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# Banking Regulation in the Wake of Basel III: A Theoretical Framework

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**Abstract**

This thesis analyzes the interdependence of capital regulation and liquidity regulation in the context of a three-period banking model. By endogenizing the optimal behaviour of a profit-maximizing bank for liquid assets it is shown that for low levels of equity, liquid assets are decreasing in leverage whereas for high levels of equity, they are increasing in leverage. Due to financial externalities, a regulator who internalizes the costs from illiquidity chooses a higher degree of liquidity holdings for every level of bank capitalization. In an environment with both liquidity requirements as well as capital requirements, the two are complements for banks with low amounts of capital, while they are independent of each other for highly equity financed banks.

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# 1 Introduction

The 2007/08 financial crisis marks a turning point in international standards for banking regulation. Although already prior to the outbreak and the subsequent spreading of the crisis capital requirements had been implemented in many countries, the early phase of the recent financial crisis showed that capital regulation alone could not bear the burden of financial stress in the context of a global financial market. In fact, as seen from an ex-post view, it is evident that the regulatory regime that was set up and operating at that time was insufficient and could neither prevent negative spill-over effects from reaching the real economy nor re-stabilize the financial sector itself.

The socio-economic implications of this crisis led policy makers and supervisory authorities around the globe to discuss possible adjustments of the system of banking regulation in order to prevent future crises from happening. One of the leading institutions in this field, the Basel Committee on Banking Supervision (BCBS), a board composed of central bankers and bank supervisors of 45 countries and based in the Bank for International Settlements, began to revise its reports on banking regulation, known as the Basel Accords. The Basel II Accord, at the time of the outbreak of the crisis the current regulatory framework, proved to be insufficiently designed to prevent the global economy from a stress scenario as experienced after the beginning of the crisis in 2007. The revised and redesigned framework that was published under Basel III in 2010 seeks to prepare the financial sector better for situations of such severe distress. Among qualitative as well as quantitative changes in capital requirements, the BCBS turned away from a capital-centred view and introduced new instruments that aim to strengthen the resilience against liquidity stress of financial institutions. These are the *Net Stable Funding Ratio* (NSFR) and the *Liquidity Coverage Ratio* (LCR). While the former promotes the resilience of the institution's funding sources over the time span of one year, the latter aims to improve the quality of the stock of liquid assets in order to withstand significant outflows over a 30-day stress scenario.

This thesis analyzes the interconnectedness between capital requirements and liquidity requirements in the context of a 3-period banking model with a representative bank. Considering a simplified balance sheet, the bank's optimal choice for both liquid assets and equity are endogenized. The thesis raises a number of questions of interest: Can liquidity requirements help to strengthen financial stability in the presence of liquidation costs? What are the effects of an increase in the strictness of capital regulation on the optimal design of liquidity regulation in a situation in which both liquidity and capital requirements are implemented? It is the aim of this thesis to analyze the implications of such a regulatory regime with regards to financial stability.

The thesis shows that in a context where authorities implement both liquidity requirements and capital requirements, the design of such a regulatory regime crucially depends on the degree of banks' equity funding. While it is optimal from a regulator's perspective to treat both requirements as complements for poorly capitalized banks, they are set independently of each other if banks are highly capital financed. The different effect of a stricter capital requirement on the liquidity requirement is driven by the fact that banks with sufficiently high levels of equity financing do not face default. It is shown that there exists a threshold for equity that leads to the two cases mentioned above. Therefore, the design of such a regulatory framework, as this thesis concludes, depends on the banks' equity ratio.

The remainder of this paper proceeds as follows. Section 2 gives an overview of the recent literature on liquidity regulation. Section 3 discusses the rationale behind regulatory intervention. Section 4 analyzes the changes in the international framework for banking regulation focusing on Basel II and Basel III. Section 5 presents the setting and the timing of the model. In Section 6, the liquidity effect and the solvency effect are defined. Section 7 gives the bank's optimal choice for liquid assets. In Section 8 the optimal amount of liquidity holdings that is chosen by the regulator is presented. Furthermore, the relation between capital regulation and liquidity regulation for different degrees of bank leverage is discussed. Finally, Section 9 concludes. All proofs are given in the appendix.

## 2 Related Literature

This paper contributes to a growing field of literature studying the potential ambiguous effects of liquidity regulation. The model is based on the work of Eisenbach et al. (2014) in which the authors establish a framework that allows analyzing effects of changes in different balance sheet parameters on bank stability. In their paper, a threshold for the minimum return on long-term assets is defined which gives helpful insights to understanding the interaction of different balance sheet parameters. Its relevance for this paper also lies in the analysis of the bank's asset structure. The authors find, very similarly as in this work, that increasing the liquidity holdings of the representative bank increases bank stability only under certain conditions. That is, a balance sheet with more liquid assets unambiguously reduces the risk of conditional insolvency (or liquidity risk), whereas they show that it can either reduce or increase the probability of fundamental insolvency.[1] Similar to this paper, they find that the ambiguity critically depends on the leverage ratio of the bank. For poorly capitalized banks more liquid assets will increase the probability of fundamental insolvency, while it will decrease the probability if the bank is highly equity financed. These findings are very similar to the ones presented in Section 6. Contrary to this paper, however, the Eisenbach et al. (2014) model does not present an analysis of the bank's profit maximizing behaviour but sticks to exogenous changes in balance sheet characteristics. By endogenizing the choice for both liquid assets and equity, this model helps to understand better potential effects of changes in the regulatory framework on the bank's profit-maximizing behaviour.

Another important work is König (2015). Here, the author extends the model by Rochet and Vives (2004), and points out the existence of the liquidity effect and the solvency effect. Defined in a very similar manner as in Section 6, these two effects are shown to be of opposing directions. Only if the liquidity effect dominates the solvency effect, larger liquidity holdings will reduce the default risk of a bank. As it is shown in Eisenbach et al. (2014) as well as in this model, König (2015) shows

[1] The definitions of conditional insolvency and fundamental insolvency are the same as in this paper and are given in Section 6.

that which effect dominates the other depends on the equity financing of the bank. This finding stands in contrast to the Rochet and Vives (2004) model in which an increase in the liquidity ratio unambiguously decreases the default probability of the bank.[2]

Contrary to the above mentioned literature, Macedo (2017) does not only discuss the interdependence of liquidity and capital regulation but also endogenizes the bank's profit-maximizing decision for liquidity holdings. Similarly as in Section 7, the author shows that for each level of capital there exists a unique value for liquidity. Endogenizing the bank's decision for liquid assets makes it possible to study the interdependence of capital regulation and liquidity regulation. This discussion is also made by Carletti et al. (2018) in a global game framework. Similarly to this paper, the authors show that an optimally designed regulatory framework should consider the balance sheet characteristics of a bank and argue that a liquidity requirement as the *Liquidity Coverage Ratio* satisfies this property. Furthermore, as it is done in this thesis, the authors show that banks choose inefficient amounts of both capital and liquidity that justify regulatory intervention. However, while Carletti et al. (2018) also adopt a reduced-form approach to model banks' liquidity choice, this thesis explicitly addresses the "pecking order" between liquid and illiquid assets.

# 3 The Rationale behind Regulation

Banking regulation is nowadays considered to be an integral part of the financial sector. While the topic of capital regulation has been discussed both academically and politically for now over four decades to a great extent, the discussion about liquidity regulation is a rather new one. However, asking for a justification for such regulatory interventions is not a trivial question and neither is its answer.

In principle, interventions follow the identification of some kind of market failure that is sought to be eliminated as its implications are considered economically

[2] In their paper, Rochet and Vives (2004) only consider the liquidity effect to be existent, while they, as König (2015) argues, neglect the solvency effect. Therefore, their model concludes that higher levels of liquidity unambiguously decrease the probability of bank default.

harmful. In the case of regulating the activities of banks, it is important to highlight the reason why they, when left unregulated, do not design their actions in a way that leads to an efficient outcome. One explanation for this is closely related to the presence of deposit insurance and the therewith accompanied monitoring frictions. As Cooper and Ross (2002) point out, deposit insurance, in theory, is able to widely eliminate the occurrence of bank runs. At the same time, however, it weakens depositors' incentive to monitor their bank's activities. This in turn may lead banks to hold riskier assets and destabilize financial markets. Resulting costs of potential default are not taken into consideration by banks but affect other agents in the economy. Cooper and Ross (2002) argue that an efficient level of risk taking can be established if both deposit insurance and capital requirements are implemented. Since banks will, so the reasoning, reduce their level of risk taking as higher capital levels transfer part of the risk back to its shareholders, there is a beneficial accompanying effect: reduced default risk.[3]

The most recent crisis, however, showed that even the combination of capital regulation and deposit insurance has not been sufficient to make the financial sector withstand such a severe shock. As proposed by the Basel Committee on Banking Supervision (2010) and discussed in this thesis, liquidity regulation may play a complementary role in strengthening financial stability. As the following section illustrates, the last financial crisis can be seen as a turning point in the design of the global regulatory framework.

# 4 Banking Regulation after 2007/08

After the 2007/08 financial crisis regulatory authorities in many countries came to the conclusion that the regulatory framework for banking supervision had not been sufficiently designed in order to prevent the economic downturn from spreading globally. Although the regulation of bank capital has been one of the main focuses already prior to 2007/08, the issue of excessive leverage is nowadays considered to have been

[3]See Allen et al. (2015) for a broader discussion of deposit insurance and the risk-shifting behaviour of banks.

a driving factor that promoted bank insolvency even more. In 2004, three years before the outbreak of the crisis, the Basel Committee on Banking Supervision published its report *"International Convergence of Capital Measurement and Capital Standards"*, often referred to as Basel II, in which it presented a detailed framework for capital regulation that should help overcome national policies even more and harmonize them into one single framework. In 2010, as a response to the recent crisis, the BCBS published its newest report, fostering the role of capital requirements in international banking supervision and adjusting the instruments that were already in use since Basel II. In its report, published under the title *"Basel III: A global regulatory framework for more resilient banks and banking systems"*, the Committee gave up its capital centered view and recommended a stronger focus on liquidity. While many countries already had implemented some weaker form of liquidity monitoring, the BCBS argued, again, in favor of a harmonization for liquidity regulation. In its report from 2010 it states: *"A survey of Basel Committee members conducted in early 2009 identified that more than 25 different measures and concepts are used globally by supervisors".*[4] These, as a reaction to the liquidity dry-up in the early stage of the crisis, either prior or as a response to the crisis, had been implemented in most cases only on a national basis and did not show strong international coordination. In order to promote the harmonization process, the BCBS introduced two newly formulated regulatory ratios for liquidity, the *Liquidity Coverage Ratio* (LCR) and the *Net Stable Funding Ratio* (NFSR). Addressing the sudden liquidity stress experienced by many banks in that period, the LCR aims to strengthen the short-term resilience by making the bank withstand a 30 calendar day stress scenario of funding outflows. On the other hand, the NFSR promotes resilience over the time span of one year and aims on the stability of the bank's funding sources. It is these two ratios for liquidity, as introduced in Basel III, that constitute the transition in international banking regulation from the capital centred to a more complementary view of both capital and liquidity requirements.

