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„Limits of Liquidity Hedging“

verfasst von / submitted by

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Abstract: *Liquidity management has become increasingly important for banks after the 2008 financial crisis. The aim of my master thesis is to explain why liquidity risk is not perfectly hedged by small banks while other risks are. For this purpose I will develop a model that shows that it is not optimal that an umbrella institution pools without limit all the liquidity needs of many small banks that are subject to different liquidity shocks from deposits. Such a model is important to understand the reasons for the limits of liquidity hedging, clarify whether this leads to a pareto-efficient situation or what causes the inefficiency and propose potential solutions.*

German Abstract: *Die Bedeutung von Liquiditätsmanagement hat nach der Finanzkrise von 2008 zugenommen. Das Ziel meiner Masterarbeit ist es zu erklären warum es Regionalbanken nicht möglich ist sich vollständig gegen Liquiditätsrisiko zu versichern wie das bei anderen Risiken üblich ist. Für diesen Zweck entwickle ich ein Modell, das zeigt, dass es für Dachinstitute nicht ideal ist das komplette Risiko von den Mitgliedern zu übernehmen. Ein deraritges Modell ist wichtig um die Gründe für diese limitierte Versicherung gegen Liquiditätsschocks zu verstehen und es soll helfen zu klären ob sich daraus eine pareto-optimale Risikoverteilung ergibt und gegebenenfalls Lösungen aufzeigen.*

Contents

1 Introduction

The idea for my master thesis developed after talking to the director of a small Austrian union bank about liquidity management in general, which is a topic that has ,for several reasons, become increasingly important for managing banks after the 2008 financial crises for several reasons. One obvious explanation is that during the crises markets in which banks used to satisfy their liquidity needs dried up and even now banks themselves are reluctant to rely on those markets as much as before. Another cause of course is regulation as also the regulators attention shifted to liquidity issues. Last but not least, the lower margins, that banks earn due to the low-interest environment in the aftermath of the crises, might have increased their awareness of costs, like liquidity cost, which before the crisis seemed negligible.

In this talk one issue came up that caught my attention in particular: While the bank eliminated almost any interest rate risk by engaging in interest swaps with its umbrella-organisation, the liquidity risk was not completely hedged with that organisation.

This means that while there are credit lines etc. offered by the umbrella organisation, these are so costly that the bank only engages in such practices to a limited extent, such that even large transactions by single customers, e.g. real estate lawyers transferring money after house purchases, may become a serious issue for the bank. The question I asked myself was why the umbrella organisation was apparently unwilling to completely hedge the small banks against this risk as apparently such transactions would be more or less uncorrelated and therefore by the law of large numbers a relatively smaller liquidity buffer would be needed by a larger institution.

One plausible reason for this apparently inefficient risk allocation, is that the umbrella organisation is not able to distinguish this ordinary liquidity volatility from a systemic liquidity component that affects all banks at the same time. Thus, the idea would be that this systemic component of liquidity risk could be more efficiently borne by the small banks.

According to the bank's director limited liquidity hedging has only become an issue after the 2008 financial crises. This may be due to an growing importance of the costs of liquidity provision in a low interest environment or due to an increased awareness about the risk associated with liquidity events by regulators as well as bank managers.

In my thesis I would like to model this situation by basically showing that ,in principle, it would be more efficient to transfer the risk stemming from idiosyncratic liquidity needs to a larger institution, but that this result breaks down once I allow for liquidity shocks that effect all banks simultaneously.

A related issue is particularly important for small banks operating in regions that rely on few seasonal industries, such as tourism in alpine Austria: such banks face seasonal liquidity cycles as the money tourists spend in those regions is held as deposits until the need for investments during off-seasons arises. However, such reoccurring cycles should in theory be pretty predictable and anticipated liquidity needs should not really cause severe problems.

The thesis will be structured as follows. First, there will be a literature review about liquidity management as well as models of banks in general. Thereafter, I will introduce the model and present the results based on the assumptions used. Then, I will discuss those results and assumptions in the context of the current literature. Finally, I will end my thesis with a conclusion about the contribution and relevance of the model.

2 Literature Review

There is no notable academic literature discussing the relationship between small banks and their umbrella-organisations in the context of liquidity risk. Therefore, I will focus my attention in this section at other papers that discuss the relationship between two sides in the context of liquidity provision.

The classical paper by Diamond and Dybvig (1983) analyses the role of banks as liquidity insurance for the customer. They basically find that banks can increase utility by offering deposit contracts compared to customers investing in isolation and also compared to a bond market. They explain the possibility of bank runs by the existence of a second equilibrium that involves all customers to withdraw their deposits early. The key difference to my thesis is that I explicitly model the possibility of a bank run and try to figure out how this risk is optimally shared between the small banks and their umbrella-organisation.[1]

The paper by Allen and Gale (1998) is an extension of the Diamond-Dybvig model and basically analyses how bank runs occur and under which circumstances they cause inefficiencies. They basically achieve this by linking the bank runs to the business cycle. The idea is that depositors get information about the future return of the assets (deposits) and show that if liquidation is not costly bankruns are actually efficient, while there is room for central bank intervention in case liquidating assets early is costly.[2]

For their analysis of 2000, Allen and Gale extend their model even further to discuss the contagion between banks. For this purpose they separate the banks in different regions, where they operate, and allow them to channel liquidity from one region to another via interbank markets. The aggregate demand for liquidity is constant, while it fluctuates within the regions. The idea is that the banks hold deposits in other banks. Similar to my setting the problem arises when aggregate demand increases and the analysis examines the contagion between the banks and how the crises spreads through the system. They also analyse the effect of capital buffers (additional liquidity) for the banks but do not model this decision explicitly and instead assume it as exogenously given.[3]

Bhattacharya and Gale (1987) discuss a situation where banks fail to optimally coordinate their liquidity through inter-bank lending as this would result in under-investing in liquidity due to asymmetric information as all banks would claim to need liquidity. Therefore, liquidity must be costly.[4]

Bhattacharya and Fulghieri (1994) discuss a similar issue in the Diamond-Dybvig model. There interbank borrowing also must have limits as otherwise the incentives

[1]Diamond, D. W., and P. H. Dybvig. "Bank Runs, Deposit Insurance, and Liquidity." Journal of Political Economy 91.3 (1983).

[2]Allen, F., and D. Gale. "Optimal financial crises." The journal of finance 53.4 (1998).

[3]Allen, F., and D. Gale. "Financial contagion." Journal of political economy 108.1 (2000).

[4]Bhattacharya, S., and D. Gale. "Preference shocks, liquidity and central bank policy, New Approaches to Monetary Economics." W. Barnett, and K. Singleton, eds., Ch. 4 (1987).

to invest in liquidity in the first place are disturbed and later the borrowing must have limits to incentivice the banks to lend at the inter-bank market.[5]

Contrary to these two papers I try to explain limited liquidity hedging not by asymmetric information or the setting of incentives but by the fact that systemic risk is efficiently located at the small banks and not the umbrella organisation.

Rochet and Vives (2004) provide a rational for the central bank to intervene in the interbank market as a lender of last resort and correct for coordination failure in this market such that the failure of solvent banks is prevented. The role of a lender of last resort and the umbrella institution I want to discuss differs in the way that the umbrella institution is itself liquidity constrained, which is particularly relevant in case of a systemic risk event.[6]

The last paper I want to mention here is the work by Eisenbach et. al (2014). They analyse in their paper the optimal capital structure of banks and how this can be regulated. In their paper they show the effects of changes to the capital structure on the asset and liability sides of the balance sheet. I instead will only allow my banks to adjust the asset side and ask what the optimal liquidity holding is when liquidity can be hedged to some extent.[7]

3 Model

In this section I would like to explain how the model is structured. I first want to model and analyse the effect of liquidity shocks to small banks that operate in isolation. Then I want to allow this small banks to hedge their liability with an umbrella institution. There, I first would like to analyse a situation where the small banks are subject to idiosyncratic liquidity shocks only and then allow for systemic components of liquidity shocks that effect all banks the same way. Finally, the

[5] Bhattacharya, S., and P. Fulghieri. "Uncertain liquidity and interbank contracting." Economics Letters 44.3 (1994).

