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"Competition, Transfer Prices and Observability"

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List of abbreviations

Symbol	Description
a	A constant, Income shift
b	A constant, Price/Quantity elasticity
$F_a(a)$	Distribution function of a
$f_a(a)$	Density function of a
p_i	Price chosen by firm i
q_i	Quantity chosen by firm i
q_i^*	Equilibrium quantity of firm i
r	Interest rate
t_i	Transfer price chosen by firm i
δ	Discount rate
θ	Intensity of competition
μ_{q_i}	Expectations of the equilibrium conjectures about the quantity
μ_a	Mean of a
Π^{Col}	Profit of collusion
Π^{Dev}	Profit of deviation
Π_i	Profit of firm i
π_D	Profit of the downstream division
π_U	Profit of the upstream division
σ_a	Standard deviation of a

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1 Introduction

The choice of transfer prices has been discussed by economists since Jack Hirshleifer.[1] The main idea and the solution presented by Hirshleifer are to transfer goods at marginal cost which is equal to marginal revenue within an organization.[2] In his paper, “Economics of the Divisionalized firm“, he points out, that the transfer price can be used as an administrative tool to coordinate the division‘s production behaviour. [3] Hirshleifer’s model is based on assumptions such as perfect information and a tax free environment. Nonetheless, empirical evidence show that most firms rather transfer above marginal cost than at marginal cost as suggested in Hirshleifer’s theory.[4] In order to address this phenomenon, many papers focus on transfer price issues that deal with intra-firm resource allocation and performance evaluation.[5] Since intra-firm resource allocation and asymmetric information were covered in my seminar paper in Controlling, I was very curious about an another approach that solves the phenomenon of above marginal cost: Competition.

In general, I have a high interest in situations in which competition takes place, therefore I was wondering, how transfer prices in a competitive environment can be modelled (at all). Could that model also be applicable in other fields of competition such as games, sport or relationships? From an intuitive point of view, a competitive approach would be an extension of the isolated view on transfer prices, since it considers not only the variables within a firm but also the external variables. As I first glanced over the papers that presented the models, I was confronted with keywords such as “Cournot“, “Bertrand“ and “Nash equilibrium“. Since those words were mentioned in past courses, in game theory and in a movie[6], but were not explained in detail, I wanted to know more. And since I also wanted to challenge myself with the mathematical models in the papers, I made my decision to work on this topic in my master thesis.

[1] See Narayanan et al. (2012), p. 135
[2] See Hirshleifer (1956), p. 183
[3] See Hirshleifer (1957), p. 98, pp. 102ff
[4] See Tang (1992), pp. 22-26 quoted from Alles et al. (1998), p. 451, see also Kaplan et al. (1998), p. 454 and Horngren et al. (1994), p. 872 quoted from Göx (2000), p. 327
[5] See Alles et al. (1998), pp. 452-453
[6] The movie “A beautiful Mind“ depicts the life and the work of John F. Nash Jr..

Therefore, following the paper of Alles et al. I want to leave the internal role of transfer prices within an organization and move towards a model that considers a competitor's action in a competitive environment. [7] Within a competition, Hirshleifer's optimal transfer price at marginal cost may not be the optimal strategy since he does not consider any participants and competitors in his calculation. It may be more profitable for the own firm and for the competitor's firm to choose a transfer price that lies above marginal cost.

In my master thesis, I want to give the readers an intuition how strategical choices on transfer prices in a competitive environment are determined and why it is profitable to transfer above marginal cost. Furthermore I want to show in my thesis, how unobservable transfer prices can be integrated in the model. After reading the thesis, the reader will also understand the impact of the intensity of competition on the choice of strategy. When both firms in a duopoly lack the incentive to transfer goods at above marginal cost, even when they could earn higher profit by doing that, they find themselves in a prisoner's dilemma. This can be solved with different mechanisms, which are presented in the last section of the thesis.

2 Related literature

Alles et al. model a firms pricing decision that is based on transfer prices above marginal cost rather than on transfer prices at marginal cost.[8] They find support in empirical studies in which a huge amount of firms use above marginal cost as the transfer price.[9] Their main objective therefore is to determine a strategic transfer price based on the competitive environment which also explains the empirical evidences of above marginal cost transfer prices.[10] They show that an optimal equilibrium exists in a strategic pricing decision.[11] The model used by Alles et al. is a differentiated Bertrand game, in which two firms compete in two different product markets.

[7] See Alles et al. (1998), p. 453
[8] See Alles et al. (1998), p. 451, p.452
[9] See Alles et al. (1998), p. 451
[10] See Alles et al. (1998), p. 452
[11] See Alles et al. (1998), p. 452

The reason for the choice of Bertrand game rather than Cournot game is that prices are strategic complements.[12] As Narayanan et al. discuss it in their paper: “Thus, each firm’s reaction curve for its transfer price increases in the transfer price of its rival. The coordinates of the intersection of these two reaction curves are the equilibrium transfer prices. An increase in transfer price by one firm triggers an increase in transfer price by its rival, and together these increase the price that the firms charge consumers.“[13] Furthermore, a differentiated Bertrand game is based on imperfect substitutes in which it does not include marginally undercutting the rival’s price in order to capture the entire market.[14]
Assumptions are made that simplify several issues in the model. One of the assumptions is the observable transfer price which is criticized by Göx.[15] Another assumption that Alles et al. make is the certainty of cost.[16] This issue is dealt by Narayanan et al. in their paper “Competition and Cost Accounting“.[17] A third assumption is the fixed organizational strategy. Both firms are assumed to decentralized.[18]

One of the topics Göx discusses in his paper is the importance of being able to observe transfer prices.[19] As Göx describes in his paper: “(...), Alles and Datar (1998) do not only assume observable transfer prices but also claim ‚Each firm need not observe its competitor’s choice of transfer price, and consequently, there is no necessity for each firm to commit to its transfer price. Rather, given the common knowledge that some transfer prices must be chosen, each firm will react to its predictions of what that transfer price will be.“[20] He further stresses: “(...) unobservable contracts cannot serve as credible precommitments unless they are employed for other than strategic reasons. This observation limits the direct use of strategic transfer pricing to the case of observable transfer prices.“[21] If each firm has a headquarter and a division, then the divisions do not have access to the transfer price information of the competitor: “Each agent can only observe the

[12] See Narayanan et al. (2000), p. 507
[13] Narayanan et al. (2000), p. 507
[14] See Göx (2000), p. 331
[15] See Göx (2000), p. 330
[16] See Alles et al. (1998), p. 454
[17] See Narayanan et al. (2012), p. 138
[18] See Alles et al. (1998), p. 454
[19] See Göx (2000), pp. 336-338
[20] Alles et al. (1998), p. 454 quoted from: Göx (2000), p. 330
[21] Göx (2000), p. 328

transfer price of his own firm, not the transfer price of the competitor, when deciding on his pricing strategy. Hence, neither firm's headquarters can expect the other firm's manager to react directly to its transfer price (...).“[22] According to his results, when transfer prices are not observable, above marginal costing is not an optimal equilibrium.[23] In his model, two firms compete in one product market in a differentiated Bertrand game. Like Alles et al., Göx assumes that both firms have a decentralized organization.[24]

Another paper that follows after the publication from Alles et al. is the paper “Impact of Competition and Taxes on Responsibility Center Organizations and Transfer Prices“ from Narayanan et al.[25] They remain consistent with the model of Alles et al. with a differentiated Bertrand duopoly but add tax issues and unobservable transfer prices to the model. Narayanan et al. also let firms choose their organizational strategy. They extend the model with asymmetric market information which is known by the divisional manager but not known by the headquarter. A transfer price set by a centralized firm does not include the market information of the divisional manager while a decentralized firm does involve the specific market knowledge in its calculation of transfer prices.[26]
In contrast to Göx, Narayanan et al. find the equilibrium in the above marginal cost pricing even when transfer prices are not observable.[27] Their main finding is “(...) that unobservable transfer prices can still play a strategic role as long as sales offices tax rates are higher than head office tax rate.“[28] It should be noted that Narayanan et al. use the mechanism of asymmetric information in order to achieve a strategic pricing equilibrium. In our model, we follow Narayanan et al. and their method to calculate unobservable transfer prices in competition.

Shor et al. also offer a perspective on the explanation of above marginal cost in firms.[29] Their model differs from the previous models in several ways: First of all, it is based on an undifferentiated Cournot game rather than on a differentiated

[22] Göx (2000), p. 337
[23] See Göx (2000), p. 327
[24] See Göx (2000), p. 330
[25] See Narayanan et al. (2000), pp. 497-529
[26] See Narayanan et al. (2000), pp. 504-505
[27] See Narayanan et al. (2000), p. 511
[28] Narayanan et al. (2000), p. 514
[29] See Shor et al. (2009), pp. 581-604

Bertrand game. The second difference is an n-firm competition instead of a duopoly. Furthermore, Shor et al. distinguish between decentralization and centralization. While in a centralized organization there is a central planner transferring goods at zero marginal cost, a decentralized organization allows divisions to maximize their profits.[30] They find collusion on organizational form to be more advantageous than collusion on prices or quantity.[31] As they argue in their paper: “The first advantage concerns the sustainability of collusion on organizational form. With traditional price and quantity collusion, the set of discount rates which support collusion vanishes as the number of firms becomes large. (...) A second advantage of colluding on organizational form concerns enforcement. Agreeing to set prices or quantities is per se illegal, while the selection of organizational form is not only less regulated but is commonly discussed at industry conferences without raising antitrust concerns.“[32]

Shor et al. find the equilibrium in a non-cooperative outcome which is the result of marginal cost decisions.[33] They show that all the firms in the industry profit from above marginal cost decision, but it is only a stable equilibrium in a repeated game with a certain trigger strategy.[34] In our model we follow Shor et al. and their concept of decentralization and centralization.

3 Model

The model of an undifferentiated Cournot competition from pp. 8-17 is based on Shor et al.[35] In the model of differentiated Cournot competition from pp. 26-35 we follow the model of Singh et al.[36] The calculation of the unobservable transfer prices from pp. 18-25 and from pp. 36-43 is based on the calculation from Narayanan et al.[37]

[30] See Shor et al. (2009), p. 584
[31] See Shor et al. (2009), pp. 582-583
[32] Shor et al. (2009), pp. 582-583
[33] See Shor et al. (2009), pp. 582-584
[34] See Shor et al. (2009), pp. 586-588
[35] See Shor et al. (2009), pp. 584-586
[36] See Singh et al. (1984), pp. 547-553
[37] See Narayanan et al. (2000), pp. 523-525

The model consists of a duopoly where two firms compete against each other in the market with a homogeneous product in the first scenario and with a differentiated product in the second scenario. The more differentiated the products are, the more the consumers are willing to buy at a higher Cournot price, since the product cannot be bought elsewhere. The competition is low in a differentiated Cournot competition, since two distinguishable products are preferred by different customers. Imagine for example the choice of buying blue jeans or red jeans. One firm specializes in blue jeans, the other in red jeans. When the products are undifferentiated, only one industrial price exists. The consumers are indifferent in the choice of the firm they buy from. When both firms specialize on exactly the same blue jeans, the two firms face perfect competition.