[4] Basel Committee on Banking Supervision (2010)

## 4.1 Basel III

After it became clear to supervisors in many countries that the international system of banking regulation, set by the Basel II standards, did not fulfill its task satisfactorily, the Committee revised and broadened the regulatory framework and finally published the Basel III Accords in 2010. These, incorporating many experiences made during the crisis up to this point in time, were to adjust and redesign the international banking regulation framework by increasing both the quality and quantity of the capital base as well as promoting the role of bank liquidity for the first time this strongly.[5] Combining both capital and liquidity requirements in one single system of regulation the BCBS intended to tackle many problems that arose during the most severe years of the last crisis. Among these problems, the Committee found:

> One of the main reasons the economic and financial crisis, which began in 2007, became so severe was that the banking sectors of many countries had built up excessive on- and off-balance sheet leverage. This was accompanied by a gradual erosion of the level and quality of the capital base. At the same time, many banks were holding insufficient liquidity buffers. The banking system therefore was not able to absorb the resulting systemic trading and credit losses [...].[6]

The growing awareness that the capital centred design of the international system for banking regulation was not sufficient, ultimately led the BCBS to design instruments that could help to prevent another crisis of a similar form. Complementing the already specified, and in the Basel III Accords again revised, capital regulation, the *Net Stable Funding Ratio* and the *Liquidity Coverage Ratio* have been developed to make banks more resilient against liquidity dry-ups.

[5] Basel Committee on Banking Supervision (2010)
[6] Basel Committee on Banking Supervision (2010)

## 4.2 The *Liquidity Coverage Ratio - LCR*

The LCR aims to ensure that banks build a sufficiently high stock of liquid assets so that it, when needed, will help to prevent banks suffering from illiquidity. This regulatory ratio forces banks to hold as much high-quality liquid assets (HQLA) as to withstand outflows during a 30-day stress scenario. Formally, the LCR is defined as:

$$LCR \equiv \frac{\textit{Stock of unencumbered HQLA}}{\textit{Total net cash outflows over the next 30 calendar days}} \geq 100\%$$

In order to be included in the stock of unencumbered HQLA, and, therefore, be counted as liquid, assets must satisfy certain characteristics. The Committee defines a liquid asset as one that "*can be easily and immediately converted into cash at little or no loss of value.*"[7] Furthermore, an asset that is to be considered as HQLA has to be of low risk, in the sense of low duration, low legal risk as well as low inflation risk, easily valuated, i.e. the pricing formula for this asset has to be transparent and publicly accessible and should show low correlation with risky assets. An asset is allowed to be included in the stock of HQLA if it can be traded in an active and sizable market. These are defined as markets that show "*historical evidence of market breadth and market depth.*"[8] Also, these markets should show low bid-ask spreads, high trading volumes and a large and diverse number of market participants. Finally, assets should show relatively stable prices over time with historical evidence that there exist markets for these assets that are of low volatility. HQLA can either be part of Level 1 assets, Level 2A assets or Level 2B assets. Level 1 assets should consist mainly of coins and banknotes as well as central bank reserves that can be liquidated immediately if needed. Level 2A assets should consist, among others, mainly of securities guaranteed by sovereigns or central banks that were assigned a 20% risk weight under Basel II and "*have a proven record as a reliable source of liquidity in the markets (repo or sale) even during stressed market conditions [...].*"[9]

[7] Basel Committee on Banking Supervision (2013)
[8] Basel Committee on Banking Supervision (2013)
[9] Basel Committee on Banking Supervision (2013)

Level 2B assets may consist of residential mortgage backed securities that satisfy certain conditions. These include that these securities are traded in large and active repo or sale markets that show a low degree of market concentration. While to Level 1 assets no haircut is applied due to their higher level of liquidity as defined above, to Level 2A assets a haircut of 15% is applied and to Level 2B assets a maximum haircut of 50% may be applied. Furthermore, the stock of HQLA may consist to an unlimited degree of Level 1 assets, while the sum of Level 2A and 2B assets is limited to 40% of the total stock of liquid assets. This cap aims to ensure that the stock of HQLA underlies as little risk as possible and that it shows a high level of liquidity.[10]

In absence of a stress scenario, the LCR must not fall under 100%. When needed, however, supervisory authorities may allow the individual bank to lower its ratio below 100% and, therefore, use its liquidity buffers to withstand moments of severe stress. The circumstances under which authorities may grant a decline in the LCR are to be defined by the supervisors in charge.

# 5 The Model

This thesis proposes a model to study the interaction between capital and liquidity regulation. To analyze this interdependence, consider a three-period model ($t = 0, 1, 2$) with a representative bank that is subject to limited liability and where retail deposits (short-term debt) are insured by a deposit insurance fund. The Bank has a simplified balance sheet as illustrated in Table 1.

| Assets | Liabilities |
|---|---|
| Liquid assets ($m$) | Short-term debt ($s$) |
| Long-term assets ($y$) | Equity ($e$) |

**Table 1: The bank's simplified balance sheet**

It can either hold long-term illiquid assets, $y$, or short-term liquid assets, $m$. Illiquid long-term assets in this model can be interpreted as, for example, corporate

[10] Basel Committee on Banking Supervision (2013)

loans that cannot be traded without loss of value in a stress scenario.[11] It finances these investments by holding two types of liabilities: short-term debt (insured retail deposits), $s$, and equity, $e$. For simplicity, both the asset side and the liability side are normalized to unity, such that $y = 1 - m$ and $s = 1 - e$.

While investing in long-term assets yields a positive return in $t = 2$, denoted by $\theta > 1$, the return from holding liquid assets is assumed to follow only a storage technology in each period, i.e. $r_m = 1$. Ceteris paribus, this makes the bank hold as few liquid assets as possible, so that the total returns on its assets are maximized.

**Assumption 1** *The return on liquid assets is normalized to $r_m = 1$. The return on short-term debt is such that it equals the return on liquid assets, i.e. $r_s = 1$.*

Assumption 1 states that the return on short-term debt is given by $r_s = r_m$. This is justified by the assumption that deposits in this model are insured by a deposit insurance and that depositors, therefore, do not demand any kind of risk premium. Intuitively, this drives down the return to the rate of return of the depositors' alternative, which is not holding deposits, i.e. holding cash.[12]

**Assumption 2** *The return on bank capital exceeds the return on short-term debt as capital is subject to risk while deposits are insured. Therefore, $\rho > 1$.*

The return from providing capital to the bank, $\rho$, is such that it exceeds the deposit rate, i.e. $\rho > 1$. This is motivated by the assumption that investors who provide capital are subject to risk and demand compensation for it. It is worth noting that this risk is not coming from the asset side of the balance sheet as long-term assets per se are not risky, but from the underlying risk of the bank's cash flows and its solvency state as discussed below.

The timing in this model is as follows: In $t = 0$, the bank chooses its balance sheet structure. That is, it decides how much liquid assets and equity to hold. Given the fixed length of the balance sheet, thereby, it determines also the amount invested

[11] Although these assets are not subject to risk themselves, it is important for the discussion to assume that they are not safe to trade without losses under any circumstances.

[12] Depositors in this model are assumed to prefer holding deposits rather than long-term assets as deposits are insured while long-term assets are subject to "liquidity risk".

in long-term assets as well as the amount of short-term debt. In $t = 1$, a certain fraction of depositors, denoted by $\alpha$, decide not to roll over their short-term debt claims and withdraw. To these, $r_s$ is paid out only in $t = 1$. To depositors who decide not to withdraw in $t = 1$ and decide to roll over their claims, the return on short-term debt, $r_s$, is paid out by the bank in both $t = 1$ and $t = 2$. The presence of withdrawals can be intuitively justified by some kind of liquidity shock experienced by depositors.[13] In this model, the fraction of withdrawing depositors is assumed to be a random variable, uniformly distributed over the unit interval. Formally, one can write $\alpha \sim \mathcal{U}[0, 1]$.

Finally, in $t = 2$ the return on long-term assets is realized and the rate on the remaining part of short-term debt is paid out to depositors. Note that long-term assets yield a positive net return, $\theta > 1$, in $t = 2$ but nothing in $t = 1$. Liquidating assets early, therefore, implies lowering the total returns of the bank, as realized in $t = 2$. The illiquidity discount that determines the loss of value when assets are liquidated and traded on a secondary market is denoted by $\tau \in [0, 1]$.[14] Long-term assets that are liquidated and sold on a secondary market in $t = 1$ only lead to a return of $\tau\theta$.

**Assumption 3** *The cost of illiquidity exceeds the return on long-term assets, i.e.* $1/\tau > \theta$.[15]

Since in $t = 1$ the bank has already chosen its amount of liquid assets, the amount of withdrawn debt may or may not exceed the bank's holdings of liquid assets. The

[13] Diamond and Dybvig (1983) model these shocks by the assumption that there is a positive probability for a consumer to be *impatient* and to want to consume early. Impatient depositors want to withdraw their claims and, under certain conditions, cause a bank run. Goldstein and Pauzner (2005) extend this model by deriving the probability of such bank runs.

[14] The fact that $\tau < 1$ can be justified by the existence of capital-constrained outside buyers. Stein (2012) argues that marginal returns on assets liquidated by the bank that are bought by outside investors have to be as high as the returns on their investment alternative. As their capital endowment is scarce and the outside option's return structure follows a concave technology, the costs of fire-sales are increasing in the amount liquidated. Although in Stein (2012) these costs are endogenized, the mechanism that justifies $\tau < 1$ is similar. For a detailed analysis of the liquidation value in a general equilibrium context see Shleifer and Vishny (1992).

[15] Assuming otherwise would imply that $\tau\theta \geq 1$, meaning that liquidation is not costly.

