MASTERARBEIT / MASTER'S THESIS

Titel der Masterarbeit / Title of the Master's Thesis

„Green paradox effects in models with endogenous resource stock"

verfasst von / submitted by

Werner Schmid BSc

angestrebter akademischer Grad / in partial fulfilment of the requirements for the degree of

Master of Science (MSc)

Wien, 2023 / Vienna 2023

Studienkennzahl lt. Studienblatt / degree programme code as it appears on the student record sheet:	A 066 913
Studienrichtung lt. Studienblatt / degree programme as it appears on the student record sheet:	Masterstudium Volkswirtschaftslehre UG2002
Betreut von / Supervisor:	ao. Univ.-Prof. Mag. Dr. Andreas Novak

Abstract

After Sinn’s (2008) groundbreaking article on the green paradox effect and its potential consequences for efficient environmental policies a large body of research developed to analyze different model assumptions and their consequences for the existence of green paradox effects. Given that many countries today try to implement carbon taxes that start low and increase over time one of the central driving forces of the green paradox it’s seems prudent to have a look at a neglected assumption concerning the green paradox that of an endogenous resource stock. This paper starts with a baseline model based on Stiglitz (1976) then extends the model based on Österle (2015) / Sinn (2008) to a new market structure and reintroduces a basic form of uncertainty. It is then extended considerably to a model based on Van d. Ploeg (2013) that also includes a backstop with or without uncertainties concerning its arrival time. It is shown that the model results concerning an increasing carbon tax as the environmental policy are robust to these modification of the original models in almost all cases giving certain necessary new assumptions, that the effects of uncertainty in our extended model are ambivalent and that a constant tax doesn´t cause a green paradox under the specific assumptions.

Nach Sinn’s (2008) bahnbrechendem Artikel über den Grünen Paradox Effekt und seine potentiellen Konsequenzen für die Umweltpolitik entstand eine umfangreiche Literatur zum Thema um unterschiedliche Modellannahmen und ihre Konsequenzen für die Existenz des Effekts zu analysieren. Gegeben das zahlreiche Staaten heute versuchen CO_2 Steuern einzuführen, welche auf niedrigem Niveau starten und danach über die Zeit ansteigen, scheint es angemessen sich einer vernachlässigten Annahme bezüglich des Grünen Paradoxons zu widmen, der eines endogenen Ressourcenbestands. Diese Arbeit startet mit einem Grundmodel basierend auf Stiglitz (1976) erweitert das Model dann basierend auf Österle (2015) / Sinn (2008) in Bezug auf die Marktstruktur und führt eine einfache Form von Unsicherheit ein. Das Model wird danach wesentlich erweitert basierend auf Van d. Ploeg (2013) um eine Backstop-Technologie mit und ohne Unsicherheiten bezüglich der Ankunftszeit. Es wird gezeigt, dass die Resultate bezüglich einer steigenden CO_2 Steuer als der Umweltpolitik robust bezüglich der Modifikation der originalen Modelle sind in nahezu allen Fällen, gegeben gewissen notwendigen Annahmen, dass die Effekte der zusätzlichen Unsicherheiten in unserem erweiterten Modell ambivalent sind und das eine konstante Steuer nicht zu einem grünen Paradoxon führt gegeben die spezifischen Annahmen.

Contents

1 Introduction

Anthropogenic climate change might be one of the greatest environmental challenges mankind has ever faced (IPCC, 2018) as Earth´s climate and in particular it´s atmosphere should be understood as a global public good and the past and current rate of resource extraction by resource owners solving their intertemporal allocation problems creates huge external effects in the absence of effective regulation. These are difficult to internalize because of commitment and enforcement problems of international climate treaties and the still essential role non-renewable resources play as energy and production inputs.

While afforestation and with technological progress sequestration might play auxiliary roles successful environmental policy mainly needs to flatten the extraction path of non-renewable resources especially fossil fuels ideally keeping them in situ. While the optimal carbon emission regulations are well studied and modelled Chichilnisky & Heal (1994), Nordhaus & Yang (1996) Sinclair (1994), Van der Ploeg & Withagen (2012a) the failure to successfully implement them has led to the study of imperfect environmental policies as well. Unintended consequences of well-intended but suboptimal demand reducing policies might under specific conditions lead to a failure of reducing climate damages from resource extraction or even see an increase in global emissions often caused by an insufficient understanding of the dynamics of the supply side reaction.

1.1 The Green Paradox problematic

A green paradox refers to a situation in which – well intended – climate policy designed to lower climate damages actually leads to a worsening of climate damages versus a baseline scenario, because the supply side reaction of the resource owners is not correctly understood or taken into account by the policy maker. Future and anticipated policies that aim to reduce the demand for a non-renewable extracted resource increase the present (and near future) equilibrium rate of extraction of the resource, causing a frontloading in the extraction path as it distorts the relative marginal profits such that future extraction becomes less attractive. It is quasi the intertemporal form of the spatial import leakage. This intertemporal substitution of resource extraction quantities caused by demand reducing policies might be a consequence of the perfectly or near perfectly inelastic total supply (and reserves to be exploited). For non-renewable resources this intertemporal dimension of the extraction decision is of great importance (Di Maria & van der Werf, 2012).

Sinclair (1992) presented a model that showed the "paradoxical" finding that carbon taxes rising over time might actually worsen the problem by frontloading the extraction of non-renewable resources and also causing an exhaustion effect as resource production has to fall faster subsequently capital formation is reduced in the model. He argued that a dynamic approach that takes the supply side and thereby the time profile of the extraction decision sufficiently into account is needed.

The main contribution that got the Green Paradox discussion started is Sinn´s (2008) paper, where he stated that the policies used by the Kyoto abatement countries might actually worsen the climate change problem because of their focus on the demand side. According to Sinn the problem is fundamentally caused by a stronger downward pressure on future non-renewable resource prices caused by the climate policies than on the current prices, which as in the case of Sinclair creates an incentive to frontload extraction by the forward looking resource owners. So the relative net value of the resource is changed for example by an increasing tax and the producers tilt there production toward the present. Therefore we see an accelerated accumulation of greenhouse gases in the atmosphere and the climate policy leads to even faster global warming, a phenomenon he called "the green paradox".

Sinn shows in his model - based on Hotelling's theory of exhaustible resource extraction (Hotelling, 1931) – that focusing on an intertemporal supply side perspective such a green paradox occurs in the case of an increasing cash-flow tax as well as in the case of an increasing sales tax (where the growth rate of the tax is higher than the social discount rate) and does not happen for constant taxes. Importantly Sinn also argues (p. 381) that such a problem might also arise for the majority of policies that aim to reduce the demand for non-renewable resources via various forms of subsidies for replacement technology, quantity constraints or "green products" as they will shift the carbon demand curve downward at each point in time. If the resource owners are able to anticipate this they will again extract more in the present and less in the future in his model and so the climate policies will not mitigate the damaging effects on the climate as they were intended to do. Since then a number of papers have been published on the topic.

An important differentiation of the Green Paradox that is going to be used in this paper is that developed by Gerlagh (2011) into a weak form that only sees initial or short term increases in the climate damage from the implementation of the environmental policy while the strong form means that the present value of the sum of all future climate damages increases vs the baseline scenario. For an overview on the subsequent green paradox literature I would suggest Di Maria & v. d. Werf (2012). Which give a classification of the literature into several categories.

One of the assumptions widely used concerning the modelling of the Green Paradox problematic is that of a fixed and known resource stock given at the beginning of the planning horizon of the resource owners. In reality firms have to invest in exploratory activities and geographical studies which are costly and risky (Bhattacharyya, 2011). This paper intends to expand the analysis of the green paradox phenomenon under the assumption of endogenous resource exploration as an initial outlay and look at a different market structure (monopoly) for a case without a backstop technology and a case with a backstop technology with and without a certain arrival time.

1.2 Exploration: an endogenous resource stock

Non-renewable resources (NRR) have a unique status among the factors of production as they need to be discovered and once they are their utilization rate is limited by geological and economic factors. So is for example the recovery rate often inversely related to the speed of extraction. Generally it holds that as known resource deposits decline extraction costs increase as labor and capital use of extraction intensifies as less favorable resource reservoirs with lower grades and which are less accessible are used. The smaller the reserves get the higher the chance that undiscovered resource deposits might exist that have lower extraction costs. (Lambertini, 2013)

The resource producers balance at the margin the cost of unknown deposits vs the costs of known deposits (Adelman, 1970). Exploration further is used to maintain and increase reserves as like inventories reserves are also held to reduce the risk of supply shortages or uncertainties of future costs of producing them. In any case the exploration process – with and without uncertainty – plays an important part in the debate on non-renewable resources and its effects on resource extraction. See Peterson (1978), Arrow and Chang (1982), Mohr (1988), Quyen (1988 and 1991) and Cairns & Quyen (1998).

Most models of resource extraction and basically no models concerning the green paradox with the exception of Österle (2015) a paper by Van der Ploeg (2013) that focuses on efficiency effects and subsidies and a new model by Day & Day (2022) that shows that for a specific form of upfront tax deductions for the development of resource reserves downward sloping tax profiles cause weak and may cause strong green paradox effects assume a fixed resource stock with which the extracting firm is already endowed. In reality exploration and development effort is needed to develop this resource stock which is a central aspect of resource supply. Exploration can be understood as investment in capital in the form of resource reserves. Policies and market conditions have been shown to influence extraction and exploration differently in certain models. If the resource owners for example fear expropriation of their resource stocks they have an incentive to speed up extraction but exploration might be discouraged Bohn & Deacon (2000). Peterson (1978) shows that increasing discount rates might cause decreased extraction rates in the long run rather than accelerated ones.

1.3 Significance of modeling endogenous resource stocks

There are a number of arguments why exploration of natural resources might still play a big role in resource economics. Vast amounts are still spent by mining companies and oil producers on the discovery and the development of new resource deposits which for many non-renewable resources have significantly increased in the last decades (Arezki, van der Ploeg, & Toscani, 2019). New geographic areas have become available for extraction like the Arctic and now potentially even the Antarctic. New technology becomes available enabling future large-scale marine mining

(Marvasti, 2000) and eventually even space will be a new frontier for exploitation of resources. Therefore many resources that are termed exhaustible might be actually better called non-renewable instead (Stürmer & Schwerhoff, 2012). Including exploration in some form into models of resource extraction when analyzing potential green paradox problems seems therefore to be a useful exercise.

Investment in new technology enables firms to increase their reserves but with diminishing returns as progressively lower grade resource deposits are unlocked or access to the resources becomes more difficult as they are deeper down in mines etc. On the other hand the resource quantities unlocked by these new technologies increase exponentially which means the diminishing returns from investment in exploration technology are offset by the geological distribution of these resources (Stürmer & Schwerhoff, 2012). While some authors like Sinn (2012) argue that the majority of discoveries have already been made and exploration might not be as important to be modeled and firms can just be endowed with their resource stocks at least in certain fields like shale oil and gas but also for certain minerals and their associated technology it is clear that exploration still has significance for the non-renewable resource markets to play in the coming decades (Kim & Lee, 2018; Luderer et al., 2018).

The reality of still rising non-renewable resource stocks that could cause additional failures in environmental policy gives then also an argument for additional intervention. If treaties like the Paris Agreement want to limit future profitability of non-renewable resource sources sufficiently a good understanding of potential resource stocks that can still be developed is of great importance. While such policies that set clear and strict limits to the amount of the carbon that can be emitted clearly reduce the revenues that can be created from new discoveries at the same time such policies especially if they have long lead times could also cause “black gold rush” like effects where resource owners and developers speed up the exploration process to still get the resources developed and out of the ground before this no longer is profitable (McGlade & Ekins, 2015 and Cairns, 2014).

Furthermore in the future depending on economic development more conservative administrations might come into power and reverse or at least postpone some of the climate policies which could mean that exploration might play a more important role again. Given the current situation of energy insecurity in Europe as a consequence of the conflict in Ukraine this might be the case even with less conservative and even green parties in power. Finally it is interesting to note that one of the explanations for the inconsistency of the implications of the basic Hotelling framework and also of empirical studies on the price path of non-renewable resources might be the assumption of a fixed and known resource stock (Kronenberg, 2008) and that to better understand the reaction of resource owners to demand reducing policies might actually require a look at the costs and uncertainties of exploratory efforts.

With the total extraction amount given an endogenous change in the price at any date causes the intertemporal substitution effect as it modifies the relative marginal profits between extractions now and later which is the main argument for Sinn's (2008) green paradox effect. But this price change might change the total extraction amount given as exploration might become more or less profitable and therefore the resource stock available might change. This stock effect can be dominated by a substitution effect. The price of a NRR is affected by its price at another date even if the costs between dates are independent. This is because of the scarcity rent included in the marginal extraction costs so the resource rent from extraction instead of keeping it in situ that means that the extraction costs at all dates are connected. So we have to deal with this intertemporal substitution relative to "normal" goods that don´t have this problem since they are not affected by eventual exhaustion. Exploration is affected by these resource rents from extraction and so depends on the future prices which are affected by taxes and therefore the resource stock that will be developed.

1.4 Monopoly in Resource extraction

Since we extend the model in 3 different forms in a monopoly market structure a few words on the Literature here. For a general overview of the implications of a monopolistic market structure in resource extraction models of non-renewable resources see Dasgupta and Heal (1979) and

Krautkraemer (1998). Concerning the green paradox literature most models deal with competitive markets but a few confirm their findings for the case of monopolistic extraction like Grafton et al. (2010). As to my knowledge it hasn´t been analyzed specifically for a green paradox model that includes an endogenous resource stock and a backstop technology given our specific form of demand function and an increasing cash flow tax.

1.5 Summary of the following chapters

Chapter 2 develops a baseline model of resource extraction to later refer to where the market form is set to a monopoly the resource stock is kept exogenous and the different model assumption are explained in detail. The model shows the basic mechanism behind the 2 forms of green paradox effects. It is then expanded to an endogenous resource stock in chapter 3 which changes the findings concerning the existence of the weak and strong green paradox as a consequence of the climate policy (an increasing cash flow tax). We then take a small look at a basic form of uncertainty concerning property rights which were included in Sinn's (2008) original paper. After that we expand to a model in which there is the potential that a backstop technology becomes available and how it influences initial and total extraction differently in chapter 4 based on Van der Ploeg (2013) where we will take a look how a constant and an increasing tax will affect the existence of a green paradox. After that we have a small discussion about two of the assumptions used the damage function and the modelling of the initial resource stock decision in chapter 5. In the following Chapter 6 conclusions about the model results are discussed and summarized to finally be followed by the appendix with the derivations used.

2 The baseline model with given resource stock and a monopolistic firm

We start with a baseline model based on Stiglitz (1976) and then expand from there to see if and how this affects the existence of weak and strong green paradox effects. This is done to better isolate the effects specific assumptions might have as quite a few papers on the topic stress that the occurrence of the weak and strong green paradox effects strongly depends on the specific underlying assumptions and it is often hard to assess in advance what might be relevant or not (Di Maria & van der Werf, 2012). Furthermore this basic model is used to explain the different components used in detail and potential alternatives and finally illustrates the mechanism that causes green paradox effects in a simple and clear way but still in a dynamic setup instead of the often used 2-period models.

Limited supply sources in resource markets make it significantly more common that there exists market power than in traditional goods markets. It seems natural to take a closer look at the extreme case where the resource is provided by a single supplier which is done here and continued to be used in all the following chapters as well. This model will then be extended in the next chapters to include a resource stock that is chosen based on an investment decision / cost function, uncertainty and a backstop technology.

2.1 Assumptions

2.1.1 The optimal control problem

The monopolistic firm maximizes its profit from extracting a homogenous, single, known and absolutely scarce non-renewable resource (NRR) with a price that depends on the amount extracted such that $p_t = p(R_t)$. There are no extraction costs (other than the scarcity cost of extraction of course) and there are also no capacity restriction for the extraction process (costless, immediate).

This gives for the case with a given initial resource stock the following constraint optimal control problem:

$$\max_{R} \int_{t=0}^{\infty} p(R_t)R_t\theta_t e^{-rt}dt \tag{2.1.1}$$

subject to:

$$\dot{S}_t = -R_t, t \in [0, \infty], S_0 > 0 given, \int_{t=0}^{\infty} R_t dt \leq S_0 \tag{2.1.2}$$

2.1.2 Notation

We have that t denotes time, t ≥ 0. The market price of the NRR is $p_t = p(R_t)$ with R_t as the supply of the resource which equals the quantity extracted and consumed in t (which is our control) S_t is the remaining reserves at time t with $S_t > 0$ (and therefore our state variable). S_0 is the initial stock of an exhaustible, homogenous resource in t = 0 with $S_0 > 0$. The constant exogenously given discount rate is denoted by r. θ_t is the tax factor and we will use λ_t as the current value costate variable used which is the shadow price of resource in situ. A "^" above a variable indicates a growth rate and a "·" above a variable means a time derivate.

2.1.3 Taxation

In general any taxation of discovered deposits reduces the private value of these deposits and so reduces the profitability and scale of exploration. This can be complicated to model if uncertainties about exploration are taken into account. A thorough analysis of the effects of taxation on resource extraction was given by Long & Sinn (1985). We will analyze the case of an increasing tax here since one of the main reasons why green paradoxes might occur is that such taxes start "small" and then increase over time too slow or too fast as we will see which facilitates green paradox effects. Here no uncertainty is assumed and the tax is an increasing cash flow tax as was used by Sinn (2008) with:

$$\theta_t = \theta_0 e^{\hat{\theta} t} \tag{2.1.3}$$

Where θ_t is the tax factor (the percentage of the revenue the firm can keep), θ_0 is the initial tax factor and with $\hat{\theta} < 0$ as the constant given growth (decline) rate of the tax factor. Therefore the tax factor will decrease slower / the tax rate increase slower for higher growth rates $\hat{\theta}$.The tax rate therefore is $1 - \theta_t$. This means that if there is no tax we will have $\theta_t = 1 \ \forall t \geq 0$ and $\hat{\theta} = 0$. We will have a look at the impact of a constant tax factor in Chapter 4.

2.1.4 The demand function

The demand function is assumed to be:

$$R_t = f(p_t) = p_t^{-1/\mu} \tag{2.1.4}$$

With $f'(p_t) < 0$, isoelastic with $1/\mu$ as the constant, absolute value of the elasticity of demand. Marginal revenue is therefore $(1-\mu)p_t$.

According to Dasgupta and Heal (1979, p. 159+) this specific form ensures given our model assumption that the optimal time horizon of extraction is infinity for this model, which makes the optimization process easier as the optimal time horizon for the extraction part is thereby already given and if there is no reason to assume that the economy does terminate at a specific future date this might actually be the most appropriate way to model it. Of course there is always the possibility that the world will end at some future date which rationally should be accounted for. See Dasgupta and Heal (1979, p. 155) for a discussion about this. This also means that we have $S_t > 0, \forall t \geq 0$.

2.1.5 The damage function

The following damage function is used to model the climate damage caused by extraction of the resource which equals utilization in our model in a simple way which originally was taken from Gerlagh (2011):

$$\Omega = \int_0^\infty \chi_t R_t e^{-rt} dt \tag{2.1.5}$$

Ω is the NPV of the climate damage caused by resource extraction and χ_t is to be understood as the shadow price on emissions, according to Gerlagh (p. 87). With $\chi_t = \chi_0 e^{\hat{\chi} t}$ and its growth rate positive indicating that the marginal climate damage is increasing over time. This reflects an assumption in climate change literature (see for example Ulph & Ulph, 1994) that an increase in the concentration of greenhouse gases in the atmosphere leads to increasing marginal damages caused to the climate. For a further discussion about this assumption please look at chapter 5.1. This is also why it is so important how the intertemporal distribution of extraction looks like even if the amount that is extracted over the whole time horizon stays the same or diminishes. Furthermore it is assumed that the growth rate of the marginal climate damages $\hat{\chi}$ will be lower than the discount rate r.

