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„Asymmetric Fair Divisions in Rubinstein Bargaining"

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1   Introducion

In the context of bargaining, experimental evidence suggests that fairness regarding preferences have a significant influence on the bargaining outcome. These preferences are possibly in the form of one's own payoff relative to the payoff of the opponent, or relative to some (not necessarily equal) reference division which the individual considers fair according to a principle. The latter case is particularly common in some scenarios where each player has to provide some input to take part in the bargain (such as bargaining over a jointly produced amount or player specific prices to enter the bargain). While the majority of the theoretical literature primarily focuses on purely self-interested agents, fairness motivation is a subject of growing interest and importance in the field.

The theory of other-regarding preferences in bargaining consists predominantly of models of inequality-aversion, where the fairness judgment of the outcome by a player is based on their share relative to those of the other players. This can also be interpreted as the players considering an equal division fair. Such agents are studied in a highly relevant work to the model here by Kohler and Schlag (2016). They propose a solution for inequality averse bargainers in infinite horizon alternating offer bargaining and show the effects of weights (guilt and envy) given to fairness on the outcome division. However, as stated above, there often exists a focal point different than the equal division as a fairness reference (such as the ratio of input in a jointly produced good), which cannot be accounted for by inequality aversion. A more general utility model is required for explaining the consequences of these asymmetric fair points. Hence, the model in Kohler and Schlag (2016) corresponds to a special case of the model in this paper where the variable fair point is set to the equal division. The solution presented

in Kohler and Schlag (2016) will be utilized in this paper to solve for bargainers that consider asymmetric divisions fair.

There is some recent theoretical work that investigates the effect of conceived fair divisions that are non-equal. These papers, however, explain the effects within the framework of axiomatic solution concepts (particularly Nash Bargaining Solution) or finite horizon protocols.

The contribution of this paper is the application of asymmetric (i.e. non-equal) fairness conceptions to the infinite-horizon alternating offer framework of Rubinstein (1982), while seperately accounting for disutilities suffered for receiving a higher share (guilt) and lower share (envy) than in the fair division. This is done utilizing a modified version of the preferences from Fehr and Schmidt (1999). It is shown that the general solution to the Rubinstein bargaining is applicable with these modified utilities, and the resulting subgame perfect equilibrium is analyzed.

While the majority of the work examining the influence of varying fair points on the bargaining outcome focus on the effect of different and often conflicting (incompatible - see Birkeland & Tungodden 2014) subjective fairness principles of the bargainers, the motivation in this paper is understanding the effects of variation in the driving factor of the conceived fair point (e.g. production relative to the opponent) on bargaining power and outcome. Thus, in this model, the bargainers agree on what constitutes a fair division for each given state of the world. The model allows any commonly observed fairness principal such as egalitarianism, laissez-faire, and accountability (Konow 1996, 2000) as long as the two bargainers share a particular principle.

As mentioned above, the examined bargaining protocol is that proposed by Rubinstein (1982). Here, the bargainers take turns making offers to the other regarding the division of the pie, a process which continues until an offer is accepted. The critical assumption is that they are impatient, meaning that the individuals derive a lower utility from an agreement share if it is realized at a later stage. Osborne and Rubinstein (1994) show for the general case, that there exists a unique subgame perfect equilibrium for the problem, where each bargainer is indifferent between accepting the opponent's offer and making their own offer in the next round at any given stage of the bargaining process.

To reflect the fairness motivation of the bargainers, this study modifies the inequality averse preferences proposed by Fehr and Schmidt (1999) where players experience utility diminishing guilt when they receive a higher share than opponents, and envy when receiving less. In a two player setting, this corresponds to utility diminishment based on the difference between a proposed share and the half of the pie. Degrees of guilt and envy are used to capture the weight each player attaches to fairness in case of receiving more and less than half respectively, which in turn determine the magnitude of the disutility suffered from inequality. The difference between the standard Fehr & Schmidt utility and the one that is used here is that while the degrees of guilt and envy are kept, the utility diminishment is based on the difference between the share from an offer and

an exogenously determined conceived fair share instead of the equal division. Consequently, guilt is experienced by the players if they receive more than their conceived fair share, and envy if less. This study follows the argument of Kohler and Schlag (2016) that the solution proposed by Osborne and Rubinstein (1994) holds only for the degrees of guilt for which a larger share of the pie is always desirable (i.e. the utility of both players remain strictly increasing in their own share of the pie). If the degree of guilt is high enough to make a player worse off when receiving a larger share than in the fair point, the agreement is immediate and on the fair division. Thus, the conclusions regarding the influence of the fair point on the bargaining outcome are valid for low enough degrees of guilt.

In the bargaining process, the offers that are similar to the fair point become more preferable relative to the offers that are far away from it if the players give higher weight to fairness (i.e. higher guilt and envy). Conversely with given degrees of guilt and envy, the conceived fair point coming closer to a given division increases the utility that both players derive from this division, increasing its relative preferability. So an increase in a player's fair share (which means the conceived fair point moving closer to divisions where this player receives a high share), may give him incentive to reject a low offer by the opponent today in favor of offering to take a higher share tomorrow.

The comparative statics in the subgame perfect equilibrium show that whether a higher fair share leads to a higher monetary payoff for a player structurally depends on how symmetric (close to equal) the conceived fair division is. More specifically, the effect differs between moderate fair points, which assign the two bargainers similar shares, and extreme fair points, which assign one player most of the pie. These classes of fairness conceptions are distinguished by thresholds determined by how much weight the bargainers give to fairness (guilt and envy) and how patient they are. With moderate fair points, players feel guilty with their own offers, and envious with the opponent's offers (where they receive a lower share than with their own offers). With extreme fair points, one player always feels guilty and the other always envious.

As long as the fair division remains moderate according to the distinction above, the higher a players fair share is, the higher a share he receives in agreement. With moderate fair points, Player 2 (player who makes the second offer) compares accepting an offer lower than his fair share to offering to take more than it tomorrow. An increase in his fair share makes his own offer better and accepting Player 1s offer worse than before. As a result, a higher offer is required to convince him to accept. However, if the fair point assigns one player very little or very much of the pie (i.e. with extreme fair points), a small increase in a players fair share can actually result in a decrease in his share in agreement. With extreme fair points, the low player (the player with the lower share in fair point) compares accepting today to making own offer tomorrow, both of which make the low player feel guilty by giving him a larger share than in the fair point. An even lower fair share increases the guilt from both offers. As the additional guilt is

discounted for tomorrow, the additional disutility is higher today. This makes tomorrow better relative to today and gives the low player incentive to reject. As a result, a higher share is given to the low player in agreement than before. The decision of the high player between accepting or rejecting the offer of the low player is affected in the same way trough envy. So if the guilt of the low player is sufficiently high relative to the envy of the high player, the outcome share of a player is decreasing in his fair share and increasing otherwise. From these effects, one can conclude that for moderate fair points, the outcome division becomes more equal as the fair point becomes more symmetric, and for extreme fair points, the outcome division becomes less(more) equal as the fair point becomes more symmetric if the guilt is high(low) relative to the envy. It is also shown that for moderate fair points, the outcome division converges to the fair division as players become more patient. The outcome share of players increase in their own envy and the opponent's guilt, and decrease in their own guilt and opponent's envy.