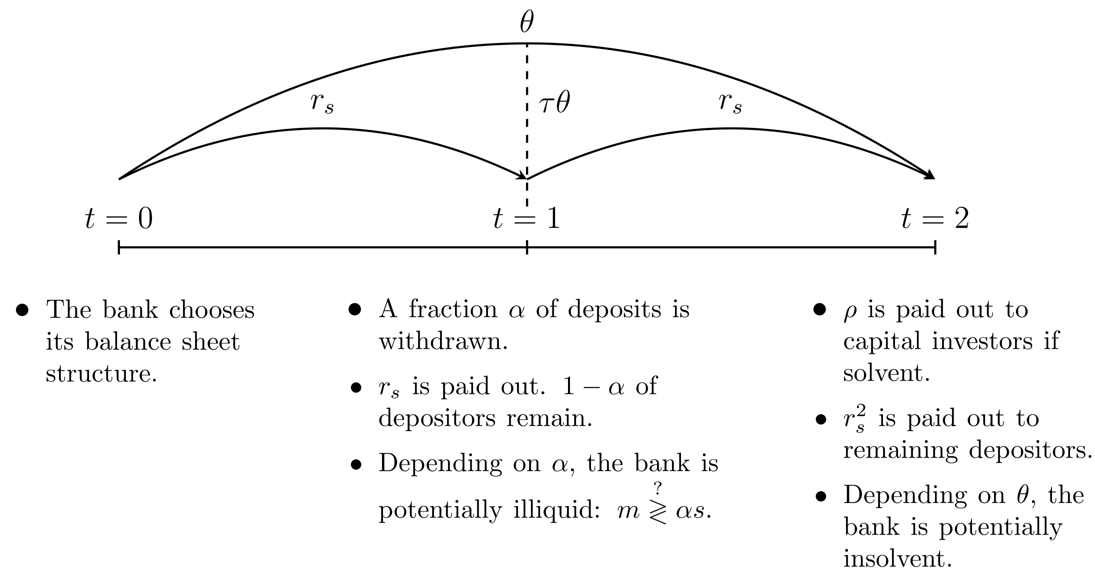


**Figure 1: Timing of the model.**

withdrawn face value of short-term debt is given by $\alpha s$. Due to the assumptions on the return structure, the bank meets depositors' claims by using its liquid assets as long as it is able to. Hence, there exists a clear pecking order of the liquidity source chosen by the bank to pay out depositors. In $t = 1$ it is crucial whether the bank holds more liquid assets than what is withdrawn: $m \gtrless \alpha s$. Hence, two cases arise: In the first one, the bank has chosen a balance sheet structure in $t = 0$ such that it holds enough liquid assets in $t = 1$ and does not need to liquidate long-term assets early. In the second case the bank does not hold enough liquid assets in $t = 1$ to meet withdrawals in full. Since the amount of liquid assets is insufficient, it has to liquidate long-term assets early and sell them at a discount. The bank sells long-term assets on a secondary market equal to the remaining amount of withdrawn claims after having paid out depositors with liquid assets as long as it has been able to. That is, given that $m < \alpha s$, it liquidates long-term assets in order to obtain $\alpha s - m$ units of liquidity.

The equity value of the bank in $t = 2$ depends on the realization of the random variable $\alpha$, or, put differently, on the mass of depositors that decide to withdraw early. The equity value function can be formally written as:

$$\nu(m,e) = \begin{cases} \max\{\theta y - s + m, 0\}, & if \quad m > \alpha s, \\ \max\left\{\theta \max\{y - \frac{\alpha s - m}{\tau\theta}, 0\} - (1-\alpha)s, 0\right\}, & if \quad m < \alpha s. \end{cases} \tag{1}$$

In the first case, in which the bank has enough liquid assets such that it does not have to liquidate assets, the equity value is simply given by the return on long-term assets minus the face value of its short-term debt plus the amount of liquid assets. In the second case, the equity value of the bank is equal to the return on long-term assets minus the costs associated with the liquidation of long-term assets in $t = 1$ minus the remaining face value of short-term debt that has to be met in $t = 2$. Clearly, this function depends on the balance sheet structure of the bank chosen in $t = 0$ as well as on the realization of $\alpha$. It is worth noting, however, that the realization of $\alpha$ is observable for the bank only in $t = 1$, when it has already made its profit maximizing decisions about liquid assets. The *max*-operators in (1) represent the limited liability assumption since they rule out negative values for the bank's equity value.

The other agent in this model is a regulator that is able to intervene indirectly in the bank's choice of balance sheet structure by introducing a $LCR$-like minimal liquidity ratio that is defined as:

$$\gamma \equiv \frac{m}{r_s s} = \frac{m}{1-e}. \tag{2}$$

That is, the regulator forces the bank to meet a certain ratio of its liquid assets to the face value of short-term debt. Note that the bank can meet this regulatory ratio either by increasing its holdings of liquid assets, $m$, *or* by increasing its equity, $e$.[16] Hence, (2) shows the substitutability with regards to regulation between equity and long-term assets from an accounting perspective.

[16] It is worth noting that the regulator in this model minimizes the bank's risk of illiquidity and not social welfare in a broader sense. Carletti et al. (2018) follow a different approach by arguing that equilibrium total output is not maximized if the bank stays unregulated.

# 6 Liquidity and Solvency Effects

This section focuses on the two effects on the solvency state of the bank that arise when it increases its holdings of liquid assets. These are the *liquidity effect* and the *solvency effect*.[17] The former is described by the fact that increasing the bank's holdings of liquid assets makes the bank withstand a greater mass of withdrawing depositors, or, put differently, a higher value of $\alpha$ before it has to liquidate long-term assets. The latter, the *solvency effect*, arises since increasing liquid assets $m$ not only makes the bank withstand a higher fraction of withdrawn face value of short-term debt but also decreases overall profits due to a reduction in profitable long-term illiquid assets. Since the bank's only source of profits is holding illiquid assets, $y = 1 - m$, the bank ends up with lower total profits. Depending on whether the bank holds a sufficient amount of liquid assets in order to prevent liquidation of long-term assets, the overall effect on the solvency state of the bank when increasing $m$ is ambiguous.

Given that the bank holds enough liquid assets in order to meet early withdrawals, it still might default due to insolvency in $t = 2$ for low values of $\theta$. That is, there exists a unique threshold, denoted in what follows by $\tilde{\theta}$, below which the bank defaults although it is able to pay out withdrawing depositors in $t = 1$. Formally, the bank becomes insolvent in $t = 2$ if

$$\theta y - \alpha s + m < (1 - \alpha)s. \tag{3}$$

That is, even though $m > \alpha s$ holds, meaning that the bank holds a sufficient amount of liquid assets, the remaining face value of debt exceeds the equity value of the bank in $t = 2$. From (3) the threshold value for the return on long-term assets, $\tilde{\theta}$, can be derived. This is the minimum return the bank has to achieve in order to avoid default due to insolvency. From (3), it follows that the threshold can be written as

$$\tilde{\theta} = \frac{s - m}{1 - m}. \tag{4}$$

[17] König (2015)

Assuming that a regulatory authority is able to make the bank adjust its balance sheet structure in a way that complies with a regulatory minimum like the one defined in (2), the threshold value in (4) can be reformulated as

$$\tilde{\theta} = \frac{s - \gamma s}{1 - \gamma s} = \frac{(1-\gamma)s}{1 - \gamma s}. \tag{5}$$

The threshold value, when expressed as a function of the regulatory ratio, $\gamma$, is an increasing function of $\gamma$, i.e. $\frac{\partial \tilde{\theta}}{\partial \gamma} > 0$. In other words, if the regulator decides to make the regulatory ratio stricter, this results in an increase of the threshold value for the return on long-term assets that is needed for the bank to remain solvent. Intuitively, it is clear that, given the bank holds enough cash to withstand withdrawals in $t = 1$, increasing $m$ makes the bank default more often. That is due to the fact that, considering a fixed size of the bank's balance sheet, when being forced to increase its holdings of liquid assets, it necessarily has to decrease its holdings of long-term assets. Hence, if the bank holds already enough liquid assets in order to prevent liquidation, the overall effect is only driven by the *solvency effect*. The counter-acting *liquidity effect* in this case is non-existent because no increase in $m$ is needed to meet all withdrawing depositors.

If the bank does not hold enough liquid assets in order to meet all of the withdrawn deposits, it has to liquidate long-term assets and sell them on a secondary market below their fundamental value, incurring a per unit loss of $1/\tau$. In this case, the inequality $m < \alpha s$ holds and the bank becomes insolvent in $t = 2$ if

$$\theta\big(y - \frac{\alpha s - m}{\tau \theta}\big) < (1-\alpha)s. \tag{6}$$

That is, the remaining face value of short-term debt exceeds the return on long-term assets minus the incurred losses due to illiquidity. Note that the fraction in the brackets is indeed positive since the withdrawn face value of debt is larger than the bank's holdings in liquid assets. From (6) it follows that the threshold value for the return on long-term assets is given by

$$\hat{\theta} = \frac{s + (\frac{1}{\tau} - 1)\alpha s - \frac{1}{\tau}m}{y}. \tag{7}$$

In this case one additional unit of liquid assets decreases the threshold by $1/\tau > 1$ since this is the otherwise incurred loss due to the liquidation of long-term assets that is now prevented. In contrast to before, holding liquid assets given that liquidation occurs is more beneficial in terms of avoiding insolvency in $t = 2$ than when no liquidation occurs. Note that whereas $\tilde{\theta}$ does not depend on $\alpha$, this threshold, $\hat{\theta}$, does depend on the mass of withdrawing depositors, as a higher realization of $\alpha$ has to come with a higher return on long-term assets if insolvency is to be avoided. Therefore, the threshold $\hat{\theta}$ increases in the amount of withdrawing depositors. Again, it is interesting to analyze how the threshold value, as described in (7), reacts to changes in the regulatory ratio, $\gamma$. Using the balance sheet identity together with (2), the threshold value for the return on long-term assets, $\hat{\theta}$, in this case can be rewritten as:

$$\hat{\theta} = \frac{s + (\frac{1}{\tau} - 1)\alpha s - \frac{1}{\tau}\gamma s}{1 - \gamma s}. \tag{8}$$

The overall effect of an increase in the strictness of the regulatory ratio, i.e. an increase in $\gamma$, in this second case is ambiguous as such a change in regulation policy comes with two effects. The first effect, the *liquidity effect*, consists in a decrease of the threshold value $\hat{\theta}$. Since holding more liquid assets, $m$, in a situation where liquidation occurs lowers the incurred costs of illiquidity, this reduces the risk of default due to illiquidity in $t = 1$ and, therefore, also lowers the risk of default due to insolvency in $t = 2$. The second effect, the *solvency effect*, leads to an increase of the threshold value for the return on long-term assets, $\hat{\theta}$, as $m$ increases. If the bank is forced to increase its holdings in liquid assets, given the fixed length of its balance sheet, it necessarily has to lower its holdings in long-term assets with a positive net return.

