Hasenöhrl

Gastheorie

IV

Hans Thirring

[illegible]

$$F(a_1,\dots a_n) = \int_x^{x_1(a_1\dots a_n)}\int^{x_2(a_1\dots a_n)}\dots\int^{x_n(a_1\dots a_n)} dx_1\dots dx_n$$

$$= \int^{\alpha_1}\int^{\alpha_2}\dots\int^{\alpha_n}\frac{\partial(x_1\dots x_n)}{\partial(\alpha_1\dots a_n)}\,da_1\dots da_n$$

[illegible] Phasendichte [illegible] Bewegungsgleichungen [illegible] $D(p\,q\,t)$ [illegible] wegende Fläche [illegible] Repräsentativpunkt [illegible] $D = C$ dieselben Relationen [illegible]

Die Substitution der Momente durch die Momentvariable ist linear mit Koeff. von q abhängig daher kann die Determinante aus dem Integral herausgehoben werden

24. II 1910. W 35-196

$\bar{Q}$ [illegible] $\quad N\bar Q = \int dw\, f Q \quad \frac{\partial}{\partial t}\int f\, C\dots$

$$\frac{\partial(\rho\bar Q)}{\partial t} = \rho\frac{D\bar Q}{Dt} - \left[\frac{\partial\rho\overline{uQ}}{\partial x} + \frac{\partial\rho\overline{vQ}}{\partial y} + ''\right] + \rho\left[\overline{X\frac{\partial Q}{\partial u}} + \overline{Y\frac{\partial Q}{\partial v}} + ''\right]$$

$$\frac{\partial\rho}{\partial t} = -\left(\frac{\partial\rho\bar u}{\partial x} + '' \; ''\right)$$ $Q = 1$ Kontinuitätsgleichung

$u = \bar u + \xi \quad v = \bar v + \eta \quad w = \bar w + \zeta$

$\bar u\ \bar v\ \bar w$ beobachtbar $\xi\ \eta\ \zeta$ molekular.

$$\rho\frac{\partial\bar Q}{\partial t} - \bar Q\left(\frac{\partial\rho\bar u}{\partial x} + \frac{\partial\rho\bar v}{\partial y} + ''\right) = \rho\frac{D\bar Q}{Dt} - \left[\frac{\partial}{\partial x}(\rho\bar u\bar Q + \rho\overline{\xi Q}) + \frac{\partial}{\partial y}(\rho\bar v\bar Q + \rho\overline{\eta Q}) \; ''\right]$$
$$+ \rho\left[\overline{X\frac{\partial Q}{\partial u}} + \overline{Y\frac{\partial Q}{\partial v}} + \overline{Z\frac{\partial Q}{\partial w}}\right]$$

$$\overline{uQ} = \overline{(\bar u + \xi)Q} = \underbrace{\overline{\bar u Q}}_{\bar u\bar Q} + \overline{\xi Q} \qquad \frac{\partial Q}{\partial u} = \frac{\partial Q}{\partial \xi} \qquad \overline{\frac{\partial Q}{\partial u}} = \frac{\partial\bar Q}{\partial \xi}$$

$$\frac{\partial}{\partial x}(\rho\bar u\bar Q) = \rho\bar u\frac{\partial\bar Q}{\partial x} + \bar Q\frac{\partial(\rho\bar u)}{\partial x}$$ [illegible] $\bar Q\frac{\partial\rho\bar u}{\partial x}$

$$\varrho\left[\frac{\partial\bar Q}{\partial t}+\bar u\frac{\partial \bar Q}{\partial x}+\bar v\frac{\partial\bar Q}{\partial y}+\bar w\frac{\partial \bar Q}{\partial z}\right] = \varrho\frac{D\bar Q}{Dt} - \left[\frac{\partial}{\partial x}\varrho\overline{u'Q'}+\frac{\partial}{\partial y}\varrho\overline{v'Q'}\right]+\varrho\left[\overline{X\frac{\partial Q}{\partial u}}\ldots\right]$$

$$\underbrace{\qquad}_{\frac{d\bar Q}{dt}}$$

[illegible] $Q = u$

[illegible] $u_1\ u_2\ \ldots\ u_1'\ u_2'$

$\overline{uu} = \overline{u(\bar u + u')} = \bar u^2$ $\quad u_1+u_2 = u_1'+u_2'$ [illegible] $\frac{D\bar u}{Dt}=0$

$$\varrho\frac{d\bar u}{dt} = -\frac{\partial(\varrho\overline{u'^2})}{\partial x} - \frac{\partial\varrho\overline{u'v'}}{\partial y} - \frac{\partial\varrho\overline{u'w'}}{\partial z}+\varrho X$$

$$\varrho\frac{d\bar v}{dt} = -\frac{\partial(\varrho\overline{u'v'})}{\partial x} - \frac{\partial(\varrho\overline{v'^2})}{\partial y} - \frac{\partial(\varrho\overline{v'w'})}{\partial z}+\varrho Y$$

$$\varrho\frac{d\bar w}{dt} = -\frac{\partial(\varrho\overline{u'w'})}{\partial x} - \frac{\partial(\varrho\overline{v'w'})}{\partial y} - \frac{\partial(\varrho\overline{w'^2})}{\partial z}+\varrho Z$$

[illegible]

$$\varrho\frac{du}{dt} = \varrho X - \underbrace{\frac{\partial X_x}{\partial x} - \frac{\partial X_y}{\partial y} - \frac{\partial X_z}{\partial z}}$$

[illegible] $\frac{du}{dt}$ [illegible] $\frac{Du}{Dt}$.

[illegible] Eulerschen [illegible]

$X_x = \varrho\overline{u'^2} \quad Y_y = \varrho\overline{v'^2} \quad Z_z = \varrho\overline{w'^2}$ $\quad -\frac{\partial p}{\partial x} + \eta\Delta u$

wenn man diese Annahmen macht [illegible] = [illegible] Euler's

$X_y = Y_x = \varrho\overline{u'v'} \quad Y_z = Z_y = \varrho\overline{v'w'} \quad Z_x = X_z = \varrho\overline{w'u'}$

[illegible]

[illegible] Hydromechanik [illegible] $X_x = Y_y = Z_z = p$

$Y_x = \ldots Z_x = 0$

[illegible] nicht reibendes [illegible] also $\overline{u'v'} = \overline{v'w'} = \overline{w'u'} = 0$

$$\varrho\overline{u'^2} = \varrho\overline{v'^2} = \varrho\overline{w'^2} = p = \frac{1}{3}\varrho\overline{c^2}$$

$$p = \frac{Nm\overline{c^2}}{3}$$ Gasdruckformel.

[illegible] $\frac{D\bar Q}{Dt} = 0$ [illegible] $u'^2+v'^2+w'^2$

[illegible]

[illegible] Hydrodyn. [illegible] $X_y = \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$ [illegible]

[illegible] $\overline{u'v'}$ [illegible]

[illegible]

[illegible]

[illegible] $\varphi(r)=0 \quad r>\sigma$

$\varphi(r) \gg 1 \quad r \leqq \sigma$

[illegible] ~ [illegible]

Zusammenstoß [illegible] merkliche Wechselwirkung stattfindet.

