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Philipp Hergovich, BSc MSc

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Betreut von / Supervisor:	Dr. Michael Reiter
	Univ.-Prof. Dipl.-Vw. Monika Gehrig-Merz,PhD

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1 Introduction

Economics is a diverse field, with potential research questions covering a variety of topics. This dissertation combines three papers which are diverse in the subject studied, but linked in the consideration of distribution. In all 3 papers, the issue studied is how ex ante heterogeneous agents are affected by changes in their respective economic environments.

The heterogeneity among the agents varies from different household characteristics and abilities to hold certain types of assets in the first paper, over the distinction between firms and workers in the second paper, to members of different social groups in a network of relationships in the third paper. Naturally, these diverse forms of heterogeneity allow for a wide range of changing economic aspects to be studied in the respective papers. While it is on different asset structures in the second chapter of the dissertation, the focus shifts more towards technological change in the later chapters, taking the form of improvements in production processes in the third and online dating in the final chapter of this dissertation.

Tools that have been developed in the economics literature can be employed to analyze all of these environments and many more. Therefore this economics dissertation is cumulative in the truest meaning of the word, as it combines numerous topics and methods and looks at the question of equity and distribution from 3 very distinct angles. It consists of three essays, each of which is a self contained contribution to the literature.

1.1 The Redistributive Effects of Monetary Policy

Motivated by intense debates about monetary policy in the wake of the Great Recession, the second chapter (joint with Michael Reiter), studies the redistributive effects of monetary policy.[1] We use a large scale overlapping generations model with housing and New-Keynesian elements in order to determine how heterogeneity affects the conduct of monetary policy and how this depends on the underlying asset structure. We calibrate the model to qualitatively match empirical life cycle paths and key macroeconomic indicators.

Our model creates strong and long lasting redistributive effects of monetary policy and demand shocks. While in a baseline New-Keynesian model the central bank is able to perfectly stabilize output and inflation when facing demand shocks (a fact known as "divine coincidence"), this is no longer true in the presence of heterogeneity. We show that there are significant trade-offs from pure inflation targeting, especially in the long run. These long lasting effects are caused by the Central Bank adjusting the interest rates to counteract inflation, thus causing redistribution between debtors and lenders, but also between young and old generations. We put special emphasis on the structure

[1] This chapter builds and greatly extends my master thesis (Hergovich, 2014).

of the bonds that are traded in the economy. We consider 4 distinct cases, in which the bonds can be short term or long term with respect to how long the interest rate remains constant, and nominal or inflation protected. These different bond structures turn out to be important at the individual level, but rather innocuous for economic aggregates.

1.2 The Price of Capital, Factor Substitutability and Corporate Profits

The third chapter (joint work with Monika Merz) of the dissertation studies the effects of cheaper capital and higher substitutability on key macroeconomic variables like the labor share and corporate profits. There is an ongoing debate about the possibility of human labor being at least partially replaced by machines and computer programs, which will have consequences in how we organize societies and politics. This paper aims at separating the effects of two trends that each could cause firms to employ more capital instead of labor. In reality, these two developments occur together and often get mixed in discussion. One feature is that capital goods are increasingly able to perform tasks that previously required a human being. The other feature of production we study in that chapter is the decreasing price of investment goods, such that it gets relatively cheaper to employ capital instead of labor. Our analysis shows that while the first channel generally lowers employment and depresses wages, the other effect might actually counter these adverse developments for workers.

To analyze our research question, we build a model of the firm which faces a frictional labor market. The theoretical nature of the analysis allows us to explicitly vary the price of capital and the degree of factor substitutability and study the results separately. We find that firms will choose a more capital intensive input mix if capital becomes cheaper or better at substituting labor. However, employment and wages increase when capital becomes cheaper and fall with higher input substitutability. The reason is that capital and labor are complements in production, thus with cheaper capital goods, firms also employ more labor while at the same expanding its capital stock. However, if the inputs become less complementary, the firm chooses to substitute labor for capital, thus lowering employment and wages. The results for profit shares are vice-versa, rising in substitutability and falling in cheaper prices of capital. Increased substitutability also increases variability in the economy, as the firm can react more flexibly to shocks. We contrast our findings to empirical evidence and find that the variability of profits has increased over time, an effect our model predicts when capital and labor become easier to substitute. Also, labor share and profit share are negatively correlated over time, which hints at the redistribution detected by our theoretical model.

1.3 The Strength of Absent Ties: Social Integration via Online Dating

People used to marry within their close social networks, like schoolmates, neighbors, colleagues. The emergence of online dating has changed this pattern. It enables new social interactions and allows relationships to be formed among individuals who otherwise would not have had the chance to interact. As private networks tend to be less diverse than society as a whole, there are potential gains in the equality of outcomes, as more diverse marriages can have economic consequences, when it comes to connections and social capital.

The final chapter of this dissertation studies these phenomena. Together with my coauthor Josue Ortega, I build a theoretical model of the marriage market to study how Online Dating affects the outcome of such markets and test some of its predictions against empirical data. The model is closely related to the *Stable Marriage Problem*, which is heavily studied in the field of matching. An equal number of female and male agents are present in a society and each of them is looking for a spouse. Marriages are defined as the outcome of a matching algorithm, details of which are provided in the main chapter. We separate the agents into various groups, which for the sake of exposition and to develop testable predictions we often associate with race. The novel feature is the network structure in the society, which implies that agents can only select from the pool of partners to which they are connected. Agents are well connected within their own group, but links to these other groups are scarce. We model online dating as an increase in the probability of being connected to agents from another race.

The outcome of these marriage markets is judged against 3 welfare indicators, the number of marriages, the goodness of fit among a couple and the share of interracial marriages. We show that each of these measures can be decreased when additional connections are established. However, when considering the average effects, online dating has positive effects on welfare. We are able to provide analytic results for the share of interracial marriages and rely on simulation results for the other two welfare indicators.

Using US data, we find that the relationship between online dating and interracial marriages remains significantly positive, even after controlling for a variety of confounding factors. We use micro level data on the socioeconomic status of individuals and regress a dummy variable whether or not a person is in an interracial marriage on the availability of broadband internet in the state of residence 3 years prior to the observations. The positive result gives weight to our theory. We use broadband availability as a proxy variable for online dating, as we are not able to observe this variable on a wide scale. Thomas (2018) uses a survey specifically designed to study dating and marriage behavior and is able to explicitly relate online dating and the propensity of a marriage being interracial. His results are well aligned with the findings of chapter 4, which provides further support in favor of the mechanism.

2 Housing and the Redistributive Effects of Monetary Policy

2.1 Introduction

Through changes in the nominal interest rate, monetary policy redistributes wealth between lenders and borrowers of nominal assets. Heterogeneity in households' asset position therefore matters for the monetary transmission mechanism, which is the topic of a fast growing literature. We contribute to this literature looking at the distributional effects of monetary policy from a somewhat different perspective: does household heterogeneity in asset positions limit the ability of the central bank to stabilize the economy against demand shocks? More specifically, we address two concerns. Does redistribution affect the macroeconomic aggregates in a way that counteracts the stabilization policy? Does redistribution increase instability at the household level, even if the aggregate economy gets stabilized?

To analyze these questions we build a general equilibrium model that combines three features that we think are important: the life-cycle structure of households, owner-occupied housing with a down payment constraint on mortgages, and differential access to asset markets across household types. The life-cycle structure together with a housing choice helps to generate a realistic degree of gross asset positions. For most middle class households, gross positions are primarily given by a house and a mortgage. Households hold a long position in housing, which is a long-lived real asset, and a short position in the form of a mortgage, which is denoted in nominal terms. Such a household can have a small net worth and nevertheless be heavily exposed to interest rate risk. This risk then depends on whether the mortgage has a variable interest rate (what we call a "short-term" asset) or a fixed interest rate ("long-term asset"). The firm side of the model has a standard New Keynesian structure with Calvo price rigidity, which makes it a type of HANK (heterogeneous agent New Keynesian) model.

We model the differential access to asset markets by assuming three types of households:

1. Low-skill hand-to-mouth consumers with a finite life, who live in rented housing and hold no assets. In old age, they live from the benefits of a pay-as-you-go pension system.
2. Middle class households with a finite life, who decide about rented versus owner-occupied housing, and save in bonds for retirement, in addition to the pension system.

3. A representative, infinitely lived dynasty of "capitalists", who all live in owner-occupied houses, own the firms as well as the rental houses.

Since the type of assets traded is essential for the redistributive effect of monetary policy, we study not only short-term versus long-term debt, but also nominal versus real (inflation indexed) debt. This gives four variants of the model, depending on whether bonds are short-term nominal, long-term nominal, short-term real or long-term real. Mostly for technical reasons, we have only one type of debt in any version of the model.[1] This may not be as restrictive as it sounds. The data (Bouyon, 2017, Figure 2) show large variations in the rate of variable-rate mortgages across European countries, ranging between 15 percent in Germany and almost 100 percent in Spain (until 2012). This suggests that the choice between fixed and variable rate is more determined by institutional factors in each country than by individual portfolio considerations.

Our main finding is that household heterogeneity makes monetary stabilization policy harder. This is true even if the economy is only hit by demand shocks, and the demands shocks are modeled such that the "divine coincidence" (simultaneous stabilization of both inflation and output under demand shocks) holds in the representative household setup. More specifically, we find that the policy trade-offs are roughly the same in the benchmark model as in a representative agent version of the model, as long as they are measured in terms of second moments of *detrended* variables. In particular, the "divine coincidence" continues to hold approximately for detrended variables. However, it fails significantly if we also consider fluctuations at lower than business cycle frequencies, for reasons different from the ones already noted in the literature Alves (2014). These fluctuations have strong effects on fluctuations in welfare. For example, strict inflation targeting reduces the variability of lifetime welfare if assets are long-term, but not if they are short-term. Interest rate smoothing reduces the variability of lifetime welfare. Underlying these findings is the fact that demand shocks, similar to monetary policy shocks, generate substantial redistribution between different cohorts and household types. This redistribution depends on the type of bond traded as well as on the concrete policy rule. Again, household heterogeneity and asset type have a small impact on the impulse responses of aggregate output and inflation to both shocks, much smaller than the width of the confidence bands in the empirical estimates of these impulses responses. However, some significant differences arise if one considers total fluctuations, not just fluctuations of conventionally detrended variables.

That the redistribution channel has such a limited effect on macroeconomic aggregates at business cycle frequencies may be the consequence of some crucial model assumptions. With several dimensions of heterogeneity, the effects of redistribution become very complex, and we want to clarify the discussion by abstracting from some potentially important mechanisms. In contrast to Kaplan et al. (2018), who stress the importance of fiscal

[1] To study monetary policy, we need a quarterly model period. With an economic life of 60 years, we have 240 cohorts, and in total the model has more than 1400 variables, so that we solve the model by linearization. The approximate solution is therefore of the certainty-equivalence type, and we cannot study portfolio choice among different types of financial assets such as short-term versus long-term debt.

policy, we reduce the role of fiscal policy to a minimum: there is no government debt, the government only runs a pay-as-you-go pension system. There are no trading frictions in our model for physical capital or housing. We also abstract from financial frictions that would give a role to the net worth of entrepreneurs in investment, a mechanism that is prominent since Carlstrom and Fuerst (1997).

Although we do not try to compute optimal policy, several policy implications emerge from our analysis. Fighting inflation aggressively is the right policy against demand shocks if long-term debt positions, such as mortgages, have fixed rather than variable interest rates, because this reduces the distributional impact of nominal interest rate movements. With variable interest rates, aggressive monetary policy generates large random redistributions. Regulatory changes that motivate banks and households to move towards fixed rate mortgages are therefore welcome from a monetary policy point of view. Since we find that the stabilization of macroeconomic aggregates does not go hand in hand with the stabilization of individual utility or welfare, our results raise the question of what is the exact objective of monetary policy.

2.1.1 Related Literature

The redistributive consequences of inflation are first described in Doepke and Schneider (2006a), where the effects of inflation through the channel of nominal assets are studied. The biggest beneficiary of inflation is the government, since it usually has a relatively large and negative asset position. Next to the government, young households gain from surprise inflation, while elderly households lose. This is because younger households are usually more indebted, either by student loans or mortgages, and thus inflation reduces their real debt burden. Our model replicates this empirical feature. For an extensive empirical study for the euro area countries see Adam and Zhu (2016). A theoretical model motivated by these observations is presented in Doepke and Schneider (2006b), where the redistribution by inflation is modeled exogenously.

Following up on these empirical findings, there is now a growing literature on the monetary transmission mechanism in the presence of heterogeneous agents. The model in the literature that is closest to ours is probably Garriga et al. (2017). This is a model of monetary policy with two types of households, house owners and capital owners. Like us, they analyze different mortgage types, namely fixed and adjustable rate mortgages. The authors consider a standard monetary policy shock as well as a shock to the target level. Their model has a more elaborate treatment of mortgages and down payment constraints. Redistribution happens between house and capital owners. In contrast, most of the redistribution in our model takes place between middle class households of different ages. The main focus of our analysis is not the transmission of monetary shocks, but the ability of monetary policy to insulate the economy from demand shocks. To the best of our knowledge, this differentiates our paper from all the papers in this literature. The empirical importance of the mortgage type was driven home by Di Maggio et al. (2017), who find that a reduction in interest rate has much stronger effects on consumption (mostly on durables) when mortgages are adjustable-rate.

Our paper fits into the HANK (Heterogeneous Agent New Keynesian) literature pioneered by Kaplan et al. (2018). Our model is somewhat orthogonal to theirs in that we have features that they miss (finite lifetime, large gross positions with housing and mortgages), but abstract from features that they stress as important such as fiscal policy. Perhaps the key difference to their model as well as the model of Luetticke (2018) is that real assets are "illiquid" in our model only at the aggregate level, in the form of capital adjustment costs, but costlessly tradeable at the individual level. This is clearly unrealistic for housing, but not so unrealistic for stocks. In the other papers, individual trades are subject to adjustment costs, which makes them illiquid at an individual level. For this reason, the redistribution channel that we highlight here is substantially different from the mechanisms stressed in those two papers. Interestingly, Luetticke (2018) also finds that the output response to a monetary shock is similar to the representative agent model, also the composition between consumption and investment changes substantially.

Another paper that can be considered as complementary to ours is Gornemann et al. (2016). While we focus on the role of the life cycle and housing choice, their model features infinitely lived households with heterogeneous skill levels facing unemployment risks. They study the output-inflation stabilization trade-off under a mixture of aggregate shocks. We focus on stabilizing demand shocks, where the relevant trade-off is not output versus inflation, but rather the stabilization of aggregates versus the stability of individual welfare.

Auclert (2017) decomposes the effects of monetary policy in what stems from the revaluation of nominal assets and the change in real assets and liabilities (which include consumption and wage income). A major result of Auclert, which is in line with our model is that longer maturities insure the agents better against unhedged interest rate exposure, which in the language of our model means that longer maturity structures or lower degree of asset nominality leads to lower variances in the consumption responses to a shock. Also for us the indirect effects brought about by heterogeneity are large, although the aggregate remains relatively unaffected. A central feature of Auclert's analysis are the UREs, the unhedged interest rate exposures, which he argues are the most important measure when talking about redistributive effects of monetary policy. UREs are defined as difference between maturing assets and liabilities. It will remain true in our model, that when households hold short term bonds where the entire value matures each period, redistributive effects tend to be larger. In the eyes of our model, maturity is essentially the same as an occasion to adjust interest rates on a bond, as there are no financial frictions.

A main difference of our paper to other general equilibrium models in this literature is the assumption of life cycle households.[2] Using US data, Wong (2018) finds that the bulk of the consumption response to a monetary policy shock comes from young households. Our model is in line with these findings. She also shows that the consumption response is greater for households that refinance their mortgage after a decrease in interest rates. In our model, all households readjust their mortgage after a shock, since there are no adjustment costs to doing this.

[2]Existing life cycle NK models focus on other questions, such rational asset price bubbles Gali (2014, 2017) or long-run real interest rates Eggertsson and Mehrotra (2017).

A paper that focuses on the redistributive effects of inflation in a life cycle model is Doepke et al. (2015). In this paper the authors identify the asset positions from the Survey of Consumer finances, and calculate how various inflation shocks (anticipated versus unanticipated) affect the real wealth of agents. They find that unexpected inflation generates large losses for older households who hold positions in long-term nominal assets, and large gains for middle-class homeowners with outstanding mortgages. They do not study the causes of inflation, but rather feed the distributional effects into a life cycle model, by directly altering the assets of each cohort, and then study the aggregate implications in the housing market. Our model generates similar redistribution effects as theirs, but our focus is on what this means for monetary policy.

Brunnermeier and Sannikov (2012) go a step further than the rest of the literature by emphasizing not only the redistribution of wealth, but also the redistribution of risk, with important consequences for financial stability. This is something we cannot do in our large model solved by linearization, which only gives certainty equivalence policies.

Our model is in a certain way similar to TANK (two-agent New Keynesian) models. Debortoli and Gali (2017) show that many of the insights about aggregate dynamics from the HANK model already emerges with only two types of agents, one being always constrained while the other is unconstrained. We add to this the life cycle and the housing component as well as a third type of household, but the linearized solution of our model shares with TANK the feature that households are either always constrained or always unconstrained. From a technical point of view, our model is similar to Heer and Scharrer (2018), who also uses a big scale OLG model and solve it by linearization around the steady state. The study the redistributive effects of fiscal policy and find that debt-financing can harm old and retired households, by reducing economic activity and thus the price of capital held by the elders. Finally, our model relates to the literature on housing over the life cycle, cf. for example Iacoviello and Pavan (2013). We use the results form this literature to inform our calibration in several ways.

The plan of the paper is as follows. Section 2.2 presents the model. In Section 2.3, we discuss the calibration and what this implies for the steady state of the model. After analzing the monetary transmission mechanism in Section 2.4, we turn in Section 2.5 to the main results of the paper, about the stabilizing role of monetary policy in an economy facing demand shocks. Section 2.6 concludes.

2.2 The Model

2.2.1 Overview

Our model economy is inhabited by three types of agents: poor households, middle class households, and capitalists. While the first two are assumed to be households with a finite life cycle, capitalists are modeled as a representative infinitely lived dynasty.

The three types of agents differ in their labor productivity, the assets that they can invest in, and the housing options available to them. Poor households are excluded from asset markets and live in rental housing, the middle class chooses between owner-occupied and rental housing, and save or dis-save in bonds. Both types participate in a pay-as-you-go pension system. Capitalists own the houses they live in as well as the rental houses, trade bonds with the middle class and the central bank, and own all the firms. They hold most of the wealth in the economy.

The firm side is New Keynesian, where firms face monopolistic competition subject to Calvo pricing. The central bank conducts monetary policy according to a Taylor Rule. We introduce two types of shocks into the model. To study the monetary transmission mechanism, we consider a monetary policy shock, as is standard in the New Keynesian literature. To analyze the ability of monetary policy to stabilize the economy, we assume that the economy is hit by demand shocks.

Next we discuss firms and households in greater detail.

2.2.2 Firms

2.2.2.1 Final good producers

Production in the economy takes place in a final goods sector with monopolistic competition and pricing of the Calvo (1983) type. Each firm produces a differentiated good, using a Cobb-Douglas gross production function with capital and labor as inputs, subject to a fixed cost of production $\bar{\kappa}$. Net production is then

$$Y_t = F(K_{t-1}, L_t) = K_{t-1}^{\alpha} L_t^{1-\alpha} - \bar{\kappa}$$

The fixed cost will be chosen such that firms make zero profit in steady state.

Factor markets are assumed to be frictionless, therefore the optimal combination of production factors implies

$$\frac{F_K(K_{t-1}, L_t)}{r_t^K} = \frac{F_L(K_{t-1}, L_t)}{w_t} \equiv RMC_t \tag{2.1}$$

with RMC denoting real marginal costs. The pricing problem of the firm is standard. Under Calvo pricing, the first order condition for a price-setting firm is

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k|t} \left(P_t^* - Y_{t+k|t} \frac{\varepsilon}{\varepsilon - 1} P_{t+k} RMC_{t+k} \right) = 0 \tag{2.2}$$

where P_t^* denotes the optimal price of a price-setting firm, θ is the Calvo Parameter, and ϵ is the demand elasticity. $Q_{t,t+k}$ is the nominal stochastic discount factor given in Equ. (2.23) below. As usual, linearization around the zero-inflation steady state leads to the following dynamic equation for inflation:

$$\pi_t = \hat{\beta} \mathbb{E}_t \pi_{t+1} + (1 - \hat{\beta}\theta) \frac{(1-\theta) \log RMC_t}{\theta \log RMC^*} \tag{2.3}$$

2.2.2.2 Investment and housing sector

We assume that new capital goods and new houses are produced by competitive firms under constant returns to scale subject to convex adjustment costs on the stock of these variables. Defining the investment ratio for physical capital as

$$\iota_t^K = \frac{I_t^K}{K_{t-1}}$$

we assume that capital evolves as

$$K_t = (1-\delta)K_{t-1} + \Phi(\iota_t^K, \phi_K)K_{t-1} \tag{2.4}$$

where

$$\Phi(\iota, \phi) = \iota - \frac{(\iota - \delta)^2}{\phi\delta}$$

Adjustment costs as well as marginal adjustment costs are zero in steady state, where the investment ratio is equal to the depreciation rate. This implies the standard Q-theory of investment, where the value of installed capital in equilibrium is given by

$$p_t^K = 1/\Phi_I(\iota_t^K, \phi_K) \tag{2.5}$$

The housing sector is analogous. Defining $\iota_t^H = \frac{I_t^H}{H_{t-1}}$, the law of motion is

$$H_t = (1-\delta_H)H_{t-1} + \Phi(\iota_t^H, \phi_H)H_{t-1}$$

and the price of housing is $p_t^H = 1/\Phi_I(\iota_t^H, \phi_H)$.

2.2.3 Households

Before describing each household type in detail, we discuss four important elements of the household problem, each of which applies to several household types: the different types of bonds, wage rigidity, demand shocks and demographics.

2.2.3.1 Bonds

Next to owner-occupied housing middle class households have access to one financial asset, which we call a bond.A short position in the bond we will interpret as a mortgage, because borrowing is restricted to a constant fraction of the value of owned housing. Although in each variant of the model there is only one type of bond available, we model bonds in a more general way than usual, allowing for different maturities of the bond as well as for a distinction between nominal and real (inflation-protected) bonds. For tractability, we model maturity such that each period a constant fraction of the bond matures, as has been used already in the literature (e.g. (Krause and Moyen, 2013)). We assume the gross return of a bond in period t is given by

$$R_t^B = (\mu + r^B)v_t^B + (1-\mu)p_t^B \tag{2.6}$$

where p_t^B denotes the price of the bond, r^B is the nominal coupon. The value μ is the fraction of the bond that matures each period. If $\mu = 1$, it is a one period bond; a ten-year bond can be approximated by $\mu = 0.025$, so that each quarter, 1/40 of the bond matures. Notice that even though it is modeled as a one period bond, it is probably best to think of it as a mortgage that needs to be refinanced every period at a potentially different interest rate, thus resembling certain characteristics of an Adjustable Rate Mortgage (ARM). The variable v_t^B denotes the real face value of the bond, following the dynamic equation

$$\log(v_t^B) = \log(v_{t-1}^B) - \chi \log(\pi_t/\pi^*) \tag{2.7}$$

The case $\chi = 1$ characterizes a nominal bond, where inflation reduces the real value one by one, and $\chi = 0$ characterizes a real bond, whose value is not affected by inflation. Intermediate values of χ are possible, but we do not consider them in this paper.

2.2.3.2 Wage rigidity

New Keynesian models where prices are rigid but wages are flexible have the well known problem that profits become counter-cyclical, which is clearly at odds with the data, at least in absolute terms. It is therefore common in the literature to introduce wage as well as price rigidity. We avoid the usual Calvo wage setting, because we have a large number of heterogeneous agents, and proceed with a simple short-cut. We replace the first order condition $w_t^h = \frac{u_{l,t}^h}{u_{c,t}^h}$ of any household h by

$$w_t^h = \left(w_{stst}^h\right)^{\rho_W} \left(\mu_W \frac{u_{l,t}^h}{u_{c,t}^h}\right)^{1-\rho_W} \tag{2.8}$$

where w_{stst}^h denotes the steady state wage for household h, and ρ_W measures the degree of wage rigidity. μ_W is the wage markup over the marginal disutility of labor, to make sure that workers gain from an increase in labor demand even when wages are rigid. We set this parameter to $\mu_W = 1/0.9$. The formula (2.8) has the desired effect of allowing labor to fluctuate strongly with small variations in the real wage, without implying a large income elasticity of labor supply.

2.2.3.3 Demand shocks

Medium-sized DSGE (see e.g. (Smets and Wouters, 2007)) models contain several shocks that one can call "demand shocks". We introduce the demand shock as a wedge in the household Euler equations, designed specifically so as to allow the central bank to completely offset the effect the shock through interest rate policy in the framework of a representative agent model.[3] This only works if two conditions are met:

1. The wedge affects the Euler equation relating to bonds, but not to houses or capital investment, such that an increase in bond rates completely offsets the wedge. This is distinct from a shock to the discount factor, which cannot be compensated by a change in the interest rate. The reason is that the difference between the return on bonds and on capital would trigger a change in investment.

[3] This is very similar to the shock ϵ^b in (Smets and Wouters, 2007, page 589), which "represents a wedge between the interest rate controlled by the central bank and the return on assets held by the households."

2. The bond, through which monetary policy is conducted, is not traded, as is the case in a representative agent model. Then the change in the return does not create wealth effects. In the heterogeneous agent model, the interest rate change causes a redistribution between households that hold a long position and those who hold a short position in bonds, which can have long-lasting consequences.

This is a very special way of modelling a demand shock, but it serves to isolate the redistribution channel, which can be seen from the deviation from the divine coincidence. We can expect the mechanisms that we describe in Section 2.5) to be active when other forms of demand shocks come into play.

We assume the demand shock follows an AR(1) process:

$$D_t = \rho_D D_{t-1} + \epsilon_t^D$$

It will affect those Euler equation that refer to bonds, not those that refer to real assets, cf. Equs. (2.18) and (2.22).

2.2.3.4 Demographics of worker households

Workers are assumed to live for 60 years, which we interpret as adult live from age 20 to age 80. Since the model period is a quarter, the model age ranges from $s = 1$ to $s = I = 240$. Households work for the first 40 years of their live, and retire after age $s = I_R = 160$.

The lifetime profile of individual labor productivity of poor households is denoted by ζ_s for $s = 1, \ldots, I_R$. For middle class households, this profile is shifted up by a constant factor $\bar{\zeta}$ such that their individual productivity is given by $\bar{\zeta}\zeta_s$.

2.2.3.5 Poor households

We identify "poor" households as the lowest two deciles of the net wealth distribution. According to the data (SCF 2013), the median poor household has negative net worth over all age bins (cf. Table 2.6 in Appendix 3.D). It does not live in owner-occupied housing, except for one age class (Table 2.8), and net financial assets are almost always negative (Table 2.7). We therefore model this class of agents as hand-to-mouth consumers, who live in rented housing and have no access to asset markets. Their income is given by

$$\tilde{y}_{s,t} = \begin{cases} w_t \tilde{l}_{s,t} \zeta_s & \text{for } s = 1, \ldots, I_R \\ \psi_t & \text{for } s = I_R + 1 \ldots, I \end{cases} \tag{2.9}$$

After retirement, they receive a lump sum pension benefit ψ_t.

The poor households' optimization problem is therefore reduced to a sequence of static labor-leisure-housing choices. With the utility function

$$u(c, l, h^R) = \log(c_t) + \eta \log(1 - l_t) + \eta_H \log(h_t^R) \tag{2.10}$$

subject to the budget constraint

$$r_t^H \tilde{h}_{s,t}^R + \tilde{c}_{s,t} = \tilde{y}_{s,t} \tag{2.11}$$

this leads to the following first order condition for consumption versus housing

$$\frac{\tilde{h}_{s,t}}{\tilde{c}_{s,t}} = \frac{\eta_H}{r^H_{s,t}} \tag{2.12}$$

Applying the rigid wage equation (2.8), we get the following first order condition for labor supply of working age households:

$$\eta \frac{c_{s,t}}{1 - l_{s,t}} = \bar{w} \left(\frac{w_t}{\bar{w}}\right)^{1/(1-\rho_W)} \zeta_s \tag{2.13}$$

In (2.13), the marginal rate of substitution varies one for one with labor productivity ζ_s, but more elastically w.r.t. cyclical wage fluctuations w_t in case $\rho_W > 0$. With flexible wages ($\rho_W = 0$), Equs. (2.11)–(2.13) imply constant labor supply, which is a consequence of log utility, were income and substitution effect exactly cancel. With wage rigidity, labor supply is still constant over the life cycle, for any given aggregate wage w_t, but responds positively to cyclical fluctuations in the wage.

2.2.3.6 Middle Class Households

The representative household of each middle class cohort owns a part and rents the remaining part of its housing. It can save in bonds, and borrow up to a certain limit against owned housing. A household born at time $t-1$ solves

$$\max \mathbb{E}_t \sum_{s=1}^{I} \beta^s u(c_{s,t+s}, l_{s,t+s}, h^O_{s,t+s}, h^R_{s,t+s}) + \beta^I MUB \cdot R^B_{t+I} b_{I,t+I} \tag{2.14}$$

subject to the per period the budget constraint

$$p^B_t b_{s,t} + p^H_t \left(h^O_{s,t} - (1-\delta_H) h^O_{s-1,t-1}\right) + c_{s,t} + r^H_t h^R_{s,t} = (1-\tau) w_t \zeta_s l_{s,t} + \mathcal{I}^R_s \psi_t + (1 - \mathcal{I}^R_s)\omega_{s,t} + R^B_t b_{s-1,t-1} \tag{2.15}$$

and the borrowing constraint

$$v^B_t b_{s,t} \geq -\nu \mathbb{E}_t p^H_{t+1} h^O_{s,t} \tag{2.16}$$

In (2.14), households receive a constant marginal utility from bequests MUB, which leads to a bequest $\Omega_t = R^B_{t+I} b_{I,t+I}$. This bequest Ω_t is then distributed evenly among working age middle class cohorts, such that their bequest is $\omega_{s,t} = \Omega_t / I_R$. The left hand side of the budget constraint (2.15) represents the spending of cohort s in period t. It buys bonds at price p^B_t, purchases new owned housing $h^O_{s,t}$, rents housing $h^R_{s,t}$ and consumes $c_{s,t}$. The right hand side gives the available resources of household at the beginning of period t, which consists of labor income, pension income (if the person is retired), bequests and the return on last period's bond holdings $b_{s,t-1}$ as described in Section 2.2.3.1. Here w_t is the hourly wage and ζ_s is the age-dependent idiosyncratic productivity of the household. The indicator function $\mathcal{I}^R_s$ is one if the household is retired.

The down payment constraint (2.16) relates the real face value of the bond to the expected real value of owned housing. It states that a household can only borrow up to the fraction ν of the value of their house. This parameter is commonly referred to as the Loan to Value Ratio (LTV). We set $\nu = 0.8$, which means that 20% of the mortgage of a house have to be financed by savings, prior to the purchase. Notice that we value $b_{s,t}$ on the lhs of (2.16) by its face value v_B rather than the market price p_B. The reason is the following. Consider a household that holds a certain amount of long-run nominal debt. If inflation unexpectedly decreases, this increases the real value of future coupon payments, which reduces the net worth of the household and diminishes its ability to repay debt. This effect is reflected in a higher real value v_B of its debt. If expected future real interest rates decrease, it also drives the market price of bonds p_B up, and thereby increase the market value of household debt, but this does not imply higher debt repayments in the future, and therefore does not reduce the ability of the household to repay debt. We therefore think that our formulation is a better approximation to the down payment constraints in real world contracts.

For the utility function of the middle class household we choose

$$\begin{aligned} u(c, l, h^R, h^O) = &\log(c_t) + \eta \log(1 - l_t) \\ &+ \eta_H \log\left[\left((h_t^R)^{(\sigma-1)/\sigma} + (\xi_s h_t^O)^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}\right] \end{aligned}$$

The parameter σ measures the elasticity of substitution between owned and rented housing. The relative efficiency of owned housing ξ_s is supposed to capture the pros and cons of home ownership versus rental. The pros are reduced moral hazard, and the ability to make alterations and adjustments. The cons are reduced geographical flexibility, and capital risk. To match the observed pattern of the ownership rate, we postulate a linear relationship in age:

$$\xi_s = \bar{\xi} + \hat{\xi} \cdot s \tag{2.17}$$

The first order condition for labor supply is again given by (2.8). The first order conditions for asset choice are given by the following three expressions, the derivation of which can be found in Appendix 2.B

$$u_{h_{i,t}^R} = r_t^H u_{ci,t} \tag{2.18}$$

$$u_{c_{i,t}} p_t^B = \beta(1 + D_t)\mathbb{E}_t \left[R_{t+1}^B u_{c_{i,t+1}}\right] \tag{2.19}$$

$$u_{c_{i,t}}[p_t^H - \frac{p_t^B}{v_t^B}\nu\mathbb{E}p_{t+1}^H] = u_{h_{i,t}^O} - \beta\mathbb{E}_t\left[u_{c_{i,t+1}}\left(\frac{R_{t+1}^B}{v_t^B}\nu\mathbb{E}p_{t+1}^H - (1 - \delta_H)p_{t+1}^H\right)\right] \tag{2.20}$$

As was described in Section 2.5, households face an aggregate demand shock D, which acts as a wedge between the returns of bonds and physical assets.

2.2.3.7 Capitalists

Capitalists own most of the real assets in the economy. They own the firms and thus are the beneficiaries of any profits accruing to them. Additional to their own housing, they own the houses which are rented out at rental rate r_t^H.

We assume the utility function of the capitalists takes the following form.

$$\hat{U}(\hat{c},\hat{l},\hat{h}^O) = \log(\hat{c}) + \eta \log(\bar{L}^C - \hat{l}) + \eta_H \log(\hat{h}^O)$$

Being infinitely lived, they solve

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \hat{\beta}^t u(\hat{c}_t, \hat{l}_t, \hat{h}^O_t)$$

subject to the budget constraint

$$\begin{aligned} Y_t - I^K_t - w_t L^W_t + r^H_t H^R_t + p^B_t B_t + p^H_t (H_t - (1-\delta_H)H_{t-1}) - I^H_t + w_t \hat{l}_t + R^B_t \hat{B}_{t-1} + \frac{R_{t-1}}{\pi_t} B^{CB}_{t-1} \\ = \hat{c}_t + p^H_t(\hat{h}^O_t + H^R_t - (1-\delta_H)(\hat{h}^O_{t-1} + H^R_{t-1})) + p^B_t \hat{B}_t + B^{CB}_t \end{aligned} \quad (2.21)$$

The income of capitalists (lhs of 2.21) has the following components. They receive the profits of the production sector, which equals output minus wage payments minus investment into physical capital. They also earn money from renting out part of the housing stock to the other types of households, and they earn the profits of the housing and capital construction sectors. They receive labor income, and they receive the returns of their bond holdings, which in equilibrium are negative since they hold a short position. Notice that the bond holdings of the capitalists are given by $\hat{B}_t = -B_t$, since they hold the offsetting position to the bonds of workers. Additionally, they can invest in a one-period nominal bond issued by the Central Bank B^{CB}_t at the interest rate R_t. This bond, which in equilibrium is in zero net supply, is the channel via which the central bank conducts monetary policy, as capitalists need to be indifferent between holding these type of bonds and any other asset. Notice that capitalists do neither contribute nor benefit from the social security systems. They face no borrowing constraints. On the spending side, the distribute these resources between their consumption, purchases on their own housing and the housing that they rent out, and the two types of bonds.

The first order condition for labor supply is again given by (2.8). The first order conditions for asset choice are

$$\begin{aligned} \hat{U}_{\hat{h}^O_t} &= p^H_t \hat{U}_{\hat{c}_t} - \hat{\beta}(1-\delta_H)\mathbb{E}_t(p^H_{t+1}\hat{U}_{\hat{c}_{t+1}}) \\ \hat{U}_{\hat{c}_t}[p^H_t - r^H_t] &= \hat{\beta}(1-\delta_H)\mathbb{E}_t(p^H_{t+1}\hat{U}_{\hat{c}_{t+1}}) \\ \hat{U}_{\hat{c}_t} p^B_t &= \hat{\beta}(1+D_t)\mathbb{E}_t(R^B_{t+1}\hat{U}_{\hat{c}_{t+1}}) \\ \hat{U}_{\hat{c}_t} &= \hat{\beta}(1+D_t)\mathbb{E}_t\left(\frac{R_t}{\pi_{t+1}}\hat{U}_{\hat{c}_{t+1}}\right) \end{aligned} \quad (2.22)$$

Capitalists are affected by the demand shock D_t just like middle class households, which operates on the first order condition with respect to bonds. Since capitalists own the firms, the relevant nominal stochastic discount factor is

$$Q_{t,t+k} = \hat{\beta}^k \frac{\lambda_{t+k}}{\lambda_t} \frac{P_t}{P_{t+k}} \frac{\hat{U}_{\hat{c}_{t+k}}}{\hat{U}_{\hat{c}_t}} \quad (2.23)$$

2.2.4 Closing the model

2.2.4.1 Aggregate Variables

Define per capita (better: per cohort) labor input of poor and middle class households as

$$\tilde{L}_t = \sum_{s=1}^{I} \zeta_s \tilde{l}_{s,t}/I \tag{2.24}$$

$$L_t = \sum_{s=1}^{I} \bar{\zeta}\zeta_s l_{s,t}/I \tag{2.25}$$

$$\tag{2.26}$$

respectively. Total labor input is then

$$L_t = 0.2\tilde{L}_t + 0.7L_t + 0.1\hat{L}_t \tag{2.27}$$

Notice that the labor efficiency of capitalists is normalized to 1. Similarly for consumption

$$\tilde{C}_t = \sum_{s=1}^{I} \tilde{c}_{s,t}/I \tag{2.28}$$

$$C_t = \sum_{s=1}^{I} c_{s,t}/I \tag{2.29}$$

$$C_t = 0.2\tilde{C}_t + 0.7C_t + 0.1\hat{C}_t \tag{2.30}$$

Bonds held by workers are given by

$$B_t = 0.7 \sum_{s=1}^{I} b_{s,t}/I$$

The bond position of capitalists is then $-B_t$. Rented and owner-occupied housing of workers is defined analogously to consumption, and then total housing given by

$$H_t = H_t^R + H_t^H + \hat{H}_t$$

The aggregate resource constraint is

$$Y_t = C_t + \hat{C}_t + I_t^K + I_t^H \tag{2.31}$$

Finally, the total housing stock H is given by adding up total rental housing H^R, aggregate housing owned by the middle class H^H and the housing owned by the capitalists $\hat{H}$. Real GDP is defined as production Y_t plus the imputed value of housing rents, evaluated at steady state price r^{H*}:

$$GDP_t = Y_t + r_t^H H_t$$

2.2.4.2 Government

The government in this model has only a passive role. It takes the form of a pay-as-you-go pension system, which taxes the labor earnings of the workforce and rebates it lump sum and equally to all retired agents, which then receive an amount ψ_t in period t. We assume that benefits are indexed to the real wage:

$$\psi^* = \tau_t w_t \frac{0.2\tilde{L}^* + 0.7L^*}{0.9} \frac{I}{I - I_R}$$

Over the business cycle, the benefit level fluctuates with the wage, but not with the number of hours. The payroll tax τ_t has to adjust so as to balance the budget of the pension system:

$$\psi_t = \tau w_t \frac{0.2\tilde{L}_t + 0.7L_t}{0.9} \frac{I}{I - I_R}$$

The adjustment factor on the right hand side of these equations accounts for the fact that labor input is measured per capita, but benefits are only received by the retirees.

2.2.4.3 The Monetary Authority

Monetary Policy is implemented by controlling the interest rate on a one period nominal bond, which is offered to capitalists. This bond is not traded in equilibrium, but linked to the other assets via a no-arbitrage condition. Under the short-term nominal bond regime, this bond is identical to the bond traded with middle class households, but in other asset regimes it is not.

In the benchmark model, the central bank follows the Taylor rule

$$\log(R_t/R^*) = \rho_R \log(R_{t-1}/R^*) + (1-\rho_R)\left(\gamma_\pi \log(\pi_t/\pi^*) + \gamma_y \log(Y_t/Y^*) + \gamma_H \log(p^H_{t+1}/p^{H^*})\right) + \epsilon^M_t \quad (2.32)$$

In general, we allow the interest rate react to inflation, to the output gap, and to deviations of the house price from its steady state. The shock ϵ^M_t is assumed to be i.i.d., but gets propagated by interest smoothing with parameter ρ_R.

Under strict inflation targeting, the nominal interest is chosen so as to get $\pi_t = 0$ always.

2.3 Calibration and Deterministic Steady State

The time period of the model is one quarter, and the economic lifetime of a worker agent is $I = 240$ quarters. In the data, we identify the three types of households according to their position in the net worth distribution of the Survey of Consumer Finances 2013 (SCF). Poor households are the poorest 20 percent, middle class households are the next 70 percent, and capitalists are identified as the top 10 percent of households in terms of net worth.