The derivative of $\hat{\theta}$ with respect to $\gamma$ is negative for positive amounts of equity, i.e. $\frac{\partial \hat{\theta}}{\partial \gamma} < 0 \ \forall e > 0$. That is, the liquidity effect dominates the solvency effect if the bank holds a positive amount of bank capital.[18]

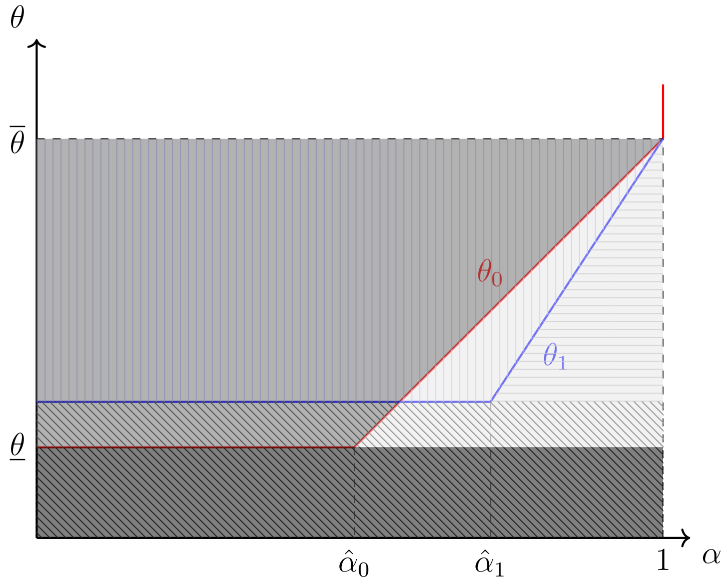


**Figure 2: Effect of a change in $\gamma$ on the threshold values in an $\alpha$-$\theta$ space, with solvency regions as in Eisenbach et al. (2014). Filled areas are for $\theta_0$ (red line) and lined areas are for $\theta_1$ (blue line).**

Figure 2 shows the effect of an increase in the strictness of the regulatory liquidity ratio, $\gamma$, on the threshold values $\tilde{\theta}$ and $\hat{\theta}$ in the $\alpha$-$\theta$ space given the bank holds a positive amount of equity. The red line, $\theta_0$, represents the threshold function before an increase in $\gamma$, while the blue line, $\theta_1$, represents the threshold function after the increase. As seen from (5), for realizations $\alpha < \hat{\alpha}$, the threshold value, represented by the function $\tilde{\theta}$ in the interval $[0,\hat{\alpha}]$, unambiguously increases in $\gamma$. Second, due to the increase in liquid assets, $m$, the threshold value $\hat{\alpha}$ also increases and, therefore, shifts to the right from $\hat{\alpha}_0$ to $\hat{\alpha}_1$.[19] Since $\hat{\alpha}$ represents the threshold value for $\alpha$ above which the bank starts to liquidate assets, this shift can be interpreted as the bank

[18]The derivative is given by: $\frac{\partial \hat{\theta}}{\partial \gamma} = \frac{s}{1-\gamma s}(\hat{\theta} - \frac{1}{\tau})$. If $e > 0$, then it follows that $\hat{\theta} - \frac{1}{\tau} < 0$ and the derivative is negative.

[19]The derivations of the thresholds for $\alpha$ will be discussed in Section 7.

withstanding a higher fraction of withdrawing depositors before it has to liquidate assets. Third, the stricter regulatory ratio increases the slope of the threshold function in the interval $[\hat{\alpha}_0, 1]$, i.e. the cross-derivative is strictly positive ($\frac{\partial^2 \hat{\theta}}{\partial \alpha \partial \gamma} > 0$). Note that the area above the threshold function represents the combinations of $\theta$ and $\alpha$ for which the bank is solvent. For pairs below the threshold, the bank is insolvent. For example, if a very small fraction of short-term debt is withdrawn in $t = 1$ but the bank has a very low return to its long-term assets, the bank would be still insolvent. The same is true in a situation in which the bank experiences a high liquidity stress scenario in which many depositors decide not to roll over their claims, i.e. a value of $\alpha$ close to 1. If the return on its long-term assets, $\theta$, is below the blue or red line, the bank will default due to insolvency. Furthermore, an additional subdivision of the solvency and insolvency areas into *conditional* solvency and *fundamental* solvency can be made. In the area above the threshold function $\theta_0$ but below $\overline{\theta}$ the bank is conditionally solvent, in the sense that for a given value of $\theta \in [\underline{\theta}, \overline{\theta}]$ the bank does not default for sufficiently low values of $\alpha$. For values $\theta > \overline{\theta}$, the bank is solvent independently of the realization of $\alpha$ because of the high return to long-term assets. Put differently, even if all of its short-term debt is withdrawn in $t = 1$, the bank still remains solvent. For any pair of $\alpha$ and $\theta$ in this area, the bank is said to be fundamentally solvent.

# 7 The Bank's Optimal Choice for Liquid Assets

This section discusses the relevant thresholds for $\alpha$ as well as the bank's profit-maximizing choice for liquid assets.

## 7.1 Threshold values for $\alpha$

For given values of $m$ and $e$, depending on the realization of $\alpha$ in $t = 1$, the bank faces three thresholds with different implications for its solvency state, denoted by

$\hat{\alpha}$, $\alpha_B$ and $\alpha_A$.[20]

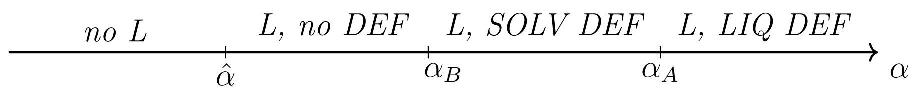


**Figure 3: The three thresholds and the corresponding implications for the solvency state of the bank.** $L \mathrel{\widehat{=}}$ **Liquidation of long-term assets,** $DEF \mathrel{\widehat{=}}$ **Default,** $SOLV\ DEF \mathrel{\widehat{=}}$ **Solvency Default in** $t = 2$**,** $LIQ\ DEF \mathrel{\widehat{=}}$ **Liquidity Default in** $t = 1$**.**

The first threshold, $\hat{\alpha}$, is the maximum value for $\alpha$ below which the bank holds always enough liquid assets in order to meet early withdrawals. This means, that for values of $\alpha$ below $\hat{\alpha}$ no liquidation of long-term assets occurs and, therefore, the bank does not incur any losses due to illiquidity. The second threshold, denoted by $\alpha_B$, gives the maximum value for early withdrawals below which the bank liquidates assets but does not default *due to insolvency* in $t = 2$. The third threshold, $\alpha_A$, denotes the highest realization of $\alpha$ below which the bank liquidates assets and does not default *due to illiquidity* in $t = 1$.

The solvency state of the bank critically depends on how many depositors decide not to roll over their claims and on the face value of withdrawn debt. Formally, the thresholds can be expressed as:[21]

$$\hat{\alpha}(m) = \min\left\{\frac{m}{1-e}, 1\right\} \tag{9}$$

$$\alpha_B(m) = \min\left\{\frac{\tau\theta(1-m)+m}{(1-e)(1-\tau)} - \frac{\tau}{1-\tau}, 1\right\} \tag{10}$$

$$\alpha_A(m) = \min\left\{\frac{\tau\theta(1-m)+m}{1-e}, 1\right\}. \tag{11}$$

As with the equity value, all three thresholds depend on the balance sheet structure of the bank. Note that here the thresholds, due to the balance sheet identity, are expressed as functions of $m$ and $e$ only. The threshold value $\hat{\alpha}$ below which the bank faces withdrawals that do not force it to liquidate long-term assets is derived from

[20] Note that all three thresholds are functions of $m$ and $e$. However, for reasons of legibility, the notation $\hat{\alpha}, \alpha_B, \alpha_A$ will be widely used instead of $\hat{\alpha}(e,m), \alpha_B(e,m), \alpha_A(e,m)$.

[21] All derivations can be found in the appendix.

the expression that describes the situation in which the bank holds just as much liquid assets as the face value of short-term debt that depositors decide to withdraw. Formally,

$$m = \alpha s. \tag{12}$$

Solving this equation for $\alpha$ yields the expression for $\hat{\alpha}$ as given in (9). A *min*-operator is used in order to comply with the fact that $\alpha$ is bounded between 0 and 1. Note that in (9) the face value of short-term debt is expressed as $1-e$, which follows from the balance sheet identity. This threshold is simply interpreted as the threshold value for realizations of $\alpha$ above which the bank has to liquidate long-term assets early due to its insufficient endowment of liquid assets to meet withdrawals. Since using liquid assets is the cheaper way of paying out depositors, the threshold value $\hat{\alpha}$ represents also the border above which the bank incurs losses due to the liquidation of long-term assets.

The threshold value $\alpha_B$ above which the bank defaults due to illiquidity in $t = 1$ or due to insolvency in $t = 2$ can be derived from setting the return on long-term assets equal to the incurred costs due to illiquidity in $t = 1$ plus the remaining face value of debt in $t = 2$. This expression is given by

$$\theta y = \frac{1}{\tau}(\alpha s - m) + (1 - \alpha)s. \tag{13}$$

On the left hand side of this equation is the return on long-term assets. The first term on the right hand side is the missing amount of liquid assets needed to meet early withdrawals in $t = 1$ times the cost factor for illiquidity. The second term is the remaining face value of short-term debt, i.e. debt from depositors that chose not to withdraw and roll over their claims to $t = 2$. Again, when solving this expression for $\alpha$ and using the balance sheet identity, one gets the expression as formulated in (10).

The third threshold, $\alpha_A$, above which the bank defaults due to illiquidity, is the solution to the expression in which the return on long-term assets equals the face value withdrawn in $t = 1$. That is, formally:

$$\theta y = \frac{1}{\tau}(\alpha s - m). \tag{14}$$

Note that in (14) only the costs from liquidating assets in $t = 1$ appear whereas, contrary to equation (13), the remaining face value of debt that has to be repaid in $t = 2$ does not. Since $\alpha_A$ is a threshold for liquidity default, it only takes into account period $t = 1$. Similarly to the two other thresholds, solving for $\alpha$ and using a *min*-operator yields the expression as in (11).

**Lemma 1** *The thresholds formulated in (9), (10) and (11) unambiguously increase both in $e$ and $m$.*

As Lemma 1 states, all of them increase in the amount of equity held by the bank. That is, ceteris paribus, highly capitalized banks always have higher thresholds than highly levered banks. This implication is quite intuitive with regards to the threshold that is associated with the default risk due to insolvency, $\alpha_B$. Nevertheless, also the threshold that gives the maximum value for $\alpha$ below which the bank does not default due to illiquidity, $\alpha_A$, is increasing with the amount of equity. Put differently, the bank can reduce the risk of illiquidity by decreasing its leverage. Since the cost of liquidating assets exceeds the return on long-term assets, so that $1/\tau > \theta$, all three thresholds are also increasing in the amount of liquid assets, $m$.

**Lemma 2** *Default due to illiquidity always implies default due to insolvency. Default due to insolvency does not necessarily imply default due to illiquidity:*
$\alpha_A(e, m) > \alpha_B(e, m)\ \forall\ e, m.$

In the proof of Lemma 2 it is shown that the inequality indeed holds, implying that for realizations of $\alpha > \alpha_B$ the bank always defaults (either due to illiquidity in $t = 1$ or due to insolvency in $t = 2$). Therefore, from the bank's perspective, $\alpha_B$ is the relevant threshold. Moreover, the threshold above which the bank starts to liquidate assets, $\hat{\alpha}$, can be shown to lie below the threshold above which the bank always defaults, $\alpha_B$, so that the inequality $\hat{\alpha} < \alpha_B$ holds for all values $m \in [0, 1)$ and $e \in [0, 1]$.