2.2 Optimal temporal distribution

The (current value) Hamiltonian is given by using our demand function (2.1.4) as:

$$H = \theta_t p(R_t) R_t - \lambda_t R_t = \theta_t R_t^{1-\mu} - \lambda_t R_t$$

The maximum principle is:

$$H_R = \theta_t(1-\mu)R_t^{-\mu} - \lambda_t = 0 \qquad (2.2.1)$$
$$\Leftrightarrow R_t = \left[\lambda_t / \theta_t(1-\mu)\right]^{-1/\mu}$$

the dynamic constraint:

$$\dot{\lambda}_t = -H_S + r\lambda_t \Leftrightarrow \dot{\lambda}_t = r\lambda_t \qquad (2.2.2)$$

The transversality condition we have is:

$$\lim_{t\to\infty} S_t \lambda_t e^{rt} = 0 \qquad (2.2.3)$$

With no extraction costs H_S is 0 and drops out of the dynamic constraint. We know therefore that $\lambda_t = \lambda_0 e^{rt}$. Using again the demand function gives us the price as: $p(R_t) = \lambda_t / (1-\mu)\theta_t$

Then using the dynamic constraint (2.2.2) as well as the definition of tax factor θ_t (2.1.3) gives us the following for the price path of the resource (the price will now just simply be written as p_t to keep it compacter if it doesn´t lead to confusion):

$$p_t = \frac{\lambda_0}{(1-\mu)\theta_0} e^{(r-\hat{\theta})t} = p_0 e^{(r-\hat{\theta})t} \qquad (2.2.4)$$

Differentiation with respect to time and dividing by the price gives us the growth rate of the price path which in our model is a modified form of the Hotelling rule for a model with a tax factor as:

$$\hat{p}_t = \dot{p}_t / p_t = r - \hat{\theta} > r.$$

The resource owner demands a higher price increase then without an increasing tax (would he only demand r then he would sell more of the resource in the current period because he knows revenue will further decline which increases supply in the current period which leads to a lower price which increases the growth rate and equilibrium then means that the equation is satisfied). We see that this form of a tax is not neutral. The right hand side gets larger and keeping the price constant the rate of change of the price must increase which means it will cause frontloading of extraction and given the resource constraint an extraction path that needs to be steeper. Furthermore (2.2.4) tells us now that initial price is:

$$p_0 = \frac{\lambda_0}{(1-\mu)\theta_0} \tag{2.2.5}$$

Which will be relevant when we expand the model to include an endogenous initial resource stock in Chapter 3. Differentiating the demand function with respect to time and plugging in our finding that the growth rate of the price is $r-\hat{\theta}$ we get the growth rate of extraction as:

$$\dot{R}_t = -1/\mu\, p(R_t)^{-1/\mu-1}\dot{p}_t = -1/\mu R_t\,\hat{p}_t \Leftrightarrow \dot{R}_t/R_t = \hat{R}_t = -\big(r-\hat{\theta}\big)/\mu < 0$$

So the optimal growth rate of extraction is constant and smaller than zero consisting of the interest rate the tax factor and the elasticity of demand. The higher the elasticity of demand the faster the decline. Solving the differential equation gives the optimal extraction path therefore as:

$$R_t = R_0 e^{\frac{-(r-\hat{\theta})t}{\mu}} \tag{2.2.6}$$

Since: $R_t = p_t^{-1/\mu} = \left(\lambda_0 / (1-\mu)\theta_0\right)^{-1/\mu} e^{\frac{-(r-\hat{\theta})t}{\mu}} = p_0^{-1/\mu} e^{\frac{-(r-\hat{\theta})t}{\mu}} = R_0 e^{\frac{-(r-\hat{\theta})t}{\mu}}$, this means that resource extraction will be positive as long as the absolute value of the elasticity of demand which is 1/ μ is greater than one. Otherwise higher prices would always lead to a higher output since there are no restrictions on maximal output per period meaning extraction would be nil in all periods with infinitely high prices. We will exclude this extreme case. This is a well-known fact in resource extraction models with monopolistic market structure (Stiglitz, 1976). So there will be at least some extraction in every period which of course will become infinitesimally small as time approaches infinity and the extraction rate goes to zero asymptotically.

As there are no extraction costs it can never be optimal not to extract all of the resource since the extraction of every last unit will be profitable. It must therefore hold that the initial resource stock equals the total sum of all extraction over the infinite time horizon: $\int_0^\infty R_t dt = S_0$. Using this fact and the equation for the optimal extraction path (2.2.6) we can rewrite the initial extraction rate as:

$$R_0 = \frac{(r-\hat{\theta})S_0}{\mu} \tag{2.2.7}$$

And therefore get for the resource extraction path:

$$R_t = \frac{(r-\hat{\theta})S_0}{\mu} e^{\frac{-(r-\hat{\theta})t}{\mu}} \tag{2.2.8}$$

Inserting into the inverse demand function $p_t = R_t^{-\mu}$ and using our optimal extraction path with our state equation gives us then also the paths for price and the resource stock:

$$p_t = \left[\frac{(r-\hat{\theta})S_0}{\mu} e^{-\frac{(r-\hat{\theta})t}{\mu}} \right]^{-\mu} = \left[\frac{\mu}{(r-\hat{\theta})S_0} \right]^{\mu} e^{(r-\hat{\theta})t} \tag{2.2.9}$$

$$S_t = S_0 e^{\frac{-(r-\hat{\theta})t}{\mu}} \tag{2.2.10}$$

2.3 Existence of weak and strong green paradox

A week green paradox exists if the initial damage increases if the increasing tax is implemented. The initial damage is just equal the initial extraction rate R_0. Remember that if there is no tax then $\hat{\theta} = 0$ and we see from (2.2.7) that in this case the extraction rate in the initial period R_0 would be minimal. A week green paradox will therefore exist always in this simple model with an exogenous resource stock if there is there is any kind of tax. The initial tax factor θ_0 is not relevant for the initial extraction but the faster the tax factor declines (lower $\hat{\theta}$) / the faster the tax rate increases the higher the initial extraction and therefore damage will be.

For the strong green paradox we have to now look at the net present value of overall damage Ω (2.1.5) and see if the implementation of the tax increases it with: $\Omega = \int_0^\infty \chi_0 e^{-(r-\hat{\chi})t} R_t dt$. Inserting the optimal time path we found for extraction (2.2.8) and integrating this gives us:

$$\Omega = \frac{(r-\hat{\theta})S_0\chi_0}{\mu(r-\hat{\chi})+r-\hat{\theta}} \tag{2.3.1}$$

For the baseline scenario total damage is therefore also independent of the initial tax factor. Next we look at how Ω depends on the growth rate of the tax factor:

$$\Omega_{\hat{\theta}} = \frac{-S_0\chi_0(\mu(r-\hat{\chi})+r-\hat{\theta})+(r-\hat{\theta})S_0\chi_0}{\left[\mu(r-\hat{\chi})+r-\hat{\theta}\right]^2} = \frac{-S_0\chi_0\mu(r-\hat{\chi})}{\left[\mu(r-\hat{\chi})+r-\hat{\theta}\right]^2} < 0$$

Which is strictly negative since we assumed: $r > \hat{\chi}$. So again the case without any taxes minimizes overall damage and any implementation of the tax will also cause a strong green paradox to occur. The higher the growth rate of the tax / the faster the decline of the tax factor (lower $\hat{\theta}$) the higher the environmental damage caused. As S_0 is given and the total extraction stays the same the increase in the growth rate of the tax makes it necessary for the firm to frontload its extraction after optimization thereby increasing total climate damage based on our damage function without the possibility to compensate by changing its investment in the resource stock. This is the core mechanism for our form of the green paradox.

In the simple baseline model based on Stiglitz (1976) with a monopolistic resource extraction firm both the weak and the strong green paradox will occur if the environmental policy is implemented since the total extraction does not change given the fixed resource stock and no extraction costs and the tax will simply lead to a frontloading of the extraction path increasing initial and total environmental damage. From this basis let us next take a look at a model with an initial investment decision for the initial resource stock S_0.

3 The model with an initial investment decision and monopolistic competition

3.1 Assumptions

The following model setup is based on Sinn (2008) concerning uncertainty, Österle (2015) concerning the idea for an analytical solution for the condition for the existence of a green paradox and Lasserre (1991) for the modelling of the exploration endogenously via an upfront investment. The market structure remains monopolistic the demand function used is slightly different than in Österles model to get a nicer condition for the existence of the strong green paradox and we will have a small look at a simple form of uncertainty of property rights at the end. Later in Chapter 4 we will expand the model by including a green alternative, a backstop technology that will change its price and for which the arrival time is not known and compare it whit this model as a special case. As discussed in chapter 1 there are many reasons why the resource stocks for firms should not be assumed as just given since taxes will effect exploratory effort via the change in optimal extraction decisions. So while the taxes set an incentive to speed up extraction they also give an incentive to reduce exploration and therefore this countervailing forces will make the existence of green paradox effects less plausible. The firm now has to optimize its initial costs for its investment in the creation of the initial resource stock S_0.

So instead of having S_0 as given it is now endogenously developed initially by the monopolistic resource extracting firm before extraction actually starts at $t = 0$ This exploratory effort creates upfront cumulative costs: $C(S_0)$ with $C'(S_0) > 0$ and $C''(S_0) > 0$ with increasing costs per unit. Or it requires stated otherwise an upfront investment for the initial resource stock with $S_0 = E(I)$ with the assumptions that $E'(I) > 0$ and $E''(I) < 0$ so that S_0 increases with investment but at a declining rate (Gaudet and Lasserre, 1988). Such increasing costs and declining returns for exploratory effort are standard assumptions in resource exploration literature see for example Pindyck (1978) or Dasgupta and Heal (1979) since it seems reasonable that the more a resource is already explored the higher the effort for the next unit becomes (given the technological level for discoveries). So the assumption is that the investment in exploration / the exploration cost needs to be paid before extraction starts which also avoids the problems of concavity of the value function appearing when exploration and extraction of the NRR are modeled to occur simultaneously see Pindyck (1978).

To keep things manageable we use the following specific form for such a rising and convex cost function which is again slightly modified from Österle to get a bit cleaner derivations than her in the end for the condition of the strong green paradox:

$$C(S_0) = \beta S_0^{\alpha+1} \tag{3.1.1}$$

With $\alpha > 0$ and $\beta > 0$. Such functional forms are for example often used for cost function in shale oil exploration see for example Henriet & Schubert (2019). Please note that although Österle states in her paper that according to Lasserre (1991, p. 105) the optimization for the exploration could be expressed as: $C(S_0) = \min \int_{-T}^{0} e^{-rt} c(s_{-t}, S_{-t}) dt$ and subject to: $\dot{S}_{-t} = s_{-t}$. With $-T < 0$ as the initial exploration date where there are no discoveries yet. The (future) resource owner now could exert exploratory effort with cost of $c(s_{-t},S_{-t})$ with s_{-t} the discovery rate and S_{-t} the accumulated total discoveries. In this way the resource stock S_0 for the extraction phase could be developed over a development period. But this is not actually used in her model where we assume the previously given form and will equate the marginal costs with the marginal profits as we will see and the exploratory effort is therefore a onetime investment at the beginning before extraction starts. We lastly assume, that the total reserves of the resource on earth are $S_E > S_0$ and $S_E - S_0$ remains untapped as it is not economically feasible to recover it in market equilibrium.

The model is therefore now expanded by an endogenously determined initial resource stock so the optimization decision of the monopolistic firm is given by:

$$\max_R \left(\int_0^\infty (p(R_t) R_t \theta_t) e^{-rt} dt \right) - C(S_0) \tag{3.1.2}$$

subject to:

$$\dot{S}_t = -R_t, t \in [0, \infty], S_0 > 0 endogenous, \int_{t=0}^\infty R_t dt \leq S_0 \tag{3.1.3}$$

In the extraction phase the net present value of the cumulative costs $C(S_0)$ for exploration are already determined (note that they are outside the integral sign) and therefore a constant so that we get the same optimal paths. The initial extraction rate as in the baseline model of the monopolistic resource owner is therefore given by (2.2.7) and the extraction path by (2.2.8).

3.2 The exploration phase

Modelling exploration is a tricky subject in resource extraction models and in order to keep the calculations reasonably complex we use a technique based on Lasserre (1991, p. 105+). He states that in the initial time period of the extraction phase the total marginal costs for the initial resource stock needs to be equal to the marginal utility that extraction of the resource offers which is represented by its shadow price λ_0. To clarify this further, the economic interpretation of the costate variable λ is that of the shadow price of the resource in situ. It gives the marginal loss in present value of the resource in situ if the current rate of extraction is increased (marginal user cost). The optimal rate of extraction is chosen such that it equates this marginal user cost with marginal profits. Which means that:

$$C'(S_0) = \lambda_0 \tag{3.2.1}$$

This is a transversality condition for $t = 0$ that tells us that the exploration cost for an additional unit of the resource needs to be equal to the value of the non-renewable resource in situ, its initial shadow price (initial scarcity rent). Up to this point the cost for exploration needs to be paid in optimum (Gaudet & Lasserre, 1988). Of course our λ_0 is different than the one we would have gotten in a competitive market. Using our specific form for the cost function (3.1.1) and the initial price from (2.2.5) we therefore get: $(\alpha+1)\beta S_0^{\alpha} = p_0\theta_0(1-\mu)$. Next we use our equation for the initial extraction rate (2.2.7) and our specific isoelastic demand function (2.1.4) and get:

$$(\alpha+1)\beta\left[\frac{R_0\mu}{(r-\hat{\theta})}\right]^{\alpha} = R_0^{-\mu}\theta_0(1-\mu) \Leftrightarrow R_0^{\alpha+\mu} = \left[\frac{r-\hat{\theta}}{\mu}\right]^{\alpha}\frac{\theta_0(1-\mu)}{(\alpha+1)\beta} \Leftrightarrow R_0 = \left[\frac{(r-\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{\theta_0(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}}$$

We now have the following expression for the initial extraction which we will use to compare the tax and the no tax scenario:

$$R_0 = \left[\frac{(r-\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{\theta_0(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}} \qquad (3.2.2)$$

3.3 The Weak Green Paradox

A weak green Paradox arises in a situation where the initial damage in the policy scenario (T) is larger than the initial damage in the scenario without taxes (NT). So we have that: $R_0^{T} > R_0^{NT}$ we insert the extraction rate we found for the initial period (3.2.2) and remember that for the case with no tax we have a tax factor for the initial period $\theta_0 = 1$ (the company can keep everything / the tax rate therefore is 0) and the growth rate of the tax factor is given by $\hat{\theta} = 0$. This gives us:

$$\left[\frac{(r-\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{\theta_0(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}} > \left[\frac{r}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}}$$

$$\Leftrightarrow \theta_0^{\frac{1}{\alpha+\mu}} > \left[\frac{\mu r}{\mu(r-\hat{\theta})}\right]^{\frac{\alpha}{\alpha+\mu}} \Leftrightarrow \theta_0 > \left(\frac{r}{(r-\hat{\theta})}\right)^{\alpha}$$

The condition for the existence of a weak green paradox therefore is:

$$\theta_0 > \left(1-\hat{\theta}/r\right)^{-\alpha} \tag{3.3.1}$$

The occurrence of a weak green paradox can now only happen for certain cases of the tax schemes. It is more likely the higher the initial tax factor θ_0 / the lower the initial tax rate and the higher the growth rate of the tax / the decline of the tax factor (lower $\hat{\theta}$). Which can be seen in Figure 1 for α = 0.5 and r = 0.03 and the absolute value of $\hat{\theta}$. Furthermore one cannot exclude the possibility of the existence of a W.G.P. for any growth rate of the tax factor which is also the case in Österle's model with a competitive market structure. Since $0 \le \theta_0 \le 1$ and the right hand side of our condition (3.3.1) is always smaller than 1 for any admissible growth rate of the tax factor other than 0 (in which case there is no tax).

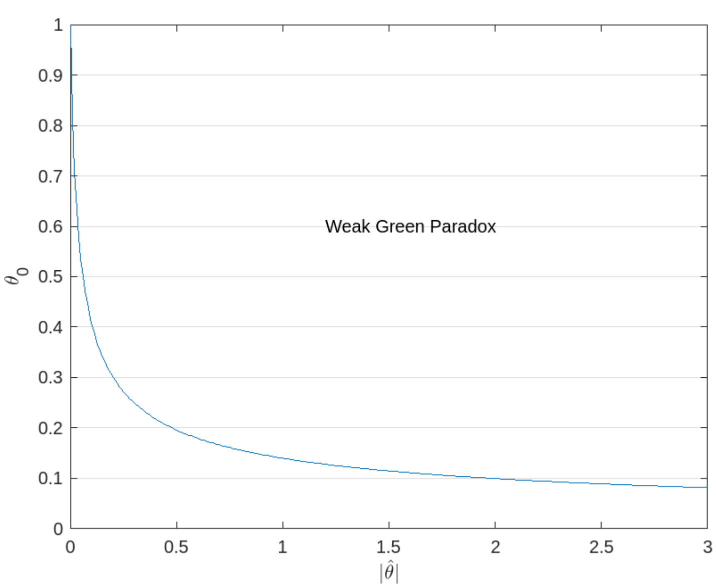

Figure 1: Shows for which combinations of the tax components a weak green paradox occurs (above the graph).

Furthermore the higher r ceteris paribus the higher the R.H.S of our condition and the less likely is a W.G. P. ceteris paribus. Lastly a high α increases the change for a weak green paradox given the tax scheme. This is the case because a high α means greater marginal discovery costs, so the reduction in discoveries caused by the tax is relative smaller therefore the price increase is also smaller and we have a lower reduction in the extraction rate. This is visualized again for the absolute value of $\hat{\theta}$ in Figures 2 and 3.