Table 2.1 lists the parameter values for the benchmark calibration. The Cobb-Douglas parameter $\alpha = 0.36$ and the depreciation rates (3 percent annually for housing, 10 percent for other fixed investment) are standard. The adjustment cost parameters for capital and housing, $\phi_K = \phi_H = 8.5$, were chosen such that total investment responds twice as much as output to a monetary policy shock on impact. We have chosen the same adjustment cost parameter for capital and for housing. This understates the historical volatility of housing investment, which varies more than business investment, but also understates the volatility of house prices. Making housing investment more volatile would make house prices even more stable.

We take the parameters for age dependent productivity ζ from Hansen (1993), who finds that labor efficiency peaks around the age of 54. The labor productivity of poor households of the cohorts $s = 1, \ldots, I_R$ follows

$$\zeta_s = 1 + 0.061329\frac{i - 0.5}{4} - 0.001011\left(\frac{i - 0.5}{4}\right)^2 \tag{2.33}$$

Middle class households have the same profile ζ, but multiplied by the constant 2.093, so as to match the differences in average earnings between the two groups.

The discount factor of the capitalists gives a real interest rate of 4 percent annually. The discount factor of middle class households was set so as to match their average bond holdings, measured as a fraction of their average labor income. The weight of leisure in workers' utility function, η, was chosen such that hours worked, averaged over all workers and weighted by labor productivity, equals one third of the labor endowment. This is a common number in the RBC literature. For capitalists, the labor endowment and weight of leisure was chosen such that they work one third of their time in steady state and their effective labor supply is 10 percent of the total, in line with their share of the population. We estimate the size of bequests by the net worth of middle class households over the age of 80 in our data, and set the marginal utility of bequests to match this target. We assume that bequests are distributed equally across all non-retired cohorts, i.e. $\omega_t = \frac{\Omega_t}{160}$. The autocorrelation of the demand shock was set to 0.95. The standard deviation of the shock was chosen to match the standard deviation of detrended log GDP for US data 1984–2017, which is 1.21 percent.

The parameters for the relative efficiency of owned housing, $\bar{\xi}$ and $\hat{\xi}$, and the weight of housing in utility, η_H, were chosen jointly to match the average home value of the middle class, as well as two statistics of the ownership rate: the average over the life cycle, which is 73.7, and the value for the 20-25 years old, which 9 percent. The result is plausible: for the youngest cohort, renting is more efficient, but home ownership becomes more efficient with age. The housing weight for capitalists is set to $\eta_H/2$ which reflects the lower share of housing in their total wealth. The elasticity of substitution between rental and owned housing was set, somewhat arbitrarily to 3.0, assuming they are close substitutes.

We set the steady state payroll tax to 18 percent. This is higher than the current US payroll tax (around 12 percent) so as to include other sources of pension income. It results in a drop of log consumption of -0.28 at retirement for poor households. This is within the range of estimates in the literature.[4]

[4]Aguiar and Hurst (2005) estimate a retirement dummy for log food consumption of -0.17. Bernheim

The parameters related to price stickiness and monetary policy are all standard in the literature. We are choosing a very high degree of wage rigidity, $\rho_W = 0.9$, to match the finding in Christiano and Evans (2005) that the maximum real wage response is about one fifth of the output response. Wage rigidity is a key determinant for the variability of inflation. Despite the strong degree of rigidity, the model still tends to exaggerate inflation in our main experiment, where fluctuations are generated by demand shocks. There the standard deviation of (annualized) inflation is about the same as that of output, while it is 64 percent in the US data since 1984, as measured by the GDP deflator.

2.3.1 Steady State Results

Our capital and investment rates are comparable to the literature. The ratio of capital to annual GDP is around 2.2 in our model and the housing stock to GDP ratio is 2.1. The corresponding values are 2.2 and 1.4 in Iacoviello and Pavan (2013) or 1.75 and 1.3 in Garriga et al. (2013). The ratio of capital investment to output is 0.22 (0.2 in Iacoviello and Pavan (2013) ,0.16 in Garriga et al. (2013)), and for housing investment the ratio is 0.06 (0.07 in Iacoviello and Pavan (2013), 0.05 in Garriga et al. (2013)). Notice that the housing stock in our calibration is somewhat higher than what is found in other studies, this is the consequence of the housing wealth in our SCF data. With a depreciation rate of 3 percent annually, this nevertheless translates into a realistic housing investment rate.

Figure 2.1 depicts some life cycle paths in the deterministic steady state for worker households. We have used the productivity values from Hansen (1993) for both poor and middle class households, and these numbers still fit the data relatively well. Our linear trend for housing efficiency in (2.33) gives an almost perfect fit for the home ownership rate. We have a somewhat larger discrepancy with the mean data in terms of financial wealth of the middle class. Our model somewhat overstates the amplitude of the life cycle path of assets, compared to the median holdings of each cohort. We have not tried to dampen this pattern. Overstating the inequality over the life cycle partially compensates for the lack of intra-cohort inequality. Taking a cross-section over the whole economy, our model still underestimates inequality in earnings, net worth and financial wealth, cf. Table 2.2. Notice that the Gini coefficient can be larger than 1 if some households hold negative wealth, which many households do with financial assets. The slow dissaving of retired people is considered a puzzle in the microeconomics literature and varies between countries, see e.g. Nakajima and Telyukova (2016). In our model, it is generated by a strong bequest motive.

The middle right panel summarizes the information about assets in the model. Middle class households start out at their borrowing constraint and accumulate bigger housing and bigger debts over time. Before the age of 50, they leave the borrowing constraint and

et al. (2001) find a change in log consumption of -0.24 after the first and -0.566 after the second year for the lowest wealth quartile. Aguiar and Hurst (2005) and Aguiar and Hurst (2013) point out that this is largely compensated by home production, but for our purpose it is market consumption that matters.

Parameter	Target	Symbol	Value
Technology			
production elasticity capital	output share of capital	α	0.360
depreciation rate for capital	$\frac{I}{K}$	δ	0.025
depreciation rate for housing	Housing investment	δ_H	0.007
adjustment cost parameter capital	IR investment	ϕ_K	8.500
adjustment cost parameter housing	IR investment	ϕ_H	8.500
labor efficiency middle class	wage differential		2.093
Utility			
discount factor of capitalists	4 % ann. interest	$\hat{\beta}$	0.990
discount factor of workers	workers' bond holdings	β	0.984
weight of leisure middle class	labor supply	η	2.638
weight of leisure capitalists	labor supply	η	0.761
labor endowment capitalists	labor supply	$\bar{L}^C$	0.231
Marg.Util.Bequest	size of bequests	MUB	1.282
autocorrelation demand shock		ρ_D	0.950
StandDev demand shock, ·100	output volatility		0.175
Utility related to housing			
weight of housing in utility	housing wealth	$\bar{\eta}_H$	0.311
intercept efficiency owner occupied	path ownership rate	$\bar{\xi}$	0.217
slope efficiency owner occupied	path ownership rate	$\hat{\xi}$	0.017
elasticity of subst. rental vs. owner		σ	3.000
Taxes			
payroll tax	Consumption old age	τ	0.180
Inflation and monetary policy			
steady state inflation		π^*	1.000
demand elasticity		ε	7.000
prob. keeping the price		θ	0.750
Taylor rule parameter inflation		γ_π	1.500
Taylor rule parameter output gap		γ_y	0.125
Influence of past interest rate		ρ_R	0.700
Wage rigidity		ρ_W	0.900

Table 2.1: Parameter values benchmark calibration

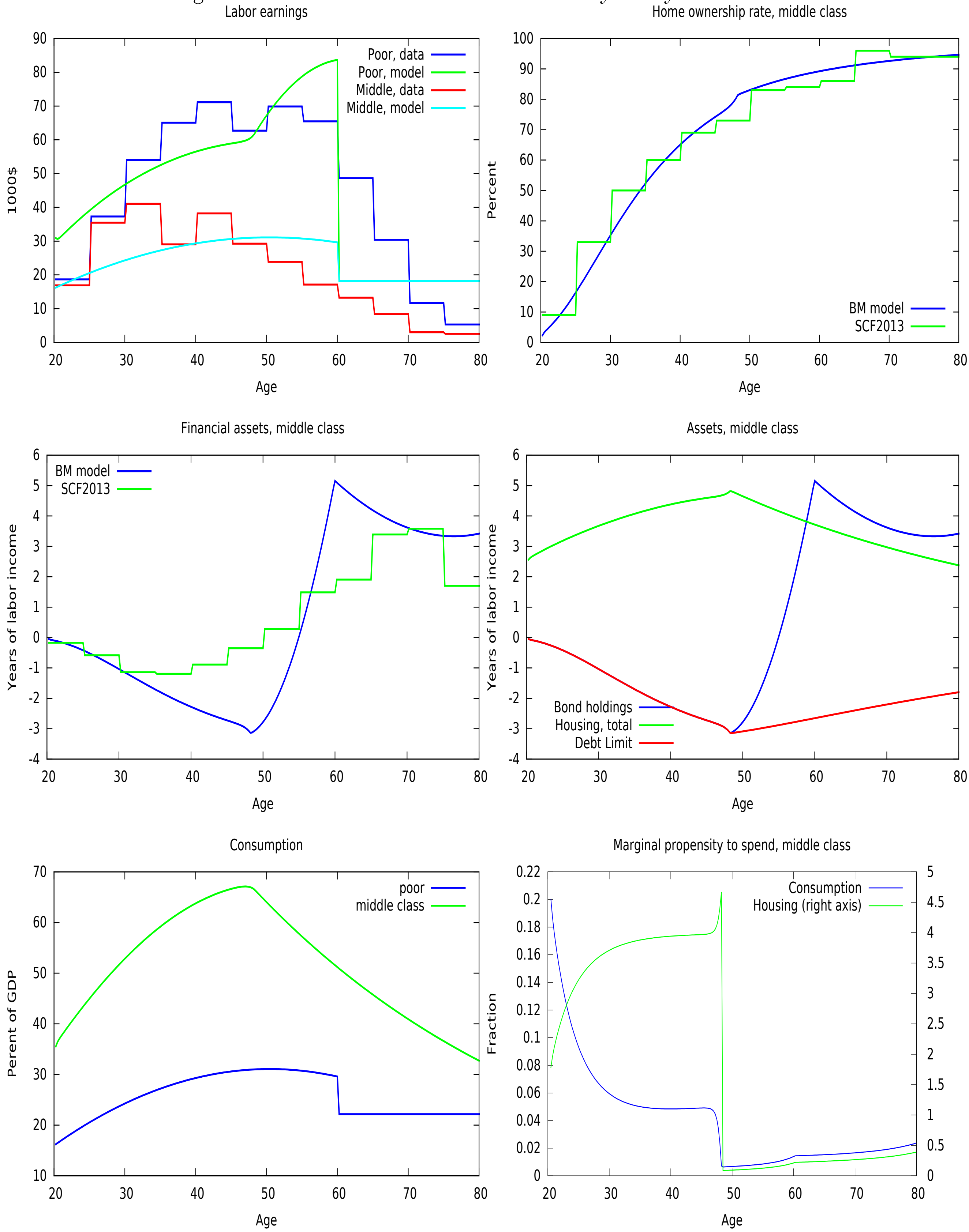


Figure 2.1: Life cycle paths in Steady State

	Financial assets	Net worth	Earnings
Data	1.41	0.85	0.65
Model	1.10	0.73	0.44

Table 2.2: Gini coefficients

start to accumulate savings for retirement. Their assets peak at the time of retirement, after which they run down the assets until the bequest motive is met at the period of 80.

Consumption (lower left panel) exhibits the hump shape commonly found in life cycle models (see e.g. Fernández-Villaverde and Krueger (2011)), and for poor households also the drop at the time of retirement. In contrast, middle class households smooth out their consumption over the whole unconstrained part of their life cycle, but consumption and housing demand (sum of owned and rented) both peak before the actual retirement, in order to build up the financial asset position and smooth consumption during retirement.

The blue line in the lower right panel shows the marginal propensity to consume (consumption goods and rental housing). Since the MPC is falling while both earnings and net worth are rising until retirement, this picture qualitatively fits the empirical findings in (Auclert, 2017, Figure 2). The green line shows the marginal propensity to buy now homes. This number is large: since households are constrained, a dollar saved allows to buy another four dollars of housing. The expansion in demand stemming from these households is therefore much greater than what the MPC suggests.

2.4 The Monetary Transmission Mechanism

Figure 2.2 contains impulse response functions to an expansionary monetary policy shock of 0.25 percentage points (1 percentage point at annualized rate) which lasts for one quarter. Remember that monetary policy shocks are uncorrelated, but under the baseline policy, the central bank has an interest smoothing motive, $\rho_R = 0.7$. Responses are shown for inflation and the nominal interest rate, as well as for the four big macroeconomic aggregates (output, consumption, investment and housing investment), and wages and house prices. In each case, there are four lines for the four different asset regimes, and for comparison, a fifth line for the representative agent (RA) version of the economy.[5] Obviously, the difference in aggregate responses across asset regimes are rather small, at least compared to the width of the confidence intervals in empirical estimates of the MP transition mechanism. Even the differences to the RA model are only moderate. The graph shows the typical picture of an expansionary monetary policy shock in a New Keynesian model: The nominal, and even more so the expected real short-term interest rates go down. Because of this expansionary effect, inflation goes up, and due to the

[5]The RA economy consists of capitalists only. It uses the same calibration as the benchmark model, and was obtained by setting the weight of the worker household to zero. Notice that capitalists have a lower utility of housing.

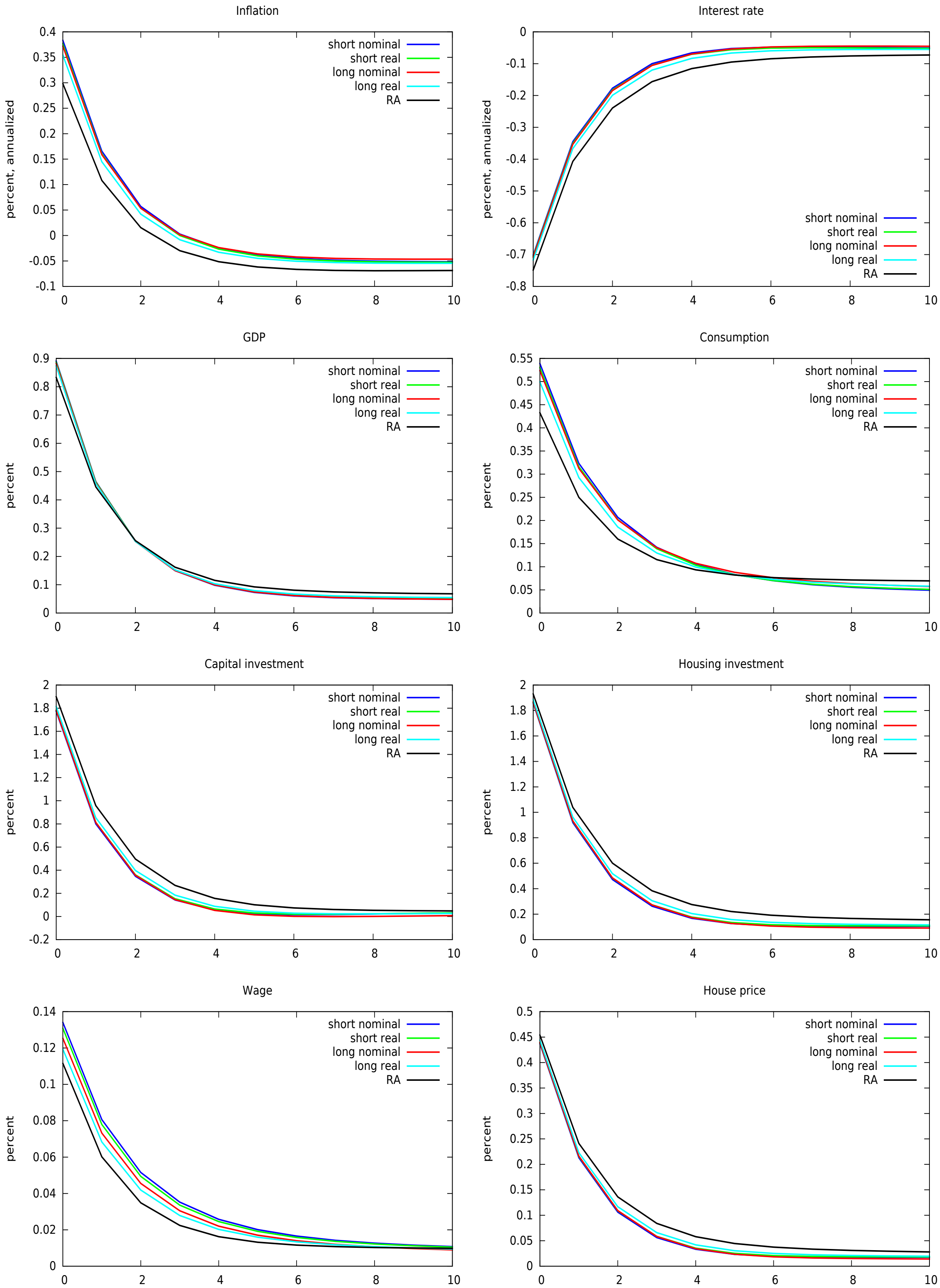


Figure 2.2: Impulse responses to monetary shock

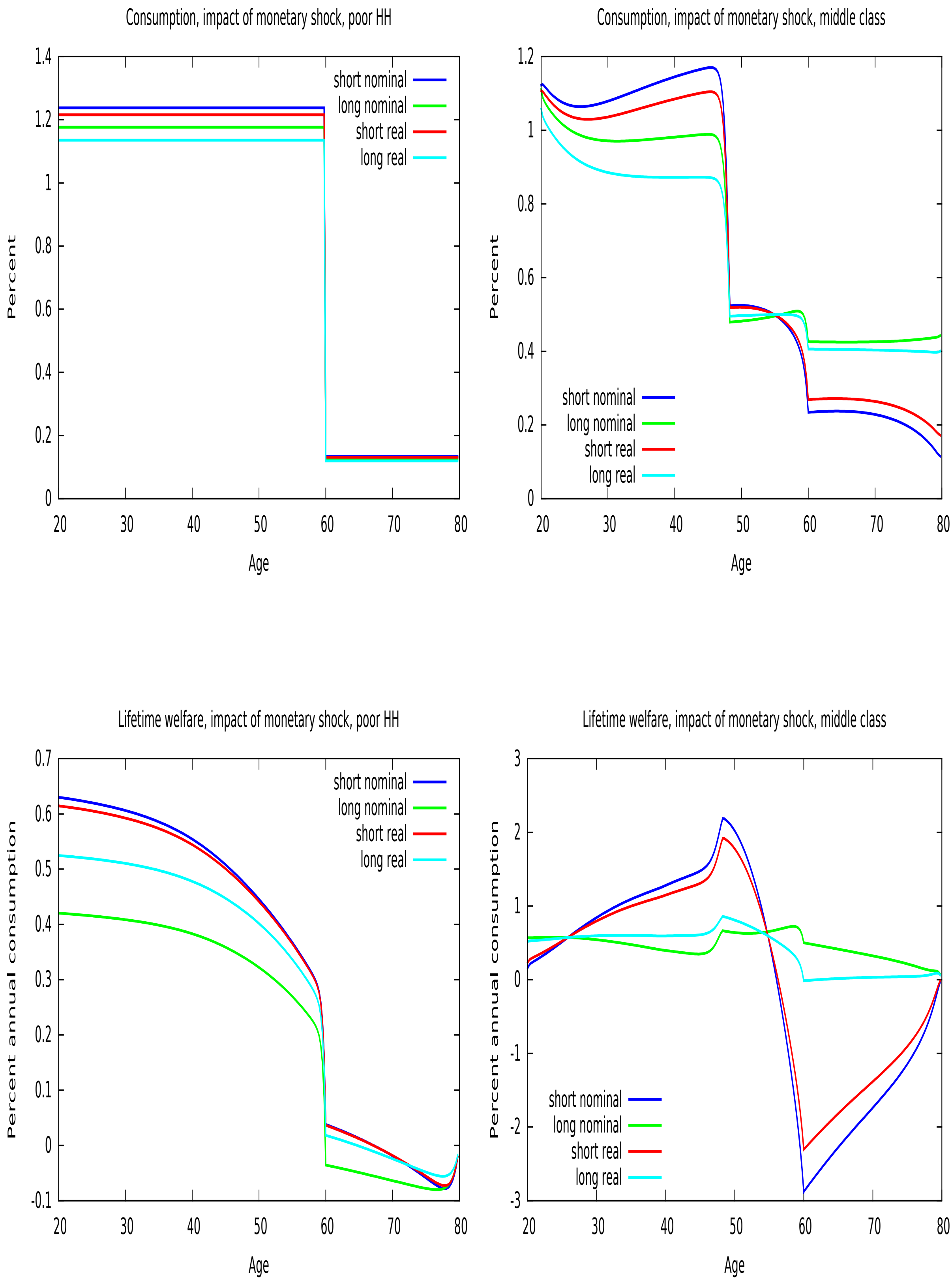


Figure 2.3: Impact effect of MP shock

immediate endogenous response of the central bank, the interest rate decreases by less than the shock. All the four macroeconomic aggregates jump up on impact due to the interest rate decrease. The increase in housing investment also leads to a increase in house prices, and the increase in economic activity raises the real wage. The effect on real wages is small because of wage rigidity. This dampens the reaction of real marginal cost and therefore inflation, in line with the empirical evidence for example in Christiano and Evans (2005).

Knowing the effect of monetary policy on prices, we can now analyze how it affects the different cohorts. The upper panels of Figure 2.3 display the effect of a monetary policy shock on the consumption of poor and middle class households, under different assumptions about the asset structure. With poor households, the consumption effect is the same for all working age households, and the same for all retired households. This is the consequence of hand-to-mouth behavior in combination with log utility. A more interesting picture arises for the middle class. Most young cohorts are constrained and can now increase consumption significantly. Middle age households are unconstrained and smooth consumption. Their response comes from an intertemporal substitution effect and a wealth effect that results both from their asset position (discussed below) and their labor earnings (or retirement income). Since the effect of the asset position is minimized with long-term real bonds, the corresponding line in the diagram approximately measures the effect from economic activity. The age pattern in the consumption response is in line with empirical findings in Wong (2018), where the response is strongest for young households and weakest for old households. Table 3 of that paper finds that the young contribute 72 percent of the total consumption response. Our model does not reproduce this rather extreme result, but it goes a long way in that direction. Although the consumption of the households of age 20 to 40 accounts for only 33 percent in steady state, it accounts for 50 percent of the reaction to a monetary policy shock.

The wealth effect of a monetary policy shock can be seen in the bottom panels of Figure 2.3. In each case, the effect is measured in percent of annual consumption. For example, a value of 2 means that the household loses the equivalent of 2 percent of consumption during one year. The effect on poor households (bottom left panel) is relatively small, both before and after retirement. These households own no assets, their welfare is only affected through the change in wages. The welfare effect on middle class households[6] (bottom right panel) is big: an expansionary monetary policy shock of 0.25 percentage points which lasts for one quarter causes utility gains and losses of up to 3 percent of annual consumption. Gains and losses vary greatly across cohorts, being mostly driven by their asset positions. To understand these effects, notice fist that the temporary increase in house prices does not seriously affect home owners. The house prices have only increased because the decrease in the expected real return on bonds requires an decrease in the expected return on housing in equilibrium. The current increase in market price, which appears as a capital gain in the books, is basically offset by the

[6] See Section 2.5.3 for how we compute welfare.

decrease in future returns on housing. One can also see it from a different angle: a temporary rise in the house price does not much affect households who hold on to the house for a long time.

Consider the blue line in that panel, which shows the results for short-term nominal bonds. Since the wealth effect is determined by the nominal asset position, this line is the mirror image of the line of steady state bond holdings in Figure 2.1. Young households with a mortgage gain from the reduction in the real interest rate, while the biggest losers are households shortly before retirement, which hold a substantial amount of bonds. This effect is strongest when bonds are short-term nominal: households with a long position in bonds suffer both from the persistent decrease in the nominal interest rate and from the increase in inflation. When bonds are short-term and real (inflation-protected), they do not suffer from the impact effect of inflation. The welfare effect should be minimized when bonds are long-term and inflation-protected. In this case, all both bonds and houses are long-term and real, so that households are largely insulated from the direct effect of monetary policy, namely interest rates and inflation. When bonds are long-term and nominal, they suffer little from the reduction in the nominal rate, because they have locked in the interest rate on their assets for ten years on average. They are still affected by inflation, but since the inflation response is small in our model due to wage rigidity, there is little difference between long-term nominal and real bonds. Of course, everybody gains from the expansion of economic activity following a monetary policy shock, but this effect is much smaller than the redistribution effects and rather evenly spread.

The above analysis has revealed enormous differences in the consumption response and in the welfare consequences of monetary shocks, both between cohorts and between different asset regimes. It is remarkable that this makes so little difference for the aggregate variables. Nevertheless, we will see in the next section that heterogeneity and redistribution can have important consequences even for the aggregate.

2.5 Economic Stabilization in the Face of Demand Shocks

2.5.1 Impulse Responses

We now analyze the implications of household heterogeneity for what is arguably the main task of monetary policy, namely stabilizing the economy in the face of demand shocks. Figures 2.4 presents impulse responses to an expansionary demand shock. The graphs show that our shock has the properties that we usually expect from a "demand shock": output, inflation, consumption and investment all go up on impact. Only housing investment goes down, as a consequence of the monetary policy reaction, which counteracts the demand shock by an increase in the nominal interest rate. The key difference between monetary and demand shocks is the behavior of interest rates, going in the same direction as inflation in the case of a demand shock, while going in the opposition direction in the case of a monetary shock. We now notice somewhat stronger differences between the different asset regimes, and a stronger difference to the representative agent benchmark, in particular with respect to housing. To understand these responses, the upper panels of Figure 2.5 report consumption and welfare effects for middle class households. The welfare consequences of a demand shock are quite different from those of a monetary policy shock, mainly because of a stronger inflation reponse. Households are best insured against a shock if they invest in long-term real assets, but large welfare variations appear with long-run nominal assets. The households that are negatively affected by changes in inflation and interest rates are again the households with large nominal asset holdings, shortly before retirement. The response to a demand shock makes both inflation and the real rate increase. The increase in inflation hurts long-run bond holders. The increase in the real rate benefits short-term bond holders.

The lower panels contain an important message: aggressive monetary policy ($\gamma_\pi = 4.5$) reduces the variability of welfare under long-term, but not under short-term nominal assets. The intuition is clear. With short-term nominal assets, fighting inflation aggressively is costly because households are negatively affected by the short-term fluctuations in real interest rates that are implied by this policy. With long-term nominal assets, households are largely protected against fluctuations in the nominal rate, and mostly care about the variability of inflation, which is reduced by aggressive monetary policy. Keep in mind that what matters here is not whether a change in utility is positive or negative, because shocks have expectation zero, and a gain to a positive shock is outweighed by the loss in response to a negative shock. Important is the absolute value of the utility change, because it indicates larger fluctuations of utility in response to a shock. In other words, we focus on second, not first moments.

2.5.2 Policy Trade-offs

A central issue in the theory of monetary policy is the trade-off between output stabilization and inflation stabilization. In the textbook model (cf. for example Clarida et al. (1999)) this trade-off arises in the face of cost-push shocks, but not in the case of demand shocks. In the latter case, the monetary authority can perfectly stabilize both output and inflation, a result which has been named the "divine coincidence". We focus on demand shocks, to see whether this favorable situation continues to hold. Table 2.3 lists statistics for 6 aggregate variables, conditional on the assumption that all fluctuations are caused by demand shocks. We report results for the benchmark Taylor rule, which is characterized by ($\gamma_\pi = 1.5$, $\rho_R = 0.7$, $\gamma_y = 0.125$ and $\gamma_H = 0$), as well as four alternatives, where in each case one of those parameters is varied. We show all results for the four combinations of asset structures, nominal versus real, and long- vs. short-run. The variables we report are GDP (Y), nominal interest rate (R), inflation (Π), the ex-post real interest rate ($Rreal$), and the percentage changes in the price of bonds (Δp^B), and housing (Δp^H). All variables except output are expressed as annual rates. The results shown come from a simulation of the model for 100,000 periods, detrended by a Hodrick-Prescott filter with smoothing weight 1600. The shock size was chosen such that the standard deviation of output in the benchmark case (short-term bonds, benchmark policy) is 1.21 percent, the number for US GDP in the period 1984-2017.

The numbers in the first part of table Table 2.3 confirm standard results. With demand shocks, there seems to be very little trade-off. A more aggressive policy, both in the form

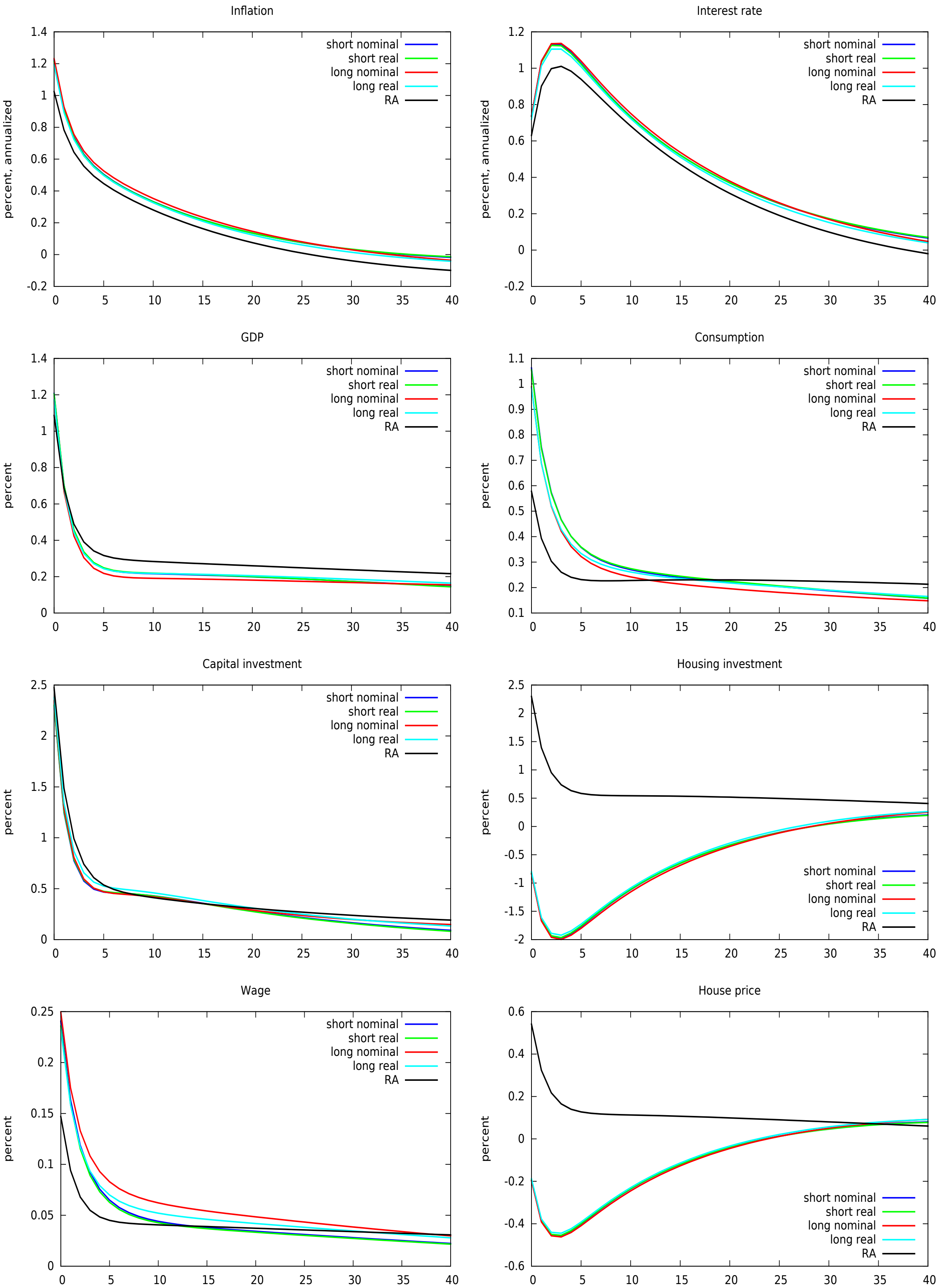


Figure 2.4: Impulse responses to demand shock

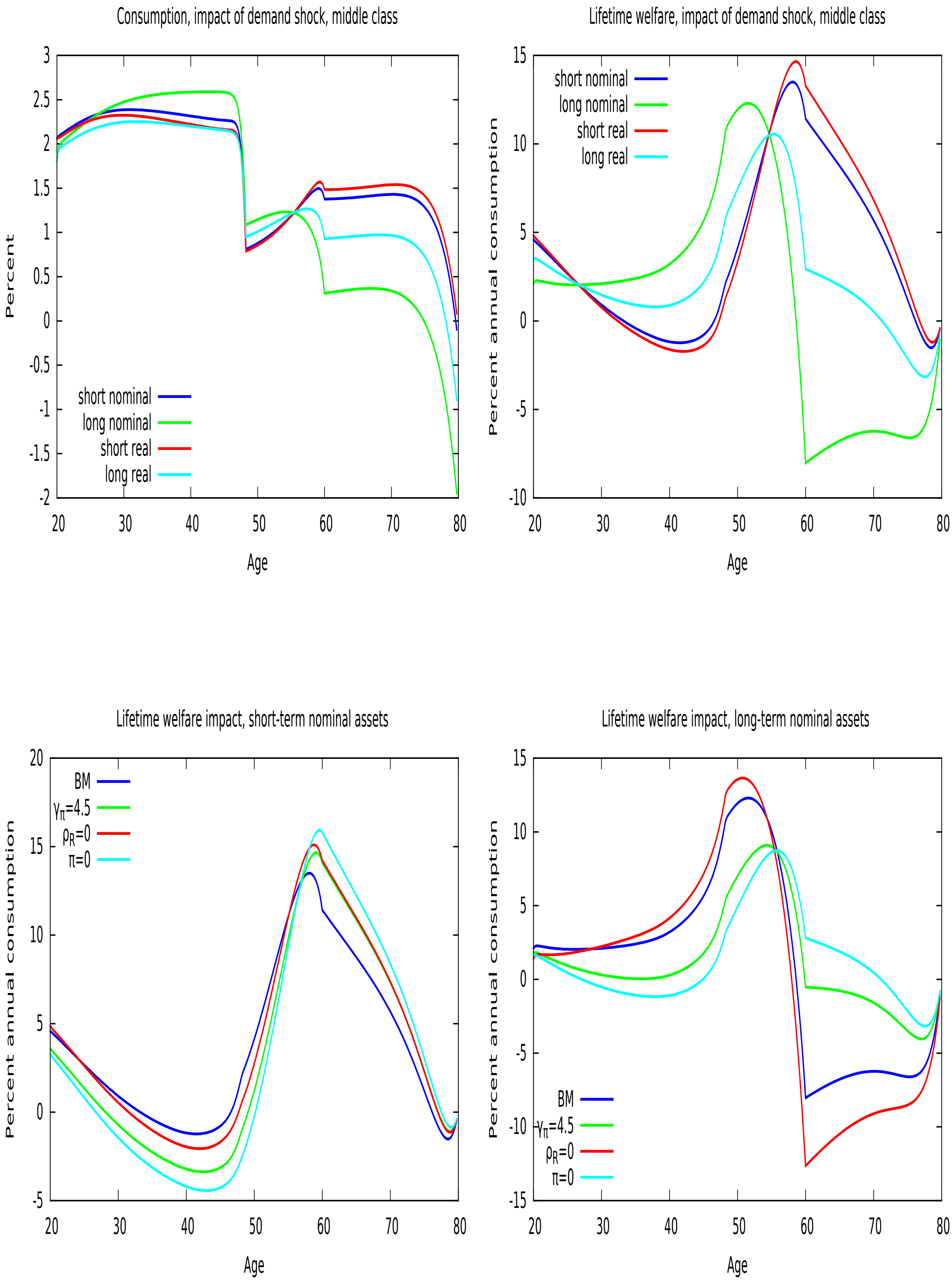


Figure 2.5: Impact effect of demand shock

	BM	$\gamma_\pi = 4.5$	$\rho_R = 0$	$\gamma_H = 0.1$	$\pi = 0$
Short-run nominal assets					
Y	1.21 (1.00)	0.42 (1.00)	0.39 (1.00)	1.37 (1.00)	0.10 (1.00)
Π	1.35 (0.95)	0.37 (0.90)	1.43 (0.99)	1.63 (0.96)	0.00 (-)
R	1.50 (0.56)	1.02 (0.42)	2.37 (0.99)	1.69 (0.63)	1.05 (-0.99)
$Rreal$	1.59 (-0.74)	0.98 (-0.48)	1.72 (0.14)	1.79 (-0.73)	1.05 (-0.61)
Δp^B	0.51 (-1.00)	0.23 (-1.00)	0.68 (-0.77)	0.59 (-1.00)	0.21 (0.47)
Δp^H	0.61 (-0.23)	0.80 (-0.26)	0.78 (-0.91)	0.56 (-0.23)	0.92 (0.88)
Long-run nominal assets					
Y	1.17 (1.00)	0.40 (1.00)	0.33 (1.00)	1.33 (1.00)	0.12 (1.00)
Π	1.37 (0.94)	0.37 (0.88)	1.47 (0.99)	1.68 (0.94)	0.00 (-)
R	1.50 (0.54)	1.01 (0.38)	2.39 (0.99)	1.73 (0.60)	1.04 (-1.00)
$Rreal$	1.59 (-0.76)	0.96 (-0.53)	1.73 (0.13)	1.82 (-0.76)	1.04 (-0.64)
Δp^B	3.43 (-0.85)	2.46 (-0.93)	4.63 (-0.44)	3.46 (-0.84)	2.14 (0.46)
Δp^H	0.61 (-0.21)	0.80 (-0.22)	0.79 (-0.91)	0.58 (-0.20)	0.92 (0.90)
Short-run real assets					
Y	1.21 (1.00)	0.42 (1.00)	0.40 (1.00)	1.37 (1.00)	0.10 (1.00)
Π	1.34 (0.95)	0.37 (0.91)	1.43 (0.99)	1.61 (0.96)	0.00 (-)
R	1.49 (0.56)	1.02 (0.43)	2.36 (0.99)	1.68 (0.63)	1.05 (-0.99)
$Rreal$	1.58 (-0.74)	0.97 (-0.48)	1.72 (0.14)	1.77 (-0.73)	1.05 (-0.61)
Δp^B	0.13 (-0.13)	0.10 (-0.94)	0.20 (-0.37)	0.15 (-0.07)	0.21 (0.47)
Δp^H	0.60 (-0.23)	0.80 (-0.27)	0.78 (-0.91)	0.56 (-0.23)	0.92 (0.88)
Long-run real assets					
Y	1.17 (1.00)	0.40 (1.00)	0.38 (1.00)	1.33 (1.00)	0.12 (1.00)
Π	1.32 (0.95)	0.37 (0.89)	1.44 (0.99)	1.58 (0.96)	0.00 (-)
R	1.47 (0.56)	1.01 (0.39)	2.37 (0.99)	1.65 (0.63)	1.04 (-1.00)
$Rreal$	1.55 (-0.74)	0.96 (-0.51)	1.72 (0.14)	1.74 (-0.73)	1.04 (-0.64)
Δp^B	1.68 (-0.85)	1.93 (-0.92)	2.14 (-0.38)	1.65 (-0.83)	2.14 (0.46)
Δp^H	0.59 (-0.23)	0.79 (-0.24)	0.78 (-0.91)	0.55 (-0.23)	0.92 (0.90)
Only capitalists, short-run nominal assets					
Y	1.21 (1.00)	0.45 (1.00)	0.46 (1.00)	1.08 (1.00)	0.00 (-)
Π	1.28 (0.97)	0.36 (0.95)	1.44 (0.98)	1.13 (0.97)	0.00 (-)
R	1.48 (0.61)	1.03 (0.53)	2.41 (0.98)	1.42 (0.63)	1.02 (-)
$Rreal$	1.53 (-0.66)	0.97 (-0.33)	1.73 (0.22)	1.41 (-0.61)	1.02 (-)
Δp^B	0.48 (-1.00)	0.21 (-1.00)	0.68 (-0.73)	0.44 (-1.00)	0.19 (-)
Δp^H	0.50 (0.85)	0.19 (0.83)	0.14 (0.90)	0.44 (0.85)	0.00 (-)

Table 2.3: Standard deviations for model driven by demand shocks, detrended (λ^{HP}=1600)

	BM	$\gamma_\pi = 4.5$	$\rho_R = 0$	$\gamma_H = 0.1$	$\pi = 0$
Short-run nominal assets					
Y	2.06 (1.00)	1.31 (1.00)	1.62 (1.00)	2.46 (1.00)	1.72 (1.00)
Π	2.34 (0.78)	0.67 (0.08)	2.99 (0.68)	2.71 (0.68)	0.00 (-)
R	3.74 (0.74)	2.68 (0.17)	5.13 (0.76)	3.83 (0.73)	2.38 (-0.14)
$Rreal$	2.38 (0.21)	2.19 (0.10)	2.65 (0.67)	2.50 (0.19)	2.38 (-0.14)
Δp^B	0.71 (-0.80)	0.27 (-0.25)	0.96 (-0.56)	0.82 (-0.70)	0.21 (0.02)
Δp^H	1.37 (-0.41)	1.75 (-0.34)	1.55 (-0.43)	1.24 (-0.38)	1.94 (-0.08)
Long-run nominal assets					
Y	2.04 (1.00)	0.76 (1.00)	1.68 (1.00)	2.38 (1.00)	1.09 (1.00)
Π	2.46 (0.64)	0.64 (0.45)	3.21 (0.43)	3.04 (0.57)	0.00 (-)
R	3.81 (0.63)	2.64 (0.37)	5.28 (0.56)	4.12 (0.62)	2.36 (-0.23)
$Rreal$	2.37 (0.17)	2.17 (0.18)	2.65 (0.57)	2.52 (0.13)	2.36 (-0.22)
Δp^B	3.57 (-0.52)	2.54 (-0.50)	4.79 (-0.09)	3.61 (-0.49)	2.20 (0.05)
Δp^H	1.47 (-0.23)	1.78 (-0.28)	1.71 (-0.12)	1.38 (-0.27)	1.96 (0.03)
Short-run real assets					
Y	2.06 (1.00)	1.34 (1.00)	1.63 (1.00)	2.47 (1.00)	1.72 (1.00)
Π	2.33 (0.77)	0.67 (0.07)	2.99 (0.67)	2.68 (0.69)	0.00 (-)
R	3.73 (0.74)	2.68 (0.17)	5.12 (0.76)	3.80 (0.73)	2.38 (-0.14)
$Rreal$	2.37 (0.21)	2.19 (0.10)	2.65 (0.67)	2.48 (0.20)	2.38 (-0.14)
Δp^B	0.14 (-0.11)	0.10 (-0.30)	0.21 (-0.06)	0.16 (-0.07)	0.21 (0.02)
Δp^H	1.35 (-0.43)	1.75 (-0.34)	1.54 (-0.44)	1.22 (-0.40)	1.94 (-0.08)
Long-run real assets					
Y	2.09 (1.00)	0.87 (1.00)	1.70 (1.00)	2.49 (1.00)	1.09 (1.00)
Π	2.30 (0.70)	0.64 (0.36)	3.00 (0.59)	2.72 (0.57)	0.00 (-)
R	3.65 (0.70)	2.64 (0.33)	5.10 (0.69)	3.76 (0.66)	2.36 (-0.23)
$Rreal$	2.34 (0.23)	2.17 (0.17)	2.64 (0.64)	2.44 (0.22)	2.36 (-0.22)
Δp^B	1.74 (-0.47)	1.98 (-0.43)	2.20 (-0.05)	1.72 (-0.43)	2.20 (0.05)
Δp^H	1.38 (-0.29)	1.76 (-0.32)	1.60 (-0.24)	1.25 (-0.28)	1.96 (0.03)
Only capitalists, short-run nominal assets					
Y	2.86 (1.00)	0.89 (1.00)	2.59 (1.00)	2.53 (1.00)	0.00 (-)
Π	2.60 (0.25)	0.65 (0.71)	3.34 (0.21)	2.18 (0.28)	0.00 (-)
R	3.93 (0.46)	2.85 (0.67)	5.49 (0.43)	3.58 (0.52)	2.53 (-)
$Rreal$	2.47 (0.35)	2.36 (0.49)	2.82 (0.59)	2.42 (0.41)	2.53 (-)
Δp^B	0.76 (-0.30)	0.27 (-0.69)	1.03 (-0.17)	0.66 (-0.33)	0.20 (-)
Δp^H	1.05 (0.70)	0.31 (0.67)	0.88 (0.67)	0.93 (0.71)	0.00 (-)

Table 2.4: Standard deviations for model driven by demand shocks, undetrended

of a higher coefficient on inflation in the Taylor rule ($\gamma_\pi = 4.5$ versus $\gamma_\pi = 1.5$), and in the form of no interest rate smoothing ($\rho_R = 0$), reduces the volatility of both output and inflation. In fact, stricter monetary policy reduces the variance of all variables under all asset structures, with the exception of Δp^B under real long-run assets. Including housing prices in the policy function stabilizes the housing market, but at the cost of destabilizing both output and inflation. The last column reports results for strict inflation targeting. Completely eliminating inflation reduces the standard deviation of output by a factor of about ten, so that the "divine coincidence" continues to hold approximately.

