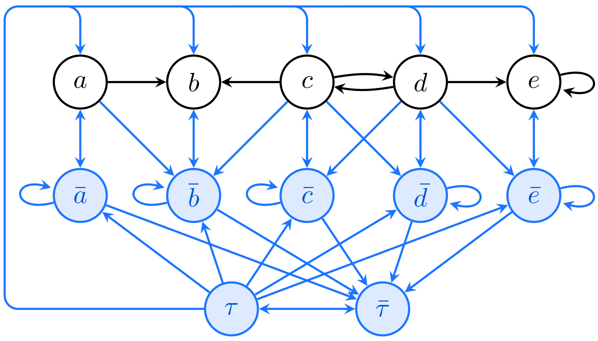


Figure 3.1.67: Translation 3.1.66 ($stb \Rightarrow \sigma$) as applied to Example 2.1.6.

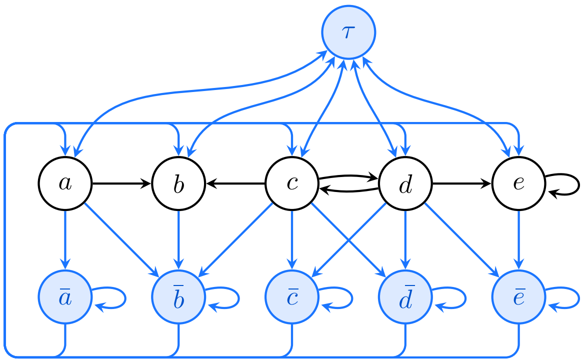


Figure 3.1.70: Translation 3.1.69 ($stb \Rightarrow prf$) as applied to Example 2.1.6.

We have $\{a\} \in stb(F_a)$ and thus $\{a\} \in adm(Tr(F_a))$. Furthermore a can not be member of any remainder set. By definition of 1-component locality a can be attacked in $Tr(F)$ by some argument x only if a is already attacked by x in $Tr(F_a)$. Thus $\{a\}$ is admissible also in $Tr(F)$ yielding a contradiction. □

The question arises whether for possibly empty argumentation framework there is a weakly exact translation from stable to preferred semantics. We respond to this question with an actual translation.

Translation 3.1.69 (E_2^e: $stb \Rightarrow prf$)**.** For an arbitrary argumentation framework F we have that the transformation Tr with

$$Tr = Tr_{id} \cup Tr_{emph0} \cup Tr_{emph1} \cup Tr_{\overline{emph}} \cup Tr_{stb0} \cup Tr_{stb1}$$

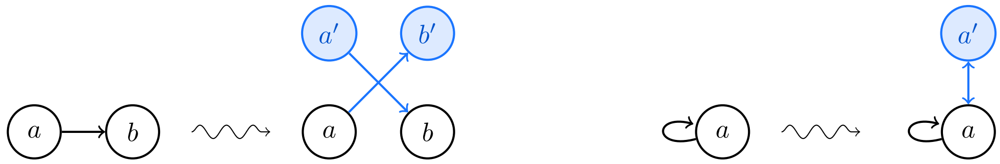
is an embedding 2-component local weakly exact translation for $stb \Rightarrow prf$ with remainder set $\{\tau\}$.

Proof. When looking at $prf(Tr(F))$ we observe that $\{\tau\} \in prf(Tr(F))$. Consider some preferred extension $E \in prf(Tr(F))$ with $E \neq \{\tau\}$ then, since E attacks τ, the proof for Translation 3.1.57 applies. □

3.1.5 Complete Semantics

Prior results were mostly very straightforward. In the following we will present a translation which is not that simple in motivation, yet manages to translate complete semantics faithfully to stage, stable, semi-stable and preferred semantics. We observe that since $com(F) \neq \emptyset$ for any argumentation framework F it suffices to show applicability for stable and preferred semantics.

Transformation 3.1.71 (Parallelism)**.** With respect to $\mathscr{L} = \{(\{a,b\}, \{(a,b)\}), (\{a\}, \{(a,a)\})\}$ we define a modular parallelizing transformation Tr_{par}:



Translation 3.1.72 ($\mathbb{F}_0^e$: $com \Rightarrow (stg|stb|sem|prf)$)**.** For an arbitrary argumentation framework F we have that the transformation Tr with

$$Tr = Tr_{id} \cup Tr_{par}$$

is an embedding modular faithful translation for $com \Rightarrow (stg|stb|sem|prf)$.

Remark 3.1.73. This translation serves similar purpose as the modular faithful embedding Translation Tr_7 from [39], however it is more simple in definition and additionally translates $com \Rightarrow prf$.

Proof. We take an argumentation framework F as given and investigate $Tr(F)$. For empty argumentation frameworks the assumption proves right, thus further on we focus on non-empty frameworks. Observe that for any conflict-free set $E' \subseteq A_{Tr(F)}$, $E = E' \cap A_F$ we have that E' attacks a' in $Tr(F)$ if and only if E attacks a in F and therefor E' defends some argument a (and a') in $Tr(F)$ if and only if E defends a in F.

- $E \in com(F) \implies E \cup \{a' \mid E \not\rightarrowtail a\} = E' \in stb(Tr(F))$: There are no attacks between any arguments of the form a' and thus E' is conflict-free. For $a \in E$ we have $a' \in E'$ and thus $\{a, a' \mid a \in E_F^+\} \subseteq E'^{+}_{Tr(F)}$. Any argument a' such that $a \in A_F \setminus E_F^+$ is not attacked by E and thus member of E'. Due to definition of completeness any argument $a \in A_F \setminus E_F^+$ is attacked by some $b \in A_F \setminus E_F^+$, $b \rightarrowtail a$ and thus $b' \rightarrowtail a$. It follows that $E'^{+}_{Tr(F)} = A_{Tr(F)}$ and thus E' is a stable extension of $Tr(F)$.
- $E' \in prf(Tr(F)) \implies E' \cap A_F = E \in com(F)$: Due to the embedding property E is conflict-free in F since it is conflict-free in $Tr(F)$. Furthermore by construction the relationship between E and E' is unique. Since $a \in E$ is defended by E' in $Tr(F)$ if and only if it is defended by E in F we conclude admissibility and with maximality of the preferred extension E' also the identity $E = \mathcal{F}_F(E)$ and thus completeness of E.

We close this proof with a reference to the subset relations of semantics as discussed in Section 2.3 ($stb \subseteq sem \subseteq prf$) and Lemma 2.3.14 (existence of a stable extension), implying that proposed properties hold for Tr. □

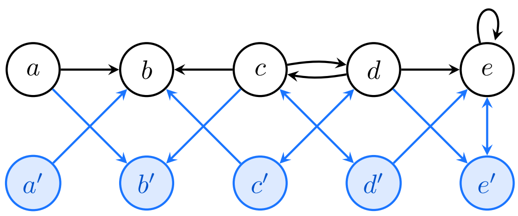


Figure 3.1.74: Translation 3.1.72 ($com \Rightarrow \sigma$) as applied to Example 2.1.6.

3.1.6 Concatenations

To spare time and energy concatenation of translations proves to be a useful tool. Intuitively with $Tr := Tr_2 \circ Tr_1$ for any argumentation framework F we refer to $Tr(F) = Tr_2(Tr_1(F))$. If translation Tr_1 happens to be exact for $\sigma_0 \Rightarrow \sigma_1$ and Tr_2 happens to be exact for $\sigma_1 \Rightarrow \sigma_2$ then obviously also Tr turns out to be exact for $\sigma_0 \Rightarrow \sigma_1$. Transitiveness for other attributes however turns out to be not that reliable.

Definition 3.1.75 (Concatenation)**.** For arbitrary domains D_0, D_1, D_2 and arbitrary mappings $f_1 : D_0 \to D_1$ and $f_2 : D_1 \to D_2$ the concatenation of f_1 and f_2, $f = f_1 \circ f_2$ is defined by

$$f : D_0 \to D_2, f(d) = f_2(f_1(d)) \textit{ for all } d \in D_0.$$

Definition 3.1.76 (Transitivity)**.** For arbitrary mappings $f_1, f_2 \in F$ an attribute α evaluating any $f \in F$ is called transitive if from $\alpha(f_1)$ and $\alpha(f_2)$ it follows that also $\alpha(f_2 \circ f_1)$.

Lemma 3.1.77. *For any argumentation framework and any translation the attributes of being exact, weakly exact, faithful, modular, monotone, finitely local, efficient, embedding and covering are transitive.*

Remark 3.1.78*.* Except for efficiency these proofs are very straightforward and therefor left as an exercise to the interested reader. As far as efficiency is concerned we only hint that the idea is to compute any data from f_1 needed for f_2 newly every time it is needed.

Example 3.1.79*.* Consider the modular weakly exact Translation 3.1.54 as $Tr_1 : stb \Rightarrow stg$ and a hypothetical strictly local faithful translation $Tr_2 : stg \Rightarrow \sigma$ with respect to $\mathscr{L}$. We assume

$$(a, b) \in R_{Tr_2(L)} \qquad \textit{for } L = (\{a, b, c\}, \{(a, c), (b, c)\})$$

We investigate the argumentation framework $F = (\{a, b\}, \emptyset)$ and the strictly local transformation $Tr(F) = (A', R')$ with:

$$Tr = Tr_2 \circ Tr_1$$

Now $(a, b) \in R'$ and thus although both transformations, Tr_1 and Tr_2 are strictly local there might be no finite k such that Tr is k-component local.

We assume Tr_2 to be introducing a new argument $\bar{a}$ for each argument a and for any argumentation framework F we assume:

$$E \in stg(F) \quad \Longleftrightarrow \quad (E \cup \{\bar{a} \mid a \in A_F \setminus E\}) \in \sigma(Tr_2(F))$$

Now with predictability (Definition 2.6.2) Tr can not be weakly exact any more since Tr_2 translates remainder set $\{\tau\}$ into a proper class of sets of arguments consisting mostly of arguments of the form $\bar{A}$.

Lemma 3.1.80. *In general the attributes of being weakly faithful, strictly local, local with diameter or k-component local are not transitive.*

Remark 3.1.81. Example 3.1.79 points out what kind of problems occur. We observe that for 1-component local translations transitivity depends on appearance of framework-independent arguments in the translation.

Lemma 3.1.82. *If for some translations Tr_1 and Tr_2 for any distinct argumentation frameworks F_1 and F_2 ($A_{F_1} \cap A_{F_2} = \emptyset$) we have*

$$A_{Tr_1(F_1)} \cap A_{Tr_1(F_2)} = \emptyset \quad \text{and} \quad A_{Tr_2(F_1)} \cap A_{Tr_2(F_2)} = \emptyset$$

then the attributes of being 1-component local, strictly local and local with finite diameter are transitive, respectively hold for $Tr = Tr_1 \circ Tr_2$ if they hold for Tr_1 and Tr_2 separately.

Lemma 3.1.83. *If for some translations Tr_1, Tr_2 we have Tr_1 is faithful and Tr_2 is weakly faithful, then $Tr_2 \circ Tr_1$ is weakly faithful.*

Proof. We take into account some argumentation framework F, $Tr_1(F) = F_1$ and $Tr_2(F_1) = F_2$. Furthermore Tr_1 is a faithful translation for $\sigma \Rightarrow \sigma_1$ and Tr_2 is a weakly faithful translation for $\sigma_1 \Rightarrow \sigma_2$ with remainder sets $\mathcal{S}$. We receive:

$$\sigma_1(F_1) = A_{F_1} \cap (\sigma_2(F_2) \setminus \mathcal{S}) \qquad \sigma(F) = A_F \cap \sigma_1(F_1)$$

Apparently $Tr = Tr_2 \circ Tr_1$ is a weakly faithful translation for $\sigma \Rightarrow \sigma_2$. ☐

Corollary 3.1.84. *We present the following translations without attaching proofs, for above Lemmata and referenced translations immediately imply the result.*

Translation 3.1.85 ($\mathbb{F}_0^e$: $adm \Rightarrow (stg|stb|sem|prf)$)**.** By referring to Translation 3.1.30 as $Tr_{adm,com}$ and to Translation 3.1.72 as $Tr_{com,\sigma}$ the transformation Tr given by

$$Tr = Tr_{com,\sigma} \circ Tr_{adm,com}$$

is an embedding modular faithful translation for $adm \Rightarrow (stg|stb|sem|prf)$.

Remark 3.1.86. Observe that in [39] with Tr_6 a strictly local embedding faithful translation for $adm \Rightarrow (stg|stb|sem)$ is introduced.

Translation 3.1.87 (F_0^e: $com \Rightarrow adm$)**.** Referring to Translation 3.1.72 as $Tr_{com,stb}$ and to Translation 3.1.66 as $Tr_{stb,adm}$ the transformation Tr given by

$$Tr = Tr_{stb,adm} \circ Tr_{com,stb}$$

is an embedding modular weakly faithful translation for $com \Rightarrow adm$.

Translation 3.1.88 ($\mathbb{E}_0^c$: $cf \Rightarrow com, naive \Rightarrow sem$)**.** Referring to Translation 3.1.22 as $Tr_{cf,adm}$ and to Translation 3.1.30 as $Tr_{adm,com}$ the transformation Tr given by

$$Tr = Tr_{adm,com} \circ Tr_{cf,adm}$$

is a covering modular exact translation for $cf \Rightarrow com$ and $naive \Rightarrow sem$.

Translation 3.1.89 (F_0^e: $naive \Rightarrow (com|adm)$)**.** Referring to Translation 3.1.51 as $Tr_{naive,stb}$ and to Translation 3.1.66 as $Tr_{stb,adm}$ the transformation given by

$$Tr = Tr_{stb,adm} \circ Tr_{naive,stb}$$

serves as an embedding modular weakly faithful translation for $naive \Rightarrow (com|adm)$. Observe that for a concrete application, arguments have to be renamed before concatenation. This is due to both translations referring to some argument τ in different context.

3.2 Advanced Relations between Conditional Semantics

As presented in Definition 2.3.24 we intuitively partition the introduced semantics into unconditional and conditional semantics, the later covering naive, stage, semi-stable and preferred semantics. In this section we focus on relations between conditional semantics.

Before presenting impossibility results, we recall already known characteristics. Naive and stage semantics focus on conflict-free sets of arguments, while semi-stable and preferred semantics require admissible sets. Naive and preferred semantics maximize extensions, while stage and semi-stable semantics maximize the range of extensions.

Table 3.2.1 summarizes relations between conditional semantics. We start this section with the assumption of the following results:

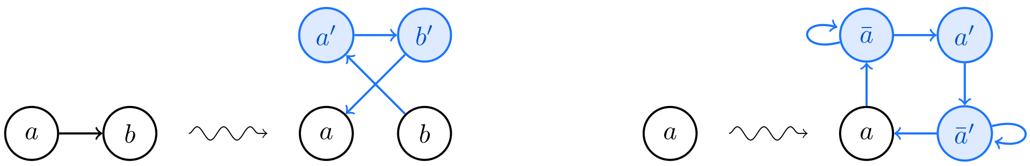
- Lemma 3.1.33: impossibility of efficient faithful translations for $(stg|sem) \Rightarrow prf$,
- Translation 3.1.22: covering modular exact for $naive \Rightarrow prf$,
- Translation 3.1.30: embedding modular exact for $prf \Rightarrow sem$ and $naive \Rightarrow stg$,
- Translation 3.1.88: covering modular exact for $naive \Rightarrow sem$,
- Subsection 3.1.3 conquers the task of translating $stg \Rightarrow sem$,
- Translation Tr_5 from [39]: embedding modular faithful for $stg \Rightarrow sem$,
- Lemmata 3.1.24, 3.1.36: impossibility of embedding exact translations $(naive|stg) \Rightarrow (sem|prf)$.

$\Rightarrow$	*naive*	*stg*	*sem*	*prf*
naive	$\mathbb{E}^e_0$: 3.1.12	$\mathbb{E}^e_0$: 3.1.30	$\mathbb{E}^c_0$: 3.1.88 $\mathbb{F}^e_0$: 3.2.2 E^e: 3.1.25	$\mathbb{E}^c_0$: 3.1.22 $\mathbb{F}^e_0$: 3.2.2 E^e: 3.1.25
stg	F: 3.2.4	$\mathbb{E}^e_0$: 3.1.12	$\mathbb{E}^c_1$: 3.1.42 $\mathbb{F}^e_0$: 3.1.39 ($\mathbb{E}^c_0$: 3.1.34) E_0: 3.1.38 E^e: 3.1.36	$\mathbb{F}^e_6$: 3.4.5 $\mathbb{E}^c_6$: 3.4.20 F_e: 3.1.33 E^e: 3.1.37
sem	F: 3.2.4	$\mathbb{F}^e_6$: 3.4.5 ($\mathbb{F}^e_2$: 3.2.27) E: 3.2.6 F_4: 3.2.24	$\mathbb{E}^e_0$: 3.1.12	$\mathbb{E}^e_6$: 3.4.17 F_e: 3.1.33
prf	F: 3.2.4	$\mathbb{F}^e_6$: 3.4.5 ($\mathbb{F}^e_2$: 3.2.27) E: 3.2.8 F_4: 3.2.24	$\mathbb{E}^e_0$: 3.1.30	$\mathbb{E}^e_0$: 3.1.12

Table 3.2.1: A summary of results regarding intertranslatability between conditional semantics with references inside this thesis.

As shown in Lemma 3.1.24 there is no embedding exact translation for *naive* $\Rightarrow$ *prf*. We attach this impossibility with an embedding modular faithful translation for *naive* $\Rightarrow$ *prf*.

Translation 3.2.2 **(**$\mathbb{F}^e_0$: $cf \Rightarrow (com|adm)$,$naive \Rightarrow (sem|prf)$**).** With respect to $\mathscr{L} = \{L_1, L_2\}$ where $L_1 = (\{a, b\}, \{(a, b)\})$ and $L_2 = (\{a\}, \emptyset)$, inspired by Tr_{par} we define a parallel inverting transformation Tr_{inv}:



We claim that the transformation $Tr = Tr_{id} \cup Tr_{inv}$ is an embedding modular faithful translation for $cf \Rightarrow (com|adm)$ and $naive \Rightarrow (sem|prf)$.

Proof. Consider some argumentation framework $F = (A, R)$. As a result of $Tr(L_2)$ we observe that for any $E \in adm(Tr(F))$ and any $a \in A$ we have $a \in E$ if and only if $a' \in E$. Furthermore, as a result of $Tr(L_1)$ we observe that for any $a \in A$ we have a defending a' and vice versa. It follows that $E \subseteq A$ is conflict-free in F if and only if $E \cup \{a' \mid a \in E\}$ is admissible in $Tr(F)$, immediately the same holds for $naive(F)$ and $prf(Tr(F))$.

Now for $cf \Rightarrow com$ observe that a (respectively a') is defended by E in $Tr(F)$ if and only if a' (respectively a) is a member of E. Thus any admissible set in $Tr(F)$ is already complete. For $naive \Rightarrow sem$ observe that for any $E, E' \in prf(Tr(F))$ with $E \neq E'$ (due to $Tr(L_2)$) immediately also $E^+_{Tr(F)} \neq E'^+_{Tr(F)}$. Thus any preferred extension in $Tr(F)$ is already a semi-stable extension. □

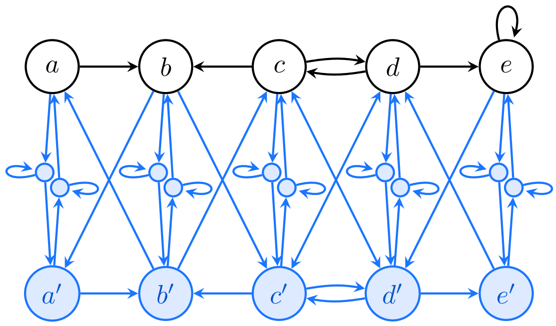


Figure 3.2.3: Translation 3.2.2 as applied to Example 2.1.6.

3.2.1 Conflict-Free Inaccessibilities

In this subsection we will present two counter-examples regarding translations approaching conflict-free, naive and stage semantics and their implications. It appears that conflict-free and naive semantics are very limited in their ability of describing extensions even for faithful approaches, stage semantics reveals difficulties for exact extensional matchings.

