universität wien

MASTERARBEIT / MASTER'S THESIS

Titel der Masterarbeit / Title of the Master's Thesis

"Application of the
uncapacitated multiple allocation p-hub median problem
to the Norwegian air transportation network."

verfasst von / submitted by

Mag.phil. Markus Zobel, Bakk.phil. BSc

angestrebter akademischer Grad / in partial fulfilment of the requirements for the degree of

Master of Science (MSc)

Wien, 2016 / Vienna 2016

Studienkennzahl lt. Studienblatt / Degree programme code as it appears on the student record sheet:	A 066 914
Studienrichtung lt. Studienblatt / Degree programme code as it appears on the student record sheet:	Masterstudium Internationale Betriebswirtschaft / Master's degree programme International Business Administration
Betreut von / Supervisor:	o. Univ.-Prof. Dipl.-Ing. Dr. Richard Hartl

„*Lite og Stort*“

https://youtu.be/Tcqxd1SmZgg

Statutory declaration

I declare that I have authored this thesis independently, that I have not used other than the declared sources / resources, and that I have explicitly marked all material which has been quoted either literally or by content from the used sources.

Moreover, I declare that I do not have any personal or professional relations to the companies mentioned in the case study of this thesis.

Vienna, May 2016

Markus Zobel

Table of Contents

List of Abbreviations

B737	Boeing 737 (aircraft)
CAB	Civil Aeronautics Board
CRPK	Cost per Revenue Passenger Kilometer
DAT	Danish Air Transport
DH1	de Havilland Dash-8-100 Series (aircraft)
DH2	de Havilland Dash-8-200 Series (aircraft)
DH3	de Havilland Dash-8-300 Series (aircraft)
DH4	de Havilland Dash-8-400 Series (aircraft)
DH8	de Havilland Dash-8 (aircraft); now: Bombardier Q-Series
EC	European Commission
EEA	European Economic Area
EUR	Euro
HLP	Hub Location Problem
IATA	International Air Transport Association
LP	Linear Program
NOK	Norwegian Kroner
OD	Origin-Destination
PSO	Public Service Obligation
SAS	Scandinavian Airlines System
STOL	Short Take-Off and Landing (runway)
UMApHMP	Uncapacitated Multiple Allocation p-Hub Median Problem
USA	United States of America
USD	United States Dollar
v.v.	vice versa

List of Tables

List of Figures

1 Introduction

The following section unrolls the motivation that underlies this master's thesis and presents an introduction into the Hub Location Problem and the area of research where this topic has its origins.

1.1 Motivation

During the past decades the Hub Location Problem (HLP) has strongly developed in the area of location science. Many researchers generated competing models to approach the problem and continuously improved the model formulations to reduce the computation power and increase the feasible network size of the models. The area of transportation logistics is a classical example for applications of the HLP. In addition, more specific models that meet the requirements of air transportation networks can be found in the literature. However, most if these models have been assessed with a single set of data from the 1970ies that reflects a case on a mainland transportation network. That raises the question, if these models are appropriate to handle a current real-world air transportation case. Thereby the focus lies intentionally on applying an easy-to-use model that works with a minimum of input data and computational power such that the hub location decision can be performed with reasonable effort.

A further aspect is that the real-world case plays in a special network setting: Norway's rugged landscape is a good example for an airline network in a remote region. In a country with about 5 million inhabitants three considerable airlines operate flights to more than 40 airports. Many of these airports are situated in villages that have not more than 2000 inhabitants, but are served with several daily flights to one of the regional capitals. Due to low demand, many of the routes cannot be operated profitable such that the government subsidizes them through the Public Service Obligation (PSO) scheme.

One of the main operators of PSO-routes is Widerøe, a Norwegian airline serving more than 40 domestic destinations connected via 5 hub-airports. Other actors in this network are SAS and Norwegian Air Shuttle, both operating one of their major hubs at Oslo airport (beside of their hubs located in other countries). In times of cost pressure and emerging point-to-point connections this network set-up looks inefficient at the first glance.

The aim of this thesis is to apply a state-of-the-art formulation of the Hub Location Problem to the Norwegian air transportation network. Thereby an exact solution should be found. The results of the model are then to be compared with the current state of the real-world case. In a further step the public subsidies will be implemented into the model and the effects of this extension are to be observed. Special attention will be given on the analysis of the route allocation. Furthermore, computation experiments with the number of hub locations to be placed will be analyzed. Finally, an in-depth analysis of the output data should reveal if the model is suitable to analyze a possible cross-subsidizing of commercial routes with public subsidies received from the PSO routes.

1.2 The Hub Location Problem

This section first explains the role of a hub in a network. In a second stage, the Hub Location Problem (HLP) is introduced and its relation to location science is explained. In a further step a detailed view on the different aspects of the HLP is then followed by a presentation of different solution methods of this model.

1.2.1 Characteristics of a Hub-and-Spoke network

In opposite to a point-to-point network (Figure 1), where all origins and destinations are connected with each other, a hub-and-spoke network connects transshipment points (hubs) with the origins and destinations. At these points flows from the incoming connections (spokes) are collected and reassigned to an outgoing spoke to reach the designated destination. For example, in an airline network a hub airport is used to collect passengers from multiple origins and to send them to their final destination. With this network design, capacities and range (aircraft size) as well as frequencies can be set according to passenger demand. Similar to an airline network, we can find hub-and-spoke networks at land- and sea-based public transportation networks, in postal delivery services and telecommunication (Mattfeld and Vahrenkamp 2014: 191ff).

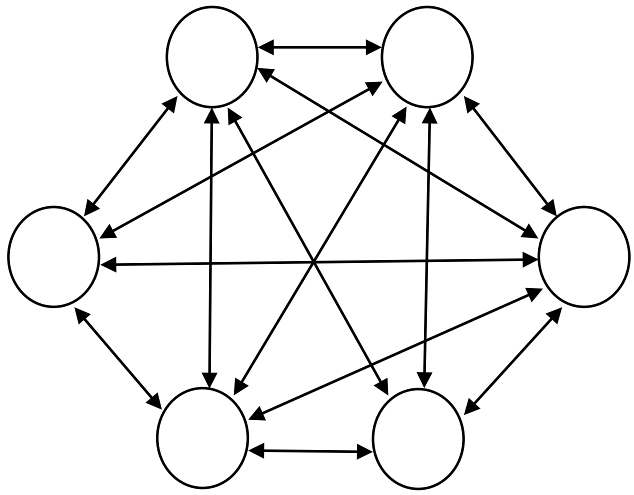

Figure 1: Point-to-point network

The following advantages of a hub-and-spoke airline network can be recognized (Mattfeld and Vahrenkamp 2014: 195):

- Multiplier effect: more destinations can be connected with the same amount of aircraft as with point-to-point traffic (with n connections to the hub $n(n+1)/2$ city-pairs can be connected).
- Economies of densities: cost reduction due to higher service densities on inter-hub routes (higher load factor).
- Economies of scope: cost reduction due to the use of centralized handling, maintenance, staffing.
- Economies of scale: cost reduction due to the use of bigger aircraft on high traffic routes.
- Online connections: passengers can change to connection flights within the same airlines (this brings a time advantage because the airlines optimize their schedules to get low connection times).
- Higher frequencies: more attractive to fly on one route due to multiple flights a day (time flexibility).
- Dominant hubs and routes: a strong hub network builds high market entry barriers.
- Hub premium: higher market prices due to market dominance.

In a 1-hub network two types of hubs are possible (Figure 2): The hourglass hub collects the traffic from one region and forwards it to another region. Thereby long distances (where a technical break is necessary, e.g. fueling) on both sides of the hub are most likely. For instance, Singapore serves as a major hub for connections between Europe and Australia/Oceania. The hinterland hub collects and forwards passengers in the same regions, for instance a classical setting of some smaller regional airports that are connected with a major airport, from where passengers get forwarded to either bigger or smaller airports again (Mattfeld and Vahrenkamp 2014: 196).

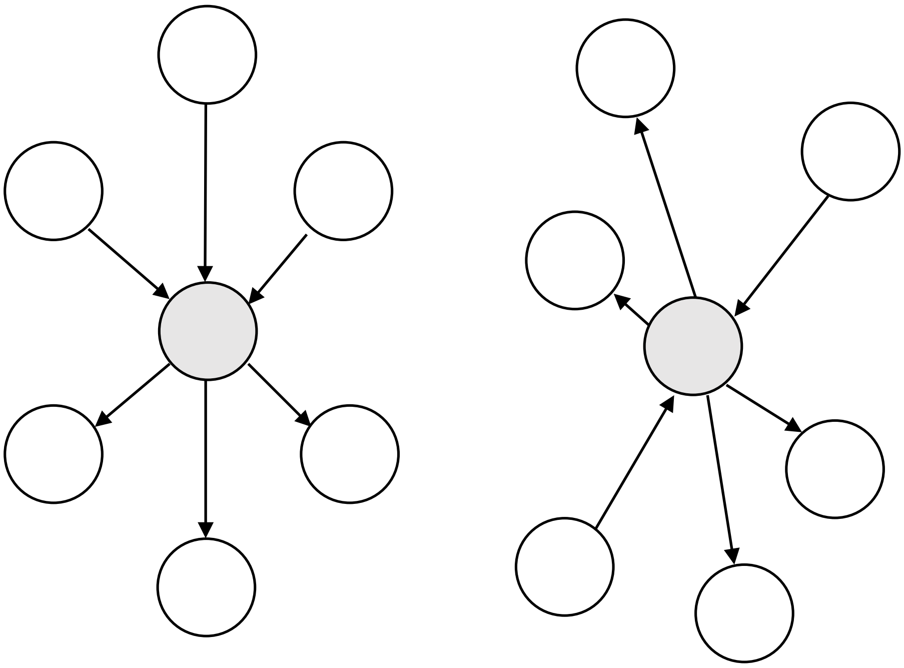

Figure 2: Hourglass and hinterland hub

Often, a network with two hubs is used (Figure 3). Thereby both hubs have a collecting and forwarding function. There is a connection between both hubs with a high volume of traffic. This allows the airline to use a bigger aircraft with lower costs-per-seat to do service on the inter-hub connection (Mattfeld and Vahrenkamp 2014: 194).

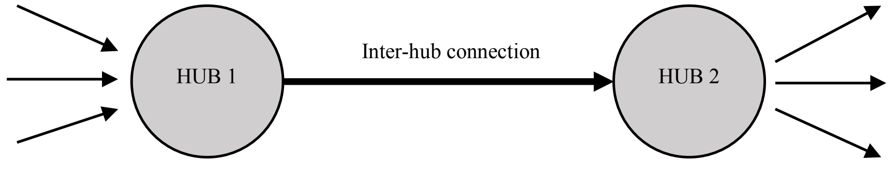

Figure 3: Network with 2 hubs

1.2.2 The HLP as part of location science

Farahani and Hekmatfar (2009: 1) define a location problem as "*modeling, formulation and solution of a class of problems that can best be described as locating facilities in some given spaces*". Thereby, four basic aspects are considered in this problem: a *space* with locations of customers and facilities, where the *customers* are assumed to be already located at points/routes and *facilities* have be located. Furthermore, we have a *metric* that indicates distances/time between customers and facilities (Farahani and Hekmatfar 2009: 1-2).

Already in the 17th century scientists addressed location science as part of analytical studies. It is said that Pierre de Fermat, Evagelistica Torricelli (a student of Galileo) and Battista Cavallieri individually proposed – or even solved – the basic Euclidean spatial median problem. Formally, Alfred Weber started the study of location theory in 1909 by addressing a single warehouse location problem to minimize total distance between warehouse and some customers. Only a few applications have been studied until Hakimi (1964) developed a model to locate switching centers for a telecommunication network as well as police stations in a highway network (Farahani and Hekmatfar 2009: 1).

These problems can be divided into three parts: location problems, allocation problems and location-allocation problems.

Location problems deal with the basic question in location science, namely where to optimally locate a facility. For instance, they search for an optimal location for a machine in a shop or items inside a warehouse. Thereby, they are able to deliver a fast decision analyses with a minimum of input variables. In case we would like to locate a new facility on the plane, the main objective is to minimize the travel distance (costs) to already existing facilities. By using for example the Euclidean distance, this problem is quick and easy to solve. The location of a single facility on the plane is the basic problem, however in reality different factors determine the scope of a problem and the model has to be extended accordingly (Moradi and Bidkhori 2009: 37-38).