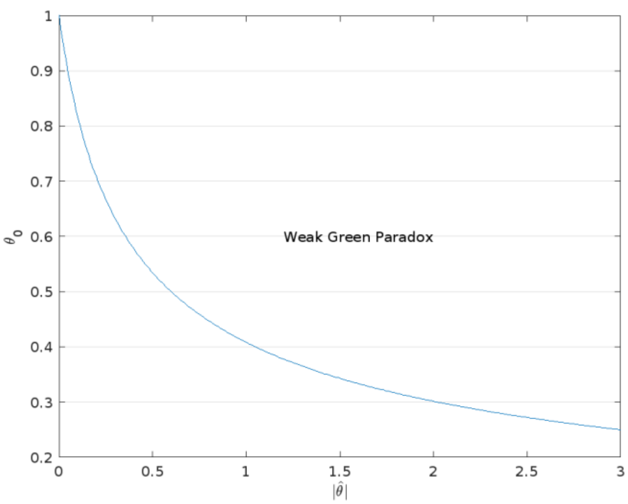

Figure 2: The effect of the discount rate with r = 0.2, a = 0.5

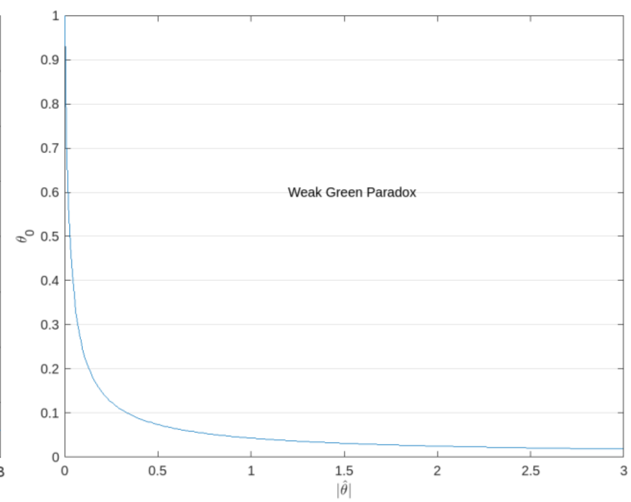

Figure 3: The effect of α with r =.0.03, α = 0.8

3.4 The Strong Green Paradox

We again have to look at the NPV of total damage Ω now to see if the policy scenario causes a strong green paradox to occur. Starting with the definition of the damage function, $\chi_t = \chi_0 e^{\hat{\chi} t}$ and inserting our expression for the optimal extraction path (2.2.8) and finally our expression for the optimal extraction in the initial period R_0 (3.2.2) we find:

$$\Omega = \int_0^\infty e^{-rt}\chi_t R_t dt = \int_0^\infty e^{(\hat{\chi}-r)t}\chi_0 R_t dt = \int_0^\infty e^{(\hat{\chi}-r)t} e^{-\frac{t(r-\hat{\theta})}{\mu}}\chi_0 R_0 dt =$$

$$= \int_0^\infty e^{-t(\frac{(r-\hat{\theta})}{\mu}+r-\hat{\chi})}\chi_0 \left[\frac{(r-\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{\theta_0(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}} dt = \frac{\mu\chi_0}{r-\hat{\theta}+\mu(r-\hat{\chi})}\left[\frac{(r-\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{\theta_0(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}}$$

So the reformulated total damage is given by:

$$\Omega = \frac{\mu\chi_0}{r-\hat{\theta}+\mu(r-\hat{\chi})}\left[\frac{(r-\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{\theta_0(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}} \tag{3.4.1}$$

With this expression for the NPV of total damage we can compare the policy scenario and the no-tax scenario to see when a strong green paradox might occur. Formally the condition is $\Omega^T > \Omega^{NT}$ (again with T standing for the tax case and NT for the case without taxes). Inserting the expression we found again remembering that $\theta_t = 1$ and $\hat{\theta} = 0$ for the case without taxes:

$$\frac{\chi_0}{\frac{r-\hat{\theta}}{\mu}+r-\hat{\chi}}\left[\frac{(r-\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{\theta_0(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}} > \frac{\chi_0}{\frac{r}{\mu}+r-\hat{\chi}}\left[\frac{r}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}}$$

$$\Leftrightarrow \frac{\theta_0^{\frac{1}{\alpha+\mu}}}{\frac{r-\hat{\theta}}{\mu}+r-\hat{\chi}}\left[\frac{(r+\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}} > \left[\frac{r}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\frac{1}{\frac{r}{\mu}+r-\hat{\chi}}$$

$$\Leftrightarrow (\theta_0)^{\frac{1}{\alpha+\mu}} > \frac{r+r\mu-\hat{\chi}\mu-\hat{\theta}}{r+r\mu-\hat{\chi}\mu}\left[\frac{r}{(r-\hat{\theta})}\right]^{\frac{\alpha}{\alpha+\mu}}$$

$$\Leftrightarrow \theta_0 > \left[1-\frac{\hat{\theta}}{r+r\mu-\mu\hat{\chi}}\right]^{\alpha+\mu}\left[1-\frac{\hat{\theta}}{r}\right]^{-\alpha}$$

Which gives us the following condition for the existence of the strong green paradox as:

$$\theta_0 > \left[1-\frac{\hat{\theta}}{r+\mu(r-\hat{\chi})}\right]^{\alpha+\mu}\left[1-\frac{\hat{\theta}}{r}\right]^{-\alpha} \tag{3.4.2}$$

We see that a high initial tax factor / low initial tax rate makes the existence of a strong green paradox more likely (as always ceteris paribus). So if the tax rate starts out relatively low and the growth rate of the tax is held the same total taxation will therefore be lower and exploration relatively higher than in a case with a high initial tax burden (and constant held growth rate). Which means that the frontloading of extraction might be the dominant effect.

The effect of the growth rate of the tax factor depends on other parameters, but it can be seen that if it goes toward 0 then the right hand side of the inequality will go towards 1 and it therefore would become less and less likely that a strong green paradox will occur. If on the other hand the growth rate of the tax goes toward minus infinity then the right hand side of the equation would go toward infinity and so a strong green paradox would then also no longer appear. So we have a range and if the growth rate of the tax factor gets too big or too small it can no longer exists. In between it still depends on the other parameters if a strong green paradox can exist or not (most strongly but not alone on α). Again matching the results of the competitive model. This is visualized in Figure 4 as always for absolute values of the growth rate of the tax factor for the case where the other parameters allow for the existence.

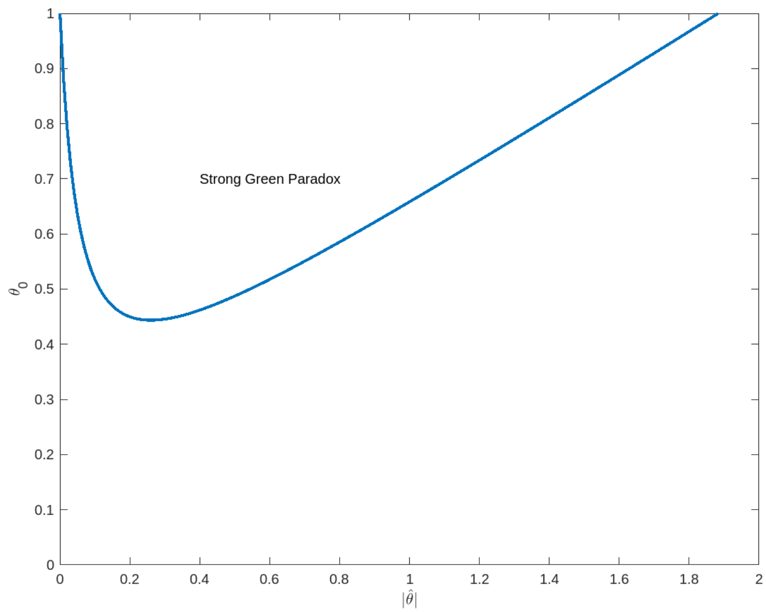

Figure 4: The range of growth rates of the tax factor that allow for a strong green paradox with: $r = 0.1$, $\chi = 0.01$, $\alpha = 4$, $\mu = 1$

Further we see that if the growth rate of the damage function factor $\hat{\chi}$ is small then the S.G.P. is more likely for given parameters. The smaller the growth rate of damage function factor is, the more influence the frontloading of extraction has and the more climate damage is caused. We see this in Figure 5 (low in blue).

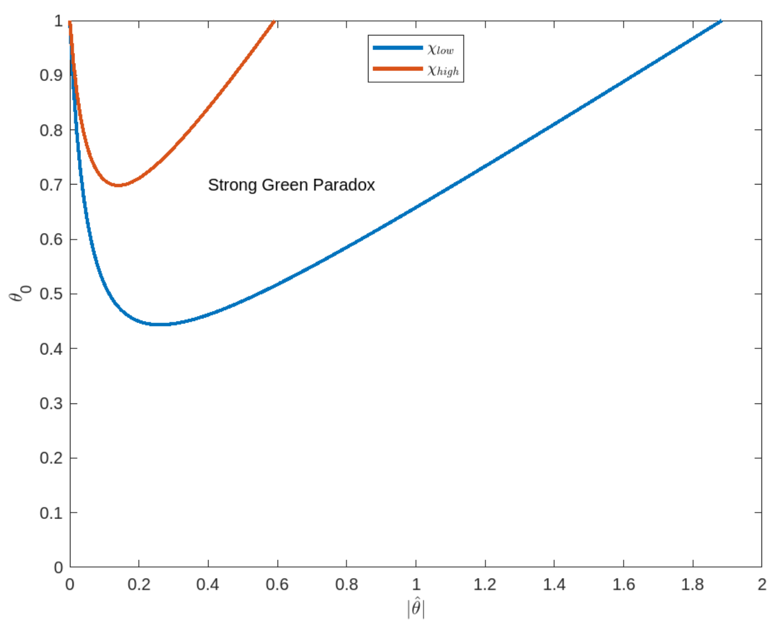

Figure 5: Effects of a low (blue) and high (red) growth rate of damage factor.

The discount rate r plays a role insofar as a higher discount rate ceteris paribus will allow green paradox effects to exist for higher growth rates of the tax which of course would be logical as the real cost of taxation would then be lower. This is shown in Figure 6 (note the scale of the x axis). For the parameter α from the exploration cost function we have (see 7.1.1) that an increasing α which means greater marginal exploration cost in absolute terms ceteris paribus increases the chance for a strong green paradox ceteris paribus, like it did in the case of the weak green paradox.

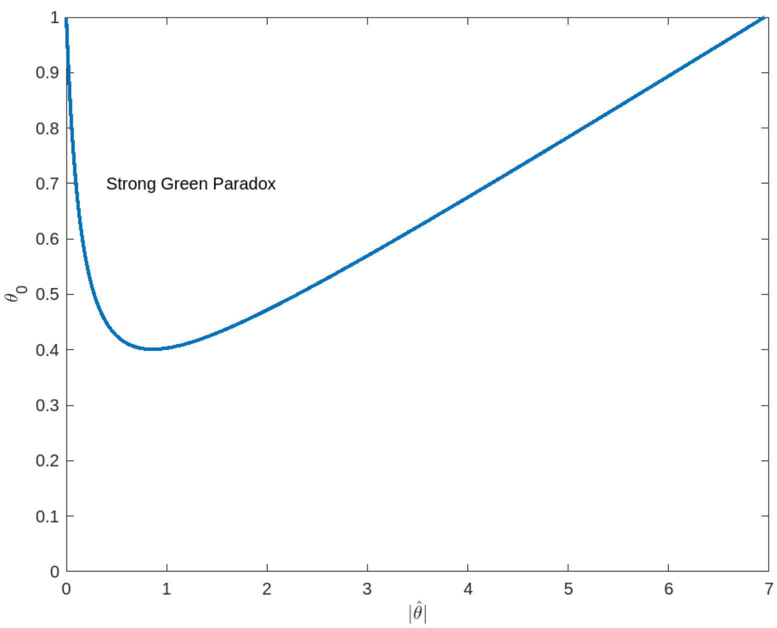

Figure 6: The effects of the discount rate on the existence of the S.G.P.

We can now see that the existence of the weak and the strong green paradox depends on the form (initial tax factor and the growth rate of the tax) of the implementation of the specific tax schedule. Ceteris paribus the weak green paradox is more likely to occur if the initial tax factor is high and if the growth rate of the tax is high. Again for holding the other parameters constant the strong green paradox will be less likely to occur if the initial tax factor is low and furthermore it can only potentially occur for a growth rate of the tax that isn´t too high or too low but even in this range must not occur depending on the other parameters. We found that this range increases in the discount rate.

Despite the change in market structure and the use of a slightly different demand and cost function these findings match the findings of Österle's model with a perfect competitive resource extracting firm given that the elasticity of demand is assumed as bigger than one (which is needed to guarantee a solution where prices don´t become infinite and extraction 0 in all periods as we stated in Chapter 2). So we see that given our assumptions the market form makes no relevant difference when we analyze the existence of weak and strong green paradoxes in this particular model specification. Of course such a finding still means that we now know that we need not worry if the observed market that resembles the assumption of the model is of competitive or monopoly form when we want to know if our specific climate policy could cause a weak or strong form green paradox.

The reason for this result might be the specific combination of no extraction costs which we require for an analytical solution and the use of an isoelastic demand function which can be considered nearly a standard assumption in resource extraction literature as it makes derivation far easier keeping our model an (quasi)autonomous one. We therefore will expand the model now further and take a look at uncertainty next.

3.5 Uncertainty

While we get for our model that the price or in case for a model that would include extraction costs the net price grows with the rate of interest and the growth rate of the tax factor this has not been observed as the actual development path of the prices of non-renewable resources in reality (Halvorsen & Smith, 1991). One explanation for this that has been proposed are insecure property rights.

Sinn accounted in a basic form for uncertain property rights in one variant of his original model by letting: $e^{-\rho t}$ be the probability for the survival of the property rights until time t. With $\rho > 0$ and constant as the instantaneous expropriation probability (Sinn, 2008, p. 12). Therefore our basic model (2.1.2) would now look like this:

$$\max_{R} \int_{t=0}^{\infty} p(R_t) R_t \theta_t e^{-rt} e^{-\rho t} dt \quad (3.5.1)$$

subject to:

$$\dot{S}_t = -R_t, t \in [0, \infty], S_0 > 0 given, \int_0^{\infty} R_t dt \leq S_0 \quad (3.5.2)$$

With all other assumption equal the net price now rises with:

$$\frac{\dot{p}_t}{p_t} = r - \hat{\theta} + \rho \quad (3.5.3)$$

Let us ignore the tax for now and focus on ρ. The resource owner that sells a unit of the resource at time t and invests it on the capital market for one period in a model without uncertainty will have: $p_t(r+1)$. Keeping it in situ gives him $p_t + \dot{p}_t$ instead. If however we have uncertainty then the resource owner that sells today still gets the same in return $p_t r$ but the one that doesn't extract will now only get $\dot{p}_t - \rho p_t$ in (expected) return indicating a steeper price path and faster extraction (Sinn, 2008). This means the resource owner now chooses a steeper extraction path taking into account this uncertainty.

Exogenous Resource Stock:

If the resource Stock is given and we know like in our first model that full extraction will happen over the infinite time horizon such uncertainty will cause additional frontloading of the extraction path and therefore has the same effect as an increase in the interest rate. The derivation of the conditions for the weak and strong green paradox is of course straight forward as it closely follows the previous one but can be found in (7.1.2) for completeness and easier traceability. No matter how small the uncertainty is it will always increase initial extraction as:

$$R_0 = \frac{(r+\rho-\hat{\theta})S_0}{\mu} \tag{3.5.4}$$

The NPV of total climate damage will increase as well with the inclusion of uncertainty since:

$$\Omega = \frac{(r+\rho-\hat{\theta})S_0\chi_0}{\mu(r-\hat{\chi})+r+\rho-\hat{\theta}} \tag{3.5.5}$$

and therefore: $d\Omega / d\rho > 0$.

Endogenous Resource Stock:

For the model with an endogenous resource stock we then get as the condition for the weak green paradox:

$$\theta_0 > \left[1-\frac{\hat{\theta}+\rho}{r}\right]^{-\alpha} \tag{3.5.6}$$

which shows that the effect of this simple form of uncertainty are identical to the effects of the growth rate of the tax factor in chapter 3. The condition for the strong green paradox is now:

$$\theta_0 > \left[1 - \frac{\hat{\theta}}{r + \rho + \mu(r - \hat{\chi})}\right]^{\alpha+\mu} \left[1 - \frac{\hat{\theta}}{r + \rho}\right]^{-\alpha} \tag{3.5.7}$$

Which means ρ extends the space in which the strong green paradox exists for high growth rates of the tax (remember that it only could exist if the growth rate of the tax rate wasn´t too high or too low). So while the findings don´t change much if we include insecure property rights in this form having it in the model reminds us of one of the reasons why reality often doesn´t match the results of models that don´t include it at all and see resources extracted faster than by the classical Hotelling rule. We will now turn to a more elaborate form of modelling uncertainty and see how it affects the extraction paths and the decision for the optimal initial resource stock.

4 The model with a substitute technology

If the demand for the non-renewable resource depends also on the price of a green alternative and the NRR is taxed increasingly or equivalent would be an increasing subsidy of the green alternative (Sinn, 2008) then more of the NRR should stay in situ which is the goal. Instead of modelling this with extraction costs we are looking at how this effects the exploration decision if it has to be costly chosen based on a certain and uncertain arrival time for a backstop technology. The idea for this modification is based on a paper by Van d. Ploeg (2013) who looks at a model with backstop with uncertain arrival time in the context of the green paradox and how its arrival changes the optimal paths of extraction. The term backstop was coined by Nordhaus (1973) for a substitute technology for the exhaustible resource which is not constrained by exhaustibility. Like nuclear fusion these are often not quite ready to replace NRR at a larger scale (Gerlagh & Lise, 2003).

We first follow the methods to derive the optimal paths and will then change this model by again using our increasing cash flow tax and a constant one and take a closer look at a case where the backstop date is announced in advance. Van d. Ploeg (2013) focuses on the welfare aspects and suggests subsidies to deal with them, while we will take a quick look on the endogenous resource stock part and give a specific form to the investment function to make our numerical simulations based on optimized resource stocks. Our findings concerning the green paradox will differ therefore from his paper slightly and confirm our findings for our previously used model.

4.1 Assumptions

The prospect of such a backstop technology changes the optimal paths for extraction and prices as we will see and also effects the incentive for exploration of the NRR and the initial investment decision for the resource stock. There are now two phases: before and after the backstop. The backstop technology that at a random time T will become available reduces the cost of a green substitute for the NRR by a fixed amount Δ (see for example Dasgupta and Heal, 1979). The price for the green alternative which is ψ drops therefore to $\psi - \Delta$ after the backstop becomes available (with ψ ≥ Δ > 0). The resource owner chooses his extraction path and total investment in exploration in the beginning once the optimal decisions of the two later phases are calculated.

To get the resource stock in time t = 0 S_0 the monopolist choses a level of Investment in exploration in the beginning with ε as the price of exploratory activity and I as the quantity of investment which gives us the optimal investment into developing the initial resource stock. For our exploratory investment we have that $S_0 = E(I), E'(I) > 0, E''(I) < 0.$ This is further explained and applied when we make the exploration decision chapter 4.6. The uncertainty component will be explained in chapter 4.4 where the optimal path decisions are derived for the phase before the backstop arrives with uncertainty. The probability distribution is thereby chosen to be an exponential distribution. See Hoel (1978) for a discussion on this. The rest of the notation stays the same. The problem of the monopolist is therefore now (we will add the tax later):

$$\max_{I,R} Exp\left(\int_{t=0}^{\infty} p_t R_t e^{-rt} dt\right) - I\varepsilon \tag{4.1.1}$$

subject to:

$$\dot{S}_t = -R_t, t \in [0, \infty], S_0 > 0, endogenous, \int_{t=0}^{\infty} R_t dt \le S_0 \tag{4.1.2}$$

So we maximize the expected value of the (discounted) profits minus the investment for exploration made in the beginning. The demand function (for the NRR) is now given for the two time intervals before and after the backstop comes online with:

$$\begin{aligned} R_t &= \psi^{\sigma} c p_t^{-\mu}, t \in [0, T) \\ R_t &= (\psi - \Delta)^{\sigma} c p_t^{-\mu}, t \in [T, \infty] \end{aligned} \tag{4.1.3}$$

Therefore the price for the NRR now depends also on the price for the green alternative and the inverse demand function for the NNR is accordingly given for our 2 phases by:

$$p(R_t,\psi) = (\psi^{\sigma} c / R_t)^{1/\mu} \qquad (4.1.4)$$
$$p(R_t,\psi - \Delta) = ((\psi - \Delta)^{\sigma} c / R_t)^{1/\mu}$$

With c > 0 constant as autonomous consumption of the resource. We again assume that μ the elasticity of demand is bigger than 1 with marginal revenue now (1- 1/μ)p_t. Therefore prices will stay finite and we will have full depletion over the infinite time horizon as explained before. So again the stock constraint in (4.1.2) will hold with equality. The green substitute can replace the NRR with σ as the constant cross elasticity, bigger than 0 that shows how good the NRR can be replaced by the green one. The optimization problem therefore has 3 sub problems: First an exploration decision is made, where the optimal investment is chosen, then the phase where the backstop isn´t available yet and the green substitute is relative expensive and lastly the post backstop phase where the price of the green alternative is relatively cheaper. We start in reverse order with the last phase where the backstop is already available. Prices, extraction rates, stocks and value functions for the post backstop phase will be indicated by an "A" (since we start with this phase in our calculations, or as "after" the substitute becomes available) and for the pre backstop phase we use a "B" as in the "before" the price gets cheaper. After these both phases the optimal resource stock is then determined based on the results in chapter 4.6.