The body of work regarding possibly non-egalitarian fairness conceptions in bargaining consists mostly of experiments which find a considerable effect on bargaining behavior for various protocols, although evidence regarding the nature of this influence is mixed. The experiments which suggest the sensitivity of the outcome to the division considered fair by the bargainers include Königstein (2000), Gantner, Güth, and Königstein (2001), Gächter and Riedl (2005, 2006), Cappelen, Hole, Sørensen, and Tungodden (2007), Birkeland (2013), Cappelen, Moene, Sørensen, and Tungodden (2013), Bolton and Karagozoglu (2016), and Gantner, Horn, and Kerschbamer (2016). There are relatively few theoretical studies that model this influence. Birkeland and Tungodden (2014) propose a model that assigns the same weight to utility diminishment caused by receiving more and receiving less than in the fair division determined by their fairness principle. Here, they examine the effects of changing fairness views on the Nash Bargaining Solution (Nash, 1950). It is also shown that with incompatible fairness views the bargaining set can become the empty set, resulting in disagreement. Bolton and Karagozoglu (2016) study the effects appealing to focal points (which are based on what they conceive as a fair division) and the possibility to commit to a certain division in an experimental setting. They then apply these concepts to the Nash Bargaining Solution and a finite Zeuthen-Harsanyi bargaining game.

In Section 2, the formal bargaining protocol and the corresponding general case equilibrium will be presented. In Section 3, a utility model of fairness motivation will be introduced, which then will be used to derive the unique SPE for such agents. In Section 4, the comparative statics of the solution will be shown and the effects of interest will be discussed. A brief summary of these findings will be presented and concluding remarks regarding the applications of the model and the possible further steps in the area will be made in Section 5.

2 Bargaining Environment and General Case Solution

In deriving the subgame perfect equilibrium alternating offer bargaining game for fairness motivated bargainers, this study uses the version of the general solution by Osborne and Rubinstein (1994) presented in Kohler and Schlag (2016), which directly uses bargainers' utility functions (as opposed to their preference over divisions) to measure optimal behavior and to determine the corresponding strategies.

Following their notation, let $U = \{v_1, f(v_1) : v_1 \in [\omega_1, \omega_2] \wedge \omega_1 \leq 0\}$ denote the bargaining set (set of all possible agreements), where v_1 is the utility Player 1 (first offer-maker) derives from her outcome share x, $f(v_1)$ is the utility Player 2 (the second offer maker) derives from her outcome share $1 - x$ as a function of v_1, and ω_1 (ω_2) is the minimum (maximum) possible utility Player 1 can get from an agreement. $\delta \in (0, 1)$ will denote the discount factor, which here will be assumed equal for both players (the solution is analogous for individual degrees of impatience). Let the pair of utilities in disagreement be $d = (0, 0)$. Starting with Player 1 in the first round, players alternately make offers regarding the division of the pie, which the other player can accept or reject. In case of rejection, the other player makes an offer in the next round. This process continues until an offer is accepted. In case of permanent rejection in every round, both players get their disagreement utility (here, (0,0)).

Now let $f(v_1)$ be such that:

(F1) *f is continuous in v_1*

This implies that there exists a small change in strategy that leads to an arbitrarily small change in agreement.

(F2) *f is strictly decreasing in v_1*

This implies a conflict of interest between the two bargainers. Furthermore, given that v_1 is strictly increasing in x, it implies that $f(v(x))$ is strictly increasing in $1 - x$, meaning a larger share of the pie is always strictly preferred by both players. When this property holds, a division that makes one player relatively better off makes the other player worse off. This implies Pareto-Efficiency of all possible agreements, as it rules out the possibility of switching a different agreement where both players are better off.

(F3) $f(0) \geq 0$
(F4) $f(\omega_2) \leq 0$

These two properties imply that the subset of the bargaining set U consisting of agreements that provide a higher utility than in disagreement for both players is non-empty. That is, there are divisions of the pie that are better than disagreement for both players.

Based on the preference based solution by Osborne and Rubinstein (1994), Kohler and Schlag (2016) show that $(v_1^*, f(v_1^*))$ is the pair of utilities derived from the unique subgame perfect equilibrium of the above bargaining game if $f(v_1)$ satisfies properties $(F1)$ through $(F4)$, and v_1^* is the only solution for $f(v_1) = \delta f(\delta v_1)$. The SPE strategies are such that Player 1 always offers v_1^* and accepts iff offered v_1 with $v_1 \geq \delta v_1^*$, and Player 2 always offers δv_1^* and accepts iff offered $v_1 \leq v_1^*$. The resulting agreement takes place in the first round, and is on the division $(v_1^{-1}(v_1^*), 1 - v_1^{-1}(v_1^*))$.

The line of arguments is as follows. Suppose $\underline{v}_i$ ($\overline{v}_i$) is the lowest (highest) payoff for player i in a subgame perfect equilibrium where i makes the first offer. Then the highest utility that Player 2 can get by making her own offer is $\overline{v}_2$. In the preceding round, Player 1's offer is sure to be accepted if it provides Player 2 with at least the discounted value of this utility, namely $\delta\overline{v}_2$. Consequently, Player 1 can receive any payoff v_1 with $v_1 \leq f^{-1}(\delta\overline{v}_2)$ in a SPE. This implies $\underline{v}_1 \geq f^{-1}(\delta\overline{v}_2)$. That is, the minimum payoff of the utility maximizing Player 1 in a SPE is at least as much as the one that makes Player 2 indifferent between accepting or getting the highest possible utility by making an offer.

In the SPE that gives Player 2 $\overline{v}_2$, Player 1 cannot receive a payoff higher than $f^{-1}(\delta\overline{v}_2)$, as these offers with $v_1 > f^{-1}(\delta\overline{v}_2)$ provide Player 2 a lower utility than making her own offer and will be rejected. Thus, $\underline{v}_1 \leq f^{-1}(\delta\overline{v}_2)$. That is, the minimum payoff of Player 1 is no more than the one that makes Player 2 indifferent between accepting and getting the highest possible utility by making own offer.

Assuming that the players accept in case of indifference, these two weak inequalities imply that Player 1's minimum payoff in a SPE is the one that makes Player 2 indifferent between accepting and rejecting in the SPE where Player 2 receives the highest payoff. Thus, $\underline{v}_1 = f^{-1}(\delta\overline{v}_2)$. Converse holds for the minimum SPE payoff of Player 2 ($\delta\overline{v}_1 = f^{-1}(\underline{v}_2)$).

In a SPE, Player 2 will accept only offers that give her at least the discounted value of the minimum utility she can secure by making her own offer ($\delta\underline{v}_2$). Thus, SPE rules out Player 1 making offers where she gets more than the share that makes Player 2 indifferent between accepting or making own offer tomorrow. This means that the maximum payoff Player 1 can receive in a SPE is no more than the one that gives Player 2 the discounted value of her minimum utility tomorrow ($\overline{v}_1 \leq f^{-1}(\delta\underline{v}_2)$).

In the SPE where Player 2 gets $\underline{v}_2$ by making her own offer, any offer by Player 1 that gives Player 2 at least the discounted value of $\underline{v}_2$ is accepted. Thus, a utility maximizing Player 1 makes an offer where Player 2 receives no more than her discounted minimum SPE utility. ($\overline{v}_1 \geq f^{-1}(\delta\underline{v}_2)$).

These two weak inequalities imply that the maximum payoff Player 1 can receive in a SPE is the one that makes Player 2 indifferent between accepting or making an offer tomorrow that gives her her lowest SPE utility. That is, $\overline{v}_1 = f^{-1}(\delta\underline{v}_2)$. Converse holds for the maximum SPE payoff of Player 2, meaning $\overline{v}_2 = f(\delta\underline{v}_1)$.

