

Leibniz and Bolzano on conceptual containment

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Abstract

Philosophers often rely on the notion of conceptual containment and apply mereological terminology when they talk about the *parts* or *constituents* of a complex concept. In this paper, I explore two historical approaches to this general notion. In particular, I reconstruct objections Bernard Bolzano puts forward against a criterion that played a prominent role in the history of philosophy and that was endorsed, among others, by Leibniz. According to this criterion, a concept that represents objects contains all and only the concepts that represent properties the objects must have in order to be represented by the former concept. Bolzano offers several counterexamples and arguments against the criterion. I argue that while some of them presuppose a strongly mereological understanding of containment, which Leibniz is not committed to, one of them also succeeds without relying on demanding mereological principles.

1 | INTRODUCTION

Talk about the composition of concepts is ubiquitous in philosophy. This talk is commonly an expression of the idea that not all concepts are on a par; some are simple and some complex. According to this idea, complex concepts, unlike simple ones, have other concepts as constituents. Consequently, conceptual analysis, arguably one of the main tasks of philosophy, consists in revealing the constituents of a complex concept and in identifying the way in which the former constitute the latter. Unsurprisingly, this idea has been spelled out in a variety of frameworks and has been invoked over and over again for many important claims in the history of philosophy.¹ To give but a few examples:

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- Locke believes that “all our complex *Ideas* are ultimately resolvable into simple *Ideas*, of which they are compounded, and originally made up” (Locke, 1690 [1975], II xxii §9).
- Hume claims that our impressions and ideas can be divided into simple and complex ones. “The complex are contrary to [the simple ones], and may be distinguished into parts” (Hume, 1739 [1978], 2).
- Moore claims that the concept *good* cannot be defined because definitions break down things into their parts and this concept lacks constituents (Moore, 1903, 7f).

And often the notion of conceptual *containment* is applied in these contexts as a philosophical key notion. Most famously, Kant characterizes analyticity in terms of conceptual containment, namely of the predicate concept's being contained in the subject concept (Kant, 1781/1787 [1998], A6/B10).

While philosophers often rely on such a picture when they discuss the composition of a particular concept, they often remain unclear on what their general account of concept composition is. For this reason, the notion of conceptual containment has been influentially attacked as all too often being “left at a metaphorical level” (Quine, 1951, p. 21). Laying out a general account of concept composition still is an important task and currently, the notion is in fact enjoying an important revival (see, e.g., Fine, 2016).

In this paper, I am concerned with two philosophers who formulated general theories of concept composition in terms of conceptual containment. Leibniz discusses the notion in detail because he takes it to be the key to the notion of truth. Bolzano, the so-called “Bohemian Leibniz,”² developed in great detail a theory of concepts as structured mereological compounds. While Bolzano's philosophical views were heavily influenced by Leibniz, he parts company when it comes to conceptual containment.³ Bolzano argues against a criterion for conceptual containment Leibniz proposes, namely THE PROPERTIES CRITERION, according to which a complex concept contains all and only the concepts that represent properties objects must have in order to be represented by the former concept.

By reconstructing and assessing Bolzano's arguments against this criterion, I reveal that Leibniz and Bolzano are guided by two distinct approaches to conceptual containment. In particular, I show to which extent these approaches comply with the intuitive idea that the relation of conceptual containment is indeed, *pace* Quine, not to be understood metaphorically but as genuine part-whole relation. According to this idea, conceptual containment has to satisfy certain general constraints that are usually assumed to hold for such relations and are studied in mereology. As it will turn out, Bolzano shows that Leibniz's containment relation cannot be a mereological one. While Leibniz might concede this, it will emerge that a serious threat to Leibniz's conception remains. One of Bolzano's arguments succeeds without relying on mereological principles.

I proceed as follows: I first spell out the necessary preliminaries and establish the common ground for the disagreement between Leibniz and Bolzano (Section 2). Then I formulate THE PROPERTIES CRITERION (Section 3) before I reconstruct a selection of counterexamples and arguments we can find in Bolzano (Sections 4 and 5).

2 | CONCEPTS AND CONTAINMENT

2.1 | Concepts

To fruitfully discuss the disagreement between Leibniz and Bolzano, one has to carve out some shared central assumptions about concepts. I will do so in terms of the theoretical role concepts play in their theories. Leibniz and Bolzano agree that concepts serve as the constituents of propositions, Bolzano even thinks that the notion *concept* is definable as: *non-propositional part of a proposition* (P 52ff [C 362ff], L 291 [G IV 422]; WL §48 [I, 216], §128 [II, 18], CE 145). Both agree, and Bolzano praises Leibniz for this, that concepts and propositions are not dependent on being thought by human minds, though for Leibniz they need to be thought by God (NE III iv §17; WL §27 [I, 120f], §51 [I, 226]).⁴ For Leibniz, concepts can represent things (L 291 [G IV 422]), things are “comprehended under” them (P 136 [A VI 4839]). Similarly for Bolzano, who calls concepts that indeed *do* represent things “objectual.”

Objectual concepts have an extension, which is the collection of objects (or the single object)⁵ represented by the concept – the objects that “stand under” it (WL §66 [I, 296ff]) or, as it is more customary since Frege, “fall under” it. Importantly, both also agree that some concepts are simple while others are analyzable (P 52 [C 361], AG 99 [C 1], 217 [G VI 612]; WL §61 [I, 263ff], §557 [IV, 335ff]).

Bolzano and Leibniz also share a related terminology. Leibniz uses the French “*idée*” and the Latin “*idea*” as well as “*notio*” and “*conceptus*.”⁶ Bolzano uses the German “*Vorstellung an sich*.” While “*Vorstellung*” has been established as a standard translation of “*idée*” and “*idea*” by Wolff (Halbfass & Onnasch, 2001, cols. 1227f), the modification “*an sich*” (“in itself”) is used to stress that Bolzano is not talking about mental occurrences. He reserves “*Begriff*” (“concept”) for a subclass of ideas, namely those that are not intuitions according to his way of drawing the intuition/concept-distinction (WL §72f [I, 325ff]). Since intuitions play no important role in my discussion, I will not follow this terminological choice but use “*idea*” and “*concept*” interchangeably in the wider sense.

What Leibniz and Bolzano also agree on is that most concepts that represent something do so by *characterizing* the things they represent. Because of two aspects on which they disagree this thesis has to be spelled out in slightly different ways. First, for Bolzano concepts have a certain *structure*. The order in which their constituents occur is not arbitrary; changing it would often yield another concept or no concept at all. Ideas that represent objects typically have the form [something, which has *b*, *b'*, *b''*],⁷ and contain certain ideas that represent properties, which Bolzano calls “attribute ideas” (WL §60 [I, 259ff]).⁸ Besides the attribute ideas, which are indicated by the “*b*”s, the ideas [something], [which] and [has] are genuine constituents of the complex idea. However, as we will see in more detail in a moment, for Leibniz complex ideas lack structure in this sense. They do not require *logical glue*, that is, syncategorematic concepts that structure complex ideas, such as [which] or the logical copula [has].