The organizational structure of a firm consists of two divisions: An upstream division and a downstream division. The upstream division produces intermediate goods and transfers them to the downstream division. The downstream division receives the goods at a transfer price set by the upstream division and transforms them into end products which are sold on the market.

The transfer price set by the upstream division depends on the choice of the organizational form. If the firm has a decentralized organizational structure, then the transfer price is generated by profit maximization of the divisions. If the firm is centralized then there is a central planer who sets the transfer price at marginal cost. We assume that there is no external market. Furthermore we do not include transaction cost or agency cost in the model.

In an undifferentiated Cournot competition the downstream divisions choose the quantity to be sold in the market. The price is dependent on the choice of the quantity set by both firms i and j:

$$p = a - bq_i - bq_j \tag{3.1}$$

$$\Pi_i = p * q_i, \ \Pi_j = p * q_j \tag{3.2}$$

q_i and q_j are the quantities produced by firm i and firm j, while Π_i and Π_j depicts the profit made by firm i and firm j. a and b are constants and $a > 0$, $b > 0$. a represents a shift in the slope. For example when the income level a of the consumers is high, then they can afford a high price for goods. b on the other hand describes how steep the slope of the demand curve is. It affects the elasticity of the quantity and how the industry price changes when the quantity is reduced or increased. A high b means that a short reduction of quantity leads to a jump in prices.

The equation in (3.1) is the typical price-quantity function in which a high quantity chosen by firm i and firm j leads to a large decrease in the price p of the product. On the other hand, when both firms choose a low quantity that generates a high price, why shouldn't one firm boost its profit by increasing the amount of production while the other firm faces a fall in profit? And what would be the response of the other firm on the production increase of the first firm? How would a firm choose its quantity, if it has no information about the competitor's choice and vice versa? What would be the final result, if both firms try to outperform each other regarding the profit maximization?

This equation is based on the idea of Augustine Cournot, who creates a dynamic game in which one's choice of quantity affects the choice of the other one and vice versa. In the Chapter 4 and 5, the dynamics and the optimal choices of the two firms in an undifferentiated Cournot competition are explained.

In a differentiated Cournot competition the demand function consists of

$$p_i = a - q_i - \theta q_j \tag{3.3}$$

$$p_j = a - q_j - \theta q_i \tag{3.4}$$

θ is the measure for intensity of the competition.[38] Following Singh et al., we have quadratic consumer preferences in the differentiated Cournot competition with $\theta \epsilon [0, 1)$.[39] When product differentiation is high, the measure for the intensity of the competition is low. In the extreme case, when two products are unique, there is no competition in the market, therefore both firms find themselves in their own monopoly. If both products are the same with no differentiation, we find ourselves in the undifferentiated Cournot competition.

[38] See Narayanan et al. (2012), p. 138

[39] See Sing et al. (1984), p. 547; See also Singh et al. (1984) quoted from: Narayanan et al. (2012), p. 138

The game in the model is divided into three stages and I follow the model of Shor et al., in which both firms act simultaneously to choose the organizational form, the transfer price and the quantity[40]:

- First, each firm chooses the organization form
- Then the upstream divisions set transfer prices for intermediate goods that are sold to the downstream divisions
- At last the downstream divisions set quantities/set prices to maximize profit

4 Undifferentiated Cournot competition with observable transfer prices

4.1 Downstream division

The downstream division chooses quantities to maximize profit. $p_i * q_i$ represents the revenues, $t_i * q_i$ are the input costs for the downstream division of firm i. t_i is the transfer price set by the upstream division. π_D is the total profit of the downstream division D when revenues are reduced by input costs:

$$\pi_D = p_i * q_i - t_i * q_i \tag{4.1}$$

We substitute p_i from the price-quantity function (3.1) into the profit function (4.1) and search for the first order condition of π_D. The optimal quantity for the downstream division of firm i is:

$$\max_{q_i} \pi'_D = \left(a - bq_i - bq_j - t_i\right) * q_i \tag{4.2}$$

[40] See Shor et al. (2009), p. 584

The result is:

$$q_i = \frac{a - bq_j - t_i}{2b} \tag{4.3}$$

Firm i's optimal quantity depends on the quantity chosen by the competitor and by the transfer price set by the upstream division. The competitor firm j wants to maximize its profit as well and chooses the optimal quantity q_j in the same way as firm i chooses q_i. It is the best response to any action of firm i and vice versa.

$$q_j = \frac{a - bq_i - t_j}{2b} \tag{4.4}$$

When both firms follow the best responses, the equilibrium quantity is established. If we substitute q_j from (4.4) into (4.3) then we get the equilibrium quantity q_i^* of firm i:

$$q_i^* = \frac{a - 2t_i + t_j}{3b} \tag{4.5}$$

The equilibrium quantity q_j^* is also calculated by firm j which substitutes q_i from (4.3) into (4.4):

$$q_j^* = \frac{a - 2t_j + t_i}{3b} \tag{4.6}$$

In the equilibrium, firm i and firm j are not interested in deviating from their equilibrium quantities q_i^* and q_j^* , because setting a quantity other than the equilibrium quantity results in lower revenues. The relationship between the equilibrium quantity and the transfer prices can be described as: The higher the transfer price set by firm i's upstream, the lower is the equilibrium quantity of firm i. The higher the transfer price of the competitor, the higher the equilibrium quantity of firm i.

4.2 Upstream division

4.2.1 Set transfer price

4.2.1.1 Set transfer price – centralized/centralized

The upstream division of firm i sets its transfer price according to the organizational form it has chosen before. If the organizational form is centralized then the upstream division acts as a central planer who transfers the goods at marginal cost.[41] In our model, the marginal cost is assumed to be 0.[42] The transfer of goods at zero marginal cost is consistent within a Cournot model.[43]

$$t_i = 0,\, t_j = 0 \tag{4.7}$$

In our case, both firms choose a transfer price of 0 since both firms are organized in a centralized form and transfer at marginal cost.

4.2.1.2 Set transfer price – decentralized/decentralized

If the organizational forms are decentralized, then the transfer price is the result of the profit maximization from both upstream and downstream divisions. Since the upstream division is the supplying division, the quantity q_i^* sold to the downstream division needs to be charged with the transfer price t_i. The upstream division is interested in maximizing the transfer price t_i in order to maximize its profit π_U.

$$\max_{t_i} \pi_U = q_i^* * t_i \tag{4.8}$$

We substitute q_i^* from (4.5) into the equation and use the first order condition:

$$\max_{t_i} \pi'_U = \frac{a - 2t_i + t_j}{3b} * t_i \tag{4.9}$$

[41] See Shor et al. (2009), p. 584
[42] See Shor et al. (2009), p. 584
[43] See Shor et al. (2009), p. 585

which results in upstream division‘s optimal transfer price of

$$t_i = \frac{1}{4}(a + t_j) \tag{4.10}$$

As both upstream divisions will maximize their profits, the upstream division of firm i can predict that the upstream division of the competitor maximizes $\pi_U = q_i^* * t_j$. The competitor chooses a transfer price, that is the best response to the transfer price set by firm i:

$$t_j = \frac{1}{4}(a + t_i) \tag{4.11}$$

When both firm‘s respond to each other‘s transfer price with the optimal solution, then the equilibrium is established. If we substitute t_j into the equation (4.10) and vice versa t_i into the equation (4.11) the results in the equilibrium transfer prices in a decentralized/decentralized scenario are

$$t_i = \frac{a}{3}, \;\; t_j = \frac{a}{3} \tag{4.12}$$

4.2.1.3 Set transfer price – decentralized/centralized

If firm i chooses to set a decentralized organizational form while firm j selects a centralized strategy, we use the equation $t_i = \frac{1}{4}(a + t_j)$ from (4.10) and equate $t_j = 0$. The optimal transfer price t_i is therefore

$$t_i = \frac{a}{4}, \;\; t_j = 0 \tag{4.13}$$

And if firm j is decentralized while firm i is centralized, then the results are reversed:

$$t_i = 0, \;\; t_j = \frac{a}{4} \tag{4.14}$$

4.2.1.4 Set transfer price - summary

To sum up the results, the transfer prices of both firms are put into a table in Fig. 1

Firm i	Competitor: Centralized	Competitor: Decentralized
Centralized	$\mathbf{0}$, $\mathbf{0}$	$\mathbf{0}$, $\boldsymbol{\frac{a}{4}}$
Decentralized	$\boldsymbol{\frac{a}{4}}$, $\mathbf{0}$	$\boldsymbol{\frac{a}{3}}$, $\boldsymbol{\frac{a}{3}}$

Fig. 1: *Transfer prices chosen in undifferentiated Cournot competition with observable transfer prices*

On the left side of the table, firm i can opt for a centralized organizational form or a decentralized organizational form. When both firms elect a centralized strategy, both firms transfer at zero marginal cost. When firm i is centralized and the competitor is decentralized, firm i chooses a transfer price of zero marginal cost and the competitor selects a transfer price at $\frac{a}{4}$. If firm i chooses "centralized", then firm i's transfer price is always 0, regardless of the choice of the competitor. If firm i sets its organizational form to "decentralized", its transfer price is either $\frac{a}{4}$ or $\frac{a}{3}$, depending on the organizational form elected by the competitor. The transfer prices are the highest, when both firms decide to decentralize. With the transfer prices given above, we can now calculate the profits and search for the dominant organizational strategy in the game.

4.2.2 Choose strategy

4.2.2.1 Choose strategy – centralized/centralized profit

The strategic choices of a firm, whether to centralize or to decentralize, depend on the comparison of profits that the two organizational strategies generate. In the calculation, the profit of firm i is a result of the equilibrium quantity from (4.5) and the price from demand function in (3.1). We assume that the choices of both firms are centralized strategies with transer prices at zero marginal cost. Therefore we put $t_i = 0$ and $t_j = 0$ into the equation (4.5). The equilibrium quantities are:

$$q_i^* = \frac{a}{3b}, \; q_j^* = \frac{a}{3b} \tag{4.15}$$

We substitute the equilibrium quantities into the demand function in (3.1) and yield:

$$p = \frac{a}{3} \tag{4.16}$$

The profit of firm i is generated by $q_i^* * p$ and results in:

$$\Pi_i = \frac{a^2}{9b} \tag{4.17}$$

It is the result for firm i when both firms follow the marginal cost strategy as described in Hirshschleifer[44]. The higher the income level a is, the more profit firm i can generate. When the elasticity of quantity b is high, the profit is decreasing. In order to underline the difference between different strategic choices, we will compare this profit with the other profits in the section “4.2.2.5 Dominant strategy and Nash equilibrium“.