The *allocation problem* refers to the optimal allocation of routes to the hubs, in case there exist more than one hub. Often algorithms combine both problems to solve the *location-allocation problem*. Typically, this problem wants to locate a set of new facilities such that the transportation cost from facilities to customers is minimized and an optimal number of facilities is placed in an area of interest in order to satisfy the customer demand (Azarmand and Jami 2009: 93-94).

The three main research problems in location science are: the center problem, the covering problem and the median problem.

The *center problem* searches for the node that is the most central in a network of several nodes. Thereby the maximum distance between each of the nodes is determined. In a second step, the node where the maximum distance is minimal is elected to be the center. Possible application areas are the location of public services like hospitals or fire brigades. Furthermore, services where the demand is equal to zero after a certain distance to the customer (banks, gas stations, fast food restaurants) can also determine an optimal location with this model (Mattfeld and Vahrenkamp 2014: 103f).

The *covering problem* is similar to the center problem: A node is selected such as the other nodes can be served within a given maximum distance around this location. It can be applied for locating warehouses or outlets. The service quality in the covering problem can be determined by service levels (Mattfeld and Vahrenkamp 2014: 108).

The two problems described above basically want to minimize the maximum distance from one node to another node that serves as distribution point. The *median problem*, however, uses the weighted distances to determine the optimal location of the distribution point. Each of the nodes gets weighted in form of a demand at that node. The model tries to find a trade-off between the distance (time) to the distribution point and the demand that occurs at the node (Mattfeld and Vahrenkamp 2014: 122f).

Hakimi (1964) was the first to present the (1)-median problem to locate hubs in a network. He worked together with the Air Force Office of Scientific Research and the U.S. Army Research Office to develop a model that searches the optimal point for a switching center (hub) in a telecommunication network. He suggested to add weights – that represent the number of wires that must be connected to the vertices in order to handle the information flows – to the vertices. The problem is to find the exact location of the switching center such that the total length of wires is minimum. In his follow-up paper Hakimi (1965) describes the possibility to have movement between two switching centers, like an inter-hub connection in a hub network. However, this idea was rejected because Hakimi assumed the cost to connect both switching centers is negligible.

Based on the ideas by Hakimi (1964, 1965), Goldman (1969) applied the p-median problem to a transportation topic. He considered different costs for collection and for processing and transfer for two successive facilities. Thereby, the transport of material with multistage processing at facilities is considered in the model. The author already mentions the advantages of economies of scale when operating a bulk/long haul connection among the facilities. However, this topic did not get much interest in the following years.

O'Kelly (1986) was then the first to address the Hub Location Problem as we think of it today. He defines hubs as "*central facilities which act as switching points in networks connecting a set of interacting nodes*". In this paper O'Kelly develops a model of two hubs in a plane. He also addresses different assumptions on scale effects on the inter-hub connections.

1.2.3 The different aspects of the HLP

The Hub Location Problem (HLP) helps to reduce total transportation costs by routing connections between origin-destination (OD) pairs over transshipment points (hubs). For instance, a fully connected network with k nodes in a non-hub network has $k(k-1)$ OD links. If a hub node is allocated to connect all other nodes with each other we get only $2(k-1)$ connections to serve the OD pairs. The links between hub and non-hub nodes are called spokes. In real-world application we can consider the movement of people, commodities and information for this problem. The cost of a network depends on the network structure. It might be more efficient in terms of costs to serve OD pairs via hubs. However, travel distances and times will increase and so, a trade-off between costs and distance/time is the objective of this problem (Farahani et al. 2013).

The structure of the network also defines the different models of the HLP. The basic (quadratic) formulation of the HLP was developed by O'Kelly (1987). There the assignment of a given number of hubs (p) in a network is discussed. This first formulation considers only the case where a non-hub node can only be connected to a single hub node and so the model is named *single allocation p-hub median problem*. In the case that one non-hub is linked to more than one hub the respective model is called *multiple allocation p-hub median problem*, first mentioned by Campbell (1992).

Both formulations have in common that they are based on the *p-median problem*. This fundamental discrete facility location problem not only has a long history, but also works with a minimum of assumptions. Thereby the objective is to locate *p* facilities at candidate sites to minimize total travel costs by serving a set of given demand nodes. The travel distance (time) and the demand at the nodes are put together to a demand-weighted sum of the distance between demand node and facility (Campbell and O'Kelly 2012).

Over the past 25 years this topic raised importance in the literature. Alumur and Kara (2008) showed that the number of publications dealing with HLPs almost doubled every 5 years since the first formulations in 1987. Furthermore, the problem got enriched by an increasing number of situations, where it can be applied.

Farahani et al. (2013) categorized the variations of this model: They distinguish whether the solution domain is the network (hubs can be located at all nodes), it is discrete (hubs can be located at some given nodes) or it is continuous (the domain of hub nodes is a plane or a sphere). Moreover, different objectives are included in the model. For example, there exist models where the maximum transportations cost between the OD pairs is minimized (Mini-Max). In comparison, it is possible to minimize the total costs incurred by locating hubs and the allocation of non-hub nodes to hubs (Mini-Sum).

The number of hubs can either be set exogenously or determined endogenously as part of the solution of the model. Moreover, some models cover the situation where only one hub is located in a network, others deal with multiple hubs. It is common that restrictions are used in the models to better simulate real-life situations. These models are called "capacitated" models – in opposite to the unlimited "uncapacitated" models. Many models also incorporate different cost structures like variable costs, fix costs or no costs for locating a hub. The same costs can be shifted to the cost for opening a connection between a non-hub node and a hub. The above-mentioned question whether a non-hub can be connected to only one or to multiple hubs (single vs. multiple allocation) completes this overview over the different aspects of the HLP.

The HLP can be applied to various settings. The most research is done in the areas of transportation and telecommunication. Thereby the variations in the field of transportation science reach from models that incorporate overnight restrictions and time zones in air transportation networks, over problems that locate master-hubs and smaller mini-hubs to scenarios that try to find best consolidation points for less-than-truckload carriers. HLPs in telecommunication differ slightly from the traditional HLP due to the different cost structures in computer networks (Alumur and Yara 2008).

Yang and Chiu (2016), for example, present a contemporary problem of the HLP that deals with stochastic demand and hub congestion.

Other problems in this field of research are: the p-hub center problem, where the objective is to minimize the maximum/costs distance between each pair of nodes under the assumption of given hub locations. The hub covering problem demands the origin and destination to be within a particular distance from the hub. The hub arc problem locates a given number of *q* arcs instead of locating hubs (Sender and Clausen 2011).

1.2.4 Solution methods

The HLP (based on the p-median problem) belongs to the problems that are classified as *NP*-hard (Non-deterministic Polynomial-time hard). This class of problems is among the hardest problems to solve. The difficulty arises from the fact that the HLP includes elements of facility location problems and quadratic assignment problems. Both of these problems are difficult to solve themselves, however, the combination of the problems makes them at least as hard to solve as comparable regular facility location problems (Campbell and O'Kelly 2012).

Farahani et al. (2013) show a comprehensive overview over the solution methods used in this field of study. Thereby the solutions methods are divided into exact methods and heuristic methods. To be able to compare the respective solution methods amongst each other, a set of data consisting of relevant data of 25 US cities issued by the Civil Aeronautics Board (CAB) in 1970 is used in most of the studies.

Campbell and O'Kelly (2012) state that the use of the CAB data set for air transportation problems was beneficial for the evolution of the hub location research due to the simulation of a real-world case and the consistence in assessing different models. However, it would be interesting to see if this standard model also performs well in case of an airline network that is situated in a remote region, in opposite to the CAB data that includes destinations on mainland USA.

Heuristic and meta-heuristic approaches are used by researchers to solve large instances of the problem. Most of the solution methods are based on models from network location problems. The tabu search, genetic algorithms, dual ascent approach as well as greedy heuristics are among the popular heuristic approaches. Thereby the emphasis in the past years was to solve large problems with restrictions (for instance: capacitated problems). Hub-median problems are much more in the focus of interest as hub-covering problems. Moreover, different approaches in terms of location and allocation of the initial solution are discussed in the literature (Farahani et al. 2013; Mattfeld and Vahrenkamp 2014: 204ff).

For an exact solution of the HLP integer or mixed-integer programming is used by most researchers. Combinations with branch-and-cut algorithms and forms of linear-, stochastic- or quadratic programming are also described in the literature (Farahani et al. 2013; Mattfeld and Vahrenkamp 2014: 210ff).

2 Literature Review

This literature review presents an overview on the research on the uncapacitated multiple allocation p-hub median problem (UMApHMP), which represents the formulation of the HLP to be used in this thesis.

2.1 *Early development*

As mentioned above, Campbell (1994) was the first to present a linear formulation for the UMApHMP. Since he used a formulation similar to the p-median problem his model also got named accordingly. Developed by Hakimi (1964), the p-median problem is one of the standard models in location science (see section 1.2.2).

Campbell (1994) states that an UMApHMP can be viewed as embedded in an undirected network $N = (V, A)$. The set of nodes (vertices) of the network $V = \{v_1, v_2, \ldots, v_a\}$ corresponds to the origins and destinations and the potential hub locations. Hubs are restricted to be located at a subset of the vertices. The link $(a, b) \in A$, which connects vertices v_a and v_b, is associated with a non-negative weight $d(a, b) \equiv d(b, a)$ that represents the length (travel time, distance, cost, etc.).

The following variables are used in this model: $\mathrm{X_{ijkm}}$ as fraction of flow from location (origin) *i* to location (destination) j, routed via hubs *k* and *m*; Y_k equals 1 if location *k* is a hub, 0 otherwise. Hence, the decision variable $\mathrm{X_{ijkm}}$ determines the allocation, while Y_k defines the location of a hub.

The input data is as follows: *n* is the number of locations; W_{ij} the flow from location *i* to location j; C_{ijkm} the cost per unit of flow from location *i* to location *j* routed via *k* and *m*; α is the discount factor for an inter-hub connection; p is the required number of hubs to be located. The total cost from traveling from location i to location j via hubs k and m given by $C_{ijkm} = c_{ik} + \alpha c_{km} + c_{mj}$. Moreover, $c_{ii} = 0$ is assumed to keep the formula valid when i and/or j is a hub. The indexes in the summations of the following formulations go from 1 to n.

Campbell (1994)

$$\min \sum_{i,j,k,m} W_{ij} C_{ijkm} X_{ijkm} \quad (1)$$

s.t. $$\sum_{k \in V} Y_k = p \ , \quad (2)$$

$$\sum_{k,m} X_{ijkm} = 1 \ \forall i,j \in V \ , \quad (3)$$

$$X_{ijkm} \leq Y_k \ \forall i,j,k,m \in V \ , \quad (4)$$

$$X_{ijkm} \leq Y_m \ \forall i,j,k,m \in V \ , \quad (5)$$

$$Y_k \ \in \{0,1\} \ \forall k \in V \ , \quad (6)$$

$$X_{ijkm} \geq 0 \ \forall i,j,k,m \in V \quad (7)$$

The objective is to minimize the total transportation cost (1). Moreover, constraint (2) makes sure that the amount of p hubs is located. The constraint (3) ensures, that the OD-flow should be routed via some hub pair, and (4) and (5) guarantee that these locations are hubs. (6) makes Y_k a binary variable and (7) guarantees non-negativity for X_{ijkm} (Campbell 1994).

Skorin-Kapov et al. (1996) state that Campbell's algorithm is a very large mixed integer problem with $n + n^4$ variables and $1 + n^2 + 2n^4$ constraints. Consequently, they did some experiments with this algorithm. They found out that the lack of fixed costs lead to lots of partial hubs when relaxing the integrality of the Y variables. They proposed a new formulation, where (4) and (5) are replaced by their aggregate forms. Compared with Campbell's (1994) formulation, they argue to having reduced the constraints of the model by $2n^3(n\text{-}1)$.

Skorin-Kapov et al. (1996)

$$\min\ (1)$$

$$\text{s.t.}\quad (2), (3), (6), (7)$$

$$\text{and}\quad \sum_{m} X_{ijkm} \leq Y_k \ \forall i,j,k \in V\ , \tag{8}$$

$$\sum_{k} X_{ijkm} \leq Y_m \ \forall i,j,m \in V \tag{9}$$

The new constraints not only guarantee that the flow is routed through hubs and reduce the number of linear constraints, but also give a better lower bound as the previous formulation since constraints (8) and (9) imply constraints (4) and (5) even when (6) is not present (Marín et al. 2006).