Second, while for Bolzano ideas such as [drake], that is, [something, which has maleness and (the property of) being a duck],⁹ contain as constituents representations of properties, for Leibniz they contain representations of the things that have the properties instead. In fact, Leibniz does not believe in the existence of properties conceived as abstract universals (A VI 4994–6). [Drake] contains [male] and [duck], the former of which represents all male things and the latter all ducks. For the criterion I will discuss, this disagreement will be taken care of by showing that containment claims are to a large extent translatable into the other framework.

Let me mention one caveat: One cannot appeal to the ontological status of concepts as common ground. What kind of things concepts are is neither settled between Leibniz and Bolzano nor in contemporary debates, in which their positions still are two of the main contenders. For Bolzano concepts are abstract entities (WL §54 [I, 237f]), akin to senses in the Fregean tradition. They are, as Bolzano stresses several times, not to be confused with mental processes (WL §48 [I, 217f], §270 [III, 6]). Leibniz, contrary to Bolzano, does not accept irreducibly abstract objects. For Leibniz, concepts are dispositions to think (L 207 [G VII 263]). In particular, they are God’s dispositions to think (NE Preface 52, L 488 [G VII 305]), “ideas are in God from all eternity” (NE III iv §17). But ideas are not *only* in God’s mind: God “impresses” his divine ideas, his power of thinking, to finite minds (L 208 [G VII 264], NE III iv §2). So, in this particular respect, one has to be cautious when comparing Leibniz’s and Bolzano’s views on concepts. I will presume that this disagreement is independent from the theoretical roles concepts are supposed to play and does not crucially affect the principles that govern conceptual containment.

2.2 | Containment

I have claimed that Leibniz and Bolzano agree in using the language of containment in describing complex concepts. This might seem problematic since containment-talk is not unequivocal. There are at least two different ways of understanding “containment,” which give rise to two ways of understanding the technical term “conceptual containment,” namely a part- and a container-reading.¹⁰ According to the former, something contains another thing *as a part*, the former *consists of* the latter. According to the latter, something contains some other thing in the sense that the latter *is enclosed by* the former. The two readings are independent of each other. My body contains my

kidneys in both senses; they are parts of it as well as in it. A glass can contain lemonade in the sense that the lemonade is enclosed by the glass but the lemonade is not a part of the glass. A molecule of water contains two hydrogen atoms as parts but it is not a container for them.

Bolzano's conception of concepts leaves little room for doubt that he conceives of concepts as mereological compounds and takes containment to be a part-whole relation.¹¹ Let me give two brief examples that illustrate how serious Bolzano takes mereological considerations when talking about concepts. First, when he argues for the existence of simple ideas, he relies on mereological atomism, that is, the mereological principle "that every object, even the most complex, must have parts that are not themselves complex, but simple" (WL §61 [I, 263f]; see Schnieder, 2020 for discussion). Second, because of certain mereological considerations, he was troubled by the question of whether there can be multiple occurrences of the same idea in a given complex idea. In the concept [married son of a married mother], the concept [married] seems to occur twice; in cases of multiple negations the concept of negation is combined with itself (WL §78 [I, 355]). Later Bolzano saw a problem because he believes that one and the same part cannot occur more than once in a given whole or be combined with itself in order to constitute a new whole (BBGA 2A 12/2,148f, see Künne, 1997b, 223f, and Künne, 2001 [2008] for discussion). In this context, I will not discuss possible solutions of this problem. What matters is that for Bolzano it arises against the background of a mereological conception of conceptual containment.

It is less clear, which understanding we should attribute to Leibniz. There are two important kinds of passages in which conceptual containment plays a crucial role for Leibniz. First, he relies on the notion of conceptual containment when he states his theory of truth. I will discuss this in connection with THE PROPERTIES CRITERION in the next section. Second, in several drafts of *logical calculi* from the 1680s Leibniz states the principles governing containment and his general mereology (see Lenzen, 2000; Mugnai, 2019 for expositions).¹² These drafts also reveal the formal features that will be important in due course:

Definition 3. That A "is in" L, or that L "contains" A, is the same as that L is assumed to be coincident with several things taken together, among which is A.

Definition 4. All things in which there is whatever is in L will together be called "components" in respect of L, which is "composed" or "constituted."

' $B \oplus N = L$ ' means that B is in L, or that L contains B, and that B and N together constitute or compose L. The same holds for a larger number of things. [...]

Postulate 2. Any plurality, such as A and B, can be taken together to compose one thing $A \oplus B$ or L. (P 132 [A VI 4832–834])

So we can combine any two things A and B into the complex thing L, which is $A \oplus B$, that is, Leibniz assumes what we nowadays know as the unrestricted composition principle of classical mereology. *A fortiori* we can combine any two concepts A and B into a complex concept. The \oplus operator is idempotent, that is, $A \oplus A = A$; and it is commutative and associative (P 132 [A VI 4832–834], cf. Zalta, 2000, Mugnai, 2019, 54f), so the order of the entities is irrelevant.¹³ Containment is reflexive (P 133 [A VI 4835]) – though we might distinguish between containment and proper containment (cf. Mugnai, 2019, p. 61) – and anti-symmetric (P 136 [A VI 4839]). Finally, which will be most important in due course, for Leibniz, as for Bolzano (WL §58 [I, 251ff]), containment is a transitive relation, that is, "[i]f A is in B and B is in C, then A is in C. A content of a content is a content of the container" (P 135 [A VI 4838]).

Commenting on the definitions cited, Leibniz occasionally talks about wholes and parts (P 141f [A VI 4832f]), and on other occasions on which he talks about containment, he also sometimes uses mereological vocabulary. Talking about numbers, the participants of a *Dialogue on Continuity and Motion*, dated 1676, agree that "to be

contained in something is certainly to be a part of it" (*Pacidius* 179 [A VI 3551]).¹⁴ It is not entirely clear what exactly Leibniz has in mind when he uses "part" in these contexts as they occur only occasionally; he may not use them with mereological seriousness. Neither the formal features nor the occasional use of mereological expressions already suffice to decisively attribute to Leibniz one of the two readings of "containment" when applied to concepts in particular.

However, we can also find in Leibniz an explicit conception of parthood as containment⁺ (cf. Mugnai, 2019, pp. 57–62), and conceptual containment appears to meet the further condition that distinguishes part-whole relations from *mere* containment. The crucial difference between the two is for Leibniz whether the contained and the container are *homogeneous*, that is, similar in a specific respect. Containment between homogeneous objects amounts to a part-whole relation, containment between heterogeneous objects does not. Points, for instance, are contained in a line but they are not parts of the line because points and lines are not homogeneous; a line that is contained in another line, however, is a part of it (L 667f [GM VII 19]). Similarly for bodies and the monads they contain on the one hand and other bodies on the other hand (see Mugnai, 2019, pp. 64–66 on A VI 41669–1671). Leibniz has different ways of spelling out *homogeneity*. The proper understanding of "homogeneous" in this context is close to the composite meaning of "ὁμογενής," from which it derives, and which is built up from "ὁμός" ("same") and "γενής" ("descent," "family," "kind"): things are homogeneous if and only if they are things of the same *kind*.¹⁵ I will not delve into the question of how finely kinds are to be individuated for this criterion. It suffices that among Leibniz's examples of homogeneous objects are circles and squares, that is, the relevant kind is broader than *circle* but rather *two-dimensional geometrical object*. Since conceptual containment is a relation that holds primarily between ideas, it is plausibly a relation between homogeneous objects. If this is the case, then, for Leibniz, conceptual containment would indeed be a mereological relation. So the understanding of conceptual containment as a part-whole relation in Leibniz at least has to be taken seriously.