## 7.2 Equilibrium values for liquid assets $m$

This subsection focuses on the optimal choice of liquid assets, $m$, as the bank's behaviour and capacity to withstand runs is crucial for understanding the interdependence of equity and liquid assets. Therefore, this section analyzes the profit-maximizing decision for $m$ in $t = 0$, given that the bank treats withdrawals in $t = 1$ as a uniformly distributed random variable over the unit interval. As it was already stated above, the representative bank in this model is subject to limited liability, meaning that it will never incur negative profits, independently of the fraction of withdrawing depositors, its balance sheet structure or the return structure of its assets and liabilities. Hence, the profit function of the representative bank takes the following form:

$$\max_{m} \ \pi(m) = \int_{0}^{\hat{\alpha}} \pi_1 dF(\alpha) + \int_{\hat{\alpha}}^{\alpha_B} \pi_2 dF(\alpha) - \rho e, \tag{15}$$

where

$$\pi_1(e, m) = \theta(1 - m) + m - (1 - e)$$

and

$$\pi_2(e, m) = \theta(1 - m) - \frac{\alpha(1 - e) - m}{\tau\theta} - (1 - \alpha)(1 - e).$$

The first integral in (15) gives the bank's expected profits if it holds enough liquid assets so that it does not incur losses due to illiquidity. Therefore, the integral ranges from 0 to $\hat{\alpha}$, which is the highest possible realization of $\alpha$ such that the bank's liquid holdings exceed the withdrawn face value of short-term debt. This first profit function, $\pi_1$, is simply the return on long-term assets plus return on liquid assets (with $r_m = 1$) minus the face value of short-term debt. The second integral gives the expected profits if the bank needs to liquidate and sell long-term assets and ranges from $\hat{\alpha}$ to the maximum possible realization of $\alpha$ to avoid default, $\alpha_B$. The profits for a given value of $\alpha$, denoted by $\pi_2$, is the return on long-term assets minus the incurred losses from liquidating assets minus the remaining face value of short-term debt. Note that in the interval [$\hat{\alpha}$,$\alpha_B$] the bank is liquidating long-term assets, whereas in the interval [0,$\hat{\alpha}$] it is not. However, as the integral does not exceed the maximum value $\alpha_B$ since the bank is subject to limited liability, the crucial interval of realizations of $\alpha$ for the representative bank is $[0, \alpha_B]$. The bank's optimal decision for liquid assets depends on the costs associated with liquidation, $1/\tau$, the return on long-term assets, $\theta$, as well as the thresholds $\hat{\alpha}$ and $\alpha_B$. Note that these thresholds themselves depend on the balance sheet structure of the bank, i.e. on liquid assets and equity. Letting $G(m)$ represent the slope of the profit function, it is set to 0 in order to find the profit-maximizing value for $m$. The function $G(m)$ is the derivative of (15) with respect to $m$. It is given by:

$$G(m) = \Big(\frac{1}{\tau} - 1\Big)\Big(F(\alpha_B) - F(\hat{\alpha})\Big) - (\theta - 1)F(\alpha_B). \tag{16}$$

Setting $G(m) = 0$ and rearranging terms yields

$$(\theta - 1)F(\alpha_B) = \Big(\frac{1}{\tau} - 1\Big)\Big(F(\alpha_B) - F(\hat{\alpha})\Big). \tag{17}$$

Equation (17) shows that the bank sets the marginal costs equal to the marginal benefits from holding liquid assets. On the left hand side, the term $(\theta - 1)$ represents the foregone net profit due to a marginal increase in $m$. That is, for every unit the bank is investing in liquid assets, $m$, instead of long-term assets, its return on this unit of investment is $r_m = 1$ rather than $\theta > 1$. The difference between these returns gives exactly the foregone return due to investment in liquid instead of long-term assets. Since $\theta > 1$, this difference is positive by assumption, meaning that the marginal costs of holding liquid assets is positive because long-term assets are more profitable than liquid assets. The foregone return is multiplied by the term $F(\alpha_B)$ which represents the cumulative distribution function of the random variable $\alpha$ at $\alpha_B$ and is, therefore, the probability that $\alpha \leq \alpha_B$. Thus, the product $(\theta - 1)F(\alpha_B)$ represents the expected foregone return from investing in liquid assets instead of long-term profitable assets. On the right hand side, the term $(\frac{1}{\tau} - 1)$ represents the net gain in profits resulting from the fact that when holding one unit more of liquid assets the bank is able to pay out one unit more to withdrawing depositors and, therefore, avoids liquidating one unit of long-term assets, incurring the loss $1/\tau$. The second term on the right hand side of (17), the difference $F(\alpha_B) - F(\hat{\alpha})$, is the probability that the realization of $\alpha$ falls in the interval $[\hat{\alpha}, \alpha_B]$. Note that in this interval, the withdrawn face value of short-term debt exceeds the bank's holdings in liquid assets and the bank incurs the per unit loss $1/\tau$ due to the liquidation of long-term assets. The product on the right hand side of the equation, therefore, gives the expected marginal benefit from investing in liquid assets instead of long-term assets. Equation (17) depends on whether the threshold value for $\alpha$ above which the bank defaults either due to illiquidity or insolvency, $\alpha_B$, is strictly below 1. If $\alpha_B = 1$, the bank never defaults, independently of the realization of $\alpha$, or, put differently, even if all depositors withdraw in $t = 1$. Such a situation would mean that $F(\alpha_B) = 1$ and that (17) simplifies to

$$(\theta - 1) = \left(\frac{1}{\tau} - 1\right)\left(1 - F(\hat{\alpha})\right). \tag{18}$$

At the optimum, either (17) or (18) has to be satisfied, meaning that the expected marginal benefit from investing in liquid assets has to equal the expected marginal costs from doing so. The following proposition states that the optimal choice for liquid assets, denoted by $m^*(e)$, is the solution to the maximization problem given in (15).

**Proposition 1** *Let $\hat{\hat{e}}$ denote the value of $e$ such that $m_1^*(e) = m_2^*(e)$. For values $e < \hat{\hat{e}}$, the model has a unique equilibrium value for $m$, denoted by $m_1^*$, such that $G(m) = 0$ and $\alpha_B < 1$. For values $e > \hat{\hat{e}}$, there exists a unique equilibrium value for $m$, denoted by $m_2^*$, such that $G(m) = 0$ and $\alpha_B = 1$.*

The profit-maximizing amount of liquid assets when the amount of withdrawn face value of short-term debt is assumed to be uniformly distributed, $m^*(e)$, is implicitly given by

$$m^*(e) = \begin{cases} m_1^*(e) = (1-\beta)\,\tilde{m}, & if \quad \alpha_B(m^*(e)) < 1, \\ m_2^*(e) = (1-\beta)(1-e), & if \quad \alpha_B(m^*(e)) = 1, \end{cases} \tag{19}$$

where

$$\beta = \frac{\theta - 1}{\frac{1}{\tau} - 1} \qquad \text{and} \qquad \tilde{m} = \frac{\tau\big(\theta - (1-e)\big)}{(1-\tau) - (1-\beta)(1-\tau\theta)}.$$

As (19) shows, the bank chooses a fixed fraction $1-\beta$ of $\tilde{m}$, a term that is dependent upon the amount of bank capital $e$ if the bank runs the risk of default, whereas it chooses the fixed fraction $1-\beta$ of the face value of short-term debt if $\alpha_B = 1$ and there is no default even if all of the depositors withdraw.

**Corollary 1** *In presence of default risk, i.e. $\alpha_B < 1$, the function $m_1^*(e)$ depends positively on the amount of bank capital, while in absence of default risk, i.e. $\alpha_B = 1$, the function $m_2^*(e)$ depends negatively on bank capital.*

Figure 4 shows the bank's optimal choice for liquid assets in the $e$-$m$ space. As illustrated, the bank's profit-maximizing decision for liquid assets as a function of equity, $m^*(e)$, is increasing in $e$ up to the point where $e = \hat{\hat{e}}$. That is, a poorly capitalized bank will increase equity and liquid assets simultaneously up to a certain level of bank capital $\hat{\hat{e}}$. Above this capital level, the function decreases with the amount of equity held until $e = 1$.

In other words, in the presence of default risk, i.e. $\alpha_B < 1$, liquid assets and bank capital complement each other. This is due to the fact that while an additional unit of equity increases both $\hat{\alpha}$ and $\alpha_B$, the interval between them becomes larger as $\alpha_B$ increases at a faster rate than $\hat{\alpha}$. Hence, ceteris paribus, for higher values of equity, the bank faces higher expected liquidation losses and will, therefore, simultaneously increase its holdings of liquid assets.

Given that the bank does not face default risk, i.e. $\alpha_B = 1$, liquid assets and equity are perfect substitutes. As further increasing $e$ above the threshold $\hat{\hat{e}}$ increases the bank's funding costs without the beneficial effect of lowering its default risk, it lowers its holdings of liquid assets, or, put differently, increases the amount of profit generating long-term assets.

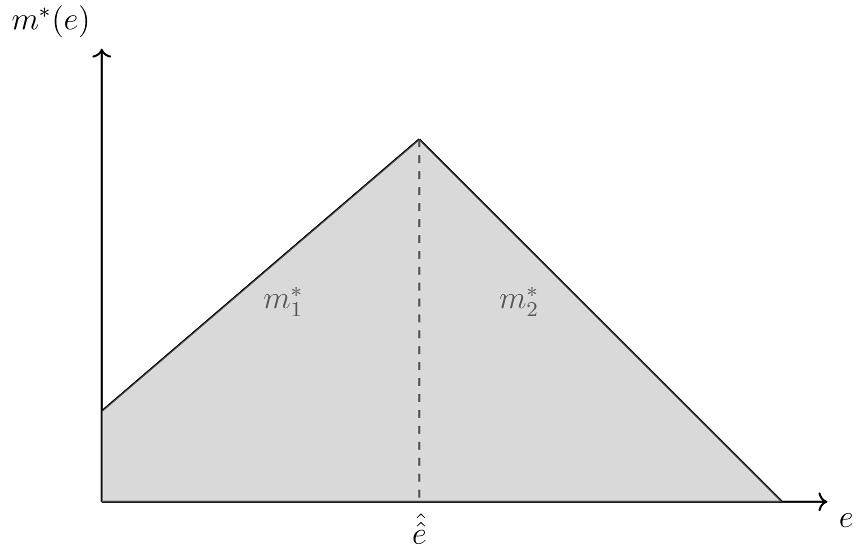


**Figure 4: Bank's optimal choice for liquid assets. For values $e < \hat{\hat{e}}$, equity and liquid assets complement each other. For values $e > \hat{\hat{e}}$ they are perfect substitutes.**

# 8 Regulation and Policy Implications

In the previous section, the bank's profit-maximizing decision for liquid assets has been analyzed without paying attention to the potential discrepancy to an optimum formulated by a regulator. This section aims to discuss this very mismatch in optimal holdings of liquid assets. To do so, it is assumed that the bank complies to an exogenously given capital ratio that is treated as a binding constraint. Hence, it will hold just the minimum amount of equity such that the regulatory ratio is satisfied. The bank's degree of capitalization can, therefore, be interpreted as the strictness of the capital requirement. The regulator's objective is to minimize the costs associated with the liquidation of long-term assets and to set the liquidity ratio accordingly. It is discussed how this choice reacts to changes in the strictness of the capital requirement.

