Energy Economics 131 (2024) 107347

Contents lists available at ScienceDirect

Energy Economics

journal homepage: www.elsevier.com/locate/eneeco

Market equilibrium strategies under learning by doing and spillovers

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ARTICLE INFO

JEL classification:
C71
C72
C73
D25
D43
P48

Keywords:
Learning by doing
Differential game
Competition
Spillovers
Solar PV
Linear Markov perfect equilibrium

ABSTRACT

We investigate intertemporal strategic interactions if monopolies, cartels, or oligopolies benefit from firm internal as well as external *learning by doing*. Our analysis is carried out for a linear learning cost curve, which allows the derivation of the linear Markov perfect equilibrium (LMPE). The model yields surprising properties: First and highly policy-relevant is the non-existence of equilibria except for very few firms and sufficiently large spillovers, although the corresponding open loop as well as the collusive (cartel) equilibria and the monopoly solution exist. This analytical result corroborates the empirical evidence on the many bankruptcies in the solar photovoltaic market. Second, from a policy perspective, learning could justify a restriction on the number of competitors in the marketplace, in particular if it is very effective. Third, surprising and of theoretical interest is that the linear (and symmetric) Markov perfect equilibrium need not be unique, which is a novel outcome for meaningful economic models.

1. Introduction

The phenomenon of improving performance in a task by simply exercising it over and over again, i.e., learning by doing, is known at least since Aristotle's famous lore:

> "*For the things we have to learn before we can do them, we learn by doing them*".

It features prominently in production – from airplanes (with the pioneering work of (Wright, 1936; Asher, 1956)) to microchips (related to Moore's law of doubling memory capacity every two years)[1] and automobiles (Ford's Model T). At present, one of the most important industries where learning curves are crucial is the renewable and alternative energy sector, as the production of sustainable energy is *the* major instrument for mitigating climate change and its effects. In this area learning by doing is key since the major technologies have been well known for a long time, e.g., wind turbines, the electric motor (invented before the combustion engine), photovoltaics (PV, a by-product from space travel) and batteries with major efficiency improvements stemming from learning by doing. There is also significant scientific evidence to support the notion that learning is effective within this industry. Bollinger and Gillingham (2019) and Sagar and Van der Zwaan (2006), emphasize the importance of learning by doing and of R&D, Wand and Leuthold (2011) integrate learning by doing and technology diffusion and apply it to private solar PV installations in Germany, and Reichelstein and Sahoo (2015, 2018) argue that learning is still going strong today. This is also supported by the data: The evolution of the average module sales prices versus cumulative PV module shipments is shown in Fig. 1(a). Prices came down from above 100 2019-US$ per W (peak) in 1976 to around 0.2 at present, and the learning effect has even increased during the more recent period (to 40% for 2006–2019 compared to 23.8% for 1976–2019 according to VDMA (2020), an industry organization. Fig. 1(b) shows the market share by geographic region of the production of PV cells since 1995, highlighting the large and increasing volumes of Chinese producers – although starting from a very low base relative to the US and Japan.

The above-mentioned papers as well as the chart in Fig. 1(a) are commonly used to analyze and understand the relationship between output and costs in a given industry: The reduction of world market prices (as a proxy of the costs of a competitive industry) is linked to learning by doing, proxied by cumulative global production. Although

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[1] And this now for decades with no end in sight, compare Brynjolfsson and Mcafee (2017). We note that an alternative to the *fundamental causes* of cost reductions in the semiconductor industry is postulated in Hatch and Mowery (1998).

https://doi.org/10.1016/j.eneco.2024.107347
Received 8 April 2023; Received in revised form 21 December 2023; Accepted 22 January 2024
Available online 4 February 2024

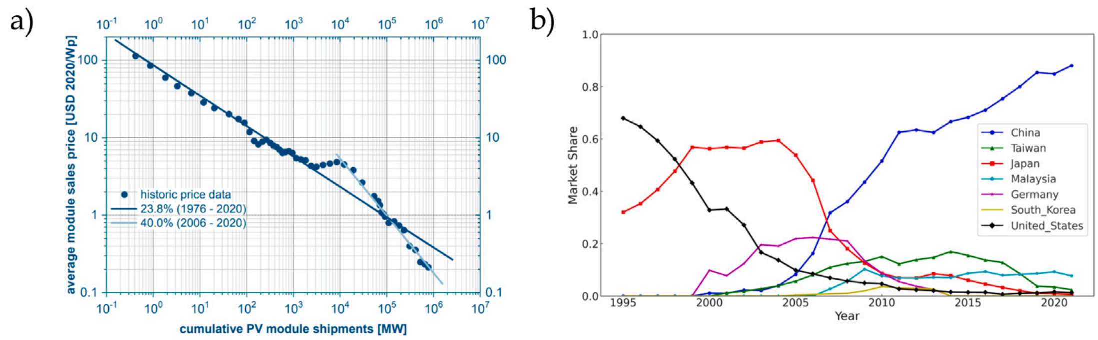

Fig. 1. (a) Relation of module prices and cumulative production (VDMA, 2020). (b) Market share in PV cell manufacturing per geographic region over time. The data for the latter was retrieved and merged from two publicly available sources: Earth Policy Institute (2023) for the periods 1995–2013 and from Helveston et al. (2022) (who in turn rely on Jäger-Waldau (2021) for the periods 2014–2021.

this linkage is common in the empirical literature, and in particular, in the literature on renewable energy (since the good, one unit of electricity, is perfectly homogeneous; see Egelman et al. (2017) for a study containing heterogeneity), this assumption raises a few questions: How to differentiate learning from autonomous technological improvement? How can competitive markets support learning? Or, how to separate price reductions from effects other than learning? For example, from declining input prices[2] or from a reduction of Ricardian rents: As new firms enter or existing firms expand production, marginal producers are eliminated. This reduces the market price even absent any cost degression at the technical level. Indeed, the PV industry is littered with bankruptcies, e.g., in the German market, key players became insolvent despite strong local demand.[3] Not to speak about the fate of the Japanese producers that dominated at the beginning and are now reduced to a small share today (see Fig. 1b). More importantly, how could the now insolvent German, Japanese, or other panel producers reduce their costs due to the large volumes produced by their competitors in China? Similarly, for the American panel producers from faraway, transatlantic as well as transpacific, production? According to this logic, if indeed global cumulative output determines all the firms' learning, then the spillover effects from production must equal 100%. However, large spillovers in turn can lead to a tragedy of the commons: Each producer wants its competitors to undertake the costly learning, at least at the beginning. Of course, spillovers exist and are indeed observed, e.g., see Lieberman (1984) who found that cumulative industry output is strongly related to short-term prices or Levitt et al. (2013), who report that new shifts entering the production of a new car model start already at fairly efficient levels — for some almost miraculous reason, as they have no experience of their own. Thus, an analysis of this interplay between learning from own production and from those of others combined with the interaction in the good's market requires to account for dynamic *and* strategic interactions.

In spite of the long-standing interest in applications, it was Arrow (1962) who provided the first theoretical analysis, which was later used in the endogenous growth literature as a potential generator of growth (among many others see (Young, 1991)). Many seminal contributions in the theoretic literature on dynamic learning by doing followed: First came the discrete two-period models of Spence (1981), Fudenberg and Tirole (1983) and Cabral and Riordan (1994). Closely related is the literature stream on theoretic R&D, e.g., d'Aspremont and Jacquemin (1988), who do account for spillovers and of which Kobayashi (2015) offers a dynamic extension. One important issue at hand from a theoretical point-of-view is the property of the market equilibrium, describing the evolution of prices/output volumes over time and in steady state. From a dynamic perspective, one can differentiate between an open loop equilibrium, which is, however, generally not subgame-perfect (a property often sought for in economic models) and the Markov perfect equilibrium, which is strongly time-consistent and thus exhibits subgame-perfection. Numerous economic models have been developed to identify these equilibria.[4] The latter is preferred, but the solution process is often constrained by tractability, in particular due to non-linearities.

In this paper we present a full-scale continuous time analysis of strategic supply-side interactions accounting for spillovers, the analysis of feedback strategies,[5] and a comparison of different industrial organizations (often only duopolies are studied): Monopoly, oligopoly, a cartel, and perfect competition.[6] We explore several questions, including whether these markets can feature equilibria and the specific conditions necessary for them to occur. And if so, is the outcome unique? Do such markets allow for competition in the long run, in particular if some, possibly only weak, asymmetries exist (like one firm having a head start)? Is knowledge accumulation at all possible in (perfectly) competitive markets, as many empirical papers assume, at least implicitly?

The standard choice for learning in the empirical literature, namely the power function, renders it impossible to derive the Markov perfect equilibrium due to the non-linearity and the high dimension of the state space.[7] Assuming instead that the production costs decline linearly with respect to learning, leads to a linear-quadratic game that allows to compute the linear Markov Perfect Equilibrium (LMPE). This (and the comparison with the open loop equilibrium) allows investigating how the more intense intertemporal interaction between firms affects the market outcome, and whether these interactions can compensate for less individual learning compared to the monopoly. We also address the bias if the degree of learning is estimated from prices observed in non-competitive markets.

The major results are:

1. Strategic intertemporal interaction (i.e., competition in an LMPE) leads to more output and learning compared to an equilibrium in open loop strategies. The economic reason is that increasing own

[2] Currently, rising input prices led to an uncharacteristic increase in the price of solar PV panels according to Bloomberg (2021).

[3] See many reports, e.g., Handelsblatt (2018).

[4] For example, in a recent paper, Sweeting et al. (2022) following Besanko et al. (2014, 2019), analyzes Markov perfect equilibria in a repeated game where strategic buyers (from which firm to buy) foster learning by doing.

[5] Indeed, Spence (1981) provided open loop solutions in a continuous model but had to revert to a two-period model in order to obtain closed-loop solutions.

[6] In this sense, our continuous time model of strategic suppliers facing (positive) spillovers is complementary to the recent analysis in Sweeting et al. (2022).

[7] We summarize the analysis of the corresponding non-linear model in open loop strategies very briefly in Section 4.4. The analysis of which can be found in the Online Appendix.

production benefits one's learning and lowers the competitors' output, despite the spillover effect. A Markov perfect equilibrium allows, *per definitionem*, to start from different initial conditions all converging to the same steady state.

