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Optimal Order Placement

In Limit Order Book Markets

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Abstract

Due to prevailing price and time priority rules in limit order book markets, a standing limit buy order of size n, submitted at time t_0 at price level p faces an order volume $Q^p_{t_0}$ with higher priority in execution.

This volume is often referred to as queue. The queue size Q_t obviously changes over time, as new sell market orders and cancellations of buy limit orders may reduce the queue volume while new buy limit order submissions may increase the queue size again. This queuing process has a direct link to the execution probability of a submitted order. We sample this process using micro market data of certain stocks to estimate the expected fill rates of submitted buy limit orders. The fill rate is given by the executed part of a submitted order as percentage of the entire order size.

Based on the fill rates and the evolution of the limit order book, the optimal price level can be derived for given time of order submission, order size and trading time horizon. We analyze, whether the order placement strategy given by choosing this optimal price level can out-perform ad-hoc trading strategies that are determined by submitting at a constant price level.

Overall an out-performance can be achieved. Especially for increasing trading time horizons and compared to submissions at or near the best bid, significant gains up to 2.32 basis points are realized. Taking into account, that this is the first work in this direction, these are quite convincing results, especially when thinking about the various possibilities to refine and extent the analysis.

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1 Introduction

In limit order book markets, the likelihood of trade-execution of outstanding limit orders is a crucial and challenging issue. In practice, most order placement strategies follow an ad-hoc strategy, such as fixed order sizes and fixed pre-determined submission price levels. This is partly because order execution is a highly non-trivial process, involving several degrees of freedom, which have to be captured simultaneously in order to handle it properly. A more refined analysis of order execution requires the precise modeling of the involved order book flow dynamics. A key advantage of micro-modeling execution processes is that it is perfectly amenable to high-frequency data and thus can be of particular help in designing more efficient order placement strategies.

Most modern equity markets are organized as electronic limit order book markets. In these markets each market participant can submit market orders and limit orders at any price and at any time the market is operating. Gould at al. (2013) [10] made a survey on various theoretical and empirical studies on limit order book markets. This market micro-structure caused a huge increase in trading volume and speed. As there are so many trades happening, the transaction costs of a trade play an important role. Minor savings per trade can already cause a significant impact. The transaction costs strongly depend on the submission size, time and price of the order. This lead to increasing interest on algorithmic trading, aiming to automate the process of trade execution.

Looking at the literature on optimal trade execution, early formulations like Bertsimas and Lo (1998) [5] and Almgren and Chriss (2000) [3] analyze optimal trading schedules, i.e., how to optimally split a large order over a certain time horizon. They did not explicitly model the mechanism that actually fills the order. More recent papers started to take different order filling mechanisms into account. One approach considers only market orders as for example in Alfonsi et al. (2010) [2], Obizhaeva and Wang (2012) [17] and Predoiu et al. (2011) [18]. The costs of the market orders are determined by an idealized function for the shape of the order book. Another stream of the literature, for instance Cont (2011) [6] and Cont and De Larrard (2013) [8], models the order filling mechanism through a dynamic random process that leads to a formulation of the optimal execution problem as a stochastic control problem. Based on that there are different approaches. Some authors, such as Bayraktar and Ludkovski (2011) [4] and Gueant and Lehalle (2015) [11] tackled the problem by only considering limit orders while Guilbaud and Pham (2015) [12], Huitema (2014) [14], Guo et al. (2013) [13] and Li (2013) [15] consider both market and limit orders. Another approach formulated by Ma and Zhang (2015) [16] determines the shape of the order book by a competitive equilibrium and the trader's utility function. The complexity of these models often demands certain assumptions on the order book and price dynamics to make them computationally tractable.

Most of the literature on optimal trade execution considers a trader who wants to purchase or liquidate a large order in a given period of time. The output is often an optimal trade schedule, telling you how to split the initial large order in several small orders. In this thesis, however, we assume that this decision has already been made. We are now interested in the optimal placement strategy of one of the small orders. Hence, the order size and the time horizon are given. Still, the trader can decide to submit limit orders at different price levels and market orders to actually execute the trade. As we are only considering small orders, the price impact and the general impact of the trade on the order book are neglected. The goal is to use available limit order book information to find the optimal order placement strategy in terms of submission price level for limit orders. Stoikov and Waeber (2012) [19] study similar problems. However, they consider only market orders and try to find optimal times when to submit them. Cont and Kukanov (2014) [7] try to find optimal placement of limit or market orders across different exchanges based on the fee structure and the state of the order book at the respective exchange. Agliardi (2015) [1] analysis the optimal timing of limit orders by modelling the probability of execution via an exponential function and the price changes by a Wiener process.

Our approach is to find the optimal price level for submitting a limit order of given order size at the start of a given time horizon. In general, a part of the submitted order is executed within the time horizon at the submission price and the rest of the order has to be executed via a market order at a worse price. As described in the abstract, an order of size n faces the initial queue size $Q^p_{t_0}$, determined by the submission time t_0 and the submission price level p. Obviously the queue Q_t changes over time as new orders arrive at the market. Let thus x_t denote the volume of all sell market(-able) orders plus the volume of cancelled limit orders with higher priority and y_t the volume of buy limit orders with higher priority than the submitted order arriving at t. Then the net supply-demand equals $\zeta_t = x_t - y_t$. It is straight-forward that the running maximum process of

$$\zeta_t^{max} = \sup_{t\in[t_0,t]} \zeta_t \tag{1.1}$$

is closely related to the execution problem of the initial limit order. In particular, one can show that the number of executed shares of the initial limit order obeys

$$V_t = \begin{cases} (\zeta_t^{max} - Q^p_{t_0})^+ & \text{if} \quad \zeta_t^{max} - Q^p_{t_0} < n, \\ n & else. \end{cases} \tag{1.2}$$

Hence, the likelihood of execution depends on the initial order book shape and the order flow imbalance between incoming liquidity supply y_t and liquidity demand x_t. The expected fill rate reads

$$\mathbb{E}_{t_0}\left[\frac{V_t}{n}\right] = \mathbb{E}_{t_0}\left[\left(\frac{\zeta_t^{max}}{n} - \frac{Q^p_{t_0}}{n}\right)^+ |\Omega_t\right]\mathbb{P}[\Omega_t] + (1-\mathbb{P}[\Omega_t]) \tag{1.3}$$

with $\Omega_t := \{\zeta_t^{max}|\zeta_t^{max} - Q^p_{t_0} < n\}$. This formula determines how much of the initial limit order is executed in the given time horizon at the submission price level and how much has to be transformed into a market order. The cost of the market order strongly depends on the bid-ask spread at the end of the given time period. Formula (1.3) can be used to estimate the transaction costs of a limit order for different submission price levels. Hence, an optimal order placement strategy can be derived.

This strategy is tested against the ad-hoc strategies of constantly submitting at the same price level. We will analyze whether the optimal strategy can beat the ad-hoc strategies and how significantly it can out-perform them.

Most of the existing papers use stochastic processes or specific functions to model the order flow dynamics and to find an optimal execution strategy. This thesis, in contrast, is solely based on the above defined running maximum process and empirical order flow dynamics directly obtained from actual market data. In this regard it is a first step in the direction of using complete limit order book data to find optimal order placement strategies.

The objective is to find optimal placement strategies that significantly out-perform ad-hoc trading strategies. A further interesting topic outside the scope of this thesis is the robustness of this approach cross-asset. In particular, which types of markets are more suitable for this modeling approach. It is not clear, whether liquid or less liquid markets require a more sophisticated handling in order-executions. A related question is whether formula (1.3) allows a uniform trading rule across assets. If we know how the running maximum process scales with the underlying and observable liquidity characteristics, optimal trading rules can be more or less re-used for other asset classes as well without the need for fully-fledged re-computation.

Section 2 starts with a short overview of limit order book markets. The focus lies on how the trading is organized in such a market and how different trader's actions effect the evolution of the order book. After that, we introduce the model, which is based on the idea that the running maximum process can be used to estimate the expected fill rate of a submitted order given the order size, a time horizon, the submission price level and the state of the order book at the time of order submission. The fill rate determines, how much of a submitted order is executed within the given time horizon. The un-executed part gets transformed into a market order. The mechanism that determines the cost (execution price) of this market order is described. Finally the fill rate and the cost of the market order determine the expected transaction costs (average execution price) of the trade. The optimal order placement strategy is derived from minimizing the resulting transaction costs. The empirical analysis is done in section 3. We start by introducing the specific data used and a short analysis of the order flow dynamics within the data. Then the estimation and sampling methodologies are discussed as well as the approach for testing the optimal order placement strategy. The results of the empirical analysis for six selected stocks are presented in section 4. Section 5 gives an outlook for refining the analysis and for further related research questions. Section 6 concludes. In the appendix there is a table of the stock characteristics of the stocks in scope (100 NASDAQ-traded stocks), as well as a German summary of the thesis.

2 The Model

2.1 The Limit Order Book

Before the model is introduced, the theoretic framework of a limit order book market is discussed. The majority of modern stock markets are organized as continuous limit order book markets. This means that any market participant can basically submit two different types of orders, namely limit and market orders. A limit order states a certain amount of shares and a certain price for which one is willing to buy or sell the shares. After submitting a limit order, say a buy limit order, there are two possibilities. Either there are shares available that can be traded at this price (marketable limit order) or not. If there are, the trade is executed immediately. If not, the limit order stays in the order book and waits for execution. In that way the order book can be build up by a sequence of limit order submissions that cannot be matched. At a certain point of time, the order book states the amount of shares available at each tick. One tick is the smallest price increment, usually one cent. This means that for every price level, there are different amounts of shares submitted by different traders. There is always a best ask price, which is the currently lowest price, a trader is willing to sell shares and a best bid price, which is the highest available price for buying respectively. It is obvious that the best ask is always higher than the best bid, because otherwise the shares would have been matched and vanished from the book. The difference between best ask and best bid is called bid-ask spread.

A market order on the other hand, specifies only the amount of shares a trader wants to buy or sell. This order will then be matched immediately with shares at the best price available. To get a better understanding of the underlying dynamics of the order book, refer to Foucault et al. (2013) [9] and Gould et al. (2013) [10].

It is important to notice that limit orders can arrive either exactly on the best bid or best ask respectively or at a lower or higher price. For a buy order, arriving at a lower price than the best bid, means that the spread does not change. One says the submission is in the book. Whereas, if a buy order arrives at a higher price than the best bid, the spread improves, which means that it gets smaller. Such a submission is called submission in the spread. The same logic holds of course for sell orders. So while a limit order can improve the spread a market order can only lead to an equal or higher spread. If the size of the market order is larger or equal to the amount of available shares at the best bid for a sell order, all shares at the best bid are matched and the spread widens. From this perspective, one can say that limit orders supply liquidity while market orders demand it.

There are three priority rules stating which orders are executed first. The first one is price priority. Orders with better price have always priority. If the price is equal, the second priority rule applies. It states that displayed orders have priority over hidden orders. There are several ways to submit hidden orders. The most common one is a so-called iceberg order. Only a certain part of a bigger order shows up in the limit order book while the rest of the order enters invisibly to other market participants as hidden order. The motivation for such an order can be to hide the actual size of a very big order that would have a significant market impact when fully seen. If displayed orders have the same price the third priority rule applies. In this case always the order that was submitted first is also executed first. Therefore pricing, timing and the type of the order (displayed or hidden) determine the order of execution.

Looking at the order book at a certain point in time one can observe several characteristics. First of all there is the bid-ask spread defined as the difference between best ask price and best bid price. It can be measured in absolute terms, for example in ticks or cents, or relative to the mid-quote. The mid-quote is the arithmetic mean of the best bid and the best ask and can be interpreted as the 'fair' price of the stock. Besides the spread one can also observe the depth of the order book. The depth always depends on the number of levels that are considered. As traders are basically allowed to submit orders at any positive price level, it can happen that the order book for a certain stock consists of many price levels. The depth for a certain amount of levels is always the entire volume of shares that stands at these levels. So for example the depth at five levels is the sum of volumes standing at the five best price levels at the buy and the sell side respectively.

As stocks are priced at very different levels and as the prices constantly vary, it often makes more sense to talk about relative price levels. The relative price is defined as the difference between the best bid or ask before submission and the submission price. On the buy side the relative price is determined as best bid minus submission price while on the sell side it is submission price minus best ask. Hence, a positive relative price means a submission in the book while a negative relative price means a submission in the spread. A relative price of zero means a submission at the best bid or best ask.

2.2 Order Execution Problem

Consider an agent who has to buy a certain amount of shares in a given period of time. The objective for such an agent is clearly to trade at the best price possible or in other words to minimize transaction costs. Transaction costs can be viewed in different ways. In this thesis we consider trading the entire amount of shares at the best bid as the benchmark execution price. The transaction costs can then be calculated as the difference to this benchmark price.

Let us assume for example that a trader submits the entire amount of shares at the best bid. Obviously it can happen that in the given period of time none or only a part of the order is executed. In this case the trader is forced to submit a market order at the end of the given time period to make sure the rest of the order is executed. As the market order is matched against the other side of the book, i.e., against the volume at the best ask, second best ask, etc., the overall execution price can rise significantly. Obviously, the lower the submission price the lower is the expected fill rate. On the other hand, the part of the order that is filled achieves an even better price than the best bid. Hence, we can see a trade-off between submitting at different price levels. A relatively high price leads to a higher expected fill rate and therefore to a smaller part of the order that has to be traded as a market order. But the part of the order that is filled achieves only a minor or no benefit (submission at best bid) compared to the benchmark price. It could be also interesting to consider submissions in the spread, which is not part of the analysis applied in this thesis.