The two equations above can also be written as $f(\underline{v}_1) = \delta \overline{v}_2$ and $f(\overline{v}_1) = \delta \underline{v}_2$. Replacing $\overline{v}_2$ ($\underline{v}_2$) with $f(\delta \underline{v}_1)$ ($f(\delta \overline{v}_1)$) yields:

$$f(\underline{v}_1) = \delta f(\delta \underline{v}_1)$$

$$f(\overline{v}_1) = \delta f(\delta \overline{v}_1)$$

Recall that the above proposed solution is under the assumption that the function f has a unique solution v_1^* for $f(v_1) = \delta f(\delta v_1)$. For this equation system this assumption implies $\underline{v}_1 = \overline{v}_1 = v_1^*$, which then becomes the unique SPE payoff of Player 1 for this bargaining game. The only credible strategy profile that yields these utilities in agreement is the one described in the proposition. Here, both players make offers that make the other indifferent between accepting and rejecting (making own offer in the next round) when it is their turn to make an offer. As a result, the offer in the first round by Player 1 is accepted.

Continuity of f ensures that the solutions from the above maximum and the minimum value equations have a common solution. Decreasing f ensures that v_1' is unique for any given $f(v_1')$, and that there is a strict preference relation between any two offers in the same round. The decreasing nature of f implies that both players make the offer that gives them the maximum payoff among acceptable offers. $F3$ and $F4$ ensure that the bargaining set is non-empty.

The next section shows that this solution can be applied to a specific utility function that regards asymmetric fair points.

3   Solution for Fairness Motivated Bargainers

3.1   The Utilities

To capture the fairness motivation in bargainer preferences, utilities similar to the ones in the inequality aversion model proposed by Fehr and Schmidt (1999) will be used.

Fehr and Schmidt (1999) suggest that in addition to their absolute share in agreement x_i, bargainers also take their shares relative to that of the opponent into consideration. In the 2 player case, if player i receives more than her opponent she suffers some utility loss, the magnitude of which measured by the difference between the payoffs and a weight $\beta_i \in [0, 1)$. Similarly, if player i receives less than her opponent, the utility loss is measured by the difference between payoffs weighted by $\alpha_i \geq 0$. $\alpha_i(\beta_i)$ captures the envy (guilt) experienced by player i as a result of an unequal division. The resulting utility function of player $i \in \{1, 2\}$ is:

$$u_i = x_i - \alpha_i max\{0, x_j - x_i\} - \beta_i max\{0, x_i - x_j\}, i \neq j$$

Denoting the share of Player 1 (the first offer-maker) by x allows us to denote the share of Player 2 by $1 - x$. Simplifying the resulting functions yields:

$$u_1 = x - \alpha_1 max\{0, 1 - 2x\} - \beta_1 max\{0, 2x - 1\}$$

and

$$u_2 = 1 - x - \alpha_2 max\{0, 2x - 1\} - \beta_2 max\{0, 1 - 2x\}$$

One difference between the utilities used in this paper and the utilities above is that in this paper, the guilt and envy are experienced based on the difference between a player's outcome share and the share that she would receive in what she considers to be the fair division (instead of the difference between her share and the share of the opponent in the outcome). If the bargainers consider the equal division fair, the model in this study becomes conceptually analogous to the utilities above. In the case where the conceived fair point is the equal division, the corresponding functional form to this description is:

$$u_i = x_i - \alpha_i max\{0, \frac{x_i + x_j}{2} - x_i\} - \beta_i max\{0, x_i - \frac{x_i + x_j}{2}\}, i \neq j$$

Now, as the objective of this study is to capture the effect of different divisions as conceived fair points on bargaining behavior, the share that player i considers to be her fair share is generalized and denoted by $\lambda_i \in [0, 1]$. Furthermore, the assumption that both players consider the same division fair implies that $\lambda_1 = 1 - \lambda_2$. As a result, the fair share of Player 1 can be denoted by λ and the fair share of Player 2 will be denoted by $1 - \lambda$. In the above utility function, replacing x_1 by x, x_2 by $1 - x$, replacing $1/2$ by λ in Player 1's utility function and by $1 - \lambda$ in Player 2's utility function yields:

$$u_1 = x - \alpha_1 max\{0, \lambda - x\} - \beta_1 max\{0, x - \lambda\}$$

and

$$u_2 = 1 - x - \alpha_2 max\{0, x - \lambda\} - \beta_2 max\{0, \lambda - x\}$$

Note that with this functional form, λ has no interaction term with x. Thus, the value of λ has no effect on the slope of the utility function for $x < \lambda$ or $x > \lambda$. However, λ determines the regions of x where guilt or envy are experienced, and the magnitude of the utility loss suffered from a given x.

An important feature of these functions is that Player i's utility is strictly increasing in her own share (for any given λ) if and only if $\beta_i < 1$. This is a necessary condition for the solution presented in Section 2 to be applicable. If $\beta_i = 1$ for Player i, then that player is indifferent between any share not smaller than her fair share. If $\beta_i > 1$, then the utility of player i has a maximum at $x_i = \lambda_i$, and decreases with higher shares than

that. In both cases, "pie is desirable" is not true, which violates the assumption $(F2)$ from Section 2. Thus, the Rubinstein solution is not applicable to bargaining with these utilities if $\beta_i \geq 1$ for any $i \in \{1, 2\}$.

3.2 The Subgame Perfect Equilibrium

Using the above described utilities to capture the fairness motivation as well as their self-interest, one can derive the unique Subgame Perfect Equilibrium for the bargaining problem.

Proposition 1. *There exists a unique Subgame Perfect Equilibrium for the infinite horizon alternating offer bargaining game with fairness motivated bargainers who agree on what constitutes a fair division if $\beta_i < 1, \forall i \in \{1, 2\}$, where agreement is reached in the first round. The strategies are such that Player 1 always offers x^* and accepts offer x if and only if $x \geq \delta x^*$, and Player 2 always offers δx^* and accepts offer x if and only if $x \leq x^*$. Outcome share x^* of Player 1 as follows.*

$$
x^* = \begin{cases}
\frac{1}{1+\delta}\left(\frac{1+\lambda\alpha_2}{1+\alpha_2} - \frac{\lambda\delta\beta_1}{1-\beta_1}\right); & \lambda \leq \underline{\lambda} \\
\frac{(1+\alpha_1)(1-\delta)+\lambda(\delta\alpha_1+\alpha_1\alpha_2+\alpha_2+\delta\beta_2+\delta^2\beta_1-\delta^2\beta_1\beta_2)}{(1+\alpha_1)(1+\alpha_2)-\delta^2(1-\beta_1)(1-\beta_2)}; & \lambda \in (\underline{\lambda}, \overline{\lambda}) \\
\frac{1}{1+\delta}\left(\frac{1-\lambda\beta_2}{1-\beta_2} + \frac{\delta\alpha_1\lambda}{1+\alpha_1}\right); & \lambda \geq \overline{\lambda}
\end{cases}
$$

From which Player 1 derives utility v_1^* with:

$$
v_1^* = \begin{cases}
\frac{1-\beta_1+\lambda(\alpha_2+\beta_1)}{(1+\alpha_2)(1+\delta)}; & \lambda \leq \underline{\lambda} \\
\frac{(1-\delta)(1-\beta_1)(1+\alpha_1)+\lambda[(1+\alpha_1)(\alpha_2+\beta_1)+\delta(1-\beta_1)(\alpha_1+\beta_2)]}{(1+\alpha_1)(1+\alpha_2)-\delta^2(1-\beta_1)(1-\beta_2)}; & \lambda \in (\underline{\lambda}, \overline{\lambda}) \\
\frac{1+\alpha_1-\lambda(\alpha_1+\beta_2)}{(1+\delta)(1-\beta_2)}; & \lambda \geq \overline{\lambda}
\end{cases}
$$

With $\underline{\lambda} = \frac{\delta(1-\beta_1)}{1+\alpha_2+\delta(1-\beta_1)}$ and $\overline{\lambda} = \frac{1+\alpha_1}{1+\alpha_1+\delta(1-\beta_2)}$.