2. Although an LMPE leads to more learning, it need not exist, even if all the corresponding other solutions (monopoly, cartel, and in particular the open loop) do exist. This lack of an equilibrium is puzzling, the more so if one accounts also for the similarity with many dynamic models of oligopolistic competition in industrial organization that started with Reynolds (1987, 1991).[8] We argue that this is a consequence of competition, which limits the viability of many firms in a race driven by learning by doing effects, in particular, if spillovers are small and/or a firm starts with a disadvantage or makes erroneous moves (e.g., of producing too little at any point in time). Time-consistency in the strong sense (i.e., subgame-perfection) requires convergence to a steady state also off the equilibrium path. Hence, any head start of one firm, an erroneous move or simply a production shortfall renders oligopolistic competition in Markov strategies infeasible unless the spillovers are sufficiently large such that they compensate for any arising cost disadvantage. As a consequence, a Markov perfect oligopolistic equilibrium in industries characterized by learning by doing can only exist if the spillovers are sufficiently large (relative to own learning) and the number of firms is very small. This has important consequences for economic policies, in particular, with respect to climate change mitigation.
3. While it is well known since Tsutsui and Mino (1990) and Dockner and Van Long (1993) that symmetric Markov strategies can include a family of uncountable many non-linear equilibria, the restriction to linear strategies (or more general to singular strategies, compare Rowat, 2007; Wirl, 2007) renders uniqueness in economic applications of differential game theory. Not in the model presented in this paper, and to our surprise, multiple equilibria in linear Markov strategies can and do exist in economic applications.
4. Perfect competition cannot support intertemporal knowledge accumulation.

We proceed as follows: First, we present our dynamic framework including the introduction of the full suite of industrial organizations in Section 2, which is followed by a brief presentation of the static outcomes of these market structures (Section 3) and the dynamic outcomes (Section 4). In Section 5 we compare the computed results. Finally, we conclude, discuss limitations and mention possible avenues for further research.

2. Framework

Market demand is determined by the consumers' marginal willingness to pay such that the price (for reasons of simplicity linear and normalized) of the homogeneous good,

$$p(t) = 1 - U(t), \tag{1}$$

is determined by the aggregate supply of n firms,

$$U(t) := \sum_{i=1}^{n} u_i(t). \tag{2}$$

Following Arrow (1962), an individual firm's knowledge (l_i) increases with respect to its own production (u_i) and, due to spillover effects, the (aggregate) production of its competitors $(u_j,\ \ j \neq i)$,

$$\dot{l}_i(t) = a u_i(t) + b \sum_{j \neq i}^{n} u_j(t) - \delta l_i(t),\ l_i(0) = l_{i0},\ \delta > 0; \tag{3}$$

$\dot{l}_i(t)$ denotes the learning of firm i in period t, δ is the rate at which knowledge (l_i) deteriorates in the absence of production and a is the marginal learning effect from own production, i.e., increasing production by one (infinitesimal) unit increases the firm's knowledge by a. The coefficient b describes how much firm i learns from the aggregate volume produced by its competitors. We assume a symmetric setup throughout the article but allow the firms to start from different initial conditions (l_{i0}). Learning from own production exceeds the learning from others, i.e., $b < a$. However, what about the aggregate effect? We impose

Assumption 1. *Learning from own production exceeds what can be learned from the production of all others. More precisely,*

$$b < \frac{a}{n-1}, \tag{4}$$

i.e., assuming identical production of all firms, then each firm learns more from its own production than from the production of all its competitors.

After all, it is called learning by doing and not learning by copying. One important consequence is that the spillover parameter b cannot be varied independently of the number of firms. However, Assumption 1 does not rule out that firm i can learn more from its competitors than from its own production in an asymmetric setup in which i's own production is small relative to the sum of its competitors, e.g., due to $l_i(0) < l_{-i}(0)$. Normalizing,

$$b(n) := \frac{\bar{b}}{n-1} \quad \text{with } \bar{b} < a, \tag{5}$$

ensures that the spillover effect depends only on the aggregate volume produced by the competitors. In order to stress that b cannot be chosen independently of n, the remaining analysis uses the parameter $\bar{b}$ with b determined according to (5). Nevertheless, it is conceivable that $\bar{b}$ depends on n (of course, always satisfying the above inequality), e.g., the knowledge spillover $\bar{b}$ decreases with the number of firms and in particular with the distance (geographic or measured on any other scale) between few firms. Alternatively, more firms could mean an increase in the generation of new ideas or that more experimental production takes place, which could counter the above effect somewhat.

Learning lowers the production costs. Although the use of the (scale invariant) power function dominates the literature,

$$c_i(t) = \kappa l_i(t)^{-\beta} \text{ with } \beta > 0,\ \kappa > 0, \tag{6}$$

we assume a linear cost function,

$$c_i(t) = k - \gamma l_i(t) \text{ with } \gamma > 0,\ k > 0, \tag{7}$$

for the rest of the paper. The reason is that the linear cost function allows computing the then linear Markov perfect equilibrium, which the power function does not. However, we briefly address the outcomes for the power function (restricted to the open loop equilibrium for the oligopoly) in Section 4.4 focusing on the implication of multiple history dependent long run outcomes that are separated by thresholds.[9] The linear cost function, in contrast to the power function, allows starting with virgin production, $l_0 = 0$, if we impose

Assumption 2.

(i) The initial costs are below the choke price, $k < 1$;
(ii) the costs remain non-negative along the evolution and in a steady state, which constrains the learning curve coefficient (γ) from above and the initial costs (k) from below;
(iii) the market clearing price stays below the choke price, $p < 1$.

[8] See also Lambertini (2018).

[9] All derivations can be found in the Online Appendix.

2.1. Different market structures

Each of the n firms of an oligopoly maximizes its individual net present value of profits (all using the same discount rate $r > 0$) by choosing its production,

$$\max_{u_i(t)\geq 0} \int_0^{\infty} e^{-rt}\left(1-\left(\sum_{j=1}^{n} u_j(t)\right)-c_i(t)\right)u_i(t)\,dt,\ i=1,\dots,n, \tag{8}$$

subject to intertemporal learning (3) in each firm.

The cooperative solution of the oligopolistic firms results from maximizing their aggregate profit (from now on omitting time arguments),

$$\max_{\{u_i\geq 0, i=1,\dots,n\}} \sum_{i=1}^{n} \int_0^{\infty} e^{-rt}\left(1-\left(\sum_{j=1}^{n} u_j\right)-c_i\right)u_i\ dt, \tag{9}$$

subject to the same n dynamic constraints (3). The solution represents a cartel of n firms if aspects of internal and external cartel stability (see d'Aspremont et al., 1983) are ignored.

In most differential games, the cooperative or cartel outcome coincides with the monopoly's. However, the two outcomes are different under learning by doing, because a monopoly cannot learn from competitors and the cartel members learn only imperfectly from the production of their peers. Therefore, maximizing a monopoly's net present value leads to a different objective that is subject to a single dynamic constraint,

$$\max_{U\geq 0} \int_0^{\infty} e^{-rt}(1-U-c)\,U\,dt, \tag{10}$$

$$s.t.\ \ \dot{l} = aU - \delta l,\ l(0) = l_0. \tag{11}$$

Even before solving and comparing the above outcomes, one may conjecture that a cartel provides the least efficient market form because it charges monopoly prices but falls short of the learning experienced by the monopoly (due to splitting the output among the n cartel members) and of an oligopoly with its higher output shared among the n firms. However, a cartel can lead to more learning in the long run in rare cases of few firms and very large spillovers.[10] Interpreting the cooperative solution as that of a cartel maximizing the joint profit of its n members makes only sense in a completely symmetric setting, i.e., a setup with identical initial conditions. The reason is that the slightest head start of one cartel member, say $l_i(0) > l_j(0)$ for all $j \neq i$, renders it optimal that firm i becomes the sole supplier, because of its lower costs today and then also in all future periods. Of course, real world cartels consisting of different firms with different owners have more complex objectives, which require, however, an entirely different type of analysis (e.g., applying bargaining concepts of cooperative game theory).

In contrast to a cartel, different initial conditions may allow for a stable oligopolistic outcome. However, if a firm starts with the by far lowest costs, it can out-produce its competitors and will therefore run down its learning cost curve ahead of them. This includes the possibility of driving one or all of them out of the market in order to seize the monopoly prize. Since we focus on interior outcomes, we make

Assumption 3. All firms produce at all times, $u_i(t) > 0$ for all $i = 1,\dots,n$ and all $t \geq 0$.

This assumption excludes the trivial case in which an interior solution (stationary or in the long run) implies negative profits so that immediately stopping production would be optimal (for small l_{i0}).[11] Nevertheless, the corresponding domains in which such a learning equilibrium is absent will be characterized. Assumption 3 also excludes outcomes in which laggards quit or decide to enter later after having sufficiently learned from the production of the incumbents, or where an incumbent adjusts its production in order to deter potential entrants.[12]

2.2. Choice of parameters

Throughout the paper, numerical examples complement the theoretical findings and partially compensate for the lack of closed-form solutions; bifurcation analyses and comparative statics are applied in order to check the robustness of our conclusions. The reference parameter values are

$$\gamma = 0.05,\ a=1,\ \delta = 0.1,\ b=\bar{b}/(n-1),\ \bar{b}=1/3,\ k=1/2,\ r=0.05. \tag{12}$$

Although γ does not allow for the same scale invariant interpretation as the learning curve slope of a power function, the choice in (12) has similar implications: A 25% reduction in unit costs if evaluated along the doubling from halfway to the steady states (around $l = 4$), which approximates the rule of thumb from the power function (20% at each doubling of output). The other parameters determine that learning depreciates at 10% (i.e., a time constant of ten periods) at twice the level of discounting, 5% per period, and the spillover effect is less than one third from own production (normalized at $a = 1$). Demand is normalized and assumes perfect substitutes, which makes sense for renewable energy technologies. In the absence of any empirical evidence, and to pay tribute to the many empirical applications, we set the (aggregate) spillover effect to one third and then use this parameter for bifurcations. Own learning a is normalized at unity. This leaves the cost coefficient k, which determines the costs at $t = 0$, which we set to half of the choke price.

3. Static outcomes

In order to contextualize our results for the dynamic case in Section 4, we now summarize the results of the corresponding static model (see Appendix A for the mathematical computation).