As you can see the submission price level plays an important role with regard to transaction costs. Keep in mind that modern trading is often based on very short time intervals and high frequency. Therefore a minor difference in transaction costs can create a significant impact. From the discussion of the transaction costs we see that there are two very important determinants. The first one is the expected fill rate and the second one is the state of the order book at the end of the time horizon relative to the state at the beginning. The cost of a market order at the end of the time horizon strongly depends on the difference between the best ask at the end and the best bid at the beginning of the time horizon. We call this difference effective spread.

2.3 Assumptions

We assume that a trader wants to buy a certain amount of shares n in a given time horizon $T-t_0$. The choice the trader has to make is on which price level she submits the entire order. At the end of the time period a certain part or the entire order may or may not be executed. The unexecuted part has to be transformed into a market order. Therefore the executed volume V, together with the state of the sell side order book at the end of the time period determine the transaction costs of the trade. After submitting an order of size n at a certain price level p at time t_0, there is a certain amount of shares in the book with higher execution priority, the initial queue $Q^p_{t_0}$. We want to find the optimal submission price level depending on the initial queue size $Q^p_{t_0}$ for given time horizons. Note that the initial queue size $Q^p_{t_0}$ can be observed at the time of order submission. Further we assume that the trade has no impact on the state of the order book and especially no price impact.

2.4 Model Approach

The idea of the model is to estimate the transaction costs for different time horizons and for different submission price levels. To do that we need to look at two stochastic determinants. The first one is the expected fill rate, i.e., the expected share of the submitted volume that is executed within the respective time horizon. The second one is the expected cost of the market order at the end of the time period. This cost is determined by the effective spread which we define as the difference between the best ask at time T and the best bid at time t_0. Actually, not only the effective spread determines the cost of the market order, but the entire state of the sell side order book at time T relative to the best bid at time t_0. The reason is, that the un-executed part of the order that is transformed into a market order, might be larger than the volume at the best ask at time of trade execution. If this is the case a part of the order is executed at the best ask price. The other part, that exceeds the volume at the best ask is executed at the second best ask price. Of course the market order can also be larger than the first two levels of the sell side order book. Then the process obviously goes on. This execution mechanism is formally described in section 3.4.3.

2.5 The Running Maximum Process

Due to price and time priority rules a standing limit buy order of size n, submitted at time t_0 at price level p faces order volumes $Q^p_{t_0}$ that have higher priority in execution. The queue size Q_t obviously changes over time, as new sell market orders and cancellations of buy limit orders with higher priority may reduce the queue volume while new buy limit order submissions with higher priority than the submitted order may increase the queue size again. Let thus x_t denote the volume of all sell market(-able) orders plus the volume of cancelled orders with higher priority and y_t the volume of buy limit orders with higher priority arriving at t. Then the net supply-demand equals $\zeta_t = x_t - y_t$. It is straight-forward that the running maximum process of

$$\zeta^{max}_t = \sup_{t \in [t_0, t]} \zeta_t \tag{2.4}$$

is closely related to the execution problem of the initial limit order. In particular, one can show that the number of executed shares of the initial limit order submitted at the price level p obeys

$$V_{t,p} = \begin{cases} (\zeta^{max}_t - Q^p_{t_0})^+ & \text{if} \quad \zeta^{max}_t - Q^p_{t_0} < n, \\ n & else, \end{cases} \tag{2.5}$$

Hence, the likelihood of execution depends on the initial order book shape and the order flow imbalance between higher priority incoming liquidity supply y_t and liquidity demand x_t. The expected fill rate at time t_0 reads

$$\mathbb{E}_{t_0}\left[\frac{V_{t,p}}{n}\right] = \underbrace{\mathbb{E}_{t_0}\left[\left(\frac{\zeta^{max}_t}{n} - \frac{Q^p_{t_0}}{n}\right)^+ |\Omega_t\right] \mathbb{P}[\Omega_t]}_{\text{Black-Scholes-Type Evaluation}} + (1 - \mathbb{P}[\Omega_t]) \tag{2.6}$$

with $\Omega_t := \{\zeta^{max}_t | \zeta^{max}_t - Q^p_{t_0} < n\}$. It stands out that the evaluation of the fill rate of a limit order has a striking resemblance to the pricing of vanilla options in terms of the Black-Scholes formula. Observe, that Q/n serves as symbolic strike K, while the running maximum process acts as an underlying.

In other words this process tells us, how much of the initial queue Q vanishes in a certain period of time and more than that, how many additional shares are hit after the queue is gone. Remember that the queue increases with submissions and decreases with executions and cancellations. The problem is that there is no distinction between cancellations and executions. This means that an order sitting atop the initial queue is also considered as executed when it is actually hit by a cancellation. In a real market setting, this is obviously not possible. Later, when we discuss the sampling methodology of the running maximum process we deal with this problem.

2.6 Costs of the Market Order

We use the running maximum process to estimate fill rates. The fill rates tell us how many shares of the submitted order are executed at the submission price and how many have to be transformed into a market order. The market order is traded against the sell side order book at the end of the time horizon. Obviously, at time t_0 we do not know the state of the sell side order book at time T. The transaction costs are compared to the benchmark case in which all shares are traded at the best bid price. Therefore we are interested in the execution price relative to the best bid at t_0. Let thus bb_{t_0} denote the best bid at t_0, $ap_t^1, \ldots, ap_t^k$ and $as_t^1, \ldots, as_t^k$ the ask prices and respective volumes at t and $AS_t^l := \sum_{j=1}^{l} as_t^j, l = 1, \ldots, k$ the cumulated ask volumes. k denotes the last level the market order is traded with. This means that k is the smallest natural number, that fulfils $AS_t^k \geq n - V_t$. Let $AS_t^0 := 0$. Then the expected cost of the market order cm_t at time t_0 relative to the best bid at submission is given by

$$\begin{aligned}\mathbb{E}_{t_0}[cm_t] =&\mathbb{E}_{t_0}[(ap_t^1 - bb_{t_0})(n - V_t)]\mathbb{P}[n - V_t < AS_t^1]+\\ &\sum_{m=1}^{2}\Big\{\mathbb{E}_{t_0}[(ap_t^m - bb_{t_0})(n - V_t - AS_t^{m-1})]\Big\}\mathbb{P}[AS_t^1 \leq n - V_t < AS_t^2]+\\ &\vdots\\ &\sum_{m=1}^{k}\Big\{\mathbb{E}_{t_0}[(ap_t^m - bb_{t_0})(n - V_t - AS_t^{m-1})]\Big\}\mathbb{P}[AS_t^{k-1} \leq n - V_t < AS_t^k]\end{aligned} \tag{2.7}$$

2.7 Transaction Costs

We defined the transaction costs as the difference between the actual trading price and the trading price at the best bid price. Therefore the transaction costs consist of two parts. The first part is what you expect to win by submitting in the book. The part of the submitted order that is executed in the given time horizon achieves at least the best bid price or a better price. Remember that we do not consider submissions in the spread. This part is given by $\mathbb{E}_{t_0}[V_{t,p} \cdot (bb_{t_0} - p)]$ and enters the transaction costs with a negative sign as it reduces them. The second part are the expected costs of the market order $\mathbb{E}_{t_0}[cm_t]$, which we derived above. Hence the total expected transaction costs are given by

$$\mathbb{E}_{t_0}[TC_{t,p}] = \mathbb{E}_{t_0}[cm_t - V_{t,p} \cdot (bb_{t_0} - p)]. \tag{2.8}$$

Note that the actual transaction costs can also be negative, if for example the entire order is executed for a submission in the book. This would mean, that transaction costs are below the benchmark of total order execution at best bid.

2.8 Optimal Order Placement

Remember that the trader chooses the submission price level at time t_0. As the state of the order book is given at t_0, the trader's choice of p determines the initial queue size $Q_{t_0}^p$. Assuming that we know the distributions of the running maximum process and the sell side order book at the end of the trading horizon, the expected transaction costs $\mathbb{E}_{t_0}[TC_{t,p}]$, as defined in formula (2.8), are fully determined by the submission price level p. The explanation is the following. The submission price level and the state of the order book at time t_0 (which is observable at t_0) determine the initial queue size $Q^p_{t_0}$. The distribution of the running maximum process and the initial queue size give the expected fill rate as outlined in section 2.5. The expected fill rate determines the size of the market order. Then the distribution of the sell side order book at the end of the trading horizon gives the cost of the market order as described in section 2.6, eventually determining the overall trading costs (section 2.7). Therefore, by assuming to know the distributions of the running maximum process and the state of the sell side order book, for each choice of p the expected transaction costs can be computed. The minimum determines the optimal submission price level. Formally, the optimal price level p^* for a submission at time t_0 is given by

$$p^* = \underset{p}{\operatorname{argmin}}\, \mathbb{E}_{t_0}[TC_{t,p}]. \tag{2.9}$$

To sum things up, at time of order submission t_0, the optimal submission price p can be derived a priori. Choosing the submission price following this logic determines the optimal order placement strategy.

3 Empirical Analysis

3.1 Data

The entire empirical analysis in this thesis is based on LOBSTER data. LOBSTER is an online limit order book tool to provide easy access to granular limit order book data. The data is reconstructed for all stocks traded at NASDAQ. The target group are academic researchers. LOBSTER is run by financial econometricians affiliated to Universtät Wien and Humboldt Universität zu Berlin.

For each active trading day and each demanded title a message and an order book file is generated. The message file contains one dataset for each activity (e.g. a submission of a limit order) that causes an update of the order book during a day. These activities are time-stamped with accuracy of at least milliseconds and up to nanoseconds, depending on the requested period. The order book file contains the evolution of the book itself. Each activity corresponds to a line in the message file. In the same line of the order book file the updated state (after the activity) of the order book is displayed. Figure 1 shows the structure of the message file.

Time (sec)	Event Type	Order ID	Size	Price	Direction
⋮	⋮	⋮	⋮	⋮	⋮
34713.685155243	1	206833312	100	118600	-1
34714.133632201	3	206833312	100	118600	-1
⋮	⋮	⋮	⋮	⋮	⋮

Figure 1: Message File, *Source: https://lobsterdata.com/info/DataStructure.php*

Explanation of variables

- Time: The time is given as seconds after midnight with precision up to nanoseconds. Note that on one time stamp, so within one nanosecond there can be several activities, each getting a separate line in the message file.

- Event type: The event type specifies the event that causes an update of the order book. There are five different types:

  - Submission (type 1): A submission means that a trader submits a new limit order, either buy or sell.

  - Cancellation (type 2): A cancellation is a partial deletion of an existing limit order that has been submitted earlier that day.

  - Deletion (type 3): A deletion is a total deletion of an existing limit order.

  - Execution (type 4): This means a visible execution (in total or in parts) of an existing limit order that has been submitted earlier that day. Note that there has to arrive a marketable order that matches with an existing limit order. For example, consider a buy limit order has been submitted and is now standing at the best bid. The 'activity' of an execution is actually caused by another trader's submission of a marketable sell order.

– Hidden execution (type5): A hidden limit order is executed.

- Order ID: The order ID is a unique reference number for each limit order during a day. Note that order IDs can be assigned again on later days. This means that orders with the same order ID on different days do not belong to each other.

- Size: This is the number of shares that is submitted, cancelled, deleted or executed.

- Price: This is the dollar price times 10000 (i.e. a stock price of $ 91.44 is given by 911400)

- Direction:

  – Buy (1): A submission, cancellation, deletion or execution happens on the buy side.

  – Sell (-1): An activity happens on the sell side.

Figure 2 shows the structure of the order book file.

Ask Price 1	Ask Size 1	Bid Price 1	Bid Size 1	Ask Price 2	Ask Size 2	Bid Price 2	Bid Size 2	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1186600	9484	118500	8800	118700	22700	118400	14930	...
1186600	9384	118500	8800	118700	22700	118400	14930	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Figure 2: Order Book File, *Source: https://lobsterdata.com/info/DataStructure.php*

Explanation of variables

- Ask price 1: the best ask price (level 1)

- Ask size 1: the volume of shares at the best ask price

- Bid price 1: the best bid price (level 1)

- Bid size 1: the volume of shares at the best bid price

- Ask price 2: the second best ask price (level 2)
- Ask size 2: the volume at the second best ask price
- ...

Levels

The number of requested levels determines the number of occupied price levels depicted in the order book file. A price level is occupied, if there is some volume at this level. Especially for high-priced stocks, the occupied price levels are often several ticks away from each other. In other words, there are holes in the order book, i.e., price levels without any volume. Hence, it is always important to keep in mind, that in general, price levels do not correspond to ticks. If, for example, five levels are requested, the order book file shows the first five levels on which there is some volume of shares. That are not necessarily the first five ticks.

Interplay of Message and Order Book File

Obviously the number of rows of both files is equal. The activity in row k corresponds to the change in the order book file from row k-1 to k. The limit order deletion (event type 3) in the second line of the message file (figure 1) reduces the number of shares at the price level 118600 from 9484 to 9384. This change can be seen in the second column of the order book file (Ask size 1) in figure 2.

Evolution of the Limit Order Book

It is important to understand which events cause the evolution of the order book in general and how this translates in the different activities according to LOBSTER data. Consider a new limit order is submitted and enters the book. The size (number of shares) is added to the existing volume at the respective submission price level. A submission in the spread improves the bid-ask spread.