[6] Rochet, J.-C., and X. Vives. "Coordination Failures and the Lender of Last Resort: was Bagehot right after all?" Journal of the European Economic Association 2.6 (2004).

[7] Eisenbach, T. M., T. Keister, J. McAndrews, and T. Yorulmazer. "Stability of funding models: an analytical framework." Economic Policy Review, 20.1 (2014).

decision of the umbrella institution, how to set the limits for credit-lines, will be modelled in more detail.

Small Banks

The model of the situation the individual small banks face and their decision is derived from a simple balance sheet. In this one-period model the idea is that the bank holds liquid assets, which yield no return, and long term loans, which provide an interest return of r. The small banks finance themselves via deposits and equity or long-term funding. While equity, respectively long-term financing is stable and not subject to any shocks, on-demand deposits are subject to exogenous shocks. On the other hand, equity providers demand a return on their investment, whereas depositors do not. Moreover, the profit of the bank has to be reduced in case the bank has insufficient liquidity to cover the withdrawals of deposits.

The small bank therefore faces exogenous shocks on the liability side and only sets the amount of liquid assets it holds optimally. The trade-off is between lower risk of illiquidity and higher return. The balance sheet is normalized to one.

Assets	Liabilities
Liquidity= L_s	Deposits
Loans= $1 - L_s$	Equity= E
1	1

The profit function for the small bank is similar to the one used in the book "Microeconomics of Banking" by Freixas and Rochet chapter 8.2.1:[8]

$$\Pi_{SB} = (1 - L_s)r - E\frac{r}{1 - Pr(L_s \leq D_o)} + Pr(L_s \leq D_o)\frac{ES - L_s}{1 - HC}$$

1. Here, L_s denotes the short term liquidity the bank holds.
2. As the balance sheet is normalized to 1, $1 - L_s$ represents the loans the banks have outstanding and which yield the return r.

[8] Freixas, Xavier, and Jean-Charles Rochet. "Microeconomics of banking." MIT press, (2008). Ch. 8.2.1, p.228 f.

3. E denotes the banks' equity respectively, long-term financing.

4. Equity and long-term financiers have to be compensated by the rate r adjusted by the probability of insufficient liquidity. As I assume those financiers to be risk neutral, they only demand the rate r, which can be seen as their outside option, adjusted for the risk.

5. This risk is represented by $Pr(L_s \leq D_o)$ which is the probability that the outflow of deposits exceeds the liquidity reserves of the bank. In the thesis I will discuss different possibilities about the distribution of deposit withdrawals.

6. The last term consists of the probability of liquidity shortfall as just discussed.

7. Further, it contains the ES, which is the expected shortfall. This is a common measure for risk, that captures the expected return of a portfolio in the defined worst quantiles. In this context, the expected shortfall measure is used to capture the expected need for liquidity in case the withdrawals exceed the short term liquidity the banks decided to hold.

8. This additional liquidity need leads to a liquidation of loans. However, these loans can only be liquidated accepting a haircut, HC.

9. Therefore, the last term represents the expected cost of holding insufficient liquidity.

The whole equation can be interpreted in the following way: The bank generates revenue by lending out loans and has two options to finance these loans. The first is to use equity or long-term financing, this source is risk-free but demands the same return as the loans generate, adjusted for the possibility that the bank is illiquid. Thus, long-term financiers have access to the same investments than the bank and are risk neutral.

In this setting, in case of a liquidity shortfall, the banks cannot pay any interest to their long-term financiers, for which they need to be compensated by higher rates. However, the possibility of defaulting on the long-term financing or the loss of equity is not considered explicitly and therefore there is no explicit compensation for this. Instead, the bank is penalized only by the lower liquidation value of loans in case of illiquidity that directly reduces the bank's profit without linking it to the cost of equity or long-term financing.

For simplicity, I will assume that the banks' decision is only about the asset side of the balance sheet, i.e. the banks maximize their profit deciding whether to invest in liquid assets or lend out loans. The liability side of the bank is seen as exogenous. This seems pretty plausible when we take a short-term view about the banks balance sheet adjustments as especially for small banks it is easier to raise or lower liquidity than issuing new equity.

The Umbrella Organisation

The idea of an umbrella organisation in this context is straight forward. The small banks fund an institution that aggregates all the liquidity shocks the small banks face. The idea is that while ordinary daily deposit volatility forces small banks to hold a lot of short-term liquidity, the umbrella organisation is able by aggregating these idiosyncratic shocks to hold much lower amounts of liquidity without increasing the cost of liquidating loans. In this sense the umbrella organisation allows its members to hedge at least the idiosyncratic deposit volatility.

In its most basic form the umbrella organisation is not a decision maker, respectively profit maximizer, but simply a superior technology that the small banks get access to. Assuming independence of the small banks deposit shocks the new technology would allow them to completely eliminate this noise as the number of banks goes to infinity.

In such a setting, perfect liquidity hedging would of course be optimal. To break this result, I would like to introduce systemic deposit shocks that affect all banks the same way, e.g. bank runs at the whole sector. If the risk of such events is explicitly modelled the result depends on how the umbrella institutions liquidated in case of default and how exactly the function the institution maximizes looks like.

At first I want the umbrella organisation not to be a decision maker but keep being simply a superior technology for small banks as before. The advantage of this approach is that this allows the small banks to simply maximize their profit choosing liquidity and credit-line at the same time. In the last part I want to analyse the situation when the credit-line is set before the small banks set the liquidity.

Deposit Outflows

In my master thesis liquidity is modelled as a combination of two probability distributions. The first distribution assigns a high probability to a no-liquidity event (0) and a low probability of a liquidity event ($-\mu$). This process affects all banks the same way and defines the mean of the second part. Therefore, liquidity events are regarded as something completely exogenous. In order to be able to compare the results I corrected the mean of the joint distribution such that it is zero no matter the shock size. The second part I would like to model by a symmetric probability distribution around the mean defined by the first part. This distribution is identical for the individual banks but for each bank an independent variable defining the deposit withdrawals is drawn. For this part I consider the normal distribution as it is continuous and differentiable.

The Relationship between Small Banks and the Umbrella Institution

The main idea is that the umbrella organisation is set up by the small banks to allow them to hedge the liquidity volatility. The idea is to model this relationship in a different way than the other relationships that have been analysed by previous papers. On the one hand, there is the bank's relationship with depositors. There are definitely some similarities of a bank's relationship to the customer as modelled by Diamond and Dybvig, while here the bank is the umbrella organisation's customer. However, while in the Diamond-Dybvig model the banking solution increases welfare by insuring depositors against liquidity shocks and bank runs are explained by a second equilibrium, in my model bank runs are explicitly modelled as exogenous and the model is based on the idea how this bank run risk is optimally shared between the bank and its umbrella institution. On the other hand, I want to discuss the difference of a bank's relationship to central banks or the bond market. The key difference to central banks as a lender of last resort is that the umbrella organisation is itself constraint and can only channel funds from lending to borrowing banks. Bond markets on the other hand are not making any decisions as umbrella organisations may do.[9]

[9] Diamond, D. W., and P. H. Dybvig. "Bank Runs, Deposit Insurance, and Liquidity." Journal of Political Economy 91.3 (1983).

Modelling Idea

The idea of the model is to explain the limits of liquidity hedging, meaning that not just the umbrella organisation holds liquid assets but also the small banks do to some extent. This is explained by the higher resilience of small banks to systemic events, rather than asymmetric information or moral hazard. The latter may be alternative explanations for the following phenomena: at the one hand asymmetric information may be a problem if the umbrella organisation cannot distinguish between banks that need liquidity and those that are insolvent or for other reasons use the credit-lines as a funding source. The moral hazard problem arises when the bank changes its financing structure due to the complete hedge. For example, the bank increases its dependency on deposits, which would lead to an increase in deposit outflow volatility.

3.1 Results

In this section I want to present the results of the model. I want to start with looking at individual banks in isolation and than further complicate the model step by step. Due to the complexity of the involved mathematics it may be necessary to evaluate the models numerically.