Looking across the four different asset regimes, the numbers are very similar, which one would expect after having seen the impulse responses. The only exception is the bond price dynamics, simply because bonds are a different type of asset under different asset structures. There are some small differences, in particular output is slightly more stable under long-run assets, both nominal and real. From this picture, it appears that all the heterogeneity in the economy has no important implications for monetary policy, at least if it is concerned with economic aggregates.

Table 2.4 provides the same information as Table 2.3, but now for the undetrended series. Remember that our model is a stationary model, so the Hodrick-Prescott filter is not necessary to stationarize the data, but the detrending partially filters out the low-frequency movements. The total variance of the undetrended series is of course higher. What about the policy trade-offs? Aggressive monetary policy ($\gamma_\pi = 4.5$) is still the right way to counteract demand shocks. However, being "aggressive" in the sense of raising interest rates immediately (no interest rate smoothing, $\rho_R = 0$), is now much less effective. In particular, it raises the variability of inflation. Most surprisingly, the divine coincidence now fails to hold by a wide margin. If the central bank sets the interest rate so as to perfectly stabilize inflation, more than two thirds of output fluctuations remain. We will take a closer look at this in Section 2.5.4.

Since redistribution matters more for the long-run movements, there are now larger differences across asset regimes. Aggressive monetary policy reduces output fluctuations much more in the case of long-term bonds. In this regime, we are also closer to the divine coincidence. Since households are less affected by short-term variations in interest rates, their use for inflation stabilization causes less redistribution.

2.5.3 The variability of welfare

The reason why long-run fluctuations matter is that they affect household utility. To shed more light on this issue, Table 2.5 reports the variability of period utility and of lifetime welfare measures under different asset and policy regimes. We compute an approximation to welfare by evaluating the individual utility function at the linearized solution of the model for different monetary policies around the same deterministic steady state. Notice that this procedure is not adequate for optimal policy exercises, where different policies would lead to different stochastic steady states (see Benigno and Woodford (2006) for a discussion of optimal policy in linearized models). We therefore make only limited use of these welfare measures: they give the utility equivalent of the generated fluctuations in consumption, leisure etc., conditional on a given mean of all variables. We compute

	BM		$\gamma_\pi = 4.5$		$\rho_R = 0$		$\gamma_H = 0.1$		$\pi = 0$	
	Poor	Middle	Poor	Middle	Poor	Middle	Poor	Middle	Poor	Middle
	Current utility, perc.of consumption									
shortnom	3.44	6.09	3.12	5.66	3.36	5.95	3.57	6.33	3.06	5.86
longnom	3.67	6.57	3.26	4.07	3.64	7.11	3.75	7.39	3.12	3.67
shortreal	3.43	6.29	3.12	5.76	3.35	6.31	3.56	6.53	3.06	5.86
longreal	3.60	5.00	3.23	3.94	3.51	4.64	3.73	5.34	3.12	3.67
	Lifetime welfare, perc.of lifetime consumption									
shortnom	1.29	1.35	1.10	1.25	1.26	1.46	1.38	1.42	1.14	1.36
longnom	1.49	1.21	1.14	0.71	1.53	1.35	1.55	1.32	1.08	0.73
shortreal	1.28	1.42	1.11	1.28	1.24	1.54	1.36	1.49	1.14	1.36
longreal	1.41	1.12	1.13	0.76	1.40	1.12	1.50	1.22	1.08	0.73

Table 2.5: Variability of utility and welfare, OLG households, model with demand shocks

welfare as the realized value of the objective function in Equ. (2.14). We ignore the demand shocks D_t for this purpose, which we do not interpret as shocks to utility, but rather as a wedge between different assets, similar to Smets and Wouters (2007).

Table 2.5 lists results separately for poor and for middle class households. The first part of the table measures the variability of period utility, averaged over all cohorts. Being aggressive on inflation reduces this variability for both types of households, but the improvement is small under short-run nominal assets. Especially for the middle class, the reduction in volatility is much more pronounced under long-term nominal assets. In that case, households are protected against variations in the nominal rate, but benefit from the decrease of inflation variability.

The second part of the table measures the variability of lifetime welfare, wich depends not just on the variability of period utility, but on its correlation over time and cohorts. If cohorts are hit by a distributional shock, they cannot expect to be compensated in the future, therefore distributional changes add up. For welfare, the asset structure matters. If bonds are short-term, the interest rate movements that are necessary to stabilize inflation generate random redistributions such that the welfare variability of middle class households is more or less unaffected (whether the variance of welfare goes up or down depends on the details of the calibration). Not so with long-term assets: households are largely shielded from interest rate movements, therefore inflation stabilization also reduces welfare variability. A further interesting result is that interest smoothing also smooths welfare. Abandoning it (the case $\rho_R = 0$) increases the variability of lifetime welfare of the middle class. This provides a new rationale for interest rate smoothing, different from the one in Woodford (2004).

These results force us to reconsider the rationale for inflation stabilization. The textbook argument why unexpected inflation is bad is that it causes random redistribution. However, the interest rate movements to stabilize inflation also cause redistribution. Under a realistic degree of household heterogeneity, it depends on the asset structure whether inflation stabilization benefits households to a substantial degree.

2.5.4 Why the divine coincidence fails

Figure 2.6 illustrates the behavior of the economy under strict inflation targeting. The upper left panel shows the response of the main economic aggregates to a demand shock. For comparison, the upper right panel shows the same response under the benchmark policy. Strict inflation targeting shifts all the curves downward. In order to keep inflation at zero, the interest rate must be set such that output goes slightly down, for the following reason. To compensate for the decrease in D_t in the Euler equations (cf. (2.18) and (2.22)), the nominal rate R_t has to increase. This leads to a redistribution towards the middle class, which holds positive nominal assets on average, and within this group from the young to the middle aged. The redistribution to the middle class means that, for a given level of labor input, wages have to increase because of a wealth effect on labor supply. To keep real marginal cost constant, the increase in wages must be matched by an increase in labor productivity, which requires a reduction in labor input and therefore output. We would like to add, without showing the details, that this effect would be much larger with flexible wages, leading to a much stronger deviation from the divine coincidence.

The fall in output is driven almost entirely by a reduction in housing investment. This comes from several mechanisms. First, the reduction in real wealth of young households who face the down payment constraint leads to a sharp reduction in their demand for housing. Furthermore, to generate a reduction in output, the interest rate has to increase by more than the wedge for several quarters (cf. below), which provides an incentive to shift assets away from housing into bonds.

The middle left panel shows the interest rates under the benchmark policy rule and under inflation targeting. The blue line gives, as a reference, the interest rate that keeps $R_t(1 + D_t) = R^*(1 + D^*)$ satisfied for all t, which is the interest rate that is implied by strict inflation targeting in the representative agent model. In our model, only a slightly higher interest rate is required to achieve this (cyan line), and only in the first few periods. One year after the shock and later, $R_t(1 + D_t) = R^*(1 + D^*)$ is satisfied quite well. In contrast, under the benchmark policy, the nominal interest (green line) rate is higher, but the real interest (red line) rate is lower than under inflation targeting.[7]

The other three panels in the graph show very long sample paths for GDP, capital stock and housing stock under the benchmark policy with short-term nominal interest rates (blue line), inflation targeting with short-term nominal interest rates (red line), and inflation targeting with long-term nominal interest rates (green line). The same

[7] It may be surprising that inflation targeting is achieved by a nominal interest rate that is lower than in the benchmark Taylor rule for almost all periods over the first 40 quarters. Part of the explanation is that inflation targeting keeps interest rate somewhat higher in later years, not shown here. This reflects again the power of forward guidance in this kind of models.

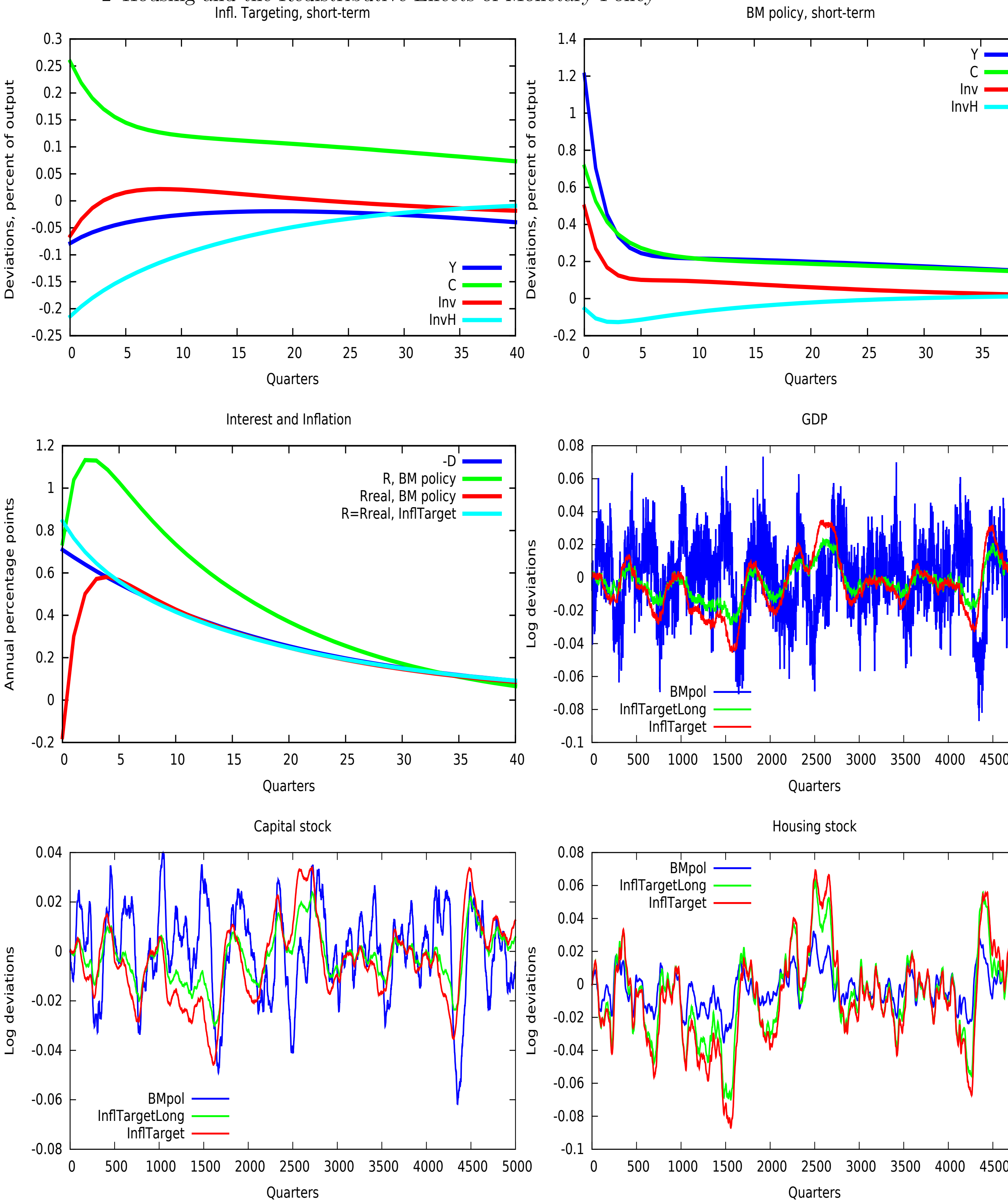


Figure 2.6: Model simulations under inflation targeting

realization of the shock series was used in the three cases. The graph for GDP shows that inflation targeting eliminates most of the high-frequency fluctuations, but leaves a lot of low-frequency fluctuations. This is because the fluctuations in the capital stock are not much dampened, and the fluctuation in the housing stock are even amplified. As the impulse responses have shown, the redistribution implied by the interest rate movements affects investment in both capital and housing. Those responses are not large, but very persistent, which leads to non-negligible fluctuations at low frequencies. Since long-term bonds reduce the size of this redistribution, it also reduces the amplitude of these fluctuations.

2.5.5 Discussion

The analysis above has highlighted a number of channels through which interest rate movements generate redistribution, which in turn affect the aggregate economy. To what extent these channels are operating in the real world will depend on a number of market imperfections.

Most obviously, asset market frictions would matter. In our model, the only friction on asset trade is the down payment constraint of worker households. Changes in housing demand come from a continuous adjustment along the intensive margin. With fixed costs of house purchases and mortgage contracts, adjustment would mainly take place at the extensive margin, such as the time of switching from rental to owner occupied housing. It is an open questions whether, in general equilibrium, those adjustments are similar in magnitude to the intensive margin adjustments in our model.

A further important factor is the labor market. We have assumed a frictionless labor market, on which wage rigidity was imposed in an ad-hoc way, similar to many papers in the New Keynesian literature. With perfectly flexible wages, the wealth effects from redistribution generate large movements in the real wage, and therefore in real marginal costs and inflation. Real wage rigidity dampens the effect on inflation, and generally affect the redistribution between worker and capitalist households. The labor literature has stressed the difference between wage rigidity for new hires versus and wage rigidity for continuing job matches Haefke et al. (2013); Pissarides (2009); Gertler and Trigari (2006). The latter has no allocative effects in standard models of frictional labor markets. Whether wages for new hires are more flexible is still disputed. In our model, the wages of continuing matches affect the redistribution between workers and firm owners. But to dampen the variability of marginal costs, it is also necessary that wages at the margin (for new hires, overtime work etc.) are rigid. Moreover, search frictions in the labor market would affect the welfare calculations. In a perfectly flexible labor market, a marginal increase in labor input does not increase welfare; the welfare effect comes from the change in wages. We have imposed a markup of wages over the marginal rate of substitution of ten percent, which generates some welfare gain from higher labor input. In a model with search frictions, a decrease in unemployment might potentially increases the welfare of worker households much more. The model in Gornemann et al. (2016) combines search frictions with wage rigidity. It would be interesting to see how this interacts with households having owner-occupied housing and mortgages.

2.6 Conclusions

In this paper, we have investigated how household heterogeneity affects the ability of the central bank to stabilize the economy. For this purpose, we have developed a New Keynesian model with strong heterogeneity across households along several dimensions: skill level, access to bond markets, home ownership, and age. This generates diversity in the exposure of households to variations in the nominal interest rate and the inflation rate. The marginal effect of wealth on expenditures, both consumption and housing, differ widely across households.

In this environment, we find that household heterogeneity makes it harder for the central bank to stabilize the economy in the face of demand shocks. At the aggregate level, this becomes apparent if one considers not just conventionally detrended time series, but total fluctuations. Since the effect of redistribution is small but very persistent, monetary policy can generate welfare-relevant fluctuations at frequencies lower than business cycle frequencies. If monetary shocks have large and persistent redistributional effects, the goals of stabilizing macroeconomic aggregates and stabilizing individual welfare are not necessarily aligned. However, they are much better aligned if the assets traded have a fixed rather than an adjustable nominal interest rate. There has been a widespread decrease in the use of variable rate mortgages cross European countries over the last five to ten years (Bouyon, 2017, Figure 2). From the view point of conventional monetary policy, this is highly welcome and allows the monetary authority to fight inflation more aggressively.

From the issues raised in this paper, we want to point out three areas for future research. The first one is the endogenous determination of the asset structure. For reasons of tractability, we have imposed the asset structure exogenously. In each version of the model, there was only one type of bond available. If asset choice were endogenized and the contracting parties chose the type of the asset that is optimal for them, what does this imply for the stabilization of the economy? Are there important externalities from asset choice? The second one is the role of labor market frictions. Distribution effects depend crucially on the behavior of wages, both wages of new hires and wages in ongoing employment relationships. Understanding the nature of wage rigidities is important not just for the analysis of the labor market, but also for monetary policy. The third point is the design of optimal policy. Our results above indicate that the following points are important. To what extent is the maturity of assets such as mortgages an individual choice, and to what extent is it determined by institutional or regulatory factors? What is the policy objective? Is it separable across cohorts and over time? Do we consider redistributions across lifetime? Following up on McKay et al. (2016), another interesting question is what the heterogeneity of our model implies for the effectiveness of forward guidance.

2.A Data from the Survey of Consumer Finances 2013

We present the data we used for calibrating our model in greater detail. These tables report the medians and means of important variables for five-year age bins. The categorization has been done by selecting the lowest 20 percent, the next 70 percent and the top 10 percent in terms of net worth in each age bin we consider. Notice that the data does not exhibit any panel dimension, so we cannot follow households over their life cycle.

Table 2.6: Median Networth

	<20	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	>84
Poor	-3010	-20900	-22400	-16299	-6050	-5870	-2300	0	0	1410	16200	4700	2151	8680	5000
Middle	4810	6160	13330	26670	47300	76700	112400	158000	193750	230100	319800	253800	241700	236100	180700
Capitalists	44400	100050	232000	397300	919200	1381050	1414500	1917700	3349800	2929000	3906800	3384000	1635900	1516299	1840600

Table 2.7: Median Net Financial Assets

	<20	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	>84
Poor	-12610	-25250	-35910	-43000	-16480	-30864	-18790	-2540	-3960	-520	-3900	0	-45	200	1150
Middle	110	-89	-1640	-6744	-7700	-5700	-6830	-910	6830	23980	44140	60640	35420	27500	28000
Capitalists	400	5220	-33920	-17400	80200	470000	400000	625500	1367500	1139000	2092005	1860000	831530	993000	1070000

Table 2.8: Median Value of Owner Occupied Housing

	<20	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	>84
Poor	0	0	0	0	0	0	0	0	0	0	7500	0	0	0	0
Middle	0	0	0	3000	68000	108000	124000	140000	150000	150000	167000	165000	128000	150000	100000
Capitalists	44000	50000	201000	200000	485000	550000	675000	470000	580000	500000	750000	600000	500000	450000	221000

Table 2.9: Median Ownership Rate

	<20	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	>84
Poor	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
Middle	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
Capitalists	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 2.10: Mean Networth

	<20	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	>84
Poor	-9007	-31874	-44878	-28772	-18931	-22055	-17314	-10674	-7222	-25202	19676	5001	-4688	12187	8194
Middle	7027	9149	25556	52836	101999	148365	185919	263208	327759	346359	468285	429670	293594	290608	244753
Capitalists	44896	173835	562604	866459	1882061	3322961	3061251	4319746	5550647	5716305	7369097	7200125	4605141	3184188	4617706

Table 2.11: Mean Net Financial Assets

	<20	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	>84
Poor	-11660	-37687	-67120	-92664	-61263	-90165	-65382	-40865	-39821	-67013	-43919	-18492	-31522	-12886	2722
Middle	1149	-7674	-26205	-51218	-53734	-40004	-15782	12887	66903	85774	152718	161172	76589	85030	93537
Capitalists	400	83971	100811	82380	356744	768185	880397	1528859	2351971	2607131	3779191	3693598	2120120	1682842	2733825

Table 2.12: Mean Housing

	<20	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	>84
Poor	0	0	10893	44543	32784	55146	29995	21591	26790	29298	50668	16103	20539	20333	3111
Middle	1299	7531	34008	81306	113113	138733	149355	180119	179708	190177	219727	200590	163108	167089	130069
Capitalists	44496	58321	241399	293654	673941	746081	741126	676932	919926	789654	1085852	850153	613624	633374	416659

Table 2.13: Mean Ownership Rate

	<20	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	>84
Poor	0	0	0.07	0.3	0.25	0.38	0.28	0.2	0.24	0.21	0.52	0.34	0.26	0.41	0.12
Middle	0.03	0.09	0.33	0.5	0.6	0.69	0.73	0.83	0.84	0.86	0.96	0.94	0.94	0.92	0.91
Capitalists	1	0.7	0.9	0.81	0.97	0.92	0.96	0.92	0.99	0.97	0.94	0.99	0.99	0.95	0.98

2.B First order conditions middle class households

The utility function of the household is given by

$$
\begin{aligned}
u(c_t, l_t, h_t^R, h_t^O) =& \log(c_t) + \eta \log(1 - l_t) \\
&+ \eta_H \log\left[\left((h_t^R)^{(\sigma-1)/\sigma} + (\xi h_t^O + \kappa)^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \right]
\end{aligned}
$$

Marginal utilities are

$$
\begin{aligned}
u_c =& \frac{1}{c_t} \\
u_l =& -\frac{\eta}{1 - l_t} \\
u_{h^R} =& \frac{\eta_H}{\left((h_t^R)^{(\sigma-1)/\sigma} + (\xi h_t^O + \kappa)^{(\sigma-1)/\sigma}\right)} (h_t^R)^{\frac{\sigma-1}{\sigma}-1} \\
u_{h^O} =& \frac{\eta_H}{\left((h_t^R)^{(\sigma-1)/\sigma} + (\xi h_t^O + \kappa)^{(\sigma-1)/\sigma}\right)} (\xi h_t^O + \kappa)^{\frac{\sigma-1}{\sigma}-1} \xi
\end{aligned}
$$

We set up the Lagrangian, using λ and $\tilde{\lambda}$ to denote the Lagrange multipliers. To simplify notation, we drop the age subscript s.

$$
\begin{aligned}
&\mathcal{L} = \max \mathbb{E}_0 \sum_{t=0}^{I-1} \beta^t u(c_t, l_t, h_t^R, h_t^O) - \\
&\lambda_t [p_t^B b_t + h_t^O p_t^H + c_t + r_t^H h_t^R - (1-\tau) w_t \zeta_t l_t - \mathcal{I}_t^R \psi_t - \\
&((\mu + r^B) v_t^B + (1-\mu) p_t^B) b_{t-1} - (1 - \delta_H) h_{t-1}^O p_t^H] \\
&+ \tilde{\lambda}_t [v_t^B b_t + \nu \mathbb{E} p_{t+1}^H h_t^O]
\end{aligned}
$$

Now taking the F.O.C.s yields (for brevity, omit the expectation operator $\mathbb{E}_t$)

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_t} &: u_{c_t} = \lambda_t \\
\frac{\partial \mathcal{L}}{\partial l_t} &: u_{l_t} + \lambda_t (1-\tau) w_t \zeta_t = 0 \\
\frac{\partial \mathcal{L}}{\partial h_t^R} &: u_{h_t^R} - \lambda_t r_t^H = 0 \\
\frac{\partial \mathcal{L}}{\partial h_t^O} &: u_{h_t^O} - \lambda_t p_t^H + \tilde{\lambda}_t \nu \mathbb{E} p_{t+1}^H + \beta \lambda_{t+1} (1 - \delta_H) p_{t+1}^H = 0 \\
\frac{\partial \mathcal{L}}{\partial b_t} &: -\lambda_t p_t^B + \tilde{\lambda}_t v_t^B + \beta \lambda_{t+1} ((\mu + r^B) v_{t+1}^B + (1-\mu) p_{t+1}^B) = 0
\end{aligned}
$$

Expressing $\tilde{\lambda}_t$ gives

$$
\tilde{\lambda}_t = \frac{\lambda_t p_t^B - \beta \lambda_{t+1} ((\mu + r^B) v_{t+1}^B + (1-\mu) p_{t+1}^B)}{v_t^B} \tag{2.34}
$$

Plugging (2.34) into the FOC for owned housing and using $u_{c_t} = \lambda_t$ we get

$$u_{h_t^O} - u_{c_t} p_t^H + \frac{u_{c_t} p_t^B - \beta u_{c_{t+1}} ((\mu + r^B) v_{t+1}^B + (1 - \mu) p_{t+1}^B)}{v_t^B} \nu \mathbb{E} p_{t+1}^H + \beta u_{c_{t+1}} (1 - \delta_H) p_{t+1}^H = 0 \tag{2.35}$$

or

$$u_{h_t^O} - u_{c_t} [p_t^H - \frac{p_t^B}{v_t^B} \nu \mathbb{E} p_{t+1}^H] - \beta \beta u_{c_{t+1}} \left[\frac{((\mu + r^B) v_{t+1}^B + (1 - \mu) p_{t+1}^B)}{v_t^B} \nu \mathbb{E} p_{t+1}^H - (1 - \delta_H) p_{t+1}^H \right] = 0$$

3 The Price of Capital, Factor Substitutability and Corporate Profits

3.1 Introduction

The stock of physical capital that is used per employed worker for the production of output has steadily risen in the post-WWII period in the United States and many other industrialized countries. Since the 1970s, this positive trend has been accompanied by a steady decline in the labor share of income – a phenomenon that has received much attention recently, since it contradicts conventional wisdom regarding constant factor shares of income that was first presented in Kaldor (1961). Recent evidence further suggests that during the same period, the ratio of corporate profits to GDP has risen and become more volatile in the past two decades. Put differently, it seems that the traditionally close tie between corporate profits and labor income has disappeared.[1] Figure 3.1 depicts these trends for the U.S. economy during the post-WWII period.

In this paper we investigate whether these developments are possibly connected in that they can be explained by a common determinant. In particular, we ask whether and to which extent they can simultaneously be explained by the observed decline in the relative price of new capital goods that Gordon (1990) documented for the U.S., or rather by the change in the production technology that slowly, but steadily has increased the substitutability of labor by capital.[2] *A priori* either of these two fundamental changes has the potential to have contributed to the rise in the capital-to-labor share and also to the declining labor share of income. But what about their respective implication for the dynamics of firms' profits?

We address these questions in the context of a dynamic stochastic equilibrium model of competitive search in the labor market. We extend the standard model by allowing firms to use physical capital in addition to labor for producing output. By assumption capital is easier to adjust than labor. We take this view because of structural change that has transformed the U.S. economy during the period we consider towards one where services

[1] This recent phenomenon is emphasized also by the FRED blog of the Federal Reserve Bank of St. Louis on August 8, 2018. `https://fredblog.stlouisfed.org/2018/08/corporate-profits-versus-labor-income/`

[2] Gordon's analysis focuses on the change in the price of equipment rather than structures and documents that the price decline of equipment was extraordinarily strong. For the sake of our analysis we do not distinguish between the various components of physical capital, but look at total physical capital and the associated weighted average price.

have become increasingly important for GDP production. In addition to labor, services require equipment rather than structures, and equipment is relatively easy to adjust. Moreover, the production technology allows for factor substitutability by permitting firms to employ multiple workers. Thus, we effectively abandon the Leontief production function of fixed factor proportions which is commonly used in models of labor market search. Doing so is necessary for studying the implications that varying degrees of factor substitutability – in addition to a change in the relative price of capital – have for our variables of interest.

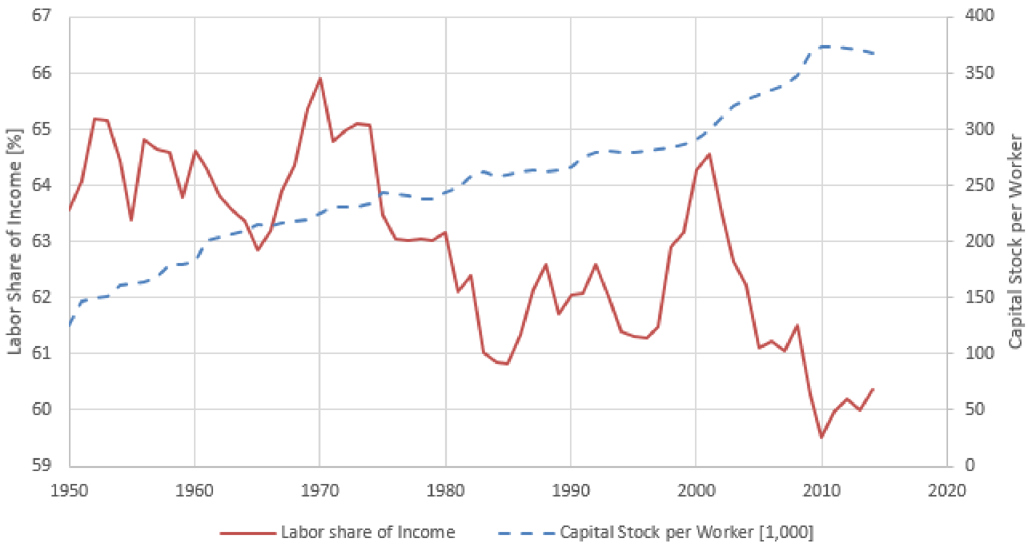
We calibrate the model to the U.S. economy in the post-WWII period, solve and simulate it. We use the model as a lab to disentangle the role that a steady decline in the relative price of new capital goods as opposed to an increase in the factor substitutability of the output production play in simultaneously explaining a declining labor share and a rise in the capital-to-labor ratio and in the level and volatility of corporate profits.

Our results show that when labor is relatively more costly to adjust than capital and the two production factors are (partial) complements, a rise in the degree of factor substitutability lets firms choose a more capital-intense input mix. The implied decrease in labor demand causes wages, employment and subsequently the labor share of income to fall. This fall in the labor share translates into rising corporate profits. When firms face shocks to total factor productivity, increased factor substitutability raises the volatility of investment and capital, but dampens that of wages and employment. In sum, corporate profits relative to output become more volatile. A decline in the relative price of capital generates identical reactions except that the labor share of income rises. Hence, our model suggests that quantitatively speaking, the implications of a change in the production technology towards increased factor substitutability have outweighed those of a steady decline in the relative price of physical capital.

Our paper contributes to the macro literature in several respects. First, we study the declining labor share in the U.S. in conjunction with the related rise in the capital-to-labor ratio and the level and volatility of firms' profits. So, rather than looking at one trend in isolation, we study several trends that we expect to be interrelated and identify a common determinant. Second, we augment a labor market model with competitive search with a production technology that uses physical capital in addition to labor and allows for factor substitutability. Abandoning the more standard fixed-proportion input type of production function is a prerequisite for exploring the role of factor substitutability. Lastly, we can explain long-run changes in the volatility of corporate profits using changes in real economic variables only, thereby creating a bridge between a standard economic setup and finance where the dynamics of firms' profits are essential for dividends and stock price movements.

The remainder of this paper is structured as follows. In section 3.2 we link our work to the closely related literature. In section 3.3 we present our dynamic equilibrium model of a frictional labor market. In section 3.4 we calibrate the model to U.S. data and perform simulation exercises to explore the implications of a change in the relative price of capital, and in the degree of factor substitutability, respectively. In section 3.5 we use aggregate time-series data from the U.S. on key model variables for a simple regression analysis to check for the empirical plausibility of our main arguments. Section 3.6 concludes.

Figure 3.1: Aggregate Trends in the U.S. Economy



Notes: Labor share relates to nonfarm business income. The capital stock is defined in millions of real U.S. Dollars (base year 2011), while employment is the total nonfarm payroll. All series were downloaded from FRED database.

3.2 Related Literature

This paper relates to several strands of the literature. First it relates to the work on factor substitutability in output production and its implications for the total economy. The distinction between short-run and long-run substitutability among input factors and the interaction with factor prices has received renewed interest in the macro literature and is discussed, e.g., in connection with increased digitization.[3] Acemoglu and Restrepo (2017) empirically study the competition between robots and workers for executing various tasks. In their environment, robots have a large negative effect on employment and wages. We do not consider tasks, but rather look at the implications of increased factor substitutability, or a declining price of capital for aggregate employment or wages in an environment where labor is subject to search frictions and, by assumption more, costly to adjust than capital. Also, wages are determined endogenously, but the price of capital is treated as a parameter. We use a CES production function and vary the parameter that reigns the degree of factor substitutability. In our environment a rise in substitutability decreases employment and wages, because firms *ceteris paribus* substitute towards the more flexible factor capital. Shim (2015) explores the implications that varying degrees of factor substitutability have for corporate profits, the associated operational risk and average stock returns of firms. His setup bridges real economic considerations and finance. Shim uses a firm valuation model that features partial capital irreversibility and external financing constraints, but treats labor as fully flexible. For the Compustat panel of U.S. firms he shows that rising factor substitutability is associated with less variable corporate profits. Shim proxies substitutability by firms' capital-labor ratios and works with a Cobb-Douglas production function that exhibits a constant unit-elasticity of substitution. Our setup nests that of Shim, but we allow for a varying degree of substitutability by altering the respective parameter in a CES production function and consider a representative firm rather than a cross-section of firms. One of our main results is that in an environment where capital is easier to adjust than labor, a rise in factor substitutability increases the volatility of firms' profits.[4]

Second, it relates to the literature on labor market search when firms can hire and employ multiple workers. In order to study the relationship between factor substitutability and firm profits, we abandon the Leontief-type production commonly used in search and matching models where a firm has one job which can be filled with one worker. We use a competitive search framework and allow firms to hire multiple workers. When firms use capital in addition to labor, competitive search with wage posting does not suffer from inefficiencies arising from the hold-up problem faced by firms under bilateral

[3] A cohesive summary of this literature is beyond the scope of this paper, but readers may want to look at Brynjolfsson and Afee (2014) for a general discussion. We instead cover a selection of examples, which all closely relate to our paper.

[4] This finding is consistent with what Danthine and Donaldson (2002) report when treating firms' labor costs as predetermined. In that case the volatility primarily affects dividends, which are defined as sales and profits net of labor costs.

wage bargaining and continues to render an efficient labor market equilibrium.[5] Hawkins (2013) is among the first to model firms that commit to a posted wage and hire multiple workers. His model has no physical capital. The same holds true for Schaal (2017) who allows for multiple workers per firm when analyzing the role of uncertainty for business cycle dynamics, and Kaas and Kircher (2015) who explore the business cycle dynamics of a model with heterogeneous firms that can employ multiple workers. Our paper differs in that it focuses on the interplay of several long-run trends, and that our model features firms that use labor and physical capital in the output production. Our setup is – to the best of our knowledge – the first to allow for physical capital in a multi-worker firm environment with competitive search in the labor market.

Lastly, our paper adds to the literature that explores alternative reasons for the decline in the labor share of income that has been observed in many OECD member countries since the mid-1970s. This observation stands in stark contrast to a supposedly constant labor share – one of the empirical facts presented in Kaldor (1961).[6] We study the declining labor share in conjunction with closely related trends, i.e. the rise in the capital-to-labor ratio and in the level and volatility of firms' profits and look for a common determinant. We use a setting with frictional labor markets and a production technology that incorporates factor substitutability to ask whether all trends can simultaneously be explained by a decline in the relative price of capital, or rather by a change in the production technology towards increased factor substitutability. Our work is linked to that of Karabarbounis and Neiman (2014) who find that lower prices of capital lead to a decline in the labor share. When estimating their model, they find an elasticity of substitution between capital and labor equal to 1.25. Compared to existing estimates by Chirinko (2008), or León-Ledesma et al. (2010), this value is high, but crucial for their results, as it implies that the inputs are substitutes rather than complements. We instead consider an elasticity of substitution less than one for our simulation exercises. With inputs being complements, a decrease in the price of capital leads to a rise in the labor share, whereas rising substitutability lets the share decline.

[5] Firms with multiple workers and physical capital have been studied when labor market matching is assumed to happen randomly. A recent example is Gertler et al. (2016).

[6] Blanchard (1997) was among the first to address diverging trends in unemployment and the labor share of income between some Anglo-Saxon countries including the U.S. and selected countries in continental Europe. He used a static general equilibrium model with frictional labor markets and monopolistic competition in the goods market to explore the role of supply vs. demand forces at work. He identifies alternative wage-setting mechanisms as key sources for observed cross-country differences in long-run trends. Recent contributions have examined alternative explanations, including sectoral concentration (Autor et al., 2017), automation and digitization (Arntz et al., 2016), increased markups (Loecker and Eeckhout, 2017) or international trade (Elsby et al., 2013). The view of a declining labor share is not unambiguosly held among economists. For a discussion on potential measurement issues see Gomme and Rupert (2004).

3.3 A Model of Competitive Search

Our model economy is populated by a unit mass of identical firms and a unit mass of identical workers. Firms post vacancies and invest in physical capital in order to maximize their profits. Due to labor market frictions, firms cannot hire workers directly, but have to post vacancies at a cost a and a corresponding wage $\tilde{w}$ that is fixed as long as the employment relationship lasts. The transition from vacancies to a filled job and from unemployed to employed depends on the number of workers applying to a vacancy and the number of vacancies posted by the firms. Firms can post vacancies in various submarkets, characterized by a wage and the ratio of jobs and jobseekers. Unemployed workers direct their search towards one of those markets, trading off the wage and the chance of getting hired. The interplay of the firms' posting behavior and the workers' application decisions generates the labor market tightness, which is defined as the ratio of vacancies to the number of applicants in a market. For ease of exposition the actual matching is governed by a standard matching function, as opposed to a specific matching algorithm.