[44]See Hirshleifer (1956), p. 183

4.2.2.2 Choose strategy – decentralized/centralized profit

If firm i is decentralized while firm j is centralized then the transfer prices are set to $t_i = \frac{a}{4}$, $t_j = 0$ according to Fig. 1. The equilibrium quantities and the industrial price are:

$$q_i^* = \frac{a}{6b}, \ q_j^* = \frac{5a}{12b}, \ p = \frac{5a}{12} \tag{4.18}$$

The profit of firm i is:

$$\Pi_i = \frac{5a^2}{72b} \tag{4.19}$$

From an initial comparison $\frac{a^2}{9b} > \frac{5a^2}{72b}$ between the profit of a centralized and a decentralized strategy, firm i is worse off with a decentralized strategy under the condition, that the competitor chooses to remain centralized.

4.2.2.3 Choose strategy – centralized/decentralized profit

Firm i is now centralized and its competitor selects the decentralized organizational form. The transfer prices are $t_i = 0$, $t_j = \frac{a}{4}$. The equilibrium quantities and the industrial prices are calculated as:

$$q_i^* = \frac{5a}{12b}, \ q_j^* = \frac{a}{6b}, \ p = \frac{5a}{12} \tag{4.20}$$

The profit of firm i is:

$$\Pi_i = \frac{25a^2}{144b} \tag{4.21}$$

When firm i stays centralized while the competitor chooses to decentralize, firm i's profit increases compared to the centralized/centralized scenario $\frac{a^2}{9b} < \frac{25a^2}{144b}$.

4.2.2.4 Choose strategy – decentralized/decentralized profit

When both firms are decentralized, the transfer prices are set to $t_i = \frac{a}{3},\ \ t_j = \frac{a}{3}$. Therefore the equilibrium quantities and the industrial price are:

$$q_i^* = \frac{2a}{9b},\ \ q_j^* = \frac{2a}{9b},\ \ p = \frac{5a}{9} \tag{4.22}$$

The profit of this scenario is:

$$\Pi_i = \frac{10a^2}{81b} \tag{4.23}$$

The strategical choices are compared in the next section in order to find the profit maximizing strategy .

4.2.2.5 Choose strategy – dominant strategy & Nash equilibrium

In Fig. 2, we put all four profit results together:

Firm i \ Competitor	Centralized	Decentralized
Centralized	$\boldsymbol{\frac{a^2}{9b}}$	$\boldsymbol{\frac{25a^2}{144b}}$
Decentralized	$\boldsymbol{\frac{5a^2}{72b}}$	$\boldsymbol{\frac{10a^2}{81b}}$

Fig. 2: *Profits for firm i in undifferentiated Cournot competition with observable transfer prices*

Let us assume that our competitor chooses a centralized strategy. Firm i can opt for a centralized strategy as well with a profit of $\frac{a^2}{9b}$ or a decentralized strategy with a profit of $\frac{5a^2}{72b}$.

Since the inequation below is correct,

$$\frac{a^2}{9b} > \frac{5a^2}{72b} \equiv 8a^2 > 5a^2 \tag{4.24}$$

firm i always prefer a centralized strategy rather than a decentralized strategy when the competitor chooses a centralized organization with zero marginal cost. If firm i assumes that the competitor will choose a decentralized organizational form, it once again compares the profits of strategic choices:

$$\frac{25a^2}{144b} > \frac{10a^2}{81b} \equiv 25a^2 > 17\frac{7}{9}a^2 \tag{4.25}$$

As the competitor chooses to be decentralized, the best move is once again to elect a centralized strategy. In conclusion, we see that it does not matter what organizational form the competitor chooses, firm i is always better off, if it stays with the centralized strategy. Therefore the centralized organizational form is a dominant strategy in this game for firm i. Since there is symmetry, the competitor, finds itself in the same situation, as no matter what strategy firm i selects, the competitor would always prefer a centralized strategy. When both firms elect the centralized strategy as the dominant strategy, then it is a Nash equilibrium where it does not pay off for either firm to deviate from this equilibrium. For example, if firm i leaves the Nash equilibrium and chooses to decentralize while firm j remains centralized, firm i would face a decrease in profit while the competitor's profit increases. Therefore the equilibrium in which both firm chooses to centralize is a stable equilibrium.

We now compare the profit between a centralized/centralized scenario and a decentralized/decentralized constellation. And as we see the inequation below,

$$\frac{10a^2}{81b} > \frac{a^2}{9b} \equiv 10a^2 > 9a^2 \tag{4.26}$$

the decentralized/decentralized scenario earns more profit for both firms if they both decide to cooperate and commit themselves to a decentralized form. The scenario “centralized/centralized“ is therefore **dominated** by the cooperative scenario “decentralized/decentralized“ where both firms set transfer prices above marginal cost. The problem with cooperation is that it is not incentive compatible and it is not a stable equilibrium . There is always an incentive to deviate from the decentralized/decentralized strategy. For example, firm i would earn more profit, if it deviates to a centralized organization while the competitor stays in a decentralized organizational form. When both firms have incentives to deviate from the cooperation form (decentralized/decentralized), then both firms will find the equilibrium in the non-cooperative scenario (centralized/centralized). The situation can also be described as a prisoner’s dilemma. Even if both firms know that they can generate more profit by cooperating than by being non-cooperative and transfer at above marginal cost, they would not collude, because the profit gain by cheating and by being non-cooperative is greater than cooperative profit. The other reason to be non-cooperative is because of the monetary loss of being cheated by the other firm prevents them from cooperating.

4.3 Result & summary

The centralized organizational form is a dominant strategy that both firms will pursue. Both firms will therefore transfer their goods at marginal cost, regardless of the organizational choice and the choice of transfer price by their competitor. The centralized/centralized situation is a Nash equilibrium where none of the two firms has an incentive to deviate from it since it would lead to less profit. Interestingly, if both firms would cooperate and set their organizational form to decentralized/decentralized, then the profit would be higher than the centralized/centralized scenario. But because of incentive issues, the decentralized/decentralized outcome is not a stable equilibrium. The result is equal to the outcome of a prisoner’s dilemma, in which cooperation is rewarded, but cheating generates even more profit.

5 Undifferentiated Cournot competition with unobservable transfer prices

5.1 Unobservable transfer prices and random variable a

In the model it is assumed that full information about the transfer price is available. Now the assumption breaks and the transfer prices are not observable. It is discussed by Narayanan et al. and Göx in their models.[45] We can find unobservable transfer prices in cases where it is for example illegal to share the transfer price information with the competitor.[46] We follow the approach from Narayanan et al. in our model as we do not allow the quantity q_i of firm i to depend on the transfer price t_j. Vice versa, the quantity q_j of firm j is not dependent on t_i.[47]

Like in Narayanan et al., we also form the constant a to a random variable, with the density function $f_a(a)$, the distribution function $F_a(a)$, mean μ_a, standard deviation σ_a and support $[\,\overline{a}, \underline{a}\,]$.[48] In a Cournot competition, a generally stands for a shift in the price which is not dependent on the decision variable q. For example a could be a change in the income situation of the consumers. With higher income, buyers are more willing to pay more for the same quantity of a good. When a is not observable, we use the distribution function $F_a(a)$ and calculate the expectation of a with the mean μ_a. The variance σ_a^2 is the additional information how much a would deviate from the expected mean μ_a. We want to find the dominant strategy and see if the prisoner's dilemma still exists in the undifferentiated Cournot game even if the transfer price of the competitor is not observable.

[45] See Narayanan et al. (2000), pp. 510 - 513 and Göx (2000), pp. 336 - 338
[46] See Narayanan et al. (2000), pp. 510-511
[47] See Narayanan et al. (2000), p. 523;Since Narayanan et al. use a Bertrand game, it is price *p* which does not depend on the transfer price of the competitor. In our Cournot game, it is quantity *q* that does not depend on the transfer price of the competitor.
[48] See Narayanan et al. (2000), p. 504

5.2 Downstream division

The downstream division solves the optimal quantity as shown in (4.3) and (4.4):

$$q_i = \frac{a - bq_j - t_i}{2b}, \quad q_j = \frac{a - bq_i - t_j}{2b} \tag{5.1}$$

The downstream division has no information about the transfer price chosen by the competitor. It can not take the decision variable t_i into its calcualtion of the quantity. Therefore the downstream division of firm i does not substitute (4.4) into (4.3), since it does not want q_i to depend on t_j. The optimal quantity q_i depends on t_i and q_j but not on t_j.

5.3 Upstream division

5.3.1 Set transfer price

5.3.1.1 Set transfer price - centralized

The transfer prices set by the central planners are at zero marginal cost:

$$t_i = 0, \quad t_i = 0 \tag{5.2}$$

5.3.1.2 Set transfer price - decentralized

When there is a decentralized organizational form, the upstream division is interested in maximizing the transfer price t_i in order to maximize its profit π_U.

$$\max_{t_i} \pi_U = q_i * t_i \tag{5.3}$$

We substitute q_i from (4.3) into the equation and use the first order condition:

$$\max_{t_i} \pi'_U = \int_{\underline{a}}^{\bar{a}} \frac{a - bq_j - t_i}{2b} * \; t_i \;\; f_a(a)\, d\, a \tag{5.4}$$

which results in firm i's optimal transfer price by integrating with respect to a

$$t_i = \frac{1}{2}(\mu_a - bq_j) \tag{5.5}$$

Firm j solves the symmetric transfer price:

$$t_j = \frac{1}{2}(\mu_a - bq_i) \tag{5.6}$$

In the observable scenario, the transfer price of firm i depends on the transfer price chosen by the competitor and vice versa. The constant a is observable. In the unobservable case, firm i's transfer price depends on the quantity chosen by firm j. The constant a changes to mean μ_a.

5.3.2 Choose strategy

5.3.2.1 Choose strategy – centralized/centralized profit

We substitute $t_i = 0$ and $t_j = 0$ into the equations in (5.1):

$$q_i = \frac{a - bq_j}{2b}, \;\; q_j = \frac{a - bq_i}{2b} \tag{5.7}$$

The quantities chosen by each firm depend on the quantity chosen by the other firm.