As mentioned above, there are different kinds of hidden submissions. For example it is possible to submit a limit order of size 1000, but only 100 shares of the order are visible. Hence, only a submission of size 100 is displayed in the message and order book file of the LOBSTER data. When the hidden part of 900 shares is executed, this shows up as hidden execution (event type 5) in the data. Hidden executions always occur on positive relative price levels. The reason is that they either stand in the spread, which is possible because they are not visible in the book or they stand atop the visible volume on occupied price levels due to less priority compared to visible orders. If a marketable order arrives that is large enough to match the entire volume at the best bid or ask respectively, this order will be split up in several activities in the LOBSTER data. First there will be as many executions (event type 4) until the volume at the best bid (ask) is gone. This leads to a new best bid (ask). When the time for the hidden order has come, because all the orders at the same price level, but with higher priority are matched, the bid (ask) has already moved and the hidden execution occurs again on a positive price level.

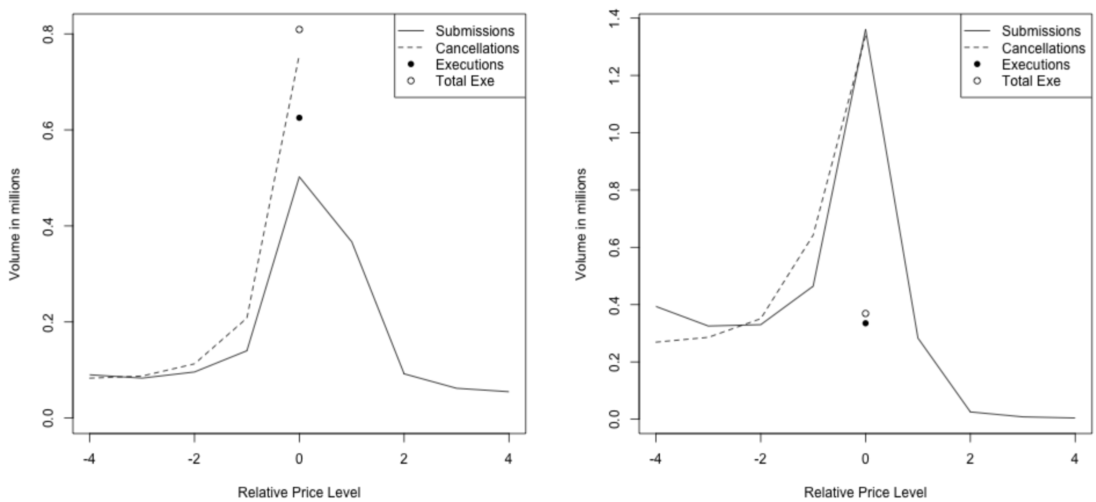
It is important to understand that market(able) orders are not shown as market orders but as executions at the other side of the book. Therefore a market buy order of say 500 shares may translate in one or more executions of sell limit orders that have been submitted earlier and wait for execution.

Sample Size

For the following analysis 100 stocks are considered in the period of January 1st to March 31st 2014 (61 days). It is important to note that only the first five levels of the order book are taken into account. For thick books this is quite sufficient while for thin books it would be advantageous to consider more levels. An order book is called thick if most of the volume stands at the top levels and if there are no big holes between levels, i.e., price levels without any volume. Usually the stocks that have a low absolute price have thick order books because the tick size is relatively large relative to the price. An order book is considered thin, if there is significant volume outside the top levels and if there are holes occurring over longer periods of time. The higher the absolute price of a stock, the lower is the relative tick size, as the absolute tick size is one cent for each stock, and the more likely it is, that the order book is thin. In the appendix there is a table stating the general characteristics of all stocks such as average spread in ticks and in basis points of the price, average execution price (mid-quote), average daily trade volume, submission volume and cancellation volume and average depth at the top level and at all five levels.

3.2 Order Flows

As the model is based on order flow dynamics, we analyze them here. Therefore we discuss which activities usually happen at different price levels and in which relation the different order types, i.e., submissions, cancellations and executions stand to each other. Practically we plot the average daily volumes of these types for the buy activities of 61 trading days. Only five price levels in the book and in the spread are considered, as the data only includes five price levels. Figure 3 shows the average daily submission and cancellation volumes at the respective relative price levels indicated by the two lines and the displayed and total execution volumes, indicated by the two points.



(a) TSLA (b) AKAM

Figure 3: Average daily submission, cancellation and execution volumes at different price levels

The left figure shows the order flows of Tesla Motors, Inc. (TSLA). Akamai Technologies (AKAM) is depicted in the right figure. Comparing the two stocks, TSLA has a much higher price and average daily trading volume than AKAM. Looking at the two lines you see that there is a co-movement of submissions and cancellations until a relative price level of 0 (best bid). This means that in the book the average daily volumes of submissions and cancellations are balanced. The volumes definitely peak at the best bid. Submissions can of course also take place in the spread. As discussed in section 2.1 these submissions improve the spread. Observe that most of these submissions occur close (1-2 ticks) to the best bid. Cancellations can of course only occur at the best bid or in the book, as there cannot be any volume in the spread. Note that a submission in the spread, i.e., on a positive relative price level, immediately improves the spread. If this submission is then cancelled again, the cancellation occurs on the relative price level 0. As submissions and cancellations are quite balanced in the book, the picture suggests that many of the executed orders are actually submitted in the spread. The full point indicates the average daily execution volume of displayed orders. These executions always occur at the best bid due to the nature of the LOBSTER data as mentioned in section 3.1. The empty point is the average daily volume of all executions, displayed and hidden. Note that hidden executions happen on positive relative price levels. A striking characteristic of the data is that submission volumes are much higher than execution volumes. On average for the 100 stocks, submission volumes are 9 times as high as execution volumes. This means that most of the activity stems actually from submitting and cancelling orders and only a small part is actually executed.

3.3 Estimation and Sampling Methodology

The purpose of the model is to find the optimal submission price level at time t_0 with information that is available at t_0. As we already know, there are two stochastic determinants for the transaction costs of a trade. The first one is the expected fill rate and the second one is the state of the sell side order book at the end of the given time period. We sample the running maximum process for a certain period of trading days. The further analysis is based on the assumption that the distribution of the running maximum stays roughly constant over time. Hence, this distribution is considered stationary. Therefore we use it to get an estimate of the execution volume for each initial queue size. To estimate the cost of the market order we do not only sample the running maximum process but also the state of the sell side order book at the end relative to the best bid at the start of the time horizon. Hence, we actually assume that the joined distribution of the running maximum process and the state of the sell side order book is stationary. The estimates are computed straightforward by taking the mean over a certain sampling period.

We define a sampling period of 20 trading days to derive the empirical distributions for the running maximum process and the sell side order book. Based on these distributions we can estimate the fill rates and the costs of the market order for different time horizons and different submission price levels. Hence, we can derive the optimal submission price level for each order submission. This gives us an optimal order placement strategy. Then we define a testing period of two trading days. In this period, starting every five seconds, we test the obtained order placement strategy against ad-hoc strategies, i.e., submitting at a constant price level. We want to know whether the resulting cost savings are significant and how they vary with different time horizons and different order sizes.

3.3.1 Data Preparation

Above we learned that the considered data is time-stamped with an accuracy of nanoseconds. Still it happens that there are several activities at the same time stamp, i.e., happening in the same nanosecond. Looking at the message files and the order book files of the LOBSTER data, we also know that in the same line of the two files we get the information of a trader's activity in the message file and the updated order book in the order book file. Hence, the trader's activity is already incorporated in the order book. However, from the perspective of the trader, only the state of the order book before her activity can be observed.

Let us assume we are at time t and the trader wants to submit an order. In a real trading setting the trader can only observe the order book at time $t-1$, which can be only a nanosecond away. Still it can be wrong to simply look at the line before in the order book, because this line can still belong to the time stamp t.

Therefore we have to go back in the order book until we get to a line corresponding to $t-1$. Now consider the case that there are several lines in the order book time-stamped at t, as well as at $t-1$ and $t-2$. Then a trader can only observe the last line of $t-2$ at time $t-1$ and again only the last line of $t-1$ at time t. Hence, we create a data frame with the information of the message file and link the order book file as such, that each line in the message file time-stamped at t corresponds to the last line of the order book file time-stamped at $t-1$. In that way we only use actually available information when making trading decisions. Practically we create data frames that contain the message file information linked with the corresponding order file information of the last time stamp for each day and for each stock. All further steps of the analysis are operated on these data frames.

3.3.2 Running Maximum Process

Now we discuss how the running maximum process is sampled for a given time horizon $T-t_0$ and a given relative submission price level i. Remember that the relative submission price level is given by the best bid price minus the actual price p. So a submission price level of 2 for example means a submission two ticks behind the best bid. Let us denote the submission price level as reference price. Note that this price level determines the initial queue size $Q^i_{t_0}$. The queue size is simply the volume at the submission price level and all better (higher) price levels at the time of submission. As the trader wants to buy a certain amount of stocks, we do our analysis only for the buy side. The results for the sell side could be derived analogously.

For the computation of ζ_t^{max} the order flow imbalance $x_t - y_t$ plays an important role. That is why we start by defining the net order flow. In the given time window $T-t_0$ we have a certain amount J of activities in our daily trading data. For each activity $j, j = 1 \dots J$ we define the net order flow NOF^j as the order size, if the activity is an execution or a cancellation and as the negative order size, if the activity is a submission. Remember that executions and cancellations reduce the queue size and submissions increase it. While executions always influence the queue size, cancellations and submissions only do, if they have higher priority than the trader's submission at t_0. For a submission arriving after t_0 this means, that the price level has to be better, i.e., higher than the reference price level. On the same price level the submission would have lower priority due to the time priority rule. Cancellations on better price levels obviously reduce the initial queue, while cancellations on the same price level are more sophisticated. Whenever an order gets cancelled that has been already there before t_0, the priority is higher and it reduces the queue size. However, when an order gets submitted at the reference price level after t_0 and then gets cancelled again, this cancellation does not affect the queue. Therefore we remember the order IDs of order submissions that arrive at the reference price levels and only take cancellations on the reference price level into account, if they have different order IDs than the remembered ones.

Let $NOF_{t,i}^j, j = 1 \dots J$ denote the net order flows for the specific time window t and relative price level i. Then the running maximum process is given by

$$\zeta_{t,i}^{max} = \max_{K:=\{1\dots L, L=1\dots J\}} (\sum_{j \in K} (NOF_{t,i}^j), 0). \tag{3.10}$$

In section 2.5 we discussed the problem, that there is no distinction between executions and cancellations. In a real market setting, obviously a cancellation cannot lead to the execution of the submitted order. It is important to consider cancellations as well when we compute the cumulated sums, as they do reduce the queue size. But to find the actual amount of executed orders, the maximum must be taken only over the cumulated sums that built up through executions. More formally, the actual running maximum process is given by

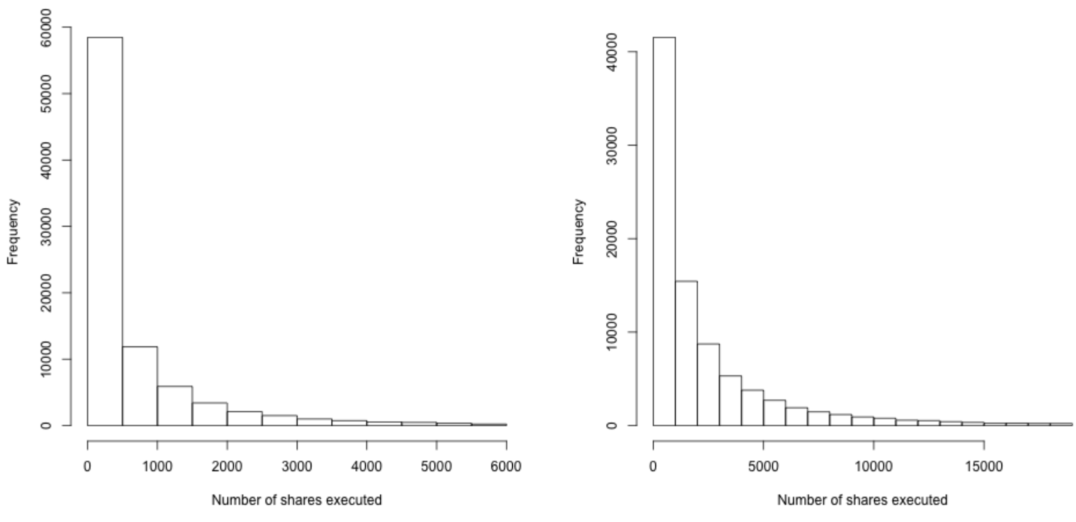
$$\zeta_{t,i}^{max} = \max_{K:=\{1\dots L, L=1\dots J, NOF_{t,i}^L \text{ is not a cancellation}\}} (\sum_{j \in K} (NOF_{t,i}^j), 0). \tag{3.11}$$

Remember that the types 4 and 5 are executions and hidden executions. Note that the running maximum process which is defined as the supremum of the order flow imbalances, eventually gives us one particular value for each time window and for each relative submission price level. This value corresponds to the amount of shares that would have been executed in the given time window and for the given submission price level. Let n denote the size of the submitted order, then the execution volume is given by

$$V_{t,i} = \max((\zeta_{t,i}^{max} - Q_{t_0}^{i})^{+}, n). \tag{3.12}$$

We calculate these $\zeta_{t,i}^{max}$ values for time windows of 10, 30, 60 and 120 seconds starting every 5 seconds and for relative submission price levels of 0-4. Assuming continuous trading from 9:30 to 16:00, i.e., for 6.5 hours this gives a sample size of $6.5 \cdot 60 \cdot 60/5 = 4680$ time windows per trading day. This gives us 20 statistics (4 time horizons, 5 submission price levels) of running maximum values. We view these statistics as empirical distributions for fill rates. The sampling period is 20 trading days starting at January 2nd, 2014. Therefore the sample size of these statistics is given by $4680 \cdot 20 = 93600$. Let us denote these statistics by $rm_{t,i}$.