4.2 The post backstop phase

Since the backstop has already arrived there is no uncertainty here. The Hamiltonian of the problem is given by:

$$\mathrm{H} = \mathrm{R}_t \mathrm{p}_t - R_t \lambda_t \qquad (4.2.1)$$

Using the maximum principle and our findings from the previous chapters gives us that $p_t = \lambda_t = \lambda_0 e^{-rt}$. Therefore we find for the growth rate of the price the same way as before for phase A (remember we include both forms of the tax factor later):

$$\hat{p}_t^A = r > 0 \tag{4.2.2}$$

Taking the time derivative of the demand function (4.1.3) and using the inverse demand function (4.1.4) to substitute in for $p_t^{-\mu}$ gives us:

$$\dot{R}_t = -\mu(\psi - \Delta)^{\sigma} c p_t^{-\mu-1} \dot{p}_t = -\mu \hat{p}_t R_t \tag{4.2.3}$$

Which gives us with (4.2.2) the optimal growth rate of extraction for phase A:

$$\hat{R}_t^A = -\mu r < 0 \tag{4.2.4}$$

And therefore the optimal extraction path as:

$$R_t^A = R_T e^{-(t-T)\mu r} \tag{4.2.5}$$

Using the stock constraint in (4.1.2) which as we said holds with equality and inserting our extraction value for time T and integrating gives for the resource stock at the beginning of the phase A:

$$S_T = \int_T^{\infty} R_t^A dt = \int_T^{\infty} R_T e^{-\mu r(t-T)} dt = R_T e^{\mu r T} \int_T^{\infty} e^{-\mu r t} dt = \frac{R_T}{\mu r} \Leftrightarrow R_T = S_T \mu r .$$

We can now state all three paths for the resource stock, the extraction rate and the price for the post backstop phase. We insert the extraction path into the state equation and get for the optimal path of the resource stock:

$$S_t^A = S_T e^{-\mu r(t-T)} \tag{4.2.6}$$

Which needs to obviously be smaller or equal to S_T. Therefore the optimal extraction rate with S_T is:

$$R_t^A = \mu r S_t = \mu r S_T e^{-\mu r(t-T)} \tag{4.2.7}$$

Now we just use the optimal extraction path again with the inverse demand function (4.1.4) and get the optimal price path for t > T:

$$p_t^A = \left[\frac{c(\psi-\Delta)^\sigma}{R_t^A}\right]^{1/\mu} = \left[\frac{c(\psi-\Delta)^\sigma}{\mu r S_t}\right]^{1/\mu} = \left[\frac{c(\psi-\Delta)^\sigma}{\mu r S_T}\right]^{1/\mu} e^{(t-T)r} \tag{4.2.8}$$

As in chapter 3 the higher the elasticity of demand and the higher the interest rate the faster the decline in extraction (and resource stock). How they are affected via the change of the resource stock in t = T will be seen in the next phase. The price is now also influenced by how good the substitute can replace the NRR and also on how much cheaper the backstop makes the green alternative (the price path will be lower in a cheaper substitute and in a better substitutability).

The optimal value function w^A for the monopolist showing the discounted profits after the backstop for the resource Stock S_T can be calculated by plugging in (4.2.7) and (4.2.8) we just derived into our objective function integral and discounting it for our time interval [T,∞]:

$$w^A(\psi-\Delta,S_T)=\int_T^\infty R_t^A p_t^A e^{-r(t-T)}dt=$$

$$=\int_T^\infty \left[\frac{c(\psi-\Delta)^\sigma}{S_T\mu r}\right]^{1/\mu} S_T\mu r e^{r(t-T)}e^{-\mu r(t-T)}e^{-r(t-T)}dt=$$

$$=\left[\frac{c(\psi-\Delta)^\sigma}{S_T\mu r}\right]^{1/\mu} S_T\mu r e^{\mu rT}\int_T^\infty e^{-\mu rt}dt$$

Which gives us:

$$w^A=S_T^{\,1-1/\mu}\left[\frac{c(\psi-\Delta)^\sigma}{\mu r}\right]^{1/\mu} \tag{4.2.9}$$

The optimal value function therefore decreases in the reduction in the price of the green alternative and the cross elasticity (cheaper green alternatives and better substitutability decrease the price of the resource and therefore profits which means the discounted value of w^A is lower) and it is higher the higher the remaining resource stock in t = T. Now we go back one phase from this deterministic part and look at the optimal decisions before the backstop became available.

4.3 The pre backstop phase with known arrival time

We first look at a simpler special case and assume that the time when the new technology becomes available is known and given by T = 1/b (which will be the expected value for it in the next subchapter). In phase A our resource extracting firm uses the extraction, resource and price paths we just calculated and there is no uncertainty. Since the date is known exactly there can be no sudden jump in the price path and it has to have the derived growth rate r like before (4.2.2). The initial price p_0 is therefore now given via (4.2.8) by:

$$p_0 = \left[\frac{c(\psi - \Delta)^{\sigma}}{\mu r S_{1/b}} \right]^{1/\mu} e^{-r/b} \quad (4.3.1)$$

Inserting this into our demand function (4.1.3) gives us the initial extraction rate:

$$R_0 = \left[\frac{c(\psi - \Delta)^{\sigma}}{\mu r S_{1/b}} \right]^{-1} c\psi^{\sigma} e^{\mu r/b} = \mu r S_{1/b} \left[\frac{\psi}{\psi - \Delta} \right]^{\sigma} e^{\mu r/b} \quad (4.3.2)$$

The extraction path before the backstops arrival is therefore given by:

$$R_t^B = \mu r S_{1/b} \left[\frac{\psi}{\psi - \Delta} \right]^{\sigma} e^{-\mu r(t-1/b)}. \quad (4.3.3)$$

To get the resource stock in time t = 1/b we insert the extraction path in our state equation (4.1.2) giving:

$$\dot{S}_t = -\mu r S_{1/b} \left[\frac{\psi}{\psi - \Delta} \right]^{\sigma} e^{-\mu r(t-1/b)}$$

Integrating both sides over the timespan until the backstop is available at 1/b and solving for $S_{1/b}$ gives (appendix (7.2.1)):

$$S_{1/b} = \frac{S_0}{(\frac{\psi}{\psi - \Delta})^{\sigma}(e^{\mu r/b} - 1) + 1} \leq S_0 \quad (4.3.4)$$

Therefore the price, extraction and the resource stock paths for both phases can be calculated. We get the optimal resource stock, extraction rate and price path after the backstop directly by using the found resource stock in (4.3.4) and just plugging it into our equation for the optimal paths for phase A (4.2.6) to (4.2.8). For the pre backstop phase we use our equations (4.3.1) and (4.3.3) to get the optimal paths before the backstop arrives. The extraction paths for phase B and A therefore are given by:

$$R_t^B = \frac{\mu r S_0 (\frac{\psi}{\psi - \Delta})^{\sigma} e^{-\mu r(t-1/b)}}{(\frac{\psi}{\psi - \Delta})^{\sigma}(e^{\mu r/b} - 1) + 1} \tag{4.3.5}$$

$$R_t^A = \frac{\mu r S_0 e^{-\mu r(t-1/b)}}{(\frac{\psi}{\psi - \Delta})^{\sigma}(e^{\mu r/b} - 1) + 1} \tag{4.3.6}$$

We see the frontloading effect on extraction caused by the backstop even if there is no uncertainty about the arrival time. For the resource stocks we therefore have:

$$S_t^A = S_0 \frac{e^{\mu r(1/b-t)}}{(\frac{\psi}{\psi - \Delta})^{\sigma}(e^{\frac{\mu r}{b}} - 1) + 1} \tag{4.3.7}$$

$$S_t^B = S_0 \frac{(\frac{\psi}{\psi - \Delta})^{\sigma}(e^{\mu r(1/b-t)} - 1) + 1}{(\frac{\psi}{\psi - \Delta})^{\sigma}(e^{\frac{\mu r}{b}} - 1) + 1} \tag{4.3.8}$$

We see that as prices increase at r and extraction and the resource stock declines at μr the path in (4.3.8) satisfies this. If we would set Δ to 0 meaning there is no backstop / the backstop has no effect then the depletion rates of the NRR are not influenced by the price of the green substitute. For the general case where the change in price from the backstop is bigger than 0 the resource stock at t = 1/b will be smaller than in the case without a backstop. The extraction rate then has to be higher before the backstop makes the RR cheaper as $(\psi / (\psi - \Delta))^{\sigma} > 0$ and the effect is stronger the better the substitutability (given by a higher parameter σ).The optimal price path will have a growth rate of r for the whole time span. There are no jumps in the case with a certain date for the price path which will change when we introduce uncertainty about the arrival time. Of course it is not very realistic to assume that we know exactly when for example fusion energy will become available. Let us turn to the case with an uncertain arrival time T. We visualize the paths later together.

4.4 The pre backstop phase with unknown arrival time

Now we look at the more complicated case where we don´t know when time T when the backstop comes online will be and get our optimal paths and the value function w^B for this phase. Uncertainty is hereby modeled by giving us the following probability that the substitution technology comes online before time t:

$$\Pr.(T \le t) = 1 - Exp(-bt); \forall t \ge 0, b \ge 0 \tag{4.4.1}$$

Where E(T) = 1/b gives us the expected time at which the backstop makes the green substitute cheaper. The properties of the exponential distribution of memorylessness and a constant b give us: $\Pr.(T > x + t / T > x) = \Pr.(T > t); \forall x, t \ge 0$. Which means that if the backstop hasn´t arrived after for example 20 periods the conditional probability that it will take at least 10 more periods is equal to the unconditional probability to see the arrival at least 10 more periods after the starting time which makes our calculations easier.

The idea is that before the backstop is discovered the resource owner has to deal with the uncertainty at each point in time t < T that at x > t it might happen and reduce demand for the NRR by making the green alternative cheaper. We now have a value function for this phase of the following form that incorporates this uncertainty and which describes the modified discounted profit maximization decision of the firm:

$$w^B(S_t)e^{-rt} = \max_{R^B}\left(\int_t^{t+\Delta} R_x^B p(R_x^B)e^{-rx}dx + (1-b\Delta t)w^B(S_{t+\Delta t})e^{-r(t+\Delta t)} + b\Delta t w^A(S_{t+\Delta t})e^{-r(t+\Delta t)}\right) \quad (4.4.2)$$

In order to solve this phase of the model we need to derive the H.J.B. equation. We look at what happens before time T when we go from t to t+Δ (Δ → 0).The resource owner is confronted in this very small time span with the uncertainty that the backstop might come online and how he optimizes for that for optimal extraction. The "hazard" for the resource owner is that the cheap alternative comes online. The "survival" function is given by 1 – F(t) = e^{-bt} for our probability distribution used. The hazard rate b(t) is given by the growth rate of the survival function and therefore is given by $-\hat{B}_t = -\left[-be^{-bt} / e^{-bt}\right] = b$ So the exponential distribution gives us a constant rate b (which is multiplied by Δt). We now look at this value function (4.4.2) before the backstop comes online over this infinitesimally small time period and start our reformulation:

$$w^B(S_t)e^{-rt} = \max_{R^B}\left(\int_t^{t+\Delta} p(R_x^B)R_x^B e^{-rx}dx + (1-b\Delta t)w^B(S_{t+\Delta t})e^{-r(t+\Delta t)} + b\Delta t w^A(S_{t+\Delta t})e^{-r(t+\Delta t)}\right)$$

$$\Leftrightarrow w^B(S_t) = \max_{R^B}\left(\int_t^{t+\Delta} p(R_x^B)R_x^B e^{-r(x-t)}dx - b\Delta t w^B(S_{t+\Delta t})e^{-r\Delta t} + w^B(S_{t+\Delta t})e^{-r\Delta t} + b\Delta t w^A(S_{t+\Delta t})e^{-r\Delta t}\right)$$

$$\Leftrightarrow \frac{w^B(S_t) - w^B(S_{t+\Delta t})}{\Delta t} = \max_{R^B}\left(\frac{\int_t^{t+\Delta} p(R_x^B)R_x^B e^{-r(x-t)}dx}{\Delta t} - bw^B(S_{t+\Delta t})e^{-r\Delta t} + \frac{w^B(S_{t+\Delta t})\left(e^{-r\Delta t}-1\right)}{\Delta t} + bw^A(S_{t+\Delta t})e^{-r\Delta t}\right)$$

Our approach is as follows. We try to get a term that for Δt → 0 as the derivative of the value function with respect to time on the left hand side. So what we did here is to first multiply by the discount factor and open up the 2nd term in the bracket. Then we subtracted $w^B(S_{t+\Delta t})$ from both sides and then divided by Δt and therefore have the time derivative for the change in t infinitesimally small. Now on the right hand side we can use the rule of Bernoulli to evaluate the limits of the indeterminate forms for the 2 fractures with:

$$\lim_{\Delta t\to 0}\frac{R_{t+\Delta t}^B p(R_{t+\Delta t}^B)e^{-r(t+\Delta t-t)}}{1} = R_t^B p(R_t^B), \lim_{\Delta t\to 0}\frac{-w^B(S_{t+t\Delta})re^{-r\Delta t}}{1} = -w^B(S_t)r$$

Next we substitute for the time derivative of the value function $\dot{w}^B$ that we got on the left hand side with a factor times the time derivative of the resource stock $w_S^B \dot{S}$. So the marginal change in the value function $\dot{w}^B$ is equal to the marginal change in the resource stock times the factor w_S^B. Therefore we get (and using our state equation) the following HJB equation (suppressing the time index to keep it compacter):

$$\max_{R^B}\left(R^B p(R^B,\psi)-R^B w_S^B(S,\psi,\Delta,b)\right)-b\left(w^B(S,\psi,\Delta,b)-w^A(S,\psi-\Delta)\right)=rw^B(S,\psi,\Delta,b) \quad (4.4.3)$$

This means the optimal profit value of the extracted resource minus the expected reduction in value from the possibility of a backstop technology showing up is equal to revenue with interest rate r. We will now leave out the arguments of the value functions and only add them in when we should need them. Differentiating the profit with respect to our control the extraction rate R^B using the inverse demand function and setting equal to 0 yields as the optimal price path for phase B:

$$(1-\frac{1}{\mu})p_t^B - w_S^B = 0 \Leftrightarrow p_t^B = \frac{w_S^B}{1-1/\mu} \quad (4.4.4)$$

So marginal revenue of extraction of the NRR has to equal marginal value in situ in optimum.
We now get the optimal extraction path for t < T by inserting this into the demand function (4.1.3):

$$R_t^B = \left(\frac{1-1/\mu}{w_S^B}\right)^{\mu} c\psi^{\sigma} \quad (4.4.5)$$

Next we can insert this extraction path and price path into (4.4.3) and get therefore the following equation:

$$\frac{c\psi^{\sigma}}{\mu}\left(\frac{w_S^B}{1-1/\mu}\right)^{1-\mu} - b\left(w^B - w^A\right) = rw^B \qquad (4.4.6)$$

We used $\dot{w}^B = w_S^B \dot{S}$ which means this a FODE with the value function $w^A = w^A(S_t, \psi - \Delta)$ as the inhomogeneity. Since $w^A = \left[c(\psi-\Delta)^{\sigma} / \mu r\right]^{1/\mu} S_T^{\;1-1/\mu}$ (4.2.9) we therefore guess that the value function we are looking for is $w^B = \mathrm{P}S_t^{1-1/\mu}$. The inhomogeneous part often has the same functional form but with different coefficients so we have to find a way to determine it and can find via the method of undetermined coefficients a P with $P = P(b, \psi - \Delta)$ that satisfies the following equation:

$$P(b+r) - \frac{c\psi^{\sigma} P^{1-\mu}}{\mu} = b\left(\frac{c(\psi-\Delta)^{\sigma}}{r\mu}\right)^{\frac{1}{\mu}} \qquad (4.4.7)$$

Therefore we indeed found an expression for the value function in phase B with:

$$w^B(S_t, \psi, \Delta, b) = S_t^{1-1/\mu}\mathrm{P}(b, \psi - \Delta) \qquad (4.4.8)$$

We can now enter this expression in our equations for the price path (4.4.4) and the extraction rate (4.4.5) and using that: $w^B = PS_t^{1-1/\mu}$, which means $\dot{w}^B = (1-1/\mu)PS_t^{-1/\mu}\dot{S}_t$ and therefore $w_S^B = (1-1/\mu)PS_t^{-1/\mu}$ this gives us for the price path and the extraction path before the backstop:

$$p_t^B = PS_t^{-1/\mu} \tag{4.4.9}$$

$$R_t^B = c\psi^\sigma \left(\frac{(1-1/\mu)PS_t^{-1/\mu}}{1-1/\mu} \right)^{-\mu} = c\psi^\sigma P^{-\mu} S_t \tag{4.4.10}$$

We then can plug in the extraction path in our state equation (4.1.2) and get directly from the differential equation the resource extraction path:

$$S_t^B = S_0 e^{-c\psi^\sigma P^{-\mu} t} \tag{4.4.11}$$

Let us also define $D = D(\psi, \Delta, b) = c\psi^\sigma P^{-\mu}$ as an inversely related factor to P. It measures extraction speed in phase B (the decline rate of the extraction rate). If we use $D = c\psi^\sigma P^{-\mu}$ and insert our resource stock path just calculated we get the following optimal paths for extraction, price and the resource stock in terms of the initial S_0 for phase B as:

$$p_t^B = S_0^{-1/\mu} P e^{Dt/\mu}, R_t^B = S_0 D e^{-Dt}, \; S_t^B = S_0 e^{-Dt} \tag{4.4.12}$$

Therefore our growth rates for the price and extraction (and therefore resource stock) in phase B can be directly seen as D/μ and –D. For comparison here are our optimal paths from phase A from before:

$$p_t^A = \left[\frac{c(\psi - \Delta)^\sigma}{\mu r S_T} \right]^{1/\mu} e^{(t-T)r}, \; R_t^A = S_T \mu r e^{-\mu r(t-T)}, \; S_t^A = S_T e^{-\mu r(t-T)}$$

Specific cases for the model parameters b and ψ are discussed in appendix (7.2.2) to appendix (7.2.4) like for example our model from the previous chapter without a backstop. The results from this will be used in the following. To see exactly what happens at time T when we observe a jump in the extraction and price paths when the backstop is introduced please take look at appendix (7.2.5). There will be a summary at the end of this chapter however. Let us now take a look at how our two factors, P and D are influenced by the change in their parameters by looking at the total differential for the P function and also get therefore insights into the extraction speed factor D. The partial derivatives are derived in appendix (7.2.6). Using (4.4.7) gives us the total differential for the P function with:

$$(r+b+(1-1/\mu)c\psi^{\sigma}P^{-\mu})dP = \frac{b\sigma}{\mu(\psi-\Delta)}\left(\frac{(\psi-\Delta)^{\sigma}c}{r\mu}\right)^{1/\mu} d(\psi-\Delta) - \frac{P}{\mu b}(P^{-\mu}c\psi^{\sigma}-\mu r)db$$

For b = 0 see appendix (7.2.2) $D = P^{-\mu}c\psi^{\sigma} = \mu r$ and for b → ∞ see appendix (7.2.3) we have $D = P^{-\mu}c\psi^{\sigma} = (\mu r)\frac{\psi^{\sigma}}{(\psi-\Delta)^{\sigma}} > \mu r$ for Δ > 0 which we have by assumption. Therefore:

$$D > \mu r; \forall b > 0 \tag{4.4.13}$$

We see that that D is higher the higher b (the chance for a backstop occurring). The extraction rate today and in the near future is increased (frontloading) to counter the disadvantage of the backstop for the resource owner which creates a price wedge which reduces the profit factor P and therefore the NPV of profits. We have therefore:

$$D_b(\psi,\Delta,b) > 0, P_b(\psi-\Delta,b) < 0; \forall b > 0 \tag{4.4.14}$$

We also see that $P_{\psi-\Delta} > 0$ for b > 0 for all admissible values of the cost reduction Δ and therefore if the green alternative gets marginally cheaper through the backstop the profit factor P declines and D will increase. In short the cheaper the backstop makes the alternative the lower profits will be and the faster extraction will be (frontloading) for a given resource stock:

$$D_{\psi-\Delta} < 0, P_{\psi-\Delta} > 0; \forall \Delta \in (0,\psi) \tag{4.4.15}$$

We further see that $\mu r < D < \mu(r+b)$ see appendix (7.2.4) for all b > 0 and that D is highest if the backstop technology means an elasticity of supply that is infinite for the green substitute which means that $\Delta = \psi$ i.e. there is no more need for the NRR in this extreme case once the backstop has arrived.