Let

$$\mu_{q_i} \equiv \int_{\underline{a}}^{\bar{a}} q_i\, f_a(a)\, d\, a \tag{5.8}$$

and

$$\mu_{q_j} \equiv \int_{\underline{a}}^{\bar{a}} q_j\, f_a(a)\, d\, a \tag{5.9}$$

be the expectations of the equilibrium conjectures about the quantity.[49] After integrating with respect to a, we receive the following two equations

$$\mu_{q_i} = \frac{\mu_\mathrm{a} - b\mu_{q_j}}{2b}, \quad \mu_{q_j} = \frac{\mu_\mathrm{a} - b\mu_{q_i}}{2b} \tag{5.10}$$

If the condition $\mu_{q_i} = \mu_{q_j}$ is met, the conjectures of each firm are fulfilled in the equilibrium.[50] The results of solving two equations with two unknown variables are:

$$\mu_{q_i} = \frac{\mu_\mathrm{a}}{3b}, \quad \mu_{q_j} = \frac{\mu_\mathrm{a}}{3b} \tag{5.11}$$

We substitute the expected quantities into the demand function (3.1) and yield:

$$p = \frac{\mu_\mathrm{a}}{3} \tag{5.12}$$

The profit generated by firm i is:

$$\Pi_i = \frac{{\mu_\mathrm{a}}^2}{9b} \tag{5.13}$$

[49] See Narayanan et al. (2012), pp. 145-149
[50] See Narayanan et al. (2012), p. 146

If $\mu_\mathrm{a} = a$, the profit outcome is the same in the unobservable transfer price scenario compared to the observable transfer price scenario. Although the transfer price of the competitor is unknown to the downstream division of firm i, the quantity set by the downstream division does not differ from its choice in the observable scenario.

5.3.2.2 Choose strategy – decentralized/centralized profit

The goods are transferred at $t_i = \frac{1}{2}(\mu_a - bq_j)$ and $t_j = 0$, because firm i chooses to decentralize instead of being centralized. The transfer prices are substituted into the equations (4.3) and (4.4).

Let $\mu_{q_i} \equiv \int_{\underline{a}}^{\bar{a}} q_i\, f_a(a)\, d\, a$ and $\mu_{q_j} \equiv \int_{\underline{a}}^{\bar{a}} q_j\, f_a(a)\, d\, a$. By integrating with respect to a, the expected quantities are:

$$\mu_{q_i} = \frac{\mu_\mathrm{a} - b\mu_{q_j}}{4b}, \quad \mu_{q_j} = \frac{\mu_\mathrm{a} - b\mu_{q_i}}{2b} \tag{5.14}$$

After solving the two equations we receive:

$$\mu_{q_i} = \frac{\mu_\mathrm{a}}{7b}, \quad \mu_{q_j} = \frac{3\mu_\mathrm{a}}{7b} \tag{5.15}$$

The corresponding price is:

$$p = \frac{3\mu_\mathrm{a}}{7} \tag{5.16}$$

Firm i yields:

$$\Pi_i = \frac{3{\mu_\mathrm{a}}^2}{49b} \tag{5.17}$$

When the result is compared to $\frac{{\mu_{\mathrm{a}}}^2}{9b}$, it shows that when transfer prices are not observable, the decentralized strategy is again less profitable if the competitor chooses to remain centralized. The outcome is the same as in the observable transfer price scenario.

5.3.2.3 Choose strategy – centralized/decentralized profit

Firm i is centralized while the competitor firm j is decentralized. The expected quantities are the same as in the decentralized/centralized scenario, but reversed.

$$\mu_{q_i} = \frac{3\mu_{\mathrm{a}}}{7b},\ \ \mu_{q_j} = \frac{\mu_{\mathrm{a}}}{7b},\ \ p = \frac{3\mu_{\mathrm{a}}}{7} \tag{5.18}$$

The profit is:

$$\Pi_i = \frac{9{\mu_{\mathrm{a}}}^2}{49b} \tag{5.19}$$

5.3.2.4 Choose strategy – decentralized/decentralized profit

In the decentralized/decentralized case, both firms transfer goods at $t_i = \frac{1}{2}(\mu_a - bq_j)$ and $t_j = \frac{1}{2}(\mu_a - bq_i)$. Let $\mu_{q_i} \equiv \int_{\underline{a}}^{\bar{a}} q_i\, f_a(a)\, d\, a$ and $\mu_{q_j} \equiv \int_{\underline{a}}^{\bar{a}} q_j\, f_a(a)\, d\, a$. By substituting them into the equation (4.3) and (4.4) and by integrating with respect to a, the expected quantities are:

$$\mu_{q_i} = \frac{\mu_{\mathrm{a}} - b\mu_{q_j}}{4b},\ \ \mu_{q_j} = \frac{\mu_{\mathrm{a}} - b\mu_{q_i}}{4b} \tag{5.20}$$

By solving the two equations

$$\mu_{q_i} = \frac{\mu_{\mathrm{a}}}{5b},\ \ \mu_{q_j} = \frac{\mu_{\mathrm{a}}}{5b} \tag{5.21}$$

firm i yields a price of

$$p = \frac{3\mu_a}{5} \tag{5.22}$$

and a profit of

$$\Pi_i = \frac{3{\mu_a}^2}{25b} \tag{5.23}$$

5.3.2.5 Choose strategy – dominant strategy & Nash equilibrium

Again we compare the profits calculated by firm i in the non-observable transfer price scenario:

Firm i \ Competitor	Centralized	Decentralized
Centralized	$\frac{{\mu_a}^2}{9b}$	$\frac{9{\mu_a}^2}{49b}$
Decentralized	$\frac{3{\mu_a}^2}{49b}$	$\frac{3{\mu_a}^2}{25b}$

Fig. 3: *Profits for firm i in undifferentiated Cournot competition with unobservable transfer prices*

If the competitor chooses to centralize, firm i always opts for the centralized strategy, since

$$\frac{{\mu_a}^2}{9b} > \frac{3{\mu_a}^2}{49b} \equiv 5\frac{4}{9}{\mu_a}^2 > 3{\mu_a}^2 \tag{5.24}$$

And if the rival elects the decentralized strategy, firm i still chooses the centralized strategy, since

$$\frac{9{\mu_a}^2}{49b} > \frac{3{\mu_a}^2}{25b} \equiv 9{\mu_a}^2 > 5\frac{22}{25}{\mu_a}^2 \tag{5.25}$$

Therefore the centralized organizational strategy with a transfer price at marginal cost remains the dominant strategy in the scenario with unobservable transfer prices. It is a Nash equilibrium in which no firm has an incentive to deviate from that equilibrium. Even when both firms cannot observe each other's transfer price, a stable Nash equilibrium can be established, in which both firms transfer at marginal cost. The condition $\mu_{q_i} = \mu_{q_j}$ needs to be met, in which the conjectures of both firms are fulfilled in equilibrium.[51]

As we compare the centralized/centralized outcome with the decentralized/decentralized outcome, the profit of a cooperation is higher than the profit of a non-cooperative result.

$$\frac{3{\mu_a}^2}{25b} > \frac{{\mu_a}^2}{9b} \equiv 3{\mu_a}^2 > 2\frac{7}{9}{\mu_a}^2 \tag{5.26}$$

Nonetheless the prisoner's dilemma still exists in the unobservable transfer pricing scenario. There is always an incentive problem and the cooperative outcome is not stable.

5.3 Result & summary

The result in the scenario with unobservable transfer prices is the same compared to the scenario with observable transfer prices: Centralization remains the dominant strategy and both firms transfer at marginal cost. The non-cooperative equilibrium (centralized/centralized) is the Nash equilibrium. The decentralized/decentralized equilibrium with a transfer price above marginal cost earns more profit than the non-cooperative equilibrium, but it is not stable and not incentive compatible. We again face a prisoner's dilemma in a scenario with unobservable transfer prices.

[51]Narayanan et al. (2012), p. 146

6 Differentiated Cournot competition with observable transfer prices

6.1 Downstream division

Except from the different price-quantity function in (3.3) the calculation process in the differentiated Cournot competition stay the same as compared to the undifferentiated Cournot competition. In the case of the differentiated Cournot competition, we use the equation (4.1) as the profit maximization function for the downstream division and substitute (3.3) into it. The downstream division of firm i solves the following equation:

$$\max_{q_i} \pi'_D = \left(a - q_i - \theta q_j - t_i\right) * q_i \tag{6.1}$$

We use the first order condition of π'_D and yield:

$$q_i = \frac{1}{2}(a - \theta q_j - t_i) \tag{6.2}$$

Since firm j maximizes its profit as well, the quantity produced by the downstream division of firm j is:

$$q_j = \frac{1}{2}(a - \theta q_i - t_j) \tag{6.3}$$

The optimal quantity set by the downstream division of firm i depends on the quantity chosen by firm j. It is the best response to any action of firm j and vice versa. When both firms predict the optimal response of each other correctly, the equilibrium quantity is established. We substitute (6.3) into (6.2) and vice versa. The equilibrium quantities are:

$$q_i^* = \frac{a(-2+\theta) + 2t_i - \theta t_j}{\theta^2 - 4} \tag{6.4}$$

and

$$q_j^* = \frac{a(-2+\theta) - \theta t_i + 2t_j}{\theta^2 - 4} \tag{6.5}$$

The results of both equilibrium quantities depend on the transfer price of the own upstream division and the transfer price of the competitor's upstream division.

6.2 Upstream division

6.2.1 Set transfer price

6.2.1.1 Set transfer price – centralized/centralized

When both firms are centralized, the transfer price set by both firms are:

$$t_i = 0,\ t_j = 0 \tag{6.6}$$

6.2.1.2 Set transfer price – decentralized/decentralized

The upstream division of firm i maximizes the following equation:

$$\max_{t_i} \pi'_U = q_i^* * t_i \tag{6.7}$$

Substituting (6.4) into (6.7), we yield:

$$\max_{t_i} \pi'_U = \frac{a(-2+\theta) + 2t_i - \theta t_j}{\theta^2 - 4} * t_i \tag{6.8}$$

After using the first order condition, the optimal transfer price t_i is defined as

$$t_i = \frac{1}{4}(2a - a\theta + \theta t_j) \tag{6.9}$$

Firm j's optimal transfer price t_j is:

$$t_j = \frac{1}{4}(2a - a\theta + \theta t_i) \tag{6.10}$$

In order to calculate the equilibrium transfer prices in a decentralized/decentralized scenario, we need to substitute (6.10) into (6.9) and vice versa. In equilibrium, both transfer prices are optimal in the sense that no firm has an incentive to deviate from those transfer prices:

$$t_i = \frac{a(2-\theta)}{4-\theta}, \;\; t_j = \frac{a(2-\theta)}{4-\theta} \tag{6.11}$$

Compared to the undifferentiated Cournot Competition, the transfer price chosen in a decentralized/decentralized scenario depends on the intensity of competition. A lower grade of competition means higher transfer prices.