Understanding conceptual containment along the lines of the part-reading, however, comes with the obligation that it must not violate plausible mereological principles. I will show, first that for this reason proponents of THE PROPERTIES CRITERION, including Leibniz, are ill-advised to adopt the part-reading of "containment"; second, I will show that one of Bolzano's arguments is successful independently of whether one adopts the part-reading or the container-reading.

3 | THE PROPERTIES CRITERION FOR CONCEPTUAL CONTAINMENT

In WL §64 Bolzano discusses and rejects a criterion that he takes to be common among his contemporaries.¹⁶ It appears to provide a compelling method for deciding questions of conceptual containment. It tells us to scrutinize the objects represented by an idea, which is why it is primarily designed for objectual ideas, that is, ideas that represent something. To begin with, we can state it as follows:

THE PROPERTIES CRITERION An objectual idea C is composed exactly of the ideas of the properties a thing must have in order to fall under C.

So we are dealing with a necessary and sufficient condition for conceptual containment that appeals to some, arguably, more accessible feature. Its proponent would tell us that [maleness] is a part of [drake] because in order to be a drake something must be male.

According to Lapointe, Bolzano attributes this criterion to Leibniz. Referring to WL §64 [I, 272], she claims that Bolzano "takes [Leibniz] to be the paradigm in this respect" (Lapointe, 2011, p. 24). While I am unable to find a direct mentioning of Leibniz in this context, it is reasonable to assume that Bolzano has Leibniz in mind. Bolzano was more than familiar with Leibniz's writings and the claim that Leibniz endorses this criterion is justified. It naturally flows

from one of Leibniz's most important principles as we can see by some formulations of his "conceptual containment theory of truth" (Adams, 1994, p. 57):

In every proposition, the predicate is said to be in the subject, that is, the notion of the predicate is contained [*involvitur*] in the notion of the subject. For, in a universal affirmative proposition, when I say "every man is an animal" I mean "the concept of animal is contained in the concept of man" (for the concept of man is to be a rational animal). (AG 11 [C 85], *Samples of the Numerical Characteristic*, 1679, editors' addition)

[T]he predicate or consequent is always in [*inest*] the subject or antecedent, and the nature of truth in general or the connection between the terms of a statement, consists in this very thing, as Aristotle observed. (AG 31 [C 518f], *Primary Truths*, 1686, my addition)

[I]n every true affirmative proposition, whether necessary or contingent, universal or particular, the notion of the predicate is in some way included [*comprise*] in that of the subject. *Praedicatum inest subjecto*; otherwise I do not know what truth is. (L 337 [G II 56], *Correspondence with Arnauld*, 1686, my addition)

Not only has Leibniz stated the underlying *Predicate-In-Subject-Principle* quite often.¹⁷ It lies at the heart of his system (see Adams, 1994, 57ff; Mates, 1986, 84ff). In every truth, it tells us, the predicate concept is in or is contained in the subject concept. It is true that vixens are female, so the concept [female] is contained in the concept [vixen]. Caesar crossed the Rubicon, so the concept [crossed the Rubicon] is contained in the concept [Caesar] (L 311 [G IV 438]). Concepts of individuals like [Caesar] are for Leibniz *complete*. It is true that Caesar was fond of cats or it is true that Caesar was not fond of cats, so either [fond of cats] or [not fond of cats] has to be contained in the concept [Caesar]. The same holds *mutatis mutandis* for every other property (AG 73 [G II 43], L 307 [G IV 433]). General concepts are not complete in this sense. Some humans are and some are not fond of cats, which is why neither [fond of cats] nor [not fond of cats] is contained in [human]; "the notion of a *species* includes only eternal or necessary truths" (AG 70 [G II 39]).

Now let me make THE PROPERTIES CRITERION more precise. Bolzano states it in the following way:

Every idea of an object is a mere collection of the ideas of all the attributes this object necessarily has as an object of this idea. [...] [M]ost logicians actually seem to believe that the idea of any object is composed of nothing but the ideas of its attributes [...] (WL §64 [I, 270])

The second sentence already shows that Bolzano sometimes omits the "necessary" as well as the careful clause "as an object of this idea." As is clear from the first sentence and his arguments, Bolzano always means to talk about properties something *must have as an object of the idea in question*. According to this criterion, the idea [equilateral triangle] has [equality of all sides] as constituent, for every equilateral triangle must have equal sides. In order to be a golden watch, something must be golden, so [goldenness] is contained in [golden watch] (WL §72 [I, 325f]).

Because of Bolzano's remarkable departure from the tradition, his discussion of the criterion has received some attention. Since properly understanding the criterion is not only important to understand Bolzano but also illuminating for the interpretation of a number of related authors, and since providing a more precise formulation is not trivial, let me briefly comment on earlier attempts. I will put aside the exposition in Morscher (2004, pp. 89–92), which explicates the criterion in terms of Bolzano's account of essential properties instead of relying on contemporary modal notions. For the purposes of my investigation, that is, a fruitful comparison with Leibniz and a systematic assessment of the arguments, this is an unnecessary complication.

According to Centrone's semi-formal reconstruction, THE PROPERTIES CRITERION states that an idea [G] is part of an idea [F] if and only if necessarily, everything that falls under [F] has the attribute of being G (cf. Centrone, 2010b, p. 318). Because of the schematic formulation with predicate letters within square brackets, it remains a bit unclear what exactly may be inserted for “F” and “G,” for which some restrictions have to be in place, and what insertions for [F] and [G] then represent.¹⁸ The ideas she refers to as conceptual components are often ideas such as [triangle], which, for Bolzano, represent objects such as triangles rather than attributes such as triangularity. Concerning necessary restrictions, if no further restriction is posed, unrealizable ideas such as [round square] would contain, according to this semi-formalization, all attribute ideas. Below, I will formulate the criterion more transparently by first offering two distinct formulations, one faithful to Bolzano in drawing the aforementioned distinction, and by second excluding unrealizable ideas. But before that let me briefly comment on another reconstruction.

Lapointe, improving on her 2007, 221, offers the following semi-formal reconstruction:

If $\beta_1 \ \& \ \beta_2 \ \& \ \dots \ \& \ \beta_n$ are essential properties of object α , then the concepts $[\beta_1] \ \& \ [\beta_2] \ \& \ \dots \ \& \ [\beta_n]$ are included in the concept of α , i.e. $[\alpha] = \{[\beta_1] \ \& \ [\beta_2] \ \& \ \dots \ \& \ [\beta_n]\}$ (Lapointe, 2011, p. 24).

This formulation is problematic for two reasons. First, the criterion is formulated only as the conditional that if β_i is an essential property of α , then $[\beta_i]$ is a part of $[\alpha]$. Not including the converse is misleading, in particular, if one claims that the set of components of $[\alpha]$ is “determined by the application of [this] rule” (Lapointe, 2011, 24). One starts to wonder, which further concepts besides the ones captured by Lapointe's conditional might be constituents of $[\alpha]$. In fact, it renders her formulation inconsistent: the part before the “i.e.” allows other concepts to be parts of $[\alpha]$, the use of the equality sign after the “i.e.” does not.