To get a better intuition of the dynamics of the running maximum process let us take a look at the histograms of TSLA for two different time horizons, shown in figure 4.



(a) 30 seconds (b) 120 seconds

Figure 4: Histograms of running maximum values for TSLA for a relative price level of 2

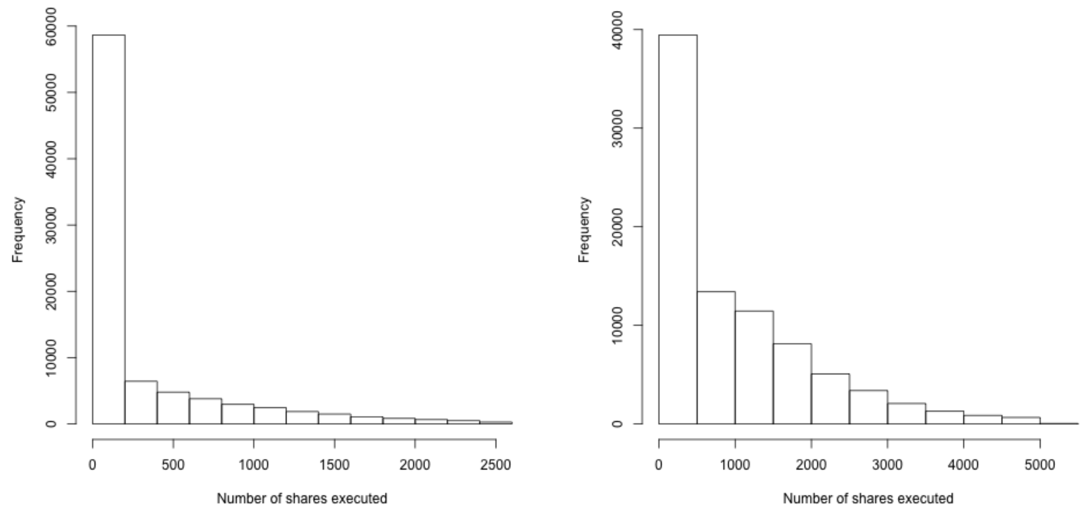
These histograms show how often a certain amount of shares is hit by the running maximum process, i.e., executed. For example, the left histogram tells us that nearly 60,000 times 0 to 100 shares, submitted at a relative price level of 2, are executed within 30 seconds. Comparing the two different time horizons, we observe that larger amounts of shares are executed the longer the time horizon. We also see a spike around zero. This means that in many cases, the initial queue size does not decrease at all in the given time horizon. This is obviously the case when the market is moving away from the submission price level. On the other hand it stands out that also very large amounts of stocks have a significant execution probability. The probability that at least n shares, submitted at a relative price level of i, are executed within t seconds is given by

$$\frac{|S_n|}{|S|}, \tag{3.13}$$

where S is the set of all indices of $rm_{t,i}$ and $S_n = \{s \in S | rm^s_{t,i} \geq n\}$ with $rm^s_{t,i}$ denoting the s-th element of the statistic $rm_{t,i}$. For $n = 1000$ and $i = 2$ this is around 21% for $t = 30$ and around 53% for $t = 120$ seconds. Comparing TSLA to AKAM, the latter has a much lower price and daily trading volume. We observe that the execution probability for a large amount of stocks is much lower for AKAM. However the general shape of the distribution is similar. Here the probabilities that at least 1000 shares are executed for submission at a relative price level of two are around 13% for 30 and 40% for 120 seconds.

3.3.3 Costs of the Market Order

We sample the state of the sell side order book simultaneously to the running maximum process. This means that we consider the same time windows and submission price levels and the same sampling period. For each time window t we save the best bid price at the beginning of the time window and the entire sell side order book (5 levels) at the end of the time window. The resulting statistics we denote by cm_t and view them as empirical distributions for the sell side order book. Combining the statistics $rm_{t,i}$ and cm_t we get for each time horizon and for each relative submission



(a) 30 seconds

(b) 120 seconds

Figure 5: Histograms of running maximum values for AKAM for a relative price level of 2

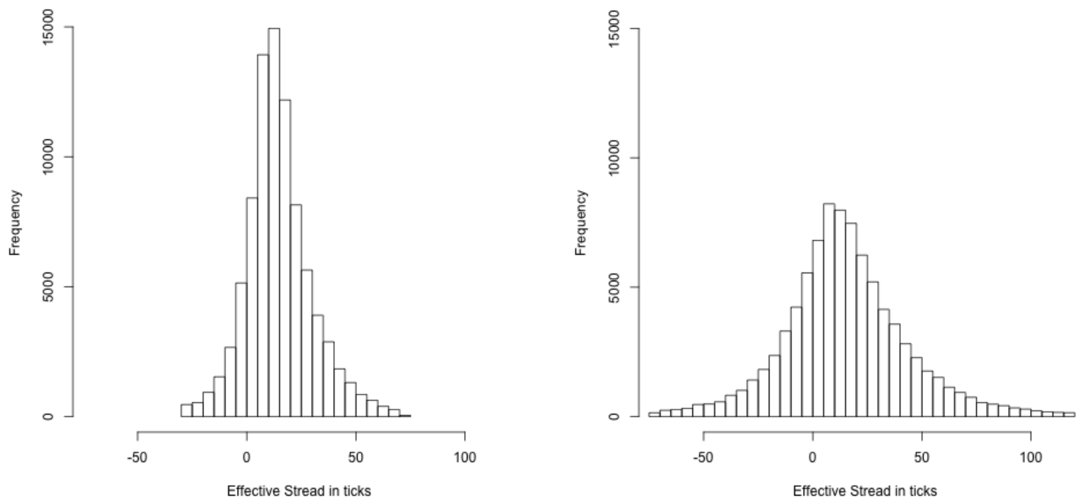
price level a joined empirical distribution to first derive the execution volume $V_{t,i}$ and then the cost of the resulting market order of size $n - V_{t,i}$. Let us define the size of the market order as $mo_{t,i} := n - V_{t,i}$. As we only consider 5 levels the cost of the market order for given t and i is given by

$$cm_{t,i} := \begin{cases} cm^1_{t,i} := (ap^1_T - bb_{t_0}) \cdot mo_{t,i} & \text{if} \quad n - V_{t,i} \leq AS^1_T \\ cm^2_{t,i} := cm^1_{t,i} + (ap^2_T - bb_{t_0})(mo_{t,i} - AS^1_T) & \text{if} \quad AS^1_T < mo_{t,i} \leq AS^2_T \\ cm^3_{t,i} := cm^2_{t,i} + (ap^3_T - bb_{t_0})(mo_{t,i} - AS^2_T) & \text{if} \quad AS^2_T < mo_{t,i} \leq AS^3_T \\ cm^4_{t,i} := cm^3_{t,i} + (ap^4_T - bb_{t_0})(mo_{t,i} - AS^3_T) & \text{if} \quad AS^3_T < mo_{t,i} \leq AS^4_T \\ cm^5_{t,i} := cm^4_{t,i} + (ap^5_T - bb_{t_0})(mo_{t,i} - AS^4_T) & \text{if} \quad AS^4_T < mo_{t,i}, \end{cases} \tag{3.14}$$

where bb_{t_0} denotes the best bid at the start of the time window t, $ap^1_T, \ldots, ap^5_T$ and $as^1_T, \ldots, as^5_T$ denote the ask prices and corresponding ask volumes at the end of the time window and $AS^l_T := \sum_{j=1}^{l} as^j_T, l = 1, \ldots, 5$ the cumulated ask volumes. Note that the market order can possibly exceed the cumulated ask volume AS^5_T. In this case, the market order would dig deeper into the book and match with further levels. As we only consider five levels, we do as if the volume at level five would be infinite.

This means that the remaining part of the market order after matching with the first four levels is always matched with the price at the fifth level.

A good indicator for the cost of the market order is the effective spread, i.e., the difference between the best ask at the end and the best bid at the start of the respective time horizon. To get a better intuition, figure 6 shows the effective spread histograms of TSLA for a time horizon of 30 and 120 seconds.

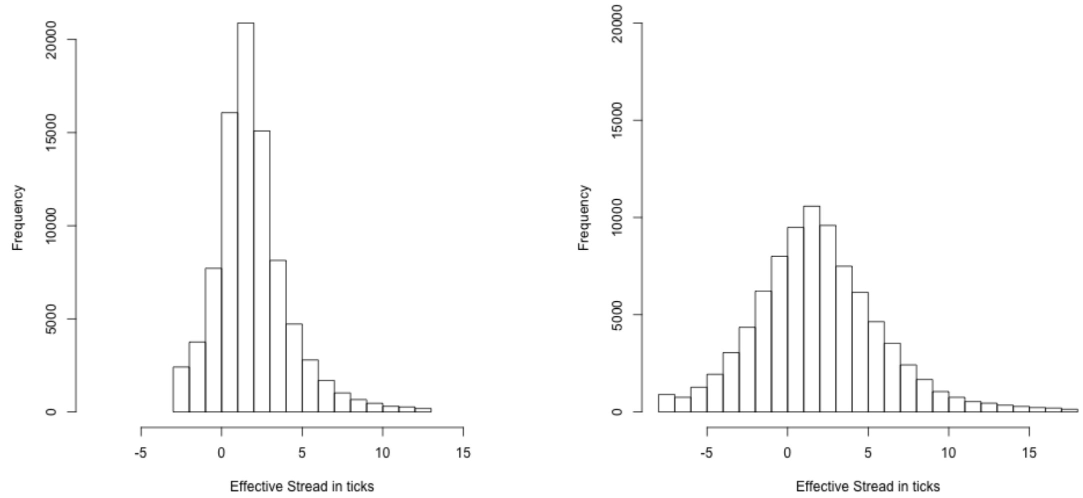


(a) 30 seconds (b) 120 seconds

Figure 6: Histograms of effective spreads for TSLA

These histograms tell us in how many cases the effective spread has a certain size in ticks. Obviously, the longer the time horizon, the broader the distribution. Figure 7 shows the effective spread histograms of AKAM for time horizon of 30 and 120 seconds.

Again we observe a similar shape, although the spreads are much smaller in absolute terms.



(a) 30 seconds (b) 120 seconds

Figure 7: Histograms of effective spreads for AKAM

3.3.4 Optimal Order Placement Strategy

Consider now an order of size n submitted at time t_0. The trader's objective is to minimize the transaction costs by choosing the optimal submission price level. In this application the trader has five options, i.e., submitting at the relative price levels $i = 0, \ldots, 4$. The relative submission price level i determines the queue size $Q^i_{t_0}$. So we get five different queue sizes. Now we want to estimate the transaction costs for the different price levels using the statistics $rm_{t,i}$ and cm_t. Let S denote the sample size of the two statistics. For given time horizon t and for given relative submission price level i we compute for each $s \in S$

$$V^s_{t,i} = \max((rm^s_{t,i} - Q^i_{t_0})^+, 0) \tag{3.15}$$

and

$$TC^s_{t,i} = cm^s_{t,i} - V^s_{t,i} \cdot i. \tag{3.16}$$

Taking the mean gives us an estimate for the transaction costs

$$\hat{TC}_{t,i} = \sum_{s \in S} \left(\frac{TC^s_{t,i}}{S} \right). \tag{3.17}$$

After computing the transaction costs for the different relative submission price levels, the optimal level for given t is given by

$$i^* = \underset{i}{\operatorname{argmin}}(\hat{TC}_{t,i}). \tag{3.18}$$

Later, when the results are discussed, we are also interested in the estimates for the fill rates as we want to know, how much of a submitted order is executed on average. An estimate for the fill rate for given t and i is given by

$$\hat{V}_{t,i} = \sum_{s \in S} \left(\frac{V^s_{t,i}}{S} \right). \tag{3.19}$$

3.3.5 Testing the Optimal Placement Strategy

For testing two trading days for different stocks and different submission sizes n are considered. Starting every five seconds the optimal placement strategy is tested against ad-hoc strategies (submission at constant price levels) for the different time horizons. For given starting time t_0 and given time horizon t the considered submission price levels are given by the best bid at t_0 and 1-4 ticks in the book (lower than the best bid). These are the reference prices. For these submission price levels the initial queue sizes $Q^i_{t_0}, i = 0, \ldots, 4$ are determined by the cumulated volumes standing at the respective reference price level and before. Contingent on the queue sizes the optimal relative submission price level i^* is derived as described in section 3.4.5. Now the actual transaction costs are computed for the five different submission price levels. Therefore we take the running maxima $rm_{t,i}, i = 0, \ldots, 4$ for the specific time window that is analyzed. Then the actual execution volumes are given by

$$V_{t,i} = \max((rm_{t,i} - Q^i_{t_0})^+, 0), i = 0, \ldots, 4 \tag{3.20}$$

and the cost of the market order of size $n - V_{t,i}$ is given by $cm_{t,i}, i = 0, \ldots, 4$ from formula (3.14), evaluated at the actual state of the sell side order book at the end of the respective time window. Hence the actual transaction costs are given by

$$TC_{t,i} = cm_{t,i} - V_{t,i} \cdot i, i = 0, \ldots, 4. \tag{3.21}$$

As i^* is the optimal submission level, the savings of the optimal strategy compared to the constant submission strategies for that particular time window are computed by

$$savings_{t_0,t}(i) = TC_{t,i} - TC_{t,i^*}, i = 0, \ldots, 4. \tag{3.22}$$

As we test the optimal placement strategy starting every five seconds for two trading days, we get a certain sample of savings against each constant submission strategy. The sample size is $4680 \cdot 2 = 9360$. To estimate the average gain of the optimal strategy we compute the mean

$$avgsavings(i) = \sum_{t_0,t} \frac{savings_{t_0,t}(i)}{S} \tag{3.23}$$

and the standard deviation is given by

$$stdsavings(i) = \frac{1}{\sqrt{(S)}} \sum_{t_0,t} \frac{(savings_{t_0,t}(i) - avgsavings(i))^2}{S-1}, \tag{3.24}$$

where S denotes the sample size.