6.2.1.3 Set transfer price – decentralized/centralized

In the case of a decentralized/centralized scenario, firm i chooses to decentralize while firm j elects a centralized strategy. We subsitute $t_j = 0$ into (6.9) and receive:

$$t_i = \tfrac{1}{4}(2a - a\theta), \;\; t_j = 0 \tag{6.12}$$

If the scenario is centralized/decentralized, then the values of the transfer prices are reversed:

$$t_i = 0, \;\; t_j = \tfrac{1}{4}(2a - a\theta) \tag{6.13}$$

6.2.1.4 Set transfer prices – summary

Fig. 4 summarizes the transfer prices chosen in a differentiated Cournot competition with observable transfer prices:

Firm i \ Competitor	Centralized	Decentralized
Centralized	0 , 0	0 , $\frac{1}{4}(2a - a\theta)$
Decentralized	$\frac{1}{4}(2a - a\theta)$, 0	$\frac{a(2-\theta)}{4-\theta}$, $\frac{a(2-\theta)}{4-\theta}$

Fig. 4: *Transfer prices chosen in differentiated Cournot competition with observable transfer prices*

Compared to the transfer prices chosen in an undifferentiated Cournot competition, the transfer prices are slightly higher in the differentiated case as long as $\theta < 1$. For example, when $\theta = \frac{1}{2}$, then the transfer price in the decentralized/centralized case is $t_i = \frac{3a}{8}$ while in the decentralized/decentralized case it is $t_i = \frac{3a}{7}$ (compared to the transfer prices $t_i = \frac{a}{4}$ and $t_i = \frac{a}{3}$ in the undifferentiated case). In the extreme case, when the products are totally differentiated and the intensity of the competition is $\theta = 0$, then the transfer price in the decentralized strategy is always $t_i = \frac{a}{2}$ for any election of the competitor's strategy. The intuition behind a zero competition is that both firms find themselves in a monopoly, where the choice of transfer price is not affected by the competitor's choice. When both firms face a perfect competition with $\theta = 1$, the scenario becomes an undifferentiated Cournot competition.

6.2.2 Choose strategy

6.2.2.1 Choose strategy – centralized/centralized profit

When both firms decide on the centralized organizational form, then the central planners transfer goods at $t_i = 0, t_j = 0$. If we substitute those transfer prices into (6.4) and (6.5), then the quantity sold by the downstream division is:

$$q_i^* = \frac{a}{2+\theta}, \;\; q_j^* = \frac{a}{2+\theta} \tag{6.14}$$

The corresponding prices p_i and p_j can be calculated by substituting the quantities into (3.3):

$$p_i = \frac{a}{2+\theta}, \;\; p_j = \frac{a}{2+\theta} \tag{6.15}$$

By multiplying $p_i * \; q_i$, the result of firms i's profit is:

$$\Pi_i = \frac{a^2}{(2+\theta)^2} \tag{6.16}$$

The result shows that the higher intensity of the competition, the lower the profit. The prices and quantities chosen in a differentiated Cournot competition are both higher than those in an undifferentiated Cournot competition when $\theta < 1$. With higher product differentiation, the competitor cannot satisfy the demand of those consumers, who would like to buy for example red jeans (instead of blue jeans), therefore the production quantites of red jeans increase. At the same time, the prices rise as well, since firm i faces less competition. In the case of zero competition with $\theta = 0$, the profit is the highest, since firm i can choose its price and quantity as if it is in a monopoly. The following graph shows the relationship between the intensity of competition and the profit in a centralized/centralized scenario, when we assume that $a = 1$:

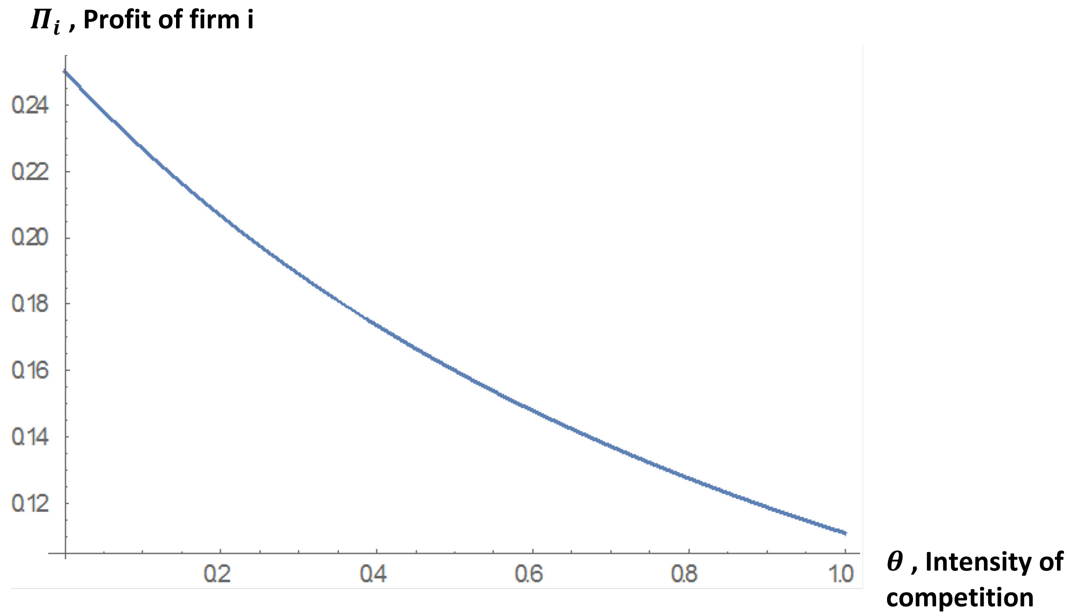

Fig. 5: *The higher the intensity of competition, the lower the profit of firm i*

The profit is more than doubled between a zero competition and a perfect competition in a centralize/centralized scenario.

6.2.2.2 Choose strategy – decentralized/centralized profit

We analyze the second case, in which firm i opts for a decentralized strategy while the competitor firm j chooses to stay centralized. The transfer prices in this scenario are calculated in (6.12): $t_i = \frac{1}{4}(2a - a\theta),\ t_j = 0$. We substitute those transfer prices again into (6.4) and (6.5):

$$q_i^* = \frac{a}{2(2+\theta)},\ q_j^* = \frac{4a + a\theta}{4(2+\theta)} \quad (6.17)$$

The corresponding prices are:

$$p_i = \frac{6a - a\theta^2}{4(2+\theta)},\ p_j = \frac{4a + a\theta}{4(2+\theta)} \quad (6.18)$$

The profit in the decentralized/centralized scenario equals:

$$\Pi_i = \frac{a^2(6-\theta^2)}{8(2+\theta)^2} \tag{6.19}$$

6.2.2.3 Choose strategy – centralized/decentralized profit

In this scenario the transfer prices and the quantities are reversed compared to the previous case. The transfer prices are $t_i = 0,\ t_j = \frac{1}{4}(2a - a\theta)$ while the quantities sold on the market are:

$$q_i^* = \frac{4a + a\theta}{4(2+\theta)},\ q_j^* = \frac{a}{2(2+\theta)} \tag{6.20}$$

We again calculate the prices for firm i and firm j which are reversed as well:

$$p_i = \frac{4a + a\theta}{4(2+\theta)},\ p_j = \frac{6a - a\theta^2}{4(2+\theta)} \tag{6.21}$$

The profit generated by firm i yields:

$$\Pi_i = \frac{(4a + a\theta)^2}{16(2+\theta)^2} \tag{6.22}$$

6.2.2.4 Choose strategy – decentralized/decentralized profit

In the last of the four scenarios, the transfer prices chosen by the upstream divisions are taken from (6.11): $t_i = \frac{a(2-\theta)}{4-\theta},\ t_j = \frac{a(2-\theta)}{4-\theta}$. By substituting the transfer prices into (6.4) and (6.5) we receive:

$$q_i^* = \frac{2a}{8 + 2\theta - \theta^2},\ q_j^* = \frac{2a}{8 + 2\theta - \theta^2} \tag{6.23}$$

The corresponding prices are:

$$p_i = \frac{a(-6+\theta^2)}{(-4+\theta)(2+\theta)},\ p_j = \frac{a(-6+\theta^2)}{(-4+\theta)(2+\theta)} \tag{6.24}$$

The following profit is generated by firm i in the decentralized/decentralized scenario:

$$\Pi_i = \frac{2a^2(6-\theta^2)}{(-4+\theta)^2(2+\theta)^2} \tag{6.25}$$

6.2.2.5 Choose strategy – dominant strategy and Nash equilibrium

Fig. 6 depicts the profit matrix of firm i. The profit depends on both firm i's and firm j's choice of the organizational form.

Firm i \ Competitor	Centralized	Decentralized
Centralized	$\frac{a^2}{(2+\theta)^2}$	$\frac{(4a+a\theta)^2}{16(2+\theta)^2}$
Decentralized	$\frac{a^2(6-\theta^2)}{8(2+\theta)^2}$	$\frac{2a^2(6-\theta^2)}{(-4+\theta)^2(2+\theta)^2}$

Fig. 6: *Profits for firm i in differentiated Cournot competition with observable transfer prices*

In order to find the dominant strategy, we need to compare the profits in the profit matrix. First, we assume that the competitor firm j remains centralized, while firm i elects the centralized or decentralized strategy.

$$\frac{a^2}{(2+\theta)^2} > \frac{a^2(6-\theta^2)}{8(2+\theta)^2} \equiv 1 > -\frac{(-6+\theta^2)}{8} \tag{6.26}$$

The inequation is correct, if we remember that $\theta \in [0\,,1)$. Therefore the centralized strategy is preferred when the competitor chooses a centralized strategy as well. In the next step we compare the profits when the competitor chooses to decentralize:

$$\frac{(4a+a\theta)^2}{16(2+\theta)^2} > \frac{2a^2(6-\theta^2)}{(-4+\theta)^2(2+\theta)^2} \equiv a^2(-4+\theta)^2(2+\theta)^2(64+\theta^4) > 0 \tag{6.27}$$

For any level of θ, the inequation is always correct since the expression on the left side is always greater than zero. We can sum up, that the centralized strategy always dominates the decentralized strategy for any strategy the competitor elects in the differentiated Cournot competition. When both firms follow the dominant strategy, it results in a centralized/centralized scenario, which is the Nash equilibrium of this game. Furthermore we compare the profit of the centralized/centralized scenario with the profit the decentralized/decentralized scenario in order to find out whether there is a prisoner's dilemma:

$$\frac{2a^2(6-\theta^2)}{(-4+\theta)^2(2+\theta)^2} > \frac{a^2}{(2+\theta)^2} \equiv a^2(-4+\theta)^2(-2+\theta)(2+\theta)^2(-2+3\theta) < 0 \tag{6.28}$$

The inequation is only true, if $\theta > \frac{2}{3}$. Only when the intensity of competition is high enough, then the decentralized/decentralized scenario dominates the centralized/centralized scenario and the prisoner's dilemma exists. If there is a low intensity of competition in a differentiated Cournot competition, the centralized strategy with transfer prices at marginal cost is always the dominant strategy.