Second, it does not capture Bolzano's “as an object of this idea” but simply refers to essential properties. If one understands “essential properties” in an important contemporary sense as *de re* necessary properties, properties an object must have in any case it exists (Kripke, 1980, p. 48), then the thesis is false. A broken watch is not essentially or necessarily broken in this sense, but [broken watch] certainly contains the concept of being broken. It is not entirely clear that applying Bolzano's understanding of essential properties would yield the correct result. At least, as Morscher points out in his reconstruction (2004, 90f), Bolzano's notion of essential properties contains a relativization to ideas. However, since Bolzano himself does not rely on the notion of essential properties in this particular discussion, I will stick to the transparent formulation I quoted above.

So let me offer my own semi-formal reconstruction of THE PROPERTIES CRITERION, which I take to be more transparent than the ones discussed:

THE PROPERTIES CRITERION

For all ideas x such that $\Diamond \exists z(z \text{ falls under } x)$ and all ideas y : x contains $y \leftrightarrow$
 y is an attribute idea $\wedge \Box \forall z(z \text{ falls under } x \rightarrow z \text{ has something } y \text{ represents})$

While Bolzano only formulates the criterion for objectual ideas, allowing *possibly* objectual ideas is unproblematic and a criterion that covers more kinds of ideas is more attractive. Without *any* restriction, as this formulation makes clear, unrealizable ideas, that is, ideas that necessarily fail to represent, would serve as counterexample against the criterion. As nothing can fall under these ideas, the right hand side of the biconditional would be true for every attribute idea. Therefore, every unrealizable idea would contain every attribute idea, which is implausible; indeed, for reasons I will come to in a moment, there would be only one unrealizable idea. As unrealizable concepts are well-known trouble-makers for intensional contexts, we should grant the proponent of THE PROPERTIES CRITERION the restriction. The fact that we have to restrict the criterion shows nevertheless that THE PROPERTIES CRITERION does not inform us *what it is* for an idea to be included in another idea since it is not completely general.

If one assumes that parts of ideas do not represent properties but the things that have these properties (see Section 2.1), the criterion can be formulated in a less complicated way:

THE PROPERTIES CRITERION*

For all ideas x such that $\diamond \exists z(z \text{ falls under } x)$ and all ideas y : $x \text{ contains } y \leftrightarrow \square \forall z(z \text{ falls under } x \rightarrow z \text{ falls under } y)$ ¹⁹

Leibniz would approve of this formulation because, contrary to the previous one, it does not mention the existence of properties, that is, attribute ideas that represent something. Which formulation one prefers, including the issue of whether there are properties, makes no substantial difference for Bolzano's arguments. I will usually prefer Bolzano's way of construing it because I quote him, but switch between the formulations when I also refer to Leibniz.

The position Bolzano characterizes in the quotation above actually is a bit stronger than THE PROPERTIES CRITERION. The latter gives us a necessary and sufficient condition for concepts to be contained in other concepts. But Bolzano also claims that a concept, according to the position he wants to attack, is "a mere collection" ("ein bloßer Inbegriff") of the relevant attribute ideas. Accordingly, there is nothing more to a complex concept than to be this collection, for example, an order in which its constituents appear. So concepts are identical if and only if they are collections of the same attribute ideas, that is,

CONCEPT IDENTITY For all ideas x and y :
 $x = y \leftrightarrow \square \forall z(z \text{ is a constituent of } x \leftrightarrow z \text{ is a constituent of } y)$

CONCEPT IDENTITY is the reason why there would be only one unrealizable idea if THE PROPERTIES CRITERION were correct and not restricted to possibly objectual ideas. All unrealizable ideas would contain all attribute ideas and nothing else; so, all unrealizable ideas would have exactly the same constituents, and therefore, be identical. Presumably, Bolzano has something even stronger in mind: at least in one of his arguments, he also likely assumes that for proponents of CONCEPT IDENTITY the *function* of a complex concept can be stated in terms of its constituents alone. If two complex concepts differ, this difference can be explained in terms of distinct constituents.

The picture of concepts that is drawn suggests that a complex concept is the unordered collection of its constituents. The concept [drake], for instance, would have [(the property of) being a duck] and [maleness] as its constituents. It would not contain connecting concepts and the constituents would not appear in a particular order. This assumption reveals an important difference between this account of concepts and Bolzano's own. While for Bolzano the concepts [learned son of an ignorant father] and [ignorant son of a learned father] share in an important sense the same components (WL §56 [I, 244]), for proponents of THE PROPERTIES CRITERION they do not. Only the former would have [learnedness] and [(the property of) being a son of an ignorant father] as components while only the latter would have [ignorance] and [(the property of) being a son of a learned father] as components.

As the proponents of THE PROPERTIES CRITERION, Bolzano also believes that objectual ideas often describe the things that fall under them, "[a]n idea is said to have objects when there is something constituted as the idea describes" (CE 169). However, what he agrees with is merely that they *often* describe the things that fall under them in terms of *some* properties they must have in order to be represented; he disagrees that they *always* contain *all* of them and *only* them. The proponent of THE PROPERTIES CRITERION can be considered to suffer from a one-sided diet of examples (WL §64 [I, 270]). Bolzano's aim is to show the inadequacy of both directions of THE PROPERTIES CRITERION:

[T]here are various constituents of an idea which express anything but the attributes of the object that correspond to it, but also that every object has attributes which, although it must have them in order to stand under a given idea, are nonetheless not thought as constituents of the idea. (WL §64 [I, 270])

4 | AGAINST THE NECESSARY CONDITION (§64.1)

Let me split up THE PROPERTIES CRITERION into the properties-to-constituents and the constituents-to-properties direction. We will now be briefly concerned with the latter, that is, with

CONTO_{PRO} For all ideas x such that $\diamond \exists z(z$ falls under $x)$ and all ideas y :

x contains $y \rightarrow$

y is an attribute idea $\wedge \square \forall z(z$ falls under $x \rightarrow z$ has something y represents).

Bolzano launches two arguments against CONTO_{PRO}. The first one is only convincing if one accepts something along the lines of his theory of connecting concepts. As already mentioned, according to Bolzano, [drake] does not only contain [(the property of) being a duck] and [maleness] but also [something] and the syncategorematic concepts [which] and [has]. If CONTO_{PRO} were correct, then drakes would have to have properties which are represented by [which] and [has]. For Bolzano, these concepts do not represent anything. So not every constituent of a concept represents a property something must have in order to fall under this concept (WL §64 [I, 270f]). As noted earlier, proponents of CONTO_{PRO} do not take ideas to contain connecting concepts.

The other counterexample does not rely on Bolzano's account of connecting concepts but on plausible ways in which attribute ideas are composed. It shows that there would be surprising omissions if CONTO_{PRO} were correct. There are concepts that, despite apparently being contained in a complex concept, are ruled out by CONTO_{PRO}. The concept [equilateral triangle] contains, Bolzano and his opponents agree, [equilaterality]. Now it seems plausible that this concept is itself complex and contains, because equilaterality is the equality of all sides, the concepts [equality] and [(the property of) being a side], which form, perhaps together with other concepts, [equilaterality]. If this is conceded and if the transitivity of the containment relation is assumed, it would follow that [equality] and [(the property of) being a side] also are contained in [equilateral triangle].