Theorem 3.2.4 (F: $(stg|stb|sem|prf|com|adm) \Rightarrow (cf|naive)$)**.** *There is no weakly faithful translation for* $\sigma \Rightarrow \sigma'$ *where* $\sigma \in \{stg, stb, sem, prf, com, adm\}$ *and* $\sigma' \in \{cf, naive\}$*.*

Proof. For a contradiction we assume that such a translation Tr exists. Consider the argumentation framework $F = (A, R)$ as shown in Figure 3.2.5 with

$$\begin{aligned} A &= \{a_1, a_2, a_3, b_1, b_2, b_3\} \\ R &= \{(a_1, a_2), (a_2, a_1), (a_2, a_3), (a_3, a_2), (a_1, a_3), (a_3, a_1)\} \\ &\quad \cup \{(a_1, b_1), (a_2, b_2), (a_3, b_3)\} \end{aligned}$$

Now observe that for

$$E_1 = \{a_1, b_2, b_3\} \qquad E_2 = \{b_1, a_2, b_3\} \qquad E_3 = \{b_1, b_2, a_3\}$$
$$B = \{b_1, b_2, b_3\}$$

we have that E_1, E_2, E_3 are σ-extensions while $B \notin \sigma(F)$. So for any E_i there has to be some $E_i' \in \sigma'(Tr(F))$ such that $E_i \subseteq E_i'$. Thus immediately[6] $B \in cf(Tr(F))$, since pairwise conflict-freeness of the b_i is granted by E_1', E_2' and E_3'. For any conflict-free set B in any argumentation framework F' there has to be some extension $B' \in naive(F')$ such that $B \subseteq B'$, yielding a contradiction also for naive semantics. □

What follows is a presentation of the impossibility of exact translations $(sem|prf) \Rightarrow stg$. The related counter example is already part of [39], nonetheless it has its origins in the work for this thesis.

[6]Recall that we require the remainder sets to form a proper joint set $\mathcal{S}$ and thus disallow arguments of original frameworks to be members of remainder sets.

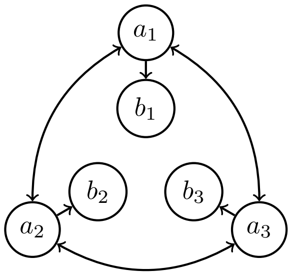


Figure 3.2.5: Argumentation framework serving as a counter example for Theorem 3.2.4.

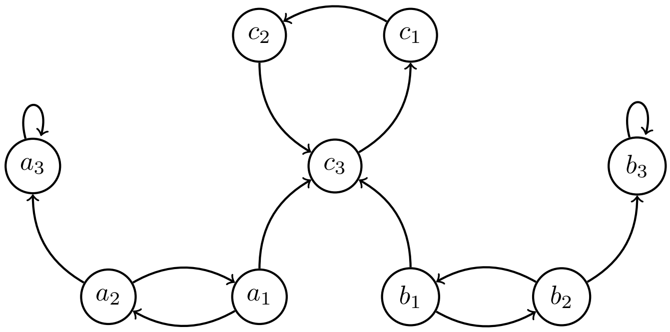


Figure 3.2.7: The counter example used for exact translations in Theorem 3.2.6.

Theorem 3.2.6 (E: $sem \Rightarrow stg$)**.** *There is no weakly exact translation for $sem \Rightarrow stg$.*

Proof. Take into account the argumentation framework $F = (A, R)$ from Figure 3.2.7:

$$A = \{a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3\} \qquad R = \{(a_1, a_2), (a_2, a_1), (a_2, a_3), (a_3, a_3)\} \cup \{(b_1, b_2), (b_2, b_1), (b_2, b_3), (b_3, b_3)\} \cup \{(a_1, c_3), (b_1, c_3), (c_1, c_2), (c_2, c_3), (c_3, c_1)\}$$

Consider the sets $E_1 = \{a_2, b_2\}$, $E_2 = \{a_1, b_2, c_1\}$ and $E_3 = \{a_2, b_1, c_1\}$. We have

$$sem(F) = \{E_1, E_2, E_3\}.$$

We assume for a contradiction that there is a (weakly) exact translation $Tr : sem \Rightarrow stg$. According to our assumption and regardless of whether looking at a weakly exact or at an exact translation, $\{E_1, E_2, E_3\} \subseteq stg(Tr(F))$. Since $c_1 \in E_2$ there is no conflict between c_1 and b_2 in $Tr(F)$. Since $c_1 \in E_3$ there is no conflict between c_1 and a_2 in $Tr(F)$. It follows that the set $B = \{a_2, b_2, c_1\}$ is conflict-free in $Tr(F)$ and not only $E_1 \subsetneq B$ but since c_1 can not be in the range of E_1 in $Tr(F)$ even $E_1{}^+_{Tr(F)} \subsetneq B^+_{Tr(F)}$ yielding a contradiction with our assumption. □

We observe that as far as the counter example from the preceding proof is concerned the crucial part was the difference between conflict-freeness and admissibility. Thus a generalization for a wider field of semantics can easily be obtained.

Corollary 3.2.8 (E: $(sem|prf|com|adm) \Rightarrow (cf|naive|stg)$). *There is no weakly exact translation for* $\sigma \Rightarrow (cf|naive|stg)$ *where* $\sigma \in \{sem, prf, com, adm\}$.

Proof. We take a look at the argumentation framework F from the previous proof again. Due to subset relations of semantics incorporating admissibility the sets E_1, E_2 and E_3 are extensions also for σ. The set $B = \{a_2, b_2, c_1\}$ is not admissible in F and therefor neither a complete nor a preferred extension. Consequently B happens to contradict the possibility of weakly exact translations $\sigma \Rightarrow (cf|naive|stg)$ with just the same arguments. □

Corollary 3.2.9 (E: $(sem|prf) \Rightarrow stb$). *There is no weakly exact translation* $(sem|prf) \Rightarrow stb$.

Proof. Considering Translation 3.1.54 (weakly exact for $stb \Rightarrow stg$) as Tr_0 we take into account the transformation built by concatenating with the hypothetical weakly exact translation Tr for $(sem|prf) \Rightarrow stb$. Now $Tr_0 \circ Tr$ forms a previously shown to be impossible weakly exact translation for $(sem|prf) \Rightarrow stg$. □

3.2.2 Empowering Finite Diameter Locality

Thanks to the previous counter-examples we are left with investigating possible faithful translations $(sem|prf) \Rightarrow stg$. We start by presenting two example frameworks indicating that modular faithful translations are not possible and follow up with upgrading this example to showing that already local with finite diameter translations are not possible.

Example 3.2.12 ($\mathbb{F}_0$: $sem \Rightarrow stg$). Investigate the circular frameworks $\mathcal{O}_3$ and $\mathcal{O}_4$ also shown in Figure 3.2.10:

$$\mathcal{O}_3 = (\{a_1, a_2, a_3\}, \{(a_1, a_2), (a_2, a_3), (a_3, a_1)\})$$
$$\mathcal{O}_4 = (\{a_1, a_2, a_3, a_4\}, \{(a_1, a_2), (a_2, a_3), (a_3, a_4), (a_4, a_1)\})$$

Assume there is a modular faithful translation $Tr : sem \Rightarrow stg$. Then $stg(Tr(\mathcal{O}_3)) = \{E\}$ and $stg(Tr(\mathcal{O}_4)) = \{E_1, E_2\}$ with $E \cap A_{\mathcal{O}_3} = \emptyset$, $E_1 \cap A_{\mathcal{O}_4} = \{a_1, a_3\}$ and $E_2 \cap A_{\mathcal{O}_4} = \{a_2, a_4\}$. Now the idea is that for predictable translations $E \cap Tr((\{a_i, a_j\}, \{(a_i, a_j)\}))$ must be strictly isomorphic for all attacks $(a_i, a_j) \in \mathcal{O}_3$. But then we can move E in an isomorphic extending way to $Tr(\mathcal{O}_4)$ and receive an unwanted extension.

The intuition behind this example can be generalized to local with finite diameter translations. We recall that if a translation is local with respect to $\mathscr{L}$ for any argumentation framework F we have

$$Tr(F) = \bigcup_{L \in \mathscr{L}, L \cong F' \subseteq F} Tr(F').$$

If a translation is local with finite diameter d, it is local with respect to $\mathscr{L}$ and for each $L \in \mathscr{L}$ we have that $dia(L) \leq d$.

For the definition of diameter we refer to Definition 2.1.28, for the corresponding definitions and implications of locality we refer to Definitions 2.6.20, 2.6.23 and 2.6.25 and Lemma 2.6.24.

We also recall Definition 2.1.18 (Isomorphism). Between isomorphic frameworks $F \cong F'$ the isomorphism φ respectively $\varphi_{F,F'}$ is defined by $\varphi_{F,F'}(F) = F'$.

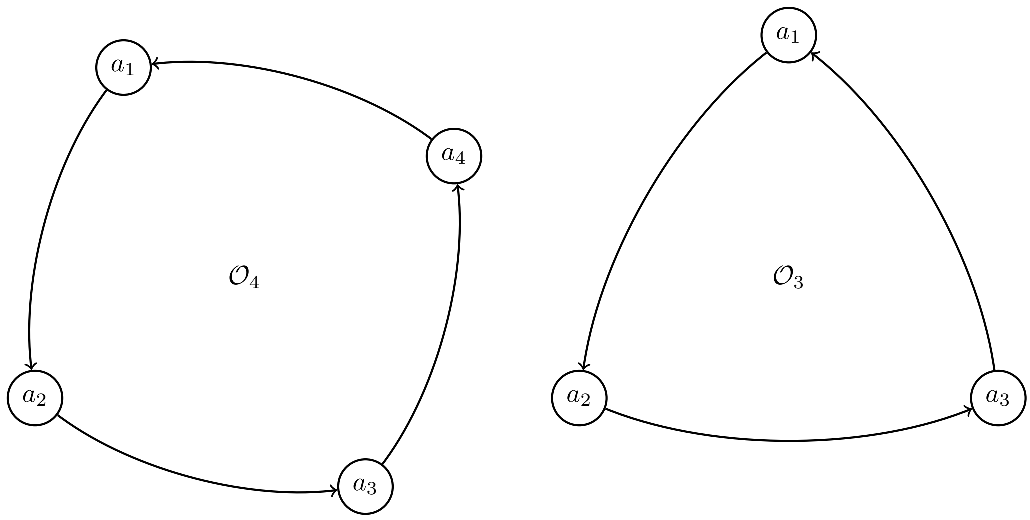


Figure 3.2.10: Circles of Example 3.2.12

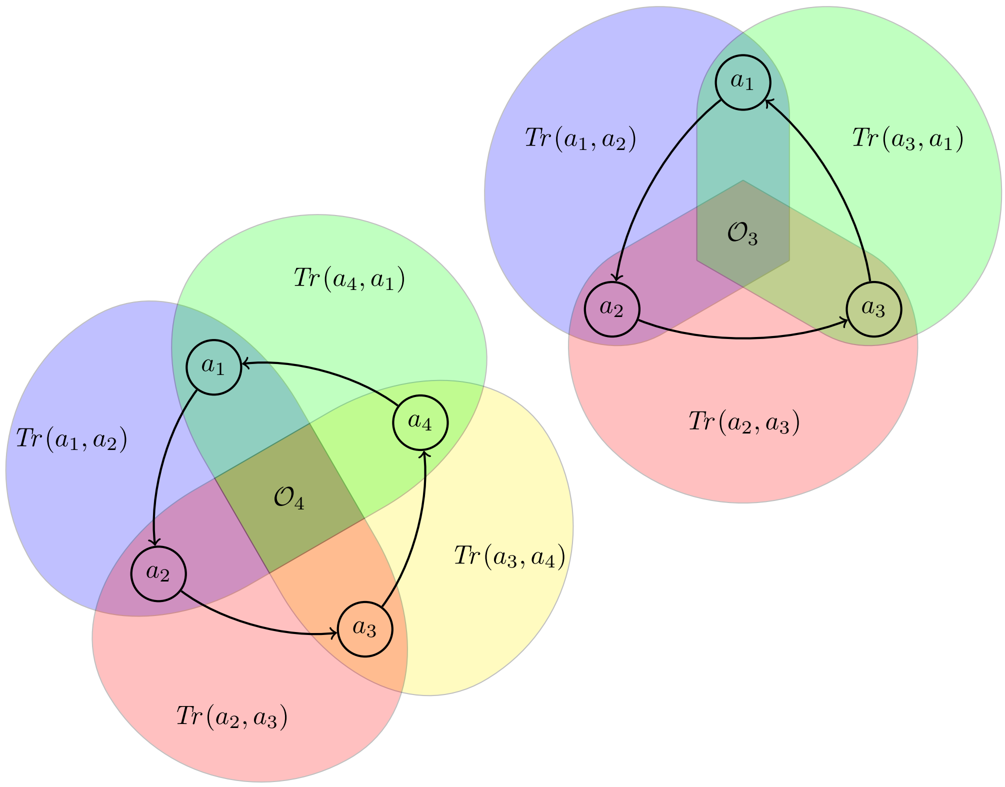


Figure 3.2.11: Circles with indication of modular translation from Example 3.2.12

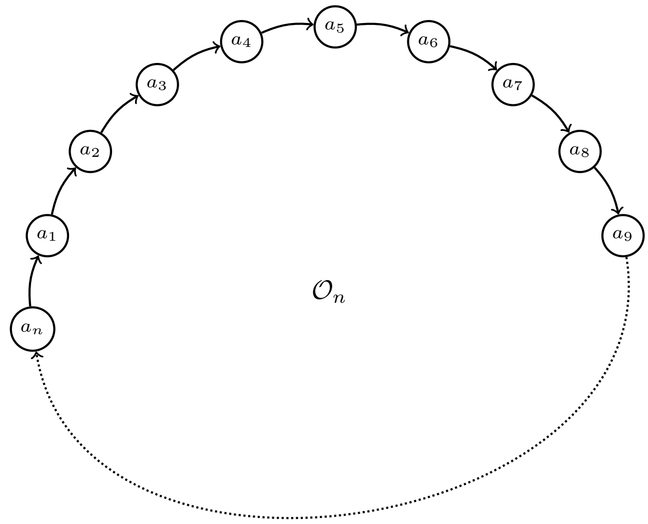


Figure 3.2.13: A circular argumentation framework $\mathcal{O}_n$.

Definition 3.2.14 (Circular and Linear Frameworks)**.** We define the standardized circular framework of size n, $\mathcal{O}_n$ as shown in Figure 3.2.13 and the standardized linear framework of length n, $\mathcal{I}_n$ by

$$\mathcal{O}_n = (\{a_1, a_2 \dots a_n\}, \{(a_1, a_2), (a_2, a_3) \dots (a_n - 1, a_n), (a_n, a_1)\})$$
$$\mathcal{I}_n = (\{a_1, a_2 \dots a_n\}, \{(a_1, a_2), (a_2, a_3) \dots (a_n - 1, a_n)\})$$

Example 3.2.15. For any natural number n we have $sem(\mathcal{O}_n) = prf(\mathcal{O}_n)$. For odd numbers $n = 2k+1$ we have $|sem(\mathcal{O}_{2k+1})| = 1$ and $sem(\mathcal{O}_{2k+1}) = prf(\mathcal{O}_{2k+1}) = \{\emptyset\}$, for even numbers $n = 2k$ we have $|sem(\mathcal{O}_{2k})| = 2$ and

$$sem(\mathcal{O}_{2k}) = prf(\mathcal{O}_{2k}) = \{\{a_1, a_3 \dots a_{2k-1}\}, \{a_2, a_4 \dots a_{2k}\}\}.$$

Example 3.2.16. Observe that for any natural number n we have $dia(\mathcal{O}_n) = \lfloor \frac{n}{2} \rfloor$. In other words for even respectively odd n

$$dia(\mathcal{O}_{2k}) = k = dia(\mathcal{O}_{2k+1}).$$

Lemma 3.2.17. *For any natural number n we look at some 1-component argumentation framework $F \neq \mathcal{O}_n$ being a true subframework of $\mathcal{O}_n$: $F \subseteq \mathcal{O}_n$, $dia(F) < \infty$ and $F \neq \mathcal{O}_n$. Then F is isomorphic to $\mathcal{I}_m$ for some natural number $m \leq n$. Furthermore for any $m \leq n$ we have $\mathcal{I}_m \subseteq \mathcal{I}_n$.*

$$F \subseteq \mathcal{O}_n, F \neq \mathcal{O}_n, dia(F) < \infty \implies \exists (m \leq n) : F \cong \mathcal{I}_m \subseteq \mathcal{I}_n$$

Theorem 3.2.18 ($\mathbb{F}_4$: $(sem|prf) \Rightarrow stg$)**.** *There is no local with finite diameter faithful translation for* $(sem|prf) \Rightarrow stg$*.*

Remark 3.2.19. We will show this theorem for argumentation frameworks $\mathcal{O}_n$ and left hand semantics sem only. Of course if a translation is impossible for specialized frameworks it is just as much impossible for general frameworks. As touched in Example 3.2.15 concerning frameworks of the form $\mathcal{O}_n$ to show the impossibility of translations $prf \Rightarrow \sigma$ it suffices to show the same result for $sem \Rightarrow \sigma$.

Furthermore by Lemma 3.2.17 and predictability (Definition 2.6.2) for Tr being local with respect to $\mathscr{L}$ and $n > 2d + 1$ without loss of generality we can use $\mathscr{L} = \{\mathcal{I}_{d'} | d' \leq d + 1\}$ and even $\mathscr{L} = \{\mathcal{I}_{d+1}\}$. Thus we will show that there is no local with respect to $\{\mathcal{I}_{d+1}\}$ faithful translation $sem \Rightarrow stg$ for argumentation frameworks of the form $\mathcal{O}_n$.

Proof. Assume for a contradiction that such a translation Tr happens to exist. Then for any natural number $n > 2d + 1$ we have $|sem(\mathcal{O}_n)| = |stg(Tr(\mathcal{O}_n))|$ and

$$sem(\mathcal{O}_n) = \{E \cap A_{\mathcal{O}_n} \mid E \in stg(Tr(\mathcal{O}_n))\}.$$

We now restrict the realm of observation to frameworks $\mathcal{O}_n$ and $\mathcal{O}_{2n}$ and a transformation Tr which claims to be a local with respect to $\mathscr{L} = \{\mathcal{I}_{d+1}\}$ faithful translation for $sem \Rightarrow stg$ where $n = 2d + 3$. Thus we have the following:

$$Tr(\mathcal{O}_n) = \bigcup_{\mathcal{I}_{d+1} \cong F' \subseteq \mathcal{O}_n} Tr(F') \qquad\qquad Tr(\mathcal{O}_{2n}) = \bigcup_{\mathcal{I}_{d+1} \cong F' \subseteq \mathcal{O}_{2n}} Tr(F')$$

Since n is odd, as mentioned in Example 3.2.15, there has to be a unique extension $E \in stg(Tr(\mathcal{O}_n))$. We recall the proper definition of translations (Remark 2.6.4) with whom every argument $a \in A_{Tr(F)} \setminus A_F$ can be defined by some $a_i^{F'}$ with $F' \subseteq F$. Due to predictability and the very restricted frameworks we are working with, for each $F' \cong \mathcal{I}_{d+1}$ with $F' \subseteq \mathcal{O}_n$ or $F' \subseteq \mathcal{O}_{2n}$ this implies one unique isomorphism

$$\varphi_{Tr(F')} : Tr(\mathcal{I}_{d+1}) \to Tr(F').$$

We observe that for any two argumentation frameworks $F, F' \subseteq \mathcal{O}_n$ with $F \cong F' \cong \mathcal{I}_{d+1}$ for any $a \in A_{Tr(\mathcal{I}_{d+1})}$ we have:

$$\varphi_{Tr(F)}(a) \in (A_{Tr(F)} \cap E) \qquad \iff \qquad \varphi_{Tr(F')}(a) \in (A_{Tr(F')} \cap E)$$

Otherwise due to symmetry reasons the extension $E \in stg(Tr(\mathcal{O}_n))$ would not be unique in $Tr(\mathcal{O}_n)$.

We now construct a stage extension E' for $\mathcal{O}_{2n}$ such that $E' \cap A_{\mathcal{O}_{2n}} = \emptyset$. For this we observe $\mathcal{I}_{d+1} \subseteq \mathcal{O}_n$ and obviously $\mathcal{I}_{d+1} \cong \mathcal{I}_{d+1}$ and thus we get:

$$E = \left\{\varphi_{Tr(F)}(a) \mid \mathcal{I}_{d+1} \cong F \subseteq \mathcal{O}_n, a \in (E \cap A_{Tr(\mathcal{I}_{d+1})})\right\}$$
$$E' = \left\{\varphi_{Tr(F)}(a) \mid \mathcal{I}_{d+1} \cong F \subseteq \mathcal{O}_{2n}, a \in (E \cap A_{Tr(\mathcal{I}_{d+1})})\right\}$$

Obviously $E' \cap A_{\mathcal{O}_{2n}} = \emptyset$ and due to locality of Tr and uniqueness of E we have that E' is also conflict-free in $Tr(\mathcal{O}_{2n})$. We take a look at the range of E' now. Since E is the only stage extension of $Tr(\mathcal{O}_n)$ and due to locality of Tr for each $b \in A_{Tr(\mathcal{O}_{2n})} \setminus E'^{+}_{Tr(\mathcal{O}_{2n})}$ we retrieve that

- b is self-attacking in $Tr(\mathcal{O}_{2n})$ and
- each c attacking b is also self-attacking in $Tr(\mathcal{O}_{2n})$.