The relationship between the intensity of the competition and the choice of strategy is shown in Fig. 7:

Fig. 7: *Strategic choice and intensity of competition*

When the competition is $\theta = 0$, we are at Hirshleifer's Marginal cost strategy. Until $\theta = \frac{2}{3}$, it is the best choice to stay centralized with a transfer price at marginal cost. As the intensity of competition exceeds $\theta = \frac{2}{3}$, a decentralized strategy with a transfer price above marginal cost earns higher profit, but both firms find themselves in a prisoner's dilemma, since deviating from the cooperative outcome is rewarded with a even higher profit.

6.3 Result & summary

In a differentiated Cournot competition the transfer prices set above marginal costs differ from the transfer prices in the undifferentiated Cournot competition. When the intensity of competition is $\theta < 1$, then the transfer prices are higher in the differentiated case. In the choice of strategy, the result remains the same as in the undifferentiated case. Both firms opt for the centralized strategy since it generates more profit. Therefore the Nash equilibrium is in the centralized/centralized scenario. Furthermore the prisoner's dilemma does not always exist in the differentiated Cournot competition. When the intensity of competition is $\theta < \frac{2}{3}$, then there is no incentive for any of the two firms to cooperate in the differentiated Cournot game.

7 Differentiated Cournot competition with unobservable transfer prices

The assumptions stay the same as in the section “5 Undifferentiated Cournot Competition with unobservable transfer prices“.

7.1 Downstream division

The downstream division of firm i solves the following maximization problem as in (6.1):

$$\max_{q_i} \pi'_D = \left(a - q_i - \theta q_j - t_i\right) * q_i \tag{7.1}$$

After the first order condition with respect to q_i , the optimal quantity chosen by the downstream division results in:

$$q_i = \frac{1}{2}(a - \theta q_j - t_i) \tag{7.2}$$

If firm i predicts correctly, the competitor firm j’s best response is to choose the optimal quantity:

$$q_j = \frac{1}{2}(a - \theta q_i - t_j) \tag{7.3}$$

Instead of substituting (7.3) into (7.2), we do not allow q_i to depend on t_j, since the transfer price of the competitor is not observed by the downstream division of firm i.[52]

[52] See Narayanan et al. (2000), p. 523

7.2 Upstream division

7.2.1 Set transfer prices

7.3.2.1 Set transfer prices – centralized

The transfer price in a centralized organization is set at zero marginal cost:

$$t_i = 0, \;\; t_i = 0 \tag{7.4}$$

7.3.2.2 Set transfer prices – decentralized

The upstream division maximizes its profit function:

$$\max_{t_i} \pi_U = \; q_i * \; t_i \tag{7.5}$$

We substitute the equation (7.2) into the profit equation (7.9) and solve the first order condition. Other than in the scenario with observable transfer prices, the downstream division does not rely on t_j in the calculation of its profit maximizing function.

$$\max_{t_i} \pi'_U = \; \int_{\underline{a}}^{\bar{a}} \left(\frac{1}{2}(a - \theta q_j - t_i)\right) * \; t_i \;\; f_a(a) \, d\, a \tag{7.6}$$

We integrate with respect to a and receive the following transfer price for firm i:

$$t_i = \frac{1}{2}(a - \theta q_j) \tag{7.7}$$

Consequently, firm j faces the same problems with unobservable transfer prices and calculates the optimal transfer price t_j:

$$t_j = \frac{1}{2}(a - \theta q_i) \tag{7.8}$$

The higher the quantity chosen by the competitor, the lower firm i's transfer price. If the intensity of competition is $\theta = 0$, the transfer price is not affected by the choice of firm j's quantity.

7.2.2 Choose strategy

7.2.2.1 Choose strategy – centralized/centralized Profit

We substitute $t_i = 0$ and $t_j = 0$ into the equations (7.2) and (7.3) and get the following quantities for both firms:

$$q_i = \frac{1}{2}(a - \theta q_j), \;\; q_j = \frac{1}{2}(a - \theta q_i) \tag{7.9}$$

The quantity chosen by the downstream division does not depend on the transfer prices, but rather on the quantity amount chosen by the competitor. Let

$$\mu_{q_i} \equiv \int_{\underline{a}}^{\bar{a}} q_i \, f_a(a) \, d\, a \tag{7.10}$$

and

$$\mu_{q_j} \equiv \int_{\underline{a}}^{\bar{a}} q_j \, f_a(a) \, d\, a \tag{7.11}$$

be the expectation of the equilibrium conjectures about the quantity.[53] When integrating with respect to a, the equations turn into:

$$\mu_{q_i} = \frac{1}{2}(\mu_\mathrm{a} - \theta \mu_{q_j}), \;\; \mu_{q_j} = \frac{1}{2}(\mu_\mathrm{a} - \theta \mu_{q_i}) \tag{7.12}$$

If the condition $\mu_{q_i} = \mu_{q_j}$ is met, the conjectures of each firm are fulfilled in the equilibrium.[54]

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[53] See Narayanan et al. (2012), pp.145-149

We solve two equations with two unknown variables and receive:

$$\mu_{q_i} = \frac{\mu_a}{2+\theta}, \quad \mu_{q_j} = \frac{\mu_a}{2+\theta} \tag{7.13}$$

The corresponding prices from (3.3) depend on the expected quantity we have calculated above:

$$p_i = p_j = \frac{\mu_a}{2+\theta} \tag{7.14}$$

The profit generated by firm i in a centralized/centralized scenario is:

$$\Pi_i = \frac{{\mu_a}^2}{(2+\theta)^2} \tag{7.15}$$

If $\mu_a = a$, we have the same result, as presented in the differentiated Cournot competition with observable transfer prices.

7.2.2.2 Choose strategy – decentralized/centralized profit

Firm i's organizational form is decentralized and it transfers goods at $t_i = \frac{1}{2}(a - \theta q_j)$, while the competitor chooses a centralized strategy and transfers at marginal cost $t_j = 0$. The transfer prices are substituted in (7.2) and (7.3) which result in quantities of

$$q_i = \frac{1}{4}(a - \theta q_j), \quad q_j = \frac{1}{2}(a - \theta q_i) \tag{7.16}$$

Following (46) and (47) we integrate with respect to a and we receive the expected quantities:

$$\mu_{q_i} = \frac{1}{4}(\mu_a - \theta q_j), \quad \mu_{q_j} = \frac{1}{2}(\mu_a - \theta q_i) \tag{7.17}$$

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[54] See Narayanan et al. (2012), p. 146

If the condition $\mu_{q_i} = \mu_{q_j}$ holds, the expected quantities are

$$\mu_{q_i} = \frac{\mu_{\mathrm{a}}(-2+\theta)}{-8+\theta^2}, \quad \mu_{q_j} = \frac{\mu_{\mathrm{a}}(-4+\theta)}{-8+\theta^2} \tag{7.18}$$

The corresponding prices are therefore

$$p_i = \frac{3(-2\mu_{\mathrm{a}} + \mu_{\mathrm{a}}\theta)}{-8+\theta^2}, \quad p_j = \frac{-4\mu_{\mathrm{a}} + \mu_{\mathrm{a}}\theta}{-8+\theta^2} \tag{7.19}$$

By multiplying $\mu_{q_i} * \ p_i$, we yield a profit of

$$\Pi_i = \frac{3{\mu_{\mathrm{a}}}^2(-2+\theta)^2}{(-8+\theta^2)^2} \tag{7.20}$$

7.2.2.3 Choose strategy – centralized/decentralized profit

In the centralized/decentralized case, the quantities and the prices generated by the downstream divisions are reversed compared to the decentralized/centralized scenario:

$$\mu_{q_i} = \frac{\mu_{\mathrm{a}}(-4+\theta)}{-8+\theta^2}, \quad \mu_{q_j} = \frac{\mu_{\mathrm{a}}(-2+\theta)}{-8+\theta^2} \tag{7.21}$$

The corresponding prices are:

$$p_i = \frac{-4\mu_{\mathrm{a}} + \mu_{\mathrm{a}}\theta}{-8+\theta^2}, \quad p_j = \frac{3(-2\mu_{\mathrm{a}} + \mu_{\mathrm{a}}\theta)}{-8+\theta^2} \tag{7.22}$$

The profit generated by firm i is:

$$\Pi_i = \frac{\mu_{\mathrm{a}}(-4+\theta)^2}{(-8+\theta^2)^2} \tag{7.23}$$

7.2.2.4 Choose strategy – decentralized/decentralized profit

When both firms elect the decentralized strategy, the upstream divisions set transfer prices at $t_i = \frac{1}{2}(a - \theta q_j), t_j = \frac{1}{2}(a - \theta q_i)$. The corresponding quantities chosen by the downstream division are:

$$q_i = \frac{1}{4}(a - \theta q_j),\ \ q_j = \frac{1}{4}(a - \theta q_j) \tag{7.24}$$

Let $\mu_{q_i} \equiv \int_{\underline{a}}^{\bar{a}} q_i\, f_a(a)\, d\, a$ and $\mu_{q_j} \equiv \int_{\underline{a}}^{\bar{a}} q_j\, f_a(a)\, d\, a$ be the expectation of the equilibrium conjectures about the price.[55] By integrating with respect to a, we receive the following equations:

$$\mu_{q_i} = \frac{1}{4}(\mu_{\mathrm{a}} - \theta q_j),\ \ \mu_{q_j} = \frac{1}{4}(\mu_{\mathrm{a}} - \theta q_i) \tag{7.25}$$

If the condition $\mu_{q_i} = \mu_{q_j}$ holds, then the conjectures of each firm are fulfilled in the equilibrium.[56] The solution to two equations with two unknown variables is:

$$\mu_{q_i} = \frac{\mu_{\mathrm{a}}}{4 + \theta},\ \ \mu_{q_j} = \frac{\mu_{\mathrm{a}}}{4 + \theta} \tag{7.26}$$

The prices generated by the two firms are:

$$p_j = \frac{3\mu_{\mathrm{a}}}{4 + \theta},\ \ p_j = \frac{3\mu_{\mathrm{a}}}{4 + \theta} \tag{7.27}$$

The profit generated by firm i in a decentralized/decentralized scenario is:

$$\Pi_i = \frac{3{\mu_{\mathrm{a}}}^2}{(4 + \theta)^2} \tag{7.28}$$

[55] See Narayanan et al. (2012), pp.145-149
[56] See Narayanan et al. (2012), p.146

7.2.2.5 Choose strategy – dominant strategy and Nash equilibrium

To summarize the results from the four scenarios:

Firm i	Competitor: Centralized	Competitor: Decentralized
Centralized	$\frac{\boldsymbol{\mu_a}^2}{(\mathbf{2}+\boldsymbol{\theta})^2}$	$\frac{\boldsymbol{\mu_a}^2(-\mathbf{4}+\boldsymbol{\theta})^2}{(-\mathbf{8}+\boldsymbol{\theta}^2)^2}$
Decentralized	$\frac{\mathbf{3}\boldsymbol{\mu_a}^2(-\mathbf{2}+\boldsymbol{\theta})^2}{(-\mathbf{8}+\boldsymbol{\theta}^2)^2}$	$\frac{\mathbf{3}\boldsymbol{\mu_a}^2}{(\mathbf{4}+\boldsymbol{\theta})^2}$

Fig. 8: *Profits for firm i in differentiated Cournot competition with unobservable transfer prices*

Even when transfer prices are not observable, the centralized organizational form is still the dominant strategy in the differentiated Cournot competition, if $\theta \epsilon [0\,,1)$. For any strategic election of the competitor, firm i does always earn more profit, if it decides to choose the centralized strategy. For example, if the competitor chooses a centralized strategy, then firm i would prefer a centralized strategy instead of a decentralized strategy. Since for any election of $\theta \epsilon [0\,,1)$, $a^2(2+\theta)^2(-8+\theta^2)^2(-8-4\theta^2+\theta^4)$ in the inequation below is always smaller than zero, the inequation is correct.

$$\frac{{\mu_a}^2}{(2+\theta)^2} > \frac{3{\mu_a}^2(-2+\theta)^2}{(-8+\theta^2)^2} \tag{7.29}$$

$$\equiv a^2(2+\theta)^2(-8+\theta^2)^2(-8-4\theta^2+\theta^4) < 0$$

If the competitor chooses a decentralized strategy, firm i would still make more profit by choosing a centralized organizational form:

$$\frac{{\mu_a}^2(-4+\theta)^2}{(-8+\theta^2)^2} > \frac{3{\mu_a}^2}{(4+\theta)^2} \tag{7.30}$$

If both firms select the dominant strategy then the Nash equilibrium is stabilized in the centralized/centralized scenario. The game remains a prisoner's dilemma as long as $\theta > -1 + \sqrt{3}$:

$$\frac{3{\mu_{\mathrm{a}}}^2}{(4+\theta)^2} > \frac{{\mu_{\mathrm{a}}}^2}{(2+\theta)^2} \tag{7.31}$$

When the condition $\theta > -1 + \sqrt{3}$ holds, the centralized/centralized scenario is the stable Nash equilibrium, but it is dominated by the decentralized/decentralized scenario. Therefore it is a "Win-Win" situation, when both firms decide to decentralize.

7.3 Result & summary

The results in the differentiated Cournot competition with unobservable transfer prices are consistent with those in the undifferentiated Cournot competition. Centralization remains the dominant strategy. When both firms choose the dominant strategy, the Nash equilibrium is established in the centralized/centralized scenario. Both firms find themselves in a prisoner's dilemma when the intensity of competition is high enough. The outcome of a prisoner's dilemma does not depend on observable transfer prices but rather on the intensity of competition in a Cournot Competition.

8 Mechanisms to solve the prisoner's dilemma

8.1 Introduction

In the model I distinguished between an undifferentiated and a differentiated Cournot competition, in which firms compete in quantities simultaneously. Within the competitions I analyzed a scenario with observable transfer prices and a scenario in which transfer prices are not observed by the competitor. The result in an undifferentiated Cournot competition is a prisoner's dilemma, in which a cooperation would generate more profit than a non-cooperative outcome for both firms. In the differentiated Cournot competition, the condition of a high level of intensity in the competition must hold in order to have a prisoner's dilemma. If the condition holds, there is always an incentive problem with cooperation in a decentralized/decentralized scenario since a firm which deviates from the cooperative outcome would be rewarded with an even higher profit. In this section I analyze the possibilities to overcome this incentive problem in an undifferentiated and in a differentiated Cournot competition with unobservable transfer prices.

8.2 Asymmetric information

From pp. 44-46 I follow the concept of asymmetric information described in the model from Narayanan et al..[57] Until now I have assumed that the random variable a is not observable, neither in a centralized organization nor in a decentralized organization. But there could also be the case when the downstream division has local information about the sales. Those information are restricted to the upstream division. For example Narayanan et al. discusses the asymmetric information of the sales office and the head office: "The sales office observes the realization of *a,* but the head office knows only its distribution. Thus, *a* captures the sales office's specific knowledge about local market conditions, such as fashion and the economy."[58] In my model, the specific knowledge of the downstream division can

[57] See Narayanan et al. (2000), pp. 504ff

[58] Narayanan et al. (2000), p. 505

be expressed as the variance σ_a^2 and it shows, how much a would deviate from the expected mean μ_a.

Let us analyze the specific knowledge of the downstream division in the undifferentiated Cournot competition with unobservable transfer prices:

Firm i	Competitor: Centralized	Competitor: Decentralized
Centralized	$\frac{\mu_a{}^2}{9b}$	$\frac{9\mu_a{}^2}{49b}$
Decentralized	$\frac{3\mu_a{}^2 + 3\sigma_a^2}{49b}$	$\frac{3\mu_a{}^2 + 3\sigma_a^2}{25b}$

Fig. 9: *Profits for firm i in an undifferentiated Cournot competition with unobservable transfer prices and asymmetric information*

In comparison to Fig. 3, the downstream division in a decentralized firm sets quantities based on the specific knowledge about the market σ_a^2.[59] Therefore the profit in the decentralized strategy includes this specific knowledge in the calculations as shown in Fig. 9. In the centralized strategy, a central planner transfers goods at marginal cost, but without taking the information about the market into account. When we compare the profits we receive the following results: As long the value of the specific knowledge exceeds $\sigma_a^2 > \frac{22\mu_a{}^2}{27}$ firm i would always choose to decentralize for any organizational election of its competitor. Decentralization would be the dominant strategy and when both firms select the decentralized strategy, the decentralized/decentralized is the Nash equilibrium. If the condition $\sigma_a^2 > \frac{22\mu_a{}^2}{27}$ holds then firm i has no incentive to deviate from the Nash equilibrium, because it would decrease the profit.

[59] See Narayanan et al. (2000), p. 505

In the case of the differentiated Cournot competition, we have the following matrix:

Firm i \ Competitor	Centralized	Decentralized
Centralized	$\frac{\mu_a{}^2}{(2+\theta)^2}$	$\frac{\mu_a{}^2(-4+\theta)^2}{(-8+\theta^2)^2}$
Decentralized	$\frac{3(\mu_a{}^2+\sigma_a^2)(-2+\theta)^2}{(-8+\theta^2)^2}$	$\frac{3\mu_a{}^2+3\sigma_a^2}{(4+\theta)^2}$

Fig. 10: *Profits for firm i in a differentiated Cournot competition with unobservable transfer prices and asymmetric information*

By comparing the results, as long the value of the specific knowledge exceeds $\sigma_a^2 > \frac{16a^2+8a^2\theta^2-2a^2\theta^4}{48-24\theta^2+3\theta^4}$, the decentralized strategy always dominates the centralized strategy, since the profit is higher. If the condition holds, the Nash equilibrium can be found in the decentralized/decentralized scenario and the prisoner's dilemma is solved by the mechanism of asymmetric information.

In the undifferentiated as well as in the differentiated Cournot competition with unobservable transfer prices and asymmetric information, the specific knowledge of the market has an impact on the choice of organizational strategy. In Cournot competition, we can successfully come out of the prisoner's dilemma when the specific knowledge of the market is high enough.

8.3 Infinitely repeated competition

8.3.1 Introduction

From pp. 46-51, I follow the concept of game continuation described in the model from Shor et al..[60] Consider the case when the Cournot game is played not only once but twice. "If player's actions are observed at the end of each period, it becomes possible for players to condition their play on the past play of their

[60] See Shor et al. (2009), pp. 587-588

opponents, which can lead to equilibrium outcomes that do not arise when the game is played only once."[61] For example we take the profit outcome in the Cournot competition with unobservable transfer prices:

		Competitor	
Firm i		Centralized	Decentralized
	Centralized	$\frac{\mu_a^2}{9b}$	$\frac{9\mu_a^2}{49b}$
	Decentralized	$\frac{3\mu_a^2}{49b}$	$\frac{3\mu_a^2}{25b}$

Fig. 11: *Profits for firm i in an undifferentiated Cournot competition with unobservable transfer prices*

Let us assume that both firms decide to collude in organizational form and both firms transfer above marginal cost in the first period. The profit outcome is $\frac{3\mu_a^2}{25b}$ for each firm. As we have shown in Cournot competition, there is always an incentive to deviate from the collusion and set a centralized strategy instead of a decentralized strategy. If firm i always defects, it would earn a profit of $\frac{9\mu_a^2}{49b}$ while firm j would yield $\frac{3\mu_a^2}{49b}$, if we assume that firm j stays at a decentralized organizational form. At the beginning of the second period, both firms have observed the profit outcome of the previous period. Firm j can either stick to the agreement of cooperating although it has observed that firm i has defected or firm j can opt to defect as well, since firm i has deviated from the agreement. Let us assume, that regardless of the choice of the competitor, firm i would always defect again. It's profit would be either $\frac{9\mu_a^2}{49b} + \frac{\mu_a^2}{9b}$, when firm j also defects or it would be $\frac{9\mu_a^2}{49b} + \frac{9\mu_a^2}{49b}$ if firm j decides once again to stick to the agreement. In both cases, the strategy to deviate has earned higher profit since $\frac{9\mu_a^2}{49b} + \frac{\mu_a^2}{9b} > \frac{3\mu_a^2}{25b} + \frac{3\mu_a^2}{25b}$ and $\frac{9\mu_a^2}{49b} + \frac{9\mu_a^2}{49b} > \frac{3\mu_a^2}{25b} + \frac{3\mu_a^2}{25b}$.