Proof. First, it will be shown that the utility assumptions necessary for the Rubinstein solution hold for the above utilities if $\beta_{1,2} < 1$. Then it will be shown that $f(v_1) = \delta f(\delta v_1)$ has a unique solution. This will be followed by the presentation of the solution and the corresponding conditions on λ.

We start by defining the bargaining set $U = \{v_1, f(v_1) : v_1 \in [\omega_1, \omega_2] \wedge \omega_1 \leq 0\}$ for the given utility functions. As the utilities are assumed to be the ones described above, let $v_1 = u_1(x) = x - \alpha_1 max\{0, \lambda - x\} - \beta_1 max\{0, x - \lambda\}$ (the utility Player 1 gets from share x). As $\beta_1 < 1$ by assumption, we know that $\frac{dv_1}{dx} = \frac{du_1}{dx} > 0, \forall x \in [0, 1]$. Thus, it can be seen that among the possible divisions, Player 1 gets her maximum

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utility if $x = 1$ and minimum if $x = 0$. So, we can set $\omega_1 = u_1(0) = -\lambda\alpha_1$ and $\omega_2 = u_1(1) = 1 - \beta_1(1 - \lambda)$.

Finally, since $f(v_1)$ is Player 2's utility as a function of Player 1's utility from a given division x, we define $f(v_1)$ as $u_2(u_1^{-1}(x))$ (or simply as $u_2(x(v_1))$). Now, as v_1 is diminished via α_1 if $x < \lambda$ and via β_1 if $x > \lambda$, it has different slopes for these ranges. As $x < \lambda(x > \lambda)$ implies $v_1 < \lambda(v_1 > \lambda)$, function $x(v_1)$ is given by:

$$x = \begin{cases} \frac{v_1 + a_1\lambda}{1 + a_1}; & v_1 < \lambda \\ \frac{v_1 - \beta_1\lambda}{1 - \beta_1}; & v_1 \geq \lambda \end{cases}$$

Replacing x with $x(v_1)$ in $u_2(x)$ yields $u_2(x(v_1)) = f(v_1)$ as we defined above:

$$f(v_1) = \begin{cases} 1 - \beta_2\lambda - (v_1 + \alpha_1\lambda)\frac{1 - \beta_2}{1 + \alpha_1}; & v_1 < \lambda \\ 1 + \alpha_2 - (v_1 - \beta_1\lambda)\frac{1 + \alpha_2}{1 - \beta_1}; & v_1 \geq \lambda \end{cases}$$

Note that because v_1 is greater(smaller) than λ, Player 1 feels guilt(envy), and Player 2 feels envy(guilt). Thus, β_1 and α_2 (α_1 and β_2) enter the utility of Player 2.

Now that we have fully defined $U = \{v_1, f(v_1) : v_1 \in [-\lambda\alpha_1, 1 - \beta_1(1 - \lambda)]\}$, we can check if the four assumptions necessary for the applicability of the Rubinstein solution hold for $f(v_1)$.

$(F1)$: $f(v_1)$ *is continuous*. It is clear that $f(v_1)$ is continuous for $v_1 \in [\omega_1, \lambda)$ and $v_1 \in (\lambda, \omega_2]$. To check the continuity at $v_1 = \lambda$, we compute the left and right limits. This yields $\lim_{v_1 \nearrow \lambda} f(v_1) = \lim_{v_1 \searrow \lambda} f(v_1) = 1 - \lambda$. Thus, we can conclude that (v_1) is continuous for all $v_1 \in [\omega_1, \omega_2]$.

$(F2)$:$f(v_1)$ *is strictly decreasing*. Taking the derivative of $f(v_1)$ with respect to v_1 yields:

$$\frac{df(v_1)}{dv_1} = \begin{cases} -\frac{1 - \beta_2}{1 + a_1}; & v_1 < \lambda \\ -\frac{1 + \alpha_1}{1 - \beta_1}; & v_1 \geq \lambda \end{cases}$$

Which, since we assume $\beta_{1,2} < 1$, is strictly negative for all $v_1 \in [\omega_1, \omega_2]$. So $f(v_1)$ is strictly decreasing.

$(F3)$ and $(F4)$: Since $1 \geq \lambda \geq 0$, $f(v_1)$ is continuous, and $\beta_2 < 1$, $f(0) = \frac{\alpha_1(1 - \lambda) + 1 - \beta_2\lambda}{1 + \alpha_1} > 0$ (F3 is satisfied). Since $1 \geq \omega_2 \geq \lambda$, $f(\omega_2) = \alpha_2(\lambda - 1) \leq 0$ (F4 is satisfied).

Now that we have shown that all the necessary and sufficient conditions hold for $f(v_1)$, we know by the proposition presented in Section 2 that if a unique solution exists for $f(v_1) = \delta f(\delta v_1)$, then that solution is the utility Player 1 gets from the agreement

in the unique subgame perfect equilibrium. We start by defining the function $h(v_1) := f(v_1) - \delta f(\delta v_1)$:

$$
h(v_1) = \begin{cases} 1 - \beta_2\lambda - (v_1 + \alpha_1\lambda)\frac{1-\beta_2}{1+\alpha_1} - \delta[1 - \beta_2\lambda - (\delta v_1 + \alpha_1\lambda)\frac{1-\beta_2}{1+\alpha_1}]; & v_1 \leq \lambda \\ 1 + \alpha_2\lambda - (v_1 - \beta_1\lambda)\frac{1+\alpha_2}{1-\beta_1} - \delta[1 - \beta_2\lambda - (\delta v_1 + \alpha_1\lambda)\frac{1-\beta_2}{1+\alpha_1}]; & v_1 \in (\lambda, \frac{\lambda}{\delta}) \\ 1 + \alpha_2\lambda - (v_1 - \beta_1\lambda)\frac{1+\alpha_2}{1-\beta_1} - \delta[1 + \alpha_2\lambda - (\delta v_1 - \beta_1\lambda)\frac{1+\alpha_2}{1-\beta_1}]; & v_1 \geq \frac{\lambda}{\delta} \end{cases}
$$

Note that the region $v_1 \leq \lambda$ corresponds to v_1 with $f(v_1), f(\delta v_1) \geq \lambda$, $v_1 \in (\lambda, \frac{\lambda}{\delta})$ corresponds to v_1 with $f(v_1) < \lambda \wedge f(\delta v_1) > \lambda$, and $v_1 \geq \frac{\lambda}{\delta}$ corresponds to v_1 with $f(v_1), f(\delta v_1) \leq \lambda$.

To show that a unique solution v_1^* exists for $h(v_1) = 0$ for any given combination of $\lambda, \beta_{1,2}$ and $\alpha_{1,2}$ that satisfy the conditions in the proposition, it is sufficient to show that $h(v_1)$ is strictly decreasing, continuous, and there exist $v_1', v_1'' \in [\omega_1, \omega_2]$ with $h(v_1') \geq 0$ and $h(v_1'') \leq 0$.

It is easy to see that as $f(v_1)$ is continuous for all $v_1 \in [\omega_1, \omega_2]$, $h(v_1)$ is also continuous. It is also shown above that $\frac{df(v_1)}{dv_1} < 0$ for all v_1 (which also implies $\frac{d[\delta f(\delta v_1)]}{dv_1} < 0$ as $\delta > 0$). Furthermore, as $\delta < 1$, we can see that $|\frac{df(v_1)}{dv_1}| > |\frac{d[\delta f(\delta v_1)]}{dv_1}|$. That is, the added term in $h(v_1)$ decreases faster with v_1 than the subtracted term. Thus, $h(v_1)$ is strictly decreasing.