In $Tr(\mathcal{O}_{2n})$ there might be stage extensions different from E', but for sure $E' \in stg(Tr(\mathcal{O}_{2n}))$, a contradiction.

□

Remark 3.2.20. Observe that as far as this proof is concerned for minimality reasons we could also look at $\mathcal{O}_{2d+2}$ and $\mathcal{O}_{2d+3}$. It does not make any substantial difference but we believe that $\mathcal{O}_n$ and $\mathcal{O}_{2n}$ are more easy to read.

Remark 3.2.21. Observe that the gap between impossibility and intertranslatability results for $sem \Rightarrow stg$ is tight. If we want to enable the correct stage extensions for $\mathcal{O}_{2n}$ we could for instance take Translation 3.1.54 ($stb \Rightarrow stg$), with the problem of $\emptyset = stg(Tr(\mathcal{O}_n))$, as used in the proof of Theorem 3.2.24. We could also take Translation 3.1.72 ($com \Rightarrow stg$) where the problem of an unwanted stage extension for $Tr(\mathcal{O}_{2n})$ occurs, as used in the proof of Theorem 3.2.18.

If we try to augment Theorem 3.2.18 to cover also weakly faithful translations, especially the last step of the preceding proof turns out to be harder. For weakly faithful translations we have that $stg(Tr(\mathcal{O}_n))$ is not necessarily unique and thus conditions change. Anyhow most of the used steps are still adaptable. We present some results for local and weakly faithful translations first.

Lemma 3.2.22. *If a translation Tr is weakly faithful with remainder sets $\mathcal{S}'$ (with possibly $|\mathcal{S}'| = \infty$) and finitely local with respect to $\mathscr{L}$ then there is some $\mathcal{S} \subseteq \mathcal{S}'$ with $|\mathcal{S}| < \infty$ such that Tr is weakly faithful with remainder sets $\mathcal{S}$.*

Proof. Take into account the set $E^{\cup}$ of all arguments possibly about to appear in remainder sets:

$$E^{\cup} = \bigcup \mathcal{S}'$$

Due to predictability (Definition 2.6.2) and finite locality (Definition 2.6.25) we have that all arguments of interest in $E^{\cup}$ are member already of the from $\mathscr{L}$ translated argumentation frameworks $Tr(L)$ for $L \in \mathscr{L}$. We therefor shrink $E^{\cup}$ to $E^{\cap}$ with:

$$E^{\cap} = E^{\cup} \cap \bigcup_{L \in \mathscr{L}} A_{Tr(L)}$$

Now since $\mathscr{L}$ is finite and we only deal with finite argumentation frameworks we have $|E^{\cap}| < \infty$. As touched before only remainder sets $E \subseteq E^{\cap}$ are matter of interest, since any $s \in E^{\cup} \setminus E^{\cap}$ can not be reached with a predictable finitely local translation. It follows that Tr is weakly faithful with finitely many remainder sets $S \in \mathcal{S}$ where $\mathcal{S} = \{E \in \mathcal{S}' \mid E \subseteq E^{\cap}\}$.

□

We merely underline the important facts of this lemma with an additional corollary.

Corollary 3.2.23. *For any finitely local with respect to $\mathscr{L}$ weakly faithful translation Tr with remainder sets $\mathcal{S}$ without loss of generality we have that $\mathcal{S}$ is finite and each $E \in \mathcal{S}$ is finite and for each $E \in \mathcal{S}$ we have:*

$$E \subseteq \bigcup_{L \in \mathscr{L}} (A_{Tr(L)} \setminus A_L)$$

Theorem 3.2.24 (F_4: $(sem|prf) \Rightarrow stg$)**.** *There is no local with finite diameter weakly faithful translation for $(sem|prf) \Rightarrow stg$.*

Remark 3.2.25. For this proof we follow for great parts the proof of Theorem 3.2.18, knowledge of prior proofs is assumed in the following.

Proof. Again we can restrict the proof to argumentation frameworks $\mathcal{O}_n$ and $\mathcal{O}_{2n}$ and a weakly faithful with remainder sets $\mathcal{S}$ translation $Tr : sem \Rightarrow stg$ which is local with respect to $\mathscr{L} = \mathcal{I}_{d+1}$ with $n = 2d+3$. Observe that due to the structure of $\mathscr{L}$ for the cases of interest this translation reduces to a finitely local translation and thus Lemma 3.2.22 (respectively Corollary 3.2.23) can be applied.

This time there is one unique $E \in stg(Tr(\mathcal{O}_n))$ such that $E \notin \mathcal{S}$. Again we can construct E' for $Tr(\mathcal{O}_{2n})$:

$$E' = \left\{ \varphi_{Tr(F)}(a) \mid \mathcal{I}_{d+1} \cong F \subseteq \mathcal{O}_{2n}, a \in E \cap A_{Tr(\mathcal{I}_{d+1})} \right\}$$

Now due to uniqueness of E in $Tr(\mathcal{O}_n)$ and locality of Tr we have that E' is conflict-free in $Tr(\mathcal{O}_{2n})$. As pointed out in Corollary 3.2.23, E' can not be a remainder set without being a remainder set in $Tr(\mathcal{O}_n)$ (and even in $Tr(\mathcal{I}_{d+1})$). Furthermore $E \cap \mathcal{O}_n = \emptyset$ and per definition of E' we have $E' \cap \mathcal{O}_{2n} = \emptyset$. Now for Tr being the translation of desire we need $E' \notin stg(Tr(\mathcal{O}_{2n}))$. Thus there is some $T' \in stg(Tr(\mathcal{O}_{2n}))$ such that $E'^{+}_{Tr(\mathcal{O}_{2n})} \subsetneq T'^{+}_{Tr(\mathcal{O}_{2n})}$.

Assuming that T' is a remainder set, as pointed out in Corollary 3.2.23 we have $T' \subseteq A_{Tr(\mathcal{I}_{d+1})}$, then T' is conflict-free also in $Tr(\mathcal{O}_n)$ and $E^{+}_{Tr(\mathcal{O}_n)} \subsetneq T'^{+}_{Tr(\mathcal{O}_n)}$, a contradiction with E being a stage extension of $Tr(\mathcal{O}_n)$.

It follows that T' is no remainder set, thus $(T' \cap A_{\mathcal{O}_{2n}}) \in sem(\mathcal{O}_{2n})$, T' is one of the two "normal" stage extensions of $Tr(\mathcal{O}_{2n})$ and in relation with $sem(\mathcal{O}_{2n})$. We now construct T_{d+1} and E_{d+1}, restrictions of the respective range to $\mathcal{I}_{d+1}$:

$$T_{d+1} = T'^{+}_{Tr(\mathcal{O}_{2n})} \cap A_{Tr(\mathcal{I}_{d+1})} \qquad E_{d+1} = E'^{+}_{Tr(\mathcal{O}_{2n})} \cap A_{Tr(\mathcal{I}_{d+1})} = E^{+}_{Tr(\mathcal{O}_n)} \cap A_{Tr(\mathcal{I}_{d+1})}$$

Now for symmetry reasons, locality and predictability we have

$$E_{d+1} \subsetneq T_{d+1}.$$

By construction of $n = 2d+3$ there is some $T \in cf(Tr(\mathcal{O}_n))$ with $T \cap A_{Tr(\mathcal{I}_{d+1})} = T' \cap A_{Tr(\mathcal{I}_{d+1})}$ and $T_{d+1} \subseteq T^{+}_{Tr(\mathcal{O}_n)}$. Observe that we are not yet finished for we might not be able to construct a stage extension for $Tr(\mathcal{O}_n)$ out of T. Without loss of generality we have $a_1 \in T$,

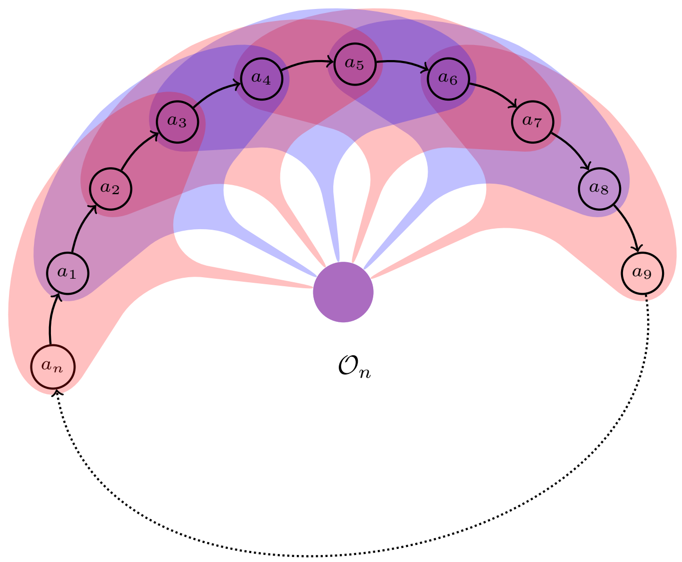


Figure 3.2.26: Illustration of a local with diameter 3 translation on circular framework $\mathcal{O}_n$.

thus T is not allowed to be a stage extension of $Tr(\mathcal{O}_n)$. But then there has to be some stage extension $R \in stg(Tr(\mathcal{O}_n))$ with $T_{d+1} \subseteq T^+_{Tr(\mathcal{O}_n)} \subsetneq R^+_{Tr(\mathcal{O}_n)}$, immediately

$$E_{d+1}{}^+_{Tr(\mathcal{O}_n)} \subsetneq R^+_{Tr(\mathcal{O}_n)}.$$

Since R has to be in the remainder sets and as pointed out in Corollary 3.2.23 and for symmetry reasons $E^+_{Tr(\mathcal{O}_n)} \subsetneq R^+_{Tr(\mathcal{O}_n)}$, thus E is no stage extension of $Tr(\mathcal{O}_n)$ □

Let us take a look at semantics which allow non-admissible extensions, i.e. conflict-free, naive and stage semantics. As pointed out in Theorem 3.2.4 conflict-free and naive semantics are highly inaccessible. In contrast to this result stage semantics appears to be more flexible in translational terms. As far as translations $(sem|prf) \Rightarrow stg$ are concerned this subsection however makes an interesting point.

Previously presented translations intuitively operated and were motivated in a local way. This subsection, culminating in Theorem 3.2.24, essentially shows that this approach will not work for $(sem|prf) \Rightarrow stg$. In other words finite diameter weakly faithful translations are not possible; investigation of a limited number of connected argumentation frameworks will not result in a general translation.

With this we leave the domain of impossibility results for translations $(sem|prf) \Rightarrow stg$ and head over to more speculative relations and enlightenments for possibility results. The following Subsection gives a partial finitely local translation for circles and sheds light on implications of possible translations $sem \Rightarrow stg$.

3.2.3 Further Relations and Partial Translations

In this subsection we will give hints on implications of and possible ways for translating $(sem|prf) \Rightarrow stg$. As a first result we point out that an inefficient embedding 1-component local faithful translation happens to exist (Translation 3.4.5). In fact the gap between above impossibility results and this translation seems to be as small as the gap between 1-component locality and finite-diameter locality. In the following however we will observe that the technique used above to show impossibility of local with finite diameter translations will not work for k-component finite locality since the frameworks $\mathcal{O}_n$ can be transformed correctly with a 2-component local translation. Then we will reflect on similarities and differences of semi-stable and stage extensions. To end this section we will take a short look at implications of existence of an efficient translation for $(sem|prf) \Rightarrow stg$.

Example 3.2.27 ($(\mathbb{F}_2^c)$: $(sem|prf) \Rightarrow stg$). For argumentation frameworks F with $F \cong \mathcal{O}_n$ for some n there is a 2-component finitely local faithful translation for $(sem|prf) \Rightarrow stg$. We present a graphical representation in Figure 3.2.28.

We take into account the set $\mathscr{L} = \{L_1, L_2\}$ and the transformation Tr' with:

$$L_1 = (\{a,b\}, \{(a,b)\}), \qquad Tr'(L_1) = (\{a_a, b_a, a_b, b\}, \{(a_a, b_a), (a_b, b)\}),$$
$$L_2 = (\{a,b,c\}, \{(b,c)\}), \qquad Tr'(L_2) = (\{b_a, c_a\}, \{(b_a, c_a)\}).$$

For argumentation frameworks F with $F \cong \mathcal{O}_n$ for some natural number n we have that

$$Tr = Tr_{id} \cup Tr'$$

is a 2-component finitely local faithful translation for $(sem|prf) \Rightarrow stg$.

In fact Tr creates copies F_a of the original framework F for each single argument a. The only difference between these copies and the original framework is that for any $(b,a) \in R_F$ we add (b_a, a) and not (b_a, a_a). Thus a_a is not attacked at all in F_a.

By looking at Figure 3.2.28 it becomes obvious that for circular frameworks $Tr(\mathcal{O}_n)$ adds a stable extension. If n is odd then any argument $a \in A_{\mathcal{O}_n}$ can be attacked by the grounded extension of $(\mathcal{O}_n)_a$. If n is even the grounded extension of $(\mathcal{O}_n)_a$ defends a as far as $(\mathcal{O}_n)_a$ is concerned, thus we can choose between the two stable extensions of $\mathcal{O}_n$.

Example 3.2.29 (Differences of Semi-Stable and Stage Extensions). As already touched at the end of Section 2.3 semi-stable and stage semantics can be completely different for some argumentation frameworks. In Figure 3.2.30 we again present the graphical representation of $F = (A, R)$ with:

$$A = \{a,b,c,d\}, \qquad R = \{(a,b),(b,a),(d,d),(d,b),(b,c),(c,c)\}$$

We have $prf(F) = sem(F) = \{\{a\}\}$ while $stg(F) = \{\{b\}\}$. Thus there are argumentation frameworks such that $\emptyset \notin (sem(F) \cup stg(F))$ but still for any two extensions $E_1 \in sem(F)$, $E_2 \in stg(F)$ we have $E_1 \cap E_2 = \emptyset$, contradicting the somewhat intuitive (yet wrong) hypothesis of any semi-stable extension being extended by some stage extension.

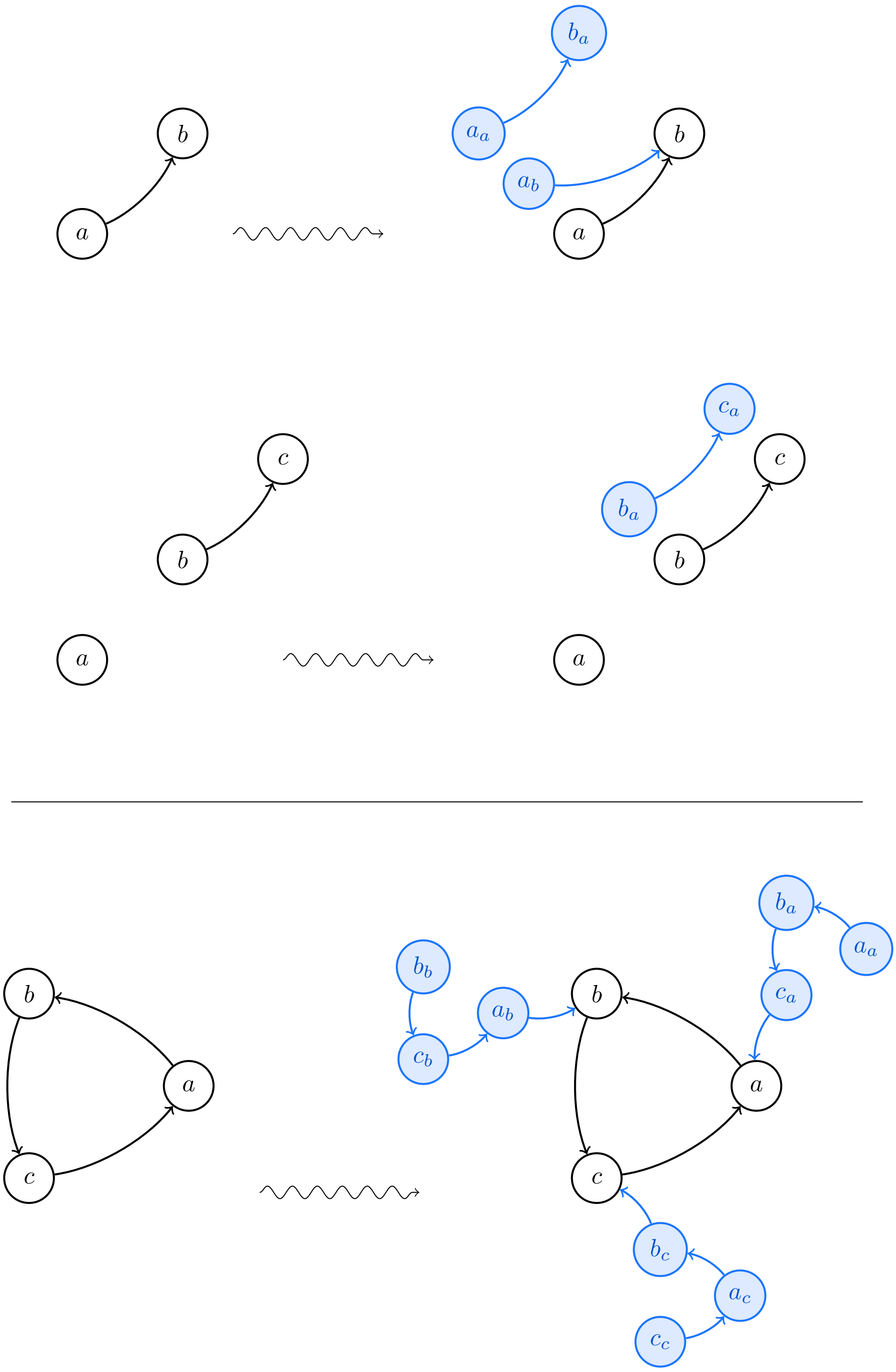
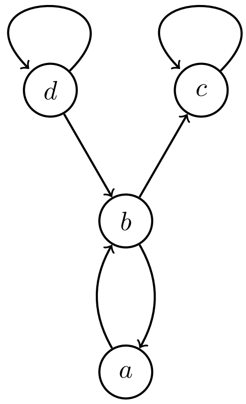


Figure 3.2.28: A transformation showing that odd length cycles can be identified by a finitely local faithful transformation for $(sem|prf) \Rightarrow stg$ as mentioned in Example 3.2.27.



$stg(F) = \{\{b\}\}$

$sem(F) = \{\{a\}\}$

Figure 3.2.30: An illustration of differences between semi-stable and stage semantics.

We recall that as far as the definitions of semi-stable (Definition 2.2.36) and stage (Definition 2.2.35) semantics are concerned the only difference is the replacement of admissibility in semi-stable extensions by conflict-freeness in stage extensions. Both semantics choose those extensions which are maximal with respect to range. We recall that admissible sets by definition are conflict-free and that there provably exists an efficient exact translation $cf \Rightarrow adm$ (Translation 3.1.22) and an efficient exact translation $stg \Rightarrow sem$ (Translation 3.1.42) as well as an inefficient faithful translation $sem \Rightarrow stg$ (Translation 3.4.5). We have proven that there is no exact (Theorem 3.2.6) and no local with finite diameter weakly faithful (Theorem 3.2.24) translation $sem \Rightarrow stg$. We know that semi-stable and stage extensions coincide for symmetric frameworks (Remark 2.3.22) as well as acyclic frameworks (Remark 2.3.21). As pointed out in Example 3.2.29 there are frameworks F such that $\emptyset \notin (sem(F) \cup stg(F))$ but for any $E_1 \in sem(F)$, $E_2 \in stg(F)$ we have $E_1 \cap E_2 = \emptyset$.

Nonetheless one might still wonder whether some efficient faithful translation $Tr : sem \Rightarrow stg$ exists and if for some argumentation framework F this possibly existing translation results in $sem(Tr(F)) = stg(Tr(F))$. For the remaining of this section we focus on implications of this potential equality. As a first step we observe that for any efficient weakly faithful translation $Tr : (sem|prf) \Rightarrow stg$ we have $stb(Tr(F)) \neq stg(Tr(F))$ for infinitely many efficiently not distinguishable argumentation frameworks F unless $\Sigma_2^P = \text{NP}$ (compare Lemma 3.1.33).