The disparity between the bank's choice and the regulator's optimal choice for liquid assets evolves from the assumption that the regulator ascribes a higher cost to illiquidity than the bank. This discrepancy in valuation can be justified by the existence of externalities in the financial market. One may think of them as effects on financial stability associated with bank default.[22] Since the regulator takes these negative externalities into account, it ascribes a higher value to the illiquidity discount, $\tau$, such that $\tau_{SP} < \tau$ and, consequently, $1/\tau_{SP} > 1/\tau$. The mechanism to force the bank to increase its holdings of liquid assets is a minimum liquidity ratio as formulated in (2). Consequently, the function that describes the optimal decision

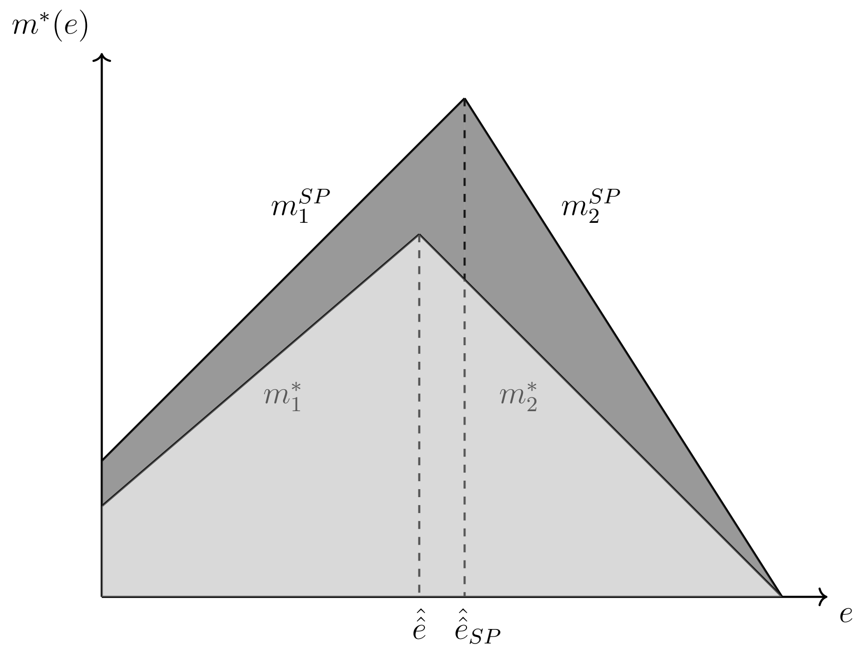


**Figure 5: Bank's optimal choice for liquid assets. Dark shaded areas are the deviations of the regulator's choice from the bank's profit-maximizing decision.**

[22]Perotti and Suarez (2011) model these externalities via the marginal costs of using short-term funding. By relying too much on short-term debt, banks in their model contribute to the possible occurrence of a liquidity crisis to a socially inefficient extent. As it is done in a similar way in this model, Perotti and Suarez (2011) argue that this inefficient individual contribution to systemic risk is a justification for liquidity regulation.

chosen by the regulator is given by:

$$m^*_{SP}(e) = \begin{cases} m^*_{SP,1}(e) = (1-\beta_{SP})\tilde{m}_{SP}, & if \quad \alpha_B(m) < 1, \\ m^*_{SP,2}(e) = (1-\beta_{SP})(1-e), & if \quad \alpha_B(m) = 1, \end{cases} \tag{20}$$

with

$$\beta_{SP} = \frac{\theta - 1}{\frac{1}{\tau_{SP}} - 1} \qquad \text{and} \qquad \tilde{m}_{SP} = \frac{\tau_{SP}\big(\theta - (1-e)\big)}{(1-\tau_{SP}) - (1-\beta)(1-\tau_{SP}\theta)}.$$

In Figure 5 one can see that the regulator chooses strictly higher values for the bank's liquid assets for every degree of capitalization. The dark shaded area in Figure 5 is the missing amount of liquid assets as seen from the regulator's perspective. The fact that $m_1^{SP} > m_1^*$ and $m_2^{SP} > m_2^*$ for all values of $e \in [0,1)$ gives a justification for regulatory intervention, since the regulator is internalizing the social costs that arise due to the higher default risk the bank exposes itself by choosing lower amounts of liquid assets. Given the higher values for liquid assets chosen by the regulator, it is worth analyzing the implications for the regulatory ratio.

As described in (2), the ratio can be complied by the bank either by adjusting $m$ or $e$. Since $m^*_{SP}(e) > m^*(e)$ for all values of $e$, the resulting regulatory ratio for liquid assets, $\gamma_{SP}$, makes the bank hold more liquid assets or equity. In a context where both liquidity regulation and capital regulation coexist, the amount of equity held by the representative bank can, as it was mentioned before, be interpreted as the value that satisfies an exogenously specified capital requirement. Since $\gamma_{SP}$ is increasing in $e$ and because of the assumption that the bank only increases its equity holdings if it is forced to do so by capital requirements, the minimum liquidity ratio set by the regulator increases with a stricter capital requirement. Formally, this can be written as:

$$\frac{\partial \gamma(m^*_{SP,1}(e))}{\partial e} > 0. \tag{21}$$

Therefore, given that $\rho > \hat{\rho}$ (Case 3 in Figure 6, see Appendix), liquidity requirements and capital requirements are *complements* in the interval $[0, \hat{\hat{e}}]$.[23] In the interval $[\hat{\hat{e}}, 1]$, however, the regulatory ratio for liquidity, $\gamma$ is *unaffected* by changes in the strictness of capital requirements. To see this, note that $\gamma$ is a function of $m^*_{SP}$, which itself is increasing in equity $e$. However,

$$\gamma(m^*_{SP,2}(e)) = (1 - \beta_{SP}) \tag{22}$$

is a constant. The minimum ratio for liquid assets is, therefore, set independently of capital requirements.

**Proposition 2** *The regulator chooses $m^*_{SP}(e) > m^*(e)$ for all values of $e \in [0, 1)$. The threshold $\hat{\hat{e}}_{SP}$ is such that $\hat{\hat{e}}_{SP} > \hat{\hat{e}}$. For values $e < \hat{\hat{e}}_{SP}$, liquidity requirements and capital requirements are complements. For values $e > \hat{\hat{e}}_{SP}$, they are set independently of each other.*

Proposition 2 states that in the presence of default risk, liquidity requirements are set optimally such that they increase as capital requirements become stricter. This is due to the fact that for $e < \hat{\hat{e}}_{SP}$, the liquidity effect dominates the solvency effect. To see this, note that in this region $m^*_{SP,1}(e)$ is the corresponding function for the regulator's optimal choice for liquidity. Increasing $e$ makes the bank unambiguously default less often but at the same time it is increasing the risk of potential liquidation losses.[24] In order to decrease expected losses due to liquidation of long-term assets, the regulator increases the minimum requirement for liquid assets. This reduces these losses on the one hand, but, on the other hand, decreases profits. However, due to Assumption 3, the foregone liquidation losses exceed the reduction in profits. Therefore, in the case where $e < \hat{\hat{e}}_{SP}$, liquidity requirements and capital requirements are *complements*.

In the absence of default risk, i.e. for values of equity such that $e > \hat{\hat{e}}_{SP}$, the regulator chooses the requirements independently of each other, meaning that an increase in $e$ does not come with an increase in $\gamma(m^*_{SP,2}(e))$. In this region it holds

[23] The threshold value for the return on capital, denoted by $\hat{\rho}$, is such that the first-order condition for equity is equal to 0. Its derivation is given in the Appendix.

[24] Note that $\frac{\delta \alpha_B}{\partial e} > \frac{\partial \hat{\alpha}}{\partial e}$.

that $\alpha_B = 1$. Increasing $e$ further, unambiguously decreases the risk of illiquidity since the interval between $\hat{\alpha}$ and $\alpha_B$ becomes smaller. Due to this reduction in the risk of illiquidity through $e$, the regulator does not change the liquidity requirement anymore and fixes it at $\gamma(m^*_{SP,2}(e)) = (1 - \beta_{SP})$.

As the regulator is the only agent in this thesis whose objective is to minimize costs of illiquidity, the model abstracts from other potential corrective actions and institutions such as discount-window or open-market operations by a central bank. In particular, one could argue that monetary policy might be sufficient in situations in which otherwise solvent banks face problems of illiquidity such that there is no reason for *ex-ante* interventions. Rochet and Vives (2004) discuss the interaction between a Lender of Last Resort (LoLR) and liquidity (and solvency) requirements and find that *ex-post* discount-window loans should be complemented with *ex-ante* requirements.[25] The finding of this model that liquidity requirements can help to mitigate the costs of illiquidity is in line with Rochet and Vives (2004) even though in their model setting, liquidity requirements should also come with discount-window activities.

From a theoretical standpoint, both LoLR activities and macroprudential regulation may lead to undesired effects. First, expectations about *ex-post* interventions by a LoLR potentially cause moral hazard frictions and might act as an incentive for increasing risk-taking behaviour of banks as they know that, when needed, regulatory institutions will give assistance.[26] Macroprudential regulation, on the other hand, always comes with the problem of how to design regulatory ratios. Since these must be set to a specific value, the questions arises if the necessary information is at hand in order to find the optimal degree of strictness of such ratios. While *ex-ante* interventions might help to prevent excessive risk-taking from the outset, LoLR activities come with the useful advantage of the easier identification of illiquid but otherwise solvent banks.[27]

[25]See also Jeanne and Korinek (2016) for an analysis of the interplay between macroprudential regulation and monetary policy.
[26]Jeanne and Korinek (2016)
[27]Jeanne and Korinek (2016)

# 9 Conclusion

This paper develops a theoretical framework that aims to understand better the interdependence of capital and liquidity regulation. While in the presence of deposit insurance capital regulation may be able to correct the market failure regarding the bank's risk-shifting incentives, the combination of these regulatory instruments proved to be insufficiently designed to maintain financial stability under liquidity stress scenarios as severe as during the last financial crisis. Therefore, two minimum ratios for liquidity have been developed by the Basel Committee on Banking Supervision (2010) in order to better tackle the issue of liquidity risk. These are the *Net Stable Funding Ratio* (NSFR) and the *Liquidity Coverage Ratio* (LCR). This paper analyzes potential ambiguous effects of a liquidity ratio on bank stability.