To show that there $h(v_1)$ can obtain both positive and negative values within domain $[\omega_1, \omega_2]$, it is wise to use $v_1 = 0$ and $v_1 = \omega_2$, since $h(v_1)$ is shown to be strictly decreasing. It is shown above that $f(0) > 0$. This implies $\delta f(0) < f(0)$ and thus, $h(0) > 0$. As we know that $\omega_2 \geq 0$, we can infer that $\delta\omega_2 \leq \omega_2$. This implies (with decreasing f) $f(\omega_2) \leq f(\delta\omega_2)$. Also shown above is $f(\omega_2) \leq 0$, These two inequalities then lead to $f(\omega_2) \leq \delta f(\delta\omega_2)$ and hence, $h(\omega_2) \leq 0$.

Function $h(v_1)$ is continuous, strictly decreasing, and maps to both positive and negative values from domain $[\omega_1, \omega_2]$. From this we can conclude that $h(v_1) = 0$ for exactly one value of v_1.

Now that we know a unique solution for $h(v_1) = 0$ exists for any given combination $\lambda, \beta_i, \alpha_1$, we can solve for each region of v_1. Note that different λ values mean different borders for the three regions of $h(v_1)$, so a change in λ results in a shift in the points where the slope of $h(v_1)$ changes. Consequently, in which of these three regions $h(v_1) = 0$ occurs will be conditional on the value of λ.

We will start with the region where $v_1 \geq \frac{\lambda}{\delta}$.

Solving $h(v_1^*) = 1 + \alpha_2\lambda - (v_1^* - \beta_1\lambda)\frac{1+\alpha_2}{1-\beta_1} - \delta[1 + \alpha_2\lambda - (\delta v_1^* - \beta_1\lambda)\frac{1+\alpha_2}{1-\beta_1}] = 0$ for v_1^* yields:

$$
v_1^* = \frac{1 - \beta_1 + \lambda(\alpha_2 + \beta_1)}{(1 + \alpha_2)(1 + \delta)}
$$

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For the solution to be in this region, $v_1^* \geq \frac{\lambda}{\delta}$ must hold. Replacing v_1^* with the expression above, then solving this condition for λ we find:

$$\lambda \leq \frac{\delta(1 - \beta_1)}{1 + \alpha_2 + \delta(1 - \beta_1)} = \underline{\lambda}$$

Similarly, solving $h(v_1^*) = 0$ for $v_1 \in (\lambda, \frac{\lambda}{\delta})$ yields:

$$v_1^* = \frac{(1 - \delta)(1 - \beta_1)(1 + \alpha_1) + \lambda[(1 + \alpha_1)(\alpha_2 + \beta_1) + \delta(1 - \beta_1)(\alpha_1 + \beta_2)]}{(1 + \alpha_1)(1 + \alpha_2) - \delta^2(1 - \beta_1)(1 - \beta_2)}$$

Replacing v_1^* with this expression in $v_1^* \in (\lambda, \frac{\lambda}{\delta})$ and solving for λ yields:

$$\lambda \in \left(\frac{\delta(1 - \beta_1)}{1 + \alpha_2 + \delta(1 - \beta_1)}, \frac{1 + \alpha_1}{1 + \alpha_1 + \delta(1 - \beta_2)} \right) = (\underline{\lambda}, \overline{\lambda})$$

Solving $h(v_1^*) = 0$ for $v_1 \leq \lambda$ yields:

$$v_1^* = \frac{1 + \alpha_1 - \lambda(\alpha_1 + \beta_2)}{(1 + \delta)(1 - \beta_2)}$$

Replacing v_1^* in this expression in $v_1^* \leq \lambda$ and solving for λ yields:

$$\lambda \geq \frac{1 + \alpha_1}{1 + \alpha_1 + \delta(1 - \beta_2)} = \overline{\lambda}$$

Note that the conditions on λ are the same for the upper border of Region 1 and the lower border of Region 2 ($\underline{\lambda}$) as well the upper border of Region 2 and the lower border of Region 3 ($\overline{\lambda}$). This implies that there is exactly one value $v_1^*(\lambda)$ for all $\lambda \in [0, 1]$. Furthermore, $\lim_{\lambda \nearrow \underline{\lambda}} v_1^* = \lim_{\lambda \searrow \underline{\lambda}} v_1^*$ and $\lim_{\lambda \nearrow \overline{\lambda}} v_1^* = \lim_{\lambda \searrow \overline{\lambda}} v_1^*$. This implies that $v_1^*(\lambda)$ is continuous.

Inserting the above functions for v_1^* in $x(v_1)$ yields:

$$x^* = \begin{cases} \frac{1}{1+\delta}\left(\frac{1 + \lambda\alpha_2}{1 + \alpha_2} - \frac{\lambda\delta\beta_1}{1 - \beta_1} \right); & \lambda \leq \underline{\lambda} \\ \frac{(1+\alpha_1)(1-\delta) + \lambda(\delta\alpha_1 + \alpha_1\alpha_2 + \alpha_2 + \delta\beta_2 + \delta^2\beta_1 - \delta^2\beta_1\beta_2)}{(1+\alpha_1)(1+\alpha_2) - \delta^2(1-\beta_1)(1-\beta_2)}; & \lambda \in (\underline{\lambda}, \overline{\lambda}) \\ \frac{1}{1+\delta}\left(\frac{1 - \lambda\beta_2}{1 - \beta_2} + \frac{\delta\alpha_1\lambda}{1 + \alpha_1} \right); & \lambda \geq \overline{\lambda} \end{cases}$$

$\square$

4 Comparative Statics and Discussion

Based on the solution presented above, this section will focus on pointing out the ceteris-paribus effects of the conceived fair division on the bargaining outcome x^*. Also, the

outcome division in the limit of patience and the ceteris-paribus effects of degrees of guilt and envy on the outcome division will be examined. For any partial derivative or limit value not explicitly shown, please refer to *Section 6: Appendix A*.

Following the terminology explained in the introduction, "moderate" fair points will correspond to $\lambda \in (\underline{\lambda}, \overline{\lambda})$, and "extreme" fair points will correspond to $\lambda \leq \underline{\lambda}$ or $\lambda \geq \overline{\lambda}$. Note that as $\underline{\lambda} = \frac{\delta(1-\beta_1)}{1+\alpha_2+\delta(1-\beta_1)} \in (0, 1/2)$ and $\overline{\lambda} = \frac{1+\alpha_1}{1+\alpha_1+\delta(1-\beta_2)} \in (1/2, 1)$ for all $\delta, \beta_i, \alpha_i$ allowed in the model, all of the three regions of λ are always nonempty intervals of real numbers. The moderate region $\lambda \in (\underline{\lambda}, \overline{\lambda})$ becomes smaller as $\delta \to 1$, but doesn't disappear.

Under the SPE strategies, if λ is in the moderate region, Player 1's own offers v_1^* give her a higher share than her fair share λ, and Player 2's offers δv_1^* give Player 1 a lower share than her fair share (as here $v_1^* \in (\lambda, \frac{\lambda}{\delta})$). Converse holds for Player 2. So with each offer Player 1 makes, Player 1(Player 2) feels guilt(envy), and with each offer Player 2 makes Player 1(Player 2) feels envy(guilt). For the extreme region where Player 1 has the larger fair share ($\lambda \geq \overline{\lambda}$), Player 1(Player 2) receives less(more) than her fair share in both her own offers v_1^* and the offers δv_1^* of Player 2 (as then $v_1^* \leq \lambda < \frac{\lambda}{\delta}$). Thus Player 1(Player 2) always feels envy(guilt) on the equilibrium path. Switching the roles of Players 1 and 2 shows that Player 1 (Player 2) always feels guilt (envy) for the low extreme region of $\lambda \leq \underline{\lambda}$. This is the underlying dynamic for the difference in solutions across the three regions. Consequently, the effect of λ on x^* will also depend on which region λ is in.