4 Results

The ultimate goal of our model is to find optimal placement strategies for given order sizes and given time horizons. The strategy indicates on which relative price level the order has to be submitted, contingent on the initial queue size. The obtained strategy is tested against order submission on a constant relative price level as outlined in section 3.4.5. Therefore the ultimate results show the average savings of the optimal strategy against the constant submission strategies for each of the time horizons of 10, 30, 60 and 120 seconds. As discussed above, fill rates and effective spreads play an important role for the results. Therefore, in this section we also elaborate on that. To keep things simple, we first discuss the results for an order size of 200 shares. After that, we analyze how the results change for different order sizes.

In total we have access to 100 stocks traded at NASDAQ. As we want to have a very detailed discussion on the results, we only consider six different stocks. Two of them have a rather high spread, two a medium and two a small spread. One stock of each spread category has a rather high average daily trading volume while the other has a rather low one. Table 1 shows the stock characteristics of the considered stocks.

Stock	Spread (ticks)	Spread (bps)	Price	Trade Vol	Sub Vol	Can Vol	Depth Top	Depth 5 Level
BRCM	1	3.98	30.11	1.69	31.35	29.66	2488	19111
AKAM	2	4.22	55.22	0.77	6.52	5.87	407	3401
GMCR	7	6.98	99.33	1	4.45	3.65	208	989
SRCL	9	8.22	115.56	0.16	1.33	1.19	154	652
TSLA	19	9.62	201.04	1.78	5.23	4.04	224	1056
ALXN	21	13.22	158.96	0.51	2.32	1.82	153	590

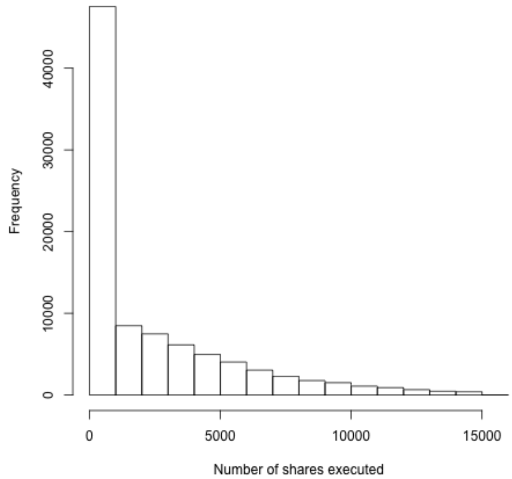
Table 1: Stock Characteristics

The stocks with rather small spreads are Broadcom Corp. (BRCM) and Akamai Technology (AKAM), the stocks with medium spreads are Keurig Green Mountain, Inc. (GMCR) and Stericycle, Inc. (SRCL) and the stocks with rather high spreads are Tesla Motors, Inc. (TSLA) and Alexion Pharmaceuticals, Inc. (ALXN).

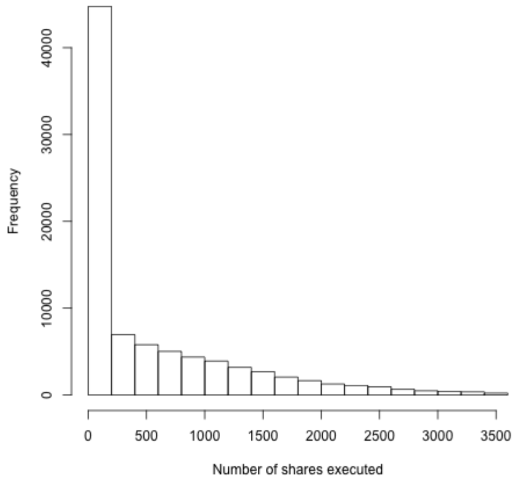
4.1 Small Spread Stocks

We start by discussing the two stocks with a rather small absolute spread. The stocks are AKAM and BRCM. In table 1 you see that the trading volume of BRCM is more than twice as high as the one of AKAM. Let us start by looking at the running maximum processes of the two stocks. Figure 8 shows the histograms of the distributions of the running maximum process for a time horizon of 60 seconds and a relative submission price level of 2.

As discussed in section 3.4.2, the running maximum process of the stock with the higher daily trading volume digs much deeper into the book. This means that the



(a) BRCM: 60 seconds



(b) AKAM: 60 seconds

Figure 8: Histograms of the running maximum process at a relative price level of 2

probability that a large amount of stocks is executed is much higher. This observation should be consistent with the obtained fill rates. But this conclusion does not take into account that the fill rates also depend on the initial queue size. This determinant in turn depends on the market depth at the different relative price levels. As the depth of BRCM is much higher, this has a contrary effect on the fill rates. Tables 2 and 3 show the estimated fill rates for the different time horizons and relative submission price levels for a submission size of 200 shares. Column 1 contains the time horizons in seconds, columns 2-6 show the respective fill rates for submissions at best bid (column 2) and 1-4 ticks in the book (columns 3-6). A fill rate of 0.5 means that 50 percent of the order is executed.

Time	**Fill Rate 0**	**Fill Rate 1**	**Fill Rate 2**	**Fill Rate 3**	**Fill Rate 4**
10	0.21 (0.0041)	0.08 (0.0028)	0.04 (0.0019)	0.02 (0.0014)	0.01 (0.001)
30	0.45 (0.0051)	0.24 (0.0044)	0.13 (0.0034)	0.07 (0.0027)	0.05 (0.0022)
60	0.63 (0.0049)	0.4 (0.005)	0.25 (0.0044)	0.16 (0.0038)	0.1 (0.0032)
120	0.78 (0.0043)	0.58 (0.0051)	0.42 (0.0051)	0.31 (0.0048)	0.21 (0.0042)

Table 2: Average fill rates (standard deviation in brackets) of BRCM for different relative price levels and time horizons

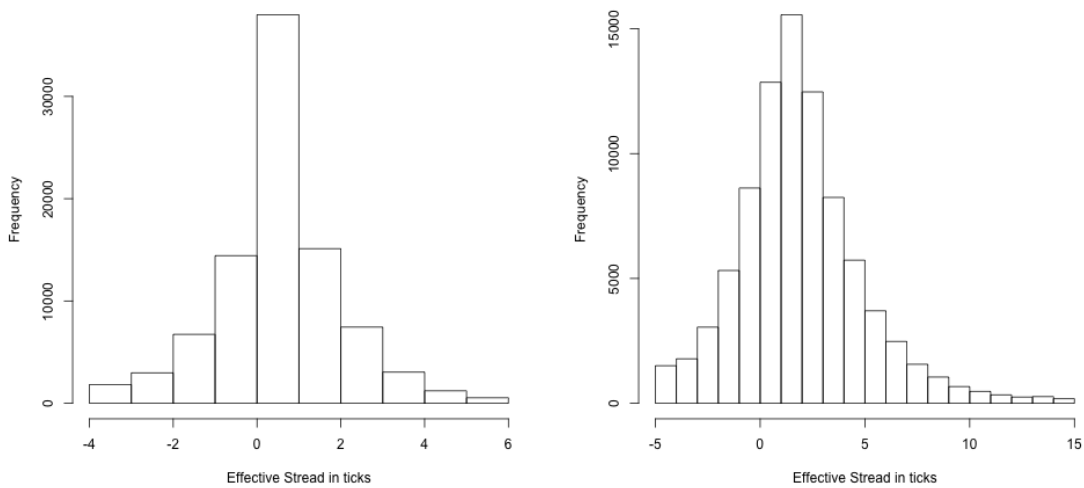
Obviously the fill rates increase with the time horizon and decrease with the relative submission price level. We observe that the fill rates of BRCM for the relative

Time	Fill Rate 0	Fill Rate 1	Fill Rate 2	Fill Rate 3	Fill Rate 4
10	0.16 (0.0036)	0.09 (0.0029)	0.06 (0.0024)	0.04 (0.0021)	0.03 (0.0018)
30	0.35 (0.0047)	0.23 (0.0042)	0.16 (0.0037)	0.12 (0.0033)	0.09 (0.003)
60	0.51 (0.005)	0.38 (0.0049)	0.29 (0.0046)	0.22 (0.0042)	0.17 (0.0039)
120	0.68 (0.0047)	0.56 (0.005)	0.47 (0.0051)	0.39 (0.005)	0.33 (0.0048)

Table 3: Average fill rates (standard deviation in brackets) of AKAM for different relative price levels and time horizons

price levels 0 (best bid) and 1 are higher and for the price levels 2-4 are lower than those of AKAM except for a time horizon of 10 seconds and a relative price level of 1. One possible explanation is the following. On the relative price levels 0 and 1 the positive effect on the fill rates stemming from the running maximum process of BRCM is higher than the negative effect of the higher initial queue size. For the relative price levels 2-4 this changes. A direct consequence is that the fill rates of BRCM vary more between the different relative price levels.

As outlined in section 3.4.3 the effective spread is a good indicator for the cost of the market order. Therefore we will now take a look at the distribution thereof. Figure 9 shows the histograms of effective spreads for a time horizon of 60 seconds. The histograms show how often a certain effective spreads (in ticks) occurs.



(a) BRCM: 60 seconds

(b) AKAM: 60 seconds

Figure 9: Histograms of effective spreads for a relative price level of 2

Note that the means of the effective spreads for BRCM and AKAM lie at 1.05 and 2.48 basis points while the standard deviations are 1.85 and 3.64 basis points respectively. The effective spreads have a direct link to the eventual results in savings, as they are a good indicator for the cost of the market order. Now that we have considered all the determinants for our final results, we take a look at the savings obtained from the optimal strategy compared to the constant submission strategies. Table 4 shows the results for BRCM for an order size of 200 shares.

Time	Φ_0	Φ_1	Φ_2	Φ_3	Φ_4
10	0.07 (0.0218)	0.04 (0.0142)	0.03 (0.012)	0.02 (0.0128)	**0.02 (0.0143)**
30	0.19 (0.0466)	**0.01 (0.0361)**	0.04 (0.0249)	0.05 (0.0219)	0.05 (0.0251)
60	0.47 (0.0695)	0.21 (0.0544)	0.13 (0.0415)	**0.08 (0.0328)**	0.1 (0.0298)
120	1.04 (0.0977)	0.66 (0.0772)	0.44 (0.0562)	**0.2** (0.0416)	0.23 (0.0398)

Table 4: BRCM: Average savings in basis points (standard deviation in brackets) achieved by the optimal strategy compared to constant submission strategies at different price levels for different time horizons

The first column shows the time horizon. Columns 2 to 6 contain the mean of the savings in basis points obtained by the optimal placement strategy compared to constant submission at relative price level 0 to 4. In brackets you see the standard deviations. The minimum values of each line are written in bold letters. The minimum tells us how much the optimal strategy wins compared to the best constant submission strategy. Note that a priori, the best constant submission strategy is not known. Therefore, it makes sense to compare the optimal strategy to all the constant submission strategies. Nevertheless, the comparison to the minimum is interesting, because it tells us, how much we win in any case. In table 4 we observe that for all relative price levels, the savings increase the longer the time horizon. Therefore, for this stock, it is interesting to see what happens, if we extent the time horizon. Hence, we look at the savings of BRCM for time horizons of 180, 240, 300 and 360 seconds, as shown in table 5.

On the relative price levels close to the best bid, the savings further increase. For submissions deeper in the book, the average savings increase until a certain point in time and than begin to fall. This behavior is quite natural. The longer the trader is able to wait, the higher is the probability that the order is executed, even if submitted deeper in the book. Therefore, the higher the time horizon, the more

Time	Φ_0	Φ_1	Φ_2	Φ_3	Φ_4
180	1.42 (0.1235)	0.98 (0.0996)	0.69 (0.0733)	**0.29 (0.0561)**	0.3 (0.0476)
240	1.68 (0.1415)	1.16 (0.1146)	0.84 (0.085)	**0.33 (0.0654)**	0.34 (0.0549)
300	1.93 (0.159)	1.26 (0.1317)	0.98 (0.0955)	0.31 (0.0745)	**0.3 (0.0559)**
360	2.32 (0.1687)	1.33 (0.1405)	0.97 (0.1037)	0.32 (0.079)	**0.27 (0.0558)**

Table 5: BRCM: Average savings in basis points (standard deviation in brackets) achieved by the optimal strategy compared to constant submission strategies at different price levels for different time horizons

beneficial it gets to submit at higher relative price levels. For a submission at a relative price level of 4, the gain compared to the benchmark price for each executed share is 4 ticks. That is why the savings of the optimal strategy compared to constant submission at low relative price levels gets quite high, even 2.32 basis points for 360 seconds. The standard deviation lies at 0.1687. This is a quite persuasive result.