2.2 Progress in research

The following part shows the progress in research on the UMApHMP, including the most important literature of the past years.

Ernst and Krishnamoorthy (1998) made further progress in the reduction of variables and constraints. They presented a very different and more efficient formulation compared to Campbell (1994) and Skorin-Kapov et al. (1996). However, the problem still remained hard to solve and so Ernst and Krishnamoorthy (1998) presented an LP based branch-and-bound method. They also developed an algorithm based on the shortest path problem that significantly outperformed their previous model. It runs 500 times faster and needs less memory. With this model they were able to obtain exact solutions for large problems (n = 200, p = 3 in approx. 632 seconds) that none else could solve at that time (Alumur and Kara 2008).

Boland et al. (2004) developed preprocessing techniques and tightening constraints to improve the computational times and reduce memory. While applying their model on the multiple allocation p-hub median problem they significantly improved some results. However, they conclude that their model is still not capable for practical-sized problems with n = 200+.

With the adaptation of a polyhedral formulation from the uncapacitated facility location problem to the multiple allocation p-hub problem Hamacher et al. (2004) where able to solve also large instances of the HLP.

Recent progress on solving the UMApHMP with an exact method came from García et al. (2012) who presented a new formulation as well as a branch-and-cut algorithm to the problem. They reported to having solved an instance with up to n = 200 while the possible number of hub locations was p = 190. With a further development of pre-processing techniques (similar as presented in section 3.2) and improved formulations for the model inequalities, they are the first to present a formulation that only requires $O(n^2)$ variables also for larger instances.

The most recent work on heuristic approaches for the UMApHMP to be found in the literature is an electromagnetism-like metaheuristic by Kratica (2013). The author states that his approach reaches all instances known in the literature so far with optimal solutions. Moreover, he demonstrates the solution of large-scale instances of n = 1000 with the number of hubs to be placed p = 20.

2.3 Triangle inequality

All models mentioned above have in common that they assume that the distances (costs) between the nodes satisfy the triangle inequality. This means that if we construct a triangle over any three locations in the model, the sum of the lengths of any two sides must be greater than or equal to the length of the remaining side. However, in real-world cases, distances among locations not always satisfy this inequality. For example, in an air transportation network, where many researchers propose the Euclidian distance as basis for calculations, the triangle inequality does not necessarily hold in the real-world. This is due to various take-off and landing procedures, changing weather, less aerodynamic drag in higher altitude and other local restrictions (Marín et al. 2006).

The formulation introduced by Campbell (1994) limits paths to three arcs, hence for a cost matrix that satisfies the triangle inequality the maximum number of hubs is two (Campbell and O'Kelly 2012). However, if a different cost structure is used, the maximum number of intermediate hubs is not restricted by the model and consequently for example four hubs could be traversed at the same route. Marín et al. (2006) therefore propose a formulation which ensures that at most two hubs are traversed at one route, while it is not necessary that the data set meets the requirements of the triangle inequality:

Marín et al. (2006)

min (1)

s.t. (2), (3), (6), (7)

and
$$Y_i + \sum_{k \neq i} \sum_{m} X_{ijkm} \leq 1 \; \forall i, j \neq i \in V \;, \tag{10}$$

$$Y_j + \sum_{k \neq j} \sum_{m} X_{ijmk} \leq 1 \; \forall i, j \neq i \in V \;, \tag{11}$$

$$Y_i + \sum_{(k,m) \neq (i,i)} \sum_{s} X_{iikm} \leq 1 \; \forall i \in V \tag{12}$$

Constraints (10) – (12) are the improved formulation for (8) and (9) and still insure that the flow is routed through hubs. Moreover, these constraints insure that at most two hubs are traversed on a route and if a hub is an origin or a destination itself, the routing is either through only one more hub or directly to the respective origin or destination. An adaption of the model by Marín et al. (2006) is the basis for the formulation in this work, which will be presented in the next section.

Note that Marín et al. (2006) state that if the hubs are not capacitated, X_{ijkm} can be considered as binary variables. So, constraint (7) is adapted accordingly:

$$X_{ijkm} \in \{0,1\} \; \forall i, j, k, m \in V \tag{13}$$

3 Mathematical Formulation

Based on the literature review, this section presents the mathematical formulation used in this thesis in order to compute the case study.

3.1 *Basic formulation*

Campbell and O'Kelly (2012) recognize the model of Marín et al. (2006) as "efficient formulation" and propose a further simplification of the model:

Campbell and O'Kelly (2012)

min (1)

s.t. (2), (3), (6), (13)

and $$X_{ijkk} + \sum_{m \neq k} \left(X_{ijkm} + X_{ijmk} \right) \leq Y_k \ \forall i, j, k \in V \quad (14)$$

Constraint (14) is a tighter formulation for (10) – (12) stated by Campbell and O'Kelly (2012).

3.2 *Size reduction*

Marín et al. (2006) propose a pre-processing method in order to reduce the size of the linear program (LP). They elaborated an idea of Hamacher et al. (2004), which states that it is useless to consider variables in the model that have costs that are not competitive. The goal is to compare the costs of one route C_{ijkm} with the costs of alternative routings (C_{ijmk}, C_{ijkk}, C_{ijmm}) and to determine the least cost route already in the pre-selection, such that the more expensive routes will not be processed in the LP. Therefore, for each i, j, k, m a preselected cost $\hat{C}_{ijkm} = min\{C_{ijkm}, C_{ijmk}, C_{ijkk}, C_{ijmm}\}$ is introduced. And the model is expanded with:

$$X_{ijkm} = 0 \ if \ C_{ijkm} > \hat{C}_{ijkm} \quad (15)$$

3.3 *Considering the subsidies*

To be able to include the public subsidies into the model, a new variable is introduced. The subsidy S_{ij} reflects the amount of money the airlines receives per passenger traveling from *i* to *j* on a subsidized route.

S_{ij} is be part of the cost function stated in section 3.4.2.

3.4 *Final formulation*

This section provides an overview of all used sets, input parameters and decision variables as well as the formulation to summarize the information given above. If not else stated, all notations $1..n$.

3.4.1 Notations

The following tables show an overview of the notations used in the model.

Abbreviation	**Description**
V	Set of origins, destinations and potential hub locations (nodes or vertices)
A	Set of connections between origins and destinations (links or arcs)

Table 1: Notations for used sets

Abbreviation	**Description**
$\mathbf{X_{ijkm}} \in [\mathbf{0}, \mathbf{1}]$	Fraction of flow from *i* to *j* via *k* and *m*
$\boldsymbol{Y_k} \in [\mathbf{0}, \mathbf{1}]$	Indicator if a hub is opened at *k* (1)

Table 2: Notations for used decision variables

Abbreviation	Description
$\boldsymbol{i, j, k, m} \in \mathbf{V}$	Indices used to describe the routing
$\boldsymbol{n} \in \mathbf{V}$	Number of airports
$\boldsymbol{W_{ij}} \in \boldsymbol{V}$	Number of passengers traveling from *i* to *j*
$\boldsymbol{C_{ijkm}} \in \boldsymbol{V}$	Cost for traveling from *i* to *j* via *k* and *m*
$\boldsymbol{\alpha}$	Discount factor for inter-hub connection
$\boldsymbol{S_{ij}} \in \boldsymbol{V}$	Amount of subsidy paid per passenger from *i* to *j*
$\boldsymbol{T_{ij}} \in \boldsymbol{V}$	Flight-time between *i* and *j*
var	Variable cost per flight minute
p	Number of hubs to be located

Table 3: Notations for used (input) parameters

3.4.2 Cost definitions

Model with subsidized routes:

$$C_{ijkm} = T_{ik}var + \alpha T_{km}var + T_{mj}var - S_{ij}$$

Model without subsidized routes:

$$C_{ijkm} = T_{ik}var + \alpha T_{km}var + T_{mj}var$$

In both cases, we pre-process the cost matrix by eliminating routings that are not cost competitive:

$$\hat{C}_{ijkm} = min\{C_{ijkm}, C_{ijmk}, C_{ijkk}, C_{ijmm}\}$$

3.4.3 Problem formulation

Marín et al. (2006), adapted by Campbell and O'Kelly (2012)

$$\min \sum_{i,j,k,m} W_{ij} C_{ijkm} X_{ijkm} \tag{1}$$

s.t.

$$\sum_{k \in V} Y_k = p \quad , \tag{2}$$

$$\sum_{k,m} X_{ijkm} = 1 \ \forall i,j \in V \ , \tag{3}$$

$$X_{ijkk} + \sum_{m \neq k} \left(X_{ijkm} + X_{ijmk} \right) \leq Y_k \ \forall i,j,k \in V \ , \tag{14}$$

$$X_{ijkm} = 0 \ if \ C_{ijkm} > \hat{C}_{ijkm} \ \forall i,j,k,m \in V, \tag{15}$$

$$Y_k \ \in \{0,1\} \ \forall k \in V \ , \tag{6}$$

$$X_{ijkm} \in \{0,1\} \ \forall i,j,k,m \in V \tag{13}$$

3.4.4 Solution

In this thesis the focus lies on finding an exact solution to the problem using integer programming. In section 1.2.4 we saw that the HLP in the 4-indexed formulation is very hard to compute, however Marín et al. (2006) reported to having solved their model with instances of 30 locations. Lately, researchers solved the algorithm with instances of up to 40 locations (Farahani et al. 2013). This number is sufficient to solve the case study presented below.

4 Case study

The following section provides the information on the case study used in this thesis. The first part focuses on the domestic Norwegian airline market and its network design, the remainder of the section describes the scheme for subsidizing routes in remote areas.

4.1 The domestic airline market in Norway

Norway's unique domestic airline market is mainly caused by its topographical structure: In a country with a size of about 385 thousand square kilometers two thirds of its 5 million inhabitants have access to an airport within one hour. That makes 52 airports that are served by a commercial operator. Since the capital city, Oslo, has more than double the inhabitants than any other Norwegian city, the air traffic is mainly concentrated on the city's main airport located in Gardermoen (OSL). From OSL other domestic airports are serviced in a hub-and-spoke manner: Regional capitals like Stavanger (SVG), Bergen (BGO), Trondheim (TRD), Bodø (BOO), Tromsø (TOS) and Kirkenes (KKN) are mainly connected with mid-size aircraft (B737 or similar; 120+ seats) and the routes function as a back-bone. Many of the small regional airports are then connected to a regional capital or to OSL with small aircraft (DH8 or similar; 39+ seats). The air transportation system plays a major role for the countries' economic strength and allows maintaining the decentralized settlements (Bråthen et al. 2012: 17ff).

In 1994 the Norwegian airline market was deregulated, which started a price war on airfares on popular routes and resulted in consolidation of two existing airlines and the bankruptcy of a new entrant (Lian 2009). The entrance of Norwegian Air Shuttle in 2002 formed the market situation as it is today: SAS and Norwegian Air Shuttle operate a dense network between Oslo and the regional capitals, as well as between the regional capitals with frequent departures. The market share of SAS is between 50 and 60 percent on most routes, however Norwegian Air Shuttle has more than 50 percent market share on some routes like from Oslo to Molde (MOL) or from Oslo to Harstad/Narvik (EVE). Both carriers also operate international flights mainly through the hub in OSL, but there are also some international routes served from the regional capitals (Bråthen 2012: 17ff).

The majority of the Norwegian airports are small regional airports with services to a close regional capital or to OSL. These airports are also restricted by aircraft size, since many of them only have a short take-off and landing (STOL) runway. The majority of the small regional airports are serviced by the dense network of Widerøe's Flyveselskap ASA (short: Widerøe). Danish Air Transport (DAT) operates one domestic route from OSL to Stord (SRP), Air Norway has services from OSL to Fagernes (VDB) and Lufttransport AS operates one helicopter route between Bodø (BOO) and Værøy (VRY) (Mathisen and Solvoll 2012; DAT 2015; Air Norway 2015).

Most of the citizens in the remote areas have to take one – or more – connecting flights to reach the capital, or another destination in Norway. Due to the low demand and the STOL runway restrictions, only small aircraft are in use on regional routes. The size of the country lead to the creation of multiple airport hubs, where the regional routes have connections to regional and major routes and vice versa. Consequently, we find the highest share of domestic network traffic in the remote areas of Northern Norway. The highest share of domestic direct connections can be found in the South and along the West coast, where we also have the highest population density (Lian 2010).