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1. dw_1
2. dw_2 } [illegible]

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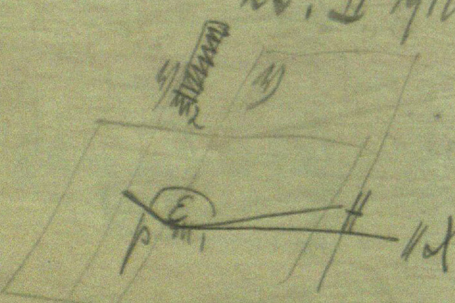


Klasse K_1 dw_1 } [illegible] | [illegible] Gattung I: K_1 [illegible] K_2

K_2 dw_2 $\varepsilon+\delta\varepsilon$

$p+\delta p$

[illegible] I

[illegible] 1 2 primmatinken [illegible] $p\,dp\,d\varepsilon$ [illegible]

$p\,dp\,d\varepsilon\, \mathfrak{N}\, f_1 dw_1 f_2 dw_2$ [illegible] I

$Q_1 = Q(u_1, v_1, w_1)$ [illegible]

[illegible] Q_1+Q_2 [illegible] $Q_1'+Q_2'$

[illegible] ΣQ [illegible] $[Q_1'+Q_2'-Q_1+Q_2]\, p\,dp\,d\varepsilon\, \mathfrak{N}\, f_1 dw_1 f_2 dw_2$

[illegible] u, v, w u_1, v_1, w_1

$$\frac{\partial \bar{Q}}{\partial t}\cdot N = \frac{1}{2}\int_0^\infty dw_1 \int_0^\infty dw_2 \int_0^\sigma p\,dp \int_0^{2\pi} d\varepsilon\, \mathfrak{N}\, f_1 f_2 \left[Q_1'+Q_2'-Q_1+Q_2\right]$$

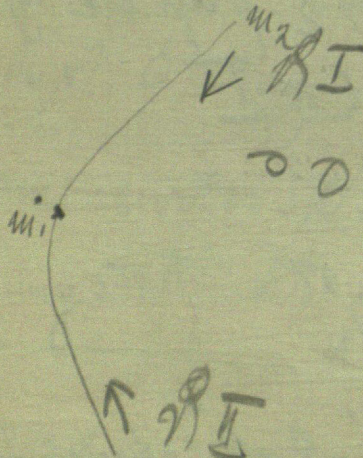
[illegible]

Klassen K_1' dw_1' f_1' [illegible]

" K_2' dw_2' f_2'

$C_1, C_2 \to C_1', C_2'$ $\quad C_1', C_2' \to C_1, C_2$

$$N\frac{d\bar{Q}}{dt} = \frac{1}{2}\iiint d\omega_1' \iiint d\omega_2' \int p' dp' \int d\varepsilon\, n\, f_1' f_2' \left[Q_1 + Q_2 - (Q_1' + Q_2')\right]$$



[illegible] m_2 [illegible] Perihel symmetrisch.

[illegible] Akzente [illegible]

[illegible] $d\omega_1' d\omega_2'$ [illegible] $d\omega_1 d\omega_2 = d\omega_1' d\omega_2'$

[illegible] Energieprinzip [illegible] Flächensatz [illegible]

$$\varrho\frac{d\bar{Q}}{dt} = \frac{m}{4}\iiint d\omega_1 \iiint d\omega_2 \int_0^{\sigma} p\,dp \int_0^{2\pi} d\varepsilon\, n \left[Q_1 + Q_2 - (Q_1' + Q_2')\right](f_1' f_2' - f_1 f_2)$$

[illegible]

$\varphi(r)$ [illegible]

$$m_1 \frac{d^2 x_1}{dt^2} = \varphi(r)\frac{x_1 - x_2}{r} \qquad m_2 \frac{d^2 x_2}{dt^2} = \varphi(r)\frac{x_2 - x_1}{r}$$

[illegible]

$$\frac{d^2(x_2 - x_1)}{dt^2} = \underbrace{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}_{\frac{1}{M}} \varphi(r)\frac{x_2 - x_1}{r}$$

Zweikörperproblem.

$$x_2 - x_1 = \xi$$

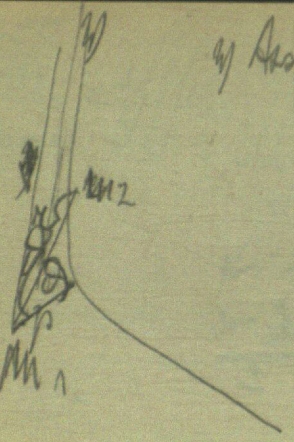
$$M\frac{d^2\xi}{dt^2} = \varphi(r)\frac{\xi}{r} \qquad M\frac{d^2\eta}{dt^2} = \varphi(r)\frac{\eta}{r}$$

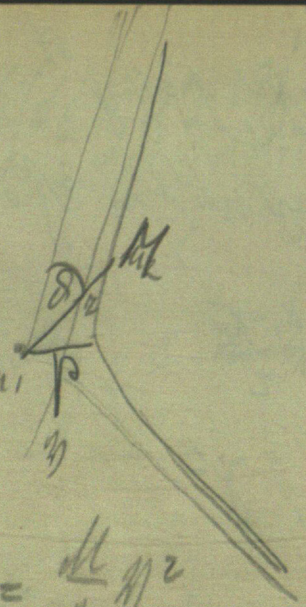
[illegible] M.

~~[illegible]~~

[illegible] ξ.

a) Asymptote





$$\frac{M}{2}\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\vartheta}{dt}\right)^2\right] + \Phi = \frac{M}{2} v^2$$

$$\frac{1}{2} r^2 \frac{d\vartheta}{dt} = \text{Const}$$

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$$\Psi = -\int_\infty^r \varphi(r)\,dr \qquad \varphi = -\frac{\partial \Psi}{\partial r} \qquad \Psi \text{ pot. Energ.}$$

$$\frac{M}{2}\left[\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\vartheta}{dt}\right)^2\right] + \Psi = \frac{M}{2} v^2 \qquad \Psi(\infty) = 0$$