3.3.1 Firms

We start the detailed description of the model at the firm as it is our core unit of analysis. There exists a unit mass of identical firms in this economy. They use capital k and labor l to produce a homogeneous output good y. The inputs are transformed into the output good according to a constant elasticity of substitution (CES) production function:

$$y(k_t, l_t, z_t) = z_t \left(\alpha k_t^\sigma + (1-\alpha) l_t^\sigma\right)^{\frac{1}{\sigma}}, \quad \text{with } \alpha \in (0,1),\ \sigma \in (-\infty, 1]$$

We choose this functional form for two reasons. First, it is more general than the commonly used Cobb-Douglas function, which it nests as a special case. Second and more importantly, this functional form allows us to explicitly vary the substitutability of input factors, which enables us to address our research question. The elasticity of substitution between k and l depends on the parameter σ and is given by $\frac{1}{1-\sigma}$. As σ is a key model parameter, it is important to understand its effects on the production function. The parameter σ can vary between $-\infty$ and 1. For the limiting case of $-\infty$ the elasticity of substitution converges to zero and the production function approaches the Leontief production function with a fixed ratio of input factors. This implies that inputs are perfect complements. For $\sigma = 1$ input factors are perfect substitutes. At $\sigma = 0$ the CES nests the Cobb-Douglas case.[7] The other parameter entering the production function is α, which governs the capital intensity of production. We also include a standard Hicks-neutral TFP process z_t, which enables us to consider the variability of economic quantities.

Firms can purchase capital at a fixed price p^k per unit. Capital depreciates at rate δ every period. Because of frictional labor markets, firms can expand their labor force

[7] For further discussions on the CES function and its properties see Klump et al. (2012).

only by posting vacancies v_t together with a wage rate $\tilde{w}_t$ in a particular submarket, which is characterized by its respective tightness, θ_t.[8] For each vacancy posted, the firm has to pay a vacancy posting cost, a. This cost can be thought of as advertising and training newly hired employees. By assumption, a constant fraction ν of matches breaks up every period. This is the only possibility for a match to end. The firms cannot decide which workers to fire. Thus, the stock of employment l_t is a state variable for the firm in period t.

The fact that firms decide on the wage offered for a posted vacancy in every period potentially generates a distribution of wages. Since we do not focus on wage dynamics *per se* in this paper, we choose to simplify the wage setting process. New hires formed during period t become productive in period $t+1$. These new hires h_t will be paid the posted wage $\tilde{w}_t$. The wage bill that a firm has to pay in period t is given by $l_t w_t$, where w_t denotes a weighted average of the wage paid to continuing workers and new hires from the previous period. In brief, $l_{t+1}w_{t+1} = (1-\nu)l_t w_t + h_t\tilde{w}_t$. We calculate the wage bill in a recursive way, which is described in greater detail in Appendix 3.A. We show that our recursive formulation is equivalent to keeping track of the entire history of hires and wages. Therefore, w_t is an additional state variable for the firm.

The firm discounts future profits at rate $0 < \beta < 1$. The firm's problem can be summarized as follows:

$$\max_{v_t,\theta_t,\tilde{w}_t,i_t} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t [y(k_t, l_t, z_t) - w_t l_t - p^k i_t - a v_t]$$

subject to

$$\begin{aligned} h_t &= v_t q(\theta_t) \\ l_{t+1}w_{t+1} &= (1-\nu)l_t w_t + h_t\tilde{w}_t \\ l_{t+1} &= (1-\nu)l_t + h_t \\ k_{t+1} &= (1-\delta)k_t + i_t \\ z_{t+1} &= \rho z_t + \epsilon_t,\ \epsilon_t \sim \mathcal{N}(0, Var_\epsilon) \end{aligned}$$

Firms maximize the expected present discounted value of future profits. Profits consist of revenue minus wage payments, investment expenditures and hiring costs. The firm takes as given that the number of newly hired employees equals the posted vacancies multiplied by the job filling rate, the recursive formulation of the wage bill, and the laws of motion for capital, labor and exogenous total factor productivity, z_t. As we elaborate below, in equilibrium two additional constraints must be satisfied, i.e. the

[8] We choose wage-posting plus directed search – rather than random search – to avoid the holdup problem a firm would face when making investment decisions. In our competitive search setting, a higher capital stock implies higher wages and also a higher job filling rate. See Acemoglu and Shimer (1999) for more details.

optimal application rule for searching workers and the requirement that the ratio of all job-vacancies to searching workers indeed equals labor market tightness in a given submarket.

3.3.2 Households

Workers are part of a big family, consisting of a continuum of members normalized to measure 1. Each worker can be employed or unemployed. If unemployed, she chooses to apply to a particular submarket that is characterized by vacancies and the corresponding wage-rate $\tilde{w}_t$. The worker's chances of getting matched depend on the ratio of vacancies posted to the measure of job seekers in that submarket, i.e. the labor market tightness. If employed, a worker inelastically supplies one unit of labor to the firm and receives a wage w_t in exchange. When unemployed, a worker receives the unemployment compensation b. At the end of each period the family pools all income. This implies that for each individual neither the actual labor market status, nor the individual wage rate in case of employment matter, since all equally share the family's total earnings. We effectively assume full risk-sharing. Moreover, we assume that all agents are risk-neutral and do not save. This is necessary for our recursive wage formulation to be an exact description of earnings over time.

Unemployed workers will apply for a job only if it is optimal compared to all other jobs or remaining unemployed. This implies they will select the best combination of job-finding rate and wage among all the ones offered in equilibrium. Denoting by U the value for an unemployed worker of getting a job the following condition holds:

$$U_t \leq p(\theta_t)\tilde{w}_t + (1 - p(\theta_t))b \tag{3.1}$$

The value U_t is the value to an unemployed individual who can apply for a job which promises the wage $\tilde{w}$ and a job-finding rate $p(\theta)$. U_t exceeds the value of the unemployment benefit b, because firms internalize this condition in their decision problem. If they were to offer just b, one firm could offer a slightly higher wage, thereby attracting all searching workers. Thus, each firm takes U_t as given, although this variable is determined endogenously in equilibrium.[9]

3.3.3 Matching

In each submarket, job vacancies and searching workers are randomly matched. We capture this process by a standard Cobb-Douglas matching function $m(u_t, v_t)$, which we assume to exhibit constant returns to scale:

$$m(u, v) = Bv^{\gamma}u^{(1-\gamma)} \ , \ B > 0 \tag{3.2}$$

where $\gamma \in [0, 1]$ is the elasticity of total matches with respect to vacancies, and B governs the efficiency of the matching process.

[9] We simplify the problem by abstracting from a continuation value for the unemployed. This makes the worker care only about current wages. However, not applying for a job will decrease the earnings by the household by an entire quarter of the annual wage bill. This loss is big, compared to the chance of a shock that would make it worthwhile for the workers to wait an entire period.

Dividing the number of matches by the measure of searching workers yields the job-finding rate $p(\theta)$, whereas dividing it by the number of vacancies delivers the job filling rate for the firm, $q(\theta)$. A firm posting vacancies v_t can expect to attract $h_t = v_t q(\theta_t)$ new workers.

3.3.4 Labor Market Equilibrium

Each firm enters period t with its stock of capital k_t, its workforce l_t, the average firm-level wage w_t, and the realization of the exogenous aggregate productivity process z_t. Those variables form its state vector (k_t, l_t, w_t, z_t).

When maximizing the expected present discounted value of future profits, the firm takes into account the laws of motion for each of its state variables and also the job application rule for searching workers given by equation (3.1). Substituting in the laws of motion for capital, employment and wages, we can summarize the firm's problem with the help of the following Lagrangian.[10]

$$\mathcal{L} = \max_{\theta_t, k_{t+1}, l_{t+1}, w_{t+1}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \{ y(k_t, l_t, z_t) - w_t l_t - [k_{t+1} - (1-\delta)k_t] p^k - a \frac{l_{t+1} - (1-\nu) l_t}{q(\theta_t)} \}$$
$$+ \lambda_t \left[U_t - (1 - p(\theta_t))b - p(\theta_t) \frac{l_{t+1} w_{t+1} - (1-\nu) l_t w_t}{l_{t+1} - (1-\nu) l_t} \right]$$

The first-order-necessary conditions that need to be satisfied in equilibrium are given by

$$\begin{aligned}
\frac{\partial}{\partial \theta_t} &: a(l_{t+1} - (1-\nu)l_t) \frac{q'(\theta_t)}{q(\theta_t)^2} + \lambda_t p'(\theta_t) \left[b - \frac{l_{t+1} w_{t+1} - (1-\nu) l_t w_t}{l_{t+1} - (1-\nu) l_t} \right] = 0 \\
\frac{\partial}{\partial k_{t+1}} &: p^k = \beta \left[\frac{\partial y(k_{t+1}, l_{t+1}, z_{t+1})}{\partial k_{t+1}} + p^k (1-\delta) \right] \\
\frac{\partial}{\partial l_{t+1}} &: - \frac{a}{q(\theta_t)} + \lambda_t(-p(\theta_t)) \frac{(1-\nu) l_t [w_t - w_{t+1}]}{(l_{t+1} - (1-\nu) l_t)^2} \\
&\quad + \beta \left[\frac{\partial y(k_{t+1}, l_{t+1}, z_{t+1})}{\partial l_{t+1}} - w_{t+1} + a \frac{(1-\nu)}{q(\theta_{t+1})} + \lambda_{t+1} \{ -p(\theta_{t+1}) \frac{(l_{t+2}(1-\nu)[w_{t+2} - w_{t+1}]}{(l_{t+2} - (1-\nu) l_{t+1})^2} \} \right] = 0 \\
\frac{\partial}{\partial w_{t+1}} &: \lambda_t (-p(\theta_t)) \frac{l_{t+1}}{l_{t+1} - (1-\nu) l_t} + \beta \left[-l_{t+1} + \lambda_{t+1} \{ p(\theta_{t+1}) \frac{(1-\nu) l_{t+1}}{l_{t+2} - (1-\nu) l_{t+1}} \} \right] = 0
\end{aligned} \tag{3.3}$$

As all firms are identical and so are all workers, their respective behavior can be summarized by that of a representative agent. Note that the representative firm continues to react to changes in the economy in a competitive way. Our competitive search setup in this particular environment reduces the many possible submarkets to a single market.

[10] For an alternative complete formulation of the problem see Appendix 3.B.

We close the model by enforcing that in equilibrium, the ratio of posted vacancies to the measure of unemployed workers needs to equal labor market tightness, $\frac{v}{1-l} = \theta$. Substituting v_t by $\frac{l_{t+1}-(1-\nu)l_t}{q(\theta_t)}$, and exploiting algebraic properties of our matching function, we get the following expression as additional equilibrium condition:

$$\theta_t = \left(\frac{l_{t+1} - (1-\nu)l_t}{B(1-l_t)} \right)^{\frac{1}{\gamma}} \tag{3.4}$$

In order to reach a steady state, we need a vector $(l^*, k^*, w^*, \theta^*, \lambda^*)$[11] which solves the system given by the 4 F.O.N.C.s in (3.3) plus equation (3.4). In equilibrium the value U is determined by the optimal values for wages and labor market tightness plugged into condition 3.1 with equality.[12] We solve the model around the deterministic steady state by second-order perturbation using Dynare.

3.4 Quantitative Analysis

3.4.1 Calibration

As the model cannot be solved analytically, calibration becomes an important matter. The model has a variety of parameters which need to be determined. We take certain values from the literature and perform robustness checks to ensure that these values are not driving the results. The crucial parameters are calibrated in order to match empirical targets, which are important when talking about factor substitutability and its implications for firms and workers.

We calibrate the model to quarterly data from the U.S. economy. Table 3.1 contains the full parametrization of the model.

One of our central questions is what happens to firm profit, employment and investment if a firm is able to substitute more easily among capital and labor. To address this question, we vary the parameter σ, which directly relates to the elasticity of substitution between capital and labor. As a baseline value, we pick $\sigma = -\frac{3}{2}$, which corresponds to an elasticity of substitution of 0.4. This value lies at the lower end of what the literature deems plausible.[13] We will change the parameter σ to $-\frac{2}{3}$ to model increased substitutability and study its effects. We use the range provided by Chirinko (2008) as a guideline for one of the experiments we perform in the context of our model.

The parameter α which governs the efficiency of capital in the production function is central to the problem, as the technology available to the firm is key to our analysis. This parameter amounts to an additional degree of freedom in the production function, which we have to tackle in our analysis.[14] We calibrate α to ensure that the model outcomes

[11] Stars denote equilibrium values.

[12] For further discussion on the solution process of labor-search models see Rogerson et al. (2005).

[13] For a survey of these values see Chirinko (2008). He argues that empirical estimates of the elasticity of substitution range from 0.4 to 0.6.

[14] For a discussion of the issue of normalizing a CES production function see e.g. León-Ledesma et al. (2010).

Table 3.1: Baseline Calibration

Parameter	Interpretation	Value	Target
α	Capital intensity	0.7914	Labor share 60%
σ	Substitutability parameter	-3/2	Elasticity of substitution 0.4
p^k	Price of capital	1	Normalization
γ	Matching function elasticity	0.5	Standard
B	Matching efficiency	0.8	Unemployment rate 7%
b	Unemployment benefit	0.9	Replacement ratio 60%
a	Vacancy posting cost	4	$p(\theta) = 0.99$
β	Discount factor	0.975	Standard
δ	Depreciation rate of capital	0.026	Depreciation rate of capital
ν	Separation rate	0.075	Labor turnover

are comparable across alternative specifications. In a standard neoclassical model with a Cobb-Douglas production function and no frictions, the parameter α corresponds to the income share of capital. We first target a labor share of of income equal to 60% to inspect the key mechanism of our model. When exploring the implications of a changing price of capital, or a varying degree of factor substitutability on this share, we adjust α such that the output level remains constant across various regimes.

We normalize the price of capital, p^k, to one. This price governs the rate at which a firm can turn its output good into next period's capital. In our comparative statics exercises, we will consider what happens when we lower this price, thereby rendering investment of the firm more productive. At a price equal to one, the output good produced by the firm can simply be used as next period's capital. When lowering p^k, we implicitly make the technology via which output can be turned into capital more efficient. A falling relative price of investment goods might cause similar effects as increased factor substitutability. Whether it is cheaper to invest in capital, or whether capital can more easily be substituted for labor is hard to distinguish in reality, as both effects occur simultaneously. In our model, we can separate these two effects and study their respective effects on our variables of interest.

We set the efficiency parameter B of the matching function to target an unemployment rate of 7% and choose the unemployment benefit to match a replacement ratio equal to 0.6. The replacement ratio is defined as unemployment benefit b relative to the equilibrium wage. The vacancy posting cost a is chosen such that a worker's job-finding rate of the worker is close to 0.99, the rate implied by the monthly rate of 0.34 which Shimer (2005) reports.

The remaining parameters are taken from the literature. Many have a clear economic interpretation. Shimer (2005) shows that around 3.42 % of workers in the U.S. labor force leave their jobs each month. So we set ν equal to 0.075 for a period of three months, to also account for workers finding a job within the same quarter. The quarterly depreciation rate of 0.026 reflects the empirical equivalent. Although not explicitly targeted, our set of calibrated parameter values implies a plausible value for the cost of hiring. Blatter et al. (2012) report this value to lie between 10 to 17 weeks of wage payments. The value in our baseline-calibration is 16.7 weeks, which we calculate by dividing the expected cost to hire a worker by the yearly wage.

3.4.2 Results

We numerically solve the model for our benchmark calibration. Table 3.2 reports the corresponding results in column 2. Column 3 states the results when the parameter σ is increased from $-\frac{3}{2}$ to $-\frac{2}{3}$. This parameter change corresponds to a rise in the elasticity of substitution among input factors from 0.4 to 0.6.

Table 3.2: Steady State Results

Variable	$\sigma = -3/2$ $p^k = 1$	$\sigma = -2/3$ $p^k = 1$
k	7.0470	8.0188
l	0.9299	0.8984
w	1.5260	1.3196
θ	1.5475	0.6875
$q(\theta)$	0.6431	0.9648
$p(\theta)$	0.9952	0.6633
v	0.1085	0.0698
y	2.3651	1.9724
π	0.3290	0.3011
u	0.07	0.10
profit share	0.1391	0.1527
labor share	0.6	0.6
investment share	0.0775	0.1057
hiring cost share	0.1834	0.1416
α	0.7914	0.5298

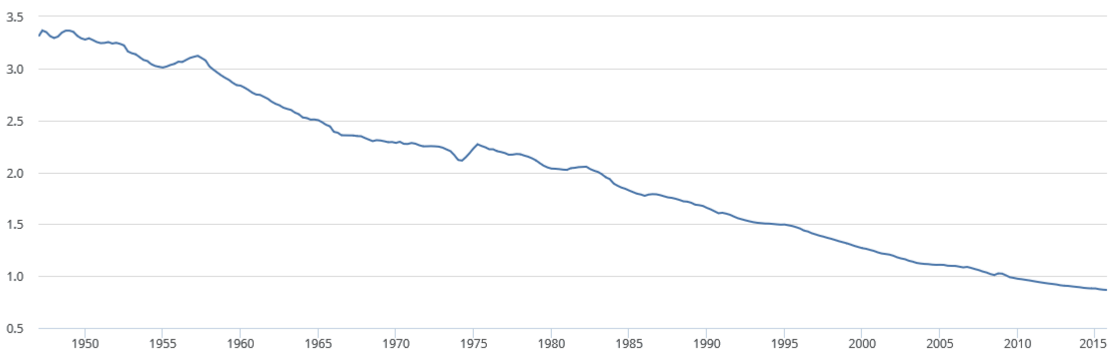
Such a rise makes production more capital-intensive while conditions for workers worsen. The job-finding rate $p(\theta)$ declines, and so do employment l and wages w. The firm spends more on investment and less on hiring, which can be seen by the decrease in the hiring cost share, which equals the costs of hiring divided by output. As the firm produces with a greater capital intensity it uses less labor and also posts fewer vacancies v. At the same time output y declines. By construction, the labor share, which is defined as the wage bill wl divided by output, remains constant, but the profit share increases. The profit share of 13% slightly exceeds what we observe in the data and increases further when factor substitutability rises.

Overall increased factor substitutability benefits firms via higher profits, while it hurts workers. They experience lower wages and a higher risk of unemployment.

3.4.2.1 A Lower Price of New Capital

As documented in detail by Gordon (1990) and Krusell et al. (2000), the relative price of investment goods has steadily declined for decades. Figure 3.2 illustrates this trend. In this section, we explore the quantitative effects of a decline in p^k for our baseline scenario ($\sigma = -\frac{3}{2}$), and also for an increased degree of factor substitutability ($\sigma = -\frac{2}{3}$). Table 3.3 reports the results from our numerical experiment. For both values of σ under consideration, capital and labor exhibit an elasticity of substitution less than 1 and thus are complements.

Figure 3.2: Relative Price of Investment Goods



Notes: Investment deflator divided by consumption deflator. The base year is 2009, seasonally adjusted. Downloaded from FRED database.

Table 3.3 separately reports the effects of each of these changes. Comparing the entries form the second to those from the third column, we see the implications of a decline of the price of capital, which are a lower profit share for firms and an increase of employment and wages as overall output production expands. Increased factor substitutability, on the other hand, again increases the profit share, as can be seen in the last column. These two effects push all variables in opposite directions, except for capital. In sum, when increased factor substitutability occurs together with lower prices of capital in a world of frictional labor markets, the only reliable statement we can make is that the extent of

Table 3.3: Steady State Results

Variable	$\sigma = -3/2$ $p^k = 1$	$\sigma = -3/2$ $p^k = 0.7$	$\sigma = -2/3$ $p^k = 0.7$
k	7.047	9.0327	10.8953
l	0.9299	0.9415	0.9152
w	1.5260	1.6596	1.34089
θ	1.5475	2.2787	1.0228
$q(\theta)$	0.6431	0.53	0.791
$p(\theta)$	0.9952	1.2076	0.8091
v	0.1085	0.1332	0.0868
y	2.3651	2.6043	2.149
π	0.329	0.3443	0.3142
u	0.07	0.0585	0.0848
profit share	0.1391	0.1322	0.1462
labor share	0.6	0.6	0.6
investment share	0.0775	0.0631	0.0923
hiring cost share	0.1834	0.2074	0.1615
α	0.7914	0.8099	0.5351

capital in use increases. However, when looking at the implied increase in profit shares, our model suggests that increased factor substitutability outweighs the cheaper price of capital.

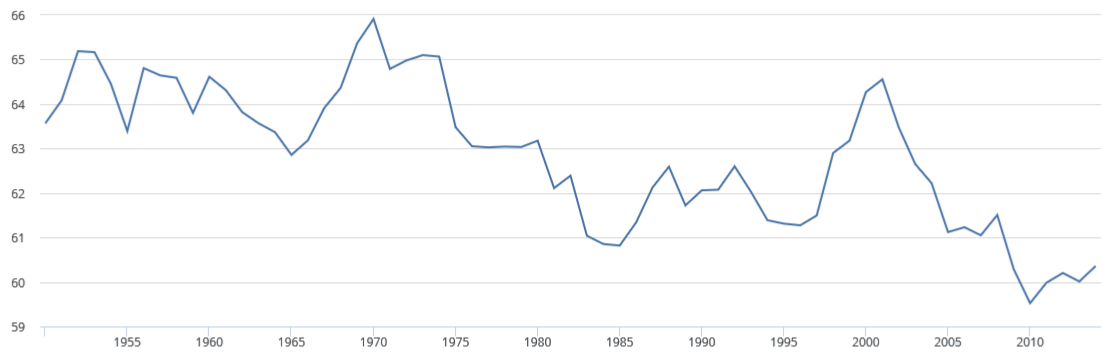
3.4.2.2 Decline in the Labor Share of Income

In all previous experiments, we recalibrated the parameter α to keep the labor share at 60% when we varied the degree of substitutability. This was done in accordance with the well-known empirical facts presented in Kaldor (1961). One of these facts states that the labor share is constant over long periods of time. As can be seen in Figure 3.3, the labor share has been on the decline since the 1970s.[15] Of course, a declining share of GDP accruing to labor implies that other factors benefit.

In what follows we explore how the labor share of income reacts to a decline in the price of capital, and to an increase in factor substitutability. We recalibrate α to keep steady-state output constant when varying our parameter of interest, σ. First, we consider a

[15] The same holds true for other OECD countries (compare Autor et al. (2017)).

Figure 3.3: Labor Share in the U.S. [%]



Notes: Downloaded from the FRED database.

change in the relative price of capital, and illustrate the implications for a firm's demand for production factors in Figure 3.4. The slope of the straight cost lines (dashed) equals the negative ratio of input factor prices, i.e. the ratio between the wage rate w and the price of capital, p^k. A drop in the price of capital increases the steepness of the cost line which we mark in red. That is because cheaper capital increases the firm's demand for capital and also for labor. A drop in p^k let's the resulting equilibrium wage rate rise, as a higher wage is needed to attract more workers. As we keep output constant, the new equilibrium lies on the same isoquant. We observe that the point of tangency moves to the left, resulting in a higher capital-labor ratio and a more capital-intensive production.[16] This rise in the overall capital intensity in production is consistent with evidence from U.S. data.[17]

The full quantitative results of this exercise are given in Table 3.4. The first column of results is again the steady state obtained under our baseline calibration. We repeat the type of numerical experiments from above holding output, y, constant, because we want to study the reaction of the labor share. A decrease in the price of capital to $p^k = 0.7$ causes the firm to use more capital and renders production more capital-intensive.[18] The price decline by 30% dominates the additional investment such that the investment share decreases. Employment and wages increase, which results in a rise of the labor share.

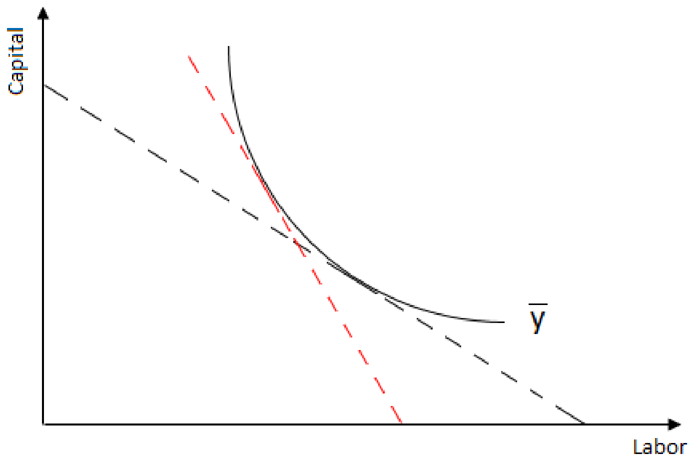
The last column in Table 3.4 shows what happens when substitutability increases. Due to frictions in the labor market, the firm decides to increasingly replace labor by capital. The decline in labor demand lets wages decrease. The labor share subsequently

[16] We refrain form illustrating the case of increased substitutability, because it would alter the shape of the production function too much, since σ and α change substantially.

[17] See Appendix 3.D.

[18] As robustness checks we used other values, the results remain qualitatively similar.

Figure 3.4: A Change in the Relative Price of Capital



drops by around 4 percentage points. The drop in the job-finding rate for unemployed workers adds to the worsened situation for the factor labor.

We conclude that cheaper investment goods cannot be the sole source for the empirically observed decrease in the labor share in many countries, since it would imply an increase in employment and wages, and thus in the labor share. On the other hand, increased factor substitutability tends to reduce this share. Separating these two effects is important when trying to understand which aspect of of the two forces under consideration leads to the observed outcomes.

3.4.3 Changes in Variability

In what follows, we will investigate whether increased factor substitutability and a lower price of capital *per se* dampens or increases the variability of profits. We therefore consider a stochastic environment where the firms face shocks to total factor productivity (TFP). We assume TFP to follow an AR-(1) process with a persistence parameter of 0.9. Increments are normally distributed with mean zero and a standard deviation equal to 0.007, a standard value in the business cycle literature.

We do a second-order approximation around the deterministic steady state of our model and compute the fluctuations of the model variables. Table 3.5 reports the ratio of each variable's coefficient of variation, i.e. the standard deviation normalized by the mean of the variable, relative to that of output.

Again, column 2 depicts the results under our baseline calibration. While capital is more volatile than employment, the volatility of profit and the profit share are an order of magnitude larger than that of capital. Also investment and the investment share are very volatile, which is consistent with empirical evidence, as investment is the most

Table 3.4: Steady State Results with Constant Output

Variable	$\sigma = -3/2$ $p^k = 1$	$\sigma = -3/2$ $p^k = 0.7$	$\sigma = -2/3$ $p^k = 1$
k/l	7.5782	8.7749	11.0589
w	1.5260	1.5603	1.4487
θ	1.5475	1.7220	1.1889
$q(\theta)$	0.6431	0.6096	0.7337
$p(\theta)$	0.9952	1.0498	0.8723
v	0.1085	0.1148	0.0941
y	2.3651	2.3651	2.3651
π	0.3290	0.3023	0.3898
profit share	0.1391	0.1278	0.1648
labor share	0.6	0.6157	0.564
investment share	0.0775	0.0623	0.1119
hiring cost share	0.1834	0.1942	0.1592
α	0.7914	0.7828	0.5827

volatile component of GDP.[19]

Columns 3 and 4 report the results when we vary the price of capital and factor substitutability, respectively. A lower relative price of new capital causes firms to maintain a more stable capital stock and employment by increasing the variability of investment. It can do so, because the price of new capital has decreased. The fluctuations in employment and wages are dampened relative to output, which lets the variability of profits increase. As the payments to the workers become more stable relative to output, the excess variability in output drives up the variability of profits. This mechanism is reminiscent of Danthine and Donaldson (2002), where wage payments are viewed as contractual obligations with the residual of the firms' earnings being paid out as dividends to the owners.

According to the results reported in the last column, increased factor substitutability causes firms to react more flexibly to stochastic fluctuations in aggregate productivity and to primarily adjust the factor which is less costly to vary. Since our model features no adjustment friction in capital, the firm reacts more strongly in capital. The volatility of investment increases in the degree of factor substitutability as does profit. At the same time, increased factor substitutability dampens fluctuations in wage and employment.

[19] See e.g. `https://fredblog.stlouisfed.org/2015/08/gdp-components-volatility/`

Table 3.5: Relative Variabilities under Alternative Specifications

Variable	$\sigma = -3/2$ $p^k = 1$	$\sigma = -3/2$ $p^k = 0.7$	$\sigma = -2/3$ $p^k = 1$
y	1	1	1
k	0.4693	0.4281	0.6414
l	0.0805	0.0552	0.0708
w	0.3648	0.2257	0.2221
θ	2.2872	1.6335	1.7962
v	1.3178	0.9991	1.1157
i	5.7196	7.3915	10.8463
π	4.7561	5.9914	7.9557
profit share	4.1141	5.2581	7.6517
labor share	0.6615	0.8038	0.7875
investment share	5.6135	7.2268	10.6993
hiring cost share	0.4681	0.3874	0.4492

Notes: Ratio of coefficient of variation relative to output.

In sum, we observe that profits and investment become more volatile as the degree of factor substitutability rises and the relative price of investment goods declines.

3.5 Empirical Evidence

We are now in a position to subject our model to an additional plausibility check and contrast its main predictions to their real world counterparts. While our model replicates the empirically observed negative relationship between the profit share and the labor share it has difficulties explaining the behavior of investment. This is because we abstract from financing issues and corporate debt while focusing on the effects of factor substitutability on firms' profits and the labor market. Before continuing a few words of caution are in order. This section is meant as an illustration of our model, which goes beyond reporting second moments of the data and their model counterparts.

We take U.S. time-series data on key economic variables and compare their statistical moments to their counterparts generated by our model. A central equation in all of our discussion is firms' profits defined as follows:

$$\begin{aligned}\pi_t =& y_t - w_t l_t - i_t p^k - v_t a \\ \frac{\pi_t}{y_t} =& 1 - \frac{w_t l_t}{y_t} - \frac{i_t p^k}{y_t} - \frac{v_t a}{y_t}\end{aligned}$$

The second line is just a normalization by output. Once we allow for errors, ϵ_t, that we assume to be normally distributed, we can estimate the following econometric model:

$$\frac{\pi_t}{y_t} = \alpha_0 + \alpha_1 \frac{w_t l_t}{y_t} + \alpha_2 \frac{i_t p^k}{y_t} + \alpha_3 v_t a + \epsilon_t \tag{3.5}$$

Most of our data originate from the FRED database.[20] We take the aggregate time series of GDP, non-financial corporate profits, investment and labor share of income directly from this database.[21] Each series comes at a quarterly frequency and covers the period from the first quarter of 1947 to the last quarter of 2016. We construct investment share and profit share by dividing the respective variables by contemporaneous GDP.

For vacancies we use an updated version of the data constructed by Barnichon (2010), which we downloaded directly from the author's website.[22] The data are an index of open vacancies relative to the labor force and have been constructed from the "Help-Wanted-Index" which only relies on job openings printed in newspapers and the online Help-Wanted Index.[23]

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[20] For a detailed description see Appendix 3.D.

[21] We take GDP instead of non-financial value added to enable comparision with our discussion on the dividend share in Appendix 3.E, because dividends cannot be decomposed in financial and non-financial companies.

[22] `https://sites.google.com/site/regisbarnichon/data`

[23] As we do not have any data for the vacancy posting costs a, which we assume to be constant, the estimate of α_3 will actually be $\frac{\alpha_3}{a}$. However, we will also not divide vacancies by GDP, because

Table 3.6: Regression Results, 1947Q1-2016Q4

	(1)	(2)	(3)
Labor share	-0.635***	-0.640***	-0.108***
	(0.0304)	(0.0291)	(0.0222)
Investment share	0.261***		0.0250
	(0.0341)		(0.0146)
L.Investment share		0.265***	
		(0.0335)	
Job openings	-0.249***	-0.248***	-0.0506
	(0.0745)	(0.0745)	(0.0260)
L.Profit share			0.864***
			(0.0331)
Constant	41.33***	41.55***	7.205***
	(2.171)	(2.085)	(1.475)
Observations	264	263	263
Adjusted R^2	0.859	0.863	0.980

Notes: The dependent variable is profit share. L. denotes the first lag of a variable. Standard errors are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

We run OLS regressions and present the results in Table 3.6. The table displays the following specifications. In column (1), we estimate the regression model from equation (3.5). The coefficients of the labor share and the job openings each are negative. The coefficient of the investment share is significantly positive, which is not expected, given the definition of profits in our model. In the model, investment directly reduces profits. The coefficient of the investment share remains positive when we use its first lag in column (2). This is done to control for potential lags between actual investment and the implied increase in revenue.

We detect autocorrelation in the residuals using the Breusch Godfrey-Test and therefore include the first lag of the profit share in column (3). The coefficient of the investment share becomes insignificant, while the coefficients of labor share remains strongly negative, and the hiring cost share barely fails to be significant at the 5% level.[24] This is in line with the predictions of our model. Investment share and profit share are empirically highly positively correlated because of two reasons. Firstly, there is a discrepancy between the definition of profit in the model and in the data. Profits in the model represent economic profits accruing from rents, while in the data corporate profits are defined as revenues minus costs. Investment expenditures do not constitute costs in this sense, because the firm still owns the capital and only the depreciation of capital lowers profits.[25] Secondly, and perhaps more importantly, our model assumes that firms' current period's retained earnings are uset to cover investment expenditures. This stands in sharp contrast to how firms in reality pay for their investments, which might include debt or additional equity. This is in line with the arguments made by Danthine and Donaldson (2002), who use the idea that wage payments enjoy seniority over dividend and other payment, which is why the labor share and profits are negatively correlated.

Following standard practice in the business cycle literature we compare the simulation results from our model to the correlations observed in the data in Table 3.7. When targeting first moments, second moments are used to determine the goodness of fit of our model. Even though our model was not primarily designed to explain the business cycle, but rather to study the effects of different degrees of input substitutability on long-run trends in corporate profits and labor market variables, it performs quite well.

When we compare the correlations over the full length of our time series we get a similar picture as in the data. Investment and profit share are positively related. Our model closely matches the correlation between the labor share and the investment share. These two variables are key elements of the firm's decision of their input mix. It also replicates a positive correlation between the hiring cost share and the investment share, although the correlation is higher than in the data. A reason for this may be lumpy investment, due to fixed costs, which are not present in the model.

Since the data and our model use different definitions for profit, the discrepancies are

normalizing the relatively constant index of vacancies by GDP would impose downward trends in this variable.

[24]It is, however, significantly negative if we use GDP instead of non-financial GDP.

[25]We control for this by using dividends as dependent variable in Appendix 3.E. The positive correlation remains.

Table 3.7: Correlations Between Various Shares

	Model $\sigma = -3/2$				U.S. Data			
	Profit	Labor	Hiring	Investm.	Profit	Labor	Hiring	Investm.
Profit	1				1			
	(0)							
Labor	-0.1365	1			-0.2731	1		
	(0.0477)	(0)						
Hiring	-0.8016	-0.4786	1		0.4351	-0.0157	1	
	(0.0289)	(0.0055)	(0)					
Investm.	-0.912	-0.2807	0.9745	1	0.6040	-0.2725	0.7105	1
	(0.0119)	(0.0191)	(0.0044)	(0)				

Notes: The model has been simulated 100 times for the same number of periods as data points are available (264). All data are HP-filtered with a smoothing factor of 1,600.

little surprising [26]. In reality, firms tend to invest and hire new employees in good times when profits are high. In our model, hiring more people will decrease contemporaneous profits, while the gains only materialize in the next period. In reality firms can use debt or issue new equity to finance investments, a possibility our model does not capture.

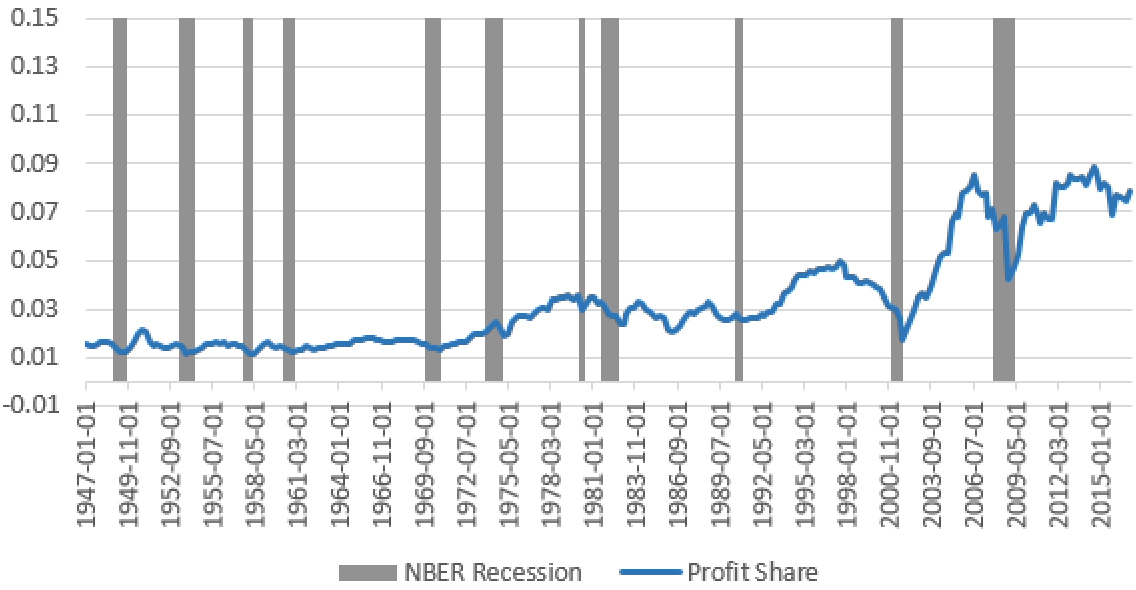
3.5.1 Sub-Periods

When inspecting the time series of profit shares presented in Figure 3.5, different regimes stand out. Between 1947 and 1969 the share is almost flat, but rises at the beginning of the 1970s. From 2000 onwards, we see strong variability in the rate. We divide the entire period accordingly. The first period ranges from 1951 to 1970, where the start is determined by data availability and the end coincides with the end of the NBER recession in 1970. The second period lasts until the burst of the dotcom bubble in 2000, while the last sub-period ranges from 2001 to the end of 2016.

All estimation results are reported in Appendix 3.C. Each table relates to a specific sub-period. We briefly summarize the main findings below. The coefficient associated with the labor share remains consistently negatively correlated with profit share and even increases in magnitude. This means that the tradeoff between profit share and labor share becomes stronger over time. While the investment share has a significantly negative effect on profit share in the period 1971-2000, this effect turns positive in the period 2001-2016. The variable job openings is not significant when running regressions

[26] We amend our definition of profits to match their empirical counterparts more closely in the appendix 3.E.0.1. While it improves the fit of the model, it does not eliminate the discrepancies.

Figure 3.5: Non-Financial Profit Share in the U.S.



Notes: Corporate non-financial profits divided by non-financial GDP, seasonally adjusted. Downloaded from FRED database. The shaded areas indicate NBER recessions.

per period. We now use fewer observations for each regression, thus standard errors tend to be bigger.

A different way to control for changes in the underlying regimes is to use dummy variables. We therefore run a regression over the entire length of the sample and control for the different regimes with time period dummies. The results are presented in Table 3.8. We observe that the labor share of income has a significant negative effect on the profit share, while the investment share is insignificant. The negative coefficient on the job openings is significant at the 10% level. The time dummies do not enter significantly, indicating that the observed relationships are stable over the entire time period.

To sum up, there is a clear negative relationship between the labor share and the profit share. This result is robust across alternative specifications and is consistent with the results generated by our theoretical model.

3.5.2 Correlations

In addition to performing regression analyses, we can compare the correlations between the time series we observe in the data to their model counterparts. If we split up the time series into the three periods previously described, we get the correlation matrices

Table 3.8: Regression over the Full Sample Period with Time Dummies

	(1)
L.Profit share	0.860***
	(0.0322)
Labor share	-0.103***
	(0.0249)
Investment share	0.0184
	(0.0179)
Job openings	-0.0388
	(0.0208)
Period1	0.0354
	(0.0492)
Period2	0.0997
	(0.130)
Constant	6.932***
	(1.624)
N	263
adj. R^2	0.980

Notes: See Table 3.6. We use 1951-1970 as our reference period. Period1 represents the period from 1971-2000 and Period2 stands for 2001-2016.

Table 3.9: Empirical Correlations by Sub-Periods

	Profit	Labor	Hiring	Investment
	1951Q1 - 1970Q4 (80 obs.)			
Profit	1			
Labor	-0.6285	1		
Hiring	0.5649	-0.1566	1	
Investment	0.8161	-0.5914	0.5154	1
	1971Q1- 2000Q4 (120 obs.)			
Profit	1			
Labor	-0.2550	1		
Hiring	0.5688	0.0026	1	
Investment	0.4732	-0.198	0.8027	1
	2001Q1-2016Q4 (64 obs)			
Profit	1			
Labor	-0.3476	1		
Hiring	0.6789	0.1798	1	
Investment	0.8022	-0.1161	0.827	1

Notes: All variables except for hiring are expressed relative to output.

observed in Table 3.9. We focus on the correlation between profit share and labor share. While it is strongly negative in the beginning, it grows less negative in the second period, only to become negative again from 2000 onwards.

A similar pattern can be observed for our model. When the degree of substitutability increases, the correlation between the profit share and the labor share becomes more negative. This is because a higher wage bill lowers the profit of the firm, but then the firm can more easily rely on capital in output production. However, these results should be taken with a grain of salt, because the post 2000 sample period is relatively short and includes the Great Recession.