The thesis puts together important insights from König (2015) and Eisenbach et al. (2014) and extends the scope of their models by endogenizing the bank's optimal decision for liquid assets and equity. Facing the possibility of liquidity default due to withdrawing depositors, the representative bank in this model exposes itself to a level of liquidity risk that lies above the regulator's optimum. This regulator internalizes potential costs of bank default and chooses higher equilibrium values for liquidity holdings. Given the discrepancy between the two equilibria, the regulator imposes a regulatory ratio for liquidity similar to the *Liquidity Coverage Ratio* defined by the Basel Committee of Banking Supervision in its publication *"Basel III: A global regulatory framework for more resilient banks and banking systems"*. Due to the fixed length of the balance sheet, the bank is able to comply with the minimum ratio for liquidity by either adjusting the asset side or the liability side of its balance sheet. Assuming the liability side has been chosen optimally by the bank in order to comply with existing capital requirements, it is possible to analyze the interdependence of capital and liquidity regulation. This paper shows that for relatively highly levered banks, liquidity regulation and capital regulation are *complements*, while they are *independent* of each other when imposed to highly capitalized banks.

In a context where liquidity requirements and capital requirements are implemented by authorities, this thesis shows that an optimal regulatory policy should consider both the asset side and the liability side of the bank's balance sheet and that possible side effects may arise when tightening one requirement.

# 10 Appendix

## 10.1 Equilibrium values for equity $e$

This section presents the bank's profit-maximizing choice for equity, treating the choice for liquidity as fixed. In order to do so, the derivative of (15) with respect to $e$ is given by the following function:

$$H(e) = F(\alpha_B) + \left(\frac{1}{\tau} - 1\right) \int_{\hat{\alpha}}^{\alpha_B} \alpha dF(\alpha) - \rho. \tag{23}$$

Setting $H(e) = 0$ and rearranging terms gives:

$$F(\alpha_B) + \left(\frac{1}{\tau} - 1\right)\Big(F(\alpha_B) - F(\hat{\alpha})\Big)E[\alpha \mid \hat{\alpha} < \alpha < \alpha_B] = \rho. \tag{24}$$

That is, when optimally choosing the amount of equity, the bank sets the marginal benefits of increasing $e$ equal to its marginal costs. The first term on the left-hand side of (24) represents the amount of interest that the bank does not have to pay out to depositors due to a one-unit decrease of short-term debt, $s$, times the probability that the realization of $\alpha$ lies below $\alpha_B$. As in (17), the term $(\frac{1}{\tau} - 1)$ on the left-hand side of the equation represents the net gain in profits resulting from a smaller face value of short-term debt. This term is multiplied by the interest rate on short-term debt due to the reduction in short-term debt, $s$. Finally, the two terms are multiplied by the probability that the realization of $\alpha$ falls between $\hat{\alpha}$ and $\alpha_B$. Altogether, the term represents the expected reduction in the cost of fire-sales times the short-term interest rate due to an increase in equity. The right-hand side of (24) represents the marginal costs. These are simply the increase in the debt burden resulting from the bank's capital holdings, $\rho$.

**Proposition 3** *Let $\hat{\rho}$ be the threshold for $\rho$ where $H(e \mid e^* > \hat{\hat{e}}) = 0$ and $\hat{\hat{\rho}}$ be the threshold for $\rho$ that is implicitly defined by $\pi(e = 1 \mid \hat{\hat{\rho}}) = \pi(e = 0 \mid \hat{\hat{\rho}})$. Then, $\rho < \hat{\rho}$ implies $H(e \mid e > \hat{\hat{e}}) > 0$, and the equilibrium value for equity is $e^* = 1$ for all $\rho < \hat{\hat{\rho}}$ as $H''(e) > 0$. Given that $\rho > \hat{\rho}$, implies that $H(e) < 0$ $\forall e$ and the equilibrium value is $e^* = 0$.*

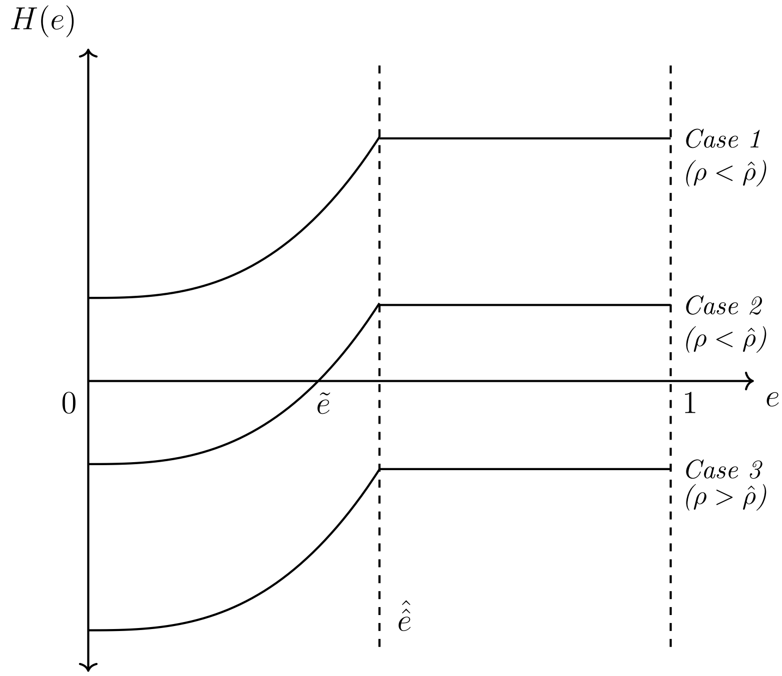


**Figure 6:** ***First-order condition for three different cases. In Case 1, the profit function is always increasing in $e$. In Case 2, the profit function has a minimum at $\tilde{e} < \hat{\hat{e}}$. In Case 3, profits are always decreasing in $e$.***

Proposition 3 claims that the profit-maximizing choice for bank capital depends on the optimality threshold for the cost of equity $\hat{\rho}$. It is given by

$$\hat{\rho} \equiv \beta\Big(1 - \frac{\beta}{2}\Big)\Big(\frac{1}{\tau} - 1\Big) + 1 \tag{25}$$

and is derived from combining the two first-order conditions for both liquidity, (17), and equity, (24). In the first case of Figure 6, it holds that $\rho < \hat{\rho}$. That is, the cost of equity is below the threshold $\hat{\rho}$ such that it is optimal for the bank to only hold capital. Note that in Case 1 the profit function is always increasing in $e$ so that profits are maximized at $e^* = 1$. In Case 2, $\rho < \hat{\rho}$ also holds. Here, the profit function has a minimum value at $\tilde{e} < \hat{\hat{e}}$. Although the cost of equity is sufficiently low, it might be, due to the shape of the profit function, that $\pi(e = 0) > \pi(e = 1)$, meaning that it is still optimal for the bank to hold no equity. If, however, the cost of equity is above $\hat{\rho}$, then the bank seeks to be fully levered. Case 3 in Figure 6 shows a profit function that is always decreasing in $e$. Here, the bank chooses $e^* = 0$ in order to maximize its profits.

## 10.2 Proof of Lemma 1

The derivatives of the thresholds formulated in (9), (10) and (11) with respect to $m$ and $e$, respectively are:

$$\frac{\partial \hat{\alpha}}{\partial m} = \frac{1}{1-e} > 0, \qquad \frac{\partial \hat{\alpha}}{\partial e} = \frac{m}{(1-e)^2} > 0,$$
$$\frac{\partial \alpha_B}{\partial m} = \frac{1-\tau\theta}{(1-e)(1-\tau)} > 0, \qquad \frac{\partial \alpha_B}{\partial e} = \frac{\tau\theta(1-m)+m}{(1-\tau)(1-e)^2} > 0,$$
$$\frac{\partial \alpha_A}{\partial m} = \frac{1-\tau\theta}{1-e} > 0, \qquad \frac{\partial \alpha_A}{\partial e} = \frac{\tau\theta(1-m)+m}{(1-e)^2} > 0.$$

□

## 10.3 Proof of Lemma 2

From equation (13) the expression for $\alpha_B$ follows:

$$\alpha_B = \frac{\tau\theta(1-m)+m-\tau(1-e)}{(1-e)(1-\tau)}.$$

Rearranging terms allows to formulate the above expression such that $\alpha_A$ appears:

$$\alpha_B = \frac{\alpha_A}{(1-\tau)} - \frac{\tau}{(1-\tau)} < \alpha_A.$$

It is easily seen that the left-hand side of this inequality is strictly smaller than $\alpha_A$ for all values of $\tau \in [0,1]$. □

## 10.4 Proof of Proposition 1

The first-order condition coming from the derivative of the bank's profit function with respect to $m$ is:

$$G(m) = \Big(\frac{1}{\tau} - 1\Big)\Big(F(\alpha_B) - F(\hat{\alpha})\Big) - (\theta - 1)F(\alpha_B).$$

Controlling the FOC at the boundaries gives:

$$G(0) = \Big(\frac{1}{\tau} - 1\Big)\Big(F\big(\alpha_B(0)\big) - 0\Big) - F\big(\alpha_B(0)\big)(\theta - 1) = F\big(\alpha_B(0)\big)\big(\frac{1}{\tau} - \theta\big) > 0$$

and

$$\begin{aligned} G(1) &= \Big(\frac{1}{\tau} - 1\Big)\Big(F\big(\alpha_B(1)\big) - F\big(\hat{\alpha}(1)\big)\Big) - F\big(\alpha_B(1)\big)(\theta - 1) \\ &= \underbrace{\Big(\frac{1}{\tau} - 1\Big)}_{>0}\underbrace{(\theta - 1)}_{>0}\frac{\overbrace{\tau - 1}^{<0}}{\underbrace{(1-e)}_{>0}\underbrace{(1-\tau)}_{>0}} < 0. \end{aligned}$$

The second-order condition with respect to $m$ gives:

$$\begin{aligned} G'(m) &= \Big(\frac{1}{\tau} - 1\Big)\big(f(\alpha_B)\alpha_B' - f(\hat{\alpha})\hat{\alpha}'\big) - (\theta - 1)f(\alpha_B)\alpha_B' \\ &= \Big(\frac{1}{\tau} - 1\Big)(\alpha_B' - \hat{\alpha}') - (\theta - 1)\alpha_B' \\ &= \Big(\frac{1}{\tau} - 1\Big)\Big(\frac{1 - \tau\theta - 1 + \tau}{(1-e)(1-\tau)}\Big) = \Big(\frac{1}{\tau} - 1\Big)\Big(\frac{\tau(1-\theta)}{(1-e)(1-\tau)}\Big) < 0 \end{aligned}$$