Small Banks in Isolation

In this first step I analyse the optimal holding of liquid assets of small banks in isolation. There each bank maximizes it's profit function with respect to L_s.

$$\max_{L_s} \quad \Pi_{SB}\left\{L_s, r, E, D_o, HC\right\}$$

This maximization problem is solved by the first order condition:

$$\frac{\partial \Pi_{SB}(L_s)}{\partial L_s} = 0$$

In order to solve this problem and be able to calculate the optimal liquidity holding and the corresponding expected profit, it is necessary in this case to use a specific distribution function for the deposit outflows. I decided to use a normal distribution, which is on the one hand is relatively simple to use for calculation and on the other hand its density function is continuous and differentiable.

The profit function of a small bank as discussed above in addition to the new assumptions would look as follows:

$$\Pi_{SB}\left\{L_s, r, E, D_o, HC\right\} =$$

$$(1 - L_s)r - E\frac{r}{1-\int\limits_{L_s}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{D_o^2}{2\sigma^2}}dD_o} - \frac{1}{1-HC}\int\limits_{L_s}^{\infty}(D_o - L_s)\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{D_o^2}{2\sigma^2}}dD_o$$

For the calculations and the graphics I also used specific parameter specifications and further I assume that the bank holds a positive amount of liquid assets to a maximum of 100%, i.e. $0 \leqslant L_s \leqslant 1$:

- equity: $E = 0.2$
- interest rate: $r = 0.05$
- deposit volatility: $\sigma = 0.15$
- hair cut:$HC = 0.3$

This specification would result in the following first order condition:

$$-r + \frac{E*r*e^{-\frac{L_S^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma(1+erf(\frac{L_S}{\sqrt{2}\sigma}))^2} + \frac{1}{1-HC}\frac{(1-erf(\frac{L_S}{\sqrt{2}\sigma}))}{2} = 0$$

The solution is given by solving the above equation for L_s. This is where the first derivative of the profit function with respect to L_s intersects with the x-axis as shown in the graph below:

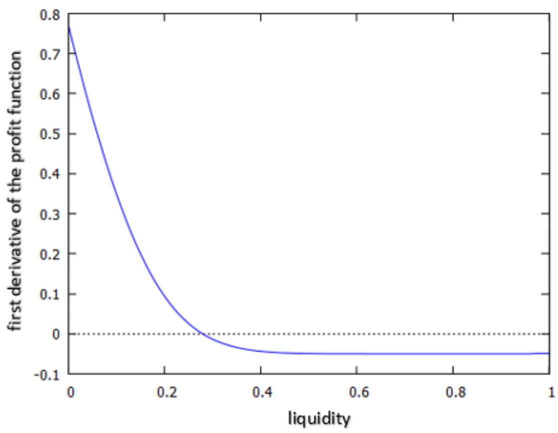

figure 1: optimal liquidity

As one can see the problem has one interior solution candidate and thus three in total:

1. If no liquidity is held by the bank the profit would be about -0.0555 or about -5.6%
2. If the bank only holds liquid assets the profit would be -1%.
3. For $L_s =$ 0.279 the corresponding profit would be 0.0231 or about 2.3%. This is the maximum of the defined range as the profit in the tails and the surroundings is lower.

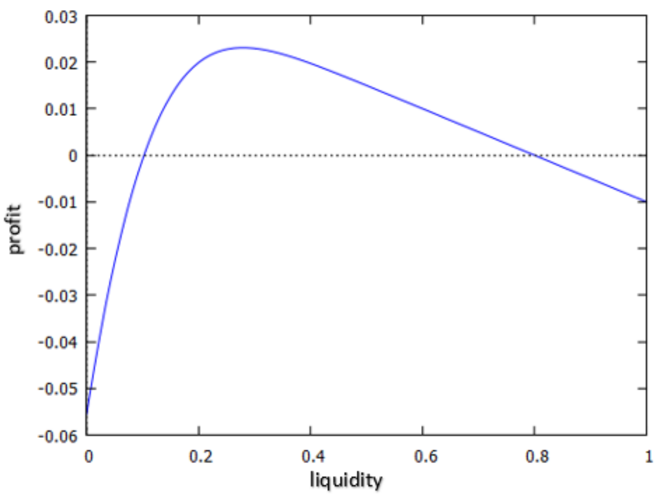

figure 2: profit function

The Umbrella Organisation with only Idiosyncratic Risk

When the small banks are only affected by idiosyncratic risk this risk can be diversified by an umbrella organisation that hedges those risk. In a way such an institution can pool all the excess liquidity and liquidity needs by offering the small banks a deposit account with an infinite credit line. The idea is that the small banks can commit to put deposits at this account whenever they experience a below average deposit outflow while the umbrella organisation commits to distribute this excess liquidity to the banks that need liquidity as they experience above average outflows.

According the law of large numbers the total liquidity needed converges almost surely to the expected value when the number of independent draws, i.e. the number of small banks that participate, converges towards infinity. As a consequence this would allow the banks to hold much less liquidity and still be able to cover the deposit outflows.[10]

If it is assumed that the mean liquidity outflow is zero and there is an infinite number of small banks this would allow the small banks to only invest into loans and not to hold any liquidity. Any liquidity excess by one bank is channelled via the umbrella institution to the banks that need liquidity. It is important to note that in this setting the small banks are committed to transfer excess funds and they are also not allowed to use the credit-lines as a funding source to over-invest.

In this simple version of the model the umbrella-institution is just a more advanced technology, that the small banks have excess to and not a player in a game. Thus, the profit function for the small firms would look as follows:

$$\Pi_{SB} = (1 - L_s)r - E * r \text{ with the constraint } 0 \leqslant L_s \leqslant 1$$

As this profit function is strictly decreasing in L_s it is intuitive that the optimal level of L_s is at the lower bound of the constraint where $L_s = 0$.

For the above defined parameter specification this would result in the following profit:

[10] Rosenthal, J. S. "A first look at rigorous probability theory." World Scientific Publishing Company, (2006). Theorem.3.4.2, p.35

$$\Pi_{SB} = r - E * r = 0.05 - 0.2 * 0.05 = 0.04$$

As the balance sheet is normalized to one it can be interpreted as a return of 4%.

The Umbrella Organisation for two Banks

Before analysing systemic shocks, I want to see how an umbrella institution for only two banks would look like to get a better insight into the underlying mechanism. Here the idea is that the umbrella institution channels the funds of just two banks. As the two deposit shocks the banks face are independent, there is certainly room for a pareto improvement in the cases where one institution has excess funds while the other has a liquidity need. The mechanism via which this is achieved is basically that a credit-line is granted by the umbrella institution to the banks such that the institution that needs liquidity draws the credit-line while the other institution deposits the excess liquidity at the umbrella institution.

However, as there are just two institutions it is not unlikely that both of them are in need of liquidity at the same time or in general that the demand for liquidity is higher than the supply. Therefore, I have to specify what happens in those cases. A reasonable approach is to assume that the umbrella institution has excess to outside financing. This outside financing should be relatively costly. One idea behind the source of financing would be that the umbrella institution could access funds only by pledging collateral. The more liquidity the umbrella institution needs, the more collateral is required. Arguably, the institution would first pledge the collateral that is cheapest before excessing collateral of worse quality. To reflect this, I introduce a cost function for the outside funding that is increasing exponentially in relation to the amount of liquidity that is needed.

For simplicity, in this case I only consider the case where the umbrella organisation is operating on behalf of the small banks and making zero profits in expectation.

The model consists of two parts. First, the umbrella institution sets the price for any combination of credit-line the small banks may demand and the liquidity they will hold in order to in expectation break even. Then the small banks decide how much of the credit-line they demand and how much liquidity they hold. Finally, the liquidity withdrawals are realized and the pay-offs assigned.