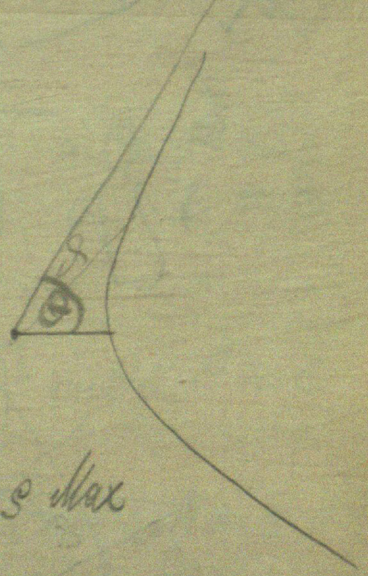
$$\frac{1}{2} r^2 \frac{d\vartheta}{dt} = \frac{1}{2} v p$$

$$dt = \frac{r^2 d\vartheta}{v p} \qquad \frac{M}{2}\left[\frac{dr^2 + r^2 d\vartheta^2}{r^4 d\vartheta^2} v^2 p^2\right] + \Psi = \frac{M}{2} v^2$$

$$p^2\left[\frac{1}{r^4}\left(\frac{dr}{d\vartheta}\right)^2 + \frac{1}{r^2}\right] + \frac{2\Psi}{M v^2} = 1 \qquad \frac{p}{r} = \varrho$$

$$\frac{p^2}{r^4} dr^2 = \left(+1 - \frac{2\Psi}{M v^2} - \frac{p^2}{r^2}\right) d\vartheta^2 \qquad -\frac{p\,dr}{r^2} = d\varrho$$

$$d\varrho^2 = \left(1 - \varrho^2 - \frac{2\Psi}{M v^2}\right) d\vartheta^2$$



$$\vartheta = \int_0^\varrho d\varrho \frac{1}{\sqrt{1 - \varrho^2 - \frac{2\Psi}{M v^2}}}$$

$$\Theta = \int_0^{\varrho_{max}} \frac{d\varrho}{\sqrt{\ldots}} \qquad \sqrt{1 - \varrho^2 - \frac{2\Psi}{M v^2}} = 0 \quad \varrho \text{ Max}$$

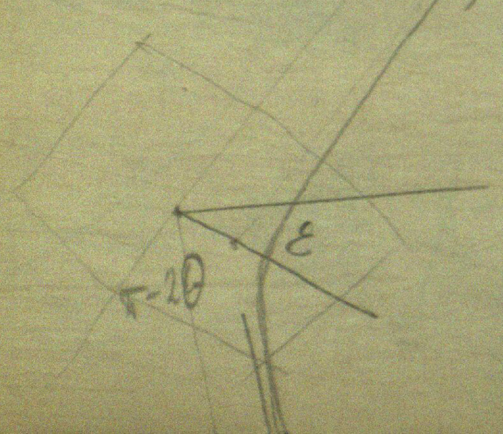
Spezialisierung $\varphi = \frac{K}{r^{n+1}}$

$\phi = \frac{k}{mr^n} = \frac{k\varrho^n}{mp^n}$

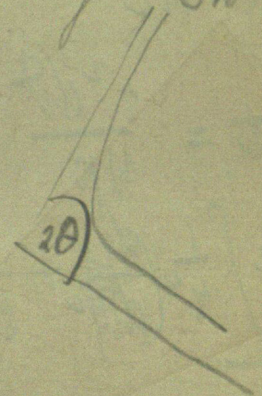
$\Theta = \int_0^{\varrho_{max}} \frac{d\varrho}{\sqrt{1-\varrho^2-\frac{2k\varrho^n}{mv^2 p^n}}}$

Ablenkung $\pi - \Theta$

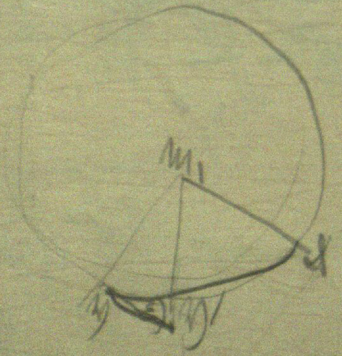
$\varepsilon = \sphericalangle\{p, (m\,o)\}$

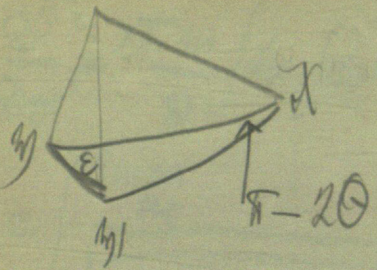


e cos Θ [illegible]



[illegible]





$\cos(v'x) = \cos(vx)\cos(\pi-2\Theta) + \sin(vx)\sin(\pi-2\Theta)\cos\varepsilon$

$|v'|\cos(v'x)$ [illegible] relativen [illegible]

$= u_2'-u_1' = (u_2-u_1)(-\cos 2\Theta) + \sqrt{v^2-(u_2-u_1)^2}\,\sin 2\Theta\cos\varepsilon$

$-\cos 2\Theta = 1-2\cos^2\Theta$

$u_2'-u_1' = (u_2-u_1) - 2(u_2-u_1)\cos^2\Theta + 2\sqrt{\dots}\,\sin\Theta\cos\Theta\cos\varepsilon$

$u_2'+u_1' = u_2+u_1$

$u_2' = u_2 - (u_2-u_1)\cos^2\Theta + \sqrt{v^2-(u_2-u_1)^2}\,\sin\Theta\cos\Theta\cos\varepsilon$

$u_1' = u_1 + (u_2-u_1)\cos^2\Theta - \sqrt{v^2-(u_2-u_1)^2}\,\sin\Theta\cos\Theta\cos\varepsilon$

[illegible] $\frac{dQ}{d\vartheta}$ $u = u_0 + v$

[illegible] $Q = \mathring{u}^2$

$$\underbrace{\mathring{u}_1'^2 - \mathring{u}_1^2 + \mathring{u}_2'^2 - \mathring{u}_2^2}$$

$$= (\mathring{u}_2 - \mathring{u}_1)^2 \cos^4\theta + [\eta^2 - (\mathring{u}_2 - \mathring{u}_1)^2] \sin\theta\cos\theta \cos\varepsilon$$

$$+ 2\mathring{u}_1(\mathring{u}_2 - \mathring{u}_1)\cos^2\theta + (\ldots)\cos\varepsilon$$

[illegible] $\cos\varepsilon$ [illegible]

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$$\varrho \frac{D\bar{Q}}{Dt} = \frac{m}{2}\int d\omega_1 \int d\omega_2 f_1 f_2 \eta \int p\,dp \int d\varepsilon (Q_1' + Q_2' - Q_1 - Q_2)$$

$$\varrho \frac{D\overline{\mathring{u}^2}}{Dt}$$

$$\mathring{u}_1'^2 + \mathring{u}_2'^2 - \mathring{u}_1^2 - \mathring{u}_2^2 = 2\sin^2\theta\cos^2\theta\left[-(u_2 - u_1)^2 + (\eta)^2 - \overline{(\mathring{u}_2 - \mathring{u}_1)^2}\right]\cos\varepsilon$$

$$+ (\ldots)\cos\varepsilon$$

$$\int d\varepsilon(\mathring{u}) = 2\sin^2\theta\cos^2\theta\,\pi\left[(\eta)^2 - 3(u_2 - u_1)^2\right]$$

$$\varrho \frac{D\overline{\mathring{u}^2}}{Dt} = \frac{2\pi m}{m}\int d\omega_1 \int d\omega_2 f_1 f_2 \,\eta\left[(\eta)^2 - 3(\mathring{u}_1 - \mathring{u}_2)^2\right]\int_0^{\infty} p\,dp \sin^2\theta\cos^2\theta$$

$$\Theta = \int_0^{\varrho_{max}} \frac{d\varrho}{\sqrt{1 - \varrho^2 - \frac{2k\varrho^n}{M(\eta)^2 p^n}}}$$

ϱ_{max} [illegible] $\sqrt{\ldots} = 0$

$$q^n = p^n \frac{M\eta^2}{k} \qquad \Theta = \int_0^{\varrho} \frac{d\varrho}{\sqrt{1 - \varrho^2 - \frac{2}{n}\frac{\varrho^n}{q^n}}}$$

$$p\,dp = q\,dq\left(\frac{k}{M\eta^2}\right)^{\frac{2}{n}}$$

$$\varrho\frac{D\overline{\mathring{u}^2}}{Dt} = \pi m \int d\omega_1 \int d\omega_2 f_1 f_2 \,\eta\left[\eta^2 - 3(\mathring{u}_1 - \mathring{u}_2)^2\right]\left(\frac{k}{M\eta^2}\right)^{\frac{2}{n}} \underbrace{\int_0^{\infty} q\,dq \sin^2\theta\cos^2\theta}_{\text{keine Zahl}}$$

weiter Spezialisierung $n = 4$

$$= \frac{1{,}3682}{4\pi} = \frac{A}{4\pi}$$

[illegible] $\left(\frac{1}{\eta^2}\right)^{\frac{2}{n}}$ sehr unbequem.