Lemma 3.2.31 ($stg = sem$). *If for some argumentation framework F we have $stg(F) = sem(F)$ then for any $E \in sem(F)$ and any $a \notin E^+$ we have $(a, a) \in R_F$.*

Proof. Consider F with $stg(F) = sem(F)$, $E \in sem(F)$ and $a \notin E^+$. We have that E does not attack a since a is not in the range of E. Then a does not attack E, otherwise E would not be admissible. Since $E^+ \subsetneq E^+ \cup \{a\}$ we need $E \cup \{a\}$ to possess some conflict. Now E is conflict-free, E is not attacked by a and E is not attacking a and therefor a itself needs to possess a conflict and thus we need a to be self-attacking. □

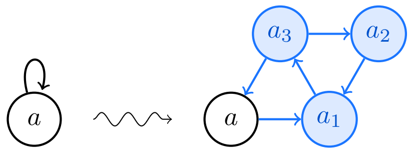


Figure 3.2.34: An illustration of the transformation for removal of self-attacks for semi-stable semantics as used in the proof of Corollary 3.2.33.

We consider argumentation frameworks which do not contain any self-attacks. Applied to these frameworks the previous lemma tells us something about stable semantics too.

Corollary 3.2.32 ($stg = sem$ and stb)**.** *Consider some self-attack-free argumentation framework $F = (A, R)$ (there is no $a \in A$ such that $(a, a) \in R$). If semi-stable and stage semantics coincide ($sem(F) = stg(F)$) then also stable semantics produces the same extensions ($stb(F) = sem(F) = stg(F)$).*

Proof. Observe that $stg(F) \neq \{\emptyset\}$ as soon as at least one argument is not self-attacking. Thus with Lemma 3.2.31 we have that for any $E \in sem(F)$ it follows that $E^+ = A_F$. □

We recall that as of Lemma 3.1.33 there is no efficient weakly faithful translation $sem \Rightarrow stb$ unless $\Sigma_2^P = \text{NP}$. Now if we take into account weakly faithful translations $sem \Rightarrow stg$ and suppose that $Tr(F)$ equalizes semi-stable and stage extensions then Lemma 3.2.31 tells us one more thing.

Corollary 3.2.33 ($stg = sem$ for $Tr(F)$)**.** *Assuming $\Sigma_2^P \neq \text{NP}$ and for some efficient weakly faithful translation $Tr : sem \Rightarrow stg$ we have $sem(Tr(F)) = stg(Tr(F))$ for any argumentation framework F then $Tr(F)$ is in general not self-attack-free, in other words there is some $a \in A_{Tr(F)}$ with $(a, a) \in R_{Tr(F)}$. This especially affects argumentation frameworks F which are self-attack-free before translating.*

Proof. Any argumentation framework F containing self-attacking arguments is as far as semi-stable semantics is concerned not distinguishable from some out of F efficiently constructible argumentation framework F'. For arguments $a \in A_F$ with $(a, a) \in R_F$ we simply need to remove (a, a) and include three new arguments a_1, a_2 and a_3 and five new attacks (a, a_1), (a_2, a_1), (a_1, a_3), (a_3, a) and (a_3, a_2). A graphical representation of this transformation is shown in Figure 3.2.34. Now a (and a_2) are in conflict with a_1 which is the only argument defending a (and a_2) against a_3, thus there is no admissible set containing any of a and a_2. In similar ways there is no admissible set containing a_1 or a_3. It follows that F and F' have the same admissible and thus semi-stable extensions. □

Although the complexity tables are similar, an efficient translation $(sem|prf) \Rightarrow stg$ has not been found yet. One might wonder if there is a reasoning problem one can think of that makes such a translation impossible, or if there is a very tricky way to efficiently faithfully translate frameworks. However the results of this chapter give a glimpse of what surely is impossible, namely exact and local with finite diameter faithful translations.

3.3 Efficient Moves on the Grounded Extension

As far as grounded semantics is concerned we have to deal with some very specific problems. First of all for any argumentation framework there is exactly one grounded extension. Thus for all of the other semantics σ introduced in this work there will be no useful translation $\sigma \Rightarrow grd$. In this chapter we put focus on translations $grd \Rightarrow \sigma$. For a warming up we begin with impossibility results from [39] and append an impossibility result for efficient translations $grd \Rightarrow naive$.

Dvořák and Woltran presented an embedding efficient faithful translation (Tr_8 in [39]) for $grd \Rightarrow (stg|stb|sem|prf|com|grd)$. Interestingly enough this translation is not local, anyhow by renumbering the arguments of the translated framework it is possible to acquire monotonicity and thus locality, which we will do with Translation 3.3.5. Furthermore we will present Translation 3.3.11, a new embedding modular faithful translation for $grd \Rightarrow sem$. We will show impossibility of various efficient translations $grd \Rightarrow stg$, impossibility of faithful translations $\sigma \Rightarrow cf$ (Lemma 3.3.4) and impossibility of finite-diameter local weakly faithful translations $grd \Rightarrow \sigma$ (Theorem 3.3.18). We refer to Section 3.4 for inefficient translations $grd \Rightarrow \sigma$.

Lemma 3.3.1 (F: $\sigma \Rightarrow grd$)**.** *There is no weakly faithful translation $Tr : \sigma \Rightarrow grd$ for any semantics σ introduced in this work with $\sigma \neq grd$. As already mentioned above and in [39] this is due to singularity of grounded extensions in general.*

Lemma 3.3.2 (E_e: $grd \Rightarrow (stb|com|adm)$)**.** *There is no efficient weakly exact translation for $grd \Rightarrow (stb|com|adm)$ unless $\mathrm{L} = \mathrm{P}$. See [39] for a proof based on the fact that Ver_{grd} is P-hard while Ver_{σ} is in L.*

Observe that although we could extend the following impossibility to translations $\sigma \Rightarrow (cf|naive)$ with $\sigma \in \{stg, stb, sem, prf, com, adm, grd\}$ easily, we only present it for $grd \Rightarrow naive$. Anyhow Lemma 3.3.4 states that there is no weakly faithful translation $\sigma \Rightarrow cf$ for $\sigma \neq cf$ and Theorem 3.2.4 shows that there is no weakly faithful translation $\sigma' \Rightarrow naive$ with $\sigma' \notin \{cf, naive, grd\}$.

Lemma 3.3.3 (F_e: $grd \Rightarrow naive$)**.** *There is no efficient weakly faithful translation $grd \Rightarrow naive$ unless $\mathrm{L} = \mathrm{P}$.*

Proof. This is simply due to complexity of credulous acceptance, which is in L for naive semantics and P-complete for grounded semantics. For naive semantics any argument not being self-attacking is in some extension. If there was a predictable efficient weakly faithful translation $grd \Rightarrow naive$ we would have access to an efficient algorithm deciding credulous acceptance of grounded semantics. □

Lemma 3.3.4 (F: $\sigma \Rightarrow cf$)**.** *For $\sigma \in \{naive, stg, stb, sem, prf, com, adm, grd\}$ there is no weakly faithful translation $\sigma \Rightarrow cf$.*

Proof. Take into account the argumentation framework $F = (\{a, b, c\}, \{(a, b), (b, c)\})$. We have $\{a, c\} \in \sigma(F)$ and $\{c\} \notin \sigma(F)$. But for standard argumentation frameworks c can only be in some conflict-free set if c is not self-conflicting. Thus any translation yields a conflict-free extension $\{c\}$, which can not be taken care of with remainder sets. □

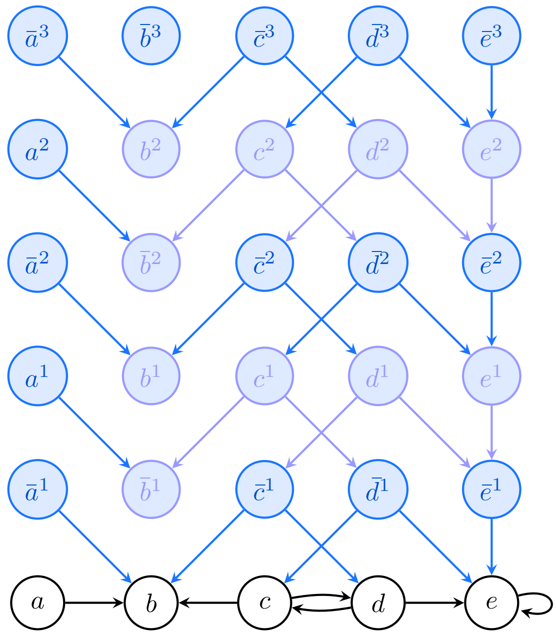


Figure 3.3.6: Translation 3.3.5 as applied to Example 2.1.6

In the following we present a slightly modified version of the efficient faithful translation $grd \Rightarrow \sigma$ from [39], enabling monotonicity by renumbering of arguments. Observe that both translations produce isomorphic frameworks.

Translation 3.3.5 ($\mathbb{F}_5^e$: $grd \Rightarrow \sigma$)**.** For an arbitrary argumentation framework $F = (A, R)$ with $\left\lceil \frac{|A|}{2} \right\rceil = m$, take into account the transformation Tr with $Tr(F) = (A', R')$ and:

$$\begin{aligned} A' &= A \cup \left\{\bar{a}^i \mid a \in A, i \in \{1 \dots m\}\right\} \\ &\quad \cup \left\{a^i \mid a \in A, i \in \{1 \dots m-1\}\right\} \\ R' &= R \cup \left\{(\bar{a}^i, b^{i-1}) \mid (a,b) \in R \textit{ with possibly } a = b, i \in \{2 \dots m\}\right\} \\ &\quad \cup \left\{(a^i, \bar{b}^i) \mid (a,b) \in R \textit{ with possibly } a = b, i \in \{1 \dots m-1\}\right\} \\ &\quad \cup \left\{(\bar{a}^1, b) \mid (a,b) \in R \textit{ with possibly } a = b\right\} \end{aligned}$$

We have that Tr is an embedding monotone faithful translation for $grd \Rightarrow \sigma$ with $\sigma \in \{stg, stb, sem, prf, com, grd\}$.

Proof. This transformation is a slightly modified version of Tr_8 from [39], therefor a detailed proof by structural induction can be derived immediately from the proof in there. Note however that this translation is based on imitation of fixed point iteration of the characteristic function. The single advantage of this modification is that numbering of additional arguments is inversed as far as attacks are concerned, thus adding monotonicity to the properties of the translation. □

Translation 3.3.7 (F_5^e: $grd \Rightarrow adm$)**.** Referring to Translation 3.3.5 as Tr and to Translation 3.1.66 as $Tr_{stb,adm}$ we have that $Tr_{stb,adm} \circ Tr$ forms an embedding monotone weakly faithful translation for $grd \Rightarrow adm$.

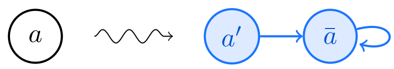
Remark 3.3.8 ($\mathbb{F}_0^e$, F_0^e: $grd \Rightarrow \sigma$)**.** Observe that by restricting Translations 3.3.5 and 3.3.7 to a union of 1-component argumentation frameworks with maximum cardinality n ($F = \bigcup_i F_i$, $|F| \leq n$, $A_{F_i} \cap A_{F_j} = \emptyset$), we can derive a modular translation. In other words, modularity of this translation depends on intentional size of argumentation frameworks. Anyhow we can obviously restrict this translation to 1-component subframeworks, thus receiving a 1-component local translation.

Translation 3.3.9 ($\mathbb{F}_3^e$,F_3^e: $grd \Rightarrow \sigma$)**.** We label Translation 3.3.5 (respectively Translation 3.3.7 for $grd \Rightarrow adm$) as Tr'. For some argumentation framework $F = F_1 \cup F_2 \ldots$ with $A_{F_i} \cap A_{F_j} = \emptyset$ for $i \neq j$ and $dia(F_i) < \infty$ for any i we get an embedding efficient 1-component local (weakly for $grd \Rightarrow adm$) faithful translation $grd \Rightarrow (stg|stb|sem|prf|com|adm)$ by

$$Tr(F) = \bigcup_i Tr'(F_i).$$

We observe that the preceding translation is the only known useful efficient translation which appears to be not finitely local so far. One might wonder if some finitely local translation can serve the same purpose. We can not answer this question in general but in the following we present a modular faithful translation for $grd \Rightarrow sem$.

Transformation 3.3.10 (Emphasizing Additional Arguments)**.** We define a modular, respectively local with respect to $\mathscr{L} = \{(\{a\}, \emptyset)\}$, transformation $Tr_{emph'}$ emphasizing additional arguments of the form A':



Translation 3.3.11 ($\mathbb{F}_0^e$: $grd \Rightarrow sem$)**.** For an arbitrary argumentation framework $F = (A, R)$ we have that $Tr = Tr_{id} \cup Tr_{par} \cup Tr_{emph'}$ forms an embedding modular faithful translation for $grd \Rightarrow sem$.

Proof. To validate this statement we recall that due to Translation 3.1.72 we have that $Tr_0 = Tr_{id} \cup Tr_{par}$ is an embedding modular faithful translation for $com \Rightarrow (stb|prf)$. For a given set $E \subseteq A$ we also recall the derived set $E' = E \cup \{a' \mid E \not\rightarrowtail^F a\}$ as used in the proof of Translation 3.1.72. Furthermore we use $A' = \{a' \mid a \in A\}$ and $\bar{A} = \{\bar{a} \mid a \in A\}$ to denote the additional arguments of $Tr(F)$.

We observe that $Tr_{emph'}$ does not affect preferred extensions, in other words we have:

$$prf(Tr(F)) = prf(Tr_0(F))$$

Since Tr_0 represents a faithful translation for $com \Rightarrow (stb|prf)$ also for any $E \in prf(Tr(F))$ we have:

$$(A \cup A') \subseteq E^+_{Tr(F)}$$

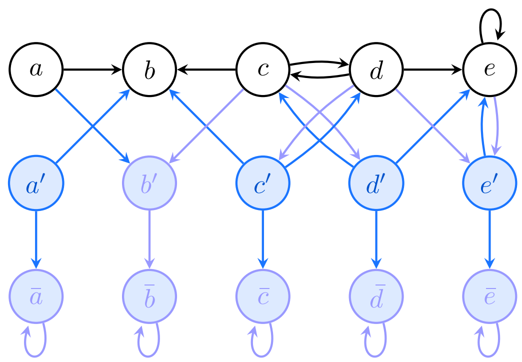


Figure 3.3.12: Translation 3.3.11 as applied to Example 2.1.6.

As pointed out in Lemma 2.3.11 any semi-stable extension is a preferred extension, thus it suffices to look for complete extensions $E \in com(F)$ such that E' is range-maximal in $Tr(AF)$ or to be precise among $\bar{A}$.

We look back at the definition of E' which for any $E \in com(F)$ gives a preferred extension of $Tr(F)$. We observe that $\bar{a} \in \bar{A}$ is in the range of E' if and only if $a' \in A'$ is a member of E'. Furthermore any $E \in com(F)$ extends the grounded extension $E_0 = grd(F)$. Thus any range of any $E' \in prf(Tr(F))$ is extended by the range of E'_0.

$$E_0 \subsetneq E \qquad \Longrightarrow \qquad (E')^+_{Tr(F)} \subsetneq (E'_0)^+_{Tr(F)}$$

□

If we are looking for efficient translations starting from the grounded extension we are thus left with (weakly) exact translations $\Rightarrow stg, sem, prf$. We will put focus on stage semantics first, presenting an interesting result incorporating some unique stage extension and stable semantics followed up by impossibility of efficient exact, embedding weakly exact and finite-diameter weakly exact translations. We will continue with impossibility of finite diameter weakly exact translations for $grd \Rightarrow (sem|prf|com)$ and finite diameter weakly faithful translations for $grd \Rightarrow (stg|stb|prf|com|adm)$. We will close this section with thoughts about weakly exact translations $grd \Rightarrow (sem|prf)$.

Translation 3.3.13 (($\mathbb{E}_e$): $stg \Rightarrow stb$)**.** Consider some argumentation framework $F = (A, R)$ with $|stg(F)| = 1$. The transformation $Tr(F) = (A', R')$ with

$$A' = A \setminus \{a \in A \mid a \rightarrowtail^R a, b \rightarrowtail^R a \Rightarrow b \rightarrowtail^R b\} \qquad R' = R \cap A' \times A'$$

is an efficient exact translation $stg \Rightarrow stb$.

Remark 3.3.14. Observe that this translation is in general not covering and can for general argumentation frameworks with the desired property not acquire the covering property. This is due to the fact that F might consist of self-attacking arguments only, thus yielding an empty stage extension for non-empty argumentation framework, while the empty set is a stable extension for empty argumentation frameworks only.

Proof. If $F = (A, R)$ has one stage extension $E \in stg(F)$ only, then the only arguments not in the range of E are self-attacking and attacked only by self-attacking arguments:

$$a \in (A \setminus E^+) \qquad \Longrightarrow \qquad (a \rightarrowtail a), (b \rightarrowtail a \implies b \rightarrowtail b)$$

Otherwise there would have to be another stage extension $E' \neq E$ such that a is in the range of E'. Observe that for argumentation frameworks with self-attacking arguments only ($a \in A \Rightarrow a \rightarrowtail^R a$) this yields an empty argumentation framework.

Now for any conflict-free set $E' \in cf(F)$ we have $E' \in cf(Tr(F))$ and furthermore $E'^+_F = E'^+_{Tr(F)}$. The stage extension E is still unique and for the reasons explained before we have $E^+_{Tr(F)} = A_{Tr(F)}$ and therefor a stable extension $E \in stb(Tr(F))$. □

Corollary 3.3.15 ($\mathbb{E}_e$: $grd \Rightarrow stg$)**.** *There is no efficient exact translation Tr taking care of $grd \Rightarrow stg$ unless* $\mathrm{L} = \mathrm{P}$.

Proof. Otherwise Tr together with Translation 3.3.13 (now referred to as Tr') provides us with an efficient exact translation $Tr' \circ Tr : grd \Rightarrow stb$ which due to Lemma 3.3.2 is impossible unless $\mathrm{L} = \mathrm{P}$. □

Lemma 3.3.16 (E^e: $grd \Rightarrow stg$)**.** *There is no embedding weakly exact translation $grd \Rightarrow stg$.*

Proof. For a contradiction assume that such a translation Tr exists. Take into account the argumentation framework $F = (\{a, b, c\}, \{(b, c), (c, b)\})$.

Taking a look at $Tr(F) = F'$ we observe that since Tr is an embedding transformation $a \not\rightarrowtail^{F'} c$ and $c \not\rightarrowtail^{F'} c$, in other words $\{a, c\} \in cf(F')$. But then $\{a\} \notin stg(F')$, a contradiction. □

Besides the question whether remainder sets can be of use for efficient translations $grd \Rightarrow stg$ or not, arbitrary weakly exact translations in this case can not be handled that easy. Practical translations, making use of remainder sets so far, consisted of exactly one remainder set, however theoretically remainder sets provide some challenging possibilities. In the following we examine several kinds of restricted local translations $grd \Rightarrow \sigma$.

Theorem 3.3.17 (E_4: $grd \Rightarrow stg$)**.** *There is no local with finite diameter d weakly exact translation $grd \Rightarrow stg$.*

Proof. For a contradiction assume that such a translation Tr exists. Take into account an argumentation framework F built by the union of a circular framework $\mathcal{O}_n$ with[7] $n > 2d + 1$ and a singular framework B, $F = \mathcal{O}_n \cup B$ with:

$$B = (\{b\}, \emptyset) \qquad \mathcal{O}_n = (\{a_1, a_2 \ldots a_n\}, \{(a_n, a_1), (a_1, a_2), (a_2, a_3) \ldots (a_{n-1}, a_n)\})$$

Observe that $grd(F) = \{\{b\}\}$. We further observe that in $Tr(F)$ due to monotonicity we have $a_2 \not\rightarrowtail^{Tr(F)} a_2$ and $\{a_2, b\} \in cf(Tr(F))$. It follows that $\{b\} \notin stg(Tr(F))$, a contradiction. □

[7] Compare Example 3.2.16 for the choice of n.

We admit that this last result turns obsolete with Lemma 3.4.13 where we will be working with monotone translations. However the previous proof is adaptable also for modified concepts of locality, while the upcoming proof strongly relies on monotonicity.

The following Theorem encompasses also complete semantics although we have discussed efficient weakly exact translations $grd \Rightarrow com$ in Lemma 3.3.2. We have presented efficient translations not being local already. Finite-diameter local translations not being efficient are possible yet we do not know about any concrete.