Proposition 1. *Imposing Assumption 2 on the static outcomes, i.e., using the fixed points of (3), of a monopoly (l^m), of cooperation (or cartel, l^c) and of an oligopoly (l^o),*

$$l^m = \frac{a(1-k)}{2(\delta - a\gamma)} > l^c = \frac{(1-k)\left(a+\bar{b}\right)}{2\left(n\delta - \left(a+\bar{b}\right)\gamma\right)} \wedge l^0 = \frac{(1-k)\left(a+\bar{b}\right)}{(n+1)\delta - \left(2a+\bar{b}\right)\gamma}, \tag{13}$$

implies in terms of parameters,

$$\delta > a\gamma,\ 1 > k > \max\left\{\frac{a\gamma}{\delta + (\delta - a\gamma)}, \frac{\left(a+\bar{b}\right)\gamma}{3\delta - a\gamma}\right\},\ 2\delta > a(\gamma+\delta). \tag{14}$$

The choice of parameters (12) satisfies Assumption 2 in the narrow sense, so that all inequalities in (14) hold and the costs are positive for all our relevant results. The last restriction in (14) is redundant for the normalization, $a = 1$. The lower bound for the initial costs is

[10] A condition and a corresponding example is given in Section 4.

[11] A necessary condition can be obtained from the condition that there must exist a level of knowledge ($l > 0$) such that the resulting stationary market price exceeds the costs,

$$1 - \frac{n\delta l}{a+\bar{b}} > c(l) \text{ for at least some } l.$$

[12] Entry deterrence may not be easy, at least for large spillovers: if the incumbent produces little to keep the competitor's learning from spillovers small, then this raises the price that can render the competitor's production profitable; producing high volumes lowers the price and thus provides an entry barrier but reduces the future costs of the potential entrant. For related details on such a model with predatory pricing, see Cabral and Riordan (1994). Those aspects are also analyzed, e.g., Malueg and Tsutsui (1997) or Greiner and Bondarev (2017). Here, Assumption 3 implicitly leads to a similar setting as in Kamien and Zang (2000), where benefiting from spillovers requires individual production.

determined by the maximum since $l^0 > l^m$ is possible (using $n = 2$, because that implies the highest level of learning for an oligopoly). The inequalities in (14) are sufficient if the dynamic analysis leads to less (stationary) learning.[13] Hence, they could be replaced by weaker conditions. However, if the oligopolistic outcome exceeds $\max\{l^m, l^o\}$, which cannot be ruled out for the Markov perfect equilibrium, then one must check explicitly for the feasibility of the solution based on the criteria in Assumption 2.

4. Dynamic outcomes

4.1. Cartel and monopoly

A cooperative solution (i.e., of a cartel) requires perfect symmetry if all firms should produce, i.e., $l_{i0} = l_0\ \forall i$, so that the state space is reduced to one state l. The monopolistic outcome is the special case of $n = 1$ and $\bar{b} = 0$. Let $W(l)$ denote the value function of the cartel, which must satisfy the Hamilton–Jacobi–Bellman (HJB) equation,

$$rW(l) = \max_{u>0}\left\{nu(1 - nu - k + \gamma l) + nW'(l)\left(au + \bar{b}u - \delta l\right)\right\}. \tag{15}$$

The maximization on the right-hand side of (15) implies,

$$u = \frac{(1 - k + \gamma l)}{2n} + \frac{\left(a + \bar{b}\right)}{2n}W'. \tag{16}$$

Since the intercept of (16) represents the myopic maximization of the profits, intertemporal output is larger, because $W' > 0$, according to economic intuition and also arithmetically as shown below. Substitution of (16) into (15) yields an ordinary (but implicit) differential equation for the functional Eq. (15). A linear-quadratic functional specification W,

$$W = w_0 + w_1 l + \frac{w_2}{2}l^2,$$

solves the HJB-Eq. (15). Substituting this guess together with (16) into the left- and right-hand side of (15), we obtain the solution W by comparing coefficients (ignoring the strategy-irrelevant intercept),

$$rw_1 = \frac{1}{2}\left(1 - k + \left(a + \bar{b}\right)w_1\right)\left(\gamma + \left(a + \bar{b}\right)w_2\right) - n\delta w_1,$$

$$r\frac{w_2}{2} = \frac{\left(\gamma + \left(a + \bar{b}\right)w_2\right)^2}{4} - n\delta w_2.$$

Substituting the resulting coefficients, more precisely, $W' = w_1 + w_2 l$, into (16) leads to

Proposition 2. *The cartel policy (indicated by the superscript c) is given by the linear feedback law,*

$$u^c = A^c + B^c l \tag{17}$$

with

$$A^c = \frac{(1-k)(r + \delta n)\left(\sqrt{}^c - r\right)}{2n\left(2\delta n(r + \delta n) - (r + 2\delta n)\left(a + \bar{b}\right)\gamma\right)}, \tag{18}$$

$$B^c = \frac{r + 2\delta n - \sqrt{}^c}{2(a + \bar{b})n} > 0, \tag{19}$$

$$\sqrt{}^c := \sqrt{(r + 2\delta n)^2 - 2\left(a + \bar{b}\right)(r + 2n\delta)\gamma}, \tag{20}$$

which converges to the steady state,

$$l_\infty^c = \frac{(1-k)(a + \bar{b})(r + n\delta)}{\left(2(r + \delta n)\delta n - \left(a + \bar{b}\right)(r + 2\delta n)\gamma\right)},$$

that increases with respect to own learning (i.e., a larger a), spillovers (i.e., a larger $\bar{b}$) and the effectiveness of learning (i.e., a larger value of γ) but decreases with respect to initial costs (k), discounting (r), depreciation (δ) and the number of firms (n) with $\lim_{n\to\infty} l_\infty^c = 0$.

Setting $n = 1$ and $\bar{b} = b = 0$ yields the profit-maximizing strategy of a monopoly.

Proposition 3. *The production strategy of a monopoly (indicated by the superscript m) exceeds the one of a monopoly maximizing its static profit (based on current costs) and is given by the linear feedback rule,*

$$U^m = A^m + B^m l \tag{21}$$

with

$$A^m = \frac{(1-k)(r + \delta)\left(\sqrt{}^m - r\right)}{2\left(2\delta(r + \delta) - (r + 2\delta)a\gamma\right)}, \tag{22}$$

$$B^m = \frac{r + 2\delta - \sqrt{}^m}{2a}, \tag{23}$$

$$\sqrt{}^m := \sqrt{(r + 2\delta)^2 - 2(r + 2\delta)a\gamma}. \tag{24}$$

This policy converges to the steady state $\left(l_\infty^m\right)$,

$$l^m = \frac{a(1-k)}{2(\delta - a\gamma)} > l_\infty^m = \frac{(1-k)(r + \delta)a}{2\delta(r + \delta) - a\gamma(r + 2\delta)}, \tag{25}$$

which is below its static counterpart and exceeds the one of the cartel unless

$$a \geq \bar{b} \geq \frac{a(n-1)\left[2\delta(n\delta + (n+1)r) - r(ac - 2r)\right]}{2\delta(n\delta + (n+1)r) - (r(n+1)ac - 2r)}. \tag{26}$$

The comparative static properties are the same as those of the cartel.

Remark 1. The intertemporal optimization of the monopoly falls short of the static one, $l_\infty^m < l^m$, because one unit of additional production today – in order to benefit from lower production costs in the future – incurs interest costs. The possibility, $l_\infty^m < l_\infty^c$, is quite surprising given the comparison of the static outcomes, $l^m > l^c$ (proven in Appendix A), and the argument that splitting the monopolistic output among firms must harm learning. And indeed, $l_\infty^m > l_\infty^c$ is true in almost all but not in all the cases. Using the above inequality (26) one can find a large but still admissible $\bar{b} \leq a$ so that $l_\infty^m > l_\infty^c$, in particular, for $n = 2$ and all the other parameters at their reference values (12) as long as $\bar{b} > 0.935$, see Fig. 2. The reason is that such large spillovers and very few (two) firms justify to increase the output in order to benefit from the then large learning spillovers.

Remark 2. Assumption 2 (i) is actually an implication of our model. Nevertheless, a monopoly may produce and sell at a loss, $p(0) = 1 - U < k$, if it is sufficiently patient, the cost degression is close to depreciation, and the initial costs are high, more precisely,

$$k > \frac{a\gamma(r + \delta)}{\delta(2(r + \delta) - a\gamma)}.$$

4.2. Oligopoly

4.2.1. Open loop equilibrium

The open loop equilibrium requires solving simultaneously the n first-order conditions of the n firms' intertemporal optimization problems. Therefore, we define the Hamiltonian of player i,

$$H_i = \left(1 - \sum_{j=1}^{n}u_j - c_i\right)u_i + \mu_i^i\left(au_i + b\sum_{j\neq i}u_j - \delta l_i\right) + \sum_{j\neq i}\mu_j^i\left(au_j + b\sum_{f\neq j}u_f - \delta l_j\right), \tag{27}$$

where μ_j^i represents the costate that player i associates with state l_j. The first-order conditions for interior solutions (now substituting the linear cost function (7) for c_i) are given by,

$$H_{u_i}^i = 1 - \sum_{j\neq i}u_j - \gamma l_i - 2u_i + a\mu_i^i + b\sum_{j\neq i}\mu_j^i \overset{!}{=} 0,$$

[13] As it does for the monopoly and the open loop equilibrium of an oligopoly (shown below).

$$\Longrightarrow u_i = \frac{1 - \sum_{j\neq i} u_j - \gamma l_i + a\mu_i^i + b\sum_{j\neq i}\mu_j^i}{2}, \text{ for } i = 1,\dots,n, \tag{28}$$

with the costate equations,

$$\dot{\mu}_i^i = (r+\delta)\,\mu_i^i - \gamma u_i,\ \lim_{t\to\infty} e^{-rt}\mu_i^i l_i = 0,\ \text{for } i = 1,\dots,n, \tag{29}$$

$$\dot{\mu}_j^i = (r+\delta)\,\mu_j^i,\ \lim_{t\to\infty} e^{-rt}\mu_j^i l_j = 0,\ \text{for } i = 1,\dots,n,\ j \neq i, \tag{30}$$

which must hold simultaneously in a Nash equilibrium. The $n(n-1)$ costates μ_j^i, $i = 1,\dots,n$, $j \neq i$, vanish for all t due to the limiting transversality condition, which captures the open loop spirit that the impact on the opponents' learning rates is ignored.