Only 17 out of 51 airports meet the requirements to serve as hub airport in terms of runway length and terminal infrastructure. These are marked with "Hub" in Table 4.

IATA	Airport	City served	County	Type	RWY (m)	Remarks[1]
AES	Ålesund Airport, Vigra	Ålesund	Møre og Romsdal	Primary	2,314	Hub
ALF	Alta Airport	Alta	Finnmark	Primary	2,087	Hub
ANX	Andøya Airport, Andenes	Andenes	Nordland	Joint	2,468	Hub
BDU	Bardufoss Airport	Bardufoss	Troms	Joint	2,443	Del:Net
BGO	Bergen Airport, Flesland	Bergen	Hordaland	Joint	2,990	Hub
BJF	Båtsfjord Airport	Båtsfjord	Finnmark	Regional	1,000	
BNN	Brønnøysund Airport, Brønnøy	Brønnøysund	Nordland	Regional	1,199	
BOO	Bodø Airport	Bodø	Nordland	Joint	3,394	Hub
BVG	Berlevåg Airport	Berlevåg	Finnmark	Regional	919	Del:Pax
EVE	Harstad/Narvik Airport, Evenes	Harstad/Narvik	Nordland	Primary	2,815	Hub
FDE	Førde Airport, Bringeland	Førde	Sogn og Fjordane	Regional	940	
FRO	Florø Airport	Florø	Sogn og Fjordane	Regional	1,199	
HAA	Hasvik Airport	Hasvik	Finnmark	Regional	970	
HAU	Haugesund Airport, Karmøy	Haugesund	Rogaland	Primary	2,120	Hub
HFT	Hammerfest Airport	Hammerfest	Finnmark	Regional	882	
HOV	Ørsta–Volda Airport, Hovden	Ørsta/Volda	Møre og Romsdal	Regional	866	
HVG	Honningsvåg Airport, Valan	Honningsvåg	Finnmark	Regional	800	
KKN	Kirkenes Airport, Høybuktmoen	Kirkenes	Finnmark	Primary	1,905	Hub
KRS	Kristiansand Airport, Kjevik	Kristiansand	Vest-Agder	Joint	1,990	Hub
KSU	Kristiansund Airport, Kvernberget	Kristiansund	Møre og Romsdal	Primary	1,84	Hub
LKL	Lakselv Airport, Banak	Lakselv	Finnmark	Joint	2,784	Hub
LKN	Leknes Airport	Leknes	Nordland	Regional	878	
LYR	Svalbard Airport, Longyear	Longyearbyen	Svalbard	Primary	2,323	Del:Net
MEH	Mehamn Airport	Mehamn	Finnmark	Regional	880	
MJF	Mosjøen Airport, Kjærstad	Mosjøen	Nordland	Regional	919	
MOL	Molde Airport, Årø	Molde	Møre og Romsdal	Primary	1,980	Hub
MQN	Mo i Rana Airport, Røssvoll	Mo i Rana	Nordland	Regional	841	
NTB	Notodden Airport, Tuven	Notodden	Telemark	Regional	1,393	Del:Net
NVK	Narvik Airport, Framnes	Narvik	Nordland	Regional	909	
OLA	Ørland Airport	Brekstad	Sør-Trøndelag	Joint	2,714	Del:Net
OSL	Oslo Airport, Gardermoen	Oslo	Akershus	Joint	3,600	Hub
OSY	Namsos Airport, Høknesøra	Namsos	Nord-Trøndelag	Regional	838	
RET	Røst Airport	Røst	Nordland	Regional	880	Del:Pax
RRS	Røros Airport	Røros	Sør-Trøndelag	Regional	1,720	
RVK	Rørvik Airport, Ryum	Rørvik	Nord-Trøndelag	Regional	880	
RYG	Moss Airport, Rygge	Moss	Østfold	Joint	2,900	Del:Net
SDN	Sandane Airport, Anda	Sandane	Sogn og Fjordane	Regional	840	Del:Pax
SKE	Skien Airport, Geiteryggen	Skien	Telemark	Regional	1,400	
SKN	Stokmarknes Airport, Skagen	Stokmarknes	Nordland	Regional	886	
SOG	Sogndal Airport, Haukåsen	Sogndal	Sogn og Fjordane	Regional	943	
SOJ	Sørkjosen Airport	Sørkjosen	Troms	Regional	919	
SRP	Stord Airport, Sørstokken	Leirvik	Hordaland	Regional	1,460	Del:Net
SSJ	Sandnessjøen Airport, Stokka	Sandnessjøen	Nordland	Regional	1,086	
SVG	Stavanger Airport, Sola	Stavanger	Rogaland	Joint	2,556	Hub
SVJ	Svolvær Airport, Helle	Svolvær	Nordland	Regional	857	
TOS	Tromsø Airport	Tromsø	Troms	Primary	2,392	Hub
TRD	Trondheim Airport, Værnes	Trondheim	Nord-Trøndelag	Joint	2,759	Hub
TRF	Sandefjord Airport, Torp	Sandefjord	Vestfold	Primary	2,950	Hub
VAW	Vardø Airport, Svartnes	Vardø	Finnmark	Regional	1,130	Del:Pax
VDB	Fagernes Airport, Leirin	Fagernes	Oppland	Regional	2,060	Del:Pax
VDS	Vadsø Airport	Vadsø	Finnmark	Regional	877	

Table 4: List of airports in Norway[2]

The Norwegian air network pattern also determines the fare structure. On the main routes – or commercial routes – market forces decide fares. In fact, average fares on routes from Oslo to the northern capitals Bodø and Tromsø declined 15-20 percent when Norwegian

[1] Hub = possible hub location; Del:Net = deleted from model because not part of the regular network; Del:Pax = deleted from model because number of yearly passengers < 10,000
[2] Source: Wikipedia 2016; own edit

Air Shuttle entered the market in 2002. Full-flex business fares even declined by 25-30 percent on these routes. In contrast, many of the regional routes have to be subsidized by the state because they are less likely to be operated profitably by a private company. These routes are called Public Service Obligation (PSO) routes. The Norwegian Ministry of Transport sets the fares on PSO-routes. In 2006 the maximum fares on these routes where reduced by 20 percent by the ministry (Lian 2010).

4.2 PSO routes

In the early 1990s the air transport in the European Union (EU) has been liberalized. To guarantee further air services to small and remote communities that would not be served in a liberalized market due to the lack of profitability, the EU introduced the scheme of Public Service Obligation (PSO) routes. The current legal framework can be found in Articles 16, 17, 18 of Regulation (EC) No 1008/2008. Under these regulations each EU member state can offer a PSO on a route and can award financial compensations if necessary (Santana 2009). Provided that a route serves a peripheral airport or a development region, each member state can outline details like frequency, capacity, maximum travel time or maximum number of connecting flights in a tender. Often maximum ticket prices and special fares for some interest groups (children, students, military, etc.) are outlined in the tender. Furthermore, the tender most likely includes a guarantee that the operator has to serve the route for a given period. The member state may also limit the access to a single carrier for a given period, if there is little interest to bid on this route (Pita et al. 2013).

Beside eight EU countries also the European Economic Area (EEA) members Norway and Iceland offer PSO routes. There were about 260 PSO routes registered in 2010, with Norway and France having the most routes. However, those two countries have the most different route structure in Europe - with average legs of about 600 km in France and 200 km in Norway. Germany offers the highest subsidy level with EUR 120 per passenger, followed by Norway, Sweden and Scotland with about EUR 60. France and Portugal subsidize the routes with only about EUR 20. Although PSO routes can be offered on both, domestic services and services between member states, 90 percent of all PSO routes count for the first (Bråthen 2011).

In 1997 the first Norwegian PSO mechanism was introduced by the Ministry of Transport and Communications (Det Kongelige Samferdselsdepartement) tendering routes for the period of three or four years. In the first round all affected routes where tendered in one single bid. However, in the following bidding rounds (2000, 2003, 2006, 2009, 2012) the routes were divided in up to 22 bundles (plus one helicopter route) reflecting geographical the conditions. This was done in order to enhance the bidding competition for the routes. (Santana 2009 and DKS 2014).

Bråthen (2011) states that PSO routes are often charged with too high prices due to the lack of competitors. In countries like Norway, where there is only one airline serving all of the PSO routes (exempt of one subsidized helicopter route served by Lufttransport AS) the operating airline has in fact the power to set the prices (DKS 2014). Evidence is that the PSO subsidies in Norway are constantly rising, while the costs of airlines in the commercial market are decreasing (Bubalo 2012). The reason for the low amount of bidders for PSO routes is partly due to technical restrictions, like availability of aircraft that are permitted to operate on STOL runways or operation in remote places with extreme weather conditions. This combination leads to high entry barriers for new airlines. Mathisen and Solvoll (2012) suggest considering the closure of some airports by simultaneously building up the road network to alternative airports and extending the runways and capacities at these alternatives. This would make it possible for more airlines to bid for PSO routes and enhances competition.

Bubalo (2012) raises even more critique on the Norwegian PSO system. He argues that the real costs of air transport in the remote areas of Norway are hidden behind a system of cross-subsidizing the loss making small airports with the profit of the bigger ones by the government (that is operating most of the airports in Norway through the wholly-owned company AVINOR). Furthermore, the government is paying a large sum to compensate the PSO routes, about NOK 690 million in 2011. Due to the lack of competitors Widerøe can in fact set the prices. Bubalo further argues that it is most likely that Widerøe cross-subsidizes its commercial routes with subsidies received for the PSO routes. In a research on social costs of PSO routes in Northern Norway he calls the state-financed air transport in North-Troms and Finnmark a “social luxury”.

4.3 PSO restrictions and subsidies

The Norwegian Ministry of Transport and Communications sets distinctive requirements in their tender for PSO routes. Depending on the route, the number of seats provided per (working) day, the maximum number of stops and/or plane changes, the minimum frequency of each route and even the time that the first and last connection of the day has to reach a certain airport is stated in the tender. Moreover, the maximum price for a fully flexible one-way ticket is set by the government. The state compensates the operator meeting these requirements with certain amount of money, paid yearly per route or bundle (DKS 2014).

There is no fixed amount of subsidy paid to the flight operator per PSO-route by the Norwegian Ministry of Transport and Communications. In fact, the airline can set the amount of subsidy it needs to operate a bundle of routes when bidding for a tender. Most PSO-routes are either tendered in a bundle or get merged to a bundle by the bidding airline. Therefore, the real amount of subsidy needed for each route is not shown in any official statistics. Some researchers tried to calculate the amount of subsidy per PSO-route during evaluations of the bidding process (Bubalo 2012, Lian et al. 2010, Bråthen et al. 2015). The following list provides an overview of the subsidies paid per passenger on each PSO-route during the period 2009-2012:

From	To (v.v.)	Geography	Subsidy p. Pax (USD)
Lakselv	Tromsø	Outermost territory	97
Andenes	Bodø	Outermost territory	277
Andenes	Tromsø	Outermost territory	36
Harstad/Narvik	Tromsø	Outermost territory	102
Svolvær	Bodø	Outermost territory	72
Leknes	Bodø	Outermost territory	72
Røst	Bodø	Island	105
Narvik	Bodø	Outermost territory	102
Brønnøysund	Bodø	Outermost territory	85
Brønnøysund	Trondheim	Outermost territory	85
Sandnessjøen	Bodø	Outermost territory	85
Sandnessjøen	Trondheim	Outermost territory	85
Mo i Rana	Bodø	Outermost territory	84
Mo i Rana	Trondheim	Outermost territory	84
Mosjøen	Bodø	Outermost territory	84
Mosjøen	Trondheim	Outermost territory	84
Namsos	Trondheim	Outermost territory	103
Rørvik	Trondheim	Outermost territory	94
Florø	Oslo	Mainland	29
Florø	Bergen	Mainland	29
Førde	Oslo	Mainland	37
Førde	Bergen	Mainland	37
Sogndal	Oslo	Mainland	117
Sogndal	Bergen	Mainland	117
Sandane	Oslo	Mainland	146
Sandane	Bergen	Mainland	146
Ørsta-Volda	Oslo	Mainland	69
Ørsta-Volda	Bergen	Mainland	69