On the other hand, the savings compared to constant submission at level 4 decrease at a certain point in time, because for long time horizons it is better to submit deep in the book, as execution probabilities are still high. Note that our strategy out-performs the constant submission strategies at all price levels and for all time horizons considered. Even if the best constant submission strategy would be known a priori, we would still win something. In this case, a time horizon of 240 seconds would be optimal, leading to savings of at least 0.33 basis points with a standard deviation of 0.0654. Table 6 now shows the savings of AKAM, the second small spread stock.

Time	Φ_0	Φ_1	Φ_2	Φ_3	Φ_4
10	**0.01 (0.0223)**	0.06 (0.0176)	0.08 (0.0154)	0.07 (0.0161)	0.1 (0.017)
30	0.2 (0.0361)	0.22 (0.0282)	0.18 (0.0252)	**0.13 (0.0241)**	0.16 (0.0272)
60	0.42 (0.0514)	0.36 (0.0406)	0.23 (0.0366)	**0.21 (0.0331)**	0.23 (0.0389)
120	0.75 (0.0748)	0.56 (0.0591)	0.35 (0.0509)	0.27 (0.0419)	**0.08 (0.0437)**

Table 6: AKAM: Average savings in basis points (standard deviation in brackets) achieved by the optimal strategy compared to constant submission strategies at different price levels for different time horizons

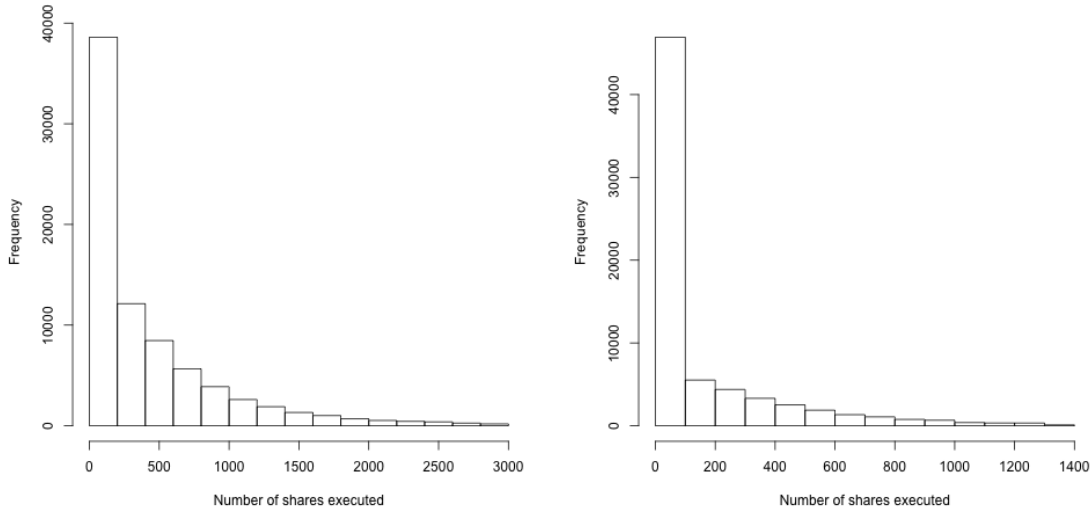
For a time horizon of 10 seconds, significant savings are clearly not obtained. However, already for a time horizon of 30 seconds, the average savings rise significantly to at least 0.13 basis points. The higher the time horizon gets, the more can be saved compared to submitting close to the best bid. If the trader is able to wait for two minutes, the savings are already 0.75 basis points with a standard deviation of 0.0748. On the other side the savings at level 4 climb until a 60 seconds time horizon and then fall again. Hence, this stock shows the same behavior as BRCM, but for smaller time horizons.

Let us take a closer look at the fill rates to understand the underlying dynamics. For 30 seconds the fill rate at level 0 is around 4 times as high as the fill rate at level 4. For 60 and 120 seconds this ratio falls to around 3 and 2 respectively. Note that the average spread for this stock lies at 2 ticks. Hence, the costs for a market order are on average around 2 ticks higher than the benchmark costs, while the savings in case of order execution from level 4 to level 0 are 4 ticks per executed share. A further analysis of the fill rates shows that in many cases the fill rates are either zero or one. A fill rate of zero for all levels means that it makes no difference where to submit, as the order is always transformed entirely into a market order. However, when all shares or at least a big part is executed, submitting deeper in the book pays off significantly. This explains why for longer time horizons, the savings against submissions close to the best bid are quite high. For this stock the maximum of the four minima, i.e., the time horizon, for which the optimal strategy wins most compared to the best constant strategy, is met in line 3 (60 seconds). The minimal savings for this time horizon are 0.21 basis points with a standard deviation of 0.0331.

4.2 Medium Spread Stocks

Now we look at the two stocks GMCR and SRCL with average spreads of 7 and 9 ticks. In table 1 we see that GMCR has approximately six times the trading volume of SRCL and about 30 percent more depth, while the average price of SRCL is around 15 percent higher. As above we analyze the running maximum processes first. Figure 10 shows the respective histograms.

The histograms are consistent with the findings in section 3.4.2. The stock with the higher trading volume also has higher execution probabilities. Despite the higher depth of GMCR, we expect higher fill rates compared to SRCL. Remember that



(a) GMCR: 120 seconds (b) SRCL: 120 seconds

Figure 10: Histograms of the running maximum process at a relative price level of 2

higher depth essentially means higher initial queue sizes, resulting in lower fill rates. However, the difference in the running maximum process is more significant. Tables 7 and 8 show the fill rates of GMCR and SCRL for the different time horizons and relative submission price levels.

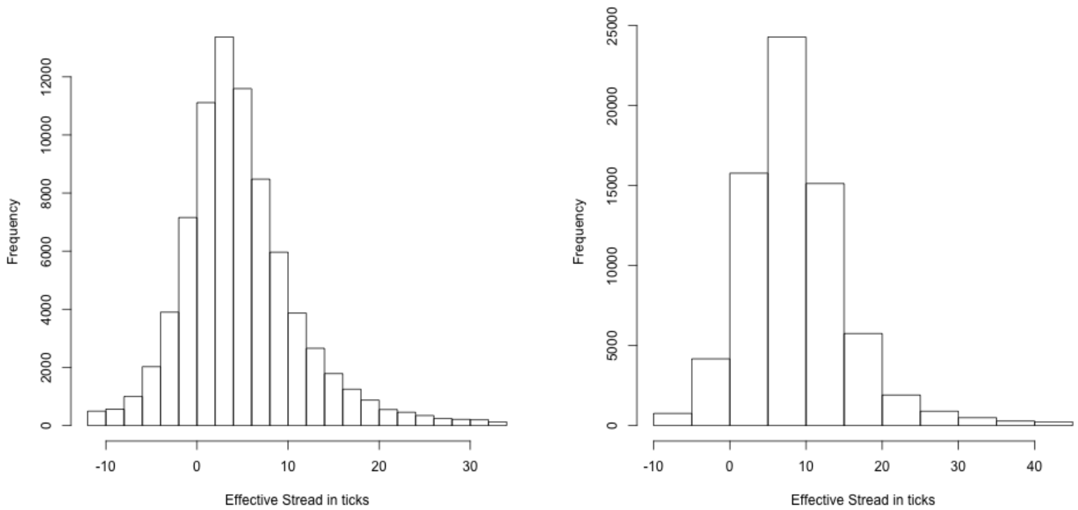
Time	**Fill Rate 0**	**Fill Rate 1**	**Fill Rate 2**	**Fill Rate 3**	**Fill Rate 4**
10	0.21 (0.0041)	0.15 (0.0036)	0.12 (0.0032)	0.09 (0.0029)	0.07 (0.0026)
30	0.45 (0.005)	0.38 (0.0049)	0.32 (0.0047)	0.27 (0.0045)	0.23 (0.0043)
60	0.64 (0.0049)	0.58 (0.0051)	0.52 (0.0051)	0.47 (0.0051)	0.42 (0.0051)
120	0.79 (0.0041)	0.75 (0.0045)	0.71 (0.0047)	0.67 (0.0049)	0.62 (0.005)

Table 7: Average fill rates (standard deviation in brackets) of GMCR for different relative price levels and time horizons

Time	**Fill Rate 0**	**Fill Rate 1**	**Fill Rate 2**	**Fill Rate 3**	**Fill Rate 4**
10	0.05 (0.0021)	0.03 (0.0018)	0.03 (0.0017)	0.02 (0.0015)	0.02 (0.0014)
30	0.13 (0.0034)	0.1 (0.0031)	0.09 (0.0028)	0.07 (0.0027)	0.06 (0.0024)
60	0.23 (0.0043)	0.19 (0.004)	0.17 (0.0038)	0.15 (0.0036)	0.13 (0.0034)
120	0.38 (0.005)	0.33 (0.0048)	0.3 (0.0047)	0.28 (0.0046)	0.25 (0.0044)

Table 8: Average fill rates (standard deviation in brackets) of SRCL for different relative price levels and time horizons

As expected, the fill rates for GMCR are much higher. Especially for small time horizons the difference is large. Due to the small trading volume of SRCL, the fill rates for short time horizons are pretty low. As above, we continue by analyzing the effective spreads, shown in figure 11.



(a) GMCR: 60 seconds

(b) SRCL: 60 seconds

Figure 11: Histograms of effective spreads at a relative price level of 2

We observe that the effective spreads for the two stocks are quite similar. Finally we look at the savings of the optimal placement strategy, starting with GMCR, shown in table 9.

Time	Φ_0	Φ_1	Φ_2	Φ_3	Φ_4
10	**0.1 (0.0143)**	0.16 (0.0133)	0.18 (0.0152)	0.21 (0.0171)	0.25 (0.0196)
30	0.33 (0.0266)	0.3 (0.0242)	**0.23 (0.0251)**	0.26 (0.0275)	0.31 (0.0325)
60	0.68 (0.0362)	0.48 (0.0318)	0.25 (0.0327)	0.22 (0.0364)	**0.17 (0.0421)**
120	1.16 (0.0475)	0.8 (0.0412)	0.39 (0.0414)	0.15 (0.0466)	**-0.09 (0.0541)**

Table 9: GMCR: Average savings in basis points (standard deviation in brackets) achieved by the optimal strategy compared to constant submission strategies at different price levels for different time horizons

For a time horizon of 10 seconds the smallest savings are achieved against constant submission at the relative price level 0 (best bid). The reason is that within 10 seconds, execution probabilities are quite low. Therefore it is better to submit at best bid to increase chances of getting executed. For 30 seconds the minimal savings are already at relative price level 2. Due to the higher expectation of execution after 30 seconds, it pays off to submit deeper in the book. For level 3 and 4 the minima are at level 4. We also observe, that for 120 seconds, the optimal strategy is out-performed by submitting constantly at level 4. Hence, for this particular stock, this time horizon is simply too long. It is always better to submit deep in the book. It would be interesting, however, to see what happens, if more price levels are considered. This possibility to extent the analysis is discussed later. Table 10 shows the results for SCRL, the second medium spread stock.

Time	Φ_0	Φ_1	Φ_2	Φ_3	Φ_4
10	**-0.01 (0.0067)**	0.05 (0.0068)	0.06 (0.0075)	0.08 (0.0084)	0.09 (0.0092)
30	**-0.02 (0.0133)**	0.09 (0.0118)	0.12 (0.0121)	0.15 (0.0133)	0.2 (0.0148)
60	**0.02 (0.0179)**	0.16 (0.016)	0.18 (0.0158)	0.21 (0.0173)	0.24 (0.0194)
120	**0.15 (0.0238)**	0.25 (0.0206)	0.24 (0.0218)	0.25 (0.0223)	0.3 (0.0258)

Table 10: SRCL: Average savings in basis points (standard deviation in brackets) achieved by the optimal strategy compared to constant submission strategies at different price levels for different time horizons

As for BRCM in the section about the small spread stocks, we see that the savings are increasing for all price levels for the considered time horizons. Therefore, it makes sense to take a look at higher time horizons again. The average savings for time horizons of 180, 240, 300 and 360 seconds we see in table 10.

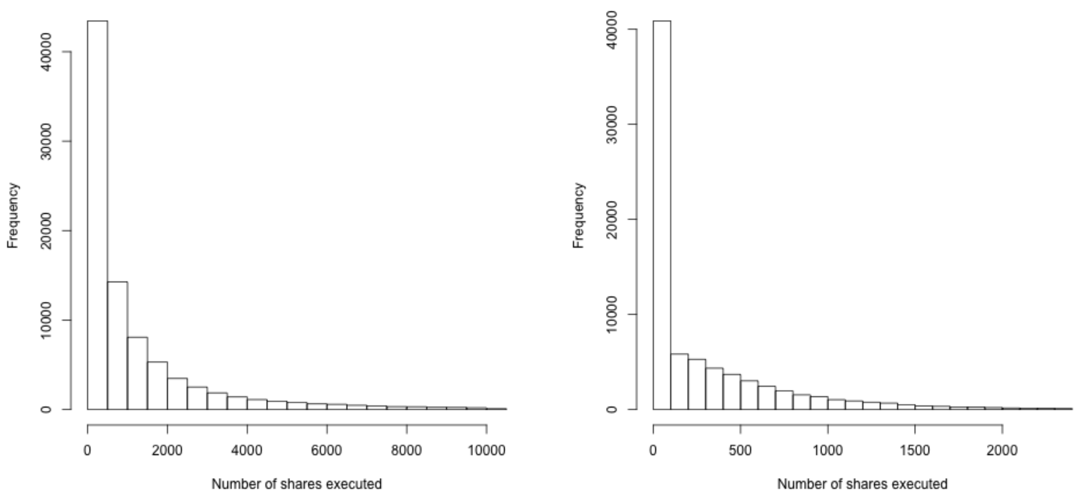
Time	Φ_0	Φ_1	Φ_2	Φ_3	Φ_4
180	0.32 (0.0271)	0.35 (0.0232)	0.31 (0.0235)	**0.26 (0.0242)**	0.3 (0.0284)
240	0.36 (0.0346)	0.34 (0.028)	0.33 (0.0266)	0.24 (0.0263)	**0.22 (0.0308)**
300	0.36 (0.0411)	0.36 (0.0321)	0.33 (0.0284)	0.23 (0.0272)	**0.16 (0.0304)**
360	0.42 (0.0443)	0.4 (0.0339)	0.34 (0.0298)	0.23 (0.0276)	**0.15 (0.0313)**

Table 11: SRCL: Average savings in basis points (standard deviation in brackets) achieved by the optimal strategy compared to constant submission strategies at different price levels for different time horizons

Again we see that the minimal savings move towards the higher relative price levels for longer time horizons. For small price levels, the average savings are increasing while for the higher levels, again there is a certain time horizon for which the savings start to decrease. This behavior seems to apply for all stocks, leading to an optimal time horizon in terms of a maximization of the minimal savings. For this stock, this time horizon is 180 seconds. Comparing this stock to BRCM, we see that the increase in savings on the levels close to best bid is really slow. Looking at the fill rates of SCRL and BRCM, we see that the latter are much higher, resulting in higher savings.