[illegible] Maxwell [illegible]

w: $\rho \frac{D\overline{\overset{\circ}{u}^2}}{Dt} = \frac{\pi m}{4\pi}\sqrt{\frac{k}{M}}\, A \int dw_1 \int dw_2\, f_1 f_2 \left[\eta^2 - 3(u_1 - u_2)^2\right]$

$\eta^2 = (u_1 - u_2)^2 + (v_1 - v_2)^2 + (w_1 - w_2)^2$ [illegible]

$\int dw_1 f_1 \int dw_2 f_2\, \overset{\circ}{u}_1^2 = N \int dw_1 f_1\, u_1^2 = N^2\, \overline{\overset{\circ}{u}^2}$

$\int dw_1 f_1 \int dw_2 f_2\, u_1 u_2 = N\bar{u}\, N\bar{u} = 0$ $M = \frac{m}{2}$ [illegible]

$$\boxed{\rho \frac{D\overline{\overset{\circ}{u}^2}}{Dt} = \frac{m}{4}\sqrt{\frac{2k}{M}}\, A \left[\overline{c^2} - \overline{\overset{\circ}{u}^2}\right]}$$

$c^2 = \overset{\circ}{u}^2 + v^2 + w^2$

$u = \bar{u} + \overset{\circ}{u}$ I

$$\rho \frac{d\bar{u}}{dt} = \rho X - \frac{\partial}{\partial x}\left(\rho\, \overline{\overset{\circ}{u}^2}\right) - \frac{\partial}{\partial y}\left(\rho\, \overline{\overset{\circ}{u} v}\right) - \frac{\partial}{\partial z}\left(\rho\, \overline{\overset{\circ}{u} w}\right)$$ $\overset{\circ}{u}$ u

nicht reibende Medien: $\overline{uv} = \overline{uw} = \overline{vw} = 0$

$\overline{\overset{\circ}{u}^2} = \overline{v^2} = \overline{w^2} = \frac{\overline{c^2}}{3} = \frac{p}{\rho}$

wir wollen die Gleichungen [illegible] reibende Medien:

$$\rho \frac{DQ}{Dt} = \rho \frac{dQ}{dt} + \frac{\partial}{\partial x}\left(\rho\, \overline{uQ}\right) + \frac{\partial}{\partial y}\left(\rho\, \overline{vQ}\right) + \dots - \rho\left(X \frac{\partial Q}{\partial u} + Y\frac{\partial Q}{\partial v}\right.$$

f_ξ [illegible] exakt

[illegible] $\rho \frac{D\bar{Q}}{Dt}$ [illegible] $Q = u^2$ [illegible] $\overline{\overset{\circ}{u}^2} = \dots \frac{\overline{c^2}}{3}$ 8

$Q = u^2 = \bar{u}^2 + 2\bar{u}\overset{\circ}{u} + \overset{\circ}{u}^2$ $\bar{Q} = \bar{u}^2 + \overline{\overset{\circ}{u}^2}$

$$\overline{\overset{\circ}{u} Q} = \overline{\underbrace{\overset{\circ}{u}\bar{u}^2}_{=0} + 2\bar{u}\overset{\circ}{u}^2 + \underbrace{\overset{\circ}{u}^3}_{0}} = 2\bar{u}\,\overline{\overset{\circ}{u}^2}$$

$\overline{vQ} = 0$ $\overline{wQ} = 0$ $\frac{\partial \bar{Q}}{\partial \bar{u}} = 2\bar{u}$

$$\rho \frac{D}{Dt}\overline{\overset{\circ}{u}^2} = \rho \frac{d}{dt}\left(\bar{u}^2 + \overline{\overset{\circ}{u}^2}\right) + \frac{\partial}{\partial x}\left(2\bar{u}\rho\, \overline{\overset{\circ}{u}^2}\right) - \rho X\, 2\bar{u}$$

[illegible] I [illegible]

$$\rho \frac{D\overline{\overset{\circ}{u}^2}}{Dt} = 2p \frac{\partial \bar{u}}{\partial x} + \rho \frac{d}{dt}\frac{p}{\rho}$$ [illegible]

$$\underbrace{\rho \frac{D}{Dt}\left(\overline{\overset{\circ}{u}^2} + \overline{v^2} + \overline{w^2}\right)}_{=0} = 2p\left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z}\right) + 3\rho \frac{d}{dt}\frac{p}{\rho}$$

[illegible]

[illegible] $\varrho \frac{d}{dt}\frac{p}{\varrho}$ [illegible]

$$\varrho \frac{D\overline{u^2}}{Dt} = \frac{2}{3} p\left[2\frac{\partial \bar u}{\partial x} - \frac{\partial \bar v}{\partial y} - \frac{\partial \bar w}{\partial z}\right] + [\varepsilon]$$

[illegible] $\varrho \frac{D\overline{u^2}}{Dt}$ [illegible]

$$\left[c^2 - 3\overline{u^2}\right] = \frac{4}{m}\sqrt{\frac{m}{2k}}\,\frac{1}{A}\,\frac{2}{3} p\left\{\left[2\frac{\partial \bar u}{\partial x} - \frac{\partial \bar v}{\partial y} - \frac{\partial \bar w}{\partial z}\right] + [\varepsilon]\right\}$$

[illegible] $= \theta$

[illegible]

$$\varrho\overline{u^2} = p - \underbrace{\frac{2p}{A_1 q \varrho}\sqrt{\frac{2m^3}{k}}}_{\frac{2R}{2}}\left\{2\frac{\partial \bar u}{\partial x} - \frac{\partial \bar v}{\partial y} - \frac{\partial \bar w}{\partial z}\right\}$$

$$\varrho\frac{du}{dt} = \varrho X - \frac{\partial p}{\partial x} - R\left[\Delta \bar u + \frac{1}{3}\frac{\partial}{\partial x}\left(\frac{\partial \bar u}{\partial x} + \frac{\partial \bar v}{\partial y} + \frac{\partial \bar w}{\partial z}\right)\right]$$

Statistische Mechanik

[illegible] Maxwell, Boltzmann [illegible] Gibbs.