Røros	Oslo	Mainland	155
Alta	Kirkenes	Outermost territory	456
Hammerfest	Vadsø	Outermost territory	456
Hammerfest	Kirkenes	Outermost territory	399
Vadsø	Kirkenes	Outermost territory	46
Vadsø	Alta	Outermost territory	453
Vardø	Kirkenes	Outermost territory	136
Båtsfjord	Kirkenes	Outermost territory	151
Båtsfjord	Vadsø	Outermost territory	94
Båtsfjord	Hammerfest	Outermost territory	421
Berlevåg	Kirkenes	Outermost territory	217
Berlevåg	Vadsø	Outermost territory	162
Berlevåg	Hammerfest	Outermost territory	391
Mehamn	Kirkenes	Outermost territory	316
Mehamn	Vadsø	Outermost territory	234
Mehamn	Hammerfest	Outermost territory	266
Honningsvåg	Hammerfest	Outermost territory	123
Honningsvåg	Vadsø	Outermost territory	456
Hasvik	Tromsø	Outermost territory	245
Hasvik	Hammerfest	Outermost territory	104
Sørkjosen	Tromsø	Outermost territory	122

Table 5: PSO-routes and subsidies paid per passenger[3]

4.4 The regional airline Widerøe

Widerøe Flyveselskap AS was founded in 1934. Initially it operated air taxi, ambulance, school transport services and aerial photo flights from its bases at Ingierstrand, just outside Oslo. Over the time Widerøe developed to an important pillar of the Norwegian transportation system. It is the biggest regional airline in Scandinavia today, operating over 400 flights each day. Widerøe serves 43 domestic airports in Norway and four international destinations, carrying more about 3 million passengers a year. The company headquarters is located in Bodø, while the airline operates a big administrative unit in Oslo. Widerøe has about 3,000 employees and an operating revenue of NOK 3.8 bn (2014). About 40 percent of the routes served by Widerøe are PSO routes, the other routes are operated commercially (Widerøe 2015).

4.4.1 Hubs

Widerøe operates five hub-airports across Norway. Thereby a hub not only functions as a connection point for passengers. Due to the high frequency and number of routes at hub-

[3] Source: Bubalo (2012), Lian et al. (2010), Bråthen et al. (2015); Calculated with average 2012 NOK/USD exchange rate (5.817596)

airports it is logical that they also function as base for crew and aircraft. From Oslo-Gardermoen (OSL) Widerøe has services to regional airports in Western Norway. Bergen-Flesland (BGO) serves as hub for “Fjord-Norway”. From Bodø Airport (BOO) the airline operates services to Helgeland, Lofoten Islands and Vesterålen. The basis for operations in Northern Norway is Tromsø Airport (TOS). A specialty in Widerøe’s network is its basis in Sandefjord-Torp (TRF). This airport mainly serves as a base for point-to-point connections to regional capitals in Western Norway rather than being a classical hub airport (Widerøe 2015; Torp 2015). The following illustration shows Widerøe’s route network and the respective hub airports (square). Note that the route map also includes some seasonal routes, stop-over connections, as well as international flights.

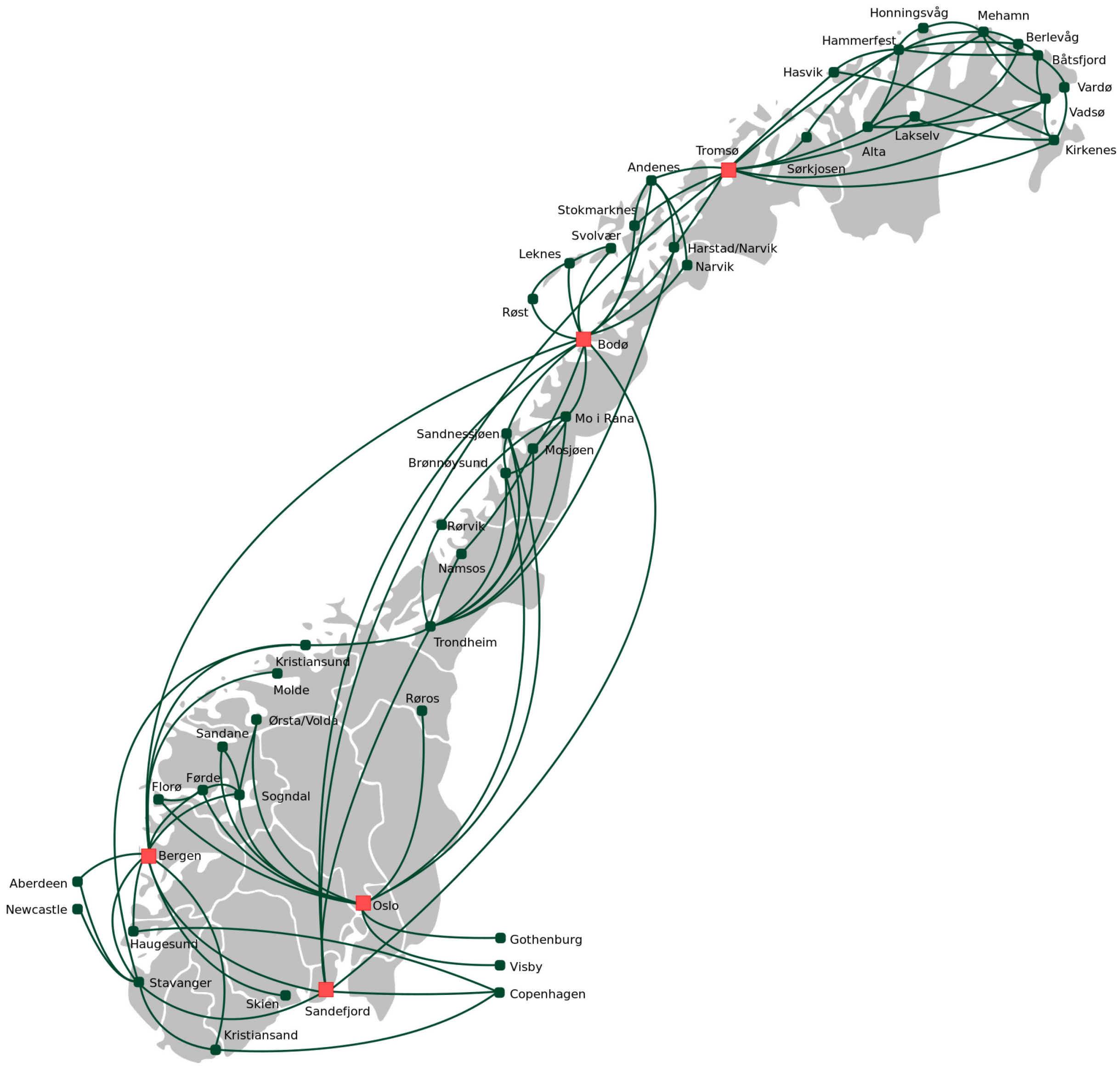

Figure 4: Widerøe's route network and hub airports[4]

4.4.2 Fleet

Widerøe operates a fleet of 41 aircraft that entirely consists of (de Havilland) Bombardier Dash-8 aircraft. Thereby the DH1/DH2-series (23 aircraft) is the smallest version with space for 39 passengers. The DH3-series (8 aircraft) has 50 seats while the DH4 (11

[4] Source: Wikimedia (https://upload.wikimedia.org/wikipedia/commons/1/14/Wideroe_routes.svg); own edit

aircraft) accommodates 78 passengers. The average fleet age is 17.3 years (Widerøe 2015).

4.4.3 Interline-agreement with SAS

Being part of SAS Group for several years, Widerøe ever had a close relation to SAS and also operated some services on behalf SAS in the past. Moreover, Widerøe serves as feeder for domestic SAS flights between regional capitals and OSL. However, in 2013 SAS decided to sell 80 percent of its Widerøe shares to a Norwegian investment group, consisting of the transportation companies Torghatten ASA and Fjord1 AS as well as the Nordland County. SAS plans to divest the remaining shares in 2016 (SAS 2013).

Despite the separation from SAS, Widerøe still holds an interline-agreement with SAS. This allows the passenger to book connecting flights with both companies on one ticket. That agreement builds a symbiotic relationship between both companies in the Norwegian air travel network, since many passengers have to use both companies to travel from remote areas to regional capitals or to Oslo. In comparison to that, passengers wishing to travel on a connecting service with Norwegian Air Shuttle have to buy a separate ticket for each flight section operated by another company (Widerøe 2014).

As described in section 4.1, SAS and Norwegian Air Shuttle operate flights to the regional capitals out of their hub at Oslo-Gardermoen (OSL) with B737 aircraft. Since some of these regional capitals serve as hubs for Widerøe's operations, these flights form the inter-hub connections of the network as described in the model. Despite the fact that Norwegian Air Shuttle has no interline-agreement with both, SAS and Widerøe, the whole domestic passenger volume (including all three airlines) is taken into account in the model. This is because it is possible that passengers buy connecting flights on separate tickets.

5 Computational experiments

For the computation of the model the software FICO® Xpress Optimization Suite was used. Moreover, a computer equipped with an Intel i5 3.10 GHz CPU and 8GB RAM was in use. All input data sets, as well as the Xpress output data sets are digitally available.

First computation experiments with the algorithm showed that the originally intended number of 45 airports could not be computed. The 4-index formulation used in the algorithm created too much variables such that the computer ran out of memory. Since the literature indicated that the algorithm works with up to 40 instances, it was decided to remove five airports from the data set. As indicated in Table 4 ("Del:Pax"), airports that do not reach at least 10.000 passengers a year were removed. Since the whole model includes more than 25 million passengers traveling in the network, the number of 10.000 passengers only accounts for 0.04 percent of the yearly traffic and therefore these airports are negligible for the model. The airports deleted are: Fagernes (VDB), Sandane (SDN), Røst (RET), Vardø (VAW) and Berlevåg (BVG).

Other airports were removed from the data set beforehand: Bardufoss (BDU), Notodden (NTB), Ørland (OLA) and Stord (SRP) due to the lack of traffic data. Moss (RYG, Rygge) because only international flights are operated from there and Svalbard (LYR, Longyearbyen) because the location is too far away from the mainland.

5.1 Data Sets

The following section describes the data sets used for the model.

5.1.1 OD-matrix

The passenger data indicating domestic air traffic of all OD-pairs in Norway is the basis for the calculations in this work. However, there is no official statistics that includes all the data from the Norwegian domestic air network. A request directed to Widerøe in order to retrieve their passenger data was declined with reference to company secrets and possible misuse of the data by competitors.

Not only the lack of data from the airline, but also other considerations make it more vulnerable for this work to use a self-composed data set: First, the company data set of Widerøe would give us an insight only into this specific company. However, we have to consider also the hub airports in terms of the passengers connecting to SAS flights due to the interline-agreement. Moreover, it makes sense to include data from the whole market, which includes also the passengers traveling with Norwegian Air Shuttle. The reason is that people still might travel via a hub to an onward destination booked on separate tickets. Since this might include two separate companies, we cannot rely on company data only to calculate hub locations for the whole market.

The passenger data set is composed out of different sources:

Primary data

This primary data comes from official tenders for PSO routes by the Norwegian Ministry of Transportation and Communication[5]. The tenders include passenger data of all OD-pairs on the respective PSO-routes from April 2010 to March 2011. The second source of primary data is the Eurostat statistics[6] on traffic on the most important routes in Norway. This statistics delivers good data also for some regional routes. However, it does not include data from transfer passengers traveling through a hub to reach their final destinations.

Secondary data

To fill the OD-Matrix also with the information on connecting passengers it is necessary to use secondary data. First we determine the number of transfer passenger at the current hub location. This can be done with the monthly statistics of AVINOR[7], the state-owned operator of most airports in Norway. Second, we determine the share of passenger traveling from the hub to other destinations.

For example, we want to determine the number of passengers traveling from Alta (ALF) to Bergen (BGO). There is no direct flight connection, so people have to use a connection flight via a hub airport. On this route, the connection via Oslo (OSL) is most convenient. From Eurostat we know the number of passengers traveling from ALF to OSL (79543 passengers). Since 45 percent of all passengers traveling to OSL have a connection flight and 17 percent of all passengers at OSL are traveling to BGO, we multiply both with the number of passenger traveling from ALF to OSL (79543 * 0,45 * 0,17 = 7462). This makes a demand of 7462 passengers on route ALF-BGO.