We also see that the correlation between the labor share and the investment share has turned less negative over time, which can be interpreted as evidence for skill-biased technological growth.[27] As firms invest more, the labor share does not decline by as much as it used to, because firms still need better qualified workers with higher wages to handle the newly installed technologies.[28] Our model replicates the positive correlation between

[27] See Krusell et al. (2000).

[28] As different skill levels are beyond the scope of this paper, we will refrain from exploring these results in greater detail.

hiring and investment. This happens because of the complementarities between capital and labor. The correlations between these two empirical series increases over time, which is consistent with what happens in our model under increased substitutability. With higher substitutability, the firm chooses a more capital-intensive input mix, thereby increasing the marginal product of an additional worker. Following positive productivity shocks, it pays to hire more workers.

3.6 Conclusions

We have developed a dynamic stochastic equilibrium model of a frictional labor market where firms search for suitable workers by posting vacancies and wages, and unemployed workers search for jobs. Firms use capital and labor for producing output with the help of a technology that exhibits a constant elasticity of substitution at any point in time. This elasticity can be varied over time. Firms can flexibly adjust capital, but expanding labor is subject to search frictions. We have calibrated this model to the U.S. economy and used it to disentangle the role that a steady decline in the relative price of new capital goods or a change in production technology towards increased factor substitutability play in explaining the following empirical trends: a rise in the capital-to-labor ratio and in the level and variability of firms' profit-to-output ratio as well as a decline in the labor share of income.

Our quantitative results underline the importance of studying the decline in the price of capital and increased factor substitutability separately, but in an integrated framework. While each change can help explain the observed upward trends, only the rise in factor substitutability generates the observed decline in the labor share. Hence, a possible interpretation of the empirical facts seen through the lense of our model is that the implications of increased factor substitutability have quantitatively outweighed those of a decline in the relative price of new capital goods.

Our model of firms using capital and labor for output production while operating in frictional labor markets is rich yet tractable enough to lend itself to various extensions so that it can help study closely related issues in macro/labor, or labor/finance. The implicit assumption that firms use retained earnings to pay for investment renders a counterfactual negative correlation between investment expenditures and profit shares. Therefore, a natural next step could be to allow firms to take on debt, thereby choosing their capital structure and make this choice dependent on the structure of the labor market. When combined with firm heterogeneity, this framework can be the analytical basis for studying the cross-sectional implications for the level and variability of return on equity as examined by Shim (2015).

3.A Recursive Wages

In a competitive search framework where firms post wage contracts, firms can decide to offer different wages in different periods. This can be caused by shocks, which will affect the optimal wage posted by the firm and can create a wage dispersion within a firm. To avoid keeping track of the entire wage distribution, we use the following recursive formula:

$$w_{t+1}l_{t+1} = w_t l_t(1-\nu) + \tilde{w}_t h_t$$

To show that this formulation is equivalent in terms of the total wage bill to keeping track of the entire wage history of wages posted by the firm, consider a firm in period t with l_t employees at a wage rate w_t. It hires h_t new employees at a wage rate $\tilde{w}_t$, while ν of the existing workforce leave the firm. For the firm it doesn't make a difference whether it pays a new wage rate w_{t+1} to all of its employees in period $t+1$, which are made up by $l_t(1-\nu)+h_t$ or whether it pays $(1-\nu)l_t$ of its employees a wage w_t and the other h_t receive $\tilde{w}_t$. As all earnings are pooled due to the big family assumption, also the household only cares about the total wage bill. We can now simply shift back the time index by one period, and are in the same situation as before, because w_t and l_t are state variables for the firm. We thus have shown that the recursive formulation of wages allows us to calculate the posted wages in a consistent way.

3.B An Alternative Formulation of the Firm's Problem

This is an alternative formulation of the problem, where all the laws of motion are written as constraints. It makes for a nice distinction between the choice variables of the firm in period t, $(v_t, \theta_t, \tilde{w}_t, i_t)$, and the endogenous state variables in the next period. However, the resulting system of equations is more complicated, but eventually determines the same equilibrium.

$$\begin{aligned}
\mathcal{L} = \max_{v_t,\theta_t,\tilde{w}_t,i_t} \mathbb{E}_t \sum_{t=0}^{\infty} & \beta^t [z_t y(k_t, l_t) - w_t l_t - i_t p^k - a v_t] \\
& + \lambda_1 \left[U_t - (1-p(\theta_t))b - p(\theta_t)\tilde{w}_t\right] \\
& + \lambda_2 \left[l_{t+1}w_{t+1} - (1-\nu)l_t w_t - v_t q(\theta_t)\tilde{w}_t\right] \\
& + \lambda_3 \left[l_{t+1} - (1-\nu)l_t - v_t q(\theta_t)\right] \\
& + \lambda_4 \left[k_{t+1} - (1-\delta)k_t - i_t\right]
\end{aligned}$$

Differentiating with respect to the 4 choice variables and next period's endogenous state variables leads to the following nonlinear system of equations. As we have 4 Lagrange multipliers we denote their time indices by superscripts rather than subscripts.

$$
\begin{aligned}
\frac{\partial}{\partial v_t} &: -a - \lambda_2^t q(\theta_t)\tilde{w}_t - \lambda_3^t q(\theta_t) = 0 \\
\frac{\partial}{\partial \theta_t} &: \lambda_1^t [p'(\theta_t)b - p'(\theta_t)\tilde{w}_t] - \lambda_2^t v_t q'(\theta_t)\tilde{w}_t - \lambda_3^t v_t q'(\theta_t) = 0 \\
\frac{\partial}{\partial \tilde{w}_t} &: -\lambda_1^t p(\theta_t) - \lambda_2^t v_t q(\theta_t) = 0 \\
\frac{\partial}{\partial i_t} &: -p^k - \lambda_4^t = 0 \\
\frac{\partial}{\partial w_{t+1}} &: \lambda_2^t l_{t+1} + \beta[-l_{t+1} - \lambda_2^{t+1}(1-\nu)l_{t+1}] = 0 \\
\frac{\partial}{\partial l_{t+1}} &: \lambda_2^t w_{t+1} + \lambda_3 + \beta\left[\frac{\partial y(k_{t+1}, l_{t+1}, z_{t+1})}{\partial l_{t+1}} - w_{t+1} - \lambda_2^{t+1}(1-\nu)w_{t+1} - \lambda_3^{t+1}(1-\nu)\right] = 0 \\
\frac{\partial}{\partial k_{t+1}} &: \lambda_4^t + \beta\left[\frac{\partial y(k_{t+1}, l_{t+1}, z_{t+1})}{\partial k_{t+1}} - \lambda_4^{t+1}(1-\delta)\right] = 0
\end{aligned}
$$

The equilibrium conditions are the same, although there are 4 Lagrange multipliers, where only λ_4 can be substituted. The other have co-dependencies, which is why we decided to present the other formulation in the main part of the paper.

3.C Estimation Results by Period

We now present in greater detail the empirical analysis in each sub-period, which we briefly described in the main text. Each regression table is structured in the following manner. In column (1) we estimate the regression model described in equation (3.5). We see - as we expect - that the coefficients of labor share and hiring cost share enter with a negative coefficient. The coefficient of investment share is significantly positive, which is surprising. This result remains when we include lagged investment in (2). We detect autocorrelation in the residuals using the Breusch Godfrey-Test and therefore include the first Lag of profitshare in (3). We see that the coefficient of investment share changes signs in the interim period, consistent with our model predictions. However, this change is reversed in the post-2000 period.

3.D Data Appendix

The data we use are of quarterly frequency. They relate to the United States and cover the period from 1951Q1 to 2016Q4. All data were downloaded from the FRED database unless noted otherwise.[29]

Output and Profit

We use the *Real Gross Domestic Product* in billions of Dollar, which is seasonally adjusted and has 2009 as base year for chaining, along with nominal GDP where appro-

[29] FRED, Federal Reserve Bank of St. Louis; `https://fred.stlouisfed.org/series`

Table 3.10: Regressions 1951Q1-1970Q4

	(1)	(2)	(3)
Labor share	-0.0314**	-0.0558***	-0.0308**
	(0.0108)	(0.00942)	(0.0101)
Investment share	0.122***		0.0875***
	(0.0148)		(0.0207)
Job openings	0.0304*	0.0409*	0.0151
	(0.0126)	(0.0159)	(0.0139)
L.Investment share		0.0750***	
		(0.0152)	
L.Profit share			0.247*
			(0.119)
Constant	2.265**	4.418***	2.252**
	(0.827)	(0.674)	(0.798)
N	80	79	79
Adj. R^2	0.815	0.745	0.841

Notes: The dependent variable is profit share. L. denotes the first lag of a variable. Standard errors are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.11: Regressions 1971Q1-2000Q4

	(1)	(2)	(3)
Labor share	-0.394***	-0.461***	-0.0591*
	(0.0662)	(0.0604)	(0.0231)
Investment share	0.423***		-0.0390*
	(0.0376)		(0.0172)
Job openings	-0.481***	-0.465***	0.0366
	(0.0933)	(0.0906)	(0.0297)
L.Investment share		0.413***	
		(0.0360)	
L.Profit share			0.980***
			(0.0294)
constant	24.83***	29.30***	4.377**
	(4.409)	(3.940)	(1.495)
N	120	119	119
adj. R^2	0.638	0.639	0.970

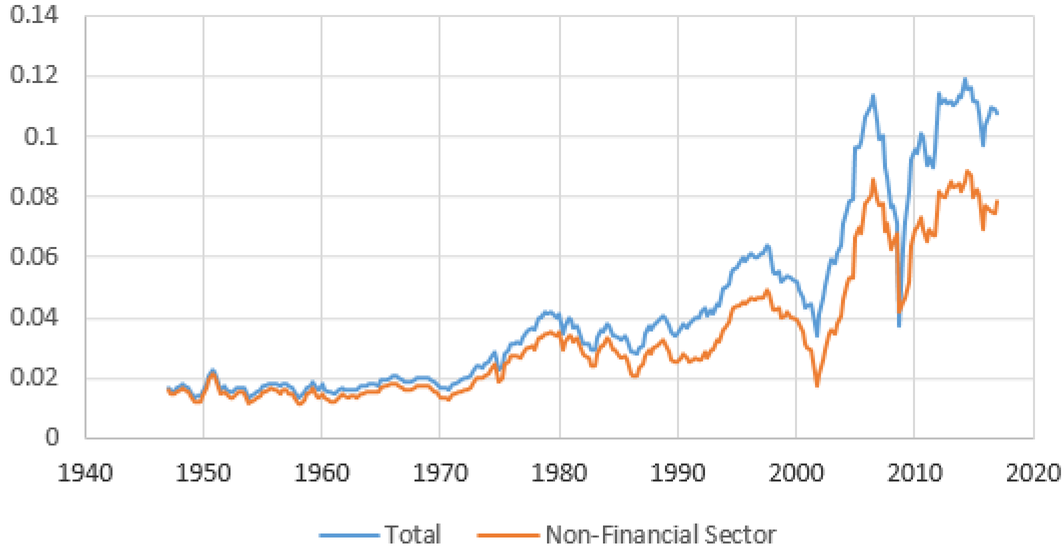
Notes: See Table 3.10.

Table 3.12: Regressions 2001Q1-2016Q4

	(1)	(2)	(3)
Labor share	-1.184***	-1.263***	-0.540***
	(0.0682)	(0.0744)	(0.149)
Investment share	0.469***		0.260*
	(0.113)		(0.116)
Job openings	0.710**	0.568	-0.0425
	(0.253)	(0.298)	(0.209)
L.Investment share		0.514***	
		(0.135)	
L.Profit share			0.596***
			(0.0955)
constant	71.62***	76.01***	32.43***
	(3.599)	(3.719)	(8.300)
N	64	63	63
adj. R^2	0.852	0.841	0.926

Notes: See Table 3.10.

Figure 3.6: Profit Shares in the U.S.



Notes: Seasonally adjusted. Profits were normalized by GDP. Downloaded from FRED database.

priate.

For profit, we take non-financial corporate profit, which is seasonally adjusted. We exclude the financial sector because we analyze a real model and therefore have no role for a financial sector. However, we also perform the empirical analysis using the entire corporate profit time series. The results are virtually unchanged. To illustrate this, we plot the resulting profit shares in Figure 3.6.

The two series track each other quite closely, but start to diverge around 1971. At this time the difference increases, meaning that the financial sector has become relatively more profitable. An interesting observation is the last quarter of 2008. In this quarter, the financial sector in total is making negative profits. Thus, the total profit in the U.S. is below the non-financial profit.

Investment, Price of Capital and Labor Share

As investment we use *Gross Private Domestic Investment*, which is seasonally adjusted and chained in 2009.

The price of capital which is depicted in Figure 3.2 is calculated by dividing the investment deflator by the consumption deflator. This is precisely the definition of the price of capital in our model and the rate at which output goods can be transformed into capital.

The labor share of income is constructed by normalizing the index of the non-financial corporate sector to its 2009 value of 60%.

Job Vacancies

For this time series we rely on the work by Barnichon (2010), who carefully combines the traditional Help-Wanted-Index taken from the print version of newspapers with the Job Openings and Labor Turnover Survey (JOLTS), which is available from 2000 onwards. The author publishes updates on his website.[30] The data are available at a monthly frequency from 1951 to 2016. We time-aggregate them to a quarterly frequency using the mean. In this way, we obtain a time series which is consistent for a long time horizon.

Capital and Labor

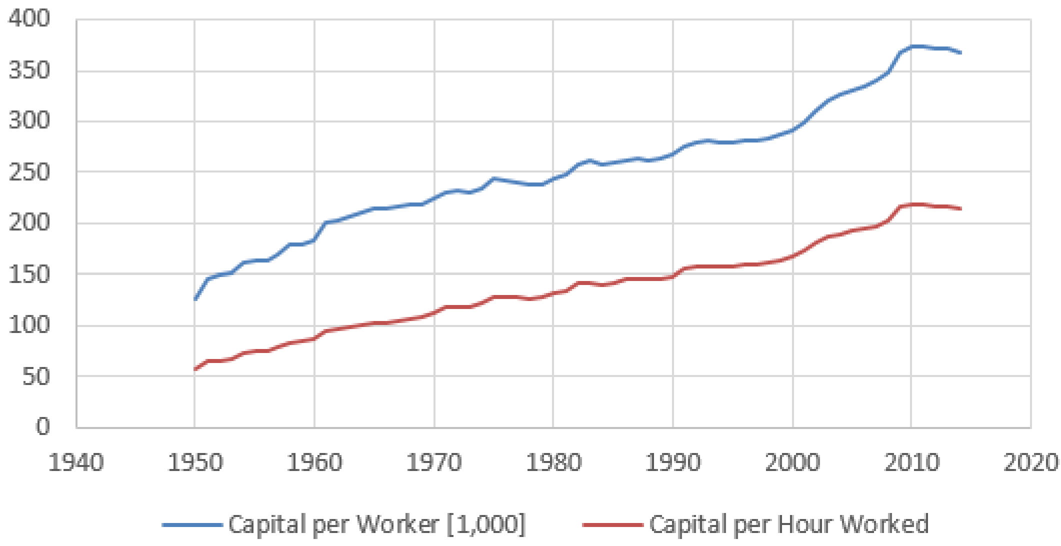
When comparing capital intensities, we are restricted to using yearly data due to the availability of data on the U.S. capital stock.

We use data on the capital stock at constant national prices. For employment we use two distinctive variables. One is the hours worked by full-time and part-time employees, and the other one is the employees who are on a non-farm payroll.

When calculating the capital-to-labor ratio, i.e. the capital intensity of production, we get two different series because we use different denominators. However, both series are increasing in the period from 1950 to 2014, as can be seen in Figure 3.7. It depicts the ratios of capital to the number of workers, and the one to total hours worked, respectively. Both ratios are steadily increasing during the period of observation.

[30] https://sites.google.com/site/regisbarnichon/data

Figure 3.7: Capital Intensities in the U.S.



Notes: Capital at constant national prices in 2011 US-Dollars. Workers include those from the non-farm sectors. Hours worked are by full-time and part-time employees.

3.E Using Dividends and Corporate Profits

One important distinction between our model-based definition of profits and the corporate profits we observe in the data is the treatment of investment. Investment reduces profits in our model, but does not affect corporate profits which are defined according to legal accounting standards. We try to tackle this issue in two ways. First, we perform the regression analysis using dividend share rather than profit share as dependent variable. Second, we define as corporate profit in our theoretical model the sum of profit and investment and compute its correlation with the other variables.

Regressions on Dividend Share

We construct the dividend share by using FRED data on dividends and divide it by GDP. We then run regressions for the full sample and for each sub-period, corresponding to the regressions in the text.

The main changes in the full sample regressions, presented in Table 3.13 are not in the scaling of the coefficients. Now the number of job openings also enters with a significant negative sign, which arguably points to the fact that new hires are financed by current revenues, thus reducing profits.

Table 3.13: Dividend Shares

	(1)	(2)	(3)
Labor share	-0.00404***	-0.00403***	-0.000253**
	(0.000277)	(0.000247)	(0.0000821)
Investment share	0.00353***		0.000329*
	(0.000222)		(0.000136)
Job openings	-0.00566***	-0.00574***	-0.000447**
	(0.000545)	(0.000517)	(0.000156)
L.Investment share		0.00370***	
		(0.000206)	
L.Dividend Share			0.935***
			(0.0255)
Constant	0.247***	0.244***	0.0144*
	(0.0194)	(0.0171)	(0.00592)
Observations	264	263	263
Adjusted R^2	0.861	0.880	0.987

Notes: The dependent variable is dividend share. L. denotes the first lag of a variable. Standard errors are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.14: Regressions on Dividend Share by Sub-Period

	(1)	(2)	(3)
Labor share	0.00000207	-0.0000610	-0.00106***
	(0.0000128)	(0.0000419)	(0.000297)
Investment share	0.0000152	-0.000143**	0.000553
	(0.0000256)	(0.0000459)	(0.000523)
Job openings	-0.0000160	0.000233***	0.00163
	(0.0000242)	(0.0000601)	(0.00196)
L.Dividend Share	0.980***	1.027***	0.770***
	(0.0288)	(0.00626)	(0.0949)
Constant	-0.000167	0.00507	0.0624**
	(0.00104)	(0.00304)	(0.0190)
Period	1951Q1-1970Q4	1971Q1-2000Q4	2001Q1-2016Q4
Observations	79	119	63
Adjusted R^2	0.966	0.999	0.873

Notes: See Table 3.13

3.E.0.1 Splitting up the Periods

This exercise corresponds to the one presented in Appendix 3.C, where we divide our sample into three sub-periods, with dividend share as dependent variable. We will only report the results for the model including one lag in the dividend share, due to autocorrelation in the other variants of the regression model.

For the period 1951Q1-1970Q4, we see that the only significant variable is lagged dividend share, which suggests that dividends in that time were not very volatile and are best explained by an AR-(1) process. In the intermediate period, the coefficient on labor share is not significant, but investment enters with a negative coefficient. Although this is in line with the predictions of our model, this result disappears again in the period following 2000, when the coefficient on labor share turns significantly negative. Overall, no clear picture emerges when looking at dividends as proxy for economic rents, as dictated by our model. Our data span a long time period, and it is likely that changes in the governance of dividends have appeared over time.

Table 3.15: Correlations

	Profit	Labor	Hiring	Investment	Corp. Profit
Profit	1				
Labor	-0.1288	1			
Hiring	-0.8052	-0.4793	1		
Investment	-0.9137	-0.2834	0.9748	1	
Corp. Profit	0.3914	-0.9623	0.2239	0.0146	1

Notes: Approximated correlation of the model, including corporate profits.

All data are HP-filtered, with a smoothing factor of 1600.

Correlations of Corporate Profits

A different way to bridge the differences in the definition of profit between our model and the data is to define a variable *Corp. Profit*, which is revenue minus wage payments and hiring costs, and calculate its share. We present the obtained correlations in Table 3.15. The strong negative correlation between investment share and profit share resulting from our model makes this newly constructed share virtually uncorrelated with investment. Qualitatively, correlations now are the same as what we report in Table 3.7 for the U.S. economy, since all signs match their empirical counterparts. Quantitatively, there are still discrepancies, due to our model abstracting from the financing decisions of firms and other real world phenomena.

4 The Strength of Absent Ties

4.1 Introduction

In the most cited article on social networks,[1] Granovetter (1973) argued that the most important connections we have may not be with our close friends but our acquaintances: people who are not very close to us, either physically or emotionally, help us to relate to groups that we otherwise we would not be linked to. For example, it is from acquaintances that we are more likely to hear about job offers (Rees, 1966; Corcoran et al., 1980; Granovetter, 1995). Those weak ties serve as bridges between our group of close friends and other clustered groups, hence allowing us to connect to the global community in a number of ways.[2]

Interestingly, the process of how we meet our romantic partners in at least the last hundred years closely resembles this phenomenon. We would probably not marry our best friends, but we are likely to end up marrying a friend of a friend or someone we coincided with in the past. Rosenfeld and Thomas (2012) show how Americans met their partners in recent decades, listed by importance: through mutual friends, in bars, at work, in educational institutions, at church, through their families, or because they became neighbors. This is nothing but the weak ties phenomenon in action.[3][4]

But in the last two decades, the way we meet our romantic partners has changed dramatically. Rosenfeld and Thomas argue that *"the Internet increasingly allows Americans to meet and form relationships with perfect strangers, that is, people with whom they had no previous social tie"*. To this end, they document that in the last decade online dating[5] has become the second most popular way to meet a spouse for Americans (see Figure 4.1).

[1] "What are the most-cited publications in the social sciences according to Google?", *LSE Blog*, 12/05/2016.

[2] Although most people find a job via weak ties, it is also the case that weak ties are more numerous. However, the individual value from an additional strong tie is larger than the one from an additional weak tie (Kramarz and Skans, 2014; Gee et al., 2017).

[3] Backstrom and Kleinberg (2014) reinforce the previous point: given the social network of a Facebook user who is in a romantic relationship, the node which has the highest chances of being his romantic partner is, perhaps surprisingly, not the one who has most friends in common with him.

[4] Similarly, most couples in Germany met through friends (32%), at the workplace or school (21%), and bars and other leisure venues (20%) (Potarca, 2017).

[5] We use the term online dating to refer to any romantic relationship that starts online, including but not limited to dating apps. We use this terminology throughout the text.

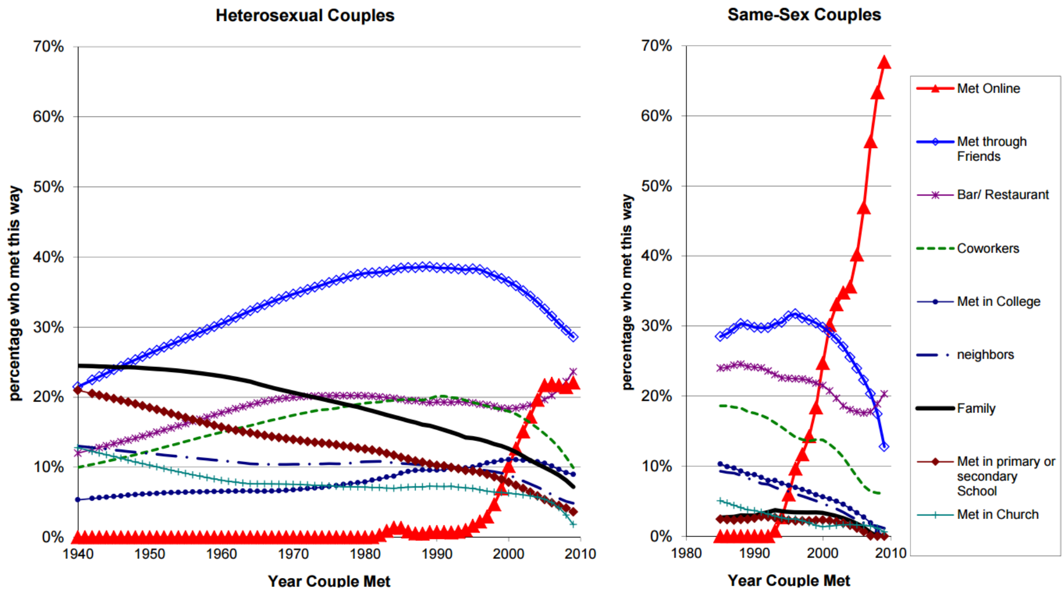


Figure 4.1: How we met our partners in previous decades.

Online dating has changed the way people meet their partners not only in America but in many places around the world. As an example, Figure 4.2 shows one of the author's Facebook friends graph. The yellow triangles reveal previous relationships that started in offline venues. It can clearly be seen that those ex-partners had several mutual friends with the author. In contrast, nodes appearing as red stars represent partners he met through online dating. These individuals have no contacts in common with him, and thus it is likely that, if it were not for online dating, they would have never interacted with him.

Because a third of modern marriages start online (Cacioppo et al., 2013), and up to 70% of homosexual relationships, the way we match online with potential partners shapes the demography of our communities, in particular its racial diversity. Meeting people outside our social network online can intuitively increase the number of interracial marriages in our societies, which is remarkably low. Only 6.3% and 9% of the total number of marriages are interracial in the US and the UK, respectively.[6] The low rates of interracial marriage are expected, given that up until 50 years ago these were illegal in many parts of the US, until the Supreme Court outlawed anti-miscegenation laws in the famous *Loving vs. Virginia* case.[7]

This paper aims at improving our understanding of the impact of online dating on racial diversity in modern societies. In particular, we intend to find out how many more

[6] "Interracial marriage: Who is marrying out", *Pew Research Center*, 12/6/2015; and "What does the 2011 census tell us about inter-ethnic relationships?", *UK Office for National Statistics*, 3/7/2014.

[7] Interracial marriage in the US has increased since 1970, but it remains rare (Arrow, 1998; Kalmijn, 1998; Fryer, 2007; Furtado, 2015). It occurs far less frequently than interfaith marriages (Qian, 1997).

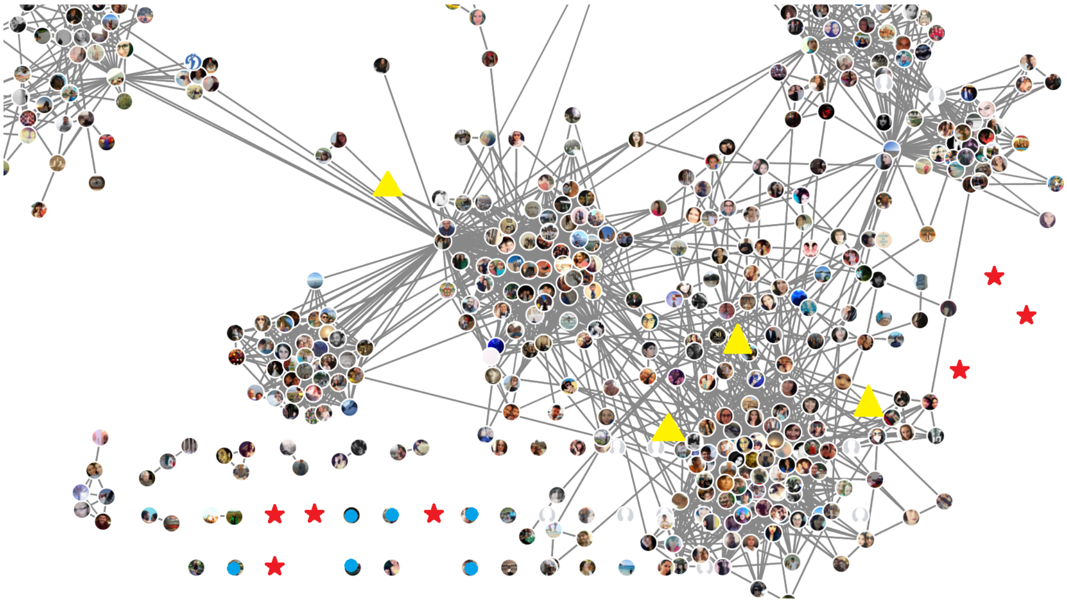


Figure 4.2: How one of us met his partners in the last decade.

Note: Triangles are partners met offline, whereas starts are partners met online.

interracial marriages, if any, occur after online dating becomes available in a society. In addition, we are also interested in whether marriages created online are any different from those that existed before.

Understanding the evolution of interracial marriage is an important problem, for intermarriage is widely considered a measure of social distance in our societies (Wong, 2003; Fryer, 2007; Furtado, 2015), just like residential or school segregation. Moreover, the number of interracial marriages in a society has important economic implications. It increases the social network of both spouses who intermarry by connecting them to people of different race. These valuable connections translate into a higher chance of finding employment (Furtado and Theodoropoulos, 2010).[8] This partially explains why the combined income of an White-Asian modern couple is 14.4% higher than than the combined income of an Asian-Asian couple, and 18.3% higher than a White-White couple (Wang, 2012). Even when controlling for factors that may influence the decision to intermarry, Gius (2013) finds that all interracial couples not involving African Americans have higher combined incomes than a White-White couple.[9]

[8]There is a large literature that analyzes the effect of marrying an immigrant. This literature is relevant because often immigrants are from different races than natives. This literature has consistently found that an immigrant who married a native often has a higher probability of finding employment (Meng and Gregory, 2005; Furtado and Theodoropoulos, 2010; Goel and Lang, 2009). Marrying a native increases the probability of employment, but not the perceived salary (Kantarevic, 2004).

[9]In some cases, intermarriage may even be correlated with poor economic outcomes. Examining the population in Hawaii, Fu (2007) finds that White people are 65% more likely to live in poverty if

Interracial marriage also affects the offspring of couples who engage in it. Duncan and Trejo (2011) find that children of an interracial marriage between a Mexican Latino and an interracial partner enjoy significant human capital advantages over children born from endogamous Mexican marriages in the US.[10] Those human capital advantages include a 50% reduction in the high school dropout rate for male children.[11]

4.1.1 Overview of Results

This article builds a novel theoretical framework to study matching problems under network constraints. Our model is different to the previous theoretical literature on marriage in that we explicitly study the role of social networks in the decision of whom to marry. Consequently, our model provides new testable predictions regarding how changes in the structure of agents' social networks impact the number of interracial marriages and the quality of marriage itself. In particular, our model combines non-transferable utility matching *à la* Gale and Shapley (1962) with random graphs, first studied by Gilbert (1959) and Erdős and Rényi (1959), which we use to represent social networks.[12]

We consider a society composed of agents who belong to different races. All agents want to marry the potential partner who is closest to them in terms of personality traits, but they can only marry people who they know, i.e. to whom they are connected. As in real life, agents are highly connected with agents of their own race, but only poorly so with people from other races.[13] Again inspired by empirical evidence (Hitsch et al., 2010; Banerjee et al., 2013), we assume that the marriages that occur in our society are those predicted by game-theoretic stability, i.e. no two unmarried persons can marry and make one better off without making the other worse off. In our model, there is a unique stable marriage in each society (Proposition 1).

After computing the unique stable matching, we introduce online dating in our soci-

they marry outside their own race.

[10] Although Hispanic is not a race, Hispanics do not associate with "standard" races. In the 2010 US census, over 19 million Latinos identified themselves as being of "some other race". See "For many Latinos, racial identity is more culture than color", *New York Times*, 13/1/2012.

[11] Pearce-Morris and King (2012) examines the behavioral well-being of children in inter and intraracial households. They find no significant differences between the two groups.

[12] Most of the literature studying marriage with matching models uses transferable utility, following the seminal work of Becker (1973, 1981). A review of that literature appears in Browning et al. (2014). Although our model departs substantially from this literature, we point out similarities with particular papers in Section 4.2.

[13] The average American public school student has less than one school friend of another race (Fryer, 2007). Among White Americans, 91% of people comprising their social networks are also White, while for Black and Latino Americans the percentages are 83% and 64%, respectively (Cox et al., 2016). Patacchini and Zenou (2016) document that 84% of the friends of white American students are also white. For high school students, less than 10% of interracial friendships exist (Shrum et al., 1988). Furthermore, only 8% of Americans have anyone of another race with whom they discuss important matters (Marsden, 1987).

eties by creating previously absent ties between races and compute the stable marriage again.[14] We compare how many more interracial marriages are formed in the new expanded society that is more interracially connected. We also keep an eye on the characteristics of those newly formed marriages. In particular, we focus on the average distance in personality traits between partners before and after the introduction of online dating, which we use as a proxy for the strength of marriages in a society (ideally, all agents marry someone who has the same personality traits as them).

Perhaps surprisingly, we find that making a society more interracially connected may decrease the number of interracial marriages. It also may increase the average distance between spouses, and even lead to less married people in the society (Proposition 2). However, this only occurs in rare cases. Our main result affirms that the expected number of interracial marriages in a society increases rapidly after new connections between races are added (Result 1). In particular, if we allow marriage between agents who have a friend in common, complete social integration occurs when the probability of being directly connected to another race is $\frac{1}{n}$, where n is the number of persons in each race. This result provides us with our first and main testable hypothesis: social integration occurs rapidly after the emergence of online dating, even if the number of partners that individuals meet from newly formed ties is small. The increase in the number of interracial marriages in our model does not require changes in agents' preferences.

Furthermore, the average distance between married couples becomes smaller, leading to better marriages (Result 2). This second result provides another testable hypothesis: marriages created in a society with online dating last longer and report higher levels of satisfaction than those created offline. We find this hypothesis interesting, as it has been widely suggested that online dating creates relationships of a lower quality.[15] Finally, the added connections in general increase the number of married couples whenever communities are not fully connected or are unbalanced in their gender ratio (Result 3). This result provides a third and final testable hypothesis: the emergence of online dating leads to more marriages.

We contrast the testable hypotheses generated by the model with US data. With regards to the first and main hypothesis, we find that the number of interracial marriages substantially increases after the popularization of online dating. This increase in interracial marriage cannot be explained by changes in the demographic composition of the US only, because Black Americans are the racial group whose rate of interracial marriage has increased the most, going from 5% in 1980 to 18% in 2015 (Livingston and Brown, 2017). However, the fraction of the US population that is Black has remained relatively constant during the last 50 years at around 12% of the population (Pew Reseach Center, 2015).

To properly identify the impact of online dating on the generation of new interracial

[14]We obtain the same qualitative results if we increase both interracial and intraracial connections, because the marginal value of interracial connections is much larger; see Appendix 4.B.

[15]"Tinder is tearing society apart", *New York Post*, 16/08/2015; and "Online dating is eroding humanity", *The Guardian*, 25/07/11.

marriages, we exploit sharp temporal and geographic variation in the pattern of broadband adoption, which we use as a proxy for the introduction of online dating. This strategy is sensible given that broadband adoption has limited correlation to other factors impacting interracial marriages and eliminates the possibility of reverse causation. Using this data from 2000 to 2016, we conclude that one additional line of broadband internet 3 years ago (marriages take time) affects the probability of being in an interracial marriage by 0.07%. We obtain this effect by estimating a linear probability model that includes a rich set of individual- and state-level controls, including the racial diversity of each state and many others. Therefore, we conclude that there is evidence suggesting that online dating is causing more interracial marriages, and that this change is ongoing.

Moreover, shortly after we first made available our paper online on September of 2017, Thomas (2018) used recently collected data on how couples meet to successfully demonstrate that couples that met online are more likely to be interracial, even when controlling for the diversity of their corresponding locations. Thomas estimates that American couples who met online since 1996 are 6% to 7% more likely to be interracial than those who met offline. His findings further establish that online dating has indeed had a positive impact on the number of interracial marriages, as predicted by our model.

With respect to the quality of marriages created online, both Cacioppo et al. (2013) and Rosenfeld (2017) find that relationships created online last at least as long as those created offline, defying the popular belief that marriages that start online are of lower quality than those that start elsewhere, and are in line with our second prediction (in fact, Cacioppo et al., 2013 finds that marriages that start online last longer and report a higher marital satisfaction).[16]

Finally, with respect to our third hypothesis that asserts that online dating should increase the number of married couples, Bellou (2015) finds causal evidence that online dating has increased the rate at which both White and Black young adults marry in the US. The data she analyzes shows that online dating has contributed to higher marriage rates by up to 33% compared to the counterfactual without internet dating. Therefore, our third prediction is consistent with Bellou's findings.

4.1.2 Structure of the Article

We present our model in Section 4.2. Section 4.3 introduces the welfare indicators underlying the further analysis. Sections 4.4 and 4.5 analyze how these measures change when societies become more connected using theoretical analysis and simulations, respectively. Section 4.6 contrasts our model predictions with observed demographic trends from the US. Section 4.7 concludes.

4.2 Model

4.2.1 Agents

There are r races or communities, each with n agents (also called nodes). Each agent i is identified by a pair of coordinates $(x_i, y_i) \in [0, 1]^2$, that can be understood as their

[16] Rosenfeld (2017) also finds that couples who meet online marry faster than those created offline.

personality traits (e.g. education, political views, weight, height, etcetera).[17] Both coordinates are drawn uniformly and independently for all agents. Each agent is either male or female. Female agents are plotted as stars and males as dots. Each race has an equal number of males and females, and is assigned a particular color in our graphical representations.

4.2.2 Edges

Between any two agents of the same race, there exists a connecting edge (also called link) with probability p: these edges are represented as solid lines and occur independently of each other. Agents are connected to others of different race with probability q: these interracial edges appear as dotted lines and are also independent. The intuition of our model is that two agents know each other if they are connected by an edge.[18] We assume that $p > q$. We present an illustrative example in Figure 4.3.

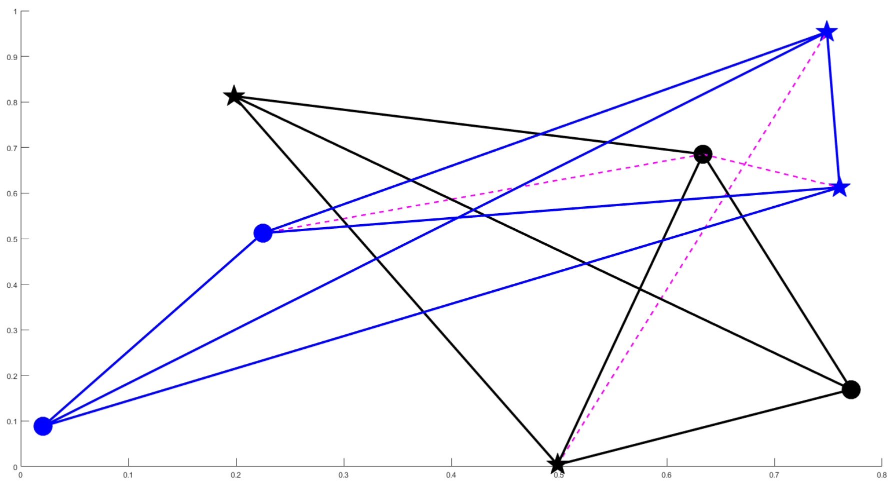


Figure 4.3: Example of a society with $n = 4$ agents, $r = 2$ races, $p = 1$ and $q = 0.2$.

Our model is a generalization of the random graph model (Erdős and Rényi, 1959; Gilbert, 1959; for a textbook reference, see Bollobás, 2001). Each race is represented by a random graph with n nodes connected among them with probability p. Nodes are connected across graphs with probability q. The r random graphs are the within-race

[17]For a real-life representation using a 2-dimensional plane see `www.politicalcompass.org`. A similar interpretation appears in Chiappori et al. (2012) and in Chiappori et al. (2016). The analysis can be easily extended to include more characteristics in a higher dimension without gaining any further intuition, and at the cost of losing its simplicity and graphic representation.

[18]This interpretation is common in the study of friendship networks, see de Martí and Zenou (2016) and references therein. Our model can be understood as the islands model in Golub and Jackson (2012), in which agents' type is both their race and gender.

set of links for each race. In expectation, each agent is connected to $n(r-1)q+(n-1)p$ persons.

A *society* S is a realization from a generalized random graph model, defined by a four-tuple (n, r, p, q). A society S has a corresponding graph $S = (M \cup W; E)$, where M and W are the set of men and women, respectively, and E is the set of edges. We use the notation $E(i, j) = 1$ if there is an edge between agents i and j, and 0 otherwise. We denote such edge by either (i, j) or (j, i).

4.2.3 Agents' Preferences

All agents are heterosexual and prefer marrying anyone over remaining alone.[19] We denote by P_i the set of potential partners for i, i.e. those of different gender. The preferences of agent i are given by a function $\delta_i : P_i \to \mathbb{R}_+$ that has a distance interpretation.[20] An agent i prefers agent $j \in P_i$ over agent $k \in P_i$ if $\delta_i(i, j) \leq \delta_i(i, k)$. The intuition is that agents like potential partners that are close to them in terms of personality traits. The function δ_i could take many forms, however we put emphasis on two intuitive ones.

The first one is the Euclidean distance for all agents, so that for any agent i and every potential partner $j \neq i$,

$$\delta^E(i, j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{4.1}$$

and $\delta^E(i, i) = \sqrt{2}\ \forall i \in M \cup W$, i.e. the utility of remaining alone equals the utility derived from marrying the worst possible partner. Euclidean preferences are intuitive and have been widely used in the social sciences (Bogomolnaia and Laslier, 2007). The indifference curves associated with Euclidean preferences can be described by concentric circles around each node.