Die lebendige Kraft eine quadratische Form der Geschwindigkeiten

$$\mathfrak{L}(q\dot q) = \bar{\mathfrak{L}}(q p)$$

$$\left.\frac{\partial \bar{\mathfrak{L}}}{\partial q_i}\right)_{q_k p} = -\left.\frac{\partial \mathfrak{L}}{\partial q_i}\right)_{q_k \dot q}$$

$$\left.\frac{\partial \bar{\mathfrak{L}}}{\partial p_i}\right)_{q p_k} = \dot q_i$$

$$\mathfrak{L} = \frac{m}{2}\left(\dot r^2 + r^2\dot\varphi^2\right)$$

$q_1 = r$ $\quad \frac{\partial \mathfrak{L}}{\partial r} = m r \dot\varphi^2$

$q_2 = \varphi$ $\quad \frac{\partial \mathfrak{L}}{\partial \varphi} = 0$

$$p_1 = \frac{\partial \mathfrak{L}}{\partial \dot r} = m\dot r$$

$$p_2 = \frac{\partial \mathfrak{L}}{\partial \dot\varphi} = m r^2 \dot\varphi$$

$\dot r = \frac{1}{m} p_1 \quad \dot\varphi = \frac{1}{m r^2} p_2$

$$\bar{\mathfrak{L}} = \frac{m}{2}\left(\frac{1}{m^2}p_1^2 + \frac{r^2}{m^2 r^4}p_2^2\right)$$

$$\bar{\mathfrak{L}} = \frac{1}{2m}\left(p_1^2 + \frac{1}{r^2}p_2^2\right)$$

$$\left.\frac{\partial \bar{\mathfrak{L}}}{\partial r}\right)_p = -\frac{1}{m r^3}p_2^2$$

$$\frac{\partial \bar{\mathfrak{L}}}{\partial \varphi} = 0$$

$$\left.\frac{\partial \bar{\mathfrak{L}}}{\partial r}\right)_p = -\frac{1}{m r^3} m^2 r^4 \dot\varphi^2$$

$$= -m r \dot\varphi^2 = -\left.\frac{\partial \mathfrak{L}}{\partial r}\right)_{\dot r \dot\varphi}$$

$$\left.\frac{\partial \bar{\mathfrak{L}}}{\partial p_1}\right)_{p_2 q} = \frac{1}{m}p_1 = \dot r$$

$$\frac{\partial \bar{\mathfrak{L}}}{\partial p_2} = \frac{1}{m r^2}p_2 = \dot\varphi$$

[illegible]

Hamiltons[illegible]

[illegible] Konservativ

[illegible]

$Q_i = -\frac{\partial V}{\partial q_i} \quad V = V(q_i\, t)$

q_i verallgem. Koordin

p_i " Impulse

[illegible] Konfiguration [illegible]

$$-\frac{\partial V}{\partial Q} = +\frac{dp_i}{dt} - \frac{\partial L}{\partial q_i}$$

$$= +\frac{dp_i}{dt} + \frac{\partial \bar{L}}{\partial q_i}$$

$$\frac{dp_i}{dt} = -\frac{\partial(V+\bar{L})}{\partial q_i}$$

$$\frac{dq_i}{dt} = \frac{\partial \bar{L}}{\partial p_i} = \frac{\partial(\bar{L}+V)}{\partial p_i}$$

[illegible] $\frac{\partial V}{\partial p_i} \equiv 0 \qquad V+\bar{L} = E$

$$\frac{dp_i}{dt} = -\frac{\partial E}{\partial q_i} \qquad \frac{dq_i}{dt} = \frac{\partial E}{\partial p_i}$$

[illegible] $E = E(q_i\, p_i)$

Hamilton's [illegible]

[illegible] $\frac{\partial V}{\partial t} = 0$ [illegible]

$E(q\,p) = \text{const}$ [illegible]

$$\frac{dE}{dt} = \sum\left[\frac{\partial E}{\partial p_i}\frac{dp_i}{dt} + \frac{\partial E}{\partial q_i}\frac{dq_i}{dt}\right] = 0$$

[illegible]

[illegible]

[illegible] Trillionen [illegible]

[illegible]

[illegible] dimensionalen [illegible]

[illegible] $N =$ [illegible]

[illegible] Kurven [illegible]

Liouville's Satz.

[illegible] Repräsentation [illegible]

$p_1, dp_1 \quad p_2, dp_2 \ldots$

$q_1, dq_1, q_2\, dq_2 \ldots$

[illegible] Parallelepiped

[illegible]

$$g = \iiint\cdots\iiint dq_1 \ldots dq_n\, dp_1 \ldots dp_n$$

$$g' = \int\cdots\int dq_1' \ldots dp_1' \ldots dp_n'$$

[illegible]

$$\Delta = \frac{\partial(p_1 \ldots p_n q_1 \ldots q_n)}{\partial(p_1' \ldots p_n' q_1' \ldots q_n')}$$

---

3. V 1910.

$$E(q, p, t) = \bar{L} + V$$

[illegible]

$q_i^0 \quad p_i^0 \quad , t = 0$

$$q_i = q_i(t\, q_1^0 \ldots q_n^0\, p_1^0 \ldots p_n^0)$$

$$\frac{\partial|q_1 \ldots q_n p_1 \ldots p_n|}{\partial|q_1^0 \ldots q_n^0 p_1^0 \ldots p_n^0|} = \frac{\partial|q\,p|}{\partial|q_0 p_0|}$$

$p \quad q \quad$ [illegible] t

$p' \quad q' \quad \ldots \quad t+dt$

$$\Delta = \frac{\partial|p'q'|}{\partial|p\,q|}$$

$$q_i' = q_i + \frac{\partial q_i}{\partial t} dt = q_i + \dot q_i \, dt$$

$$= q_i + \frac{\partial E}{\partial p_i} dt$$

$$p_i' = p_i + \dot p_i \, dt = p_i - \frac{\partial E}{\partial q_i} dt$$

$$q_1' = q_1 + \frac{\partial E}{\partial p_1} dt$$

$$\frac{\partial q_1'}{\partial q_1} = 1 + \frac{\partial^2 E}{\partial q_1 \partial p} dt \qquad \frac{\partial q_1'}{\partial q_2} = \frac{\partial^2 E}{\partial q_2 \partial p} dt$$

$$\cdots\cdots$$

$$\frac{\partial q_1'}{\partial p_1} = \frac{\partial^2 E}{\partial p_1^2} dt, \quad \frac{\partial q_1'}{\partial p_n} = \frac{\partial^2 E}{\partial p_1 \partial p_n} dt$$

$$q_2' = q_2 + \frac{\partial E}{\partial p_2} dt$$

$$p_1' = p_1 - \frac{\partial E}{\partial q_1} dt$$

$$\Delta = \begin{vmatrix} 1 + \frac{\partial^2 E}{\partial p_1 \partial q_1} dt, & \frac{\partial^2 E}{\partial p_1 \partial q_2} dt, & \cdots & \frac{\partial^2 E}{\partial p_1 \partial p_n} dt, & \frac{\partial^2 E}{\partial p_1^2} dt & \cdots & \frac{\partial^2 E}{\partial p_1 \partial p_n} dt \\ \frac{\partial^2 E}{\partial p_2 \partial q_1} dt, & 1 + \frac{\partial^2 E}{\partial p_2 \partial q_2} dt & \cdots & \cdots & \cdots & \cdots & \frac{\partial^2 E}{\partial p_2 \partial p_n} dt \\ \frac{\partial^2 E}{\partial q_n \partial q_1} dt & \cdots & \cdots & \cdots & \cdots & \cdots & \frac{\partial^2 E}{\partial p_n} dt \\ -\frac{\partial^2 E}{\partial q_n \partial q_1} dt & \cdots & -\frac{\partial^2 E}{\partial q_n^2} dt, & -\frac{\partial^2 E}{\partial q_n \partial p_1} dt & \cdots & 1 - \frac{\partial^2 E}{\partial q_n \partial p_n} dt \end{vmatrix}$$