Theorem 3.3.18 (E_4: $grd \Rightarrow (sem|prf|com)$)**.** *There is no local with finite diameter d weakly exact translation for $grd \Rightarrow (sem|prf|com)$.*

Proof. For a contradiction we assume that such a translation Tr happens to exist. For $grd \Rightarrow (prf|com)$ we take into account[8] the argumentation frameworks $F_1, F_2, \mathcal{O}_{2n}$ where $n = 2d$ and

$$\begin{aligned}\mathcal{O}_{2n} &= (\{a_1, a_2 \dots a_{2n}\}, \{(a_1, a_2), (a_2, a_3) \dots (a_{2n-1}, a_{2n}), (a_{2n}, a_1)\})\\ F_1 &= (A_{\mathcal{O}_{2n}}, R_{\mathcal{O}_{2n}} \setminus \{(a_{2n}, a_1)\})\\ F_2 &= (A_{\mathcal{O}_{2n}}, R_{\mathcal{O}_{2n}} \setminus \{(a_n, a_{n+1})\})\end{aligned}$$

Obviously these argumentation frameworks are strongly related, $F_1 \cong F_2$ and even $E = grd(F_1) = grd(F_2) = \{a_1, a_3 \dots a_{2n-1}\}$. Furthermore due to the nature of finite-diameter local translations, the equality $dia(F_1) = 4d - 1$ and the obvious identity $\mathcal{O}_{2n} = F_1 \cup F_2$ we get

$$Tr(\mathcal{O}_{2n}) = Tr(F_1) \cup Tr(F_2).$$

Thus with E being admissible in $Tr(F_1)$ and $Tr(F_2)$ we get E being admissible also in $Tr(\mathcal{O}_{2n})$ and therefor E is at least subset of some preferred and complete extension of $\mathcal{O}_{2n}$.

As far as semi-stable semantics is concerned we reconsider the proof of Theorem 3.3.17 and thus consider the argumentation frameworks F_0, F with:

$$F_0 = (\{b\}, \emptyset) \qquad\qquad F = F_0 \cup \mathcal{O}_{2n}$$

Due to finite-diameter locality we get

$$Tr(F) = Tr(F_0) \cup Tr(\mathcal{O}_{2n})$$

Now we derive that $E \cup \{b\}$ is an admissible set in $Tr(F)$, but then due to finite-diameter locality either there is some semi-stable extension E' in $Tr(F)$ with $b \in E'$ and $|E'| > 1$ or there is no semi-stable extension in $Tr(F)$ containing b, either way contradicting the assumption. □

We observe that the previous finite-diameter proofs do not work upon faithful translations, basically due to Remark 3.3.8. The impact of some faithful translation $grd \Rightarrow \sigma$ does not depend on diameter only but also on size.

[8] For more information on properties of $\mathcal{O}_{2n}$ we refer to Definition3.2.14 and the subsequent results.

Remark 3.3.19 (Circular and Linear Frameworks). In the proof for the following theorem we will make use of modified versions of circular and linear frameworks from Definition 3.2.14. We investigate the following linear and circular frameworks:

$$\begin{aligned}\mathcal{I}_N &= (\{a_1, a_2 \ldots a_N\}, \{(a_1,a_2),(a_2,a_3)\ldots(a_{N-1},a_N)\})\\ \mathcal{I}_{i,j} &= (\{a_i, a_{i+1} \ldots a_j\}, \{(a_i,a_{i+1}),(a_{i+1},a_{i+2})\ldots(a_{j-1},a_j)\})\\ \mathcal{O}_{i,j} &= (\{b_i, b_{i+1} \ldots b_j\}, \{(b_i,b_{i+1}),(b_{i+1},b_{i+2})\ldots(b_{j-1},b_j),(b_j,b_i)\})\end{aligned}$$

We observe that for Tr a local with diameter $d < \infty$ translation, $i > d$, $j > i + 2d + 1$, $N > j + d$, due to predictability and proper definition of translations there are unique induced structure preserving mappings

$$\varphi_{\mathcal{I}_N} : Tr(\mathcal{I}_N) \to Tr(\mathcal{O}_{i,j}) \qquad \text{and} \qquad \varphi_{\mathcal{O}_{i,j}} : Tr(\mathcal{O}_{i,j}) \to Tr(\mathcal{I}_N),$$

with $\varphi_{\mathcal{I}_N} : a_{k+m(j-i)} \mapsto b_k$ and $\varphi_{\mathcal{O}_{i,j}} : b_k \mapsto a_{k+m(j-i)}$ for $i \leq k \leq j$.[9] Now $\varphi_{\mathcal{I}_N}$ is an onto function and $\varphi_{\mathcal{O}_{i,j}}$ is the corresponding inverse. The mappings operate in a bijective way, i.e. for E an extension of $Tr(\mathcal{O}_{i,j})$ we get $\varphi_{\mathcal{I}_N}(\varphi_{\mathcal{O}_{i,j}}(E)) = E$, the reverse however does not necessarily hold.

Theorem 3.3.20 (F_4: $grd \Rightarrow (stg|prf)$). *There is no finite-diameter local weakly faithful translation for $grd \Rightarrow (stg|prf)$.*

Proof. For a contradiction we assume that a local with finite diameter d transformation Tr serves as weakly faithful translation for $grd \Rightarrow \sigma$ with $\sigma \in \{stg, prf\}$. We recall that we did restrict ourselves to finite argumentation frameworks (Remark 2.1.15) and investigate linear and circular frameworks and induced mappings from Remark 3.3.19. We observe that for $2d < i$, $i + 2d + 1 < j$, $j + 2d < N$ and the argumentation frameworks of interest due to predictability and without loss of generality we can assume Tr to be local with respect to $\mathcal{I}_{d+1}$. Furthermore we observe that $\mathcal{I}_N \cap \mathcal{O}_{i,j} = (\emptyset, \emptyset)$, and thus build $F = \mathcal{I}_N \cup \mathcal{O}_{i,j}$, for which we have $Tr(F) = Tr(\mathcal{I}_N) \cup Tr(\mathcal{O}_{i,j})$.

We observe that the grounded extension is unique for any argumentation framework and we deal with finite argumentation frameworks and thus finite translated frameworks only. We choose the unique $E = \sigma(Tr(F))$ with $E \cap A_F \in grd(F)$. We observe that $E_a = E \cap A_{Tr(\mathcal{I}_N)}$ and $E_b = E \cap A_{Tr(\mathcal{O}_{i,j})}$ serve as the respective unique extensions, related to $grd(\mathcal{I}_N)$ and $grd(\mathcal{O}_{i,j})$. Furthermore for any relevant remainder set $S \in \mathcal{S}$ we have $S \subseteq A_{Tr(\mathcal{I}_{d+1})}$ and $S \subseteq A_{Tr(\mathcal{O}_{i,j})} \cap A_{Tr(\mathcal{I}_N)}$.

Now for N big enough the structure of the chosen σ-extension in $Tr(\mathcal{I}_N)$ will repeat itself. Thus we can choose i, j, N such that

$$\varphi_{\mathcal{O}_{i,j}}(\varphi_{\mathcal{I}_N}(E \cap A_{Tr(\mathcal{I}_{i-d,j+d})})) = E \cap A_{Tr(\mathcal{I}_{i-d,j+d})}.$$

We define $B = \varphi_{\mathcal{I}_N}(E \cap A_{Tr(\mathcal{I}_{i-d,j+d})})$. Clearly B is conflict-free in $Tr(\mathcal{O}_{i,j})$. For $\sigma = prf$ we observe that with E being admissible in $Tr(F)$ also B is admissible in $Tr(F)$ and thus

[9] Using the modulo operator we might also write a_{i+n} and $b_{i+(n \mod (j-i))}$. Recall that by Remark 2.1.19 with $\varphi : (A, R) \to (A', R')$ we implicitly define a mapping also for $A \to A'$, $R \to R'$, $\wp(A) \to \wp(A')$ and $\wp(R) \to \wp(R')$.

there has to be some preferred extension E^B extending B where without loss of generality $b_i \in E^B$, implying that E^B is no remainder set.

For $\sigma = stg$ we observe that B is not in conflict with E_a and thus we need the proper subset relation $B^+ \subsetneq E_b^+$. We have a look at $A = \varphi_{\mathcal{O}_{i,j}}(E_b)$ and observe that firstly there can not be any remainder set T such that $A^+_{Tr(F)} \subsetneq T^+_{Tr(F)}$ for otherwise also ${E_b}^+_{Tr(\mathcal{O}_{i,j})} \subsetneq T^+_{Tr(\mathcal{O}_{i,j})}$. Secondly E and A are not comparable in range, i.e. there is some $a \in A^+$ such that $a \notin E^+$ and vice versa, by the relation $B^+ \subsetneq E_b^+$. We conclude that there has to be some $E^A \in stg(Tr(F))$ with $A^+_{Tr(F)} \subseteq {E^A}^+_{Tr(F)}$ such that E^A is not a remainder set and $E^A \neq E$. □

We recall that there are modular weakly faithful translations $(stb|com|adm) \Rightarrow prf$. Concatenation ($Tr_2 \circ Tr_1$) of some modular ($Tr_2$) and some finite-diameter local (Tr_1) transformation yields another finite-diameter local transformation.[10] We can thus immediately conclude the following result.

Corollary 3.3.21 (F_4: $grd \Rightarrow (stb|com|adm)$)**.** *There is no finite-diameter local weakly faithful translation for* $grd \Rightarrow (stb|com|adm)$*.*

We close this section with thoughts upon relations between the latest semantics and intertranslatability. It appears that not only $grd \Rightarrow (sem|prf)$ is not possible with a finite diameter weakly exact translation, but also any possible efficient or weakly exact translation shows some remarkable characteristics.

Lemma 3.3.22 ($grd \Rightarrow (sem|prf)$, Weakly Exact Behavior)**.** *If* Tr *is a weakly exact translation for* $grd \Rightarrow \sigma$*,* $\sigma \in \{prf, sem\}$ *and* $|\sigma(Tr(F))| > 1$ *then* $grd(Tr(F)) = \{\emptyset\}$*, regardless of* $grd(F)$*.*

Proof. Since σ-semantics consist of one element only, as soon as the empty set is a σ-extension it follows that the empty set is no σ-extension: $\emptyset \notin \sigma(Tr(F))$. We consider $\{E_0\} = grd(Tr(F))$. Since any σ-extension is a complete extension for $E \in \sigma(Tr(F))$ we have $E_0 \subseteq E$. Now remainder sets consist of arguments from $A_{Tr(F)} \setminus A_F$ only. For exact translations extensions not being remainder sets consist of arguments from A_F only. It follows that $E_0 = \emptyset$. □

Lemma 3.3.23 ($grd \Rightarrow (sem|prf)$, Exact Behavior)**.** *If* Tr *is an efficient exact translation for* $grd \Rightarrow (sem|prf)$ *then for infinitely many isomorphically different argumentation frameworks* F *we have* $grd(F) \neq grd(Tr(F))$*.*

Proof. Choose $\sigma \in \{sem, prf\}$. We have $grd(F) = \{E_0\} = \sigma(Tr(F))$. If we assume $grd(F) = grd(Tr(F))$ then, since any complete extension extends the grounded extension and any σ-extension extends some complete extension, already $com(Tr(F)) = \{E_0\} = \sigma(Tr(F))$. Thus assuming L $\neq$ P and not to be contradicting Lemma 3.3.2 for infinitely many isomorphically different argumentation frameworks F we need $grd(F) \neq grd(Tr(F))$. □

So if there is an efficient weakly exact translation $grd \Rightarrow prf$ or $grd \Rightarrow sem$ most likely the grounded extensions of the original and of the translated framework differ. As suggested by Lemma 3.3.22 cutback to the empty set might be a good choice.

[10] Order is important here, $Tr_1 \circ Tr_2$ does not necessarily preserve finite-diameter locality.

Remark 3.3.24. Due to subset relations from Section 2.3 any faithful translation $grd \Rightarrow adm$ is also valid for $grd \Rightarrow com$, any faithful translation $grd \Rightarrow com$ is also valid for $grd \Rightarrow prf$, any faithful translation $grd \Rightarrow prf$ is also valid for $grd \Rightarrow sem$ and any faithful translation $grd \Rightarrow stb$ is also valid for $grd \Rightarrow (stg|sem)$.

3.4 Leaving Efficiency - Oracle Translations

In this last contributing section we will give inefficient translations with the aim of closing gaps between prior results. In some cases where provably no efficient translation exists closing of gaps seems to be the right term. In other cases we might rephrase the aim to minimizing of gaps, e.g. where provably no finite diameter translation exists, yet we do not know for sure if an efficient translation might still be possible.

Definition 3.4.1 (Oracle Translation)**.** An oracle translation Tr has access to an oracle which for any argumentation framework F and any semantics σ gives all extensions $E \in \sigma(F)$. Therefor most likely oracle translations make use of arguments from $\{a_1^F, a_2^F \dots\}$, which we hereby also call $\{E_{F'}^i \mid E^i \in \sigma(F'), F' \subseteq F\}$, where E^i refers to extensions of subframeworks of F.

In the following we will observe that monotonicity and even 1-component locality do not appear to be a problem with respect to theoretical capability for most translations. We point out that any connected argumentation framework provides a finite diameter, since we restricted ourselves to finite argumentation frameworks. However the translations presented in the following will not be finite-diameter local.

Definition 3.4.2 (Finite-Diameter Subframework)**.** To shorten the following results in definition, for argumentation frameworks F,F' we define the finite-diameter subframework relation:

$$F' \subseteq^d F \iff F' \subseteq F, dia(F') < \infty$$
$$F' \subsetneq^d F \iff F' \subsetneq F, dia(F') < \infty$$

Observe that for this definition "finite" does not actually restrict the size but only the type. To be more specific we require finite-diameter subframeworks to be subframeworks consisting of one connected component only.

Remark 3.4.3 (Non-Interference Extended). We recall Definition 2.3.23 (Non-Interference) and observe that if a non-interfering semantics σ has the empty set as only extension of the empty argumentation framework, immediately any argumentation framework obtains at least one extension. In the following we will make use of this observation, thus providing applicability of presented translations for semantics beyond the scope of this thesis.

Definition 3.4.4 (Regularity)**.** A conflict-free non-interfering argumentation semantics σ, such that for $F_0 = (\emptyset, \emptyset)$ we have $\sigma(F_0) = \{\emptyset\}$ is called a regular semantics.

Translation 3.4.5 ($\mathbb{F}^e_6$: $\sigma \Rightarrow (stg|stb|sem|prf)$)**.** For an arbitrary argumentation framework $F = (A, R)$ and any regular semantics[11] we define the 1-component local transformation $Tr(F) = (A', R')$ with:

$$\begin{aligned}
A' &= A \cup \left\{F', E^i_{F'} \mid F' \subseteq^d F, E^i_{F'} \in \sigma(F')\right\} \\
R' &= R \cup \left\{(E, F'), (F', F'), (F', a) \mid F' \subseteq^d F, E \in \sigma(F'), a \in A_{F'}\right\} && \textit{admissibility} \\
&\cup \left\{(E, b) \mid F' \subseteq^d F, E \in \sigma(F'), b \in A_{F'} \setminus E\right\} && \textit{maximality} \\
&\cup \left\{(E_1, E_2) \mid F' \subseteq^d F, E_1 \neq E_2 \in \sigma(F')\right\} && \textit{extension picking} \\
&\cup \left\{(E', E^<), (E', F^<) \mid F' \subseteq^d F, F^< \subsetneq^d F', E' \in \sigma(F'), E^< \in \sigma(F^<)\right\} && \textit{monotonicty}
\end{aligned}$$

Now Tr is an embedding 1-component local faithful translation for $\sigma \Rightarrow (stg|stb|sem|prf)$.

Proof. We observe that for disjunct argumentation frameworks F_1, F_2 with $A_{F_1} \cap A_{F_2} = \emptyset$, also $A_{Tr(F_1)} \cap A_{Tr(F_2)} = \emptyset$, furthermore Tr is supposed to possess a stable extension for every translated argumentation framework. Thus without loss of generality we assume F to consist of one connected component, $dia(F) < \infty$.

Now for $E \in \sigma(F)$ and thus $E \in cf(F)$ we take a look at $E' = E \cup \{E\}$. E' is conflict-free since E attacks only (but all) those arguments from A not being member of E. Furthermore E attacks any $E_2 \in \sigma(F), E_2 \neq E$, any $E_2 \in \sigma(F')$ for $F' \subsetneq^d F$ and any F' for $F' \subseteq^d F$. This implies that E' is a stable extension of $Tr(F)$. By being a stable extension E' is also a stage, semi-stable and preferred extension of $Tr(F)$.

We now assume E' to be some preferred extension of $Tr(F)$. Then for any $E_2 \in \sigma(F')$ with $F' \subsetneq^d F$ we have that E_2 is not a member of E' since the only arguments defending E_2 against the non-empty set $\sigma(F)$ are members of $\sigma(F)$ and thus also attacking E_2. Furthermore at most one $E \in \sigma(F)$ is member of E'. We observe that there is no admissible set E_0 in $Tr(F)$ such that $E_0 \cap \sigma(F) = \emptyset$ since all arguments from A are attacked by F. We can thus pick the unique $E \in E' \cap \sigma(F)$. But then E defends all arguments $a \in E$ and it follows immediately that $E' \cap A = E \in \sigma(F)$. □

Example 3.4.6. Take into account the argumentation framework $F = (\{a, b, c\}, \{(b, c), (c, b)\})$ with complete semantics, $com(F) = \{\{a\}, \{a, b\}, \{a, c\}\}$. The application of Translation 3.4.5 is illustrated in Figure 3.4.7, where $F_1 = (\{a\}, \emptyset)$ and $F_2 = (\{b, c\}, \{(b, c), (c, b)\})$. We receive $\left\{a, \{a\}_{F_1}, \emptyset_{F_2}\right\}$, $\left\{a, \{a\}_{F_1}, b, \{b\}_{F_2}\right\}$ and $\left\{a, \{a\}_{F_1}, c, \{c\}_{F_2}\right\}$ as stable extensions of $Tr(F)$. Observe that we simplified the illustration by collecting all non-maximal finite-diameter subframeworks $F^<$ and their extensions $E^<$ into one ignorable node. The bunch of $E^<$, $F^<$ thus refers to eight distinct arguments.

We observe that for Translation 3.4.5 some translated argumentation frameworks contain the empty set as a complete extension. Besides this of course any preferred extension is also a complete extension and since the only non-empty admissible extensions of $Tr(F)$ are extending some $E \in \sigma(F)$ and E defends all $a \in E$ in $Tr(F)$ the very same transformation also gives a weakly faithful translation for $\sigma \Rightarrow com$.

[11]As far as the semantics introduced in this work are concerned this characterization of σ excludes only stable semantics.

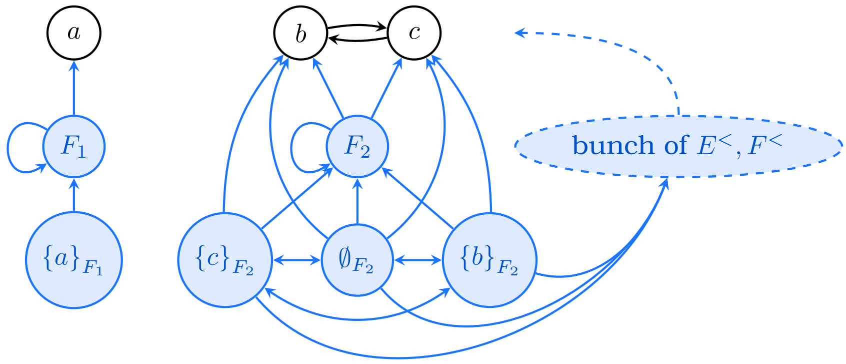


Figure 3.4.7: Illustration of Translation 3.4.5 ($com \Rightarrow stb$) as described in Example 3.4.6.

Corollary 3.4.8 (F_6^e: $\sigma \Rightarrow com$)**.** *Translation 3.4.5 serves as an embedding* 1*-component local weakly faithful translation with remainder set* $\emptyset$ *for* $\sigma \Rightarrow com$ *where* σ *refers to some regular semantics.*

Corollary 3.4.9 (F_6^e: $\sigma \Rightarrow adm$)**.** *Referring to Translation 3.4.5 as* $Tr_{\sigma,stb}$ *and to Translation 3.1.66 as* $Tr_{stb,adm}$ *the transformation* Tr *given by*

$$Tr = Tr_{stb,adm} \circ Tr_{\sigma,stb}$$

is an embedding 1*-component local weakly faithful translation for* $\sigma \Rightarrow adm$ *where* σ *refers to some regular semantics.*

We recall that by Lemmata 3.1.62, 3.1.63 and 3.1.64 there is no faithful or weakly exact translation for $(naive|stg|stb|sem|prf) \Rightarrow com$. By Lemmata 3.1.60 and 3.1.61 there is no faithful or weakly exact translation for $(naive|stg|stb|sem|prf|com) \Rightarrow adm$.