---

[5] https://www.regjeringen.no/en/dokumenter/invitation-to-tender-scheduled-regiona-2/id705230/
[6] http://ec.europa.eu/eurostat/web/transport/data/database
[7] https://avinor.no/en/corporate/about-us/statistics/archive

However, there are some routes that are fairly important (since Widerøe operates several daily flights on these routes) that cannot be found in any of the public statistics. Here the calculation of demand works as follows:

The number of seats offered per year is multiplied by the load factor (average seats sold with respect to the offered seats; SAS 2012). This is done for the routes ANX-EVE v.v., TOS-EVE v.v., BGO-HAU v.v., BGO-SKE v.v. and BGO-TRF v.v.

5.1.2 Distance matrix

As mentioned above, many earlier models use a matrix based on the Euclidean distance between the origins and destinations. This, however, is not the best way to determine the distance that makes the most sense for airline operations. Especially when it comes to short distances between origins and destinations – such as in our case – other factors of the flight have a significant influence in the time needed for a flight: Many airports have certain take-off and landing procedures due to their exposed topographical location (e.g. small islands, fjords and mountains). Weather, especially wind conditions and visibility influences the flight as well. This factors influence the flight time between two points, where, however, the Euclidean distance stays the same.

This all is evidence to rather use the flight time between the OD-pairs then using the Euclidian distance. The online-tool Flight time Calculator (2015) was used to fill the distance matrix. The Flight time calculator tries to get as close as possible to the real conditions. Since flight time needed for one nautical mile gets lower at longer total distances (e.g. faster aircraft assumed, higher altitude causes lower aerodynamic drag, etc.) the flight times in this matrix are not linear. Furthermore, time needed for take-off and landing is taken into account by the calculator.

5.1.3 Cost factor

Conklin & de Decker (2015), an airline consulter, provides the data for the calculation of the cost matrix. There, the weighted average variable cost per seat for one flight hour for Widerøe's aircraft fleet is used to determine the cost factor. Thereby, the variable costs include fuel, airframe maintenance, labor and parts, engine restoration and miscellaneous costs.

The weighted average cost per seat per flight hour will be multiplied by the passengers traveling on a distinctive route. This calculation method implies that every operated flight is fully booked. However, Widerøe reported a load factor of only 58.7 percent (SAS 2012). In order to compensate the empty seats in the model we equally distribute the costs of the seats that are available on each flight over the booked seats. This means that the passengers traveling have to cover the costs of the empty seats. Therefore, we increase the cost of every booked seat by the factor of the empty seats (1 – 0.587). The table below shows the calculation:

A/C	# of A/C	# of seats	VC/FH	VCS/FH (USD)	VCS/FH Fleet (USD)
DH1/2	23	39	2626	67.33	1548.67
DH3	8	50	2433	48.66	389.28
DH4	11	78	3867	49.58	545.35
SUM	*42*				*2483.29*
		59.13	Weighted avg. cost per seat per flight hour		
		0.413	Factor of empty seats on each flight		
(59.13*1.413)	/ 60	**1.3925**	**Avg. cost per booked seat per flight minute**		

Table 6: Calculation of the weighted average cost per seat (Widerøe)[8]

5.1.4 Discount factor

The discount factor α in this model describes the savings on an inter-hub route compared to a non-hub to hub route. We assume that all inter-hub connections in our case study are serviced by Boeing 737 (B737) aircraft. This aircraft is used by both, SAS and Norwegian Air Shuttle on routes between Oslo and the regional capitals. Due to the interline-agreement between SAS and Widerøe, we assume that most connecting passengers on inter-hub routes use SAS. The basis for the discount factor is the cost per revenue passenger kilometer (CRPK) of each airline. Bubalo (2012) states a CRPK of NOK 4.02 for Widerøe and NOK 1.4 for SAS based on company annual reports.

This makes it possible to set the discount factor $\alpha = \frac{1.40}{4.02} = 0.3483$.

5.2 Results

Although the problem with 40 instances is categorized as mid-sized problem in this area of research and an exact solution without the application of a branch and bound algorithm

[8] A/C = aircraft, VC/FH = variable costs per flight hour, VCS/FH = variable costs per seat per FH, VCS/FH Fleet = total VCS/FH for the fleet; data: https://www.conklindd.com/CDALibrary/ACCostSummary.aspx

was the goal, the model was solved with Xpress within ten seconds. In detail, we can observe that the number of hubs to be located (p) does not have a big effect on the computation time, the model that includes the subsidy, runs approximately one second faster for p=5 and p=6.

Run-times (seconds)	p=4	p=5	p=6
no subsidy	9.6	9.6	9.8
subsidy	9.8	8.7	8.7

Table 7: Run-times of the model in different settings

Xpress used the Newton-Barrier algorithm to solve the problems. It is also known as interior point algorithm that is suitable to solve linear as well as quadratic programs. The algorithm proceeds from some initial interior point in the set of feasible solutions towards an optimal solution without touching the border of the feasible set (FICO 2009: 110).

5.2.1 Hub locations

In the first step of the output analysis we focus on the core task of our algorithm: The location of the hub airports in the network. Since it is possible to identify five airports serving as a hub in the current Norwegian passenger air transportation network, we also run the model computation with the number of five hubs to be located (p=5) first.

Real-world hubs	Model hubs (no subsidy)	Model hubs (with subsidies)
Oslo-Gardermoen (OSL)	Oslo-Gardermoen (OSL)	Oslo-Gardermoen (OSL)
Bergen-Flesland (BGO)	Bergen-Flesland (BGO)	Bergen-Flesland (BGO)
Sandefjord-Torp (TRF)	Trondheim-Værnes (TRD)	Trondheim-Værnes (TRD)
Bodø Airport (BOO)	Bodø Airport (BOO)	Bodø Airport (BOO)
Tromsø Airport (TOS)	Tromsø Airport (TOS)	Tromsø Airport (TOS)

Table 8: Comparison - real-world and model hub locations

In Table 8 we see that four out of five hub locations are identical in real-world and in the model. However, the model locates a hub at Trondheim-Værnes (TRD) rather than at the real-world hub at Sandefjord-Torp (TRF). If we look deeper into the OD-matrix, we can easily see that the model prefers TRD because of the higher volume of passengers (nearly 3 million yearly passengers at TRD compared to approx. 400.000 at TRF). This is evidence that TRF is designated as hub airport because of operational reasons. Looking into the flight schedule, we can see that Widerøe operates frequent flights from TRF to

Bergen, Stavanger and Trondheim (and also to Copenhagen) starting the first services early morning. Therefore, it makes sense that crew and aircraft are based at TRF rather than at TRD. Moreover, most traffic from TRD is routed to Oslo, which also serves as a hub in this case.

When we compare the model calculated with subsidies and the model without subsidies, no change in the proposed hub locations can be observed. This means, that although there are many PSO routes in the North and West of Norway, the impact of the subsidies on the cost structure of the network is not strong enough to change the location of hub airports.

In the following computational experiment, we want to analyze what effect a change in the hub policy - plus and minus one hub - has on the hub locations.

	p=4	**p=4 (S[9])**	**p=5**	**p=5 (S)**	**p=6**	**p=6 (S)**
Hub locations	OSL BGO TRD TOS	OSL BGO TRD TOS	OSL BGO TRD BOO TOS	OSL BGO TRD BOO TOS	OSL SVG BGO TRD BOO TOS	OSL SVG BGO TRD BOO TOS
Total cost (.000 USD)	871570	778232	760140	666801	652955	559616

Table 9: Hub locations with various number of hubs (p)

At a first glance we can see that the location of the hubs does not change in the model with subsidies. If we set the number of hubs to 4 in the model, we lose Bodø Airport (BOO) as hub location. This is quite interesting, since Bodø is actually the main hub of Widerøe and the location of the company's headquarters. Instead, the model locates a hub in Trondheim-Værnes (TRD), which is not a hub location in the real-world.

In case we set the number of hub locations to 6, the model adds Stavanger-Sola (SVG) as hub location. This creates a triangle of 3 hub locations in the South-west of Norway serving approx. one third of the airports included in the model. Again, we see that the model focuses on passenger volume at the airports rather than on reducing the distances to the hubs.

The total cost in the model is declining the more hub locations are added. Since we did not include the fix cost for operating an airport hub into the model, this is a logical behavior of the model (shorter distances to hubs; more discounted inter-hub connections).

---

[9] (S) = model with subsidies

5.2.2 Route allocation

In this part of the analyses we focus on the routes that are allocated to each of the hub airports. Table 10 shows an overview over the route allocation in the model with five hubs, including only hub to non-hub routes. Since the route allocation is identical in both calculations, we can assume that the public subsidies on the PSO routes do not have an impact on the route allocation.

Hub	**OSL**	**TRD**	**BGO**	**BOO**	**TOS**	**Hub**	**OSL**	**TRD**	**BGO**	**BOO**	**TOS**
allocation	AES	AES	AES	ANX	ALF	**allocation**	AES	AES	AES	ANX	ALF
model	FDE	BNN	FDE	BNN	ANX	*model*	FDE	BNN	FDE	BNN	ANX
p=5	HOV	FRO	FRO	EVE	BJF	*p=5*	HOV	FRO	FRO	EVE	BJF
no subsidy	KRS	KSU	HAU	LKN	EVE	*with subsidy*	KRS	KSU	HAU	LKN	EVE
	KSU	MJF	HOV	MJF	HAA		KSU	MJF	HOV	MJF	HAA
	MOL	MOL	KRS	MQN	HFT		MOL	MOL	KRS	MQN	HFT
	RRS	MQN	KSU	NVK	HVG		RRS	MQN	KSU	NVK	HVG
	SOG	OSY	MOL	SKN	KKN		SOG	OSY	MOL	SKN	KKN
	SVG	RVK	SKE	SSJ	LKL		SVG	RVK	SKE	SSJ	LKL
		SSJ	SOG	SVJ	LKN			SSJ	SOG	SVJ	LKN
			SVG		MEH				SVG		MEH
			TRF		SKN				TRF		SKN
					SOJ						SOJ
					SVJ						SVJ
					VDS						VDS

Table 10: 5-hub model route allocation[10]

Figure 5 is an illustration of the model network with five hub airports. Thereby non-hub to hub routes (green lines) as well as inter-hub routes (red lines) are shown. Compared to Widerøe's original route map we can observe the biggest difference for routes out of Tromsø Airport (TOS) towards destinations in Finnmark. In the real-world many of the routes are operated with one or more stopovers. That is due to low demand and proximity of some of the destinations. Furthermore, it makes it easier to travel between two non-hub airports without changing planes at a hub. However, our model is not able to consider stopover flights and so every connection in Finnmark is routed via Tromsø. Moreover, some PSO routes require a certain routing with stopovers and direct flights to some destinations.

[10] Table shows only hub to non-hub routes

Figure 5: Illustration of model network with five hubs[11]

Table 11 shows the real world route allocation according to Widerøe's flight schedule. Since this analysis focuses on the allocation of the non-hub to hub routes, the inter-hub connections are not shown in the table.

[11] Source: Wikimedia (https://upload.wikimedia.org/wikipedia/commons/1/14/Wideroe_routes.svg); own edit

Hub	OSL	TRF	BGO	BOO	TOS
allocation	FDE	BGO	AES	ANX	ALF
Real-world	FRO	HAU	FDE	BNN	ANX
	HOV	SVG	FRO	EVE	BJF*
	RRS	TRD	HAU	LKN	EVE
	SOG		KRS	MJF	HAA
			KSU	MQN	HFT
			MOL	NVK	HVG*
			SKE	SKN	KKN
			SOG	SSJ	LKL
			SVG	SVJ	MEH*
			TRF		SKN
					SOJ
					VDS

Table 11: Real-world route allocation[12]

At the first glance the model shows almost the same route allocation to the hub airports as the real-world case. Compared to the real-world case the model adds Svolvær-Helle (SVJ) as destination to be served out of Tromsø Airport (TOS). At Bodø Airport (BOO) the route allocation is completely identical in both, the model and the real-world. For the routes allocated to the hub Bergen-Flesland (BGO) the model adds a connection to Ørsta–Volda (HOV), which is serviced by Oslo in the real-world case.

At Oslo-Gardermoen (OSL) we experience the biggest impact of the fact, that the model solution and the real-world case has a different location for one of the hubs. The model allocates Kristiansand (KRS), Kristiansund (KSU) and Molde (MOL) to the hub in Oslo, while in the real-world Oslo has an additional service to Florø (FRO).