[illegible] dt [illegible]:

$$\cancel{1 + \frac{\partial^2 E}{\partial p_1 \partial q_1} dt + \frac{\partial^2 E}{\partial p_1 \partial q_2} dt + \cdots \frac{\partial^2 E}{\partial p_1 \partial p_n} dt}$$

$$\cancel{- \frac{\partial^2 E}{\partial p_1 \partial q_1} dt - \cdots - \frac{\partial^2 E}{\partial p_1 \partial p_n} dt}$$

$$\Delta = 1 \qquad \frac{\partial(q' p')}{\partial(q, p)} = 1 \qquad \text{[illegible]} \quad \frac{\partial(q'' p'')}{\partial(q' p')} = 1$$

$$\frac{\partial(q'' p'')}{\partial(q\, p)} = \frac{\partial(q'' p'')}{\partial(q' p')} \cdot \frac{\partial(q' p')}{\partial(q\, p)} = 1 \cdot 1 = 1$$

[illegible] q, p $= 1$ [illegible]

[illegible] $\frac{\partial(q, p)}{\partial(q_0 p_0)} = 1$

[illegible]

dasselbe System [illegible]

[illegible] $E(pq)$ [illegible] $p_0 q_0$, [illegible]

$$\iiint \cdots \iiint dp_1 \ldots dp_n \, dq_1 \ldots dq_n = g_0$$

[illegible] $\rho = \iiint \dots \iiint dp_1 \dots dq_n$

[illegible] ρ_0

$p_1 \dots q_n$ [illegible] $p_1^0 \dots q_n^0$

$$\iiint \dots \iiint dp_1 \dots dq_n = \iiint \dots \iiint dp_1^0 \dots dq_n^0 \, \Delta\left(\frac{p\,q}{p_0 q_0}\right)$$

$$= \iiint \dots \iiint dp_1^0 \dots dq_n^0$$

$\rho = \rho_0$ Liouvilles [illegible]

[illegible]

[illegible]

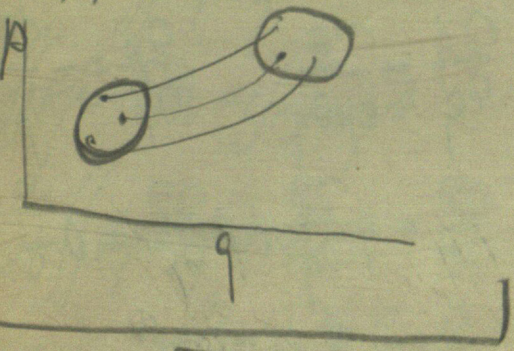
$L = \frac{m}{2}\dot{x}^2 \quad V = -mgx \qquad q = x \quad p = \frac{\partial L}{\partial \dot{x}} = m\dot{x}$

$E = \frac{1}{2m}p^2 - mgq \qquad p = p_0 + mgt$

$\frac{dp}{dt} = -\frac{\partial E}{\partial q} = mg \qquad q = q_0 + \int \frac{p}{m} dt =$

$\frac{dq}{dt} = \frac{\partial E}{\partial p} = \frac{p}{m} \qquad = q_0 + \frac{p_0}{m} t + \frac{g}{2} t^2$

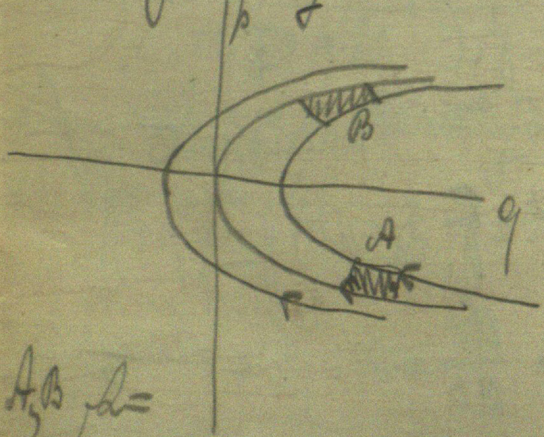
$$\frac{\partial(q\,p)}{\partial(q_0 p_0)} = \begin{vmatrix} 1 & \frac{t}{m} \\ 0 & 1 \end{vmatrix} = 1$$



9. V 1910.

$E = \frac{1}{2m}p^2 - mgq$

$q = \frac{1}{2m^2 g}p^2 - \frac{E}{mg}$



A, B [illegible] $=$

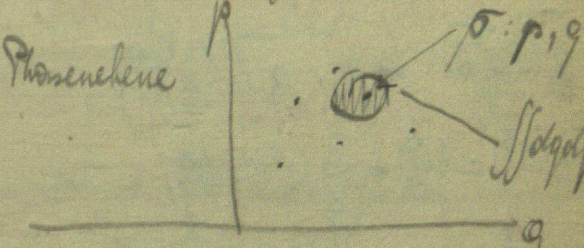
[illegible]

$L = \frac{m}{2}\dot{x}^2 \quad V = f(x)$

$q = x \quad p = m\dot{x}$

[illegible]

[illegible]



$D \iint dq\,dp$ [illegible]

$\iint dq\,dp$ [illegible]

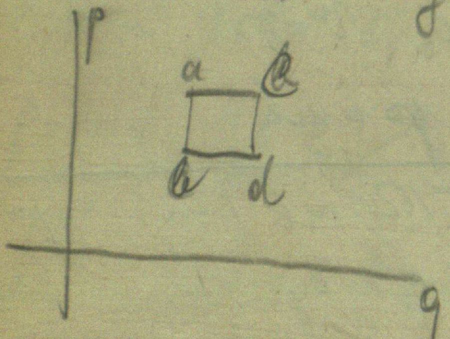
D Phasendichte $D = D(p, q, t)$

$N = \iint dp\,dq \cdot D$ [illegible]

[illegible] $\frac{\partial D}{\partial t}$ $\quad dq\,dp \frac{\partial D}{\partial t}$

[illegible]

$$\frac{\partial \varrho}{\partial t} + \frac{\partial(u\varrho)}{\partial x} + \frac{\partial(v\varrho)}{\partial y} = 0$$