In presence of default risk, i.e. $\alpha_B < 1$, it follows from (9), (10) and (19), that the first-order condition can be rewritten as

$$\begin{aligned}
\hat{\alpha} &= \alpha_B(1-\beta) \\
m &= (1-\beta)(1-e)\frac{\tau\theta(1-m)+m-\tau(1-e)}{(1-e)(1-\tau)} \\
m\big(1-\tau+(1-\beta)(\tau\theta-1)\big) &= (1-\beta)\big(\tau\theta-\tau(1-e)\big) \\
m_1^*(e) &= (1-\beta)\,\underbrace{\frac{\tau\big(\theta-(1-e)\big)}{(1-\tau)-(1-\beta)(1-\tau\theta)}}_{\equiv \tilde{m}}.
\end{aligned}$$

In the absence of default risk, i.e. $\alpha_B = 1$, it follows from (9), (10) and (19), that the first-order condition can be rewritten as

$$\begin{aligned}
\hat{\alpha} &= 1-\beta \\
m_2^*(e) &= (1-\beta)(1-e).
\end{aligned}$$

Therefore, $m^*(e)$ is given by:

$$m^*(e) = \begin{cases} m_1^*(e) = (1-\beta)\,\tilde{m}, & if \quad \alpha_B(m) < 1, \\ m_2^*(e) = (1-\beta)(1-e), & if \quad \alpha_B(m) = 1. \end{cases}$$

The intersection of these two functions, denoted by $\hat{\hat{e}}$, is such that

$$m_1^*(e) \stackrel{!}{=} m_2^*(e),$$

from which it follows that

$$\begin{aligned}
1-e &= \frac{\tau\big(\theta-(1-e)\big)}{(1-\tau)-(1-\beta)(1-\tau\theta)} \\
\tau\theta &= (1-e)\Big(1-(1-\beta)(1-\tau\theta)\Big) \\
\hat{\hat{e}} &= 1-A,
\end{aligned}$$

where

$$A \equiv \frac{\tau\theta}{1-(1-\beta)(1-\tau\theta)}.$$

□

## 10.5 Proof of Corollary 1

The derivative of $m_1^*(e)$ with respect to $e$ is given by:

$$\frac{\partial m_1^*(e)}{\partial e} = \frac{(1-\beta)\tau}{(1-\tau)-(1-\beta)(1-\tau\theta)} > 0.$$

The derivative of $m_2^*(e)$ with respect to $e$ is given by:

$$\frac{\partial m_2^*(e)}{\partial e} = -(1-\beta) < 0.$$

□

## 10.6 Proof of Proposition 3

Assuming a uniform distribution for the random variable $\alpha$, the first-order condition for equity can be written as:

$$H(e) = \alpha_B + \left(\frac{1}{\tau}-1\right)\int_{\hat{\alpha}}^{\alpha_B} \alpha dF(\alpha) - \rho.$$

Setting $H(e) = 0$ and exploiting the fact from (17) that $\alpha_B - \hat{\alpha} = \beta\alpha_B$ yields:

$$\beta\alpha_B = \frac{\rho-\alpha_B}{\left(\alpha_B - \frac{\beta}{2}\right)\left(\frac{1}{\tau}-1\right)}$$

$$\beta\alpha_B\left(\alpha_B - \frac{\beta}{2}\right)\left(\frac{1}{\tau}-1\right) = \rho - \alpha_B.$$

In the absence of default risk, i.e. $\alpha_B = 1$, it follows that the first-order condition can be rewritten as:

$$\beta\Big(1 - \frac{\beta}{2}\Big)\Big(\frac{1}{\tau} - 1\Big) + 1 = \rho.$$

Hence, in the absence of default risk, the first-order condition for equity does not depend on $e$. The optimal choice for capital depends, therefore, on its cost. Let $\hat{\rho}$ be the optimality threshold for the cost of bank capital in the absence of default risk. It is given by:

$$\hat{\rho} \equiv \beta\Big(1 - \frac{\beta}{2}\Big)\Big(\frac{1}{\tau} - 1\Big) + 1.$$

Assuming $\rho > \hat{\rho}$ implies that $H(e) < 0 \ \forall e$. In this case, the profit function is always decreasing in $e$, which makes it optimal for the bank so hold no equity, i.e. $e^* = 0$. This case corresponds to Case 3 in Figure 6.

In Case 1 of Figure 6, $\rho < \hat{\rho}$ holds and $H(e) > 0 \ \forall e$. In this case, clearly, it is optimal for the bank to be fully equity financed as the profit function is always increasing in $e$.

In Case 2, $\rho < \hat{\rho}$ holds as well but as $H''(e) > 0$, it is still not obvious which of the two corner solutions is the profit-maximizing decision. Therefore, due to the convexity of the profit function, $e^* = 1$ if and only if $\pi(e = 1) > \pi(e = 0)$. Let $\hat{\hat{\rho}}$ be the threshold for the cost of equity defined such that $\pi(e = 1) = \pi(e = 0)$. It follows that, given Case 2 applies, $e^* = 1$ if and only if $\rho < \hat{\hat{\rho}}$. This inequality rules out the case in which $\rho < \hat{\rho}$ but $\pi(e = 1) < \pi(e = 0)$. In order to find the second-order condition for equity $H(e)$ is rewritten as:

$$H(e) = \frac{1}{1 - e}A + \frac{1}{(1 - e)^2}B - \frac{\tau}{1 - \tau} + \frac{\tau}{2(1 - \tau)} - \rho,$$

where

$$A \equiv \frac{\theta + m}{\frac{1}{\tau} - 1} > 0 \quad \text{and} \quad B \equiv \frac{\big(\theta(1 - m) + m\big)\big(m(2 - \tau) + \tau\theta(1 - m)\big)}{2(1 - \tau)} > 0.$$

The second-order condition is, therefore, given by:

$$\frac{\partial H(e)}{\partial e} = \frac{1}{(1-e)^2}A + \frac{2}{(1-e)^3}B > 0.$$

□

## 10.7 Proof of Proposition 2

Since $\beta_{SP} < \beta$, it follows that $m^*_{SP,1} > m^*_1$ and that $m^*_{SP,2} > m^*_2$ for all values of $e$. To see this, one can take the derivative of $m^*_1$ with respect and show that $\frac{\partial m^*_1}{\partial \tau} < 0$ so that for values of $\tau_{SP} < \tau$ it holds that $m^*_{SP,1} > m^*_1$. In order to do so, first rewrite $m^*_1(e)$ to

$$\frac{\tau C - \tau^2 \theta C}{\tau^2(1-\theta^2) + 2\tau(\theta - 1)},$$

where $C \equiv \big(\theta - (1-e)\big)$. It follows that the derivative with respect to $\tau$ is given by:

$$\frac{\partial m^*_1}{\partial \tau} = \frac{(C - 2\tau\theta C)\big(\tau^2(1-\theta^2) + 2\tau(\theta-1)\big) - (\tau C - \tau^2\theta C)\big(2\tau(1-\theta^2) + 2(\theta-1)\big)}{\Big(\tau^2(1-\theta^2) + 2\tau(\theta-1)\Big)^2}.$$

To check whether this is positive or negative define the numerator as $D$. Then,

$$\begin{aligned} D = \tau^2\theta - \tau^2 - \tau^2\theta^2 =& \tau^2(2\theta - 1 - \theta^2) = -\tau^2(\theta^2 + 1 - 2\theta) = \\ & -\tau^2(1-\theta)^2 < 0. \end{aligned}$$

Hence, it follows that $\frac{\partial m^*_1}{\partial \tau} < 0$ and that $m^*_{SP,1} > m^*_1$ since $\tau_{SP} < \tau$. Due to the different valuation of the cost of fire-sales, the threshold value $\hat{\hat{e}}_{SP}$ does not coincide with $\hat{\hat{e}}$. To show this, the derivative of $\hat{\hat{e}}$ with respect to $\tau$ is taken:

$$\frac{\partial \hat{\hat{e}}}{\partial \tau} = \frac{2 - \frac{1}{\theta} - \theta}{\theta^2} = \frac{2 - \frac{1}{\theta} - \theta}{\theta^2}.$$

As the denominator is always positive, let the numerator be defined as $E$. It can be rewritten as:

$$E \equiv 2 - \frac{1}{\theta} - \theta < 0,$$

since

$$-(2\theta - 1 - \theta^2) < 0 \Longleftrightarrow -(\theta - 1)^2 < 0$$

holds. Therefore, $\hat{\hat{e}}_{SP} > \hat{\hat{e}}$. The regulatory ratio for liquidity is given by $\gamma = \frac{m}{1-e}$. Plugging in the socially optimal choice and taking the derivative with respect to $e$ gives:

$$\gamma(m^*_{SP,1}(e)) = \frac{\tau(\theta - 1 + e)}{1 - e} E, \qquad \gamma(m^*_{SP,2}(e)) = 1 - \beta_{SP},$$

$$\frac{\partial \gamma(m^*_{SP,1}(e))}{\partial e} = \frac{\tau\theta}{(1 - e)^2} E > 0, \qquad \frac{\partial \gamma(m^*_{SP,2}(e))}{\partial e} = 0,$$

where

$$E \equiv \frac{(1 - \beta_{SP})}{(1 - \tau) - (1 - \beta_{SP})(1 - \tau\theta)}. \qquad \square$$

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# German abstract

*Die vorliegende Masterarbeit behandelt die Interdependenz von Liquiditäts- und Kapitalreglementierungen im Kontext eines drei-Perioden bank-run-Modells mit einem Regulierer, der die auftretenden negativen Externalitäten internalisiert. Die profitmaximierende Entscheidung über das Ausmaß an von der Bank gehaltenen liquiden Aktiva, wird als endogene Variable bestimmt. Anhand dieses Modells lässt sich einerseits zeigen, dass Banken, die geringe Mengen an Eigenkapital aufweisen, mehr liquide Aktiva halten, während jene mit hohen Eigenmitteln, weniger liquide Aktiva halten. Andererseits wird gezeigt, dass ein das Illiquiditätsrisiko minimierender Akteur, höhere liquide Reserven für jeden Grad der Eigenkapitalfinanzierung der Bank anstrebt. In einem Kontext, in dem sowohl gesetzliche Anforderungen bezüglich der liquiden Aktiva als auch der Eigenkapitalmittel implementiert sind, zeigt sich, dass sich diese komplentär zueinander verhalten, ist die Bank mit wenig Eigenmitteln finanziert, während diese unabhängig voneinander gesetzt werden, wenn es sich um Banken handelt, die vergleichsweise stark durch Eigenkapital finanziert sind.*