For this setting I assumed that the small banks would first use their own liquid funds before drawing on the credit-line. An umbrella organisation that would set the price for the credit-line equal to its own cost would price the credit-line as function of the liquidity reserves of the small bank and credit-line. Therefore, the cost $i(L_s, c)$ is the ex ante expected cost for liquidity needs. This means for all the possible cases where the deposit outflow of the small banks exceeds the inflows one has to calculate the expected cost times the probability of this case to occur:

$$i(L_s, c) = (1+g)^{\int\limits_{c+L_s}^{\infty}\int\limits_{c+L_s}^{\infty} 2c\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}dydx} \int\limits_{c+L_s}^{\infty}\int\limits_{c+L_s}^{\infty} 2c\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}dydx$$

$$+(1+g)^{\int\limits_{L_s}^{c+L_s}\int\limits_{L_s}^{c+L_s} (x+y-2L_s)\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}dydx} \int\limits_{L_s}^{c+L_s}\int\limits_{L_s}^{c+L_s} (x+y-2L_s)\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}dydx$$

$$+2[(1+g)^{\int\limits_{L_s}^{c+L_s}\int\limits_{c+L_s}^{\infty} (x+c-L_s)\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}dydx} \int\limits_{L_s}^{c+L_s}\int\limits_{c+L_s}^{\infty} (x+c-L_s)\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}dydx$$

$$+(1+g)^{\int\limits_{0}^{L_s}\int\limits_{c+L_s}^{\infty} (c)\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}dydx} \int\limits_{0}^{L_s}\int\limits_{c+L_s}^{\infty} (c)\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}dydx$$

$$+(1+g)^{\int\limits_{-c}^{0}\int\limits_{c+L_s}^{\infty} (x+c)\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}dydx} \int\limits_{-c}^{0}\int\limits_{c+L_s}^{\infty} (x+c)\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}dydx$$

$$+(1+g)^{\int\limits_{0}^{L_s}\int\limits_{L_s}^{c+L_s} (y-L_s)\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}dydx} \int\limits_{0}^{L_s}\int\limits_{L_s}^{c+L_s} (y-L_s)\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}dydx$$

$$+(1+g)^{\int\limits_{-c}^{0}\int\limits_{L_s-x}^{c+L_s} (x+y-L_s)\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}dydx} \int\limits_{-c}^{0}\int\limits_{L_s-x}^{c+L_s} (x+y-L_s)\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}dydx]$$

Here c represents the credit-line, the maximum amount each bank can withdraw from the umbrella-institution. i(c) is the price the small banks have to pay up-front for the total credit line. x and respectively y are the amounts that are actually withdrawn by the two banks. g is a growth rate that represents how fast the cost of refinancing is increasing for the umbrella institution if it needs to raise more and more funds.

The first term is the expected cost if both banks draw the whole credit-line. The next part represents the situation where both draw a part of the line but neither the whole line. Then the next terms are multiplied by two as the situation is symmetric for the two banks.

So, the third term reflects one withdrawing the whole limit, while the other withdraws but not up to the limit. Then, one draws the whole line, while for the other the own liquid reserves suffice to meet the deposit outflows. Further, one uses the whole line while the other deposits some excess reserves. The next term represents one using its own liquid assets only, while the other also uses the credit-line but not to the full extent. And finally one deposits excess funds while the other draws part of the credit-line and the withdrawn amount exceeds the deposits.

The profit function of the two small banks would then be similar to the profit function in isolation, just adding the cost of the credit-line and the corresponding lower default risk:

$$\Pi_{SB}\left\{L_s, r, E, D_o, HC\right\} =$$

$$(1-L_s)r - E\frac{r}{1-\int_{L_s+c}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{D_o^2}{2\sigma^2}}dD_o} - \frac{1}{1-HC}\int_{L_s+c}^{\infty}(D_o - L_s - c)\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{D_o^2}{2\sigma^2}}dD_o - i(L_s, c)$$

The plot of the function is not entirely exact as the software is not able to plot the last term of the cost of the credit-line as a function of L_s and c. However, I approximated the absolute amount of the term in the optimum and the marginal effects of the term are negligible.

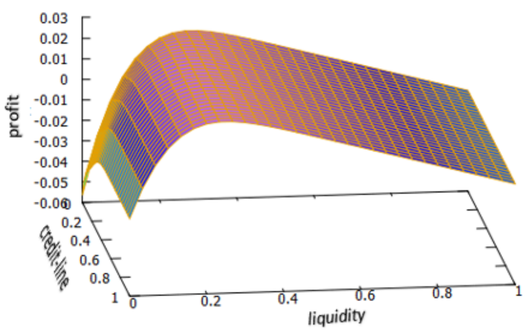

figure 3: profit as function of L_s and c

In this setting the small banks would optimally hold about 24.7% (0.2468) of their assets in short-term liquid securities and the credit-line would then be set to infinity. Extending the credit-line has a decreasing effect on the total profit as default becomes less and less likely. The total profit for the small banks would be about 2.4% (0.0235).

Small Banks in Isolation with Systemic Risk Component

For small banks in isolation there is no real difference between the system and the idiosyncratic component of the risk. The only difference is that the probability would look different if systemic risk is modelled as a mean-shift of the initial function that occurs with a small probability.

In addition to the parameters specified above the systemic liquidity shock is specified as follows:

- probability of systemic shock: $p = 0.05$
- deposit outflow through systemic shock: $k = 0.6$

The probability density function of this joint function would just be the weighted sum of the two probability density functions.

$$f(D_o) = p\frac{e^{-\frac{(D_o-k+p*k)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} + (1-p) * \frac{e^{-\frac{(D_o+p*k)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}$$

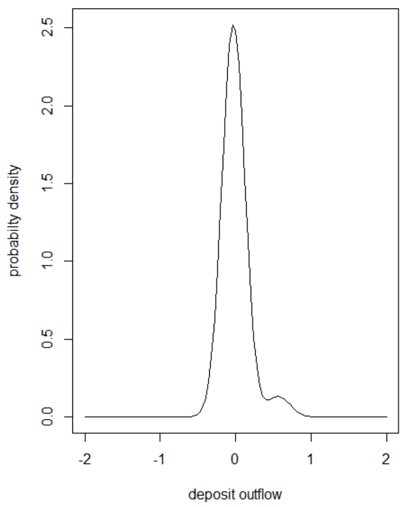

figure 4: probability density of withdrawals

Note that the density function has a mean of zero. This allows to compare different shock sizes as the mean does not change and can be interpreted that in normal times banks experience small deposit inflows, while some times there are huge withdrawals due to systemic shocks.

For the small banks in isolation nothing fundamentally changes on account of the introduction of the systemic risk component:

$$\Pi_{SB}(L_s) = (1 - L_s)r - \frac{E*r}{1 - \int_{L_s}^{\infty}(\frac{p}{\sqrt{2\pi}\sigma}e^{-\frac{(D_o-k+p*k)^2}{2\sigma^2}} + \frac{1-p}{\sqrt{2\pi}\sigma}e^{-\frac{(D_o+p*k)^2}{2\sigma^2}})dD_o}$$

$$-\frac{1}{1-HC}\int_{L_s}^{\infty}(D_o - L_s)(\frac{p}{\sqrt{2\pi}\sigma}e^{-\frac{(D_o-k+p*k)^2}{2\sigma^2}} + \frac{1-p}{\sqrt{2\pi}\sigma}e^{-\frac{(D_o+p*k)^2}{2\sigma^2}})dD_o$$

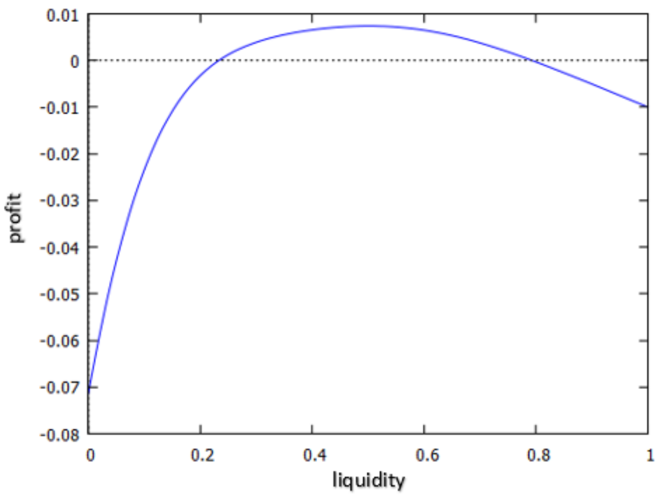

figure 5: profit function with shock

The first derivative of the profit function with respect to the choice variable L_s can be found in the appendix.