Assuming symmetry, i.e., $l_i(0) = l_0$ for all players i in addition to identical parameters in (28), yields the open loop strategy,

$$u = \frac{1 - k + \gamma l + a\lambda}{n+1}, \tag{31}$$

and consequently the following two canonical equations (using the normalization (5) for the spillover),

$$\dot{l} = \frac{(1 - k + \gamma l + a\lambda)\left(a + \bar{b}\right)}{n+1} - \delta l,\ l(0) = l_0, \tag{32}$$

$$\dot{\lambda} = (r+\delta)\,\lambda - \frac{\gamma(1 - k + a\lambda + \gamma l)}{n+1},\ \lim_{t\to\infty} e^{-rt}\lambda l = 0. \tag{33}$$

Proposition 4. *The symmetric open loop equilibrium (indicated by the superscript o) can be written as a feedback form (the derivation is found in Appendix B), $u = A^o + B^o l$, for a better comparison with the other solutions (monopoly, cartel and LMPE), where,*

$$A^o = \frac{(1-k)(r+\delta)\left(\sqrt{^o} - (n+1)r + \gamma\bar{b}\right)}{2(n+1)\left[\left(\delta(n+1) - \gamma\bar{b}\right)(r+\delta) - a\gamma(r+2\delta)\right]}, \tag{34}$$

$$B^o = \frac{(n+1)(r+2\delta) + \gamma\bar{b} - \sqrt{^o}}{2(n+1)\left(a+\bar{b}\right)} > 0, \tag{35}$$

$$\sqrt{^o} := \sqrt{\left[(n+1)(r+2\delta) - \gamma\left(2a+\bar{b}\right)\right]^2 - 4a\left(a+\bar{b}\right)\gamma^2}. \tag{36}$$

Therefore, given Assumptions 1–3, the open loop equilibrium exists (i.e., the root in (36) exists), is unique, and the strategy is positive and increasing with respect to own but is independent of the others' knowledge, while $k \geq 1$ rules out supply. The long run outcome falls below its static counterpart,

$$l^o = \frac{(1-k)\left(a+\bar{b}\right)}{(n+1)\delta - \gamma\left(2a+\bar{b}\right)} > l_\infty^o = \frac{(1-k)(a+\bar{b})}{(n+1)\delta - \bar{b}\gamma - a\gamma\dfrac{2\delta+r}{r+\delta}} > 0. \tag{37}$$

Comparative statics are similar to a cartel: Learning decreases with respect to k, r, δ and n, and increases in a, γ, and in $\bar{b}$ so that the size of the spillover dominates the induced free riding. The monopoly solution may lead to learning below the open loop outcome (as in the static case) if the spillover is sufficiently large, i.e., $l_\infty^o > l_\infty^m$, if,

$$a > \bar{b} > \frac{a(r+\delta)}{2(r+\delta) - a\gamma}. \tag{38}$$

The explicit solution (37) allows addressing how the industrial organization affects the knowledge stock and consequently costs and prices in the long run. Recall that more competitors, i.e., an increase in n, increases total output under Cournot competition without learning. While a larger learning coefficient (i.e., larger value of γ) increases (long run) knowledge, an additional competitor lowers it so that the consequences on long run output are ambiguous. As indicated by the static case (see Appendix A), more firms need not lead to an increase in (long run) aggregate output and may even lead to a decrease if learning is very effective, more precisely, if (inserting (14)),

$$\frac{\delta}{a} > \gamma > \frac{\delta(r+\delta)}{(r+\delta)\left(a+\bar{b}\right) + a\delta}. \tag{39}$$

A similar relation can be established for the spillover parameter $\bar{b}$ (see Appendix B). Since, learning *per se* does not matter from a consumer's perspective – only the price and thus the total supply does – more competition is not beneficial for consumers if the above inequality (39) holds. Since more competitors lower the producer surplus, a restriction on the number of competitors can therefore be welfare-increasing.

Fig. 2(a) compares long run learning for the different solutions – monopoly, cartel, and open loop oligopoly – with respect to the degree of spillover and the number of firms. Larger spillovers increase learning for the duopoly, surpassing the level of the monopoly at $\bar{b} = 0.6$ (see Appendix B). Fig. 2(a) also shows for the cartel the surprising case addressed above where $l_\infty^m < l_\infty^c$ for $n = 2$ and $\bar{b} > 0.935$ and thus below but very close to the upper bound, $\bar{b} < a = 1$.

4.2.2. Feedback equilibrium

Assuming Markov strategies, each profit maximizing firm i has to solve the following HJB-equation for its value function V^i,

$$rV^i(l_1,\dots,l_n) = \max_{u_i>0}\left\{\begin{array}{c}\left[\left(1 - \sum_{j=1}^{n} u_j\right) - k + \beta l_i\right] u_i + V_{l_i}^i\left(au_i + b\sum_{j\neq i} u_j - \delta l_i\right)\\ +\sum_{j\neq i} V_{l_j}^i\left(au_j + b\sum_{f\neq j} u_f - \delta l_j\right)\end{array}\right\}, i = 1,\dots,n, \tag{40}$$

which depends on all players' states. The first-order condition for the maximization on the right-hand side of (40) is,

$$1 - \sum_{j\neq i} u_j - k + \gamma l_i - 2u_i + aV_{l_i}^i + b\sum_{j\neq i} V_{l_j}^i = 0,\ i = 1,\dots,n. \tag{41}$$

All actions and all derivatives of the value functions of i's competitors enter only as sums. Therefore, considering a symmetric equilibrium in this symmetric game suggests that one could work with the reduced-form model for 2 players i and j in which player i inserts $(n-1)$ times the actions of player j and vice versa for player j's objective. However, this procedure fails to solve the n-state problem for $n > 2$ because of the difference in the optimal actions implied by the Hamilton–Jacobi–Bellman equations.[14] Therefore, the full state space must be taken into account, because, economically, each player must account for deviations by each individual competitor. The disadvantage is that no closed-form solutions can be determined for the induced high-order polynomial equations. Solving for the Markov perfect equilibria turned out to be challenging, because each player adds an additional state, which itself is interdependent with the controls of all players.

In general, the linear quadratic value function for each player i,

$$V_i = \theta_{i0} + \sum_{j=1}^{n}\phi_{ij} l_j + \frac{1}{2}\sum_{m=1}^{n}\sum_{o=1}^{n}\psi_{i,mo} l_m l_o,\ \text{with}\ \ \psi_{i,mo} = \psi_{i,om} \tag{42}$$

solves (40) and implies linear feedback strategies according to the first-order conditions (41). The ith player adds $i+1$ additional coefficients per player to the system,[15] which yields

$$N := 1 + 2n + \binom{n}{2} = \frac{2 + 3n + n^2}{2},$$

equations per player or

$$nN = \frac{n}{2}(1+n)(2+n),$$

equations in total in order to solve for the same number of coefficients. For example, the six-player game leads to 168 simultaneous equations

[14] For details, see the explanation in Wirl (2023) for a different n-state differential game.

[15] One linear term, one quadratic term and all interactions with existing states, $n-1$.

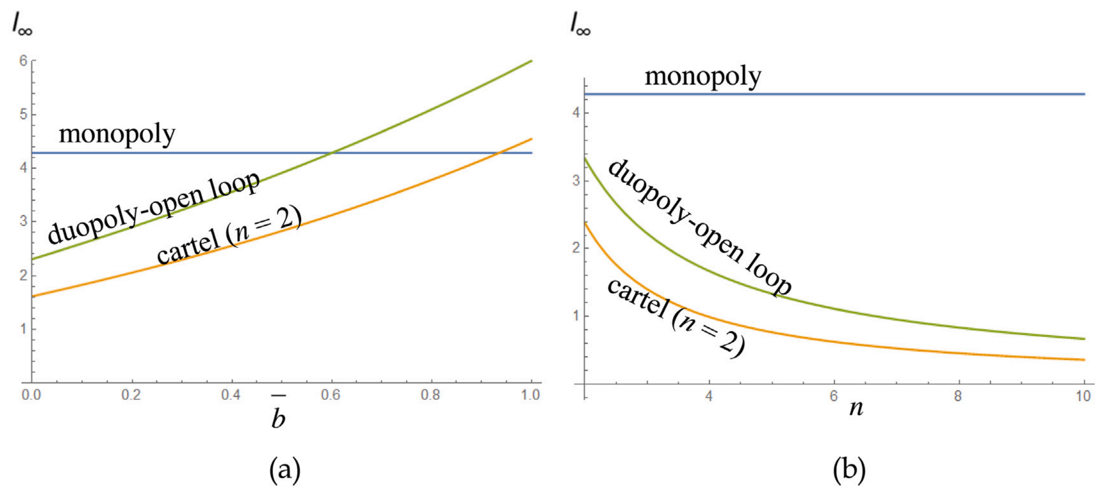

Fig. 2. Comparing, monopoly, cooperation (cartel) and oligopoly (open loop) versus (a) spillover and (b) the number of firms around the reference parameter values (11).

for comparing coefficients after substituting (42) into the left- and right-hand sides of (40) and using the strategies determined by (41) . Using symmetry reduces the functional form of (42) to,

$$V = \theta_0 + \theta_1 l + \theta_2 l^2 + \theta_3 \sum_{j\neq i} l_j + 2\theta_4 l \sum_{j\neq i} l_j + \theta_5 \left(\sum_{j\neq i} l_j\right)^2 \tag{43}$$

and therefore requires solving for six coefficients, irrespective of the number of players n. Note, however, that the structure of these six equations resulting from substitution in (40) significantly complicates, the larger the number of players, such that no closed form solutions can be obtained. Therefore, Newton's Method was used instead, which requires different initial values and iterative steps in order to approximate all roots, the number being determined by the fundamental theorem of algebra. We solve for all the real roots of this system of which only those that imply a stable system and positive production qualify as an LMPE.[16]

In the case of an oligopoly – and in contrast to the cartel – and if a linear Markov perfect equilibrium with all firms producing exists, then the same steady state results even if some firms start at a cost disadvantage. This is just a restatement of the criterion of subgame-perfection. If existing, an LMPE satisfies this condition over the entire state space (only bound by $l_i \geq 0$ and by large levels of l_i that imply negative costs or costs above the price for some player j). While the open loop equilibrium as well as the monopoly and cartel outcomes exist in all economically sensible settings, the demand for subgame-perfection limits the existence of an LMPE to a small subset of model parameters. This has important economic policy implications on what learning and competition can achieve even for very few firms.[17] The bifurcation diagrams in Fig. 3. a) — d) document this for a few crucial parameters. Fig. 3. (a) varies the spillover parameter over its entire admissible range, $0 < \bar{b} < a = 1$. At the reference level, $\bar{b} = 1/3$, an LMPE is restricted to a duopoly. In total, the market only allows for coexistence of up to four firms (but no more) even if the spillover parameter were close to the upper bound. No LMPE exists for small spillovers, not even for a duopoly. For this reason, no LMPE exists for $n > 2$ in the reference case. Also note that an increase in the number of firms flattens how the spillover parameter increases stationary learning, which confirms economic intuition.