The proponent of CONTO_{PRO}, however, must reject that they are contained in [equilateral triangle] because “[a]n equilateral triangle is neither a kind of side, nor does it have the attribute of equality” (WL §64 [I, 271]). So the concepts cannot appear as constituents of [equilateral triangle], even though they seem to be quite prominently contained in the idea that is combined with [triangle] to form the complex idea. One might consider to reject the counterexample by employing a non-transitive containment relation instead, even though for Leibniz containment is, as indicated earlier, transitive. However, the general idea does not depend on transitivity, provided that we can point out apparent constituents somewhat directly. We can, for example, consider concepts that contain the negation. Bolzano's opponent cannot assume that the idea [man who has no integrity] contains [integrity] or the negation [not] (WL §65 [I, 282]) while they are, at least *prima facie*, prominently contained in the complex concept. One reason why one might believe that [not] and [integrity] are indeed contained in [man who has no integrity] is that one, arguably, must grasp the former in order to grasp the latter, which poses a serious challenge to CONTO_{PRO}. Incidentally, this objection hints at Bolzano's own criterion for conceptual containment (WL §65 [I, 283], §281 [III, 39]):

THE COGNITIVE CRITERION A complex concept C contains all and only the concepts one must grasp in order to grasp C .

While this criterion will not be at the center of my discussion, being aware of it also helps to better understand some of Bolzano's objections against the converse of CONTO_{PRO}, to which I turn now.

5 | AGAINST THE SUFFICIENT CONDITION (§64.2)

Bolzano not only shows that the criterion *excludes plausible candidates* for constituents of complex concepts. He also points to cases in which it *includes implausible candidates* for constituents of complex concepts. The properties-to-

constituents direction of the criterion allows us to infer conceptual constituents from the properties things falling under the complex idea must have:

PROToCON For all ideas x such that $\diamond \exists z(z \text{ falls under } x)$ and all ideas y :
 y is an attribute idea $\wedge \square \forall z(z \text{ falls under } x \rightarrow z \text{ has something } y \text{ represents}) \rightarrow x \text{ contains } y.$

5.1 | Counterexamples

In order to attack PROToCON, Bolzano first advances three counterexamples, which are supposed to show what the proponent of THE PROPERTIES CRITERION is committed to (WL §64 [I, 271f]):

Unexpected Constituents The idea [square] contains [(the property of) having sides standing in the ratio $1:\sqrt{2}$ to the diagonal].

Same Content Necessarily, all and only equilateral triangles are equiangular triangles. Therefore, the idea [equilateral triangle] has the same constituents as, and is, by CONCEPT IDENTITY, identical to [equiangular triangle].

Infinitely Many Constituents The idea [$\sqrt{2}$] contains [(the property of) having 4 as its first decimal place], [(the property of) having 1 as its second decimal place], ...

Motivated by his own criterion, Bolzano here presupposes a tight connection between ideas and thinking, which is why he is primarily concerned with the psychological implausibility of these claims. It does not seem necessary to think about sides standing in the ratio $1:\sqrt{2}$ to the diagonals in order to think about squares; grasping [equilateral triangle] and grasping [equiangular triangle] do not appear to be entirely the same; the worry concerning the third counterexample is that we, finite beings, cannot possibly grasp concepts with an infinite number of constituents.

Bolzano himself suggests how his *prima facie* counterexamples might be defused. Neither are we always aware of all constituents of an idea we grasp, nor is assessing the identity of concepts a trivial matter, which is why the first two claims might be correct. Concerning the third one, one might argue that it is possible for a finite intelligence to grasp an idea with an infinite number of constituents “provided that we do not require that it be distinctly aware of all of them” (WL §64 [I, 273]). Leibniz would approve of this objection because he believes that “[e]ach soul knows the infinite – knows all – but confusedly” (AG 211 [G VI 604]).

The second counterexample is of special interest. It shows that concepts that necessarily represent the same things would have exactly the same constituents. Since, according to the picture under attack, concepts are not structured, these concepts could not be distinct. Sometimes, however, Leibniz can be read as distinguishing concepts that necessarily represent the same things:

[T]hings that are conceptually distinct, that is, things that are formally but not really distinct, are distinguished solely by the mind. Thus, in the plane, Triangle and Trilateral do not differ in fact but only in concept, and therefore in reality they are the same, but not formally. Trilateral as such mentions sides; Triangle, angles.²⁰

Leibniz himself acknowledges that his substitutivity principles for concepts break down in oblique contexts, in which not the extension of a concept but instead “our way of conceiving it” (P 56f [C 366f]) is relevant (cf. Mates, 1986, pp. 128–132). In a similar spirit, Bolzano suggests a way to account for the difference in order to defuse his own counterexample, which points into a similar direction as it also concerns different ways of thinking about necessarily the same objects. The difference between the two concepts “might simply lie in the fact that in one of them certain constituents are distinctly thought, while in others other constituents are” (WL §64 [I, 237]). So when thinking about objects as triangles and when thinking about them as trilaterals, one might grasp the very same concept but does so in two different ways such that one is in each case aware of different constituents.

However, if one assumes that necessarily co-extensional concepts coincide in the context of a logical calculus, a further problem arises. As Bolzano's example shows, [equilateral triangle] would have to be identical to [equiangular triangle]. There are at least two ways in which this concept can be built up:

$$\begin{aligned} [\text{equilateral triangle}] &= [\text{equilateral}] \oplus [\text{triangle}] \\ [\text{equiangular triangle}] &= [\text{equiangular}] \oplus [\text{triangle}] \end{aligned}$$

Now if the identification of the complex concepts is correct, that is, if

$$[\text{equilateral triangle}] = [\text{equiangular triangle}]$$

then by the transitivity of identity, we can infer, and Leibniz indeed assumes,

$$[\text{equilateral}] \oplus [\text{triangle}] = [\text{equiangular}] \oplus [\text{triangle}] \text{ (cf. P 133 [A VI 4 836])}$$

Now consider a plausible mereological principle, which I borrow from Frege (1892, p. 36), and which is also assumed in contemporary mereological frameworks (Krifka, 1998, p. 199), according to which a whole and one of its parts uniquely determine the remainder:

$$\text{UNIQUEREMAINDER} \quad (A \oplus C = B \oplus C) \rightarrow A = B$$

If we assume that in our example the part, C, is [triangle], then in the one case the uniquely determined remainder is [equilateral] while in the other case it is [equiangular]. Therefore, [equiangular] would have to be identical to [equilateral]. This is clearly wrong. All rhombi are equilateral but not all are equiangular while all rectangles are equiangular but not all are equilateral. Leibniz is careful enough not to accept the mereological background assumption. While he accepts its converse, he takes UNIQUEREMAINDER to be false (P 133 [A VI 4835f]). So, for Leibniz and proponents of THE PROPERTIES CRITERION in general, conceptual containment should not be a relation for which UNIQUEREMAINDER, a plausible mereological principle, holds; for Bolzano it may.