We close our inspection of (weakly) faithful translations at this point and turn over to exact translations. We will first be visiting $grd \Rightarrow \sigma$ and close the thesis with exact translations for $sem \Rightarrow prf$, $stg \Rightarrow prf$ and $stg \Rightarrow stb$.

We recall that by Lemmata 3.1.45 and 3.1.46 there is no covering weakly exact and no monotone weakly exact translation for $grd \Rightarrow stb$.

Translation 3.4.10 ($\mathbb{E}$: $grd \Rightarrow stb$)**.** For an arbitrary argumentation framework the transformation $Tr(F) = (grd(F), \emptyset)$ is an exact translation for $grd \Rightarrow stb$.

Remark 3.4.11 (F^e,F_7: $grd \Rightarrow naive$). Lemma 3.3.3 reflects upon efficient translations $grd \Rightarrow naive$. We extend this result to local and embedding translations. Considering the circular framework $F = (\{a,b\}, \{(a,b),(b,a)\})$, we have that obviously $grd(F) = \{\emptyset\}$. Thus immediately any weakly faithful translation Tr for $grd \Rightarrow naive$ can not be embedding, otherwise there would be some naive extension extending $\{b\}$ in $Tr(F)$. In other words any node-covering weakly faithful translation $grd \Rightarrow naive$ has to add the attacks (a,a) and (b,b). Furthermore any weakly faithful translation $grd \Rightarrow naive$ can not be monotone, since we could easily add some argument c and the attack (c,a) to F, ensuring an extension containing b.

We recall that by Lemma 3.3.16 there is no embedding weakly exact translation for $grd \Rightarrow stg$.

Translation 3.4.12 ($\mathbb{E}^c$: $grd \Rightarrow (naive|stg)$)**.** For an arbitrary argumentation framework $F = (A, R)$ the transformation $Tr(F) = (A, R \cup \{(a, a) \mid a \in A \setminus grd(F)\})$ is a covering exact translation for $grd \Rightarrow (naive|stg)$.

Lemma 3.4.13 (E_7: $grd \Rightarrow stg$)**.** *There is no monotone weakly exact translation for* $grd \Rightarrow stg$.

Proof. For a contradiction we assume that such a translation Tr happens to exist. Take into account the argumentation frameworks $F_1 = (A_1, R_1)$ and $F_2 = (A_2, R_2)$ with

$$A_1 = \{a, b, c\} \qquad\qquad A_2 = \{a, b, c\}$$
$$R_1 = \{(b, c), (c, b)\} \qquad\qquad R_2 = \{(a, b), (b, c), (c, b)\}$$

We have that $grd(F_1) = \{a\}$ and $grd(F_2) = \{a, c\}$. Observe that any monotone translation needs to copy at least all the original arguments. Now clearly $F_1 \subseteq F_2$ and thus for monotone translation Tr also $Tr(F_1) \subseteq Tr(F_2)$. Since Tr is supposed to be weakly exact we need $(a, b), (a, c) \in R_{Tr(F_1)}$ which immediately leads to a contradiction with $Tr(F_2)$. □

We recall that by Theorem 3.3.18 there is no finite-diameter local weakly exact translation for $grd \Rightarrow (sem|prf|com)$ and by Lemma 3.3.2 there is no efficient weakly exact translation for $grd \Rightarrow com$.

Translation 3.4.14 ($\mathbb{E}^e_6$: $grd \Rightarrow (sem|prf|com)$)**.** For an arbitrary argumentation framework $F = (A, R)$ we take into account the transformation $Tr(F) = (A', R')$ with:

$$\begin{aligned} A' &= A \cup \left\{F' \mid F' \subseteq^d F\right\} \\ R' &= R \cup \left\{(F', F'), (F', a) \mid F' \subseteq^d F, a \in A_{F'} \setminus grd(F')\right\} && \textit{admissibility} \\ &\quad \cup \left\{(a, F^<) \mid F^< \subsetneq^d F' \subseteq^d F, a \in A_{F'}\right\} && \textit{monotonicity} \end{aligned}$$

We claim that this transformation gives an embedding 1-component local exact translation for $grd \Rightarrow (sem|prf|com|grd)$.

Proof. We take into account that for the semantics of interest there is no difference between computing the semantics of the union of a collection of disjunct (1-connected component) argumentation frameworks and computing the semantics prior to building the union. Without loss of generality we can therefor assume $Tr(F)$ and thus F to consist of one connected component only. Observe that we can ignore all arguments $F' \subsetneq^d F$ since in $Tr(F)$ they are self-attacking and attacked by every single argument $a \in A$.

Now the argument F is attacked only by itself yet attacks any argument not being a member of the grounded extension of F, thus eliminating the arguments $A_F \setminus grd(F)$ for any admissible extension. If $\emptyset = grd(F)$ it follows that the only admissible set in $Tr(F)$ is the empty set, immediately the assumption follows. If on the other hand $\emptyset \neq grd(F)$ then there is some $a \in grd(F)$ which is not attacked in $Tr(F)$. Subsequently it follows that $grd(F) = grd(Tr(F))$ and thus the semantics of interest collapse. □

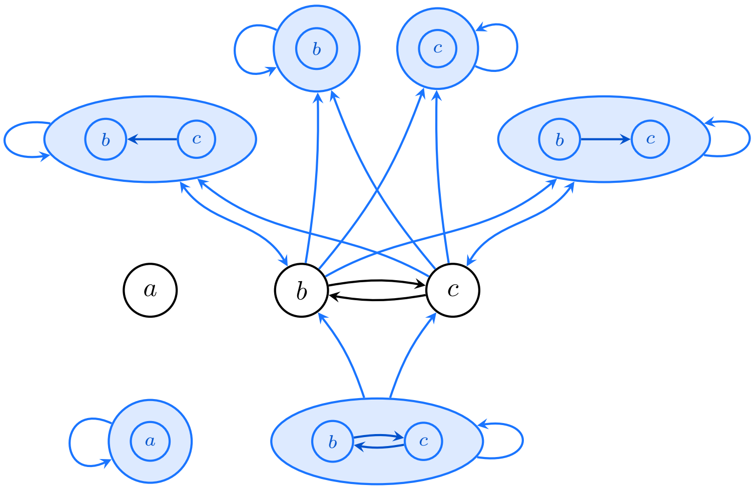


Figure 3.4.16: Illustration of Translation 3.4.12 ($grd \Rightarrow \sigma$) as described in Example 3.4.15.

Example 3.4.15. Investigate the argumentation framework $F = (\{a, b, c\}, \{(b, c), (c, b)\})$. The illustration of an application of Translation 3.4.12 is to be found in Figure 3.4.16. For $\sigma \in \{sem, prf, com\}$ we have

$$grd(F) = \{\{a\}\} = \sigma(Tr(F)).$$

In the illustration the significant arguments derived from finite-diameter subframeworks are placed below the original framework, while insignificant arguments are placed above. Obviously the arguments above are necessary only to ensure monotonicity.

Observe that enlargement of the original argumentation framework potentially results in exponential growth for the translated framework. $Tr(F)$ consists of 9 arguments and 20 attacks. Adding of the attack (a, b) to F results in overall 13 arguments and 48 attacks in the translated framework.

Translation 3.4.17 ($\mathbb{E}_6^e$: $sem \Rightarrow prf$)**.** We use $\mathscr{P}_{F'}$ to denote $\mathscr{P}_{F'} = prf(F') \setminus sem(F')$ for any argumentation framework F' in the following.

For an arbitrary argumentation framework $F = (A, R)$, consider the 1-component local transformation $Tr(F) = (A', R')$ with:

$$\begin{aligned} A' &= A \cup \left\{P^i_{F'} \mid F' \subseteq^d F, P \in \mathscr{P}_{F'}\right\} \\ R' &= R \cup \left\{(a, P), (P, P), (P, b) \mid F' \subseteq^d F, P \in \mathscr{P}_{F'}, a \in A_{F'} \setminus P, b \in P\right\} && \textit{extension picking} \\ &\quad \cup \left\{(a, P^<) \mid F^< \subsetneq^d F' \subseteq^d F, a \in A_{F'}, P^< \in \mathscr{P}_{F^<}\right\} && \textit{monotonicity} \end{aligned}$$

We claim that Tr is an embedding 1-component local exact translation for $sem \Rightarrow prf$.

Proof. As in previous translations, without loss of generality we assume $dia(F) < \infty$. Now observe that any non-empty conflict-free set in $Tr(F)$ consists of arguments $a \in A$ only. Additional arguments of the form $P \in \mathscr{P}_{F'}$ with $F' \subsetneq^d F$ are attacked by all $a \in A$, we can thus restrict our investigation to the arguments $a \in A \cup \mathscr{P}_F$.

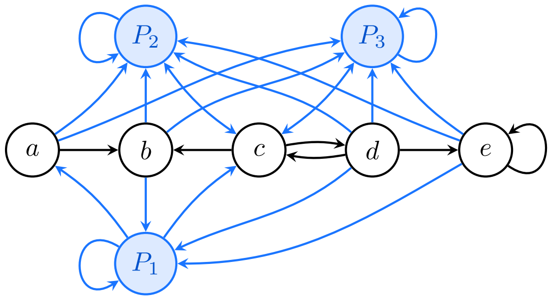


Figure 3.4.19: Illustration of Translation 3.4.17 ($sem \Rightarrow prf$) as described in Example 3.4.18.

Now assume E to be some semi-stable extension of F. Since $E \setminus P \neq \emptyset$ for any $P \in \mathscr{P}_F$ we have that E attacks all $P \in \mathscr{P}_F$ in $Tr(F)$ and thus is a preferred extension of $Tr(F)$.

On the other hand we might look at some $E \in prf(Tr(F))$ and assume for a contradiction that $E \notin sem(F)$. Then there has to be some $P \in \mathscr{P}_F$ such that $E \subseteq P$. But now E is attacked by the argument P in $Tr(F)$ and defended only by arguments $a \in A \setminus P$ and thus can not be admissible. □

Example 3.4.18. Take into account the argumentation framework F from Example 2.1.6. An application of Translation 3.4.17 is illustrated in Figure 3.4.19. Observe that this example has on the whole 38 finite-diameter subframeworks, fortunately only three of them possess differing semi-stable and preferred semantics:

- For $F = (\{a, b, c, d, e\}, \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\})$ we have that $P_1 = \{a, c\}_F$ is a preferred but not a semi-stable extension.
- For $F_2 = (\{b, c, d, e\}, \{(c, b), (c, d), (d, c), (d, e), (e, e)\})$ we have that $P_2 = \{c\}_{F_2}$ is a preferred but not a semi-stable extension.
- For $F_3 = (\{c, d, e\}, \{(c, d), (d, c), (d, e), (e, e)\})$ we have that $P_3 = \{c\}_{F_3}$ is a preferred but not a semi-stable extension.

Now P_1, P_2 and P_3 are self-attacking. Thus possible extensions of $Tr(F)$ consist of arguments from A_F only. Due to F_2 and F_3 being proper finite-diameter-subframeworks of F we have that P_2 and P_3 are attacked by all $a \in A_F$ in $Tr(F)$. Thus the only additional argument of significance is P_1.

For $Tr(F)$ we have that a can be defended against P_1 only by d, b can not be defended against a, c can not be defended against P_1 and d defends itself. Computing the semantics in question thus results in $sem(F) = \{\{a, d\}\} = prf(Tr(F))$.

We recall that by Lemmata 3.1.37 and 3.1.33 there is no embedding weakly faithful and no efficient weakly exact translation for $stg \Rightarrow prf$.

Corollary 3.4.20 ($\mathbb{E}_6^c$: $stg \Rightarrow prf$). *Considering Translation 3.4.17 as* $Tr_{sem,prf}$ *and Translation 3.1.42 as* $Tr_{stg,sem}$ *the transformation* $Tr = Tr_{sem,prf} \circ Tr_{stg,sem}$ *is a covering* 1*-component local exact translation for* $stg \Rightarrow prf$.

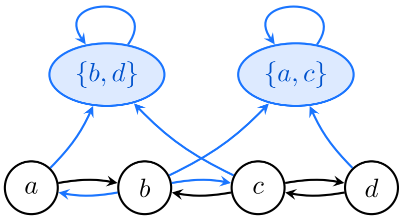


Figure 3.4.23: Illustration of Translation 3.4.21 ($grd \Rightarrow \sigma$) as described in Example 3.4.22.

Translation 3.4.21 ($\mathbb{E}$: $stg \Rightarrow stb$)**.** For an arbitrary argumentation framework $F = (A, R)$ we consider the transformation $Tr_0(F) = (A_0, R_0)$ with:

$$A_0 = \{a \mid a \in A, (a, a) \notin R\} \qquad \textit{conflict-removal}$$
$$R_0 = \{(a, b), (b, a) \mid a, b \in A_0, (a, b) \in R\} \qquad \textit{symmetry}$$

We continue by specifying the set $\mathscr{P} = naive(Tr_0(F)) \setminus stg(F)$ and by augmenting transformation Tr_0 to the extension-enhancing $Tr(F) = (A', R')$ with:

$$A' = A_0 \cup \mathscr{P} \qquad R' = R_0 \cup \{(a, P), (P, P) \mid P \in \mathscr{P}, a \in A_0 \setminus P\}$$

We claim that Tr is an exact translation for $stg \Rightarrow stb$.

Proof. We observe that $prf(Tr(F))$ and $naive(F)$ coincide. Furthermore $stg(F) \neq \emptyset$ and $stg(F) = \{\emptyset\}$ only if $(a, a) \in R$ for all $a \in A$, as a result $Tr(F) = (\emptyset, \emptyset)$. In the following we thus assume $\emptyset \notin stg(F)$.

Now if $E \in stg(F)$ then for any $P \in \mathscr{P}$ there is some $a \in E \setminus P$, $a \rightarrowtail P$ and thus E is a stable extension of $Tr(F)$. If on the other hand E is a stable extension of $Tr(F)$ it has to attack all $P \in \mathscr{P}$ and thus E has to be different from all $P \in \mathscr{P}$. Since E still needs to be a naive extension of F it follows that also $E \in stg(F)$. □

Example 3.4.22. Take into account the argumentation framework F with arguments a, b, c, d and e from Example 2.1.6. An application of Translation 3.4.21 is illustrated in Figure 3.4.23. Observe that the argument e is removed. Now $E = \{a, d\}$ is the only stage extension of F, and a stable extension of $Tr(F)$. Since $\{b, d\}$ is attacked only by a and c, one of a or c has to be a member of any stable extension of $Tr(F)$. The same holds for b and d. Since a is in conflict with b, b is in conflict with c and c is in conflict with d, we have that E is the only stable extension of $Tr(F)$.

3.5 Overview - Contributions to the Realm of Argumentation

Throughout this thesis we have presented a lot of new results, impossibilities, translations and generalizations. We have covered insights ranging from strong (embedding modular exact) to weak (oracle weakly faithful) translations, from strong (oracle) impossibilities to weak (modular) impossibilities.

Impossibilities and categorized translations are summarized in Table 3.5.1. We recall that by Definition 3.1.14, $\mathbb{E}$ denotes exactness, E denotes weak exactness, $\mathbb{F}$ denotes faithfulness and F denotes weak faithfulness; $\mathbb{E}^e$ refers to some exact embedding translation, F^c refers to some weakly faithful covering translation, E_i refers to some weakly exact translation where the value of i is resolved by:

0	modular	4	finite-diameter local
1	strictly local	6	1-component local
2	finitely local	7	monotone
3	efficient 1-component local	e	efficient

Furthermore each cell in Table 3.5.1 is composed of an upper (translations) and a lower part (impossibilities), parentheses are used to denote applicability of the respective result only to selected argumentation frameworks.

Selected intertranslatability results are visualized also by Figures 3.5.2 till 3.5.7 where an arrow represents directed translatability, sharing of some node represents undirected translatability, dotted arrows represent potential translations and impossibilities are presented by omission of other relations. We note that the represented relations are transitive, e.g. from weakly exact translations $cf \Rightarrow adm$ and $adm \Rightarrow com$ a weakly exact translation $cf \Rightarrow com$ is implied.

We would also like to hint to selected results not being covered by Table 3.5.1:

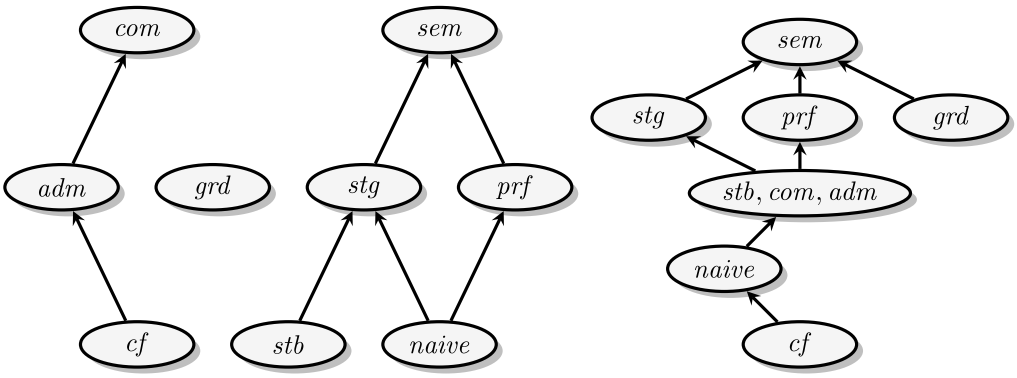
- Efficiency of finitely local (Theorem 3.1.4), strictly local and modular (Corollary 3.1.5) translations.
- Equality of stage and semi-stable semantics implies equality also of stable semantics for argumentation frameworks without self-attacks (Corollary 3.2.32).
- Possibility of $\bigcup stg(F) \cap \bigcup prf(F) = \emptyset$, even if $\emptyset \notin prf(F)$ (Example 3.2.29).