The LMPE disappears also for increasing the effectiveness of own learning beyond a certain threshold, i.e., for sufficiently large a so that $a > a_h$, where a_h is defined by the saddle-point bifurcation in a. The economic reason is similar to the one stated above, i.e., as learning from own experience gets large – relative to the spillovers – any deviation from the equilibrium path cannot be matched by the laggard in future periods. Hence, no LMPE exists in such cases too, see Fig. 3(c). Fig. 3(d) depicts the bifurcation diagram in the (a, b) parameter space for a duopoly; the blackened part violates our assumptions (of non-negative costs). Low spillovers, even $\bar{b} = 0$, allow for an LMPE, but only if a is sufficiently smaller than the (normalized reference) level of $a = 1$. This leads to a somewhat paradoxical relationship: No LMPE exists if learning is effective, but it does exist in the case of modest learning, i.e., if a is small. Any increase in the number of competitors decreases prices and therefore profits, and by extension, the feasibility of an equilibrium.

From a theoretical aspect, even more surprising is that multiple (more precisely, two) symmetric LMPEs exist over a range of sufficiently large but not too large spillover parameters, see Fig. 3(a), and other parameter ranges (see Fig. 3. b) for γ and Fig. 3(c) for a).[18] Focusing on the spillover parameter $\bar{b}$ and $n = 2$, multiple LMPEs exist for $0.22 \simeq b_l < \bar{b} < b_h \simeq 0.4$ and thus for our reference example. For $n = 4$ only multiple LMPEs exist but only over a very small set of very large spillover parameters, due to $\bar{b} < a = 1$. As a consequence, the equilibrium in linear symmetric Markov strategies need not be unique. The domain of multiple equilibria borders the domain of no equilibrium for a large and of a unique equilibrium if a is small. Of course, the possibility of (infinitely) many non-singular = non-linear strategies is well known since Tsutsui and Mino (1990). However, the non-uniqueness in linear strategies is puzzling because the so far known very few (actually two) examples in the literature, Lockwood (1996), Example 2) and an example in a series of papers first in Weeren et al. (1999) and then in Engwerda (2000, 2005, 2007,

[16] We solve for the symmetric case (43) as well as the more general asymmetric one (42) and obtain identical results, i.e., no asymmetric LMPEs were identified.

[17] This is in line with the results in Besanko et al. (2014, 2019) that learning by doing may start between competing firms but may ultimately end up in a monopoly.

[18] Although we allow for asymmetric LMPEs, a point emphasized in the works of Engwerda (2000, 2005, 2007, 2016), no asymmetric LMPEs exist in our game.

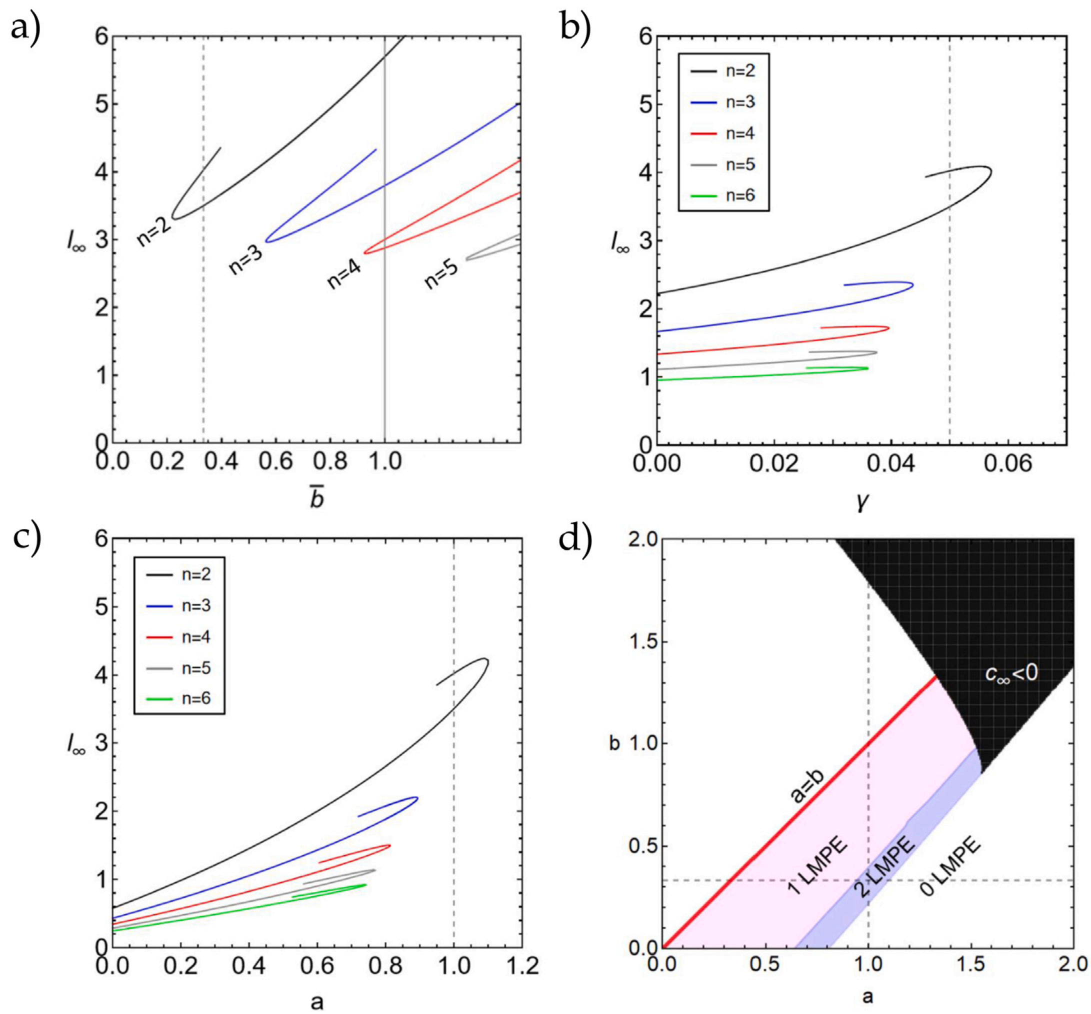

Fig. 3. (a)–(d): Bifurcation diagrams. All non-indicated parameters are at their reference levels.

2016), do not originate from economic problems and in fact have to make assumptions that are hardly fulfilled in economic models. It is further puzzling because non-uniqueness originates here in a model that is similar to many others in the industrial organization literature and has no obvious distinct feature except for the dimension of the state space.

What is the reason for a second LMPE? Why is its existence restricted to a (relatively small) subset of spillover parameters? To answer these questions, let us look at the time paths of the two LMPEs in the reference case for $n = 2$ and different initial conditions. Fig. 4(a) highlights that the long run outcome does not depend on initial conditions but that the choice among the two equilibria, either $LMPE^a$ or $LMPE^b$, matters. Of these two, the one that is less sensitive to the states (i.e., $LMPE^b$ in Fig. 4(a)) leads to less learning and exists for large spillover parameters too, while the more aggressive one (i.e., $LMPE^a$ in Fig. 4(a)) is restricted to $0.22 \simeq b_l < \bar{b} < b_h \simeq 0.4$. See also Fig. 4(b) for the respective bifurcation diagram. However, $LMPE^b$ converges much slower, in particular for the asymmetric initial conditions shown in Fig. 4(a) due to the higher sensitivity with respect to own knowledge, and this in spite of very similar adjustment speeds along $l_1 = l_2$. Higher spillovers increase learning and output, and this is true for all LMPEs despite encouraging free riding. At $\bar{b} = b_h$, the steady state of the dynamic system implied by $LMPE^a$ is turned from a stable node into a saddle point, i.e., one of the two eigenvalues of the Jacobian of $(\dot{l}_1, \dot{l}_2)$ turns from negative into positive. As a consequence, stability is restricted to perfect symmetry, but these only conditionally stable solutions cannot be subgame-perfect because any deviation from $l_1 = l_2$ would lead the outcome astray. The dynamic system remains stable for $LMPE^b$ including large spillovers, $\bar{b} \geq b_h$, which suggests that $LMPE^a$ is supplementary. This observation and the comparison in Fig. 4(a) and (b) suggest the following economic explanation:

Larger spillovers increase the prospects of catching up from behind, yet, at the same time, increase the time constant of adjustment. At some point, the process does not converge in finite time anymore. Therefore, large spillovers ensure a unique LMPE (in this example, in particular for $n = 2$), but too small spillovers lead to the non-existence of an LMPE!

Proposition 5. *(i) Asymmetry with respect to initial values is eliminated over time, i.e., convergence to the same steady states (respectively) results if starting from different initial conditions. (ii) Multiple and symmetric linear Markov perfect equilibria exist, e.g., for intermediate levels of spillovers in the reference example. (iii) The existence of an LMPE requires spillovers of a sufficient magnitude and that only few firms compete or learn very ineffectively (i.e., a is small).*

The parts (ii) and (iii) of Proposition 5 are very surprising, even puzzling. First, because the requirement of Markov perfection increases the number of equilibria but restricts the domain in which equilibria exist. Normally, subgame-perfection refines the set of equilibria. Second, because the equilibrium exists for the limiting static game based on the stationary relation between production and learning. Third, all other solutions, including the open loop one exist (see Fig. 2). Fourth, most (if not all) related games of oligopolistic competition allow for

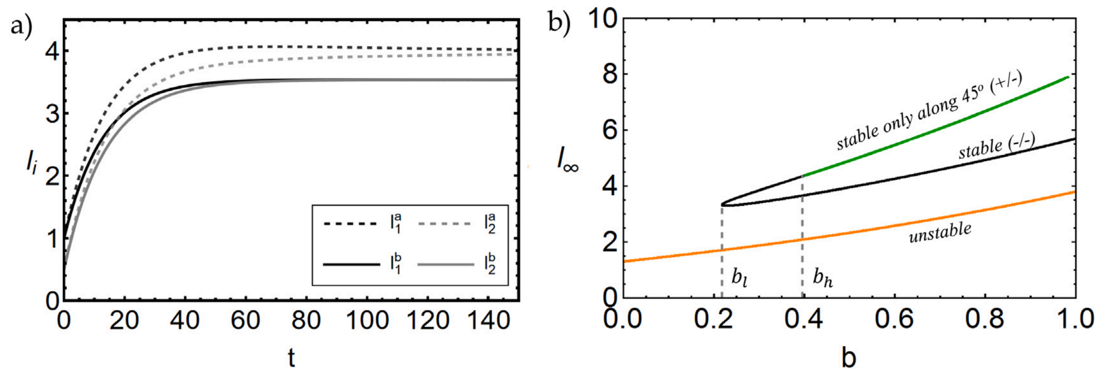

Fig. 4. (a) Time paths of learning (l) departing from different initial conditions, $l_1(0) = 1$ and $l_2(0) = \frac{1}{2}$, for the two different linear Markov perfect symmetric equilibria:
$u_1^a = 0.22 + 0.081\ l_1 - 0.062\ l_2$, $u_2^a = 0.22 - 0.062\ l_1 + 0.081\ l_2$
$u_1^b = 0.20 + 0.046\ l_1 - 0.027\ l_2$, $u_2^b = 0.20 - 0.027\ l_2 + 0.046\ l_1$
for the duopoly ($n = 2$). (b) Bifurcation diagram of the duopoly in b.

an equilibrium. In particular, the case of no spillovers at all ($\bar{b} = 0$) leads to a game that is very similar to many other games of oligopolistic competition since Reynolds (1987, 1991) in which knowledge replaces the firms' capital stocks. How can one explain this puzzle? [19] If the spillover is too weak, the firm with a disadvantage, no matter how small – either at the beginning or by an error along the evolution – grants the competitor(s) a cost advantage that cannot be eliminated in future periods. Hence, the firm will quit, rendering in our example the other firm the position of an uncontested monopoly.