$\overline{ab}\,\dot{q}\,D \quad \overline{ab}\,(\dot{q}\,D)'$

$(\dot{q}D)' = \dot{q}D + \frac{\partial}{\partial q}(\dot{q}D)\,dq$

$ab\left[\dot{q}D - (\dot{q}D)'\right] = -$

$= -dq\,dp\,\frac{\partial}{\partial q}(\dot{q}D)$

$-dq\,dp\,\frac{\partial}{\partial p}(\dot{p}D)$

$$\frac{\partial D}{\partial t} + \frac{\partial(\dot{q}D)}{\partial q} + \frac{\partial(\dot{p}D)}{\partial p} = 0$$

[illegible]

[illegible]

$$\frac{dq}{dt} = \frac{\partial E}{\partial p} \qquad \frac{dp}{dt} = -\frac{\partial E}{\partial q}$$

$$\frac{\partial \dot{q}}{\partial q} = \frac{\partial^2 E}{\partial p\,\partial q} \qquad \frac{\partial \dot{p}}{\partial p} = -\frac{\partial^2 E}{\partial p\,\partial q}$$

$$\frac{\partial D}{\partial t} + \dot{q}\frac{\partial D}{\partial q} + \dot{p}\frac{\partial D}{\partial p} = 0$$

$$D = D(q\,p\,t) = C$$

$$\frac{dD}{dt} = 0 \quad (\text{Wasser-Öl})$$

Satz der Erhaltung der Phasendichte

[illegible] Hamilt. [illegible]

$f_1(p\,q\,t) = C_1$

$f_2(\quad) = C_2$

[illegible] Phasendichte

$$D = F(f_1(q\,p\,t)\,f_2(q\,p\,t))$$

$\frac{dD}{dt} = 0$ [illegible] Liouvilles

[illegible]

$D_0 \ldots \iint dp_0\,dq_0$

$D \ldots\ldots \iint dp\,dq$

$$\iint dp_0\,dq_0\,D_0 = \iint dp\,dq\,D$$

$$\frac{d}{dt}\iint dp\,dq\,D = 0$$

[illegible]

$$\frac{d}{dt}(Dg) = 0$$

$$= \cancel{\frac{dD}{dt}}\,g + \frac{dg}{dt}\,D = 0$$

Liouville

11. V. 1910

Liouville: Satz der Erhaltung der Phasenausdehnung.

[illegible]

$$\frac{dD}{dt} = 0$$

$$\frac{D}{N} = P = e^{\eta}\,{}^{1}g$$

$$\frac{D(p\,q\,t)\,dq_1 \ldots dp_n}{N}$$

[illegible]

P [illegible]

η [illegible]

[illegible] $\frac{\partial D}{\partial t} = 0$

[illegible] Gibbs

systematisches = [illegible]

$E(q\,p) = \text{const}$

$D = F(E)$

$$-\frac{\partial D}{\partial t} = \frac{dF}{dE}\left[\sum \dot{q}\frac{\partial E}{\partial q} + \dot{p}\frac{\partial E}{\partial p}\right]$$

23. V 1910.

$$N = \iint \cdots \int D\, dq_1 \cdots dp_n$$

$$\frac{\partial D}{\partial t} = 0 \quad [illegible] \quad D = D(E)$$

[illegible] P:

$$P = e^{\frac{\psi - E}{\Theta}}$$ kanonische Phasenverteilung

$$\int \cdots \int e^{\frac{\psi - E}{\Theta}} dq_1 \cdots dp_n = 1$$

[illegible]

[illegible] N [illegible]

$$\int \cdots \int e^{\frac{\psi - E_1}{\Theta}} dq_1 \cdots dp_n$$

[illegible]

N [illegible] $q_{n+1} \cdots q_{n+m}$

$$\int \cdots \int e^{\frac{\psi_2 - E_2}{\Theta}} dq_{n+1} \cdots dp_{n+m}$$

[illegible] $n+m$ [illegible]

N [illegible] $n+m$ [illegible] $p_1 \cdots p_{n+m}$

[illegible] 3 [illegible] kanonische

[illegible] $\overline{W}$ [illegible]

$$w = w_1 \cdot w_2$$

$$= \iint \cdots \int dq_1 \cdots dp_n\, e^{\cdots}$$

$$\cdot \int \cdots \int dq_{n+1} \cdots dp_{n+m}\, e^{\cdots}$$

$$= \int \cdots \int dq_1 \cdots dp_{n+m}\, e^{\frac{\psi_1 + \psi_2 - \cdots}{\Theta}}$$

[illegible]

[illegible]:

φ [illegible] $\varphi(q\,p)$

$\bar{\varphi}$ [illegible] φ, [illegible]

[illegible]

$$\bar{\varphi}_* = \int \cdots \int \varphi\, e^{\frac{\psi - E}{\Theta}} dq_1 \cdots dp_n$$

[illegible] $m_1\, m_2 \cdots m_\nu$

$x_1\, y_1\, z_1 \cdots x_\nu\, y_\nu\, z_\nu$

$$3\nu = n$$

$$L = \sum_i \frac{m_i}{2}(\dot{x}_i^2 + \cdots)$$

$$p_i = m_i \dot{x}_i$$

$$\int \cdots \int e^{\frac{\psi - E}{\Theta}} dx_1\, dy_1 \cdots dz_\nu$$

$$\underbrace{m_1^3 \cdots m_\nu^3}_{M}\, d\dot{x}_1 \cdots d\dot{z}_\nu$$

[illegible]

$$E = \Phi + \sum \frac{m_i}{2}(\dot{x}_i^2 + \cdots)$$

$$e^{\frac{\varphi - E}{\Theta}} = e^{\frac{\varphi - \Phi}{\Theta}} \cdot e^{-\frac{m_1}{2}\frac{\dot{x}_1^2}{\Theta}} \cdots$$

$$\bar{\varphi} = M \int \cdots \int e^{\frac{\psi - \Phi}{\Theta}} dx_1 \cdots dz_\nu \int e^{-\frac{m_1 \dot{x}_1^2}{2\Theta}} d\dot{x}_1 \int e^{-\frac{m_1 \dot{y}_1^2}{\Theta}} dy_1 \cdots \int$$

[illegible]

[illegible]

$$L = \frac{m_1}{2}\dot{x}_1^2 + \cdots$$

$\frac{m_1}{2}\dot{x}^2$ [illegible]

$$= M \int \cdots \int e^{\frac{\psi - \Phi}{\Theta}} dx_1 \cdots dz_\nu$$

[illegible]

$$\cdot \int_{-\infty}^{+\infty} \frac{m_1}{2} \dot{x}_1^2\, e^{-\frac{m_1 \dot{x}_1^2}{2\Theta}} \int_{-\infty}^{+\infty} \cdots \int d\dot{z}_\nu$$

[illegible]

$$\int e^{\cdots} = \frac{1}{\nu} \; \frac{1}{2}$$

$$M\int\limits^{\infty}\cdots\int dx_1\ldots dz_\nu\, e^{\frac{\psi-V}{2\Theta}}\int\cdots\int\limits_{-\infty}^{+\infty} e^{-\frac{m\dot{z}_\nu^2}{2\Theta}}\, d\dot{z}_\nu = 1$$