Furthermore due to importance, complexity or elegance we would like to highlight the following results:

- Impossibility of weakly faithful translations $(stb|stg|sem|prf|com|adm) \Rightarrow (cf|naive)$
- Impossibility of weakly exact translations $sem \Rightarrow stg$
- Impossibility of weakly exact translations $(sem|prf|com|adm) \Rightarrow (cf|naive|stg)$
- Impossibility of finite-diameter local weakly faithful translations $(sem|prf) \Rightarrow stg$
- Impossibility of efficient exact translations $grd \Rightarrow stg$
- Impossibility of finite-diameter local weakly exact translations $grd \Rightarrow (sem)$
- Impossibility of finite-diameter weakly faithful translations $grd \Rightarrow (stg|stb|prf|com|adm)$
- Possibility of embedding modular faithful translations $com \Rightarrow (stg|stb|sem|prf)$
- Possibility of embedding modular faithful translations $grd \Rightarrow sem$
- Possibility of embedding oracle exact translations $sem \Rightarrow prf$
- Possibility of finitely local faithful translations $(sem|prf) \Rightarrow stg$ for directed circles

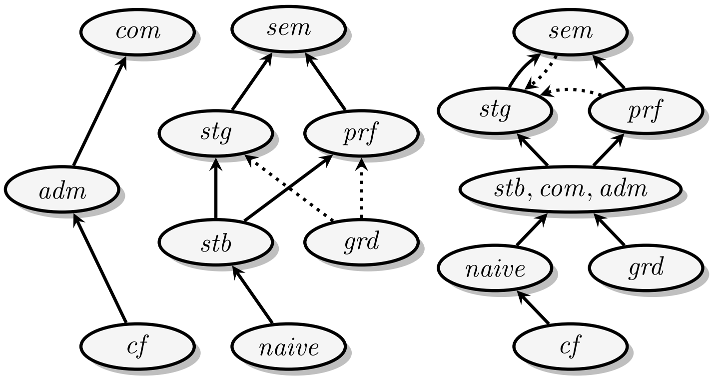
$\Rightarrow$	*cf*	*naive*	*stg*	*stb*	*sem*	*prf*	*com*	*adm*	*grd*
cf	$\mathbb{E}_0^e$: 3.1.12	$\mathbb{F}_0^e$: 3.1.27 E: 3.1.26	$\mathbb{F}_0^e$: 3.1.27 E: 3.1.26	$\mathbb{F}_0^e$: 3.1.27 E: 3.1.26	$\mathbb{F}_0^e$: 3.1.27 E: 3.1.26	$\mathbb{F}_0^e$: 3.1.27 E: 3.1.26	$\mathbb{E}_0^c$: 3.1.88 $\mathbb{F}_0^e$: 3.2.2 E^e: 3.1.25	$\mathbb{E}_0^c$: 3.1.22 $\mathbb{F}_0^e$: 3.2.2 E^e: 3.1.24	F: 3.3.1
naive	F: 3.3.4	$\mathbb{E}_0^e$: 3.1.12	$\mathbb{E}_0^e$: 3.1.30	$\mathbb{E}_e$: 3.1.47 $\mathbb{F}_0^e$: 3.1.51 E^c: 3.1.45 E_7: 3.1.46	$\mathbb{E}_0^c$: 3.1.88 $\mathbb{F}_0^e$: 3.2.2 E^e: 3.1.25	$\mathbb{E}_0^c$: 3.1.22 $\mathbb{F}_0^e$: 3.2.2 E^e: 3.1.25	F_0^e: 3.1.89 E: 3.1.63 $\mathbb{F}$: 3.1.64	F_0^e: 3.1.89 E: 3.1.60 $\mathbb{F}$: 3.1.61	F: 3.3.1
stg	F: 3.2.4	F: 3.2.4	$\mathbb{E}_0^e$: 3.1.12	$\mathbb{E}$: 3.4.21 ($\mathbb{E}_e$: 3.3.13) $\mathbb{F}_6^e$: 3.4.5 E^c: 3.1.45 E_7: 3.1.46 F_e: 3.1.33	$\mathbb{E}_1^c$: 3.1.42 ($\mathbb{E}_0^c$: 3.1.34) $\mathbb{F}_0^e$: 3.1.39 E_0: 3.1.38 E^e: 3.1.36	$\mathbb{E}_6^c$: 3.4.20 $\mathbb{F}_6^e$: 3.4.5 E^e: 3.1.37 F_e: 3.1.33	F_6^e: 3.4.8 E: 3.1.63 $\mathbb{F}$: 3.1.64 F_e: 3.1.33	F_6^e: 3.4.9 E: 3.1.60 $\mathbb{F}$: 3.1.61 F_e: 3.1.33	F: 3.3.1
stb	F: 3.2.4	F: 3.2.4	E_0^e: 3.1.54 $\mathbb{F}$: 3.1.44	$\mathbb{E}_0^e$: 3.1.12	E_0^e: 3.1.54 $\mathbb{F}$: 3.1.44	E_2^e: 3.1.69 F_0^e: 3.1.66 E_6: 3.1.68 $\mathbb{F}$: 3.1.44	(E_2^e: 3.1.59) F_0^e: 3.1.66 E: 3.1.62 $\mathbb{F}$: 3.1.64	(E_2^e: 3.1.57) F_0^e: 3.1.66 E: 3.1.60 $\mathbb{F}$: 3.1.61	F: 3.3.1
sem	F: 3.2.4	F: 3.2.4	$\mathbb{F}_6^e$: 3.4.5 ($\mathbb{F}_2^e$: 3.2.27) E: 3.2.6 F_4: 3.2.24	$\mathbb{F}_6^e$: 3.4.5 E: 3.2.9 F_e: 3.1.33	$\mathbb{E}_0^e$: 3.1.12	$\mathbb{E}_6^e$: 3.4.17 F_e: 3.1.33	F_6^e: 3.4.8 E: 3.1.63 $\mathbb{F}$: 3.1.64 F_e: 3.1.33	F_6^e: 3.4.9 E: 3.1.60 $\mathbb{F}$: 3.1.61 F_e: 3.1.33	F: 3.3.1
prf	F: 3.2.4	F: 3.2.4	$\mathbb{F}_6^e$: 3.4.5 ($\mathbb{F}_2^e$: 3.2.27) E: 3.2.8 F_4: 3.2.24	$\mathbb{F}_6^e$: 3.4.5 E: 3.2.9 F_e: 3.1.33	$\mathbb{E}_0^e$: 3.1.30	$\mathbb{E}_0^e$: 3.1.12	F_6^e: 3.4.8 E: 3.1.63 $\mathbb{F}$: 3.1.64 F_e: 3.1.33	F_6^e: 3.4.9 E: 3.1.60 $\mathbb{F}$: 3.1.61 F_e: 3.1.33	F: 3.3.1
com	F: 3.2.4	F: 3.2.4	$\mathbb{F}_0^e$: 3.1.72 E: 3.1.26	$\mathbb{F}_0^e$: 3.1.72 E: 3.1.26	$\mathbb{F}_0^e$: 3.1.72 E: 3.1.26	$\mathbb{F}_0^e$: 3.1.72 E: 3.1.26	$\mathbb{E}_0^e$: 3.1.12	F_0^e: 3.1.87 E: 3.1.60 $\mathbb{F}$: 3.1.61	F: 3.3.1
adm	F: 3.2.4	F: 3.2.4	$\mathbb{F}_0^e$: 3.1.85 E: 3.1.26	$\mathbb{F}_0^e$: 3.1.85 E: 3.1.26	$\mathbb{F}_0^e$: 3.1.85 E: 3.1.26	$\mathbb{F}_0^e$: 3.1.85 E: 3.1.26	$\mathbb{E}_0^e$: 3.1.30	$\mathbb{E}_0^e$: 3.1.12	F: 3.3.1
grd	F: 3.3.4	$\mathbb{E}^c$: 3.4.12 F_e: 3.3.3 F^e: 3.4.11 F_7: 3.4.11	$\mathbb{E}^c$: 3.4.12 $\mathbb{F}_3^e$: 3.3.9 ($\mathbb{F}_0^e$: 3.3.8) $\mathbb{E}_e$: 3.3.15 E^e: 3.3.16 E_7: 3.4.13 F_4: 3.3.20	$\mathbb{E}$: 3.4.10 $\mathbb{F}_3^e$: 3.3.9 ($\mathbb{F}_0^e$: 3.3.8) E_e: 3.3.2 E^c: 3.1.45 E_7: 3.1.46 F_4: 3.3.21	$\mathbb{E}_6^e$: 3.4.14 $\mathbb{F}_0^e$: 3.3.11 E_4: 3.3.18	$\mathbb{E}_6^e$: 3.4.14 $\mathbb{F}_3^e$: 3.3.9 ($\mathbb{F}_0^e$: 3.3.8) F_4: 3.3.20	$\mathbb{E}_6^e$: 3.4.14 $\mathbb{F}_3^e$: 3.3.9 ($\mathbb{F}_0^e$: 3.3.8) E_e: 3.3.2 F_4: 3.3.21	F_3^e: 3.3.9 (F_0^e: 3.3.8) E: 3.1.60 $\mathbb{F}$: 3.1.61 F_4: 3.3.21	$\mathbb{E}_0^e$: 3.1.12

Table 3.5.1: Summary of intertranslatability results referenced inside this thesis, see Definition 3.1.14 for decryption.



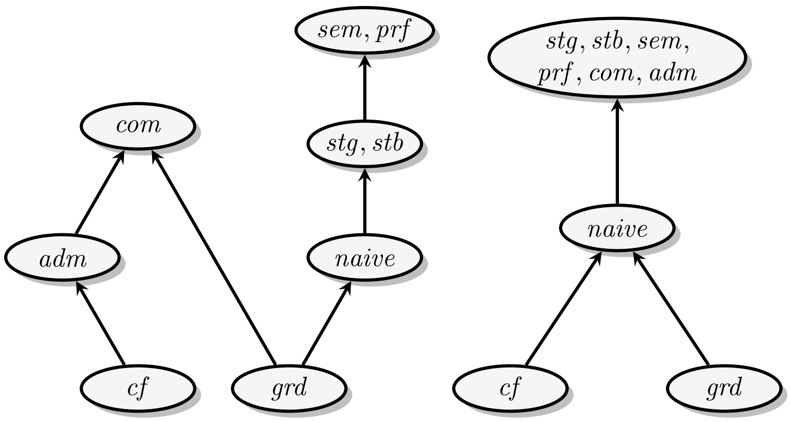
(a) (weakly) exact (b) (weakly) faithful

Figure 3.5.2: Results for strictly local intertranslatability.



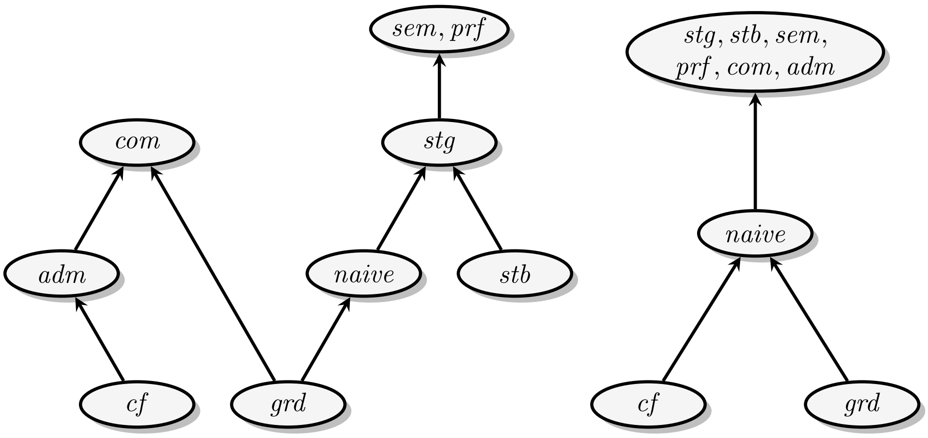
(a) (weakly) exact (b) (weakly) faithful

Figure 3.5.3: Results for efficient intertranslatability.



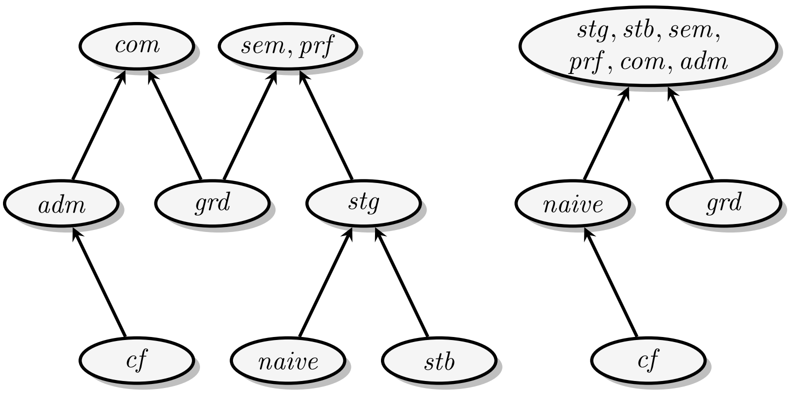
(a) (weakly) exact (b) (weakly) faithful

Figure 3.5.4: Results for inefficient intertranslatability.



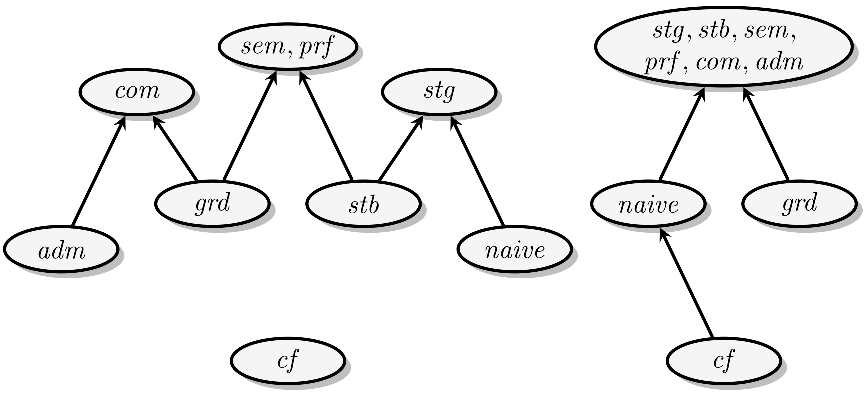
(a) (weakly) exact (b) (weakly) faithful

Figure 3.5.5: Results for covering intertranslatability.



(a) (weakly) exact (b) (weakly) faithful

Figure 3.5.6: Results for monotone intertranslatability.



(a) (weakly) exact (b) (weakly) faithful

Figure 3.5.7: Results for embedding intertranslatability.

4 Discussion

The last chapter of this thesis is dedicated to related work, a short summary of achievements, implications and thoughts about future work and open questions. We will proceed in this very order.

4.1 Related Work

4.1.1 Intertranslatability in a Very Wide Context of Argumentation

In the field of abstract argumentation and similar logical systems the use of collections of various semantics is widespread. Naturally comparison of these semantics is a common task and intertranslatability as presented in this thesis can be understood as such. Prior to [39] however comparison was mostly based on various semantical properties, quite similar to Section 2.3. Proof procedures for ideal semantics are presented in [30] with the aim of comparing abstract and assumption-based argumentation. Starting from set-inclusion properties in [7] the authors present and use partially new semantical characteristics for the single purpose of comparing the most common argumentation semantics.

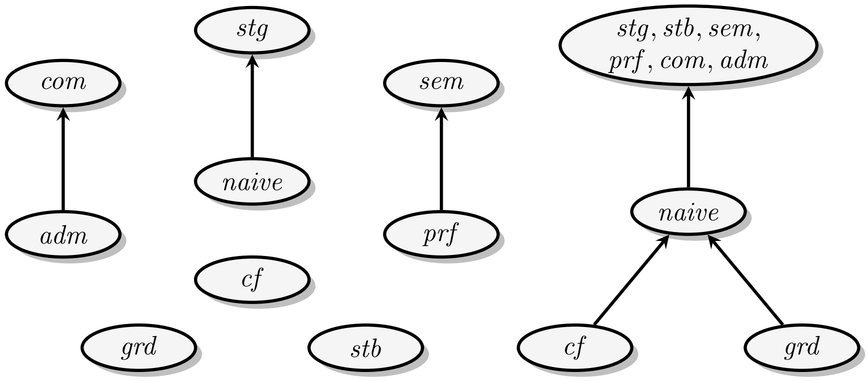
Semantics responding to strongly connected components are introduced in [9], the authors present the concept of SCC-recursiveness and transform semantical principles to express genuinely new semantics. A comparison of stable models in logic programs is to be found in [52]. In [19] a labelling approach is presented and used to interlock abstract argumentation semantics with modal logic models.

A generalization of labellings and higher level attacks is to be found in [43] introducing equational semantics. Higher level attacks also play a major role for AFRA [6] (drawing a line to Dung-style argumentation frameworks) as well as EAF [58] (formalizing logic programming).

Prior to [39] intertranslatability in the field of abstract argumentation was bound to comparison of logical systems rather than models in these systems. Thus mappings from frameworks to frameworks were mostly used to interlock different kinds of frameworks. Taking into account abstract dialectical frameworks [17], which allow arbitrary acceptance conditions, this form of coupling also applies to [16].

4.1.2 Graph Transformations

Similarities between argumentation frameworks and graph theory are obvious but not of practical nature. Similarly we can think of relations between graph transformations and



(a) embedding modular exact (b) oracle weakly faithful

Figure 4.2.1: Summary of the most restricted and the most free forms of intertranslatability.

framework translations. In plain theory a graph transformation is a mapping from graphs to graphs, applying certain rules, thus being actually close to our definition of framework transformation.

Practically spoken however graph transformations as used for various computational purposes operate more on a semantical level, while framework transformations as used in this thesis (compare Remark 2.6.3 on predictability) operate on highly abstracted graphs only. Thus e.g. a graph transformation maps UML graphs by mainly investigating relations of specifically named nodes. It seems to better suit the needs of abstract argumentation to treat each argument the same, thus we did restrict ourselves to predictable translations.

Still for the definition of locality graph transformations served as a source of inspiration. We refer to [66] and [49] for further reading.

4.2 Summary and Implications

In the preceding work we have investigated various intertranslatability results for selected argumentation semantics. We built upon [39] yet expanded semantics of interest to also include conflict-free and naive semantics. Furthermore we introduced a notion of locality, enabling classification of existing translations and we also investigated inefficient intertranslatability.

A detailed overview of gained results can be found in Section 3.5. Although there are still open questions as far as efficient translations are concerned, we managed to complete the big picture for the subclass of strictly local translations as well as for the superclass of oracle translations. As far as possible yet unknown efficient translations are concerned we managed to restrict these to transformations not being finite-diameter local and thus most likely complicated in definition.

As a side result the surprising relation of stage, semi-stable and stable semantics in self-attack-free argumentation frameworks (Corollary 3.2.32), as well as the relation of efficiency and modularity (Corollary 3.1.5) was discovered respectively pointed out.

$\Rightarrow$	stg	stb	sem	prf	com	adm
stg		$\mathbb{F}_4^e$		$\mathbb{F}_4^e$, $\mathbb{E}_4^c$	F_4^e	F_4^e
sem	$\mathbb{F}_2^e$, $\mathbb{F}_e^e$	$\mathbb{F}_4^e$		$\mathbb{E}_4^e$	F_4^e	F_4^e
prf	$\mathbb{F}_2^e$, $\mathbb{F}_e^e$	$\mathbb{F}_4^e$			F_4^e	F_4^e
grd	E_e^c, $\mathbb{F}_2^e$	$\mathbb{F}_2^e$	$\mathbb{E}_2^e$, $\mathbb{E}_e^e$	$\mathbb{E}_2^e$, $\mathbb{E}_e^e$, $\mathbb{F}_2^e$	$\mathbb{F}_2^e$	F_2^e

Table 4.3.1: Missing results, still open questions as far as focus of this thesis is concerned. See Definition 3.1.14 for decryption.

This thesis can be seen as a book of reference for intertranslatability of argumentation semantics, as well as an introduction to abstract argumentation. The concept of finite-diameter locality turned out to be an extensive tool for impossibility proofs where corresponding efficiency results are still hidden in the depths of "The Book".

4.3 Open Questions - Future Research

4.3.1 Evident Future Work

Although Table 3.5.1 grew quite big, there are still enough semantics out there to be checked against intertranslatability. CF2 [9], ideal [30] and prudent [25] semantics come to mind pretty soon, others might follow.

As mentioned in Section 4.1 mappings of different kinds of frameworks and logics have been investigated already. However we believe that the principles of intertranslatability as introduced in [39] are also adaptable to AFRA [6], EAF [58], ADF [17] as well as labelling-based approaches [19] and related logical systems.

4.3.2 Open Questions

We hereby summarize the still open intertranslatability questions for semantics cf, $naive$, stg, stb, sem, prf, com, adm, grd, the gaps between translations and impossibilities as far as our definitions are concerned. On the one hand we have cases where provably no efficient translation exists and we could come up with some 1-component local translation, thus leaving us with the question for existence of an oracle finite-diameter local translation. On the other hand we have cases where provably no finite-diameter local translation exists and we could come up with some (sometimes even efficient) 1-component local translation, thus leaving us with the question for existence of efficient or even finitely local translations.

We refer to Table 3.5.1 for a chart with actual results and Table 4.3.1 for a chart with missing results. Observe that these two tables do not leave any gaps with respect to locality and the other introduced translational properties. For instance $grd \Rightarrow stg$ might still be possible with some weakly exact efficient transformation Tr. If so Tr is not exact, not embedding and not local.

We observe that although the difference between oracle 1-component local and oracle finite-diameter local seems to be minimal, this differentiation might still be of practical use. We only need to think about sparsely populated argumentation frameworks, e.g. with a limited amount of attached attacks for each argument, where finite-diameter locality implies efficiency. Also if we think about stage semantics and an arbitrary argumentation framework F, $E \in stg(F)$, and any maximal subframework $F' \subseteq F$ with diameter 2 and center a such that $a \in A_{F'}$, $(a, a) \notin R_{F'}$ and from $b \in A_F$ with $dist(a, b) = 1$ it follows that also $b \in A_{F'}$. It follows that $A_{F'} \cap E \neq \emptyset$. Inefficient finite-diameter local translations thus might be possible and useful.

Finally, as mentioned in Remark 3.1.7, it remains an open question whether the relation between existence of finitely local and efficient 1-component local translations is bidirectional. In any case we could conclude interesting efficiency results.

4.3.3 Strong Intertranslatability

We refer to labellings as an alternative as opposed to the extensional approach. A labelling is a mapping from arguments to the set $\{in, out, undecided\}$, where intuitively "in" refers to arguments in the extension, "out" refers to arguments attacked by the extension and "undecided" refers to the remaining arguments (see also [73]). Strengthening our definitions we could think about translations preserving not only extensions but also labellings. Intuitively embedding weakly exact translations are label-preserving translations.

The concept of ideal semantics (see [30, 36, 32]) generalizes the relation of complete and grounded semantics, thus providing one unique ideal extension for any reasonable semantics. Translations preserving ideal semantics demonstrate another strengthening to our definitions. Intuitively embedding exact translations are already ideal-preserving translations.