Similar to the previous situation, the problem has one interior solution candidate:

1. $L_s =$ 0.501 this would result in a profit of 0.0074 or about 0.7% return on assets.
   After checking for the surrounding I concluded that this constitutes maximum.

2. If no liquidity is held the profit would be about -7.1% (-0.0714) and close to -1% if the bank only holds liquid assets.

The Umbrella Organisation with Systemic Risk

For the umbrella institution a problem arises when systemic risk is introduced. As all banks are effected by this shock in the same way, the umbrella organisation cannot hedge this risk. The institution would face a probability density function with two spikes one at zero with a large probability and one with at negative value with a small probability.

The result of the model depends on what happens in the case that the umbrella institution defaults and whether it is able to get some equity financing to absorb such shocks or whether it can insure against systemic shocks. If for example insurance at a fair premium was possible the advantage of an umbrella-institution would be similar to the case of no systemic risk. However, this assumption would be quite implausible.

Setting Credit-Line and Liquid Reserves simultaneously- Social Optimum

For this section I want to assume that the umbrella institution is an organisation set up to serve in the interest of the small banks. Such an institution would offer credit-lines to an infinite number of small banks and would price these credit-lines in such a way that it would break even in expectation. The idea here is that if a small bank would experience liquidity outflows above its liquid asset holdings, it would draw the credit-line before defaulting which it can up to the maximum level of c which is set in advance.

So what actually happens is that each of the infinitely many small symmetrical banks would first decided how much liquidity it holds and how large the credit-line should be. When pricing the credit-line the optimal liquidity holding is considered, as well. Then if it experiences deposit inflows it would deposit them at the umbrella institution, which would use this funds to provide liquidity to those banks that need it. If the deposit withdrawals do not exceed the liquidity the small bank holds the umbrella institution does not receive any funds but also is not required to pay out any funds. If the deposit withdrawals exceed the liquid assets a bank holds it can draw the credit-line up to the pre-specified amount c. If the withdrawals are even higher the small bank defaults as discussed in the early sections. However, in this case the umbrella institution would still experience outflows of c.

This setting also requires to talk about the case where a larger amount of liquidity is withdrawn than deposited at the umbrella organisation. In this case I assume that the organisation has access to outside financing. However, this outside funding is becoming increasingly expensive as the amount that has to be raised increases. I assume that it is increasing exponentially at the rate g.

The first question would be at what price could the umbrella institution offer the credit-lines for any given amount of liquidity the small banks would than hold in the optimum. As there are an infinite number of many small banks by the law of large numbers I can reduce the problem to analysing two cases. The first one would be that no-systemic shock occurs and thus the average deposit withdrawals would be negative. This implies that the umbrella organisation would not suffer any losses. The other situation would be a systemic shock. For this case one has to calculate the average deposit outflow per bank the umbrella organisation faces. Therefore, the umbrella institution would set the following price for the credit-line.

$$i(c, L_s) = p(1+g)^{D_{mo}} D_{mo}$$

This is the probability of a systemic shock, which is the only case the outflows exceed the inflows, times the the average credit small banks need in case of a systemic shock, where D_{mo} and times the cost which is exponentially increasing in D_{mo}. The average credit-line drawn in case of a systemic shock is:

$$D_{mo} = \int_{-\infty}^{0} D_o \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(D_o - k + p*k)^2}{2\sigma^2}} dD_o + \int_{0}^{Ls} 0 \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(D_o - k + p*k)^2}{2\sigma^2}} dD_o$$

$$+ \int_{L_s}^{L_s+c} (D_o - L_s) \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(D_o - k + p*k)^2}{2\sigma^2}} dD_o + \int_{L_s+c}^{\infty} c \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(D_o - k + p*k)^2}{2\sigma^2}} dD_o$$

- Here, c is the credit line the organisation grants to each of the small banks.
- L_s is the liquidity small banks hold.

- i is the amount the small banks have to pay for the credit-line.
- D_o are the deposit outflows.
- g is the growth rate of the external financing cost, which I set here to 0.5.

Once, we have this cost we add this decision variable to the initial profit function of the small bank in isolation and maximize the new profit function with respect to the two decision variables, c and L_s

$$\Pi_{SB}(c, L_s) = (1 - L_s)r - \frac{E*r}{1 - \frac{p}{\sqrt{2\pi\sigma}}e^{-\frac{(L_s+c-k+p*k)^2}{2\sigma^2}} - \frac{1-p}{\sqrt{2\pi\sigma}}e^{-\frac{(L_s+c+p*k)^2}{2\sigma^2}}}$$

$$-\frac{1}{1-HC}\int_{L_s+c}^{\infty}(D_o - L_s - c)\left(\frac{p}{\sqrt{2\pi\sigma}}e^{-\frac{(D_o-k+p*k)^2}{2\sigma^2}} + \frac{1-p}{\sqrt{2\pi\sigma}}e^{-\frac{(D_o+p*k)^2}{2\sigma^2}}\right)dD_o - i(c, L_s)$$

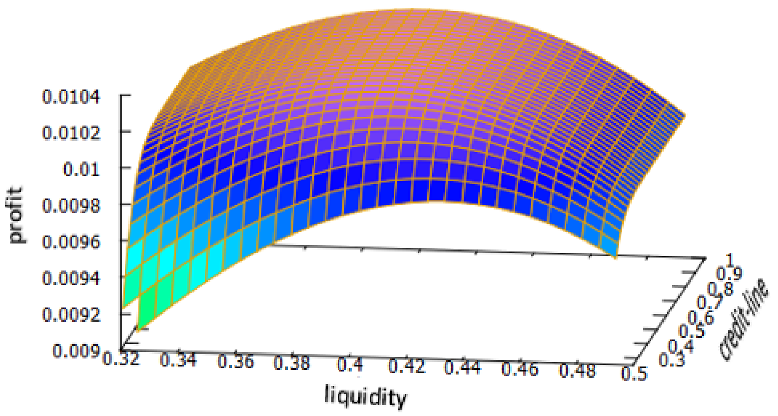

figure 6: profit as a function of L_s and c

The maximum of the profit function is found by analysing graphs of the profit function generated by the maxima software. The result can also be explained by theory. The optimal liquidity small banks should held is about 0.4024 or 40.2% of their assets. The credit-line would then be optimally set at infinity although the profit doesn't really change once c is large enough (∼0.4). The theory behind this is the following: up to a certain point it is cheaper to use liquid assets rather than the credit-lines, however, at some point the credit-lines become cheaper. Note that the higher the liquid assets the small banks hold the cheaper are the credit-lines as the risk that they a drawn decreases. When the liquid asset holding reaches

0.4024 the cost for the credit-line drops below 5%, which is the opportunity cost of holding liquid assets and thus small banks use credit-lines from than onwards. In principle credit-lines become more expensive the more they are extended, however, here apparently the cost stays below the cost for liquid assets even if the credit-line is increased to infinity. The profit in this case would be about 0.01034.

This gives rise to the question why a bank that has access to an infinite credit-line would hold any liquid assets in the first place. In this example this is the case because neither the umbrella institution nor the small banks act strategically but just combine their possibilities to reach a socially optimal solution. The small bank is maximizing its profits taking into account that the umbrella institution passes on the cost. In the next section I want this result to break down.

Fixing Credit-Line before Liquidity Reserves

In this section, I want the umbrella institution to specify a fair price for the credit-line and then the small banks decide, how much of the credit line they demand. And after agreeing on the credit-line the small banks would set their liquidity reserves given the credit-line.

So in this setting, although trying to maximize the small banks' profit while breaking even, the umbrella institution is an independent player that tries to anticipate what the small banks will do when they set the liquidity reserves. The game is solved by backward induction. Therefore, the first step would be to determine the optimal amount of liquidity a small bank should hold for any given credit-line.