Appendix (7.2.5) shows us that at the time of the arrival of the backstop the lower cost of the green alternative let extraction of the NRR R_T jump down by a fixed value. The price at t = T will drop as long as the following condition for σ (substitutability) and Δ (price reduction) holds:

$$\left(\frac{\psi}{\psi-\Delta}\right)^{\sigma} > D / r\mu \tag{4.4.16}$$

Let us visualize the P and D factors in the following figures 7 and 8 for a backstop that strongly reduces the price of the green alternative in blue and for a less effective one in red with b on the horizontal axis. The parametrization used is: r = 0.03, c = 1, ψ = 500, Δ = 400 / 200, μ = 2, σ = 1 and c = 1. We see the maximal value for P at 91.297 for the no backstop case. The cheaper the backstop makes the green alternative and the sooner the expected arrival date the lower P will be. D shows the opposite picture and has its lowest value at b = 0 with 0.06. If the backstop is more of a danger to the monopolist or is expected to arrive sooner the extraction speed will be higher for a given resource stock.

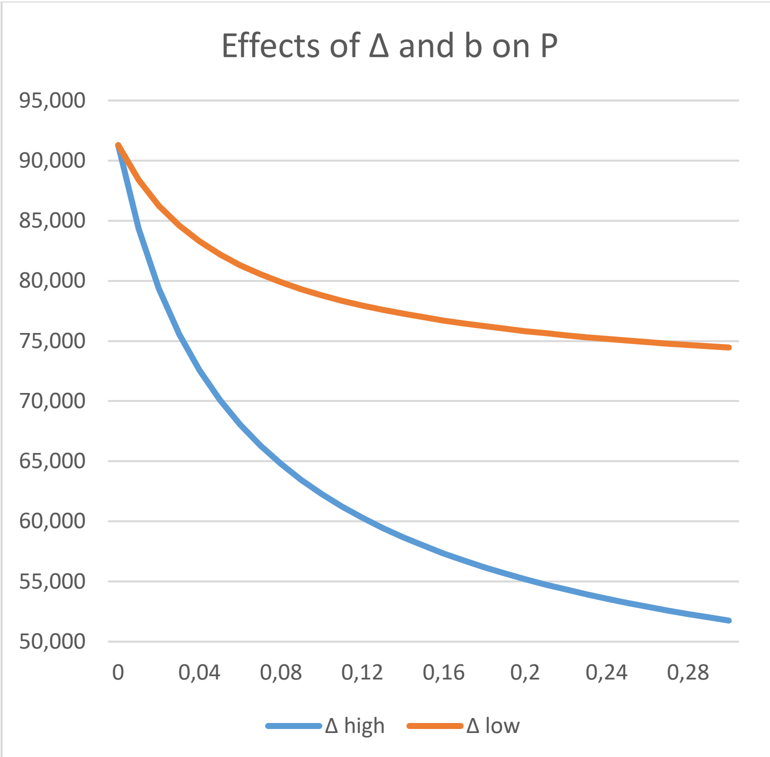

Figure 7: The Profit factor for a strong (blue) and a weak cost reduction of the green substitute

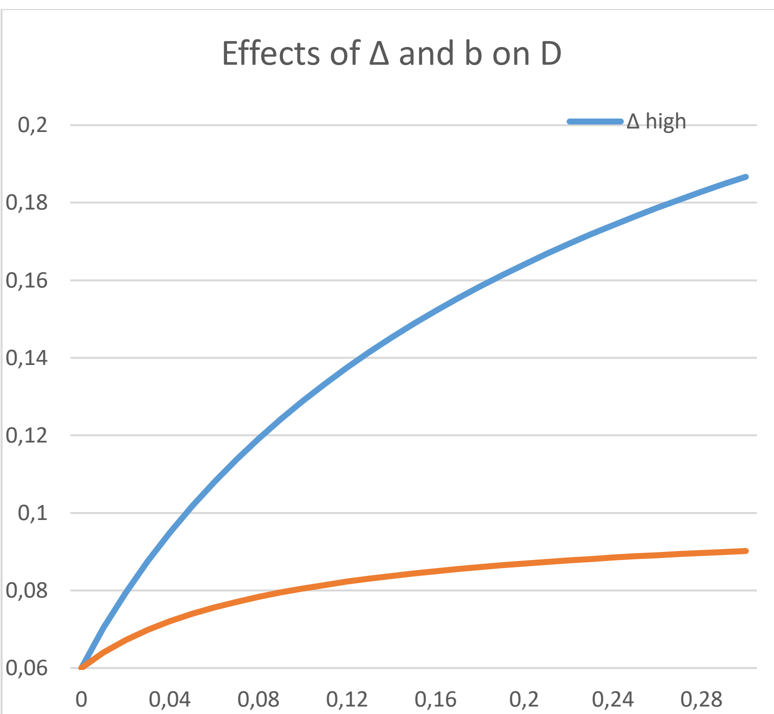

Figure 8: The extraction speed D for a high (blue) and low reduction in cost of the green substitute.

As a summary: After T, the price increases with a growth rate of r and the extraction rate / resource stock declines with μr with the paths in (4.2.6), (4.2.7) and (4.2.8). Before T, the growth rate of the price is higher with $\hat{p}_t^B = P^{-\mu} c\psi^\sigma / \mu = D / \mu$ and the extraction rate / resource stock declines faster with: $P^{-\mu} c\psi^\sigma = D$ in anticipation of the uncertain arrival of the backstop technology. This effect is stronger the higher the chance of the arrival and the higher the cost reduction from the backstop for the green alternative is. The extraction and resource stock as well as the price path before T are given by (4.4.12). As a consequence of the change to the backstop technology there is a jump downward for the extraction rate and the price will also jump down if the price of the RR drops enough and if the substitutability for the NRR is high enough in combination. This is for a given resource stock.

4.5 The case of no backstop

A few more words on the case where there is no backstop and we would be back in the model in chapter 3 except for the substitute. If b = 0 then the backstop will never be discovered or can be thought of as available from the initial period as just a cheaper green energy alternative. If we use b = 0 in (4.4.7) than we can easily solve for P with: $P = \left(c\psi^\sigma / \mu r\right)^{1/\mu}$ and therefore for $D = P^{-\mu} c\psi^\sigma = (c\psi^\sigma / \mu r)^{-1} c\psi^\sigma = \mu r$. Then we will have the optimal extraction paths after (4.2.7) and before (4.4.12) the backstop with $R_t^A = R_t^B = \mu r S_t ; \forall t \in (0, \infty)$ as can be seen by $R_t^B = S_0 D e^{-Dt} = \mu r S_0 e^{-\mu r t} = \mu r S_t e^{\mu r t} e^{-\mu r t} = \mu r S_t$. The resource stock path will accordingly also have a decline rate μr over the whole time span. The price path will over the full time scale depend on ψ or ψ – Δ for the never discovered / always there cases. Which means they will have the same growth rate and no jumps but will start from a higher or lower initial level. This was our model in chapter 3 without the green alternative (and therefore a different demand function).

4.6 The exploration decision

Now the final step is to get an optimized S_0 by choosing how much to optimally invest which was assumed given until know. We take the optimal value function w^B (4.4.8) for the time when the decision for the resource stock is made at time t = 0 and plug in our investment function E(I) with E´(I) > 0 and E´´(I) < 0 for the initial resource stock. Therefore:

$$w^B(S_0) = w^B(E(I)) = P(b, \psi - \Delta)E(I)^{1-1/\mu} \tag{4.6.1}$$

Then we use that in optimum marginal profit of investing in another unit of exploration of the resource needs to equal marginal costs as we discussed in chapter 3. Which gives for our total cost of εI that marginal costs are ε and we therefore have:

$$\varepsilon = (1 - \frac{1}{\mu})E(I)^{\frac{-1}{\mu}} E'(I)P \tag{4.6.2}$$

Taking the partial derivatives and using our findings from the total differential for P we get the total differential for I as (see appendix 7.2.6):

$$\varepsilon \left[\frac{E'(I)}{\mu E(I)} - \frac{E''(I)}{E'(I)} \right] dI = \frac{\varepsilon(P_b db + P_{\psi-\Delta} d(\psi - \Delta))}{P} - d\varepsilon \tag{4.6.3}$$

Compared to our previous model investment now depends additionally on the degree to which the price of the green substitute gets cheaper Δ and on how likely it is that the backstop becomes available (higher b means lower 1/b, means faster arrival). We therefore see that investment in exploration decreases obviously in the costs of investment ε, in a cheaper price of the substitute via the backstop and in a higher probability of a backstop occurring as P declines in b and in a bigger Δ. Compared to a baseline monopolist model that is efficient with our assumptions (7.2.7) we therefore have higher inefficiency via a lower then optimal initial resource stock caused by the uncertainty but which helps the climate by keeping more resources in situ. This means summarized:

$$I_b < 0, I_{\psi-\Delta} > 0, I_\varepsilon < 0. \tag{4.6.4}$$

4.7 Taxation and the green paradox effects

We have seen that extraction paths will be steeper initially as a consequence of the uncertainty in the backstop model. This frontloading of extraction means higher extraction rates early for a given resource stock, but the same uncertainty about the arrival time of the backstop will effect initial exploration investment and therefore the initial resource stock. We can now finally extend the model and look how taxes intended to reduce the profit of the resource owner and thereby keeping the NRR in situ effect our optimal paths with all 3 sub problems solved.

Let us start with a few words about the case $\theta_t = \theta$ since we didn´t discuss it in chapter 3 before we analyze our increasing cash flow tax again. A constant tax factor will only decrease the NPV of the profits of the resource owner but will not change the demand function or the state equation and therefore all paths of extraction, price and resource stock (4.2.6), (4.2.7) and (4.2.8) are not affected by it. We can just take the constant tax factor out of the integral sign of the optimization problem. This would be the case in our previous model without a backstop as well. So we still have that $\hat{p}_t^A = r$ therefore. The value functions w^A from (4.2.9) will be nearly identical just multiplied by the θ and the same holds therefore for w^B from (4.4.8) which is now given by $w^B(S_t, \psi, \Delta, b, \theta) = S_t^{1-1/\mu}\mathrm{P}\theta$ which reduces their discounted value. This gives us exactly the same for the optimal paths in phase B from (4.4.12) as well. For the investment decision our equation in (4.6.2) changes to:

$$\varepsilon = (1-\frac{1}{\mu})PE(I)^{\frac{-1}{\mu}}\theta E'(I) .$$

We immediately see that the partial derivative of Investment with respect to the tax factor is given by $I_\theta = \frac{\varepsilon}{\theta} > 0$ and find the total differential with:

$$dI = \frac{\frac{\varepsilon}{\theta}d\theta + \frac{\varepsilon(P_{\psi-\Delta}d(\psi-\Delta)+P_b db)}{P} - d\varepsilon}{\varepsilon\left[\frac{E'(I)}{\mu E(I)} - \frac{E''(I)}{\mu E'(I)}\right]}.$$

We see the same effects of uncertainty on the investment decision about arrival time, decrease in the price of the green substitute and costs of investment as before in (4.6.4). We find $I_\theta > 0$ unsurprisingly. The lower the fixed tax factor / the higher the fixed tax rate the lower future profits will be discouraging exploration. So with a constant tax factor / rate S_0 and S_T will be lower but the optimal paths themselves will be from these lower initial resource stocks unchanged. Therefore no relative frontloading occurs and therefore no weak and no strong green paradox effects manifest (since the total amount extracted needs to be lower now). Of course the uncertainty of the backstop date itself leads to faster extraction initially before its arrival. So while such a tax does nothing to make it worse for the environment (there of course will be effects on welfare) it also does not help prevent the frontloading caused by uncertainty in this model from the backstop itself which was not a problem in chapter 3 since as we said the paths are unchanged. In addition it was already discussed by Sinn (2008) why such a tax would be problematic to implement since it needed to be chosen very high and unannounced. We therefore look at the more realistic case next.

Increasing cash flow tax:

We will again use an increasing cash flow tax of the form: $\theta_t = \theta_0 e^{\hat{\theta} t}, 0 < \theta_0 < 1, \hat{\theta} < 0$. With θ_t as the tax factor and θ_0 the initial tax factor in period t = 0. The optimization problem therefore changes to:

$$\max_{I,R} Exp\left(\int_{t=0}^{\infty} \theta_t p_t R_t e^{-rt} dt \right) - I\varepsilon$$

With the same depletion equations as before. With such a tax our Hamiltonian looks like it did in the model in chapter 3 just with the price of the NRR now also depending on the price of the green substitute. The growth rate of the extraction rate and of the price in phase A will be therefore given by $\hat{R}_t^A = -\mu(r - \hat{\theta}) < -\mu r$ and $\hat{p}_t^A = r - \hat{\theta} > r$. Using our state equation we can find the paths for prices and extraction for phase A via the same procedure as before as:

$$p_t^A = \left(\frac{(\psi - \Delta)^{\sigma} c}{\mu(r - \hat{\theta}) S_T} \right)^{1/\mu} e^{(r-\hat{\theta})(t-T)} \tag{4.7.1}$$

$$R_t^A = S_T \mu(r - \hat{\theta}) e^{-\mu(r-\hat{\theta})(t-T)} \tag{4.7.2}$$

The optimal resource development path is then given by:

$$S_t^A = S_T e^{-\mu(r-\hat{\theta})(t-T)} \tag{4.7.3}$$

This means the extraction rate in t = T will be higher given S(T) and the extraction path will decline therefore steeper with $\hat{R}_t^A = -\mu(r-\hat{\theta}) < -\mu r$ than in the no-tax case. It also means that the price path will be steeper but will start at a lower level for the given resource stock in t = T. Lastly the resource stock path will also be steeper in its decline for a given S(T). As long as we would assume a given resource stock the effects would be clear and cause weak and strong (given the stock constraint of full depletion) green paradox effects in phase A.

The optimal value function after the backstop w^A changes from (4.2.9) and its counterpart w^B for the pre backstop phase from (4.4.8) and they now have the tax factor as an argument and are given by:

$$w^A(\theta_T, S_T, \psi-\Delta) = \left[\frac{(\psi-\Delta)^\sigma c}{\mu(r-\hat{\theta})}\right]^{1/\mu} \theta_T S_T^{\,1-1/\mu} \tag{4.7.4}$$

$$w^B(\theta_t, S_t, \psi, \Delta, b) = \mathrm{P}(\psi-\Delta, b, \hat{\theta})\theta_t S_t^{\,1-1/\mu} \tag{4.7.5}$$

Whereby the derivation of the HJB equation follows the same procedure as before and our guess for the form for w^B changed now by the tax factor θ_t to $w^B = \mathrm{P}\theta_t S_t^{\,1-1/\mu}$. For the factor P we get therefore that equation (4.4.7) changes to:

$$P(r-\hat{\theta}+b) - \frac{c\psi^\sigma P^{1-\mu}}{\mu} = b\left(\frac{(\psi-\Delta)^\sigma c}{(r-\hat{\theta})\mu}\right)^{\frac{1}{\mu}} \tag{4.7.6}$$

Which $P = P(\hat{\theta}, b, \psi-\Delta)$ satisfies now with the growth rate of the tax as an additional argument. Therefore we can have a look at the optimal paths before the backstop comes online and see that they have the same form as before the tax was introduced in (4.4.12). But as we just said of course our P factor solves now (4.7.6) and is different than before depending on the growth rate of the tax factor. The same is true for the extraction speed factor $D = D(\psi, \Delta, b, \hat{\theta})$ which now also depends on $\hat{\theta}$. The paths for phase B are therefore given by:

$$p_t^B = S_0^{-1/\mu} P(\psi - \Delta, b, \hat{\theta}) e^{D(\psi,\Delta,b,\hat{\theta})t/\mu}$$

$$R_t^B = S_0 D(\psi, \Delta, b, \hat{\theta}) e^{-D(\psi,\Delta,b,\hat{\theta})t}$$

$$S_t^B = S_0 e^{-D(\psi,\Delta,b,\hat{\theta})t}$$

Like before we can now look how the factors P and D will change in their arguments. The partial derivatives for P are again found in appendix (7.2.6). The total differential is given by:

$$dP = \frac{\left(\frac{(\psi-\Delta)^\sigma c}{\mu(r-\hat{\theta})}\right)^{\frac{1}{\mu}} \frac{b\sigma}{\mu(\psi-\Delta)} d(\psi-\Delta) - \frac{P}{\mu b}(D - \mu(r-\hat{\theta}))db + \left[P + \left(\frac{c(\psi-\Delta)^\sigma}{\mu(r-\hat{\theta})}\right)^{\frac{1}{\mu}} \left(\frac{b}{\mu(r-\hat{\theta})}\right)\right] d\hat{\theta}}{(1-1/\mu)c\psi^\sigma P^{-\mu} + r - \hat{\theta} + b} \tag{4.7.7}$$

Which gives us (remember $D(\psi, \Delta, b, \hat{\theta}) = P(\psi - \Delta, b, \hat{\theta})^{-\mu} c\psi^\sigma$):

$$D_b > 0, D_{\psi-\Delta} < 0, D_{\hat{\theta}} < 0, P_b < 0, P_{\psi-\Delta} > 0, P_{\hat{\theta}} > 0. \tag{4.7.8}$$

The profit will be smaller for a higher probability of the backstop arriving and for high Δ meaning for larger decline in the cost of the green substitute after arrival of the backstop. Which matches our findings before the tax was introduced. For D we have the opposite. Additionally now the slower the tax factor declines (higher $\hat{\theta}$) / the slower the tax rate increases the higher the P factor and the lower therefore the extraction speed D since future resource rents are reduced only slowly. Our equation for the optimal exploration decision in the beginning is now given by:

$$\varepsilon = \theta_0 (1 - \frac{1}{\mu}) P(\psi - \Delta, b, \hat{\theta}) E(I)^{\frac{-1}{\mu}} E'(I)$$

We find straight forward that the partial differential of investment with respect to the growth rate of the tax factor is: $I_{\hat{\theta}} = (\varepsilon / P) P_{\hat{\theta}}$ and the initial tax factor is just like the constant tax factor from before. The total differential therefore is:

$$dI = \frac{\frac{\varepsilon}{\theta_0} d\theta_0 + \frac{\varepsilon (P_{\psi-\Delta} d(\psi - \Delta) + P_{\hat{\theta}} d\hat{\theta} + P_b db)}{P} - d\varepsilon}{\varepsilon \left[\frac{E'(I)}{\mu E(I)} - \frac{E''(I)}{\mu E'(I)} \right]}$$

We again have:

$$I_{\psi-\Delta} > 0, I_b < 0, I_\varepsilon < 0 \qquad (4.7.9)$$

Which matches our findings in the model without and increasing cash flow tax (4.6.4). Investment will decline in its cost, in a faster arrival time of the backstop and in higher reduction of the price of the green substitute via the backstop. For the effects of the tax on exploration itself we see using (4.7.8) that:

$$I_{\theta_0} > 0, I_{\hat{\theta}} > 0 \qquad (4.7.10)$$

The higher the initial tax factor / the lower the initial tax rate, the higher investment will still be and therefore the resource stock but again not higher than without a tax of course. Any positive tax factor therefore will lead to lower overall extraction. The higher $\hat{\theta}$ so the slower the decline rate of the tax factor / increase of the tax rate the higher will be investment as future resource rents shrink slower again but not above the case without a tax. Extraction will be frontloaded. A weak green paradox therefore will only manifest like in our model without a backstop in chapter 3 if the reduction in the initial resource stock is not strong enough to compensate for the initial frontloading The existence of a strong green paradox is also ambivalent only existing for certain combinations of the initial tax factor and the decline rate of the tax factor and the specification of the other parameters as well. But the tax now affects the extraction path and the investment decision that is already affected by the uncertainty connected to the backstop. So the effects compound. Without an endogenous resource stock both green paradox effects would exist like in Chapter 2 for any form of tax and the uncertainty would just lead to frontloading of extraction as well which again shows the importance of modelling it endogenous.