We might also think of translations preserving other semantical properties or even specific graph theoretical properties such as being connected, acyclic or of limited treewidth [64].

4.3.4 Weak Intertranslatability

By restricting the argumentation frameworks or the semantical properties of interest, a weakening of the introduced concepts of intertranslatability can be achieved. Examples already used in this thesis are, for instance, restrictions to symmetric argumentation frameworks and frameworks without self-attacks.

We have presented some results applying only to non-empty argumentation frameworks. As touched in Remark 3.0.2 we abstained from thinking of these translations as being general due to the belief of principle-equality of empty semantics and empty extensions. Thus a weakening for our definitions of intertranslatability might be achieved by taking into account only argumentation frameworks for which the semantics of choice provides non-empty extensions. However argumentation frameworks of this kind most likely exceed the concept of efficiency.

As touched in Subsection 3.2.3 translations preserving only some reasoning problems might be more easy to achieve than general translations. Such partial translations might on the one hand serve special needs, on the other hand guide the path to full translations one day.

4.3.5 Extending Locality

With the definition of locality in mind we think about modifications. A first such modification might be to restrict the notion of isomorphic subframeworks to a more strict version.

$$Tr(F) = \bigcup_{L \in \mathscr{L}, A' \subseteq A_F, L \cong F|_{A'}} Tr(F|_{A'})$$

Thus allowing for instance removal of self-attacking arguments. Of course such a definition of restricted locality breaks the bidirectional relation of locality and monotonicity. Conventional local translations however can still be defined by this notion of restricted locality but $|\mathscr{L}|$ most likely blows up.

Another variation that comes to mind is the restriction to connected components, allowing 1-component local translations with finitely local definitions. Also strongly connected components, limited distance or other graph properties might be of interest.

In Definition 3.1.15 we have presented the union of transformations, allowing different transformations being applied to some argumentation framework at the same time. If we think of an ordering of these applications, subsequently applied transformations might as well remove previously added arguments, thus allowing some kind of add-remove modus operandi, operating upon one original argumentation framework.

4.3.6 Back to the Real World

Intertranslatability was introduced with the concept of efficiency in mind, thus application of encountered translations to practical problems seems to be apparent. By implementing efficient transformations, solvers for semantical issues can be reused. Comparisons of actual argumentation frameworks and corresponding semantics gain a new notion of equivalence, as put into practice in Chapter 1 and the following.

Example 4.3.2 (Strudel of the Day). Back in the days of glory Pallas Athena and Pál Erdős decided to bake a strudel. They had a look at their cookbook and soon cut down the strudels of interest to apple strudel, cream strudel and poppy-seed strudel. Erdős volunteered to check presence of necessary ingredients in the larder. For some reason on the way back he got lost and partially forgot his findings. Finally back to the kitchen he said, “We only have to choose between two strudels, for we are missing either apples or cream or poppy-seed, I just do not remember which.” “Well you know,” Athena replied, “I think it is still an option to bake any of the three strudels.”

What appears to be a strange story, actually reveals insights into intertranslatability for abstract argumentation semantics. Assuming Athena goes with naive semantics and Erdős goes with stable semantics, we only have to agree on the following

no apples	attacks	apple strudel, no cream and no poppy-seed
no cream	attacks	cream strudel, no apples and no poppy-seed
no poppy-seed	attacks	poppy-seed strudel, no apples and no cream

Upon inspection this interpretation as an abstract argumentation framework is isomorphic to the framework used for the proof of Theorem 3.2.4. Thus due to choice of semantics

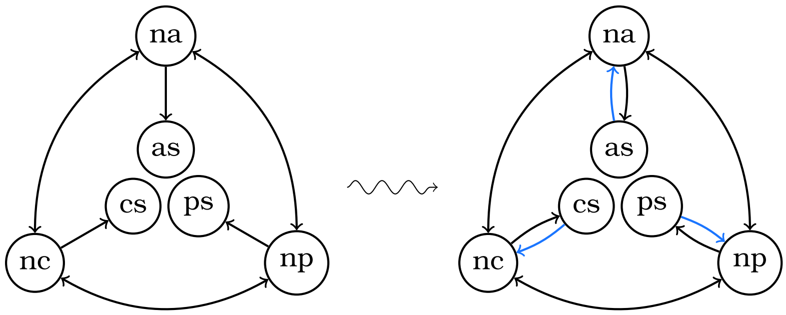


Figure 4.3.3: A graphical representation of matters from Example 4.3.2.

Athena can not agree with Erdős with the choice of extensions. Anyhow due to Translation 3.1.47 Erdős can reconceive the common knowledge base to understand Athena's point of view, in this case simply by symmetrizing the attacks.

We present an illustration of this transformation in Figure 4.3.3 where the arguments are referred by:

no apples	=	na	apple strudel	=	as
no cream	=	nc	cream strudel	=	cs
no poppy-seed	=	np	poppy-seed strudel	=	ps

To the glorious minds who keep being curious, searching and naive! To "The Book" of all strudel recipes. To the universe, spirituality and science, who keep surprising each other!

Figure 4.3.4: An Illustration of Pál Erdős and Pallas Athena. [44]

Bibliography

[1] L. Amgoud, C. Cayrol, M.-C. Lagasquie-Schiex, and P. Livet. On bipolarity in argumentation frameworks. Int. J. Intell. Syst., 23(10):1062–1093, 2008.

[2] L. Amgoud, Y. Dimopoulos, and P. Moraitis. A unified and general framework for argumentation-based negotiation. In E. H. Durfee, M. Yokoo, M. N. Huhns, and O. Shehory, editors, Proceedings of the 6th International Joint Conference on Autonomous Agents and Multiagent Systems (AAMAS 2007), pages 963–970. IFAAMAS, 2007.

[3] L. Amgoud, S. Parsons, and N. Maudet. Arguments, dialogue, and negotiation. In W. Horn, editor, ECAI 2000 - 14th European Conference on Artificial Intelligence, Frontiers in Artificial Intelligence and Applications, pages 338–342. IOS Press, 2000.

[4] P. Baroni, M. Caminada, and M. Giacomin. An introduction to argumentation semantics. Knowledge Eng. Review, 26(4):365–410, 2011.

[5] P. Baroni, F. Cerutti, P. E. Dunne, and M. Giacomin. Computing with infinite argumentation frameworks: the case of AFRAs. In Proooceedings of First International Workshop on the Theory and Applications of Formal Argumentation (TAFA 2011), 2011.

[6] P. Baroni, F. Cerutti, M. Giacomin, and G. Guida. AFRA: Argumentation framework with recursive attacks. Int. J. Approx. Reasoning, 52(1):19–37, 2011.

[7] P. Baroni and M. Giacomin. On principle-based evaluation of extension-based argumentation semantics. Artif. Intell., 171(10-15):675–700, 2007.

[8] P. Baroni and M. Giacomin. Semantics of abstract argument systems. In I. Rahwan and G. R. Simari, editors, Argumentation in Artificial Intelligence, chapter 2, pages 25–44. Springer, 2009.

[9] P. Baroni, M. Giacomin, and G. Guida. SCC-recursiveness: A general schema for argumentation semantics. Artif. Intell., 168(1-2):162–210, 2005.

[10] R. Baumann and G. Brewka. Expanding argumentation frameworks: Enforcing and monotonicity results. In P. Baroni, F. Cerutti, M. Giacomin, and G. R. Simari, editors, Proceedings of the 3rd Conference on Computational Models of Argument (COMMA 2010), volume 216 of Frontiers in Artificial Intelligence and Applications, pages 75–86. IOS Press, 2010.

[11] T. J. M. Bench-Capon and K. Atkinson. Abstract argumentation and values. In I. Rahwan and G. R. Simari, editors, Argumentation in Artificial Intelligence, chapter 3, pages 45–64. Springer, 2009.

[12] T. J. M. Bench-Capon and P. E. Dunne. Argumentation in AI and law: Editors' introduction. Artif. Intell. Law, 13(1):1–8, 2005.

[13] T. J. M. Bench-Capon and P. E. Dunne. Argumentation in artificial intelligence. Artif. Intell., 171(10-15):619–641, 2007.

[14] T. J. M. Bench-Capon, H. Prakken, and G. Sartor. Argumentation in legal reasoning. In I. Rahwan and G. R. Simari, editors, Argumentation in Artificial Intelligence, chapter 18, pages 363–382. Springer, 2009.

[15] P. Besnard and S. Doutre. Checking the acceptability of a set of arguments. In J. P. Delgrande and T. Schaub, editors, Proceedings of the 10th International Workshop on Non-Monotonic Reasoning (NMR 2002), pages 59–64, 2004.

[16] G. Brewka, P. E. Dunne, and S. Woltran. Relating the semantics of abstract dialectical frameworks and standard AFs. In T. Walsh, editor, Proceedings of the 22nd International Joint Conference on Artificial Intelligence (IJCAI 2011), pages 780–785. AAAI Press, 2011.

[17] G. Brewka and S. Woltran. Abstract dialectical frameworks. In F. Lin, U. Sattler, and M. Truszczyński, editors, Proceedings of the 12th International Conference on Principles of Knowledge Representation and Reasoning (KR 2010), pages 102–111. AAAI Press, 2010.

[18] M. Caminada. Semi-stable semantics. In P. E. Dunne and T. J. M. Bench-Capon, editors, Proceedings of the 1st Conference on Computational Models of Argument (COMMA 2006), volume 144 of Frontiers in Artificial Intelligence and Applications, pages 121–130. IOS Press, 2006.

[19] M. Caminada and D. M. Gabbay. A logical account of formal argumentation. Studia Logica, 93(2):109–145, 2009.

[20] M. Caminada and B. Verheij. On the existence of semi-stable extensions. In Proceedings of the 22nd Benelux Conference on Artificial Intelligence, 2010.

[21] C. Cayrol and M.-C. Lagasquie-Schiex. Bipolar abstract argumentation systems. In I. Rahwan and G. R. Simari, editors, Argumentation in Artificial Intelligence, chapter 4, pages 65–84. Springer, 2009.

[22] F. Cerutti. Decision support through argumentation-based practical reasoning. In T. Walsh, editor, Proceedings of the 22nd International Joint Conference on Artificial Intelligence (IJCAI 2011), pages 2786–2787. AAAI Press, 2011.

[23] C. I. Chesñevar, A. G. Maguitman, and G. R. Simari. Recommender system technologies based on argumentation 1. In I. Maglogiannis, K. Karpouzis, M. Wallace, and J. Soldatos, editors, Emerging Artificial Intelligence Applications in Computer Engineering, volume 160 of Frontiers in Artificial Intelligence and Applications, pages 50–73. IOS Press, 2007.

[24] P. J. Cohen. Set theory and the continuum hypothesis. W. A. Benjamin, Inc., New York-Amsterdam, 1966.

[25] S. Coste-Marquis, C. Devred, and P. Marquis. Prudent semantics for argumentation frameworks. In 17th IEEE International Conference on Tools with Artificial Intelligence (ICTAI 2005), pages 568–572. IEEE Computer Society, 2005.

[26] S. Coste-Marquis, C. Devred, and P. Marquis. Symmetric argumentation frameworks. In L. Godo, editor, Proceedings of the 8th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU 2005), volume 3571 of Lecture Notes in Computer Science, pages 317–328. Springer, 2005.

[27] J. de Bruijn, T. Eiter, and H. Tompits. Embedding approaches to combining rules and ontologies into autoepistemic logic. In G. Brewka and J. Lang, editors, Proceedings of the 11th International Conference on Principles of Knowledge Representation and Reasoning (KR 2008), pages 485–495. AAAI Press, 2008.

[28] Y. Dimopoulos and A. Torres. Graph theoretical structures in logic programs and default theories. Theor. Comput. Sci., 170(1-2):209–244, 1996.

[29] P. M. Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. Artif. Intell., 77(2):321–358, 1995.

[30] P. M. Dung, P. Mancarella, and F. Toni. Computing ideal sceptical argumentation. Artif. Intell., 171(10-15):642–674, 2007.

[31] P. M. Dung, P. M. Thang, and F. Toni. Towards argumentation-based contract negotiation. In P. Besnard, S. Doutre, and A. Hunter, editors, Proceedings of the 2nd Conference on Computational Models of Argument (COMMA 2008), volume 172 of Frontiers in Artificial Intelligence and Applications, pages 134–146. IOS Press, 2008.

[32] P. E. Dunne. The computational complexity of ideal semantics. Artif. Intell., 173(18):1559–1591, 2009.

[33] P. E. Dunne and T. J. M. Bench-Capon. Coherence in finite argument systems. Artif. Intell., 141(1/2):187–203, 2002.

[34] P. E. Dunne and M. Caminada. Computational complexity of semi-stable semantics in abstract argumentation frameworks. In S. Hölldobler, C. Lutz, and H. Wansing, editors, Proceedings of the 11th European Conference on Logics in Artificial Intelligence (JELIA 2008), volume 5293 of Lecture Notes in Computer Science, pages 153–165. Springer, 2008.

[35] P. E. Dunne and M. Wooldridge. Complexity of abstract argumentation. In I. Rahwan and G. R. Simari, editors, Argumentation in Artificial Intelligence, chapter 5, pages 85–104. Springer, 2009.

[36] W. Dvořák, P. E. Dunne, and S. Woltran. Parametric properties of ideal semantics. In T. Walsh, editor, Proceedings of the 22nd International Joint Conference on Artificial Intelligence (IJCAI 2011), pages 851–856. AAAI Press, 2011.

[37] W. Dvořák and C. Spanring. Comparing the expressiveness of argumentation semantics. In B. Verheij, S. Szeider, and S. Woltran, editors, Proceedings of the 4th Conference on Computational Models of Argument (COMMA 2012), volume 245 of Frontiers in Artificial Intelligence and Applications, pages 261–272. IOS Press, 2012.

[38] W. Dvořák and S. Woltran. Complexity of semi-stable and stage semantics in argumentation frameworks. Inf. Process. Lett., 110(11):425–430, 2010.

[39] W. Dvořák and S. Woltran. On the intertranslatability of argumentation semantics. J. Artif. Intell. Res. (JAIR), 41:445–475, 2011.

[40] H.-D. Ebbinghaus and J. Flum. Finite Model Theory. Springer, 2005.

[41] H.-D. Ebbinghaus, J. Flum, and W. Thomas. Mathematical Logic. Springer, 1996.

[42] D. M. Gabbay. Semantics for higher level attacks in extended argumentation frames part 1: Overview. Studia Logica, 93(2):357–381, 2009.

[43] D. M. Gabbay. Introducing equational semantics for argumentation networks. In W. Liu, editor, Proceedings of the 11th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU 2011), volume 6717 of Lecture Notes in Computer Science, pages 19–35. Springer, 2011.

[44] M.-T. Gallnbrunner. Illustrations, 2011.

[45] M. R. Garey and D. S. Johnson. Computers and Intractability: A Guide to the Theory of NP-Completeness. William H. Freeman, 1979.

[46] K. Gödel. Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. Monatsh. Math. Phys., 38(1):173–198, 1931.

[47] G. Gottlob. Translating default logic into standard autoepistemic logic. J. ACM, 42(4):711–740, 1995.

[48] C. L. Hamblin. Fallacies. Methuen [London], 1970.

[49] R. Heckel. Graph transformation in a nutshell. Electronic Notes in Theoretical Computer Science, 148(1):187–198, 2006.

[50] T. Imielinski. Results on translating defaults to circumscription. Artif. Intell., 32(1):131–146, 1987.

[51] T. Janhunen. On the intertranslatability of non-monotonic logics. Ann. Math. Artif. Intell., 27(1-4):79–128, 1999.

[52] T. Janhunen, I. Niemelä, D. Seipel, P. Simons, and J.-H. You. Unfolding partiality and disjunctions in stable model semantics. ACM Trans. Comput. Log., 7(1):1–37, 2006.

[53] A. C. Kakas, L. Amgoud, G. Kern-Isberner, N. Maudet, and P. Moraitis. ABA: Argumentation based agents. In H. Coelho, R. Studer, and M. Wooldridge, editors, ECAI 2010 - 19th European Conference on Artificial Intelligence, volume 215 of Frontiers in Artificial Intelligence and Applications, pages 1005–1006. IOS Press, 2010.

[54] K. Kunen. Set Theory An Introduction To Independence Proofs (Studies in Logic and the Foundations of Mathematics). North Holland, 1983.

[55] P. Liberatore. Bijective faithful translations among default logics. CoRR, abs/0707.3781, 2007.

[56] P. McBurney, I. Rahwan, S. Parsons, and N. Maudet, editors. Argumentation in Multi-Agent Systems, 6th International Workshop, ArgMAS 2009, Revised Selected and Invited Papers, volume 6057 of Lecture Notes in Computer Science. Springer, 2010.

[57] S. Modgil. Labellings and games for extended argumentation frameworks. In C. Boutilier, editor, Proceedings of the 21st International Joint Conference on Artificial Intelligence (IJCAI 2009), pages 873–878. AAAI Press, 2009.

[58] S. Modgil. Reasoning about preferences in argumentation frameworks. Artif. Intell., 173(9-10):901–934, 2009.

[59] M. Možina, J. Žabkar, and I. Bratko. Argument based machine learning. Artif. Intell., 171:922–937, 2007.

[60] C. M. Papadimitriou. Computational Complexity. Addison-Wesley, 1994.

[61] D. Pearce and L. Uridia. The Gödel and the splitting translations. In G. Brewka, V. W. Marek, and M. Truszczyński, editors, Nonmonotonic Reasoning: Essays Celebrating its 30th Anniversary, volume 31, pages 335–360. College Publications, 2011.

[62] A. D. Procaccia and J. S. Rosenschein. Extensive-form argumentation games. In M. P. Gleizes, G. A. Kaminka, A. Nowé, S. Ossowski, K. Tuyls, and K. Verbeeck, editors, Proceedings of the Third European Workshop on Multi-Agent Systems (EUMAS 2005), pages 312–322. Koninklijke Vlaamse Academie van Belie voor Wetenschappen en Kunsten, 2005.

[63] O. Reingold. Undirected connectivity in log-space. J. ACM, 55(4):17:1–17:24, 2008.

[64] N. Robertson and P. D. Seymour. Graph minors. II. Algorithmic aspects of tree-width. Journal of Algorithms, 7(3):309 – 322, 1986.

[65] H. Rogers. Theory of Recursive Functions and Effective Computability. MIT Press, 1987.

[66] G. Rozenberg. Handbook of Grammars and Computing by Graph Transformation. World Scientific, 1997.

[67] S. Toulmin. The Uses of Argument. Cambridge University Press, 2003.

[68] A. M. Turing. On computable numbers, with an application to the Entscheidungsproblem. Proc. London Math. Soc., s2-42(1):230–265, 1937.

[69] F. H. van Eemeren, R. F. Grootendorst, and F. S. Henkemans. Fundamentals of Argumentation Theory: A Handbook of Historical Backgrounds and Contemporary Applications. Lawrence Erlbaum Associates, 1996.

[70] B. Verheij. Two approaches to dialectical argumentation: admissible sets and argumentation stages. In J.-J. C. Meyer and L. C. van der Gaag, editors, Proceedings of the 8th Dutch Conference on Artificial Intelligence (NAIC'96), pages 357–368, 1996.

[71] D. Walton. Argumentation theory: A very short introduction. In I. Rahwan and G. R. Simari, editors, Argumentation in Artificial Intelligence, chapter 1, pages 1–22. Springer, 2009.

[72] Wikipedia. Liar paradox — Wikipedia, the free encyclopedia, 2011. [Online; accessed 27-August-2011].

[73] Y. Wu and M. Caminada. A labelling-based justification status of arguments. Studies in Logic, 3(4):12–29, 2010.

Curriculum Vitae

Personal Data	Christof Spanring
	February 22, 1981, Kirchdorf/Krems
	spanring@dbai.tuwien.ac.at

Formal Education

June, 1999	Matura, BRG Kirchdorf/Krems

Discontinued Studies

2000 – 2001	TU Wien, Wirtschaftsinformatik
2000 – 2002	TU Wien, Technische Mathematik
2000 – 2004	TU Wien, Informatik
2001 – 2002	Universität Wien, Japanologie/Philosophie
2003 – 2005	Konservatorium der Stadt Wien, Jazztrompete

Ongoing Studies

2008 – 2009	FU/HU/TU Berlin, Erasmus year abroad
2001 – 2013	Universität Wien, Mathematik, Logik und Grundlagenforschung der Mathematik

Publications

2012	W. Dvořák and C. Spanring. Comparing the Expressiveness of Argumentation Semantics. [37]