This is achieved setting the first derivative of the profit function specified above with respect to the choice variable L_s equal to zero. Here, I want to indicate this graphically.

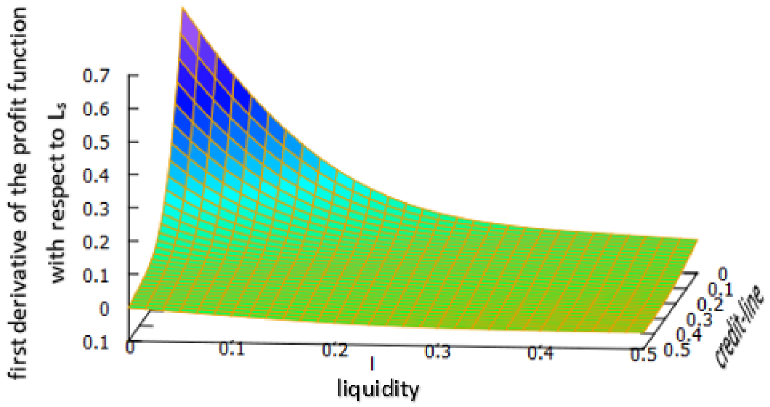

figure 7: first derivative with respect to L_s

The best response of the small banks to any c set would be to add up the sum $(L_s + c)$to 0.501. Which is also what they would choose in isolation where c would be zero.

Taking this under consideration the umbrella institution would maximize the small banks' profit while breaking even in expectation. Or in other words the umbrella institution would set the prices for the credit-line such that it would break even and the small banks would choose the optimal credit-line to maximize their profits. This would lead to the following solution of the problem:

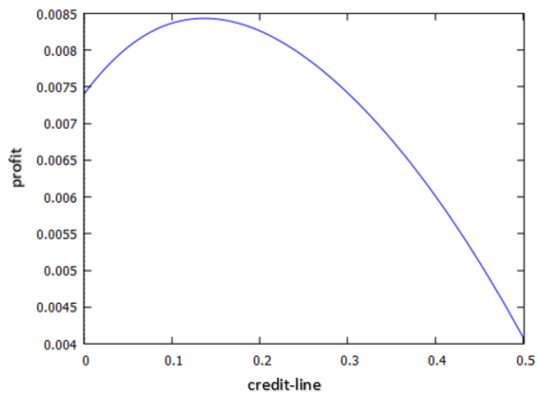

figure 8: profit as a function of c

The profit would be maximized at a credit-line of 0.137 and consequently liquid assets of 0.364. At that point the marginal cost for the credit-line would exactly equal the marginal opportunity cost of more liquidity, which is constantly the interest rate (5%). The profit would be 0.0084.

3.2 Comparison

In this part I want to compare the results of the different situations discussed above. The aspects across which the models can be compared are the optimal liquidity held by the small banks in isolation, their profits and the probability of illiquidity.

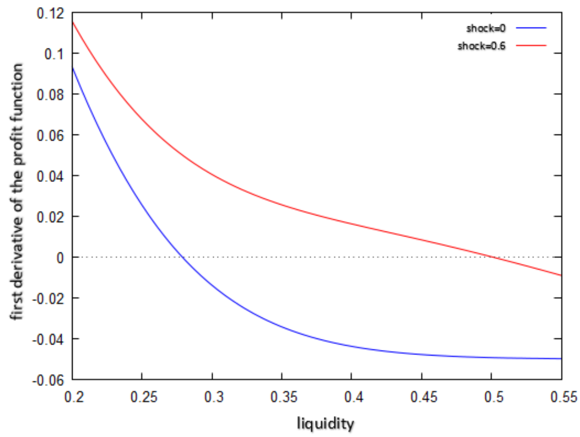

figure 9: comparison of shock and no-shock optimal liquidity

As this graph shows for this parameter specification the small banks in isolation would increase their liquidity holding in the optimum, when the systemic risk is introduced. This is quite intuitive as one would expect a bank to increase it's liquid assets when the liquidity risk increases.

In the next section, I will discuss in more detail what happens when the magnitude of the probability shocks decreases and thus the density function has just one peak but a heavy tail.

3.3 Comparative Statics

In this section I want to focus primarily at the magnitude of the systemic risk shock, as I think this might be the most interesting to analyse in this context.

The first figure is a comparison of the first derivative of the profit function of small banks' in isolation with respect to the liquidity holding depending on the magnitude of the systemic liquidity shock=k.

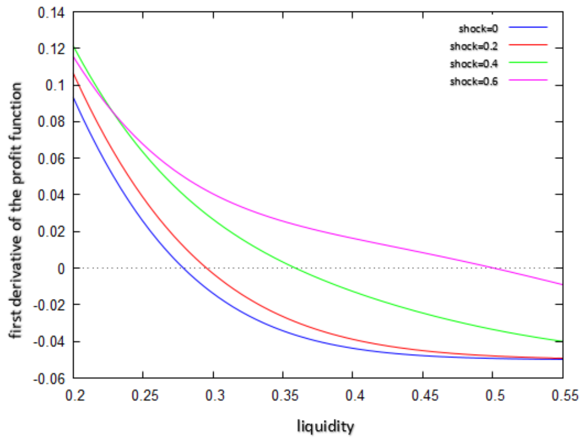

figure 10: optimal liquidity for different shock sizes r=0.05

As one can see here the optimal liquidity such isolated banks would hold decreases when the shock sizes decreases.

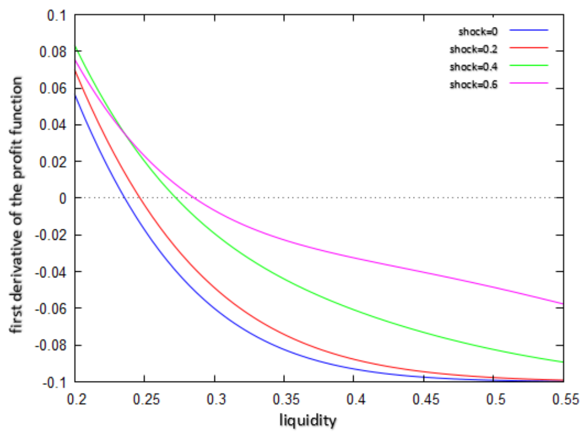

figure 11: optimal liquidity for different shock sizes r=0.1

In general a larger systemic shock drives isolated banks to hold more liquidity as the probability of default increases, the marginal benefit of holding more liquidity increases while the marginal cost, which is r, the interest rate that could be earned by lending loans, stays the same. On the other hand, as one can see here when comparing figure 10 and 11 when the interest rate increases then the optimal liquidity decreases as the opportunity cost increases.

The lower the interest rate the higher is the optimal amount of liquidity the bank should hold. Consequently, when the interest rate is increased, at some point the curve does not cross the x-axis in this case no liquidity holding is optimal. This is also intuitive as when the interest rate is high, opportunity cost of holding liquidity is increasing. On the other hand, when interest rates drop to zero there is no need to invest in interest bearing loans and the optimal liquidity holding would be 100%.

The interest effect for small banks that have access to an umbrella institution is even stronger. An higher interest rate would make credit-lines relatively more attractive. Thus when credit-line and liquidity are set simultaneously in the setting I used, the liquid assets proportion would drop to 14% if the rate increases to 7% and no liquid assets would be held at a rate of 10%.