Last but not least, the supplementary LMPE ($LMPE^a$) implies comparative statics different from all the results obtained so far. For example, while more effective learning, i.e., larger values of a or γ, increases stationary knowledge in all the other cases (static, open loop) this need not hold for $LMPE^a$. More precisely, this equilibrium implies a more aggressive strategy that is non-monotonic in a and γ, first increasing but then decreasing towards the end of the parameter ranges of a and γ that support two LMPEs; the strategy implied by $LMPE^b$ keeps the expected sign that larger parameter values monotonically increase steady state learning. This confirms the above interpretation as $LMPE^a$ being the supplementary equilibrium and $LMPE^b$ as the standard one.

4.3. Competitive equilibrium

Taking the limit of the open loop oligopoly outcome in (37) yields,

$$\lim_{n\to\infty} l_\infty^o = 0.$$

Therefore,

Proposition 6. *The limit of oligopolistic competition does not allow for the accumulation of knowledge and thus for competitive and infinitesimally small firms. This result extends to the approach that models competition directly (instead of as the limit of a particular Nash equilibrium) subject to aggregate spillovers following Krugman (1991) and the endogenous growth literature (see Appendix C).*

4.4. Power function

We briefly summarize our findings relating to the non-linear model.[20] In all the considered types of industrial organization, the long run outcomes are characterized by thresholds of initial knowledge (i.e., initial costs) so that only starting above allows for the long run supply of all firms (including the monopoly). Social welfare is generally maximized at an intermediate level of competition (more precisely, a low number of competing firms) and not by one of the extremes (i.e., a monopoly or perfect competition). Many oligopolistic firms competing is not welfare maximizing, because a monopoly's faster rate of learning does better in lowering costs and also prices despite the larger mark-up. A cartel performs worst for consumers and social welfare.

An interesting issue is whether asymmetric initial conditions persist and lead to an asymmetric long run outcome in which both produce (Assumption 3)? Restricting the analysis to a duopoly, the asymmetrical equilibrium turns out as unstable. I.e., transient cost differentials will be eliminated as the disadvantaged firm catches up and eventually converges with the other one to the same symmetric steady state, unless the initial conditions are highly asymmetric.

5. Comparison and results

Table 1 summarizes the results using the parameters (12). All solutions are feasible, as required by Assumption 2 (because the static monopoly solution implies the largest level of learning and thus even for (14)).

Since in any symmetric steady state,

$$u_\infty = \frac{\delta l_\infty}{a + \bar{b}}, \tag{44}$$

the fixed points of all industrial organizations are located on the same straight line through the origin with the slope in (44). This means that $u_\infty^{OL}/x_\infty^{OL} = u_\infty^{FB}/x_\infty^{FB}$ and thus induce similar transient dynamics as well.

At first sight puzzling is the observation for the open loop oligopoly: While the static analysis implies for the parameters in (12) that total supply decreases with respect to n due to less learning, it is constant in our example of the open loop equilibrium at $U = 1 - k = 1/2$ for all n, which corresponds to the competitive supply in the absence of learning (and the limit of the open loop equilibrium for $n \to \infty$). The reason is that our choice of the learning coefficient is at the critical level expressed in (39) so that the counteracting forces due to an increase in n, more competition but less learning, cancel each other.

Fig. 5(a) compares all strategies assuming symmetric states ($l_1 = l_2 = l$) and for aggregated production. The intersection of the aggregated strategies with the steady state condition ($\dot{l} = 0$, which is different for the monopoly) determines the stationary knowledge. Compared to the open loop outcome, the increased intensity of competition by the LMPE leads to more knowledge, more production and lower profits; this effect is larger for $LMPE^a$ from Fig. 4. The economic reason is that increasing own production does not only increase one's learning

[19] In contrast, it is economically intuitive that we lack sensible equilibria for $a, b \gg 0$, as costs will eventually be driven down into the negative domain, therefore contributing to the objective in a fictitiously positive way (γl_i).

[20] The derivation and complete analysis can be found in the Online Appendix.

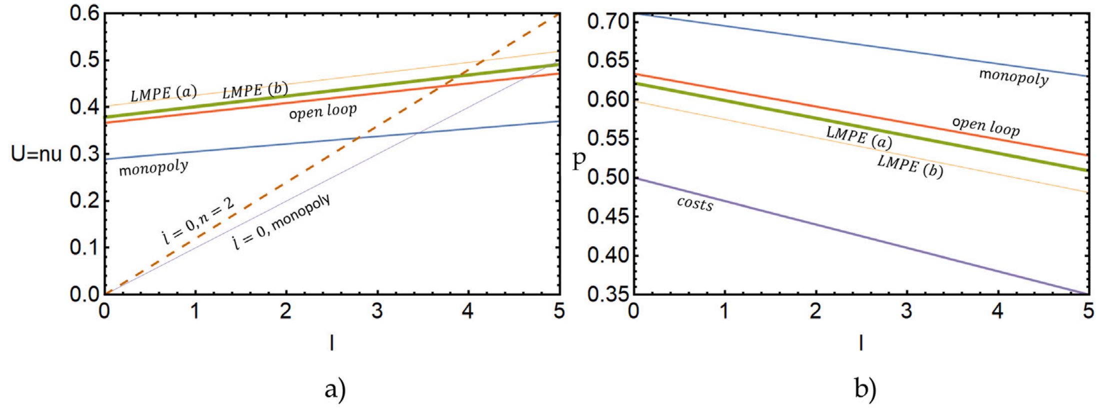

Fig. 5. (a) Comparison of all (aggregated) strategies under all market forms. (b) compares prices versus learning with costs and shows that the price degression is much less than the cost degression.

Table 1
Summary of long run outcomes.

	n	Monopoly	Cooperation/Cartel	Oligopoly		
				OL	LMPE	
					(a)	(b)
	2	|	0.164	0.250	0.302	0.262
u_∞	4	0.429	0.070	0.125		– – – –
	6	|	0.045	0.083		– – – –
	2	|	0.328	0.500	0.604	0.525
U_∞	4	0.429	0.279	0.500		– – – –
	6	|	0.267	0.500		– – – –
	2	|	2.189	3.333	4.024	3.499
l_∞	4	4.286	0.929	1.667		– – – –
	6	|	0.594	1.111		– – – –
	2	|	0.260	0.333	0.299	0.325
c_∞	4	0.286	0.457	0.417		– – – –
	6	|	0.482	0.444		– – – –
	2	|	1.904	3.515	3.932	3.623
Π	4	3.445	1.499	3.097		– – – –
	6	|	1.414	2.938		– – – –

Total discounted social welfare (for $l_{i0} = 0.5$) and steady state values for production, costs, knowledge and differing number of players. No feedback equilibria exists beyond $n = 2$, but two exist in this case and are denoted as (a) and (b), respectively. In addition to the duopoly, values for $n = 4$ as well as for $n = 6$ are shown.

(as in the open loop case), but this increase in one's learning lowers also the competitor's production, see Fig. 5(b). Indeed, and in contrast to the relationship established for the open loop equilibrium, long run learning under $LMPE^a$ exceeds the static oligopoly solution (yet the solution satisfies Assumption 2) but not so for $LMPE^b$. The monopoly produces less than an oligopoly at each level of the knowledge stock, but ultimately attains the highest level of knowledge and thus the lowest production costs.

In Fig. 5(b) we address the familiar approach of estimating the learning coefficient (γ) from price data. This procedure can be problematic in non-competitive markets, because the 'observed' price degression falls short of the cost degression. More precisely, whereas cost degression is assumed at $\gamma = 0.05$, the (absolute) slopes of the resulting prices are roughly only half of that for the monopoly and merely 0.039 for the duopoly ($LMPE^a$).

The subgame-perfect equilibrium in linear strategies can either lack uniqueness or existence for $n > 2$. The reason is that an additional firm will increase competition, lowers each firm's output so that less learning and consequently higher unit costs result. Coupled with lowered consumer prices (as total output increases) each firm's profit were substantially reduced. Therefore, quantity-choosing oligopolists will not start producing and thereby will not start the learning process because of anticipating that prices will not exceed costs sufficiently even after learning. Therefore, no LMPE with all firms ($n \geq 3$) producing exists in our example, while the open equilibrium exists for all n.

Fig. 6(a) compares the net present value of social surplus (Π) for the three market forms and different number of players (n). Fig. 6(b) and Fig. 6(c) decompose social surplus into consumer (CS) and producer (PS) surplus. Not surprisingly, consumers prefer an oligopolistic outcome due to the lower prices. A market composed of only a few oligopolists maximizes social welfare. The reason is that adding another oligopolist increases aggregate but decreases individual production, resulting in less learning and higher costs. In our example, only a duopoly is socially more efficient than a monopoly. This holds for the open loop equilibrium, because $n \geq 3$ leads to too high costs, and *a fortiori* for $LMPE^a$ and $LMPE^b$ which exist only for $n = 2$.