5.2 | Two mereological arguments

Bolzano offers several arguments he takes to be more convincing than his counterexamples. While WL §64 has been touched upon in several papers, the arguments themselves have received little attention.²¹ As I will show, Bolzano often tacitly relies on a specific background assumption in these arguments, which has not been made explicit yet. The tacit background assumption is a mereological principle, which is why the first two arguments I present presuppose an understanding of containment along the lines of the part-reading; the third one does not. Let me begin with one that has some similarity to Bolzano's first counterexample from above (WL §64 [I, 274f]):

- A1 [Equilateral triangle] is composed by combining [triangle] with [(the property of) being equilateral].²²
- A2 [Equiangularity] is neither contained in [triangle] nor in [(the property of) being equilateral].
- C [Equiangularity] is not contained in [equilateral triangle].

The conclusion is incompatible with PROTOCON, which requires that [equiangularity] is a part of [equilateral triangle]. Stated in this way, the argument clearly is not valid and we still need to reveal a tacit premise.

Bolzano justifies the conclusion by analogy with [equilateral quadrangle]. Necessarily, all equilateral triangles are equiangular; but not all equilateral quadrangles are equiangular. So [equilateral triangle], though not [equilateral

quadrangle], would have to contain [equiangularity]. However, both ideas seem to be formed in a similar way, which is by combining [(the property of) being equilateral] with either [triangle] or [quadrangle]. PROTOCON entails that none of the three concepts used as building blocks contains [equiangularity]. So we face an explanatory challenge: How does, when combining [(the property of) being equilateral] with [triangle], the concept [equiangularity] enter the resulting complex concept when it is not contained in the two salient ingredients? And why is this not so for the combination of [(the property of) being equilateral] and [quadrangle]? As noted earlier, Bolzano assumes that for proponents of THE PROPERTIES CRITERION differences between complex concepts are to be explained in terms of their constituents. So, for him, these questions must be answered in terms of the constituents of the concepts in question. Bolzano believes that this challenge cannot be met because in the combination of [(the property of) being equilateral] with [triangle] the concept [equiangularity] “does not occur in the whole that is formed of nothing but these two parts” (WL §64 [I, 274]). He apparently relies on a mereological principle, according to which complexes contain only parts that are already contained in the ingredients, which we combine:

$$\text{INGREDIENTS } \forall x \forall y \forall z ((x \text{ is in } y \oplus z) \rightarrow (x \text{ is in } y) \vee (x \text{ is in } z))$$

INGREDIENTS would indeed complete the argument. The problem is that a complex *can* have parts that are *not* contained in the ingredients out of which it is formed. When there is an apple tree in my garden and a pear tree in my neighbor's garden, then the mereological compound consisting of both our gardens contains a region that inhabits an apple tree and a pear tree. However, this part is neither contained in my nor in my neighbor's garden. So INGREDIENTS is not a general mereological principle, and it is doubtful that it should be a restricted principle governing concept formation in particular. When I combine [bigness] with [bad wolf] into [big bad wolf], then a part of the resulting complex concept contains the combination of [bigness] and [badness], but this concept is not a part of either of the two concepts combined in the first place. So we have to account for the possibility that new complex parts arise by combining a part of each of the two main parts. We can do so by adding the underlined disjunct:

$$\text{INGREDIENTS}^* \forall x \forall y \forall z ((x \text{ is in } y \oplus z) \rightarrow (x \text{ is in } y) \vee (x \text{ is in } z) \vee \exists x_1 \exists x_2 ((x = x_1 \oplus x_2) \wedge \underline{x_1 \text{ is in } y} \wedge \underline{x_2 \text{ is in } z}))$$

Now the proponent of PROTOCON can maintain that [equiangularity] is a part of [equilateral triangle] if she is prepared to assume that it is the result of combining a part of [(the property of) being equilateral] with a part of [triangle]. While being theoretically possible, this assumption differs considerably from the assumption that [big bad wolf], formed in the way described, contains the combination of [bigness] and [badness], even though this concept is not contained in [bigness] or [bad wolf]. It is clear out of which concepts the combination of [bigness] and [badness] is formed and where its parts come from. It remains obscure what parts are supposed to constitute [equiangularity] and where exactly they come from. Regardless of the theoretical possibility opened by replacing INGREDIENTS with INGREDIENTS*, Bolzano's argument puts the proponent of THE PROPERTIES CRITERION under pressure to show just how [equiangularity] is supposed to make it into [equilateral triangle].

Bolzano provides us with another argument worth considering:

According to §61, there must be simple ideas. If *a* is such a simple idea, then the proposition “the idea *a* is simple” is true. Thus the concept “simplicity” expresses an attribute which *necessarily* belongs to the object of the concept “the idea *a*,” namely, to *a*. Hence, if the premise I contest were correct, then the concept of simplicity would have to occur as a part in the just-mentioned concept. But this is not the case, for it occurs neither in the concept of an *idea*, nor in the idea *a* itself. (WL §64 [I, 275f], translation altered)

This argument relies on a subtle distinction between idea-levels, which is remarkable for Bolzano's time. As the distinction has been overlooked in the only available reconstruction and the 2014 English translation is misleading in a

related respect, let me make it explicit. Consider the following reconstruction of the argument (Centrone, 2010b, p. 321, my translation and adjusted labeling):

1. There are simple ideas. (assumption)
2. Let [has] be such a simple idea. (ekthesis)
3. All instances of the schema “x is a simple idea” express a necessary truth if they express a truth. (assumption)
- K.1** Necessarily, [has] is a simple idea. (from 2. and 3.)
4. Every property an object must have in order to be an object represented by a particular idea corresponds to a constituent of this idea. (according to **PROToCON**)
- K.2** The idea [has] contains as component the idea [simple]. (from K.1 and 4.)

Let us focus on the step from K.1 and 4. to K.2 and leave other worries aside. The reconstruction concludes from **PROToCON** and the necessary simplicity of [has] that [has] contains [simple idea]. But [has] is objectless, so **PROToCON** remains silent about its constituents. Unlike the reconstruction, Bolzano's argument is not concerned with the constituents of a simple idea such as [has] *at all*, so we also cannot simply substitute “[has]” with a designator for a simple idea such as “[object].” Bolzano states that “the concept “the idea *a*,” so [the idea [a]] or [the idea [has]] respectively, would contain [simplicity] if **PROToCON** were correct. The reconstruction confuses the higher-order idea [the idea [has]], an idea that represents the idea [has], with the idea [has] itself.²³ This confusion conceals that Bolzano in this argument relies on the same mereological background assumptions as in the previous argument. In order to see this, consider the following reconstruction of the argument:

- A1** The idea [has] is simple.
- A2** If an idea is simple, then it is necessarily simple.
- C1** The idea [has] is necessarily simple. (from A1 and A2)
- C2** The idea [the idea [has]] contains [simplicity]. (from C1 and **PROToCON**)
- A3** The idea [idea] does not contain [simplicity].²⁴
- A4** The idea [has] does not contain [simplicity].
- C3** The idea [the idea [has]] does not contain [simplicity]. (from A3, A4 and a tacit background assumption; contradicts C2, **PROToCON** is to be rejected).