4.1 Outcome with Respect to the Fair Point

In the moderate region, we see that $\frac{dx^*}{d\lambda} > 0, \forall \lambda \in (\underline{\lambda}, \overline{\lambda})$. This implies that $\frac{d(1-x^*)}{d(1-\lambda)} > 0$, meaning that both players' outcome shares are increasing with their own shares in the conceived fair point, as long as this fair point remains moderate. A more interesting finding is for the extreme regions. For low fair shares of Player 1 ($\lambda \leq \underline{\lambda}$) we have:

$$\frac{dx^*}{d\lambda} \underset{>}{\lessgtr} 0 \iff \frac{\alpha_2}{1+\alpha_2} \underset{>}{\lessgtr} \frac{\delta\beta_1}{1-\beta_1}$$

This means if Player 1's guilt relative to Player 2's envy is high enough to satisfy the above condition (note that this is the case for most combinations $\{\delta, \alpha_i, \beta_i \in (0,1)\}$), then the outcome payoff of each player is decreasing in her own fair share as long as $\lambda \leq \underline{\lambda}$ still holds.

Similarly for the extreme regions with a high fair share of Player 1 ($\lambda \geq \overline{\lambda}$) we find:

$$\frac{dx^*}{d\lambda} \underset{>}{\lessgtr} 0 \iff \frac{\delta\alpha_1}{1+\alpha_1} \underset{>}{\lessgtr} \frac{\beta_2}{1-\beta_2}$$

That is, if Player 2's guilt relative to Player 1's envy is sufficiently high, then the

outcome payoff of each player is decreasing in her own share as long as $\lambda \geq \overline{\lambda}$ still holds.

Note that the condition for decreasing x^* is stricter for $\lambda \leq \underline{\lambda}$ than for $\lambda \geq \overline{\lambda}$. This is due to δ changing sides, and is caused by the first mover advantage. This difference disappears as $\delta \to 1$, in which case we can generalize the effect described above by denoting the envy of the high fair share player in an extreme case α_h and the guilt of the low fair share player β_l as follows.

$$\lim_{\delta \to 1} \frac{dx^*}{d\lambda} \begin{cases} > 0; & \lambda \in (\lim_{\delta \to 1} \underline{\lambda}, \lim_{\delta \to 1} \overline{\lambda}) \\ \lessgtr 0; & \lambda \notin (\lim_{\delta \to 1} \underline{\lambda}, \lim_{\delta \to 1} \overline{\lambda}) \wedge \frac{\alpha_h}{1+\alpha_h} \lessgtr \frac{\beta_l}{1-\beta_l} \end{cases}$$

With $\lim_{\delta \to 1} \underline{\lambda} = \frac{1-\beta_1}{2+\alpha_2-\beta_1}$ and $\lim_{\delta \to 1} \overline{\lambda} = \frac{1+\alpha_1}{2+\alpha_1-\beta_2}$ That is, with highly asymmetric fair points conceived by highly patient bargainers, the outcome share of each player decreases (increases) with their own fair share if the guilt of the low player is sufficiently high (low) relative to the envy of the high player.

4.2 Outcome against the Fair Point

Another important finding for moderate fair points is that $x^* > \lambda$ if $\lambda \in (\underline{\lambda}, \overline{\lambda})$, but $\lim_{\delta \to 1} x^* = \lambda$ if $\lambda \in [\lim_{\delta \to 1} \underline{\lambda}, \lim_{\delta \to 1} \overline{\lambda}]$. That is, the outcome payoff of Player 1 is higher than her fair share, but the outcome division converges to exactly the fair division as the bargainers become more patient if the fair division is symmetric enough. This shows us that with moderate fair points, Player 1 receiving more than her fair share is caused solely by the first mover advantage. For extreme fair points however, this is not the case. For $\lambda \leq \underline{\lambda}$, we know that $x^* \geq \delta\lambda > \lambda$. Furthermore, $\lim_{\delta \to 1} x^* > \lambda$ for all $\lambda < \lim_{\delta \to 1} \underline{\lambda}$. This means for any $\lambda < \underline{\lambda}$, the outcome share of Player 1 is strictly greater than her fair share, and it converges to a value strictly above her fair share. Similarly for $\lambda \geq \overline{\lambda}$, we know that $x^* \leq \lambda$ (with $x = \lambda$ for $\lambda = \overline{\lambda}$). We can also compute that $\lim_{\delta \to 1} x^* < \lambda$ for all $\lambda > \lim_{\delta \to 1} \overline{\lambda}$. Thus, for any $\lambda > \overline{\lambda}$, the outcome share of Player 1 is strictly smaller than her fair share, and it converges to a value strictly below her fair share. Combining these three observations we have:

$$x^* \begin{cases} > \lambda; & \lambda < \overline{\lambda} \\ = \lambda; & \lambda = \overline{\lambda} \\ < \lambda; & \lambda > \overline{\lambda} \end{cases}$$

and

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$$\lim_{\delta \to 1} x^* \begin{cases} > \lambda; & \lambda < \lim_{\delta \to 1} \underline{\lambda} \\ = \lambda; & \lambda \in [\lim_{\delta \to 1} \underline{\lambda}, \lim_{\delta \to 1} \overline{\lambda}] \\ < \lambda; & \lambda > \lim_{\delta \to 1} \overline{\lambda} \end{cases}$$

From the above, the most important inferences we can make are that for moderate fair points, first mover receivers a slightly higher share than in the fair division, but this excess payoff disappears with patience. For extreme fair points, the player with the high fair share receives less than her fair share and the one with the low fair share receives more. For every given combination of utility parameters $\delta < 1, \alpha_i$ and β_i, the only fair point for which the outcome is exactly equal to the fair division is $\overline{\lambda}$.

Finally, we can compute that $\lambda < \lim_{\delta \to 1} \underline{\lambda} \Rightarrow \lim_{\delta \to 1} x^* < \frac{1}{2}$ and $\lambda > \lim_{\delta \to 1} \overline{\lambda} \Rightarrow \lim_{\delta \to 1} x^* > \frac{1}{2}$. Combining this with the above argument about the limits, we conclude that $\lambda < \frac{1}{2} \Rightarrow \lim_{\delta \to 1} x^* \in [\lambda, \frac{1}{2})$, $\lambda > \frac{1}{2} \Rightarrow \lim_{\delta \to 1} x^* \in (\frac{1}{2}, \lambda]$, and $\lambda = \frac{1}{2} \Rightarrow \lim_{\delta \to 1} x^* = \frac{1}{2}$. That is, in the limit of patience, the outcome division is always between the fair point and the equal division.

4.3 Outcome with Respect to Guilt and Envy

Holding all else constant, we now observe the effects of the weights that the bargainers attach to the fair point on the outcome shares of the bargainers. The expectation is that each individual weight ($\alpha_{1,2}, \beta_{1,2}$) only affects the outcome with values of λ for which it is relevant. Relevance of a weight means that it is experienced by the corresponding bargainer in some stage under the equilibrium strategies.