Subsidies, positive feedback effects and tipping points

Van der Ploeg comes to the conclusion that with appropriate subsidies the green paradox will exist initially as a consequence of environmental policies but then disappear as the effect of lowered overall extraction will most likely dominate the frontloading. An optimally designed government subsidy that helps speed up the arrival time of the backstop might then be useful (and also deal with welfare effects). But this of course strongly depends on the damage function used (we discuss the damage function a bit more in Chapter 5). There might be tipping points caused by positive feedback effects in the carbon cycle that play a role here. For example the speed up of resource extraction leads to large scale melting of the polar ice caps which reduce the reflection capacity of the planet or huge amounts of methane are released from thawing permafrost areas, changes in vegetation can release or bind huge amounts of carbon etc. so that the GHG released by extraction effect the GHG released into the atmosphere by natural causes (Winter, 2014).

We might then have the situation that the initial speed up of extraction caused by making b endogen via an environmental policy that subsidies innovation to push the date for the arrival of the backstop forward will push us over the tipping point whereas without or a milder policy the backstop would have arrived later and the tipping point could have been avoided but early enough so that enough of the resource stays in the ground to avoid too much increase in temperature. There might be a sweet spot in the middle for which it is best for the backstop to arrive and carbon pricing might need to play an important role in addition to subsidizing innovation to speed up the advent of a backstop technology in order to avoid green paradox effects (Winter, 2014).

4.8 Numerical Simulation

We will now visualize some of our findings in the following figures. We will focus on the extraction paths which most directly show the effects of the model additions and the tax on the frontloading of extraction and the initial resource stock. The parametrization used is $b = 0.05$, $r = 0.03$, $c = 1$, $\psi = 500$, $\Delta = 400$, $\mu = 2$, $\sigma = 1$ and $\varepsilon = 1$ (investment costs). The backstop is expected for $T = 1/b = 20$. If the resource stock is assumed exogenously given we set it at $S_0 = 500$. The investment function E(I) is set to $S_0 = \alpha I^{\beta}$ with $\alpha = 1$ and $\beta = 0.9$. These will be used if not otherwise stated in the following.

Let us first take a look at the extraction paths if the resource stock is exogenously given and there is only the frontloading response to isolate the effects. We compare our model without a backstop to different cases with a backstop. We see in Figure 9 in red the extreme case of $\psi = \Delta$ (NRR is obsolete after the backstop) which means that the profits will be lowest for the discussed cases with a P factor of $P = 55.90$ and the extraction rate initially will be highest accordingly with the decline rate $D = 0.16$ before the backstop arrives. This will after some time change because of the faster depletion and extraction will become slower. In green we see the "standard" backstop case with uncertainty. Using equation (4.4.7) we have that $P = 70.096$ and the decline rate of extraction before the backstop arrives as $D = 0.1018$. In magenta we have the case were the arrival time of the backstop in known. In blue we see our original model without a backstop (we set $b = 0$ as in (7.2.2)) and get $P = 91.287$, $D = \mu r = 0.06$ for the whole time horizon. Once the backstop arrives extraction rates jump down by a fixed amount and then decline at the same optimal rate $-\mu r$ although from different resource stock levels S_T as a consequence of different initial extraction rates (in case of the perfect backstop extraction rates will be 0). The initial extraction rates for the given resource stock are $R_0 = 30$ for the case without a backstop, 39 for the case with a known arrival time, 50.9 for the case with uncertainty about the arrival time of the backstop and 80 for the special case where the NRR becomes obsolete as soon as the backstop is there.

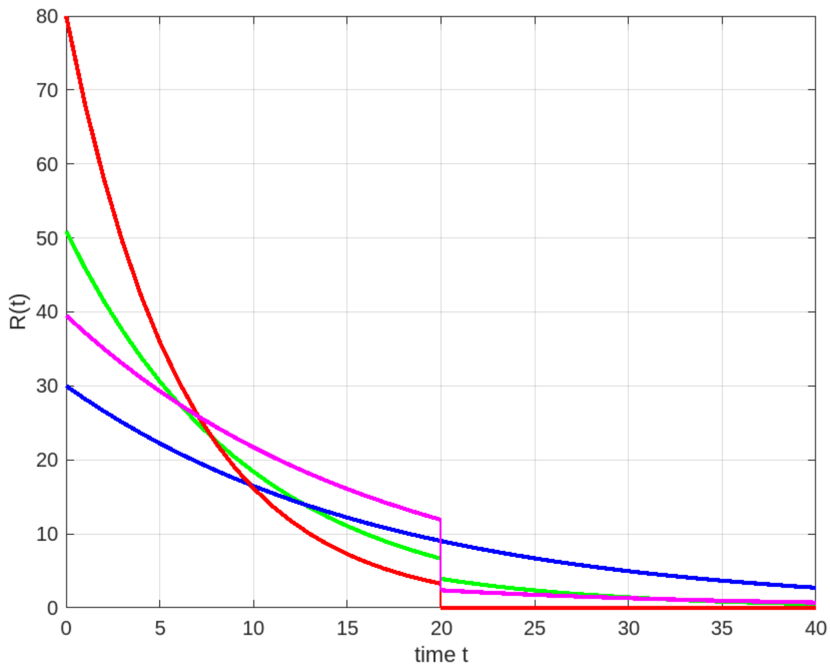

Figure 9: Extraction paths for an exogenous resource stock. In blue the no-backstop case, in magenta the case with a known arrival time, in green the backstop with unknown arrival time and in red the case with infinite elasticity of the substitute.

Let us next take a closer look at the uncertain arrival time and optimized initial resource stock S_0. We see in Figure 10 the extraction path for an optimized S_0 comparing our model without a backstop ($b = 0$) as the blue line and the paths of our model with a backstop for 2 cases. In green the case where the guess for the arrival time was right at $T = 1/b = 20$ and for a case where the guess was wrong and the backstop arrived delayed at $T = 30$ (in red). The decline rate of the extraction path of the non-backstop model is μr for the whole time horizon starting now from the significantly higher initial resource stock relative to the backstop cases. For the backstop models we have a steeper decline rate D in both cases before the backstop arrives (lines are identical until T 20) and then the jump down in extraction once the backstop arrives after which the decline rates are the same with $\mu r < D$ again from different resource stock levels meaning that in the case where the backstop arrives at $T = 30$ there will be faster extraction speed for longer and a smaller jump down meaning more frontloading.

The uncertainty about the arrival time leads to higher extraction rates for a – given - resource stock which will eventually reverse because of this higher decline rate. In the case depicted the initial extraction rates are indeed higher for the backstop cases but if the endogenous initial resource stock would decline even stronger vs the no-backstop case as a consequence of the uncertainty it could also be that initial extraction rates are lower. In our parametrization the extraction rates will get lower relatively fast for the backstop models quite some time before the backstop arrives vs the no backstop-model.

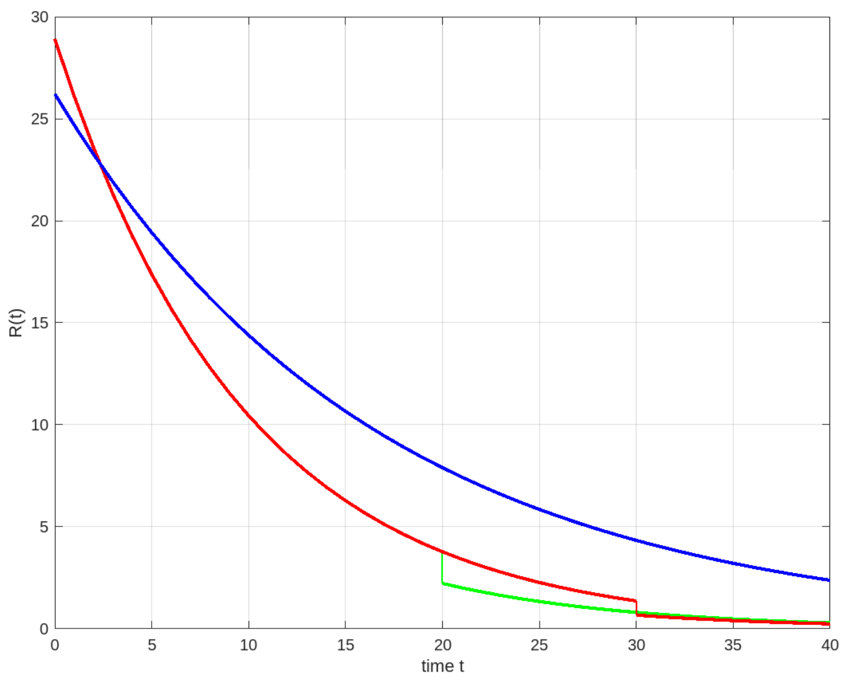

Figure 10: Extraction paths with no backstop (blue) and for a model with a backstop and a wrong guess (green) and right guess (red) for the arrival date of the backstop. The expected arrival date was T = 20.

We now take a look how the extraction paths change if we allow for an optimal exploration decision in Figure 11. The optimal initial resource stocks here are S_0 =284 from (4.6.2) and our specific form of the investment function for the standard backstop case, S_0 = 437 for the case without a backstop since there is no uncertainty here encouraging investment. Furthermore S_0 = 196 for the special case of the perfect backstop and S_0 = 350 for the case of the known arrival time. We see that for our parametrization the extraction paths still show higher initial extraction rates vs the no backstop-case but since the initial resource stocks will now be lower if endogenously determined caused by the lower P factor they are now much lower than before. If the resource stocks would have declined even more than we depicted based on our parametrization because of the uncertainty / cheaper price of the green alternative then initial extraction rates could be lower vs the no-backstop case. The case were the arrival time is known is less inefficient and the resource stock decline is relatively smaller than in the uncertain case and the extreme case of the perfect backstop shows the strongest decline in the initial resource stock. This again shows the importance of allowing an optimized initial resource stock decision in our model.

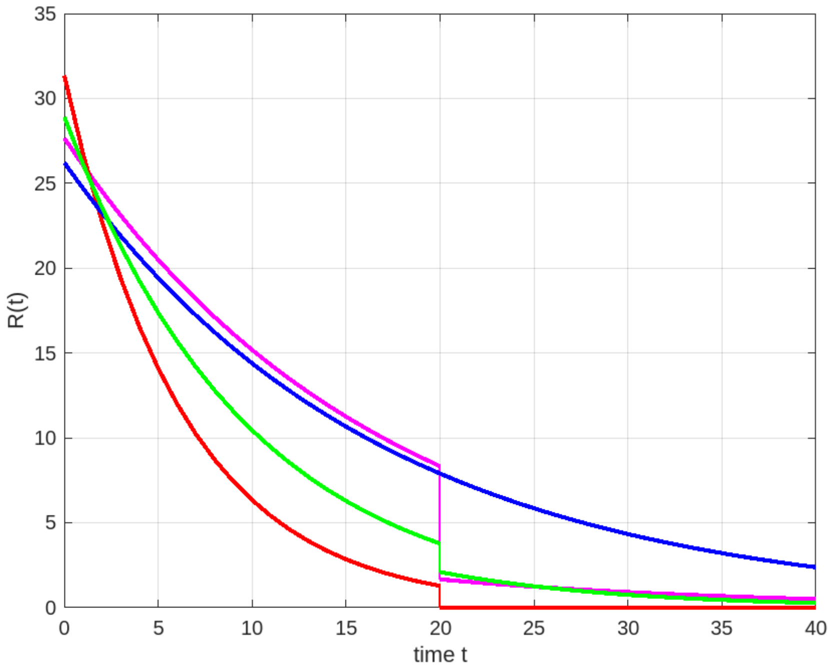

Figure 11: Extraction paths for a known arrival time (magenta) vs unknown but correctly guessed case (green) the perfect backstop case (red) and our model without a backstop (in blue).

Let us turn to the effects of taxation next. We showed that a constant tax will leave the optimal paths unchanged but lowers profits and thereby investment in exploration and so the initial resource stock. This means initial extraction rates will be lower as the tax factor decreases and total extraction will be lowered which means that both green paradox effects are avoided. This is shown exemplary for our "standard" backstop case with a solid line for $\theta = 1$ (no tax), a dashed line for $\theta = 0.9$ and a dotted line for $\theta = 0.7$ in Figure 12. It was explained in the previous chapter and by Sinn (2008) why it is unlikely that such a tax is implemented however.

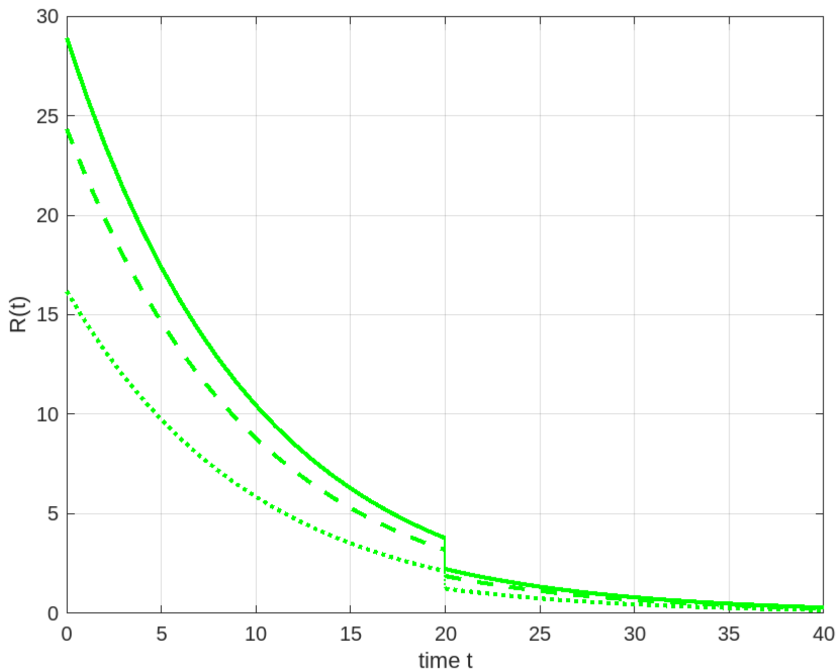

Figure 12: The effects of a constant tax. Solid means no tax, dashed for a low tax rate, dotted for a high tax rate.

We now look at the effect an increasing cash flow tax has on our model without and with a backstop. P will now satisfy (4.7.6) in the tax case and depends on the growth rate of the tax factor which we set to $\hat{\theta} = -0.05$. And for the initial tax factor we have $\theta_0 = 0.8$. The tax will reduce profits, increase extraction speed and lower the initial resource stock just like the inclusion of the backstop itself. Investment will be affected through the change in P caused by the growth rate of the tax factor and through the initial tax factor as seen in (4.7.10). Our optimal value function w^B changed to (4.7.5) and has now the tax factor as an argument which means that our condition for the optimal resource investment in t = 0 changed to $\varepsilon = (1-1/\mu)PE(I)^{-1/\mu}\theta_0 E'(I)$.

We therefore get a decline of the profit factor to P = 48.82 for our backstop model as we expected since future profits are lowered and a speed up of extraction with D = 0.21 showing the frontloading of extraction caused by declining future resource rents. Our investment decisions therefore gives us a lower $S_0 = 157$. For our model without a backstop P also declines to P = 55.9 giving us a D = 0.16 = $\mu(r-\hat{\theta})$ and a decline in the initial resource stock to $S_0 = 196$. Figure 13 depicts this with the extraction path of the model without a backstop as always in blue and dotted for the case with a tax and in green for our backstop model (again the dotted line showing the tax case). In the model without a backstop the extraction path still declines over the whole time horizon with the same now higher rate of $\mu(r-\hat{\theta})$ in the tax case from a higher starting point. For the back stop model the decline rate after the backstops arrival also changes from μr to $\mu(r-\hat{\theta})$ and the decline rate before the backstops D will be faster as we just saw as well. The effects of the tax itself are of the same nature qualitatively in both models. For this parametrization the extraction rates again increase initially for both cases after the introduction of the tax which means a weak green paradox occurs. Of course like in our model in chapter 3 the weak green paradox exists for a range of combinations of initial tax factors and growth rates of the tax and might be avoided through a higher reduction in the resource stock for different parametrizations.

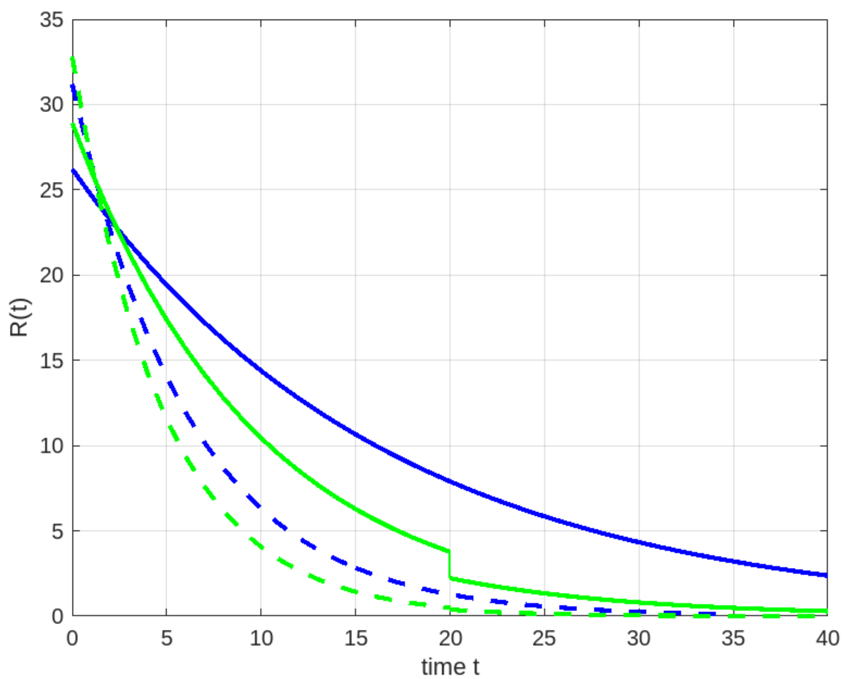

Figure 13: The effects of an increasing cash flow tax (dotted lines) on extraction paths of the model without a backstop (blue) and with a backstop (green).

Two comments on the price paths in Van der Poleg's model here. First the jump for the price path at the backstops arrival will only be downward for the NRR if the backstop technology is good enough with high enough σ and Δ in combination as is shown in appendix (7.2.5) and otherwise will jump upward. Second the price paths are different for the case of the backstop never arriving and the case of it existing from the beginning as lower prices of the green alternative. In the latter case the price paths will be lower. This is depicted in Figure 13 for the case that the backstop arrives at its expected time T = 20.

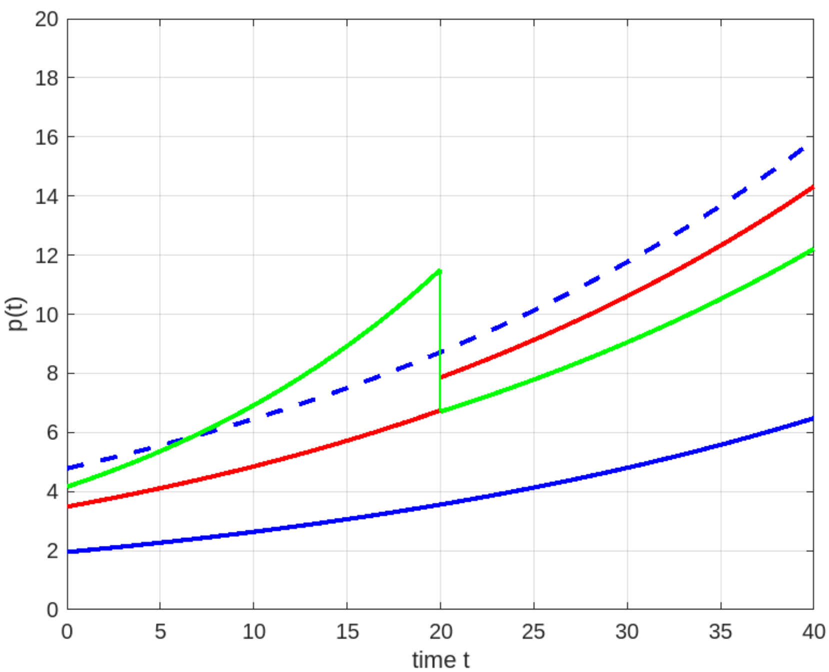

Figure 14: Price paths for weak backstop in red and strong backstop in green and for the no backstop cases (dotted for the case with high prices for the green substitute).