The last sentence of the quotation indicates the transition from A3 and A4 to C3. Bolzano assumes that [simplicity] could only be a constituent of the higher-order idea in question if it were a constituent of one of its two salient ingredients, which are [idea] and [has]. Since it is not a constituent of either, “it occurs neither in the concept of an *idea*, nor in the idea *a* itself,” [simplicity] cannot be a part of the complex idea that is the result of combining both. So again Bolzano presupposes a mereological principle that sounds like **INGREDIENTS**, which would indeed lead to a contradiction. We have seen that **INGREDIENTS** is, arguably, false but even if we assume that Bolzano merely endorses **INGREDIENTS***, this argument reveals a serious implausibility of **PROToCON**. It puts the opponent under pressure to show how [simplicity] is supposed to enter [the idea [has]] without being a part of [idea] or [has].

While there is *mutatis mutandis* the same theoretical possibility as in the previous argument, namely that [simplicity] is composed of [has] and a part of [idea], this argument is more worrisome because it generalizes and applies to all higher-order ideas. First, it applies to all ideas representing simple ideas, which may be the reason Bolzano preferred a schematic formulation. For every simple idea there would have to be a part of [idea] such that this part combined with the simple idea is identical to [simplicity]. Then the argument easily generalizes to more complex ideas. Let us assume that [donkey] has seven constituents. Then [the idea [donkey]] would have to contain [(the property of) having seven constituents], which is why there would have to be a combination of a part of [idea] and a part of [donkey] that is identical to [(the property of) having seven constituents]. And *mutatis mutandis* for every idea: every idea would have to contain a constituent that can be combined with a part of [idea] such that the result is the

concept of having the number of constituents the initial idea has. Since the problem arises systematically, Bolzano's opponent should provide a systematic solution for it.

I will not discuss in detail two further arguments Bolzano advances because in their crucial steps they also rely on the same mereological principles as the ones I have discussed. The argument in WL §64.2.g [I, 276] also has a conclusion to the effect that certain higher-order ideas representing simple concepts do not have the constituents they would have if *ProToCon* were correct. Bolzano simply states that certain containment claims would be absurd, *INGREDIENTS* and *INGREDIENTS** would plausibly account for the apparent absurdity. In the last argument offered, Bolzano schematically argues for the possibility that properties can follow from a concept for its objects without the concept containing an idea of this property (WL §64.2.h [I, 276f]). Besides further potential problems, the argument relies on assumptions on what higher-order ideas can and cannot contain in terms of their salient ingredients (WL §64.2.h [I, 277]), to which, again, *mutatis mutandis* my discussion of *INGREDIENTS* and *INGREDIENTS** applies.

5.3 | The argument from simple ideas

There is one more argument I will discuss. This argument seems to be the only one that does not rely on *INGREDIENTS**, the mereological principle presupposed by the other arguments. So it is the most promising argument for refuting *ProToCon* when conceptual containment is not understood as a mereological relation. In particular, what has gone unnoticed in the few earlier discussion of §64, it is the only one that works equally well under the container-reading of “containment” as it does under the part-reading.

Simple ideas, too, must, perhaps with a few exceptions, represent something. Let this something be of any character whatever, it must have the attribute “of being something.” Hence, if every attribute of an object, which it must have in order to be the object of a certain idea, had to occur as a constituent of this idea, then all simple ideas would have to contain the idea “something.” But in order to remain simple, they could contain no other ideas besides. Consequently, all simple ideas (with the exception of those that have no object at all) would have to be identical with one another or, to put it better, there would be only one simple idea, namely, “something.” Clearly, this is absurd. (WL §64.2.f [I, 276])

Three remarks: First, contrary to Leibniz, who takes containment to be reflexive (see Section 2.2), for Bolzano, simple concepts do not contain anything. But here he allows them, for the sake of the argument, to contain concepts in the improper sense of being identical to them.²⁵ Second, while for Bolzano the concept [something] is simple, the word “something” is composed of the words “some” and “thing,” which can be misleading. I will substitute “something” by “object,” which Bolzano takes to be a synonym of “something” (CE 109). Third, while it is not explicit in his formulation, Bolzano's argument only leads to the intended contradiction if there are at least two ideas that are both objectual and simple, which I add as an explicit premise.

A1 There are at least two objectual simple ideas.

A2 Whatever is represented by an idea necessarily has the property of being an object.

C1 [Object] is contained in every simple objectual idea. (from A2 and *ProToCon*)

A3 A simple idea can only contain [object] if it is identical to [object].

C2 [Object] is the only simple objectual idea. (from C1 and A3, contradicts A1, *ProToCon* is to be refuted)

If one is troubled by the reliance on [object] instead of [(the property of) being an object], then one can substitute the former for the latter in the argument. In order to appreciate the full force of the argument, it is important to note

that in A2 one can choose *any* necessary property of all things, such as being self-identical. It is *not* required that the idea of the property one picks is simple: a simple idea certainly does not contain a complex idea, not even in the improper sense of being identical to it.

One way to justify A1 is to appeal to Bolzano's account of intuitions, which are by definition simple and objectual (Centrone, 2010b, p. 322). Proponents of PROTOCON might be in a position to reject this account by rejecting Bolzano's account of intuitions. In Bolzano we can find several further suggestions for simple objectual ideas, for example, [compositeness] (WL §82.1 [I, 394]), [(the property of) being a ground] (WL §202 [I, 351]) or [obligation] (WL §127.2 [II, 12f]). Leibniz at least considers [extended] and [conscious] to be plausible candidates (P 52 [C 361]). Since here is not the place to argue for the simplicity of specific concepts, let me put the upshot more cautiously as a conditional. The argument shows that *if there are at least two distinct simple objectual ideas and at least one property every object has necessarily*, then PROTOCON leads to a contradiction.

6 | CONCLUSION

Bolzano successfully shows that THE PROPERTIES CRITERION faces serious difficulties. In particular, with a bit of Fregean support we can take Bolzano as having shown that this criterion fails plausible mereological principles such as UNIQUEREMAINDER. Furthermore, if a proponent of THE PROPERTIES CRITERION assumes that containment is a mereological relation, then he faces troublesome explanatory challenges. Explanations are required how exactly, first, certain concepts enter more complex concepts without their components being *prima facie* contained in the salient ingredients of the complex concept. Second, one needs to explain how a concept stating the number of constituents of a given concept always is a constituent of the higher-order concept representing it. As I have noted, at least Leibniz explicitly rejects principles such as UNIQUEREMAINDER. Despite concepts being *prima facie* homogeneous to each other, he does not seem to take conceptual containment to be a mereological relation, which is why he can also possibly avoid the explanatory challenges. However, THE PROPERTIES CRITERION is refuted after all by the argument from simple ideas.

Regardless of whether Leibniz is able to avoid contradiction, there is one further lesson to be learned. The question of whether conceptual containment should be understood in mereological terms, and the different answers we can find in Leibniz and Bolzano, reveal different ways of thinking about conceptual containment. Bolzano himself suspects that his opponents may just mean something else when talking about the composition of ideas than he does (WL §65.2 [I, 282f]). With his objections, he clarifies what this understanding is and what it can and cannot achieve in contrast to his mereological understanding, which is a valuable contribution to the still ongoing task of properly explicating the notion of conceptual containment.