4.3 High Spread Stocks

At last we look at two stocks with relatively large spreads, namely TSLA with 19 ticks and ALXN with 21 ticks. Table 1 shows that TSLA has more than three times the trading volume and around 50% more depth. Due to the higher trading volume we expect a wider running maximum histogram for TSLA. Figure 12 shows the respective running maximum processes for a time horizon of 60 seconds and a relative submission price of 2.



(a) TSLA: 120 seconds (b) ALXN: 120 seconds

Figure 12: Histograms of the running maximum process at a relative price level of 2

The difference is quite striking. The running maximum process of TSLA digs into the book about four to five times deeper than the one of ALXN. Although the higher depth of TSLA, leading to lower expected fill rates because of the higher initial queue size, we can expect the fill rates of TSLA to be significantly above the ones of ALXN. Tables 12 and 13 show the respective fill rates.

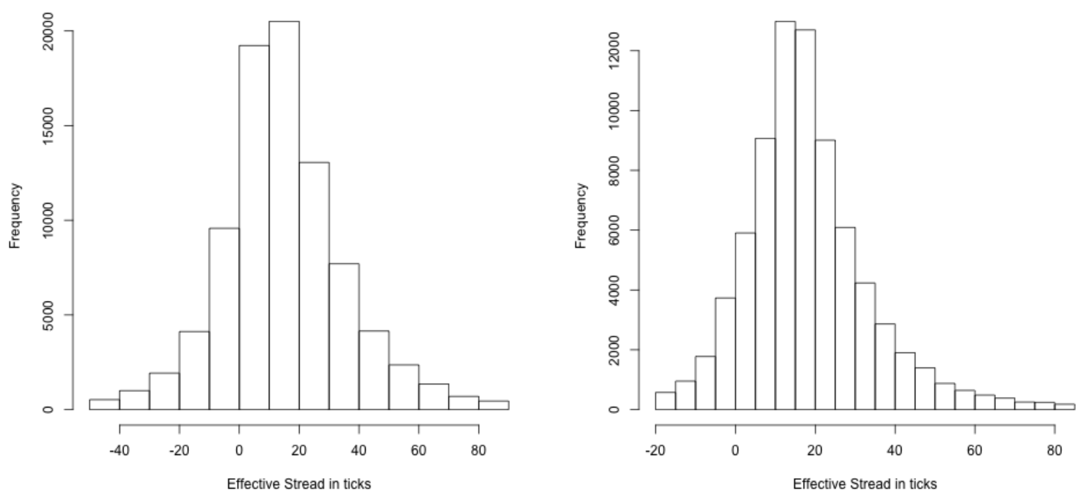
The picture meets our expectation. Figure 13 shows the respective histograms of the effective spreads.

Time	**Fill Rate 0**	**Fill Rate 1**	**Fill Rate 2**	**Fill Rate 3**	**Fill Rate 4**
10	0.25 (0.0043)	0.22 (0.0041)	0.2 (0.004)	0.19 (0.0039)	0.18 (0.0038)
30	0.51 (0.005)	0.47 (0.005)	0.45 (0.005)	0.43 (0.005)	0.41 (0.0049)
60	0.69 (0.0046)	0.66 (0.0047)	0.64 (0.0048)	0.63 (0.0049)	0.61 (0.0049)
120	0.84 (0.0037)	0.83 (0.0038)	0.81 (0.0039)	0.8 (0.004)	0.79 (0.0041)

Table 12: Average fill rates (standard deviation in brackets) of TSLA for different relative price levels and time horizons

Time	**Fill Rate 0**	**Fill Rate 1**	**Fill Rate 2**	**Fill Rate 3**	**Fill Rate 4**
10	0.15 (0.0036)	0.13 (0.0034)	0.12 (0.0032)	0.11 (0.0031)	0.1 (0.003)
30	0.34 (0.0048)	0.31 (0.0047)	0.29 (0.0046)	0.28 (0.0045)	0.26 (0.0044)
60	0.51 (0.0051)	0.47 (0.0051)	0.46 (0.0051)	0.44 (0.005)	0.42 (0.005)
120	0.69 (0.0047)	0.65 (0.0049)	0.64 (0.0049)	0.62 (0.005)	0.61 (0.005)

Table 13: Average fill rates (standard deviation in brackets) of ALXN for different relative price levels and time horizons



(a) TSLA: 60 seconds (b) ALXN: 60 seconds

Figure 13: Histograms of effective spreads at a relative price level of 2

Finally, table 14 shows the results in savings for TSLA.

Time	Φ_0	Φ_1	Φ_2	Φ_3	Φ_4
10	**0.04 (0.0171)**	0.28 (0.0184)	0.36 (0.0206)	0.39 (0.0225)	0.41 (0.0251)
30	**0.19 (0.0261)**	0.43 (0.0256)	0.52 (0.0272)	0.52 (0.0314)	0.52 (0.0349)
60	**0.33 (0.0367)**	0.46 (0.0333)	0.48 (0.0342)	0.42 (0.0369)	0.39 (0.0415)
120	0.72 (0.0428)	0.66 (0.0368)	0.53 (0.0362)	0.35 (0.0407)	**0.19 (0.0454)**

Table 14: TSLA: Average savings in basis points (standard deviation in brackets) achieved by the optimal strategy compared to constant submission strategies at different price levels for different time horizons

For short time horizons we see that the savings compared to submission at best bid are minimal. The longer the trader is able to wait, the higher the savings get. On the other hand, looking at the savings compared to constantly submitting at the relative price level 4, we see an increase from 10 to 30 seconds but then a decrease.

Table 15 shows that albeit the difference in trade volume and depth, the savings of ALXN behave pretty similar to the ones of TSLA.

Time	Φ_0	Φ_1	Φ_2	Φ_3	Φ_4
10	**0.03 (0.0118)**	0.2 (0.0173)	0.25 (0.02)	0.27 (0.0226)	0.31 (0.0256)
30	**0.11 (0.0186)**	0.34 (0.0282)	0.36 (0.031)	0.33 (0.0337)	0.35 (0.0359)
60	**0.22 (0.0312)**	0.43 (0.0249)	0.38 (0.0282)	0.32 (0.0321)	0.27 (0.0354)
120	0.41 (0.0364)	0.53 (0.0304)	0.43 (0.0307)	0.23 (0.0307)	**0.09 (0.0378)**

Table 15: ALXN: Average savings in basis points (standard deviation in brackets) achieved by the optimal strategy compared to constant submission strategies at different price levels for different time horizons

4.4 Order Sizes

So far, the average savings have been calculated for a submission size of 200 shares for all stocks. Now we want to analyze if a change in order sizes has observable effects on the results. Tables 16 and 17 show the average savings for GMCR for order sizes of 100 and 500 shares respectively.

Time	Φ_0	Φ_1	Φ_2	Φ_3	Φ_4
10	**0.13 (0.0177)**	0.17 (0.0156)	0.19 (0.017)	0.21 (0.0192)	0.26 (0.0219)
30	0.43 (0.0305)	0.36 (0.0282)	0.28 (0.0286)	**0.27 (0.0303)**	0.31 (0.035)
60	0.79 (0.0407)	0.55 (0.0359)	0.28 (0.0364)	0.21 (0.0398)	**0.15 (0.0448)**
120	1.25 (0.053)	0.86 (0.0459)	0.43 (0.0456)	0.16 (0.0497)	**-0.09 (0.0565)**

Table 16: GMCR: Average savings in basis points (standard deviation in brackets) achieved by the optimal strategy compared to constant submission strategies at different price levels for different time horizons for an order size of 100 shares

We observe that the savings for the small order size out-perform the ones of the large order size for submissions close to or at the best bid. This is quite natural, as the probability of being executed is higher for smaller orders. Therefore it pays off

Time	Φ_0	Φ_1	Φ_2	Φ_3	Φ_4
10	**0.02 (0.01)**	0.13 (0.0103)	0.17 (0.0126)	0.23 (0.0146)	0.27 (0.0163)
30	**0.12 (0.0201)**	0.21 (0.019)	0.22 (0.0214)	0.28 (0.0246)	0.35 (0.0286)
60	0.38 (0.0277)	0.31 (0.0237)	**0.2 (0.0258)**	0.21 (0.0318)	0.21 (0.0375)
120	0.81 (0.0397)	0.56 (0.0343)	0.26 (0.036)	0.04 (0.0421)	**-0.13 (0.0498)**

Table 17: GMCR: Average savings in basis points (standard deviation in brackets) achieved by the optimal strategy compared to constant submission strategies at different price levels for different time horizons for an order size of 500 shares

to submit deeper in the book, especially when the time horizon increases.

For both order sizes the generally observed behavior stays the same. The average savings increase with the time horizon for small relative submission price levels and first increase and then decrease for higher levels. The difference lies in the timing. Obviously for 100 shares the fill rates are higher than for 500 shares. Hence, the optimal time horizon in terms of maximal minimal savings for 100 shares (30 seconds) is smaller than the one for 500 shares (60 seconds).

4.5 Summary of the Savings Behavior

Summarizing we can state that the optimal strategy wins compared to constant submission strategies in most of the cases considered. Even compared to the respective optimal constant submission strategy, which is of course not known a priori, savings around 0.3 basis points are achieved for certain time horizons. However, it is rather fair to compare the optimal strategy to constant submission at best bid as this is a valid benchmark strategy. Especially for increasing time horizons the savings are quite significant with reasonably small standard deviations.

Overall we observe that for small time horizons the savings compared to constant submission at the best bid are minimal. This is quite natural as for small time horizons, the fill rates are pretty low. Therefore it is better to submit at a relatively high price. It is quite credible that submitting in the spread would even be better. Remember that we excluded this option in our analysis. With increasing time horizon the minimal savings, i.e., the best constant strategy moves away from the best bid. This means, as fill rates are rising, submissions in the book get more attractive.

On the relative price level 4 we observe that the optimal placement strategy outperforms constant level 4 submissions for small time horizons. For increasing time horizons, the extent of the out-performance increases first, but at a certain point in time it falls again.

5 Outlook

As this thesis is the first work in this direction, there are several possible extensions and improvements to the proposed model and the applied analysis. This section gives an overview over these opportunities for further elaborations.

Possible extension 1:
Extent the analysis to more than five price levels
We only consider five price levels, namely submission at the best bid and 1-4 levels in the book. Especially for small time horizons it would be very interesting to see, how the fill rates behave when submissions in the spread are also considered. As we saw in section 3.2, the submissions and cancellations in the book are quite balanced in terms of volumes. This means that a significant part of orders, that are executed, are actually submitted in the spread. To implement the possibility for the trader to submit in the spread, one has to adjust the sampling of the running maximum process. A submission in the spread means, that the initial queue size is zero. Further submissions at higher price levels, i.e., even further in the spread would increase the queue size, while cancellations of these submissions and executions would reduce it again. Hence, the implementation can be easily achieved without a fully-fledged re-write of the code. As we see that for larger time horizons, submissions in the book pay off, it would also be beneficial to consider more levels in the book. The sampling methodology stays the same. However, the amount of data and the computation time would increase.

Possible extension 2:
Condition the analysis on certain states of the market

When we sample the joined distribution for the running maximum process and for the cost of the market order, we consider a certain sampling period. However we do not condition these distributions on certain states of the market. The analysis would be more refined, if one would make a difference between states in which the market exhibits low or high spreads, low or high price volatility, low or high trading activity, low or high depths at the top order book levels, etc. As there are very different market conditions, especially with regards to spreads and price volatility, taking this into account, could actually make a significant difference.

When we look at the running maximum process and effective spread histograms, we see quite a large variation, especially for the effective spreads. Sampling conditional distributions would most probably decrease this variation. To implement that, one has to divide the market data into certain regimes, e.g., low and high spread, low and high price volatility, low and high depths, etc. before sampling the running maximum process and the sell side order book. When deriving the optimal price level for a submission at time t_0, one can observe the market at t_0, determine the regime, the market is in and then use the respective distribution.

Possible extension 3:

Consider a more refined grid of order sizes and time horizons

To keep things simple, the entire analysis is based on a universal order size of 200 shares, except for the short section with order sizes of 100 and 500 shares. A more refined analysis would take the trading volume and the depth of the respective stocks into account. One possibility would be to choose order sizes at a certain percentage of average daily trading volume or of average depth at the first five price levels. In addition, the analysis can be refined by choosing a finer grid of trading horizons.