The real-world hub airport Sandefjord-Torp (TRF) has only services to 4 bigger cities along the west coast, one of them serving as hub as well (BGO). In opposite of that, the 5th hub in the model, Trondheim-Værnes (TRD), takes over a major role in Mid-Norway since it serves as basis to 9 destinations in this area. However, most of the destinations - except Namsos (OSY) and Rørvik (RVK) - are allocated to one of the other hubs at the same time.

[12] Table shows only hub to non-hub routes (except TRF) that are also considered in the model; basis: Widerøe flight schedule
* Airports connected via stopover

Due to the high amount of multiple allocated routes in Mid-Norway, a reduction of hub airports could be an idea to create a more efficient network structure.

Route allocation with four hub airports

We saw in section 5.2.1 that hub location Bodø Airport (BOO) will be closed if we set the number of hub airports to be located to p=4, leaving Oslo-Gardermoen (OSL), Bergen-Flesland (BGO), Trondheim-Værnes (TRD) and Tromsø Airport (TOS) as hub airports. Thus, the route allocation in Mid- and Northern Norway changes severely. Table 12 shows the routes allocated to the four hub airports.

Hub	**OSL**	**BGO**	**TRD**	**TOS**
allocation	AES	AES	AES	ALF
model	FDE	FDE	BNN	ANX
p=4	KRS	FRO	BOO	BJF
no subsidy	KSU	HAU	FRO	BOO
	MOL	HOV	KSU	EVE
	RRS	KRS	LKN	HAA
	SOG	KSU	MJF	HFT
	SVG	MOL	MOL	HVG
		SKE	MQN	KKN
		SOG	OSY	LKL
		SVG	RVK	LKN
		TRF	SSJ	MEH
			SVJ	MQN
				SKN
				SOJ
				SSJ
				SVJ
				VDS

Table 12: 4-hub model route allocation[13]

We can observe that Tromsø Airports (TOS) now takes over hub responsibility not only for Troms and Finnmark, but also for airports located in Nordland, including those at Lofoten Islands, Vesterålen and even some parts of Helgeland. Also Trondheim-Værnes (TRD) increases its importance with 13 allocated routes to destinations in Nordland (parts of Helgeland), Nord-Trøndelag and Møre og Romsdal. Bergen-Flesland (BGO) and Oslo-Gardermoen (OSL) slightly increase their route network in South- and Western Norway in the model with four hub locations.

[13] Table shows only hub to non-hub routes

Figure 6: Illustration of model network with four hubs[14]

5.2.3 Cross-subsidizing

As stated in section 4.3, some researchers think that the Norwegian PSO pattern leads to cross-subsidizing of commercial routes with the money paid by the government for the operation of PSO routes.

---

[14] Source: Wikimedia (https://upload.wikimedia.org/wikipedia/commons/1/14/Wideroe_routes.svg); own edit

From	To (v.v.)	Geography	Cost p. Pax (USD)
Lakselv	Tromsø	Outermost territory	-35.73
Andenes	Bodø	Outermost territory	-215.73
Andenes	Tromsø	Outermost territory	11.35
Harstad/Narvik	Tromsø	Outermost territory	-49.09
Svolvær	Bodø	Outermost territory	-26.05
Leknes	Bodø	Outermost territory	-26.05
Røst	Bodø	Island	*
Narvik	Bodø	Outermost territory	-48.30
Brønnøysund	Bodø	Outermost territory	-25.12
Brønnøysund	Trondheim	Outermost territory	-23.73
Sandnessjøen	Bodø	Outermost territory	-32.09
Sandnessjøen	Trondheim	Outermost territory	-16.77
Mo i Rana	Bodø	Outermost territory	-38.05
Mo i Rana	Trondheim	Outermost territory	-7.41
Mosjøen	Bodø	Outermost territory	-29.69
Mosjøen	Trondheim	Outermost territory	-17.16
Namsos	Trondheim	Outermost territory	-55.66
Rørvik	Trondheim	Outermost territory	-42.48
Florø	Oslo	Mainland	21.13
Florø	Bergen	Mainland	46.35
Førde	Oslo	Mainland	35.41
Førde	Bergen	Mainland	11.74
Sogndal	Oslo	Mainland	-54.34
Sogndal	Bergen	Mainland	-66.87
Sandane	Oslo	Mainland	*
Sandane	Bergen	Mainland	*
Ørsta-Volda	Oslo	Mainland	6.20
Ørsta-Volda	Bergen	Mainland	-9.12
Røros	Oslo	Mainland	-89.55
Alta	Kirkenes	Outermost territory	-316.75
Hammerfest	Vadsø	Outermost territory	-312.57
Hammerfest	Kirkenes	Outermost territory	-255.57
Vadsø	Kirkenes	Outermost territory	123.89
Vadsø	Alta	Outermost territory	-313.75
Vardø	Kirkenes	Outermost territory	*
Båtsfjord	Kirkenes	Outermost territory	18.89
Båtsfjord	Vadsø	Outermost territory	75.89
Båtsfjord	Hammerfest	Outermost territory	-277.57
Berlevåg	Kirkenes	Outermost territory	*
Berlevåg	Vadsø	Outermost territory	*
Berlevåg	Hammerfest	Outermost territory	*
Mehamn	Kirkenes	Outermost territory	5.11
Mehamn	Vadsø	Outermost territory	-71.08
Mehamn	Hammerfest	Outermost territory	-129.53
Honningsvåg	Hammerfest	Outermost territory	5.11
Honningsvåg	Vadsø	Outermost territory	-301.43
Hasvik	Tromsø	Outermost territory	-193.48
Hasvik	Hammerfest	Outermost territory	6.01
Sørkjosen	Tromsø	Outermost territory	-78.83

Table 13: 5-hub model cost per passenger on PSO routes[15]

Table 13 shows that most of the PSO routes in the model have negative costs with average costs of -66.49 USD per passenger. This is due to relatively high subsidies on

[15] * = Airport deleted from the model

some PSO routes, especially in Northern Norway, and the fact that the model uses the weighted average costs of Widerøe’s fleet as cost factor for hub to non-hub connections. While for some routes the subsidy/cost ratio seems to be balanced, as the costs are positive at a low level, others have negative costs of more than 300 USD per passenger. We have to admit that the costs for operations in remote areas are definitely higher than the average costs on commercial routes. However, there are some routes that have nearly five times higher negative costs as the average because of the high subsidies on these routes. Even in remote areas, it is unlikely that real operating costs are five times higher than the operating costs of the average fleet. Bearing in mind that the airline can set the amount of the compensation paid by the state when applying for a tender without fearing that another airline will bid on the same tender (e.g. due to availability of suitable aircraft for STOL runways), the model shows certain evidence that commercial routes are cross-subsidized with compensation received for operating PSO routes.

6 Conclusion

In this thesis a state-of-the-arte formulation for the uncapacitated multiple allocation p-hub median problem is applied to a real-world case. The following sections discuss the outcome of the computational analysis, show the limitations of the used model and describe possible further research.

6.1 Discussion

The UMApHMP formulated by Marín et al. (2006) showed acceptable results when applied to the real-world case. It was possible to include most of the destinations of the Norwegian air transportation network into the model while maintaining an exact solution of the algorithm. Due to the simple structure of the model and the resulting low demand of input data, it was possible to get a solution with data freely available on the internet. Moreover, all computations could be done with a standard home PC within reasonable time. This is essential in case a hub location decision needs to be done with a minimum effort on market research.

The model was able to replicate the real-world scenario almost completely. Only one hub airport is located at a different place in the real-world compared to the model solution due to operational reasons. Greater differences between the model and the real-world were observed at the route allocation. Also here operational reasons as well as requirements for PSO routes are among the reasons for this variation. Computational experiments showed that the network seems to work well with a reduction of one hub location. Due to practical reasons, the 4-hub location model is unlikely to be established. This is because the model removed the hub location that serves as company headquarters in real-world. Consequently, a closure of this hub location would have a tremendous operational, financial and social impact in real-world.

Another computational experiment revealed however, that the public subsidies paid to the airline for operating PSO routes do not have any effect on the network. Neither the hub locations nor the route allocation change when subsidies are considered in the model. However, if the discussion about cross-subsidizing of commercial routes with the compensation paid for operating PSO routes is joined, the model data reveals that the subsidies paid for some routes massively exceed the average operational costs.

Considering the critique on the Norwegian PSO system raised in the literature combined with the results from the model, it is likely that commercial routes are cross-subsidized in this network.

Finally, it is notable that the formulation used in this thesis fits especially well for distance matrices where the triangle inequality between the destinations does not hold. Especially for airlines it is more suitable to calculate the flight time based on a nonlinear matrix including the time needed for take-off and landing procedures and other factors rather than using the Euclidean distance between two airports.

6.2 Limitations

The model used in this thesis requires only a minimum of input data. On the one hand this is a quick and easy tool to determine hub locations with open accessible information. However, some comparisons would require more input data, such as fix costs for opening a hub.

The absence of fix costs in this model leads to a situation in which the total costs of the calculation cannot be compared with each other when the number of hubs to be placed (p) is changed. Consequently, the model presented in this thesis can be used for cases, where the number of hubs is fixed beforehand.

Moreover, due to the strict hubbing policy of the UMApHMP it is not possible to meet all requirements of the PSO routes as stated in the official tenders. For example, there are cases where multiple destinations are to be served with stopover flights or a maximum number of transfers is described. Taking these requirements into consideration would require a different model formulation.

6.3 Future research

On a strategic hub planning level, the model could be expanded with fix costs for opening a hub as stated above. The model would then be able to endogenously determine the optimal number of hubs for this network. Moreover, the solution values for the total costs would be comparable for a different number of hubs (p) to be placed. This fix costs are a highly aggregated number that includes lots of costs that require an access to confidential data of the airline. This means that expanding the model with that kind of data requires more collaboration with involved airline companies.

An insight into real booking data of the airline could also enrich the OD-matrix with a more precise information on demand on certain routes. Furthermore, capacity restrictions that reflect the available seat capacity of the aircraft used on a route could be implemented into the model. This is crucial for operational planning when taking into account that the majority of the airports in the network is in fact restricted by aircraft size due to the STOL runways. Moreover, capacity restrictions at the hub locations reflecting the maximum passenger capacity of the respective airport could be introduced to the model. Both restrictions would simulate an even more detailed real-world scenario.

When thinking about time restrictions on the PSO routes (for some routes departure or arrival time and route frequency are already set in the tender) the model could be expanded with formulations that meet the requirements of the time-windows. This could be done when using the model for operational route planning.

7 References

Air Norway (2015): Homepage. URL: http://www.airnorway.no (retrieved: 01/21/15)

Alumur, S. and Kara, B.Y. (2008): Network hub location problems: The state of the art. *European Journal of Operational Research*, 190: 1-21.

Azarmand, E. and Jami, E.N. (2009): Location Allocation Problem. *Farahani, R.Z. and Hekmatfar, M. [eds.]: Facility Location: Concepts, Models, Algorithms and Case Studies.* Contributions to Management Science. Heidelberg: Physica-Verlag.

Boland, N., Krishnamoorthy, M., Ernst, A.T. and Ebery, J. (2004): Preprocessing and cutting for multiple allocation hub location problems. *European Journal of Operational Research*, 155(3): 638-653.

Bråthen, S. (2011): Air transport services in remote regions. *International Transport Forum Discussion Paper 2011(13)*, OECD, Paris, France.

Bråthen, S., Haplern, N. and Williams, G. (2012): The Norwegian Air Transport Market in the Future. Some possible trends and scenarios. *Rapport 1205*. Høgskolen i Molde.

Bråthen, S., Thune-Larsen, H., Oppen, J., Svendsen, H.J., Bremnes, H., Sandberg Eriksen, K., Bergem, H.J., Heen, K.P. (2015): Forslag til anbudsopplegg for regionale flyruter i Nord-Norge. *Rapport 1509*. Høgskolen i Molde. [Norwegian]

Bubalo, B. (2012): Social costs of public service obligation routes – calculating subsidies of regional flights in Norway. *Netnomics*, 13: 125-150.

Campbell, J.F. (1994): Integer programming formulations of discrete hub location problems. *European Journal of Operation Research*, 72: 387-405.