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ENDNOTES

¹ Beginning with Descartes, the term “idea” was used to refer, among many other things, to concepts, often located in the human understanding. For the different senses of “idea” in early modern philosophy see McRae (1965) and Perler (2010); for the variety of uses in Locke see Ryle (1968 [2009], p. 134). For the notion of parthood as applied to ideas in Locke and suggestions on how it is connected with the different senses of “idea,” see Stuart (2010).

- ² The label goes back to the Czech philosopher Josef Durdík (cf. Künne, 1997a, p. 27).
- ³ The intellectual proximity of Leibniz and Bolzano has been pointed out on a general level (cf. Morscher, 2003), but there are few investigations that go into detail, namely Mugnai (1992a), Centrone (2010a) and Künne (2016).
- ⁴ Bolzano takes Leibniz to be a forerunner of his theory of propositions (WL §21 [I, 85] on G VII 190–193). This assessment has been criticized by Berg (1962, 51f) and defended by Mugnai (1992a, pp. 207–215).
- ⁵ Strictly speaking, according to Bolzano's mereology, concepts that represent exactly one object do not have an extension because the extension of an idea is a mereological compound and therefore must have at least two parts (WL §82 [I, 393f], §84 [I, 400]). So adding this disjunct means taking some terminological distance from Bolzano's mereology, which he himself takes occasionally (e.g., in WL §66.2 [I, 298]); see Textor (1996, 45f) for discussion.
- ⁶ Often Leibniz uses expressions like “idea or concept” (“idée ou notion”) in a way that suggests that these expressions are interchangeable (e.g., several times in the *Discourse on Metaphysics*, see L 310, 312, 322, 323 [G IV 437, 440, 455, 457]). While Leibniz *often* uses “idea” and “concept” interchangeably, Barth points out that Leibniz' terminology is not entirely consistent and that a substantial distinction between ideas and concepts can be found. Also in the *Discourse* Leibniz explicitly mentions how one could draw such a distinction (L 321 [C 452]). Accordingly, ideas are dispositions with concepts being acts that result from their actualization (cf. Barth, 2019, 230).
- ⁷ As is customary in Bolzano scholarship, I will use square brackets in order to talk about concepts directly; [bachelor] is the concept expressed by “bachelor” (see Morscher, 2016, pp. 29–31).
- ⁸ Bolzano prefers “attribute,” reserving “property” for a subclass of what he calls “attributes,” namely intrinsic ones in contrast to relations (WL §80 [I, 378ff]). Outside of quotations and fixed expressions such as “attribute idea,” I will prefer “property” as it is the more common term.
- ⁹ The attribute idea [(the property of) being a duck] represents the property of being a duck, for which the English language lacks a noun. As Bolzano uses brackets for a similar purpose (WL §136 [II, 51]), I will usually bracket “the property of” in order to indicate that [the property of] is not a part of the whole attribute idea.
- ¹⁰ See Fine (2010, p. 560) for a similar distinction between different kinds of something being *in* another thing.
- ¹¹ For general expositions of Bolzano's mereology see Krickel (1995), Simons (1997), Casari (2016, appendix C) and Schnieder (2017).
- ¹² As Lenzen and Mugnai note, in these writings Leibniz talks about things in general, not restricted to but including concepts. Translations are altered accordingly.
- ¹³ The mereological sum operators in Krifka (1998, p. 199) and Varzi (2016) share these features.
- ¹⁴ “Contineri autem utique est partem esse” (Pacidius 178). Note that in the *logical calculus* referred to above, Leibniz also frequently uses “contingere” for the converse relation of *inesse*. The explication of the \oplus – operator in definition 4, which I quoted in translation, reads in Latin: “ $B \oplus N \infty L$ significat B esse in L , seu L continere B ” (A VI 4832).
- ¹⁵ As Leibniz puts it, things are homogeneous if and only if they “agree [...] in some form or nature” (A VI 3, 483) or share the same *genus* (NE Preface 63). They are homogeneous if and only if they are similar or can be made similar (GM VII, 30) while they are similar if and only if they “belong to the same lowest species” (A VI 4872), see Mugnai (2019, pp. 58–60).
- ¹⁶ Importantly, it is included under the label “comprehension of an idea” in the *Port-Royal Logic*, which served as the canonical logic textbook for centuries (see Arnauld and Nicole 1662 [1850], I, ch. 6, 49).
- ¹⁷ For an abundance of further passages and discussion see Sleight (1982).
- ¹⁸ See Morscher (2003, pp. LXVI–LXX) on the problems of and restrictions for using schematic symbols and variables within square brackets.
- ¹⁹ The left-to-right direction is close to Künne's rendering of the so-called *Port Royal Constraint* in Künne (2001 [2008], p. 217).
- ²⁰ As translated in Mates (1986, p. 127), who also refers to similar passages. A transcript of the original passage, manuscript LH IV viii 61r, can be found in Mugnai (1992b, p. 158):

Diversa realiter solent sensibus distingui, conceptualiter diversa, sola mente, seu diversa formalitatibus, etsi non rebus. Velut in plano Triangulum et Trilaterum re non differunt, sed solo concepto, itaque realiter sunt idem, non formaliter. Trilaterum qua tale latera dicit, triangulum angulos.

- ²¹ Morscher (2004) reconstructs the criterion and suggests how Bolzano's rejection of the criterion might have prompted Meinong to endorse it. Lapointe focuses on the criterion itself (cf. Lapointe, 2007, Lapointe, 2011, ch. 2). Künne (2001 [2008]) focuses on a related thesis, the *canon of reciprocity*, in WL §120. Only Centrone (2010b), also more concerned with §120, discusses some of the arguments.
- ²² Bolzano himself talks about [triangle] rather than [triangularity]; he also states that this concept is combined with the *proposition* [which is equilateral]. The assumption that whole propositions are involved in a vast amount of concepts (WL §58 [I, 252f], §60 [I, 260f]) is problematic for independent reasons, which is why it is paraphrased away for the sake of this paper.
- ²³ The English translation invites another confusion of idea-levels by translating “noch in der Vorstellung *a* selbst vor” with “nor in the concept of *a* itself” instead of “nor in the idea *a* itself,” as I corrected in the quotation above.
- ²⁴ Otherwise, according to CONTOPRO, all ideas would have to be simple, which both Leibniz and Bolzano deny.
- ²⁵ Referring to WL §92 [I, 433], it has been suggested (Berg, 1962, p. 74; Centrone, 2010b, 322, fn. 25; Krickel, 1995, p. 41) that also within Bolzano's own theory, one can identify the content of a simple idea, that is, the mereological sum of its constituents, with the simple idea itself. In this passage, however, Bolzano states that what *would be* the content of a simple idea is the same as the simple idea itself (“ist das, was ihr Inhalt wäre, ein und dasselbe mit ihnen selbst,” my emphasis), conveying that simple ideas do not have a content and one should not talk this way. Depending on the aims and rules one has set for one's reconstruction, it might be advantageous to assume that simple ideas have a content with which they are identical, but it is not Bolzano's official position.

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