[61] Fudenberg et al. (1991), p. 145

Since the profit generated in the future is worth less than the profit generated in the actual period, we need to consider the discounted payoff instead of the undiscounted overall profit as presented in the previous example. Let δ denote the discount rate and let r be the interest rate.[62]

$$\delta = \frac{1}{1+r} \tag{8.1}$$

In the second period, the profit is discounted by δ:

$$\Pi = \Pi_0 + \delta\Pi_1 \tag{8.2}$$

In order to play an infinite game, the equation is extended:

$$\Pi = \Pi_0 + \delta\Pi_1 + \delta^2\Pi_2 + \cdots$$

$$\equiv \Pi = \frac{1}{1+\delta}\Pi_0 \tag{8.3}$$

8.3.2 Grim trigger strategy in undifferentiated Cournot competition with unobservable transfer prices

As Fudenberg et al. describes: "One example (...) is the 'unrelenting' strategy 'cooperate until the opponent defects, if ever the opponent defects then defect in every subsequent period.' The profile where both players use this unrelenting strategy is a subgame-perfect equilibrium of the infinitely repeated game if the discount factor is sufficiently close to 1."[63] The "unrelenting" strategy is also called the grim trigger strategy.[64] For example, firm i sets a decentralized organizational form and a transfer price above marginal cost, while firm j does the same. Firm i remains decentralized as long as firm j does the same. If firm j deviates from the decentralized strategy, then firm i uses the grim punishment and chooses to stay with the centralized strategy for the rest of the game. Firm j's reaction is to remain in the centralized strategy forever. We calculate and compare the overall profit of

[62] See Shor et al. (2009), p. 587
[63] Fudenberg et al. (1991), p. 145
[64] See Shor et al. (2009), pp. 587-588

deviation and collusion of firm i. The overall profit of collusion is the sum of all profits when both firms hold the promise and stay decentralized in all periods of the infinitely repeated game:

$$\Pi^{Col} = \frac{3{\mu_a}^2}{25b} + \delta\frac{3{\mu_a}^2}{25b} + \delta^2\frac{3{\mu_a}^2}{25b} + \cdots$$

$$\equiv\ \Pi^{Col} = \frac{1}{1-\delta} * \frac{3{\mu_a}^2}{25b} \qquad (8.4)$$

The overall profit of deviation is the sum of a centralized/decentralized profit in the first period and a centralized/centralized profit in all other periods:

$$\Pi^{Dev} = \frac{9{\mu_a}^2}{49b} + \delta\frac{{\mu_a}^2}{9b} + \delta^2\frac{{\mu_a}^2}{9b} + \cdots$$

$$\equiv\ \Pi^{Dev} = \frac{9{\mu_a}^2}{49b} + \delta\frac{1}{1-\delta} * \frac{{\mu_a}^2}{9b} \qquad (8.5)$$

Whether the overall profit of collusion is higher than the overall profit of deviation depends on the choice of discount rate δ or interest rate r from (8.1):

$$\Pi^{Col} > \Pi^{Dev} \quad if\ 0.88 < \delta < 1$$

or

$$\Pi^{Col} > \Pi^{Dev} \quad if\ 0 < r < 0.14 \qquad (8.6)$$

As long as the condition holds that the interest rate r is below 0.14, the collusion is a stable equilibrium and firm i considers the decentralized strategy as the dominant strategy in the infinitely repeated games when the grim trigger strategy is applied to both firms.

8.3.3 Grim trigger strategy in differentiated Cournot competition with unobservable transfer prices

In the differentiated Cournot competition the profit of a collusion between firm i and firm j in an infinitely repeated game is:

$$\Pi^{Col} = \frac{3{\mu_a}^2}{(4+\theta)^2} + \delta \frac{3{\mu_a}^2}{(4+\theta)^2} + \delta^2 \frac{3{\mu_a}^2}{(4+\theta)^2} + \cdots$$

$$\equiv \ \Pi^{Col} = \frac{1}{1-\delta} * \frac{3{\mu_a}^2}{(4+\theta)^2} \tag{8.7}$$

The overall profit for firm i to defect and to choose a centralized strategy would trigger the grim trigger strategy of the rival. Firm i's profit of deviation would be therefore:

$$\Pi^{Dev} = \frac{{\mu_a}^2(-4+\theta)^2}{(-8+\theta^2)^2} + \delta \frac{{\mu_a}^2}{(2+\theta)^2} + \delta^2 \frac{{\mu_a}^2}{(2+\theta)^2} + \cdots$$

$$\equiv \ \Pi^{Dev} = \frac{{\mu_a}^2(-4+\theta)^2}{(-8+\theta^2)^2} + \delta \frac{1}{1-\delta} * \frac{{\mu_a}^2}{(2+\theta)^2} \tag{8.8}$$

A stable Nash equilibrium in the cooperative outcome can be established, when the collusive profit is higher than the profit from the deviation strategy. Unlike the undifferentiated Cournot competition, the discount rate δ depends on the choice of the intensity of competition variable θ. The following table depicts the relationship between the intensity of competition θ, the discount rate δ and the interest rate r. If intensity of competition becomes lower, the discount rate δ is moving towards 1.

θ	δ	R
1	0,8775	0,13960114
0,9	0,909807	0,09913421
0,8	0,956781	0,04517126
0,7	~ 1	0
0,6	~ 1	0
0,5	~ 1	0
0,4	~ 1	0
0,3	~ 1	0
0,2	~ 1	0
0,1	~ 1	0
0	~ 1	0

Fig. 12: *The discount rate* $\boldsymbol{\delta}$ *and the interest rate r depend on the intensity of competition* $\boldsymbol{\theta}$

At a certain level of intensity of competition, the Nash equilibrium can be found in the collusion between the two firms when the interest rate is below the number shown in Fig. 12. If this condition holds, the decentralized strategy with a transfer price above marginal cost is the dominant strategy in the differentiated Cournot competition. It is interesting to mention that a low intensity of competition and a low interest rate decrease the willingness to cooperate. For example, with $\theta = 0{,}8$ and with a interest rate higher than $r > 0{,}045$, it is more profitable to deviate from the cooperation.

9 Conclusion

In my master thesis, the optimal choice of strategy and the transfer price setting in a competitive environment has been shown. The reader understands that under certain conditions, it is profitable for a firm to transfer goods above marginal cost when the competitor chooses to do so as well. But both firms face the problem, that it is even more profitable to deviate from an above marginal cost strategy. Therefore a transfer price above marginal cost is not easily established.
The question with unobservable transfer prices has been answered. The approach that has been followed is to observe the quantity of the competitor rather than the transfer price. When the transfer price of the competitor cannot be observed, the choice of the optimal strategy is still the marginal cost pricing. Non-observability only leads to transfer prices which are slightly lower than those in the scenario with observable transfer prices when there is collusion.
In a differentiated Cournot competition, the result shows that the intensity of competition has impacts on the choice of strategy and on the profit. When the intensity of competition is low, it is not profitable to cooperate and choose a transfer price above marginal cost. Only when the intensity of competition is high enough, an above marginal cost can be considered. When the intensity of competition is decreased, the profit becomes higher. In the extreme case, the profit is the highest when there is zero competition.
Furthermore the mechanisms to solve the prisoner's dilemma are presented in the thesis. When a downstream manager has additional information about the market, an above marginal cost decision can be established as a stable equilibrium in a Cournot competition. When the competitive situation is repeated infinitely, a grim trigger strategy can also establish an equilibrium in which no firm has an incentive to deviate from it.
It was a very fruitful experience for me to engage with a competitive approach to the transfer price setting. The models that I have studied were exciting, but at the same time complex and hard to read. For example, I spent a lot of time on the model of Narayanan et al. (2000), in which the mathematical calculations have been difficult to understand. One advice that I followed was from Univ.-Prof. Pfeiffer, that some models take a month to understand and therefore I had a clear Idea how much time I wanted to spend on the models.

The result of my thesis was not the outstanding one that I hoped for, although I invested a lot of time on the “trial & error“ calculations of unobservable transfer prices. In the end, I could not establish my own method to calculate a scenario with unobservable transfer prices and I misunderstood models of a Bertrand competition. But I gave my best and chose, with the help of Dr. Löffler, papers from qualitative journals since a thesis is only as good as the sources it refers to. Nonetheless, my understanding of the transfer price models from different researchers has increased a lot. I have also learned, that the results of researchers may sound very good, but there are a lot of conditions and assumptions to meet in order to achieve that result (For example the model in Shor et al.). If the conditions and assumptions are fully understood, the results may not seem that overwhelming.

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Appendix

Abstract

The importance of transfer prices as an internal tool for coordination or ressource allocation have been underlined by many researchers. This master thesis turns away from the internal role of transfer prices and analyzes it from an external perspective as it focuses on transfer price setting in a competitive environment.

First, the mathematical model describes two firms competing with each other on a homogeneous product and how they set transfer prices strategically. Afterwards the thesis presents a solution to the problem when the transfer prices of the competitor can not be observed.

In the second part, the two firms compete with each other on a differentiated product. Then the condition of observable transfer prices breaks again, and the two firms choose their strategy in situations with different intensity of competition.

In the last part, mechanisms are shown that help the two firms to come out of a so called “prisoner’s dilemma“. These mechanisms lead to a “Win-Win“ situation for both firms.

Zusammenfassung

Der hohe Stellenwert des Verrechnungspreises als interner Koordinationsmechanismus zur Ressourcenverteilung wurde von vielen Forschern betont. Diese Masterarbeit wendet sich von der internen Rolle des Verrechnungspreises ab und analysiert diesen aus einer externen Perspektive, in dem sie die Setzung der Verrechnungspreise in einer Wettbewerbssituation in den Vordergrund rückt.

Im ersten Teil wird ein mathematisches Model dargestellt, in dem zwei Firmen um den Absatz eines homogenen Produktes konkurrieren. Es wird gezeigt, wie sie darin einen strategischen Verrechnungspreis setzen. Danach wird das Problem der nicht beobachtbaren Verrechnungspreise gelöst.

Im zweiten Teil konkurrieren die zwei Firmen um den Absatz eines differenzierten Produktes. Auch hier wird die strategische Entscheidung in einem unterschiedlich stark umkämpften Markt bei unbeobachtbaren Verrechnungspreise dargestellt.

Im letzten Teil werden Mechanismen gezeigt, die den zwei Firmen helfen, aus dem sogenannten “Gefangenen Dilemma“ herauszukommen. Diese Mechanismen führen zu einer “Win-Win“ Situation für beide Unternehmen.

Curriculum Vitae

Academic Experiences

2013/03 – 2015/09* (*Expected End)	Master's Degree – University of Vienna Majors in: Managerial Accounting, External Corporate Accounting Overall grade: 2,2	Vienna
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2014/09 – 2014/11 3 Months	Project Assistant – Turner & Townsend Part time: Support in Budget Planning 2014, Monthly reporting	Vienna
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2012/09 – 2012/12 4 Month	Accounting Assistant – Intervet Internship full time: Support at accounts payable/receivable, payment transaction, month-end closing	Vienna
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2011/10 – 2012/01 4 Months	Business Development Assistant – Kerkhoff Consulting Internship full time: Mailings to potential customers, Contact by phone, Translations	Shanghai/China

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Chinese	Mothertongue
German	Mothertongue
English	Fluent

IT

SAP	Working Expericence in Module FI/CO Course Experience in Module MM
MS Office	Excel, Word, Powerpoint