Campbell, J.F. and O'Kelly, M.E. (2012): Twenty-Five Years of Hub Location Research. *Transportation Science*, 46: 153-169.

Conklin & de Decker (2015): Aircraft Cost Evaluator. URL: https://www.conklindd.com/CDALibrary/ACCostSummary.aspx (retrieved: 01/27/15)

DAT (2015): Homepage. URL: http://www.dat.dk (retrieved: 01/21/15)

DKS - Det Kongelige Samferdselsdepartement (2014): Tenders. URL: http://www.regjeringen.no/en/dep/sd/documents/Other-documents/Tenders.html (retrieved: 05/14/2014)

Ernst, A.T. and Krishnamoorthy, M. (1998): Exact and heuristic algorithms for the uncapacitated multiple allocation p-hub median problem. *European Journal of Operational Research*, 104: 100-112.

Farahani, R.Z. and Hekmatfar, M. [eds.] (2009): Facility Location: Concepts, Models, Algorithms and Case Studies. Contributions to Management Science. Heidelberg: Physica-Verlag.

Farahani, R.Z., Hekmatfar, M., Arabani, A.B. and Nikbakhsh, E. (2013): Hub location problems: A review of models, classification, solution techniques, and applications. *Computers & Industrial Engineering*, 64: 1096-1109.

FICO (2009): Getting started with Xpress. Release 7. URL: http://www.fico.com/en/node/8140?file=5136 (retrieved: 02/25/16)

Flight time Calculator (2015): Homepage. URL: http://flighttime-calculator.com (retrieved: 01/27/15)

García, S., Landete, M., and Marín, A. (2012): New formulation and a branch-and-cut algorithm for the multiple allocation p-hub median problem. *European Journal of Operational Research*, 220(1): 48-57.

Goldman, A.J. (1969): Optimal location for centers in a network. *Transportation Science*, 3(4): 352-360.

Hakimi, S.L. (1964): Optimum Locations of Switching Centers and the Absolute Centers and Medians of a Graph. *Operations Research*, 12(3): 450-459.

Hakimi, S.L. (1965): Optimum Distribution of Switching Centers in a Communication Network and some related Graph Theoretic. *Operations Research*. 13(3): 462-475.

Hamacher, H.W., Labbé, M., Nickel, S. and Sonneborn, T. (2004): Adapting polyhedral properties from facility to hub location problems. *Descrete Applied Mathematics*, 145: 104-116.

Kratica, J. (2013): An electromagnetism-like metaheuristic for the uncapacitated multiple allocation p-hub median problem. *Computers & Industrial Engineering*, 66: 1015-1024.

Lian, J.I. (2010): Network dependency and airline competition – Consequences for remote areas in Norway. *Journal of Air Transport Management*, 16: 137-143.

Lian, J.I., Thune-Larsen, H. and Draagen, L. (2010): Evaluering av anbudsordningen for regionale flyruter. TØI rapport 1116/2010. Transportøkonomisk Institutt: Oslo. [Norwegian]

Marín, A., Cánovas, L. and Landate, M. (2006): New formulations for the uncapacitated multiple allocation hub location problem. *European Journal of Operational Research*, 172: 274-292.

Mathisen, T.A. and Solvoll, G. (2012): Reconsidering the regional airport network in Norway. *European Transportation Research Review*, 4: 39-46.

Mattfeld, D. and Vahrenkamp, R. (2014): Logistiknetzwerke: Modelle für Standortwahl und Tourenplanung. Wiesbaden: Springer Fachmedien. [German]

Moradi, E. and Bidkhori, M. (2009): Single Facility Location Problem. *Farahani, R.Z. and Hekmatfar, M. [eds.]: Facility Location: Concepts, Models, Algorithms and Case Studies.* Contributions to Management Science. Heidelberg: Physica-Verlag.

O'Kelly, M.E. (1986): The location of interacting hub facilities. *Transportation Science*. 20(2): 92-106.

O'Kelly, M.E. (1987): A quadratic integer program for the location of interacting hub facilities. *European Journal of Operational Research*, 32: 393-404.

Pita, J.P., Antunes, A.P., Barnhart, C., and Gomes de Menezes, A. (2013): Setting public service obligations in low-demand air transportation networks: Application to the Azores. *Transportation Research Part A*, 54: 35-48.

Santana, I. (2009): Do Public Service Obligations hamper the cost competitiveness of regional airlines? *Journal of Air Transport Management*, 16: 334-349.

SAS (2012): SAS Annual Report 2012 - The airlines' operations: Widerøe. URL: http://sasannualreport2012.com/en/Start/The+airlines%E2%80%99+operations/The+airlines%E2%80%99+operations%3A+Wider%C3%B8e (retrieved: 01/20/15)

SAS (2013): The sale of Widerøe completed. Press release. URL: http://www.sasgroup.net/en/the-sale-of-wideroe-completed (retrieved: 11/09/15)

Sender, J. and Clausen, U. (2011): Hub Location Problems with Choice of Different Hub Capacities and Vehicle Types. *Pahl, J., Reiners, T. and Voß, S. (Eds.): Network Optimization: 5th International Conference*, INOC 2011, LNCS 6701: 535-546.

Skorin-Kapov, D., Skorin-Kapov, J. and O'Kelly, M. (1996): Tight linear programming relaxations of uncapacitated *p*-hub median problems. *European Journal of Operational Research*, 94: 582-593.

Torp (2015): Torp Sandefjord Lufthavn. Homepage. URL: http://www.torp.no (retrieved: 11/09/15)

Widerøe (2014): Codeshare agreement. URL: http://www.wideroe.no/en/News-archive/Codeshare-agreement-with-Eastern-Airways (retrieved: 11/13/15)

Widerøe (2015): About the company. URL: http://www.wideroe.no/en/all-about-us/about-wideroe (retrieved: 11/13/15)

Wikipedia (2016): List of Airports in Norway. URL: https://en.wikipedia.org/wiki/List_of_airports_in_Norway (retrieved: 03/03/16)

Yang, T.-H. and Chiu, T.-Y. (2016): Airline hub-and-spoke system design under stochastic demand and hub congestion. *Journal of Industrial and Production Engineering*, 33(2): 69-76.

8 Appendix

8.1 Model program code

```
model HLP
uses "mmxprs"; !gain access to the Xpress-Optimizer solver

declarations
!indices
V = 1..40 !set of Vertices (Destinations)

!data variables
W: array(V,V) of integer !number of passenger traveling from i to j
C: array(V,V,V,V) of real !cost of traveling from i to j via k and m
CC: array(V,V,V,V) of real !pre-processed costs
T: array(V,V) of integer !flight-time between OD
alpha: real !discount factor for hub-to-hub connection
S: array(V,V) of real !subsidy paid per passenger from i to j
var: real !variable cost per flight minute per
p: integer !number of hubs to be opened
code: array(V) of string !3-letter airport code for output

!decision variables
X: array(V,V,V,V) of mpvar !fraction of flow from i to j via k and m
Y: array(V) of mpvar !indicator if a hub is openend at k (1) of not (0)

FILENAME="hlp.dat" !data file

end-declarations

!load data
 initialisations from FILENAME
    alpha W T var code S
 end-initialisations

! set the number of hubs to be located
p:= 5

!binary; 1=hub location open
forall(k in V) Y(k) is_binary

!cost function
forall(i,j,k,m in V)
    C(i,j,k,m):=  T(i,k)*var  + alpha*T(k,m)*var + T(m,j)*var !- S(i,j)

!pre-processing data
forall(i,j,k,m in V)
    CC(i,j,k,m) := minlist(C(i,j,k,m), C(i,j,m,k), C(i,j,k,k), C(i,j,m,m))

!objective function (1)
Z:= sum(i,j,k,m in V) W(i,j) * C(i,j,k,m) * X(i,j,k,m)


 !s.t.
 !(2) number of hub locations
 sum(k in V) Y(k) = p

 !(3) each OD flow is sent via some hub pair
 forall(i,j in V)
     sum(k,m in V) X(i,j,k,m) = 1

 !(13) hubs are opened for all routings of flows
 forall(i,j,k in V)
     X(i,j,k,k) + sum(m in V | m<>k ) (X(i,j,k,m) + X(i,j,m,k) ) <= Y(k)

 !(14) for pre-processing
 forall(i,j,k,m in V)
     if (C(i,j,k,m) > CC(i,j,k,m) ) then
         X(i,j,k,m) = 0
     end-if

 !(7) non-negativity
 forall(i,j,k,m in V)
     X(i,j,k,m) >= 0

 !objective
 minimize(Z)

 !output
 writeln("Total Cost: ", getobjval)

 forall(k in V |getsol( Y(k))>0 )
     writeln(code(k), " serves as hub airport")


 forall(i,j,k,m in V | getsol( X(i,j,k,m) )>0 and W(i,j) > 1 )
     writeln("route: ", code(i) , " - " , code(j), " via ", code(k), " (", code(m), ") | PAX: ", W(i,j), " | cost: ", C(i,j,k,m)  )

 end-model
```

This model shows the same solution as a model with constraint: $X_{i,j,k,m}$ is binary

8.2 *Curriculum vitae*

Personal information

Name	Markus Zobel
Place of birth	Villach
Citizenship	Austrian
Contact	zobel.markus@gmail.com

Education

2013 - 2016	Master's degree programme International Business Administration (University of Vienna)
2012 – 2015	Master's degree programme Journalism and Communication (University of Vienna)
2009 – 2013	Bachelor’s degree programme International Business Administration (University of Vienna)
2008 – 2012	Bachelors’s degree programme Journalism and Communication (University of Vienna)

Studies abroad

2013 – 2014	Norwegian Business School, Oslo (Erasmus)
2011	Carleton University, Ottawa, Canada (Joint-Study)

Work experience

2015 - 2016	Hewlett-Packard GesmbH *Internship – Social Media & Communications*
Summer 2012	Infineon Technologies Austria AG *Student worker*
Summer 2010 & 2011	Kleine Zeitung Marketing GmbH *Event marketing*
Spring 2008	UEFA EURO 2008 *Volunteer*
Summer 2006 & 2007	UniCredit Bank Austria AG *Intern*

Skills

Languages	German, English, Italien, Russian, Norwegian
IT	MS Office, SPSS, SAP, Typo3, XPress-MP, Adobe PS

8.3 Abstract

This thesis deals with the application of a model for the Hub Location Problem to a real-world case in the area of airline network planning. More specific, the uncapacitated multiple allocation p-hub median problem (UMApHMP) is applied to the Norwegian passenger airline network. Thereby, the selected model copes with special conditions such as a network located in a remote area, a nonlinear distance matrix or different levels of public subsidies on certain routes. Although the model requires only a minimum of input data, the results show that it is suitable to solve a real-world case. Both, hub location and route allocation of the model mostly correspond to the real-world case. However, there are some cases where they differ due to operational reasons. Another computational experiment reveals that the public subsidies paid for some routes do not have any impact on the model network. Finally, this thesis shows some indications for a cross-subsidizing of commercial routes with compensation paid for subsidized routes.

8.4 Deutsche Zusammenfassung

In dieser Masterarbeit wird ein Model für das Hub Location Problem auf ein reales Fallbeispiel im Bereich der Netzwerkplanung von Fluglinien angewandt. Im Detail befasst sich die Arbeit mit der Anwendung des „uncapacitated multiple allocation p-hub median problem" (UMApHMP) auf das norwegische Passagierflugnetzwerk. Dabei ist das ausgewählte Model in der Lage mit speziellen Eigenschaften, wie ein Netzwerk in einem entlegenen Gebiet, eine nicht-lineare Abstandsmatrix oder verschieden hohe öffentliche Förderungen auf bestimmten Strecken, fertig zu werden. Obwohl das Model nur wenige Eingabedaten benötigt, zeigen die Ergebnisse, dass es geeignet ist, ein reales Fallbeispiel zu lösen. Der Hub-Standort sowie die Routenzuweisung des Models gleichen dabei weitgehend dem realen Fallbeispiel. In einigen Fällen kommt es jedoch zu einer Abweichung aus operationellen Gründen. Eine weitere Berechnung ergibt, dass die öffentlichen Förderungen auf manchen Strecken keinen Einfluss auf das Modelnetzwerk haben. Schließlich werden einige Hinweise präsentiert, die auf eine Quersubventionierung kommerzieller Strecken mit den Kompensationsgeldern der geförderten Strecken hindeuten.