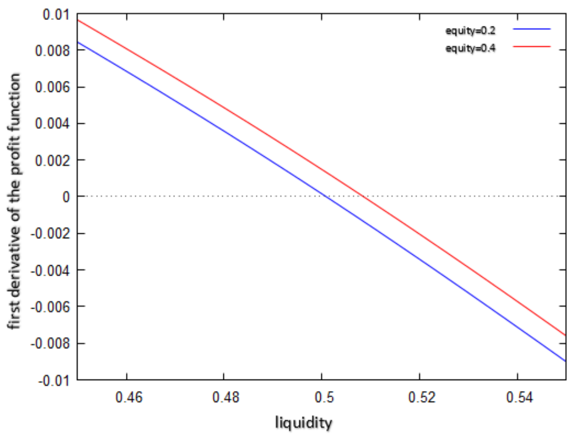

figure 12: optimal liquidity for different equity

Higher equity or long-term financing leads to slightly more liquidity that is held by isolated banks in the optimum. This is not really intuitive as one would assume that higher equity financed institutions are less likely to default due to liquidity shocks at the funding side. The reason for the effect here is that I only increased the equity of the bank and kept the size of the deposit shocks the same and did not decrease them as one would assume is the case when the bank uses less deposits as funding source. Thus, the main effect is that the penalty for not repaying the equity holders in case of illiquidity increases, which the bank adjusts for by increasing liquidity and consequently lowering the risk of illiquidity.

3.4 Interpretation

What can definitely be concluded when comparing the different model settings is definitely that small banks can increase their profits decisively if they have access to an umbrella institution. This can be seen when comparing the models without systemic risk. While an isolated bank can expect a profit of 2.31% this already increases to 2.35% when two banks pool the deposit withdrawals via an umbrella organisation and if an infinite number of small banks do so their profit would increase to 4%.

The same logic applies for banks facing systemic shocks. Here isolated banks can expect a profit of 0.7%, while in case of socially optimal hedging this increases to a bit more then 1%. However, in a more realistic setting where the small banks cannot commit to the amount of liquidity they are going to hold when setting the credit line the profit would be down to about 0.8%.

The initial idea was to show that liquidity cannot be completely hedged by umbrella organisations as systemic risk components would be better hedged by the small banks holding liquid assets on their own balance sheet. This is true as even in the social optimum small banks would hold liquid assets if their is a chance of systemic liquidity events. However, this is not a sign of inefficient risk distribution or even imperfect hedging as in this model it would be still optimal to extend the credit-lines to infinity.

At first I thought this result could not be maintained if I change the umbrella institution into a profit-maximizer itself, such that incomplete hedging could be a consequence of the umbrella institution's management to behave in the interest of their own institute rather than the small-banks'. However, even such a profit maximizing monopolist would offer the social optimal contract as a take it or leave it offer, subtracting all the additional profit, which is the difference to the small bank's profit in isolation. Hence, there would just be a transfer of profits but the situation would still be efficient and perfect hedging via infinite credit-lines possible.

Therefore, I came to the conclusion that the problem was rather a commitment problem between the small banks and the umbrella institution. If the liquidity were decided after the credit-line is fixed the small bank would consider the cost for the credit-line as fixed and would not take it into account when it decides how much liquidity it wants to hold. Such a small bank would always be able to meet the same deposit outflows than a small bank in isolation would no-matter how large the credit-line was. So the credit-line plus own liquid funds would be equal to the liquid funds a small bank in isolation would hold. Therefore, even an umbrella institution that only tries to break even would in the optimum just be able to offer credit-lines that are not infinite. If it would offer an infinite credit-line the small banks would hold no liquidity. This would make the credit-line so costly that the small bank would not agree to it but only use its own liquidity instead. Thus, in this case deposit volatility is not completely hedged by the umbrella institution via credit-lines.

4 Discussion

First, I want to discuss alternative explanations why it might be the case that liquidity is not perfectly hedged. One explanation I already mentioned before is asymmetric information. The umbrella institution may not be aware whether a given bank is actually just illiquid or insolvent when it wants to use the credit-line. However, in reality the umbrella institutions seem to be pretty aware of the economic and financial situation of the small banks as they actually help them to evaluate this information. Another potential explanation would be moral hazard. This would mean that the small banks respond to having access to the credit-line not just by lowering their own liquidity holding, which is the intention of this option, but also by other forms of over-investing. For example they might start to invest in lower quality bonds or reduce their equity and long term financing. But, given the lack of information asymmetry and the close relationship between the banks and the umbrella institution, it is plausible to argue that this is not the main reason for limited liquidity hedging, as the umbrella institution could figure out ways to discipline the small banks. Further, there are many different channels via which moral hazard could occur in the relationship between the two and thus there must be mechanisms in place that prevent or reduce this risk. The final aspect I want to mention here is regulation. Clearly, if this was the only reason why small banks inefficiently do not hedge liquidity, it would be in their own interest to lobby against those regulations and also there is no reason why the regulating authority should enforce such inefficient rules.

Of course the reality is much more complex then suggested in the model and especially as there are more than just one sort of liquid assets banks hold. Some of these assets may be used before the credit-line is drawn, while other less liquid assets or more costly liquid sources may be used after the credit-line is fully used. Nevertheless, focussing on the liquid assets that are used before the credit-line is used is the best way to analyse the small banks' relationship with the umbrella institution, as these are the assets that affect the expected use of the credit-line the most.

Another point to discuss would be the time horizon of the model. Usually asset and liability management is concerned about meeting the daily obligations, while leaving intra day liquidity provision to the bank‘s treasury. However, it is also possible to think of the model covering a long time horizon talking about weakly or even monthly deposit fluctuations.

The difference in the relationship between the umbrella institution and the small banks to other relationships (bank - customer, central bank - bank, interbank market - bank or bank - bank), also leads to different results.

While in the Diamond-Dybvig model that analyses a customer - bank relationship, the banks hold liquid assets that allow them to pay out the expected withdrawals, they do not take any precautionary measures for bank-runs, which are seen as another equilibrium. As there is no defined probability for a bank-run to occur, whether customers use bank loans or invest on their own, which would in my setting correspond to using credit-lines or liquid assets, it would entirely depend on customers believes about which equilibrium will occur at the later stage and can therefore not be properly calculated.[11]

Comparing my setting to the interbank deposit contracts banks use in the Allen and Gale (2000) paper, the key difference in the results is that they don't solve a maximization problem about how much liquidity they should hold. In general the

[11] Diamond, D. W., and P. H. Dybvig. "Bank Runs, Deposit Insurance, and Liquidity." Journal of Political Economy 91.3 (1983).

paper is more about how such shocks spread in the system rather then looking for the optimal capital structure for the banks.[12]

The central bank's role as discussed by Rochet and Vives (2004) is different to that of an umbrella institution as the central bank has itself no liquidity constraints and therefore is not pooling the liquidity demands of many banks as it is discussed in my paper.[13]

Eisenbach et al. analyse optimal balance sheet's for banks but not the engagement with a counter party such as an umbrella institution.[14]

The papers that are closest to mine in their results are Bhattacharya and Gale (1987) and Bhattacharya and Fulghieri (1994). Apart from the problem of misreporting the true liquidity demand the first also finds underinvestment in liquidity in case of an interbank market. However, in their setting the banks decide first knowing there will be an interbank market. Moreover, they do not explicitly discuss the effect of this underinvestment at the interbank lending market. The second paper talks about the effect at the later-stage inter bank market for liquidity but rather about setting the rates at this market in a way that potential lenders have an incentive to lend. I didn't talk about this mechanism in my paper as I assumed deposit inflows would be deposited anyway.[15]

5 Conclusion

The main takeaway of my thesis is that umbrella organisations are an effective way for small banks to increase their profits by pooling independent volatility at the funding side and therefore hedging this risk. However, when apart from this

[12] Allen, F., and D. Gale. "Financial contagion." Journal of political economy 108.1 (2000).

[13] Rochet, J.-C., and X. Vives. "Coordination Failures and the Lender of Last Resort: was Bagehot right after all?" Journal of the European Economic Association 2.6 (2004).

[14] Eisenbach, T. M., T. Keister, J. McAndrews, and T. Yorulmazer. "Stability of funding models: an analytical framework." Economic Policy Review, 20.1 (2014).

[15] Bhattacharya, S., and D. Gale. "Preference shocks, liquidity and central bank policy, New Approaches to Monetary Economics." W. Barnett, and K. Singleton, eds., Ch. 4 (1987).