Possible extension 4:

Extent the analysis to a larger stock universe

For the discussion of the results, only 6 out of 100 stocks are considered. The goal of further elaborations can be to find relations between observable stock characteristics and the resulting savings of the optimal strategy.

Besides these possible extensions it is also interesting to further analyze the statistical properties of the key determinants of the transaction costs, i.e., the running maximum process and the evolution of the sell side order book.

6 Conclusion

This thesis introduces a new order execution mechanism, i.e., the running maximum process of the net liquidity supply and demand. A key prerequisite to sample this process is the access to the limit order book micro data. We use this detailed information to derive optimal placement strategies for relatively small and given order sizes and for rather short and given time horizons. This thesis is a first approach in determining trading strategies by solely looking at the plane data without making any assumptions on price processes, order book shapes, etc. It is based on the idea, that the initial queue size - the volume in the order book with higher execution priority at the time of order submission - plays a crucial role for the execution probability.

Further it builds on the believe, that the execution probabilities contingent on initial queue sizes stay roughly constant over time. However, as discussed in section 5, it might be beneficial to divide the market into certain regimes and assume constant behavior within the respective regimes. Based on that stationarity of execution probabilities, fill rates can be derived, contingent on the initial queue size and the order size. The order execution problem is defined as such, that the un-executed part of the submitted order has to be transformed into a market order at the end of the given time period. Therefore the second crucial part of the introduced model (besides the fill rates) are the costs of this market order. These costs depend on the evolution of the sell side order book relative to the submission price level.

To sum things up, the model is based on a certain order execution mechanism (running maximum process) and a resulting cost function to determine expected transaction costs for different submission price levels. Therefore it can be used to derive the optimal submission price level based on entirely observable data at time of submission. Parts of the results are quite convincing, as the savings - especially compared to constant submission at best bid - significantly exceed one basis point for increasing time horizons, climbing up to 2.32 basis points. Considering that trading takes place at a very high frequency, this is quite a significant gain.

Overall this thesis gives interesting insights in the dynamics of order flows and how these dynamics can be leveraged to determine optimal order submission strategies. As this is the first work in this direction, we expect a lot of further research on the topic.

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Appendix

Stock Characteristics

	Stock	Spread (ticks)	Spread (bps)	Price	Trade Vol (mn)	Sub Vol (mn)	Can Vol (mn)	Depth Top	Depth 5 level
1	AAPL	15	2.74	532.87	2.28	13.88	11.99	195	928
2	ADBE	2	2.64	64.09	1.06	13.54	12.51	530	4662
3	ADI	2	3.19	50.43	0.62	13.83	13.27	781	6414
4	ADP	2	2.87	77.61	0.56	9.27	8.82	437	4016
5	ADSK	2	3.46	51.86	0.82	13.97	13.22	583	5069
6	AKAM	2	4.22	55.22	0.77	6.52	5.87	407	3401
7	ALTR	1	3.87	34.35	0.99	14.07	13.11	1380	11274
8	ALXN	21	13.22	158.96	0.51	2.32	1.82	153	590
9	AMAT	1	6.97	18.35	2.82	66.72	63.41	9251	56299
10	AMGN	5	4.09	121.39	0.94	8.77	7.89	278	1490
11	AMZN	23	6.02	371.26	0.98	3.77	3.02	155	579
12	ATVI	1	7.24	19	1.59	32.66	30.98	4716	30665
13	AVGO	3	4.61	58.57	0.55	4.83	4.41	365	2584
14	BBBY	2	3.17	67.74	0.78	8.25	7.55	443	3984
15	BIDU	15	9.11	167.62	1.08	3.54	2.73	241	1131
16	BIIB	46	14.05	317.59	0.46	2.75	2.34	158	678
17	BRCM	1	3.98	30.11	1.69	31.35	29.66	2488	19111
18	CA	1	3.99	32.6	0.79	17.56	16.73	1902	14099
19	CELG	12	7.32	158.59	0.93	3.63	2.78	198	838
20	CERN	3	4.62	57.73	0.51	5.43	4.93	355	2949
21	CHKP	4	5.54	66.09	0.34	3.15	2.92	284	1895
22	CHRW	2	3.46	54.45	0.57	6.06	5.52	573	4355
23	CHTR	12	10.92	130.95	0.37	2.11	1.81	170	794
24	CMCSA	1	2.66	52.09	4.16	76.25	71.65	2382	14913
25	COST	3	2.92	114.44	0.67	5.33	4.88	287	2107
26	CSCO	1	5.38	22.1	8.15	151.59	142.23	20154	113756
27	CTRX	2	5.41	48.08	0.57	13.5	12.88	385	3113
28	CTSH	3	3.86	86.78	0.69	6.49	5.8	348	2465
29	CTXS	2	3.72	59.16	0.9	8.17	7.45	416	3477
30	DISCA	4	4.84	82.61	0.48	3.85	3.47	237	1578

	Stock	Spread (ticks)	Spread (bps)	Price	Trade Vol (mn)	Sub Vol (mn)	Can Vol (mn)	Depth Top	Depth 5 level
1	DISH	2	4.19	58.19	0.78	6.25	5.58	431	3212
2	DLTR	2	2.97	53.24	0.66	7.65	7.11	567	5140
3	DTV	2	2.65	73.34	1.15	12.85	11.71	639	4997
4	EBAY	1	2.33	55.21	3.17	43.86	40.56	1383	9607
5	EQIX	23	12.45	184.34	0.23	1.54	1.36	149	641
6	ESRX	2	2.2	74.86	1.42	14.76	13.09	711	5699
7	EXPD	2	4.16	41.04	0.43	6.27	5.79	584	4903
8	EXPE	3	4.03	72.52	0.71	5.82	5.21	315	2275
9	FAST	2	3.94	46.75	0.67	7.81	7.19	572	5009
10	FB	1	2.18	63.42	12.71	131.66	121.26	1862	14204
11	FFIV	7	5.73	105.84	0.52	3.16	2.77	177	937
12	FISV	2	4.4	57.25	0.41	4.93	4.57	346	2663
13	FOXA	1	3.53	32.73	2.73	35.25	32.48	2765	18775
14	GILD	2	2.44	78.49	3.16	32.14	29.11	589	4190
15	GMCR	7	6.98	99.33	1	4.45	3.65	208	989
16	GOOG	55	4.53	1169.82	0.57	2.95	2.47	119	434
17	GRMN	2	4.86	49.5	0.4	3.65	3.25	356	2753
18	HSIC	13	11.4	116.55	0.15	1.35	1.23	170	665
19	ILMN	21	13.24	150.94	0.53	2.51	2.01	161	617
20	INTC	1	5.09	24.97	6.23	118.56	111.69	13875	81587
21	INTU	2	2.88	76.35	0.56	8.98	8.52	422	3605
22	ISRG	63	14.55	423.97	0.15	1.21	1.07	98	359
23	KLAC	3	4.14	64.69	0.42	5.43	5.1	350	2952
24	KRFT	2	2.93	54.31	0.74	10.68	10.13	660	5985
25	LBTYA	3	4.73	71.31	0.76	7.26	6.68	452	3448
26	LINTA	1	4.95	28.45	0.51	10.29	9.73	1275	9561
27	LLTC	1	3.04	46.11	0.68	11.94	11.24	889	8590
28	LMCA	16	12.48	134.58	0.24	1.49	1.33	158	720
29	MAR	2	3.13	51.66	0.68	11.04	10.4	631	5756
30	MAT	1	3.59	39.72	1.04	16.65	15.68	1407	10529
31	MDLZ	1	3.36	34.2	2.6	32.25	29.68	3102	20919
32	MNST	4	5.34	70.2	0.41	3.32	2.93	232	1278
33	MSFT	1	3.41	37.53	8.37	173.43	164.47	8182	52137
34	MU	1	5.25	23.78	6.35	89.56	83.21	6422	37835
35	MXIM	1	4.09	30.71	0.92	11.51	10.66	1597	12083

	Stock	Spread (ticks)	Spread (bps)	Price	Trade Vol (mn)	Sub Vol (mn)	Can Vol (mn)	Depth Top	Depth 5 level
1	MYL	2	3.61	48.58	1.19	15.88	14.47	721	4924
2	NFLX	35	8.53	401	0.69	4.06	3.43	134	524
3	NTAP	1	3.52	40.44	1.29	22.34	21.09	1366	10602
4	NVDA	1	7.6	17.11	1.75	39.41	37.27	6977	40897
5	NXPI	2	4.17	52.11	0.92	8.33	7.72	517	3791
6	ORLY	11	7.12	143.08	0.23	1.87	1.69	151	658
7	PAYX	1	3.17	42.46	0.71	13.2	12.47	1341	10880
8	PCAR	2	3.26	61.78	0.66	7.49	6.84	461	3826
9	PCLN	116	9.07	1234.22	0.21	1.7	1.53	87	300
10	QCOM	1	1.85	75.32	2.48	45.88	43.04	1450	11170
11	REGN	49	16.57	308.15	0.32	1.68	1.38	107	428
12	ROST	2	3.06	71.01	0.67	6.73	6.18	422	3434
13	SBAC	5	5.73	92.69	0.31	2.6	2.35	216	1150
14	SBUX	2	1.99	74.03	1.74	23.96	21.98	734	6603
15	SIAL	5	5.83	93.42	0.22	2.19	2.02	198	1191
16	SIRI	1	31.36	3.54	9.28	178.24	165.32	257534	1274538
17	SNDK	3	3.51	73.82	1	8.38	7.51	377	3106
18	SPLS	1	7.92	13.07	2.18	40.74	37.97	9630	55399
19	SRCL	9	8.22	115.56	0.16	1.33	1.19	154	652
20	STX	2	4.01	53.63	0.99	10.25	9.31	500	4190
21	SYMC	1	5.58	21.4	2.06	43.46	41.05	5506	32428
22	TRIP	8	8.76	91.2	0.58	3.51	3.04	206	1084
23	TSCO	5	7.21	70.73	0.41	3.31	2.98	233	1603
24	TSLA	19	9.62	201.04	1.78	5.23	4.04	224	1056
25	TXN	1	3.04	44.13	1.68	32.79	30.97	2291	16356
26	VIAB	2	2.9	85.18	0.79	7.23	6.63	368	3096
27	VIP	1	10.49	10.29	0.64	7.25	6.41	2984	18875
28	VOD	1	3.65	38.15	2.2	86.8	84.11	4352	34479
29	VRSK	3	5.37	63.38	0.26	2.78	2.58	284	1929
30	VRTX	9	10.65	79.26	0.47	3.15	2.72	198	976
31	WDC	4	4.51	86.37	0.65	7.24	6.8	258	1833
32	WFM	2	3.1	53.28	1.12	12.19	11.32	582	5343
33	WYNN	19	8.01	219.91	0.46	2.15	1.78	135	508
34	XLNX	1	2.84	49.67	1	17.09	16.14	832	7746
35	YHOO	1	3.49	38.21	4.51	70.45	65.8	2644	16741

Deutsche Zusammenfassung

Aufgrund vorherrschender Prioritätsregeln betreffend Zeitpunkt und Preis in Limit-Order-Book-Märkten hat eine zum Zeitpunkt t_0 und zum Preis p platzierte Buy-Limit-Order der Größe n ein gewisses Volumen an Aktien vor sich. Diese Aktien werden bevorzugt gehandelt, da sie entweder einen besseren Preis aufweisen oder bei gleichem Preis früher platziert wurden. Dieses Volumen mit höherer Ausführpriorität wird häufig als Queue $Q^p_{t_0}$ bezeichnet.

Das Volumen der Queue Q_t verändert sich im Lauf der Zeit klarerweise, da neue Sell-Market-Orders und annullierte Buy-Limit-Orders die Queue reduzieren können, während neu platzierte Buy-Limit-Orders diese erhöhen können. Wie sich herausstellt, hat dieser Prozess einer sich verändernden Queue eine direkte Verbindung zur Ausführwahrscheinlichkeit einer Order. Unter Verwendung von hochgranularen Marktmikrodaten zu ausgewählten Aktientiteln schätzen wir mithilfe dieses Prozesses die erwarteten Fill-Rates von platzierten Buy-Limit-Ordern. Die Fill-Rates bezeichnen den prozentualen Anteil einer platzierten Order, der nach einer bestimmten Zeit im Verhältnis zur gesamten Ordergröße ausgeführt worden ist.

Basierend auf den geschätzten Fill-Rates und der Entwicklung des Limit-Order-Book-Marktes, kann das optimale Preislevel zur Platzierung einer bestimmten Buy-Limit-Order bestimmt werden. Die Ordergröße, der Zeitpunkt der Platzierung und der Zeithorizont des Tradings sind dabei gegeben. Wir analysieren, ob die Strategie zum Platzieren von Orders, die sich durch die Wahl dieses optimalen Preislevels ergibt, bestimmte ad-hoc Strategien schlagen kann, bei denen die Orders immer am selben Preislevel platziert werden.

Insgesamt wird eine höhere Performance durch die optimale Strategie erreicht. Speziell für größer werdende Trading-Zeiten ergeben sich im Vergleich zu Order-Platzierungen am Best Bid signifikante Verbesserungen von bis zu 2,32 Basispunkten. Für eine erste Arbeit in diese Richtung sind das durchaus überzeugende Ergebnisse, speziell im Hinblick auf die zahlreichen Möglichkeiten, die Analyse zu verfeinern und fortzuführen.

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