Observing the derivatives of outcome x^* with respect to α_1 and β_2, we find the following.

$$\frac{dx^*}{d\alpha_1}, \frac{dx^*}{d\beta_2} \begin{cases} = 0; & \lambda \leq \underline{\lambda} \\ > 0; & \lambda > \underline{\lambda} \end{cases}$$

Outcome share of Player 1 is increasing in her own envy and Player 2's guilt if her fair share is at least moderately high. Note that if the fair point is in the lower extreme region($\lambda \leq \underline{\lambda}$), the outcome is independent of Player 1's envy and Player 2's guilt. This is because for $\lambda \leq \underline{\lambda}$ the fair share of Player 1 is so low that even Player 2's (lower) offers with her equilibrium strategy don't give Player 1 a lower share than in the fair division. As a result, Player 1 never experiences envy and Player 2 never experiences guilt on the equilibrium path.

Important to note is that α_1 and β_2 are determinant factors (enter the function) for $\overline{\lambda}$ but not for $\underline{\lambda}$. As $\frac{d\overline{\lambda}}{d\alpha_1}, \frac{d\overline{\lambda}}{d\beta_2} > 0$, an increase in α_1 or β_2 can mean that a given high extreme fair point ($\lambda \geq \overline{\lambda}$) now becomes a moderate fair point. However, since in both

of these regions x^* is increasing in α_1 and β_2, the above condition of positivity holds for all $\alpha_1, \beta_2 \in (0,1)$.

When we look at x^* with respect to α_2, β_1 we see the following.

$$\frac{dx^*}{d\alpha_2}, \frac{dx^*}{d\beta_1} \begin{cases} < 0; & \lambda \leq \overline{\lambda} \\ = 0; & \lambda > \overline{\lambda} \end{cases}$$

Outcome share of Player 1 decreases with her own guilt, and Player 2's envy. Similarly, the guilt of Player 1 and the envy of Player 2 are only relevant if the fair share of Player 1 is low enough. If the fair division assigns Player 1 a very high share ($\lambda \geq \overline{\lambda}$), Player 1 never receives more than her fair share on the equilibrium path and thus, never feels guilty (β_1 is irrelevant). Same logic applies for Player 2's envy (α_2 is irrelevant).

Also analogous to the case for α_1 and β_2, we can see that only $\underline{\lambda}$ depends on α_2, β_1 (with $\frac{dx^*}{d\alpha_2}, \frac{dx^*}{d\beta_1} < 0$). Consequently, an extremely low given fair point ($\lambda \leq \underline{\lambda}$) can become moderate with increasing α_2 or β_1. Still, as the outcome share of Player 1 decreases in both these regions, the above positivity conditions continue to hold.

Combining the two observations above, we can conclude that the outcome payoff of each bargainer is increasing in her own envy and the opponent's guilt, and decreasing in her own guilt and the opponent's envy, given that the fair point is such that each weight is relevant according to the description above. In other words, feeling a high degree of envy strengthens the bargaining position of a player and feeling a high degree of guilt weakens it.

Finally, note that in the moderate region, the effects of guilt and envy are due to the first mover advantage and disappear as $\delta \rightarrow 1$ (recall that $\lim_{\delta \to 1} x^* = \lambda$ for $\lambda \in [\lim_{\delta \to 1} \underline{\lambda}, \lim_{\delta \to 1} \overline{\lambda}]$ and λ is considered a given constant in this subsection). For extreme regions, however, $\frac{dx^*}{d\alpha_1}$ and $\frac{dx^*}{d\beta_2}$ converge to positive values and $\frac{dx^*}{d\alpha_2}$ and $\frac{dx^*}{d\beta_1}$ converge to negative values when they are relevant. Therefore, in limit of patience, the effects of weight given to fairness remain only in extreme regions.

As we know $\lim_{\delta \to 1} x^* > \lambda$ for $\lambda < \lim_{\delta \to 1} \underline{\lambda}$ with any given α_i, β_i and x^* decreasing with α_2, β_1 in this region, we can conclude that $\lambda < \lim_{\delta \to 1} \underline{\lambda} \Rightarrow \lim_{\delta \to 1} \frac{d(|x^* - \lambda|)}{d\alpha_2} < 0$, and $\lim_{\delta \to 1} \frac{d(|x^* - \lambda|)}{d\beta_1} < 0$. Both of these limits are equal to zero for any $\lambda \geq \lim_{\delta \to 1} \underline{\lambda}$ as then α_2 and β_1 have no effect on x^* in limit. With the same argument, we can find that $\lambda > \lim_{\delta \to 1} \overline{\lambda} \Rightarrow \lim_{\delta \to 1} \frac{d(|x^* - \lambda|)}{d\alpha_1} < 0$ and $\lim_{\delta \to 1} \frac{d(|x^* - \lambda|)}{d\beta_2} < 0$. Analogously to above, we see that these two limits are equal to zero in any other region.

We can generalize the statements above as follows.

$$\lim_{\delta \to 1} \frac{d(|x^* - \lambda|)}{d\alpha_i} \leq 0 \wedge \lim_{\delta \to 1} \frac{d(|x^* - \lambda|)}{d\beta_i} \leq 0, \forall i \in \{1, 2\}$$

That is, in limit of patience, higher weight of any kind attached to fairness leads to outcomes that are not further from (possibly closer to) the fair division.

5 Concluding Remarks

In searching for the effects of changes in the determinant factor for the conceived fair division, fairness motivated agents were modeled as agents who suffer utility loss based on the difference between the share they actually receive and the share that they would have received in the division they consider fair. The agents of interest were the ones who agree on what constitutes a fair division in any given scenario (i.e. judge according to the same fairness principle), and still strictly prefer a higher share of the pie with any given division.

When such two agents engage each other in an open-ended alternating offer bargaining setting, it is shown that the effect of the fair division depends structurally on the symmetry of this fair point. If the bargainers consider it fair that they receive similar shares (moderate fair points), the monetary payoff of an individual in the bargaining outcome increases with her fair share. The relative attractiveness of her own offers increases and this gives her incentive to reject the opponent's offers, making her capable of making additional credible threats. This effect continues unless her fair share is even higher than the payoff that she receives from her own offers (or even lower than the payoff she receives from the opponent's offers). With these highly asymmetric (extreme) fair divisions, the outcome payoff of a player is decreasing in her fair share if the degree of guilt that the low fair share player feels is high relative to the degree of envy that the high fair share player feels. This is because an even further decrease in the fair share of the low player decreases her utility from both own and the opponent's offers by the same absolute amount via guilt, giving the low player incentive to reject and make her own offer in the next round.

Also shown is that for highly asymmetric fair points, the high fair share player gets less than her fair share, and the low fair share player more than hers. For highly patient players (in limit), the outcome converges to exactly the fair division if the fair division is highly symmetric, and is between the fair division and 50:50 if the fair division is highly asymmetric. Feeling a high degree of guilt decreases the bargaining power of a player, and a high degree of envy increases it. For highly patient bargainers, both degrees of "attachment" to fairness (guilt and envy) lead to an outcome that is closer to the fair division.

Often in real world bargaining situations, individuals take the fairness of the outcome into consideration along with their monetary payoff. Furthermore in many situations, there exists some benchmark variable for what would be the fair division (such as how much input each player provided for a jointly produced amount, how costly it was for each individual to take part in the process in the first place, how much each individual needs the good, etc.), as opposed to simply considering it fair that all players receive the same amount. Such bargains often take place in an environment where the individuals disregard or have uncertainty about the time limit of the bargaining process (a behavior which can be captured by the infinite horizon bargaining protocol). Al-

though the evidence for the exact effect of fairness motivation is mixed in experimental literature, especially when simulating the infinite horizon, the existence of it as a determinant factor on bargaining behavior has been strongly supported. Thus, it is indeed of importance to study the behavior of fairness motivated agents in an infinite setting. Further steps in the area might include bargaining between players who vary in their fairness principles in an infinite-horizon setting (as individuals often tend to support fairness concepts that favor their position), or incomplete information regarding the fair point, as the input each bargainer has provided is rarely measurable in practice.