The second preferences we consider are such that every agent prefers a partner close to them in personality trait x, but they all agree on the optimum value in personality trait y. The intuition is that the y-coordinate indicates an attribute that is usually considered desirable by all partners, such as wealth. We call these preferences *assortative*.[21] Formally, for any agent i and every potential partner $j \in P_i$,

$$\delta^A(i, j) = |x_i - x_j| + (1 - y_j) \tag{4.2}$$

and $\delta^E(i, i) = 2\ \forall i \in M \cup W$. The δ functions we discussed can be weighted to account for the strong intraracial preferences that are often observed in reality (Wong, 2003; Fisman et al., 2008; Hitsch et al., 2010; Rudder, 2014; Potarca and Mills, 2015;

[19] Both assumptions are innocuous and for exposition only.

[20] The function δ can be generalized to include functions that violate the symmetry ($\delta(x, y) \neq \delta(y, x)$) and identity ($\delta(x, x) = 0$) properties of mathematical distances.

[21] If we keep the x-axis fixed, so that agents only care about the y-axis, we get full assortative mating as a particular case. The preferences for the y attribute are also known as vertical preferences.

McGrath et al., 2016).[22] Inter or intraracial preferences can be incorporated into the model, as in equation (4.3) below

$$\delta'_i(i,j) = \beta_{ij}\ \delta(i,j) \tag{4.3}$$

where $\beta_{ij} = \beta_{ik}$ if agents j and k have the same race, and $\beta_{ij} \neq \beta_{ik}$ otherwise. In equation (4.3), the factor β_{ij} captures weightings in preferences. The case $\beta_{ij} < 1$ implies that agent i relative prefers potential partners of the same race as agent j, while $\beta_{ij} > 1$ expresses relative dislike towards potential partners of the same race as agent j. Although our results are qualitatively the same when we explicitly incorporate racial preferences using the functional form in equation (4.3), we postpone this analysis to Appendix 4.B.

A society in which all agents have either all Euclidean or all assortative preferences will be called Euclidean or assortative, respectively. We focus on these two cases. In both cases agents' preferences are strict because we assume personality traits are drawn from a continuous distribution.

4.2.4 Marriages

Agents can only marry potential partners who they know, i.e. if there exists a path of length at most k between them in the society graph.[23] We consider two types of marriages:

1. Direct marriages: $k = 1$. Agents can only marry if they know each other.
2. Long marriages: $k = 2$. Agents can only marry if they know each other or if they have a mutual friend in common.

To formalize the previous marriage notion, let $\rho_k(i,j) = 1$ if there is a path of at most length k between i and j, with the convention $\rho_1(i,i) = 1$. A marriage $\mu : M \cup W \to M \cup W$ of length k is a function that satisfies

$$\forall m \in M \qquad \mu(m) \in W \cup \{m\} \tag{4.4}$$

$$\forall w \in W \qquad \mu(w) \in M \cup \{w\} \tag{4.5}$$

$$\forall i \in M \cup W \qquad \mu(\mu(i)) = i \tag{4.6}$$

$$\forall i \in M \cup W \qquad \mu(i) = j \text{ only if } \rho_k(i,j) = 1 \tag{4.7}$$

[22] It is not clear whether the declared intraracial preferences show an intrinsic intraracial predilection or capture external biases, which, when removed, leave the partner indifferent to match across races. Evidence supporting the latter hypothesis includes: Fryer (2007) documents that White and Black US veterans have had higher rates of intermarriage after serving with mixed communities. Fisman et al. (2008) finds that people do not find partners of their own race more attractive. Rudder (2009) shows that online daters have a roughly equal user compatibility. Lewis (2013) finds that users are more willing to engage in interracial dating if they previously interacted with a dater from another race.

[23] A path from node i to t is a set of edges $(ij), (jk), \ldots, (st)$. The length of the path is the number of such pairs.

We use the convention that agents that remain unmarried are matched to themselves. Because realized romantic pairings are close to those predicted by stability (Hitsch et al., 2010; Banerjee et al., 2013), we assume that marriages that occur in each society are *stable*.[24] A marriage μ is k-stable if there is no man-woman pair (m, w) who are not married to each other such that

$$\rho_k(m,w) = 1 \tag{4.8}$$
$$\delta(m,w) < \delta(m,\mu(m)) \tag{4.9}$$
$$\delta(w,m) < \delta(w,\mu(w)) \tag{4.10}$$

Such a pair is called a blocking pair. Condition (4.8) is the only non-standard one in the matching literature, and ensures that a pair of agents cannot block a direct marriage if they are not connected by a path of length at most k in the corresponding graph. Given our assumptions regarding agents' preferences,

Proposition 1. *For any positive integer k, every Euclidean or assortative society has a unique k-stable marriage.*

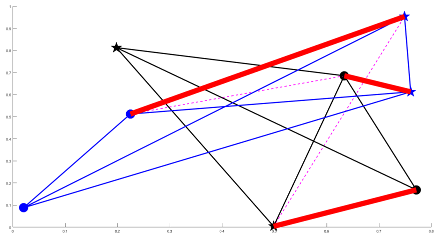
Proof. For the Euclidean society, a simple algorithm computes the unique k-stable marriage. Let every person point to their preferred partner to whom they are connected to by a path of length at most k. In case two people point to each other, marry them and remove them from the graph. Let everybody point to their new preferred partner to which they are connected to among those still left. Again, marry those that choose each other, and repeat the procedure until no mutual pointing occurs. The procedure ends after at most $\frac{rn}{2}$ iterations. This algorithm is similar to the one proposed by Holroyd et al. (2009) for 1-stable matchings in the mathematics literature[25] and to David Gale's top trading cycles algorithm (in which agents' endowments are themselves), used in one-sided matching with endowments (Shapley and Scarf, 1974) .

For the assortative society, assume by contradiction that there are two k-stable matchings μ and μ' such that for two men m_1 and m_2, and two women w_1 and w_2, $\mu(m_1) = w_1$ and $\mu(m_2) = w_2$, but $\mu'(m_1) = w_2$ and $\mu'(m_2) = w_1$.[26] The fact that both marriages are k-stable implies, without loss of generality, that for $i, j \in \{1, 2\}$ and $i \neq j$,

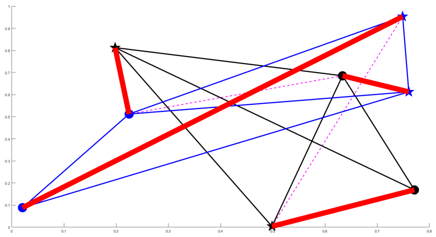
[24] We study the stability of the marriages created, following the matching literature, not of the network per se. Stability of networks was defined by Jackson and Wolinsky (1996) in the context of network formation. We take the network structure as given.

[25] Holroyd et al. (2009) require two additional properties: non-equidistance and no descending chains. The first one is equivalent to strict preferences, the second one is trivially satisfied. In their algorithm, agents point to the closest agent, independently if they are connected to them.

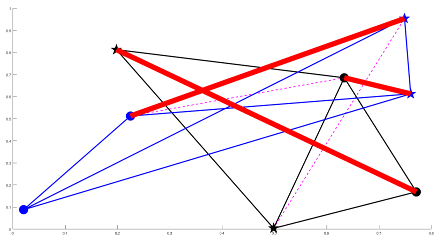
[26] It could be the case that in the two matchings there are no four people who change partner, but that the swap involves more agents. The argument readily generalizes.



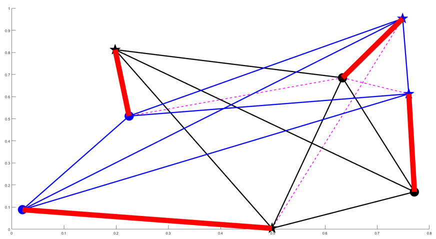
(a) Direct marriage, Euclidean preferences.



(b) Long marriage, Euclidean preferences.



(c) Direct marriage, assortative preferences.



(d) Long marriage, assortative preferences.

Figure 4.4: Direct and long stable marriages for the Example in Figure 4.3.

$\delta(m_i, w_i) - \delta(m_i, w_j) < 0$ and $\delta(w_i, m_j) - \delta(w_i, m_i) < 0$. Adding up those four inequalities, one obtains $0 < 0$, a contradiction. ■ □

The fact that the stable marriage is unique allows us to unambiguously compare the characteristics of marriages in two different societies.[27] Figure 4.4 illustrates the direct and long stable marriages for the Euclidean and assortative societies depicted in Figure 4.3. Marriages are represented as red thick edges.

4.2.5 Online Dating on Networks and Expansions of Societies

We model online dating in a society S by increasing the number of interracial edges. Given the graph $S = (M \cup W; E)$, we create new interracial edges between every pair that is disconnected with a probability ϵ.[28,29] S_ϵ denotes a society that results after

[27] In general, the set of stable marriages is large. Under different restrictions on agents' preferences we also obtain uniqueness (Eeckhout, 2000; Clark, 2006). None of the restrictions mentioned in those papers applies the current setting.

[28] Online dating is likely to also increase the number of edges inside each race, but since we assume that $p > q$, these new edges play almost no role. We perform robustness checks in Appendix 4.B, increasing both p and q but keeping its ratio fixed.

[29] We could assume that particular persons are more likely than others to use online dating, e.g. younger people. However, the percentage of people who use online dating has increased for people of all ages.

online dating has occurred in society S. S_ϵ has exactly the same nodes as S, and all its edges, but potentially more. We say that the society S_ϵ is an *expansion* of the society S. Equivalently, we model online dating by increasing q. Online dating generates a society drawn from a generalized random graph model with a higher q, i.e. with parameters $(n, r, p, q + \epsilon)$.

4.3 Welfare Indicators

We want to understand how the welfare of a society changes after online dating becomes available, i.e. after a society becomes more interracially connected. We consider three welfare indicators:

1. **D**iversity, i.e. how many marriages are interracial. We normalize this indicator so that 0 indicates a society with no interracial marriages, and 1 equals the diversity of a colorblind society in which $p = q$, where an expected fraction $\frac{r-1}{r}$ of the marriages are interracial. Formally, let $\mathcal{R}$ be a function that maps each agent to their race and M^* be the set of married men. Then

$$dv(S) = \frac{|\{m \in M^* : \mathcal{R}(m) \neq \mathcal{R}(\mu(m))\}|}{|M^*|} \cdot \frac{r}{r-1} \tag{4.11}$$

2. **S**trength, defined as $\sqrt{2}$ minus the average Euclidean distance between each married couple. This number is normalized to be between 0 and 1. If every agent gets her perfect match, strength is 1, but if every agent marries the worst possible partner, strength equals 0. We believe strength is a good measure of the quality of marriage not only because it measures how much agents like their spouses, but also because a marriage with a small distance between spouses is less susceptible to break up when random agents appear. Formally

$$st(S) = \frac{\sqrt{2} - \frac{\sum_{m \in M^*} \delta^E(m, \mu(m))}{|M^*|}}{\sqrt{2}} \tag{4.12}$$

3. **S**ize, i.e. the ratio of the society that is married. Formally,

$$sz(S) = \frac{|M^*|}{n} \tag{4.13}$$

4.4 Edge Monotonicity of Welfare Indicators

Given a society S, the first question we ask is whether the welfare indicators of a society always increase when its number of interracial edges grow, i.e. when online dating becomes available. We refer to this property as *edge monotonicity*.[30]

See: "5 facts about online dating", *Pew Research Center*, 29/2/2016. To obtain our main result, we only need a small increase in the probability of interconnection for each agent.

[30] Properties that are edge monotonic have been thoroughly studied in the graph theory literature (Erdős et al., 1995). Edge monotonicity is different from node monotonicity, in which one node, with all its

Definition 1. *A welfare measure w is edge monotonic if, for any society S, and any of its extensions S_ϵ, we have*

$$w(S_\epsilon) \geq w(S) \tag{4.14}$$

That a welfare measure is edge monotonic implies that a society unambiguously becomes better off after becoming more interracially connected. Unfortunately,

Proposition 2. *Diversity, strength, and size are all not edge monotonic.*

Proof. We show that diversity, strength and size are not edge monotonic by providing counterexamples. To show that size is not edge monotonic, consider the society in Figure 4.3 and its direct stable matching in Figure 4.4a. Remove all interracial edges: it is immediate that in the unique stable matching there are now four couples, one more than when interracial edges are present.

For the case of strength, consider a simple society in which all nodes share the same y-coordinate, as the one depicted in Figure 4.5. There are two intraracial marriages and the average Euclidean distance is 0.35. When we add the interracial edge between the two central nodes, the closest nodes marry and the two far away nodes marry too. The average Euclidean distance in the expanded society increases to 0.45, hence reducing its strength.

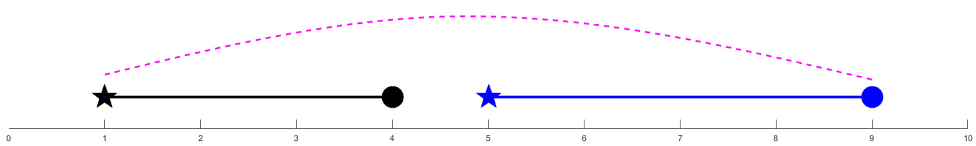
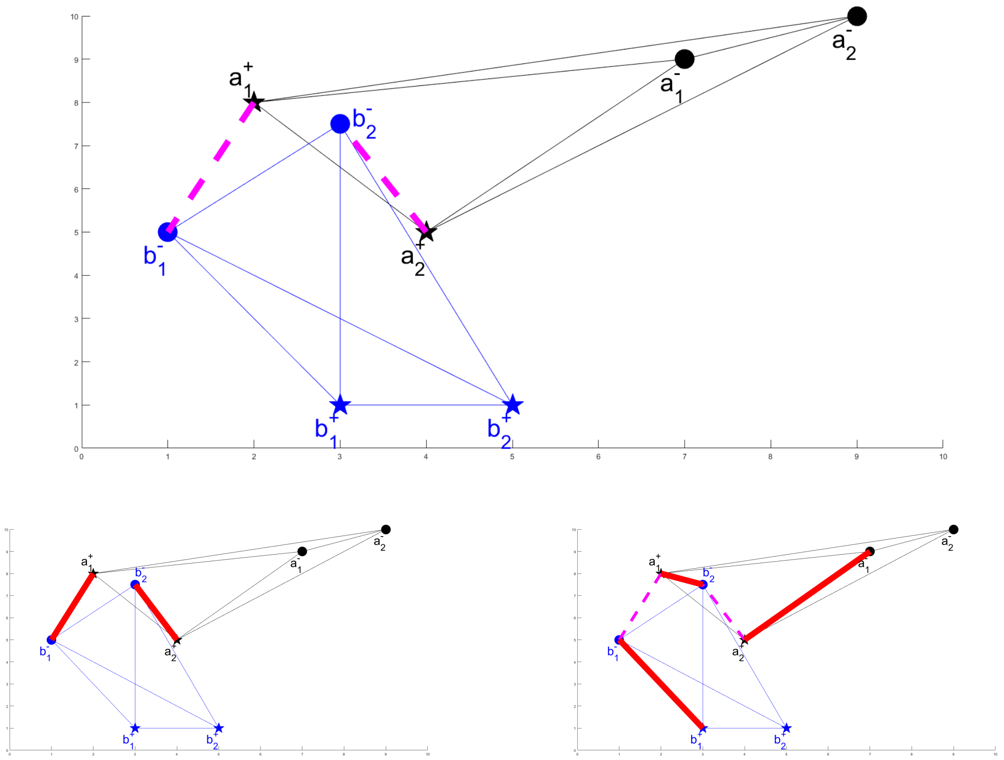


Figure 4.5: Strength is not edge monotonic.

Note: The average Euclidean distance between spouses increases after creating the interracial edge between the nodes in the center.

To show that diversity is not edge monotonic, consider Figure 4.6. There are two men and two women of each of two races a and b. Each gender is represented with the superscript $^+$ or $^-$.

corresponding edges, is added to the matching problem. It is well-known that when a new man joins a stable matching problem, every woman weakly improves, while every man becomes weakly worse off (Theorems 2.25 and 2.26 in Roth and Sotomayor, 1992).



(a) $dv(S) = 2$

(b) $dv(S_\epsilon) = 2/3$

Figure 4.6: Diversity is not edge monotonic.

Note: The diversity of this society reduces after creating the interracial edge (a_1^+, b_2^-). The top graph represents the original society. The bottom left graph show the marriages in the original society, whereas the bottom right graph shows the marriages in the expanded society.

Stability requires that $\mu(b_1^-) = a_1^+$ and $\mu(b_2^+) = a_2^-$, and everyone else is unmarried. However, when we add the interracial edge $(a_1^+ b_2^-)$, the married couples become $\mu(b_1^-) = b_1^+$, $\mu(a_2^+) = a_1^-$, and $\mu(a_1^+) = b_2^-$. In this extended society, there is just one interracial marriage, out of a total of three, when before we had two out of two. Therefore diversity reduces after adding the edge $(a_1^+ b_2^-)$. ■ □

The failure of edge monotonicity by our three welfare indicators makes evident that, to evaluate welfare changes in societies, we need to understand how welfare varies in an average society after introducing new interracial edges. We develop this comparison in the next Section.

A further comment on edge monotonicity. The fact that the size of a society is not edge monotonic implies that adding interracial edges may not lead to a Pareto improvement for the society. Some agents can become worse off after the society becomes more connected. Nevertheless, the fraction of agents that becomes worse off after adding an extra edge is never more than one-half of the society, and although it does not vanish as the societies grow large, the welfare losses measured in difference in spouse ranking become asymptotically zero. Ortega (2018b) discusses both findings in detail.

4.5 Expected Welfare Indicators

To understand how the welfare indicators behave on average, we need to form expectations of these welfare measures. We are able to evaluate this expression analytically for diversity, and rely on simulation results for the others.

4.5.1 Diversity

The expected diversity of a society with direct marriages is given by

$$\mathbb{E}[dv(S_{\text{direct}})] = \frac{q\frac{(r-1)n}{2}}{p\frac{n}{2} + q\frac{(r-1)n}{2}} \cdot \frac{r}{r-1} \tag{4.15}$$

where $q(r-1)n/2$ is the expected number of potential partners of a different race to which an agents is directly connected, and $pn/2$ is the corresponding expected number of potential partners of the same race. The term $\frac{r}{r-1}$ is just the normalization we impose to ensure that diversity equals one when $p = q$. Equation (4.15) is a concave function of q, because

$$\frac{\partial^2 \mathbb{E}[dv(S_{\text{direct}})]}{\partial q^2} = \frac{-pr(r-1)}{(pn + q(r-1)n)^3} < 0 \tag{4.16}$$

and therefore a small increase in q around $q = 0$ produces an even larger increment in the expected diversity of a society. If we consider long marriages, we observe a more interesting change. The expected diversity in a society with long marriages is given by

$$\mathbb{E}[dv(S_{\text{long}})] = \frac{P(B)\frac{(r-1)n}{2}}{P(A)\frac{n}{2} + P(B)\frac{(r-1)n}{2}} \cdot \frac{r}{r-1} \tag{4.17}$$

where $P(A)$ denotes the probability that any agent (say i) is connected to another member of his community (i') by a path of length at most 2, and $P(B)$ denotes the probability that any agent (i) is connected to any agent of another community (j) by a path of length at most two, perhaps via another agent (h) who does not share race neither with i nor with j. These are given by

$$P(A) = 1 - \underbrace{(1-p)}_{E(i,i')=0} \underbrace{(1-p^2)^{n-2}}_{E(i,i'')=E(i'',i')=0} \underbrace{(1-q^2)^{(r-1)n}}_{E(i,h)=E(h,i')=0} \tag{4.18}$$

$$P(B) = 1 - \underbrace{(1-q)}_{E(i,j)=0} \underbrace{(1-pq)^{2n-2}}_{E(i,i')=E(i',j)=0} \underbrace{(1-q^2)^{(r-2)n}}_{E(i,h)=E(h,j)=0} \tag{4.19}$$

Plugging the values computed in equations (4.18) and (4.19) into (4.17), we can plot that function and observe that it grows very fast: after q becomes positive, the diversity of a society quickly becomes approximately one. To understand the rapid increase in diversity, let us fix $p = 1$ and let $q = 1/n$. Then

$$\begin{aligned} P(B) &= 1-(1-q)^{2n-1}(1-q^2)^{(r-2)n} && (4.20)\\ &= 1-(1-q)^{rn-1}(1+q)^{(r-2)n} && (4.21)\\ &= 1-(1-\frac{1}{n})^{rn-1}(1+\frac{1}{n})^{(r-2)n} && (4.22)\\ &\underset{n\to\infty}{=} 1-e^{-2} \approx 0.86 && (4.23) \end{aligned}$$

Substituting the value of $P(B)$ into (4.17), we obtain that $\mathbb{E}[dv(S_{\text{long}})] \approx \frac{.86r}{.86r+.14}$, which is very close to 1 even when r is small ($\mathbb{E}[dv(S_{\text{long}})] \approx .92$ already for $r = 2$), showing that the diversity of a society becomes 1 for very small values of q, in particular $q = 1/n$. The intuition behind full diversity for the case of long marriages is that, once an agent obtains just one edge to any other race, he gains $\frac{n}{2}$ potential partners. Just one edge to a person of different race gives access to that person's complete race.

Although we fixed $p = 1$ to simplify the expressions of expected diversity, the rapid increase in diversity does not depend on each race having a complete graph. We also obtain a quick increase in diversity for many other values of p, as we discuss in Appendix 4.B. When same-race agents are less interconnected among themselves, agents gain fewer connections once an interracial edge is created, but those fewer connections are relatively more valuable, because the agent had less potential partners available to him before.[31]

To further visualize the rapid increase in diversity we use simulations. We generate several random societies and observe how their average diversity change when they become more connected. We create ten thousand random societies, and increase the expected number of interracial edges by increasing the parameter q. In the simulations presented in the main text we fix $n = 50$ and $p = 1$.[32]

As predicted by our theoretical analysis, a small increase in the probability of inter-

[31] This finding should not be confused with (and it is not implied by) two well-known properties of random graphs. The first one establishes that a giant connected component emerges in a random graph when $p = 1/n$, whereas the graph becomes connected when $p = \log(n)/n$; for a review of these properties see Albert and Barabási (2002). The second result is that the property that a random graph has diameter 2 (maximal path length between nodes) has a sharp threshold at $p = (2 \ln n/n)^{1/2}$ (Blum et al., 2017). Result 1 is also similar to, but not implied by, the *small world* property of simple random graphs (Watts and Strogatz, 1998), where an average small path length occurs in a regular graph after rewiring a few initial edges.

[32] We restrict to $n = 50$ and ten thousand replications because of computational limitations. The results for other values of p are similar and we describe them in Appendix 4.B.

racial connections achieves perfect social integration in the case of long marriages.[33,34] For the cases with direct marriages, the increase in diversity is slower but still fast: an increase of q from 0 to 0.1 increases diversity to 0.19 for $r = 2$, and from 0 to 0.37 with $r = 5$.[35] Figure 4.7 summarizes our main result, namely

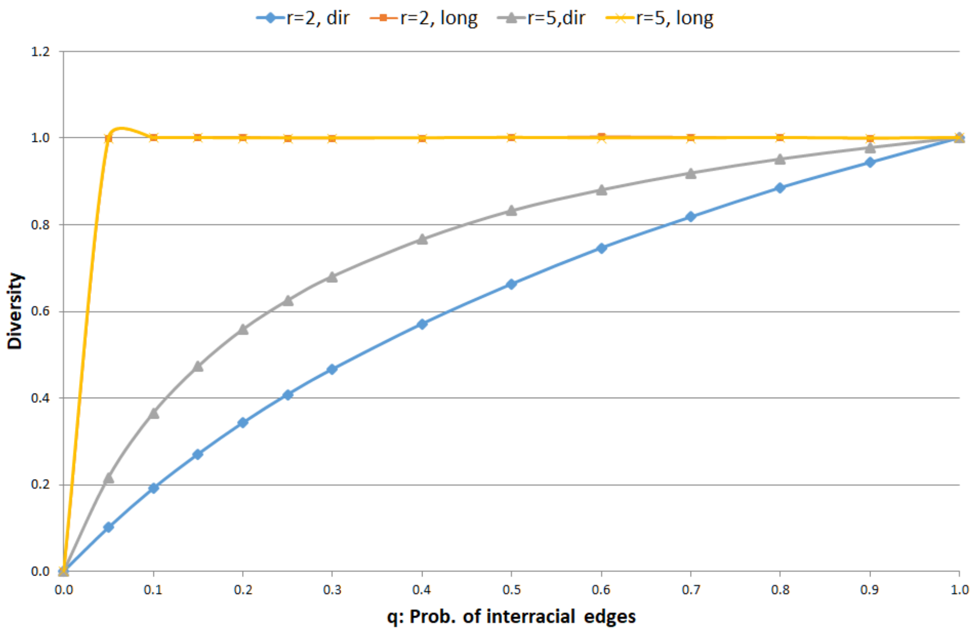


Figure 4.7: Average diversity of an Euclidean society for different values of q.

Note: The yellow and orange curves are indistinguishable in this plot because they are identical. Exact values and standard errors (which are in the order of 1.0e-04) are provided in Appendix 4.A, as well as the corresponding graph for an assortative society, which is almost identical.

Result 1. *Diversity is fully achieved with long marriages, even if the increase in interracial connections is arbitrarily small.*

[33] Perfect social integration (diversity equals one) occurs around $q = 1/n$, as we have discussed. The emergence of perfect integration is not a phase transition but rather a crossover phenomenon, i.e. diversity smoothly increases instead of discontinuously jumping at a specific point: see Figure B1 in Appendix 4.B.

[34] This result is particularly robust as it does not depend on our assumption that the marriages created are stable. Stability is not innocuous in our model, as we could consider other matching schemes that in fact are edge-monotonic.

[35] Empirical evidence strongly suggests that q is very close to zero in real life. See footnote 13.

With direct marriages, diversity is achieved partially, yet an increase in q around $q = 0$ yields an increase of a larger size in diversity.

We have showed that with either direct ($k = 1$) or long ($k = 2$) marriages diversity increases after the emergence of online dating, although at very different rates. An obvious question is whether online dating actually helps to create long marriages. We study the case of long marriages not because we expect that if a man meets a woman online, then that man will be able to date that woman's friends. Rather, we study it because it shows that when people meet their potential spouses via friends of friends ($k > 2$), a few existing connections can quickly make a difference: recall that meeting through friends of friends is the most common way to meet a spouse both in the US and Germany (around one out of every three marriages start this way in both countries (Rosenfeld and Thomas, 2012; Potarca, 2017)).[36]

Our analysis shows that immediate social integration occurs for all values of $k \geq 2$. The mechanism we consider for those larger values of k is that, once an interracial couple is created, it serves as a bridge between two different races. For example, if woman a marries man b of a different race, in the future it allows agent a', an acquaintance of woman a, to meet agent b', an acquaintance of man b, allowing a' and b' to marry. In summary, we expect that some marriages created by online dating will be between people who meet directly online, but some will be created as a consequence of those initial first marriages, and thus the increase in the diversity of societies will be somewhere in between the direct and the long marriage case.

Result 1 implies that a few interracial links can lead to a significant increase in the racial integration of our societies, and leads to optimistic views on the role that dating platforms can play in modern civilizations. Our result is in sharp contrast to the one of Schelling (1969, 1971) in its seminal models of residential segregation, in which a society always becomes completely segregated. We pose this finding as the first testable hypothesis of our model.

Hypothesis 1. *The number of interracial marriages increases after the popularization of online dating.*

4.5.2 Strength & Size

A second observation, less pronounced than the increase in diversity, is that strength is increasing in q. We obtain this result by using simulations only, given that it seems impossible to obtain an analytical expression for the expected strength of a society.[37] Figure 4.8 presents the average strength of the marriages obtained in ten thousand simulations with $n = 50$ and $p = 1$.

[36] Ortega (2018a) finds the minimal number of interracial edges needed to guarantee that any two agents can marry for all values of k.

[37] Solving the expected average distance in a toy society with just one race, containing only one man and one woman, requires a long and complicated computation "Distance between two random points in a square", *Mind your Decisions*, 3/6/2016.

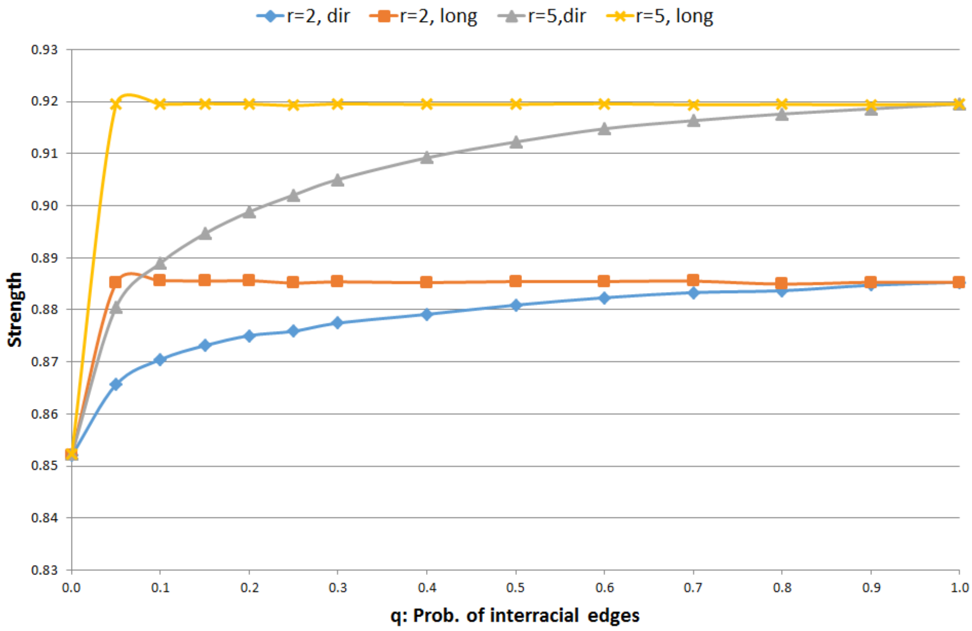


Figure 4.8: Average strength of an Euclidean society for different values of q.

Note: Exact values and standard errors (which are in the order of 1.0e-04) provided in Appendix 4.A, as well as the corresponding graph for an assortative society, which is very similar.

The intuition behind this observation is that agents have more partner choices in a more connected society. Although this does not mean that every agent will marry a more desired partner, it does mean that the average agent will be paired with a better match. It is clear that, for all combinations of parameters (see Appendix 4.B for further robustness checks), there is a consistent trend downwards in the average distance of partners after adding new interracial edges, and thus a consistent increase in the strength of the societies. We present this observation as our second result.

Result 2. *Strength increases after the number of interracial edges increases. The increase is faster with long marriages and with higher values of r.*

Assuming that marriages between spouses who are further apart in terms of personality traits have a higher chance of divorcing because they are more susceptible to break up when new nodes are added to the society graph, we can reformulate the previous result as our second hypothesis.

Hypothesis 2. *Marriages created in societies with online dating have a lower divorce rate.*

Finally, with regards to size, we find that the number of married people also increases when q increases. This observation, however, depends on $p < 1$.[38] This increase is due to the fact that some agents do not know any available potential spouse who prefers them over other agents. Figure 4.9 presents the evolution of the average size of a society with $p = 1/n$.

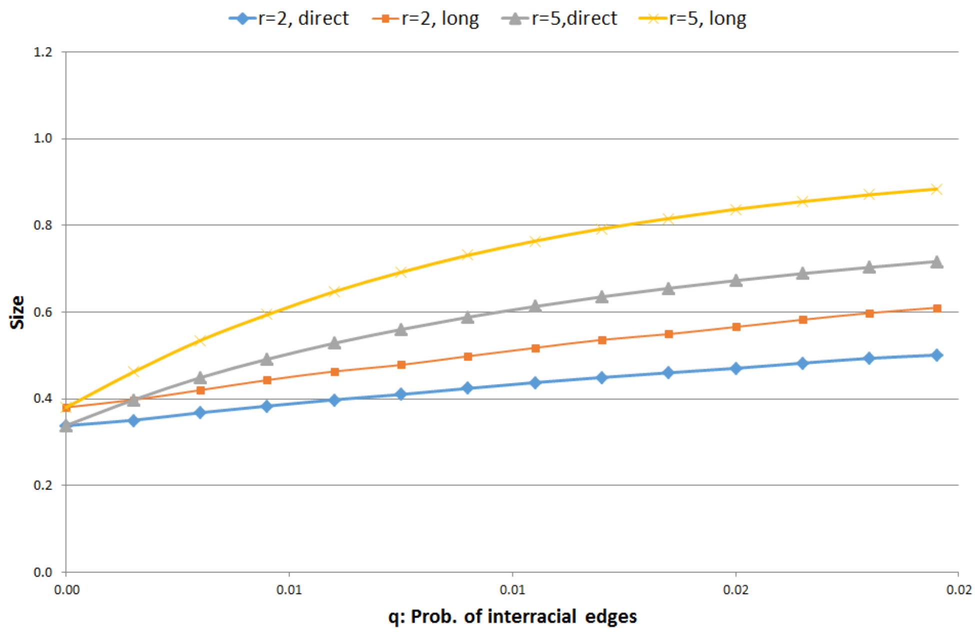


Figure 4.9: Average size of an Euclidean society for values of q up to $q = p = 1/n$.

: Note: Exact values and standard errors (which are in the order of 1.0e-04) provided in Appendix 4.A, as well as the corresponding graph for an assortative society, which is very similar.

The increase in the number of married people becomes even larger (and does not require $p < 1$) whenever i) some races have more men than women, and vice versa,[39] ii) agents become more picky and are only willing to marry an agent if he or she is sufficiently close to them in terms of personality traits, or iii) some agents are not searching for a relationship. All these scenarios yield the following result

Result 3. *Size increases after the number of interracial edges increases if either $p < 1$,*

[38] Using Hall's marriage theorem, Erdős and Rényi (1964) find that in a simple random graph ($r = 1$) the critical threshold for the existence of a *perfect matching* is $p = \log n/n$, i.e. a marriage with size 1. Even when $p = q$, this critical threshold is only a lower bound for a society to have size 1. This is because there is no guarantee that the stable matching will in fact be a perfect one.

[39] See (Ahn, 2018) for empirical evidence on how gender imbalance affects cross-border marriage.

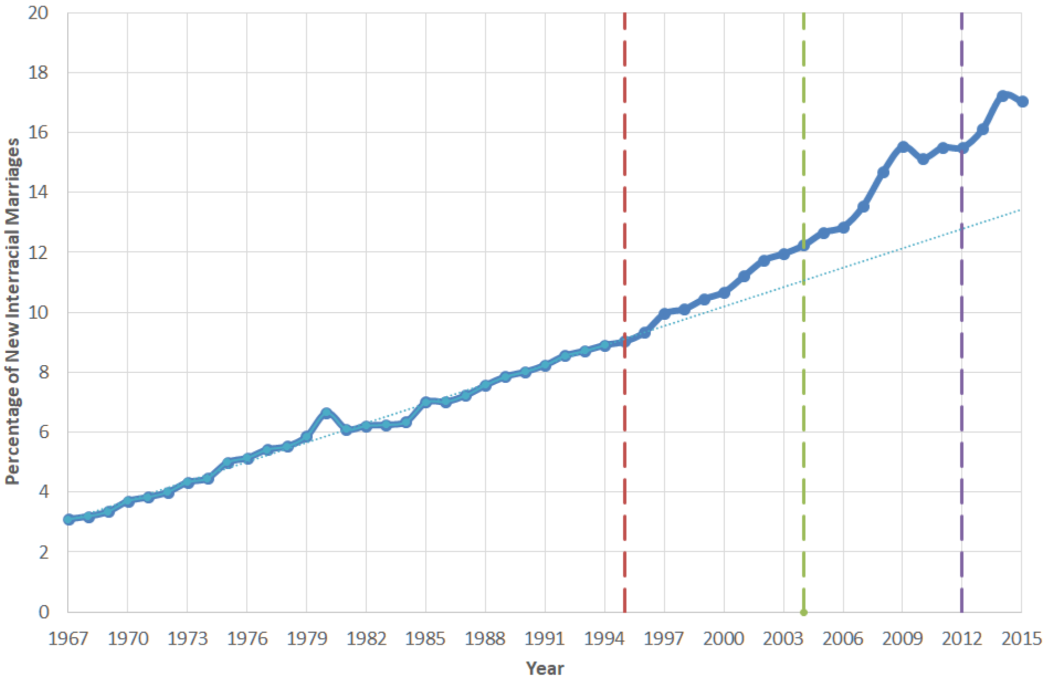


Figure 4.10: Percentage of interracial marriages among newlyweds in the US.
: Note: *Source* Pew Research Center analysis of 2008-2015 American Community Survey and the 1980, 1990 and 2000 decennial censuses (IPUMS). The red, green, and purple lines represent the creation of Match.com, OKCupid, and Tinder. The creation of Match.com roughly coincides with the popularization of broadband in the US and the 1996 Telecommunications Act. The blue line represents a linear prediction for 1996 – 2015 using the data from 1967 to 1995.

societies are unbalanced in their gender ratio, or some agents are deemed undesirable. The increase is faster with long marriages and with higher values of r.

The previous result provides us with a third and final testable hypothesis, namely

Hypothesis 3. *The number of married couples increases after the popularization of online dating.*

4.6 Hypotheses and Data

4.6.1 Hypothesis 1: More Interracial Marriages

What does the data reveal? Is our model consistent with observed demographic trends? We start with a simple preliminary observation before describing our empirical work.

Figure 4.10 presents the evolution of interracial marriages among newlyweds in the US from 1967 to 2015, based on the 2008-2015 American Community Survey and the 1980, 1990 and 2000 decennial censuses (IPUMS). In this Figure, interracial marriages include those between White, Black, Hispanic, Asian, American Indian or multiracial persons.[40]

We observe that the number of interracial marriages has consistently increased in the last 50 years. However, it is intriguing that a few years after the introduction of the first dating websites in 1995, like Match.com, the percentage of new marriages created by interracial couples increased. The increase becomes steeper around 2006, a couple of years after online dating became more popular: it is around this time when well-known platforms such as OKCupid emerged. During the 2000s, the percentage of new marriages that are interracial rose from 10.68% to 15.54%, a huge increase of nearly 5 percentage points, or 50%. After the 2009 increase, the proportion of new interracial marriage jumps again in 2014 to 17.24%, remaining above 17% in 2015 too. Again, it is interesting that this increase occurs shortly after the creation of Tinder, considered the most popular online dating app.[41]

The increase in the share of new marriages that are interracial could be caused by the fact that the US population is in fact more interracial now than 20 years ago. However, the change in the population composition of the US cannot explain the huge increase in intermarriage that we observe, as we discuss in detail in Appendix 4.C. A simple way to observe this is to look at the growth of interracial marriages for Black Americans. Black Americans are the racial group whose rate of interracial marriage has increased the most, going from 5% in 1980 to 18% in 2015. However, the fraction of the US population that is Black has remained constant at around 12% of the population during the last 40 years. Random marriage accounting for population change would then predict that the rate of interracial marriages would remain roughly constant, although in reality it has more than tripled in the last 35 years.

4.6.2 Empirical Test of Hypothesis 1

In order to rigorously test our prediction that online dating increases the number of interracial marriages we use the following strategy. Our empirical setup exploits state variations in the development of broadband internet from 2000 to 2016, which we use as a proxy for online dating. There is little concern for reverse causality, which would imply that broadband developed faster in states where there was a higher number of interracial couples. Our dependent variable is a dummy showing whether a person's marriage is interracial. We use a variety of personal and state-level covariates in order to identify the relationship between online dating and interracial marriages as precisely as possible. Figure 4.11 displays a preview of the relationship between broadband development and

[40] We are grateful to Gretchen Livingston from the Pew Research Center for providing us with the data. Data prior to 1980 are estimates. The methodology on how the data was collected is described in Livingston and Brown (2017).

[41] Tinder, created in 2012, has approximately 50 million users and produces more than 12 million matches per day. See "Tinder, the fast-growing dating app, taps an age-old truth", *New York Times*, 29/10/2014. The company claims that 36% of Facebook users have had an account on their platform.

interracial marriage by state.