Bhattacharya, S., and P. Fulghieri. "Uncertain liquidity and interbank contracting." Economics Letters 44.3 (1994).

idiosyncratic volatility there is also a systemic component to liquidity risk apart from relying on credit-lines from the umbrella institution it is efficient that each bank in addition holds own liquid assets to reduce the cost of liquidity through systemic risk. It is also reasonable to assume that there might be a commitment problem that leads to an inefficient situation. This happens when the small banks can not commit to hold a certain amount of liquidity when the credit-lines are set. This would make such credit-lines so expensive that they are only granted to a limited extent. This forces the banks to hold even more liquidity then in the efficient situation. Or in general leads to suboptimal precautions for liquidity risk.

Another interesting thing to note is the effect of low-interest rates. In my partial models that only consider bank behaviour low interest rates would actually lead banks to hold more liquid assets, e.g. government bonds, instead of giving out loans to the real economy. This may be a mechanism that works in the opposite direction of central banks' intentions when lowering interest rates. Here, lower rates would not lead to cheap money to be lend to the economy but rather driving out money from real investments to inflating the price of highly liquid assets as the opportunity cost of holding liquidity decreased.

Coming back to the issue the director of the union bank faced, the problem would be if withdrawals in normal times exceed the sum of the credit-line granted by the umbrella-institution and some of the most easily available liquid funds. It is also plausible to assume that there is a hurdle or cost to restrict the availability of the credit-line at the umbrella institution in order to reduce the commitment problem I discussed.

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7 Mathematical Appendix

first derivative the small banks' profit function in isolation:

$$\frac{\partial \Pi_{SB}(L_s)}{\partial L_s} = -r + \frac{E*r*2(\sqrt{2}(p-1)e^{-\frac{(L_s+p*k)^2}{2\sigma^2}} - \sqrt{2}p*e^{-\frac{(L_s+p*k-k)^2}{2\sigma^2}}}{\sqrt{\pi}\sigma(3-perf(\frac{L-s+p*k-k}{\sqrt{2}\sigma})-(1-p)erf(\frac{L-s+p*k}{\sqrt{2}\sigma}))^2}$$

$$+\frac{1}{1-HC}\frac{\sqrt{\pi}\left((p-1)\operatorname{erf}\left(\frac{L_s+p*k}{\sqrt{2}\sigma}\right)-p\operatorname{erf}\left(\frac{L_s+k(p-1)}{\sqrt{2}\sigma}\right)+1\right)-k(p-1)p\left(\frac{\sqrt{2}(L_s+k(p-1))\mathrm{e}^{-\frac{(L_s+k(p-1))^2}{2\sigma^2}}}{\sigma(L_s+k(p-1))}-\frac{\sqrt{2}(L_s+p*k)\mathrm{e}^{-\frac{(L_s+p*k)^2}{2\sigma^2}}}{\sigma(L_s+p*k)}\right)}{2\sqrt{\pi}}$$

$$+\frac{1}{1-HC}\frac{\sqrt{2}\sigma\left(p\left(\frac{(L_s+k(p-1))\mathrm{e}^{-\frac{(L_s+k(p-1))^2}{2\sigma^2}}}{\sigma^2}-\frac{(L_s+kp)\mathrm{e}^{-\frac{(L_s+p*k)^2}{2\sigma^2}}}{\sigma^2}\right)+\frac{(L_s+p*k)\mathrm{e}^{-\frac{(L_s+p*k)^2}{2\sigma^2}}}{\sigma^2}\right)}{2\sqrt{\pi}}$$

$$+\frac{1}{1-HC}\frac{\sqrt{\pi}L_s\left(\frac{\sqrt{2}(p-1)\mathrm{e}^{-\frac{(L_s+p*k)^2}{2\sigma^2}}}{\sqrt{\pi}\sigma}-\frac{\sqrt{2}p\mathrm{e}^{-\frac{(L_s+k(p-1))^2}{2L_s^2}}}{\sqrt{\pi}\sigma}\right)}{2\sqrt{\pi}}$$

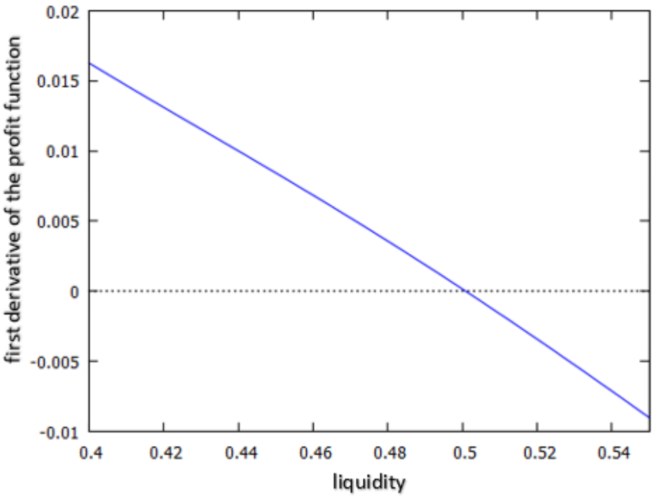

figure A: first derivative of profit function

8 Appendix

8.1 English Summary

My master thesis is about why small banks do not completely hedge against liquidity shocks on the funding side using umbrella institutions. Those shocks are modelled as deposit out- or inflows. There are basically two ways to meet those liquidity outflows, the first would be to invest into more liquid assets at the cost of foregoing more profitable loans. The alternative is to obtain credit-lines with an umbrella organisation that pools the liquidity demands of many small banks. When there are systemic shocks that affect all banks, even umbrella organisations that are expected to break even have to pass on the cost of the possibility of default to the small banks.

Thus, in the case that small banks decide simultaneously about how large the credit-line should be and how much liquidity they should hold, would invest to some degree into liquid assets to reduce the cost of the credit-line. The umbrella institution would then ideally offer an infinite credit-line, which would result in a socially optimal hedging of the liquidity risk.

However, if the small banks cannot commit to invest into liquid assets when the credit-lines are set the umbrella institution would foresee underinvestment into liquidity and thus have to price the credit-line at a higher cost such that the small banks would only demand a limited credit-line and also invest into liquid assets themselves. Compared to the previous situation before such an outcome would be quite inefficient.

8.2 German Summary/ Deutsche Zusammenfassung

Meine Masterarbeit befasst sich mit der Frage warum kleine Banken ihr Liquiditätsrisiko nicht komplett mit Dachinstituten hedgen. Das Liquiditätsrisiko wird als Schwankungen der Kundeneinlagen also auf der Passivseite der Banken modelliert. Im Grunde gibt es für kleine Banken zwei Möglichkeiten sich gegen diese Schwankungen abzusichern. Zum einem können sie selbst in liquidere Aktiva investieren, dies ist allerdings damit verbunden profitablere Investments auszulassen. Zum anderen besteht die Alternative Kreditrahmen mit Dachinstituten zu vereinbaren. Diese Organisationen haben den Vorteil Liquiditätsschwankungen von vielen Banken ausgleichen zu können. Zieht man allerdings die Möglichkeit von systemischen Liquiditätskrisen, die alle Banken gleichzeitig betreffen, in Betracht, müssen diese Dachinstitute, selbst wenn man erwartet, dass sie ausschließlich kostendeckend arbeiten, die dadurch entstehenden Kosten an ihre Banken weitergeben.

Darum würden Banken, in dem Fall, dass sie zur gleichen Zeit über den Kreditrahmen sowie die eigenen kurzfristigverfügbaren Mittel entscheiden, einen Teil liquide veranlagen um einen günstigeren Kreditrahmen zu erhalten. Der Kreditrahmen hätte in dem Fall kein Limit, was zu einer sozial optimalen Lösung führen würde.

Können die kleinen Banken allerdings nicht glaubwürdig vermitteln wie sie ihre liquiden Mittel wählen werden, wenn die Kreditrahmen vereinbart werden, können die Dachinstitute ihre Kreditrahmen, wissend dass die Banken danach nicht ausreichen liquide veranlagen, nur zu höheren Kosten anbieten. Darum veranlagen die Banken in liquide Mittel um einen Teil des Risikos selbst abfangen zu können und wählen nur einen kleineren Kreditrahmen. Dies wäre allerdings verglichen mit der obigen Situation recht ineffizient.