4.9 Findings

The addition of uncertainty in the model by Van der Ploeg via the unknown date of the arrival of a backstop technology leads to steeper extraction paths before the backstop arrives caused by lowered future profits as demand for the NRR will decline in cheaper prices of the green alternative vs a model without a backstop. The extraction rate will jump down at the arrival time of the backstop and then follow a flatter extraction path that matches the one in our model without a backstop but from a lower starting point (7.2.5). The price path will also be steeper (in its increase) before t = T and there will be a jump downward or upward in the price of the NRR depending on how good the backstop technology substitutes the NRR and how strong it reduces the price of the alternative (4.4.16). This is for a – given – initial resource stock. The steepest extraction path is seen for the case of a perfect backstop technology where the NRR becomes obsolete once the backstop arrives.

The initial resource stock decision is however also affected by the uncertainty and will now not only be lower in its cost but also in a higher decrease of the price of the green alternative and in a shorter arrival time of the backstop (4.6.4). So while the uncertainty speeds up extraction additionally it also lowers the resource stock additionally. It basically adds another layer of the same countervailing forces the tax itself causes.

When we add a constant tax to the model it lowers the net present value of profits and therefore affects initial investment in exploration which will be lowered as the tax is set higher, creating a lower initial resource stock. The optimal paths are not changed from this new lower resource stock levels in both phases however and will therefore not create either green paradox effect as no frontloading occurs and total extraction will be lower.

An increasing tax leads to an increased decline rate of extraction in the phase after the backstop given by our equation for the extraction path in (4.7.2) compared to the case without a tax (4.2.7) since $(r-\hat{\theta})>r$ which is the same finding as in chapter 3. Before the backstop comes online the paths found have the same form as in the case without a tax (4.4.12) but the growth rate of the tax factor now influences our profit factor P and the inversely related D which is a measure of extraction speed. The faster the tax factor declines / the tax rate increases (the lower $\hat{\theta}$) the faster the extraction speed D and the decline in profits as seen in (4.7.8). Furthermore any tax will decrease the initial resource stock in S_0 and consequently given our findings for the extraction rates (and resource paths) S_T given the resource constraint must hold. The higher the initial tax factor is the higher investment in exploration but since $\theta_0 \in [0,1]$ it can never be higher than without a tax. The higher the growth rate of the tax factor / the slower the tax increases the higher the initial investment remains but again always below the case of no tax which is seen in (4.7.10).

Furthermore will S_0 and S_T not be affected by the uncertainty of the backstop technology coming online in a different way through the initial tax rate or the growth rate of the tax as seen in (4.7.9). We see that the introduction of an increasing cash flow tax into our model with a backstop doesn´t show a different influence of exploratory costs, of a cheaper green alternative after the arrival of the backstop and of the probability of the arrival of a backstop vs a baseline model without the tax concerning the exploratory decision and therefore the resource stocks.

There is additional frontloading now as the backstop and the tax cause it and there is an additional reduction in the resource stock also caused by these two. The uncertainty of the backstop arrival time itself causes frontloading and therefore worsens initial climate damage for a given resource stock but the initial resource stock will be lower as this uncertainty reduces the value of exploration. The tax causes frontloading of extraction in both phases and reduces initial investment efforts. Depending on the relative size of these effects of increasing extraction speed and declined resource stocks in the 2 phases we could observe an increase or decrease in initial and total damages and therefore for a probability of a weak and strong green paradox vs a no-backstop model. It was again shown that modelling the initial resource stock endogenously is important to see that the green paradox only exists for certain specifications of the tax. Optimal climate policy that tries to avoid green paradox effects has to take care to look at the specific effects technological progress has on the frontloading of extraction and the investment in exploration to be fully effective.

5 Discussion of model assumptions

5.1 The damage function

We want to add a small discussion about two of the assumptions used in the models concerning generalizability of the results. Since the weak green paradox only looks at initial extraction it is of course irrelevant for it how the damage function we use looks like. The choice of the right damage function is on the other hand highly important for the existence of a strong green paradox effect as the function we use in Chapter 2 and 3 assumes that the marginal damage a unit extracted causes increases slower than the interest rate which means that the discounted marginal damage decreases as time progresses so damage is driven relatively stronger by the timing then by total extraction and a flattening of the extraction path is seen as preferable to just keeping more of the resource in situ. Damage functions exhibiting constant marginal environmental damage independent of timing (Allen et al., 2009) and ones that try to incorporate tipping points (Dietz et al, 2021) or lagged adjustment processes between the atmospheric accumulation of carbon and the climate damage (Houghton et al, 1992, Chap. 3) put the focus on cumulative extraction instead.

Neither is the depreciation of CO_2 (or other GHG) in the atmosphere taken into account which would probably have a decreasing rate caused by the saturation of the carbon-sink capacities of for example heating oceans although increasing CO_2 might also lead to more biomass in the form of forests that could absorb more and an increasing amount of it (Houghton et al, 1992). Since we are dealing with a model with an infinite optimization horizon at least the later should be kept in mind concerning the results. In any case it is not a simple task to know what makes the damage function more realistic and what actually doesn´t.

The assumption of our model is, that the climate damage only depends on the accumulated emissions and increases smoothly while in reality nonlinearities and tipping points may play a role (Lenton, 2008) which potentially could cause accelerated accumulation and even higher damage per unit of extraction as time progresses. There are on the other hand some arguments that the effect of intertemporal frontloading might not be that severe if cumulative extraction is the main determinant of the behavior of the system (Allen, 2009).

5.2 The Initial resource stock

Any initial tax in our models reduces S_0 since $\theta_0 \in (0,1)$. There is no initial tax rate that allows the capital stock to be bigger in a scenario with taxes than in a scenario without taxes. The models used see exploration as an initial investment based on the idea that the exploratory effort is a sunk cost. But in reality an increasing tax could theoretically affect exploration by causing a "black gold rush" (Cairns, 2014) which could be understood as a speed up of exploration much like the frontloading happening for the extraction. This could in itself cause additional environmental damage and would make resources earlier available for extraction thereby frontloading extraction potentially additionally. Or put another way the accumulation of resources could be finished earlier than t = 0 in our model and therefore extraction could start earlier. On the other hand the lower value of the explored resource stock might discourage exploration and lower S_0 like it does in our models especially since exploration costs are very often seen as sunk cost and therefore our modelling might actually be not as unrealistic as one might think. Additionally concerns are how much exploration could actually be sped up because of limitations of manpower, technology, and environmental laws etc. which effects exploration potentially even more than extraction. (Cairns, 2014).

6 Conclusions

After our motivation for using an endogenous resource stock we started with a simple classical model of resource extraction under monopolistic market structure akin to Stiglitz (1976) to have a baseline for later additions to the model, to explain in detail the assumptions and to show the countervailing forces caused by the increasing tax most clearly. This led to the existence of a weak and a strong green paradox given our model specifications for any possible tax scheme with the additional finding that the higher the decay rate of the tax factor (the faster the tax increases) the higher was total damage just like in Sinn's model since the resource stock was exogenously given.

Extending the model to the case where an optimized exploration decision is made in the beginning after which extraction starts and keeping the monopolistic market structure we found that it now is important how the environmental policy is designed. The existence of a weak green paradox is more probable for a high initial tax factor / a low initial tax rate and for a fast decreasing tax factor / a fast increasing tax rate for the other parameters held constant. Furthermore one cannot exclude the possibility of the existence of a W.G.P. for any growth rate of the tax factor. The existence of a strong green paradox also increased in probability in a high initial tax factor and the growth rate of the tax had now a range for which a strong green paradox could exist with too low and too high values excluding the possibility for its existence (but even for this range it must not exist given the other parameters). The other model parameter were analyzed for their specific effects on the two tax scheme elements. Additionally it was shown that with a simple form of uncertainty concerning property rights that were present in Sinn's (2008) original paper that those exhibits the same effect as the discount rate does for the weak and strong form of the green paradox. The results were then visualized. The findings of Österle therefore where replicated with the new market structure given an assumption about the elasticity of demand to exclude extreme cases and under a slightly modified but still isoelastic demand function.

In Chapter 4 the model was extended to a form that includes a green alternative to the NRR resource a backstop technology that will lower the price of the green substitute and uncertainty about its arrival time based on Van der Ploeg (2013). It was explained that the growth rates of extraction were now different before and after the arrival of the backstop. For a given resource stock initial extraction rates increased vs a non-backstop variant with even higher speed up if the date of the backstops arrival is not known and the stronger the backstop lowers the price of the green substitute. At the time of the arrival of the backstop extraction jumped down by a fixed amount. After the arrival of the backstop the extraction rates followed the same decline rate as in our model without a backstop but from lower (and less efficient) resource stock levels caused by lower investment caused by the uncertainties which reduce profits.

It was shown that the resource stock in both phases will further be lower if an increasing cash flow tax is introduced. The uncertainties introduced changed the structure of the optimized paths for extraction, prices and the resource stock reinforcing the effects that the backstop itself caused and showed the same reaction to the tax like in our model in chapter 3. These findings also hold for the exploratory effort which did now additionally decrease based on the other additions to the model (a cheaper price of the green substitute caused by the backstop and a higher probability of the backstop arriving). It was shown that investment declined in a decline of the initial tax factor and in a faster increase of the tax rate.

The uncertainties about the backstops arrival time itself introduced an additional speeding up of extraction which increases the chance for a green paradox effect but also reduced investment in exploration which lowers it. The findings of the original model therefore are also robust to the implementation of a backstop technology with uncertainty about its arrival time if the original assumptions of no extraction costs and the specific form of the iso-elastic demand function are kept concerning our increasing cash flow tax. It was furthermore shown that a constant tax in this specific model only effected the investment decision (by lowering investment) and did not change the optimal paths of extraction, price and the resource stock in both phases given the new lower resource stocks like in Sinn's (2008) original model. The findings were then visualized by a numeric simulation taking into account the change in the initial resource stock. We closed with a short discussion of certain model assumptions that could be further investigated.

7 Appendix

7.1 Monopoly with endogenous resource stock and no backstop

7.1.1 Effect of α on the condition for a S.G.P.

Let us define the R.H.S. of (3.4.2) as:

$$\vartheta^{\alpha}\left[1-\frac{\hat{\theta}}{r+(r-\hat{\chi})\mu}\right]^{\mu} \text{ with: } \vartheta=\frac{1-\frac{\hat{\theta}}{r+(r-\hat{\chi})\mu}}{1-\frac{\hat{\theta}}{r}}$$

We have:

$$\begin{aligned} & r+(r-\hat{\chi})\mu > r \\ \Leftrightarrow & \frac{\hat{\theta}}{r+(r-\hat{\chi})\mu} > \frac{\hat{\theta}}{r} \\ \Leftrightarrow & 1-\frac{\hat{\theta}}{r+(r-\hat{\chi})\mu} < 1-\frac{\hat{\theta}}{r} \\ \Leftrightarrow & \vartheta < 1. \end{aligned}$$

We see that the right hand side is strictly decreasing in α. Of course this could also be shown with the partial derivative with respect to α which is strict smaller than 0.

7.1.2 Model with uncertain property rights

The optimization problem is now:

$$\max_{R_t} \pi = \int_0^{\infty} (p(R_t)R_t\theta_t)e^{-(r+\rho)t}dt - C(S_0)$$

$\dot{S}_t = -R_t, S_t > 0, S_0\ endogenous, \int_0^{\infty} R_t dt \le S_0; \forall t \ge 0$

The current value Hamiltonian:

$$H = p(R_t)R_t\theta_t - \lambda_t R_t$$

Optimality conditions:

$$p(R_t)\theta_t - \mu\theta_t R_t^{-\mu} = \lambda_t$$

$$\dot{\lambda}_t = -H_S + (r+\rho)\lambda_t \Leftrightarrow \dot{\lambda}_t = (r+\rho)\lambda_t$$

$$\lim_{t\to\infty} S(t)\lambda(t)e^{(r+\rho)t} = 0$$

These give us the price path as:

$p_t = \frac{\lambda_0}{\theta_0(1-\mu)} e^{(r+\rho-\hat{\theta})t} = p_0 e^{(r+\rho-\hat{\theta})t}$ and $p_0 = \frac{\lambda_0}{\theta_0(1-\mu)}$ stays the same.

The extraction path is therefore:

$$R_t = \left[\frac{\lambda_0}{\theta_0(1-\mu)}\right]^{-1/\mu} e^{-\frac{(r+\rho-\hat{\theta})t}{\mu}} = R_0 e^{\frac{-(r+\rho-\hat{\theta})t}{\mu}}.$$

Using that the initial resource stock has to equal total extraction we find the initial extraction rate and the extraction path formulated for the initial resource stock:

$R_0 = \frac{(r+\rho-\hat{\theta})S_0}{\mu}$ and $R_t = \frac{(r+\rho-\hat{\theta})S_0}{\mu} e^{-\frac{(r+\rho-\hat{\theta})t}{\mu}}$

Plugging R_t into the damage function Ω (2.1.5) and solving the integral gives equation:

$$\Omega = \frac{(r+\rho-\hat{\theta})S_0\chi_0}{\mu(r-\hat{\chi})+r+\rho-\hat{\theta}}.$$

To make S_0 endogen we again equate the marginal cost of total exploration and the shadow price of the resource in period t = 0 and get the following expression for the initial extraction rate:

$$C'(S_0) = \lambda_0$$
$$\Leftrightarrow (\alpha+1)\beta S_0^{\ \alpha} = p_0\theta_0(1-\mu)$$
$$\Leftrightarrow (\alpha+1)\beta\left[\frac{R_0\mu}{(r+\rho-\hat{\theta})}\right]^{\alpha} = R_0^{\ -\mu}\theta_0(1-\mu)$$
$$\Leftrightarrow R_0^{\ \alpha+\mu} = \left[\frac{r+\rho-\hat{\theta}}{\mu}\right]^{\alpha}\frac{\theta_0(1-\mu)}{(\alpha+1)\beta}$$
$$\Leftrightarrow R_0 = \left[\frac{(r+\rho-\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{\theta_0(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}}$$

For the W.G.P. we compare $R_0^{\ T} > R_0^{\ NT}$ and get the following condition:

$$\left[\frac{(r+\rho-\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{\theta_0(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}} > \left[\frac{(r+\rho)}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}}$$
$$\Leftrightarrow \theta_0^{\frac{1}{\alpha+\mu}} > \left[\frac{\mu(r+\rho)}{\mu(r+\rho-\hat{\theta})}\right]^{\frac{\alpha}{\alpha+\mu}} \Leftrightarrow \theta_0 > \left(\frac{r+\rho}{(r+\rho-\hat{\theta})}\right)^{\alpha}$$
$$\Leftrightarrow \theta_0 > \left(\frac{r+\rho-\hat{\theta}}{r+\rho}\right)^{-\alpha} \Leftrightarrow \theta_0 > \left(1-\frac{\hat{\theta}}{r+\rho}\right)^{-\alpha}$$

And for the S.G.P. we again get our expression for total climate damage as:

$$\Omega = \int_0^{\infty} e^{-rt}\chi_t R_t dt = \int_0^{\infty} e^{(\hat{\chi}-r)t}\chi_0 R_t dt$$
$$= \int_0^{\infty} e^{(\hat{\chi}-r)t} e^{-\frac{t(r+\rho-\hat{\theta})}{\mu}}\chi_0 R_0 dt =$$
$$= \int_0^{\infty} e^{-t(\frac{(r+\rho-\hat{\theta})}{\mu}+r-\hat{\chi})}\chi_0\left[\frac{(r+\rho-\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{\theta_0(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}} dt =$$
$$= \frac{\chi_0}{(\frac{r+\rho-\hat{\theta}}{\mu})+r-\hat{\chi}}\left[\frac{(r+\rho-\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{\theta_0(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}} =$$
$$= \frac{\mu\chi_0}{r+\rho-\hat{\theta}+\mu(r-\hat{\chi})}\left[\frac{(r+\rho-\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{\theta_0(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}}$$

Giving us:

$$\Omega = \frac{\mu\chi_0}{r+\rho-\hat{\theta}+\mu(r-\hat{\chi})}\left[\frac{(r-\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{\theta_0(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}}$$

If we compare $\Omega^T > \Omega^{NT}$ we finally get the condition for the existence of the strong green paradox:

$$\frac{\chi_0}{r+\rho-\hat{\theta}+\mu(r-\hat{\chi})}\left[\frac{(r+\rho-\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{\theta_0(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}} > \frac{\chi_0}{r+\rho+\mu(r-\hat{\chi})}\left[\frac{r+\rho}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\left[\frac{(1-\mu)}{(\alpha+1)\beta}\right]^{\frac{1}{\alpha+\mu}}$$

$$\Leftrightarrow \theta_0^{\frac{1}{\alpha+\mu}}\left[\frac{(r+\rho-\hat{\theta})}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}} > \left[\frac{r+\rho}{\mu}\right]^{\frac{\alpha}{\alpha+\mu}}\frac{r+\rho-\hat{\theta}+\mu(r-\hat{\chi})}{r+\rho+\mu(r-\hat{\chi})}$$

$$\Leftrightarrow (\theta_0) > \left[\frac{r+\rho-\hat{\theta}+\mu(r-\hat{\chi})}{r+\rho+\mu(r-\hat{\chi})}\right]^{\alpha+\mu}\left[\frac{r+\rho}{(r+\rho-\hat{\theta})}\right]^{\alpha}$$

$$\Leftrightarrow \theta_0 > \left[1-\frac{\hat{\theta}}{r+\rho+\mu(r-\hat{\chi})}\right]^{\alpha+\mu}\left[1-\frac{\hat{\theta}}{r+\rho}\right]^{-\alpha}$$

Giving us:

$$\theta_0 > \left[1-\frac{\hat{\theta}}{r+\rho+\mu(r-\hat{\chi})}\right]^{\alpha+\mu}\left[1-\frac{\hat{\theta}}{r+\rho}\right]^{-\alpha}$$

7.2 The backstop model

7.2.1 Derivation of $S_{1/b}$ for a known arrival time of the backstop

$$\int_0^{1/b}\dot{S}_t dt = \int_0^{1/b} -\mu r S_{1/b}\left[\frac{\psi}{\psi-\Delta}\right]^{\sigma} e^{-\mu r(t-1/b)}dt$$

$$\Leftrightarrow S_{1/b} - S_0 = -\mu r S_{1/b}\left[\frac{\psi}{\psi-\Delta}\right]^{\sigma} e^{\mu r/b}\int_0^{1/b} e^{-\mu r t}dt$$

$$\Leftrightarrow S_{1/b} - S_0 = S_{1/b}\left[\frac{\psi}{\psi-\Delta}\right]^{\sigma}(1-e^{\mu r/b})$$

$$\Leftrightarrow S_{1/b}\left[1-\left[\frac{\psi}{\psi-\Delta}\right]^{\sigma}(1-e^{\mu r/b})\right] = S_0$$

$$\Leftrightarrow S_{1/b} = \frac{S_0}{\left[1-\left[\frac{\psi}{\psi-\Delta}\right]^{\sigma}(1-e^{\mu r/b})\right]}$$

$$\Leftrightarrow S_{1/b} = \frac{S_0}{\left[1+\left[\frac{\psi}{\psi-\Delta}\right]^{\sigma}(e^{\mu r/b}-1)\right]}$$

And we have: $S_{1/b} \le S_0 e^{-\mu r/b} \le S_0$

7.2.2 Case of no backstop: $b = 0$

Using (4.4.7) we have that: $P = \left((\psi^{\sigma}c)/\mu r\right)^{\frac{1}{\mu}}$ and therefore $D = c\psi^{\sigma}P^{-\mu} = r\mu$. So when there never is a backstop or think of it as just a lower price of the green alternative from the very beginning we are back in our model from chapter 3 for the decline rate of the extraction path. We see that the decline rate of extraction is the same for the whole time span of the model and the extraction paths of A and B are identical for all t.