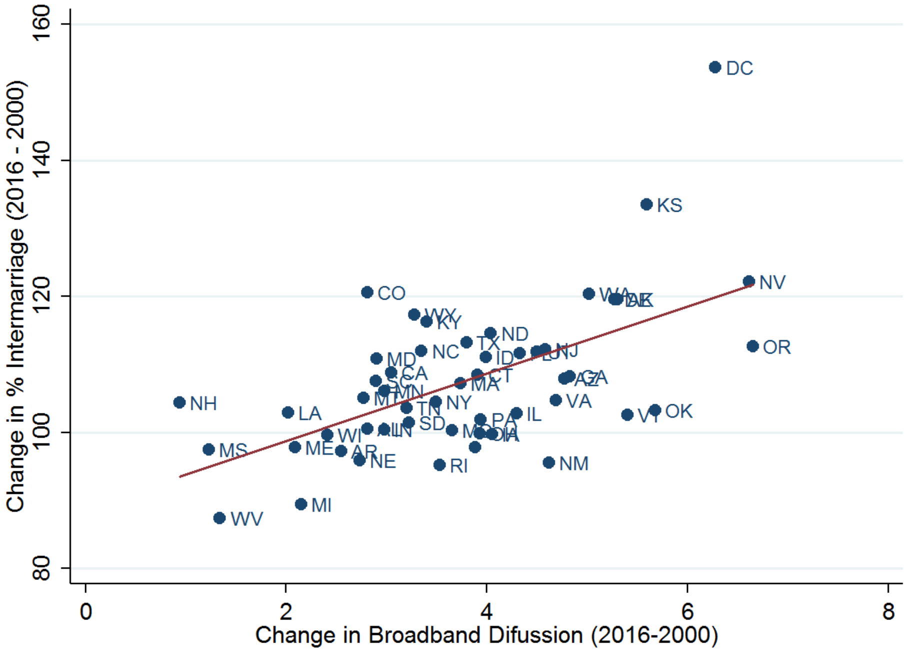


Figure 4.11: Change in percentage of marriages that are interracial in US, by state. Note: *Source*: FCC statistical reports on broadband development, US Census population estimates, and the American Community Survey (IPUMS) from 2000 to 2016 (longest period available).

We use three main data sources for our analysis. All data concerning individuals is downloaded from IPUMS, and we restrict our analysis to married individuals only. Although the data is only on the individual level, it is possible to construct marriage relationships, by employing a matching procedure described at IPUMS website. As additional control variables, we employ the total income, education and age, as these are likely to affect the marriage decision.

We construct the broadband data using information from reports by the Federal Communications Commission (FCC), which is the regulatory authority in the United States responsible for communication technology. Following Bellou (2015),[42] we use the number of residential high-speed internet lines per 100 people as our explanatory variable. Data is available for the years 2000 to 2016. However, we have to discard Hawaii from our analysis, as observations are missing up to 2005.

We download additional state controls from the Current Population Survey. Following Bellou's work, we include variables like the ratio of the male divided by female population

[42] She uses a similar specification to examine the role of internet diffusion in the creation of new marriages. Our dataset is described in detail in Appendix 4.D.

within a state, age bins and the ratio of non-white people in a state. This last explanatory variable is especially important in our context of interracial marriages.

We estimate the following reduced form equation by a linear probability model:

$$\text{Inter}_{ist} = \alpha + \beta\,\text{Broadband}_{st} + \gamma_1 X_{ist} + \gamma_2 Z_{st} + FE_s + FE_t + \epsilon_{ist} \tag{4.24}$$

where Inter_{ist} is one if a person is in an interracial marriage and 0 otherwise. The indices relate to person i, living in state s at time t. We are mostly interested in the coefficient β, as it captures the propensity of online dating. The values in X are covariates relating directly to the person, while Z represents state level variables. We additionally include state- and year fixed effects, and cluster the standard errors ϵ_{ist} at the state-year level. Our rich battery of control variables enables us to clearly identify the relationship between interracial marriages and broadband internet, which can be seen as an instrument for online dating. As marriages take a while to form, we include the broadband variable with a 3 year lag based on empirical evidence (Rosenfeld, 2017).[43]

The first column in Table 4.1 states that one additional line of broadband internet 3 years ago affects the probability of being in an interracial marriage by 0.07%. The coefficient is positive, as predicted by our theoretical model. In column (2) we include controls at the state level and find that the relationship between interracial marriages and broadband remains significantly positive. This continues to be true when including the individual covariates, all of which decrease the probability of a marriage being interracial. Perhaps surprisingly, education enters negatively. A potential underlying reason might be that education leads to more segregated friendship circles, a conjecture worth being explored in subsequent work.

Column (4) is now the specification outlined in (4.24). Even with all controls, the effect of broadband penetration on interracial marriages is highly significant and positive. This result suggests a causal relationship in the sense described by our model. As additional evidence for this claim, we see that once we replace the lagged broadband with its contemporaneous counterpart, the coefficient declines in size, which means that the state of broadband 3 years ago has a bigger effect on interracial marriages as compared to broadband today. This is because it takes time for marriages to form.

Overall, the work we have presented here, jointly with robustness checks described in Appendix 4.D, suggests that there is empirical support for our hypothesis of online dating leading to more interracial marriages.

Furthermore, the work of Thomas (2018), released shortly after we made the first version of our paper available online, has provided further evidence of the role of online dating in the creation of new interracial marriages. Using a self-collected dataset representative of the US population (known as "how couples met and stayed together" or HCMST), Thomas finds that couples who met online were more likely to be interracial,

[43] In Appendix 4.D we follow a different strategy. We construct shares of interracial marriages per state and year and estimate this with panel methods. The advantage is that the dependent variable is continuous rather than dichotomous, however we cannot use individual controls and introduce standard errors via aggregation. These standard errors should be negligible given the amount of observations we have available. The analysis there confirms our results.

	Interracial Marriage				
	(1)	(2)	(3)	(4)	(5)
Broadband (-3)	.00071***	.00058***	.00065***	.00054***	
	(.000057)	(.000066)	(.000055)	(.000063)	
Broadband					.00020***
					(0.000057)
Age			-.0031***	-.0031***	-.0031***
			(.000028)	(.00028)	(.000028)
Education			-.0024***	-.0024***	-.0024***
			(.00027)	(.00027)	(.00027)
Income			-5.9e-08***	-5.9e-08***	-5.9e-08***
			(2.68e-09)	(2.68e-09)	(2.68e-09)
State controls		x		x	x
N	17,284584	17,284584	17,284584	17,284584	17,284584
Adj. R^2	0.021	0.021	0.045	0.045	0.045

Standard errors are in parentheses and clustered at state-year level.

All regressions include state and year dummies.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4.1: Effect of broadband diffusion on the propensity of interracial marriage.

even after controlling for the racial composition of their locations and confounding factors. In particular, after analyzing information about 3,036 American couples, he finds that couples who met online since 1996 are 6 to 7 percent more often interracial than couples who met purely offline. His finding, using different methods and data, is similar to ours and provides further support for Hypothesis 1. His dataset is freely available online for replication purposes.

4.6.3 Hypothesis 2 & 3: More and Better Marriages

With regards to Hypothesis 2 and 3, which establish the creation of better and more marriages, respectively, we do not provide novel empirical work but we survey existing research from different disciplines.

There are two articles which have focused on whether relationships created online last longer than those created elsewhere. The first one is Cacioppo et al. (2013). They find that marriages created online were less likely to break up and exhibited a higher marital satisfaction, using a sample of 19,131 Americans who married between 2005 and 2012.

They write: *"Meeting a spouse on-line is on average associated with slightly higher marital satisfaction and lower rates of marital break-up than meeting a spouse through traditional off-line venues"*. The second one is Rosenfeld (2017). Analyzing the HCMST dataset from 2009 to 2015, he finds no difference in the duration of marriages that start online and offline. Besides their methodological differences, what it is clear is that both papers find that marriages created online last at least as long as those created elsewhere, disproving the common popular belief that online relationships are only casual and of lower quality (see footnote 15). This finding aligns with Hypothesis 2 of our model.

With regards to Hypothesis 3, which states that the advent of online dating leads to a higher number of marriages, there is in fact empirical evidence supporting it. Bellou (2015) examines the role that internet penetration (in the form of broadband deployment) has had in the number of White and Black young adults who decide to marry. She uses data from the Current Population Survey and the FCC from 2000 to 2010. She finds that wider internet availability has indeed *caused* more interracial marriages among people between 21 and 30 years old. In particular, she finds that marriage rates are currently higher by 13% to 33% from what they might have been if the internet had not been available, despite a pre-existing downward trend in the propensity to marry among young adults.

4.7 Final Remarks

4.7.1 Limitations of our Model

Our model does not explain three observed characteristics of interracial marriages. First, it does not explain why interracial marriages are more likely to end up in divorce (Bratter and King, 2008; Zhang and Van Hook, 2009). Second, it does not explain why some intraracial marriages from a particular race last longer than intraracial marriages from another race (e.g. Stevenson and Wolfers, 2007 document that Blacks who divorce spend more time in their marriage than their White counterparts). And third, our model does not explain why interracial marriage between specific combinations of race and gender are more common than others (marriage between White men and Asian women is much more common than marriage among Asian men and White women). A theoretical model that can account for those stylized facts is still missing (see Fryer, 2007 for a discussion of how well existing models of marriage explains observed interracial marriage trends).

4.7.2 Further Applications

The theoretical model we present discusses a general matching problem under network constraints, and hence it can be useful to study other social phenomena besides interracial marriage. Furthermore, the role of connecting highly clustered groups is also not only linked to online dating. Another example is the European student exchange program "Erasmus", which helped more than 3 million students and over 350 thousand academics and staff members to spend time at a University abroad.[44] Although it would

[44] "ERASMUS: Facts, figures and trends.", *European Commission*, 10/6/2014. Interestingly, Parey and Waldinger (2011) find that participating in ERASMUS increases the probability of working abroad

be interesting to test our model in these and other scenarios, we leave this task for further research.

4.7.3 Conclusion

We introduce a theoretical model to analyze the complex process of deciding whom to marry in the times of online dating. Our model is admittedly simple and fails to capture many of the complex features of romance in social networks, like love. However, in our view, the simplicity of our model is its main strength. It generates strong predictions with a basic and pedagogic structure. The main one is that the diversity of societies, measured by the number of interracial marriages in it, increases after the introduction of online dating. Not only is this prediction consistent with demographic trends, but an empirical analysis of interracial marriages within each US state suggests that online dating is indeed partially responsible for the observed increase in interracial marriage. And if that is the case, in words of the MIT Technology Review (2017): *"the model implies that this change is ongoing. That's a profound revelation. These changes are set to continue, and to benefit society as result"*.

Simple models are great tools for conveying an idea. Schelling's segregation model clearly does not capture many important components of how people decide where to live. It could have been enhanced by introducing thousands of parameters. Yet, it has broadened our understanding of racial segregation, and has been widely influential: according to Google Scholar, it has been quoted 3,258 times by articles in a variety of field ranging from sociology to mathematics. It has provided us with a way to think about an ubiquitous phenomenon.

Our model is a modest attempt that goes in the same direction.

4.A Simulation Results

First, we present the evolution of diversity, strength and size for assortative societies, which we omitted in the main text. The reader can check the results are almost identical to the case of Euclidean societies. First, there is a rapid increase of diversity, in particular for long marriages. Second, there is also an increase in strength and size. As in the main text, $p = 1$ for Figures 4.12 and 4.13; and $p = 1/n$ for Figure 4.14.

Table 4.2 presents the exact values of the simulations reported in Figures 7 and 8 in the main document, and Figures 4.12 and 4.13 in this Appendix.

4.B Robustness Checks

In this Appendix we conduct several robustness checks to show that the fast increase in the diversity of societies, described in Result 1, occurs for many combinations of model parameters.

by 15 percentage points. Their data suggests that a large fraction of this effect comes from marrying a foreign partner.

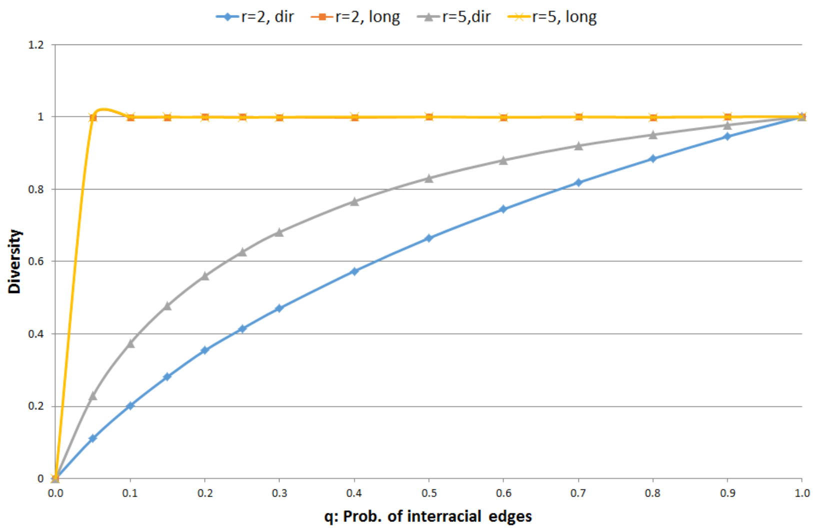


Figure 4.12: Average diversity of an assortative society for different values of q.

The yellow and orange curves are indistinguishable in this plot because they are identical. Exact values and standard errors (which are in the order of 1.0e-04) are provided in Table 4.2.

Different Values of p

The first exercise we conduct is to simulate the model again, but varying the probability of intraracial connections p. We allow q to vary between 0 and p, as we have explained in the text that $q \leq p$, because people tend to be more connected to people from their own race. In summary, we observe the same results as those documented in the main text: diversity and strength increase in a similar fashion as with $p = 1$.

With respect to diversity, long marriages always lead to an almost immediate increase to 1, meaning complete social integration. This increase is shown in Figure 4.16. As expected, a society integrates faster when the value of p is higher. With respect to strength, we also observe minor variations, which appear in Figure 4.17. A smaller p makes agents less connected to potential partners, and thus the strength of resulting marriages becomes weaker. With long marriages, strength converges very quickly to its optimal value, around 0.9, which again, is smaller in societies with low values of p and q.

The detailed results of our simulations with p equal to 0.7, 0.5, 0.3 appear in Tables 4.4, 4.5 and 4.6 at the end of this Appendix. We also present in Figure 4.15 the graph for $p = 1/n$, in which it can be clearly observed how the diversity for the case of long marriages is a crossover phenomenon rather than a phase transition. We present this graph separately to improve its exposition.

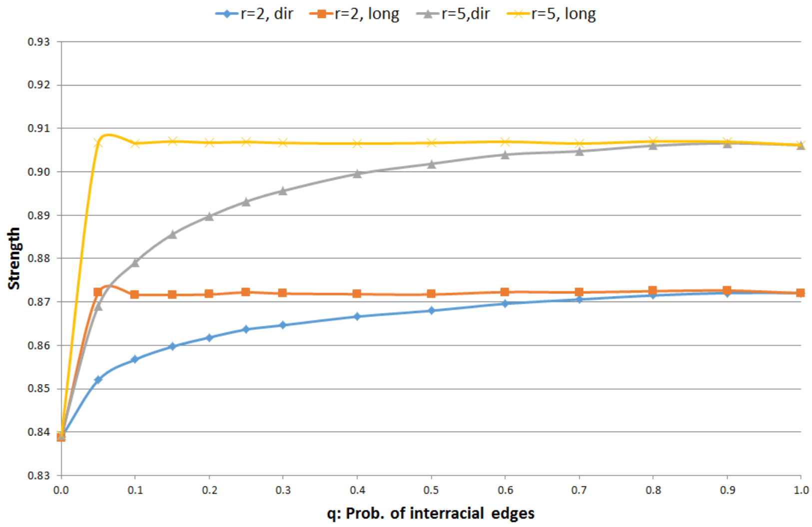


Figure 4.13: Average strength of an assortative society for different values of q.

Exact values and standard errors (which are in the order of 1.0e-04) provided in Table 4.2.

Varying p and q Simultaneously

The second robustness test we perform is to vary p and q simultaneously but keeping its ratio fixed. Both parameters indicate how connected a person is to people of his own race and to people of other races.

To find a good estimate of the ratio $\frac{p}{q}$, we use data from the American Values Survey by the Public Religion Research Institute (PRRI), a nonpartisan, independent research organization. The data is well described in the following article from the Washington Post: "Three quarters of Whites don't have any non-White friends", 25/8/2014. The PRRI data shows that, if a White American has 100 friends, 91 are expected to be of his own race, and 1 Black, 1 Latino, and 1 Asian (the rest are multiracial or of unknown race). Black Americans are more interracially connected, with 83 friends expected to be of his own race, 8 Whites, 2 Latinos, and and no Asians.

We use the ratio $p/q = 10$, based on the ratio between the expected number of Black and White friends for Black people. This ratio implies that a person is 10 times more likely to be connected to a person from her own race. We vary p from 0 to 1. We present the results for Euclidean societies only (as we have seen that Euclidean and assortative societies produce almost identical results).

A first conclusion we obtain is that, with long marriages, we observe complete integration, just as we did when increasing q alone. However, this time it does not happen as quickly as when we increase only q. With direct marriages the increase is very fast but full integration is not obtained. It only reaches values of 20% and 40% in societies with 2 and 5 races, respectively.

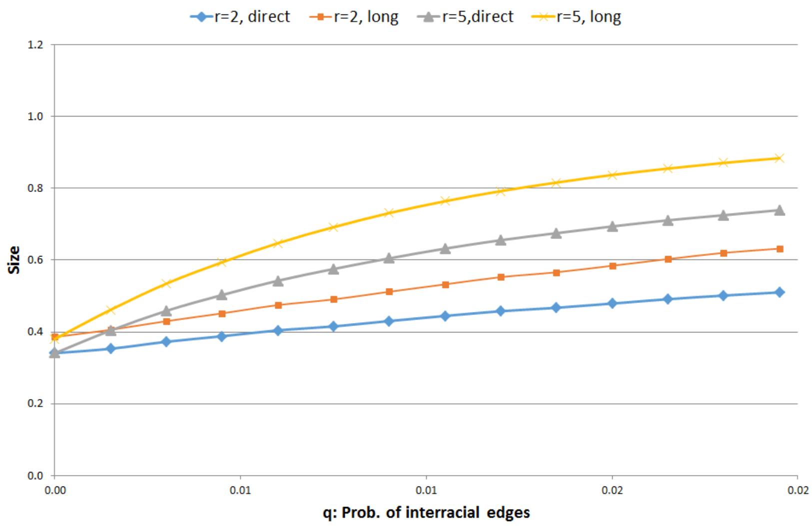


Figure 4.14: Average size of an assortative society for values of q up to $q = p = 1/n$.

Exact values and standard errors (which are in the order of 1.0e-04) provided in Table 4.3.

We could say that the diversity achieved when agents' intra and interracial circles both expand is much lower, compared to the results shown in the main text. But this this conclusion is flawed, because we compare our diversity measure to one where agents were completely connected within their own race, i.e. a complete graph. Therefore, the diversity obtained already is 20% and 40% of the diversity in a complete graph. This is a very high percentage of interracial marriages, because we fix that agents are 10 times more connected withing their own race. Notice that the results (Table 4.8) are consistent with what is displayed in Figures 7 and 8 in the main text, for the point $q = 0.1$.

Finally, the strength levels we observe with direct marriages are the lowest we have found so far, which is not surprising given the small number of potential partners that agents have. It is equally expected to observe that the strength of a society increases when p grows.

The detailed results of our simulations with $p/q = 10$ appear in Table 4.7 at the end of this Appendix.

Homophily

The third robustness test we perform is to introduce intraracial preferences, as described in equation (3). We do this in the following intuitive way. Agents prefer marrying someone from their own race β times as much as marrying someone from another race. This is, for agents i, j, k, with agents i and j being from the same race, and agent k being from another race, i is indifferent between j and k only if $\delta(i, j) = \beta\,\delta(i, k)$, where $\beta \geq 1$. We still impose that marrying any potential partner is better than remaining

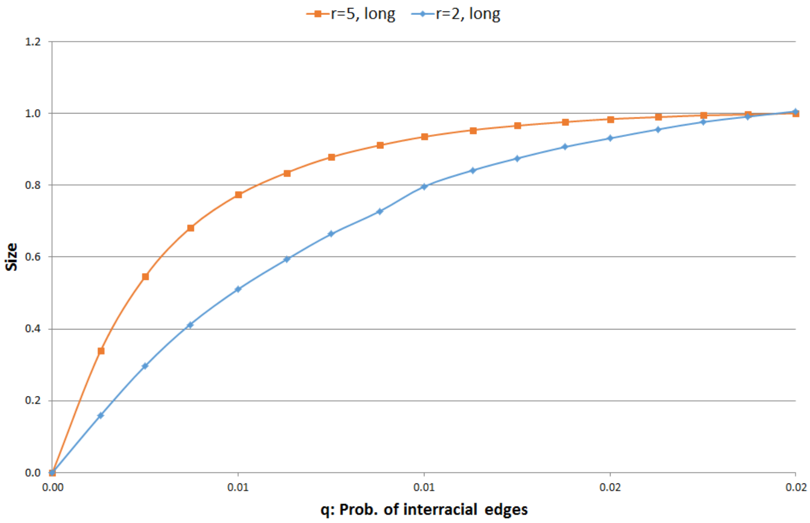


Figure 4.15: Fast increase of diversity for long marriages ($n = 50, p = 1/n = .02$).

alone for all agents.

There is evidence suggesting that persons substitute similarities in race for similarities in personality traits. See Furtado (2012) for evidence of tradeoffs in marriage choices between race and education.

Figure 4.19 presents the behavior of diversity and strength when agents prefer their own race twice and $\beta = 2$. With long marriages, we obtain a fast increase in diversity of our societies. However, only a diversity of 0.4 and 0.6 is achieved with 2 and 5 races, respectively. This is the diversity with respect to a society with $\beta = 1$. Therefore, the diversity achieved is large, even when agents have intraracial preferences.

With direct marriages, we observe that the increase converges to the same levels as with long marriages, but at a slower rate. The increase is a concave function of q, as documented in the main text when no homophily is present.

The reader may wonder how large β needs to be so that no diversity occurs in the society. Figure 4.20 shows how diversity changes as a function of β, for a society with $p = q = 1$. What we find is that even when agents prefer their own race 3.5 times as much as any other race, the society achieves 20% of the integration it would achieve without any racial preferences.

4.C Interracial Marriages and Population Composition

In this Appendix, we estimate the number of interracial marriages that would occur in 2015 if the racial composition of the U.S. population would have remained constant since 1980.

The main complication of estimating the adjusted rate of interracial marriage is that there is little data available regarding newlyweds. Only the 1980 U.S. Census and the

American Community Survey (ACS) from 2008 to 2015 allow us to identify subjects who recently married. The new marriages in 1980 can be identified using the variables *age* and *age of marriage*. Whenever those two coincide, we know that a couple married within a year of the data collection. In the ACS 2008 – 2015, married subjects were asked directly whether they married within the last year. The data is available at https://usa.ipums.org/usa. We use the 1 percent samples.

We obtained the percentage of subjects that married interracially by race in 1980 and 2015. Hispanics were not recorded as a race in 1980, so we estimate which percentage of other races are Hispanics. The races we consider are White, Black, Native Americans, Asians, and Hispanics. Also from the Census and the ACS, we obtain the racial composition of the U.S. both in 1980 and in 2015 (Table 4.9), and estimate the interracial marriages that would occur with random marriage.[45]

Random marriage is easy to compute. If 80% of Americans were White in 1980, a White American had a 0.2 probability of intermarrying. This is 5.33 times larger than the real intermarrying rate for Whites in 1980, which was 0.0375% only (Table 4.10). Our constructions of interracial rates by race are different from those by Lee and Edmonston (2005) as we estimate the race for Hispanics in 1980. Hispanic was not considered a race in the 1980 decennial census, despite the fact that Hispanics have problems identifying themselves with any of the major races.

We fix the 1980 ratio between actual and predicted interracial marriages. We use this ratio to compute the interracial marriage rate that would have occurred in 2015 with the population composition of 1980, using the prediction obtained from random marriage.[46]

Our estimates suggest that, even accounting for demographic change, the percentage of interracial marriages in our society increases by 30% to 37%. While this estimation needs to be taken with care, due to the limitations of the data available, it adds further evidence to our claim that increase in the number of interracial marriages in our societies cannot be due exclusively to changes in the composition of the U.S. population by race. Furthermore, if the increase in interracial marriage we observe in the data was due to changes in the population composition, the intermarriage rates for Black Americans should remain relatively constant over time, just like the fraction of the U.S. population that is Black. However, we observe that the intermarriage rates for Black Americans more than triplicate from 1980 to 2015, as described in the main text.

4.D Regression Analysis

This Appendix further elaborates on the empirical analysis. We start by describing the sources from which we draw our data.

The data on broadband development is obtained from the US Federal Communications Commission statistical reports on broadband deployment. These are available at www.fcc.gov/general/reports-high-speed-services-internet-access. They publish the number of high-speed lines exceeding 200 kbps in at least one direction (upload or download). This data is released twice per year in June and December. We use the

[45] The Stata code is available at www.josueortega.com.

[46] A similar estimation appears on "Why is interracial marriage on the rise?", *Priceonomics*, 1/9/2016.

December data, as the June version is missing for several years. Data ranges from 2000 to 2016.

We adjust the number of residential high-speed lines by the number of people on each state. Our variable of interest is lines per 100 inhabitants in a state. The population data by state is taken directly from the US Census estimates. This data is available at
www.census.gov/data/tables/2017/demo/popest/state-total.html, and
www.census.gov/programs-surveys/popest/technical-documentation/research/evaluation-estimates.html.

The corresponding variables are consistent with those from Bellou (2015). The data from Wyoming in 2000 and Hawaii 2000 - 2005 is missing. There is high variation in the level of broadband deployment by state. For example, by 2010 some states had above 50 lines per 100 persons (DC, Hawaii, Alaska, Connecticut, New Jersey, Maryland and Massachusetts), whereas some others had under 38 (West Virginia, Michigan, Idaho, Iowa, and South Dakota).

The data on interracial marriages comes from the Annual Community Survey (ACS), available from 2000 to 2016 at https://usa.ipums.org/usa/. We obtain from the same source additional variables like race, income, education and an identifier of the spouse (the variable *sploc* is the one that allows us to identify the partner, and the variable *stateicp* allows us to identify the state in which the subject lives). We construct the following racial categories: White, Black, Asian, Latino, Native American, and other. Whether other is included does not change the results, because there are few observations in this category. The IPUMS data is freely available online, but can be also obtained from us, as well as the FCC data, either via request or via www.josueortega.com. We compute the interracial marriage rates using the person weights provided (variable *perwt*), although the results are very similar when using either no weights or household weights (variable *hhwt*). Table 4.11 at the end of the document presents an overview of the data.

Table 4.11: Trends in Intermarriage Rates and Broadband Diffusion

State	Broadband		Intermarriage	
	2000	2016	2000	2016
Alabama	0.7	101.3	5.4	8.2
Alaska	0.1	119.7	17.8	23.1
Arizona	2.7	110.7	13.7	18.5
Arkansas	1.0	98.2	7.7	10.3
California	3.1	111.9	19.0	22.0
Colorado	2.1	122.7	14.0	16.8
Connecticut	3.0	111.5	7.3	11.2
Delaware	0.3	119.9	7.6	12.9
District of Columbia	2.5	156.2	15.5	21.8
Florida	1.6	113.2	10.4	14.7
Georgia	0.8	109.0	6.8	11.7
Hawaii	0.0	127.4	32.7	39.6
Idaho	1.0	112.1	7.7	11.7

Illinois	1.5	104.2	7.8	12.1
Indiana	0.4	100.8	6.8	9.8
Iowa	1.9	101.6	4.6	8.6
Kansas	2.4	135.9	6.9	12.5
Kentucky	0.3	116.6	5.3	8.7
Louisiana	0.9	103.8	8.7	10.7
Maine	2.0	99.8	5.8	7.8
Maryland	0.7	111.6	10.2	13.1
Massachusetts	4.0	111.3	7.9	11.7
Michigan	1.2	90.6	8.7	10.8
Minnesota	2.1	108.2	7.5	10.5
Mississippi	0.1	97.6	5.8	7.1
Missouri	1.4	101.8	6.6	10.3
Montana	0.7	105.8	8.2	10.9
Nebraska	3.0	98.8	6.7	9.5
Nevada	2.5	124.6	15.7	22.3
New Hampshire	3.2	107.5	8.1	9.1
New Jersey	2.8	115.0	8.8	13.4
New Mexico	1.2	96.8	19.1	23.7
New York	2.6	107.1	10.2	13.7
North Carolina	1.0	113.0	7.5	10.8
North Dakota	0.9	115.5	5.1	9.1
Ohio	1.5	101.4	5.9	9.8
Oklahoma	1.2	104.5	18.1	23.8
Oregon	1.9	114.6	10.6	17.3
Pennsylvania	0.9	102.8	5.3	9.2
Rhode Island	2.8	98.0	7.9	11.4
South Carolina	0.9	108.5	6.4	9.2
South Dakota	1.4	102.8	5.3	8.5
Tennessee	1.4	104.9	5.8	9.0
Texas	2.0	115.3	12.2	16.0
Utah	1.3	113.2	9.7	14.2
Vermont	1.2	103.7	4.8	10.2
Virginia	1.1	105.9	9.2	13.9
Washington	2.8	123.2	13.2	18.3
West Virginia	0.3	87.7	5.5	6.9
Wisconsin	1.1	100.8	6.9	9.3
Wyoming	0.0	117.4	9.8	13.0

Most of our control variables also come from the ACS IPUMS database. The ACS records agents income, household income, education, age, times married, age at the time of marriage, employment and migration status. From this dataset, one can construct further controls such as racial diversity of each state and availability of partners within an age frame, as well as the gender-ratio between men and women available.

In the main text we have estimated a linear probability model with fixed effects, due to the abundance of observations which is around 17 million. However, we additionally perform a panel regression, where we transform our individual observations into yearly state averages of interracial marriages. We estimate the model:

$$\text{Inter}_{st} = \alpha + \beta\,\text{Broadband}_{st} + \gamma Z_{st} + FE_s + \epsilon_{st} \tag{4.25}$$

Table 4.12 presents the result this alternative regression model. We estimate this model in addition to the linear probability model in the main text, because by transforming the variable indicating whether or not a variable is interracial into a continuous share, we improve the fit of the OLS model, indicated by a higher R^2. This comes at the cost of not being able to include individual controls and additional standard errors when aggregating, although these should be small given the number of data points is well over a thousand in all states and years.

The first specification is just a fixed effects models without any additional covariates, and we see the positive correlation. In column (2) we estimate a pooled OLS regression, and check that also the partial correlation of broadband diffusion and the interracial marriages within a state remains positive, even after controlling for other covariates. The full fledged model is estimated in column (3), where we also exploit the panel dimension of our data. We include state fixed effects and all covariates. The effect is highly positive and states that if internet in a state increases by 1 percentage point, the share of interracial marriages in this state rises by 0.03%. Keep in mind that our dependent variable is the share of interracial marriages among all marriages within a state, so even small percentage changes constitute a sizeable effect in terms of numbers of interracial partnerships.

Table 4.2: Supporting data for Figures 7, 8, A1 and A2

q	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
Panel A: Welfare on Euclidean societies														
$r = 2$, direct marriages														
Dv	0.00	0.10	0.19	0.27	0.34	0.41	0.47	0.57	0.66	0.75	0.82	0.89	0.94	1.00
St	0.85	0.87	0.87	0.87	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.89
$r = 2$, long marriages														
Dv	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
St	0.85	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.88	0.89	0.89
$r = 5$,direct marriages														
Dv	0.00	0.22	0.37	0.47	0.56	0.62	0.68	0.77	0.83	0.88	0.92	0.95	0.98	1.00
St	0.85	0.88	0.89	0.89	0.90	0.90	0.91	0.91	0.91	0.91	0.92	0.92	0.92	0.92
$r = 5$, long marriages														
Dv	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
St	0.85	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92
Panel B: Welfare on assortative societies														
$r = 2$, direct marriages														
Dv	0.00	0.11	0.20	0.28	0.35	0.41	0.47	0.57	0.66	0.75	0.82	0.88	0.95	1.00
St	0.84	0.85	0.86	0.86	0.86	0.86	0.86	0.87	0.87	0.87	0.87	0.87	0.87	0.87
$r = 2$, long marriages														
Dv	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
St	0.84	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87
$r = 5$,direct marriages														
Dv	0.00	0.23	0.37	0.48	0.56	0.63	0.68	0.77	0.83	0.88	0.92	0.95	0.98	1.00
St	0.84	0.87	0.88	0.89	0.89	0.89	0.90	0.90	0.90	0.90	0.90	0.91	0.91	0.91
$r = 5$, long marriages														
Dv	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
St	0.84	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91

*Average over 10,000 random simulations, $n = 50$, $p = 1$.

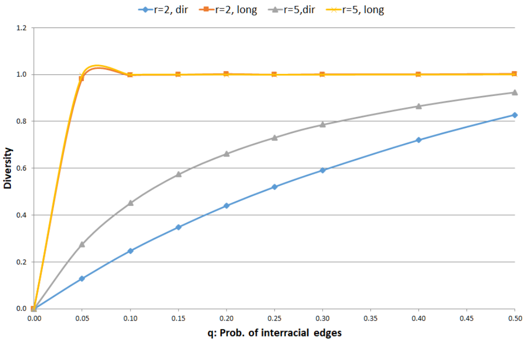
Standard errors in the order of 1.0e-04, so we do not present them.

Table 4.3: Supporting data for Figures 9 and A2

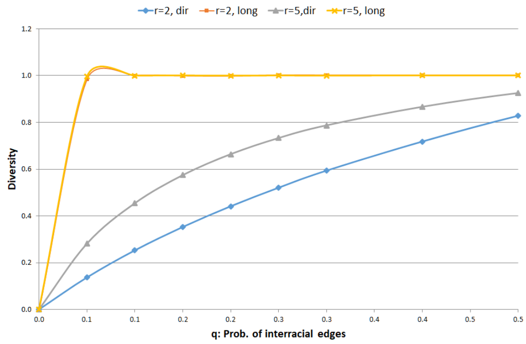
q	0		.003		.006		.009		.012		.015		.018	$1/n$
Panel A: Size on Euclidean societies														
$r = 2$, direct marriages														
Sz	0.34	0.35	0.37	0.38	0.40	0.41	0.42	0.44	0.45	0.46	0.47	0.48	0.49	0.50
$r = 2$, long marriages														
Sz	0.38	0.40	0.42	0.44	0.46	0.48	0.50	0.52	0.54	0.55	0.57	0.58	0.60	0.61
$r = 5$,direct marriages														
Sz	0.34	0.40	0.45	0.49	0.53	0.56	0.59	0.61	0.64	0.65	0.67	0.69	0.70	0.72
$r = 5$, long marriages														
Sz	0.38	0.46	0.53	0.59	0.65	0.69	0.73	0.76	0.79	0.82	0.84	0.85	0.87	0.88
Panel B: Size on assortative societies														
$r = 2$, direct marriages														
Sz	0.34	0.35	0.37	0.39	0.40	0.42	0.43	0.44	0.46	0.47	0.48	0.49	0.50	0.51
$r = 2$, long marriages														
Sz	0.39	0.41	0.43	0.45	0.47	0.49	0.51	0.53	0.55	0.57	0.58	0.60	0.62	0.63
$r = 5$,direct marriages														
Sz	0.34	0.40	0.46	0.50	0.54	0.58	0.60	0.63	0.66	0.68	0.69	0.71	0.73	0.74
$r = 5$, long marriages														
Sz	0.39	0.47	0.55	0.62	0.68	0.73	0.77	0.80	0.83	0.86	0.88	0.90	0.91	0.92

*Average over 10,000 random simulations, $n = 50$, $p = 1/n$.

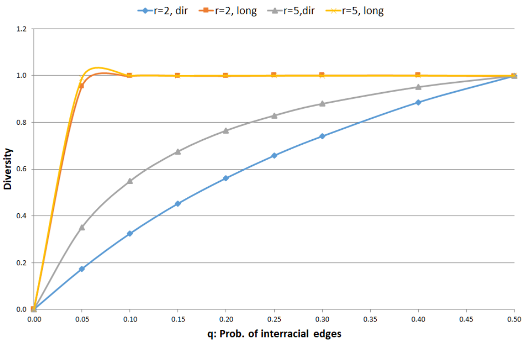
Standard errors in the order of 1.0e-04, so we do not present them.



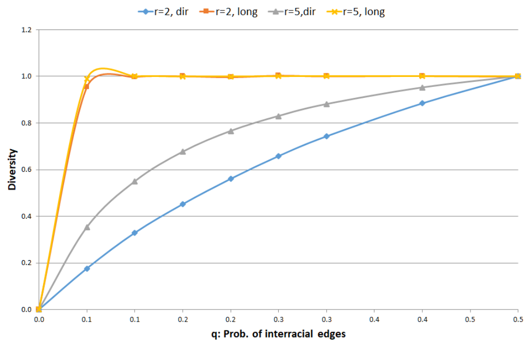
(a) Euclidean society, $p = .7$.



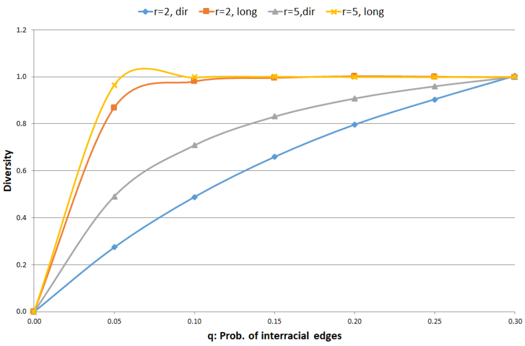
(b) Assortative society, $p = .7$



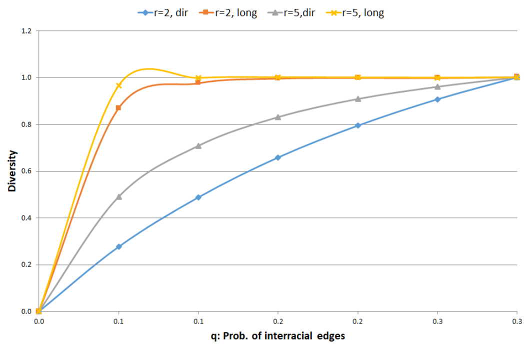
(c) Euclidean society, $p = .5$.



(d) Assortative society, $p = .5$

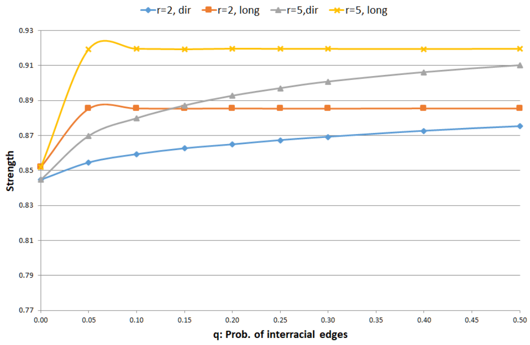


(e) Euclidean society, $p = .3$.

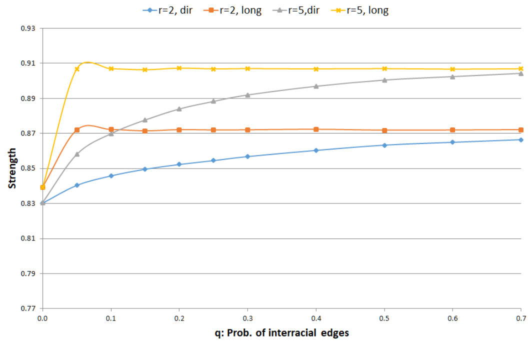


(f) Assortative society, $p = .3$

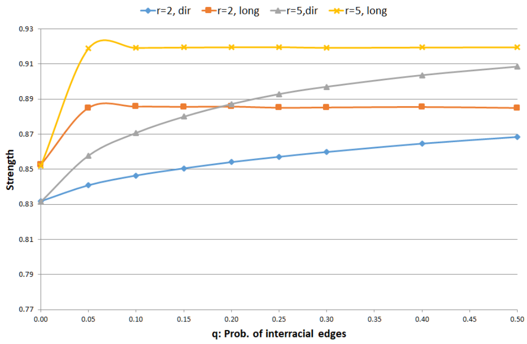
Figure 4.16: Average diversity (y-axis) of a random society for several values of p.



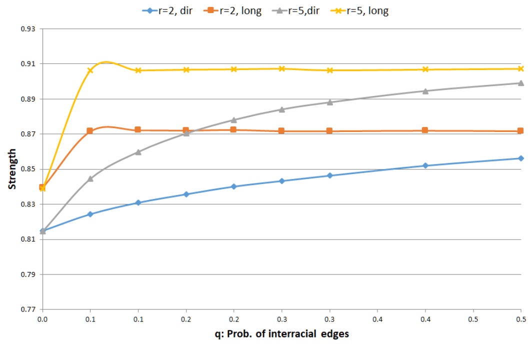
(a) Euclidean society, $p = .7$.



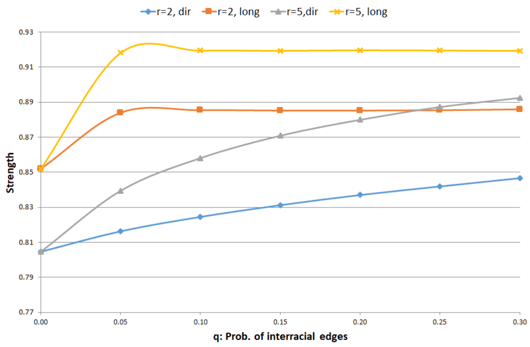
(b) Assortative society, $p = .7$



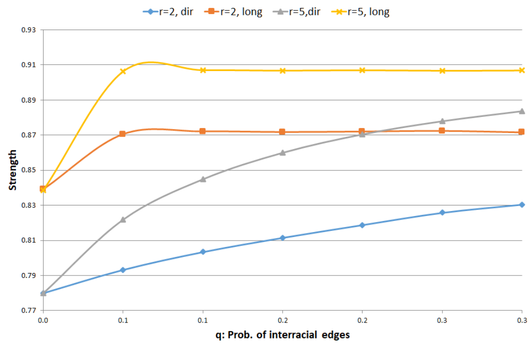
(c) Euclidean society, $p = .5$.