[illegible]

$$\frac{\overline{m_1\dot{x}_1^2}}{2} = \frac{\frac{m_1}{2}\int \dot{x}_1^2\, e^{-\frac{m_1\dot{x}_1^2}{2\Theta}}\, d\dot{x}_1}{\int\limits_{-\infty}^{+\infty} e^{-\frac{m_1\dot{x}_1^2}{2\Theta}}\, d\dot{x}_1}$$

$$\int\limits_{-\infty}^{+\infty} e^{-\lambda x^2}\, dx = \sqrt{\frac{\pi}{\lambda}} \qquad \int\limits_{-\infty}^{+\infty} x^2 e^{-\lambda x^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{\lambda^3}}$$

$$\frac{\overline{m_1\dot{x}_1^2}}{2} = \frac{\frac{m_1}{2}\,\frac{1}{2}\sqrt{\pi\left(\frac{2\Theta}{m_1}\right)^3}}{\frac{1}{2}\sqrt{\pi\frac{2\Theta}{m_1}}} = \frac{m_1}{4}\,\frac{2\Theta}{m_1} = \frac{\Theta}{2}$$

[illegible] $= \frac{3}{2}\Theta$

[illegible] $\frac{3\nu}{2}\Theta = \frac{n}{2}\Theta = L$

$$L = \frac{n}{2}\Theta$$

[illegible]

24/V 1910.

[illegible] $\int d\dot{x}$ [illegible] $-\infty$ [illegible] $+\infty$ [illegible]

[illegible]

[illegible] $\int = \left(\sqrt{\pi\frac{2\Theta}{m_i}}\right)^3$

[illegible]

$$(2\pi\Theta)^{\frac{3\nu}{2}}(m_1\ldots m_\nu)^{-\frac{3}{2}}\int\cdots\int e^{\frac{\psi-V}{\Theta}}\, dx_1\ldots dz_\nu$$

[illegible]

$E = V + L$

$$\int\cdots\int e^{\frac{\psi-E}{\Theta}}\, dq_1\ldots dp_n$$

$$= \int\cdots\int e^{\frac{\psi-V}{\Theta}}\, dq_1\ldots dq_n \int\cdots\int e^{-\frac{L}{\Theta}}\, dp_1\ldots dp_n$$

[illegible] $\int =$

$$L = A_{11}p_1^2 + A_{12}p_1p_2 \ldots\ldots$$

[illegible]

$$r_1 = a_{11}p_1 + a_{12}p_2 + \ldots a_{1n}p_n$$

$$\vdots$$

$$r_n = a_{n1}p_1 + \ldots\ldots a_{nn}p_n$$

[illegible] L [illegible]

[illegible] $L = 0$.

$$L = \frac{1}{2}(r_1^2 + r_2^2 + \ldots r_n^2)$$

[illegible]

$$dp_1\ldots dp_n = \frac{\partial(p_1\ldots p_n)}{\partial(r_1\ldots r_n)}\, dr_1\ldots dr_n$$

[illegible]

[illegible] $e^{\ldots}$ [illegible] q_i

$$\int\limits_{-\infty}\cdots\int e^{\frac{\psi-V}{\Theta}}\frac{\partial(p_1\ldots p_n)}{\partial(r_1\ldots r_n)}\int\cdots\int e^{-\frac{r_1^2+\ldots}{2\Theta}}\, dr_1\ldots dr_n$$

$$\frac{\partial p_i}{\partial r_k} = \frac{\partial}{\partial r_k}\frac{\partial L}{\partial \dot{q}_i}$$ [illegible]

$$\frac{\partial}{\partial r_k}\sum_{\lambda=1}^{n}\frac{\partial L}{\partial r_\lambda}\frac{\partial r_\lambda}{\partial \dot{q}_i} =$$

$$\sum_\lambda \frac{\partial^2 L}{\partial r_k \partial r_\lambda}\frac{\partial r_\lambda}{\partial \dot{q}_i} = \frac{\partial}{\partial \dot{q}_i}\frac{\partial L}{\partial r_k}$$

$$= \frac{\partial r_k}{\partial \dot{q}_i}$$

$$\frac{\partial(p_1\ldots p_n)}{\partial(r_1\ldots r_n)} = \frac{\partial(r_1\ldots r_n)}{\partial(\dot{q}_1\ldots \dot{q}_n)}$$

$$\left\{\frac{\partial(p_1\ldots p_n)}{\partial(r_1\ldots r_n)}\right\}^2 = \frac{\partial(p_1\ldots)}{\partial(r_1\ldots)}\cdot\frac{\partial(r_1\ldots)}{\partial(\dot{q}_1\ldots)}$$

$$= \frac{\partial(p_1 \ldots p_n)}{\partial(\dot q_1 \ldots \dot q_n)}$$

$L = L(\dot q)$

$$= \begin{vmatrix} \frac{\partial^2 L}{\partial \dot q_1^2} & \frac{\partial^2 L}{\partial \dot q_1 \partial \dot q_2} & \cdot & \cdot \\ \frac{\partial^2 L}{\partial \dot q_2 \partial \dot q_1} & \cdot & & \\ & & \cdot & \cdot \\ \cdot & \cdot & & \end{vmatrix} = |a_{ik}| = \Delta_{\dot q}$$

Hessesche Determinante

$$\frac{\partial(p \ldots)}{\partial(r \ldots)} = \Delta_{\dot q}^{1/2}$$

$\frac{\overline{r_1^2}}{2}$

Boltzmann ... Momentoide

$$\frac{\overline{r_1^2}}{2} = \frac{\int^n_{p_1} \int e^{\frac{\psi - E}{\Theta}} \frac{1}{2} r_1^2 \, dq_1 \ldots dq_n \, dp_1 \ldots dp_n}{\int^n_{p} \int e^{\frac{\psi - E}{\Theta}} \, dq_1 \ldots dp_n}$$

$$= \frac{\int^n_{\eta} \int e^{\frac{\psi - \vartheta}{\Theta}} \Delta_{\dot q}^{1/2} \int e^{-\frac{r_1^2}{2\Theta}} \frac{r_1^2}{2} dr_1}{\int^n_{\eta} \int e^{\frac{\psi - \vartheta}{\Theta}} \Delta_{\dot q}^{1/2} \, dq_1 \ldots \int dr_1 \ldots}$$

$$\frac{\frac{1}{2}\int_{-\infty}^{+\infty} r_1^2 e^{-\frac{r_1^2}{2\Theta}} dr_1}{\int_{-\infty}^{+\infty} e^{-\frac{r_1^2}{2\Theta}} dr_1} =$$

$$\frac{\frac{1}{2} \frac{1}{2} \sqrt{\pi (2\Theta)^3}}{\sqrt{\pi 2 \Theta}} = \frac{\Theta}{2} = \frac{\overline{r_1^2}}{2}$$

$$(2\pi\Theta)^{n/2} \int \ldots \int e^{\frac{\psi - \vartheta}{\Theta}} \Delta_{\dot q}^{1/2} dq_1 \ldots$$

$$= L = \frac{n}{2}\Theta$$

Die Phasendichte als Integral: $D(p_0 q_0)$ ... $\bar D(p q t) = D(p_0(p q t), q_0(p q t))$ ... $\bar D(p q t)$ ... $\sigma \varepsilon \sim \int$