6 Appendix A

6.1 Partial Derivatives

Partial derivatives of the SPE outcome share of Player 1:

$$\frac{dx^*}{d\lambda} = \begin{cases} \frac{1}{1+\delta}\left(\frac{\alpha_2}{1+\alpha_2} - \frac{\delta\beta_1}{1-\beta_1}\right); & \lambda \leq \underline{\lambda} \\ \frac{\delta\alpha_1 + \alpha_1\alpha_2 + \alpha_2 + \delta\beta_2 + \delta^2\beta_1 - \delta^2\beta_1\beta_2}{(1+\alpha_1)(1+\alpha_2) - \delta^2(1-\beta_1)(1-\beta_2)} > 0; & \lambda \in (\underline{\lambda}, \overline{\lambda}) \\ \frac{1}{1+\delta}\left(\frac{\delta\alpha_1}{1+\alpha_1} - \frac{\beta_2}{1-\beta_2}\right); & \lambda \geq \overline{\lambda} \end{cases}$$

The following are for $D := (1 + \alpha_1)(1 + \alpha_2) - \delta^2(1 - \beta_1)(1 - \beta_2)$ and $E := (1 + \alpha_1)(1 - \delta) + \lambda(\delta\alpha_1 + \alpha_1\alpha_2 + \alpha_2 + \delta\beta_2 + \delta^2\beta_1 - \delta^2\beta_1\beta_2)$. Note that $\lambda D < E$ if $\lambda \in (\underline{\lambda}, \overline{\lambda})$.

$$\frac{dx^*}{d\alpha_1} = \begin{cases} 0; & \lambda \leq \underline{\lambda} \\ \frac{(1-\delta+\lambda(\delta+\alpha_2))D - (1+\alpha_2)E}{D^2} > 0; & \lambda \in (\underline{\lambda}, \overline{\lambda}) \\ \cdot\frac{\delta\lambda}{(1+\delta)(1+\alpha_1)^2} > 0; & \lambda \geq \overline{\lambda} \end{cases}$$

$$\frac{dx^*}{d\alpha_2} = \begin{cases} \frac{\lambda-1}{(1+\delta)(1+\alpha_1)^2} < 0; & \lambda \leq \underline{\lambda} \\ \frac{(1+\alpha_1)(\lambda D - E)}{D^2} < 0; & \lambda \in (\underline{\lambda}, \overline{\lambda}) \\ 0; & \lambda \geq \overline{\lambda} \end{cases}$$

$$\frac{dx^*}{d\beta_1} = \begin{cases} \frac{-\delta\lambda}{(1+\delta)(1-\beta_1)^2} < 0; & \lambda \leq \underline{\lambda} \\ \frac{\delta^2(1-\beta_2)(\lambda D - E)}{D^2} < 0; & \lambda \in (\underline{\lambda}, \overline{\lambda}) \\ 0; & \lambda \geq \overline{\lambda} \end{cases}$$

$$\frac{dx^*}{d\beta_2} = \begin{cases} 0; & \lambda \leq \underline{\lambda} \\ \frac{\lambda\delta(1-\delta\beta_1)D - \delta^2(1-\beta_1)E}{D^2} > 0; & \lambda \in (\underline{\lambda}, \overline{\lambda}) \\ \frac{1-\lambda}{(1+\delta)(1-\beta_2)^2} > 0; & \lambda \geq \overline{\lambda} \end{cases}$$

Partial derivatives of region borders:

$$\frac{d\underline{\lambda}}{d\delta} = \frac{(1-\beta_1)(1+\alpha_2)}{(1+\alpha_2+\delta(1-\beta_1))^2} > 0$$

$$\frac{d\overline{\lambda}}{d\delta} = -\frac{(1-\beta_2)(1+\alpha_1)}{(1+\alpha_2+\delta(1-\beta_1))^2} < 0$$

$$\frac{d\underline{\lambda}}{d\alpha_2} = -\frac{\delta(1-\beta_1)}{(1+\alpha_2+\delta(1-\beta_1))^2} < 0$$

$$\frac{d\underline{\lambda}}{d\beta_1} = -\frac{\delta(1+\alpha_1)}{(1+\alpha_2+\delta(1-\beta_1))^2} < 0$$

$$\frac{d\overline{\lambda}}{d\alpha_1} = \frac{\delta(1-\beta_2)}{(1+\alpha_2+\delta(1-\beta_1))^2} > 0$$

$$\frac{d\overline{\lambda}}{d\beta_2} = \frac{\delta(1+\alpha_1)}{(1+\alpha_2+\delta(1-\beta_1))^2} > 0$$

$$\frac{d\underline{\lambda}}{d\alpha_1} = \frac{d\underline{\lambda}}{d\beta_2} = \frac{d\overline{\lambda}}{d\alpha_2} = \frac{d\overline{\lambda}}{d\beta_1} = 0$$

6.2 Limits as $\delta \to 1$

Region borders:

$$\lim_{\delta\to 1} \underline{\lambda} = \frac{1-\beta_1}{2+\alpha_2-\beta_1} \in (0, \frac{1}{2})$$

$$\lim_{\delta\to 1} \overline{\lambda} = \frac{1+\alpha_1}{2+\alpha_1-\beta_2} \in (\frac{1}{2}, 1)$$

Outcome division:

$$\lim_{\delta\to 1} x^* \begin{cases} \frac{1}{2}\left(\frac{1+\lambda\alpha_2}{1+\alpha_2} - \frac{\lambda\beta_1}{1-\beta_1}\right) > \lambda; & \lambda < \lim_{\delta\to 1} \underline{\lambda} \\ = \lambda; & \lambda \in [\lim_{\delta\to 1} \underline{\lambda}, \lim_{\delta\to 1} \overline{\lambda}] \\ \frac{1}{2}\left(\frac{1-\lambda\beta_2}{1-\beta_2} + \frac{\lambda\alpha_1}{1+\alpha_1}\right) < \lambda; & \lambda > \lim_{\delta\to 1} \overline{\lambda} \end{cases}$$

7 Appendix B (Abstract)

7.1 English

This paper presents a model of infinite horizon alternating offer bargaining with asymmetric divisions as conceived fair points, where both bargainers consider the same division fair. The behavior of the bargaining outcome with respect to varying fair points is studied. A higher fair share can lead to a higher or lower outcome share for a player, depending on how asymmetric the fair division is. With highly patient bargainers, the fair point can be achieved as the outcome if it is highly symmetric. Feeling guilt weakens and feeling envy strengthens the bargaining position of each player. With highly patient players, weight attached to fairness brings the solution closer to the fair point .

7.2 Deutsch

Diese Arbeit präsentiert ein offenes Verhandlungsmodel mit asymmetrichen Aufteilungen als erachtete Fairpunkte, wobei beide Spieler dieselbe Aufteilung fair finden. Das Verhalten des Verhändlungsergebnisses in Bezug auf verschiedene Fairpunkte ist zu beobachten. Ob ein höherer Fairanteil von einem Spieler zu einem höheren oder kleineren Ergebnisanteil führt hängt von der Symmetrie des Fairpunktes ab. Mit sehr geduldigen Spielern kann der Fairpunkt als Verhandlungsergebnis erreicht werden, wenn er sehr symmetrisch ist. Für jeden Spieler, Schuld schwächt die Verhandlungsposition und Neid stärkt die Verhandlungsposition. Mit sehr geduldigen Spielern, das Verhandlungsergebnis bewegt sich näher zum Fairpunkt mit einer Erhhung von Schuld oder Neid.

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