6. Final remarks

We have analyzed intertemporal market equilibria under learning by doing, depending on the own but also on the competitors' production volumes. The major implication of our analysis from an economic policy perspective is that the pair of competition and learning by doing does not match well. Of course, not under perfect competition, but also not for oligopolies. In particular, an LMPE exists only over a fairly restricted domain of only very few firms and requires sufficiently large knowledge spillovers. This is in stark contrast to the open loop equilibrium which exists independent of the magnitude of the spillovers and for more and even for many firms. However, a market characterized by many oligopolists is not welfare-optimal if learning is substantial, which justifies entry regulations like those famously exercised by MITI (for car companies in Japan after World War II). At present, discussions on potential import tariffs for (heavily state-subsidized) battery-powered electric vehicles (BPEV) from China are taking place in the EU. Of course, this seems impossible in a global case like renewable energy and not necessary if the firms play (linear) Markov strategies, because if an LMPE exists, then it leads to higher output (and more learning). Furthermore, it can lead to multiple symmetric equilibria, which is a phenomenon that is by and large overlooked in the literature.[21]

Our analysis can be extended or adapted into many directions. Starting with demand, one may consider unit demand by each consumer,

[21] The only two exceptions mentioned above lack an economic motivation and moreover require assumptions that are implausible from an economic perspective.

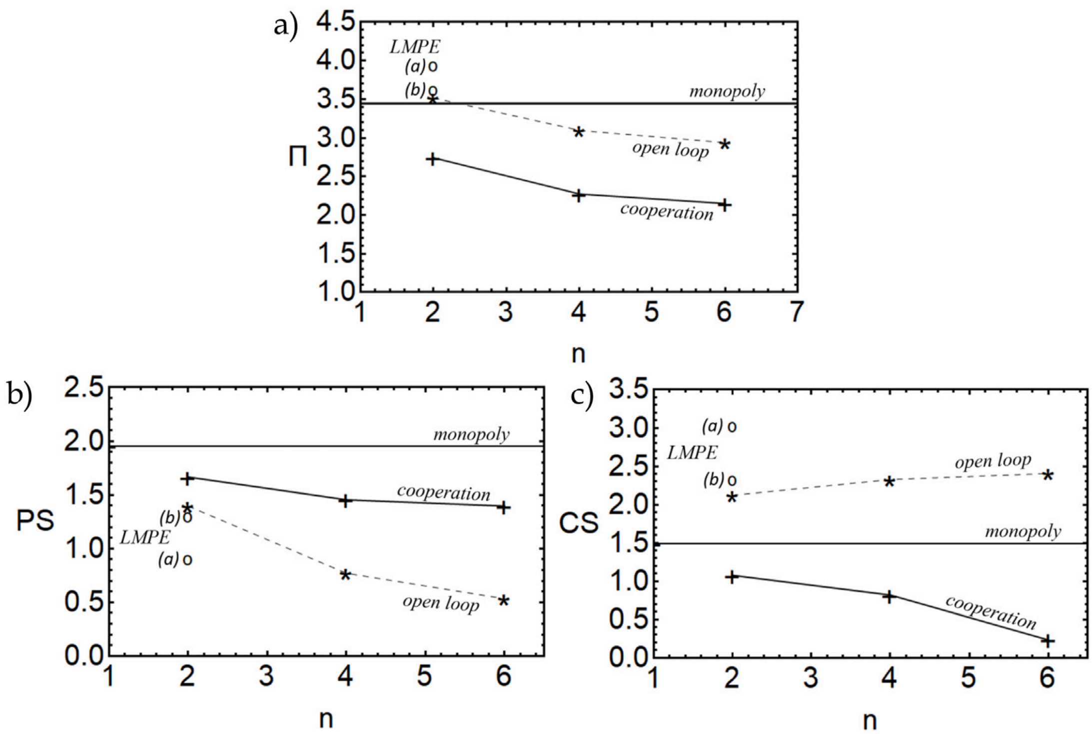

Fig. 6. (a) Net present value of social welfare (b) decomposed in consumer and (c) producer surplus for $n = 2, 4, 6$. The horizontal line indicates the monopoly, the dashed line the cartel and the dotted line the oligopoly.

accounting for saturation effects. Or to allow for endogenous matching of consumers with firms so that purchasing from firm i induces learning there and only indirectly in firms $j \neq i$ via spillovers. That is, one can add spillovers into the repeated game analysis of Sweeting et al. (2022). We model the spillover depending on aggregate output of the competitors. Therefore, one extension is to include neighborhood structures determining spillovers. Indeed, the growing dominance of China in global manufacturing suggest that the spillovers also have a spatial dimension. We assume that all firms produce, which excludes the possibility of entry, of entry-deterrence and that a firm is driven out of the market. Entry deterrence by an incumbent suggests a Stackelberg setup. However, such equilibria will not be time-consistent in the open loop case and the analysis of Stackelberg equilibria in feedback strategies (with either short run or global commitment) is challenging given the dimension of the state space. The puzzling possibility of multiple symmetric LMPEs of a LQ differential game suggests to explore the underlying economic and arithmetical reasons and conditions for such an outcome in general, as well as to check for multiple LMPEs in other economic models. Our study suggests that a larger state space and, presumably also, an interaction in the state dynamics, could be helpful to obtain multiple and symmetric LMPEs in an economic setting.

CRediT authorship contribution statement

Markus Eigruber: Conceptualization, Formal analysis, Investigation, Visualization, Writing – original draft, Writing – review & editing. **Franz Wirl:** Conceptualization, Formal analysis, Investigation, Visualization, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

We thank Herbert Dawid and Peter Kort as well as the participants of the Dynamic games in environmental economics and management workshop (organized by Florian Wagener) for feedback on an earlier draft of this paper. Special gratitude is extended towards Jacob Engwerda, whose expertise and input was especially valuable. Additionally, we thank the two anonymous reviewers for their helpful comments and constructive feedback, which significantly contributed to the improvement of this manuscript.

Appendix A. Feasibility of the static cases

Maximizing the monopoly's static choice for profit maximization,

$$\arg\max_u \left(1 - u - (k - \gamma l^m)\right) u = \frac{(1-k)\,\delta}{2\,(\delta - a\gamma)},$$

based on the stationary solution of the learning dynamics (in a symmetric outcome),

$$l = \frac{au + \bar{b}u}{\delta}$$

and thus for the monopoly $\left(\bar{b} = 0\right)$, yields,

$$l^m = \frac{a\,(1-k)}{2\,(\delta - a\gamma)}.$$

Feasibility of this solution requires that

$$l^m > 0 \wedge 0 < c = k - \gamma l^m < 1 - \delta l^m = p < 1.$$

More precisely:

1. Non-negative learning which implies for the monopoly,

$$l^m = \frac{a\,(1-k)}{2\,(\delta - a\gamma)} > 0,$$

which allows for two possibilities. First,

$$k < 1 \wedge \delta > a\gamma, \tag{45}$$

i.e., profitable start but small decline γ if we fix a (and normalize it to 1), and second,

$$k > 1 \wedge \delta < a\gamma, \tag{46}$$

i.e., start with a loss but in expectation of steep cost decline.

2. Costs must be non-negative,

$$c = k - \gamma l^m = k - \frac{a\gamma (1-k)}{2(\delta - a\gamma)} > 0, \tag{47}$$

$$\text{assuming (45)} \iff 2(\delta - a\gamma) k > a\gamma (1-k), \tag{48}$$

$$\iff (2\delta - 2a\gamma + a\gamma) k > a\gamma, \tag{49}$$

$$\text{knowing (45)} \iff k > \frac{a\gamma}{\delta + (\delta - a\gamma)}, \tag{50}$$

which is (almost) trivially fulfilled for $k > 1$ and large γ but provides a lower bound on k or respectively, on γ beyond (45). Note that condition (46) could potentially imply a violation of the non-negativity constraint in the derivation above.

3. Prices must be below the choke price, thus for

$$p = 1 - \frac{a\delta (1-k)}{2(\delta - a\gamma)} < 1 \iff \frac{a\delta (1-k)}{2(\delta - a\gamma)} > 0,$$

which is trivially met for both conditions (45) and (46).

4. Prices must exceed the costs,

$$p = 1 - \delta l > k - \gamma l \iff 1 - k > (\delta - \gamma) l,$$

which requires for $k > 1$ a large $\gamma > \delta$ irrespective whether a is normalized or not and a large stock of knowledge to overcome the high initial cost. Substituting l^m from above implies,

$$\frac{2(a\gamma - \delta)}{a(\gamma - \delta)} > 0$$

and thus no additional constraint, at least for the normalization $a = 1$.

Proposition A.1. *Assuming $a = 1$, then the case $k > 1$ does not allow for the monopoly outcome even if γ were large.*

Therefore, we impose the conditions,

$$k < 1, \tag{51}$$

$$\delta - a\gamma > 0, \tag{52}$$

and

$$2\delta > a(\gamma + \delta), \tag{53}$$

which are assumed to hold for the linear case. Note that (53) is redundant for the normalization, $a = 1$.

Analogously, computing the static solutions of the cartel (l^c) and the oligopoly (l^o) yields the following levels of learning,

$$l^c = \frac{(1-k)\left(a + \bar{b}\right)}{2\left(n\delta - \gamma\left(a + \bar{b}\right)\right)},$$

$$l^o = \frac{(1-k)\left(a + \bar{b}\right)}{(n+1)\delta - \left(2a + \bar{b}\right)\gamma}.$$

Proposition A.2. *Therefore, $l^m > l^c$ and $l^o > l^c$ and the solutions are feasible for the assumptions about the monopoly outcome in (25). While increasing n lowers total supply of the cartel, the outcome for an oligopoly depends on the parameters on how strong learning is (even within our assumptions in (14)), more precisely if*

$$\left(2a + \bar{b}\right)\gamma > \delta \iff \gamma > \frac{\delta}{2a + \bar{b}}. \tag{54}$$

then the total supply of an oligopoly decreases with respect to n and thus for our reference parameters (12).

Proof. Assuming $k < 1$, then $l^o > 0$ is ensured for the condition derived for the monopoly, $\delta - a\gamma > 0$, since

$$(n+1)\delta - \gamma\left(2a + \bar{b}\right) > 0 \iff (n+1)\delta > \gamma\left(2a + \bar{b}\right) > 3a\gamma, \tag{55}$$

$$\implies (n+1)\delta > 3\delta > 3a\gamma. \tag{56}$$

First comparing monopoly and cartel,

$$l^m > l^c \iff \frac{l^m}{l^c} = \frac{a\left(n\delta - \gamma\left(a + \bar{b}\right)\right)}{(\delta - a\gamma)\left(a + \bar{b}\right)} > 1$$

$$\iff an\delta - a\gamma\left(a + \bar{b}\right) > \delta\left(a + \bar{b}\right) - a\gamma\left(a + \bar{b}\right)$$

$$\iff \delta\left(a(n-1) - \bar{b}\right) > 0 \iff a(n-1) - \bar{b} > 0,$$

yet

$$a(n-1) - \bar{b} > a - \bar{b} > 0.$$

Comparing oligopoly and cartel,

$$l^o = \frac{(1-k)\left(a + \bar{b}\right)}{(n+1)\delta - \left(2a + \bar{b}\right)\gamma} > l^c = \frac{(1-k)\left(a + \bar{b}\right)}{2\left(n\delta - \gamma\left(a + \bar{b}\right)\right)}$$

$$\iff \frac{2\left(n\delta - \gamma\left(a + \bar{b}\right)\right)}{(n+1)\delta - \left(2a + \bar{b}\right)\gamma} > 1$$

$$\iff 2n\delta - 2\gamma a - 2\bar{b}\gamma > (n+1)\delta - 2a\gamma - \bar{b}\gamma \iff (n-1)\delta - \bar{b}\gamma > 0.$$

Similarly,

$$(n-1)\delta - \bar{b}\gamma > \delta - a\gamma > 0,$$

and the last inequality holds by assumption.