7.2.3 Case of imminent backstop: $b \to \infty$

From (4.4.7) we get that:

$P \to \left(\frac{(\psi-\Delta)^{\sigma}c}{r\mu}\right)^{\frac{1}{\mu}}$ and $D \to (r\mu)\frac{\psi^{\sigma}}{(\psi-\Delta)^{\sigma}}$ This is larger than rμ for Δ > 0 (otherwise the backstop technology would have no effect on the price). Therefore P is higher without a backstop ever arriving (b = 0) and D is higher (independent of the green substitute's price) if it is imminent that the backstop arrives.

7.2.4 Case of a perfect backstop: $\Delta = \psi$

This gives us from (4.4.7) that: $(c\psi^{\sigma} P^{1-\mu}) / \mu = P(r+b)$ and therefore:

$$P = \left(\frac{\psi^{\sigma} c}{(r+b)\mu} \right)^{\frac{1}{\mu}}$$

$$D = \psi^{\sigma} c \left(\frac{\psi^{\sigma} c}{(r+b)\mu} \right)^{-1} = (r+b)\mu > \mu r .$$

This means extraction speed will be faster and profit will be lower (for any b > 0) than the standard case since if the backstop arrives there will be no more demand for the NRR since $\psi - \Delta = 0$. The extraction path before the backstop arrives is therefore given by: $R_t^B = \mu(r+b) S_0 e^{-\mu(r+b)(t-T)}$. P is now P(0,b) and $P_b = -\frac{1}{\mu(r+b)} \left(\frac{c\psi^{\sigma}}{\mu(r+b)} \right)^{1/\mu} < 0$.

7.2.5 Jump in paths at time T:

If we look at the point in time just before the backstop technology arrives we get from (4.4.12) that at t = T- immediately before t = T we have for the price, the extraction rate and the resource stock using $D = c\psi^{\sigma} P^{-\mu}$:

$$p_{T-}^{B} = P S_0^{-1/\mu} e^{DT/\mu} = \left(\frac{c\psi^{\sigma}}{S_0 D} \right)^{1/\mu} e^{DT/\mu} ,\ R_{T-}^{B} = S_0 D e^{-DT} \text{ and } S_{T-}^{B} = S_0 e^{-DT} = S_T^A .$$

So the stock of the resource at t = T and immediately before that is the same. Inserting into our paths from phase A (4.2.7) and (4.2.8) with t = T and the current resource stock gives:

$$p_T^A = \left[\frac{c(\psi-\Delta)^\sigma}{\mu r S_0}\right]^{1/\mu} e^{DT/\mu}$$

$$p_T^A = \left[\frac{c(\psi-\Delta)^\sigma}{\mu r S_0}\right]^{1/\mu} e^{DT/\mu} < p_{T-}^B = \left[\frac{c\psi^\sigma}{DS_0}\right]^{1/\mu} e^{DT/\mu}$$

$$\Leftrightarrow \left(\frac{\psi}{\psi-\Delta}\right)^\sigma > D / r\mu$$

$$R_T^A = S_0 \mu r e^{-DT} < R_{T-}^B = S_0 D e^{-DT}$$

So the extraction rate jumps down when the backstop arrives and the price will also jump down if the combination of cost reduction and probability of arrival is high enough. Otherwise it will not change if the last term holds with equality or the price will instead jump up if the effect of the backstop is too weak.

7.2.6 Partial derivatives for the P factor and investment

P factor for the case without taxes:

For our P factor $P = P(b, \psi - \Delta)$ and $D = P^{-\mu} c\psi^\sigma$ in the no tax case (4.4.7) we find the partial derivatives as:

$$P_{\psi-\Delta} = \frac{b}{\mu}\left(\frac{(\psi-\Delta)^\sigma c}{r\mu}\right)^{1/\mu-1} \frac{\sigma(\psi-\Delta)^{\sigma-1} c}{r\mu} = \frac{b\sigma}{\mu(\psi-\Delta)}\left(\frac{(\psi-\Delta)^\sigma c}{r\mu}\right)^{1/\mu} > 0$$

$$P_b = \left(\frac{(\psi-\Delta)^\sigma c}{r\mu}\right)^{1/\mu} - P = \frac{1}{b}\left(rP - \frac{c\psi^\sigma P^{1-\mu}}{\mu}\right) = -\frac{P}{\mu b}(P^{-\mu} c\psi^\sigma - \mu r) = -\frac{P}{\mu b}(D - \mu r) < 0$$

$$P_P = r + b - (1-\mu)\frac{1}{\mu} c\psi^\sigma P^{-\mu} = r + b + \left(1 - \frac{1}{\mu}\right) c\psi^\sigma P^{-\mu} > 0$$

This gives the total derivative as:

$$(r + b + (1 - 1/\mu) c\psi^\sigma P^{-\mu}) dP = \frac{b\sigma}{\mu(\psi-\Delta)}\left(\frac{(\psi-\Delta)^\sigma c}{r\mu}\right)^{1/\mu} d(\psi-\Delta) - \frac{P}{\mu b}(P^{-\mu} c\psi^\sigma - \mu r) db$$

Investment for the case without taxes:

For our optimality condition without tax $\varepsilon = (1-\frac{1}{\mu})E(I)^{\frac{-1}{\mu}}E'(I)P(b,\psi-\Delta)$ and for E´(I)>0 and E´´(I) < 0 we find the partial derivatives with:

$$I_b = (1-\frac{1}{\mu})E(I)^{-\frac{1}{\mu}}E'(I)P_b = \frac{\varepsilon P_b}{P} < 0$$

$$I_{\psi-\Delta} = (1-\frac{1}{\mu})E(I)^{-\frac{1}{\mu}}E'(I)P_{\psi-\Delta} = \frac{\varepsilon P_{\psi-\Delta}}{P} > 0$$

$$I_\varepsilon = -1 < 0$$

$$I_I = \varepsilon\left[\frac{E'(I)}{\mu E(I)} - \frac{E''(I)}{E'(I)}\right] > 0$$

This gives the total derivative as:

$$\varepsilon\left[\frac{E'(I)}{\mu E(I)} - \frac{E''(I)}{E'(I)}\right]dI = \frac{\varepsilon(P_b db + P_{\psi-\Delta}d(\psi-\Delta))}{P} - d\varepsilon$$

<u>P factor for the case with an increasing tax:</u>

For our P factor $P = P(\hat{\theta}, b, \psi-\Delta)$ and $D = P^{-\mu}c\psi^{\sigma}$ in the no tax case (4.7.6) we find the partial derivatives as:

$$P_b = \left(\frac{(\psi-\Delta)^{\sigma}c}{(r-\hat{\theta})\mu}\right)^{1/\mu} - P = \frac{1}{b}(-\frac{1}{\mu}c\psi^{\sigma}P^{1-\mu} + (r-\hat{\theta})P) = -\frac{P}{\mu b}(P^{-\mu}c\psi^{\sigma} - \mu(r-\hat{\theta}) = -\frac{P}{\mu b}(D-\mu(r-\hat{\theta})) < 0$$

$$P_{\psi-\Delta} = \frac{b}{\mu}\left(\frac{(\psi-\Delta)^{\sigma}c}{(r-\hat{\theta})\mu}\right)^{1/\mu-1}\frac{\sigma(\psi-\Delta)^{\sigma-1}c}{(r-\hat{\theta})\mu} = \frac{b\sigma}{\mu(\psi-\Delta)}\left(\frac{(\psi-\Delta)^{\sigma}c}{(r-\hat{\theta})\mu}\right)^{1/\mu} > 0$$

$$P_{\hat{\theta}} = P + \left(\frac{c(\psi-\Delta)^{\sigma}}{\mu(r-\hat{\theta})}\right)^{\frac{1}{\mu}}\left(\frac{b}{\mu(r-\hat{\theta})}\right) > 0$$

$$P_P = -(1-\mu)\frac{1}{\mu}c\psi^{\sigma}P^{-\mu} + r - \hat{\theta} + b > 0$$

This gives the total derivative as:

$$dP = \frac{-\frac{P}{\mu b}(D-\mu(r-\hat{\theta}))db + \left(\frac{(\psi-\Delta)^{\sigma}c}{\mu(r-\hat{\theta})}\right)^{\frac{1}{\mu}}\frac{b\sigma}{\mu(\psi-\Delta)}d(\psi-\Delta) + \left[P + \left(\frac{c(\psi-\Delta)^{\sigma}}{\mu(r-\hat{\theta})}\right)^{\frac{1}{\mu}}\left(\frac{b}{\mu(r-\hat{\theta})}\right)\right]d\hat{\theta}}{(1-1/\mu)c\psi^{\sigma}P^{-\mu} + r - \hat{\theta} + b}$$

7.2.7 Efficiency of the optimal paths

Stiglitz (1976) showed that for no extraction costs and isoelastic demand the optimal growth rate of prices has to be equal to the interest rate r and the optimal decline rate of the extraction rate has to be -μr if there are no taxes and if there is no uncertainty in the case of a monopoly. If there is no backstop technology then P = P(ψ-Δ,0) and D = D(ψ,Δ,0) as b = 0 and the efficient paths are:

$p_t = \left(\frac{c\psi^{\sigma}}{\mu r S_0}\right)^{1/\mu} e^{rt}$ if the expected date for the backstop is t = ∞

$p_t = \left(\frac{c(\psi-\Delta)^{\sigma}}{\mu r S_0}\right)^{1/\mu} e^{rt}$ if the backstop exists from the start as a cheaper substitute.

$$R_t = S_0 \mu r e^{-\mu r t}$$

$$S_t = S_0 e^{-\mu r t}$$

Therefore the efficient paths of extraction and resource stock are not affected by the price of the substitute and the price path depends on the price of the substitute or the cheaper price for the substitute. After the backstop the optimal paths are efficient given a S_T that is too low and before the backstop they are too high in the backstop model. They start at too high levels and then decline / increase too fast that is why S_T is too low.

8 References

Adelman, M. A. (1970). “Economics of exploration for petroleum”, Geoexploration 8, p. 131-150.

Allen, Myles R., David J. Frame, Huntingford Chris, Jones Chris D., Lowe Jason A., Meinshausen Malte and Nicolai Meinshausen. 2009. Greenhouse-gas emission targets for limiting global warming to 2°C. Nature 458 (7242): p. 1158–62.

Arezki, R., Van der Ploeg, F and Toscani, F. 2019. “The shifting natural wealth of nations: The role of market orientation.” Journal of Development Economics Volume 138, May 2019, p. 228-245.

Arrow, K. and Chang, S. 1982. “Optimal Pricing, Use and Exploration of Uncertain Natural Resource Stocks”, J. Environ. Econ. Manag. 9, p. 1–10.

Arrow, K. 1962. "The Economic Implications of Learning by Doing", Review of Economic Studies, Oxford University Press, vol. 29(3), p. 155-173.

Bhattacharyya, S. C. 2011. “Energy Economics: Concepts, Issues, Markets and Governance.” London: Springer pp. 538.

Bohn, H., Deacon, R. T. 2000. “Ownership Risk, Investment and the Use of Natural Resources”, American Economic Review 90, pp. 526-549.

Cairns, R. D., Quyen N. V. 1998. “Optimal Exploration for and Exploitation of Heterogeneous Mineral Deposits”, Journal of Environmental Economics and Management, 35, p. 164-89.

Cairns, R. D. 2014. “The green paradox of the economics of exhaustible resources”, Energy Policy, 65(1), p 78–85.

Chichilnisky, Graciela, Geoffrey Heal, and David Starrett. 2000. “Equity and efficiency in environmental markets: Global trade in CO2 emissions. Environmental Markets: Equity and Efficiency.”

Dasgupta, P. and Heal, G. 1979. “Economic Theory and Exhaustible Resources”, Cambridge: Cambridge University Press.

Day, G. and Day, C. 2022. “The supply-side climate policy of decreasing fossil fuel tax profiles: can subsidized reserves induce a green paradox?”, Climatic Change 173, 27 (2022).

Dietz Simon, Rising James, Stoerk Thomas and Wagner Gernot. 2021. "Economic impacts of tipping points in the climate system", Proceedings of the National Academy of Sciences, Proceedings of the National Academy of Sciences, vol. 118(34), August.

Di Maria, C. and van der Werf, E. 2012. “Imperfect Environmental Policy and Polluting Emissions: The Green Paradox and Beyond”, International Review of Environmental and Resource Economics, 2012, 6: p. 153–194.

Gaudet, G. and Lasserre, P., 1988. “On comparing monopoly and competition in exhaustible resource exploitation.”, J. Environ. Econ. Manage. 15 (4), p. 412–418.

Gerlagh, R. and W. Lise. 2003. “Induced Technological Change Under Carbon Taxes”, Working Papers 2003.84, Fondazione Eni Enrico Mattei.

Gerlagh, R. 2011. “Too much oil.”, CESifo Economic Studies, 57: p. 79-102.

Grafton, Q., Kompas T., and Van Long. 2010. “Biofuels Subsidies and the Green Paradox.”, CESifo Working Paper 2960.

Halvorsen, R. and Smith T. 1991. “A Test of the Theory of Exhaustible Resources”, Quarterly Journal of Economics, February 1991, 106 (1), p. 123–140.

Henriet, F. and Schubert, K. 2019. "Is Shale Gas a Good Bridge to Renewables? An Application to Europe," Environmental & Resource Economics, Springer; European Association of Environmental and Resource Economists, vol. 72(3), p. 721-762, March.

Hoel Michael. 1978. "Resource Extraction, Uncertainty, and Learning," Bell Journal of Economics, The RAND Corporation, vol. 9(2), p. 642-645, autumn.

Hotelling, H. 1931. “The Economics of Exhaustible Resources. Journal of Political Economy”. 39, p. 137-175.

Houghton, J. T., B. A. Callander and S. K. Varney (eds.) (1992): Climate Change 1992 – The Supplementary Report to the IPCC Scientific Assessment, (Report from Working Group I, The Intergovernmental Panel on Climate Change). WMO, UNEP, IPCC. Cambridge: Cambridge University Press.

IPCC, 2018: Global Warming of 1.5°C.An IPCC Special Report on the impacts of global warming of 1.5°C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty [Masson-Delmotte, V., P. Zhai, H.-O. Pörtner, D. Roberts, J. Skea, P.R.Shukla, A.Pirani, W. Moufouma-Okia, C. Péan, R. Pidcock, S. Connors, J.B.R. Matthews, Y. Chen, X. Zhou, M.I. Gomis, E. Lonnoy, T. Maycock, M.Tignor, and T. Waterfield (eds.)]. In Press

Kim, J.-H., Lee, Y.-G. 2018. „Learning Curve, Change in Industrial Environment, and Dynamics of Production Activities in Unconventional Energy Resources”. Sustainability, 10(3322/9).

Krautkraemer, J. A. 1998. “Nonrenewable resource scarcity.” Journal of Economic Literature 36 (4), p. 2065 - 2107.

Kronenberg, T. 2008. “SHOULD WE WORRY ABOUT THE FAILURE OF THE HOTELLING RULE?” Journal of Economic surveys. Volume22, Issue4 September 2008, p. 774 – 793.

Lambertini, Luca. 2013. "Exploration for Nonrenewable Resources in a Dynamic Oligopoly: An Arrovian Result." Working Paper series 25_13, Rimini Centre for Economic Analysis.

Lasserre, Pierre. 1991. “Long-Term Control of Exhaustible Resources.” Chur: Harwood Academic Publishers.

Lenton, T. M., H. Held, E. Elmar, J. W. Hall, W. Lucht, S. Rahmstorf, and H.-J. Schellnhuber. 2008. Tipping elements in the Earth's climate system. Proceedings of the National Academy of Sciences 105: 1786–93.

Long, N. V. and Sinn, H.-W. 1985. Surprise price shifts, tax changes and the supply behaviour of resource extracting firms. Australian Economic Papers, 24, p. 278–289.

Luderer, G., Vrontisi, Z., Bertram, C., Edelenbosch, O. Y., Pietzcker, R. C., Rogelj, J., Kriegler, E. 2018. "Residual fossil CO2 emissions in 1.5–2 °C pathways". Nature Climate Change, 8(7), p. 626–633.

Marvasti, A. 2000. "Resource Characteristics, Extraction Costs, and Optimal Exploitation of Mineral Resources," Environmental & Resource Economics, Springer; European Association of Environmental and Resource Economists, vol. 17(4), pages 395-408, December.

McGlade, C. and Ekins, P. 2015. "The geographical distribution of fossil fuels unused when limiting global warming to 2 °C". Nature, 517(7533), p. 187–190.

Mohr, E. 1988 "Appropriation of Common Access Natural Resources Through Exploration: The Relevance of the Open-Loop Concept", International Economic Review, p. 29, 307-20.

Nordhaus, W. D. 1973 "The Allocation of Energy Resources," Brookings Papers on Economic Activity, 4(3), p. 529–576.

Nordhaus, W. D. and Yang, Z. 1996. „A regional dynamic general-equilibrium model of alternative climate-change strategies." The American Economic Review, p. 741-765.

Österle I. 2012. "Fossil Fuel Extraction and Climate Policy: A Review of the Green Paradox with Endogenous Resource Exploration," Climate Change and Sustainable Development 122010, Fondazione Eni Enrico Mattei (FEEM).

Österle I. 2015. „The green paradox and the importance of endogenous resource exploration" Australian Journal of Agricultural and Resource Economics, 60: p. 60 – 78

Peterson, F.M. 1978. "A Model of Mining and Exploration for Exhaustible Resources", Journal of Environmental Studies and Management, 5, p. 236-51.

Pindyck, Robert S. 1978. "The Optimal Exploration and Production of Nonrenewable Resources." Journal of Political Economy Vol. 86, No. 5 (Oct., 1978), p. 841-861.

Quyen, N.V. 1988. "The Optimal Depletion and Exploration of a Nonrenewable Resource", Econometrica, 56, p. 1467-71.

Quyen, N.V. .1991. "Exhaustible Resources: A Theory of Exploration" Review of Economic Studies, 58, p. 777-89.

Sinclair, P. J. N. 1992. "High does nothing and rising is worse: carbon taxes should keep declining to cut harmful emissions." The Manchester School, 60(1): p.41-52.

Sinclair, P. 1994. "On the Optimum Trend of Fossil Fuel Taxation." Oxford Economic Papers 46: p. 869–877.

Sinn, Hans-Werner. 2008. “Public Policies against global warming: a supply side approach.” International Tax Public Finance, 15: p. 360-394.

Sinn, H.-W. 2012. “The green paradox: A supply-side approach to global warming.” Cambridge, Mass.: MIT Press.

Stiglitz, J. E. 1976 “Monopoly and the Rate of Extraction of Exhaustible Resources” The American Economic Review, Vol. 66, No. 4 (Sep., 1976), p. 655-661.

Stürmer, M. and Schwerhoff G. “Non-renewable but inexhaustible - resources in an endogenous growth model.” Max Plank Society, 2012.

Ulph, A. and D. Ulph (1994). ‘‘The optimal time path of a carbon tax’’, Oxford Economics Papers 46, p. 857–868.

Van der Ploeg, F. and C. Withagen. 2012a. “Too Much Coal, Too Little Oil.” Journal of Public Economics 96: p. 62–77.

Van der Ploeg, R. 2013. "Cumulative Carbon Emissions and the Green Paradox," OxCarre Working Papers 110, Oxford Centre for the Analysis of Resource Rich Economies, University of Oxford.

Winter, R. 2014. "Innovation and the dynamics of global warming," Journal of Environmental Economics and Management, Elsevier, vol. 68(1), p. 124-140.