(d) Assortative society, $p = .5$

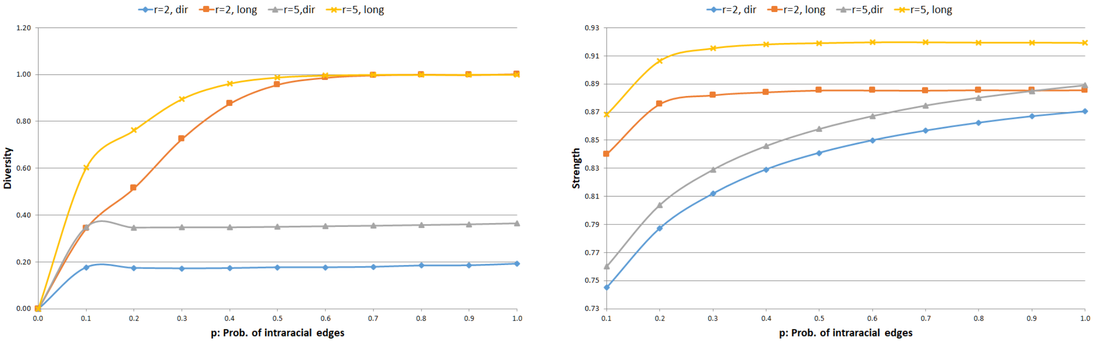


(e) Euclidean society, $p = .3$.



(f) Assortative society, $p = .3$

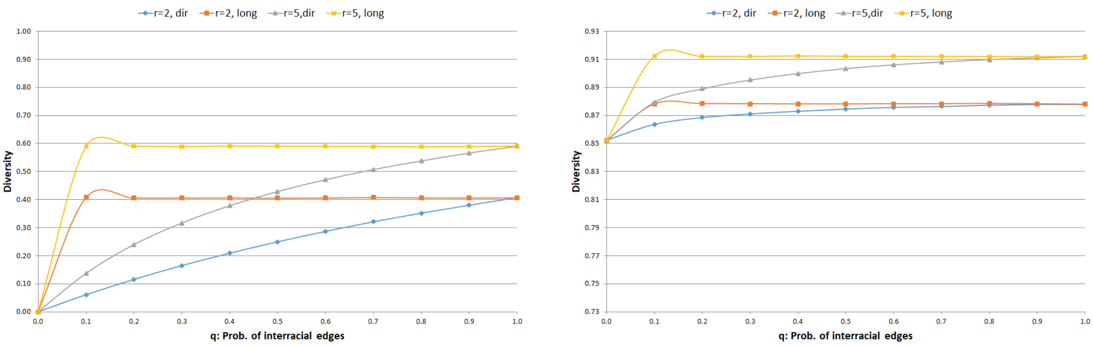
Figure 4.17: Average stregth (y-axis) of a random society for several values of p.



(a) Diversity.

(b) Strength.

Figure 4.18: Average diversity and strength of an Euclidean society for $p \in [0, 1]$ with p/q fixed.



(a) Diversity.

(b) Strength.

Figure 4.19: Average diversity and strength of a random society with $\beta = 2$.

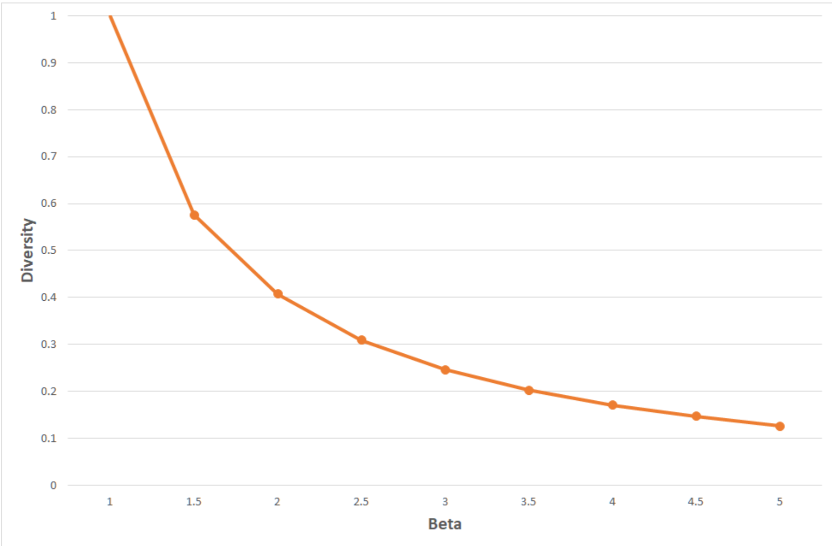


Figure 4.20: Relationship between β and diversity.

[Parameters] $n = 50$, $r = 2$, $p = q = 1$.

Table 4.4: Welfare with $p = 0.7$

q	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50	0.60	0.70
Panel A: Welfare on Euclidean societies											
$r = 2$, direct marriages											
Dv	0.00	0.13	0.25	0.35	0.44	0.52	0.59	0.72	0.83	0.92	1.00
St	0.84	0.85	0.86	0.86	0.86	0.87	0.87	0.87	0.88	0.88	0.88
Sz	0.98	0.97	0.97	0.97	0.97	0.98	0.98	0.98	0.99	0.99	0.99
$r = 2$, long marriages											
Dv	0.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
St	0.85	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89
Sz	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$r = 5$, direct marriages											
Dv	0.00	0.28	0.45	0.57	0.66	0.73	0.79	0.87	0.92	0.97	1.00
St	0.84	0.87	0.88	0.89	0.89	0.90	0.90	0.91	0.91	0.91	0.92
Sz	0.98	0.97	0.97	0.98	0.98	0.98	0.99	0.99	0.99	0.99	1.00
$r = 5$, long marriages											
Dv	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
St	0.85	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92
Sz	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Panel B: Welfare on assortative societies											
$r = 2$, direct marriages											
Dv	0.00	0.14	0.25	0.35	0.44	0.52	0.59	0.72	0.83	0.92	1.00
St	0.83	0.84	0.85	0.85	0.85	0.85	0.86	0.86	0.86	0.86	0.87
Sz	0.98	0.97	0.97	0.97	0.98	0.98	0.98	0.98	0.99	0.99	0.99
$r = 2$, long marriages											
Dv	0.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
St	0.84	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87
Sz	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$r = 5$, direct marriages											
Dv	0.00	0.28	0.45	0.57	0.66	0.73	0.79	0.87	0.93	0.97	1.00
St	0.83	0.86	0.87	0.88	0.88	0.89	0.89	0.90	0.90	0.90	0.90
Sz	0.98	0.97	0.97	0.98	0.98	0.99	0.99	0.99	0.99	1.00	1.00
$r = 5$, long marriages											
Dv	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
St	0.84	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91
Sz	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

*Average over 10,000 random simulations, $n = 50$.

Sz equals the percentage of agents married.

Standard errors in the order of 1.0e-04.

Table 4.5: Welfare with $p = 0.5$

q	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50
Panel A: Welfare on Euclidean societies									
$r = 2$, direct marriages									
Dv	0.00	0.17	0.32	0.45	0.56	0.66	0.74	0.89	1.00
St	0.83	0.84	0.85	0.85	0.85	0.86	0.86	0.86	0.87
Sz	0.96	0.95	0.96	0.96	0.96	0.97	0.97	0.97	0.98
$r = 2$, long marriages									
Dv	0.00	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00
St	0.85	0.88	0.89	0.89	0.89	0.89	0.89	0.89	0.88
Sz	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$r = 5$, direct marriages									
Dv	0.00	0.35	0.55	0.68	0.77	0.83	0.88	0.95	1.00
St	0.83	0.86	0.87	0.88	0.89	0.89	0.90	0.90	0.91
Sz	0.96	0.96	0.97	0.97	0.98	0.98	0.98	0.99	0.99
$r = 5$, long marriages									
Dv	0.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
St	0.85	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92
Sz	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Panel B: Welfare on assortative societies									
$r = 2$, direct marriages									
Dv	0.00	0.18	0.33	0.45	0.56	0.66	0.74	0.88	1.00
St	0.58	0.58	0.59	0.59	0.59	0.60	0.60	0.60	0.61
Sz	0.96	0.96	0.96	0.96	0.97	0.97	0.97	0.98	0.98
$r = 2$, long marriages									
Dv	0.00	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00
St	0.84	0.87	0.87	0.87	0.87	0.87	0.87	0.87	0.87
Sz	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$r = 5$, direct marriages									
Dv	0.00	0.35	0.55	0.68	0.77	0.83	0.88	0.95	1.00
St	0.58	0.60	0.61	0.62	0.62	0.63	0.63	0.63	0.64
Sz	0.96	0.96	0.97	0.98	0.98	0.98	0.99	0.99	0.99
$r = 5$, long marriages									
Dv	0.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
St	0.84	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91
Sz	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

*Average over 10,000 random simulations, $n = 50$.

Sz equals the percentage of agents married.

Standard errors in the order of 1.0e-04.

Table 4.6: Welfare with $p = 0.3$

q	0	0.05	1	0.15	0.2	0.25	0.3
Panel A: Welfare on Euclidean societies							
$r = 2$, direct marriages							
Dv	0.00	0.27	0.49	0.66	0.80	0.90	1.00
St	0.80	0.82	0.82	0.83	0.84	0.84	0.85
Sz	0.91	0.92	0.93	0.93	0.94	0.95	0.95
$r = 2$, long marriages							
Dv	0.00	0.87	0.98	1.00	1.00	1.00	1.00
St	0.85	0.88	0.89	0.89	0.89	0.89	0.89
Sz	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$r = 5$, direct marriages							
Dv	0.00	0.49	0.71	0.83	0.91	0.96	1.00
St	0.00	0.05	0.10	0.15	0.20	0.25	0.30
Sz	0.91	0.94	0.95	0.97	0.97	0.98	0.98
$r = 5$, long marriages							
Dv	0.00	0.96	1.00	1.00	1.00	1.00	1.00
St	0.85	0.92	0.92	0.92	0.92	0.92	0.92
Sz	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Panel B: Welfare on assortative societies							
$r = 2$, direct marriages							
Dv	0.00	0.28	0.49	0.66	0.80	0.91	1.00
St	0.78	0.79	0.80	0.81	0.82	0.83	0.83
Sz	0.92	0.93	0.93	0.94	0.95	0.95	0.96
$r = 2$, long marriages							
Dv	0.00	0.87	0.98	1.00	1.00	1.00	1.00
St	0.84	0.87	0.87	0.87	0.87	0.87	0.87
Sz	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$r = 5$, direct marriages							
Dv	0.00	0.49	0.71	0.83	0.91	0.96	1.00
St	0.78	0.82	0.84	0.86	0.87	0.88	0.88
Sz	0.92	0.94	0.96	0.97	0.98	0.98	0.98
$r = 5$, long marriages							
Dv	0.00	0.97	1.00	1.00	1.00	1.00	1.00
St	0.84	0.91	0.91	0.91	0.91	0.91	0.91
Sz	1.00	1.00	1.00	1.00	1.00	1.00	1.00

*Average over 10,000 random simulations, $n = 50$.

Sz equals the percentage of agents married.

Standard errors in the order of 1.0e-04.

Table 4.7: Welfare with $\frac{p}{q} = 10$

p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Welfare on Euclidean societies										
$r = 2$, direct marriages										
Dv	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
St	0.18	0.17	0.17	0.17	0.18	0.18	0.18	0.18	0.19	0.19
Sz	0.75	0.79	0.81	0.83	0.84	0.85	0.86	0.86	0.87	0.87
$r = 2$, long marriages										
Dv	0.75	0.87	0.91	0.94	0.95	0.96	0.97	0.97	0.98	0.98
St	0.34	0.52	0.73	0.88	0.96	0.99	1.00	1.00	1.00	1.00
Sz	0.84	0.88	0.88	0.88	0.89	0.89	0.89	0.89	0.89	0.89
$r = 5$, direct marriages										
Dv	0.91	0.98	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
St	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.36	0.36	0.36
Sz	0.76	0.80	0.83	0.85	0.86	0.87	0.87	0.88	0.88	0.89
$r = 5$, long marriages										
Dv	0.60	0.76	0.90	0.96	0.99	1.00	1.00	1.00	1.00	1.00
St	0.87	0.91	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92
Sz	0.94	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

*Average over 10,000 random simulations, $n = 50$.

Sz equals the percentage of agents married.

Standard errors in the order of 1.0e-04.

Table 4.8: Welfare with $\beta = 2$

q	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Welfare on Euclidean societies											
$r = 2$, direct marriages											
Dv	0.00	0.06	0.12	0.16	0.21	0.25	0.29	0.32	0.35	0.38	0.41
St	0.85	0.86	0.87	0.87	0.87	0.87	0.88	0.88	0.88	0.88	0.88
Sz	1.00	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00
$r = 2$, long marriages											
Dv	0.00	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41
St	0.85	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88
Sz	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$r = 5$, direct marriages											
Dv	0.00	0.14	0.24	0.32	0.38	0.43	0.47	0.51	0.54	0.57	0.59
St	0.85	0.88	0.89	0.90	0.90	0.90	0.91	0.91	0.91	0.91	0.91
Sz	1.00	0.98	0.98	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00
$r = 5$, long marriages											
Dv	0.00	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59	0.59
St	0.85	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91
Sz	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

*Average over 10,000 random simulations, $n = 50$, $p = 1$.

Sz equals the percentage of agents married.

Standard errors in the order of 1.0e-04.

Table 4.9: U.S. population composition by race, in percentage

Race	1980	2015
White	80	64
Black	11.6	12.2
Native	0.5	0.7
Asian	1.5	4.84
Hispanic	6.5	16.3
Multiracial	0	2

Source: Authors' analysis of 1980 decennial census and 2015 ACS.

Table 4.10: U.S. interracial marriage rate by race

Race	1980	2015
White	3.8	10.8
Black	5.6	20.0
Native	51.5	55.3
Asian	24.1	32.3
Hispanic	27.3	30.2

Source: Authors' analysis of 1980 decennial census and 2015 ACS.

Table 4.12: Impact of broadband diffusion on interracial marriages.

	Interracial Marriage		
	(1)	(2)	(3)
Broadband (-3)	0.0393***	0.0516***	0.0330***
	(0.00109)	(0.00816)	(0.00236)
Fixed effects	x		x
Control variables		x	x
N	700	700	700
adj. R^2	0.640	0.397	0.661

Standard errors in parentheses.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Bibliography

Acemoglu, D. and P. Restrepo (2017, March). Robots and Jobs: Evidence from US Labor Markets. NBER Working Paper 23285, National Bureau of Economic Research, Inc.

Acemoglu, D. and R. Shimer (1999, November). Holdups and Efficiency with Search Frictions. *International Economic Review 40*(4), 827–849.

Adam, K. and J. Zhu (2016, August). Price-Level Changes And The Redistribution Of Nominal Wealth Across The Euro Area. *Journal of the European Economic Association 14*(4), 871–906.

Aguiar, M. and E. Hurst (2005, October). Consumption versus Expenditure. *Journal of Political Economy 113*(5), 919–948.

Aguiar, M. and E. Hurst (2013). Deconstructing Life Cycle Expenditure. *Journal of Political Economy 121*(3), 437–492.

Ahn, S. Y. (2018). Matching across markets: Theory and evidence on cross-border marriage. *unpublished*.

Albert, R. and A. Barabási (2002). Statistical mechanics of complex networks. *Reviews of Modern Physics 74*(1), 47.

Alves, S. A. L. (2014). Lack of divine coincidence in New Keynesian models. *Journal of Monetary Economics 67*(C), 33–46.

Arntz, M., T. Gregory, and U. Zierahn (2016). The Risk of Automation for Jobs in OECD Countries: A Comparative Analysis. Technical report.

Arrow, K. (1998, June). What has economics to say about racial discrimination? *Journal of Economic Perspectives 12*(2), 91–100.

Auclert, A. (2015). Monetary Policy and the Redistribution Channel. 2015 Meeting Papers 381, Society for Economic Dynamics.

Auclert, A. (2017). Monetary Policy and the Redistribution Channel. Technical report. Manuscript.

Autor, D., D. Dorn, L. F. Katz, C. Patterson, and J. V. Reenen (2017, May). The Fall of the Labor Share and the Rise of Superstar Firms. NBER Working Paper 23396, National Bureau of Economic Research, Inc.

Backstrom, L. and J. Kleinberg (2014). Romantic partnerships and the dispersion of social ties: A network analysis of relationship status on facebook. In *Proceedings of the 17th ACM Conference on Computer Supported Cooperative Work & Social Computing*, CSCW '14, pp. 831–841.

Banerjee, A., E. Duflo, M. Ghatak, and J. Lafortune (2013, May). Marry for what? caste and mate selection in modern india. *American Economic Journal: Microeconomics 5*(2), 33–72.

Barnichon, R. (2010, December). Building a composite Help-Wanted Index. *Economics Letters 109*(3), 175–178.

Becker, G. (1973). A theory of marriage: Part i. *Journal of Political Economy 81*(4), 813–846.

Becker, G. (1981). *A Treatise on the family.* Harvard University Press.

Bellou, A. (2015, Apr). The impact of internet diffusion on marriage rates: evidence from the broadband market. *Journal of Population Economics 28*(2), 265–297.

Benigno, P. and M. Woodford (2006). Optimal taxation in an rbc model: A linear-quadratic approach. *30*(9-10), 1445–1489.

Bernheim, B. D., J. Skinner, and S. Weinberg (2001, September). What Accounts for the Variation in Retirement Wealth among U.S. Households? *American Economic Review 91*(4), 832–857.

Blanchard, O. J. (1997). The Medium Run. *Brookings Papers on Economic Activity 28*(2), 89–158.

Blatter, M., S. Muehlemann, and S. Schenker (2012). The Costs of Hiring Skilled Workers. *European Economic Review 56*(1), 20–35.

Blum, A., J. Hopcroft, and R. Kannan (2017). Foundations of data science. *https://www. cs. cornell. edu/jeh/book. pdf*.

Bogomolnaia, A. and J.-F. Laslier (2007). Euclidean preferences. *Journal of Mathematical Economics 43*(2), 87–98.

Bollobás, B. (2001). *Random graphs.* Cambridge University Press.

Bouyon, S. (2017). Recent trends and developments in european mortgage markets. European Credit Research Institute, Commentary No. 21.

Bratter, J. and R. King (2008). But will it last?: Marital instability among interracial and same-race couples. *Family Relations 57*(2), 160–171.

Browning, M., P.-A. Chiappori, and Y. Weiss (2014). *Economics of the family.* Cambridge University Press.

Brunnermeier, M. K. and Y. Sannikov (2012). Redistributive monetary policy. *Proceedings - Economic Policy Symposium - Jackson Hole*, 331–384.

Brynjolfsson, E. and A. M. Afee (2014). *The Second Machine Age: Work, Progress, and Prosperity in a Time of Brilliant Technologies.* W.W. Norton & Company.

Cacioppo, J., S. Cacioppo, G. Gonzaga, E. Ogburn, and T. VanderWeele (2013). Marital satisfaction and break-ups differ across on-line and off-line meeting venues. *Proceedings of the National Academy of Sciences 110*(25), 10135–10140.

Calvo, G. (1983). Staggered Prices in a Utility Maximizing Framework. *Journal of Monetary Economics 12*, 383–398.

Carlstrom, C. T. and T. S. Fuerst (1997, December). Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis. *American Economic Review 87*(5), 893–910.

Chiappori, P.-A., R. McCann, and B. Pass (2016, Apr). Multidimensional matching. Papers 1604.05771, arXiv.org.

Chiappori, P.-A., S. Oreffice, and C. Quintana-Domeque (2012). Fatter attraction: Anthropometric and socioeconomic matching on the marriage market. *Journal of Political Economy 120*(4), 659–695.

Chirinko, R. S. (2008, June). Sigma: The Long and Short of It. *Journal of Macroeconomics 30*(2), 671–686.

Christiano, Lawrence J., M. E. and C. L. Evans (2005). Nom- inal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Clarida, R., J. Galí, and M. Gertler (1999). The science of monetary policy: A New Keynesian perspective. *37*(4), 1661–1707.

Clark, S. (2006, December). The uniqueness of stable matchings. *The B.E. Journal of Theoretical Economics 6*(1), 1–28.

Corcoran, M., L. Datcher, and G. Duncan (1980). Most workers find jobs through word of mouth. *Monthly Labor Review 103*(8), 33–35.

Cox, D., J. Navarro-Rivera, and R. Jones (2016). Race, religion, and political affiliation of americans' core social networks. Technical report, Public Religion Research Institute, 2013 American Values Survey.

Danthine, J.-P. and J. B. Donaldson (2002). Labour Relations and Asset Returns. *Review of Economic Studies 69*(1), 41–64.

de Martí, J. and Y. Zenou (2016). Segregation in friendship networks. *Scandinavian Journal of Economics 119*(3), 656–708.

Debortoli, D. and J. Gali (2017, September). Monetary Policy with Heterogeneous Agents: Insights from TANK models. 2015 meeting papers, Universidad Pompeu Fabra.

Di Maggio, M., A. Kermani, B. J. Keys, T. Piskorski, R. Ramcharan, A. Seru, and V. Yao (2017, November). Interest rate pass-through: Mortgage rates, household consumption, and voluntary deleveraging. *American Economic Review 107*(11), 3550–88.

Doepke, M. and M. Schneider (2006a, December). Inflation and the Redistribution of Nominal Wealth. *Journal of Political Economy 114*(6), 1069–1097.

Doepke, M. and M. Schneider (2006b, June). Inflation as a Redistribution Shock: Effects on Aggregates and Welfare. NBER Working Papers 12319, National Bureau of Economic Research, Inc.

Doepke, M., M. Schneider, and V. Selezneva (2015). Distributional Effects of Monetary Policy. Technical Report 14, Hutchins Center on Fiscal & Monetary Policy.

Duncan, B. and S. Trejo (2011). Intermarriage and the intergenerational transmission of ethnic identity and human capital for mexican americans. *Journal of Labor Economics 29*(2), 195–227.

Eeckhout, J. (2000). On the uniqueness of stable marriage matchings. *Economics Letters 69*(1), 1–8.

Eggertsson, G. and N. Mehrotra (2017). Secular Stagnation and Inequality. 2017 Meeting Papers 1567, Society for Economic Dynamics.

Elsby, M., B. Hobijn, and A. Sahin (2013). The Decline of the U.S. Labor Share. *Brookings Papers on Economic Activity, Fall*.

Erdős, P. and A. Rényi (1959, 1959). On random graphs i. *Publicationes Mathematicae (Debrecen) 6*, 290–297.

Erdős, P. and A. Rényi (1964). On random matrices. *Magyar Tud. Akad. Mat. Kutató Int. Közl 8*(455-461), 1964.

Erdős, P., S. Suen, and P. Winkler (1995). On the size of a random maximal graph.

Random Structures & Algorithms 6(2-3), 309–318.

Fernández-Villaverde, J. and D. Krueger (2011, November). Consumption And Saving Over The Life Cycle: How Important Are Consumer Durables? *Macroeconomic Dynamics 15*(05), 725–770.

Fisman, R., S. Iyengar, E. Kamenica, and I. Simonson (2008). Racial preferences in dating. *Review of Economic Studies 75*(1), 117–132.

Fryer, R. (2007, June). Guess who's been coming to dinner? trends in interracial marriage over the 20th century. *Journal of Economic Perspectives 21*(2), 71–90.

Fu, X. (2007). Inter-racial marriage and family socio-economic status: A study among whites, filipinos, japanese, and hawaiians in hawaii. *Journal of Economic Perspectives 38*, 533–554.

Furtado, D. (2012). Human capital and interethnic marriage decisions. *Economic Inquiry 50*(1), 82–93.

Furtado, D. (2015). Ethnic intermarriage. In *International Encyclopedia of the Social & Behavioral Sciences*, pp. 118–122. Elsevier.

Furtado, D. and N. Theodoropoulos (2010, November). Why does intermarriage increase immigrant employment? The role of networks. *B.E. Journal of Economic Analysis & Policy 10*(1), 1–33.

Gale, D. and L. Shapley (1962). College admissions and the stability of marriage. *American Mathematical Monthly 69*(1), 9–15.

Gali, J. (2014). Monetary Policy and Rational Asset Price Bubbles. *American Economic Review 104*(3), 721–752.

Gali, J. (2017). Monetary Policy and Bubbles in a New Keynesian Model with Overlapping Generations. Technical report, CREI Working Paper.

Garriga, C., F. E. Kydland, and R. Sustek (2013, December). Mortgages and Monetary Policy. NBER Working Papers 19744, National Bureau of Economic Research, Inc.

Garriga, C., F. E. Kydland, and R. Šustek (2017). Mortgages and monetary policy. *The Review of Financial Studies 30*(10), 3337–3375.

Gee, L. K., J. Jones, and M. Burke (2017). Social networks and labor markets: How strong ties relate to job finding on facebook's social network. *Journal of Labor Economics 35*(2), 485–518.

Gertler, M., C. Huckfeldt, and A. Trigari (2016, June). Unemployment Fluctuations, Match Quality, and the Wage Cyclicality of New Hires. NBER Working Paper 22341, National Bureau of Economic Research, Inc.

Gertler, M. and A. Trigari (2006). Unemployment fluctuations with staggered nash wage bargaining. NBER Working Paper 12498.

Gilbert, E. (1959, 12). Random graphs. *Annals of Mathematical Statistics 30*(4), 1141–1144.

Gius, M. (2013). The effects of interracial marriage on individual-level earnings: An analysis using 2010 pums data. *Journal of Applied Business and Economics 14*, 37–41.

Goel, D. and K. Lang (2009, July). Social ties and the job search of recent immigrants. NBER Working Papers 15186.

Golub, B. and M. Jackson (2012). How homophily affects the speed of learning and best-response dynamics. *Quarterly Journal of Economics 127*(3), 1287–1338.

Gomme, P. and P. Rupert (2004). Measuring labor's share of income. *Policy Discussion Papers* (Nov).

Gordon, R. J. (1990). *The Measurment of Durable Goods Prices.* National Bureau of Economic Research.

Gornemann, N., K. Kuester, and M. Nakajima (2016, May). Doves for the Rich, Hawks for the Poor? Distributional Consequences of Monetary Policy. International Finance Discussion Papers 1167, Board of Governors of the Federal Reserve System (U.S.).

Granovetter, M. (1973). The strength of weak ties. *American Journal of Sociology 78*(6), 1360–1380.

Granovetter, M. (1995). *Getting a job: A study of contacts and careers.* University of Chicago Press.

Haefke, C., M. Sonntag, and T. van Rens (2013). Wage rigidity and job creation. *60*, 887–899.

Hansen, G. (1993). The cyclical and seuclar behaviour of the labour input: comparing efficiency units and hours worked. *Journal of Applied Econometrics 8*, 71–80.

Hawkins, W. B. (2013, 02). Competitive Search, Efficiency, And Multiworker Firms. *International Economic Review 54*(1), 219–251.

Heer, B. and C. Scharrer (2018). The age-specific burdens of short-run fluctuations in government spending. *Journal of Economic Dynamics and Control 90*(C), 45–75.

Hergovich, P. (2014). Housing and the redistributive efffects of monetary policy. Master's thesis, Institute for Advanced Studies/Technical University Vienna.

Hitsch, G., A. Hortaçsu, and D. Ariely (2010, March). Matching and sorting in online dating. *American Economic Review 100*(1), 130–63.

Holroyd, A., R. Pemantle, Y. Peres, and O. Schramm (2009, 02). Poisson matching. *Annales de l'Institut Henri Poincaré, Probabilités et Statistiques 45*(1), 266–287.

Iacoviello, M. and M. Pavan (2013). Housing and debt over the life cycle and over the business cycle. *Journal of Monetary Economics 60*(2), 221–238.

Jackson, M. and A. Wolinsky (1996). A strategic model of social and economic networks. *Journal of Economic Theory 71*(1), 44–74.

Kaas, L. and P. Kircher (2015, October). Efficient Firm Dynamics in a Frictional Labor Market. *American Economic Review 105*(10), 3030–3060.

Kaldor, N. (1961). Capital accumulation and economic growth. In F. Lutz and D. C. Hague (Eds.), *The Theory of Capital.* New York: St. Martin's Press.

Kalmijn, M. (1998). Intermarriage and homogamy: Causes, patterns, trends. *Annual Review of Sociology 24*(1), 395–421.

Kantarevic, J. (2004, May). Interethnic marriages and economic assimilation of immigrants. Discussion Papers 1142, IZA.

Kaplan, G., B. Moll, and G. L. Violante (2018). Monetary Policy According to HANK. *American Economic Review 108*(3), 697–743.

Karabarbounis, L. and B. Neiman (2014). The Global Decline of the Labor Share. *The Quarterly Journal of Economics 129*(1), 61–103.

Klump, R., P. McAdam, and A. Willman (2012, December). The Normalized CES Production Function: Theory And Empirics. *Journal of Economic Surveys 26*(5), 769–799.

Kramarz, F. and O. Skans (2014). When strong ties are strong: Networks and youth labour market entry. *Review of Economic Studies 81*(3), 1164–1200.

Krause, M. U. and S. Moyen (2013). Public debt and changing inflation targets. Technical report.

Krusell, P., L. E. Ohanian, J.-V. RÌos-Rull, and G. L. Violante (2000, September). Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis. *Econometrica 68*(5), 1029–1054.

Lee, S. M. and B. Edmonston (2005). *New marriages, new families: US racial and Hispanic intermarriage*, Volume 60. Population Reference Bureau Washington, DC.

León-Ledesma, M. A., P. McAdam, and A. Willman (2010, September). Identifying the Elasticity of Substitution with Biased Technical Change. *American Economic Review 100*(4), 1330–1357.

Lewis, K. (2013). The limits of racial prejudice. *Proceedings of the National Academy of Sciences 110*(47), 18814–18819.

Livingston, G. and A. Brown (2017). Intermarriage in the u.s. 50 years after loving v. virginia. *Pew Research Center Report*.

Loecker, J. D. and J. Eeckhout (2017, August). The Rise of Market Power and the Macroeconomic Implications. NBER Working Paper 23687, National Bureau of Economic Research, Inc.

Luetticke, R. (2018, June). Transmission of Monetary Policy with Heterogeneity in Household Portfolios. Technical report, UCL.

Marsden, P. (1987). Core discussion networks of americans. *American Sociological Review 52*(1), 122–131.

McGrath, A., G. Tsunokai, M. Schultz, J. Kavanagh, and J. Tarrence (2016). Differing shades of colour: Online dating preferences of biracial individuals. *Ethnic and Racial Studies 39*(11), 1920–1942.

McKay, A., E. Nakamura, and J. Steinsson (2016, October). The Power of Forward Guidance Revisited. *American Economic Review 106*(10), 3133–3158.

Meng, X. and R. Gregory (2005). Intermarriage and the economic assimilation of immigrants. *Journal of Labor Economics 23*(1), 135–174.

MIT Technology Review (2017, October). First evidence that online dating is changing the nature of society. Technical report, Massachusetts Institute of Technology.

Nakajima, M. and I. Telyukova (2016). Housing and saving in retirement across countries. In Stiglitz and Guzman (Eds.), *Contemporary Issues in Microeconomics*, Chapter 5, pp. 88–16. Palgrave Macmillian.

Ortega, J. (2018a, May). Integration in social networks. Papers 1705.08033, arXiv.org.

Ortega, J. (2018b). Social integration in two-sided matching markets. Papers 1705.08033, arXiv.org.

Parey, M. and F. Waldinger (2011). Studying abroad and the effect on international labour market mobility: Evidence from the introduction of erasmus. *Economic Journal 121*(551), 194–222.

Patacchini, E. and Y. Zenou (2016). Racial identity and education in social networks. *Social Networks 44*, 85–94.

Pearce-Morris, J. and V. King (2012). The well-being of children living with interethnic parents. *Journal of Family Issues 33*, 898–919.

Pew Reseach Center (2015). Modern immigration wave brings 59 million to u.s., driving population growth and change through 2065. Technical report, Washington, D.C.

Pissarides, C. A. (2009). The unemployment volatility puzzle: Is wage stickiness the answer? *Econometrica 77*(5), 1339–69.

Potarca, G. (2017). Does the internet affect assortative mating? evidence from the u.s. and germany. *Social Science Research 61*, 278–297.

Potarca, G. and M. Mills (2015). Racial preferences in online dating across european countries. *European Sociological Review 31*(3), 326–341.

Qian, Z. (1997). Breaking the racial barriers: Variations in interracial marriage between 1980 and 1990. *Demography 34*(2), 263–276.

Rees, A. (1966). Information networks in labor markets. *American Economic Review 56*(1/2), 559–566.

Rogerson, R., R. Shimer, and R. Wright (2005). Search-Theoretic Models of the Labor Market-A Survey. *Journal of Economic Literature*.

Rosenfeld, M. (2017). Marriage, choice, and couplehood in the age of the internet. *Sociological Science 4*(20), 490–510.

Rosenfeld, M. and R. Thomas (2012). Searching for a mate: The rise of the internet as a social intermediary. *American Sociological Review 77*(4), 523–547.

Roth, A. and M. Sotomayor (1992). *Two-sided matching: A study in game-theoretic modeling and analysis*. Cambridge University Press.

Rudder, C. (2009, October). How your race affects the messages you get. OK Cupid Blog.

Rudder, C. (2014). *Dataclysm*. Crown Publishing Group.

Schaal, E. (2017). Uncertainty and Unemployment. *Econometrica 85*, 1675–1721.

Schelling, T. (1969). Models of segregation. *American Economic Review 59*(2), 488–493.

Schelling, T. (1971). Dynamic models of segregation. *Journal of Mathematical Sociology 1*(2), 143–186.

Shapley, L. and H. Scarf (1974). On cores and indivisibility. *Journal of Mathematical Economics 1*(1), 23–37.

Shim, K. H. (2015). The Substitutability of Labor: Implications for the Cross Section of Stock Returns. Tech. rep.

Shimer, R. (2005, March). The Cyclical Behavior of Equilibrium Unemployment and Vacancies. *American Economic Review 95*(1), 25–49.

Shrum, W., N. Cheek, and S. Hunter (1988). Friendship in school: Gender and racial homophily. *Sociology of Education*, 227–239.

Smets, F. and R. Wouters (2007, June). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review 97*(3), 586–606.

Stevenson, B. and J. Wolfers (2007, June). Marriage and divorce: Changes and their driving forces. *Journal of Economic Perspectives 21*(2), 27–52.

Thomas, R. (2018). Online exogamy reconsidered: Estimating the internet's effects on racial, educational, religious, political and age assortative mating. *unpublished*.

Wang, W. (2012). The rise of intermarriage. *Pew Research Center Report*.

Watts, D. and S. Strogatz (1998). Collective dynamics of “small-world” networks. *Nature 393*(6684), 440.

Wong, A. (2018). Transmission of monetary policy to consumption and population aging. Technical report, Princeton.

Wong, L. (2003). Why do only 5.5% of black men marry white women? *International Economic Review 44*(3), 803–826.

Woodford, M. (2004). Inflation targeting and optimal monetary policy. *Review* (Jul), 15–42.

Zhang, Y. and J. Van Hook (2009). Marital dissolution among interracial couples. *Journal of Marriage and Family 71*(1), 95–107.

Abstract

This dissertations consists of three independent papers which are loosely related via aspects of distribution.

The first essay deals with questions of redistribution in a New-Keynesian Model with heterogeneous agents. Heterogeneity is brought in via an OLG structure, and it is one of the first essays to do so. The main results are as follows. First, heterogeneity makes central banking a more difficult task than in standard New Keynesian models. This is because the "divine coincidence", which states that a monetary policy that is stabilizing inflation is also stabilizing output, breaks down with heterogeneity. The reason behind this is that changing the interest rates creates redistribution among the agents (especially between borrowers and lenders). These redistributive effects are long lasting, which is why they are more prevalent in unfiltered time series of the macroeconomic aggregates, as opposed to the ones where lower frequencies are removed.

In the second paper of this dissertation, we build a model of a frictional labor market where firms can hire multiple workers, and use this setup to analyze the effects of two empirical trends; an increased degree of substitutability among input factors and a relative decrease in the price of capital. We find that both effects lead to an increase in the capital-to-labor ratio, a rise in firm profitability and higher volatility of profits. However, the labor share of income, which is an important measure for distribution in an economy, moves in opposite directions. A lower price of capital leads to a counterfactual increase of the labor share, whereas increased factor substitutability decreases this share.

In the final chapter of the dissertation the focus switches from macroeconomic markets and models to online dating. It lays out a model of a society consisting of various heterogeneous groups which take part in a matching procedure to determine who marries whom. Using a random graph setup, it analyzes the introduction of online dating into this society, modeled via an increase in the probability of establishing new links. Welfare indicators are defined and used to judge the pre and post-online dating societies. We find that the newly established connections lead to more diverse and stronger marriages. These results are contrasted to US data, and we find the key results confirmed.

Zusammenfassung

Diese Dissertation besteht aus drei eigenständigen Beiträgen, die durch das übergreifende Thema der ökonomischen Verteilung verbunden sind.

Die erste Arbeit beschäftigt sich mit Fragen zu den Umverteilungswirkungen von Geldpolitik in einem Neu-Keynesianischen Modell mit heterogenen Agenten. Heterogenität ist eine notwendige Voraussetzung, um über Verteilung und Umverteilung sprechen zu können, und wird in diesem Modell durch eine Kohortenstruktur hergestellt, ein Novum in dieser Literatur. Die Hauptresultate sind, dass Heterogenität erfolgreiche Geldpolitik erschwert, da die Bekämpfung von Inflation nicht automatisch auch den Gesamtoutput stabilisiert. Diese Verbindung, die in Standardmodellen vorherrscht, bricht durch Umverteilungseffekte zusammen, die sich aus den Änderungen der Zinssätze ergeben. Diese Effekte sind sehr persistent über die Zeit, wodurch vor allem ungefilterte Zeitreihen eine starke Abweichung von den Resultaten der Standard Modelle zeigen.

Das zweite Paper dieser Dissertation präsentiert ein Modell eines friktionalen Arbeitsmarktes, auf dem Firmen mehrere Arbeitskräfte anstellen können. Wir benutzen dieses Setup um zwei langfristige Trends darzustellen und gemeinsam zu analysieren. Einerseits hat sich die technische Möglichkeit Arbeit und Kapital zu substitutieren erhöht, andererseits ist der Preis für neue Investitionsgüter gesunken. Beide Effekte führen in unserem Modell zu einem Anstieg des Einsatzverhältnisses von Kapital zu Arbeit, zu höherer Profitabilität von Firmen und zu größerer Volatilität dieser Profite. Betrachtet man jedoch die Entlohnung des Faktors Arbeit, eine zentrale Kennzahl der Verteilung in einer Ökonomie, so führen die beiden Trends zu unterschiedlichen Effekten. Ein geringerer Preis des Kapitals führt zu einem kontrafaktischen Anstieg der Lohnquote, wohingegen die erhöhte Substituierbarkeit diese senkt.

Im finalen Papier der Dissertation wandert der Fokus weg von makroökonomischen Märkten und Modellen hin zu Online Dating. In einem Modell einer Gesellschaft, die aus mehreren Gruppen besteht, verwenden wir einen Matching Algorithmus um die Verteilung der Partnerschaften zu bestimmen. In einem Zufallsgraph-Model etablieren wir Online Dating über eine erhöhte Kontaktwahrscheinlichkeit mit Mitgliedern anderer Gruppen. Wir definieren Kennzahlen zur Wohlfahrt in dieser Ökonomie und benutzen diese um den Effekt von Online Dating zu quantifizieren. Es zeigt sich, dass Online Dating zu stabileren und durchmischteren Partnerschaften führt. Wir vergleichen diese Resultate mit US Daten und finden die Modellvorhersagen bestätigt.

Curriculum Vitae

Personal information

- Born: 27 May 1990, Eisenstadt, Austria
- Nationality: Austrian

Education

- BSc Business, Economics and Social Sciences; Major in Economics, WU Wien (2012)
- MSc Economics, TU Wien and IHS Wien (2014)
- PhD in Economics, University of Vienna (currently)

Publications

- Bericht zur Wiener Wirtschaft, Kooperationen von Wiener Unternehmen bei der Patentanmeldungen beim Europäischen Patentamt (with Huber P., et al.), 2011, *WIFO Paper*
- CENTROPE Regional Development Report, Focus Report on Technology Policy, Research, Development and Innovation in CENTROPE (with Csismadia Z. and Huber P.). 2011, *WIFO Paper*

Teaching Experience

- TA for Makroökonomie für Studierende der Volkswirtschaftslehre University of Vienna (2017)
- Lecturer for Grundzüge der Volkswirtschaftslehre University of Vienna, (2015 and 2016)
- TA for Mathematics I, IHS Wien (2013)