While it is obvious that increasing n lowers total supply of the cartel, the outcome for an oligopoly depends on the parameters, because

$$\frac{\partial}{\partial n} u^o = \frac{\partial}{\partial n} \frac{n\delta l^o}{a + \bar{b}} = \frac{(1-k)\delta\left(\delta - \left(2a + \bar{b}\right)\gamma\right)}{\left((n+1)\delta - \left(2a + \bar{b}\right)\gamma\right)^2},$$

which leads to (54).

Proposition A.3. *Learning by an oligopoly can exceed that of a monopoly.*

To the contrary, the monopoly's knowledge exceeds that of the oligopoly if and only if,

$$l^m - l^o = (1-k)\frac{\delta a(n-1) + \bar{b}(a\gamma - 2\delta)}{2(\delta - a\gamma)\left(\delta(n+1) - \gamma\left(2a + \bar{b}\right)\right)} > 0.$$

The denominator is positive, thus

$$l^m - l^o > 0 \iff \delta\left(a(n-1) - 2\bar{b}\right) + \bar{b}a\gamma = \delta\underbrace{\left(a(n-1) - \bar{b}\right)}_{+} + \bar{b}(a\gamma - \delta) > 0.$$

Since, $\partial/\partial n > 0$ for the above term of the numerator, consider the lowest value, $n = 2$. Then,

$$\delta\left(a - 2\bar{b}\right) + \bar{b}a\gamma > 0 \iff \bar{b} < \frac{a\delta}{2\delta - a\gamma}.$$

Therefore, the stationary oligopoly outcome can exceed the one of the monopoly for sufficiently large spillovers, and this takes place in our reference example for $\bar{b} > 2/3$.

Appendix B. Feedback representation of the open loop equilibrium

Departing from the canonical equations, we 'guess'

$$\lambda = \alpha + \kappa l$$

and substitute this guess into (33) on the left-hand and on the right-hand side. Then, equating the coefficients (after substituting (32)) and solving the two equations for these two coefficients yields,

$$\kappa = \frac{(n+1)(r + 2\delta) - \gamma\left(2a + \bar{b}\right) - \sqrt{^o}}{2a\left(a + \bar{b}\right)}, \tag{57}$$

$$\alpha = \frac{(1-k)\left(\gamma + \left(a+\bar{b}\right)\kappa\right)}{(n+1)(r+\delta) - a\kappa\left(a+\bar{b}\right) - a\gamma}, \tag{58}$$

with the root reported as in (36).

Considering the long run knowledge stock of n firms with normalized spillover $\bar{b}$, one can compute the necessarily higher spillover parameter, $\tilde{b} > \bar{b}$, that compensates for the decrease of l_∞^o due to an additional firm,

$$\tilde{b} = \frac{(r+\delta)\left(a+(n+2)\bar{b}\right) - a\bar{b}\gamma}{(n+1)(r+\delta) - a\gamma},$$

and of course only if $\tilde{b} < a$ according to our assumptions. This means for the reference example, (12) and $n = 2$, that $\tilde{b} \approx 0.833$ and thus much larger than $\bar{b} = 1/3$; this upgrade is reduced with increasing n ($\tilde{b} \to \bar{b}$ for $n \to \infty$).

The comparative static claims follow from elementary differentiation of (37).

The root in (36) is real if and only if,

$$(n+1)(r+2\delta) - \gamma\left(2a+\bar{b}\right) > 2\gamma\sqrt{a\left(a+\bar{b}\right)}$$
$$\Longleftrightarrow (n+1)(r+2\delta) > \gamma\left(2a+\bar{b}+2\sqrt{a\left(a+\bar{b}\right)}\right). \tag{59}$$

Since,

$$2a+\bar{b}+2\sqrt{a\left(a+\bar{b}\right)} < 2a+\bar{b}+2a+\sqrt{a\bar{b}} < 6a,$$

we get (for all $n \geq 2$),

$$(n+1)(r+2\delta) > 2(n+1)\delta > 6a\gamma > \gamma\left(2a+\bar{b}+2\sqrt{a\left(a+\bar{b}\right)}\right).$$

The resulting market price exceeds the costs since,

$$p = 1 - n\delta l_\infty^o > c = k - \gamma l_\infty^o \Longleftrightarrow 1-k > (n\delta - \gamma)\, l_\infty^o, \tag{60}$$

$$\Longleftrightarrow p - c = \frac{(1-k)\,\delta\left(\left((n+1)+\left(a-\bar{b}\right)n\right)(r+\delta)+a\gamma\right)}{\left(n\delta - \left(a+\bar{b}\right)\gamma\right)(r+\delta) + ((r+\delta)-a\gamma)\,\delta} > 0, \tag{61}$$

of which the numerator is positive and so is the denominator (for $n \geq 2$) as it is written above.

The steady state level of learning of the monopoly may fall below the one of the open loop equilibrium (as in the static case). Considering the most favorable case, $n = 2$, then $l_\infty^m < l_\infty^o$, if the spillover is sufficiently large as claimed in (38), which can be further simplified by imposing the normalization $a = 1$,

$$1 > \bar{b} > \frac{1}{2 - \frac{\gamma}{r+\delta}} > \frac{1}{2}.$$

While Cournot competition increases total supply, learning can change this rule of thumb as already demonstrated in the static case and Appendix A. Computing total long run supply,

$$\frac{n\delta l_\infty^o}{a+\bar{b}} = \frac{n\delta(1-k)}{(n+1)\delta - \bar{b}\gamma - a\gamma\frac{2\delta+r}{r+\delta}}$$

differentiating with respect to n and rearranging the crucial numerator yields,

$$-(1-k)\,\delta\,(r+\delta)\left((r+\delta)\left(\left(a+\bar{b}\right)\gamma - \delta\right) + a\gamma\delta\right).$$

This expression is negative for γ sufficiently large as expressed in (39), which is below its static counterpart in (54). Alternatively, one can compute the critical value of the spillover parameter,

$$\bar{b}^{crit} = \frac{\delta}{\gamma} - a\frac{r+2\delta}{r+\delta}, \tag{62}$$

and again higher spillovers reduce long run supply.

Appendix C. Competitive equilibrium

Because the limit of oligopolistic competition does not allow for learning of competitive and thus infinitesimal small firms, the approach applied in Krugman (1991) and in the endogenous growth literature is used below (see Wirl, 2016, for an extension of this model to strategically acting agents that, nevertheless, leads to the same history dependent outcomes as competition). Small letters, refer to firm specific and thus also controllable variables like u for the production of a competitive firm and capital letters to market aggregates, e.g., U for total production. In this setting, each competitive firm,

$$\max_u \int_0^\infty e^{-rt}\left[(1-U)\,u - uc(l)\right] dt,$$

where $c(l)$ denotes the unit production costs conditional on the stock of knowledge, here either the power or the linear or another cost function. The maximization is subject to the constraint,

$$\dot{l} = au + \bar{b}U - \delta l,\; l(0) = l_0,$$

where $\bar{b} < a$ accounts for the spillover to an individual firm from aggregate production. Assuming a symmetric competitive equilibrium of identical firms with their aggregate mass normalized to 1, then individual and market aggregates are identical,

$$U = u, \text{ and } L = l.$$

It is important that the capitalized market aggregate variables are exogenous to the individual firm's optimization, yet their evolution can be predicted perfectly in a rational expectation equilibrium.

Remark A.1. The industry is competitive but not subject to free entry, because a later entering firm lacks the learning (at least the own learning but presumably also the spillover if this is tied to producing) and thus must start at higher costs, rendering entry unprofitable. In short, this allows us to ignore the zero profit condition, yet (the net present value of) profits must be non-negative to get the business going.

The Hamiltonian of the competitive firm's intertemporal optimization problem,

$$H = (1-U)\,u - uc\,(l) + \lambda\,(au + bU - \delta l)\,,$$

is linear in the control,

$$H_u = 1 - U - c + a\lambda,$$

and

$$\dot{\lambda} = r\lambda - H_l = (r+\delta)\,\lambda + uc'.$$

A competitive equilibrium is only possible along the singular arc, $H_u = 0$, because otherwise nothing or very large volumes were produced. Using in addition from now on the condition of symmetry yields for the singular arc,

$$H_u = 0 \Longrightarrow u = 1 + a\lambda - c,$$

and thus the canonical equations,

$$\dot{l} = (a+b)(1+a\lambda-c) - \delta l,\; l(0) = l_0, \tag{63}$$

$$\dot{\lambda} = (r+\delta)\,\lambda + (1+a\lambda-c)\,c'. \tag{64}$$

A long run competitive equilibrium is only possible if prices exceed cost and this inequality can be strict and thus allows for long run profits due to the assumption of no entry. Computing the steady states,

$$\dot{l} = 0 \Longrightarrow u = \frac{\delta l}{a+b}, \tag{65}$$

$$\dot{\lambda} = 0 \Longrightarrow \lambda = -\frac{uc'}{r+\delta}, \tag{66}$$

and combining yields

$$u = 1 - c + a\lambda = 1 - c - a\frac{uc'}{r+\delta} \Longrightarrow u = \frac{(1-c)(r+\delta)}{r+\delta+ac'}. \tag{67}$$

Production requires costs below the choke price, $c < 1$. Hence,

$$r + \delta + ac' > 0.$$

Since the market price cannot fall below the costs in a viable competitive equilibrium, at least in the long run (but not necessarily transiently),

$$p = 1 - u = \frac{ac' + (r+\delta)c}{r+\delta+ac'} \geq c \Longleftrightarrow ac' \geq ac'c.$$

Therefore, $1 \leq c$ because of $c' < 0$, i.e., the costs must exceed the choke price (equal to 1). Contradiction. QED.

Appendix D. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.eneco.2024.107347.

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